N71-16785

[illegible]

NASA CR 116207

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Report 32-1508

Polynomial Expressions for Planetary Equators and Orbit Elements With Respect to the Mean 1950.0 Coordinate System

Francis M. Sturms, Jr.

CASE FILE COPY

JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

January 15, 1971

JET PROPULSION LABORATORY *California Institute of Technology • 4800 Oak Grove Drive, Pasadena, California 91103*

October 13, 1971

Recipients of Jet Propulsion Laboratory
Technical Report 32-1508

Subject: Errata

Gentlemen:

Please note the following corrections to Technical Report 32-1508, Polynomial Expressions for Planetary Equators and Orbit Elements With Respect to the Mean 1950.0 Coordinate System, by Francis M. Sturms, Jr., dated January 15, 1971. The values in Table 2 on page 7 of the report should read as follows:

$$\alpha_{50} = 98.02255$$

$$\delta_{50} = -68.98877$$

$$\Delta_{50} = 180.28229 + 0.11043\ T + 0.00062\ T^2$$

$$I = 176.54704 - 0.01507\ T + 0.00001\ T^2$$

Very truly yours,

Robert M Van Buren for

John Kempton, Manager
Publications Section

Telephone 354-4321 *Twx 213-449-2451*

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Report 32-1508

Polynomial Expressions for Planetary Equators and Orbit Elements With Respect to the Mean 1950.0 Coordinate System

Francis M. Sturms, Jr.

JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

January 15, 1971

Prepared Under Contract No. NAS 7-100
National Aeronautics and Space Administration

Preface

The work described in this report was performed by the Mission Analysis Division of the Jet Propulsion Laboratory.

Acknowledgment

The author wishes to express his thanks to Mrs. Helen Ling for her assistance in generating the data and polynomial curve fits for the report, and to Mr. Jack Hudes, a part-time UCLA student-employee, who performed the analysis and computations presented in Appendix A. Special thanks go to Miss Dorothy Babcock for her excellent typing of the manuscript.

Contents

Tables

Contents (contd)

Figures

Abstract

Expressions are presented for the mean orbital elements of the nine planets with respect to the mean equinox and ecliptic of 1950.0. Also, expressions are presented for the right ascension and declination of the north pole of each of the nine planets with respect to the mean equinox and Earth equator of 1950.0. The expressions are polynomials in time *T* measured in Julian centuries from the epoch January 1.0, 1950 E.T. The expressions are useful for coordinate transformations and approximate planetary ephemerides in astrodynamic computer programs.

Polynomial Expressions for Planetary Equators and Orbit Elements With Respect to the Mean 1950.0 Coordinate System

I. Introduction

In astrodynamic computer programs, the mean orbital elements of the planets and the planet pole vectors are often needed for purposes such as coordinate transformations or generating approximate planet ephemerides. Polynomial expressions for these quantities are given in sources such as Refs. 1 and 2. However, these expressions give the quantities with respect to the mean Earth equator or ecliptic *of-date,* whereas a common fixed coordinate system is desired for vector operations. For example, in a patched conic interplanetary computer program, the position and velocity of the launch planet at the launch date and the arrival planet at the arrival date would be in different coordinate systems if generated from available mean elements. Thus, coordinate transformations on one or both state vectors are required. A set of mean elements with respect to a common coordinate system would simplify the program logic.

An inertial reference coordinate system currently in wide use is that associated with the mean Earth equator or ecliptic of 1950.0, where the epoch notation refers to the beginning of the Besselian year and corresponds to Julian Ephemeris Date (JED) 2433282.423357. The Jet Propulsion Laboratory (JPL) ephemeris tapes (Ref. 3) and double precision trajectory program (Ref. 4), use coordinates with respect to the mean Earth equator and equinox of 1950.0.

This report presents expressions for the mean orbit elements and pole vectors of the planets and the Sun with respect to the 1950.0 coordinate systems. The derivations are described in detail, and numerical results are given. In addition, some variables useful for coordinate system transformations are given. Coordinate system transformations are discussed in Appendix C. A list of symbols and nomenclature is also provided to define the notations used in the polynomials and coordinate transformations.

Because of their rapidly varying nature, lunar angles are not suitable for representation as 1950.0 polynomials, but are best handled in terms of the mean-*of-date* polynomials. Special consideration of the Moon is discussed in Appendix D.

II. Reference Epochs and Units of Time

When coordinates are specified with respect to some system, two different times are used. The first time is that at which the coordinate reference planes are defined to exist; the second time is that at which the coordinate components are desired. In *of-date* polynomials, these two times are the same; i.e., the reference plane is defined at the same instant as the coordinate components are evaluated. The reference planes for the polynomials presented in this report are defined at the epoch 1950.0, and remain fixed. Therefore, the time in the polynomials refers only to the time at which the values of the coordinate components are measured.

Time in the polynomials is measured from some reference epoch and in some units. In selecting a reference epoch, one might first choose the epoch 1950.0, since it is also used to define the reference planes. However, this epoch is an awkward number. It has many decimal places when expressed as a Julian date or in conventional days, hours, minutes, and seconds. To avoid this disadvantage, the reference epoch is selected as a nearby even date: 1950, January 1.0, E.T. (JED 2433282.5). Thus, the beginning of any ephemeris day is counted in whole numbers of days from the reference epoch. It is important to remember in the following derivations that the time reference epoch and the coordinate system epoch are not exactly the same instant.

For these applications, it is common in technical literature to find time measured in one of the following units: (1) ephemeris days, (2) Julian centuries, or (3) tropical centuries. Time is also measured from one of the following reference epochs: (1) 1950.0; (2) 1900, January 0.5, E.T.; or (3) 1950, January 1.0, E.T.

Equations (1) and (2) define the conversion from the following two common polynomial time units:

(1) T_0, time in Julian centuries from the epoch 1900, January 0.5, E.T.

(2) T_{tr}, time in tropical centuries from the epoch 1950.0

to the standard time unit selected for the derived polynomials of this report. The equations also show the coefficients of the desired polynomial in terms of the given polynomial coefficients.

$$\left.\begin{aligned} T_0 &= T + 0.5 \\ A &= a_0 + a_1 T_0 + a_2 T_0^2 + a_3 T_0^3 \\ &= (a_0 + 0.5a_1 + 0.25a_2 + 0.125a_3) \\ &\quad + (a_1 + a_2 + 0.75a_3)\, T \\ &\quad + (a_2 + 1.5a_3)\, T^2 \\ &\quad + a_3 T^3 \end{aligned}\right\} \qquad (1)$$

where T = time in Julian centuries from the epoch 1950, January 1.0, E.T.

$$\left.\begin{aligned} T_{\text{tr}} &= \frac{36525.0}{36524.21988} T + \frac{0.076643}{36524.21988} \\ &= FT + k \\ A &= a_0 + a_1 T_{\text{tr}} + a_2 T_{\text{tr}}^2 + a_3 T_{\text{tr}}^3 \\ &= (a_0 + a_1 k + a_2 k^2 + a_3 k^3) \\ &\quad + (a_1 F + 2a_2 Fk + 3a_3 Fk^2)\, T \\ &\quad + (a_2 F^2 + 3a_3 F^2 k)\, T^2 \\ &\quad + (a_3 F^3)\, T^3 \end{aligned}\right\} \qquad (2)$$

III. Derivation of Orbit Element Polynomials

Table 13 of Ref. 2 contains polynomials in T_0 for mean orbit elements with respect to the mean equinox and ecliptic *of-date* for all planets except Pluto. These polynomials were developed from sources given in Ref. 1. (The mean anomaly, because of the more rapid variation, may have the linear time term in units of days, d_0.)

The first step in converting to the new polynomials is to convert the given expressions according to Eq. (1) to obtain polynomials in T. This step changes the reference epoch from 1900, January 0.5 to 1950, January 1.0. Since the mean anomaly M, the eccentricity e, and the semimajor axis a are independent of coordinate systems, their polynomials are now in the form desired. It remains to

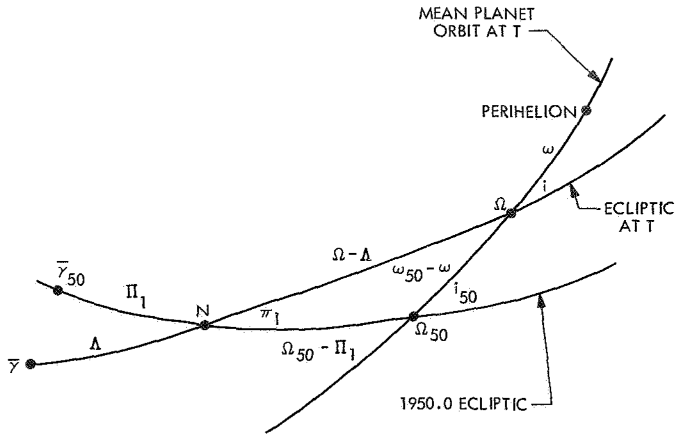


Fig. 1. Geometry of orbit elements with respect to mean-of-date and 1950.0 ecliptic planes

transform the other three elements i, Ω, and ω to the desired coordinate system.

Figure 1 illustrates the geometry of the problem. The spherical triangle $N\Omega\Omega_{50}$ has the known side $\Omega - \Lambda$ and the known angles π_1 and i. The solution for the unknown sides and angle has the form

$$\left.\begin{aligned}
\sin i_{50} \sin(\Omega_{50} - \Pi_1) &= \sin i \sin(\Omega - \Lambda) \\
\sin i_{50} \cos(\Omega_{50} - \Pi_1) &= \cos i \sin \pi_1 \\
&\quad + \sin i \cos \pi_1 \cos(\Omega - \Lambda) \\
\cos i_{50} &= \cos i \cos \pi_1 \\
&\quad - \sin i \sin \pi_1 \cos(\Omega - \Lambda) \\
\sin i_{50} \sin(\omega_{50} - \omega) &= \sin \pi_1 \sin(\Omega - \Lambda) \\
\sin i_{50} \cos(\omega_{50} - \omega) &= \sin i \cos \pi_1 \\
&\quad + \cos i \sin \pi_1 \cos(\Omega - \Lambda)
\end{aligned}\right\} \quad (3)$$

Two methods are used to obtain the desired polynomials for i_{50}, Ω_{50}, and ω_{50} from Eq. (3).

