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Department of Electrical Engineering

CONTROL OF DISTRIBUTED PARAMETER SYSTEMS AS APPLIED TO A LUNAR LANDING VEHICLE SIMULATOR

by

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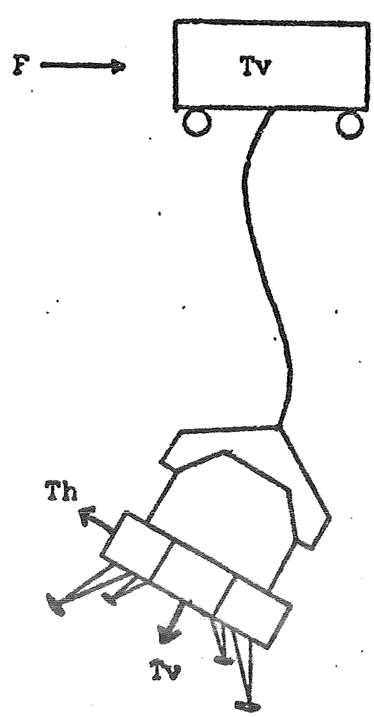
I. INTRODUCTION

This study has been undertaken for the purpose of obtaining an imporved feedback control for a lunar landing vehicle simulator. The simulator, used to train astronauts for lunar landing, consists of a cable supported, rocket propelled vehicle. The lunar gravity is simulated by maintaining tension on the cable equal to 5/6 of the vehicle weight. A drum, that the cable in wound onto, is controlled to keep the proper tension on the cable, and to allow the cable length to be changed.

The drum is mounted on a mechanism similar to an x-y plotter. A bridge moves along one axis and a cart moves along the bridge for the other axis.



Tv = tracking cart

F = control force

T_h = Horizonal Thrust

T_v = Vertical Thrust

Figure 1-1

The existing hardware simulator at the NASA Langley Reserach Center [1], uses the angle at the top of the cable with respect to vertical as an indication of vehicle position relative to the tracking cart. Unfortunately, if the piloting becomes too active, lateral and longitudinal vibrations occur in the cable which impose disturbing forces on the vehicle. If the control action is to be improved so as to attenuate these cable vibrations, it is clearly necessary to introduce additional state information to the feedback control law.

Although a complete study of this system would entail consideration of a three dimensional problem with interacting dynamics and time variable parameters, the scope of this work will be restricted to a study of vehicular motion along a horizontal coordinate axis. Since for small perturbations the responses of the system along the three axes are decoupled, this study can to a first approximation be readily generalized to include the three dimensional case. However, the rigorous analysis of the effect of time varying parameters is beyond the scope of this thesis.

Because we are considering a distributed parameter system, it is not possible to obtain complete state information about the cable from on-line measurements, since in theory there are an infinite number of states. A further restriction, imposed by practical considerations, is that information about the cable can be measured only at its ends, and nowhere else along its length. This lack of freedom of sensor location is due to the changing cable length, the difficulties in attaching suitable sensors, and cable twisting which would confuse directional orientation.

An approach to solving this problem is to design an observer which is capable of estimating a more complete state vector based on knowledge of the control input and states that are measurable [2][3]. However, the effects of parameter uncertainties and noise have not been studied in sufficient depth to justify confidence in control system design based on the use of the observer principle when estimating states of higher order systems. This is the subject of further research.

Another approach, used here, is to develop an optimal or sub-optimal control law from measurements which are available. Thus, a linear control law, consisting of available measured information multiplied by a gain vector, can be optimized in terms of an appropriate cost function by using Powell's Method [4], which is basically a modified relaxation technique. For a description of Powell's Method, see Appendix A.

It should be recognized that the solution, as found by the minimization procedure, will be sub-optimal for several reasons.

(a) All of the state information is not available.

(b) The integration time interval of a cost function producing a constant-gain solution is infinite. This interval can be used for analytical solutions, but when a cost function is used in simulation studies some practical restriction must be placed on the upper limit. This is usually fixed by observing asymptotic behavior of feedback terms as the time interval is increased. Here the practical limitation encountered was the large amounts of computer time concerned. Therefore, twenty seconds was used for the integration interval.

(c) The system is large enough so that it is impractical to find the cost as a function of gains. This leaves the possibilities open for local minima.

(d) Only one representative thrust program was used in the minimization procedure. Other thrust programs could result in a different solution.

The effects of (b), (c), and (d) could be studied with a hybrid computer. With its high speed repetitive mode, it would be possible to try different initial gain vectors, longer running times, and different thrust programs. Unfortunately, the digital program to be described was found to require solution times in the order of five hours, ruling out the possibilities for extensive simulation studies. It is expected, however, that the thrust program used is representative enough to produce satisfactory results.

The model of the system used for this study was developed by C. H. Knapp [5]. It is a segmented representation of the real cable in which the accuracy of simulation depends upon the number of sections used in the model. Six sections, as used to model the cable in this study, will support up to five harmonic modes. Experience shows that this is more than adequate for the accuracy demanded in the simulation.

II PRACTICAL STATE MEASUREMENT AND CONTROL

The distributed parameter system presents a particularly difficult problem to the control engineer. Not only are concepts of controllability and observability difficult to apply, but the stability theory of partial differential equations has not yet been developed to the same extent as for ordinary differential equations.

Athans [6] offers a set of procedures, and a philosophy leading to a reasonable approach to distributed parameter systems. A similar set of rules has been followed in this problem.

(a) Analysis of the system should remain in distributed parameters form as long as possible.

(b) The number of transducers must be limited to some practical number; in this case there is a firm restriction to the cable ends only.

(c) The number of control inputs to the system must be limited to some practical number. In this problem the control inputs must be limited to the ends of the cable.

Goodson and Klein [7] have presented a weakened definition of observability for use with systems having modal solutions. A system is defined as N-mode observable if mode amplitudes for the first N modes can be uniquely established from measured information. Higher modes constitute an error in the function. In the cable problem, with reference to Figure 3-1, and subject to the assumption that the cable ends are fixed, if the motion is defined by the first N modes,

$$y\ (x,t) = \sum_{n=1}^{N} \sin \frac{n\pi x}{\ell} \ (A_n \cos \frac{n\pi at}{\ell} + B_n \sin \frac{n\pi at}{\ell}), \qquad (2.1)$$

then the remaining terms may be defined as an error function,

$$e(x,t) = \sum_{n=N+1}^{\infty} \sin \frac{n\pi x}{\ell}(A_n \cos \frac{n\pi at}{\ell} + B_n \sin \frac{n\pi at}{\ell}). \qquad (2.2)$$

The value of N will depend on factors such as the locations and number of the transducers, and the number of derivatives that can be obtained in practice.

The concept of N-mode observability can be considered a conservative one. The measured information may actually contain information defining higher modes, with some practical consideration, such as noise problems, limiting how many modes can be observed. To show that higher mode information is available from the angles measured at the cable's ends, the following is offered.

The deflection at an arbitrary point x on the cable is

$$y(x,t) = \sum_{n=1}^{\infty} \sin \frac{n\pi x}{\ell}(A_n \cos \frac{n\pi at}{\ell} + B_n \sin \frac{n\pi at}{\ell}). \qquad (2.3)$$

Taking the first spacial derivative

$$\theta(x,t) \simeq \frac{dy(x,t)}{d\,x} = \sum_{n=1}^{\infty} \frac{n\pi}{\ell} \cos \frac{n\pi x}{\ell} \ (A_n \cos \frac{n\pi at}{\ell} + B_n \sin \frac{n\pi at}{\ell}) \qquad (2.4)$$

where ℓ is the overall cable length.

The first time derivative becomes

$$\dot{\theta}\ (x,t) = \sum_{n=1}^{\infty} (\frac{n\pi a}{\ell})(\frac{n\pi}{\ell}) \cos \frac{n\pi x}{\ell} \ (-A_n \sin \frac{n\pi at}{\ell} + B_n \cos \frac{n\pi at}{\ell}). \qquad (2.5)$$

Each successive derivative, evaluated at both ends of the cable, provides two more equations with the same unknowns, A_n and B_n. By taking n successive derivatives, and evaluating at both ends of the cable, the first through n^{th} terms of A_n and B_n can be found. By making n arbitrarily large, an arbitrarily large number of modes can be uniquely defined.

It is instersting to note that the term cos $\frac{n\pi x}{\ell}$ will always equal ± 1 for all n, when evaluated at $x=0$ and $x=\ell$. In this case, the restriction that measurements be made only at the ends of the cable is not a serious drawback in theory. However, the fact that higher mode information is available in theory does not mean that it is available in practice, since higher order derivatives are quickly obscured by the noise. Although modal amplitudes were not included explicitly in the cost function it can be argued that the higher mode information contained in the measured angles will insure that these modes will not become unstable for a set of gains obtained from a minimization procedure.

III THE SIMULATION MODEL

The simulation model used for computer simulation in this problem is a segmented model developed in [5], modified to permit the use of a force input. Thus, with reference to the mass at the top of the cable, as shown in Figure 3-1

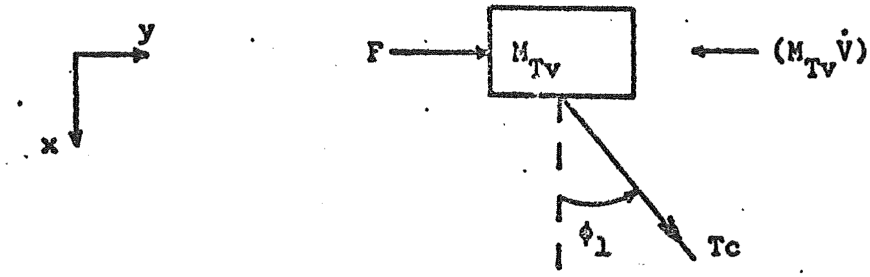


Figure 3-1

$$\dot{v} = \frac{F}{M_{Tv}} + \frac{T_c}{M_{Tv}} \sin \phi_1$$

where T_c is the tension in the cable, and ϕ_1 is the angle of the cable, measured with respect to the vertical, at the point of contact with the cart. With the small angle approximation

$$\dot{v} = \frac{T_c}{M_{Tv}} \phi_1 + \frac{F}{M_{Tv}} \quad . \tag{3.1}$$

The remaining state equations, as developed in [5] are, using small angle approximations:

$$\ddot{\phi}_1 = -\frac{T_c}{r_e}\left[\frac{1}{M_{Tv}} + \frac{1}{M_c}\right]\phi_1 + \frac{T_c}{M_c r_e}\phi_2 - \frac{F}{M_{Tv} r_e}$$

$$\ddot{\phi}_2 = \frac{T_c}{M_c r_e}\phi_1 - 2\frac{T_c}{M_c r_e}\phi_2 + \frac{T_c}{M_c r_e}\phi_3$$

$$\ddot{\phi}_3 = \frac{T_c}{M_c r_e}\phi_2 - 2\frac{T_c}{M_c r_e}\phi_3 + \frac{T_c}{M_c r_e}\phi_4$$

$$\ddot{\phi}_4 = \frac{T_c}{M_c r_e}\phi_3 - 2\frac{T_c}{M_c r_e}\phi_4 + \frac{T_c}{M_c r_e}\phi_5$$

$$\ddot{\phi}_5 = \frac{T_c}{M_c r_e}\phi_4 - 2\frac{T_c}{M_c r_e}\phi_5 + \frac{T_c}{M_c r_e}\phi_6$$

$$\ddot{\phi}_6 = \frac{T_c}{M_c r_e}\phi_5 - \frac{T_c}{r_e}\left[\frac{1}{M_c} + \frac{1}{M_w}\right]\phi_6 + \frac{T_w}{M_w r_e}\phi_7$$

$$\ddot{\phi}_7 = \frac{T_c}{M_w r_w}\phi_6 - \frac{T_w}{r_w}\left[\frac{1}{M_v} + \frac{1}{M_w}\right]\phi_7 + \frac{T_h}{M_v r_w} \qquad (3.2)$$

where

M_{Tv} = mass of tracking vehicle

T_c = cable tension

T_w = whiffletree tension

r_e = equalibrium of cable segment

r_w = whiffletree length

M_c = mass of cable segment

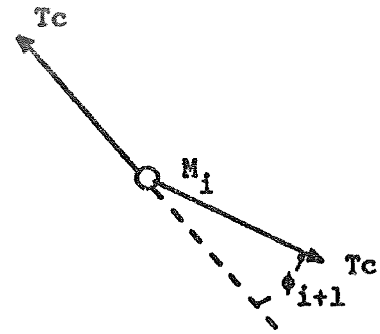
M_w = mass of whiffletree

M_v = mass of simulation vehicle

T_h = horizontal thrust component

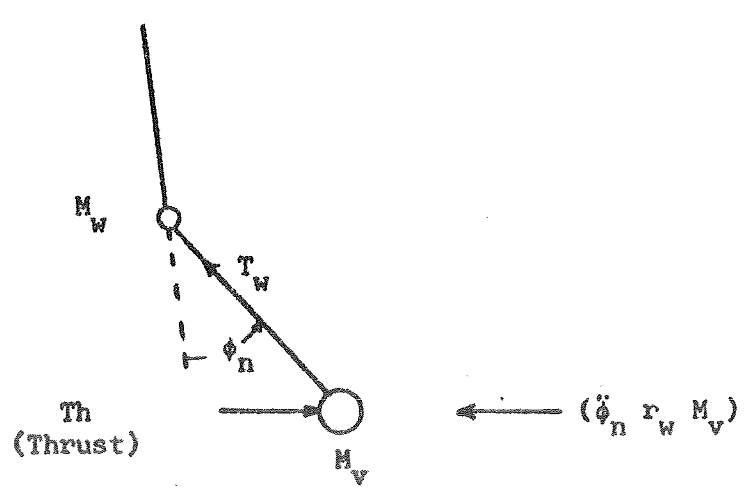
F = Force applied to tracking vehicle

To explain the coordinate system of the segmented model, Figures (3-2) and (3-3) are offered as illustrated in [5].



i^{th} cable section

Figure 3-2



n^{th} section whiffletree and vehicle

Figure 3-3

IV TRANSFORMATION OF SEGMENTED MODEL STATES TO INFORMATION MEASURABLE ON THE REAL CABLE

Since the information that is measurable from the real cable is not directly available from the segmented model, a transformation must be developed to extract this information. The initial assumptions are:

(a) The time constants of the vehicle, and of the tracking cart, are long enough compared to those of the cable, to consider the cable as constrained at both ends.

(b) Angles will be small enough to justify use of small angle approximations.

(c) Deflections in the cable are small enough so that the tension in the cable may be considered to be constant.

(d) Bending moments in the cable are negligible.

