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## COMPUTER PROGRAM FOR PRELIMINARY DESIGN AND ANALYSIS OF V/STOL TIP-TURBINE FANS

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# COMPUTER PROGRAM FOR PRELIMINARY DESIGN AND ANALYSIS OF V/STOL TIP-TURBINE FANS by Henry C. Haller, Seymour Lieblein, and Bruce M. Auer <br> Lewis Research Center 

## SUMMARY

An analysis of a tip-turbine driven lift fan was developed for use in preliminary design or parametric investigation of tip-turbine lift fan systems applicable to V/STOL aircraft. An electronic digital computer program was developed to determine flow conditions and flow areas for a specific total thrust requirement over a range of fan stage and of turbine input variables.

The program developed has provisions for treating single-stage fan rotors with either one- or two-stage tip turbines mounted on the fan periphery. Both turbine types have uncooled impulse rotors with the single-stage designs employing impulse, diffusing, or no exit stators. The program also treats tip-turbine velocity diagram determination, blade row loss analysis, partial admission (for the single-stage turbine), and consideration of turbine-to-fan leakage. The fan-stage inputs required are pressure ratio, efficiency, and inlet axial Mach number.

Because of the detailed nature of the turbine loss analysis and provision for several stage types, the program is capable of handling turbine flows over a wide range of pressure and temperature levels. This allowed for solutions for a number of generator types or supply sources for the turbine flow.

Sample calculations were made for a high-pressure turbine supply case to exhibit the potential of the program.

## INTRODUCTION

The potential of the tip-turbine lift-fan concept for V/STOL applications has been investigated in numerous research efforts (refs. 1 to 7 ) and demonstrated by the XV-5A fan-in-wing research aircraft (refs. 8 and 9 ). In the tip-turbine fan concept, turbine blades are mounted around the periphery of the rotor of a single-stage ducted fan to
provide a compact drive arrangement. Lift fans employ relatively low fan-stage pressure ratios in order to obtain low discharge noise and low specific fuel consumption.

Lift propulsion systems with tip-turbine lift fans can be used in the design of shorthaul V/STOL transports (e.g., refs. 10 and 11). An airplane of this type might employ multiple fans in the wings or in pods near the fuselage or out along the wings. The principal characteristic of the tip-turbine lift fan is its relatively short axial depth, which could constitute a favorable advantage in V/STOL aircraft design.

Tip-turbine lift fans have been powered by the hot exhaust from a gas generator (turbojet engine) as in the case of the XV-5A airplane. However, another drive possibility is a compressed-air generator powerplant that would supply relatively cold highpressure air to the tip turbine. This concept could include burners ahead of the turbine. The compressed air drive approach might be considered if it is desired to minimize the size and weight of the fan turbine scroll or if large amounts of ducting are required between the fans and the powerplants (e.g., for engine-out safety).

The heart of the tip-turbine lift fan is a single-stage ducted-fan rotor with turbine blades attached to the periphery. Other major components include the fan and turbine exit stators, the support frames, the inlet bellmouth, a scroll, which distributes the supply flow to the tip turbine nozzles, the exit louvers, which provide a means for varying the fan exhaust flow direction, and a burner, if specified.

This analysis is a cycle study that allows a determination of the gross characteristics of tip-turbine lift fans for use in preliminary design or for parametric analytical studies of lift-fan systems. In the analysis developed herein, overall fan-stage performance parameters and turbine supply flow state are program inputs. Fan component size and required turbine flow rate for a prescribed total fan thrust are the major outputs. The tip turbine is treated in detail with an analysis that includes velocity diagram determination, blade row loss analysis, turbine-to-fan leakage, and scroll duct pressure losses. Four types of tip-turbine stage arrangements are investigated: a conventional single-stage turbine with impulse exit stators, a single-stage turbine with diffusing exit stators, a single-stage turbine without exit stators, and a two-stage turbine with impulse exit stators. In all four types of turbines, the rotors are designed for impulse operation.

This report presents a description of the system analysis and a working computer program that includes input and output listings. A sample calculation is also included to point out program operating characteristics, and a comparison of the four turbine stage types is presented to illustrate the program output variations.

## ANALYSIS

## Lift-Fan Model

A schematic drawing of the basic elements of the lift-fan system showing the fan stages and the turbine-flow-path station designations is shown in figure 1. The configuration illustrated is an outlet-stator fan stage with a single-stage turbine mounted on the periphery of the fan rotor. An inlet duct and a burner (not always required) are also shown in the figure. Compressed air or hot gas at total conditions $p_{1}^{\prime}$ and $T_{1}^{\prime}$ enters the duct and passes directly, or through a burner, to the entrance of the turbine scroll, which distributes the flow around the periphery to the turbine inlet stator. The flow is accelerated in the inlet stator and directed against the turbine rotor blades. Work is extracted from the rotor to drive the fan. The flow is then discharged either directly or through the exit stators, with the axial component of outlet velocity contributing to the overall thrust of the tip-turbine fan system.

Ambient air at $p_{10}^{\prime}$ and $T_{10}^{\prime}$ enters the fan rotor with velocity $V_{11}$. The rotor produces an increase in static and total pressure from inlet to exit to accelerate the fan air flow to velocity $\mathrm{V}_{15}$, thus providing the majority of the system thrust. Louvers are considered at the discharge of the fan for thrust vectoring.

A schematic cross sectional drawing of a tip-turbine lift-fan configuration as considered in reference 6 is shown in figure 2. This figure also shows a planform view of the fan system. Symbols designating all quantities used in the description of the tipturbine fan are given in appendix $A$.

## Approach

The primary objective of the computer program development described herein was to afford a means of obtaining preliminary design characteristics of tip-turbine lift fans that could serve as thrusters for V/STOL applications. The program is capable of designing a tip-turbine fan to provide a prescribed total thrust level, fan plus turbine, with specified fan-stage inputs such as overall efficiency and pressure ratio, inlet axial Mach number, rotor tip speed, and rotor inlet hub-tip ratio. It was presumed that these inputs would be obtained from detailed fan-stage aerodynamic design procedures.

Basic turbine inputs include the supply gas total temperature and total pressure, turbine stage arrangement, and specification of burning. Several types of turbine stage design were included to allow for good performance with respect to such factors as pressure level of turbine supply flow, low discharge noise, high stage efficiency, and reduced turbine-to-fan leakage. Turbine stage efficiency was determined in the calculation from
the resultant blade row velocity diagrams (mean radius values) in conjunction with prescribed blade row loss variations for stators (nozzle), rotors (buckets), and exit stators. Loss relations for the flow in the scroll and for gas leakage from the turbine to the outer flow of the fan upstream of the fan rotor were also included (see fig. 3). Further provision for the effects of losses was made by the inclusion of loss inputs for the turbine supply flow duct, the fan inlet, the fan and turbine outlet ducts, and the exit louvers. Burner temperature and pressure drop were inputs.

The principal outputs of the calculation program were the gas flow rate into the turbine required to produce the desired total thrust, the hub and tip diameters at the various fan and turbine stations, the diameter of the scroll inlet, and the maximum overall diameter of the unit.

## Turbine Stage Design

Four turbine stage designs based on impulse rotor operation were included in the analysis. Impulse rotor design (constant static pressure across rotor) was specified in all cases for simplicity and for minimum inlet stator exit static pressure. Low inlet stator discharge pressure is desired in order to minimize gas leakage from the turbine flow path to the fan flow path in the axial clearance in front of and behind the fan-turbine rotor.

The four-turbine-stage designs included are (1) a single-stage turbine with impulse exit stators, (2) a single-stage turbine with diffusing exit stators, (3) a single-stage turbine without exit stators, and (4) a two-stage turbine with impulse exit stators. A schematic diagram of the one- and two-stage tip turbine concepts is shown in figure 3. The figure also shows the potential leakage flow paths.

A diffusing exit stator was considered for the single-stage turbine to provide for reduced discharge velocity and noise. The diffusing exit stator also results in reduced inlet stator exit pressure with corresponding increase in turbine specific work output and with reduced turbine-to-fan leakage. However, these gains are obtained at the expense of reduced turbine discharge thrust and turbine efficiency. The stage with no exit stator was included to determine configurations with the minimum number of blade rows and minimum blade-row losses. Here, too, the objective is achieved at the expense of turbine discharge thrust. The two-stage configuration was introduced to maintain high overall efficiency levels and low discharge velocity (low noise) for cases with supply gas at high pressures.

Partial admission was provided in the development for the single-stage cases as a means for increasing turbine rotor blade height, if desired, or for simulating effects of localized flow reduction for purposes of thrust control.

## Development of Equations

The development of the equations describing the design of the tip-turbine lift-fan system is presented in detail in appendixes B to $G$. The main program, which gives the overall fan design and calls the various specific turbine subroutines, is presented in appendix $B$.

The main program calculation procedure yields an overall fan design, that is, fan component diameters and station thermodynamics. All analyses, resulting design equations, and program logic are given in detail in appendix B. The main program also outputs fan performance parameters such as thrust specific fuel consumption of the burner (if used), bypass ratio (ratio of fan flow to turbine flow), and fan thrust augmentation ratio (ratio of total thrust to the thrust that would be obtained from isentropic expansion to ambient of the supply gas flow at $\mathrm{p}_{1}^{\prime}$ and $\mathrm{T}_{1}^{\prime}$ ).

Appendix $C$ gives the procedures for determining turbine losses. The losses are considered to comprise profile losses, secondary flow losses, and shock losses. Appendix $C$ details the approach to the determination of these losses by use of correlations obtained from the available literature as reported by numerous investigators (refs. 12 to 14).

The single-stage turbine design subroutines are given in appendix D. This appendix includes the detailed analysis of the three single-stage turbine types: impulse exit stators, diffusing exit stators, and no exit stators. The loss analysis for diffusing exit stators is also given in appendix D .

The two-stage turbine analysis is presented in appendix E , and the leakage analysis required by all four turbine arrangements is given by appendix $F$. The various fluid property and computational subroutines are listed in detail in appendix G. A complete program listing is included and given in appendix H .

## COMPUTATIONAL PROCEDURE

Calculations were performed using an iterative procedure programmed into an electronic digital computer. The calculations were based on the relations and procedures described in the appendixes of this report. The calculation program comprises iteration. loops within iteration loops as shown on the iteration flow chart (fig. 4) for the main computational routine.

The program as written follows the following basic steps for obtaining the desired thrust for a tip-turbine lift fan:
(1) Read in turbine flow properties, trial efficiency, initial value of flow rate, and exit stator exit Mach number.
(2) Read in fan-stage pressure ratio and efficiency, inlet axial Mach number, and rotor tip speed.
(3) Calculate the shaft power and amount of turbine thrust.
(4) Calculate fan thrust, fan size, and turbine blade height. (Note that the total thrust (fan plus turbine) does not equal the desired thrust at this point.)
(5) Calculate turbine losses, leakage, and revised efficiency.
(6) Recalculate fan geometry and total thrust (note that total thrust does not equal desired thrust).
(7) Repeat steps (1) to (6) using a second input value of turbine flow rate.
(8) Use a linear interpolation of turbine weight flow and total thrust to obtain the required turbine flow rate for the specified thrust.
(9) Repeat steps (1) to (6) to obtain the correct total thrust, final fan turbine geometry, and cycle thermodynamics.

The calculated thrust and desired thrust have been shown to agree within $1 / 2$ percent based on this calculation procedure.

The entire procedure is repeated for a range of input values of the turbine stator exit axial Mach number $M_{S}$. The program then has the option to print the results obtained for each input value of $\mathrm{M}_{\mathrm{S}}$ or enter an optimization routine.

The optimization routine first plots turbine output values of inlet stator exit angle $\alpha_{5}$, turbine tip diameter $d_{T, t}$, rotor inlet relative Mach number $M_{r, 5}$, turbine rotor blade height $\mathrm{H}_{\mathrm{T}}$, turbine weight flow required $\mathrm{w}_{1}$, and turbine louver exit axial velocity $V_{L}$, all against exit stator exit Mach number $M_{S}$. Over the range of $M_{S}$ used, the program then obtains a value of $\mathrm{M}_{\mathrm{S}}$ that yields minimum turbine flow rate $\mathrm{w}_{1}$. Before this value is accepted, a check is made on $\alpha_{5}$ to determine whether the angle is less than the limiting value (taken as $\alpha_{5}<75^{\circ}$ or 1.31 rad ). Angles larger than $75^{\circ}$ result in excessive inlet stator losses. The other outputs are visually inspected to insure that the turbine rotor blade height $\mathrm{H}_{\mathrm{T}}$ exceeds a minimum acceptable value.

A sample computer output plot of the variation of these parameters for the singlestage turbine without exit stators is given in figure 5. The parameters are normalized to the values obtained for $M_{S}=0.70$ of table $I$. Similar curves can be obtained for the other three turbine arrangements investigated.

The major inputs required for the main computational routine are listed in appendix B. Additional inputs required by the turbine subroutines are given in appendixes $D$ and $E$. A complete listing of all program inputs is given in table $I$, which is a sample program input-output listing. The definition of the variables in table I are given in the nomenclature definitions of appendix $I$.
Sample calculations were run for the tip-turbine lift fan with the four turbine stage configurations over a range of turbine stator exit Mach number $M_{S}$. A total thrust (turbine plus fan) of 10000 pounds was chosen for the representative cases. The turbines were specified as full admission and were to be within the limits of practical design, that is, reasonable velocity diagrams, turbine rotor blade height, stator diffusion, and so forth. An available turbine supply flow pressure ratio $p_{1}^{\prime} / p_{0}$ of 8.0 was specified along with a supply total temperature $T_{1}^{\prime}$ of $1080^{\circ}$ R. Burning was also specified to bring the turbine scroll inlet temperature $\mathrm{T}_{3 \mathrm{sc}}^{\prime}$ up to $1900^{\circ} \mathrm{R}$. A list of the remaining major input variables common to all four turbine types are
Inlet supply duct pressure loss ratio, $\mathrm{p}_{2}^{\prime} / \mathrm{p}_{1}^{\prime}$ ..... 0.95
Burner efficiency, $\eta_{b}$ ..... 0.98
Burner total pressure loss fraction, $\Delta \mathrm{p}^{\prime} / \mathrm{p}_{2}^{\prime}$ ..... 0.06
Fan-stage pressure ratio, $p_{13}^{\prime} / p_{11}^{\prime}$. ..... 1.20
Fan inlet axial Mach number, $\mathrm{M}_{10}$. ..... 0.55
Fan rotor hub-tip ratio, $Y_{F}$ ..... 0.35
Air flow supply duct inlet Mach number, $M_{1}$ ..... 0.20
Fan rotor tip speed, $U_{F, t}, f t / s e c$ ..... 700
Fan-stage efficiency, $\eta_{F}$ ..... 0.86
Scroll Mach number, $\mathrm{M}_{3 \mathrm{sc}}$ ..... 0.30The remaining inputs for the four turbine types are given along with a sample of the cor-responding output listings in table $I$ which is for the specific case listed previously.
The sample sets of data for the specific cases investigated were used to exhibit the comparative variations in geometry and performance for the four turbine stage configurations. Figure 6 shows a plot of the variation in required turbine flow rate with stator exit axial Mach number. It is seen from the figure that turbine flow rate does not vary greatly with stator exit axial Mach number. The figure also indicates the practical design limitation imposed on inlet stator exit angle. For the inlet conditions investigated, the two-stage turbine yielded the lowest flow rate because of the inherently higher turbine total efficiency for these inlet conditions. The single-stage turbine with an impulse stator is shown as a single point because realistic turbine velocity diagrams ( $\alpha_{5}<75^{\circ}$ ) are not obtainable for stator exit Mach numbers less than 0. 80.
Another important aspect of turbine design is shown in figure 7 which is a plot of turbine exit flow velocity against stator exit axial Mach number. Because of noise considerations, it is desirable to reduce the turbine exit flow velocity to meet the requirement for low discharge noise. Thus, designing a turbine to the maximum nozzle exit angle of around $75^{\circ}$ yields minimum exit flow velocity in all cases. The two-stage turbine has the lowest velocity (fig. 7).

The velocities for the two-stage and the single-stage with impulse or diffusing exit stators all fall on essentially a single line because of the specification of axial discharge. The slight variation is due to varying efficiency and thus exit temperature. The exit velocity for the single-stage turbine without an exit stator is higher than the other three turbine designs because of the residual tangential velocity. Also, the higher axial velocity across the rotor is inherent with this configuration.

The single-stage turbine without exit stators and the single-stage turbine with impulse exit stators would probably be unacceptable designs for the supply gas pressure level because of the high discharge velocities.

The calculated variation of turbine-tip diameter with exit stator exit Mach number $\mathrm{M}_{\mathrm{S}}$ for the four turbine arrangements is shown in figure 8. The limit for $\alpha=75^{\circ}$ is shown for each case.

## CONCLUDING REMARKS

The program described herein is capable of handling turbine flows over a wide range of pressure and temperature levels because of the detailed nature of the turbine loss analysis and provision for several stage types. This capability allows for a wide range of generator types or sources for the turbine flow. The potential of the program was exhibited by a sample calculation for a high-pressure turbine supply case. Calculations performed with the program compared favorably with the tip-turbine fan designs for use with an exhaust gas generator system presented in reference 6. Thrust, geometry and turbine performance agreed very closely with the values of the reference cases when specified for equal turbine flow rate.

It is hoped that this working program will serve as a useful tool in tip-turbine fan preliminary design and for the comparison of various lift-fan propulsion systems for V/STOL aircraft applications.

Lewis Research Center,
National Aeronautics and Space Administration, Cleveland, Ohio, September 24, 1970, 721-03.

## APPENDIX A

## SYMBOLS

A flow area, in. ${ }^{2}$
AR aspect ratio, $\mathrm{H} / \mathrm{c}$
AUG thrust augmentation ratio
a arc length of turbine admission, in.
b exponent in seal leakage (eq. (F5))

B labyrinth tooth width
BPR bypass ratio
CC turbine rotor axial velocity multiplier

C constant
$\mathrm{C}_{\mathrm{f}} \quad$ flow coefficient
cl seal clearance, in.
$C_{p} \quad$ specific heat at constant pressure, Btu/(lbm) ${ }^{0} R$ )
$\mathrm{C}_{\mathbf{T}} \quad$ thrust coefficient
c
blade chord, in.
D diffusion factor
d diameter, in.
E energy loss, Btu/lb
e expansion function
FF flow factor, $(w / A)\left(\sqrt{R^{\prime}} / p^{\prime}\right)$
F thrust, lbf
f/a fuel-to-air ratio
G fuel-to-air ratio function, $\left[\eta_{b}(f / a) / 1+(f / a)\right]$
g gravitational constant, $32.174 \mathrm{ft} / \mathrm{sec}^{2}$

H height, in.
HV lower heating value of fuel, Btu/lbm
$\mathrm{H}^{\mathrm{O}} \quad$ inlet enthalpy of the liquid fuel, Btu/lbm
h
$\Delta \mathrm{h}$

IPR

J

K
L
M
n

P
$\Delta p$
p
q
R

Re
r

SFC
enthalpy, Btu/lbm
specific work or enthalpy change, Btu/lbm
ideal incompressible pressure recovery (eq. (B26))
mechanical equivalent of heat, $778 \mathrm{ft}-\mathrm{lb} / \mathrm{Btu}$
loss coefficient
length, in.
Mach number
number of (blades, burners, etc.)
power, Btu/sec
pressure drop, psi
pressure, psia
veiocity head, $p^{\prime}-p$, psia
gas constant, $1545 /$ molecular weight, $\mathrm{ft}-\mathrm{lbf} /(\mathrm{lbm})\left({ }^{\circ} \mathrm{R}\right)$

Reynolds number
radius, in.
specific fuel consumption, lbm/(hr)(lbf)


| N | turbine stator blade row | 3 | burner exit |
| :--- | :--- | :--- | :--- |
| p | profile | 3 sc | scroll inlet |
| pa | partial admission | 4 | turbine inlet |
| r | relative | 5 | first turbine stator exit |
| S | exit stator exit | $5^{,}$ | first turbine rotor inlet |
| s | sonic | 6 | first turbine rctor exit |
| sc | scroll | 7 | second turbine stator exit |
| se | secondary | 8 | second turbine rotor exit |
| sh | shock | 10 | fan inlet |
| st | stoichiometric | 11 | fan rotor inlet |
| T | turbine | 12 | fan rotor exit |
| t | tip | 13 | fan stator exit |
| u | circumferential component | 14 | fan louver inlet |
| x | axial component | 15 | fan louver exit |
| 0 | ambient air | Superscripts: |  |
| 1 | air or gas generator supply | , | stagnation condition |
| 2 | inlet duct exit | $*$ | reference |
| $2 b$ | burner inlet | - | overall |

## APPENDIX B

## FAN-TURBINE MAIN PROGRAM

The main program produces an overall fan design, that is, fan overall dimensions, total thrust, and thermodynamics. The fan-turbine design chosen for this analysis is shown in figure 1. This figure gives the station designations for both the tip turbine and the fan. For the two-stage tip turbine, figure 1 is modified by including station 7 for the interstage stator exit and station 8 for the second-stage rotor exit (fig. 9).