A. Least-Squares Curve-Fit Method

Steps for using the least-squares curve-fit method are listed below:

(1) Evaluate the known polynomials for i, Ω, ω, Λ, Π_1, and π_1 at selected values of T over the time period of interest. The values used are $T = 0.0$, 0.1, 0.2, 0.3, 0.4, and 0.5 for a total of six points.

(2) Compute the right hand sides of Eq. (3) at each value of T.

(3) Solve Eq. (3) to obtain values of i_{50}, Ω_{50}, and ω_{50} at each value of T.

(4) Perform a least-squares curve-fit to the data from step (3). In this step, the elements are assumed to be representable as power series in T to an appropriately low order; i.e., 2 or 3. The curve-fit yields the coefficients of the desired polynomials.

In step (1), the known quantities Λ, π_1, and Π_1, pertaining to earth precession, are obtained from the Ref. 5 expressions that are given in the tropical century form:

$$\left.\begin{aligned}
\sin \pi_1 \sin \Pi_1 &= 4''.585765\, T_{\mathrm{tr}} + 0''.19435\, T_{\mathrm{tr}}^2 \\
&\quad - 0''.00022\, T_{\mathrm{tr}}^3 \\
\sin \pi_1 \cos \Pi_1 &= -46''.84975\, T_{\mathrm{tr}} + 0''.0525\, T_{\mathrm{tr}}^2 \\
&\quad + 0''.00032\, T_{\mathrm{tr}}^3 \\
\Lambda - \Pi_1 &= 5026''.7515\, T_{\mathrm{tr}} + 1''.1116\, T_{\mathrm{tr}}^2 \\
&\quad + 0''.00014\, T_{\mathrm{tr}}^3
\end{aligned}\right\} \quad (4)$$

In the first two expressions of Eq. (4), the use of seconds-of-arc to represent the product of two trigonometric functions may be confusing. The actual numerical value of the product is obtained by converting seconds-of-arc to radians.

In the least-squares curve-fit method, each expression of Eq. (4) is evaluated numerically in the form it appears. In the series manipulation method, which is described in the next subsection, these expressions are converted to direct polynomials in T for the quantities Λ, Π_1, and π_1 themselves, rather than their trigonometric functions. These converted expressions are actually those desired for the mean elements of Earth, since by definition (for Earth only)

$$\left.\begin{aligned}
i_{50} &= \pi_1 \\
\Omega_{50} &= \Pi_1 \\
\omega_{50} &= \tilde{\omega} - \Lambda
\end{aligned}\right\} \quad (5)$$

The results of the least-squares curve-fit method agree very closely with the results of the series manipulation method and are summarized in Section V of this report.

The JPL double precision trajectory program, DPTRAJ (Ref. 4), provides an interesting alternative that is equivalent to the least-squares curve-fit method. The link

TRIC of DPTRAJ provides transformations between various coordinate systems. The link TRIC was used to rotate the elements i, Ω, and ω (with respect to the mean equinox and ecliptic of-date) to the elements i_{50}, Ω_{50}, and ω_{50} (with respect to the mean equinox and ecliptic of 1950.0). The results were then fit with polynomials as previously mentioned in step (4). The TRIC coordinate transformation uses the Cartesian transformation matrix

$$\mathbf{Q} = \{\bar{\epsilon}_{50}\}_x \{\zeta_0 - 90°\}_z \{-\theta\}_x \{90° + Z\}_z \{-\bar{\epsilon}\}_x$$

involving the equatorial precession angles ζ_0, θ, and Z and the mean obliquity, $\bar{\epsilon}$ (Refs. 2 and 5). The least-squares curve-fit method, although solving the spherical triangle equations of Eq. (3) directly, is effectively using the Cartesian transformation matrix

$$\mathbf{R} = \{-\Pi_1\}_z \{-\pi_1\}_x \{\Lambda\}_z$$

which involves the ecliptic precession angles. When the matrices $\mathbf{Q}$ and $\mathbf{R}$ are equated, equations result for Π_1, π_1 and Λ in terms of $\bar{\epsilon}$, ζ_0, θ, and Z. The solution of these equations gives a check on the consistency between Eq. (4) and the more commonly used expressions for $\bar{\epsilon}$, ζ_0, θ, and Z. (See Eqs. 11 and 16.) This subject is explored more fully in Appendix A.

B. Series Manipulation Method

Steps for using the series manipulation method are listed below:

(1) Consider the known polynomials for i, Ω, ω, Λ, Π_1, and π_1 to have the form of a leading term plus a small increment. Expand the sines and cosines of the polynomials as power series in the small increment.

(2) Perform series multiplications, additions, and divisions indicated in Eq. (3) to obtain series expressions for the tangents of i_{50}, $\Omega_{50} - \Pi_1$, and $\omega_{50} - \omega$.

(3) Expand the arc tangents as power series in a small increment.

(4) Make final series additions to yield the desired polynomials for i_{50}, Ω_{50}, and ω_{50}.

Preparatory to step (1), the expressions of Eq. (4) are converted by similar series manipulations to obtain the starting polynomials in Π_1, π_1, and Λ.

The series expressions used in this method are developed in detail in Appendix B. The numerical results of the series manipulation method agree very closely with the results of the least-squares curve-fit method, and are summarized in Section V.

As stated at the beginning of this section, the elements of Pluto were not obtained by the preceding methods. The values of i_{50}, Ω_{50}, and ω_{50} for Pluto are adopted as constants and are equal to the osculating values from Table 23 of Ref. 2. (Both the values in Ref. 2 and those adopted here are from the JPL ephemeris DE 19, Ref. 3).

IV. Derivation of the Planet Equator Equations

Reference 2 gives expressions for the right ascension and declination of the mean north pole with respect to the mean equinox and equator of-date for the Sun, Venus, Mars, Jupiter, Saturn, Uranus, and Neptune (the mean pole neglects nutation). The expressions are linear in time and are measured from a variety of reference epochs. The rate term accounts for the combined precession of the Earth equator and the planet equator, using expressions derived in Ref. 6. The form of the expressions is

$$\begin{aligned} \alpha &= \alpha_{t_0} + \dot{\alpha}\,(t - t_0) \\ \delta &= \delta_{t_0} + \dot{\delta}\,(t - t_0) \end{aligned} \tag{6}$$

where $\dot{\alpha}$ and $\dot{\delta}$ are evaluated by the terms of Eq. (7) at the reference epoch t_0

$$\begin{aligned} \dot{\alpha} &= m + n \sin\alpha\, \tan\delta + \mu \sin I \cos\Delta \sec\delta \\ \dot{\delta} &= n \cos\alpha + \mu \sin I \sin\Delta \end{aligned} \tag{7}$$

The first term in Eq. (7) accounts for the precession of the Earth and the second term accounts for the precession of the planet. To get α_{50} and δ_{50} (with respect to the mean equinox and equator of 1950.0), perform the following steps:

(1) Substitute $t = 1950.0$ in Eq. (6) to obtain α_{50} and δ_{50} at the epoch 1950.0.

(2) Evaluate the rates using only the second term of Eq. (7):

$$\begin{aligned} \dot{\alpha}_{50} &= \mu \sin I \cos\Delta_{50} \sec\delta_{50} \\ \dot{\delta}_{50} &= \mu \sin I \sin\Delta_{50} \end{aligned} \tag{8}$$

(3) The planet precession rate μ is commonly available as a rate per tropical century. Therefore, steps (1) and (2) give expressions that can be transformed by Eq. (2) to the desired form

$$\begin{aligned} \alpha_{50} &= \alpha_0 + \dot{\alpha}_{50}\, T \\ \delta_{50} &= \delta_0 + \dot{\delta}_{50}\, T \end{aligned} \tag{9}$$

Earth, Mercury, and Pluto require special treatment, separate from that for the other planets and the Sun. For Earth, the precession angles yield the desired forms directly, since by definition

$$\begin{aligned} \alpha_{50} &= -\zeta_0 \\ \delta_{50} &= 90^\circ - \theta \end{aligned} \tag{10}$$

References 2 and 5 give the following:

$$\begin{aligned} \zeta_0 &= 2304''.9516\, T_{tr} + 0''.3022\, T_{tr}^2 + 0''.0180\, T_{tr}^3 \\ \theta &= 2004''.2573\, T_{tr} - 0''.4268\, T_{tr}^2 - 0''.0418\, T_{tr}^3 \end{aligned} \tag{11}$$

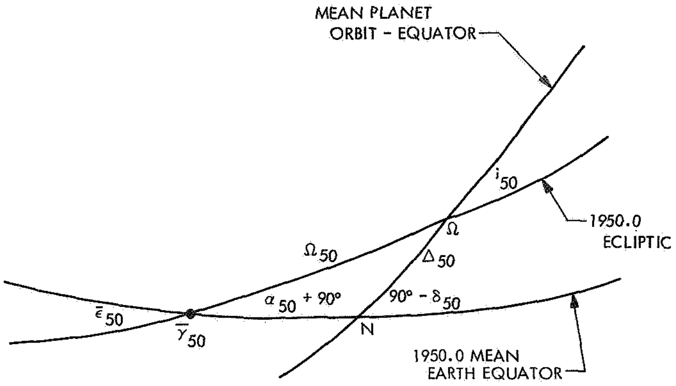


Fig. 2. Geometry for planet north pole normal to orbit

For use in Eq. (10), first transform Eq. (11) according to Eq. (2).