We now look at the classical vibrating string problem. The partial differential equation describing the string is

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}, \quad a^2 = \frac{Tg}{w'} \tag{4.1}$$

where

y = deflection of a point on the cable perpendicular to cable length direction

x = coodinate axis along cable length

T = cable tension

g = gravitational acceleration

w'= cable weight per unit length.

Assume a solution of the form

$$y(x,t) = (C \cos \frac{\lambda}{a} x + D \sin \frac{\lambda}{a} x)\ (A \cos \lambda t + B \sin \lambda t). \qquad (4.2)$$

With the cable constrained at both ends, and defining ℓ as the cable length, we have

$$y(0,t) = y(\ell,t) = 0.$$

Thus at $x=0$

$$x(0,t) = 0 = \overset{0}{\cancel{C}}\ (A \cos \lambda t + B \sin \lambda t) \qquad (4.3)$$

and at $x=\ell$

$$\sin \frac{\lambda}{a} \ell = 0 \quad \text{or} \quad \frac{\lambda \ell}{a} = n\pi.$$

Therefore

$$\lambda_n = \frac{n\pi a}{\ell}, n= 1,2,3,\ldots$$

and

$$y(x,t) = \sum_{n=1}^{\infty} \sin \frac{n\pi x}{\ell} (A_n \cos \frac{n\pi at}{\ell} + B_n \sin \frac{n\pi at}{\ell}) \qquad (4.4)$$

Deflection along the cable will vary periodically between extreme values proportional to

$$\sum_{n=1}^{\infty} \sin \frac{n\pi x}{\ell}. \qquad (4.5)$$

The amplitude term is

$$(A_n \cos \frac{n\pi at}{\ell} + B_n \sin \frac{n\pi at}{\ell}). \qquad (4.6)$$

The restriction imposed by the nature of the cable is that information is measurable only at the ends of the cable. Another restriction is that it is not practical to consider higher derivatives than the first, because of noise problems. These restrictions impose a limitation on mode observability. Considering the first two vibration modes, we have

$$y(x,t) = A(t) \sin \frac{\pi x}{\ell} + B(t) \sin \frac{2\pi x}{\ell} \tag{4.7}$$

where A(t) and B(t) are the peak mode amplitudes of the first and second modes respectively.

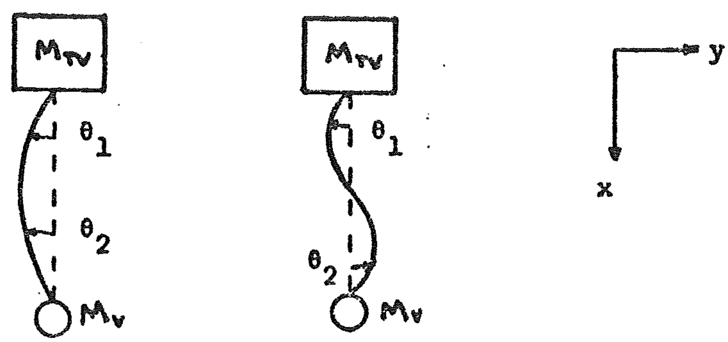


Figure 4-1

Taking the first derivative with respect to the spacial coordinate x

$$\frac{dy}{dx}(x,t) = A(t) \ \frac{\pi}{\ell} \cos \frac{\pi x}{\ell} + B(t) \ \frac{2\pi}{\ell} \ \cos \frac{2\pi x}{\ell} . \tag{4.8}$$

Using the small angle approximation,

$$\frac{dy}{dx}(x,t) \doteq \theta \ (x,t) \tag{4.9}$$

Evaluating $\theta(x,t)$ at $x=0$ and $x=\ell$

$$\theta_1(0,t) = A(t) \ \pi/\ell + B(t) \ 2\pi/\ell \tag{4.10}$$

$$\theta_2(\ell,t) = - A(t) \ \pi/\ell + B(t) \ 2\pi/\ell \tag{4.11}$$

We now have two equations and two unknowns, showing the relationship between the measured angles at the ends of the cable, and estimated amplitudes of the first two modes.

Taking the first time derivative

$$\dot{\theta}_1 = \dot{A}(t)\ \pi/\ell + \dot{B}(t)\ 2\pi/\ell, \tag{4.12}$$

$$\dot{\theta}_2 = -\dot{A}(t)\ \pi/\ell + \dot{B}(t)\ 2\pi/\ell. \tag{4.13}$$

Within the stated assumptions and restrictions, and given the angles and first derivatives, the amplitudes of the first two modes are uniquely defined. These amplitudes are an estimate with higher mode amplitudes constituting the error in the estimate. By definition [7] the cable is two-mode observable.

In addition to the vibration modes, there is also a pendulum mode, at a much lower frequency as illustrated by Figure 4-2.

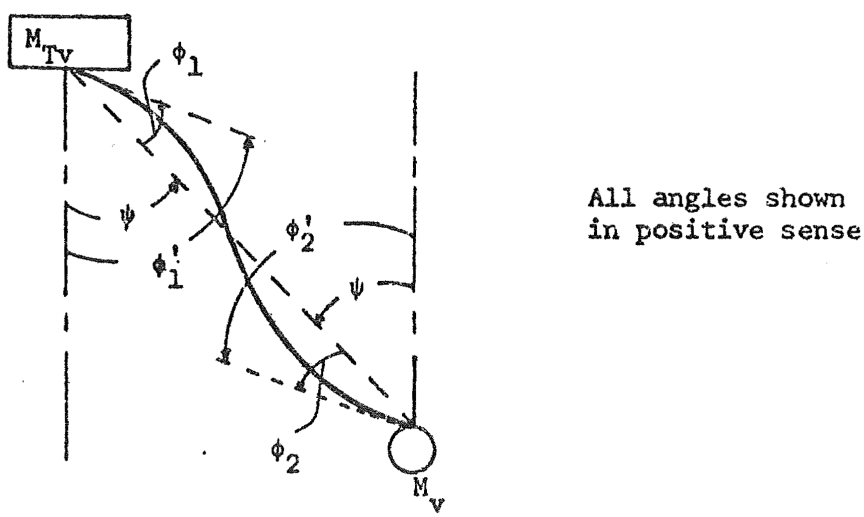


Figure 4-2

Thus the pendulum and vibration mode information can be easily separated if desired. From Figure 4-2 we see that

$$\theta_1 = \theta_1' - \psi \qquad (4.14)$$

$$\theta_2 = \theta_2' - \psi$$

with derivatives

$$\dot{\theta}_1 = \dot{\theta}_1' - \dot{\psi} \qquad (4.15)$$

$$\dot{\theta}_2 = \dot{\theta}_2' - \dot{\psi}$$

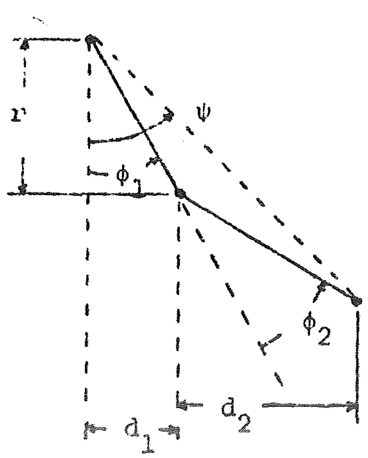
The angles θ_1', $\dot{\theta}_1'$, θ_2', $\dot{\theta}_2'$, ψ and $\dot{\psi}$ are assumed to be measurable.

While Knapp's segmented model [5] is well suited to computer simulation, it has yet to be shown that the angles defining the orientation of the segments can be used to define motion of the real cable in terms of mode amplitudes.

Using the small angle approximation for the two-segment model it is seen that angle ψ defined in Figure 4.3 can be expressed as

$$\psi = \frac{d_1 + d_2}{2r} = \frac{\phi_1 r + (\phi_1 + \phi_2)r}{2r}$$

$$= \frac{2\phi_1 + \phi_2}{2} \qquad (4.16)$$



$d_1 = \phi_1 r$ (r=cable segment length)

$d_2 = (\phi_1 + \phi_2)r$

Figure 4.3

By carrying this on to n-1 sections:

$$\psi = \frac{(n-1)\phi_1 + (n-2)\ \phi_2 + \ldots . + \phi_{n-1}}{(n-1)} . \qquad (4.17)$$

Here (n-1) is used as the last cable section, the n^{th} section being reserved for the whiffletree.

Other angles, corresponding to measured angles on the real cable, can be found through simple geometric relations.

$$\theta_2^s = \alpha - \psi \qquad (4.18)$$

$$\theta_1^s = \phi_1 - \psi \qquad (4.19)$$

$$\alpha = \sum_{k=1}^{n-1} \phi_k \qquad (4.20)$$

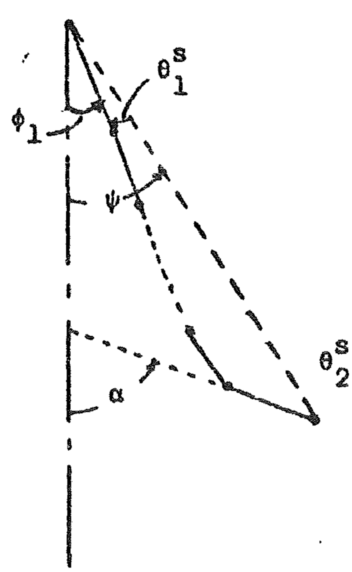


Figure 4-4

Note that θ_1^s, θ_2^s are defined relative to the segmented model. As shown below, with reference to Figure 4-5, a transformation can be found relating these angles to θ_1, θ_2 as measured at the ends of the actual cable.

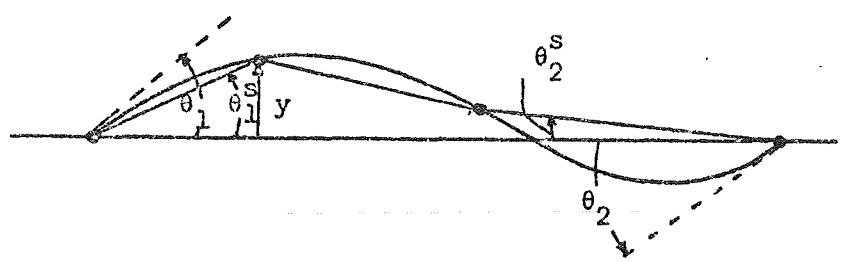


Figure 4-5

The deflection of an arbitrary point on the cable is a sum of the deflections caused by the two modes,

$$y\ (x) = a \sin \frac{\pi x}{\ell} + b \sin \frac{2\pi x}{\ell} = y_1 + y_2 \tag{4.21}$$

with a and b representing the instantaneous amplitudes.

Looking at the contribution from the first mode (see Figure 4-6) where $\ell/(n-1)$ is the length of the first segment and

$$\theta_{ij} = \theta_{\text{angle, mode}}$$

we have

$$\theta_{11} = \frac{dy_1}{dx} = \frac{a\pi}{\ell}\Big|_{x=0}$$

and at the hinge of the first segment

$$y_1 = a \sin \frac{\pi x}{\ell}$$

$$= a \sin \frac{\pi}{n-1} \quad .$$

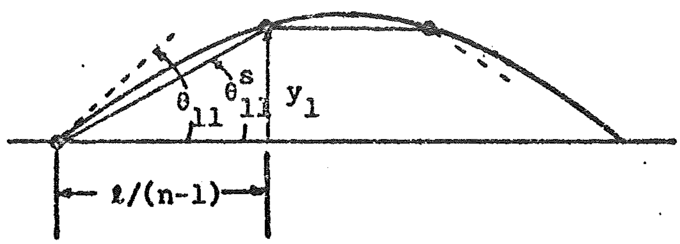


Figure 4-6

from which it follows that

$$\theta^s_{11} \approx \frac{a \sin(\frac{\pi}{n-1})}{\ell/n-1} = \frac{a(n-1) \sin(\frac{\pi}{n-1})}{\ell} . \tag{4.22}$$

The correction factor for the first mode is:

$$K_1 = \frac{\theta_{11}}{\theta^s_{11}} = \frac{a\pi/\ell}{\frac{a(n-1) \sin(\frac{\pi}{n-1})}{\ell}} = \frac{\pi}{(n-1) \sin(\frac{\pi}{n-1})} . \tag{4.23}$$

Following the same development for the second mode

$$K_2 = \frac{\theta_{12}}{\theta^s_{12}} = \frac{2\pi}{(n-1) \sin(\frac{2\pi}{n-1})} . \tag{4.24}$$

The angles at the ends of the cable are also a sum of the contributions from two modes,

$$\begin{aligned} \theta_1 &= \theta_{11} + \theta_{12} , \quad \theta^s_1 = \theta^s_{11} + \theta^s_{12} , \\ \theta_2 &= \theta_{21} + \theta_{22} , \quad \theta^s_2 = \theta^s_{21} + \theta^s_{22} . \end{aligned} \tag{4.25}$$

From the first and second mode correction factors

$$\theta_{11} = \frac{\pi\theta^s_{11}}{(n-1)\sin\left(\frac{\pi}{n-1}\right)}, \qquad \theta_{21} = \frac{\pi\,\theta^s_{21}}{(n-1)\sin\left(\frac{\pi}{n-1}\right)},$$

$$\theta_{12} = \frac{2\pi\theta^s_{12}}{(n-1)\sin\left(\frac{2\pi}{n-1}\right)}, \qquad \theta_{22} = \frac{2\pi\,\theta^s_{22}}{(n-1)\sin\left(\frac{2\pi}{n-1}\right)}. \tag{4.26}$$

and from (4.10) and (4.11) relating to the real cable, it follows that

$$a = \frac{\ell(\theta_1-\theta_2)}{2\pi}, \qquad b = \frac{\ell(\theta_1+\theta_2)}{4\pi} \tag{4.27}$$

Because harmonic modes are symetric the following relations exist between the mode angles at the cable ends:

$$-\theta_{11} = \theta_{21}, \qquad \theta_{12} = \theta_{22},$$

$$-\theta^s_{11} = \theta^s_{21}, \qquad \theta^s_{12} = \theta^s_{22}. \tag{4.28}$$

Therefore, eliminating the second angle,

$$\theta_1 = \theta_{11} + \theta_{12}, \quad \theta^s_1 = \theta^s_{11} + \theta^s_{12},$$

$$\theta_2 = -\theta_{11} + \theta_{12}, \quad \theta^s_2 = -\theta^s_{11} + \theta^s_{12}. \tag{4.29}$$

Substituting (4.29) into (4.27) we find that

$$a = \frac{\ell(\theta_{11} + \theta_{12} + \theta_{11} - \theta_{12})}{2\pi} = \frac{2\ell\theta_{11}}{2\pi}, \tag{4.30}$$

$$b = \frac{\ell(\theta_{11} + \theta_{12} - \theta_{11} + \theta_{12})}{4\pi} = \frac{2\ell\theta_{12}}{4\pi}.$$

Now (4.30), (4.31), (4.23), (4.24) yield

$$a = \frac{\ell\, \theta_{11}^{s}}{(n-1)\sin\frac{\pi}{n-1}} , \qquad b = \frac{\ell\, \theta_{12}^{s}}{(n-1)\sin\frac{2\pi}{n-1}} . \tag{4.31}$$

From (4.29) it follows that

$$\theta_1^s - \theta_2^s = 2\theta_{11}^s , \qquad \theta_1^s + \theta_2^s = 2\theta_{12}^s . \tag{4.32}$$

Substituting for θ_{11}^s and θ_{12}^s, the mode amplitudes become

$$a = \frac{\ell(\theta_1^s - \theta_2^s)}{2(n-1)\sin\frac{\pi}{n-1}} , \qquad b = \frac{\ell\,(\theta_1^s + \theta_2^s)}{2(n-1)\sin\frac{2\pi}{n-1}} . \tag{4.33}$$

In (4.33) we have the model amplitudes of real cable expressed in terms of angles derived from the segmented model.