The Fan-Turbine Design is initiated by assuming the following major inputs:
(1) Inlet duct mass flow rate $w_{1}$
(2) Inlet duct inlet total pressure $p_{1}^{\prime}$
(3) Inlet duct inlet total temperature $\mathrm{T}_{1}^{\prime}$
(4) Turbine stage total-to-total efficiency $\eta_{T}$
(5) Fan-stage pressure ratio $p_{13}^{\prime} / p_{11}^{\prime}$
(6) Fan inlet axial Mach number $\mathrm{M}_{11}$
(7) Fan tip speed $U_{F, t}$
(8) Fan-stage adiabatic efficiency $\eta_{\mathrm{F}}$

The program nomenclature required for the main program and the turbine subroutines are included in appendix I.

This appendix details all the equations used to define the thermodynamics of the fan and turbine streams along with the descriptive equations for the tip turbine fan radial dimensions. The calculational procedure is initiated by first analyzing the turbine stream. Then, on obtaining the shaft power output, the fan stream calculations are made.

## Turbine Calculations

The inlet duct, as shown in figure 1 accepts flow from a gas or air generator source and directs it to the turbine at which point power is extracted. The turbine exit flow is then passed through an exit stator and louvers and ejected to supply a portion of the tipturbine fan thrust. The turbine flow path is thus composed of a supply duct, burner, scroll, turbine, exit stator, and louvers.

Inlet duct. - The inlet duct accepts flow from a gas or air source at an inlet total pressure and temperature, $\mathrm{p}_{1}^{\prime}$ and $\mathrm{T}_{1}^{\prime}$, respectively. A trial value of flow rate $\mathrm{w}_{1}$ is required to initiate the calculations to obtain the flow area $A_{2}$.

The total pressure at the exit of the supply duct is obtained from

$$
\begin{equation*}
\mathrm{p}_{2}^{\prime}=\mathrm{p}_{1}^{\prime}\left(\frac{\mathrm{p}_{2}^{\prime}}{\mathrm{p}_{1}^{\prime}}\right) \tag{B1}
\end{equation*}
$$

where $p_{2}^{\prime} / p_{1}^{\prime}$ is a duct loss factor and an input. With provision for heat loss in the supply duct, where $\Delta T_{d}^{\prime}$ is a program input,

$$
\begin{equation*}
T_{2}^{\prime}=T_{1}^{\prime}-\left(\Delta T_{d}^{\prime}\right) \tag{B2}
\end{equation*}
$$

With no bleed flow from the duct,

$$
\begin{equation*}
w_{2}=w_{1} \tag{B3}
\end{equation*}
$$

The flow factor at the duct exit is

$$
\begin{equation*}
\mathrm{FF}_{2}=\frac{\mathrm{M}_{2} \sqrt{\gamma_{2} \mathrm{~g}}}{\left(1+\frac{\gamma-1}{2} \mathrm{M}^{2}\right)_{2}^{\left(\gamma_{2}+1\right) / 2\left(\gamma_{2}-1\right)}}=\left(\frac{\mathrm{w}}{\mathrm{~A}} \frac{\sqrt{\mathrm{RT}}}{\mathrm{p}^{\prime}}\right)_{2} \tag{B4}
\end{equation*}
$$

where $\mathrm{M}_{2}$ is a program input and $\gamma_{2}$ is obtained from

$$
\begin{equation*}
\gamma_{2}=\frac{C_{p, 2}}{\left(C_{p, 2}-\frac{R_{2}}{778.16}\right)} \tag{B5}
\end{equation*}
$$

with the fluid property subroutines (appendix G) used to provide

$$
\mathrm{C}_{\mathrm{p}, 2}=\text { function }\left(\mathrm{T},\left.\frac{\mathrm{f}}{\mathrm{a}}\right|_{1}, \eta_{\mathrm{b}}\right)
$$

and

$$
\mathrm{R}_{2}=\text { function }\left(\left.\frac{\mathrm{f}}{\mathrm{a}}\right|_{1}\right)
$$

If high pressure air is supplied, $\left.\frac{f}{\mathrm{a}}\right|_{1}$ is set equal to zero. For both the air and hot-gas cases the burner efficiency $\eta_{b}$ is equal to 0 , since the gas generator burner is not considered in these calculations. The inlet duct exit area is then obtained from

$$
\begin{equation*}
A_{2}=\left(\frac{w \sqrt{T^{\prime} R}}{p^{\prime} F F}\right)_{2} \tag{B6}
\end{equation*}
$$

Burner. - The diffuser section is between the inlet duct exit and the burner. The diffuser section flow is treated as an isentropic flow with the resulting equation for burner inlet area dependent on the number of burners $n_{b}$ and the burner reference Mach number $M_{2 b}$, both of which are inputs. All losses in the diffuser section are lumped in the input burner pressure loss fraction. From continuity,

$$
\begin{equation*}
A_{2 b}=\frac{A_{2}}{n_{b}}\left(\frac{M_{2}}{M_{2 b}}\right)\left(\frac{1+\frac{\gamma_{2}-1}{2} M_{2 b}^{2}}{1+\frac{\gamma_{2}-1}{2} M_{2}^{2}}\right)^{\left(\gamma_{2}+1\right) / 2\left(\gamma_{2}-1\right)} \tag{B7}
\end{equation*}
$$

and the diameter for a can type of burner is then

$$
\begin{equation*}
d_{2 b}=\sqrt{\left(\frac{4}{\pi}\right) A_{2 b}} \tag{B8}
\end{equation*}
$$

The burner exit temperature (station 3) is set equal to the desired inlet stator inlet temperature, thus

$$
\begin{equation*}
\mathrm{T}_{3}^{\prime}=\mathrm{T}_{4}^{\prime} \tag{B9}
\end{equation*}
$$

The required burner fuel-to-air ratio is obtained from the expression

$$
\begin{equation*}
\left.\frac{\mathrm{f}}{\mathrm{a}}\right|_{\mathrm{b}}=\frac{\left(\mathrm{h}_{3}^{\prime}-\mathrm{h}_{2}^{\prime}\right)\left[1+\left(\frac{\mathrm{f}}{\mathrm{a}}\right)_{1}\right]}{H V \eta_{\mathrm{b}}-\left(\mathrm{h}_{3}^{\prime}-\mathrm{H}^{\mathrm{O}}\right)} \tag{B10}
\end{equation*}
$$

where $H^{\circ}$ is the inlet enthalpy of the liquid fuel, and $h_{2}^{\prime}$ and $h_{3}^{\prime}$ are based on $T_{2}^{\prime}$ and $T_{3}^{\prime}$, respectively. This equation is given in the fuel-to-air ratio subroutine (appendix G).

The total mass flow of the gas mixture is then

$$
\begin{equation*}
w_{3}=w_{2}\left(T+\left.\frac{f}{a}\right|_{4}\right) \tag{B11}
\end{equation*}
$$

where the burner exit flow state is given by the following expressions:

$$
\begin{equation*}
\gamma_{3}=\frac{c_{p, 3}}{\left(C_{p, 3}-\frac{R_{3}}{778.16}\right)} \tag{B12}
\end{equation*}
$$

where, from the fluid property subroutines,

$$
\begin{equation*}
\mathrm{C}_{\mathrm{p}, 3}=\text { function }\left(\mathrm{T}_{3}^{\prime},\left.\frac{\mathrm{f}}{\mathrm{a}}\right|_{4}, \eta_{\mathrm{b}}\right) \tag{B13}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{R}_{3}=\text { function }\left(\left.\frac{\mathrm{f}}{\mathrm{a}}\right|_{4}\right) \tag{B14}
\end{equation*}
$$

In equations (B11), (B13), and (B14), the total fuel-to-air ratio is defined as

$$
\left(\frac{\mathrm{f}}{\mathrm{a}}\right)_{4}=\left(\frac{\mathrm{f}}{\mathrm{a}}\right)_{1}+\left(\frac{\mathrm{f}}{\mathrm{a}}\right)_{\mathrm{b}}
$$

The burner outlet total pressure is obtained from the expression

$$
\begin{equation*}
\mathrm{p}_{3}^{\prime}=\mathrm{p}_{2}^{\prime}\left(1-\frac{\Delta \mathrm{p}^{\prime}}{\mathrm{p}_{2}^{\prime}}\right) \tag{B15}
\end{equation*}
$$

where burner inlet pressure $p_{2}^{\prime}$ is obtained from equation (B1). The value of $\Delta p^{\prime} / p_{2}^{\prime}$ is an input and is generally obtained from experimental results for specific applicable burner designs and is expected to be functions of reference Mach number $M_{2 b}$ and
temperature ratio $T_{3}^{\prime} / T_{2}^{\prime}$. The flow factor at the burner exit (constant area burner) is

$$
\begin{equation*}
\mathrm{FF}_{3}=\frac{\mathrm{w}_{3} \sqrt{\mathrm{~T}_{3}^{\prime} \mathrm{R}_{3}}}{\mathrm{p}_{3}^{\prime} \mathrm{A}_{2 \mathrm{~b}}} \tag{B16}
\end{equation*}
$$

The burner exit Mach number $\mathrm{M}_{3}$ is obtained from $\mathrm{FF}_{3}$ and the function FMM subroutine, where $\mathrm{M}_{3}$ and $\mathrm{FF}_{3}$ are related by an expression of the form of equation (B4).

The burner exit also has a transition duct which accelerates the flow and connects to the scroll inlet. The friction pressure loss in this duct is also included in the overall burner pressure-drop fraction. Thus, for the calculation, $p_{3 s c}^{\prime}=p_{3}^{\prime}$. If a nonburning case is to be analyzed, the burner pressure-drop fraction is set equal to zero, and temperature $\mathrm{T}_{4}^{\prime}$ is set equal to $\mathrm{T}_{2}^{\prime}$. This in effect bypasses the burner entirely.

Scroll. - The flow, on leaving the burner, enters the turbine scroll. The scroll could have from one to four duct inlets where the number of duct inlets equals the number of burners $n_{b}$ with one or two scroll arms per inlet $n_{A}$, depending on the ducting configuration used. Some typical scroll configurations are shown on figure 10. Thus, the number of scroll arms $n_{s c}$ for a full admission turbine scroll is given by the expression

$$
\begin{equation*}
n_{s c}=n_{b}\left(n_{A}\right) \tag{B17}
\end{equation*}
$$

Then, with the number of scroll arms $n_{s c}$ and the prescribed scroll inlet Mach number $\mathrm{M}_{3 \mathrm{sc}}$ known, the inlet area of a scroll arm is calculated using continuity given as (the scroll is assumed to be isothermal, $\mathrm{T}_{4}^{\prime}=\mathrm{T}_{3 \mathrm{sc}}^{\prime}=\mathrm{T}_{3}^{\prime}$ )

$$
\begin{equation*}
A_{3 s c}=A_{2 b}\left(\frac{1}{n_{s c}}\right)\left(\frac{M_{3}}{M_{3 s c}}\right)\left(\frac{1+\frac{\gamma_{3}-1}{2} M_{3 s c}^{2}}{1+\frac{\gamma_{3}-1}{2} M_{3}^{2}}\right)^{\left(\gamma_{3}+1\right) / 2\left(\gamma_{3}-1\right)} \tag{B18}
\end{equation*}
$$

The inlet diameter of each scroll arm is then

$$
\begin{equation*}
\mathrm{d}_{3 \mathrm{sc}}=\sqrt{\frac{4 \mathrm{~A}_{3 \mathrm{sc}}}{\pi}} \tag{B19}
\end{equation*}
$$

The loss in total pressure in the scroll is obtained as a function of the inlet dynamic
head $q_{3 \text { sc }}$. Thus, the total pressure $p_{4}^{\prime}$ at the scroll exit is

$$
\begin{equation*}
\mathrm{p}_{4}^{\prime}=\mathrm{p}_{3 \mathrm{sc}}^{\prime}-\mathrm{K}_{\mathrm{sc}} \mathrm{q}_{3 \mathrm{sc}} \tag{B20}
\end{equation*}
$$

where the loss coefficient $\mathrm{K}_{\mathrm{Sc}}$ is an input and is approximated by the momentum loss associated with the number of turns the fluid is required to make. For a typical scroll that has three fluid turns, a $\mathrm{K}_{\mathrm{Sc}}$ value of 0.6 was selected. The dynamic head is defined in terms of Mach number as

$$
\begin{equation*}
q_{3 \mathrm{sc}}=\mathrm{p}_{3 \mathrm{sc}}^{\prime}\left[1-\frac{1}{\left(1+\frac{\gamma_{3}-1}{2} \mathrm{M}_{3 \mathrm{sc}}^{2}\right)^{\gamma_{3} /\left(\gamma_{3}-1\right)}}\right] \tag{B21}
\end{equation*}
$$

Turbine exit stator. - A knowledge of the thermodynamic state of the gas flow entering the turbine stators along with the fact that the turbine is not treated in detail in the main program, requires only turbine exit state conditions to determine the ideal specific power output of the turbine. Thus, it is necessary to supply the flow thermodynamics at the turbine exit stator exit.

The exit stator exit axial Mach number $\mathrm{M}_{\mathrm{S}}$ is a program input, and along with a trial value of $\gamma_{S}=1.35$ (A better value will be obtained when the stator exit total temperature is determined.) allows the calculation of the flow factor at the stator exit from the expression

$$
\begin{equation*}
\mathrm{FF}_{\mathrm{S}}=\frac{\mathrm{M}_{\mathrm{S}} \sqrt{\gamma_{S^{\mathrm{g}}}}}{\left(1+\frac{\gamma-1}{2} \mathrm{M}^{2}\right)_{\mathrm{S}}^{\left(\gamma_{\mathrm{S}^{+1}}\right) / 2\left(\gamma_{S^{-1)}}\right.}} \tag{B22}
\end{equation*}
$$

or for $\gamma_{S}=1.35$

$$
\begin{equation*}
\mathrm{FF}_{\mathrm{S}}=\frac{6.591 \mathrm{M}_{\mathrm{S}}}{\left(1+0.175 \mathrm{M}_{\mathrm{S}}^{2}\right)^{3.3571}} \tag{B23}
\end{equation*}
$$

Then it is desired to determine the total pressure at the stator exit $p_{S}^{\prime}$, which then sets
the turbine pressure ratio and the total-to-total enthalpy drop across the turbine. It is specified first that

$$
\begin{equation*}
\mathrm{p}_{\mathrm{D}}=\mathrm{p}_{\mathrm{L}}=\mathrm{p}_{0} \tag{B24}
\end{equation*}
$$

Next, it is required to obtain the exit Mach number in the duct $M_{D}$. The duct is allowed to vary in flow area to provide flow deceleration if desired. Thus, the duct acts as a diffuser with the duct loss in total pressure given by the expression

$$
\begin{equation*}
\frac{p_{D}^{\prime}}{p_{S}^{\prime}}=1-\left(\frac{\Delta p_{D}^{\prime}}{q_{S}}\right)\left(\frac{q}{p^{\prime}}\right)_{S} \tag{B25}
\end{equation*}
$$

where the loss term $\Delta p_{D}^{\prime}$ is defined as the loss in total pressure. Using the data of reference 15 for an effective diffuser included angle of $10^{\circ}$ (an angle that yields a 10 -percent loss in static pressure recovery) the loss term is taken as

$$
\begin{equation*}
\frac{\Delta \mathrm{p}_{\mathrm{D}}^{\prime}}{\mathrm{q}_{\mathrm{S}}}=0.10(\mathrm{IPR}) \tag{B26}
\end{equation*}
$$

The ideal pressure recovery $I P R$ is defined in terms of area ratio $A_{D} / A_{S}$ as

$$
\begin{equation*}
\operatorname{IPR}=1-\left(\frac{1}{\frac{A_{D}}{A_{S}}}\right)^{2} \tag{B27}
\end{equation*}
$$

The term $q_{S} / p_{S}^{\prime}$ in equation (B25) is defined in terms of an input value of stator exit axial Mach number as

$$
\begin{equation*}
\frac{\mathrm{q}_{\mathrm{S}}}{\mathrm{p}_{S}^{\prime}}=1-\frac{1}{\left(1+\frac{\gamma-1}{2} \mathrm{M}^{2}\right)_{\mathrm{S}}^{\gamma_{S} /\left(\gamma_{S}-1\right)}} \tag{B28}
\end{equation*}
$$

Substitution of equations (B26) to (B28) into (B25) yields the expression for duct total pressure ratio

$$
\begin{equation*}
\frac{p_{D}^{\prime}}{p_{S}^{\prime}}=1-0.1\left[1-\frac{1}{\left(\frac{A_{D}}{A_{S}}\right)^{2}}\right]\left[1-\frac{1}{\left(1+\frac{\gamma-1}{2} M^{2}\right)^{\gamma /(\gamma-1)}}\right]_{S} \tag{B29}
\end{equation*}
$$

The duct annular area ratio $A_{D} / A_{S}$ is defined for a $10^{\circ}$ included angle and approximated by a two-dimensional diffuser due to the very high hub-to-tip ratio as

$$
\begin{equation*}
\frac{A_{D}}{A_{S}}=1+0.1744\left(\frac{L}{H}\right)_{D} \tag{B30}
\end{equation*}
$$

where the duct length to height ratio $(\mathrm{L} / \mathrm{H})_{\mathrm{D}}$ is an input. The height H is defined at the stator exit. If no duct diffusion is specified, the value of ( $\mathrm{L} / \mathrm{H})_{\mathrm{D}}$ is set equal to zero.