North pole vectors have not been determined for Mercury and Pluto; however, the common assumption will be made that the pole is normal to the orbit. The geometry for this is shown in Fig. 2. Solving the spherical triangle $\gamma_{50} N \Omega$ in the same manner for Eq. (3) gives

$$\left.\begin{aligned} \sin(90^\circ - \delta_{50}) \sin(\alpha_{50} + 90^\circ) &= \sin i_{50} \sin \Omega_{50} \\ \sin(90^\circ - \delta_{50}) \cos(\alpha_{50} + 90^\circ) &= \cos i_{50} \sin \bar{\epsilon}_{50} + \sin i_{50} \cos \bar{\epsilon}_{50} \cos \Omega_{50} \\ \cos(90^\circ - \delta_{50}) &= \cos i_{50} \cos \bar{\epsilon}_{50} - \sin i_{50} \sin \bar{\epsilon}_{50} \cos \Omega_{50} \\ \sin(90^\circ - \delta_{50}) \sin \Delta_{50} &= \sin \bar{\epsilon}_{50} \sin \Omega_{50} \\ \sin(90^\circ - \delta_{50}) \cos \Delta_{50} &= \sin i_{50} \cos \bar{\epsilon}_{50} + \cos i_{50} \sin \bar{\epsilon}_{50} \cos \Omega_{50} \end{aligned}\right\} \tag{12}$$

Equations (12) are evaluated for Mercury and Pluto, and the numerical results are given in Section V. For Pluto, the pole vector is fixed because the orbit elements are constant, but for Mercury, the rates on α_{50} and δ_{50} are those required to keep the pole normal to the changing orbit. These rates cannot be related to a Mercury precession, as in Eq. (8), since the obliquity I is zero.

The specification of a planetary orbit and equator allows the definition of a *planet vernal equinox* in a manner analogous to that for Earth; i.e., the ascending node of the orbit on the equator. The planet equinox is used as a reference direction in planetary coordinate systems. The angle along the planet equator, measured from the ascending node on the mean 1950.0 Earth equator to the *autumnal* equinox Δ_{50} is useful in coordinate transformations. Another useful angle is that between the orbit and equator of the planet; i.e., the planet obliquity I. The solutions for Δ_{50} and I are obtained from spherical triangles and appear in Ref. 1 (p. 332). They are reproduced here with subscript 50 to denote the use of the mean 1950.0 ecliptic and equator, rather than the mean-*of-date* in the Ref. 1 equations.

First, a set of intermediate angles are defined:

$$\left.\begin{aligned}
\sin z \sin x &= \sin \bar{\epsilon}_{50} \cos \alpha_{50} \\
\sin z \cos x &= -\cos \bar{\epsilon}_{50} \cos \Omega_{50} \cos \alpha_{50} - \sin \Omega_{50} \sin \alpha_{50} \\
\cos z &= \cos \bar{\epsilon}_{50} \sin \Omega_{50} \cos \alpha_{50} - \cos \Omega_{50} \sin \alpha_{50} \\
\sin z \sin y &= \sin \bar{\epsilon}_{50} \sin \Omega_{50} \\
\sin z \cos y &= \cos \bar{\epsilon}_{50} \sin \Omega_{50} \sin \alpha_{50} + \cos \Omega_{50} \cos \alpha_{50}
\end{aligned}\right\} \tag{13}$$

Then

$$\left.\begin{aligned}
\sin I \sin \Delta_{50} &= \sin z \sin (x - i_{50}) \\
\sin I \cos \Delta_{50} &= -\cos (x - i_{50}) \cos (y - \delta_{50}) + \sin (x - i_{50}) \sin (y - \delta_{50}) \cos z \\
\cos I &= \cos (x - i_{50}) \sin (y - \delta_{50}) + \sin (x - i_{50}) \cos (y - \delta_{50}) \cos z
\end{aligned}\right\} \tag{14}$$

For Mercury and Pluto, $I = 0$, and the equinox is undefined in the standard sense. A substitute equinox is defined at the ascending node of the 1950.0 *ecliptic* on the planet equator. This definition explains the use of the notation Δ_{50} in Fig. 2 and Eq. (12).

For accurate applications requiring consistency between various coordinate system definitions (e.g., DPTRAJ, Ref. 4), solutions for I and Δ_{50} may be obtained by Eqs. (13) and (14) for each time point. For other applications, Eqs. (13) and (14) may be used to generate data for curve-fits similar to the least-squares curve-fits in Section III, resulting in polynomials for I and Δ_{50}.

For Earth, these polynomials are in terms of the standard angles (Refs. 2 and 5):

$$\begin{aligned}
I &= \bar{\epsilon} \\
\Delta_{50} &= 90^\circ - Z
\end{aligned} \tag{15}$$

and from Ref. 2

$$\begin{aligned}
\bar{\epsilon} &= 23^\circ 26' 44''.84 - 46''.850\, T_{tr} - 0''.0034\, T_{tr}^2 \\
&\quad + 0.0018\, T_{tr}^3 \\
Z &= 2304''.9516\, T_{tr} + 1''.0951\, T_{tr}^2 + 0.0183\, T_{tr}^3
\end{aligned} \tag{16}$$

which can be transformed by Eq. (2) to obtain the desired forms of Eq. (15).

The numerical results for I and Δ_{50} polynomials are given in the next section, and the use of these and other angles in coordinate transformations is explained in Appendix C.

V. Numerical Results

The numerical results from the least-squares curve-fit and series manipulation methods for obtaining the mean orbit element polynomials agree consistently to five decimal places. The results shown in the following tables show at least five decimal places, but more are given for some Earth polynomials to preserve consistency with widely used expressions with a different time unit. In the least-squares curve-fit method, the quadratic curve-fits produced rms errors from the fitted data to the order 10^{-8} deg or smaller. The cubic curve-fits produced errors of the order 10^{-11} deg or smaller. Since the quadratic fits are sufficiently accurate, the numerical results are shown only to order T^2 for the mean elements.

The numerical results for the north pole vectors vary in the order of the polynomials. For the Sun, Venus, and all planets from Jupiter out, the north pole vector is constant, which results from adopting $\mu = 0$ for these planets (except Pluto, which has a constant pole normal to the constant orbit plane). For Mars and Earth, the pole vectors are given to order T and T^3, respectively, to account for the known planetary precession. For Mercury, the pole vector is given as a curve-fit to order T^2 to keep the pole normal to the changing orbit plane.

Additional angles useful for coordinate transformations are I, Δ_{50}, and V. Curve-fits to computed values of I and Δ_{50} are listed in Tables 1–10. The right ascension of the prime meridian V (also known as the hour angle of the equinox) is derived as outlined in Ref. 1 (p. 336).

The convention of measuring longitude positive East, which is opposite to that in Ref. 1, is used by JPL. Therefore, in JPL convention, the signs are reversed in the expression for the longitude of the central meridian LCM. The JPL expression is

$$\mathrm{LCM} = A_E - V + \frac{r\omega}{\tau} \tag{17}$$

For more detail, see Ref. 7.

For the Sun, Venus, Earth, Mars, and Jupiter, expressions from Ref. 2 are converted to the standard time unit d, which is measured from January 1.0, 1950 E.T. For Mercury, zero longitude is defined as that through the subsolar point at the perihelion on May 1, 1968 (JED 2439978.30246).

For Saturn, Uranus, Neptune, and Pluto, V is adopted as zero on the reference epoch January 1.0, 1950 E.T.

Table 1. Mean orbit elements and north pole for Mercury

Independent of plane

$a = 0.3870986$ AU $= 57.9091 \times 10^6$ km

$e = 0.2056244325 + 0.00002043\,T - 0.000000030\,T^2$

$M = 318.537027 + 4.0923344366\,d + 0.0000066667\,T^2$

With respect to mean equinox and ecliptic of 1950.0

$i_{50} = 7.00381 - 0.00597\,T + 0.000001\,T^2$

$\Omega_{50} = 47.73859 - 0.12559\,T - 0.00009\,T^2$

$\omega_{50} = 28.93892 + 0.28439\,T + 0.00007\,T^2$

With respect to mean equinox and Earth equator of 1950.0

$\alpha_{50} = 280.86554 - 0.03289\,T - 0.00001\,T^2$

$\delta_{50} = 61.39767 - 0.00471\,T + 0.00001\,T^2$

$\Delta_{50} = 37.95923 - 0.09577\,T - 0.00008\,T^2$

Obliquity and prime meridian

$I = 0.0$

$V = 343.54720 + 6.136\,d$

Table 2. Mean orbit elements and north pole for Venus

Independent of plane

$a = 0.7233316$ AU $= 108.2089 \times 10^6$ km

$e = 0.00679684275 - 0.000047649\,T + 0.000000091\,T^2$

$M = 311.505478 + 1.6021301892\,d + 0.0012860555\,T^2$

With respect to mean equinox and ecliptic of 1950.0

$i_{50} = 3.39413 - 0.00086\,T - 0.00003\,T^2$

$\Omega_{50} = 76.22967 - 0.27785\,T - 0.00014\,T^2$

$\omega_{50} = 54.63793 + 0.28818\,T - 0.00115\,T^2$

With respect to mean equinox and Earth equator of 1950.0

$\alpha_{50} = 97.97745$

$\delta_{50} = -69.01123$

$\Delta_{50} = 179.97237 + 0.11088\,T + 0.00062\,T^2$

Obliquity and prime meridian

$I = 176.52462 - 0.01504\,T + 0.00001\,T^2$

$V = 317.695584 + 1.483924\,d$

Table 3. Mean orbit elements and north pole for Earth

Independent of plane

$a = 1.00000023$ AU $= 149.597927 \times 10^6$ km

$e = 0.0167301085 - 0.000041926\,T - 0.000000126\,T^2$

$M = 358.000682 + 0.9856002628d - 0.0001550000\,T^2 - 0.0000033333\,T^3$

With respect to mean equinox and ecliptic of 1950.0

$i_{50} = 0.013076\,T - 0.000009\,T^2$

$\Omega_{50} = 174.40956 - 0.24166\,T + 0.00006\,T^2$

$\omega_{50} = 287.67097 + 0.56494\,T + 0.00009\,T^2$

With respect to mean equinox and Earth equator of 1950.0

$\alpha_{50} = -0.0000013435 - 0.6402780091\,T - 0.0000839481\,T^2 - 0.50003 \times 10^{-5}\,T^3$

$\delta_{50} = 89.9999988317 - 0.5567500297\,T + 0.0001185607\,T^2 + 0.0000116119\,T^3$

$\Delta_{50} = 89.9999986565 - 0.6402780100\,T - 0.0003042075\,T^2 - 0.0000050837\,T^3$

Obliquity and prime meridian[a]

$I = 23.4457888616 - 0.0130141669\,T - 0.09445 \times 10^{-5}\,T^2 + 0.05000 \times 10^{-5}\,T^3$

$$V = \frac{\mathrm{E.T.} - (1.002737909294 + 0.589 \times 10^{-10}\,T)(\mathrm{E.T.} - \mathrm{UT1})}{240} + 100.0755426042 + 36000.7693120833\,T + 0.0003870833\,T^2$$

[a] E.T. and UT1 are expressed in seconds from zero hours on current day.