It is also convenient to write from (4.27)

$$\begin{aligned} \theta_1 &= \frac{\pi}{\ell}\ (2b + a) \\ \theta_2 &= \frac{\pi}{\ell}\ (2b - a) \end{aligned} \tag{4.34}$$

Two more measurable states are available from the whiffletree. Since the whiffletree is taken to be an inflexible, inextensible metal rod, a suitable sensor attached at the hinge can be used to characterize its angular deflection and rate. As described by equations (4.35) and (4.36), this information is obtainable from the segmented model.

$$\theta_w = \alpha + \phi_n \tag{4.35}$$

$$\dot{\theta}_w = \dot{\alpha} + \dot{\phi}_n \tag{4.36}$$

Here α is defined by equation (4.20), ϕ_n is the angle of the whiffletree with respect to the $(n-1)^{th}$ cable segment, as illustrated in Figure 3-3, and θ_w is the angle of the whiffletree from vertical.

The velocity of the simulation vehicle can be related to the variables used to define the segmented model. In Figure 4-7, the velocity V is expressed in terms of variables as defined in equations (4.17), (4.20), (3.1), as

$$V = V_{Tv} + \ell\dot{\psi} + r_w (\dot{\alpha} + \dot{\phi}_n). \tag{4.37}$$

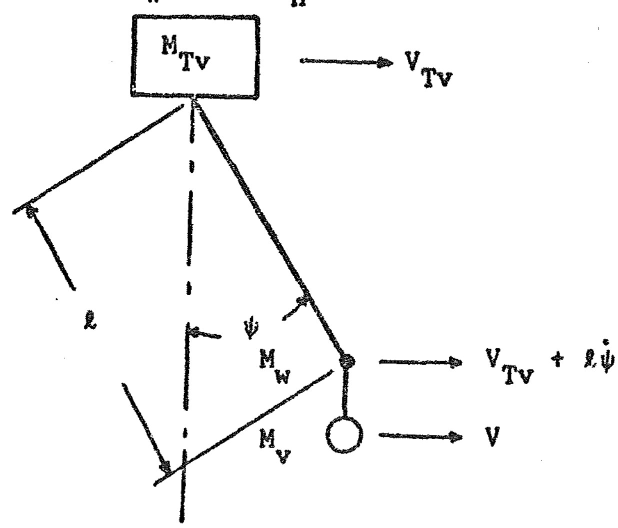


Figure 4.7

On the real system this velocity should be measurable.

V PERFORMANCE EVALUATION OF THE CLOSED LOOP SYSTEM

With the information that is now available from the segmented model, the closed loop system of Figure 5-1 is proposed.

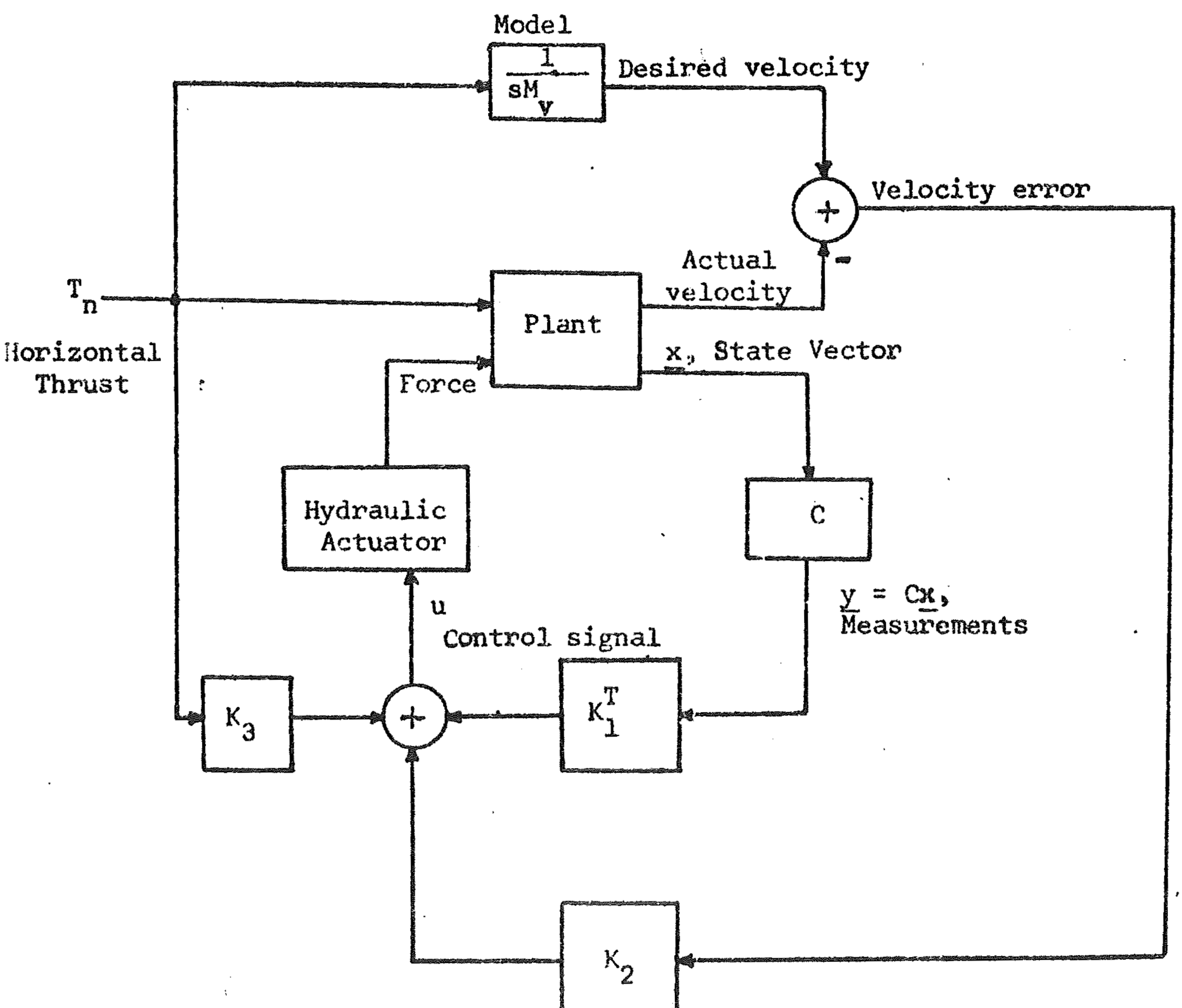


Figure 5-1

Here the plant is described by equations (3.1) through (3.5), and (4.37. C is the linear transformation developed in section IV, with $\underline{y}$ consisting of variables measurable from the real system. The vector $\underline{x}$ represents all of the state information of the plant.

A low order approximation, shown by equation (5.1), describes the hydraulic actuator used to drive the tracking cart,

$$\frac{F}{u} = \frac{w_d^2}{S^2 + 1.2\, w_d\, S + w_d^2}, \tag{5.1}$$

with $w_d = 2.5$ rad/sec.

The states of this actuator will not be used for feedback information in this study.

In summary, the following terms are included in the feedback control law:

V_{Tc}, ψ, $\dot{\psi}$, θ_1, $\dot{\theta}_1$, θ_2, $\dot{\theta}_2$, θ_w, $\dot{\theta}_w$, velocity error, and Th.

A minimization procedure based on Powell's Method [4] is used to obtain an optimal set of feedback gains. This leads to the question of a suitable cost function. The ultimate goal of this problem is to produce the best possible simulation of a free vehicle using the information that is available.

A useful measure of system performance is a functional comparing some dynamic property of the plant with that of a model whose response is considered to be ideal. In this case, the measure of error between the plant and the model could be a combination of position, velocity, or acceleration. The relative weights to be given to the costs on any of these quantities would depend upon the exact specifications that should be met. For example, if position or acceleration errors are less important than velocity error, the velocity error should be given the greatest weighting in the cost functional. In this study, only velocity error has been considered. The level of the control signal at the cart must also be included so as to insure that unreasonable demands are not placed on the control force. A suitable cost function is therefore presented in equation (5.2).

$$J = \int_0^{t_f} \left[\exp\left(\frac{F}{F_{max}}\right)^2 + \beta\,(\text{error})^2 - 1 \right] dt \qquad (5.2)$$

When an analytical approach to optimal control is used, there are advantages to using quadradic terms in the cost function. In this case however, because of the complex system and the distributed parameter problem, no analysis is anticipated. This permits some latitude in choosing different forms for terms in the cost function. The exponential term was chosen because it imposes a relatively large penalty on control forces larger than F_{max}. This puts a firm constraint on the magnitude of the control signal. The exponential form also puts a relatively small penalty on forces smaller in magnitude than F_{max}.

Note that vibration mode amplitudes of the cable are not included in the cost function, and are therefore not directly penalized. Any attempt to include mode amplitude terms in the cost function should result in a degradation of system performance at the minimum cost point. However, if, in the running of the system, mode amplitudes were to exceed some specified limit, mode terms for the first two modes, as shown in equations (4.10) through (4.13), could be included.

For a complete listing, flow diagrams, and discussion of the computer programs used to simulate the system and find an optimal set of gains, see Appendix B.

VI RESULTS AND CONCLUSIONS

The system was optimized under the following conditions:

(a) Cable length set to 200 feet.

(b) The thrust program used was a two second, +500 lbs. burst at t=0 secs , and a -500 lbs. burst at t=10 secs of two second duration.

(c) Cost function calculated over a 20 second interval.

The optimal feedback gains are shown in Table 5-1 along with the associated measured states, and indications of the changes in cost and RMS error resulting from adjusting the individual gain terms. The overall reduction in cost was from 1543.36 to 24.18, with a drop in RMS error from 8.787 to .797.

The initial value of K_3 was set to 1.0 to keep the first-run cost and RMS error down to a reasonable level. Preliminary results showed that thrust, or the linearly related acceleration of the vehicle, is an important term in the feedback law.

Table 5-1 clearly shows that only ψ, $\dot{\psi}$, $\dot{\phi}_w$, and Th are really necessary for near optimal control. Slight improvements can be made with θ_w, $\dot{\theta}_1$, V_{tc} and Error, while the rest have negligible effect.

Operation with a 200 foot cable length is satisfactory, with maximum values of ψ at .006 radians, and cable vibration amplitudes not greater than three inches peak. At shorter cable lengths, the pendulum and vibration frequencies become high with respect to the hydraulic actuator frequency, and the system becomes unstable. With

Gain Terms	Intial Gains	Optimal Gains	Measured State	Decrease in Cost	Decrease in RMS Error
K_1	0.	88386.38	ψ	917.65	3.200
K_2	0.	61295.43	$\dot{\psi}$	479.61	2.998
K_3	1.0	6.52	Th	35.77	.396
K_4	0.	-28609.35	ϕ_w	2.11	.009
K_5	0.	141062.75	$\dot{\phi}_w$	71.09	1.211
K_6	0.	736.00	Error	4.60	.049
K_7	0.	32622.52	θ_1	.15	.001
K_8	0.	87164.19	$\dot{\theta}_1$	3.14	.019
K_9	0.	9773.14	θ_2	1.22	.017
K_{10}	0.	162.87	$\dot{\theta}_2$	.003	0.
K_{11}	0.	-104.68	V_{Tc}	6.13	.070

Table 5-1

constant gain terms, the cost and RMS error remain nearly constant down to about 75 feet, where the cost begins to increase rapidly. By making gain terms associated with cable states length dependent, the stable control extended down to 50 feet. However, this change was made on an intuitive basis; better results could be obtained by finding optimal gains at several cable lengths, and making the control law length dependent using curve fitting techniques.

APPENDIX A MODIFIED POWELL'S METHOD

Powell's method is an efficient technique for minimizing a function of several variables. It is especially useful when it is not possible or practical to use gradient methods. An especially desirable feature of Powell's Method is its ability to develop search directions along long narrow troughs. This insures rapid convergence to a minimum with quadradic or nearly quadradic functions.

An interation is as follows:

(i) For $r = 1,2,\ldots,n$ calculate λ_r so that $f(P_{r-1} + \lambda_r \xi_r)$ is a minimum, and define $P_r = P_{r-1} + \lambda_r \xi_r$.

Step (i) is a search in n directions for minimum points. A good initial direction matrix $[\xi]$ is a row of ones along the diagonal. This insures an initial round of n orthagonal search directions.

(ii) Find the integer m, $1 \leq m \leq n$, so that $\{f(P_{m-1}) - f(P_m)\}$ is a maximum, and define $\Delta = f(P_{m-1}) - f(P_m)$.

This step identifies the direction which produces the largest change in the functional value.

(iii) Calculate $f_3 = f(2P_n - P_0)$ and define $f_1 = f(P_0)$ and $f_2 = f(P_n)$.

This and the next step are to prevent nearly dependent search directions from being introduced. Powell states that when minimizing a function of more than five variables, these steps may be necessary to achieve convergence.

(iv) If either $f_3 > f_1$ and/or

$(f_1 - 2f_2 + f_3) \cdot (f_1 - f_2 - \Delta)^2 > \frac{1}{2}\Delta(f_1 - f_3)^2$,

use the old directions $\xi_1, \xi_2, \ldots, \xi_n$ for the next direction, and use P_n for the next P_0, otherwise

(v) Defining $\xi = (P_n - P_0)$, calculate λ so that $f(P_n + \lambda\xi)$ is a minimum, use $\xi_1, \xi_2, \ldots, \xi_{m-1}, \xi_{m+1}, \xi_{m+2}, \ldots, \xi_n, \xi$ as the directions, and $P_n + \lambda\xi$ as the starting point for the next iteration.

This step introduces a new conjugate search direction.