With $p_{D}^{\prime} / p_{S}^{\prime}$ from equation ( B 29 ) and $A_{D} / A_{S}$ from equation (B30), the duct flow factor is calculated from

$$
\begin{equation*}
\mathrm{FF}_{\mathrm{D}}=\frac{\mathrm{FF}_{\mathrm{S}}}{\left(\frac{\mathrm{p}_{\mathrm{D}}^{\prime}}{\mathrm{p}_{\mathrm{S}}^{\prime}} \frac{\mathrm{A}_{\mathrm{D}}}{\mathrm{~A}_{\mathrm{S}}}\right)} \tag{B31}
\end{equation*}
$$

Once the value of $F F_{D}$ is known, the duct exit Mach number $M_{D}$ is obtained using the function FMM subroutine given in appendix G. After the Mach number at the duct exit is determined, the total pressure is obtained from the expression (for $\gamma_{S}=1.35$ ):

$$
\begin{equation*}
\mathrm{p}_{\mathrm{D}}^{\prime}=\mathrm{p}_{0}\left(1+0.175 \mathrm{M}_{\mathrm{D}}^{2}\right)^{3.86} \tag{B32}
\end{equation*}
$$

Then, from equations (B29) and (B32), the total pressure at the stator exit is obtained as

$$
\begin{equation*}
\mathrm{p}_{\mathrm{S}}^{\prime}=\mathrm{p}_{\mathrm{D}}^{\prime}\left(\frac{1}{\frac{p_{\mathrm{D}}^{\prime}}{\mathrm{p}_{\mathrm{S}}^{\prime}}}\right) \tag{B33}
\end{equation*}
$$

The values of turbine inlet and exit total pressures ( $p_{4}^{\prime}$ and $p_{S}^{\prime}$ ) obtained from equations (B20) and (B33), respectively, together with the total-to-total turbine efficiency $\eta_{T}$ (an input for the initial calculation and an output of the turbine subroutine on subsequent cases) and the inlet total temperature $\mathrm{T}_{4}^{\prime}$ enable the determination of the actual total enthalpy drop across the turbine $\Delta h_{T}^{\prime}$ and the stator exit total temperature $T_{S}^{\prime}$. This calculation is performed in the subroutine called ADIABT given in appendix G.

The gas constant at the stator exit $\mathrm{R}_{\mathrm{S}}$ is obtained from the GASCON subroutine (appendix G) and $\gamma_{S}$ at the stator exit is obtained from the GAMMA subroutine (appendix $G$ ) based on the determined value of stator exit total temperature $T_{S}^{\prime}$. From the new value of $\gamma_{S}$, a value of $F F_{S}$ is recalculated in equation (B22). The flow factor at the duct exit is then calculated from the total pressure and duct area ratio given previously according to equation (B31). Then, a new value of duct exit Mach number $M_{D}$ is obtained from the FMM subroutine. The total pressure at the duct exit is then recalculated from the expression

$$
\begin{equation*}
p_{D}^{\prime}=p_{0}\left(1+\frac{\gamma-1}{2} M^{2}\right)_{D}^{\gamma_{D} /\left(\gamma_{D}-1\right)} \tag{B34}
\end{equation*}
$$

where $\gamma_{D}=\gamma_{S}$. The stator exit total pressure $p_{S}^{\prime}$ is recalculated in equation (B33) using the new value of $p_{D}^{\prime}$ and the previous value of $p_{D}^{\prime} / p_{S}^{\prime}$ from equation (B29).

With the new value of $\mathrm{p}_{S}^{\prime}$ and with $\mathrm{p}_{4}^{\prime}, \mathrm{T}_{4}^{\prime}$, and $\eta_{\mathrm{T}}$, the calculation proceeds to the ADIABT subroutine to obtain new values of $\Delta h$ and $T \dot{S}$. The entire procedure is repeated until the stator exit temperature exhibits no further change.

The static pressure at the stator exit is then recalculated as

$$
\begin{equation*}
\mathrm{p}_{\mathrm{S}}=\frac{\mathrm{p}_{\mathrm{S}}^{\prime}}{\left(1+\frac{\gamma-1}{2} \mathrm{M}^{2}\right)_{\mathrm{S}}^{\gamma_{S} /\left(\gamma_{S}-1\right)}} \tag{B35}
\end{equation*}
$$

where $M_{S}$ is the input value. It should be noted that $p_{S}$ does not necessarily equal $\mathrm{p}_{\mathrm{D}}$ or $\mathrm{p}_{0}$ (it depends on $\mathrm{A}_{\mathrm{D}} / \mathrm{A}_{\mathrm{S}}$ ).

Turbine power output. - The turbine power output is obtained from

$$
\begin{equation*}
\mathrm{P}_{\mathrm{T}}=\mathrm{w}_{4} \Delta \mathrm{~h}_{\mathrm{T}}\left(1-\mathrm{Z}_{5}\right) \tag{B36}
\end{equation*}
$$

where $\mathrm{w}_{4}$ is set equal to $\mathrm{w}_{3}$ and $\Delta \mathrm{h}_{\mathrm{T}}$ is obtained from the ADIABT subroutine. In
the first pass through the main program the turbine leakage flow fraction $Z_{5}=0$. On subsequent passes, the value of $Z_{5}$ that is generated in the applicable subroutine is used.

Turbine thrust calculation. - Thrust is calculated at the louver exit. It is assumed that the louver is in the flow passage of constant annular area. It is first required to determine the drop in total pressure across the exit louvers. The expression used is

$$
\begin{equation*}
\mathrm{p}_{\mathrm{L}}^{\prime}=\mathrm{p}_{\mathrm{D}}^{\prime}\left\{1-\bar{\omega}_{\mathrm{L}}\left[1-\frac{1}{\left(1+\frac{\gamma_{S}-1}{2} \mathrm{M}_{\mathrm{D}}^{2}\right)^{\gamma_{S} /\left(\gamma_{S}-1\right)}}\right]\right\} \tag{B37}
\end{equation*}
$$

where the total pressure loss coefficient across the louvers $\bar{\omega}_{\mathrm{L}}$ is a program input. This value of $\bar{\omega}_{L}$ is also assumed to be applicable to the exit louver of the fan. In equation (B37) it is assumed that $\gamma_{D}=\gamma_{L}=\gamma_{S}$. This is valid because $\gamma$ is dependent on the fuel-to-air ratio (which does not vary) and the total temperature (which is constant). Then from the louver total- to static-pressure ratio $p_{L}^{\prime} / p_{0}$, the louver exit Mach number is calculated from the expression

$$
\begin{equation*}
M_{L}=\left\{\frac{2}{\gamma_{S}-1}\left[\left(\frac{p_{L}^{\prime}}{p_{0}}\right)^{\left(\gamma_{S}-1\right) / \gamma_{S}}-1\right]\right\}^{1 / 2} \tag{B38}
\end{equation*}
$$

in which the positive root (absolute value) of equation (B38) is used. The exit velocity is then

$$
\begin{equation*}
V_{L}=M_{L} \sqrt{\gamma_{S} g R_{S} T_{L}} \tag{B39}
\end{equation*}
$$

where $T_{L}$ is obtained from

$$
\begin{equation*}
\mathrm{T}_{\mathrm{L}}=\frac{\mathrm{T}_{\mathrm{L}}^{\prime}}{\left(1+\frac{\gamma_{\mathrm{S}}-1}{2} \mathrm{M}_{\mathrm{L}}^{2}\right)} \tag{B40}
\end{equation*}
$$

The turbine thrust is

$$
\begin{equation*}
\mathrm{F}_{\mathrm{T}}=\frac{\mathrm{w}_{4}\left(1-\mathrm{Z}_{5}\right) \mathrm{V}_{\mathrm{L}} \mathrm{C}_{\mathrm{T}}}{\mathrm{~g}} \tag{B41}
\end{equation*}
$$

where $C_{T}$ is the thrust coefficient, a program input.
Turbine duct areas. - The annular area at the stator exit is

$$
\begin{equation*}
A_{S}=w_{4}\left(1-Z_{5}\right)\left(\frac{\sqrt{T^{\prime} R}}{p^{\prime} F F}\right)_{S} \tag{B42}
\end{equation*}
$$

Then when $A_{D} / A_{S}$ from equation (B30) is known, the flow area at duct exit is obtained from

$$
\begin{equation*}
A_{D}=A_{S}\left(\frac{A_{D}}{A_{S}}\right) \tag{B43}
\end{equation*}
$$

Fan Calculations

Once the turbine calculations are completed yielding the turbine station thermodynamics, turbine thrust, and the turbine power output, the fan is then analyzed. As previously mentioned, only fan station thermodynamics will be considered because fan aerodynamic details are not handled in this program. The fan calculations required the prescribed inputs of fan-stage pressure ratio $p_{13}^{\prime} / p_{11}^{\prime}$, fan average inlet Mach number $M_{11}$, fan tip speed $U_{F, t}$, and fan-stage adiabatic efficiency $\eta_{F}$. The fan discharge flow is considered axial in this analysis.

Fan flow rate. - The fan upstream total temperature and total pressure are set as

$$
\left.\begin{array}{l}
\mathrm{T}_{10}^{\prime}=\mathrm{T}_{0}  \tag{B44}\\
\mathrm{p}_{10}^{\prime}=\mathrm{p}_{0}
\end{array}\right\}
$$

The pressure drop in the inlet (effect of struts, bellmouth friction, etc.) is read in as a total-pressure-loss coefficient

$$
\begin{equation*}
\bar{\omega}_{10}=\frac{\Delta \mathrm{p}^{\prime}}{\mathrm{p}_{11}^{\prime}-\mathrm{p}_{11}} \tag{B45}
\end{equation*}
$$

The total-pressure ratio of the rotor inlet is then

$$
\begin{equation*}
\frac{\mathrm{p}_{11}^{\prime}}{\mathrm{p}_{10}^{\prime}}=1-\bar{\omega}_{10}\left[1-\frac{1}{\left(1+0.2 \mathrm{M}_{11}^{2}\right)^{3.5}}\right] \tag{B46}
\end{equation*}
$$

so that the total pressure at the fan inlet is

$$
\begin{equation*}
p_{11}^{\prime}=\left(\frac{p_{11}^{\prime}}{p_{10}^{\prime}}\right) p_{10}^{\prime} \tag{B47}
\end{equation*}
$$

With fan-stage pressure ratio an input, the total pressure at the fan-stage exit is

$$
\begin{equation*}
p_{13}^{\prime}=\left(\frac{p_{13}^{\prime}}{p_{11}^{\prime}}\right) p_{11}^{\prime} \tag{B48}
\end{equation*}
$$

The fan actual enthalpy rise is now obtained using the ADIABT subroutine and the inputs $\mathrm{T}_{11}^{\prime}, \mathrm{p}_{11}^{\prime}, \mathrm{p}_{13}^{\prime}$, and $\eta_{\mathrm{F}}$ to obtain

$$
\begin{equation*}
\Delta h_{F}=h_{13}^{\prime}-h_{11}^{\prime} \tag{B49}
\end{equation*}
$$

The fan exit total temperature $\mathbf{T}_{13}^{\prime}$ is also an output of the subroutine. Neglecting rotor bearing friction and windage losses, the fan power is determined directly from the previous final value of turbine power as

$$
\begin{equation*}
P_{F}=P_{T} \tag{B50}
\end{equation*}
$$

The required fan air mass flow rate is then obtained from

$$
\begin{equation*}
\mathrm{w}_{11}=\frac{\mathrm{P}_{\mathrm{F}}}{\Delta \mathrm{~h}_{\mathrm{F}}^{\prime}} \tag{B51}
\end{equation*}
$$

Fan state conditions. - Rotor average inlet static pressure (for $\gamma=1.4$ ) is

$$
\begin{equation*}
\mathrm{p}_{11}=\frac{\mathrm{p}_{11}^{\prime}}{\left(1+0.2 \mathrm{M}_{11}^{2}\right)^{3.5}} \tag{B52}
\end{equation*}
$$

The rotor inlet static temperature is

$$
\begin{equation*}
\mathrm{T}_{11}=\frac{\mathrm{T}_{11}^{1}}{\left(1+0.2 \mathrm{M}_{11}^{2}\right)} \tag{B53}
\end{equation*}
$$

and the average axial inlet velocity is

$$
\begin{equation*}
\mathrm{v}_{11}=49.1 \sqrt{\mathrm{~T}_{11}}\left(\mathrm{M}_{11}\right) \tag{B54}
\end{equation*}
$$

The total pressures at each station are obtained next. The duct total pressure loss is an input to the program as the ratio $p_{14}^{\prime} / p_{13}^{\prime}$. Then, the duct exit total pressure is

$$
\begin{equation*}
p_{14}^{\prime}=\left(\frac{p_{14}^{\prime}}{p_{13}^{\prime}}\right) p_{13}^{\prime} \tag{B55}
\end{equation*}
$$

where $p_{13}^{\prime}$ is given in equation ( $B 48$ ). The pressure drop across the louvers is

$$
\begin{equation*}
\frac{\mathrm{p}_{15}^{\prime}}{\mathrm{p}_{14}^{\prime}}=1-\bar{\omega}_{\mathrm{L}}\left[1-\frac{1}{\left(1+0.2 \mathrm{M}_{14}^{2}\right)^{3.5}}\right] \tag{B56}
\end{equation*}
$$

where the total pressure loss coefficient across the louvers $\bar{\omega}_{L}$ is the same value used for the turbine flow stream.

The Mach number at the duct exit $\mathrm{M}_{14}$ is obtained from the assumption that the static pressure at stations 15 and 14 is equal to ambient pressure ( $p_{15}=p_{14}=p_{0}$ ). Then, with the total- to static-pressure ratio at the duct exit $p_{14}^{\prime} / \mathrm{p}_{14}, \mathrm{M}_{14}$ is obtained from the expression.

$$
\begin{equation*}
\mathrm{M}_{14}=\left[\frac{2}{\gamma-1}\left(\frac{\mathrm{p}_{14}}{\mathrm{p}_{0}}\right)^{(\gamma-1) / \gamma}-\frac{2}{\gamma-1}\right]^{1 / 2} \tag{B57}
\end{equation*}
$$

in which $\gamma=1.40$. Now that $\mathrm{p}_{15}^{\prime} / \mathrm{p}_{14}^{\prime}$ is known, the total- to static-pressure ratio at the louver exit can be obtained as

$$
\begin{equation*}
\frac{\mathrm{p}_{15}^{\prime}}{\mathrm{p}_{15}}=\left(\frac{\mathrm{p}_{15}^{\prime}}{\mathrm{p}_{14}^{\prime}}\right) \mathrm{p}_{14}^{\prime}\left(\frac{1}{\mathrm{p}_{14}}\right) \tag{B58}
\end{equation*}
$$

Then, from equation (B57) the expression for louver exit Mach number (for $\gamma=1.40$ ) is

$$
\begin{equation*}
\mathrm{M}_{15}=\left[5.0\left(\frac{\mathrm{p}_{15}^{\prime}}{\mathrm{p}_{0}}\right)^{0.286}-5.0\right]^{1 / 2} \tag{B59}
\end{equation*}
$$

The louver total temperature is obtained by assuming adiabatic flow through the duct and louvers; that is,

$$
\begin{equation*}
T_{15}^{\prime}=T_{14}^{\prime}=T_{13}^{\prime} \tag{B60}
\end{equation*}
$$

where $\mathbf{T}_{13}^{\prime}$ is obtained from the output of the ADIABT subroutine (eq. (B49)). The static temperature at the louver exit is then

$$
\begin{equation*}
\mathrm{T}_{15}=\frac{\mathrm{T}_{15}^{\prime}}{\left(1+0.2 \mathrm{M}_{15}^{2}\right)} \tag{B61}
\end{equation*}
$$

and the discharge velocity is

$$
\begin{equation*}
\mathrm{V}_{15}=49.1 \mathrm{M}_{15} \sqrt{\mathrm{~T}_{15}} \tag{B62}
\end{equation*}
$$

For the fan stage, axial discharge is assumed, so that the louver exit velocity in equation ( B 62 ) is the axial discharge velocity.

The stator exit axial Mach number $\mathrm{M}_{13}$ is taken to be some function of the difference between $M_{11}$ and $M_{15}$ in the form

$$
\begin{equation*}
M_{13}=M_{11}+K_{F}\left(M_{15}-M_{11}\right) \tag{B63}
\end{equation*}
$$

where $K_{F}$ is an input. The stator exit static conditions are then

$$
\begin{equation*}
p_{13}=\frac{p_{13}^{\prime}}{\left(1+0.2 \mathrm{M}_{13}^{2}\right)^{3.5}} \tag{B64}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{T}_{13}=\frac{\mathrm{T}_{13}^{\prime}}{\left(1+0.2 \mathrm{M}_{13}^{2}\right)} \tag{B65}
\end{equation*}
$$

The average exit axial velocity is

$$
\begin{equation*}
\mathrm{V}_{13}=49.1 \mathrm{M}_{13} \sqrt{\mathrm{~T}_{13}} \tag{B66}
\end{equation*}
$$

The static temperature at the duct exit is

$$
\begin{equation*}
\mathrm{T}_{14}=\frac{\mathrm{T}_{14}^{\mathrm{t}}}{\left(1+0.2 \mathrm{M}_{14}^{2}\right)} \tag{B67}
\end{equation*}
$$

And the duct exit axial velocity is calculated from the expression

$$
\begin{equation*}
\mathrm{V}_{14}=49.1 \mathrm{M}_{14} \sqrt{\mathrm{~T}_{14}} \tag{B68}
\end{equation*}
$$

Thrust. - The fan thrust is given by the expression (since $\mathrm{p}_{15}=\mathrm{p}_{0}$ )

$$
\begin{equation*}
F_{F}=\frac{w_{11} V_{15} C_{T}}{g} \tag{B69}
\end{equation*}
$$

where the fan thrust coefficient $C_{T}$ is an input taken to be equal to the turbine thrust coefficient.

The tip-turbine fan total thrust is obtained as the sum of the fan and turbine thrusts (eqs. (B41) and (B69)) as

$$
\begin{equation*}
\mathbf{F}=\mathbf{F}_{\mathbf{T}}+\mathbf{F}_{\mathbf{F}} \tag{B70}
\end{equation*}
$$

This value of total thrust, which is the result of the initial choice of turbine mass flow rate $\mathrm{w}_{1}$, will generally not be equal to the value of thrust desired.