Table 4. Mean orbit elements and north pole for Mars

Independent of plane
$a = 1.5236915 \text{ AU} = 227.9410 \times 10^6 \text{ km}$ $e = 0.09335891275 + 0.000091987\,T - 0.000000077\,T^2$ $M = 169.458720 + 0.5240207716\,d$ $\quad + 0.0001825972\,T^2 + 0.0000011944\,T^3$
With respect to mean equinox and ecliptic of 1950.0
$i_{50} = 1.85000 - 0.00821\,T - 0.00002\,T^2$ $\Omega_{50} = 49.17193 - 0.29470\,T - 0.00065\,T^2$ $\omega_{50} = 285.96668 + 0.73907\,T + 0.00047\,T^2$
With respect to mean equinox and Earth equator of 1950.0
$\alpha_{50} = 316.8538 - 0.0996\,T$ $\delta_{50} = 53.0066 - 0.0566\,T$ $\Delta_{50} = 43.34526 - 0.09181\,T - 0.00010\,T^2$
Obliquity and prime meridian
$I = 24.76883 + 0.01220\,T + 0.00006\,T^2$ $V = 148.672501 + 350.891962\,d$

Table 5. Mean orbit elements and north pole for Jupiter

Independent of plane
$a = 5.202803 \text{ AU} = 778.3284 \times 10^6 \text{ km}$ $e = 0.04841911 + 0.00016302\,T$ $M = 302.650461 + 0.0830898769\,d$
With respect to mean equinox and ecliptic of 1950.0
$i_{50} = 1.30592 - 0.00205\,T + 0.00003\,T^2$ $\Omega_{50} = 99.94335 - 0.16728\,T + 0.00055\,T^2$ $\omega_{50} = 273.57374 + 0.04756\,T - 0.00086\,T^2$
With respect to mean equinox and Earth equator of 1950.0
$\alpha_{50} = 268.0447$ $\delta_{50} = 64.5528$ $\Delta_{50} = 317.92374 + 0.08006\,T - 0.00019\,T^2$
Obliquity and prime meridian
$I = 3.06959 + 0.00060\,T + 0.00003\,T^2$ $V = 239.751 + 877.90\,d$

Table 6. Mean orbit elements and north pole for Saturn

Independent of plane
$a = 9.538843 \text{ AU} = 1426.9908 \times 10^6 \text{ km}$ $e = 0.055716475 - 0.00034705\,T$ $M = 66.251797 + 0.0334442397\,d$
With respect to mean equinox and ecliptic of 1950.0
$i_{50} = 2.49036 + 0.00186\,T - 0.00003\,T^2$ $\Omega_{50} = 113.22015 - 0.25973\,T + 0.00002\,T^2$ $\omega_{50} = 338.84837 + 0.82257\,T - 0.00033\,T^2$
With respect to mean equinox and Earth equator of 1950.0
$\alpha_{50} = 38.41314$ $\delta_{50} = 83.31049$ $\Delta_{50} = 46.06929 + 0.01624\,T - 0.00010\,T^2$
Obliquity and prime meridian
$I = 26.73305 + 0.00880\,T + 0.00004\,T^2$ $V = 844.30\,d$

Table 7. Mean orbit elements and north pole for Uranus

Independent of plane
$a = 19.181996 - 0.000570\,T \text{ (AU)}$ $= (2869.5862 - 0.0853\,T) \times 10^6 \text{ km}$ $e = 0.04718232 + 0.00027204\,T$ $M = 288.465359 + 0.0117258558\,d$
With respect to mean equinox and ecliptic of 1950.0
$i_{50} = 0.77300 - 0.00186\,T - 0.00004\,T^2$ $\Omega_{50} = 73.74521 + 0.06671\,T - 0.00068\,T^2$ $\omega_{50} = 96.10329 + 0.16097\,T + 0.00037\,T^2$
With respect to mean equinox and Earth equator of 1950.0
$\alpha_{50} = 76.761$ $\delta_{50} = 14.920$ $\Delta_{50} = 6.05795 - 0.00182\,T - 0.00004\,T^2$
Obliquity and prime meridian
$I = 97.97862 - 0.00100\,T + 0.00001\,T^2$ $V = 798.767\,d$

Table 8. Mean orbit elements and north pole for Neptune

Independent of plane
$a = 30.057658 + 0.001210\,T$ (AU) $= (4496.5623 + 0.1810\,T) \times 10^6$ km $e = 0.008566995 + 0.00007701\,T$ $M = 150.769275 + 0.0059952644\,d$
With respect to mean equinox and ecliptic of 1950.0
$i_{50} = 1.77467 + 0.00037\,T + 0.00001\,T^2$ $\Omega_{50} = 131.22959 - 0.00574\,T - 0.00029\,T^2$ $\omega_{50} = 272.95650 - 0.51258\,T - 0.00002\,T^2$
With respect to mean equinox and Earth equator of 1950.0
$\alpha_{50} = 295.5712$ $\delta_{50} = 41.46635$ $\Delta_{50} = 17.21185 - 0.00078\,T - 0.00002\,T^2$
Obliquity and prime meridian
$I = 28.80226 - 0.00016\,T - 0.00001\,T^2$ $V = 617.143\,d$

Table 9. Mean orbit elements and north pole for Pluto

Independent of plane
$a = 39.3736414$ AU $= 5890.2138 \times 10^6$ km $e = 0.2488033$ $M = 301.68757 + 0.0039892964\,d$
With respect to mean equinox and ecliptic of 1950.0
$i_{50} = 17.16987$ $\Omega_{50} = 109.68346$ $\omega_{50} = 114.33841$
With respect to mean equinox and Earth equator of 1950.0
$\alpha_{50} = 313.89136$ $\delta_{50} = 66.36420$ $\Delta_{50} = 69.13587$
Obliquity and prime meridian
$I = 0.0$ $V = 56.338\,d$

Table 10. Mean orbit elements and north pole for Sun

Independent of plane
$\left.\begin{array}{r} a = \\ e = \\ M = \end{array}\right\}$ Same as Earth (Table 3)
With respect to mean equinox and ecliptic of 1950.0
$\left.\begin{array}{r} i_{50} = \\ \Omega_{50} = \\ \omega_{50} = \end{array}\right\}$ Use Earth elements or, for apparent geocentric orbit of Sun, add 180° to ω_{50}
With respect to mean equinox and Earth equator of 1950.0
$\alpha_{50} = 286.0193$ $\delta_{50} = 63.7718$ $\Delta_{50} = 60.38908 - 0.10213\,T + 0.00003\,T^2$
Obliquity and prime meridian
$I = 7.25547 + 0.00215\,T - 0.00004\,T^2$ $V = 0.4254225 + 14.18439716\,d$

Appendix A

Consistency of Adopted Expressions for Precession Angles

From planetary theory and the adopted values of the precession constants (see Ref. 5), expressions for the motion of the ecliptic are obtained that are reproduced in this report as Eq. (4). Additional expressions for the equatorial precession angles and the mean obliquity are also obtained; these are shown in Eqs. (11) and (16). The latter expressions for ζ_0, θ, Z, and $\bar{\epsilon}$ are widely published (e.g., Refs. 1, 2, and 5) and used in computer programs to perform coordinate transformations.

The coordinate transformation from mean equinox and ecliptic of-date to ecliptic of 1950.0 may be done in terms of the commonly available equatorial precession angles by the matrix

$$\mathbf{Q} = \{\bar{\epsilon}_{50}\}_x \{\zeta_0 - 90^\circ\}_z \{-\theta\}_x \{90^\circ + Z\}_z \{-\bar{\epsilon}\}_x \qquad \text{(A-1)}$$

Alternately, this transformation may be done more directly in terms of the ecliptic angles by the matrix

$$\mathbf{R} = \{-\Pi_1\}_z \{-\pi_1\}_x \{\Lambda\}_z \qquad \text{(A-2)}$$

When matrices **Q** and **R** are equated, equations are produced that give Π_1, π_1, and Λ as functions of ζ_0, θ, Z, and $\bar{\epsilon}$.

$$\left.\begin{aligned}
&\text{(1,3)[1]} & \sin\pi_1 \sin\Pi_1 &= -\sin\zeta_0 \cos Z \sin\bar{\epsilon} - \cos\zeta_0 \cos\theta \sin Z \sin\bar{\epsilon} + \cos\zeta_0 \sin\theta \cos\bar{\epsilon} \\
&\text{(2,3)} & -\sin\pi_1 \cos\Pi_1 &= -\cos\bar{\epsilon}_{50} \cos\zeta_0 \cos Z \sin\bar{\epsilon} + \cos\bar{\epsilon}_{50} \sin\zeta_0 \cos\theta \sin Z \sin\bar{\epsilon} - \cos\bar{\epsilon}_{50} \sin\zeta_0 \sin\theta \cos\bar{\epsilon} \\
&&&\quad + \sin\bar{\epsilon}_{50} \sin\theta \sin Z \sin\bar{\epsilon} + \sin\bar{\epsilon}_{50} \cos\theta \cos\bar{\epsilon} \\
&\text{(3,3)} & \cos\pi_1 &= \sin\bar{\epsilon}_{50} \cos\zeta_0 \cos Z \sin\bar{\epsilon} - \sin\bar{\epsilon}_{50} \sin\zeta_0 \cos\theta \sin Z \sin\bar{\epsilon} + \sin\bar{\epsilon}_{50} \sin\zeta_0 \sin\theta \cos\bar{\epsilon} \\
&&&\quad + \cos\bar{\epsilon}_{50} \sin\theta \sin Z \sin\bar{\epsilon} + \cos\bar{\epsilon}_{50} \cos\theta \cos\bar{\epsilon} \\
&\text{(3,1)} & -\sin\pi_1 \sin\Lambda &= \sin\bar{\epsilon}_{50} \cos\zeta_0 \sin Z + \sin\bar{\epsilon}_{50} \sin\zeta_0 \cos\theta \cos Z - \cos\bar{\epsilon}_{50} \sin\theta \cos Z \\
&\text{(3,2)} & \sin\pi_1 \cos\Lambda &= -\sin\bar{\epsilon}_{50} \cos\zeta_0 \cos Z \cos\bar{\epsilon} + \sin\bar{\epsilon}_{50} \sin\zeta_0 \cos\theta \sin Z \cos\bar{\epsilon} + \sin\bar{\epsilon}_{50} \sin\zeta_0 \sin\theta \sin\bar{\epsilon} \\
&&&\quad - \cos\bar{\epsilon}_{50} \sin\theta \sin Z \cos\bar{\epsilon} + \cos\bar{\epsilon}_{50} \cos\theta \sin\bar{\epsilon}
\end{aligned}\right\} \qquad \text{(A-3)}$$

When the adopted values of ζ_0, θ, Z, and $\bar{\epsilon}$ are substituted from Eqs. (11) and (16) into Eq. (A-3), the results are the values for π_1, Π_1 and Λ that are listed in Table A-1. If these values are compared with values computed directly from Eq. (4), there are disagreements of as much as 0.00009 deg.