APPENDIX B COMPUTER PROGRAMS

The computer programs, used to apply the optimization procedure, were based on techniques developed by R. J. Kochenburger [8]. Although the system, with its twenty states, is relatively complex, the program runs in nearly real time when compiled with the Fortran H compiler, run on an IBM 360/65 computer, and using a Δt of .005 seconds.

A desireable feature of subroutine SYSTEM, which simulates the system, is that changes can be easily made without extensive modifications to the system equations. Changes in the control law, the hydraulic actuator, or other portions can be made by just changing a few lines of program. Also, the thrust program is written as a subroutine, allowing easy changes without changing the system equations.

SUBROUTINE LINKAGE

Named Common Block Table

LINK	M/PROG	POWELL	MINIMA	SYSTEM	INT	THRUST
A	FSTPAS			FSTPAS		
A	ERMS			ERMS		
B	FINISH	FINISH				
B	FAIL	FAIL				
B	EXRNDS	EXRNDS				
B	EXCESS	EXCESS				
D	J	J		J		
D	ROUNDS	ROUNDS		ROUNDS		
D	GAINS	GAINS		GAINS		
D	K(20)	K(20)		K(20)		
E	TRIALS	TRIALS	TRIALS	TRIALS		
E	COST	COST	COST	COST		
F		SUBFIN	SUBFIN			
F		SUBEXC	SUBEXC			
F		MAXTLS	MAXTLS			
F		A	A			
F		DA	DA			
F		TOLMIN	TOLMIN			
G				NEWDT	NEWDT	
G				NEWTIM	NEWTIM	
G				LSTPAS	LSTPAS	
G				ITERAT	ITERAT	
G				STATES	STATES	
G				DT	DT	
G				X(20)	X(20)	
G				Y(20)	Y(20)	
H				T	T	T
I				CAL		CAL
I				TH		TH

Table B-1

DEFINITIONS OF BRANCHING VARIABLES

CAL - Causes subroutine thrust to read data and set initial conditions of thrust during system initialization

DYNOUT - With DYNOUT is set to true, system dynamics can be printed out with the time increment of PRNDEL

EXFRTR - Used to store information that an excessive number of trials was required in the j direction

EXMIN - Signals that a search is being made in an orthogonal direction as the last step in an iteration of Powell's Method

EXRNDS - Set to true when the maximum number of iterations, or rounds, has been exceeded

FAIL - Indicates a failure of the optimizing procedure, when set to true either for excessive rounds or excessive trials.

FINISH - When set to true, the procedure is terminated, either successfully or not.

FSTPAS - Routes subroutine SYSTEM through the initialization branch on the first pass through SYSTEM

LSTPAS - Causes subroutine INT to go through the first branch of Fourth Order Runga-Kutta integration at the beginning of each Δt.

MAXTLS - The maximum allowable number of trials, or attempts to find a minimum along any one direction vector.

MXRNDS - The maximum allowable number of rounds (see ROUNDS)

NEWDT - Initially set to true. This causes an adjustment in the time increments to suit the integration subroutine on the first pass through subroutine INT

NEWTIM - Signals for a new value of thrust from subroutine thrust whenever time is incremented

NXTPAS - Routes subroutine INT through the correct branch

ROUNDS - Counts the number of iterations of the optimization procedure. One round is a minimization in all directions plus possibly in an orthogonal direction.

RUNPOW - When RUNPOW is set to false, the procedure stops after one pass through subroutine SYSTEM. This is useful when system dynamics for only one set of conditions is desired.

SEARCH - Starts the search for the minimum point on a quadratic curve in subroutine MINIMA after the minimum has been passed by the regular steps of $\Delta\alpha$

SUBEXC - Excessive number of trials in subroutine MINIMA will cause this to be set to true.

SUBFIN - Signals that a minimum has been found in subroutine MINIMA

T_F - Total running time of system dynamics

TOLMIN - A change in the cost function for two successive trials of less than the specified value of Tolmin shows that a minimum has been found. SUBFIN is then set to true.

TOLPOW - When the change in cost in each direction is less than the specified value of TOLPOW for an entire round, FINISH is set to true and the optimum parameters have been found.

TPT - Sets the time for the next print-out of system dynamics

TRIALS - Counts the number of trials in one direction; reset to one for each new direction

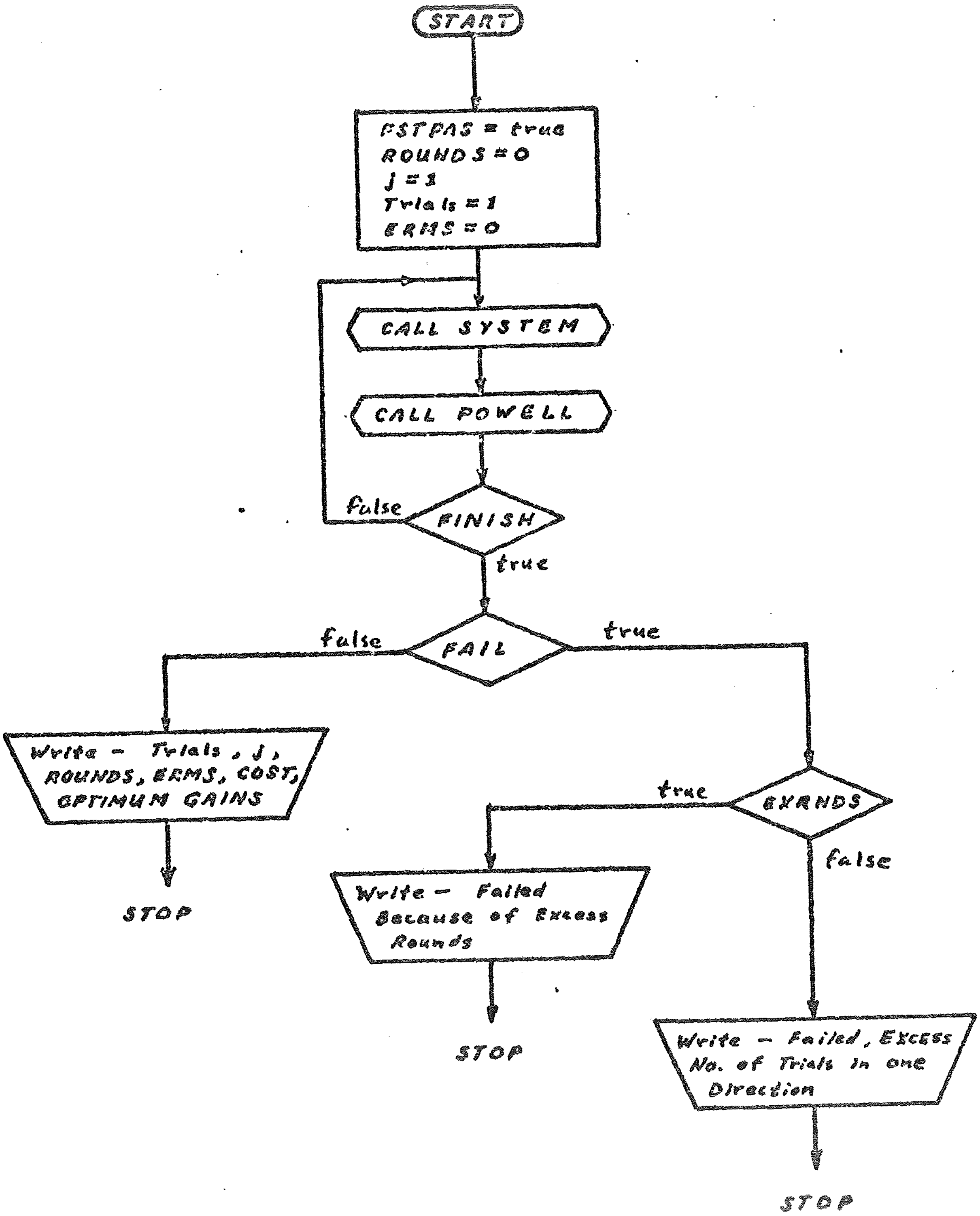


Figure B-1 Main Program

```
C     MAIN PROGRAM
      COMMON /LINKA/FSTPAS,ERMS
      COMMON /LINKB/FINISH,FAIL,EXRNDS,EXCESS
      COMMON /LINKD/J,ROUNDS,GAINS,K(20)
      COMMON /LINKE/COST,TRIALS
      LOGICAL FSTPAS,FINISH,FAIL,EXRNDS
      INTEGER EXCESS,TRIALS,ROUNDS,GAINS
      REAL K
C
C     MAIN PROGRAM INITIALIZATION
      FSTPAS=.TRUE.
      ROUNDS=0.
      J=1
      TRIALS=1
      ERMS=0.
C
C     *****************************************************
C
C     OUTSIDE PARAMETER OPTIMIZATION LOOP
C
   10 CALL SYSTEM
      CALL POWELL
      IF(.NOT.FINISH)GO TO 10
C
C     *****************************************************
C
      IF(FAIL)GO TO 11
      WRITE(6,20)
      WRITE(6,21)TRIALS,J,ROUNDS,ERMS,COST
      WRITE(6,22)(K(I),I=1,GAINS)
      STOP
C
   11 IF(.NOT.EXRNDS)GO TO 12
      WRITE(6,23)
      STOP
C
   12 WRITE(6,24)EXCESS
      STOP
C
   20 FORMAT(//6X,'OPTIMUM GAINS FOUND'/)
   21 FORMAT(1X,3I5,2F12.2)
   22 FORMAT(1X,13F10.2)
   23 FORMAT(//6X,'FAILED BECAUSE OF EXCESSIVE ROUNDS')
   24 FORMAT(//6X,'TOO MANY TRIALS AT DIRECTION VECTOR',I5)
      END
```

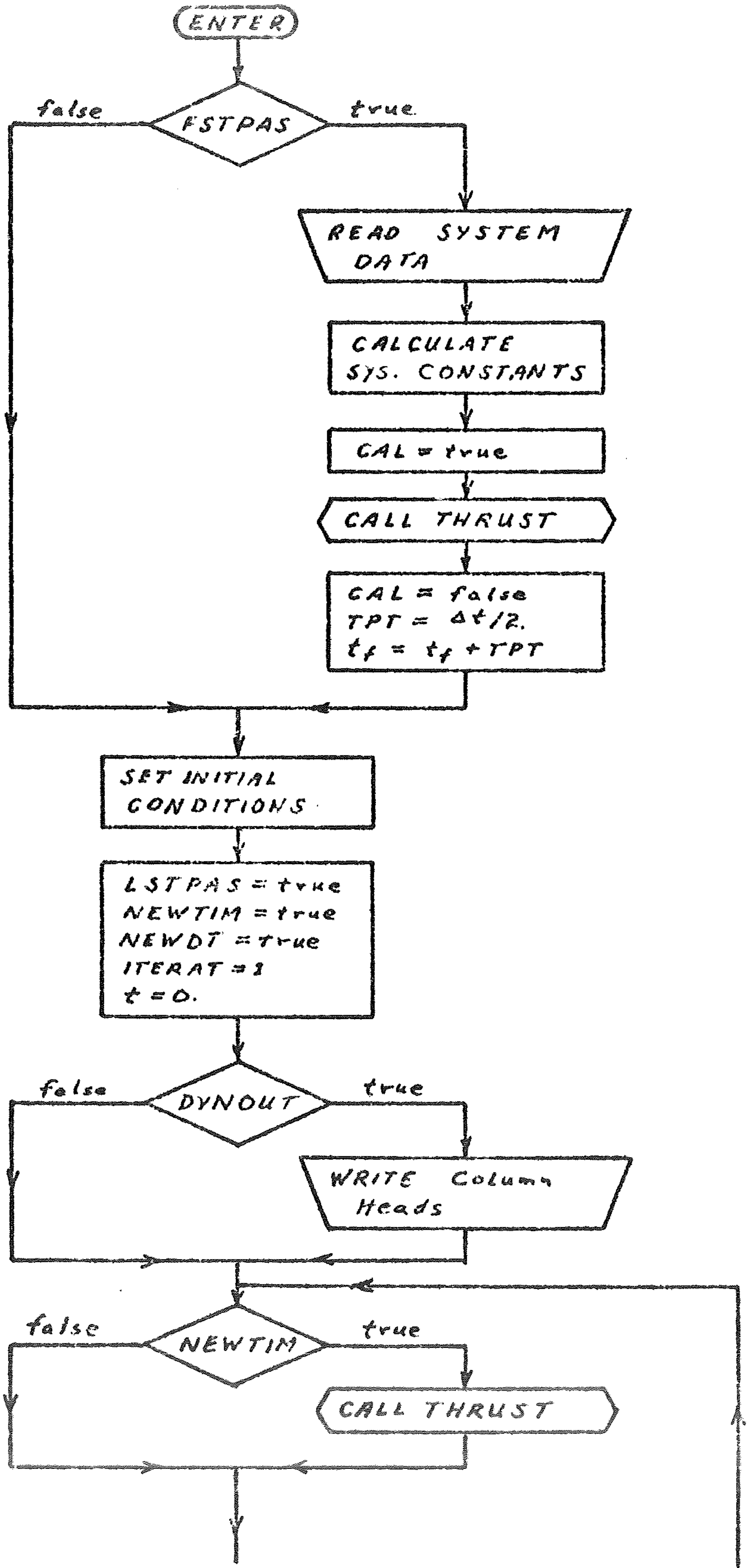
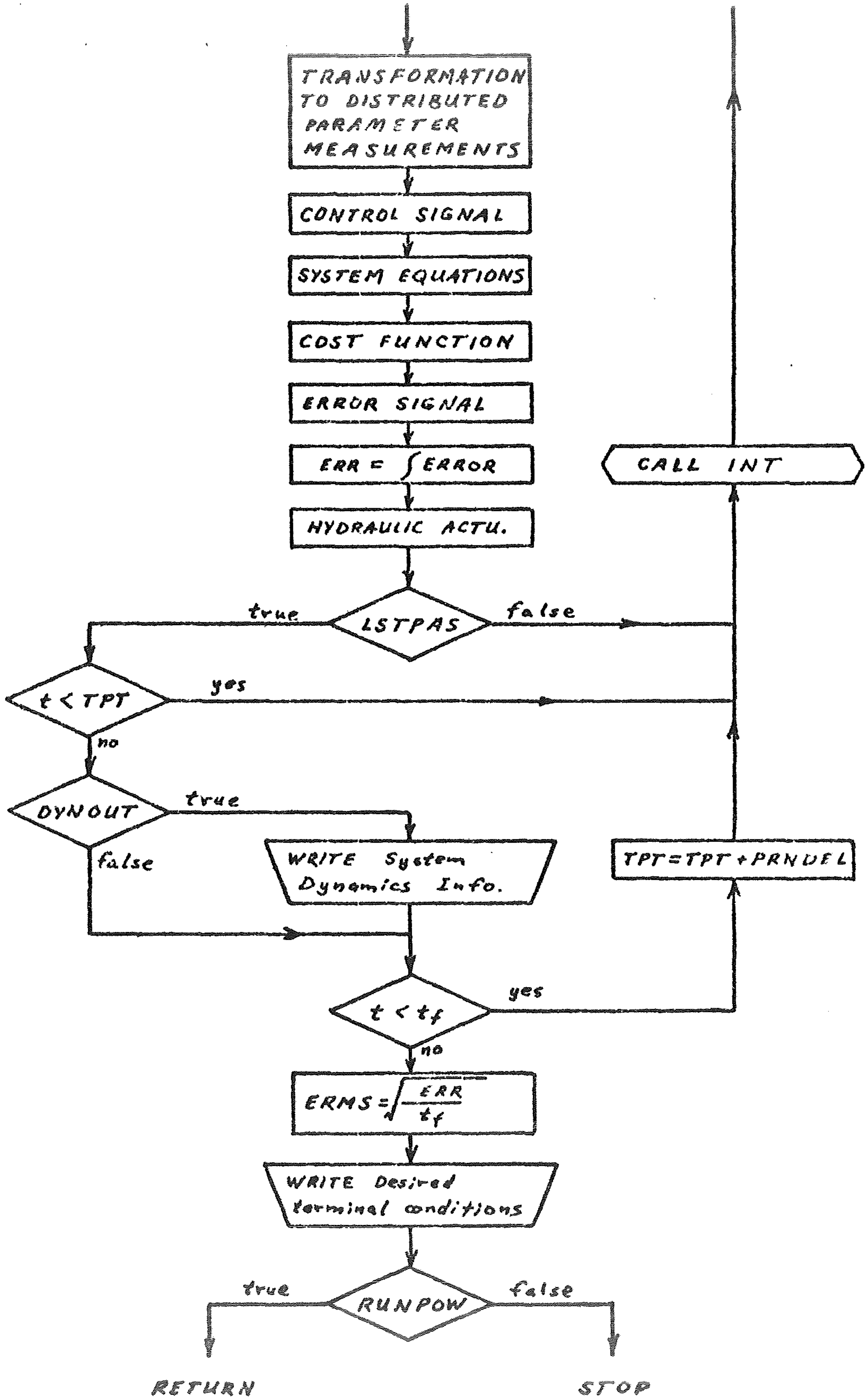