## Fan and Turbine Geometry

After the calculation of the initial total thrust using the first value of turbine mass flow rate, fan and turbine areas and geometry are calculated. For the fan stage, at the rotor inlet (station 11), the flow area is

$$
\begin{equation*}
\mathrm{A}_{11}=\left(\frac{\mathrm{wRT}}{\mathrm{pV}}\right)_{11} \tag{B71}
\end{equation*}
$$

The stator exit area is (station 13)

$$
\begin{equation*}
\mathrm{A}_{13}=\frac{\mathrm{w}_{11} \mathrm{RT}_{13}}{\mathrm{p}_{13} \mathrm{~V}_{13}} \tag{B72}
\end{equation*}
$$

and the duct exit area is (station 14)

$$
\begin{equation*}
\mathrm{A}_{14}=\frac{\mathrm{w}_{11} \mathrm{RT}_{14}}{\mathrm{p}_{14} \mathrm{~V}_{14}} \tag{B73}
\end{equation*}
$$

and the louver exit (station 15)

$$
\begin{equation*}
\mathrm{A}_{15}=\frac{\mathrm{w}_{11} \mathrm{RT}_{15}}{\mathrm{p}_{15} \mathrm{~V}_{15}} \tag{B74}
\end{equation*}
$$

The fan rotor assembly is shown in figure 11. The fan rotor tip diameter is calculated from the expression

$$
\begin{equation*}
\mathrm{d}_{\mathrm{F}, \mathrm{t}}=1.128 \sqrt{\left(\frac{\mathrm{~A}_{11}}{1-\mathrm{Y}_{\mathrm{F}}^{2}}\right)} \tag{B75}
\end{equation*}
$$

where the fan hub-tip ratio at the rotor inlet $\left(Y_{F}=\left(d_{h} / d_{t}\right)_{F}\right)$ is a program input. The fan hub diameter is then

$$
\begin{equation*}
d_{F, h}=d_{F, t^{\prime}} Y_{F} \tag{B76}
\end{equation*}
$$

and the fan rotor blade height

$$
\begin{equation*}
H_{F}=\frac{1}{2} d_{F, t}\left(1-Y_{F}\right) \tag{B77}
\end{equation*}
$$

Between the fan rotor blade and the turbine blades is a shroud which acts as the carrier for the turbine blades. The carrier must be sized to provide an adequate structure to withstand inherent thermal stresses and allow for fabrication and possible cooling requirements. In general, the ring length increases somewhat with increasing fantip diameter. The value of ring height $\Delta r_{c}$ will be an input. Then the turbine hub diameter is

$$
\begin{equation*}
d_{T, h}=d_{F, t}+2 \Delta r_{c} \tag{B78}
\end{equation*}
$$

The turbine tip diameter for both full and partial admission is

$$
\begin{equation*}
\mathrm{d}_{\mathrm{T}, \mathrm{t}}=\left[\mathrm{d}_{\mathrm{T}, \mathrm{~h}}^{2}+\frac{4 \mathrm{~A}_{5}}{\pi}\left(\frac{360}{\theta}\right)\right]^{1 / 2} \tag{B79}
\end{equation*}
$$

where the turbine rotor annular area $A_{5}$ is obtained by first setting $A_{5}=A_{S}$. (On subsequent iterations, using the value calculated in the turbine subroutines, $A_{5}$ equals the area at the turbine rotor inlet normal to the axial component of velocity.) The arc of admission, $\theta$, in equation (B79) is in degrees and is an input.

The turbine hub-to-tip diameter ratio is then defined as

$$
\begin{equation*}
\mathrm{Y}_{\mathrm{T}}=\frac{\mathrm{d}_{\mathrm{T}, \mathrm{~h}}}{\mathrm{~d}_{\mathrm{T}, \mathrm{t}}} \tag{B80}
\end{equation*}
$$

and the turbine rotor blade height is

$$
\begin{equation*}
\mathrm{H}_{\mathrm{T}}=\frac{1}{2}\left(\mathrm{~d}_{\mathrm{T}, \mathrm{t}}-\mathrm{d}_{\mathrm{T}, \mathrm{~h}}\right) \tag{B81}
\end{equation*}
$$

At this point in the main program, the appropriate turbine subroutine (appendix D or $E$ ) is called to obtain an accurate value of turbine efficiency. The subroutine is called at this point because it requires the turbine tip diameter to obtain the turbine mean line rotational speed $U_{T}$; that is

$$
\begin{equation*}
\mathrm{U}_{\mathrm{T}}=\left(\mathrm{d}_{\mathrm{T}, \mathrm{~h}}+\mathrm{H}_{\mathrm{T}}\right) \frac{\mathrm{U}_{\mathrm{F}, \mathrm{t}}}{\mathrm{~d}_{\mathrm{F}, \mathrm{t}}} \tag{B82}
\end{equation*}
$$

The turbine subroutine goes into the detail of internal turbine analysis including blade loss and leakage flow considerations. After obtaining the new value of efficiency, the calculation returns to subroutine ADIABT to obtain a new value of $\Delta \mathrm{h}$ and total temperature $T_{S}^{\prime}$. The calculations then commence with equation (B36). This procedure is repeated until

$$
\left|\left(\left.\eta_{\mathrm{T}}\right|_{\mathrm{n}+1}-\eta_{\mathrm{T}} \mid \mathrm{n}\right)\right| \leqq 0.001
$$

This is done for each input choice of exit stator exit Mach number $M_{S}$ and each value of inlet duct mass flow $\mathrm{w}_{1}$ used.

## Performance Parameters

Corrected fan mass flow is obtained as

$$
\begin{equation*}
\mathrm{w}_{\mathrm{eq}}=\mathrm{w}_{11} \frac{\left(\frac{\mathrm{~T}_{11}^{\prime}}{518.7}\right)^{1 / 2}}{\frac{\mathrm{p}_{11}^{\prime}}{14.7}} \tag{B83}
\end{equation*}
$$

The corrected total thrust is

$$
\begin{equation*}
\frac{\mathrm{F}}{\delta}=\frac{\mathrm{F}}{\frac{\mathrm{p}_{11}^{1}}{14.7}} \tag{B84}
\end{equation*}
$$

The burner fuel flow is

$$
\begin{equation*}
\mathrm{w}_{\mathrm{f}, \mathrm{~b}}=\left.\frac{\mathrm{f}}{\mathrm{a}}\right|_{\mathrm{b}}\left(\mathrm{w}_{2}\right) \tag{B85}
\end{equation*}
$$

and the burner thrust specific fuel consumption is

$$
\begin{equation*}
\mathrm{SFC}=\left(\frac{\mathrm{w}_{\mathrm{f}, \mathrm{~b}}}{\mathrm{~F}}\right) 3600 \tag{B86}
\end{equation*}
$$

where total thrust $F$ is from equation (B70).
The fan bypass ratio is defined as the ratio of fan stage flow rate to turbine flow rate or

$$
\begin{equation*}
\mathrm{BPR}=\frac{\mathrm{w}_{11}}{\mathrm{w}_{4}} \tag{B87}
\end{equation*}
$$

The fan thrust augmentation ratio is defined as the ratio of lift fan thrust to ideal thrust obtainable from the expansion of the supply gas to ambient pressure. This expression is

$$
\begin{equation*}
A U G=\frac{F}{F_{i d, g g}} \tag{B88}
\end{equation*}
$$

where

$$
\begin{equation*}
F_{i d, g g}=\frac{w_{1} V_{i d, g g}}{g} \tag{B89}
\end{equation*}
$$

and the ideal expanded discharge velocity is

$$
\begin{equation*}
\mathrm{V}_{\mathrm{id}, \mathrm{gg}}=\left(2 \mathrm{gJ} \Delta \mathrm{~h}_{\mathrm{id}}\right)^{1 / 2} \tag{B90}
\end{equation*}
$$

The ideal enthalpy drop $\Delta h_{i d}$ is obtained using subroutine ADIABT with the inputs $T_{1}^{\prime}$, $\mathrm{p}_{1}^{\prime}, \mathrm{p}_{0},\left.\frac{\mathrm{f}}{\mathrm{a}}\right|_{1}, \eta_{\mathrm{b}}$, and $\eta$ where $\eta$ is set equal to one.

## Desired Total Thrust Determination

On completion of all program calculations for an initial choice of turbine mass flow rate which yields a value of tip-turbine fan total thrust, the program repeats the calculations using another choice of inlet duct mass flow $w_{1}$. This procedure yields two values of total thrust corresponding to the two choices of inlet duct mass flow. The program then uses a linear interpolation to obtain the correct value of $\mathrm{w}_{1}$ corresponding to the desired total thrust. The program is then rerun using the new value of $w_{1}$ to obtain the turbine and fan designs. This procedure has been shown to yield less than $1 / 2$ percent discrepancy in total thrust.

## APPENDIX C

## TURBINE LOSS ANALYSIS

The determination of turbine stage efficiency and turbine state conditions (appendixes $\mathbf{D}$ and E ) in parametric studies that involve a relatively large variation in overall pressure ratio, inlet temperature, stage velocity diagram parameters, and turbine geometry requires a realistic estimation of the loss factors involved (rotors and stators). In addition, the magnitude of the overall efficiency of the turbine will help determine whether one or two stages is desirable for the application.

A typical set of velocity diagrams for a single-stage impulse turbine are given in figure 12 (station designations correspond to those used in the program). Although the flow entering the inlet stator from the scroll may not be axial and may vary in approach direction around the periphery, for simplicity, the velocity at the inlet stator will be assumed to be axial. The error in inlet-stator-loss coefficient that this assumption causes is assumed to be negligible.

If a two-stage turbine is used, the interstage stator will generally have a greater amount of turning than the inlet stator because the gas flow into the interstage stator is not axial. The velocity diagram of the inlet and exit absolute gas velocities for an interstage stator is given in figure 13. Thus, the turning angle of the interstage stator is $\Delta \alpha=\alpha_{7}-\alpha_{6}$.

The loss analysis is divided into three parts: (1) the stator, which has a sizable static pressure drop (2) the impulse rotor blades, and (3) impulse exit stators. Diffusing exit stators are discussed in appendix D.

## Inlet and Interstage Stator Efficiency

The efficiency of the turbine inlet stator is defined as

$$
\begin{equation*}
\eta_{\mathrm{N}}=1-\frac{\text { Energy loss }}{\text { Ideal energy }} \tag{C1}
\end{equation*}
$$

where the ideal energy is obtained from the enthalpy change $\Delta h_{N}$, id in the isentropic expansion from inlet total pressure to outlet static pressure. The ratio of energy loss to ideal energy is referred to as the energy-loss coefficient or

$$
\begin{equation*}
\xi_{\mathrm{N}}=\left(\frac{\text { Energy loss }}{\text { Ideal energy }}\right)_{\mathrm{N}} \tag{C2}
\end{equation*}
$$

The loss coefficient of a row of stator blades is recognized, in general, to be the summation of three principal loss components: (1) the profile loss $\left(\xi_{\mathrm{p}}\right)_{\mathrm{N}}$, (2) secondaryflow losses $\left(\xi_{\mathrm{se}}\right)_{\mathrm{N}}$, and (3) shock losses $\left(\xi_{\mathrm{sh}}\right)_{\mathrm{N}}$. Thus, for a tip-turbine stator

$$
\begin{equation*}
\xi_{\mathrm{N}}=\left(\xi_{\mathrm{p}}\right)_{\mathrm{N}}+\left(\xi_{\mathrm{se}}\right)_{\mathrm{N}}+\left(\xi_{\mathrm{sh}}\right)_{\mathrm{N}} \tag{C3}
\end{equation*}
$$

The profile loss (loss due to basic blade element) is depenent on a large number of factors, such as, blade shape, turning angle, blade maximum and trailing edge thicknesses, solidity, incidence angle, and Reynolds number. For the subject stator, it will be assumed that the following conditions are met in the stator design:
(1) Optimum solidity (e.g., Zweifel loading coefficient)
(2) Optimum incidence angle ( $\sim 0^{\circ}$ )
(3) Optimum blade profile shape

For these conditions, the profile loss will depend primarily on the air turning angle and Reynolds number:

$$
\begin{equation*}
\left(\xi_{\mathrm{p}}\right)_{\mathrm{N}}=\mathrm{f}(\Delta \alpha, \mathrm{Re}) \tag{C4}
\end{equation*}
$$

The variation of profile-loss coefficient with air turning angle has been determined experimentally for typical stator blades, as shown in figure 14 (from refs. 12 and 13). A wide spread of loss values is indicated. For the range of calculations to be covered for varying pressure ratio across the turbine, the inlet stator outlet angle $\alpha_{5}$ is not expected to vary by more than around $15^{\circ}$ (e.g., from around $60^{\circ}$ to $75^{\circ}$ ). Equation (C4) then becomes

$$
\begin{equation*}
\left(\xi_{\mathrm{p}}\right)_{\mathrm{N}}=\left(\xi_{\mathrm{p}}^{*}\right)_{\mathrm{N}, 5}[\mathrm{f}(\mathrm{Re})] \tag{C5}
\end{equation*}
$$

In this case for simplicity, a single value of loss coefficient can be adopted to be representative of the range of $\alpha_{5}$ and thus eliminate the need for an iteration on $\alpha_{5}$. The reference value of $\left(\xi_{\mathrm{p}}^{*}\right)_{\mathrm{N}, 5}$ used in equation (C5) for the inlet stator is equal to 0.030 .

For the two-stage turbine, the interstage stator will have a greater turning angle depending on the value of absolute rotor exit angle $\alpha_{6}$ (fig. 13). The range of turning angle for this case should also be relatively small (from around $75^{\circ}$ to $85^{\circ}$ ). Once again a single representative $\left(\xi_{\mathrm{p}}^{*}\right)_{N, 7}$ value of 0.040 was chosen for the range of turning angle considered. Here again, the error associated with this assumption is assumed small. Thus, in both the inlet and interstage stators $\left(\xi_{p}\right)_{N}$ is only a function of Reynolds number.

Inasmuch as the calculation will include variations in turbine size, provision should
be included for reflecting an effect of Reynolds number on the profile loss. This will be done in the form

$$
\begin{equation*}
\left(\xi_{\mathrm{p}}\right)_{\mathrm{N}}=\left(\xi_{\mathrm{p}}^{*}\right)_{\mathrm{N}}\left(\frac{R \mathrm{Re}}{R e^{*}}\right)^{-0.2} \tag{C6}
\end{equation*}
$$

where the reference value of Reynolds number $R e^{*}$ used in the program is $1.5 \times 10^{6}$. For this analysis, if a relatively constant value of blade aspect ratio is considered as turbine size is varied, then Reynolds number can be defined in terms of the blade passage height and expressed as in reference 14 as

$$
\begin{equation*}
\mathrm{Re}=\frac{\mathrm{w}}{\mu \mathrm{r}_{\mathrm{T}, \mathrm{~m}}} \tag{C7}
\end{equation*}
$$

where $w$ is the turbine flow rate, $r_{T, m}$ is the mean radius, and $\mu$ is viscosity. For simplicity, the viscosity can be expressed as a function of the total temperature at the stator inlet. Values of viscosity are obtained using the VISCON subroutine (appendix G).

Experimental data for stators indicates that the loss coefficient for a given blade row stays essentially constant with outlet Mach number up to sonic velocity. However, as shock formation increases with supersonic Mach numbers, the loss will increase. Thus, a shock-loss coefficient can be established as a function of outlet Mach number, such that for $M_{5}>1.0$

$$
\begin{equation*}
\xi_{s h}=f\left(M_{5}\right) \tag{C8}
\end{equation*}
$$

The shock-loss coefficient given by equation (C8) for values of Mach number greater than 1.0 can be established from experimental data. Such data are currently unavailable. Because of the difficulty in obtaining valid experimental data, a preliminary estimate was established from a plot of stator efficiency against stator exit Mach number shown in figure 15. The curve presented is based on limited analytical estimates available from unpublished Lewis calculations for stator designs with high exit Mach numbers.

A representative shock-loss coefficient variation was obtained from this plot from the relation $\xi=1-\eta$ such that

$$
\begin{equation*}
\xi_{\text {sh }}=\left(\xi_{\mathrm{N}}\right)_{\text {supersonic }}-\left(\xi_{\mathrm{N}}\right)_{\text {sonic }}=\left(\eta_{\mathrm{N}}\right)_{\text {sonic }}-\left(\eta_{\mathrm{N}}\right)_{\text {supersonic }} \tag{C9}
\end{equation*}
$$

The corresponding plot of $\xi_{s h}$ is shown in figure 16 for which an analytical variation was obtained as

$$
\begin{equation*}
\xi_{\mathrm{sh}}=0.0075\left(\mathrm{M}^{2}-\mathrm{M}\right)_{\mathrm{N}} \tag{C10}
\end{equation*}
$$

where $M_{N}$ is inlet stator outlet Mach number ( $M>1$ ).
The losses due to secondary flows and casing friction are fairly complex and difficult to express as a general functional relation. However, for simplicity and for the purposes of this analysis, the secondary-loss coefficient was accounted for by taking it to be proportional to the profile-loss coefficient. Thus,

$$
\begin{equation*}
\left(\xi_{\mathrm{se}}\right)_{\mathrm{N}}=\left(\mathrm{K}_{\mathrm{se}}\right)_{\mathrm{N}}\left(\xi_{\mathrm{p}}\right)_{\mathrm{N}} \tag{C11}
\end{equation*}
$$

But, since secondary losses are not sensitive to blade Reynolds number effects, equation (C11) becomes

$$
\begin{equation*}
\left(\xi_{\mathrm{se}}\right)_{\mathrm{N}}=\left(\mathrm{K}_{\mathrm{se}}\right)_{\mathrm{N}}\left(\xi_{\mathrm{p}}^{*}\right)_{\mathrm{N}} \tag{C12}
\end{equation*}
$$

Values of $\left(\mathrm{K}_{\mathrm{se}}\right)_{\mathrm{N}}$, which is a function of turning, are indicated to vary from around 0.5 to somewhat over 1.0 in the literature. The value selected for $\left(\mathrm{K}_{\mathrm{se}}\right)_{\mathrm{N}}$ was 0.75 .

Then the total loss coefficient, from equations (C3), (C6), (C10), and (C12), becomes

$$
\begin{equation*}
\xi_{\mathrm{N}}=\left(\xi_{\mathrm{p}}^{*}\right)_{\mathrm{N}}\left[\left(\frac{\mathrm{Re}}{\mathrm{Re}^{*}}\right)^{-0.2}+\left(\mathrm{K}_{\mathrm{se}}\right)_{\mathrm{N}}\right]+\left(\xi_{\mathrm{sh}}\right)_{\mathrm{N}} \tag{C13}
\end{equation*}
$$

## Rotor Blade Efficiency

The rotor blade is designed for impulse flow and a shrouded tip. The efficiency of the turbine blades, which are treated as a cascade, is defined in terms of a kinetic energy loss by the expression

$$
\begin{equation*}
\eta_{\mathrm{B}}=1-\xi_{\mathrm{B}} \tag{C14}
\end{equation*}
$$

where the energy-loss coefficient $\xi_{B}$ for a row of rotor blades is defined as the ratio of the loss in kinetic energy to the ideal outlet kinetic energy. As in the case of the stator, the energy loss comprises the profile loss, secondary flow losses, and shock losses. The loss coefficient is then

$$
\begin{equation*}
\xi_{\mathrm{B}}=\left(\xi_{\mathrm{p}}\right)_{\mathrm{B}}+\left(\xi_{\mathrm{se}}\right)_{\mathrm{B}}+\left(\xi_{\mathrm{sh}}\right)_{\mathrm{B}} \tag{C15}
\end{equation*}
$$

The variation of profile loss coefficient with air turning angle has been determined experimentally for rotor blades, as shown in figure 17 (from refs. 12 and 13). A wide range of values has been obtained. Only the data of Ainley are definitely known to be for impulse blades. Thus, the selected representative relation for $\left(\xi_{\mathrm{p}}\right)_{\mathrm{B}}$ against $\Delta \beta$ will be influenced primarily by these data. Figure 18 shows the selected curve used for turning angles up to $140^{\circ}$. In equation form, for Re*,

$$
\begin{equation*}
\left(\xi{\underset{\mathrm{p}}{\mathrm{~B}}}_{*}\right)=0.025+0.035\left(\frac{\Delta \beta}{100}\right)^{2.5} \tag{C16}
\end{equation*}
$$

where $\left(\xi_{\mathrm{p}}^{*}\right)_{\mathrm{B}}$ corresponds to $R \mathrm{e}^{*}=1.5 \times 10^{6}$ with Re defined as before in equation (C7). The turning angle $\Delta \beta$ for simplicity and in order to avoid an iteration, is assumed to be equal to twice the inlet angle; that is,

$$
\begin{equation*}
\Delta \beta \cong 2 \beta_{\text {inlet }} \tag{C17}
\end{equation*}
$$

The change in the value of profile-loss coefficient due to this assumption is expected to be small compared with the spread in the data variation as indicated in figure 17.

