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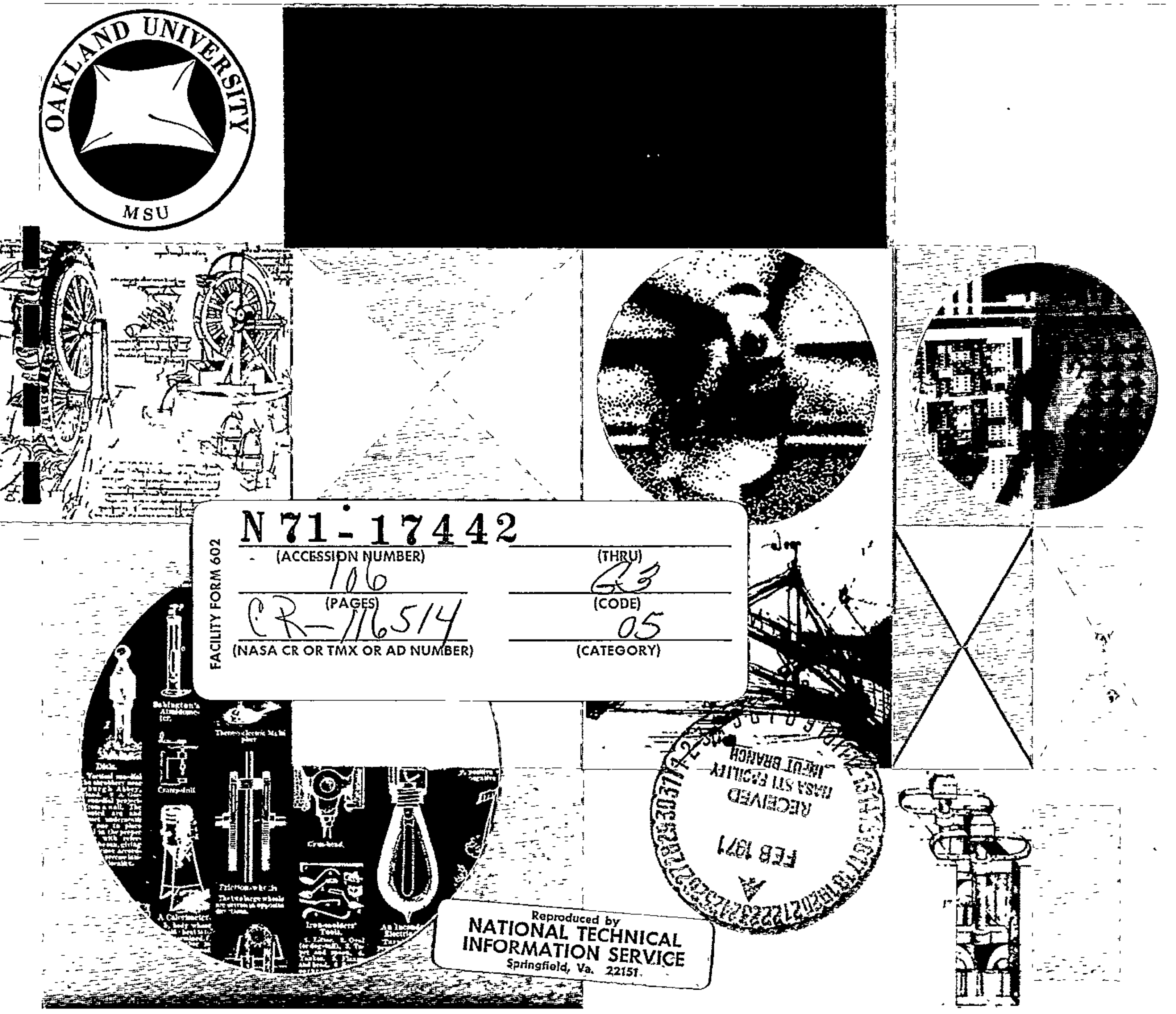

# FINAL REPORT

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## BIOSYSTEMS ENGINEERING RESEARCH

Volume II An Investigation of Two Hybrid Computer Identification Techniques For Use In Manual Control Research.

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VOLUME II

# AN INVESTIGATION OF TWO HYBRID COMPUTER IDENTIFICATION TECHNIQUES FOR USE IN MANUAL CONTROL RESEARCH

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## FOREWORD

This report was prepared under NASA Contract NGR 23-054-003, which provided institutional support for biosystems research conducted by the Oakland University School of Engineering, and is one of four volumes reporting research conducted during the period March 15, 1969 to June 31, 1970.

The four volumes of the series, together with their principal authors are listed below:

| | |
|---|---|
| Volume I | A Dynamic Model of the Human Postural Control System (J.C. Hill). |
| Volume II | An Investigation of Two Hybrid Computer Identification Techniques for Use in Manual Control Research (G.A. Jackson). |
| Volume III | Pattern Recognition of Biological Photomicrographs Using Coherent Optical Techniques (R.E. Haskell). |
| Volume IV | Augmentation of the Focus Control System of the Human Eye through Index of Refraction Variation of Liquid Crystal Materials (R.H. Edgerton). |

Professors J.C. Hill and J.E. Gibson, Dean of the School of Engineering, were principal investigators on the contract.

## ABSTRACT

Two hybrid computer techniques for the identification of the gain and time delay parameters of the crossover model of a compensatory control system are discussed. These techniques were specifically developed to perform accurate parameter identification when limited amounts of input-output data are available from the system being identified. The methods are thus useful in those cases where it is desired to know the system parameters that existed over a short time span, and there is not enough input-output data for identification by pure analog, or pure digital, means.

One method is a hybrid version of the continuous parameter tracking method, while the second is a modified stochastic approximation method. With proper programming either method can be used as an on-line device which will continually yield those average parameter values which existed over the last fifteen seconds of operation.

ACKNOWLEDGMENT

The research discussed in this report was aided by the efforts of two Oakland University undergraduate research assistants: Mr. James Kenning and Mr. Robert White. Their assistance and constructive comments on computer programming and experimental testing is greatly appreciated.

AN INVESTIGATION OF TWO HYBRID COMPUTER
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# I. INTRODUCTION

One area of active research in the broad discipline called manual control is compensatory tracking. In this type of tracking task the human operator serves as the controller in a conventional closed loop system [11]*.

The subject is asked to manipulate the signal forcing a dynamic signal in such a manner that the system output follows a given input. The magnitude of the signal forcing the dynamic system is determined by the position of a control stick which the operator moves with his hand, arm, or wrist, depending on the particular type of control stick being used. The error $\varepsilon_1(t)$ between the input $\theta_i(t)$ and the actual system output $\theta_o(t)$ is displayed on an oscilloscope located in front of the subject. This error signal is used by the subject to determine present and future control stick action. The system configuration is shown in Fig. 1(a).

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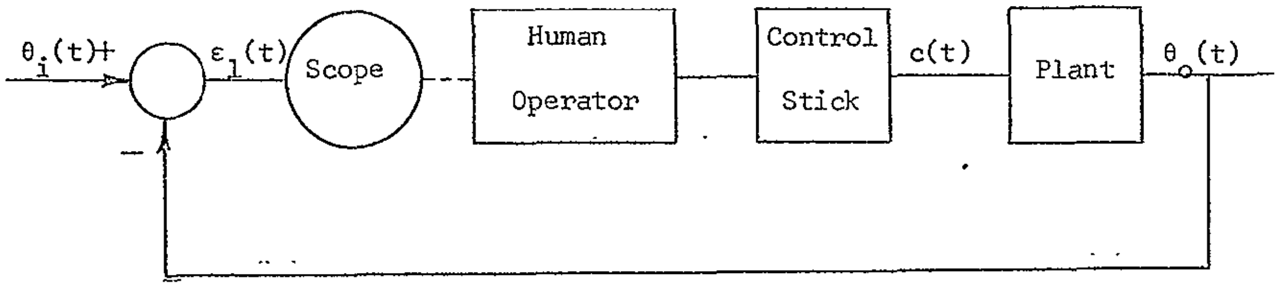
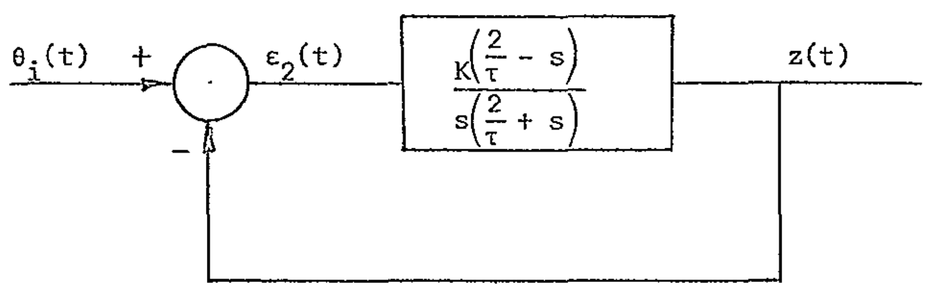
* Numbers in brackets refer to references listed at the end of the report.




(a) Block Diagram of the Compensatory Control System.




(b) Approximate Crossover Model of the Compensatory Control System.

Fig. 1 The Closed Loop Compensatory Control System.

The input signal used in this type of tracking task is some unpredictable signal, such as low frequency filtered gaussian white noise, or a sum of sinusoids designed to approximate this type of signal. The subject is simply told to keep the error as small as possible at all times and is free to move the control stick in any manner he deems best.

Research related to this type of manual control task started during World War II when men in the armed forces were being trained to manually sight and fire automatic weapons [14]. The research continued in the 1950's and has been stimulated in recent years by the presence of manual control systems in space vehicles.

## 1.A. The Crossover Model

A large portion of the total effort expended in compensatory tracking task research has been spent developing the mathematical equations which describe the way in which the human operator controls the compensatory system. One of the more important results in this particular area of research was the postulation by McRuer and his associates [10] of the "crossover model". It was discovered that for all subjects controlling first and second order plants, the best linear equation for modeling the entire forward loop of the compensatory system was always the same form. This form, called the crossover model, consisted of a gain K, a time delay $\tau$, and a single integration. One form of this model, which is often used, is given in Fig. 1(b). Here the first order Padé time delay

$$e^{-\tau s} \cong \frac{\frac{2}{\tau} - s}{\frac{2}{\tau} + s}$$

is used in place of a pure time delay.

A distinct advantage that the crossover model has over more complicated models proposed by other researchers is the fact that only two parameters, K and $\tau$, are needed to completely determine the model and the action of the subject being tested. This fact has led to its wide acceptance, and it has proven useful to many researchers [4,5,8].

Since the crossover model is completely defined for a given subject once the parameters K and $\tau$ are known, considerable effort has been placed on the development of techniques which can be used to determine these parameters from actual test data. The most commonly used methods are continuous parameter tracking [1,2,13], which is an analog computer technique, and random input describing functions [9], which is a digital computer technique. Although both of these methods are quite accurate, they suffer from the fact that two or three minutes of subject data is necessary in order for K and $\tau$ to be determined. This means that these methods cannot be used to determine those parameter values which exist over short intervals of time. Since there are practical situations which arise where only short lengths of subject data are available for evaluation, an investigation was made on two techniques which can be used to identify crossover model parameters when limited compensatory tracking data is available. This report outlines the results of this investigation.

## 1.B. Description of the Present Research Effort

Research emphasis in the manual control systems area has been placed on the development of two Hybrid computer techniques for the identification of the gain and time delay parameters of the crossover model of the compensatory control system. These techniques were specifically developed to perform accurate parameter identification when limited amounts of input-output data are available from the system being identified. The methods are thus useful in those cases where it is desired to know the K and $\tau$ parameters which existed over a short time span and there is not enough input-output data for the identification to be determined accurately and/or quickly by pure analog or pure digital means.

The first method developed and tested is called The Hybrid parameter tracking method and is a Hybrid version of the continuous parameter tracking method. The second method is called the modified stochastic approximation method, and is a variation of a method proposed by Neal and Bekey [12]. Both methods are shown to be accurate, stable identification techniques that should prove valuable to those involved in research on compensatory tracking systems.

A general outline of the research discussed in the remainder of this report is as follows:

1. A discussion of the Hybrid version of the continuous parameter tracking method.
2. An analysis of the statistical characteristics of a periodic-random signal obtained by repeating a short length of signal taken from a continuous random process. In particular, the spectral density of the periodic signal is related to the spectral density of the continuous process, as a function of the sample length T. This type of signal is encountered in the first identification method.
3. A discussion of a Hybrid routine using a modified stochastic approximation method.
4. A discussion of possible extensions of the methods developed.

The results of the research to date indicate that either of the Hybrid methods proposed can identify crossover model parameters using 15 (or less) seconds of input-output data from the compensatory control system. Both methods are capable of being programmed so as to perform the parameter identification calculations in very short periods of computation time.

The Hybrid system used in the tests discussed in this report consists of an IBM 1130 digital computer, an EAI 680 analog, and an EAI 693 interface.

## II. HYBRID VERSION OF CONTINUOUS PARAMETER TRACKING

Continuous parameter tracking [1,2,6,13,16] is a well documented analog method of parameter identification which has been used effectively in identifying the parameters of various human operator models. This method assumes that the equations describing the unknown system are known, but that the coefficients have to be determined. A brief review of the basic method is given below.

## 2.A Continuous Parameter Tracking

In the standard application of the output error method of continuous parameter tracking to the identification of the true crossover model parameters, $K_o$ and $\tau_o$, the model matching error is defined as [5]

$$e(t,K,\tau) = z(t,K,\tau) - \theta_o(t,K_o,\tau_o) \tag{1}$$

where

$z(t,K,\tau)$ = model output.

$\theta_o(t,K_o,\tau_o)$ = actual compensatory system output.

$K_o,\tau_o$ = true values of the compensatory system gain and time-delay parameters.

$K,\tau = K(t),\ \tau(t)$ = the parameter tracking values which should converge to $K_o$ and $\tau_o$.

The index of performance used to evaluate the accuracy of the model is

$$I(t,K,\tau) = \frac{1}{2}\, e^2(t,K,\tau) \tag{2}$$

and the parameters $K(t)$ and $\tau(t)$ are adjusted continuously in time via the parameter adjustment equations

$$\frac{dK}{dt} = -k_o \frac{\partial I}{\partial K} = -k_o\ e \frac{\partial z}{\partial K} = -k_o\ eu_o \tag{3}$$

$$u_o = u_o(t,K,\tau) = \frac{\partial z}{\partial K}$$

and

$$\frac{d\tau}{dt} = -k_1 \frac{\partial I}{\partial \tau} = -k_1 \frac{\partial z}{\partial \tau} = -k_1\ eu_1 \tag{4}$$

$$u_1 = u_1(t,K,\tau) = \frac{\partial z}{\partial \tau} .$$

$k_o$ and $k_1$ are the parameter adjustment gains and are constant for a given run. A block diagram of the complete system is given in Figure 2. $\theta_i(t)$, the input signal, is generally low frequency noise obtained by filtering white noise.

It has been shown [5,6] that this particular system can be used to generate accurate on-line estimates for the parameters $K_o$ and $\tau_o$. However, one drawback of the method is that the convergence rate is fairly slow. Jackson [5] found that when tracking K and $\tau$ values for actual subjects, meaningful results required two minutes of test data. One minute of data was needed to let the parameters converge, and one additional minute of data was used to generate average values of K and $\tau$ for estimating $K_o$ and $\tau_o$.

Figure 3 is a reproduction of one two-minute run of a continuous parameter tracking system showing K and $\tau$ converging to 5.0 and 0.14 respectively. This run used an actual

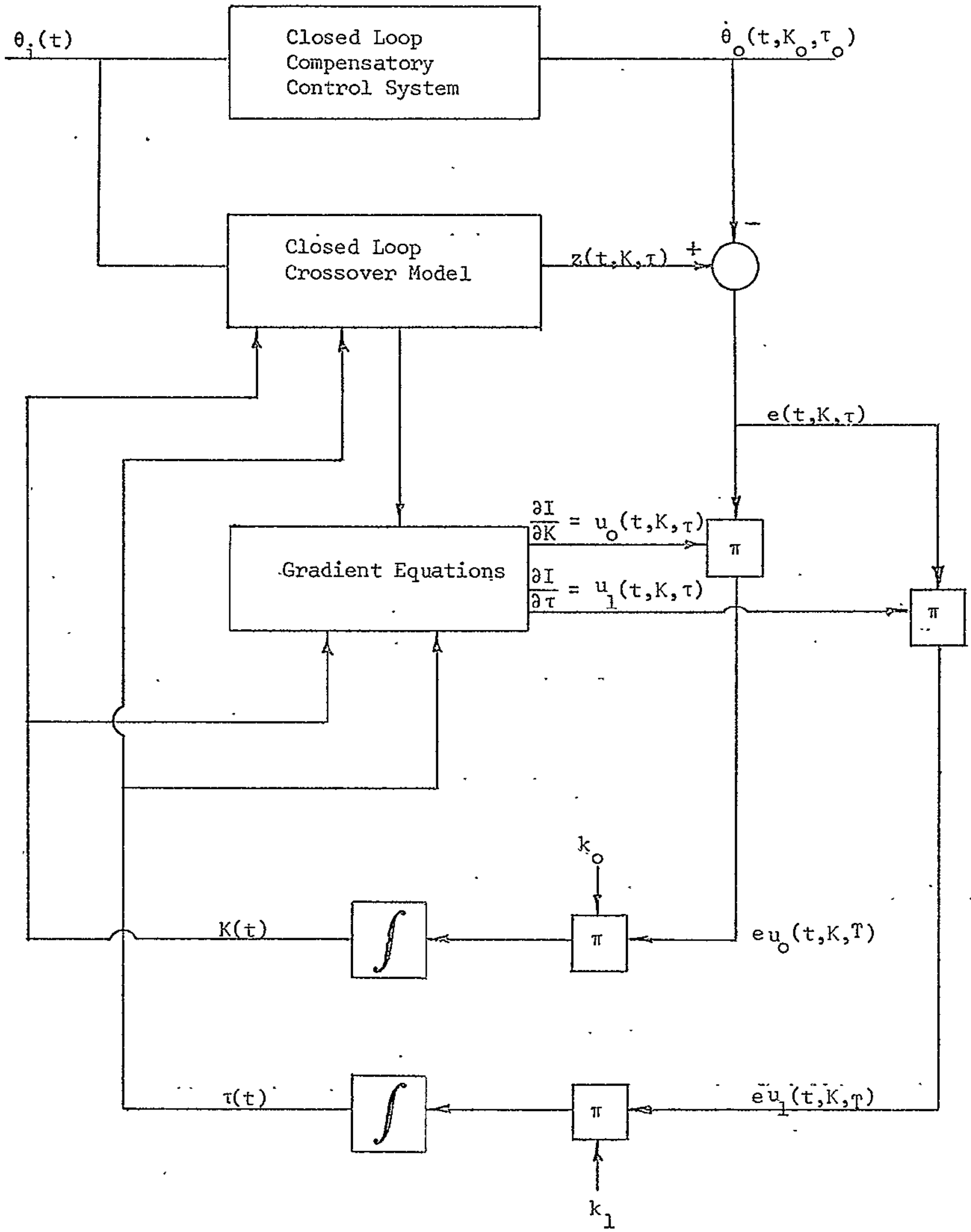



Fig. 2 Block Diagram of the Continuous Parameter Tracking System.

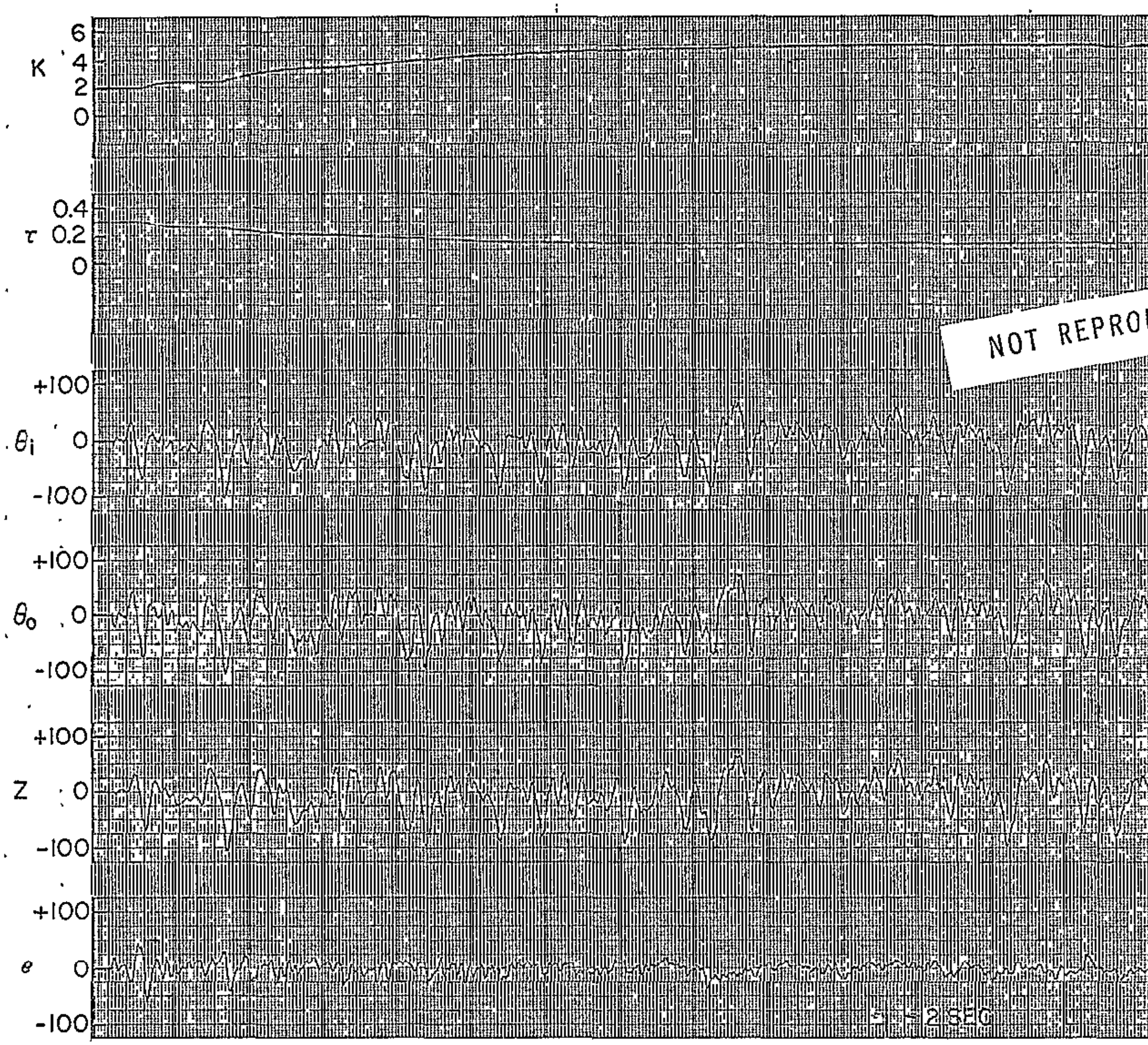





Fig. 3 Time History of a Two Minute Continuous Parameter Tracking Test.

subject and is typical of continuous parameter tracking operation [5].

2.B Hybrid Parameter Tracking

As its name implies, this method is a Hybrid adaptation of the continuous parameter tracking technique that uses short lengths of input-output data from the system being identified. In the method developed the compensatory system input and output, $\theta_i(t)$ and $\theta_o(t)$, are sampled for a short period of time ($T \leq 15$ seconds) with the samples stored in the IBM 1130. During the next time interval, the stored values are fed into a fast-time, analog, continuous parameter tracking model, exactly like the one discussed above and developed in reference [5]. However, since 15 seconds of data are not sufficient for the parameter tracking system to converge in the presence of noise, the fast time model iterates through the same input-output data several times. To allow smooth convergence, the initial values of $K(t)$ and $\tau(t)$ used at the start of each analog iteration are continually updated. For the $(n + 1)^{st}$ iteration the $K(o)$ and $\tau(o)$ values are the average values of $K(t)$ and $\tau(t)$ which were present over the last half of the $n^{th}$ iteration, i.e.:

$$K_{n+1}(0) = \int_{T/2}^{T} K_n(t)dt = \overline{K}_n \tag{5}$$

$$\tau_{n+1}(0) = \int_{T/2}^{T} \tau_n(t)dt = \overline{\tau}_n$$

Additionally, the parameter adjustment gains, $k_o$ and $k_1$, used in the adjustment networks for K(t) and $\tau(t)$ are systematically reduced, increased and reduced again in the following manner:

$$
\begin{aligned}
k_o &= \frac{A}{\sqrt{n}} \;;\quad n = 1,\ldots,5 & k_1 &= \frac{B}{\sqrt{n}} \;;\quad n = 1,\ldots,5 \\
&= \frac{A}{\sqrt{n-5}} \;;\quad n = 6,\ldots,10 & &= \frac{B}{\sqrt{n-5}} \;;\quad n = 6,\ldots,10 \\
&= \frac{A}{\sqrt{n-10}} \;;\quad n = 11,\ldots,15 & &= \frac{B}{\sqrt{n-10}} \;;\quad n = 11,\ldots,15 \\
&= \frac{A}{\sqrt{n-15}} \;;\quad n = 16,\ldots,20 & & \frac{B}{\sqrt{n-15}} \;;\quad n = 16,\ldots,20
\end{aligned}
\tag{6}
$$

where A and B are empirically determined constants. This gain manipulation was empirically shown to give smooth convergence even when considerable noise was present in the output of the system being identified. $\overline{K}_{20}$ and $\overline{\tau}_{20}$, the average values of K(t) and $\tau(t)$ over the last half of the 20$^{th}$ iteration were used as the estimates of $K_o$ and $\tau_o$. Twenty iterations were usually sufficient for satisfactory parameter convergence.