Figure B-2 Subroutine System


TRANSFORMATION TO DISTRIBUTED PARAMETER MEASUREMENTS
CONTROL SIGNAL
SYSTEM EQUATIONS
COST FUNCTION
ERROR SIGNAL
ERR = ∫ERROR
HYDRAULIC ACTU.
LSTPAS
true
false
CALL INT
t < TPT
yes
no
DYNOUT
true
false
WRITE System Dynamics Info.
TPT=TPT+PRNDEL
t < tf
yes
no
ERMS = √(ERR/tf)
WRITE Desired terminal conditions
RUNPOW
true
false
RETURN
STOP

```
      SUBROUTINE SYSTEM
      COMMON /LINKA/FSTPAS,ERMS
      COMMON /LINKD/J,ROUNDS,GAINS,K(20)
      COMMON /LINKE/COST,TRIALS
      COMMON /LINKG/NEWDT,NEWTIM,LSTPAS,ITERAT,STATES,DT,
     1             X(20),Y(20)
      COMMON /LINKH/T
      COMMON /LINKI/CAL,TH
      REAL LENGTH,K
      LOGICAL CAL,FSTPAS,LSTPAS,NEWTIM,NEWDT,DYNOUT,RUNPOW
      INTEGER STATES,ITERAT,GAINS,TRIALS,ROUNDS
      IF(.NOT.FSTPAS)GO TO 1
C
C     ******************************************************
C
C     INITIALIZATION BRANCH
C
      STATES=20
      GAINS=11
C     SET NON-VARYING SYSTEM PARAMETERS
      TVM=2360.
      RW=10.
      VM=310.
      WM=62.
C
C     READ STATEMENT NO. 1
      IE=1
      READ(5,30,ERR=50)DT,TF,PRNDEL,FMAX,BETA
      WRITE(6,32)DT
      WRITE(6,40)
      WRITE(6,41)TF,PRNDEL,FMAX,BETA
C
C     READ STATEMENT NO. 2
      IE=2
C     READ INITIAL GAIN VECTOR
      DO 2 I=1,GAINS
    2 READ(5,31,ERR=50)K(I)
C     WRITE INITIAL GAIN VECTOR
      WRITE(6,42)
      WRITE(6,34)(K(I),I=1,GAINS)
C
C     READ STATEMENT NO. 3
      IE=3
C     READ MODE CONTROL LOGICAL VARIABLES
C     PRINT OUT SYSTEM DYNAMICS WITH DYNOUT = .TRUE.
C     STOP AFTER FIRST PASS THROUGH SYSTEM IF RUNPOW = .FALSE.
      READ(5,36,ERR=50)DYNOUT,RUNPOW
C     WRITE MODE CONTROL LOGICAL VARIABLES
      WRITE(6,44)
      WRITE(6,45)DYNOUT,RUNPOW
C
C     READ STATEMENT NO. 4
      IE=4
C     READ CABLE LENGTH
      READ(5,30,ERR=50)LENGTH
C     WRITE CABLE LENGTH
      WRITE(6,37)LENGTH
C
```

```
C
C     SET SYSTEM PARAMETERS THAT COULD VARY
C
C     THIS SECTION SHOULD BE MOVED INSIDE THE
C     INTEGRATION LOOP FOR TIME VARYING CABLE
C     LENGTH
C
      RE=LENGTH/6.
      CM=.02485*LENGTH/5.
      TC=10333.
      TW=8333.
      C1=TC/2360.
      C3=TC/(CM*RE)
      C2=C1/RE+C3
      C4=C3+.01612*TC/RE
      C5=.01612*TW/RE
      C6=.001612*TC
      C7=.001945*TW
      C8=1./TVM
      C9=C8/RE
C
C     INITIALIZE THRUST PROGRAM
      CAL=.TRUE.
      CALL THRUST
      CAL=.FALSE.
      FSTPAS=.FALSE.
      TPT=-.5*DT
      TF=TF+TPT
C
C     ***************************************************************
C
C     SET PLANT STATES TO INITIAL CONDITIONS
    1 DO 3 N=1,STATES
    3 Y(N)=0.
      LSTPAS=.TRUE.
      NEWTIM=.TRUE.
      NEWDT=.TRUE.
      ITERAT=1
      T=0.
      IF(.NOT.DYNOUT)GO TO 24
C
C     WRITE COLUMN HEADS
      WRITE(6,38)
C
C
C     ITERATIVE PORTION STARTS HERE - INTEGRATION LOOP
   24 IF(.NOT.NEWTIM)GO TO 20
      CALL THRUST
C
C     TRANSFORMATION TO STATES MEASURABLE FROM
C     DISTRIBUTED PARAMETER SYSTEM  (REAL CABLE)
   20 PSI=Y(3)+.833*Y(5)+.667*Y(7)+.5*Y(9)+.333*Y(11)
     1     +.167*Y(13)
      PSID=Y(4)+.833*Y(6)+.667*Y(8)+.5*Y(10)+.333*Y(12)
     1     +.167*Y(14)
      ALPHA=Y(3)+Y(5)+Y(7)+Y(9)+Y(11)+Y(13)
      ALPHAD=Y(4)+Y(6)+Y(8)+Y(10)+Y(12)+Y(14)
      ANGIS=Y(3)-PSI
```

```
      ANG1SD=Y(4)-PSID
      ANG2S=ALPHA-PSI
      ANG2SD=ALPHAD-PSID
      A=LENGTH*.1667*(ANG1S-ANG2S)
      AD=LENGTH*.1667*(ANG1SD-ANG2SD)
      B=LENGTH*.09615*(ANG1S+ANG2S)
      BD=LENGTH*.09615*(ANG1SD+ANG2SD)
      POL=3.1416/LENGTH
      ANG1=(A+B+B)*POL
      ANG2=(-A+B+B)*POL
      ANG1D=(AD+BD+BD)*POL
      ANG2D=(-AD+BD+BD)*POL
      WTA=ALPHA+Y(15)
C
C     CALCULATE ERROR SIGNAL
      ERROR=Y(1)-Y(2)-LENGTH*PSID-10.*(ALPHAD+Y(16))
      ESQ=ERROR*ERROR
C
C     CALCULATE CONTROL SIGNAL
      U=LENGTH*(PSI*K(1)+PSID*K(2)+ANG1*K(7)
     1 +ANG1D*K(8)+ANG2*K(9)+ANG2D*K(10))
     2 +(ALPHA+Y(15)-PSI)*K(4)+(ALPHAD+Y(16)-PSID)
     3 *K(5)+TH*K(3)+ERROR*K(6)+Y(2)*K(11)
C
C     CALCULATE FORCE APPLIED TO TRACKING
C     VEHICLE BY HYDRAULIC ACTUATOR
      FORCE=Y(19)+U
C
C     SYSTEM EQUATIONS
      X(1)=.003226*TH
      X(2)=C1*Y(3)+C8*FORCE
      X(3)=Y(4)
      X(4)=-C2*Y(3)+C3*Y(5)-C9*FORCE
      X(5)=Y(6)
      X(6)=C3*(Y(3)-Y(5)-Y(5)+Y(7))
      X(7)=Y(8)
      X(8)=C3*(Y(5)-Y(7)-Y(7)+Y(9))
      X(9)=Y(10)
      X(10)=C3*(Y(7)-Y(9)-Y(9)+Y(11))
      X(11)=Y(12)
      X(12)=C3*(Y(9)-Y(11)-Y(11)+Y(13))
      X(13)=Y(14)
      X(14)=C3*Y(11)-C4*Y(13)+C5*Y(15)
      X(15)=Y(16)
      X(16)=C6*Y(13)-C7*Y(15)+.000322*TH
C
C     COST FUNCTION
      X(17)=EXP((FORCE/FMAX)**2)+BETA*ESQ-1.
C
C     ERROR SQUARED INTEGRATED
      X(18)=ESQ
C
C     HYDRAULIC ACTUATOR
      X(19)=Y(20)-U
      X(20)=-6.25*Y(19)-3.*(Y(20)-U)
C
      IF(.NOT.LSTPAS)GO TO 21
      IF(T.LT.TPT)GO TO 21
```

```
C     SYSTEM DYNAMICS PRINT-OUT FROM HERE
      IF(.NOT.DYNOUT)GO TO 25
      WRITE(6,35)TH,FORCE,ERROR,Y(17),A,B,PSI,WTA,T
   25 IF(T.LT.TF)GO TO 22
      ERMS=SQRT(Y(18)/TF)
      COST=Y(17)
      WRITE(6,33)TRIALS,J,RCUNDS,ERMS,COST
      WRITE(6,34)(K(I),I=1,GAINS)
      IF(.NOT.RUNPOW)STOP
      RETURN
C
   22 TPT=TPT+PRNDEL
   21 CALL INT
      GO TO 24
   50 WRITE(6,51)IE
      STOP
C
   30 FORMAT(6F10.3)
   31 FORMAT(F20.4)
   32 FORMAT(6X,'INTEGRATION INTERVAL = ',F6.4,' SECS'//)
   33 FORMAT(1H0,1X,3I6,2F12.3)
   34 FORMAT(1X,13F10.2)
   35 FORMAT(1X,9F12.3)
   36 FORMAT(2L10)
   37 FORMAT('OCABLE LENGTH = ',F6.2,' FT.')
   38 FORMAT(5X,'THRUST',8X,'FORCE',7X,'ERROR',
     1 8X,'COST',11X,'A',11X,'B',8X,'PSI',10X,
     2 'WTA',8X,'TIME')
   40 FORMAT(4X,'TF',6X,'PRNDEL',6X,'FMAX',6X,'BETA')
   41 FORMAT(4F10.3)
   42 FORMAT('OINITIAL GAIN VECTOR')
   44 FORMAT('OLOGICAL MODE CONTROL TERMS')
   45 FORMAT(2L10)
   51 FORMAT(' READ DATA ERROR AT READ STATEMENT NO. ',I3)
      END
```