However, the more fundamental quantities are π_1, Π_1, and Λ. A reverse solution can be obtained by equating the two matrices

$$\left.\begin{aligned}
\mathbf{Q}^* &= \{\zeta_0 - 90^\circ\}_z \{-\theta\}_x \{90^\circ + Z\}_z \\
\mathbf{R}^* &= \{-\bar{\epsilon}_{50}\}_x \mathbf{R} \{\bar{\epsilon}\}_x
\end{aligned}\right. \qquad \text{(A-4)}$$

---

[1] Numbers in parentheses refer to the row and column, respectively, of the expanded **Q** and **R** matrices.

Table A-1. Adopted and computed values of precession angles

T	Adopted values (Eq. 3)			Computed from adopted values of ζ_0, θ, Z, and $\bar{\epsilon}$ (Eq. A-3)		
	π_1	Π_1	Λ	π_1	Π_1	Λ
0.0	0.000000	174.40956	174.40956	0.000000	174.40947	174.40947
0.1	0.001308	174.38540	174.52504	0.001308	174.38532	174.52496
0.2	0.002615	174.36123	174.64052	0.002615	174.36117	174.64046
0.3	0.003922	174.33707	174.75600	0.003922	174.33702	174.75596
0.4	0.005229	174.31290	174.87150	0.005229	174.31287	174.87147
0.5	0.006536	174.28875	174.98700	0.006536	174.28873	174.98698

T	Adopted values (Eqs. 11 and 16)			Computed from adopted values of π_1, Π_1, Λ, and $\bar{\epsilon}$ (Eq. A-5)		
	ζ_0	θ	Z	ζ_0	θ	Z
0.0	0.000001	0.000001	0.000001	0.000008	0.000001	−0.000006
0.1	0.064030	0.055675	0.065032	0.064037	0.055675	0.064025
0.2	0.128060	0.111346	0.128069	0.128068	0.111346	0.128062
0.3	0.192092	0.167015	0.192112	0.192100	0.167015	0.192105
0.4	0.256126	0.222681	0.256162	0.256134	0.222681	0.256154
0.5	0.320162	0.278345	0.320217	0.320169	0.278345	0.320210

The results give ζ_0, θ, and Z as functions of π_1, Π_1, Λ, and $\bar{\epsilon}$.

$$\left.\begin{aligned}
&(2,3) && -\sin\theta\sin\zeta_0 = \sin\bar{\epsilon}\,\{\cos\epsilon_{50}\,[\sin\Pi_1\sin\Lambda + \cos\Pi_1\cos\pi_1\cos\Lambda] - \sin\bar{\epsilon}_{50}\sin\pi_1\cos\Lambda\}\\
& && \qquad + \cos\bar{\epsilon}\,\{-\cos\bar{\epsilon}_{50}\cos\Pi_1\sin\pi_1 - \sin\bar{\epsilon}_{50}\cos\pi_1\}\\
&(1,3) && \sin\theta\cos\zeta_0 = \sin\bar{\epsilon}\,\{\cos\Pi_1\sin\Lambda - \sin\Pi_1\cos\pi_1\cos\Lambda\} + \cos\bar{\epsilon}_{50}\sin\Pi_1\sin\pi_1\\
&(3,3) && \cos\theta = \sin\bar{\epsilon}\,\{\sin\bar{\epsilon}_{50}\,[\sin\Pi_1\sin\Lambda + \cos\Pi_1\cos\pi_1\cos\Lambda] + \cos\bar{\epsilon}_{50}\sin\pi_1\cos\Lambda\}\\
& && \qquad + \cos\bar{\epsilon}\,\{-\sin\bar{\epsilon}_{50}\cos\Pi_1\sin\pi_1 + \cos\bar{\epsilon}_{50}\cos\pi_1\}\\
&(3,2) && -\sin\theta\sin Z = \cos\bar{\epsilon}\,\{\sin\bar{\epsilon}_{50}\,[\sin\Pi_1\sin\Lambda + \cos\Pi_1\cos\pi_1\cos\Lambda] + \cos\bar{\epsilon}_{50}\sin\pi_1\cos\Lambda\}\\
& && \qquad + \sin\bar{\epsilon}\,\{\sin\bar{\epsilon}_{50}\cos\Pi_1\sin\pi_1 - \cos\bar{\epsilon}_{50}\cos\pi_1\}\\
&(3,1) && -\sin\theta\sin Z = \sin\bar{\epsilon}_{50}\,[\sin\Pi_1\cos\Lambda - \cos\Pi_1\cos\pi_1\sin\Lambda] - \cos\bar{\epsilon}_{50}\sin\pi_1\sin\Lambda
\end{aligned}\right\}\quad \text{(A-5)}$$

Substitution of the adopted values of π_1, Π_1, Λ, and $\bar{\epsilon}$ from Eqs. (3) and (16) into Eq. (A-5) gives the values for ζ_0, θ, and Z that are listed in Table A-1. If these values are compared with adopted values from Eqs. (11) and (16), there are disagreements of up to 0.000008 deg.

These inconsistencies are *sometimes* troublesome in high-precision computations. For most users, however, the small differences can be neglected.

It is not clear to what extent the differences in Table A-1 are due to numerical errors in performing the computations in Eqs. (A-3) and (A-5). The values in Table A-1 are rounded from values computed in double precision on the Univac 1108, producing 18 figures. Not all 18 figures may be *significant*, however, because of small differences of nearly equal quantities.

When consistency is a necessity for the Earth angles, the procedure used in Section IV for the other planets can also be used for the Earth. To review, the steps of this procedure are as follows:

(1) Noting the equivalences of Eq. (5), evaluate i_{50} and Ω_{50} from formulas for the mean orbit elements for Earth (Table 3).

(2) Noting the equivalences of Eq. (10), evaluate α_{50} and δ_{50} from formulas for the north pole for Earth (Table 3).

(3) Use Eqs. (13) and (14) to compute I and Δ_{50}.

The equivalences of Eq. (15) indicate that the steps listed above provide computed values of $\bar{\epsilon}$ and Z as functions of adopted values of $\bar{\epsilon}_{50}$, Π_1, Λ, π_1, ζ_0, and θ.

Appendix B

Formulas for Series Manipulations

I. Basic Expansions

For basic expansions of small angle x, to the order x^3, the following equations are used:

$$\sin x = x - \frac{x^3}{6} \tag{B-1}$$

$$\cos x = 1 - \frac{x^2}{2} \tag{B-2}$$

$$\tan x = x + \frac{x^3}{3} \tag{B-3}$$

II. Sine and Cosine Series

Given

$$\alpha = \alpha_0 + \alpha_1 T + \alpha_2 T^2 + \alpha_3 T^3$$

Find

$$\sin \alpha = S_0 + S_1 T + S_2 T^2 + S_3 T^3$$

$$\cos \alpha = C_0 + C_1 T + C_2 T^2 + C_3 T^3$$

Let

$$\alpha = \alpha_0 + x$$

where x is a small angle:

$$x = \alpha_1 T + \alpha_2 T^2 + \alpha_3 T^3$$

$$\sin \alpha = \sin \alpha_0 \cos x + \cos \alpha_0 \sin x$$

$$\cos \alpha = \cos \alpha_0 \cos x - \sin \alpha_0 \sin x$$

Using Eqs. (B-1) and (B-2) and retaining terms of order T^3, we determine

$$\begin{aligned}\sin \alpha = \sin \alpha_0 &+ [\alpha_1 \cos \alpha_0]\, T \\ &+ \left[\alpha_2 \cos \alpha_0 - \frac{1}{2}\alpha_1^2 \sin \alpha_0\right] T^2 \\ &+ \left[\left(\alpha_3 - \frac{1}{6}\alpha_1^3\right)\cos \alpha_0 - \alpha_1 \alpha_2 \sin \alpha_0\right] T^3\end{aligned} \tag{B-4}$$

$$\begin{aligned}\cos \alpha = \cos \alpha_0 &+ [-\alpha_1 \sin \alpha_0]\, T \\ &+ \left[-\alpha_2 \sin \alpha_0 - \frac{1}{2}\alpha_1^2 \cos \alpha_0\right] T^2 \\ &+ \left[-\left(\alpha_3 - \frac{1}{6}\alpha_1^3\right)\sin \alpha_0 - \alpha_1 \alpha_2 \cos \alpha_0\right] T^3\end{aligned} \tag{B-5}$$