The influence of Reynolds number will be taken to be the same as for the inlet stator loss or

$$
\begin{equation*}
\left(\xi_{\mathrm{p}}\right)_{\mathrm{B}}=\left(\xi_{\mathrm{p}}^{*}\right)_{\mathrm{B}}\left(\frac{\mathrm{Re}}{\mathrm{Re}^{*}}\right)^{-0.2} \tag{C18}
\end{equation*}
$$

The losses due to secondary flows are treated in a manner similar to the inlet stator procedure. The secondary-loss coefficient is taken to be proportional to the reference profile-loss coefficient. Thus,

$$
\begin{equation*}
\left(\xi_{\mathrm{se}}\right)_{\mathrm{B}}=\left(\mathrm{K}_{\mathrm{se}}\right)_{\mathrm{B}}{ }^{\left(\xi_{\mathrm{p}}^{*}\right)_{\mathrm{B}}} \tag{C19}
\end{equation*}
$$

Values of $\left(\mathrm{K}_{\mathrm{se}}\right)_{\mathrm{B}}$ that are a function of turning, are indicated to vary from around 0.30 to 0.50 (ref. 12). A ( $\left.\mathrm{K}_{\mathrm{se}}\right)_{\mathrm{B}}$ value of 0.40 was used in the program.

The shock-loss contribution can be established from an estimated variation of blade efficiency with rotor relative inlet Mach number. Since no data were available for this variation, an assumed shock-loss variation three times the magnitude of the stator shock loss was adopted as shown in figure 19. The resulting expression takes the form

$$
\begin{equation*}
\left(\xi_{s h}\right)_{B}=K_{s h}\left(M_{r}^{2}-M_{r}\right)_{B} \tag{C20}
\end{equation*}
$$

where the constant $K_{s h}$ is equal to 0.0225 . Thus, the total loss coefficient, from equations (C15) and (C18) to (C20), is

$$
\begin{equation*}
\xi_{\mathrm{B}}=\left(\xi_{\mathrm{p}}^{*}\right)_{\mathrm{B}}\left[\left(\frac{\mathrm{Re}}{R \mathrm{e}^{*}}\right)^{-0.2}+\left(\mathrm{K}_{\mathrm{se}}\right)_{\mathrm{B}}\right]+\left(\xi_{\mathrm{Sh}}\right)_{\mathrm{B}} \tag{C21}
\end{equation*}
$$

## Exit Stator Total-Pressure-Loss Coefficient

The loss coefficient for the impulse exit stator is obtained from the approach used for the rotor blades. The profile loss is composed of mostly surface friction and diffusion. In terms of a kinetic energy loss, the profile-loss coefficient is

$$
\begin{equation*}
\left(\xi_{\mathrm{p}}\right)_{\mathrm{S}}=\mathrm{f}(\Delta \alpha, \mathrm{Re}) \tag{C22}
\end{equation*}
$$

The relation of $\left(\xi_{\mathrm{p}}^{*}\right)_{\mathrm{S}}$ and $\Delta \alpha$ is taken to be the same as for the rotor blade (eq. (C16))

$$
\begin{equation*}
\left(\xi_{\mathrm{p}}^{*}\right)_{\mathrm{S}}=0.025+0.035\left(\frac{\Delta \alpha}{100}\right)^{2.5} \tag{C23}
\end{equation*}
$$

Since the primary purpose of the exit stator is to straighten the flow to axial, the amount of turning is just equal to the exit stator inlet angle $\alpha_{6}$ or $\alpha_{8}$; that is,

$$
\begin{equation*}
\Delta \alpha=\left|\alpha_{6}\right| \tag{C24}
\end{equation*}
$$

is used with equation (C23) for the exit stator. Since Mach numbers in the stator are always less than one, the shock loss term is not needed. Thus, with Reynolds number and secondary losses,

$$
\begin{equation*}
\xi_{\mathrm{S}}=\left(\xi_{\mathrm{p}}^{*}\right)_{\mathrm{S}}\left[\left(\frac{\mathrm{Re}}{\mathrm{Re}^{*}}\right)^{-0.2}+\left(\mathrm{K}_{\mathrm{Se}}\right)_{\mathrm{S}}\right] \tag{C25}
\end{equation*}
$$

where $\left(\mathrm{K}_{\mathrm{Se}}\right)_{\mathrm{S}}=\left(\mathrm{K}_{\mathrm{se}}\right)_{\mathrm{B}}=0.40$ and $\mathrm{Re}^{*}=1.5 \times 10^{6}$.
The exit stator total-kinetic-energy-loss coefficient $\xi_{\mathrm{S}}$ is converted to an exit stator total-pressure-loss coefficient $K_{S}$ for ease of calculation. Conversion of the kinetic-energy-loss coefficient to a total-pressure-loss coefficient is developed in reference 12 (section 2.7). This expression in general form is given as

$$
\begin{equation*}
\mathrm{K}_{\mathrm{S}}=\xi_{\mathrm{S}}\left(\frac{1+\gamma \mathrm{M}^{2}}{2}\right)_{\mathrm{S}} \tag{C26}
\end{equation*}
$$

where $M_{S}$ is exit stator outlet Mach number and the total-pressure-loss coefficient $K_{S}$ in the preceding expression is defined as $\Delta p^{\prime} /\left(p^{\prime}-p\right)_{S}$. Then, the final expression for total-pressure-loss coefficient is obtained from equations (C25) and (C26) as

$$
\begin{equation*}
\mathrm{K}_{\mathrm{S}}=\left(\xi_{\mathrm{p}}^{*}\right)_{\mathrm{S}}\left[\left(\frac{\mathrm{Re}}{1.5 \times 10^{6}}\right)^{-0.2}+\left(\mathrm{K}_{\mathrm{Se}}\right)_{\mathrm{B}}\right]\left(\frac{1+\gamma \mathrm{M}^{2}}{2}\right)_{\mathrm{S}} \tag{C27}
\end{equation*}
$$

Since

$$
\left(p^{\prime}-p\right)_{S} \cong\left(p^{\prime}-p\right)_{6 \text { or } 8}
$$

then

$$
M_{S} \cong M_{6} \text { or } 8
$$

for an impulse stator. Thus, the total-pressure loss factor was applied to the stator inlet, so that, for the exit stator,

$$
\begin{equation*}
\Delta p_{S}^{\prime}=K_{S}\left(p^{\prime}-p\right)_{6} \text { or } 8 \tag{C28}
\end{equation*}
$$

## APPENDIX D

## SINGLE-STAGE TURBINE SUBROUTINES

This subroutine calculates the value of turbine total-to-total efficiency that is required by the main program. The single-stage turbine analysis presented investigates three types of turbines: (1) with impulse exit stators, (2) without exit stators, and (3) with diffusing exit stators. These three types of turbines are described in subroutines, TURB-1, TURB-2, and TURB-4, respectively. The turbine design-point efficiency includes the effects of blade-row losses, partial admission and its associated losses, and turbine seal leakage which is included as a reduction in turbine mass flow. It is assumed that the velocity diagrams are not affected by either partial admission or leakage flow losses.

The turbine calculations are for the mean line since the turbine hub-to-tip radius ratio is of the order 0.90 or greater. The turbine mean line velocity diagrams are determined by the choice of exit stator exit Mach number, wheel speed, inlet stator spouting velocity, and the designation of turbine type. The turbine station designations used in the analysis for the single-stage turbines are given in figure 1.

The inputs required from the main program for all three cases are
Turbine mean line radius
Turbine mean line velocity . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . $\mathrm{U}_{\mathrm{T}}$
Total temperature of the gas into the inlet stator . . . . . . . . . . . . . . . . . . . . T ${ }_{4}^{\prime}$
Total pressure of the gas into the inlet stator . . . . . . . . . . . . . . . . . . . . . . $\mathrm{p}_{4}^{\prime}$
Total mass flow of gas into the inlet stator. . . . . . . . . . . . . . . . . . . . . . . . $\mathrm{w}_{4}$
Exit stator exit axial Mach number . . . . . . . . . . . . . . . . . . . . . . . . . . . . $\mathrm{M}_{\mathrm{S}}$
Exit stator exit static pressure. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . p $\mathrm{p}_{\mathrm{S}}$
Fan tip diameter. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . $\mathrm{d}_{\mathrm{F}, \mathrm{t}}$
Fuel-to-air ratio at the burner exit. . . . . . . . . . . . . . . . . . . . . . . . . . . $\left.\frac{\mathrm{f}}{\mathrm{a}}\right|_{4} ^{\mathrm{t}}$
In addition to these inputs, several geometry and loss-coefficient inputs are required for the leakage and blade-row efficiency determinations, respectively.

## Subroutine TURB-1 - Impulse Exit Stators

Assumptions. - The velocity diagram for this stage is shown in figure 10. The diagram is determined by the choice of inputs. The design assumptions for the impulse exit stator case are constant static pressure through the rotor and exit stator ( $\mathrm{p}_{\mathrm{S}}=\mathrm{p}_{6}=\mathrm{p}_{5}$ ) and constant annular area across the rotor. In addition, the rotor inlet
axial velocity is initially prescribed by the condition $M_{x, 5}=M_{S}$, and a trial value of inlet stator efficiency $\eta_{N}$ is prescribed.

Rotor inlet conditions. - The inlet stator exit velocity is obtained from the actual total- to static-enthalpy drop across the inlet stator, $\Delta h_{N}=h_{4}^{\prime}-h_{5}$ as illustrated in the enthalpy-entropy diagram of figure 20. The value of $\Delta h_{N}$ is determined in the ADIABT subroutine described in appendix $G$. The specific inputs required for the determination of $\Delta h_{N}$ are $T_{4}^{\prime}, p_{4}^{\prime}, \eta_{\mathrm{N}},\left.\frac{f}{\mathrm{a}}\right|_{4}, p_{5}$, and $\eta_{\mathrm{b}}$. The ADIABT subroutine also determines the value of the static temperature at the inlet stator exit $\mathrm{T}_{5}$.

The velocity at the inlet stator exit is then calculated from the expression

$$
\begin{equation*}
\mathrm{V}_{5}=\sqrt{2 \mathrm{gJ} \Delta \mathrm{~h}_{\mathrm{N}}} \tag{D1}
\end{equation*}
$$

Values of $\mathrm{R}_{5}$ and $\gamma_{5}$ at the inlet stator exit are obtained from the subroutines of appendix $G$. Inputs to the gas property subroutines are temperature and fuel-to-air ratio. Thus the value for sonic velocity is

$$
\begin{equation*}
\mathrm{V}_{\mathrm{S}, 5}=\sqrt{(\gamma \mathrm{gRT})_{5}} \tag{D2}
\end{equation*}
$$

The inlet stator exit Mach number is then calculated from

$$
\begin{equation*}
M_{5}=\frac{\mathrm{V}_{5}}{\mathrm{~V}_{\mathrm{s}, 5}} \tag{D3}
\end{equation*}
$$

Once $\mathrm{M}_{5}$ is calculated, the new value of inlet stator efficiency is obtained from

$$
\begin{equation*}
\eta_{\mathrm{N}}=1-\xi_{\mathrm{N}} \tag{D4}
\end{equation*}
$$

where the loss coefficient $\xi_{N}$ is obtained from equation (C13) of appendix C. With the value of $\eta_{\mathrm{N}}$ known, the subroutine iterates on $\mathrm{M}_{5}$ until

$$
\begin{equation*}
\left|\left(\left.M_{5}\right|_{n}-\left.M_{5}\right|_{n+1}\right)\right| \leq 0.002 \tag{D5}
\end{equation*}
$$

The rotor inlet velocity diagram is then established in the following manner: The inlet stator exit angle $\alpha_{5}$ (measured from the axis) is calculated as

$$
\begin{equation*}
\alpha_{5}=\cos ^{-1}\left(\frac{V_{x}}{V}\right)_{5}=\cos ^{-1}\left(\frac{M_{x}}{M}\right)_{5} \tag{D6}
\end{equation*}
$$

where $M_{x, 5}$ is the initial prescribed value $\left(=M_{S}\right)$. The rotor inlet axial velocity is then

$$
\begin{equation*}
\mathrm{V}_{\mathrm{x}, 5}=\mathrm{V}_{5} \cos \alpha_{5} \tag{D7}
\end{equation*}
$$

and the tangential velocity component is

$$
\begin{equation*}
\mathrm{v}_{\mathrm{u}, 5}=\mathrm{v}_{5} \sin \alpha_{5} \tag{D8}
\end{equation*}
$$

Then, when the rotor mean line velocity $\mathrm{U}_{\mathbf{T}}$ is known, the tangential component of the gas may be determined from

$$
\begin{equation*}
\mathrm{w}_{\mathrm{u}, 5}=\mathrm{V}_{\mathrm{u}, 5}-\mathrm{U}_{\mathrm{T}} \tag{D9}
\end{equation*}
$$

The relative velocity at the rotor inlet is

$$
\begin{equation*}
\mathrm{w}_{5}=\sqrt{\mathrm{v}_{\mathrm{x}, 5}^{2}+\mathrm{w}_{\mathrm{u}, 5}^{2}} \tag{D10}
\end{equation*}
$$

The rotor relative inlet blade angle measured from the axis is

$$
\begin{equation*}
\beta_{5}=\tan ^{-1}\left(\frac{W_{u}}{V_{x}}\right)_{5} \tag{D11}
\end{equation*}
$$

The total pressure at the rotor inlet is obtained from $p_{5}$ and $M_{5}$ as

$$
\begin{equation*}
\mathrm{p}_{5}^{\prime}=\mathrm{p}_{5}\left[1+\frac{1}{2}(\gamma-1) \mathrm{M}^{2}\right]_{5}^{\gamma_{5} /\left(\gamma_{5}-1\right)} \tag{D12}
\end{equation*}
$$

and the density is

$$
\begin{equation*}
\rho_{5}=\left(\frac{144 \mathrm{p}}{\mathrm{RT}}\right)_{5} \tag{D13}
\end{equation*}
$$

Rotor blade efficiency and exit conditions. - The blade efficiency $\eta_{\mathrm{B}}$ is obtained using the loss coefficient approach developed in appendix C. The blade efficiency is

$$
\begin{equation*}
\eta_{\mathrm{B}}=1-\xi_{\mathrm{B}} \tag{D14}
\end{equation*}
$$

where the rotor blade loss coefficient $\xi_{B}$ is calculated from equation (C21) of appendix $C$. The relative gas velocity at the rotor exit is then, by definition

$$
\begin{equation*}
\mathrm{w}_{6}=\mathrm{w}_{5} \sqrt{\eta_{\mathrm{B}}} \tag{D15}
\end{equation*}
$$

The energy loss in the rotor blade is calculated from the expression

$$
\begin{equation*}
\mathrm{E}_{\mathrm{B}}=\frac{\mathrm{w}_{5}^{2}-\mathrm{w}_{6}^{2}}{2 \mathrm{gJ}} \tag{D16}
\end{equation*}
$$

or in terms of rotor blade efficiency

$$
\begin{equation*}
\mathrm{E}_{\mathrm{B}}=\frac{\mathrm{W}_{5}^{2}}{2 \mathrm{gJ}}\left(1-\eta_{\mathrm{B}}\right) \tag{D17}
\end{equation*}
$$

From this value of the energy loss, the rotor exit fluid properties are calculated by assuming that the gas specific heat $C_{p, 6}$ is obtained using rotor inlet conditions as the inputs to subroutine SPHTCP (appendix G). This assumption is valid because a relatively small variation in static temperature exists across the impulse rotor. Thus,

$$
\begin{equation*}
\mathrm{T}_{6}=\mathrm{T}_{5}+\frac{\mathrm{E}_{\mathrm{B}}}{\mathrm{C}_{\mathrm{p}, 6}} \tag{D18}
\end{equation*}
$$

The static density is, since $p_{6}=p_{5}$,

$$
\begin{equation*}
\rho_{6}=\left(\frac{144 \mathrm{p}}{\mathrm{TR}}\right)_{6} \tag{D19}
\end{equation*}
$$

For constant annular area across the rotor, the rotor exit axial velocity from continuity is

$$
\begin{equation*}
\mathrm{V}_{\mathrm{x}, 6}=\mathrm{V}_{\mathrm{x}, 5}\left(\frac{\mathrm{~T}_{6}}{\mathrm{~T}_{5}}\right) \tag{D20}
\end{equation*}
$$

The relative tangential component of the outlet flow is

$$
\begin{equation*}
\mathrm{w}_{\mathrm{u}, 6}=\sqrt{\mathrm{w}_{6}^{2}-\mathrm{v}_{\mathrm{x}, 6}^{2}} \tag{D21}
\end{equation*}
$$

and the component $V_{u, 6}$ is

$$
\begin{equation*}
\mathrm{V}_{\mathbf{u}, 6}=\mathrm{W}_{\mathbf{u}, 6}-\mathrm{U}_{\mathbf{T}} \tag{D22}
\end{equation*}
$$

Then the relative flow angle at the exit is

$$
\begin{equation*}
\beta_{6}=\tan ^{-1}\left(\frac{\mathrm{~W}_{\mathrm{u}}}{\mathrm{~V}_{\mathrm{x}}}\right)_{6} \tag{D23}
\end{equation*}
$$

and the absolute angle is

$$
\begin{equation*}
\alpha_{6}=\tan ^{-1}\left(\frac{\mathrm{~V}_{\mathrm{u}}}{\mathrm{~V}_{\mathrm{x}}}\right)_{6} \tag{D24}
\end{equation*}
$$

The absolute flow exit velocity is then

$$
\begin{equation*}
\mathrm{V}_{6}=\left(\frac{\mathrm{V}_{\mathrm{x}}}{\cos \alpha}\right)_{6} \tag{D25}
\end{equation*}
$$

and the absolute exit Mach number $\mathrm{M}_{6}$ is obtained as

$$
\begin{equation*}
\mathrm{M}_{6}=\left(\frac{\mathrm{V}}{\sqrt{\gamma \mathrm{gRT}}}\right)_{6} \tag{D26}
\end{equation*}
$$

The relative exit Mach number is

$$
\begin{equation*}
\mathrm{M}_{\mathbf{r}, 6}=\left(\frac{\mathrm{W}}{\sqrt{\gamma \mathrm{gRT}}}\right)_{6} \tag{D27}
\end{equation*}
$$

The total temperature at the rotor exit is then

$$
\begin{equation*}
\mathrm{T}_{6}^{\prime}=\mathrm{T}_{6}\left[1+\frac{1}{2}(\gamma-1) \mathrm{M}^{2}\right]_{6} \tag{D28}
\end{equation*}
$$

and the total pressure is

$$
\begin{equation*}
\mathrm{p}_{6}^{\prime}=\mathrm{p}_{6}\left[1+\frac{1}{2}(\gamma-1) \mathrm{M}^{2}\right]_{6}^{\gamma_{6} /\left(\gamma_{6}-1\right)} \tag{D29}
\end{equation*}
$$

Exit stator. - The exit stator is specified as impulse, that is, constant static pressure. The total pressure at the stator exit is determined from

$$
\begin{equation*}
\mathrm{p}_{\mathrm{S}}^{\prime}=\mathrm{p}_{6}^{\prime}-\mathrm{K}_{\mathrm{S}}\left(\mathrm{p}^{\prime}-\mathrm{p}\right)_{6} \tag{D30}
\end{equation*}
$$

where the exit stator total-pressure-loss coefficient $K_{S}$ is calculated as described in appendix $C$.