To insure convergence of the basic Hybrid method in twenty iterations, one further modification of the continuous parameter tracking method was necessary. This modification stems from the fact that the initial conditions which were actually present in the compensatory control system when $\theta_i(t)$ and $\theta_o(t)$ were recorded are unknown. Thus, the initial conditions on the model at the start of each iteration are probably incorrect, and introduce transients into model which will cause a model matching error which is not due to the parameters being incorrect. If these initial transient errors are allowed to affect the parameter adjustment integrators, an incorrect adjustment at the start of each analog iteration will result. Since the total analog run time is short, the adjustment loop may not recover in the time allotted. This has experimentally been shown to produce erroneous estimates of $K_o$ and $\tau_o$.

To eliminate this effect, the parameter adjustment integrators are left in the IC mode for the first five seconds of real-time data at the start of each iteration. It is easily shown that, with K and $\tau$ fixed at nominal values, all transients in the analog system will decay to less than 5% of their initial values in this period of time. It was felt this was sufficient for practical operation of the Hybrid system, and experimental data appears to substantiate this assertion.

With these facts in mind, the basic time-logic pattern of the Hybrid parameter tracking system will be outlined. This is given in Table 1 on the following page. The basic FORTRAN program, which is well documented with comment statements is found in Appendix A. The analog circuit diagram may also be found in that Appendix.

Table 1. Time-Logic Outline for One Hybrid Parameter Tracking Sequence

| Time t | Operating Status |
|---|---|
| $< 0$ | 1. Compensatory system to be identified is in operation.<br>2. Analog tracking system is in IC mode. |
| $0$ | 1. Sampling of $\theta_i(t)$ and $\theta_o(t)$ begins at a rate of 20 sample/second using AD converters. Samples are taken for 15 seconds and stored in the IBM 1130.<br>2. Analog parameter tracking system remains in IC mode. |
| $T$ | 1. Sampling stops. |
| $T + t_o$ | 1. First analog iteration begins with analog tracking system being placed into OPERATE at time scale of 10 times real-time.* (Parameter adjustment integrators, 00 and 95 in Figure A4, are left in IC).<br>2. $\theta_i(t)$ and $\theta_o(t)$ samples are fed into the analog circuit via DAC's at 10 times sampling rate. |
| $T + t_1$ | 1. $t_1 - t_o = 0.5$ seconds $= 1/3 \times T/10$<br>2. Parameter adjustment integrators are placed into OPERATE. |
| $T + t_2$ | 1. $t_2 - t_o = 0.75$ seconds $= 1/2 \times T/10$<br>2. Samples of $K(t)$ and $\tau(t)$ are begun in order to calculate $\overline{K}_1$ and $\overline{\tau}_1$, the average values of $K(t)$ and $\tau(t)$ present over the last half of the first iteration.** |
| $T + t_3$ | 1. $t_3 - t_o = 1.5$ seconds $= T/10$.<br>2. First analog iteration ends.<br>3. Analog is placed into IC, except for the parameter adjustment integrators which are placed in HOLD.<br>4. The average parameter values $\overline{K}_1$ and $\overline{\tau}_1$ are determined on digital computer.** |
| $T + t_4$ | 1. $\overline{K}_1$ and $\overline{\tau}_1$ are typed on typewriter.<br>2. Parameter adjustment integrators are placed into IC with values of $\overline{K}_1$ and $\overline{\tau}_1$.<br>3. Parameter adjustments gains $k_o$ and $k_1$ are adjusted via Equation (6). |
| $T + t_5$ | 1. Second analog iteration begins.<br><br>Twenty analog iterations are made, at which time the system goes back to the start of the Table for a new set of data. $\overline{K}_{20}$ and $\overline{\tau}_{20}$ are the estimates of $K_o$ and $\tau_o$, the true parameter values. |

*The method presented was limited to a fast-time operation of 10 times real-time due to the use of FORTRAN programming. Through the use of Assembly language, the iterations could be accomplished at least 10 times faster.

**These averages should be calculated on the analog computer. Lack of analog equipment would not allow this on the tests being reported.

## III. THE DATA LENGTH/CONVERGENCE PROBLEM

Before presenting some experimental results of the Hybrid parameter tracking technique, the effect of the parameter tracking length of the input-output data sample used in the iteration mode will be discussed.

### 3.A Convergence Properties of A Continuous Parameter Tracking System

For a continuous parameter tracking system to identify accurately, the input signal must have measurable power in the band pass region of the system being identified. Simply stated, this means that for the model matching error $e(t,K,\tau)$ to have meaning, the unknown system must be tested over those frequencies which will be passed by the system. This region is approximately 0-10 radians/second for a compensatory system.

Another aspect of the input signal which must be considered in a continuous parameter tracking system is the effect of the input spectrum on parameter convergence rate. Jackson has shown [5,7] that in the neighborhood of the correct solution ($K \cong K_0$ and $\tau \cong \tau_o$) the convergence of the parameters is essentially first order, providing the parameter adjustment gains $k_o$ and $k_1$ are relatively small. The time constant of convergence for one parameter, say $K(t)$, is

$$\text{Time Constant of } K(t) \cong \frac{1}{k_o E\{u_o^2(t,K_o,\tau_o)\}} \qquad (7)$$

where $E\{\cdot\}$ is the expected value, or statistical average. But

$$E\{u_o^2(t,K_o,\tau_o)\} = \frac{1}{\pi}\int_0^\infty S_{u_o u_o}(\omega)\,d\omega \qquad (8)$$

$$= \text{average power in } u_o(t,K_o,\tau_o)$$

$$= \frac{1}{\pi}\int_0^\infty S_{\theta\theta}(\omega)\left|\frac{U_o(\omega,K_o,\tau_o)}{\theta_1(\omega)}\right|^2 d\omega \qquad (9)$$

where

$S_{u_o u_o}(\omega)$ = PSD of $u_o(t,K_o,\tau_o)$

$S_{\theta\theta}(\omega)$ = PSD of $\theta_i(t)$

$\dfrac{U_o(\omega,K_o,\tau_o)}{\theta_1(\omega)}$ = transfer function relating $u_o(t,K_o,\tau_o)$ and $\theta_i(t)$.

Similar results are found for $\tau(t)$.

Eqs. (7) and (9) indicate that the convergence rate depends explicitly on the spectrum of the input signal. If this spectrum is altered, the convergence properties will change, possibly drastically.

## 3.B Hybrid Parameter Tracker Convergence Properties

To relate the above continuous parameter tracking characteristics to the Hybrid parameter tracker, it is merely noted that during the fast-time iteration process the analog input signal is a section taken from $\theta_i(t)$ during the sampling operation, and the effective period is directly related to the sample length T. The particular values appearing in this signal are, of course, random, and will vary from sample to sample. A pictorial example of one possible outcome for $\theta_i(t)$ which might be used to test the unknown system is shown in Figure 4. Also shown are several periodic signals, which were obtained by using T-length sections of $\theta_i(t)$. The $x(t)$'s are several of a family of signals which might be seen by the fast-time parameter tracking model.

Note that over one time interval T seconds in length $x(t) = \theta_i(t)$. Outside of this interval $x(t)$ is periodic. Note also that $x(t)$ is being discussed as though it were a real-time signal, when in fact it is a fast-time signal. This can be done in this case, since both $x(t)$ and the analog system are time scaled equally. Thus, any convergence properties developed in real-time can be directly translated into fast-time information.

The question which now arises is as follows: How small can T be and still have the expected Hybrid convergence rate with $x(t)$ as an input be close to the continuous convergence rate expected from $\theta_i(t)$? To gain insight into this question,

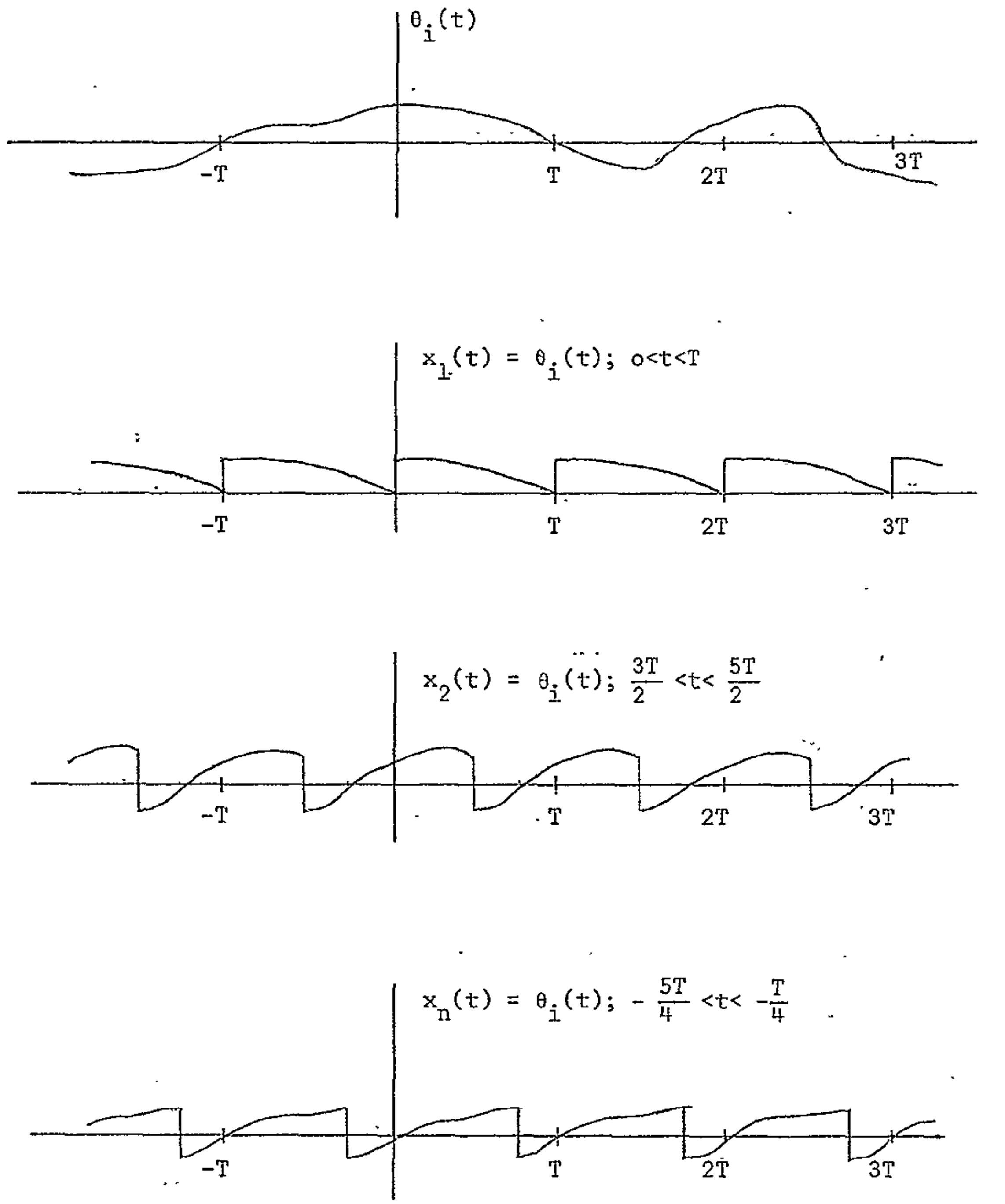



Fig. 4 Generation of Several x(t)'s from One Representation of $\theta_i(t)$.

an expression is developed in the following sections which gives the difference in the average powers in $x(t)$ and $\theta_i(t)$ over a given frequency band, That is, the expression

$$\int_{\omega_1}^{\omega_2} S_{\theta\theta}(\omega)d\omega - \int_{\omega_1}^{\omega_2} S_{xx}(\omega)d\omega$$

is derived as a function of T, where $S_{xx}(\omega)$ is the power spectral density of $x(t)$. Although this difference is not specifically the convergence rate, it is a quantity which is directly related to it, as seen by Eq. (9). If this difference is small for values of $\omega_1$ and $\omega_2$ in the pass band of the compensatory system, then the convergence properties of the continuous and Hybrid systems will be comparable.

### 3.C An Analysis of the Periodic-Random Signal x(t)

In Section B it was noted that the signal seen by the analog model in the Hybrid parameter tracking scheme is essentially a periodic signal that is generated from a continuous random process. The purpose of this section is the generation of the equations that describe the periodic signal, in terms of the original process. The problem will be stated as follows:

Assume that $\theta_i(t)$ is a continuous, wide sense stationary, random process with known autocorrelation and power spectral density functions, $R_{\theta\theta}(\alpha)$ and $S_{\theta\theta}(\omega)$. From $\theta_i(t)$ a new random process is generated by randomly choosing a section of $\theta_i(t)$ T seconds long and assuming that this is one period of a periodic signal, $x(t)$. The following properties of $x(t)$ will be shown:

1. That $R_{xx}(\omega)$, the autocorrelation function for $x(t)$, can be given in terms of $R_{\theta\theta}(\alpha)$ and T.
2. That a simple expression relating $S_{xx}(\omega)$, the power spectral density of $x(t)$, and $S_{\theta\theta}(\omega)$ can be obtained if $\theta_i(t)$ is restricted to a given class of signals.

### 3.C.1 Development of $R_{xx}(\alpha)$.

To develop the statistical autocorrelation function, $R_{xx}(\alpha)$, for the signal $x(t)$, use will be made of the time auto-correlation function

$$\overline{R_{xx}}(\alpha) = \lim_{\gamma\to\infty} \frac{1}{2\gamma} \int_{-\gamma}^{\gamma} x(t)x(t+\alpha)dt \tag{11}$$

and the fact that

$$E\{\overline{R_{xx}}(\alpha)\} = R_{xx}(\alpha). \tag{12}$$

If it is assumed that one set of outcomes for $\theta_i(t)$ and $x(t)$ are as given in Figure 4, then $x(t)$ and $x(t+\alpha)$ will be as shown in Figure 5, where $x_1(t)$ from Figure 4 has been used. From Figure 5, several things can be deduced about $\overline{R_{xx}}(\alpha)$. First, since $x(t)$ is periodic, $x(t)x(t+\alpha)$ is periodic with period T, for any $\alpha$. Second, the time average value of

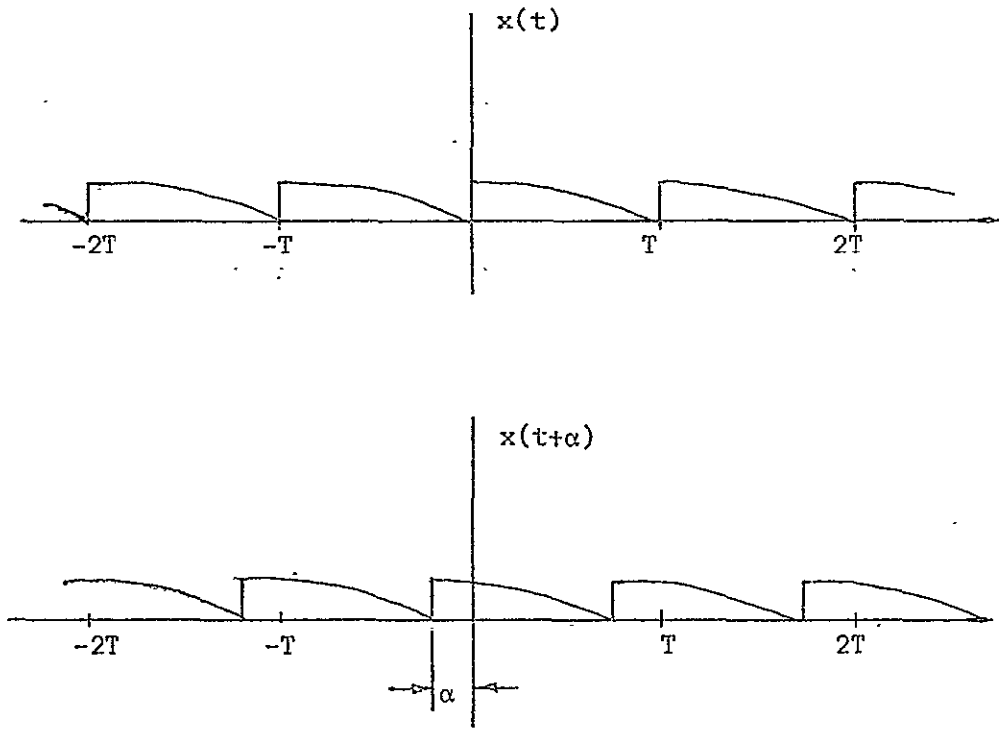



Fig. 5 One Possible Set of $x(t)$ and $x(t+\alpha)$ for Determining $\overline{R}xx(\alpha)$.

$x(t)x(t+\alpha)$ over all time is equal to the average value over one period. Therefore,

$$\overline{R_{xx}}(\alpha) = \frac{1}{T} \int_{0}^{T} x(t)x(t+\alpha)d\alpha \ , \ 0 \le \alpha \le T \tag{13}$$

This equation can be placed in terms of $\theta_i(t)$ by noting the following relationships from Figure 5.

$$\begin{aligned} x(t) &= \theta_i(t) && 0 \le t \le T \\ x(t+\alpha) &= \theta_i(t+\alpha) && 0 \le t \le T - \alpha \\ &= \theta_i(t+\alpha-T) && T - \alpha \le t \le T \end{aligned}$$

Equation (13) can thus be written as

$$\overline{R_{xx}}(\alpha) = \frac{1}{T} \int_{0}^{T} \theta_i(t)x(t+\alpha)dt \ , \ 0 \le \alpha \le T$$

$$= \frac{1}{T} \left\{ \int_{0}^{T-\alpha} \theta_i(t)\theta_i(t+\alpha)dt + \int_{T-\alpha}^{T} \theta_i(t)\theta_i(t+\alpha-T)dt \right\} \ , \ 0 \le \alpha \le T \tag{14}$$

For a given $\alpha$, $\overline{R_{xx}}(\alpha)$ is a random variable which depends on four factors: (a) the particular realization of $\theta_i(t)$ used in evaluation of the integral; (b) the value of $T$; (c) the particular section of $\theta_i(t)$ used to generate $x(t)$; and (d) the value of $\alpha$.

However, from Eq. (12)

$$E\{\overline{R_{xx}}(\alpha)\} = R_{xx}(\alpha)$$

$$= \frac{1}{T}\int_{0}^{T-\alpha} E\{\theta_i(t)\theta_i(t+\alpha)\}\,dt + \frac{1}{T}\int_{T-\alpha}^{T} E\{\theta_i(t)\theta_i(t+\alpha-T)\}dt$$

$$= \frac{1}{T}\int_{0}^{T-\alpha} R_{\theta\theta}(\alpha)dt + \frac{1}{T}\int_{T-\alpha}^{T} R_{\theta\theta}(\alpha-T)dt$$

$$= \frac{T-\alpha}{T}\; R_{\theta\theta}(\alpha) + \frac{\alpha}{T}R_{\theta\theta}(\alpha-T) \qquad 0 \leq \alpha \leq T \tag{15}$$

Since $R_{\theta\theta}(\alpha)$ is an even function, $R_{\theta\theta}(\alpha-T) = R_{\theta\theta}(T-\alpha)$ and Eq. (15) can be written

$$R_{xx}(\alpha) = \frac{T-\alpha}{T} R_{\theta\theta}(\alpha) + \frac{\alpha}{T} R_{\theta\theta}(T-\alpha)\;, \quad 0 \leq \alpha \leq T \tag{16}$$

which is the desired result.*

By inspection of Figure 5,

$$R_{xx}(\alpha) = R_{xx}(\alpha \pm kT) \quad k = 0,\, 1,\, 2,\, \ldots \tag{17}$$

### 3.C.2 A Comparison of $S_{\theta\theta}(\omega)$ and $S_{xx}(\omega)$ as a Function of T.

Before developing $S_{xx}(\omega)$, two assumptions relating to the random process $\theta_i(t)$ will be made. It will be shown later

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*This result can be obtained by directly evaluating $R_{xx}(\alpha) = E[x(t)x(t+\alpha)]$. This derivation was completed, but is considerably longer than the method presented.

---

that these assumptions are valid for the type of signals encountered in human operator work.

Assumption 1: It is assumed that $\theta_i(t)$ is obtained in such a manner that

$$|R_{\theta\theta}(o)| >> |R_{\theta\theta}(\alpha)| \cong 0; \quad |\alpha| > \beta$$

for some finite value of $\beta$. For simplicity of presentation, it is assumed for the present that $R_{\theta\theta}(\alpha)$ is as shown in Figure 6a. Assumption 2: T is chosen so that $\frac{T}{2} > \beta$.

Under these assumptions and through Eqs. (16) and (17), $R_{xx}(\alpha)$ will be shown in Figure 6b. Note that under the assumptions given, the portion of $R_{xx}(\alpha)$ around $\alpha = o$ does not overlap those portions centered at $\alpha = \pm T$.