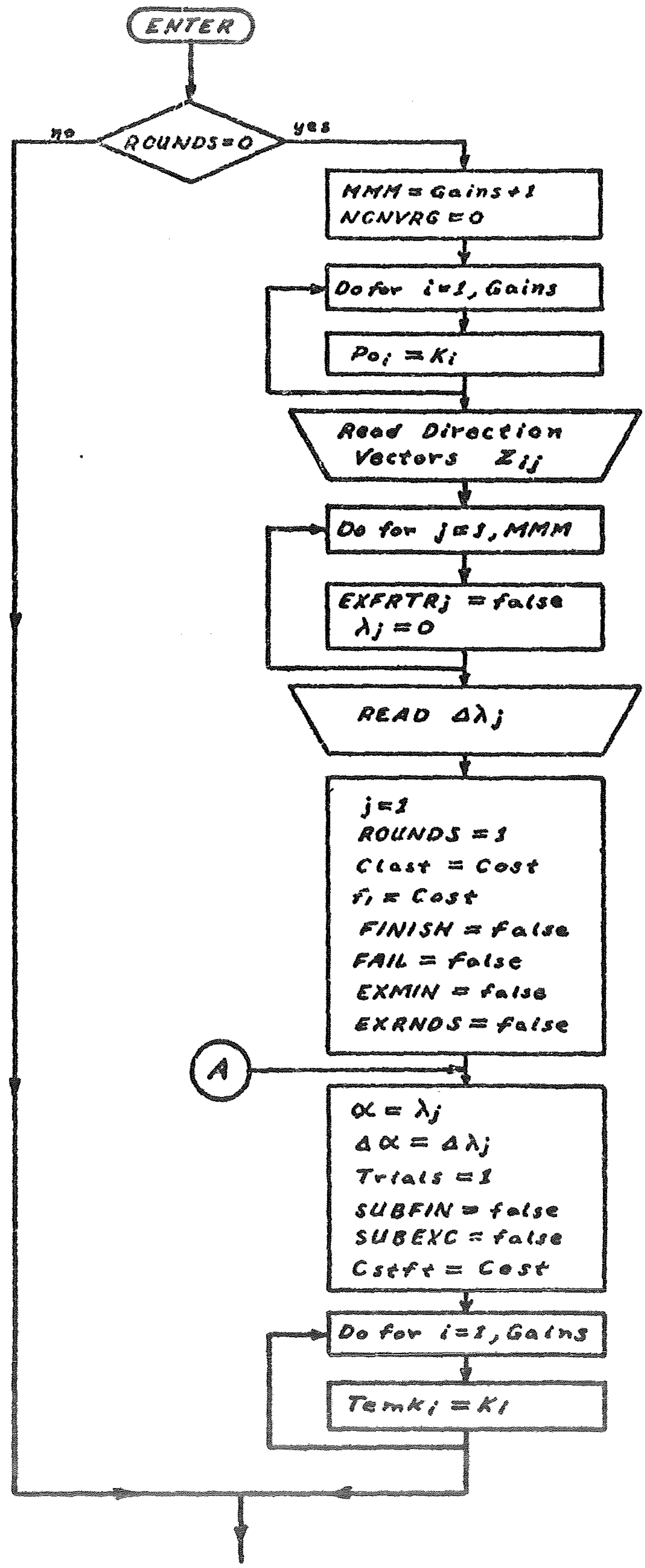
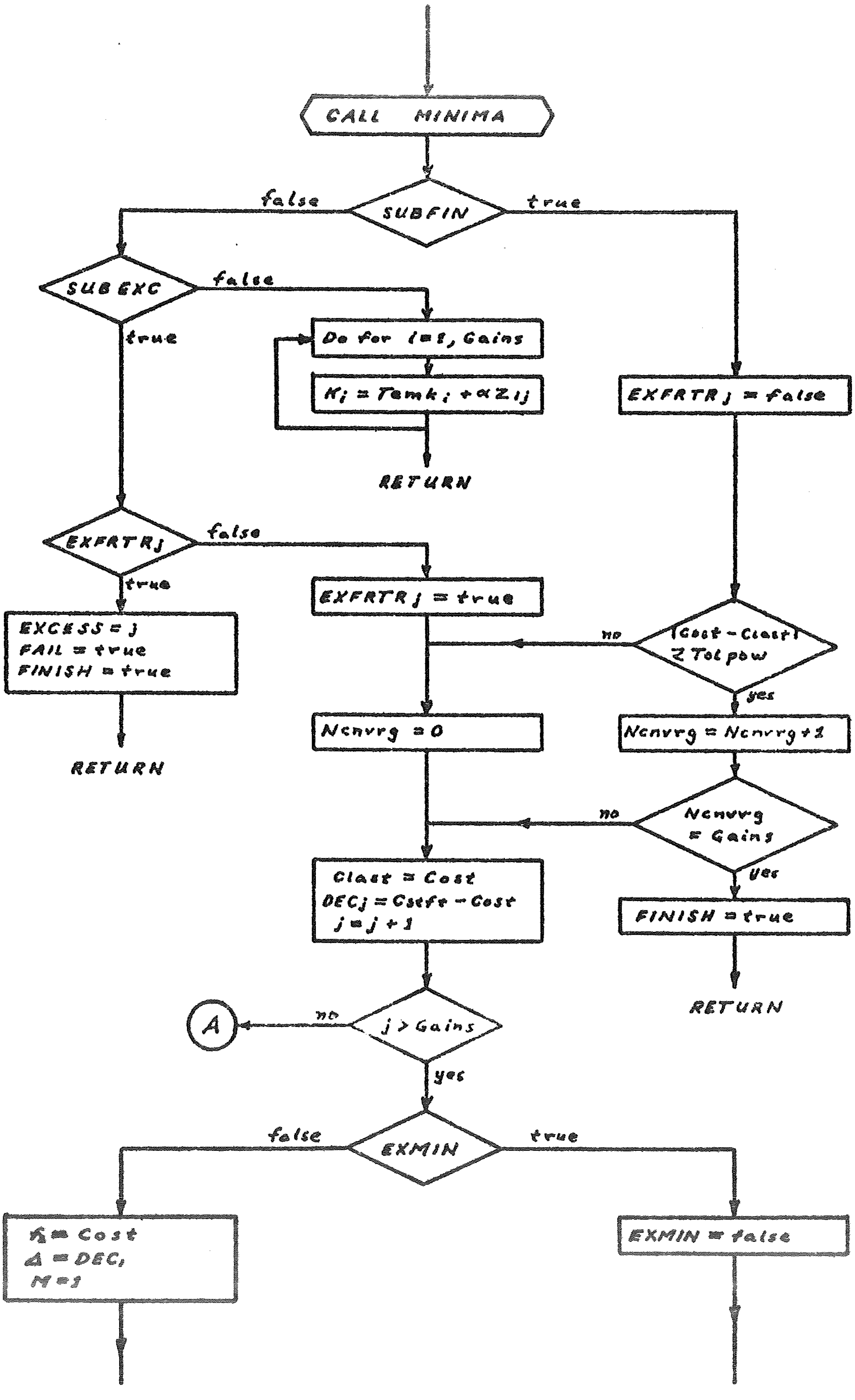
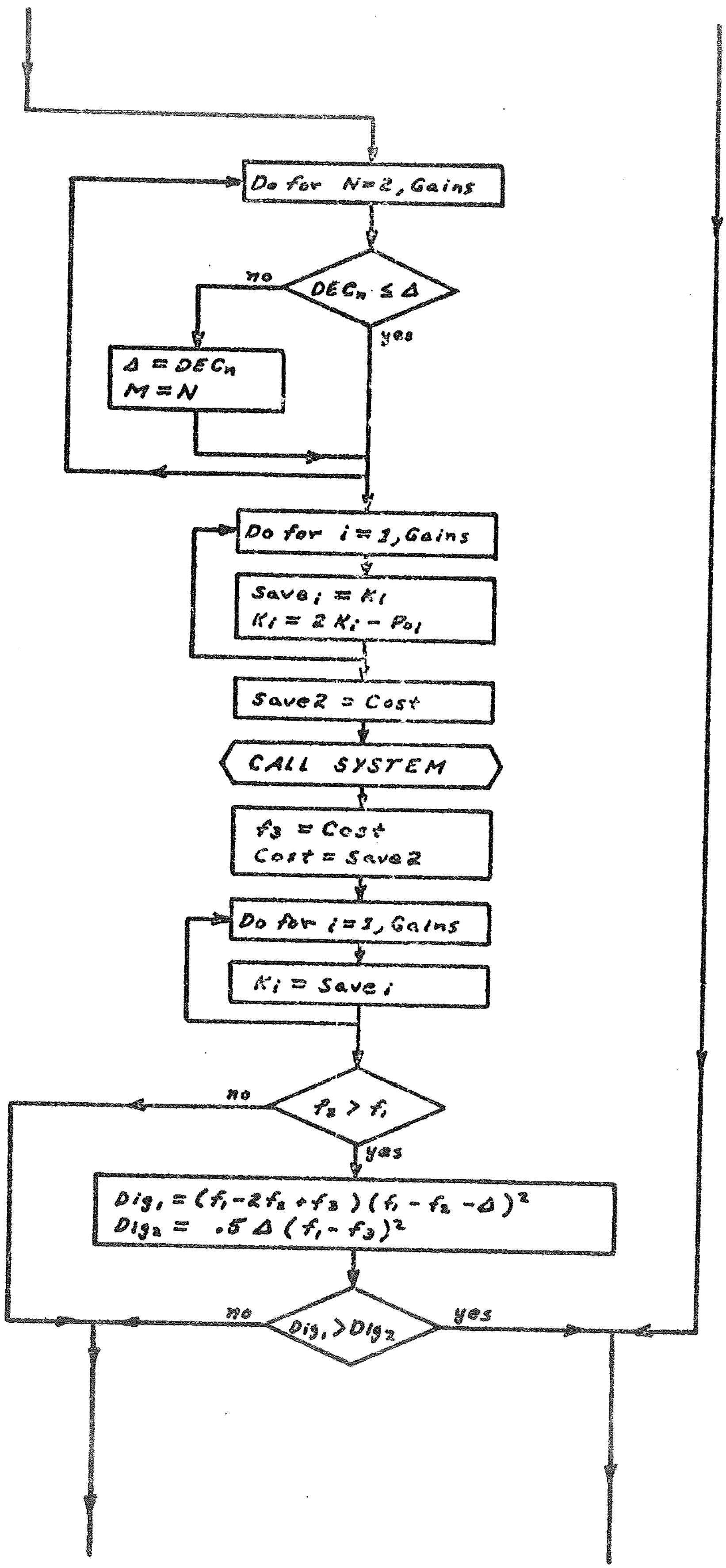
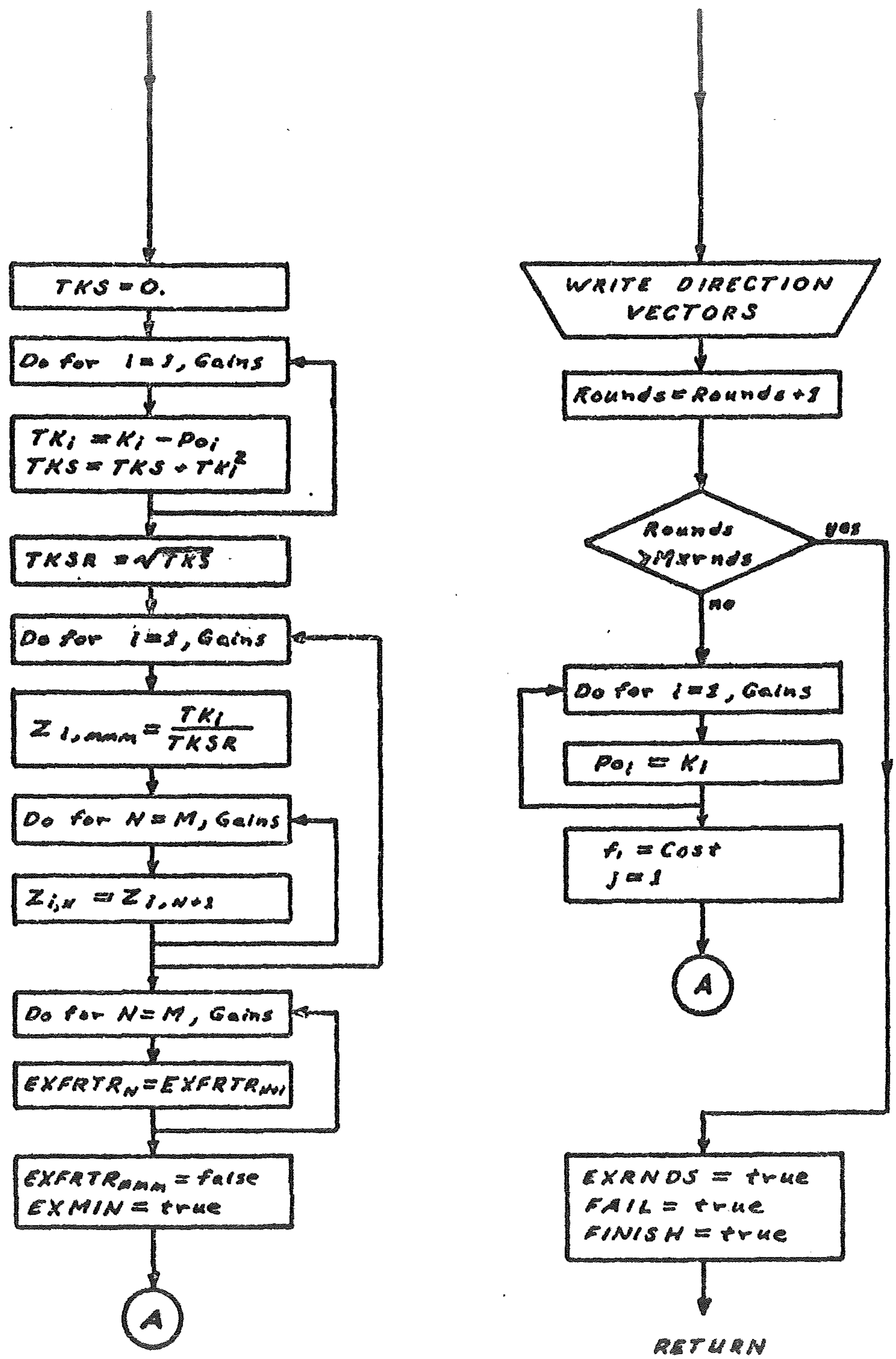


Figure B-3 Subroutine Powell


CALL MINIMA
SUBFIN
false
true
SUBEXC
false
true
Do for l=1, Gains
$K_l = Temk_l + \alpha Z_{lj}$
RETURN
EXFRTR j = false
EXFRTR j
false
true
EXCESS = j
FAIL = true
FINISH = true
RETURN
EXFRTR j = true
|Cost - Clast| ≥ Tol pow
no
yes
Ncnvrg = 0
Ncnvrg = Ncnvrg + 1
Ncnvrg = Gains
no
yes
Clast = Cost
DECj = Cstfe - Cost
j = j + 1
FINISH = true
RETURN
A
no
j > Gains
yes
EXMIN
false
true
f_0 = Cost
Δ = DEC_1
M = 1
EXMIN = false


Do for N=2, Gains
DECn ≤ Δ
no
yes
Δ = DECn
M = N
Do for i = 1, Gains
Save i = Ki
Ki = 2 Ki − P0i
Save2 = Cost
CALL SYSTEM
f3 = Cost
Cost = Save2
Do for i = 1, Gains
Ki = Save i
f2 > f1
no
yes
Dig1 = (f1 − 2f2 + f3)(f1 − f2 − Δ)²
Dig2 = .5 Δ (f1 − f3)²
Dig1 > Dig2
no
yes