The exit stator exit axial Mach number is calculated by noting that $p_{S}=p_{6}$ so that

$$
\begin{equation*}
M_{S}=\left\{\frac{2}{\gamma-1}\left[\left(\frac{p^{\prime}}{p}\right)^{(\gamma-1) / \gamma}-1\right]\right\}_{S}^{1 / 2} \tag{D31}
\end{equation*}
$$

The value of $M_{S}$ calculated is then compared with the value of $M_{S}$ specified in the main program. If the values are different, the starter value of $M_{x, 5}$, which is initially obtained as $M_{x, 5}=M_{S}$, is adjusted according to

$$
\begin{equation*}
M_{x, 5} \ln _{n+1}=M_{x, 5} \ln \left[\frac{M_{S(\text { prescribed })}}{M_{S(\text { calculated })}}\right] \tag{D32}
\end{equation*}
$$

The entire calculation is repeated until convergence is obtained according to

$$
\begin{equation*}
\left|\left[\left(M_{S}\right)_{n+1}-\left(M_{S}\right)_{n}\right]\right| \leq 0.001 \tag{D33}
\end{equation*}
$$

After the final stator exit Mach number is determined, the stator exit fluid properties are calculated. The static temperature is, since $T_{S}^{\prime}=T_{6}^{\prime}$,

$$
\begin{equation*}
\mathrm{T}_{S}=\frac{\mathrm{T}_{\mathrm{S}}^{\prime}}{\left[1+\frac{1}{2}(\gamma-1) \mathrm{M}^{2}\right]_{\mathrm{S}}} \tag{D34}
\end{equation*}
$$

and the stator exit velocity, which is prescribed as axial, is then calculated as

$$
\begin{equation*}
V_{S}=(M \sqrt{g \gamma R T})_{S} \tag{D35}
\end{equation*}
$$

The total pressure at the stator exit is calculated knowing the value of $p_{S}$ and the input value of $\mathrm{M}_{\mathrm{S}}$ from the expression

$$
\begin{equation*}
p_{S}^{\prime}=p_{S}\left[1+\frac{1}{2}(\gamma-1) M^{2}\right]_{S}^{\gamma_{S} /\left(\gamma_{S}-1\right)} \tag{D36}
\end{equation*}
$$

Turbine-stage efficiency. - The turbine-stage efficiency required for the main program is a total-to-total efficiency since the main program only deals with total conditions. This value of turbine efficiency will replace the assumed value used as an initial value by the main program as discussed in appendix B. The stage efficiency is calculated initially for the case of no leakage (i.e., $w_{5}=w_{1}$ ). This calculation yields the maximum attainable efficiency for the previously calculated velocity diagrams. By definition, the turbine-stage efficiency is

$$
\begin{equation*}
\eta_{\mathrm{T}}=\frac{\mathrm{P}_{\mathbf{T}}}{\mathrm{P}_{\mathrm{T}, \mathrm{id}}} \tag{D37}
\end{equation*}
$$

where, for a constant turbine pitch diameter, the no-leakage actual power is

$$
\begin{equation*}
P_{T}=\frac{U_{T}\left(W_{u, 5}-W_{u, 6}\right) w_{4}}{g J} \tag{D38}
\end{equation*}
$$

and the ideal power based on total-to-total conditions is

$$
\begin{equation*}
\mathrm{P}_{\mathrm{T}, \mathrm{id}}=\mathrm{w}_{4}\left(\Delta \mathrm{~h}_{\mathrm{T}}^{\prime}\right)_{\mathrm{id}} \tag{D39}
\end{equation*}
$$

As illustrated in the h-S diagram of figure 20

$$
\begin{equation*}
\left(\Delta h_{T}^{\prime}\right)_{i d}=h_{4}^{\prime}-\left(h_{S}^{\prime}\right)_{i d} \tag{D40}
\end{equation*}
$$

The ideal total-to-total enthalpy drop from turbine inlet to exit stator exit is calculated using the ADIABT subroutine. Inputs required are $p_{4}^{\prime}, T_{4}^{\prime}$, and $p_{S}^{\prime}$ along with $\eta_{\mathrm{T}}=1.0$. The subroutine also issues as output the isentropic total temperature at the exit stator exit $\left(T_{S}^{\prime}\right)_{i d}$. The efficiency is then obtained using equations (D37) to (D40) as

$$
\begin{equation*}
\eta_{\mathrm{T}}=\frac{\mathrm{U}_{\mathrm{T}}\left(\mathrm{~W}_{\mathrm{u}, 5}-\mathrm{W}_{\mathrm{u}, 6}\right)}{\operatorname{gJ}\left(\Delta \mathrm{h}_{\mathrm{T}}^{\prime}\right)_{\mathrm{id}}} \tag{D41}
\end{equation*}
$$

This value of efficiency is then adjusted to account for the turbine flow leakage at the inlet stator exit to the fan passage. This leakage reduces shaft power output by reducing the mass flow across the rotor. The adjusted efficiency is then given as

$$
\begin{equation*}
\eta_{\mathrm{T}, \text { leak }}=\eta_{\mathrm{T}}\left(1-\mathrm{Z}_{5}\right) \tag{D42}
\end{equation*}
$$

The value of $Z_{5}$, which is defined as the ratio of the turbine seal leakage flow rate to turbine inlet flow rate, is given as

$$
\begin{equation*}
\mathrm{Z}_{5}=\frac{\mathrm{w}_{\text {leak }}}{\mathrm{w}_{4}} \tag{D43}
\end{equation*}
$$

Details for computing the leakage fraction are given in appendix $F$.
The total-to-total efficiency as given by equation (D42) corresponds to the definition of efficiency used in the main program (appendix B).

Partial admission effects on design-point efficiency. - Additional losses in power will arise in partial admission due to (1) windage losses (centrifuging of gas in the nonactive part of the wheel), (2) filling and emptying losses (expansion and loss of momen-
tum of the gas in the blade passages at the end of the sector of admission), and (3) mixing losses (mixing of the working gas with stagnant gas in the blade passages at the ends of the admission arc). The method developed by Stenning (ref. 16) is used to estimate these losses.

The filling and emptying loss affects the momentum of the gas leaving the rotor blades so that, in effect, the relative velocity leaving the blade is reduced. Thus equation (D15) becomes

$$
\begin{equation*}
\mathrm{w}_{6}=\mathrm{w}_{5} \sqrt{\eta_{\mathrm{B}}} \mathrm{~K}_{\mathrm{pa}} \tag{D44}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathrm{K}_{\mathrm{pa}}=\left(1-\frac{\mathrm{s}}{3 \mathrm{a}}\right) \tag{D45}
\end{equation*}
$$

In equation (D45), $a$ is the arc length of admission given by

$$
\begin{equation*}
\mathrm{a}=\pi \mathrm{d} \mathrm{~T}, \mathrm{~m}\left(\frac{\theta}{360}\right) \tag{D46}
\end{equation*}
$$

and $s$ is the blade spacing at the mean line, which is obtained from the blade solidity $\sigma=\mathrm{c} / \mathrm{s}$ and aspect ratio $\mathrm{AR}=\mathrm{H}_{\mathrm{T}} / \mathrm{c}$ to give

$$
\begin{equation*}
\mathrm{s}=\frac{\mathrm{H}_{\mathrm{T}}}{\sigma(\mathrm{AR})} \tag{D47}
\end{equation*}
$$

The values of solidity and aspect ratio are program inputs.
The effect on the actual power due to pumping and mixing of the fluid in the rotor is given as a modification of equation (D38).

$$
\begin{equation*}
P_{T}=\frac{U_{T}}{g J}\left(W_{u, 5}-W_{u, 6}\right) w_{4}-\frac{1.4 U_{T}^{3} c w_{4}}{2 g J V_{5} a \cos \alpha_{5}} \tag{D48}
\end{equation*}
$$

Simplification results in

$$
\begin{equation*}
P_{T}=\frac{U_{T} W_{4}}{g J}\left(W_{u, 5}-W_{u, 6}-\frac{1.4 U_{T}^{2} c}{2 V_{5} \mathrm{c} \cos \alpha_{5}}\right) \tag{D49}
\end{equation*}
$$

The revised turbine total-to-total efficiency expression replacing equation (D41) is, then,

$$
\begin{equation*}
\eta_{\mathrm{T}}=\frac{\mathrm{U}_{\mathrm{T}}}{\left(\Delta \mathrm{~h}_{\mathrm{T}}^{\prime}\right)_{i d} \mathrm{gJ}}\left(\mathrm{~W}_{\mathrm{u}, 5}-\mathrm{W}_{\mathrm{u}, 6}-\frac{1.4 \mathrm{U}_{\mathrm{T}}^{2} \mathrm{c}}{2 \mathrm{~V}_{5^{\mathrm{a}} \cos \alpha_{5}}}\right) \tag{D50}
\end{equation*}
$$

Turbine annulus flow areas. - After completing the velocity diagram and efficiency calculations, turbine annulus areas were obtained. The inlet stator exit annulus area for full admission is giverrby the expression

$$
\begin{equation*}
\mathrm{A}_{5}=\frac{144 \mathrm{w}_{4}}{\mathrm{~V}_{\mathrm{x}, 5} \rho_{5}} \tag{D51}
\end{equation*}
$$

Rotor inlet annulus area is for constant axial velocity assuming no change in static conditions

$$
\begin{equation*}
A_{5^{\prime}}=A_{5}\left(1-Z_{5}\right) \tag{D52}
\end{equation*}
$$

and, for prescribed constant annulus area,

$$
\begin{equation*}
A_{6}=A_{5} \tag{D53}
\end{equation*}
$$

The exit stator exit annulus area is

$$
\begin{equation*}
A_{S}=\frac{R_{S} T_{S} S_{4}\left(1-Z_{5}\right)}{p_{S} V_{S}} \tag{D54}
\end{equation*}
$$

Turbine design parameters. - The speed-work parameter (as defined in ref. 14) is given as

$$
\begin{equation*}
\lambda=\frac{\mathrm{U}_{\mathrm{T}}}{\Delta \mathrm{~V}_{\mathrm{u}}} \tag{D55}
\end{equation*}
$$

But, for constant turbine pitch diameter

$$
\Delta \mathrm{V}_{\mathrm{u}}=\Delta \mathrm{W}_{\mathrm{u}}
$$

thus,

$$
\begin{equation*}
\lambda=\frac{\mathrm{U}_{\mathrm{T}}}{\mathrm{~W}_{\mathrm{u}, 5}-\mathrm{W}_{\mathrm{u}, 6}} \tag{D56}
\end{equation*}
$$

Another parameter often used in describing turbine efficiency is the blade-jet speed ratio, defined as

$$
\begin{equation*}
\nu=\frac{\mathrm{U}_{\mathrm{T}}}{\mathrm{~V}_{\mathrm{j}}} \tag{D57}
\end{equation*}
$$

where the velocity term $\mathrm{V}_{\mathrm{j}}$ is the ideal jet velocity corresponding to the ideal expansion from inlet total to exit static conditions across the turbine

$$
\begin{equation*}
\mathrm{v}_{\mathrm{j}}=\frac{\mathrm{v}_{5}}{\sqrt{\eta_{\mathrm{N}}}} \tag{D58}
\end{equation*}
$$

Subroutine TURB-2 - No Exit Stators

In this design the whirl component of the exit velocity is lost because it is not available to produce turbine thrust. This subroutine differs from the TURB-1 subroutine in that $M_{S}=M_{x, 6}, p_{S}^{\prime}=p_{6}^{\prime}, T_{S}=T_{6}$, and $V_{S}=V_{x, 6}$. Specifying a value of $M_{S}$ in the main program sets both the rotor inlet and exit axial velocity for this case. Definitions of stage work and turbine total efficiency are not altered. The calculation procedure and sequences for this case are the same as those of the impulse stator case of TURB-1.

## Subroutine TURB-4-Diffusing Exit Stator

In order to get the maximum power from impulse turbines along with obtaining low exit velocities, the use of diffuser exit stators is required. This type of vane will be designed to, not only turn the flow, but also to permit a reduction in velocity such that $\mathrm{V}_{\mathrm{S}}<\mathrm{V}_{6}$. The velocity diagrams for this case are the same as those shown in figure 12. For this type of stage and for an ambient exit pressure $\left(p_{S}=p_{0}\right)$, the rotor inlet and exit static pressures will be subambient.

The amount of diffusion in a blade row will depend on the loss that can be tolerated.

The losses in the blade were expressed in terms of the total-pressure-loss coefficient:

$$
\begin{equation*}
\mathrm{K}_{\mathrm{S}}=\frac{\frac{1-p_{S}^{\prime}}{\mathrm{p}_{6}^{\prime}}}{\frac{1-\mathrm{p}_{6}}{\mathrm{p}_{6}^{\prime}}}=\frac{\frac{1-\mathrm{p}_{S}^{\prime}}{\mathrm{p}_{6}^{\prime}}}{1-\left(1+\frac{\gamma-1}{2} M^{2}\right)_{6}^{\gamma_{6} /\left(\gamma_{6}-1\right)}} \tag{D59}
\end{equation*}
$$

The value of the total-pressure-loss coefficient was estimated from the relation

$$
\begin{equation*}
\mathrm{K}_{\mathrm{S}}=\frac{2 \sigma_{\mathrm{S}} \mathrm{~K}_{\mathrm{S}}^{*} \mathrm{e}^{3.7 \mathrm{D}}}{\cos \beta_{6}} \tag{D60}
\end{equation*}
$$

where $D$ is the diffusion factor, $\sigma_{S}$ is the stator solidity (taken to be 1.2), and $K_{S}^{*}$ is equal to ( $\mathrm{K}_{\mathrm{S}} \cos \beta_{6} / 2 \sigma_{\mathrm{S}}$ ) ${ }_{\mathrm{D}=0}$ (taken to be 0.005 ). The diffusion factor for turning back to axial is defined as

$$
\begin{equation*}
\mathrm{D}=\left(1-\frac{\mathrm{V}_{\mathrm{S}}}{\mathrm{~V}_{6}}\right)+\frac{\sin \alpha_{6}}{2 \sigma_{\mathrm{S}}} \tag{D61}
\end{equation*}
$$

The value of exit velocity $V_{S}$ can be calculated from equation (D61) knowing rotor exit flow conditions ( $\mathrm{V}_{6}$ ), an input value of exit stator solidity, and an input value of allowable diffusion factor. In order to insure that a solution is satisfactory, the program iterates on diffusion factor $D$ until the calculated value of exit stator exit Mach number agrees with the prescribed value to within 0.001 . This procedure then yields a specific value of diffusion factor (program output) for each choice of exit stator exit Mach number. The value of diffusion factor calculated for a choice of $M_{S}$ may be beyond the practical limit thus limiting the range of exit stator exit Mach number.

## APPENDIX E

## TWO-STAGE TURBINE SUBROUTINE

For turbine designs with large pressure ratios, it may be desirable to design a twostage turbine in order to reduce the pressure ratio per stage and increase stage and overall efficiencies, thus reducing required turbine weight flow. This section provides a calculation of turbine total-to-total efficiency for use in the main program. The twostage design is based on full admission, impulse rotors, and near-equal stage work with each individual stage made up of a stator and rotor. In addition, there is an impulse exit stator to turn the gas flow to the axial direction to increase turbine thrust.

The turbine design point efficiency includes the effects of blade row losses. In addition, it is adjusted to include the reduction in turbine mass flow due to turbine-to-fan seal leakage. It is assumed that the velocity diagrams are not affected by this reduction in flow rate. Only the leakage path upstream of the first-stage rotor is considered in the analysis.

The two-stage turbine is mounted at the tip of the fan rotor blade as shown in figure 3. The turbine station designations for the two-stage turbine design are given in figure 9, and the typical velocity diagrams are shown in figure 13. Provision is made in the analysis for a variation of mean diameter between first- and second-stage rotors. Most inputs required from the main program are identical to those required for the single-stage designs given in appendix D. The rotor inlet axial Mach number for each stage is at first set equal to the exit stator exit Mach number, that is, $M_{x, 5}=M_{x, 7}=M_{S}$.

## First Stage

The static pressure at the first-stage rotor inlet was determined by the expression

$$
\begin{equation*}
\mathrm{p}_{5}=\mathrm{X} \sqrt{\mathrm{p}_{4}^{\prime} \mathrm{p}_{0}} \tag{E1}
\end{equation*}
$$

where $X$ is an input that enabled the program to obtain variable work split between the stages. For example, a value of $X$ equal to 0.90 results in a 55 -percent -45 -percent work split for the first and second stages, respectively. With the calculated value of $p_{5}$ and the inlet conditions $\mathrm{T}_{4}^{\prime}, \mathrm{p}_{4}^{\prime},\left.\frac{\mathrm{f}}{\mathrm{a}}\right|_{4}, \eta_{\mathrm{b}}$, and an initial trial input value of $\eta_{\mathrm{N}, 5}$, the subroutine ADIABT is used to obtain inlet stator outlet condition $\mathrm{T}_{5}$ and the enthalpy drop

$$
\begin{equation*}
\Delta \mathrm{h}_{\mathrm{N}, 5}=\mathrm{h}_{4}^{\prime}-\mathrm{h}_{5} \tag{E2}
\end{equation*}
$$

for the expansion from $p_{4}^{\prime}$ and $T_{4}^{\prime}$ to $p_{5}$. The inlet stator exit velocity is then

$$
\begin{equation*}
\mathrm{V}_{5}=\sqrt{2 \mathrm{gJ} \Delta \mathrm{~h}_{\mathrm{N}, 5}} \tag{E3}
\end{equation*}
$$

The values of the gas constants $R_{5}$ and $\gamma_{5}$ are obtained from the subroutine GASCON and GAMMA, respectively (appendix G). The Mach number at the inlet stator exit is then

$$
\begin{equation*}
\mathrm{M}_{5}=\left(\frac{\mathrm{V}}{\sqrt{\gamma \mathrm{gRT}}}\right)_{5} \tag{E4}
\end{equation*}
$$

A new value of inlet stator efficiency is then obtained from the relation

$$
\begin{equation*}
\eta_{\mathrm{N}, 5}=1-\xi_{\mathrm{N}, 5} \tag{E5}
\end{equation*}
$$

and the inlet stator loss relations of appendix C. The value of $\xi_{\mathrm{N}, 5}$ is obtained from equation (C13) of appendix $C$. Once a value of $\eta_{N, 5}$ has been obtained, the program iterates on $\mathrm{M}_{5}$ until

$$
\begin{equation*}
\left|\left(\left.\mathrm{M}_{5}\right|_{\mathrm{n}}-\left.\mathrm{M}_{5}\right|_{\mathrm{n}+1}\right)\right| \leq 0.002 \tag{E6}
\end{equation*}
$$

The velocity diagram calculations are conducted by initially setting

$$
\begin{equation*}
M_{x, 5}=M_{S} \tag{E7}
\end{equation*}
$$

Then

$$
\begin{align*}
& \alpha_{5}=\cos ^{-1}\left(\frac{\mathrm{M}_{\mathrm{x}}}{\mathrm{M}}\right)_{5}  \tag{E8}\\
& \mathrm{~V}_{\mathrm{x}, 5}=\mathrm{V}_{5} \cos \alpha_{5}  \tag{E9}\\
& \mathrm{~V}_{\mathrm{u}, 5}=\mathrm{V}_{5} \sin \alpha_{5} \tag{E10}
\end{align*}
$$