The power spectral density of $\theta_i(t)$ is by definition,

$$S_{\theta\theta}(\omega) = \int_{-\infty}^{\infty} R_{\theta\theta}(\alpha)e^{-j\omega\alpha}d\alpha \tag{18}$$

which under the assumptions above reduces to

$$S_{\theta\theta}(\omega) = \int_{-T/2}^{T/2} R_{\theta\theta}(\alpha)e^{-j\omega\alpha}d\alpha. \tag{19}$$

As discussed earlier, the operation of a parameter tracking system depends on the measurable power in the band pass region of the system being tested. It is of interest then to look at the total average power in $\theta_i(t)$ in the low frequency band

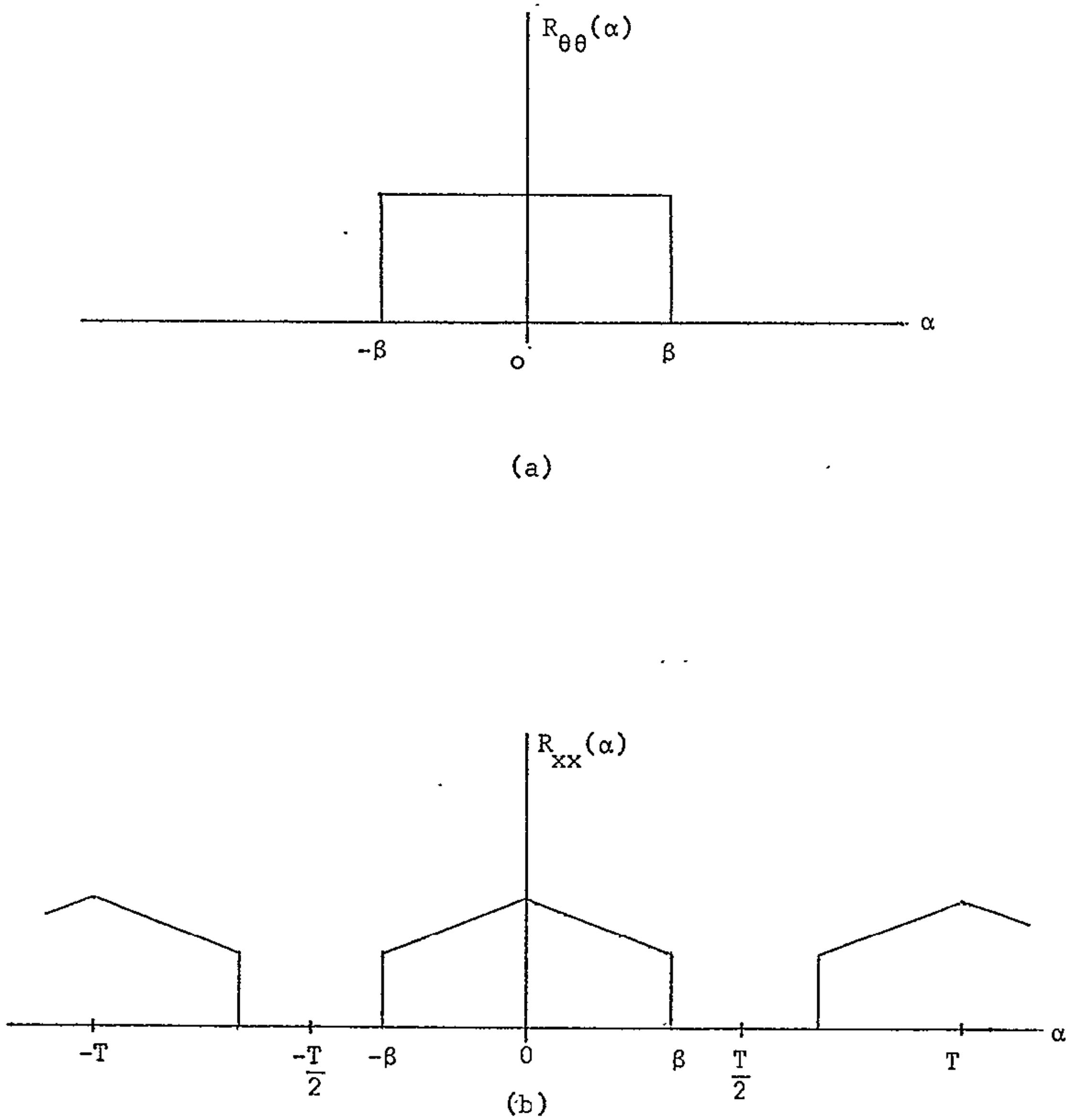


Fig. 6 Theoretical Value of $R_{\theta\theta}(\alpha)$ and the Related $R_{xx}(\alpha)$.

$$\frac{\omega_o}{2} \le \omega \le N\omega_o + \frac{\omega_o}{2} \quad ; \quad N = 1,\ 2,\ 3,\ldots$$

where $\omega_o = \frac{2\pi}{T}$. (Note that this implies that $\frac{\omega_o}{2}$ is at the low end of the band pass region of the system tested. This will be discussed later in the section). This power, $P_\theta$, is defined as

$$P_\theta = \frac{1}{\pi} \int_{\omega_1}^{\omega_2} S_{\theta\theta}(\omega)d\omega$$

$$= \frac{1}{\pi} \int_{\omega_1}^{\omega_2} d\omega \int_{-T/2}^{T/2} R_{\theta\theta}(\alpha)e^{-j\omega\alpha}d\alpha \qquad (21)$$

where

$$\omega_1 = \frac{\omega_o}{2}$$

$$\omega_2 = N\omega_o + \frac{\omega_o}{2} .$$

In the last equation, if the integral with respect to $\omega$ is approximated by Euler's method using $\Delta\omega = \omega_o = \frac{2\pi}{T}$ for the width of the rectangular sections, then

$$P_\theta \cong \frac{1}{\pi} \sum_{n=1}^{N} \left\{ \int_{-T/2}^{T/2} R_{\theta\theta}(\alpha)e^{-jn\omega_o\alpha}d\alpha \right\} \frac{2\pi}{T}$$

$$P_\theta \cong \frac{4}{T} \sum_{n=1}^{N} \int_{o}^{T/2} R_{\theta\theta}(\alpha)\cos n\omega_o\alpha d\alpha \qquad (22)$$

The reason for using the Euler approximation and the particular value for $\Delta\omega$ chosen will be apparent after the calculation of $S_{xx}(\omega)$ in the next step.

To generate an expression for $x(t)$ similar to that for $\theta(t)$ found in Eq. (22), it is first noted that since $x(t)$ is a wide sense stationary periodic random process, its autocorrelation function can be expanded in a Fourier Series.

$$R_{xx}(\alpha) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_o\alpha} \tag{23}$$

where

$$a_n = \frac{1}{T}\int_{-T/2}^{T/2} R_{xx}(\alpha) e^{-jn\omega_o\alpha} d\alpha$$

$$= \frac{2}{T}\int_{o}^{T/2} R_{xx}(\alpha)\cos n\omega_o\alpha d\alpha \tag{24}$$

since $R_{xx}(\alpha)$ is an even function.

It has been shown that for periodic processes of this type, the spectrum is discrete and, for the case being discussed, is given by [3]:

$$S_{xx}(\omega) = 2\pi\sum_{n=-\infty}^{\infty} a_n\delta(\omega-n\omega_o) \tag{25}$$

where $\delta(\omega)$ is the unit impulse function.

The average power in $x(t)$ in the frequency band $\omega_1 \leq \omega \leq \omega_2$, called $P_x$, is

$$P_x = \frac{1}{\pi} \int_{\omega_1}^{\omega_2} S_{xx}(\omega)\ d\omega \tag{26}$$

where $\omega_1$ and $\omega_2$ are defined in Eq. (21). Substituting Eq. (25) into Eq. (26)

$$P_x = \frac{1}{\pi} \int_{\omega_1}^{\omega_2} \left\{ 2\pi \sum_{n=-\infty}^{\infty} a_n \delta(\omega - n\omega_o) \right\} d\omega$$

$$= 2 \sum_{n=1}^{N} a_n$$

$$= 2 \sum_{n=1}^{N} \left\{ \frac{2}{T} \int_{o}^{T/2} R_{xx}(\alpha) \cdot \cos n\omega_o \alpha d\alpha \right\}$$

$$= \frac{4}{T} \sum_{n=1}^{N} \int_{o}^{T/2} R_{xx}(\alpha) \cos n\omega_o \alpha d\alpha \tag{27}$$

However, from Eq. (16), although in general

$$R_{xx}(\alpha) = \frac{T-\alpha}{T} R_{\theta\theta}(\alpha) + \frac{\alpha}{T} R_{\theta\theta}(T-\alpha),\quad 0 \leq \alpha \leq T$$

in the region $0 \leq \alpha \leq \frac{T}{2}$ this reduces to

$$R_{xx}(\alpha) = \frac{T-\alpha}{T} R_{\theta\theta}(\alpha) \tag{28}$$

due to the assumptions that were originally placed on $R_{\theta\theta}(\alpha)$. Therefore,

$$P_x = \frac{4}{T} \sum_{n=1}^{N} \int_{0}^{T/2} (1-\frac{\alpha}{T})\ R_{\theta\theta}(\alpha) \cos n\omega_o \alpha d\alpha \qquad (29)$$

which is, again, the average power present in x(t) in the frequency band

$$\frac{\omega_o}{2} \leq \omega \leq N\omega_o + \frac{\omega_o}{2}$$

and has the same general form as $P_\theta$ given in Equation (22).

The difference in the average powers in $\theta_i(t)$ and x(t) in this frequency band is merely the difference $P_\theta - P_x$. Using Eqs. (22) and (29),

$$P_\theta - P_x = \frac{4}{T^2} \sum_{n=1}^{N} \int_{0}^{T/2} \alpha R_{\theta\theta}(\alpha) \cos n\omega_o \alpha d\alpha \qquad (30)$$

which is the result desired.

From Eq. (30), a figure of merit for how well $P_x$ is approximating $P_\theta$ is $(P_\theta - P_x)/P_\theta$. This ratio will be evaluated for the input signal actually in testing the Hybrid parameter tracker in the next section.

### 3.C.3 $(P_\theta - P_x)/P_\theta$ for a Standard Input Signal

In the Hybrid parameter tracking method discussed in Section II, $\theta_i(t)$ was obtained by passing Gaussian white noise through a 3rd order filter of the form $1/(Bs + 1)^3$. The autocorrelation function, $R_{\theta\theta}(\alpha)$, in this case is

$$R_{\theta\theta}(\alpha) = \frac{[(\frac{\alpha}{B})^2 + 3(\frac{\alpha}{B}) + 3]e^{-\alpha/B}}{16B}, \quad \alpha > 0$$
$$= R_{\theta\theta}(-\alpha) \tag{31}$$

A plot of this function is given in Fig. 7 for $\alpha \geq 0$.

It is seen that at $\alpha = 8B$, $|R_{\theta\theta}(8B)| \cong 0.01\ |R_{\theta\theta}(0)|$ and that $R_{\theta\theta}(\alpha)$ is decreasing monotonically to zero. Thus,

if $$\frac{T}{2} \geq 8B$$

or $$T \geq 16B \tag{32}$$

the assumptions of Section 3.C.2 are valid and Eq. (30) for $P_\theta - P_x$ will be valid.

At this point it should be noted that if $x(t)$ is to have power at low frequencies, some discretion must be used in the choice of T. In particular, if it is desired that some power be in $x(t)$ at frequencies at least as low as 0.5 radians/second, then

$$\omega_o = \frac{2\pi}{T} \leq 0.5$$

or $$T \geq 4\pi \tag{33}$$

which is an additional restriction on T. Combining the constraints of Eqs. (32) and (33) the following general statements on the sample length T can be made:

1. $T \geq 16B$ in order that the low frequency power of $x(t)$ can be compared to $\theta_i(t)$ through Eq. (30).

$$R_{\theta\theta}(\alpha) = \frac{\left[\left(\frac{\alpha}{B}\right)^2 + 3\left(\frac{\alpha}{B}\right) + 3\right] e^{-\frac{\alpha}{B}}}{16B}, \quad \alpha \geq 0$$

$$= R_{\theta\theta}(-\alpha)$$

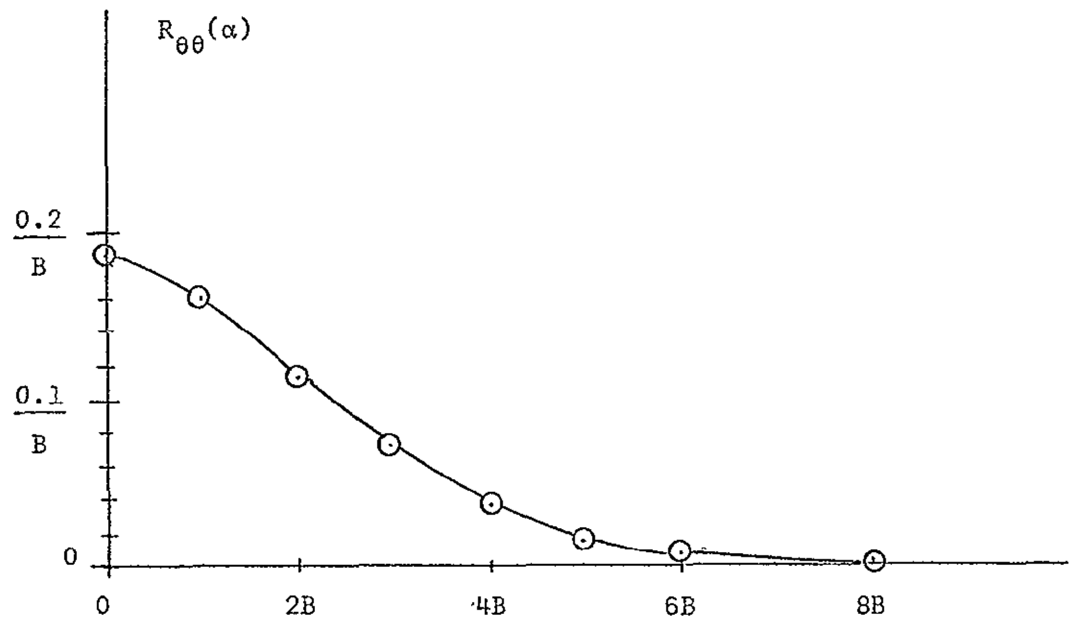


Fig. 7 $R_{\theta\theta}(\alpha)$ for the Input Signal Used to Test the Hybrid Parameter Tracker.

2. $T \geq 4\pi$ so that some low frequency power is present in $x(t)$.

For the tests discussed in Section 3.D $B = \frac{1}{2}$. Therefore,

$$T \geq 16B \text{ or } T \geq 8 \text{ seconds}$$

and

$$T \geq 4\pi \text{ or } T \geq 12.5 \text{ seconds}$$

were the constraints. To insure a sufficiently long T, T = 15 seconds was used in all tests. This can undoubtedly be shortened, and is another area for future research.

Table 2 is a table of $(P_\theta - P_x)/P_\theta$ with T = 15, $\omega_1 = 0.209$ radian/second and $\omega_2$ values up to 8.57 radians/second. It is seen that the total low frequency power in $\theta_i(t)$ and $x(t)$ are quite close and that the convergence of $K(t)$ and $\tau(t)$ in the Hybrid case should be similar to that found in the continuous case. This was indeed found to be true, as will be shown in the next section.

## 3.D Experimental Testing of the Hybrid Parameter Tracker

The convergence properties of the Hybrid parameter tracking system were evaluated in four different ways. First, the general convergence characteristics were checked by having the system determine the parameters of a fixed crossover model whose parameters were known. Second, the Hybrid parameter's convergence rate was compared with the convergence rate of a continuous parameter tracker, to substantiate the derivations of Section 3.C concerning the effect of sample length on the Hybrid parameter tracker's performance. Third,

Table 2. A Comparison of the Average Powers in $\theta_i(t)$ and $x(t)$ in the Low Frequency Region.

| N | $P_\theta$ | $P_\theta - P_x$ | $\frac{P_\theta - P_x}{P_\theta}$ |
|---|---|---|---|
| 1 | .11722 | .00608 | .05186 |
| 2 | .19931 | .00800 | .04013 |
| 3 | .24845 | .00723 | .02910 |
| 4 | .27550 | .00557 | .02021 |
| 5 | .28997 | .00395 | .01362 |
| 6 | .29774 | .00263 | .00883 |
| 7 | .30200 | .00101 | .00533 |
| 8 | .30442 | .00083 | .00272 |
| 9 | .30583 | .00022 | .00071 |
| 10 | .30668 | -.00025 | -.00081 |
| 11 | .30721 | -.00063 | -.00205 |
| 12 | .30755 | -.00095 | -.00308 |
| 13 | .30777 | -.00121 | -.00393 |
| 14 | .30793 | -.00143 | -.00464 |
| 15 | .30804 | -.00162 | -.00525 |
| 16 | .30812 | -.00179 | -.00580 |
| 17 | .30820 | -.00193 | -.00626 |
| 18 | .30827 | -.00206 | -.00668 |
| 19 | .30834 | -.00218 | -.00707 |
| 20 | .30843 | -.00228 | -.00739 |

$$P_\theta = \int_{\omega_1}^{\omega_2} S_{\theta\theta}(\omega)\, d\omega \qquad P_x = \int_{\omega_1}^{\omega_2} S_{xx}(\omega)\, d\omega \qquad \omega_1 = \frac{\omega_o}{2} = \frac{\pi}{15}, \quad \omega_2 = \frac{\omega_o}{2} + N\omega_o$$

the Hybrid parameter tracker was used to determine the parameters of an actual compensatory system. Fourth, the parameter estimates determined by the Hybrid parameter tracker were cross-checked against those estimates determined by the modified stachastic approximation method (to be discussed in Section IV). This was accomplished by having the two methods use exactly the same input-output data from an actual compensatory control system, and comparing the parameter values determined by each method. Sample results from the first three of these tests will be presented here. The comparison of the results of the two methods will be discussed in Section 4.D.

In all these tests, $\theta_i(t)$ was obtained by passing gaussian pseudo-white noise through a $1/(0.5s+1)^3$ filter. The sampling rate of $\theta_i(t)$ and $\theta_o(t)$ was 20 samples per second, and the sample length, T, was 15 seconds.

### 3.D.1 Convergence of the Hybrid Parameter Tracker on a Known Crossover Model

Tables 3 through 6 and Figure 8 present typical results as the system converges on a known crossover model. Several initial conditions on K(t) and $\tau(t)$ are used and various values for the known model are tried. In all cases tested, the parameter adjustment potentimeters P03 and P95 of Figure A4 in Appendix A, were both set at 0.600. This allowed parameter convergence rates which were usually exponential in character, rather than damped oscillations. In all cases convergence was quite good. In the cases tested, the $K_o$ and $\tau_o$ estimates were

Table 3 Convergence of the Hybrid Parameter Tracker on a Known Crossover Model with $K_o$=4.0 and $\tau_o$=0.210.

| n | Gain Estimate $\overline{K}_n$ | Time Delay Estimate $\overline{\tau}_n$ |
|---|---|---|
| | Initial K = 1.985 | Initial $\tau$ = 0.305 |
| 1 | KBAR(1,1)= 3.178 | TBAR(1,1)=0.250 |
| 2 | KBAR(1,2)= 3.497 | TBAR(1,2)=0.232 |
| 3 | KBAR(1,3)= 3.660 | TBAR(1,3)=0.224 |
| 4 | KBAR(1,4)= 3.757 | TBAR(1,4)=0.220 |
| 5 | KBAR(1,5)= 3.822 | TBAR(1,5)=0.218 |
| 6 | KBAR(2,1)= 3.918 | TBAR(2,1)=0.215 |
| 7 | KBAR(2,2)= 3.956 | TBAR(2,2)=0.213 |
| 8 | KBAR(2,3)= 3.980 | TBAR(2,3)=0.212 |
| 9 | KBAR(2,4)= 3.995 | TBAR(2,4)=0.212 |
| 10 | KBAR(2,5)= 4.007 | TBAR(2,5)=0.212 |
| 11 | KBAR(3,1)= 4.022 | TBAR(3,1)=0.211 |
| 12 | KBAR(3,2)= 4.030 | TBAR(3,2)=0.211 |
| 13 | KBAR(3,3)= 4.038 | TBAR(3,3)=0.211 |
| 14 | KBAR(3,4)= 4.043 | TBAR(3,4)=0.211 |
| 15 | KBAR(3,5)= 4.045 | TBAR(3,5)=0.211 |
| 16 | KBAR(4,1)= 4.051 | TBAR(4,1)=0.212 |
| 17 | KBAR(4,2)= 4.048 | TBAR(4,2)=0.211 |
| 18 | KBAR(4,3)= 4.047 | TBAR(4,3)=0.211 |
| 19 | KBAR(4,4)= 4.047 | TBAR(4,4)=0.211 |
| 20 | KBAR(4,5)= 4.047 | TBAR(4,5)=0.211 |

Table 4 Convergence of the Hybrid Parameter Tracker on a Known Crossover Model with $K_o$=3.0 and $\tau_o$=0.300.

| n | Gain Estimate $\overline{K}_n$ | Time Delay Estimate $\overline{\tau}_n$ |
|---|---|---|
| | Initial K = 1.985 | Initial $\tau$ = 0.305 |
| 1 | KBAR(1,1)= 3.459 | TBAR(1,1)=0.291 |
| 2 | KBAR(1,2)= 3.132 | TBAR(1,2)=0.295 |
| 3 | KBAR(1,3)= 3.053 | TBAR(1,3)=0.299 |
| 4 | KBAR(1,4)= 3.032 | TBAR(1,4)=0.301 |
| 5 | KBAR(1,5)= 3.027 | TBAR(1,5)=0.302 |
| 6 | KBAR(2,1)= 3.025 | TBAR(2,1)=0.304 |
| 7 | KBAR(2,2)= 3.024 | TBAR(2,2)=0.305 |
| 8 | KBAR(2,3)= 3.025 | TBAR(2,3)=0.305 |
| 9 | KBAR(2,4)= 3.024 | TBAR(2,4)=0.305 |
| 10 | KBAR(2,5)= 3.025 | TBAR(2,5)=0.306 |
| 11 | KBAR(3,1)= 3.025 | TBAR(3,1)=0.306 |
| 12 | KBAR(3,2)= 3.024 | TBAR(3,2)=0.306 |
| 13 | KBAR(3,3)= 3.025 | TBAR(3,3)=0.306 |
| 14 | KBAR(3,4)= 3.023 | TBAR(3,4)=0.306 |
| 15 | KBAR(3,5)= 3.024 | TBAR(3,5)=0.306 |
| 16 | KBAR(4,1)= 3.025 | TBAR(4,1)=0.306 |
| 17 | KBAR(4,2)= 3.024 | TBAR(4,2)=0.306 |
| 18 | KBAR(4,3)= 3.023 | TBAR(4,3)=0.306 |
| 19 | KBAR(4,4)= 3.023 | TBAR(4,4)=0.306 |
| 20 | KBAR(4,5)= 3.024 | TBAR(4,5)=0.306 |

Table 5 Convergence of the Hybrid Parameter Tracker on a Known Crossover Model with $K_o$=5.0 and $\tau_o$=0.250.

| n | Gain Estimate $\overline{K}_n$ | Time Delay Estimate $\overline{\tau}_n$ |
|---|---|---|
| | Initial K = 1.985 | Initial $\tau$ = 0.305 |
| 1 | KBAR(1,1)= 3.759 | TBAR(1,1)=0.260 |
| 2 | KBAR(1,2)= 4.107 | TBAR(1,2)=0.247 |
| 3 | KBAR(1,3)= 4.306 | TBAR(1,3)=0.243 |
| 4 | KBAR(1,4)= 4.437 | TBAR(1,4)=0.242 |
| 5 | KBAR(1,5)= 4.533 | TBAR(1,5)=0.242 |
| 6 | KBAR(2,1)= 4.697 | TBAR(2,1)=0.244 |
| 7 | KBAR(2,2)= 4.775 | TBAR(2,2)=0.245 |
| 8 | KBAR(2,3)= 4.821 | TBAR(2,3)=0.246 |
| 9 | KBAR(2,4)= 4.852 | TBAR(2,4)=0.246 |
| 10 | KBAR(2,5)= 4.878 | TBAR(2,5)=0.246 |
| 11 | KBAR(3,1)= 4.918 | TBAR(3,1)=0.247 |
| 12 | KBAR(3,2)= 4.938 | TBAR(3,2)=0.247 |
| 13 | KBAR(3,3)= 4.948 | TBAR(3,3)=0.247 |
| 14 | KBAR(3,4)= 4.957 | TBAR(3,4)=0.247 |
| 15 | KBAR(3,5)= 4.960 | TBAR(3,5)=0.247 |
| 16 | KBAR(4,1)= 4.970 | TBAR(4,1)=0.247 |
| 17 | KBAR(4,2)= 4.975 | TBAR(4,2)=0.247 |
| 18 | KBAR(4,3)= 4.979 | TBAR(4,3)=0.247 |
| 19 | KBAR(4,4)= 4.979 | TBAR(4,4)=0.247 |
| 20 | KBAR(4,5)= 4.981 | TBAR(4,5)=0.247 |

Table 6 Convergence of the Hybrid Parameter Tracker on a Known Crossover Model with $K_o$=5.0 and $\tau_o$=0.210.

| n | Gain Estimate $\overline{K}_n$ | Time Delay Estimate $\overline{\tau}_n$ |
|---|---|---|
| | Initial K = 1.985 | Initial $\tau$ = 0.305 |
| 1 | KBAR(1,1)= 2.713 | TBAR(1,1)=0.262 |
| 2 | KBAR(1,2)= 3.090 | TBAR(1,2)=0.238 |
| 3 | KBAR(1,3)= 3.349 | TBAR(1,3)=0.222 |
| 4 | KBAR(1,4)= 3.543 | TBAR(1,4)=0.213 |
| 5 | KBAR(1,5)= 3.700 | TBAR(1,5)=0.208 |
| 6 | KBAR(2,1)= 3.981 | TBAR(2,1)=0.201 |
| 7 | KBAR(2,2)= 4.145 | TBAR(2,2)=0.199 |
| 8 | KBAR(2,3)= 4.265 | TBAR(2,3)=0.200 |
| 9 | KBAR(2,4)= 4.354 | TBAR(2,4)=0.200 |
| 10 | KBAR(2,5)= 4.431 | TBAR(2,5)=0.201 |
| 11 | KBAR(3,1)= 4.574 | TBAR(3,1)=0.203 |
| 12 | KBAR(3,2)= 4.668 | TBAR(3,2)=0.205 |
| 13 | KBAR(3,3)= 4.735 | TBAR(3,3)=0.207 |
| 14 | KBAR(3,4)= 4.784 | TBAR(3,4)=0.207 |
| 15 | KBAR(3,5)= 4.829 | TBAR(3,5)=0.208 |
| 16 | KBAR(4,1)= 4.907 | TBAR(4,1)=0.210 |
| 17 | KBAR(4,2)= 4.953 | TBAR(4,2)=0.210 |
| 18 | KBAR(4,3)= 4.988 | TBAR(4,3)=0.211 |
| 19 | KBAR(4,4)= 5.020 | TBAR(4,4)=0.211 |
| 20 | KBAR(4,5)= 5.044 | TBAR(4,5)=0.212 |