TKS = O.
Do for i = 1, Gains
TKi = Ki − Poi
TKS = TKS + TKi²
TKSR = √TKS
Do for i = 1, Gains
Z i,mmm = TKi / TKSR
Do for N = M, Gains
Zi,N = Z i,N+1
Do for N = M, Gains
EXFRTRN = EXFRTRN+1
EXFRTRmmm = false
EXMIN = true
A
WRITE DIRECTION VECTORS
Rounds = Rounds + 1
Rounds > Mxrnds
yes
no
Do for i = 1, Gains
Poi = Ki
f1 = Cost
j = 1
A
EXRNDS = true
FAIL = true
FINISH = true
RETURN

```
      SUBROUTINE POWELL
      COMMON /LINKB/FINISH,FAIL,EXRNDS,EXCESS
      COMMON /LINKD/J,ROUNDS,GAINS,K(20)
      COMMON /LINKE/COST,TRIALS
      COMMON /LINKF/SUBFIN,SUBEXC,MAXTLS,A,DA,TOLMIN
      DIMENSION SAVE(20),PO(20),LAMDA(20),DLAMDA(20)
      DIMENSION EXFRTR(20),Z(20,20),TK(20),DEC(20),TEMK(20)
      LOGICAL FINISH,FAIL,EXRNDS,SUBFIN,SUBEXC,EXMIN
      LOGICAL START,EXFRTR
      INTEGER GAINS,EXCESS,TRIALS,ROUNDS
      REAL K,LAMDA
      IF(ROUNDS.GT.0)GO TO 10
C
C     ************************************************************
C
C     INITIALIZATION BRANCH
C
      MMM=GAINS+1
C
C     READ STATEMENT NO. 6
      IE=6
      READ(5,43,ERR=50)TOLMIN,TOLPOW,MXRNDS,MAXTLS
      WRITE(6,60)
      WRITE(6,61)TOLMIN,TOLPOW,MXRNDS,MAXTLS
C
C     READ STATEMENT NO. 7
      IE=7
C     READ INITIAL DIRECTION VECTORS
      DO 5 I=1,GAINS
    5 READ(5,44,ERR=50)(Z(I,J),J=1,GAINS)
C     WRITE INITIAL DIRECTION VECTORS
      WRITE(6,62)
      DO 6 I=1,GAINS
    6 WRITE(6,45)(Z(I,J),J=1,GAINS)
C
C     READ STATEMENT NO. 8
      IE=8
C     READ INITIAL DLAMDAS
      READ(5,46,ERR=50)(DLAMDA(J),J=1,MMM)
C     WRITE INITIAL DLAMDAS
      WRITE(6,63)
      WRITE(6,40)(DLAMDA(J),J=1,MMM)
C
      DO 1 I=1,GAINS
    1 PO(I)=K(I)
      DO 2 J=1,MMM
      EXFRTR(J)=.FALSE.
    2 LAMDA(J)=0.
      J=1
      ROUNDS=1
      NCNVRG=0
      CLAST=COST
      F1=COST
      FINISH=.FALSE.
      FAIL=.FALSE.
      EXMIN=.FALSE.
      EXRNDS=.FALSE.
C
```

```
C     RE-INITIALIZE FROM HERE FOR EACH DIRECTION
    3 A=LAMDA(J)
      DA=DLAMDA(J)
      TRIALS=1
      SUBFIN=.FALSE.
      SUBEXC=.FALSE.
      DO 4 I=1,GAINS
    4 TEMK(I)=K(I)
      CSTFT=COST
C
C     ****************************************************************
C
   10 CALL MINIMA
      IF(SUBFIN)GO TO 12
      IF(SUBEXC)GO TO 13
      DO 14 I=1,GAINS
   14 K(I)=TEMK(I)+A*Z(I,J)
C     RETURN FOR NEXT TRIAL WITH NEW GAIN VECTOR
      RETURN
C
   13 IF(.NOT.EXFRTR(J))GO TO 15
      EXCESS=J
      FAIL=.TRUE.
      FINISH=.TRUE.
C     EXCESS TRIALS IN ONE DIRECTION IN TWO SUCESSIVE ROUNDS
C     FAILED TO FIND OPTIMUM
      RETURN
C
   15 EXFRTR(J)=.TRUE.
      GO TO 16
   12 EXFRTR(J)=.FALSE.
      IF(ABS(COST-CLAST).GT.TOLPOW)GO TO 16
      NCNVRG=NCNVRG+1
      IF(NCNVRG.LT.GAINS) GO TO 17
      FINISH=.TRUE.
C     FOUND OPTIMUM GAINS - SEARCH COMPLETED
      RETURN
C
   16 NCNVRG=0
   17 CLAST=COST
      DEC(J)=CSTFT-COST
      J=J+1
      IF(J.LE.GAINS)GO TO 3
      IF(.NOT.EXMIN)GO TO 21
      EXMIN=.FALSE.
      GO TO 29
   21 F2=COST
C     FIND DIRECTION IN WHICH LARGEST CHANGE IN COST OCCURED
      DEL=DEC(1)
      M=1
      DO 22 N=2,GAINS
      IF(DEC(N).LE.DEL)GO TO 22
      DEL=DEC(N)
      M=N
   22 CONTINUE
      DO 23 I=1,GAINS
      SAVE(I)=K(I)
   23 K(I)=K(I)+K(I)-PO(I)
```

```
      WRITE(6,42)
      SAVE2=COST
      CALL SYSTEM
      F3=COST
      DO 24 I=1,GAINS
   24 K(I)=SAVE(I)
      COST=SAVE2
C     CHECK TO SEE IF NEW ORTHOGONAL DIRECTION
C     VECTOR IS NEARLY DEPENDENT
      IF(F3.LE.F1)GO TO 30
      DIG1=(F1-2.*F2+F3)*(F1-F2-DEL)**2
      DIG2=.5*DEL*(F1-F3)**2
      IF(DIG1.LE.DIG2)GO TO 30
   29 WRITE(6,41)
      DO 32 I=1,GAINS
C     WRITE CURRENT DIRECTION VECTORS
   32 WRITE(6,45)(Z(I,N),N=1,GAINS)
      WRITE(6,41)
      ROUNDS=ROUNDS+1
      IF(ROUNDS.GT.MXRNDS)GO TO 20
      DO 31 I=1,GAINS
   31 PO(I)=K(I)
      F1=COST
      J=1
C     START NEXT ROUND
      GO TO 3
   20 EXRNDS=.TRUE.
      FAIL=.TRUE.
      FINISH=.TRUE.
C     EXCESSIVE NUMBER OF ROUNDS - FAILED TO FIND OPTIMUM
      RETURN
C
   30 TKS=0.
      DO 28 I=1,GAINS
      TK(I)=K(I)-PO(I)
   28 TKS=TKS+TK(I)*TK(I)
      TKSR=SQRT(TKS)
      DO 33 I=1,GAINS
      Z(I,MMM)=TK(I)/TKSR
      DO 33 N=M,GAINS
   33 Z(I,N)=Z(I,N+1)
      DO 34 N=M,GAINS
   34 EXFRTR(N)=EXFRTR(N+1)
      EXFRTR(MMM)=.FALSE.
      EXMIN=.TRUE.
C     SEARCH FOR MINIMUM ALONG NEW DIRECTION VECTOR
      GO TO 3
   50 WRITE(6,51)IE
      STOP
C
   40 FORMAT(1X,13F10.2)
   41 FORMAT(1H0)
   42 FORMAT('OCHECK COST AT K=2KN-KO')
   43 FORMAT(2F10.2,2I10)
   44 FORMAT(11F7.2)
   45 FORMAT(1X,13F10.5)
   46 FORMAT(13F6.2)
   51 FORMAT(' READ DATA ERROR AT READ STATEMENT NO. ',I3)
```

```
60 FORMAT('0',8X,'TOLMIN',4X,'TOLPOW',7X,'MXRNDS'
  1        ,4X,'MAXTLS')
61 FORMAT(7X,2F10.3,2I10)
62 FORMAT('0INITIAL DIRECTION VECTORS')
63 FORMAT('0INITIAL DELTA LAMDAS')
   END
```

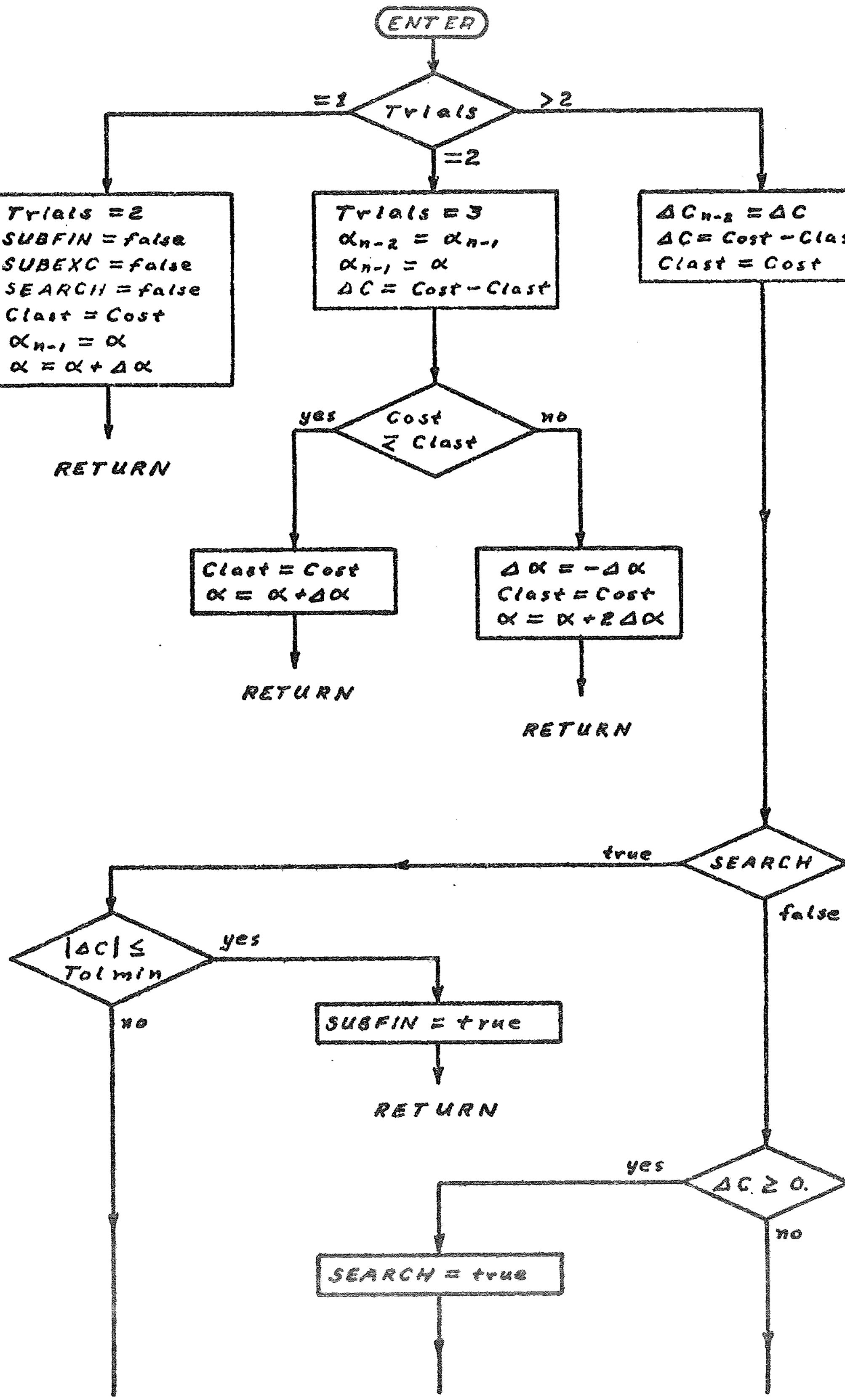
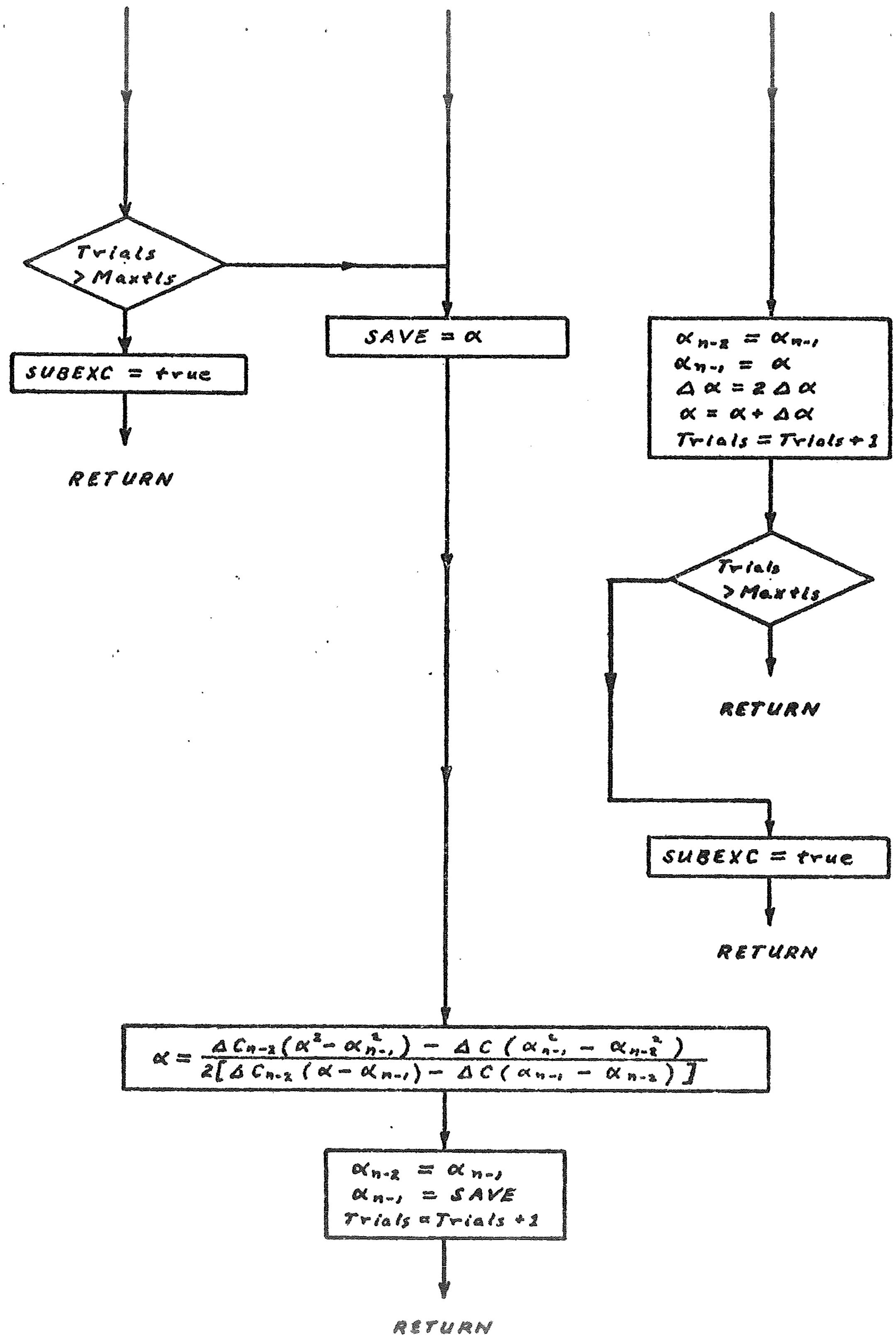
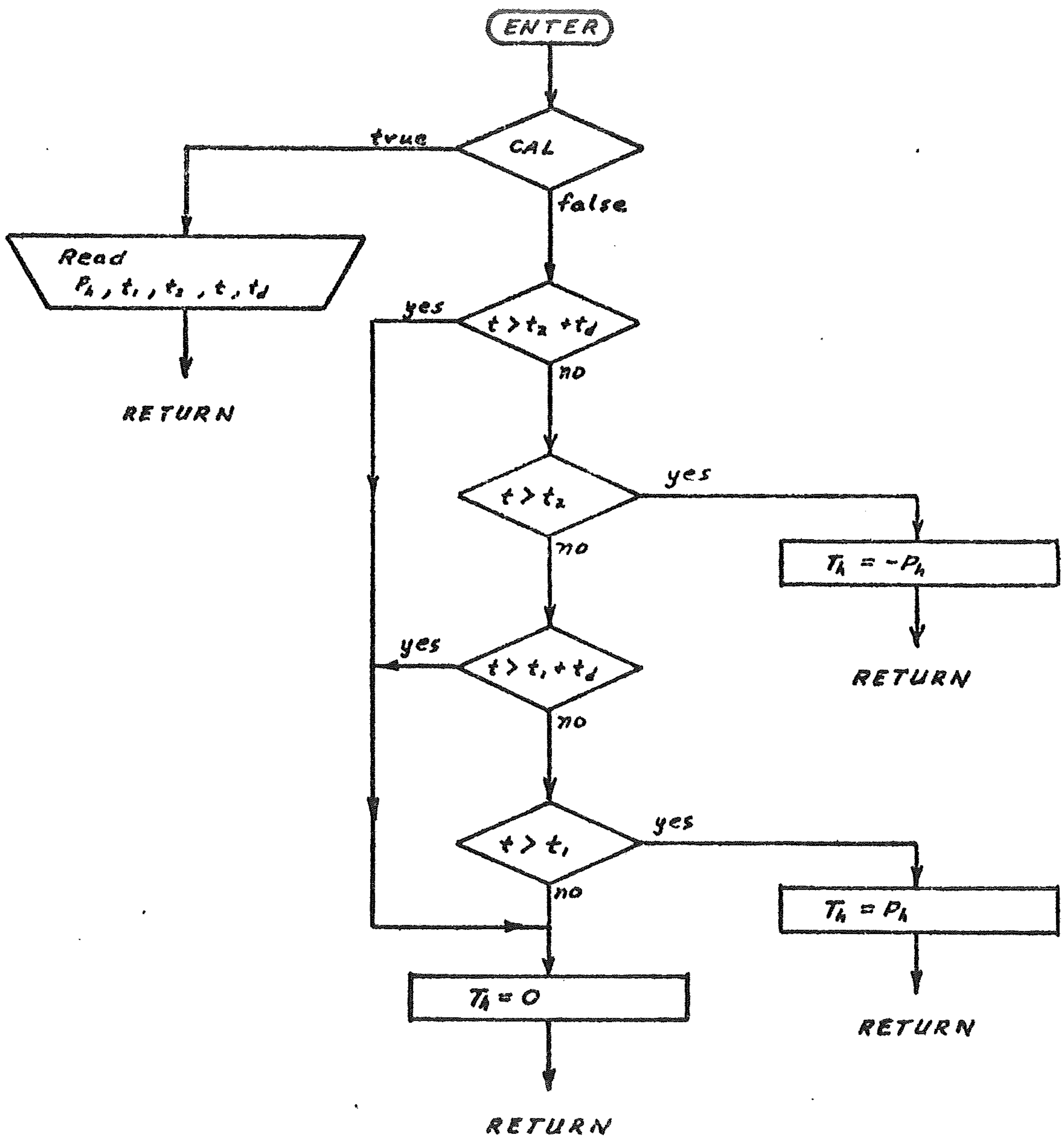