The relative tangential velocity component is

$$
\begin{equation*}
\mathrm{w}_{\mathrm{u}, 5}=\mathrm{V}_{\mathrm{u}, 5}-\mathrm{U}_{\mathrm{T}, 6} \tag{E11}
\end{equation*}
$$

and

$$
\begin{equation*}
\beta_{5}=\tan ^{-1}\left(\frac{\mathrm{~W}_{\mathrm{u}}}{\mathrm{~V}_{\mathrm{x}}}\right)_{5} \tag{E12}
\end{equation*}
$$

Then

$$
\begin{equation*}
\mathrm{W}_{5}=\sqrt{\mathrm{v}_{\mathrm{x}, 5}^{2}+\mathrm{W}_{\mathrm{u}, 5}^{2}} \tag{E13}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{M}_{\mathrm{r}, 5}=\left(\frac{\mathrm{W}}{\sqrt{\gamma \mathrm{gRT}}}\right)_{5} \tag{E14}
\end{equation*}
$$

The total pressure at the rotor inlet is

$$
\begin{equation*}
\mathrm{p}_{5}^{\prime}=\mathrm{p}_{5}\left[1+\frac{1}{2}(\gamma-1) \mathrm{M}^{2}\right]_{5}^{\gamma_{5} /\left(\gamma_{5}-1\right)} \tag{E15}
\end{equation*}
$$

and the density is

$$
\begin{equation*}
\rho_{5}=\left(\frac{144 \mathrm{p}}{R T}\right)_{5} \tag{E16}
\end{equation*}
$$

After completing the calculation for the rotor inlet, the exit conditions are initiated by calculating the losses across the rotor blades. The relative exit velocity is

$$
\begin{equation*}
\mathrm{w}_{6}=\mathrm{W}_{5} \sqrt{\eta_{\mathrm{B}, 6}} \tag{E17}
\end{equation*}
$$

where the rotor blade efficiency $\quad \eta_{B, 6}$ is calculated from

$$
\begin{equation*}
\eta_{\mathrm{B}, 6}=1-\xi_{\mathrm{B}, 6} \tag{E18}
\end{equation*}
$$

The loss coefficient is obtained from equation (C21) of appendix C. The values of $\eta_{B, 6}$ and $W_{6}$ permit the calculation of rotor energy loss:

$$
\begin{equation*}
\mathrm{E}_{\mathrm{B}, 6}=\frac{\mathrm{w}_{5}^{2}-\mathrm{w}_{6}^{2}}{2 \mathrm{gJ}} \tag{E19}
\end{equation*}
$$

Next, $C_{p, 6}$ is obtained from the SPHTCP subroutine of appendix $G$ using rotor inlet conditions, $\mathrm{p}_{5}$ and $\mathrm{T}_{5}$. Then

$$
\begin{equation*}
\mathrm{T}_{6}=\mathrm{T}_{5}+\left(\frac{\mathrm{E}_{\mathrm{B}}}{\mathrm{C}_{\mathrm{p}}}\right)_{6} \tag{E20}
\end{equation*}
$$

For an impulse rotor,

$$
\begin{equation*}
\mathrm{p}_{6}=\mathrm{p}_{5} \tag{E21}
\end{equation*}
$$

and the density is

$$
\begin{equation*}
\rho_{6}=\left(\frac{144 \mathrm{p}}{\mathrm{RT}}\right)_{6} \tag{E22}
\end{equation*}
$$

For the condition that $A_{6}=A_{5}$ and for continuity

$$
\begin{equation*}
\mathrm{V}_{\mathrm{x}, 6}=\mathrm{V}_{\mathrm{x}, 5} \frac{\mathrm{~T}_{6}}{\mathrm{~T}_{5}} \tag{E23}
\end{equation*}
$$

The tangential component of the flow is (see velocity diagram fig. 13)

$$
\begin{equation*}
\mathrm{w}_{\mathrm{u}, 6}=\sqrt{\mathrm{w}_{6}^{2}-\mathrm{v}_{\mathrm{x}, 6}^{2}} \tag{E24}
\end{equation*}
$$

and the component $\mathrm{V}_{\mathrm{u}, 6}$ is

$$
\begin{equation*}
\mathrm{V}_{\mathrm{u}, 6}=\left(\mathrm{W}_{\mathrm{u}}-\mathrm{U}_{\mathrm{T}}\right)_{6} \tag{E25}
\end{equation*}
$$

Then the relative gas angle at the rotor exit is

$$
\begin{equation*}
\beta_{6}=\tan ^{-1}\left(\frac{\mathrm{~W}_{\mathrm{u}}}{\mathrm{~V}_{\mathrm{x}}}\right)_{6} \tag{E26}
\end{equation*}
$$

and the absolute angle is

$$
\begin{equation*}
\alpha_{6}=\tan ^{-1}\left(\frac{v_{u}}{v_{x}}\right)_{6} \tag{E27}
\end{equation*}
$$

Thus,

$$
\begin{equation*}
\mathrm{V}_{6}=\left(\mathrm{V}_{\mathrm{x}} \cos \alpha\right)_{6} \tag{E28}
\end{equation*}
$$

The rotor exit absolute Mach number is calculated from the expression

$$
\begin{equation*}
\mathrm{M}_{6}=\left(\frac{\mathrm{V}}{\sqrt{\gamma \mathrm{gRT}}}\right)_{6} \tag{E29}
\end{equation*}
$$

The relative Mach number is then

$$
\begin{equation*}
\mathbf{M}_{r, 6}=\left(\frac{\mathrm{W}}{\sqrt{\gamma \mathrm{gRT}}}\right)_{6} \tag{E30}
\end{equation*}
$$

The total temperature at the rotor exit is

$$
\begin{equation*}
\mathrm{T}_{6}^{\prime}=\mathrm{T}_{6}\left(1+\frac{\gamma-1}{2} \mathrm{M}^{2}\right)_{6} \tag{E31}
\end{equation*}
$$

and the total pressure is

$$
\begin{equation*}
\mathrm{p}_{6}^{\prime}=\mathrm{p}_{6}\left[1+\frac{1}{2}(\gamma-1) \mathrm{M}^{2}\right]_{6}^{\gamma_{6} /\left(\gamma_{6}-1\right)} \tag{E32}
\end{equation*}
$$

These fluid properties now constitute the inputs to the second-stage stator.

## Second Stage

The velocity diagram for the second-stage stator and rotor are shown in figure 13. For the stage,

$$
\begin{equation*}
\mathrm{p}_{7}=\mathrm{p}_{8}=\mathrm{p}_{\mathrm{S}} \tag{E33}
\end{equation*}
$$

The enthalpy drop across the second-stage stator was first calculated with a trial starter value of stator efficiency $\eta_{\mathrm{N}, 7}$ and inputs to the ADIABT subroutine (appendix G) $\mathrm{T}_{6}^{\prime}, \mathrm{p}_{6}^{\prime},\left.\frac{\mathrm{f}}{\mathrm{a}}\right|_{4}, \eta_{\mathrm{b}}$, and $\mathrm{p}_{7}$. This yielded $\mathrm{T}_{7}$ and

$$
\begin{equation*}
\Delta h_{N, 7}=h_{6}^{\prime}-h_{7} \tag{E34}
\end{equation*}
$$

Then the stator exit velocity is

$$
\begin{equation*}
\mathrm{V}_{7}=\sqrt{2 \mathrm{gJ} \Delta \mathrm{~h}_{\mathrm{N}, 7}} \tag{E35}
\end{equation*}
$$

The Mach number is then calculated as

$$
\begin{equation*}
M_{7}=\left(\frac{V}{\sqrt{\gamma g R T}}\right)_{7} \tag{E36}
\end{equation*}
$$

A new value of stator efficiency $\eta_{\mathrm{N}, 7}$ is then obtained from the relation

$$
\begin{equation*}
\eta_{\mathrm{N}, 7}=1-\xi_{\mathrm{N}, 7} \tag{E37}
\end{equation*}
$$

The energy-loss coefficient $\xi^{\mathrm{N}, 7}$ is calculated in the same manner as the loss coefficient of equation (E5).

The relation used for adjusting the mean radius of the second-stage rotor is

$$
\begin{equation*}
\left(\mathbf{r}_{\mathrm{T}, \mathrm{~m}}\right)_{8}=(\mathrm{TRM})\left(\mathrm{r}_{\mathrm{T}, \mathrm{~m}}\right)_{6} \tag{E38}
\end{equation*}
$$

where (TRM) is an input which is generally less than 1.0 because of the fan duct contraction. The pitch line velocity is then

$$
\begin{equation*}
\mathrm{U}_{\mathrm{T}, 8}=(\mathrm{TRM}) \mathrm{U}_{\mathrm{T}, 6} \tag{E39}
\end{equation*}
$$

Once the second-stage stator efficiency $\eta_{\mathrm{N}, 8}$ has been calculated, the program iterates on stator exit Mach number until

$$
\begin{equation*}
\left|\left(\left.M_{7}\right|_{n}-\left.M_{7}\right|_{n+1}\right)\right| \leq 0.002 \tag{E40}
\end{equation*}
$$

At the second-stage rotor inlet, the axial flow Mach number $M_{x, 7}$ is taken to be equal to the axial Mach number into the first-stage rotor $M_{x, 5}$. The second-stator absolute exit angle $\alpha_{7}$ is then calculated as

$$
\begin{equation*}
\alpha_{7}=\cos ^{-1}\left(\frac{M_{x}}{M}\right)_{7} \tag{E41}
\end{equation*}
$$

And

$$
\begin{gather*}
\mathrm{V}_{\mathrm{x}, 7}=(\mathrm{V} \cos \alpha)_{7}  \tag{E42}\\
\mathrm{~V}_{\mathrm{u}, 7}=(\mathrm{V} \sin \alpha)_{7}  \tag{E43}\\
\mathrm{p}_{7}^{\prime}=\mathrm{p}_{7}\left[1+\frac{1}{2}(\gamma-1) \mathrm{M}^{2}\right]_{7}^{\gamma_{7} /\left(\gamma_{7}-1\right)} \tag{E44}
\end{gather*}
$$

In the preceding equation $p_{7}$ is obtained from equation (E33). The flow density is

$$
\begin{equation*}
\rho_{7}=\left(\frac{144 \mathrm{p}}{\mathrm{RT}}\right)_{7} \tag{E45}
\end{equation*}
$$

The remaining velocities and angles of the velocity diagram at rotor inlet are

$$
\begin{gather*}
\mathrm{W}_{\mathrm{u}, 7}=\mathrm{V}_{\mathrm{u}, 7}-\mathrm{U}_{\mathrm{T}, 8}  \tag{E46}\\
\beta_{7}=\tan ^{-1}\left(\frac{\mathrm{~W}_{\mathrm{u}}}{\mathrm{~V}_{\mathrm{x}}}\right)_{7}  \tag{E47}\\
\mathrm{~W}_{7}=\sqrt{\mathrm{V}_{\mathrm{x}, 7}^{2}+\mathrm{W}_{\mathrm{u}, 7}^{2}}  \tag{E48}\\
\mathrm{M}_{\mathrm{r}, 7}=\left(\frac{\mathrm{W}}{\sqrt{\gamma \mathrm{gRT}}}\right)_{7} \tag{E49}
\end{gather*}
$$

For the rotor outlet, the relative velocity is obtained from

$$
\begin{equation*}
\mathrm{w}_{8}=\mathrm{w}_{7} \sqrt{\eta_{\mathrm{B}, 8}} \tag{E50}
\end{equation*}
$$

In this equation, the second-stage rotor blade efficiency is obtained from the expression

$$
\begin{equation*}
\eta_{\mathrm{B}, 8}=1-\xi_{\mathrm{B}, 8} \tag{E51}
\end{equation*}
$$

for which the loss coefficient is obtained in a manner similar to the first-stage rotorblade loss coefficient. The value of second-stage rotor blade efficiency from equation (E51) enables the determination of energy loss through the rotor based on relative velocities

$$
\begin{equation*}
\mathrm{E}_{\mathrm{B}, 8}=\frac{\mathrm{W}_{7}^{2}-\mathrm{W}_{8}^{2}}{2 \mathrm{gJ}} \tag{E52}
\end{equation*}
$$

then $C_{p, 8}$ is obtained from subroutine $S P H T C P$ of appendix $G$ using $T_{7}$. Then,

$$
\begin{equation*}
T_{8}=T_{7}+\left(\frac{E_{B}}{C_{p}}\right)_{8} \tag{E53}
\end{equation*}
$$

By specification, the static pressure is

$$
\begin{equation*}
\mathrm{p}_{8}=\mathrm{p}_{7} \tag{E54}
\end{equation*}
$$

and the flow density at the rotor exit is

$$
\begin{equation*}
\rho_{8}=\left(\frac{144 \mathrm{p}}{R T}\right)_{8} \tag{E55}
\end{equation*}
$$

From continuity and the condition that $A_{8}=A_{7}$ the exit axial velocity is

$$
\begin{equation*}
\mathrm{V}_{\mathrm{x}, 8}=\mathrm{V}_{\mathrm{x}, 7}\left(\frac{\mathrm{~T}_{8}}{\mathrm{~T}_{7}}\right) \tag{E56}
\end{equation*}
$$

the tangential component is then

$$
\begin{equation*}
\mathrm{W}_{\mathrm{u}, 8}=\sqrt{\mathrm{W}_{8}^{2}-\mathrm{v}_{\mathrm{x}, 8}^{2}} \tag{E57}
\end{equation*}
$$

and the component $\mathrm{V}_{\mathrm{u}, 8}$ is

$$
\begin{equation*}
\mathrm{V}_{\mathrm{u}, 8}=\left(\mathrm{W}_{\mathrm{u}}-\mathrm{U}_{\mathrm{T}}\right)_{8} \tag{E58}
\end{equation*}
$$

The rotor exit angles are

$$
\begin{align*}
& \beta_{8}=\tan ^{-1}\left(\frac{\mathrm{w}_{\mathrm{u}}}{\mathrm{~V}_{\mathrm{x}}}\right)_{8}  \tag{E59}\\
& \alpha_{8}=\tan ^{-1}\left(\frac{\mathrm{~V}_{\mathrm{u}}}{\mathrm{~V}_{\mathrm{x}}}\right)_{8} \tag{E60}
\end{align*}
$$

and

$$
\begin{equation*}
\mathrm{V}_{8}=\left(\frac{\mathrm{V}_{\mathrm{x}}}{\cos \alpha}\right)_{8} \tag{E61}
\end{equation*}
$$

Then

$$
\begin{equation*}
M_{8}=\left(\frac{V}{\sqrt{r g R T}}\right)_{8} \tag{E62}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{M}_{\mathrm{r}, 8}=\left(\frac{\mathrm{W}}{\sqrt{\gamma \mathrm{gRT}}}\right)_{8} \tag{E63}
\end{equation*}
$$

The rotor exit fluid properties are

$$
\begin{equation*}
\mathrm{T}_{8}^{\prime}=\mathrm{T}_{8}\left(1+\frac{\gamma-1}{2} \mathrm{M}^{2}\right)_{8} \tag{E64}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{p}_{8}^{\prime}=\mathrm{p}_{8}\left(1+\frac{\gamma-1}{2} \mathrm{M}^{2}\right)_{8}^{\gamma_{8} /\left(\gamma_{8}-1\right)} \tag{E65}
\end{equation*}
$$

## Exit Stators

The velocity diagram for the exit stators is given in figure 13. The exit stator is specified as impulse, that is, $\mathrm{p}_{8}=\mathrm{p}_{\mathrm{S}}$. The exit stator total-pressure-loss coefficient $K_{S}$, is calculated in a manner similar to that of the single-stage turbine. Equation (C27) of appendix $C$ gives the expression for $K_{S}$. The total pressure at the stator exit is

$$
\begin{equation*}
\mathrm{p}_{\mathrm{S}}^{\prime}=\mathrm{p}_{8}^{\prime}-\mathrm{K}_{\mathrm{S}}\left(\mathrm{p}^{\prime}-\mathrm{p}\right)_{8} \tag{E66}
\end{equation*}
$$

Then knowing $p_{S}^{\prime}$ and $p_{S}$, the exit axial Mach number is

$$
\begin{equation*}
M_{S}=\left\{\frac{2}{\gamma-1}\left[\left(\frac{p^{\prime}}{p}\right)^{(\gamma-1) / \gamma}-1\right]\right\}_{S}^{1 / 2} \tag{E67}
\end{equation*}
$$

The calculated value of $M_{S}$ is then compared with the value of $M_{S}$ specified in the main program. If the values are different, an iteration is conducted based on an adjustment of the starter value of $M_{x, 5}$ and the second-stage axial Mach number $M_{x, 7}$ according to

$$
\begin{equation*}
\mathrm{M}_{\mathrm{x}, 5} \ln +1=\mathrm{M}_{\mathrm{X}, 5} \ln \left[\frac{\mathrm{M}_{\mathrm{S}(\text { prescribed })}}{\mathrm{M}_{\mathrm{S}(\text { calculated })}}\right] \tag{E68}
\end{equation*}
$$

and

$$
\begin{equation*}
\left.M_{x, 7}\right|_{n+1}=\left.M_{x, 5}\right|_{n+1} \tag{E69}
\end{equation*}
$$

This procedure is repeated until

$$
\begin{equation*}
\left|\left(M_{S}\left|n_{+1}-M_{S}\right| n\right)\right| \leq 0.001 \tag{E70}
\end{equation*}
$$

From the final value of exit stator exit axial Mach number, the exit fluid properties are

$$
\begin{equation*}
\mathrm{T}_{S}^{\prime}=\mathrm{T}_{8}^{\prime} \text { (adiabatic stator) } \tag{E71}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{T}_{\mathrm{S}}=\frac{\mathrm{T}_{\mathrm{S}}^{\prime}}{\left[1+\frac{1}{2}(\gamma-1) \mathrm{M}^{2}\right]_{\mathrm{S}}} \tag{E72}
\end{equation*}
$$

The exit stator exit fluid density is

$$
\begin{equation*}
\rho_{\mathrm{S}}=\left(\frac{144 \mathrm{p}}{\mathrm{RT}}\right)_{\mathrm{S}} \tag{E73}
\end{equation*}
$$

Then the exit stator exit axial velocity is calculated as

$$
\begin{equation*}
\mathrm{V}_{\mathrm{S}}=(\mathrm{M} \sqrt{\gamma \mathrm{RgT}})_{\mathrm{S}} \tag{E74}
\end{equation*}
$$

## Overall Turbine Efficiency

A calculated value of total-to-total efficiency based on blade-row losses is then developed to replace the assumed value initially used in the main program. The determination is based on the enthalpy-entropy diagram for the two-stage turbine shown in figure 21. The total-to-total efficiency is defined as

$$
\begin{equation*}
\eta_{\mathbf{T}}=\frac{\mathrm{P}_{\mathrm{T}}}{\mathbf{P}_{\mathbf{T}, \mathrm{id}}} \tag{E75}
\end{equation*}
$$