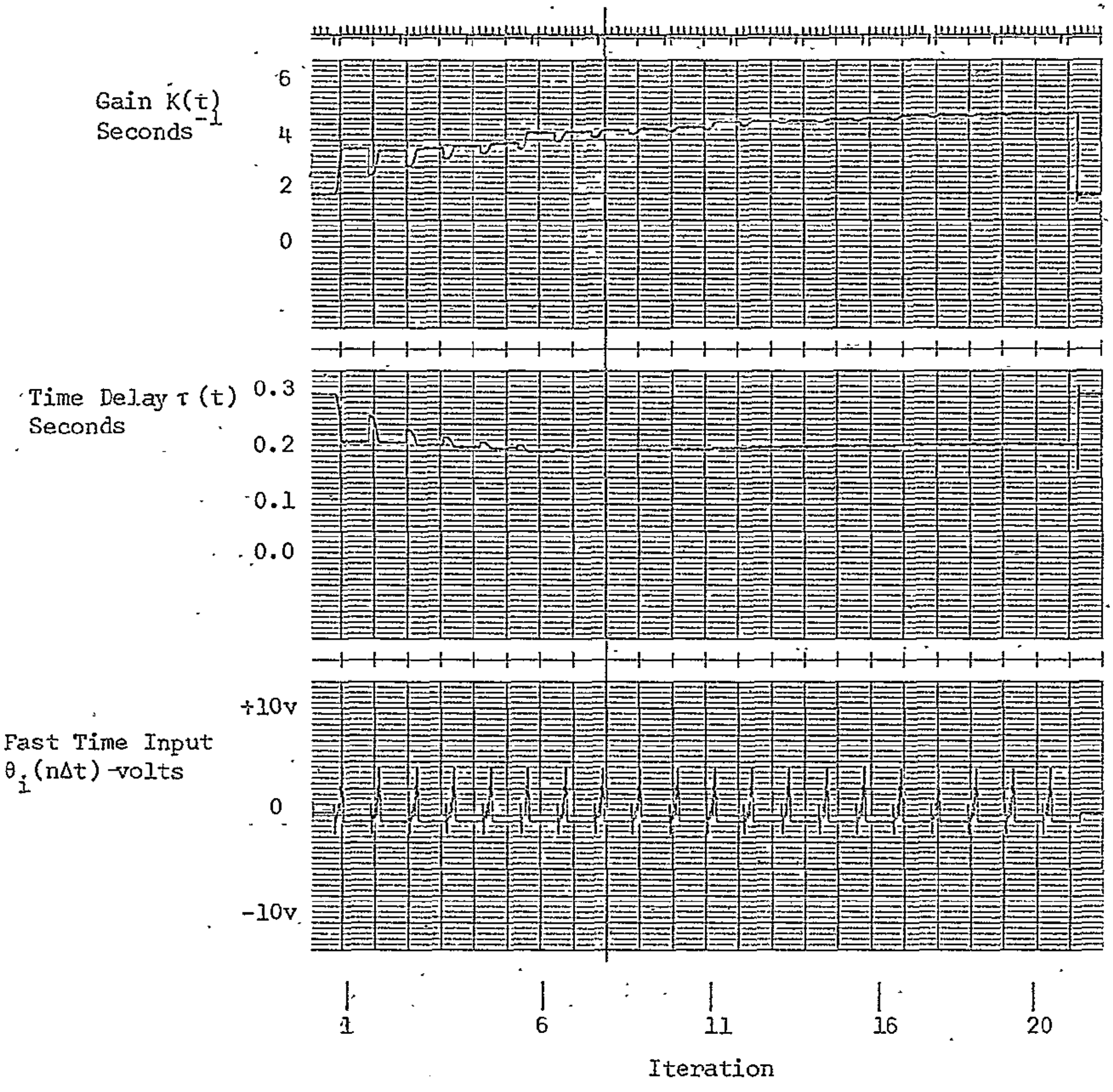


Fig. 8 Analog Parameter Values Present During the Identification Run Given in Table 6.

found to converge to within two percent of the true $K_o$ and $\tau_o$ values in the 20 iterations allowed.

### 3.D.2 A Comparison of the Convergence of the Hybrid and Continuous Parameter Tracking Systems

The analysis of Section III-C indicated that with: (a) equivalent parameter adjustment gains; (b) input signals obtained by passing white noise through a $1/(0.5s+1)^3$ filter, and (c) a sampling length of 15 seconds on the Hybrid parameter tracker, the convergence rates of continuous and Hybrid parameter trackers should be comparable. To substantiate this result, the parameter adjustment gains of the analog portion of the system were set at the values which produced a relatively slow convergence rate. The system was first allowed to run as an all analog system, and then again in the Hybrid mode. Both times the systems identified a known crossover model with $K_o = 4.0$ and $\tau_o = 0.21$. The results are given in Figures 9 and 10.

It is seen that in the continuous case K and $\tau$ converge in an exponential manner with time constants of about 10 seconds. In the Hybrid case, the convergence is also basically exponential, as predicted by Section 3.C. Sixty percent of convergence is achieved here in approximately one iteration. This one iteration has an analog OPERATE time of one second which due to the time scale factor of 10, is equivalent to ten seconds of original data. Parameter convergence after the first iteration is somewhat slower than would be found in a true exponential convergence. This is as expected though,

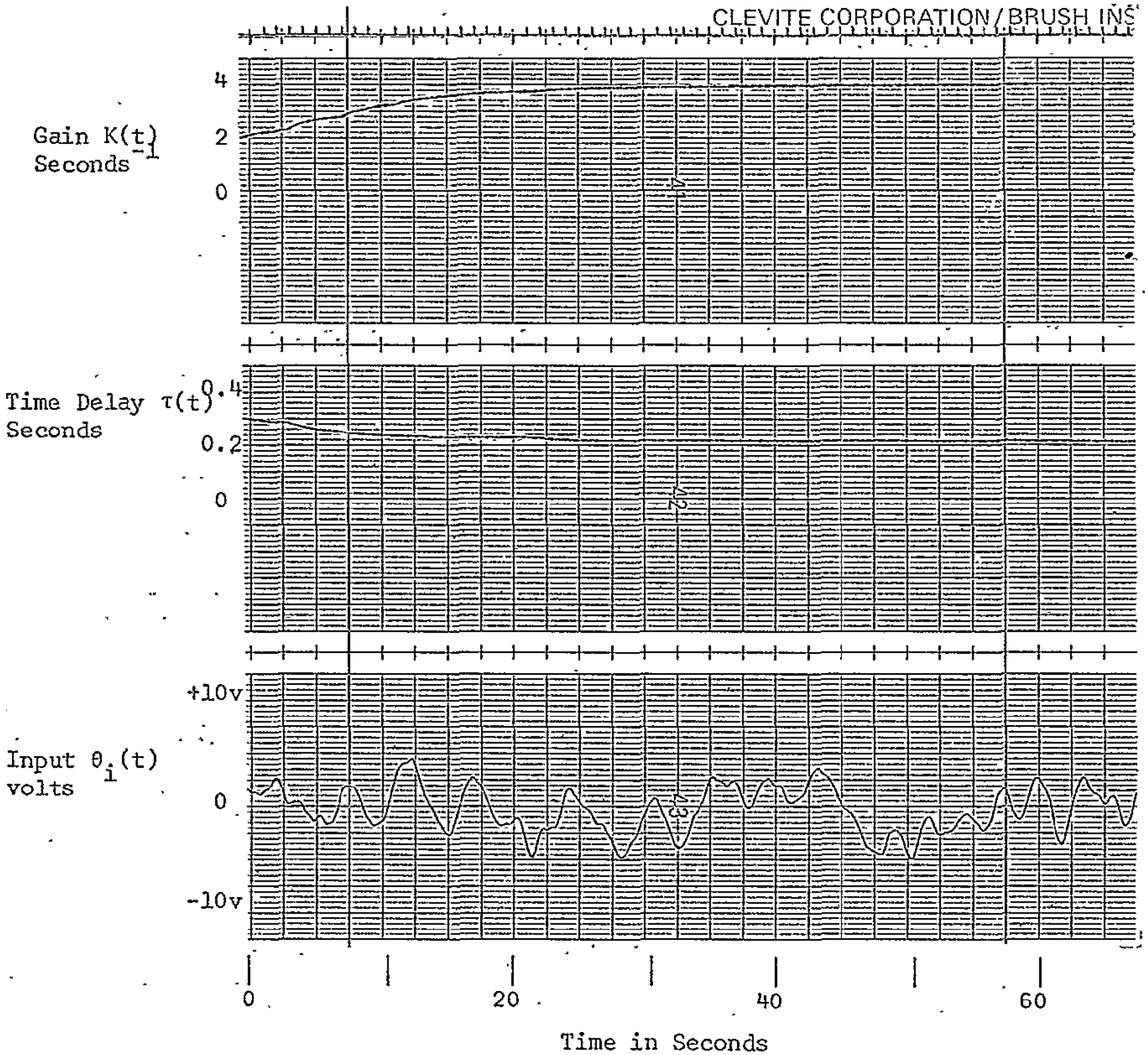


Fig. 9 Continuous Parameter Tracker Converging on a Know Crossover Model with $K_o$=4.0 and $\tau_o$=0.210.

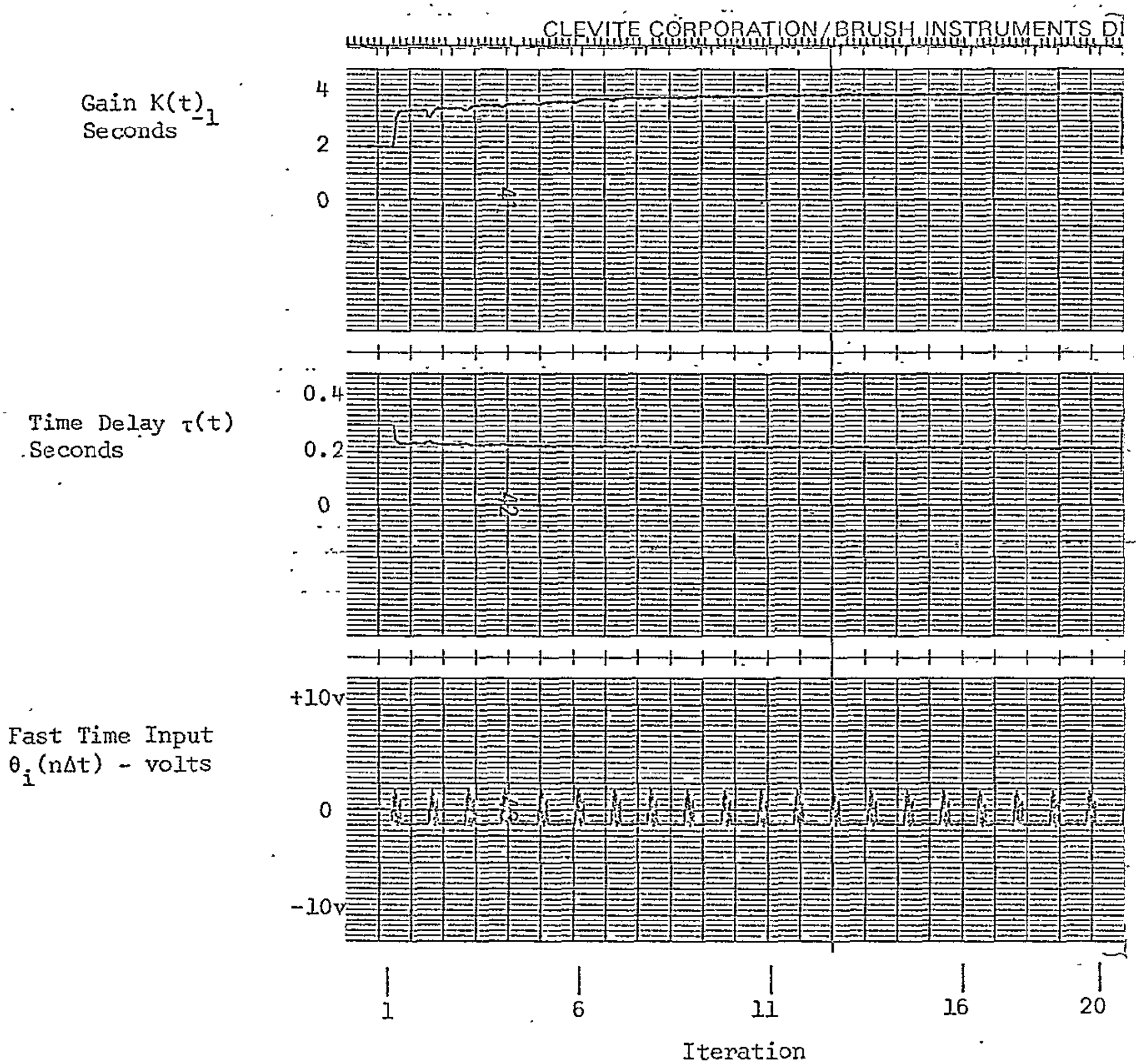


Fig. 10 Hybrid Parameter Tracker Converging on a Known Crossover Model with $K_o$=4.0 and $\tau_o$=0.210.

since the parameter adjustment gains were not fixed, but were cycled up and down, and had average values which were less than those found in the continuous case. The convergence rates are thus quite alike when T = 15 seconds as was predicted in Section 3.C.

It should be noted that the convergence time constants for $K(t)$ and $\tau(t)$ vary from run-to-run in both the continuous and Hybrid parameter tracking systems. This is due to the random nature of the forcing function which determines convergence. However repeated trials of the same experiment indicated that the two methods do have very comparable properties with T = 15 seconds, even when the parameter adjustment gains are increased.

3.D.3 <u>Convergence on an Actual Compensatory System</u>

To check the convergence of the Hybrid parameter tracker on an actual system, the Hybrid parameter tracker was used to determine the gain and time-delay parameters of a human subject in a conventional compensatory control system. The compensatory system used a horizontal arm movement control stick which had negligible inertia and a light spring restoring force. The subject was a senior male engineering student with a limited amount of compensatory tracking experience. The plant was a single integrator.

The Hybrid parameter tracking system was found to be stable and achieved convergence within the 20 iterations allowed in all cases tested. Table 7 and Fig. 11 are typical results obtained.

Table 7 Convergence of the Hybrid Parameter Tracker on an Actual Compensatory System.

| n | Gain Estimate $\overline{K}_n$ | Time Delay Estimate $\overline{\tau}_n$ |
|---|---|---|
| | Initial K = 1.985 | Initial $\tau$ = 0.305 |
| 1 | KBAR(1,1)= 3.697 | TBAR(1,1)=0.228 |
| 2 | KBAR(1,2)= 4.000 | TBAR(1,2)=0.207 |
| 3 | KBAR(1,3)= 4.152 | TBAR(1,3)=0.198 |
| 4 | KBAR(1,4)= 4.251 | TBAR(1,4)=0.193 |
| 5 | KBAR(1,5)= 4.323 | TBAR(1,5)=0.190 |
| 6 | KBAR(2,1)= 4.464 | TBAR(2,1)=0.183 |
| 7 | KBAR(2,2)= 4.529 | TBAR(2,2)=0.182 |
| 8 | KBAR(2,3)= 4.571 | TBAR(2,3)=0.181 |
| 9 | KBAR(2,4)= 4.601 | TBAR(2,4)=0.181 |
| 10 | KBAR(2,5)= 4.624 | TBAR(2,5)=0.181 |
| 11 | KBAR(3,1)= 4.675 | TBAR(3,1)=0.179 |
| 12 | KBAR(3,2)= 4.700 | TBAR(3,2)=0.179 |
| 13 | KBAR(3,3)= 4.715 | TBAR(3,3)=0.179 |
| 14 | KBAR(3,4)= 4.727 | TBAR(3,4)=0.180 |
| 15 | KBAR(3,5)= 4.735 | TBAR(3,5)=0.180 |
| 16 | KBAR(4,1)= 4.760 | TBAR(4,1)=0.179 |
| 17 | KBAR(4,2)= 4.771 | TBAR(4,2)=0.179 |
| 18 | KBAR(4,3)= 4.777 | TBAR(4,3)=0.179 |
| 19 | KBAR(4,4)= 4.782 | TBAR(4,4)=0.179 |
| 20 | KBAR(4,5)= 4.786 | TBAR(4,5)=0.179 |

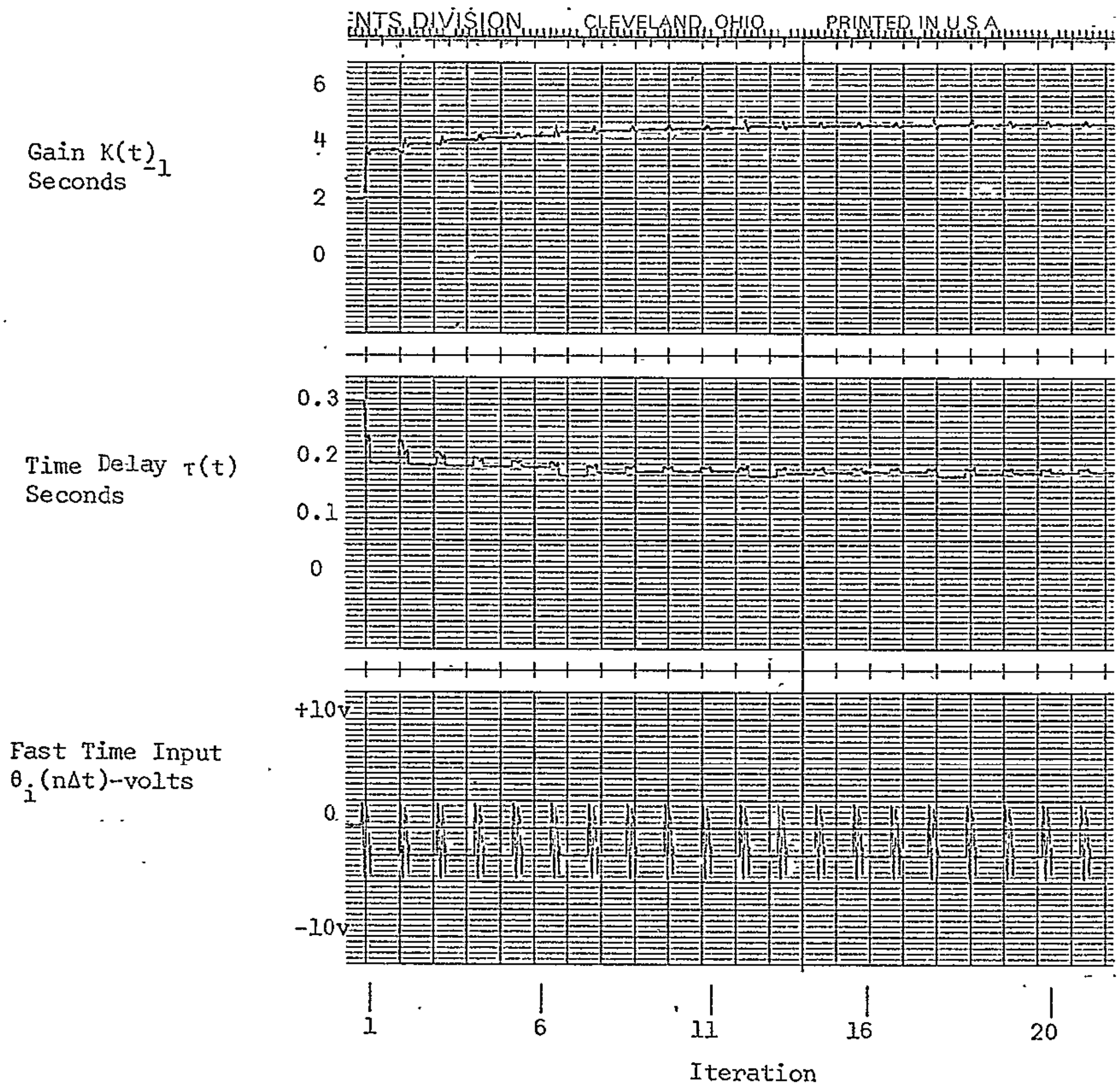


Fig. 11 Analog Parameter Values Present During the Identification Run Given in Table 7.

# IV. A MODIFIED STOCHASTIC APPROXIMATION METHOD OF PARAMETER IDENTIFICATION

The second Hybrid parameter identification method developed and tested is a modification of the stochastic approximation method proposed by Bekey and Neal at the 5th Annual Manual Control Conference [12,16]. This is an iterative, discrete step size, gradient approach that requires only short lengths of data from the input and output of the compensatory control system being identified.

## 4.A The Modified Stochastic Approximation Method

To start the identification process samples of $\theta_i(t)$ and $\theta_o(t)$ are taken from the compensatory system being identified at a rate of 20 samples per second for a period of T seconds. (The minimum value of T is again in the neighborhood of 8-15 seconds. Fifteen seconds was used for all work discussed in this report). The samples are taken through the A-D converters and stored on the IBM 1130. During the actual identification process the stored values of $\theta_i(t)$ and $\theta_o(t)$ are fed via D-A converters into a fast-time analog system. The $\theta_i(t)$ samples are used as the inputs into two closed loop crossover models, while the $\theta_o(t)$ samples are subtracted from the crossover model outputs to give two model matching error signals. Ideally, four analog models should be used, but the equipment complement of the EAI-680 used would not allow this. Using two models instead of four merely doubles the number of analog runs needed in the identification process.

To explain the basic identification process, it is best to discuss what goes on during the $n^{th}$ iteration on one set of input-output data. The $n^{th}$ iteration requires two analog runs (since only two analog models were used). During both analog runs of the $n^{th}$ iteration, the estimates of the gain and time delay parameters of the crossover model are $K_n$ and $\tau_n$. During the first analog run of the $n^{th}$ iteration, $\tau = \tau_n$ on both crossover models, while $K = K_n + c_n$ on model number one and $K = K_n - c_n$ on model number two. $c_n$ is a parameter which is defined in Eqs. (37) and (38) below. During the second analog run of the $n^{th}$ iteration, $K = K_n$ on both models while $\tau = \tau_n + d_n$ on model number one and $\tau = \tau_n - d_n$ on model number two. $d_n$ is a parameter also defined in Eqs. (37) and (38) below. Thus, in these two analog runs, the crossover model is run with four different sets of parameter values. These values are symmetrically placed about the point $K_n$, $\tau_n$ in the K-$\tau$ plane as shown in Figure 12 below.

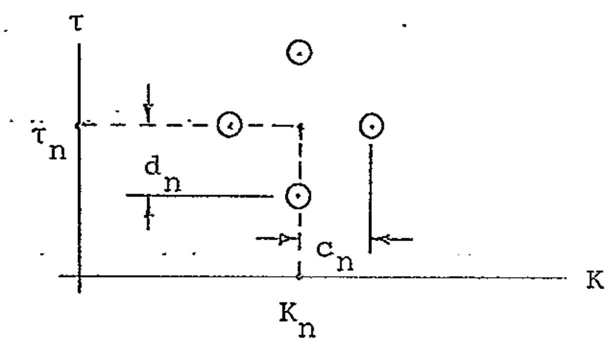


Figure 12. Crossover Model Parameter Values for $n^{th}$ Iteration.

While the two analog runs of the $n^{th}$ iteration are being made, four performance indices are calculated

$$\left.\begin{aligned} J_{n1} &= \int_{\frac{T}{3}}^{T} e_1^2(t, K_n + c_n, \tau_n)\,dt \\ J_{n2} &= \int_{\frac{T}{3}}^{T} e_2^2(t, K_n - c_n, \tau_n)\,dt \end{aligned}\right\} \begin{array}{l}\text{Calculated during}\\ \text{first analog run}\end{array} \tag{34}$$

$$\left.\begin{aligned} J_{n3} &= \int_{\frac{T}{3}}^{T} e_1^2(t, K_n, \tau_n + d_n)\,dt \\ J_{n4} &= \int_{\frac{T}{3}}^{T} e_2^2(t, K_n, \tau_n - d_n)\,dt \end{aligned}\right\} \begin{array}{l}\text{Calculated during}\\ \text{second analog run}\end{array} \tag{35}$$

$e_i$ is the model matching error of the $i^{th}$ crossover model. (See Figure 13). In Eqs. (34) and (35) the lower limit on the integral is $\frac{T}{3}$. This allows all initial transients in the analog models to die out before the performance indices are calculated.