Figure B-4 Subroutine MINIMA


Trials > Maxtls
SUBEXC = true
RETURN
SAVE = α
$\alpha_{n-2} = \alpha_{n-1}$
$\alpha_{n-1} = \alpha$
$\Delta\alpha = 2\Delta\alpha$
$\alpha = \alpha + \Delta\alpha$
Trials = Trials + 1
Trials > Maxtls
RETURN
SUBEXC = true
RETURN
$\alpha = \frac{\Delta C_{n-2}(\alpha^2 - \alpha_{n-1}^2) - \Delta C(\alpha_{n-1}^2 - \alpha_{n-2}^2)}{2[\Delta C_{n-2}(\alpha - \alpha_{n-1}) - \Delta C(\alpha_{n-1} - \alpha_{n-2})]}$
$\alpha_{n-2} = \alpha_{n-1}$
α_{n-1} = SAVE
Trials = Trials + 1
RETURN

```
      SUBROUTINE MINIMA
      COMMON /LINKE/COST,TRIALS
      COMMON /LINKF/SUBFIN,SUBEXC,MAXTLS,A,DA,TOLMIN
      LOGICAL SUBFIN,SUBEXC,SEARCH
      INTEGER TRIALS
C
      IF(TRIALS.GT.2)GO TO 3
      IF(TRIALS.GT.1)GO TO 2
C
      SUBFIN=.FALSE.
      SUBEXC=.FALSE.
      SEARCH=.FALSE.
      CLAST=COST
      AM1=A
      TRIALS=2
      A=A+DA
      RETURN
C
    2 TRIALS=3
      AM2=AM1
      AM1=A
      IF(COST.LE.CLAST)GO TO 4
C
C     COST INCREASING - DECREASE A
      DA=-DA
      DC=COST-CLAST
      CLAST=COST
      A=A+DA+DA
      RETURN
C
C     COST DECREASING - CONTINUE INCREASING A
    4 DC=COST-CLAST
      CLAST=COST
      A=A+DA
      RETURN
C
    3 DDC=DC
      DC=COST-CLAST
      IF(SEARCH)GO TO 9
      IF(DC.GE.0.)GO TO 8
      CLAST=COST
      AM2=AM1
      AM1=A
C     DOUBLE STEP SIZE AND TRY AGAIN
      DA=DA+DA
      A=A+DA
      TRIALS=TRIALS+1
      IF(TRIALS.GT.MAXTLS)GO TO 12
      RETURN
C
    8 SEARCH=.TRUE.
      GO TO 10
    9 IF(ABS(DC).GT.TOLMIN)GO TO 11
C     HAVE FOUND A MINIMUM IN THIS DIRECTION
      SUBFIN=.TRUE.
      RETURN
C
   11 IF(TRIALS.LE.MAXTLS)GO TO 10
```

```
C     EXCEEDED MAX ALLOWABLE TRIALS IN THIS DIRECTION
   12 SJBEXC=.TRUE.
      RETURN
C
   10 SAVE=A
C     FIND MINIMUM POINT ON QUADRATIC CURVE
      AS=A*A
      AM1S=AM1*AM1
      AM2S=AM2*AM2
      ANUM=DDC*(AS-AM1S)-DC*(AM1S-AM2S)
      DEN=DDC*(A-AM1)-DC*(AM1-AM2)
      A=.5*ANUM/DEN
      AM2=AM1
      AM1=SAVE
      CLAST=COST
      TRIALS=TRIALS+1
      RETURN
C
      END
```



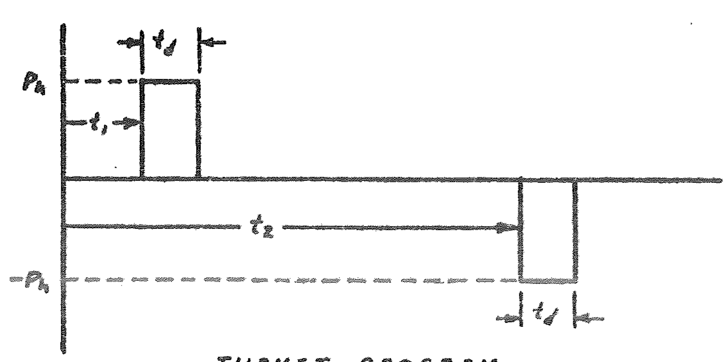
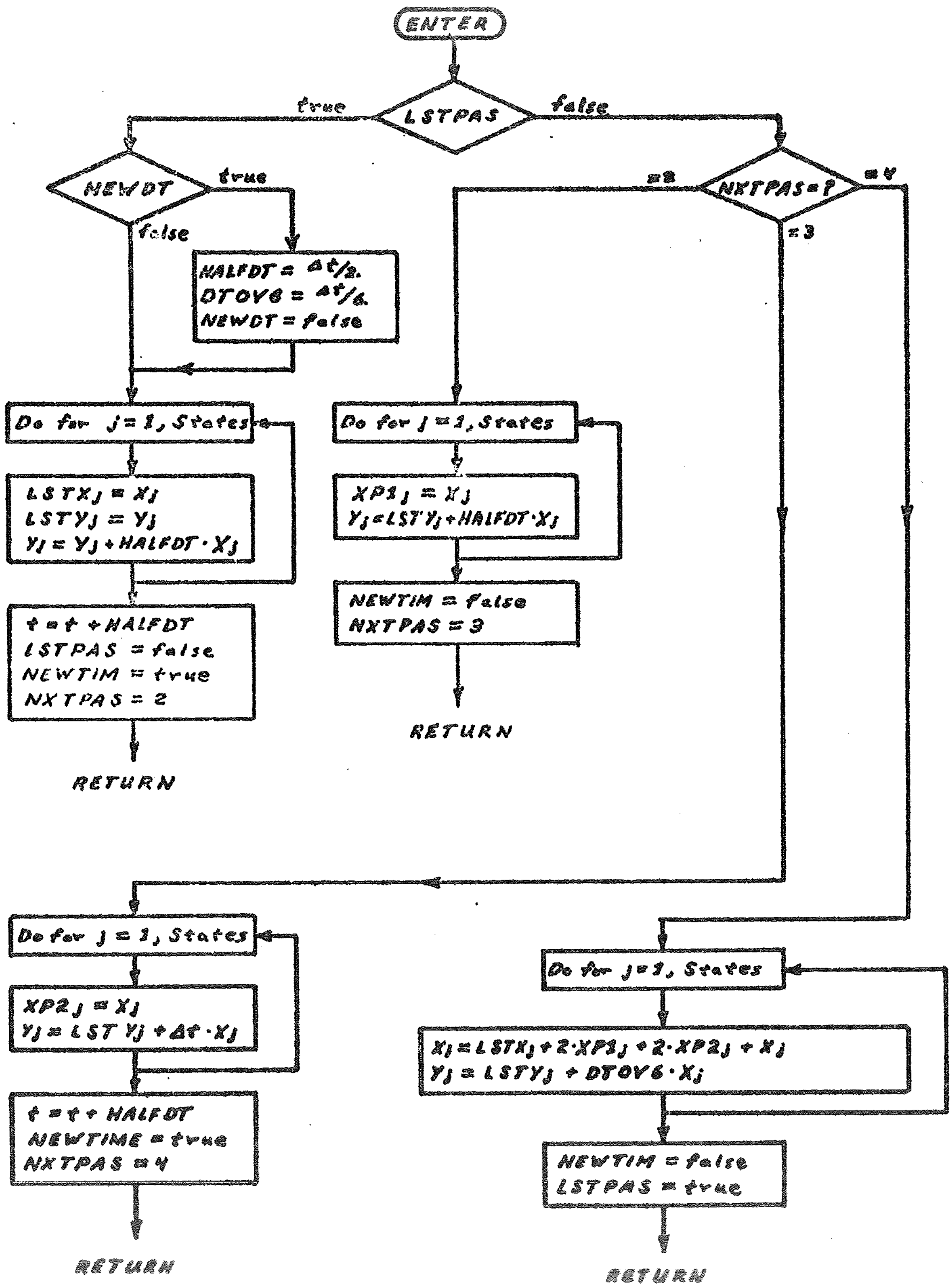


Figure B-5 Subroutine Thrust

```
      SUBROUTINE THRUST
      COMMON /LINKH/T
      COMMON /LINKI/CAL,TH
      LOGICAL CAL
      IF(CAL)GO TO 20
      IF(T.GT.T2+TD)GO TO 10
      IF(T.GE.T2)GO TO 11
      IF(T.GT.T1+TD)GO TO 10
      IF(T.GE.T1)GO TO 12
   10 TH=0.
      RETURN
C
   11 TH=-PH
      RETURN
C
   12 TH=PH
      RETURN
C
C
C     *******************************************************************
C
C     INITIALIZE THRUST PROGRAM
C
C     READ STATEMENT NO. 5
   20 IE=5
      READ(5,21,ERR=50)PH,T1,T2,TD
      WRITE(6,22)PH
      WRITE(6,23)TD
      RETURN
C
C     *******************************************************************
C
   50 WRITE(6,51)IE
      STOP
C
   21 FORMAT(4F10.4)
   22 FORMAT('0',6X,'PULSE HEIGHT OF HOZ THRUSTER = '
     1        ,F8.2,' LBS'//)
   23 FORMAT(6X,'PULSE WIDTH OF HOZ THRUSTER = '
     1        ,F5.3,' SECS'//)
   51 FORMAT(' READ DATA ERROR AT READ STATEMENT NO. ',I3)
      END
```



FOURTH-ORDER RUNGA-KUTTA INTEGRATION

Figure B-6 Subroutine INT

```
      SUBROUTINE INT
C     FOURTH-ORDER RUNGA-KUTTA INTEGRATION (4A-1)
      COMMON /LINKG/NEWDT,NEWTIM,LSTPAS,ITERAT,
     1  STATES,DT,X(20),Y(20)
      COMMON /LINKH/T
      DIMENSION LSTX(20),LSTY(20),XP1(20),XP2(20)
      INTEGER STATES
      REAL LSTX,LSTY
      LOGICAL LSTPAS,NEWTIM,NEWDT
      IF(.NOT.LSTPAS)GO TO 200
C
C     FIRST PASS
      IF(.NOT.NEWDT)GO TO 201
      HALFDT=.5*DT
      DTOV6=DT/6.
      NEWDT=.FALSE.
  201 DO 202 J=1,STATES
      LSTX(J)=X(J)
      LSTY(J)=Y(J)
  202 Y(J)=Y(J)+HALFDT*X(J)
      T=T+HALFDT
      LSTPAS=.FALSE.
      NEWTIM=.TRUE.
      NXTPAS=2
      RETURN
  200 IF(NXTPAS-3)203,204,205
C
C     SECOND PASS
  203 DO 206 J=1,STATES
      XP1(J)=X(J)
  206 Y(J)=LSTY(J)+HALFDT*X(J)
      NEWTIM=.FALSE.
      NXTPAS=3
      RETURN
C
C     THIRD PASS
  204 DO 207 J=1,STATES
      XP2(J)=X(J)
  207 Y(J)=LSTY(J)+DT*X(J)
      T=T+HALFDT
      NEWTIM=.TRUE.
      NXTPAS=4
      RETURN
C
C     FOURTH PASS
  205 DO 208 J=1,STATES
      X(J)=LSTX(J)+2.*XP1(J)+2.*XP2(J)+X(J)
  208 Y(J)=LSTY(J)+DTOV6*X(J)
      NEWTIM=.FALSE.
      LSTPAS=.TRUE.
      RETURN
      END
```

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