III. Series Multiplication and Division

Given

$$A = a_0 + a_1 T + a_2 T^2 + a_3 T^3$$

$$B = b_0 + b_1 T + b_2 T^2 + b_3 T^3$$

Find

$$C = AB = c_0 + c_1 T + c_2 T^2 + c_3 T^3$$

$$D = \frac{A}{B} = d_0 + d_1 T + d_2 T^2 + d_3 T^3$$

Elementary term-by-term algebraic multiplication gives

$$\begin{aligned}C = a_0 b_0 &+ [a_1 b_0 + a_0 b_1]\, T \\ &+ [a_2 b_0 + a_1 b_1 + a_0 b_2]\, T^2 \\ &+ [a_3 b_0 + a_2 b_1 + a_1 b_2 + a_0 b_3]\, T^3\end{aligned} \tag{B-6}$$

Let

$$\frac{1}{B} = \frac{1}{b_0}\left[1 + \left(\frac{b_1}{b_0} T + \frac{b_2}{b_0} T^2 + \frac{b_3}{b_0} T^3\right)\right]^{-1}$$

Expand $1/B$ by binomial theorem, and by the use of (B-6) for $A \cdot (1/B)$

$$\begin{aligned}D = \frac{a_0}{b_0} + \left[\frac{a_1}{b_0} - \frac{a_0 b_1}{b_0^2}\right] T + \left[\frac{a_2}{b_0} - \frac{a_0 b_2 + a_1 b_1}{b_0^2} + \frac{a_0 b_1^2}{b_0^3}\right] T^2 \\ + \left[\frac{a_3}{b_0} - \frac{a_0 b_3 + a_1 b_2 + a_1 b_1}{b_0^2} + \frac{2a_0 b_1 b_2 + a_1 b_1^2}{b_0^3} - \frac{a_0 b_1^3}{b_0^4}\right] T^3\end{aligned} \tag{B-7}$$

IV. Arc Tangent Series

Given

$$\tan \alpha = t_0 + t_1 T + t_2 T^2 + t_3 T^3 \tag{B-8}$$

Find

$$\alpha = \alpha_0 + \alpha_1 T + \alpha_2 T^2 + \alpha_3 T^3$$

Let

$$\alpha = \alpha_0 + x$$

where x is a small angle:

$$\tan \alpha = \frac{\tan \alpha_0 + \tan x}{1 - \tan \alpha_0 \tan x}$$

which by Eq. (B-3) becomes

$$\tan \alpha = \frac{t_0 + \left(x + \frac{x^3}{3}\right)}{1 - t_0\left(x + \frac{x^3}{3}\right)}$$

by expansion in powers of x

$$\tan \alpha = t_0 + x\,(1 + t_0^2) + x^2\, t_0\,(1 + t_0^2) + x^3\left(\frac{1}{3} + t_0^2\right)\left(1 + t_0^2\right)$$

by substitution for x and expansion

$$\tan \alpha = t_0 + [\alpha_1 (1 + t_0^2)]\, T + [\alpha_2 (1 + t_0^2) + \alpha_1^2\, t_0\, (1 + t_0^2)]\, T^2 + \left[\alpha_3 (1 + t_0^2) + 2\,\alpha_1\,\alpha_2\, t_0\, (1 + t_0^2) + \alpha_1^3\left(\frac{1}{3} + t_0^2\right)\left(1 + t_0^2\right)\right] T^3 \tag{B-9}$$

Equating of like coefficients of T in Eqs. (B-8) and (B-9) allows the solution for the desired coefficients:

$$\left.\begin{aligned}
\alpha_0 &= \tan^{-1}(t_0) \\
\alpha_1 &= \frac{t_1}{1 + t_0^2} \\
\alpha_2 &= \frac{t_2}{1 + t_0^2} - \alpha_1^2\, t_0 \\
\alpha_3 &= \frac{t_3}{1 + t_0^2} - 2\,\alpha_1\,\alpha_2\, t_0 - \alpha_1^3\left(\frac{1}{3} + t_0^2\right)
\end{aligned}\right\} \tag{B-10}$$

Appendix C

Coordinate System Transformations

I. Definitions

To facilitate the description of coordinate transformations, it is useful to first state several working definitions. Three characteristics of coordinate systems are used: (1) frame, (2) center, and (3) type. Thus, individual coordinate transformations involve a change in frame, center, or type, and a complete transformation generally involves a series of changes of all three. This report deals only with changes of frame.

A. Frame

A coordinate frame (Table C-1) is defined in terms of an associated set of Cartesian axes: x, y, and z. The x–y plane is a defined reference plane (e.g., the equator plane) and the x-axis is a reference direction (e.g., the equinox). The z-axis is normal to the reference plane in a direction corresponding to some physical quantity; e.g., north. The reference plane is generally associated with some body; however, the origin of the frame need not be centered at that body. Reference planes and directions are commonly defined in terms of a time at which they exist physically; however, this time is not necessarily the same as the time associated with the coordinate values.

The reference coordinate frame in this report is the Earth mean equator and equinox of 1950.0 (referred to as "mean 1950.0"). All other frames are defined as rotations from this frame.

A frame may be regarded in two ways in relation to the time designation. The first way is to regard the time as defining the *epoch* at which the frame is defined and, subsequently, is nonrotating as coordinates are measured with respect to it. The second way is to regard the time to be the same as that of the coordinate values, and is a rotating frame always aligned *of-date*. The distinction has implications regarding the velocity rotations and is discussed in more detail later. In summary, a frame is defined by the specification of the following:

(1) Body.

(2) Reference plane.

(3) Reference direction.

(4) Epoch.

B. Center

The center of a coordinate frame may be at the center of any of the nine planets, the Sun, the Moon, or a defined station on the surface of any of these bodies. Translation between body centers is accomplished in mean 1950.0 Cartesian coordinates and translation between a body center and a station is accomplished in body-fixed Cartesian coordinates.

C. Type

Given a center and coordinate frame, there are several types of coordinates commonly used; they are:

(1) Cartesian.

(2) Spherical.

(3) Classical conic orbit elements.

(4) Hyperbolic asymptote.

Table C-1. Coordinate frames

Name	x–y plane	x-axis	z-axis
Mean equator and equinox	Mean equator of body	Ascending node of mean orbit on mean equator (mean equinox)	Direction of body rotation vector (north)
Mean orbit and equinox	Mean orbit of body	Mean equinox	Direction of orbital rotation vector
True equator and equinox	True equator of body	Ascending node of mean orbit on true equator (true equinox)	North
Mean orbit and true equinox	Mean orbit of body	True equinox	Direction of orbital rotation vector
Body fixed	True equator of body	Prime meridian	North

The type used to transfer between coordinate frames is the standard Cartesian coordinate set: $(x, y, z, \dot{x}, \dot{y}, \dot{z})$.

II. Transformation Between Frames

Frame transformations are done only in Cartesian coordinates, and are shown schematically in Fig. C-1. To transfer from one frame to another, follow the block diagram from the initial frame to the mean 1950.0 block (with T = initial epoch) and then back to the desired block (with T = final epoch).

At each step, check to see if the current block is the desired final frame or if the next step should branch to one of the orbit blocks.

Figure C-1 shows the position transformation matrix in the direction of the arrow. For example,

$$(\bar{X})_{\substack{\text{Body mean equator}\\ \text{and equinox of } T}} = A(\bar{X})_{50}$$

The reverse position transformations are simply the transposes of those indicated, since all transformations are orthogonal.

The velocity transformations are more complicated. In general, the velocity transformation involves the time derivatives of the transformation matrix. For example,

$$(\dot{\bar{X}})_{\substack{\text{Body mean equator}\\ \text{and equinox of } T}} = A(\dot{\bar{X}})_{50} + \dot{A}(\bar{X})_{50}$$

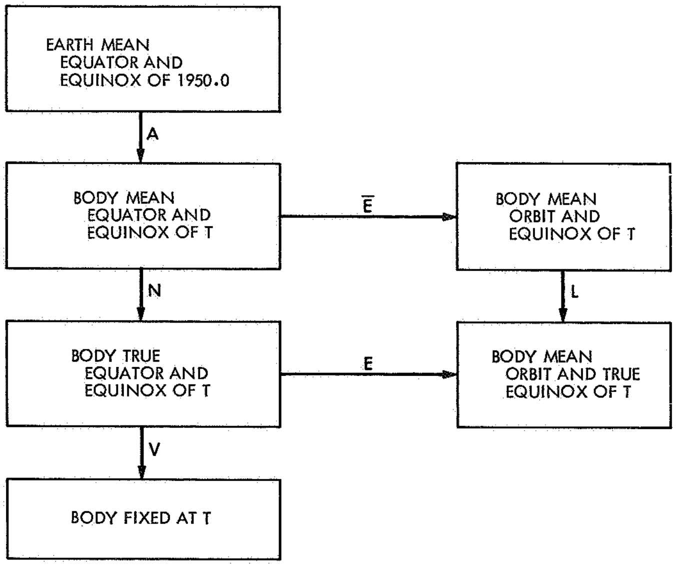


Fig. C-1. Transformations between coordinate frames

There are important exceptions to this transformation, and the following rules apply:

(1) In transforming *from* mean 1950.0 *to* any *of-date* frame, the matrix derivative is

  (a) Nonzero if the final output type is Cartesian or spherical.

  (b) Zero if the final output type is classical conic orbit elements or hyperbolic asymptote.

(2) In transforming *from* mean 1950.0 *to* any *epoch* frame, the matrix derivative is zero.

(3) In transforming *from* any frame *to* mean 1950.0, the matrix derivative is

  (a) Zero if the initial type is classical conic orbit elements or hyperbolic asymptote.

  (b) Zero if the initial conditions are relative to an *epoch* frame.

  (c) Nonzero if neither (a) nor (b) is true.

These rules are based on the philosophy that classical conic orbit elements and hyperbolic asymptotic types are related to *inertial* velocities, and that frames with designated epochs are nonrotating. It is possible with the noted rules to have input and output coordinates that are seemingly close in terms of the number of sequential transformations, but actually must follow a more elaborate sequence. For example, assume the following data:

(1) Initial coordinates: Sun-centered, Earth mean orbit and equinox *of-date* Cartesian coordinates.

(2) Final coordinates: Sun-centered, Earth mean orbit and equinox *of-date* classical conic orbit elements.