Using these indices, new estimates of the gain and time delay parameters of the compensatory system being identified are made using the following modified stochastic approximation algorithm:

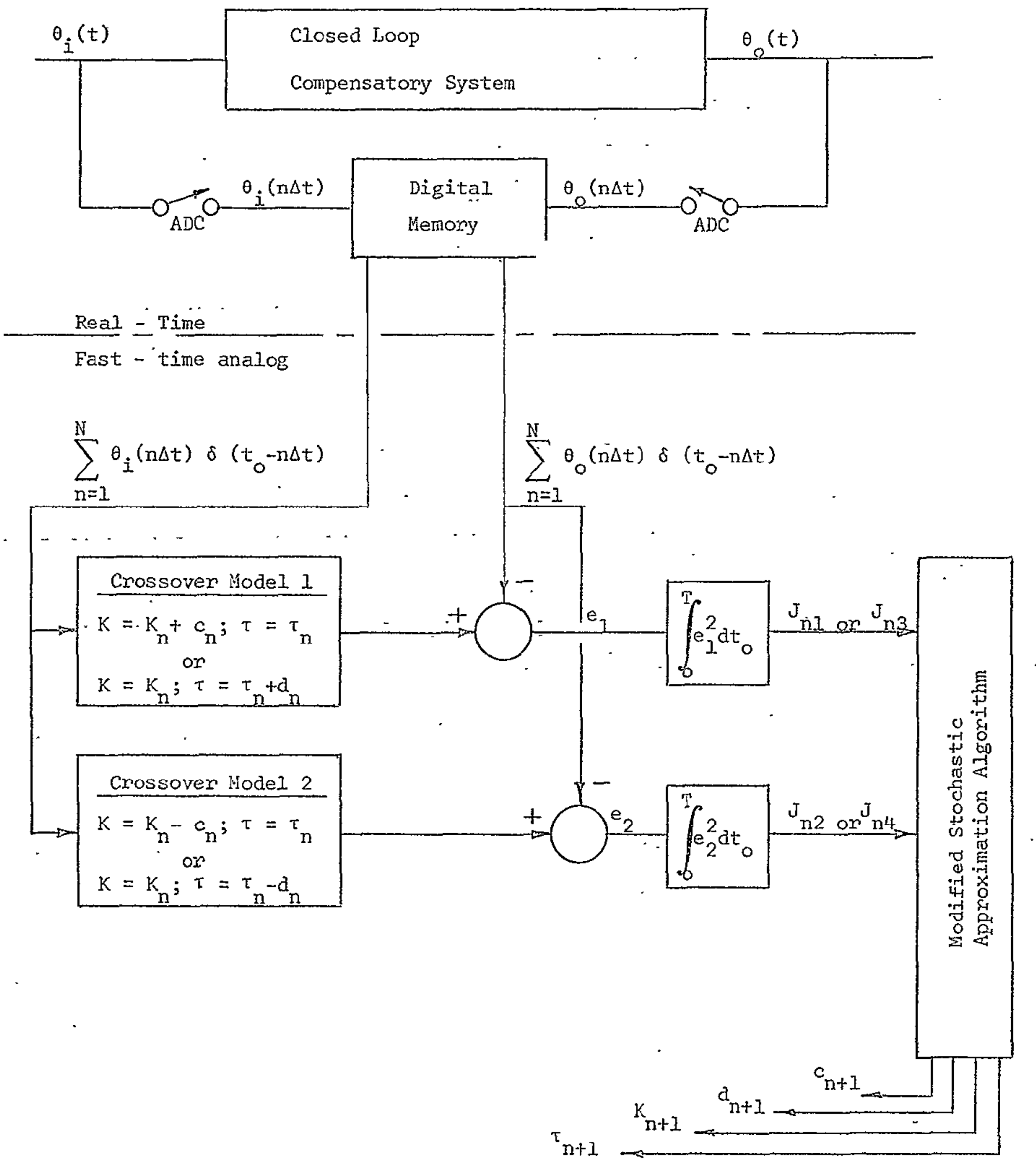


Fig. 13 Block Diagram of the Modified Stochastic Approximation Method.

$$K_{n+1} = K_n + a_n(J_{n2} - J_{n1})/c_n$$
$$\tau_{n+1} = \tau_n + b_n(J_{n4} - J_{n3})/d_n \qquad (36)$$

where

$$a_n = \frac{A}{n},\ b_n = \frac{B}{n},\ c_n = \frac{C}{\sqrt{n}} \text{ and } d_n = \frac{D}{\sqrt{n}}\ ,\ n \leq 10 \qquad (37)$$

and

$$a_n = a_{10},\ b_n = b_{10},\ c_n = c_{10} \text{ and } d_n = d_{10},\ n > 10 \qquad (38)$$

A, B, C and D are positive constants which are emperically determined. For the work discussed in this report the maximum values of $c_n$ and $d_n$ were 0.3 and 0.03, while $c_{10} = \frac{0.3}{\sqrt{10}}$ and $d_{10} = \frac{0.03}{\sqrt{10}}$. These were found to work quite well with the system tested. As n increases, $K_n \to K_o$ and $\tau_n \to \tau_o$, the true values of the gain and crossover model parameters.

A block diagram of the system discussed above is given in Figure 13 and a time-logic outline of the method is found in Table 8. A complete FORTRAN program for the system and the analog circuitry can be found in Appendix B.

The method outlined above would be the usual Keifer-Wolfowitz algorithm if Eq. (38) were eliminated and Eq. (37) used for all n. The modification was made because some problems were encountered with $c_n$ and $d_n$ getting too small before convergence was achieved. When extremely small values for $c_n$ and $d_n$ are used, convergence cannot be obtained because

Table 8. Time-Logic Outline for One Modified Stochastic Approximation Sequence.

| Time t | Operating Status |
|---|---|
| <0 | 1. Compensatory system to be identified is in operation. |
| | 2. Analog portion of the identification system is in IC mode. |
| 0 | 1. Sampling of $\theta_i(t)$ and $\theta_o(t)$ begins at a rate of 20 samples/second using A-D converters. Samples are taken for 15 seconds and stored in the IBM 1130. |
| | 2. Analog system stays in IC mode. |
| T | 1. Sampling stops. |
| $T+t_o$ | 1. First analog run of the first iteration begins with $K = K_1$ and $\tau = \tau_1$ as first estimates of $K_o$ and $\tau_o$. |
| | 2. $K = K_1 + c_1$ and $\tau = \tau_1$ on crossover model 1. $K = K_1 - c_1$ and $\tau = \tau_1$ on crossover model 2. |
| | 3. Crossover models are placed into OPERATE with a time scale of 10 times normal,* except for performance index integrators which are left in IC mode. |
| | 4. $\theta_i(t)$ and $\theta_o(t)$ are fed into the analog circuit via DAC's at 10 times the sample rate.* |
| $T+t_1$ | 1. $t_1 - t_o = 0.5$ seconds $= 1/3 \times T/10$. This time is allowed for initial analog transients to die out. |
| | 2. Performance integrators (90) and (95) in Figure <u>B5</u>, are placed into OPERATE. |
| $T+t_2$ | 1. $t_2 - t_o = 1.5$ seconds $= T/10$. |
| | 2. Analog system is placed into HOLD. $J_{11}$ and $J_{12}$, the performance index values for this run are read and stored on the IBM 1130. |
| | 3. Analog system is placed into IC with $K = K_1$ and $\tau = \tau_1 + d_1$ on crossover model 1 and $K = K_1$ and $\tau = \tau_1 - d_1$ on crossover model 2. |

Table 8 (continued)

| Time t | Operating Status |
|---|---|
| $T+t_3$ | 1. Second analog run of the first iteration begins with the crossover models being placed into OPERATE with a time scale 10 times normal. The performance index integrators remain in IC. |
| | 2. $\theta_i(t)$ and $\theta_o(t)$ are fed into the analog circuit via DAC's at 10 times the sample rate.* |
| $T+t_4$ | 1. $t_4 - t_3 = 0.5$ seconds $= 1/3 \times T/10$. |
| | 2. Performance integrators (90) and (95) in Figure B5 are placed into OPERATE. |
| $T+t_5$ | 1. $t_5 - t_3 = 1.5$ seconds $= T/10$. |
| | 2. Analog system is placed into HOLD. $J_{13}$ and $J_{14}$, the performance index values for this run are read and stored on the IBM 1130. |
| | 3. $K_2$ and $\tau_2$ are determined using $K_1$, $\tau_1$, $J_{11}$, $J_{12}$, $J_{13}$, $J_{14}$, and Eqs. (34) and (35). |
| | 4. $K_2$ and $\tau_2$ are typed on typewriter. |
| | 5. Analog system is placed in IC mode with $K = K_2 + d_2$ and $\tau = \tau_2$ on crossover model 1, and $K = K_2 - d_2$ and $\tau = \tau_2$ on crossover model 2. |
| $T + t_6$ | System is ready for the second iteration. Go back to $T + t_o$, part 3. |
| | *The method was limited to a fast-time operation of 10 times real-time due to the use of FORTRAN programming. This can be speeded up considerably through the use of Assembly language. |

0

the analog crossover models become insensitive to the very small parameter changes encountered. $c_n$ and $d_n$ were thus limited to minimum values of approximately 0.01 and 0.001, respectively. Good results were achieved when this modification vas added. This method further differs from the Bekey-Neal work in that $c_n$ and $d_n$ for $n \leq 10$ are reduced by a factor of $1/\sqrt{n}$, rather than $1/n^{1/6}$. Better convergence was obtained with the values used.

## 4.B Identification of the Parameters of a Known Crossover Model

To evaluate the convergence characteristics of the modified stochastic approximation method, the method was first used to identify a fixed crossover model with known parameters $K_o$ and $\tau_o$. The input signal used to test the system was gaussian pseudo-white noise passed through a $1/(0.5\ s + 1)^3$ filter.

It was found that accurate convergence was obtained for all values of $K_o$ and $\tau_o$ in the normal range of human operator parameter values. Tables 9 and 10 give the $K_o$ and $\tau_o$ estimates for several values of $K_o$ and $\tau_o$, and several initial K and $\tau$ values. It is seen that, as the number of iterations increases on a given test, the estimates converge to the correct values. Although accurate convergence is achieved in each case, the number of iterations required for complete convergence varies from run-to-run. This difference is undoubtedly due to the run-to-run variation in the noise signal used for $\theta_i(t)$. In most cases tested, convergence was achieved within 25 iterations.

## 4.C Identification of the $K_o$, $\tau_o$ Parameters of a Human Operator

To test the operation of the modified stochastic approximation method on a real system, a six day compensatory tracking task test of a previously untrained subject was conducted. An arm movement control stick and test situation exactly like that discussed earlier was used. The plant controlled was a single integrator, and the input signal, $\theta_i(t)$, was generated by pass-

Table 9 Two Examples of the Convergence of the Modified Stochastic Approximation Method on a Known Crossover with $K_o$=4.0 and $\tau_o$=0.300.

| n | Time Delay Estimate | Gain Estimate |
|---|---|---|
| 1. | T= 0.1700 | K= 9.9999 |
| 2. | T= 0.1602 | K= 6.4872 |
| 3. | T= 0.1577 | K= 6.0111 |
| 4. | T= 0.1602 | K= 5.5351 |
| 5. | T= 0.1651 | K= 5.1322 |
| 6. | T= 0.1748 | K= 4.7294 |
| 7. | T= 0.1919 | K= 4.3632 |
| 8. | T= 0.2090 | K= 4.1434 |
| 9. | T= 0.2285 | K= 3.9603 |
| 10. | T= 0.2456 | K= 3.9603 |
| 11. | T= 0.2603 | K= 3.9603 |
| 12. | T= 0.2701 | K= 3.9969 |
| 13. | T= 0.2823 | K= 3.9237 |
| 14. | T= 0.2896 | K= 3.9969 |
| 15. | T= 0.2969 | K= 3.9603 |
| 16. | T= 0.2969 | K= 3.9969 |
| 17. | T= 0.2969 | K= 3.9603 |
| 18. | T= 0.2969 | K= 3.9603 |
| 19. | T= 0.2993 | K= 3.9603 |
| 20. | T= 0.2993 | K= 3.9237 |
| 21. | T= 0.2969 | K= 3.9237 |
| 22. | T= 0.2993 | K= 3.9969 |
| 23. | T= 0.2993 | K= 3.9603 |
| 24. | T= 0.3018 | K= 3.9969 |

| | | |
|---|---|---|
| 1. | T= 0.2000 | K= 5.0000 |
| 2. | T= 0.2097 | K= 4.5971 |
| 3. | T= 0.2268 | K= 4.2309 |
| 4. | T= 0.2415 | K= 4.0478 |
| 5. | T= 0.2561 | K= 3.9745 |
| 6. | T= 0.2732 | K= 3.9379 |
| 7. | T= 0.2830 | K= 3.9745 |
| 8. | T= 0.2903 | K= 3.9745 |
| 9. | T= 0.2927 | K= 3.9745 |
| 10. | T= 0.2927 | K= 3.9745 |
| 11. | T= 0.2952 | K= 3.9379 |
| 12. | T= 0.2952 | K= 3.9379 |
| 13. | T= 0.2952 | K= 3.9745 |
| 14. | T= 0.2952 | K= 3.9379 |
| 15. | T= 0.2952 | K= 3.9379 |
| 16. | T= 0.2976 | K= 3.9379 |
| 17. | T= 0.3000 | K= 3.9379 |

Table 10 Convergence of the Modified Stochastic Approximation Method on a Known Crossover Model with $K_o$=5.0 and $\tau_o$=0.210.

| n | Time Delay Estimate | Gain Estimate |
|---|---|---|
| 1. | T= 0.3000 | K= [illegible] |
| 2. | T= 0.2072 | K= 4.2020 |
| 3. | T= 0.2057 | K= 4.4752 |
| 4. | T= 0.2037 | K= 4.5530 |
| 5. | T= 0.2037 | K= [illegible] |
| 6. | T= 0.2026 | K= 4.[illegible]65 |
| 7. | T= 0.2026 | K= 4.6704 |
| 8. | T= 0.2026 | K= 4.[illegible]2 |
| 9. | T= 0.2026 | K= 4.7032 |
| 10. | T= 0.2026 | K= 4.7154 |
| 11. | T= 0.2034 | K= 4.7585 |
| 12. | T= 0.2034 | K= 4.7317 |
| 13. | T= 0.2034 | K= 4.7949 |
| 14. | T= 0.2034 | K= 4.7[illegible]4 |
| 15. | T= 0.2042 | K= [illegible] |
| 16. | T= 0.2042 | K= [illegible] |
| 17. | T= 0.2049 | K= 4.8512 |
| 18. | T= [illegible] | K= [illegible] |
| 19. | T= [illegible] | K= 4.[illegible]545 |
| 20. | T= [illegible] | K= [illegible] |
| 21. | T= [illegible] | K= [illegible] |
| 22. | T= [illegible] | K= [illegible] |
| 23. | T= [illegible] | K= 4.[illegible]775 |
| 24. | T= [illegible] | K= [illegible] |
| 25. | T= 0.2049 | K= 4.[illegible]01 |
| 26. | T= [illegible] | K= [illegible] |
| 27. | T= 0.2049 | K= 4.[illegible]997 |
| 28. | T= [illegible] | K= [illegible] |
| 29. | T= [illegible] | K= [illegible]122 |
| 30. | T= [illegible] | K= [illegible] |
| 31. | T= [illegible] | K= [illegible] |
| 32. | T= [illegible] | K= [illegible] |
| 33. | T= [illegible] | K= [illegible] |
| 34. | T= [illegible] | K= [illegible] |
| 35. | T= [illegible] | K= [illegible] |
| 36. | T= [illegible] | K= [illegible] |
| 37. | T= [illegible] | K= [illegible] |

ing gaussian pseudo-white noise through a $1/(0.5\ s + 1)^3$ filter.

The subject, a male junior engineering student, made five tracking runs a day for six days. Each run lasted 60 to 75 seconds. Approximately 45 seconds into each run, the samples of $\theta_i(t)$ and $\theta_o(t)$ were started at a rate of 20 samples per second, and were taken for the next 15 seconds. These samples were used in the identification process.

The results of these tests are given in Figure 14. This figure gives the maximum, minimum, and average values of the five estimates of $K_o$ and $\tau_o$ made each day. The values are quite in line with values found by previous methods, and show the usual trends of the average values as a function of continued practice, [5,6]. Table 11 gives examples of the type of convergence achieved during these tests.

The only odd point found in the analysis was the estimate of $K_o = 8.49$ determined on the fourth day of testing. This value was generated during a time when the subject later admitted to "having experimented" with his arm movements by trying a large amount of dither in his movement. Whether the dither actually increased his apparent $K_o$, or merely biased the parameter estimate, has not been determined.

## 4.D A Crosscheck of the Two Identification Methods

Both the modified stochastic approximation method, and the Hybrid parameter tracking method have been shown to converge

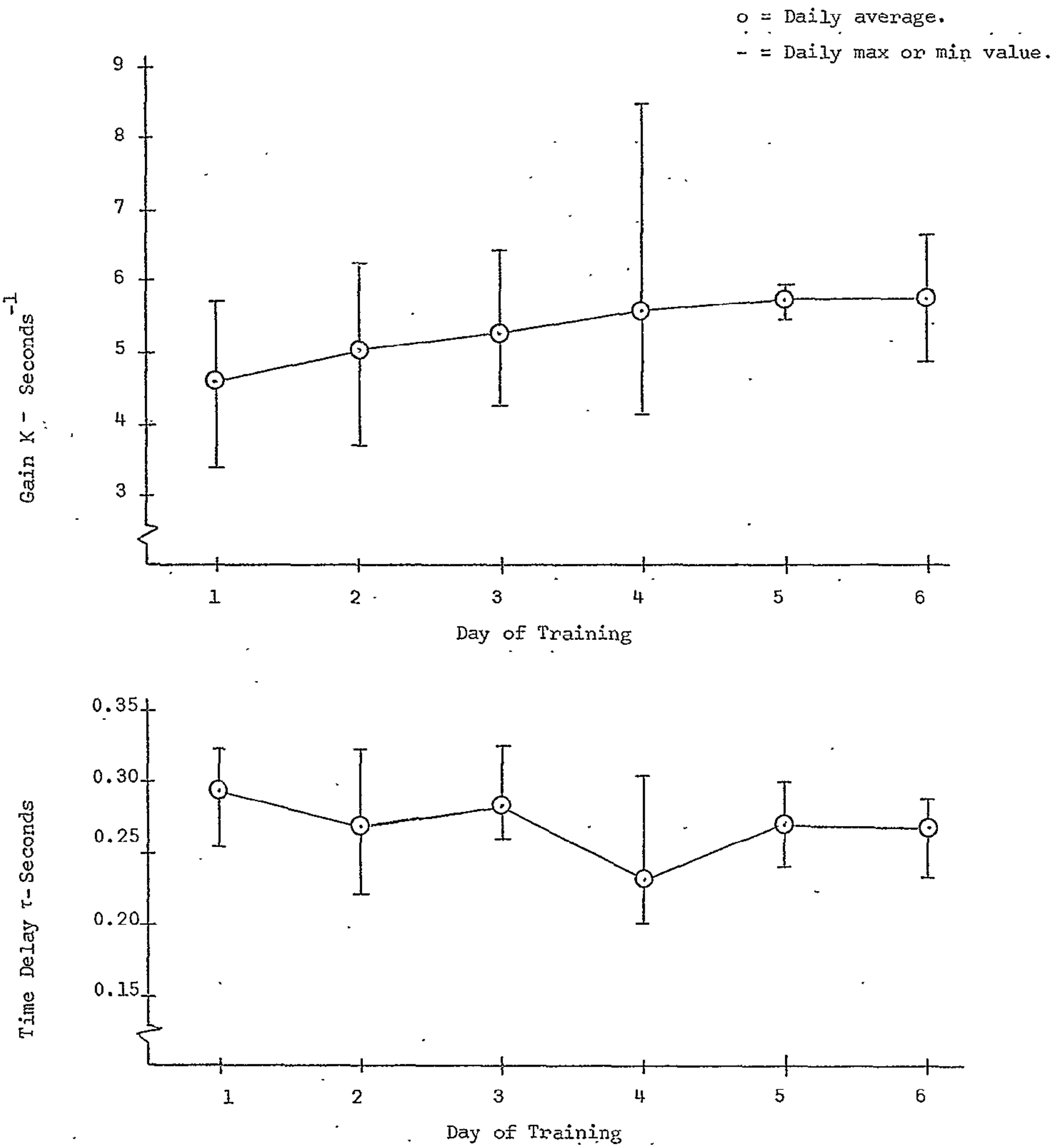


Fig. 14 $K_o$ and $\tau_o$ Estimates for one Subject. Obtained by the Modified Stochastic Approximation Method.

Table 11 Convergence of the Modified Stochastic Approximation Method on Two Runs Using an Actual Subject.

| n | Time Delay Estimate | Gain Estimate | |
|---|---|---|---|
| 1. | T= 0.2500 | K= 4.0000 | First Day of Training Run No. 4. |
| 2. | T= 0.2752 | K= 5.3183 | |
| 3. | T= 0.2927 | K= 5.2493 | |
| 4. | T= 0.2979 | K= 5.2185 | |
| 5. | T= 0.2983 | K= 5.2121 | |
| 6. | T= 0.2990 | K= 5.2175 | |
| 7. | T= 0.2990 | K= 5.2212 | |
| 8. | T= 0.2998 | K= 5.2224 | |
| 9. | T= 0.2998 | K= 5.2224 | |
| 10. | T= 0.3000 | K= 5.2244 | |
| 11. | T= 0.3002 | K= 5.2225 | |
| 12. | T= 0.3002 | K= 5.2244 | |
| 13. | T= 0.3004 | K= 5.2254 | |
| 14. | T= 0.3003 | K= 5.2244 | |

| | | | |
|---|---|---|---|
| 1. | T= 0.2000 | K= 5.0000 | Fifth Day of Training Run No. 1. |
| 2. | T= 0.2614 | K= 5.7904 | |
| 3. | T= 0.2893 | K= 5.7904 | |
| 4. | T= 0.2690 | K= 5.6512 | |
| 5. | T= 0.2799 | K= 5.7427 | |
| 6. | T= 0.2806 | K= 5.7468 | |
| 7. | T= 0.2806 | K= 5.7493 | |
| 8. | T= 0.2807 | K= 5.7493 | |
| 9. | T= 0.2808 | K= 5.7461 | |
| 10. | T= 0.2811 | K= 5.7471 | |
| 11. | T= 0.2811 | K= 5.7481 | |
| 12. | T= 0.2811 | K= 5.7481 | |
| 13. | T= 0.2810 | K= 5.7490 | |
| 14. | T= 0.2812 | K= 5.7490 | |
| 15. | T= 0.2813 | K= 5.7481 | |
| 16. | T= 0.2812 | K= 5.7490 | |

on fixed crossover models and on actual compensatory systems. The convergence of both methods on fixed models was found to be relatively quick and quite accurate. To check the parameter values obtained by the two methods while identifying an actual compensatory system, the methods were used to identify a compensatory system while using exactly the same data. To accomplish this both identification routines had to be modified.

The major changes necessary were in the modified stochastic approximation method. First, the operate times of the performance index integrators were changed so that they were in operation over the last half of each iteration ($\frac{T}{2} \leq t \leq T$). This brought this method more in line with the Hybrid parameter tracking method where the $K_o$ and $\tau_o$ estimates are obtained by averaging the $K(t)$ and $\tau(t)$ values present over the last half of each iteration. Second, the modified stochastic approximation program was altered so that the $\theta_i(t)$ and $\theta_o(t)$ samples taken from the compensatory system could be punched on cards. These cards were then used by the Hybrid parameter tracking method.

The only change in the Hybrid parameter tracking program was to allow the $\theta_i(t)$ and $\theta_o(t)$ samples to be obtained from punched cards, rather than from on-line A-D conversion.

A partially trained subject was then run on the compensatory test described previously. The modified stochastic approximation method was used to estimate $K_o$ and $\tau_o$ and the $\theta_i(t)$

samples were punched on cards. These cards were then used by the Hybrid parameter tracking system and new estimates of $K_o$ and $\tau_o$ were obtained. These estimates are given in Tables 12 and 13.

In Table 12, the modified stochastic approximation method estimates are $K_o = 4.60$ and $\tau_o = 0.269$. In Table 13, the Hybrid parameter tracker gives estimates of $K_o = 4.29$ and $\tau_o = 0.271$. The $\tau_o$ estimates are nearly identical, while the $K_o$ estimates differ by slightly less than seven percent.

The differences in the estimates of the two methods are undoubtedly due to a combination of noise and performance indices. The performance indices used in the two methods are not identical. One is the integral of the error squared, Eqs. (34) and (35), while the other is a continuous error squared, Eq. (2). The effects of noise, or remnant [9], in the two cases are probably not identical. Which identification method does the best job of minimizing the effects of noise is yet to be determined.

## V. CONCLUSIONS AND EXTENSIONS

The evaluation tests completed on the Hybrid parameter tracking and modified stochastic approximation techniques indicate that either method can be used to accurately identify crossover model parameter while using only short lengths of input-output data from the compensatory system. At their present stage of development, both methods are essentially

Table 12 Modified Stochastic Approximation Estimates Found in a Comparison of the Two Identification Methods.

| n | Time Delay Estimate | Gain Estimate |
|---|---|---|
| 1. | T= 0.3000 | K= 4.0000 |
| 2. | T= 0.2686 | K= 4.6958 |
| 3. | T= 0.2680 | K= 4.6615 |
| 4. | T= 0.2676 | K= 4.6493 |
| 5. | T= 0.2677 | K= 4.6445 |
| 6. | T= 0.2679 | K= 4.6397 |
| 7. | T= 0.2679 | K= 4.6349 |
| 8. | T= 0.2680 | K= 4.6319 |
| 9. | T= 0.2681 | K= 4.6297 |
| 10. | T= 0.2681 | K= 4.6273 |
| 11. | T= 0.2681 | K= 4.6250 |
| 12. | T= 0.2682 | K= 4.6228 |
| 13. | T= 0.2683 | K= 4.6206 |
| 14. | T= 0.2683 | K= 4.6197 |
| 15. | T= 0.2684 | K= 4.6188 |
| 16. | T= 0.2684 | K= 4.6165 |
| 17. | T= 0.2684 | K= 4.6143 |
| 18. | T= 0.2684 | K= 4.6125 |
| 19. | T= 0.2685 | K= 4.6120 |
| 20. | T= 0.2685 | K= 4.6107 |
| 21. | T= 0.2686 | K= 4.6103 |
| 22. | T= 0.2687 | K= 4.6094 |
| 23. | T= 0.2687 | K= 4.6094 |
| 24. | T= 0.2687 | K= 4.6094 |
| 25. | T= 0.2687 | K= 4.6076 |
| 26. | T= 0.2688 | K= 4.6071 |
| 27. | T= 0.2688 | K= 4.6062 |
| 28. | T= 0.2688 | K= 4.6058 |
| 29. | T= 0.2688 | K= 4.6049 |
| 30. | T= 0.2688 | K= 4.6044 |
| 31. | T= 0.2688 | K= 4.6049 |
| 32. | T= 0.2688 | K= 4.6053 |
| 33. | T= 0.2688 | K= 4.6053 |
| 34. | T= 0.2688 | K= 4.6044 |
| 35. | T= 0.2688 | K= 4.6049 |
| 36. | T= 0.2688 | K= 4.6040 |
| 37. | T= 0.2688 | K= 4.6040 |

Table 13 Hybrid Parameter Tracking Estimates Found in a Comparison of the Two Identification Methods.

| n | Gain Estimate $\overline{K}_n$ | Time Delay Estimate $\overline{\tau}_n$ |
|---|---|---|
| | Initial K = 1.985 | Initial $\tau$ = 0.305 |
| 1 | KBAR(1,1)= 3.351 | TBAR(1,1)=0.280 |
| 2 | KBAR(1,2)= 3.665 | TBAR(1,2)=0.276 |
| 3 | KBAR(1,3)= 3.824 | TBAR(1,3)=0.275 |
| 4 | KBAR(1,4)= 3.920 | TBAR(1,4)=0.275 |
| 5 | KBAR(1,5)= 3.988 | TBAR(1,5)=0.274 |
| 6 | KBAR(2,1)= 4.111 | TBAR(2,1)=0.274 |
| 7 | KBAR(2,2)= 4.160 | TBAR(2,2)=0.273 |
| 8 | KBAR(2,3)= 4.187 | TBAR(2,3)=0.273 |
| 9 | KBAR(2,4)= 4.206 | TBAR(2,4)=0.273 |
| 10 | KBAR(2,5)= 4.220 | TBAR(2,5)=0.273 |
| 11 | KBAR(3,1)= 4.255 | TBAR(3,1)=0.272 |
| 12 | KBAR(3,2)= 4.266 | TBAR(3,2)=0.272 |
| 13 | KBAR(3,3)= 4.270 | TBAR(3,3)=0.271 |
| 14 | KBAR(3,4)= 4.273 | TBAR(3,4)=0.272 |
| 15 | KBAR(3,5)= 4.276 | TBAR(3,5)=0.272 |
| 16 | KBAR(4,1)= 4.292 | TBAR(4,1)=0.271 |
| 17 | KBAR(4,2)= 4.294 | TBAR(4,2)=0.271 |
| 18 | KBAR(4,3)= 4.294 | TBAR(4,3)=0.271 |
| 19 | KBAR(4,4)= 4.294 | TBAR(4,4)=0.271 |
| 20 | KBAR(4,5)= 4.293 | TBAR(4,5)=0.271 |

batch processors that use fifteen seconds of input-output data, and take approximately one minute to generate the parameters that were present over that fifteen second interval. These methods as they stand, can certainly be valuable tools for laboratory experimentation. It is the author's opinion that these methods are superior to most methods presently being used in the measurement of human operator transfer functions.

There are, however, several modifications that can be made to either method, which will increase their utility. These are avenues for future research, and will be discussed briefly below.

## 5.A Reduction of Total Computation Time

Presently, it takes approximately one minute of computation time for the identification systems to converge on their $K_o$ and $\tau_o$ estimates. This time can be shortened by at least one, and possibly two, orders of magnitude by writing the computer programs in Assembly rather than FORTRAN language.

Another change which can be made in both programs to reduce the total computation time is a reduction in T, the length of the input-output samples used in the identification process. Fifteen seconds of data were used in all tests discussed in this report. The theoretical work of Section III-C indicates that fifteen seconds of data is longer than is necessary for convergence to be achieved. Future experimentation will be directed toward determining a practical minimum value of T. One consideration here is the effect that noise, or remnant, has on the parameter estimates. As T gets shorter, the accuracy of the estimates is likely to deteriorate. Since the gains made by going to Assembly language will be much greater than those possible by reducing T, it may be that T should not be reduced just to lower computation time. This will be evaluated.

One additional change, which would affect the convergence properties of the two methods and bring about further reductions in computation time, is the refinement of the parameter adjustment algorithms. In particular, in the Hybrid parameter tracking method, the parameter adjustment gains are cycled up and down each five iterations for a total of twenty iterations. It is doubtful that either five or twenty are optimum numbers at the present time. Further improvement here is certainly probable.

### 5.B Simultaneous Sampling and Identification

The most significant modification to be investigated in future research would appear to be the possibility of simultaneous sampling and identification. In this modification real-time sampling of the compensatory system and the fast-time iterations of the Hybrid parameter tracking model would be done simultaneously. The compensatory system under test would have its input and output sampled at a fixed rate. Call these samples $\theta_i(n\Delta t)$ and $\theta_o(n\Delta t)$, respectively. If it is assumed that N samples are used during each fast-time iteration, then the input and output signals to the fast-time parameter tracking system during the first iteration would be

$$\sum_{n=1}^{N} \theta_i(n\Delta t)\ \delta\ (t_o - n\Delta t) \text{ and } \sum_{n=1}^{N} \theta_o(n\Delta t)\ \delta\ (t_o - n\Delta t)$$

respectively. ($t_o$ is the time scaled time variable, and $\delta(t_o - n\Delta t)$ is a delta function located at $t_o = n\Delta t$.) While this first fast-time iteration is being made, d new samples of $\theta_i(t)$ and $\theta_o(t)$ from the real-time system would have been taken. Thus, the second iteration of the fast-time model would use input and output signals of

$$\sum_{n=k+d}^{N+d} \theta_i(n\Delta t)\ \delta\ (t_o - n\Delta t) \text{ and } \sum_{n=k+d}^{N+d} \theta_o(n\Delta t)\ \delta\ (t_o - n\Delta t).$$

Each iteration would therefore use d new data points while discarding an equal number of the oldest data points stored. The gradient gains in the tracking system would again by cycled up and down in order to obtain good convergence. If this modification is written in Assembly language, one iteration should be possible each second or two after the first N samples are taken in. A running estimate of the $K_o$ and $\tau_o$ parameters which were present in the system over the last T seconds would thus be continually available every second or two. This would enable the identification system to be an on-line device, where the $K_o$ and $\tau_o$ estimates could be used to monitor and/or aid in control of the system being identified. The method would also be valuable in checking the variations which occur in $K_o$ and $\tau_o$ during a normal compensatory control task.

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# VII. APPENDIX A

Appendix A is a listing of the digital computer program and analog circuitry needed to implement the Hybrid parameter tracking method for the identification of crossover model parameters.

```
// FOR
*IOCS(1132 PRINTER,DISK,CARD,TYPEWRITER,KEYBOARD)
*ONE WORD INTEGERS
*LIST ALL
C       PROGRAM FOR IDENTIFYING THE GAIN-K AND TIME DELAY-T PARAMETERS OF
C       A COMPENSATORY CONTROL SYSTEM USING HYBRID PARAMETER TRACKING.
C       15 SECONDS OF INPUT/OUTPUT DATA ARE USED IN IDENTIFICATION PROCESS.
        REAL K
        DIMENSION XBAR(4,5),TBAR(4,5)
        DIMENSION IVAL(2000,2),IVALU(15),IVALE(15),IVALK(2000),IVALT(2000)
C       SET INTEGRATORS 00 AND 95 ( K AND T INTEGRATORS) TO IC MODE
        CALL RSCL(0,IER)
        CALL SSCL(1,IER)
C       SET ALL POTS TO DESIRED VALUES.  THEN PAUSE  FOR MANUAL ANALOG RUNS.
        CALL SPOT( 003, 4500, 5, IER)
        CALL SPOT( 005, 4000, 5, IER)
        CALL SPOT( 006, 9520, 5 ,IER)
        CALL SPOT( 007, 4000, 5, IER)
        CALL SPOT( 008, 9520, 5, IER)
        CALL SPOT( 009, 1500, 5, IER)
        CALL SPOT( 030, 9000, 5, IER)
        CALL SPOT( 032, 5000, 5, IER)
        CALL SPOT( 033, 4000, 5, IER)
        CALL SPOT( 035, 2500, 5, IER)
        CALL SPOT( 036, 5000, 5, IER)
        CALL SPOT( 037, 2000, 5, IER)
        CALL SPOT( 038, 5000, 5, IER)
        CALL SPOT( 039, 1500, 5, IER)
        CALL SPOT( 060, 4500, 5, IER)
        CALL SPOT( 061, 1000, 5, IER)
        CALL SPOT( 062, 2500, 5, IER)
        CALL SPOT( 063, 4000, 5, IER)
        CALL SPOT( 065, 5500, 5, IER)
        CALL SPOT( 067, 2000, 5, IER)
        CALL SPOT( 068, 2000, 5, IER)
        CALL SPOT( 095, 4500, 5, IER)
        CALL SPOT( 097, 5000, 5, IER)
```



Hybrid Parameter Tracking Program (page 1 of 4).

```
    1 DO 2 I=1,15
    2 IVALU(I)=0
      CALL LBDA(0,7,IVALU,IER)
      CALL TLDA
C     INPUT NORMALIZED INITIAL GUESSES ON K AND T.
      IVALU(3)=6500
      IVALU(4)=10000
      CALL LBDA(2,3,IVALU,IER)
      CALL TLDA
C     SET GRADIENT GAINS TO INITIAL VALUE SO THAT MANUAL ANALOG RUNS
C     CAN BE MADE IF DESIRED.
      IVALU(1) = 20000
      IVALU(2) = 25000
      CALL LBDA(0,1, IVALU, IER)
      CALL TLDA
      PAUSE 1000
C     USER PRESS PROGRAM START TO INITIATE SAMPLING OF 15 SECONDS OF DATA
      CALL STCO(0,IER)
      CALL SAMO(4,IER)
C     SAMPLE INPUT/OUTPUT DATA OF THE COMPENSATORY CONTROL SYSTEM FOR
C     15 SECONDS AT A RATE OF 20 SAMPLES PER SECOND.
      DO 100 I = 1,300
      CALL DELAY(.044)
      CALL CRBC(1,2,IVALU,IER)
      IVAL(I,1)=IVALU(2)
      IVAL(I,2)=IVALU(3)
  100 CONTINUE
  101 CALL SAMO(6,IER)
      CALL STCO(1,IER)
      PAUSE 1200
```

```
C     ANALOG COMPUTER SET TO FAST TIME OPERATION. USER PRESS PROGRAM START
C     TO START IDENTIFICATION PROCESS.
      DO 900 M=1,4
      MM = 300
      II = 1
      NN = 151
C     INPUT / OUTPUT DATA IS PUT INTO FAST MODEL AND ITERATED 5 TIMES.
      DO 500 L=1,5
C     GRADIENT GAINS ARE REDUCED BY 1/SQRT(L)
      ZL = L
      IDUM1 = 20000./SQRT(ZL)
      IDUM2 = 25000./SQRT(ZL)
      IVALU(1) = IDUM1
      IVALU(2) = IDUM2
      CALL   LBDA(0,1,IVALU,IER)
      CALL TLDA
      CALL SAMO(4,IER)
      DO 200 I=II,MM
      IVALU(5)=IVAL(I,1)
      IVALU(6)=IVAL(I,2)
      CALL LBDA(4,5,IVALU,IER)
      CALL TLDA
         R=5.+5.
         R=2.
         R=2.
         R=2.
      CALL CRBC(3,4,IVALE,IER)
      IVALK(I)=IVALE(4)
      IVALT(I)=IVALE(5)
      IF (I-100) 200,199,200
  199 CALL RSCL(1,IER)
      CALL SSCL(0,IER)
  200 CONTINUE
```

```
      CALL SAMO(5,IER)
      CALL RSCL(0.IER)
      SUMK=0.0
      SUMT=0.0
      DO 300 I = NN,MM
      K = IVALK(I)
      T = IVALT(I)
      SUMK = SUMK + K
      SUMT = SUMT + T
  300 CONTINUE
      SUMK = SUMK/3276.7
      SUMT = SUMT/32767.
C     AVERAGE VALUE OF K AND T ARE FOUND OVER LAST HALF OF ITERATION.
      XBAR(M,L) = SUMK/150.
      TBAR(M,L) = SUMT/150.
      WRITE(1,10) M,L,XBAR(M,L),M,L,TBAR(M,L)
   10 FORMAT(/5X,'KBAR(',I1,',',I1,')=',F6.3,10X,'TBAR(',I1,',',I1,')=',
     1F5.3)
      CALL SAMO(6,IER)
C      INITIAL CONDITIONS FOR K AND T ARE SET TO THE AVERAGE VALUES OF
C     THE PREVIOUS ITERATION.
      XK=XBAR(M,L)*3276.7
      T=TBAR(M.L)*32767.
      IVALU(3)=XK
      IVALU(4)=T
      CALL LBDA(2,3,IVALU,IER)
      CALL TLDA
      CALL SSCL(1,IER)
  500 CONTINUE
  900 CONTINUE
C    PROGRAM RETURNS FOR NEW SET OF DATA.
      GO TO 1
  999 CALL EXIT
      END
```

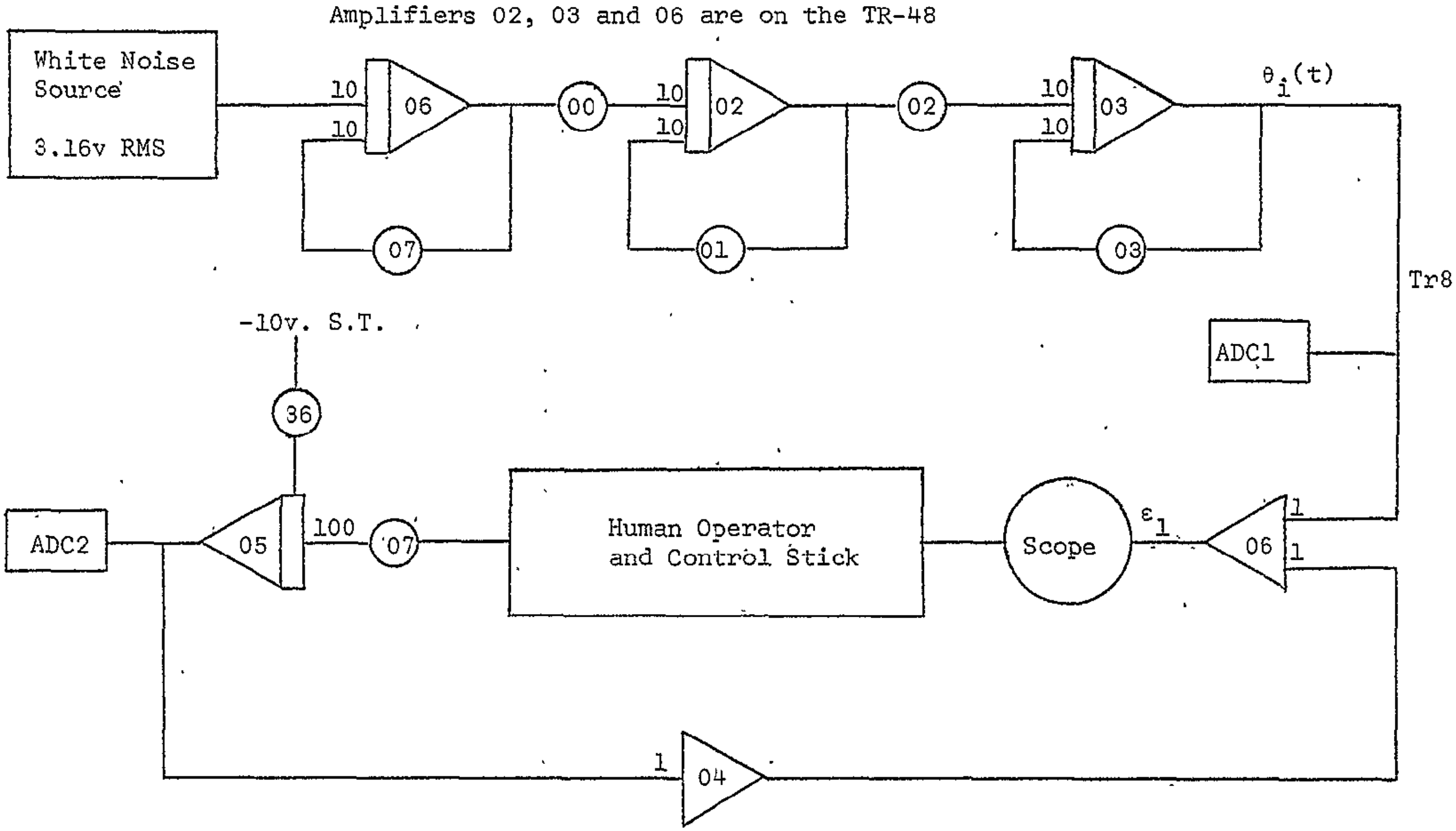


Fig. A1 Input Signal Generation and the Compensatory Control System.

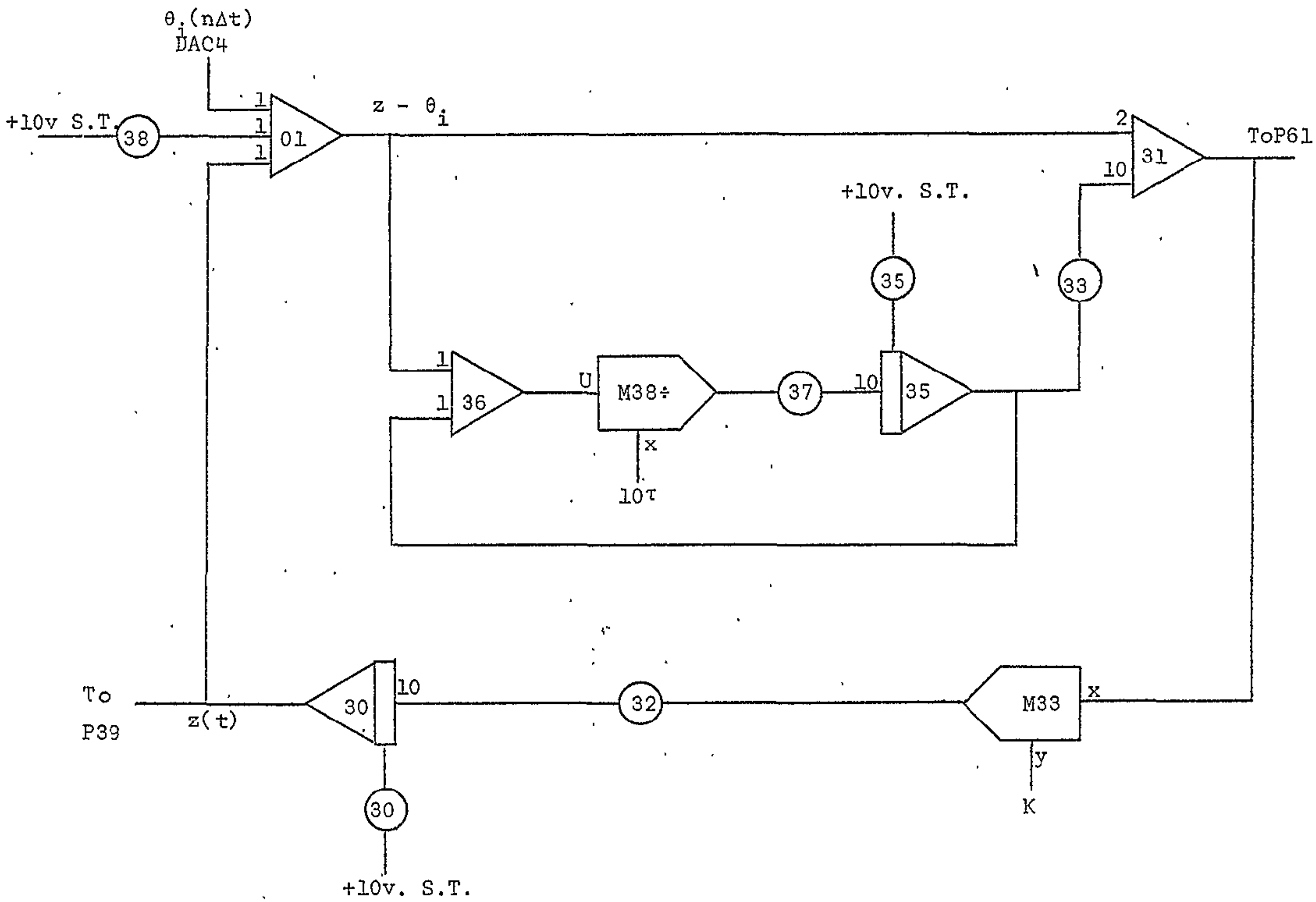


Fig. A2 Approximate Crossover Model of the Compensatory Control System.