This would appear to be simply a change of type only; however, it is necessary to rotate first to mean 1950.0 with nonzero matrix derivatives, then back to Earth mean orbit and equinox of-date Cartesians with zero matrix derivatives (to get inertial Cartesian velocities), and then change to classical conic orbit elements.

The key to developing the velocity rotation logic is whether inertial velocities are needed and where.

All of the transformations in Fig. C-1 are general for all bodies, and are specified in terms of angles, which have the *same name* but different values depending on the particular body. Equations for evaluating the angles for each body are given in Section V; each matrix remains to be defined in terms of the angles.

Each matrix is developed in terms of individual rotations about the current *x*-, *y*-, or *z*-axis. The notations are defined in the Nomenclature.

$$\mathbf{A} = \{\Delta_{50} + 180°\}_z \{90° - \delta_{50}\}_x \{\alpha_{50} + 90°\}_z$$

$$\mathbf{N} = \mathbf{E}^T \mathbf{L} \bar{\mathbf{E}} = \{-\epsilon\}_x \{-\delta\,\psi\}_z \{\bar{\epsilon}\}_x$$

$$\bar{\mathbf{E}} = \{\bar{\epsilon}\}_x$$

$$\mathbf{L} = \{-\delta\,\psi\}_z$$

$$\mathbf{E} = \{\epsilon\}_x$$

$$\mathbf{V} = \{V + \delta\,\psi \cos\epsilon\}_z$$

The nutation angles ($\delta\psi$, $\delta\epsilon$) are not specified in this report. Actually, the nutation is unknown for all bodies except the Earth and the Moon, and therefore $\delta\psi$ and $\delta\epsilon$ are adopted as zero. For Earth, expressions are given in Ref. 1 and are also tabulated on the JPL ephemeris tapes (Ref. 3). Special consideration for the Moon is discussed in Appendix D.

Appendix D

Special Treatment for Moon Angles

The angles of the Moon are rapidly varying and cannot be represented with respect to the 1950.0 system by polynomials of low order in T. To obtain the 1950.0 angles for use in standardized coordinate transformation matrices, such as those in Appendix C, the following steps are recommended:

(1) Evaluation of mean-of-date polynominals for mean orbit elements. These polynomials are found from expressions in Ref. 1 (p. 107). Converting by Eq. (1), we obtain, with respect to the mean equinox and ecliptic of T

$$\left.\begin{aligned} i &= 2\sin^{-1}(\gamma) \\ &= 5.145396 \\ \Omega &= 12.112791 - 1934.139929\,T \\ &\quad + 0.002081\,T^2 + 0.000002\,T^3 \\ \omega &= \Gamma' - \Omega \\ &= 196.731198 + 6003.163629\,T \\ &\quad - 0.012425\,T^2 - 0.000014\,T^3 \end{aligned}\right\} \qquad \text{(D-1)}$$

and independent of plane

$$\left.\begin{aligned} a &= 384399.3 \text{ km } (a_{\leftmoon} \text{ in Table 17 of Ref. 2}) \\ e &= 0.05490\,0489 \\ M &= \leftmoon - \Gamma' \\ &= 215.531463 + 477198.858310\,T \\ &\quad + 0.009214\,T^2 + 0.000014\,T^3 \end{aligned}\right\} \qquad \text{(D-2)}$$

(2) Computation of i_{50}, Ω_{50}, and ω_{50} for the Moon. This computation is accomplished in the same manner as for the planets in the least-squares curve-fit method of Section III, by the use of Eq. (3).

(3) Mean obliquity for the Moon. This is the sum of i in step (1) and I', the constant angle between the ecliptic and the mean lunar equator (see Eq. 79, p. 26, Ref. 2).

$$\left.\begin{aligned} I' &= 1.533611 \\ I &= i + I' = 6.679007 \end{aligned}\right\} \qquad \text{(D-3)}$$

This equation comes from Cassini's second and third laws of the lunar rotation (see p. 27, Ref. 8).

(4) Computation of α_{50}, δ_{50}, and Δ_{50} for the Moon. The geometry of this problem is shown in Fig. D-1. First, the spherical triangle $\Omega\ Q\ E$ is solved for the intermediate angle β, and sides x and y

$$\Omega^* = \Omega - \Lambda + 180^\circ \qquad \text{(D-4)}$$

$$\left.\begin{aligned} \sin\beta\sin x &= \sin\pi_1 \sin\Omega^* \\ \sin\beta\cos x &= \cos\pi_1 \sin I' + \sin\pi_1 \cos I' \cos\Omega^* \\ \cos\beta &= \cos\pi_1 \cos I' - \sin\pi_1 \sin I' \cos\Omega^* \\ \sin\beta\sin y &= \sin I' \sin\Omega^* \\ \sin\beta\cos y &= \sin\pi_1 \cos I' + \cos\pi_1 \sin I' \cos\Omega^* \end{aligned}\right\} \qquad \text{(D-5)}$$

Next, the spherical triangle $\bar{\gamma}_{50}NQ$ is solved for the desired angles

$$\left.\begin{aligned} \sin(90^\circ - \delta_{50})\sin(\Delta_{50} - x) &= \sin\bar{\epsilon}_{50}\sin(\Pi_1 + y) \\ \sin(90^\circ - \delta_{50})\cos(\Delta_{50} - x) &= \cos\bar{\epsilon}_{50}\sin\beta + \sin\bar{\epsilon}_{50}\cos\beta\cos(\Pi_1 + y) \\ \cos(90^\circ - \delta_{50}) &= \cos\bar{\epsilon}_{50}\cos\beta - \sin\bar{\epsilon}_{50}\sin\beta\cos(\Pi_1 + y) \\ \sin(90^\circ - \delta_{50})\sin(\alpha_{50} + 90^\circ) &= \sin\beta\sin(\Pi_1 + y) \\ \sin(90^\circ - \delta_{50})\cos(\alpha_{50} + 90^\circ) &= \sin\bar{\epsilon}_{50}\cos\beta + \cos\bar{\epsilon}_{50}\sin\beta\cos(\Pi_1 + y) \end{aligned}\right\} \qquad \text{(D-6)}$$

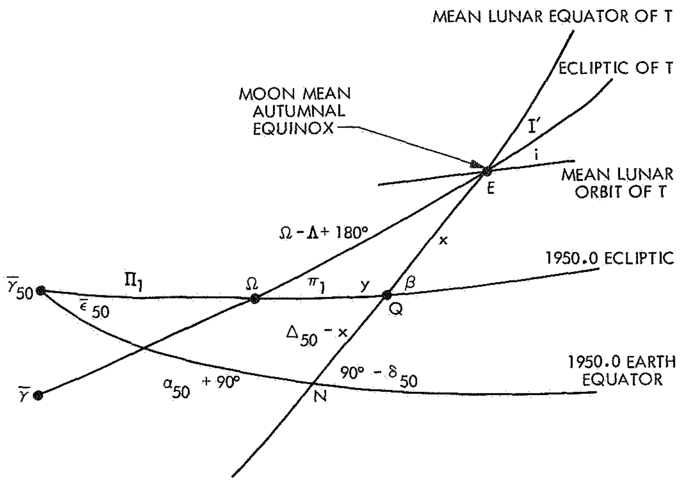
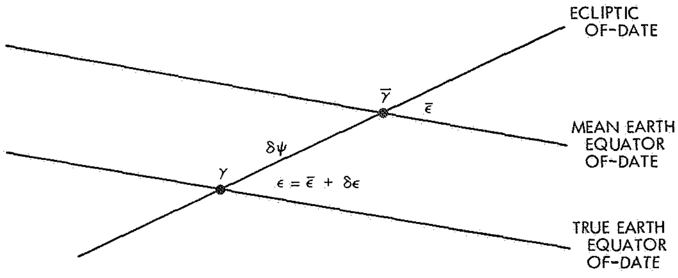


Fig. D-1. Geometry for pole angles of Moon

(5) Computation of Moon nutation angles. The nutations of the Moon are commonly represented by the physical libration angles τ, σ, ρ. The equivalent Moon nutation angles, analogous to those used for the Earth, are shown in Fig. D-2. First, evaluate the physical libration angles as specified, for example, in Ref. 2 (p. 26). Then, solve the spherical triangle $\gamma_{☾} N \gamma_{☾}$ in Fig. D-2 ($\epsilon_{☾} = I$ from Step 3).

$$\left.\begin{aligned}
\sin\epsilon_{☾}\sin(-\delta\psi_{☾}) &= \sin(I' + \rho)\sin\sigma \\
\sin\epsilon_{☾}\cos(-\delta\psi_{☾}) &= \cos(I' + \rho)\sin i \\
&\quad + \sin(I' + \rho)\cos i\cos\sigma \\
\cos\epsilon_{☾} &= \cos(I' + \rho)\cos i \\
&\quad - \sin(I' + \rho)\sin i\cos\sigma \\
\sin\epsilon_{☾}\sin x^* &= \sin i\sin\sigma \\
\sin\epsilon_{☾}\cos x^* &= \sin(I' + \rho)\cos i \\
&\quad + \cos(I' + \rho)\sin i\cos\sigma
\end{aligned}\right\} \qquad \text{(D-7)}$$

(a) EARTH NUTATION



(b) MOON NUTATION

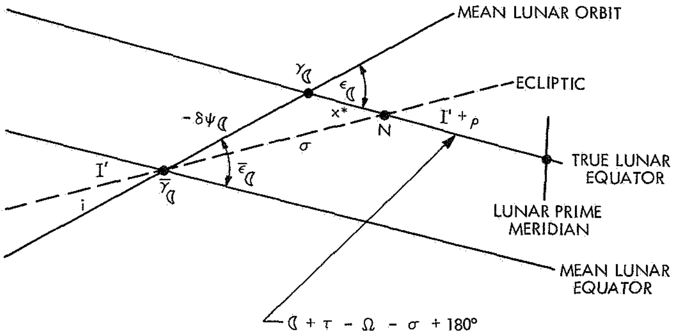


Fig. D-2. Geometry for nutation angles of Moon

$$\delta\epsilon_{☾} = \epsilon_{☾} - I \qquad \text{(D-8)}$$

(6) Computation of hour angle of true equinox. The hour angle of the true equinox is (from Fig. D-2)

$$V + \delta\psi_{☾}\cos\epsilon_{☾} = M + \omega + \tau - \sigma + 180^\circ + x^* \qquad \text{(D-9)}$$

Steps 1–6 provide a set of Moon angles equivalent to those in Section V for use in the coordinate transformations of Appendix C.