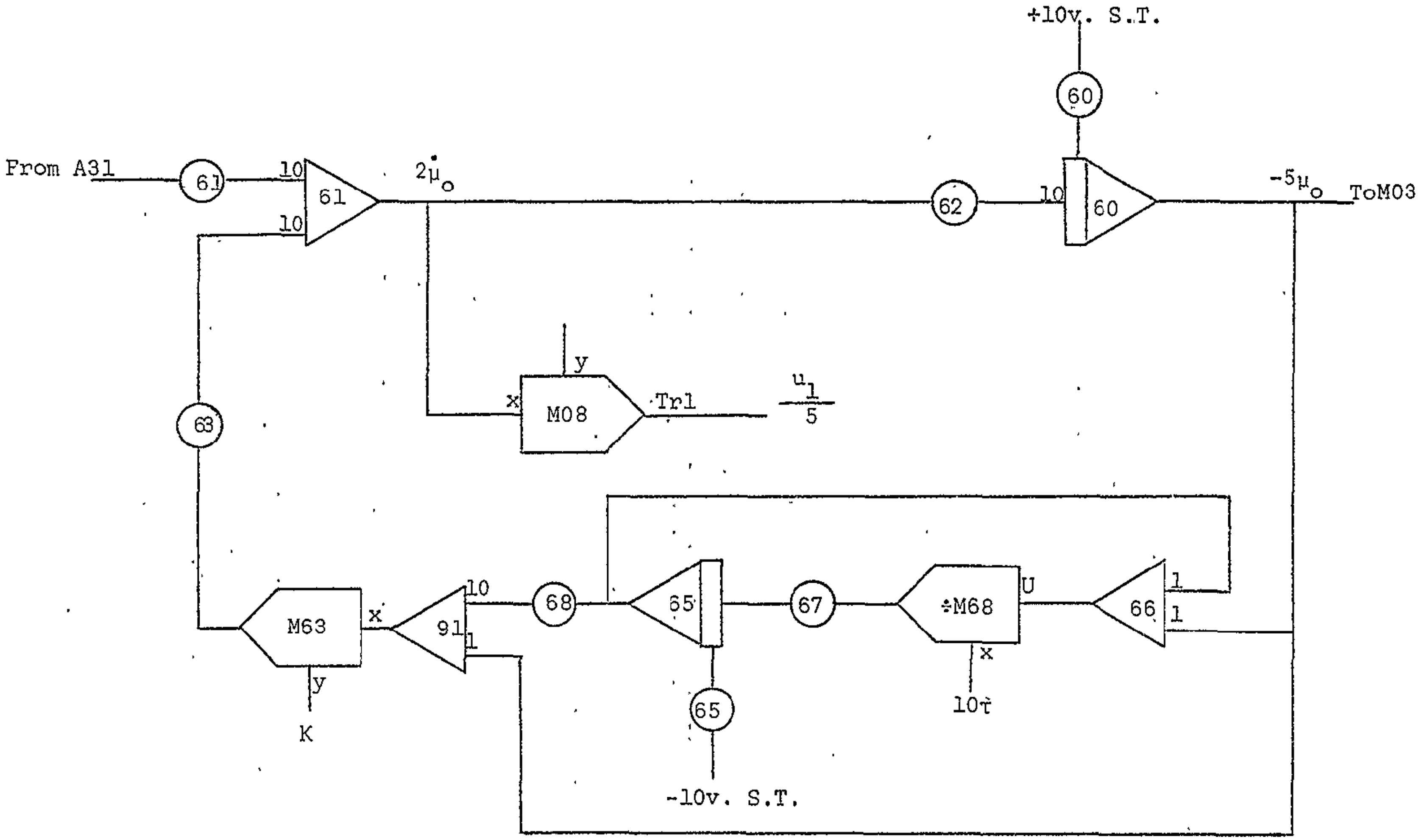


Fig. A3 Gradient Equation Circuitry.

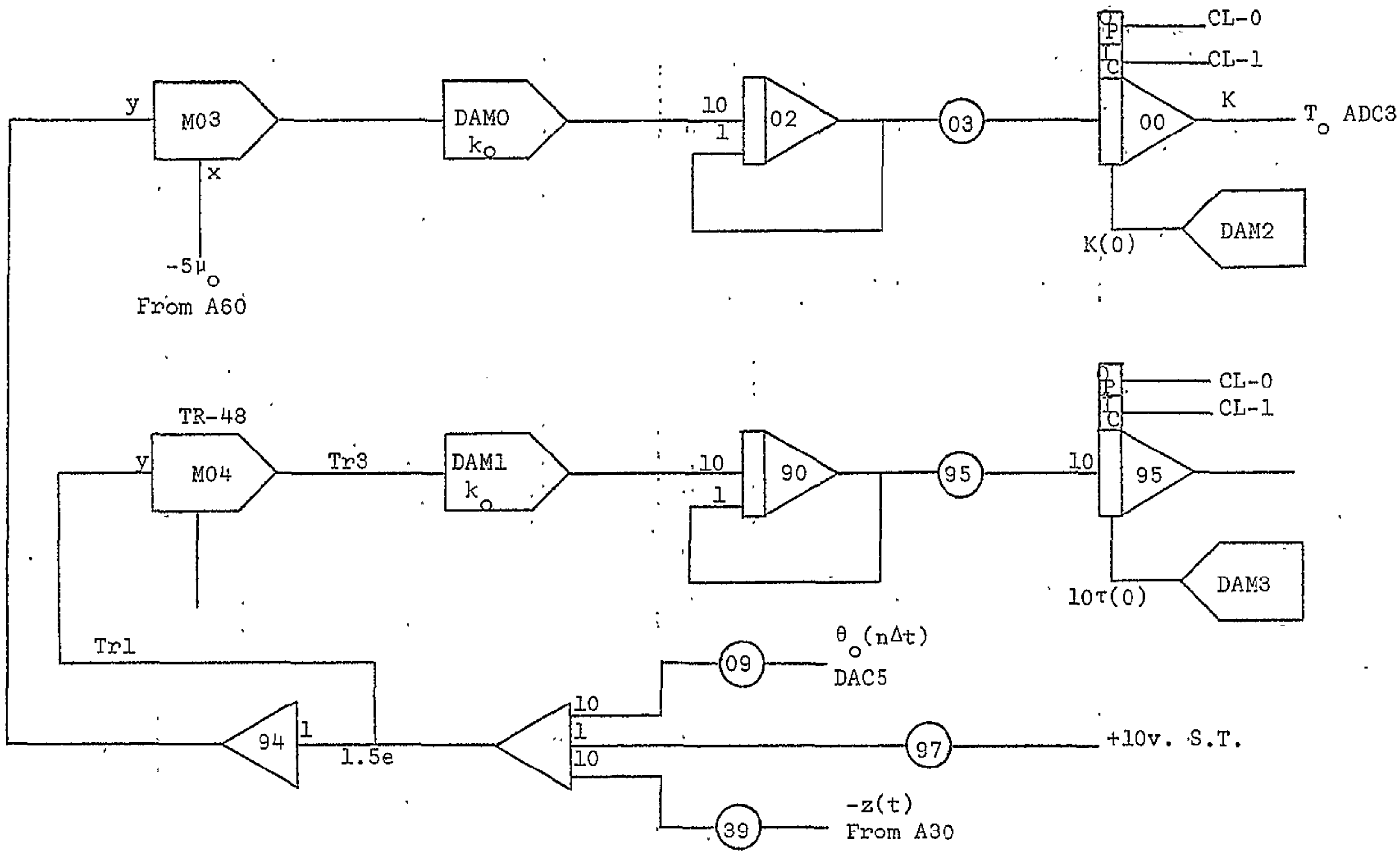


Fig. A4 Parameter Adjustment Equations

AMPLIFIER ASSIGNMENT SHEET EAI 680



DATE June 8, 1970

NAME G. Jackson

PROBLEM Hybrid Parameter Tracker

| Variable | Amp. # | Static Check | Normal IC | Peak Voltage | Comments |
|---|---|---|---|---|---|
| K | 00 | 2.0 | | | |
| $z-\theta_i(t)$ | 01 | 4.0 | | | |
| | 03 | -3.82 | | | |
| | 04 | -5.0 | | | |
| | 05 | 5.0 | | | |
| $\varepsilon 1$ | 06 | 5.0 | | | |
| $-u_{1/5}$ | 08 | 1.44 | | | |
| $-z(t)$ | 30 | -9.0 | | | |
| | 31 | 2.0 | | | |
| | 33 | -0.4 | | | |
| | 35 | -2.5 | | | |
| | 36 | -1.5 | | | |
| | 38 | 5.0 | | | |
| $-5u_o$ | 60 | -4.5 | | | |
| $2\dot{u}_o$ | 61 | -7.2 | | | |
| | 63 | 1.3 | | | |
| | 66 | -1.0 | | | |
| | 68 | 3.33 | | | |
| | 91 | -6.5 | | | |
| $-1.5e$ | 94 | -8.5 | | | |
| $10\tau$ | 95 | 3.0 | | | |
| $1.5e$ | 96 | 8.5 | | | |

AMPLIFIER ASSIGNMENT SHEET TR48

DATE 6-8-70

NAME G. Jackson

PROBLEM Hybrid Parameter Tracker

| Variable | Amp. # | Static Check | Normal IC | Peak Voltage | Comments |
|---|---|---|---|---|---|
| | 02 | | | | |
| | 03 | | | | |
| | 06 | | | | |
| | 31 | -1.224 | | | |

POTENTIOMETER ASSIGNMENT SHEET EAI680 DATE 6-8-70

NAME G. Jackson

PROBLEM Hybrid Parameter Tracker

| Pot. # | Coefficient | Pot. Setting | Amp. Gain | Amp. # |
|---|---|---|---|---|
| 03 | Parameter Adjustment Fixed Gain | 0.600 | | |
| 07 | Scale Factor | 0.400 | | |
| 09 | Scale Factor for e(t) | 0.150 | | |
| 30 | S.T. | 0.900 | | |
| 32 | | 0.500 | | |
| 33 | | 0.400 | | |
| 35 | S.T. | 0.250 | | |
| 36 | S.T. | 0.500 | | |
| 37 | | 0.200 | | |
| 38 | S.T. | 0.500 | | |
| 39 | Scale Factor for e(t) | 0.150 | | |
| 60 | S.T. | 0.450 | | |
| 61 | | 0.100 | | |
| 62 | | 0.250 | | |
| 63 | | 0.400 | | |
| 65 | S.T. | 0.550 | | |
| 67 | | 0.200 | | |
| 68 | | 0.200 | | |
| 95 | | 0.600 | | |
| 97 | | 0.500 | | |

POTENTIOMETER ASSIGNMENT SHEET TR48 DATE 6-8-70

NAME G. Jackson

PROBLEM Hybrid Parameter Tracker

| Pot. # | Coefficient | Pot. Setting | Amp. Gain | Amp. # |
|---|---|---|---|---|
| 00 | 1/5B | 0.400 | | |
| 01 | 1/10B | 0.200 | | |
| 02 | 1/10B | 0.200 | | |
| 03 | 1/10B | 0.200 | | |
| 07 | 1/10B | 0.200 | | |
| | 1. These are filter coeffcients for $\frac{1}{(BS+1)^3}$ where B = 1/2 | | | |

# VIII. APPENDIX B

Appendix B is a listing of the digital computer program and analog circuitry needed to implement the modified stochastic approximation method for identification of crossover model parameters.

```
C       DIMENSIONING VARIABLES
C       DIMENSIONS FOR CROSSOVER LOOP 1
          DIMENSION ZK1(100), ZK2(100), TAU3(100)
C       DIMENSIONS FOR CROSSOVER LOOP 2
          DIMENSION ZK3(100), TAU1(100), TAU2(100)
C       DIMENSIONS FOR ADJUSTING PARAMETERS
          DIMENSION AMOD(100), BMOD(100), CMOD(100), DMOD(100)
C       DIMENSIONS FOR INTEGRAL SQUARED ERRORS
          DIMENSION ZISE1(100,4), ZISE2(100,4)
C       DIMENSIONS FOR TRANSFERING PERFORMANCE INDEXES
          DIMENSION IPF(6)
C       DIMENSIONS FOR SAMPLING OF THE KNOWN SYSTEM
          DIMENSION KNOWN(4)
C       DIMENSIONS FOR STORING THE SAMPLED INPUT OUTPUT DATA
          DIMENSION INPUT(1202), IOUT(1202)
C       DIMENSIONS FOR SETTING THE DAMS
          DIMENSION IVALU(6)
C       DIMENSIONS FOR FEEDING THE SAMPLES INTO FAST MODE
          DIMENSION IFAST(8)
C       INITIAL VALUE OF A CONSTANT
          ZPRO=0.0
C       SELECTING DESIRED ANALOG CHANNEL
    1     CALL SACN (1, IER)
C       CONSTANTS FOR PARAMETER ADJUSTMENT
    4     A=3.
          B=1.
          C= .3
          D= .03
          ZLOO = 1.0
          ZPRO=ZPRO + 1.0
C       CONSOL INSTRUCTION ON PROGRAM OPTIONS DEALING WITH PRINT OUT
          WRITE (1, 500)
  500     FORMAT ('OPTIONS USING DATA SWITCHES'/)
```

Modified Stochastic Approximation Method Program (page 1 of 10)

```
      WRITE (1, 501)
 501  FORMAT (5X, 'SW 1   UP FOR STATIC CHECK')
      WRITE (1, 502)
 502  FORMAT (5X, 'SW 2   UP FOR EXIT FROM ITERATION LOOP')
      WRITE (1, 503)
 503  FORMAT (5X, 'SW 3   UP FOR CONSOL PRINT OUT')
      WRITE (1, 504)
 504  FORMAT (5X, 'SW 4   UP FOR NO MAIN PRINT OUT')
      WRITE (1, 506)
 506  FORMAT (5X, 'SW 5   UP FOR NO PRINT OUT AT ALL')
      WRITE (1, 507)
 507  FORMAT (5X, 'SW 6   UP FOR PUNCHING OF AN INPUT-OUTPUT DECK ')
C     SET ANALOG MODE TO POT SET
    5 CALL SAMO (7, IER)
C     SETTING THE POTS TO THE DESIRED VALUES
    7 CALL SPOT (001,6666,3,IER)
      CALL SPOT (002,6666,3,IER)
      CALL SPOT (004,2000,3,IER)
      CALL SPOT (006,2000,3,IER)
      CALL SPOT (007,5000,3,IER)
      CALL SPOT (008,2000,3,IER)
      CALL SPOT (030,2000,3,IER)
      CALL SPOT (034,5000,3,IER)
      CALL SPOT (035,5000,3,IER)
      CALL SPOT (036,2000,3,IER)
      CALL SPOT (060,2000,3,IER)
      CALL SPOT (063,0300,3,IER)
      CALL SPOT (064,5000,3,IER)
      CALL SPOT (065,5000,3,IER)
      CALL SPOT (066,2000,3,IER)
      CALL SPOT (068,0300,3,IER)
      CALL SPOT (090,5000,3,IER)
      CALL SPOT (091,1500,3,IER)
```

```
      CALL SPOT (095,5000,3,IER)
      CALL SPOT (092,4000,3,IER)
      CALL SPOT (093,4000,3,IER)
      CALL SPOT (096,1500,3,IER)
      CALL SPOT (097,4000,3,IER)
      CALL SPOT (098,4000,3,IER)
C     SET ANALOG MODE TO POT CHECK
      CALL SAMO (1, IER)
C     NOW PAUSE TO SEE IF POT SETTINGS ARE CORRECT
C     CONSOL INSTRUCTION FOR POT CHECK
 16   WRITE (1, 17)
 17   FORMAT (//'PAUSE TO CHECK POTS, TO CONTINUE PRESS THE PROGRAM S
     1TART BUTTON'//)
 20   PAUSE
C     NOW COME THE STATEMENTS CHANGING THE PARAMETER ADJUSTERS
      DO 25 I=1, 100
      ZI=I
C     IF STATEMENT TO STOP THE ADJUSTMENT AFTER THE 10 TH ITERATION
      IF (11-I) 24, 24, 23
 23   AMOD(I)= (A/ZI)*3276.7
      BMOD(I)=B/ZI
      CMOD(I)=(C/(ZI**(1./6.)))*3276.7
      DMOD(I)=D/(ZI**(1./6.))
      GO TO 25
 24   AMOD(I)= AMOD(I-1)
      BMOD(I)= BMOD(I-1)
      CMOD(I)= CMOD(I-1)
      DMOD(I)= DMOD(I-1)
 25   CONTINUE
C     SETTING THE VALUES ON THE DAMS TO ZERO FOR SAMPLING OF KNOWN
C     LOOPS
      IVALU(1)=0
      IVALU(2)=0
```

```
      IVALU(3)=0
      IVALU(4)=0
      CALL LBDA (0,3,IVALU,IER)
27    CALL TLDA
      IFAST(5)=0
      IFAST(6)=0
      CALL LBDA (4,5,IFAST,IER)
29    CALL TLDA
C     SET ANALOG MODE TO IC
      CALL SAMO (6,IER)
C     SET ANALOG TIME CONSTANT TO NORMAL
      CALL STCO (0, IER)
C     SET ANALOG LOGIC MODE TO RUN
      CALL SLMO (1, IER)
C     COLLECT 300 SAMPLES IN 15 SEC FROM THE UNKNOWN SYSTEM
32    CONTINUE
C     SET ANALOG MODE TO OPERATE
      CALL SAMO ( 4, IER )
C     CARDS TO SAMPLE DATA ( 20 SAMPLES/SEC )
      DO 30 J= 1, 300
      CALL CRBC (0,1,KNOWN,IER)
      INPUT(J)=KNOWN(1)
      IOUT(J)=KNOWN(2)
      CALL DELAY ( .049 )
30    CONTINUE
      CALL SAMO (5, IER)
C     SET ANALOG TIME CONSTANT TO NORMAL * 10
45    CALL STCO (1, IER)
C     SETTING THE PARAMETER VALUES FOR THE FIRST RUN THROUGH THE
C     ADJUSTMENT PROCEEDURE
```

```
C         CONSOL INSTRUCTION FOR COMMENT LINES
 843        WRITE (1, 844)
 844        FORMAT ('PROGRAM COMMENTS (USE ERASE FIELD KEY BEFORE COLUMN 78
     1 TO OBTAIN DESIRED NUMBER OF LINES. TO CONTINUE PRESS EOF KEY)'///
     2/)
C         READ CARDS NECESSARY FOR COMMENT LINES ON CONSOL PRINTER
            READ (6, 845 ) JUNK1
 845        FORMAT (79X, A1)
            WRITE (1, 937)
 937        FORMAT (//)
C         SET ANALOG MODE TO IC
            CALL SAMO (6, IER)
C         CHECKING TO SEE IF STATIC CHECK IS DESIRED
            CALL DATSW (1, INT)
            IF (2-INT) 47, 47, 51
C         CONSOL INSTRUCTION FOR STATIC CHECK
  51        WRITE (1, 52)
  52        FORMAT (//'PAUSE FOR AUTOMATIC STATIC CHECK'/)
            CALL STAT
C         CONSOL INSTRUCTION FOR SELECTING STARTING POINT
  47        WRITE (1, 49)
  49        FORMAT(//'TYPE IN FLOATING POINT VALUES FOR K (SPACES 1-6) AND
     1TAU (SPACES 7-12) ')
C         READ CARDS FOR STARTING POINT K AND T FROM CONSOL KEYBOARD
            READ (6, 48) AAA, BBB
  48        FORMAT (2F6.2)
  53        WRITE (1, 945)
 945        FORMAT (//)
C         CARDS SETTING INITIAL DISTANCES AWAY FROM BASE POINT
            ZK3(1)=AAA*3276.7
            TAU3(1)=BBB
            ZK1(1)=ZK3(1)+CMOD(1)
            ZK2(1)=ZK3(1)-CMOD(1)
            TAU1(1)=TAU3(1)+DMOD(1)
            TAU2(1)=TAU3(1)-DMOD(1)
C         OUTER ( MAIN ) DO LOOP, PLANED EXIT IS BY CHOICE RATHER THEN
C         BY COMPLETION OF THE DO LOOP
```

```
      DO 110 M=1, 100
C     DATA SWITCH BY WHICH EXIT FROM THE OUTER DO LOOP MAY BE OBTAINED
 220    CALL DATSW (2, LETGO)
        IF (2-LETGO) 131, 131, 842
C     CONSOL INSTRUCTION TO REMIND OPERATOR TO RETURN SWITCH TO DOWN POSIT
 842    WRITE (1, 849)
 849    FORMAT (//'RETURN DATSW 2 TO DOWN POSITION'//)
        GO TO 113
C     SET ANALOG MODE TO IC
 131    CALL SAMO (6, IER)
C     NOW USE DO LOOP TO SELECT WHICH MODEL YOU ARE GOING INTO
        DO 70 K=1,2
C     CHOOSING CORRECT PARAMETERS DEPENDING ON THE LOOP
C     FIRST LOOP ADJUSTS K, SECOND LOOP ADJUSTS T
        IF (1-K) 55, 50, 50
  50    IVALU(1)=(1./TAU3(M))*3276.7
        IVALU(2)=ZK1(M)
        IVALU(3)=IVALU(1)
        IVALU(4)=ZK2(M)
        GO TO 60
  55    IVALU(1)=(1./TAU1(M))*3276.7
        IVALU(2)=ZK3(M)
        IVALU(3)=(1./TAU2(M))*3276.7
        IVALU(4)=IVALU(2)
  60    CALL LBDA (0, 3, IVALU, IER)
  64    CALL TLDA
C     SET ANALOG MODE TO IC
        CALL SAMO (6, IER)
C     SETTING THE INDEPENDENTLY CONTROLED AMPLIFIERS TO THE IC CONDITION
C     USING CONTROL LINES
        CALL RSCL (0, IER)
        CALL SSCL (1, IER)
C     NOW HAVE WHAT EVER MODEL IT IS GO THROUGH ITS VALUES
C     SET ANALOG MODE TO OPERATE
  67    CALL SAMO (4, IER)
  73    DO 65 L= 1, 300
```

```
C       SETTING THE INDEPENDENTLY CONTROLED AMPLIFIERS TO OPERATE USING
C       CONTROL LINES
          IF ( 150 - L ) 63, 61, 63
 61       CALL SSCL (0,IER)
          CALL RSCL (1, IER)
 63       IFAST(5)=INPUT(L)
          IFAST(6)= IOUT(L)
          CALL LBDA (4, 5, IFAST, IER)
          CALL TLDA
C       CARDS FOR DELAY
          R=5.+5.+5.+5.+5.
          R=2.
          R=2.
          R=2.
          R=2.
 65       CONTINUE
C       SET ANALOG MODE TO HOLD
          CALL RSCL (0, IER)
 77       CALL SAMO (5, IER)
C       READ THE TWO PERFORMANCE INDEXES
 79       CALL CPBC (2,3,IPF,IER)
C       ZISE1 IS BASE POINT  + DELTA, ZISE2 IS BASE POINT  - DELTA
 69       ZISE1(M, K)= IPF(3)/3
          ZISE2(M, K)= IPF(4)/3
 70       CONTINUE
C       THE NEXT SET OF CARDS ARE PRINT OPTIONS USING DATA SWITHCES
C       DATA SWITCH 5 FOR NO PRINT OUT (NO PRINT OUT IN THE UP POSITION)
          CALL DATSW (5, IFAS)
          IF (2-IFAS) 833, 833, 105
 833      CONTINUE
C       DATA SWITCH 3 FOR CONSOL PRINT OUT ( UP FOR PRINT OUT )
          CALL DATSW (3,LIPR)
          IF (2-LIPR) 515, 515, 510
 510      VTAU=TAU3(M)
          VK=ZK3(M)/3276.7
```

```
      WRITE (1, 520) ZLOO , VTAU, VK
520   FORMAT (F6.0, 5X, 'T=', F8.4, 5X, 'K=', F8.4)
C     DATA SWITHC 4 FOR MAIN PRINT OUT ( WHEN UP IT DOES NOT PRINT )
515   CALL DATSW (4, NOPR)
      IF (2-NOPR) 505, 505, 105
505   CONTINUE
C     WRITE OUT ALL OF THE PERTINENT DATA ON THE MAIN PRINTER
      WRITE (3, 96) ZPRO
96    FORMAT (1X, 'PROGRAM NUMBER', F6.1 /)
      WRITE (3, 74) ZLOO
74    FORMAT (1X, 'LOOP NUMBER', F6.1/)
      VTAU=TAU2(M)
      VK=ZK3(M)/3276.7
      WRITE(3, 80) VTAU, VK
80    FORMAT(10X,'TAU(ADJUSTED)=',F8.4,5X,'K(ADJUSTED)=',F8.4//)
      ZZK1=ZK1(M)/3276.7
      ZZK2=ZK2(M)/3276.7
      WRITE (3, 83)TAU1(M), TAU2(M), ZZK1, ZZK2
83    FORMAT (10X,'T+DT=',F8.4,10X,'T-DT=',F8.4,10X,'K+DK=',F8.4,
     1 10X,'K-DK=',F8.4)
      T1=ZISE1(M, 1)/3276.7
      T2=ZISE2(M, 1)/3276.7
      T3=ZISE1(M, 2)/3276.7
      T4=ZISE2(M, 2)/3276.7
      WRITE (3, 85) T1, T2
85    FORMAT (10X,'INT. SQ. ERROR (K+DK,T)=',F8.5,10X,'INT. SQ. ERROR
     1 (K-DK,T)=',F8.5)
      WRITE (3, 90) T3, T4
90    FORMAT(10X,'INT. SQ. ERROR (K,T+DT)=',F8.5,10X,'INT. SQ. ERROR
     1(K,T-DT)=',F8.5//)
      WRITE (3, 95) A, B, C, D
95    FORMAT (10X,'A=',F8.4,10X,'B=',F8.4,10X,'C=',F7.4,10X,'D=',
     1 F7.4)
      ZA=AMOD(M)/3276.7
      ZC=CMOD(M)/3276.7
```

```
      WRITE(3,100) ZA, BMOD(M), ZC, DMOD(M)
  100 FORMAT(10X,'A(ADJUSTED)=',F8.4,10X,'B(ADJUSTED)=',F8.4,10X,
     1 'C(ADJUSTED)=',F7.4,10X,'D(ADJUSTED)=',F7.4////)
C     ADJUSTING THE PARAMETERS
  105 ZK3(M+1)=ZK3(M)+(ZISE2(M,1)-ZISE1(M,1))*AMOD(M)/CMOD(M)
      ZK1(M+1)=ZK3(M+1)+CMOD(M+1)
      ZK2(M+1)=ZK3(M+1)-CMOD(M+1)
      TAU3(M+1)=TAU3(M)*3276.7+((ZISE2(M,2)-ZISE1(M,2))*(BMOD(M)/
     1 (DMOD(M)*50. )))
      TAU3(M+1)= TAU3(M+1)/3276.7
      TAU1(M+1)=TAU3(M+1)+DMOD(M+1)
      TAU2(M+1)=TAU3(M+1)-DMOD(M+1)
C     INCREASING CONSTANT BY ONE ( FOR LABLING PROGRAM NUMBER )
      ZLOO=ZLOO+1.0
  110 CONTINUE
  113 WRITE (1, 933)
  933 FORMAT('PROGRAM COMMENTS (SEE PREVIOUS INSTRUCTIONS)'////)
      READ (6, 934) JUNK2
  934 FORMAT (79X, A1)
      WRITE (1, 935)
  935 FORMAT (////)
C     NOW CHOOSE IF PROGRAM IS TO BE RUN AGAIN
C     DATA SWITCH FOR INPUT-OUTPUT DECK OPTION ( UP FOR DECK PUNCHING )
      CALL DATSW ( 6, IDUP )
      IF ( 2-IDUP )  913, 913, 911
C     CARDS SO THAT THE CARD PUNCH WILL NOT PUNCH HOLES IN EXECUTE CARD
  911 READ (2,981) JJJ
  981 FORMAT (I1)
      READ (2,982)  KKK
  982 FORMAT (I1)
      WRITE ( 2, 912 )  ( INPUT(I), IOUT(I),  I = 1, 300)
  912 FORMAT ( 10X,I15, 10X, I15 )
  913 CONTINUE
C     CONSOL INSTRUCTION ON WHETHER OR NOT TO CONTINUE
      WRITE (1, 115)
  115 FORMAT('DO YOU WISH TO RUN THE PROGRAM AGAIN&  IF YES TYPE IN
     1 1,  IF NO TYPE IN 2')
```

```
C        READ CARD FOR PREVIOUS CONSOL INSTRUCTION
          READ (6, 120) NUMB
 120      FORMAT (I1)
          IF (1-NUMB) 125, 926, 926
 926      WRITE (1, 927)
 927      FORMAT (//'DO YOU WISH TO RE-RUN THE PROGRAM USING THE SAME DAT
     1A.  IF YES TYPE 1 IN THE FIRST SPACE, IF NO TYPE IN A 2.')
          READ (6, 929)  NYES
 929      FORMAT (I1)
          IF (1-NYES)  535, 600, 600
C        RE-SETTING VALUES ON DACS FOR RE-RUNNING OF PROGRAM
 600      IVALU(1)=0
          IVALU(2)=0
          IVALU(3)=0
          IVALU(4)=0
          CALL LPDA (0, 3, IVALU, IER)
          CALL TLDA
          IFAST(5)=0
          IFAST(6)=0
          CALL LPDA (4,5, IFAST, IER)
          CALL TLDA
C        CHANGING CONSTANTS
          ZLOO=1.0
          ZPRO=ZPRO+1.0
C        CONSOL INSTRUCTION
          WRITE (1, 873)
 873      FORMAT (/'PROGRAM IS BEING RE-RUN USING SAME SAMPLING DATA'/)
          GO TO 45
 535      WRITE (1, 969)
 969      FORMAT (//////)
          GO TO 1
 125      CALL EXIT
          END
```

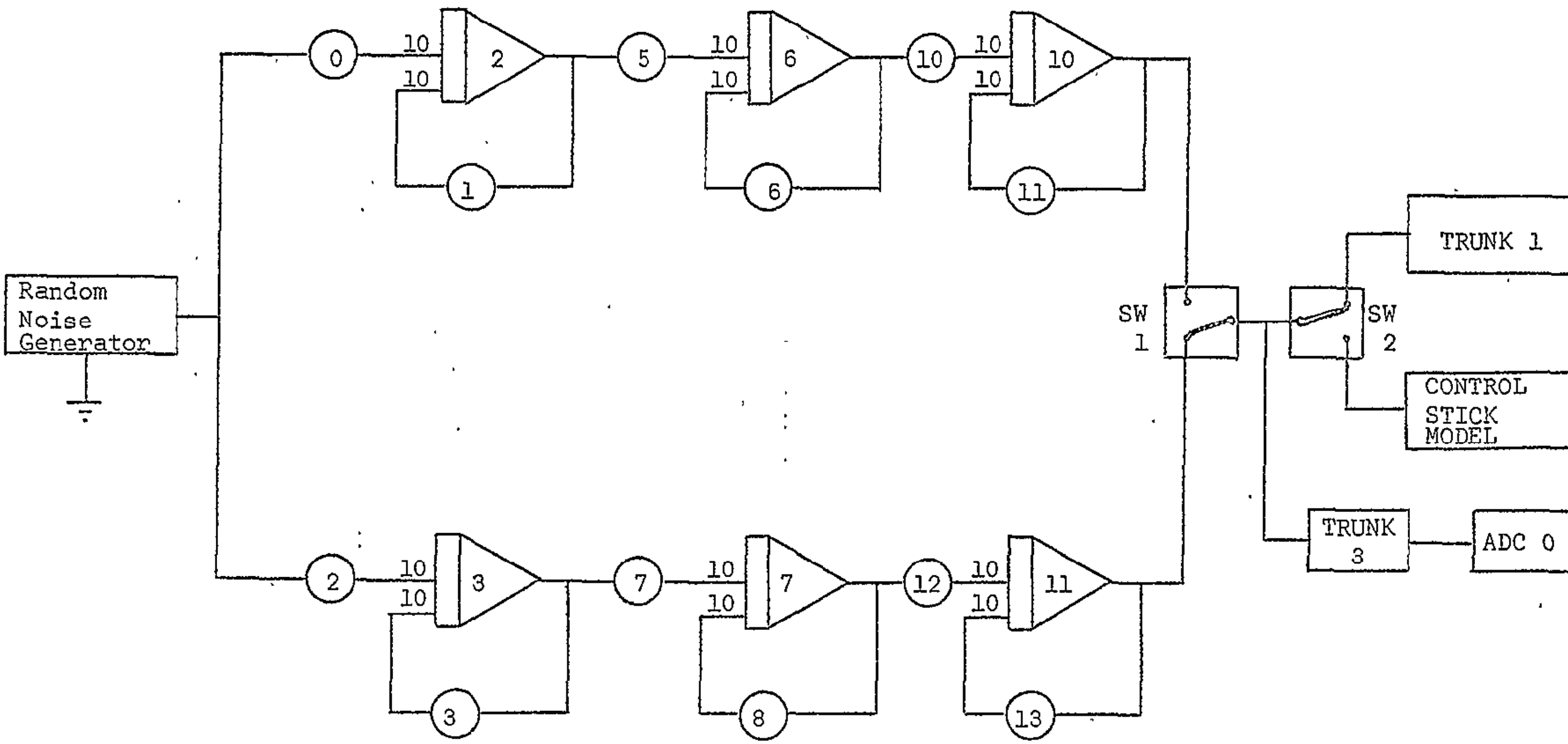


Fig.. B1 Input Signal Generation -- On the TR48.

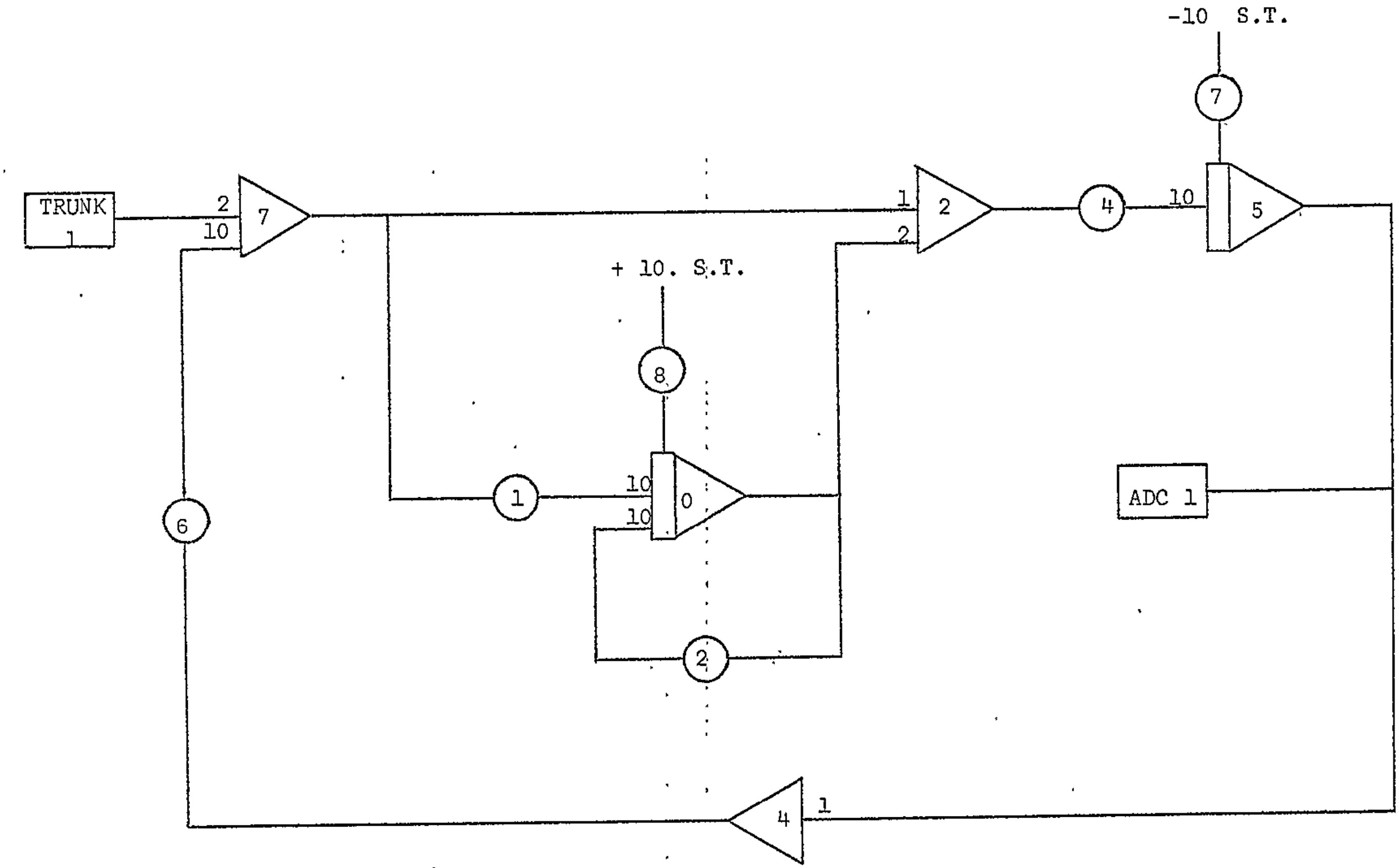


Fig. B2 A "Known" Crossover Model

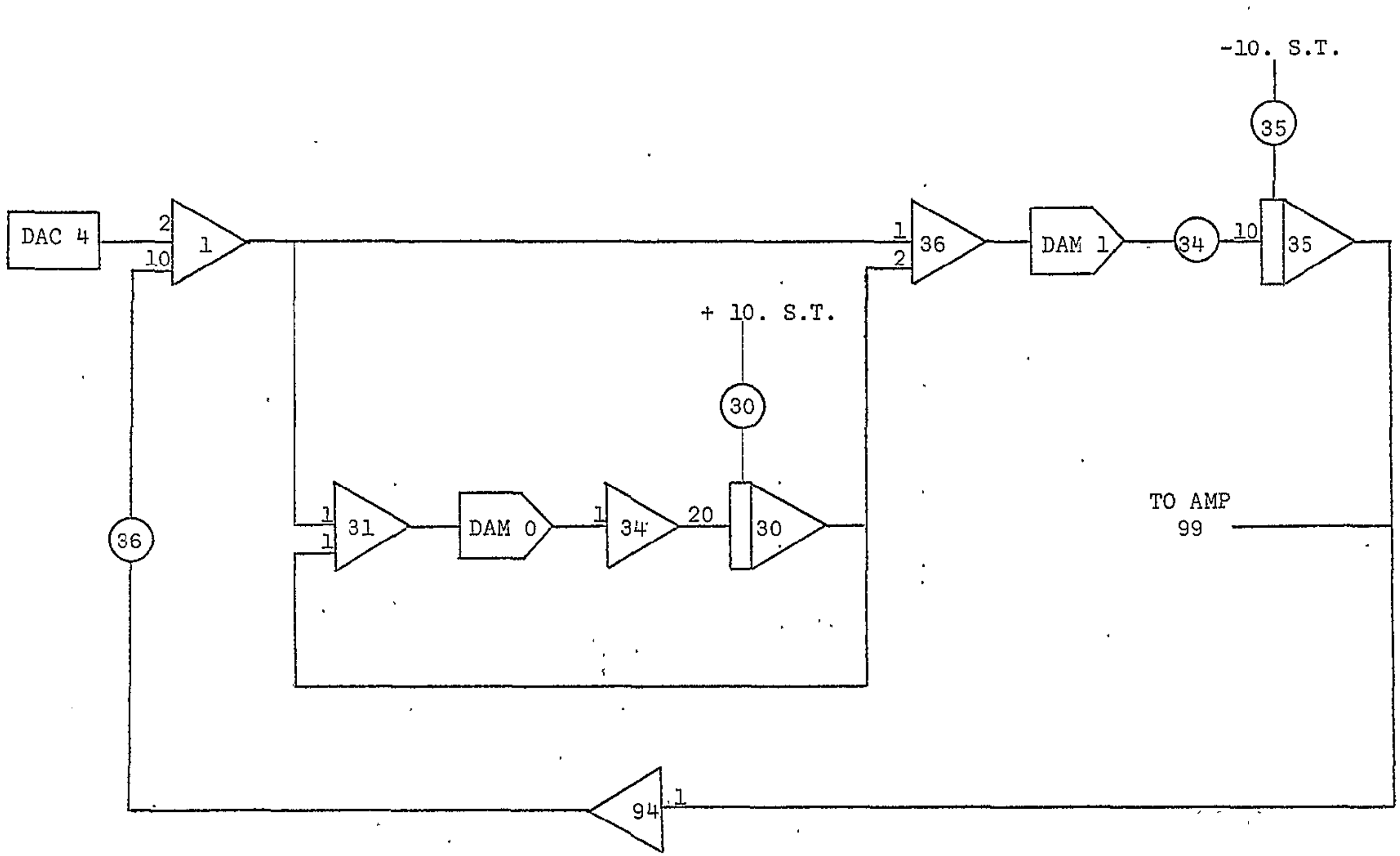


Fig. B3 Adjustable Crossover Model Number One.

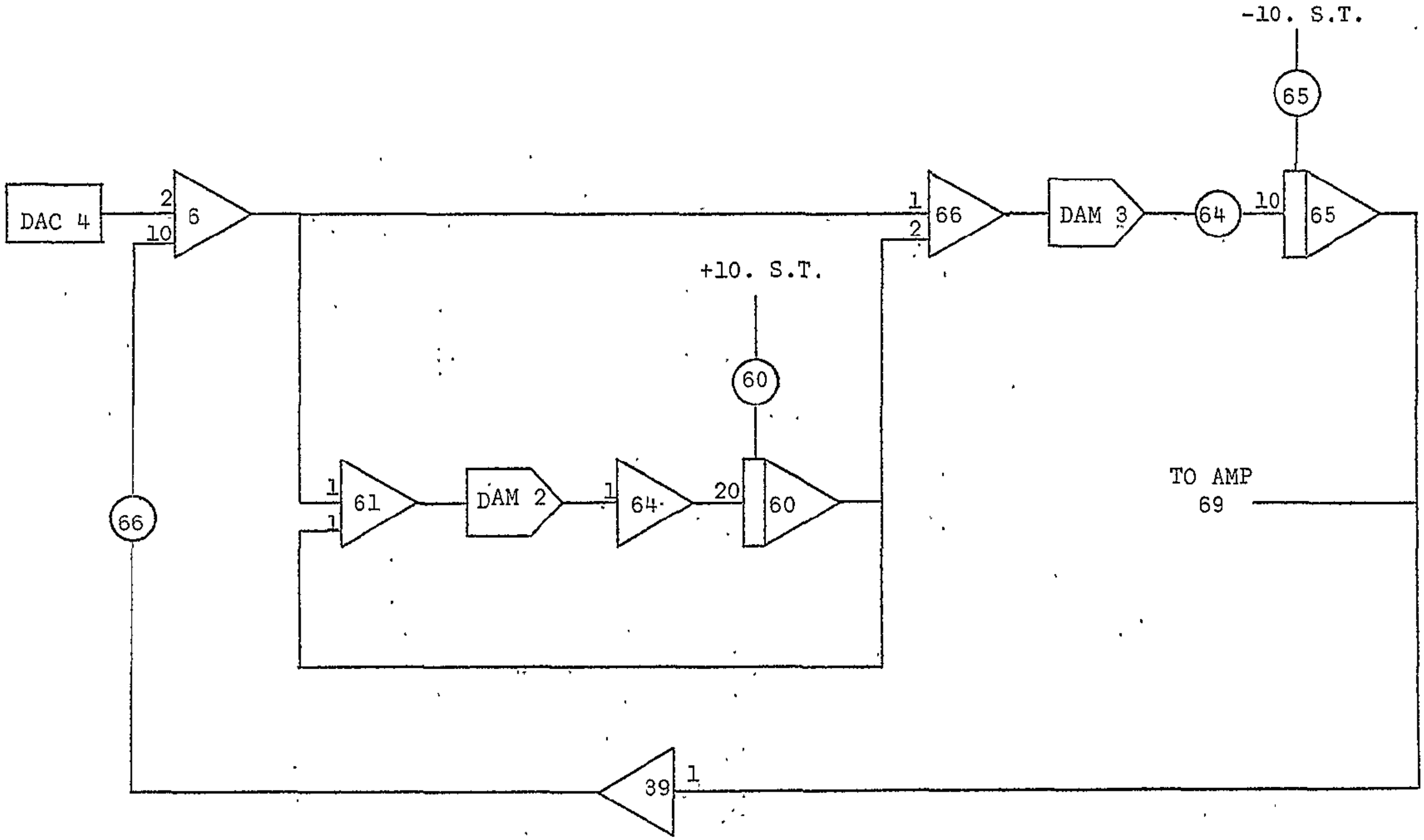


Fig. B4 Adjustable Crossover Model Number Two.

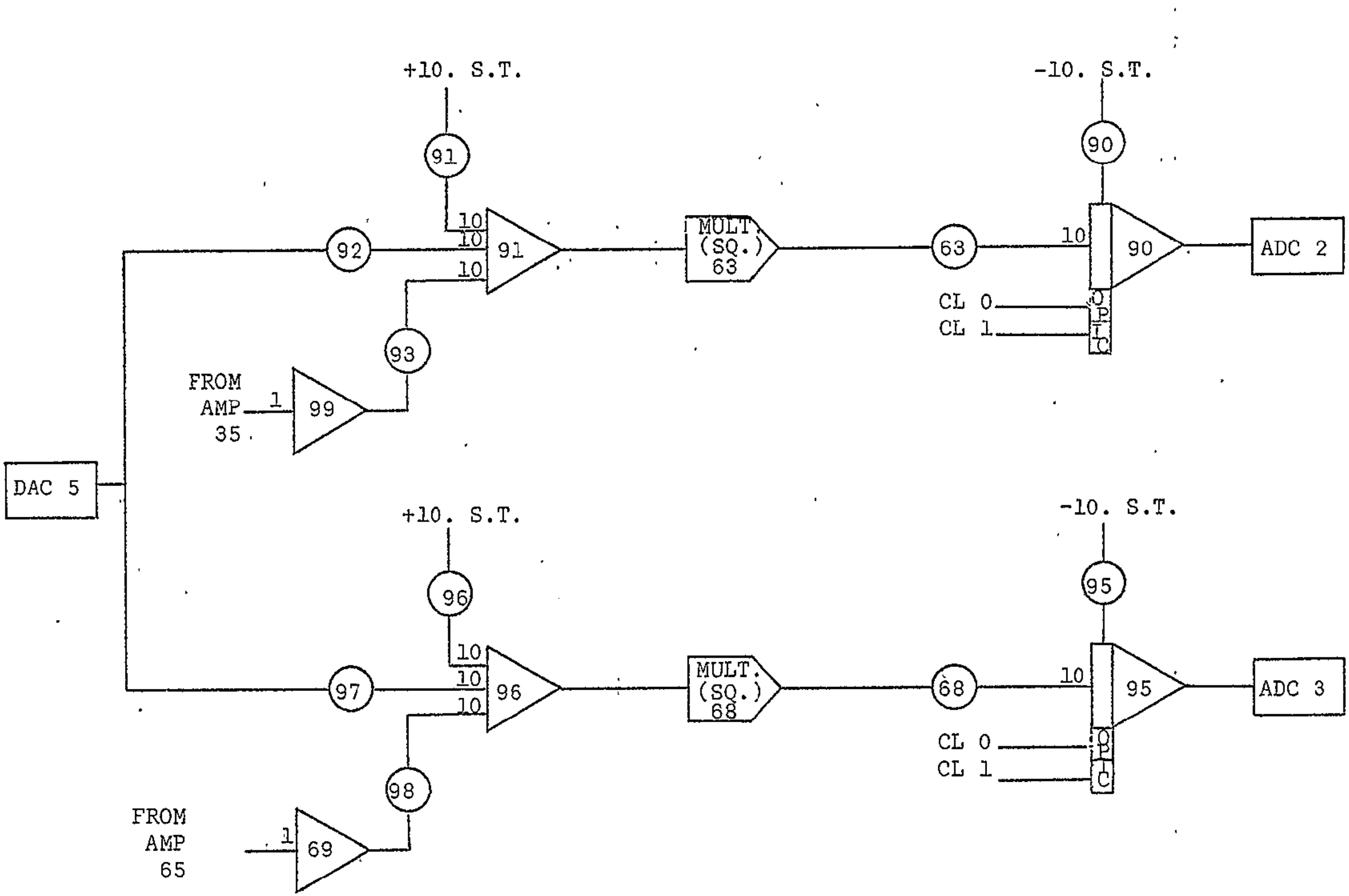


Fig. B5 Performance Index Network.

AMPLIFIER ASSIGNMENT SHEET DATE 6-8-70

NAME J. Kenning, G. Jackson

PROBLEM Modified Stochastic Approximation Method

| Variable | Amp. # | Static Check | Normal IC | Peak Voltage | Comments |
|---|---|---|---|---|---|
| $\theta_o(t)$ | 5 | 5.0 | | | |
| $-\theta_o(t)$ | 4 | -5.0 | | | Figure B 12 |
| | 7 | 10.0 | | | |
| | 0 | -2.0 | | | |
| | 2 | -6.0 | | | |
| Out. Mod1 | 35 | 5.0 | | | |
| -Out. Mod2 | 94 | -5.0 | | | |
| | 1 | 10.0 | | | Figure B 13 |
| | 31 | -8.0 | | | |
| | 34 | 2.66 | | | |
| | 30 | -2.0 | | | |
| | 36 | -6.0 | | | |
| Out.Mod2 | 65 | 5.0 | | | |
| -Out.Mod2 | 39 | -5.0 | | | |
| | 6 | 10.0 | | | Figure B 14 |
| | 61 | -8.0 | | | |
| | 64 | 2.66 | | | |
| | 60 | -2.0 | | | |
| | 66 | -6.0 | | | |
| | 99 | -5.0 | | | |
| $10e_1(t)$ | 91 | 5.0 | | | |
| $J_{n1}$ or $J_{n3}$ | 90 | 5.0 | | | Figure B 15 |
| $10e_2(t)$ | 69 | -5.0 | | | |
| | 96 | 5.0 | | | |
| $J_{n2}$ or $J_{n4}$ | 95 | 5.0 | | | |
| $10e_1^2(t)$ | 63 | -2.5 | | | |
| $10e_2^2(t)$ | 68 | -2.5 | | | |