Nomenclature

$\mathbf{A}$	coordinate transformation matrix (see Fig. C-1)
A	general polynomial variable
a_0, a_1, a_2, a_3	general polynomial coefficients
a	semimajor axis
A_E	planetocentric right ascension of Earth
d	ephemeris days from January 1.0, 1950
d_0	ephemeris days from January 0.5, 1900
$\mathbf{E}$	coordinate transformation matrix (see Fig. C-1)
$\overline{\mathbf{E}}$	coordinate transformation matrix (see Fig. C-1)
E.T.	ephemeris time
e	eccentricity
F	ratio of Julian to tropical century (see Eq. 2)
I'	angle between ecliptic and mean lunar equator of-date
I	general mean obliquity for all planets
i	inclination of mean orbit to ecliptic
JED	Julian Ephemeris Date
k	tropical centuries from 1950.0 to January 1.0, 1950 (see Eq. 2)
$\mathbf{L}$	coordinate transformation matrix (see Fig. C-1)
LCM	longitude of central meridian
M	mean anomaly
m	general precession in right ascension
$\mathbf{N}$	coordinate transformation matrix (see Fig. C-1)
n	general precession in declination
$\mathbf{Q}$	coordinate transformation matrix (see Section III)
$\mathbf{R}$	coordinate transformation matrix (see Section III)
r	Earth-planet distance at epoch defining LCM
T	time in Julian centuries from January 1.0, 1950
T_0	time in Julian centuries from January 0.5, 1900
T_{tr}	time in tropical centuries from 1950.0
$\mathbf{V}$	coordinate transformation matrix (see Fig. C-1)
V	hour angle of mean equinox from prime meridian
x,y,z	intermediate angles (see Eq. 13)
$\overline{X} = (x,y,z)^T$	Cartesian position coordinates
$\dot{\overline{X}} = (\dot{x},\dot{y},\dot{z})^T$	Cartesian velocity coordinates
x^*	angle from true lunar equinox along true lunar equator to descending node on ecliptic (see Fig. D-2)
Z	Earth equatorial precession angle (see Refs. 1, 2, and 5)
α	right ascension of planet mean north pole
$\dot{\alpha}$	linear rate of α
β	intermediate angle (see Fig. D-1)
Γ'	mean longitude of lunar perigee
γ	constant of lunar inclination (see Eq. D-1)
Δ	angle from node on Earth equator along planet equator to autumnal equinox
δ	declination of planet mean north pole
$\dot{\delta}$	linear rate of δ
$\delta\psi$	nutation in longitude (Earth)
$\delta\epsilon$	nutation in obliquity (Earth)
$\delta\psi_{☾}$	Moon nutation in longitude
$\delta\epsilon_{☾}$	Moon nutation in obliquity
$\bar{\epsilon}$	Earth mean obliquity
$\bar{\epsilon}_{☾}$	Moon mean obliquity
ϵ	Earth true obliquity
$\epsilon_{☾}$	Moon true obliquity
ζ_0	Earth equatorial precession angle (see Refs. 1, 2, and 5)

Nomenclature (contd)

θ	Earth equatorial precession angle (see Refs. 1, 2, and 5)
Λ	ecliptic precession angle (see Fig. 1)
μ	planet precession rate (see Eq. 8)
π_1	ecliptic precession angle (see Fig. 1)
Π_1	ecliptic precession angle (see Fig. 1)
ρ,σ,τ	lunar physical libration angles
τ	light time for unit distance in definition of LCM (Eq. 17)
Ω	longitude of node of mean orbit on ecliptic
Ω^*	$\Omega - \Lambda + 180°$ (for use in Eq. D-5)
ω	planet rotation rate in definition of LCM (Eq. 17)
ω	argument of perihelion of mean orbit
$\tilde{\omega}$	longitude of perihelion $= \Omega + \omega$
1950.0	beginning of Besselian year 1950 (JED 2433282.423357)
γ	true vernal equinox
$\bar{\gamma}$	mean vernal equinox
☾	mean longitude of Moon

Subscript

50	(1) with respect to the mean Earth equator (or ecliptic) and equinox of 1950.0 when used with variables $\bar{\gamma}$, i, Ω, ω, δ, α, and Δ (2) evaluated at the epoch 1950.0 when used with variables $\bar{\epsilon}$, $\dot{\alpha}$, and $\dot{\delta}$

Rotation matrices

$\{\theta\}_i$ positive rotation about i-axis by an angular amount θ

$$\{\theta\}_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$\{\theta\}_y = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\{\theta\}_z = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

References

1. *Explanatory Supplement to the Ephemeris,* Her Majesty's Stationery Office, London, 1961.
2. Melbourne, W. G., et al., *Constants and Related Information for Astrodynamic Calculations, 1968,* Technical Report 32-1306. Jet Propulsion Laboratory, Pasadena, Calif., July 15, 1968.
3. Devine, C. J., *JPL Development Ephemeris Number 19,* Technical Report 32-1181. Jet Propulsion Laboratory, Pasadena, Calif., Nov. 15, 1967.
4. Khatib, A. R., "Double-Precision Trajectory Program: DPTRAJ," in *Supporting Research and Advanced Development,* Space Programs Summary 37-49, Vol. II. Jet Propulsion Laboratory, Pasadena, Calif., Jan. 31, 1968.
5. Lieske, J., *Expressions for the Precession Quantities and Their Partial Derivatives,* Technical Report 32-1044. Jet Propulsion Laboratory, Pasadena, Calif., June 15, 1967.
6. De Vaucouleurs, G., "The Physical Ephemeris of Mars," *Icarus,* Vol. 3, 1964.
7. Sturms, F. M., Jr., "A Proposed Venus Coordinate System," in *Supporting Research and Advanced Development,* Space Programs Summary 37-51, Vol. III. Jet Propulsion Laboratory, Pasadena, Calif., June 30, 1968.
8. *Physics and Astronomy of the Moon.* Edited by Z. Kopal. Academic Press, New York, 1962.

NASA — JPL — Coml., L.A., Calif.

N71-16785

TECHNICAL REPORT STANDARD TITLE PAGE

1. Report No. 32-1508

2. Government Accession No.

3. Recipient's Catalog No.

4. Title and Subtitle
POLYNOMIAL EXPRESSIONS FOR PLANETARY EQUATORS AND ORBIT ELEMENTS WITH RESPECT TO THE MEAN 1950.0 COORDINATE SYSTEM

5. Report Date January 15, 1971

6. Performing Organization Code

7. Author(s) Francis M. Sturms, Jr.

8. Performing Organization Report No.

9. Performing Organization Name and Address
JET PROPULSION LABORATORY
California Institute of Technology
4800 Oak Grove Drive
Pasadena, California 91103

10. Work Unit No.

11. Contract or Grant No.
NAS 7-100

13. Type of Report and Period Covered
Technical Report

12. Sponsoring Agency Name and Address
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Washington, D.C. 20546

14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

Expressions are presented for the mean orbital elements of the nine planets with respect to the mean equinox and ecliptic of 1950.0. Also, expressions are presented for the right ascension and declination of the north pole of each of the nine planets with respect to the mean equinox and Earth equator of 1950.0. The expressions are polynomials in time T measured in Julian centuries from the epoch January 1.0, 1950 E.T. The expressions are useful for coordinate transformations and approximate planetary ephemerides in astrodynamic computer programs.

17. Key Words (Selected by Author(s))
Mathematical Sciences
Planetary Motion
Trajectory Analysis/Orbit Determination

18. Distribution Statement
Unclassified -- Unlimited

19. Security Classif. (of this report)
Unclassified

20. Security Classif. (of this page)
Unclassified

21. No. of Pages
22

22. Price

HOW TO FILL OUT THE TECHNICAL REPORT STANDARD TITLE PAGE

Make items 1, 4, 5, 9, 12, and 13 agree with the corresponding information on the report cover. Use all capital letters for title (item 4). Leave items 2, 6, and 14 blank. Complete the remaining items as follows:

3. Recipient's Catalog No. Reserved for use by report recipients.
7. Author(s). Include corresponding information from the report cover. In addition, list the affiliation of an author if it differs from that of the performing organization.
8. Performing Organization Report No. Insert if performing organization wishes to assign this number.
10. Work Unit No. Use the agency-wide code (for example, 923-50-10-06-72), which uniquely identifies the work unit under which the work was authorized. Non-NASA performing organizations will leave this blank.
11. Insert the number of the contract or grant under which the report was prepared.
15. Supplementary Notes. Enter information not included elsewhere but useful, such as: Prepared in cooperation with... Translation of (or by)... Presented at conference of... To be published in...
16. Abstract. Include a brief (not to exceed 200 words) factual summary of the most significant information contained in the report. If possible, the abstract of a classified report should be unclassified. If the report contains a significant bibliography or literature survey, mention it here.
17. Key Words. Insert terms or short phrases selected by the author that identify the principal subjects covered in the report, and that are sufficiently specific and precise to be used for cataloging.
18. Distribution Statement. Enter one of the authorized statements used to denote releasability to the public or a limitation on dissemination for reasons other than security of defense information. Authorized statements are "Unclassified—Unlimited," "U.S. Government and Contractors only," "U.S. Government Agencies only," and "NASA and NASA Contractors only."
19. Security Classification (of report). NOTE: Reports carrying a security classification will require additional markings giving security and downgrading information as specified by the Security Requirements Checklist and the DoD Industrial Security Manual (DoD 5220.22-M).
20. Security Classification (of this page). NOTE: Because this page may be used in preparing announcements, bibliographies, and data banks, it should be unclassified if possible. If a classification is required, indicate separately the classification of the title and the abstract by following these items with either "(U)" for unclassified, or "(C)" or "(S)" as applicable for classified items.
21. No. of Pages. Insert the number of pages.
22. Price. Insert the price set by the Clearinghouse for Federal Scientific and Technical Information or the Government Printing Office, if known.