The actual power $P_{T}$ is made up of the first- and second-stage power inputs, which for the case of no leakage and full admission, is

$$
\begin{equation*}
P_{T}=P_{4-6}+P_{6-8} \tag{E76}
\end{equation*}
$$

where

$$
\begin{equation*}
P_{4-6}=U_{T, 6}\left(W_{u, 5}-W_{u, 6}\right) \frac{W_{4}}{g J} \tag{E77}
\end{equation*}
$$

and

$$
\begin{equation*}
P_{6-8}=\frac{U_{T, 8}\left(W_{u, 7}-W_{u, 8}\right) w_{4}\left(1-\mathrm{Z}_{6}\right)}{g J} \tag{E78}
\end{equation*}
$$

where $Z_{6}$ is the interstage stator leakage factor defined as the ratio of the flow that bypasses the interstage stator (see fig. 3) to the through flow. The value of $Z_{6}$ is a program input. It is assumed that the interstage leakage flow does not affect the stage velocity diagrams or flow areas. From equation (E39), equation (E76) can be rewritten as

$$
\begin{equation*}
\mathrm{P}_{\mathrm{T}}=\mathrm{U}_{\mathrm{T}, 6}\left[\mathrm{~W}_{\mathrm{u}, 5}-\mathrm{W}_{\mathrm{u}, 6}+(\mathrm{TRM})\left(\mathrm{W}_{\mathrm{u}, 7}-\mathrm{W}_{\mathrm{u}, 8}\right)\left(1-\mathrm{Z}_{6}\right)\right] \frac{\mathrm{w}_{4}}{\mathrm{gJ}} \tag{E79}
\end{equation*}
$$

The ideal power for the two-stage turbine based on total-to-total conditions is (from fig. 21)

$$
\begin{equation*}
\mathrm{P}_{\mathrm{T}, \mathrm{id}}=\mathrm{w}_{4}\left(\Delta \mathrm{~h}_{\mathrm{T}}^{\prime}\right)_{\mathrm{id}}=\mathrm{w}_{4}\left(\mathrm{~h}_{4}^{\prime}-\mathrm{h}_{\mathrm{S}, \mathrm{id}}^{\prime}\right) \tag{E80}
\end{equation*}
$$

The isentropic total-to-total enthalpy drop is obtained using the ADIABT subroutine of appendix $G$ with the inputs, $p_{4}^{\prime}, T_{4}^{\prime}, \mathrm{p}_{\mathrm{S}}^{\prime},\left.\frac{\mathrm{f}}{\mathrm{a}}\right|_{4}, \eta_{\mathrm{b}}$, and $\eta=1.0$. Thus, the overall turbine total-to-total efficiency becomes

$$
\begin{equation*}
\eta_{\mathbf{T}}=\frac{\mathrm{U}_{\mathrm{T}, 6}\left[\mathrm{~W}_{\mathrm{u}, 5}-\mathrm{W}_{\mathrm{u}, 6}+(\mathrm{TRM})\left(\mathrm{W}_{\mathrm{u}, 7}-\mathrm{W}_{\mathrm{u}, 8}\right)\left(1-\mathrm{Z}_{6}\right)\right]}{\left(\mathrm{h}_{4}^{\prime}-\mathrm{h}_{\mathrm{S}, \mathbf{i d}}^{\prime}\right) \mathrm{gJ}} \tag{E81}
\end{equation*}
$$

The total-to-total efficiency as given by equation (E81) corresponds to the definition of efficiency used in the main program. This value of efficiency is then adjusted to account for the turbine-to-fan leakage which reduces the actual power output by reducing the mass flow.

The two-stage total-to-total turbine efficiency adjusted for leakage is then

$$
\begin{equation*}
\eta_{\mathrm{T}, \text { leak }}=\eta_{\mathrm{T}}\left(1-\mathrm{Z}_{5}\right) \tag{E82}
\end{equation*}
$$

The value of $\mathrm{Z}_{5}$ in equation (E82) is defined as

$$
\begin{equation*}
\mathrm{Z}_{5}=\frac{\mathrm{w}_{\text {leak }}}{\mathrm{w}_{4}} \tag{E83}
\end{equation*}
$$

This expression is obtained using the method of appendix $F$.

## Turbine Annulus Flow Areas

The first-stage stator exit annulus area for full admission is given by

$$
\begin{equation*}
A_{5}=\frac{144 w_{4}}{V_{x, 5} \rho_{5}} \tag{E84}
\end{equation*}
$$

The rotor inlet annulus area which includes the turbine leakage term (to give the same velocity across the station) is given by

$$
\begin{equation*}
A_{5},=A_{5}\left(1-Z_{5}\right) \tag{E85}
\end{equation*}
$$

and as previously stated, rotor outlet annulus area is

$$
\begin{equation*}
A_{6}=A_{5} \tag{E86}
\end{equation*}
$$

The second-stage stator exit annulus area is

$$
\begin{equation*}
A_{7}=\frac{w_{4}\left(1-Z_{5}\right)}{\rho_{7} V_{x, 7}} \tag{E87}
\end{equation*}
$$

The second-stage rotor inlet area is the same as the inlet stator exit annulus area, and it was also prescribed that rotor inlet area equals rotor exit area, so that

$$
\begin{equation*}
\mathrm{A}_{8}=\mathrm{A}_{7} \tag{E88}
\end{equation*}
$$

Exit stator exit annulus area is obtained from

$$
\begin{equation*}
A_{S}=\frac{w_{4}\left(1-Z_{5}\right)}{\rho_{S} V_{S}} \tag{E89}
\end{equation*}
$$

Turbine Design Parameters

For a multistage turbine, the overall speed-work parameter as defined in reference 14 is

$$
\begin{equation*}
\bar{\lambda}=\frac{U_{T}}{\Delta V_{u}} \tag{E90}
\end{equation*}
$$

Then for a two-stage turbine,

$$
\Delta \mathrm{V}_{\mathrm{u}}=\mathrm{W}_{\mathrm{u}, 5}-\mathrm{W}_{\mathrm{u}, 6}+\mathrm{W}_{\mathrm{u}, 7}-\mathrm{W}_{\mathrm{u}, 8}
$$

and equation (E90) can be rewritten as

$$
\begin{equation*}
\bar{\lambda}=\frac{U_{T}}{\left(W_{u, 5}-W_{u, 6}+W_{u, 7}-W_{u, 8}\right)} \tag{E91}
\end{equation*}
$$

The blade-jet speed ratio is (where $T R M=1$, for simplicity)

$$
\begin{equation*}
\nu=\frac{\mathrm{U}_{\mathrm{T}}}{\mathrm{~V}_{\mathrm{j}}} \tag{E92}
\end{equation*}
$$

where $V_{j}$ is the ideal jet velocity corresponding to the ideal expansion from inlet to exit static conditions across the turbine. Equation (D58) of appendix D is the expression used for obtaining $V_{j}$.

## APPENDIX F

## LEAKAGE ANALYSIS

The principal problem with leakage seals for tip-turbine fan-rotor wheels (see fig. 3) is the difficulty in maintaining close seal clearances due to poor dimensional control inherent in this configuration. Seals applicable for such use could probably be of the abradable labyrinth or honeycomb types which allow for local rubbing contact without seizing or destruction of the seal.

## Leakage Flow Factor

Leakage of the primary flow from the turbine to the tip region in front of and behind the fan rotor not only reduces turbine power output and thrust but also deteriorates fan performance. Thus, it is important to be able to predict the magnitude of the leakage flow for the geometry in question. This analysis will consider only the leakage for the first-stage stator because it has a direct flow path to the inlet tip region of the fan rotor as shown in figure 3.

The basic relation for the leakage flow parameter was obtained from the relations given in references 17 and 18. The ideal leakage flow rate $\mathrm{w}_{\text {leak }}$ is given by

$$
\begin{equation*}
\mathrm{w}_{\text {leak }}=\mathrm{AC}_{\mathrm{f}} \mathrm{e} \sqrt{\mathrm{gp} \mathrm{p}^{\prime} \rho} \tag{F1}
\end{equation*}
$$

where $A$ is the seal clearance geometry area (equal to $\pi d_{F, t} c l$ ), $\rho$ is the density of the gas at the seal inlet, $C_{f}$ is the orifice flow coefficient, $p^{\prime}$ is the seal inlet total pressure, and $e$ is the expansion function. Rewriting equation (F1) to form the leakage flow factor results in

$$
\begin{equation*}
\frac{\mathrm{w}_{\text {leak }} \sqrt{T^{\prime} R}}{A p^{\prime}}=C_{f} e \sqrt{g} \tag{F2}
\end{equation*}
$$

In this equation, e was determined from experimental data on a round sharp-edge orifice. Values of e are tabulated in table I of reference 17 and plotted against $p_{e x} / p^{\prime}$ in figure 7 of the same reference. These values of e were determined based on a constant value of flow coefficient. Also, reference books give $C_{f}$ values for sharp-edge orifices from 0.60 to 0.624 ; an average value, $C_{f}=0.615$, was used in conjunction with the expansion function $e$ for reference 17 (single labyrinth).

Comparison of resultant plots of the product $C_{f}$ and e of references 17 and 18 presented in figure 22 showed very good agreement. Thus, for calculation purposes, the flow parameter based on equation (F2) was used in which the effective orifice coefficient $C_{f}$ is set equal to 0.615 .

For computer program use, it is desirable to have an analytical variation for the expansion function. It was found that the variation of $e$ in reference 17 could be expressed reasonably well by the relation (for $\mathrm{B} / \mathrm{cl}=1.0$ )

$$
\begin{equation*}
e=e_{\max } \sqrt{1-\left(\frac{p_{e x}}{p^{\prime}}\right)^{2.3}} \tag{F3}
\end{equation*}
$$

where $e_{\max }$ varied with the number of chambers in the seal. An expression for $e_{\max }$ as a function of the number of labyrinth restrictions $n_{l e a k}$ was deduced from the data of reference 17 and given as

$$
\begin{equation*}
\mathrm{e}_{\max }=\left(\frac{0.925}{\mathrm{n}_{\text {leak }}}\right)^{0.50} \tag{F4}
\end{equation*}
$$

Variations in $C_{f}$ due to other choices of $B / c 1$ are handled by varying the pressure ratio exponent in equation (F3) in the following manner

$$
\begin{equation*}
e=e_{\max } \sqrt{i-\left(\frac{p_{e x}}{p^{\prime}}\right)^{b}} \tag{F5}
\end{equation*}
$$

The variation of the exponent $b$ with element width-to-clearance ratio $\mathrm{B} / \mathrm{cl}$ was obtained from reference 17 and is shown plotted in figure 23.

The final expression for the seal leakage flow rate function is obtained using equations (F2) to (F4) as

$$
\begin{equation*}
\frac{w_{\text {leak }} \sqrt{T^{\prime} R}}{A p^{\prime}}=5.24 \mathrm{C}_{\mathrm{f}} \sqrt{\frac{1-\left(\frac{p_{\text {ex }}}{p^{\prime}}\right)^{2.3}}{n_{\text {leak }}}} \tag{F6}
\end{equation*}
$$

## Seal Pressure Ratio

Since the approach velocity upstream of the seal orifice is expected to be quite low (relatively small flow rate and large area), the total pressure is essentially the local static pressure. Thus, for the inlet seal, in terms of the program nomenclature (see fig. 3)

$$
\begin{equation*}
\mathrm{p}^{\prime}=\mathrm{p}_{5} \tag{F7}
\end{equation*}
$$

and the temperature $\mathrm{T}^{\prime}$ in equation ( F 6 ) is the total temperature, that is, $\mathrm{T}^{\prime}=\mathrm{T}_{5}^{\prime}$.
The seal exit pressure $p_{e x}$ is defined as the local static pressure at the inlet bellmouth at the point of the seal. This pressure is given by the expression

$$
\begin{equation*}
\mathrm{p}_{\mathrm{ex}}=\mathrm{p}_{0}-\mathrm{C}_{\text {leak }}\left(\mathrm{p}_{10}^{\prime}-\mathrm{p}_{10}\right) \tag{F8}
\end{equation*}
$$

where $\mathrm{p}_{10}^{\prime}-\mathrm{p}_{10}$ is the average value of the inlet velocity head at the mean fan radius and $C_{\text {leak }}$ is a constant obtained from potential flow solutions of typical inlet bellmouth sections for lift fans (e.g., ref. 19). Calculations indicated a representative $C_{\text {leak }}$ value of 1.2 , since the axial velocity at the tip of the rotor inlet is generally larger than at the midpoint. The average static pressure in equation (F8) is obtained from (for $\lambda=1.4)$

$$
\begin{equation*}
\frac{\mathrm{p}_{10}}{\mathrm{p}_{10}^{\prime}}=\frac{1}{\left(1+0.2 \mathrm{M}_{10}^{2}\right)^{3.5}} \tag{F9}
\end{equation*}
$$

where $p_{10}^{\prime}$ and $M_{10}$ are inputs from the main program.

## Leakage Rate

Once $C_{f} e$ and expansion pressure ratio $p_{e x} / p^{\prime}$ are obtained, the leakage flow factor equation (F6) can be rewritten in the following form by inserting $A=\pi d_{F, t}{ }^{c l}$ and noting that $\mathrm{p}^{\prime}$ and $\mathrm{T}^{\prime}$ are replaced by $\mathrm{p}_{5}$ and $\mathrm{T}_{5}^{\prime}$, respectively,

$$
\begin{equation*}
\mathrm{w}_{\text {leak }}=5.24 \pi \mathrm{~d}_{\mathrm{F}, \mathrm{t}}^{\mathrm{cl}} \frac{\mathrm{p}_{5}}{\sqrt{\mathrm{~T}_{5}^{\prime} \mathrm{R}}} \mathrm{C}_{\mathrm{f}} \sqrt{\frac{1-\left(\frac{\mathrm{p}_{\mathrm{ex}}}{\mathrm{p}_{5}}\right)^{2.3}}{\mathrm{n}_{\text {leak }}}} \tag{F10}
\end{equation*}
$$

Since the actual value of running clearance will not generally be known, the value of cl was prescribed by the empirical expression

$$
\begin{equation*}
\mathrm{cl}=\mathrm{C}_{\mathrm{c} 1} \mathrm{~d}_{\mathrm{F}, \mathrm{t}} \tag{F11}
\end{equation*}
$$

The factor $\mathrm{C}_{\mathrm{cl}}$, in practice has values of around $2.5 \times 10^{-4}$ for the rub-in type of seal. The leakage ratio based on turbine flow rate is then

$$
\begin{equation*}
\mathrm{Z}_{5}=\frac{\mathrm{w}_{\mathrm{leak}}}{\mathrm{w}_{4}} \tag{F12}
\end{equation*}
$$

The turbine-to-fan leakage for the two-stage turbine design (fig. 3) was treated in he same manner as the single-stage design. Inherent in the two-stage turbine is a igher inlet stator exit static pressure and possible mechanical design problems (larger learance gap) that would increase the leakage flow rate. These conditions may warrant somewhat larger value of $C_{c l}$ for use in equation (F11).

The leakage flow not only affects the performance of the turbine by reducing the rimary weight flow, but also affects the performance of the fan. The leakage flow disurbs the fan flow at the tip region which causes a reduction in fan efficiency $\eta_{F}$. The ffect on $\eta_{F}$ is not included specifically in the program; however, the value of $\eta_{F}$ which is an input) can be changed to account for the leakage flow if the variation is snown.

## APPENDIX G

## FLUID PROPERTY SUBROUTINES

In addition to the major subroutines required by the main program, there are 12 smaller routines that describe the fluid properties of both primary (turbine) flow and secondary (fan) flow. The subroutines are written in terms of a basic routine called POLY. This subroutine is a working tool for the other routines because it describes enthalpy, entropy function, and specific heat as a function of temperature along with the reverse situation of temperature as a function of enthalpy or entropy function. The subroutines presented in this section are general in that they are valid for both fuel and air mixtures and air. A list of the fluid property subroutines described in this appendix are
(1) Function POLY
(7) Function SPHTCP
(2) Function FULAIR
(8) Subroutine ADIABT
(3) Function TFROMS
(9) Function GAMMA
(4) Function GASCON
(10) Function ENTROP
(5) Function TFROMH
(11) Function VISCON
(6) Function ENTHAL
(12) Function FMM

## Function POLY

The thermodynamic properties (specific heat, enthalpy, and entropy function) for air and for stoichiometric fuel and air mixtures for several fuels have been tabulated by Hall and Weber in reference 20. These properties for air and for a stoichiometric mixture of air with hydrocarbon fuel have been programmed as polynomials in absolute temperature ( ${ }^{\circ} \mathrm{R}$ ) in function POLY.

Formulas are also derived in reference 20 that permit the calculation of the ideal fuel-to-air ratio for any given inlet air temperature and final combustion temperature. Ideal fuel-to-air ratio corresponds to complete combustion assuming no dissociation. Procedures are also indicated for calculating the thermodynamic properties of fuel-air mixtures with fuel-to-air ratios less than stoichiometric.

POLY yields gas properties based on relations for the specific heat of a mixture, entropy function of a mixture, and enthalpy of a mixture. These relations are

$$
\begin{gather*}
C_{p}=C_{p, a}+G \psi\left(C_{p}\right)  \tag{G1}\\
\varphi=\varphi_{a}+G \psi(\varphi)  \tag{G2}\\
h=h_{a}+G \psi(h) \tag{G3}
\end{gather*}
$$

where the term $G$ is a function (modified for incomplete combustion) of the efficiency of burning and the fuel-to-air ratio. The expression for $G$ is

$$
\begin{equation*}
\mathrm{G}=\frac{\eta_{\mathrm{b}}\left(\frac{\mathrm{f}}{\mathrm{a}}\right)}{1+\left(\frac{\mathrm{f}}{\mathrm{a}}\right)} \tag{G4}
\end{equation*}
$$

The value of $\eta_{b}$ is the same as that used in the burner. The functions $C_{p, a}, \varphi_{a}$, $h_{\mathrm{a}}, \psi\left(\mathrm{C}_{\mathrm{p}}\right), \psi(\varphi)$, and $\psi(\mathrm{h})$ are all tabulated in reference 20 and are expressed as polynomials in the absolute temperature $\left({ }^{\circ} R\right)$ in function POLY. The POLY functions are:

For air

$$
\begin{aligned}
& \operatorname{POLY}(\mathrm{T}, 1)=\mathrm{h} \\
& \operatorname{POLY}(\mathrm{~T}, 2)=\varphi \\
& \operatorname{POLY}(\mathrm{T}, 3)=\mathrm{C}_{\mathrm{p}} \\
& \operatorname{POLY}(\mathrm{~h}, 7)=\mathrm{T} \\
& \operatorname{POLY}(\varphi, 8)=\mathrm{T}
\end{aligned}
$$

For JP fuel in air

$$
\begin{aligned}
& \operatorname{POLY}(\mathrm{T}, 4)=\psi(\mathrm{h}) \\
& \operatorname{POLY}(\mathrm{T}, 5)=\psi(\varphi) \\
& \operatorname{POLY}(\mathrm{T}, 6)=\psi\left(\mathrm{C}_{\mathrm{p}}\right)
\end{aligned}
$$

## Function FULAIR

This function evaluates fuel-to-air ratio as a function of burner inlet and outlet temperature and burner inlet fuel-to-air ratio. The function FULAIR gives only the ratio of fuel added in the burner to the air (not the fuel-to-air mixture) entering the burner. Then the total burner outlet fuel-to-air ratio will simply be the sum of the input and the computed fuel-to-air ratio. An additional input required is the burner efficiency $\eta_{\mathrm{b}}$. The function FULAIR is given as

$$
\text { Function FULAIR }\left(\mathrm{T}_{1}, \mathrm{~T}_{2},\left.\frac{\mathrm{f}}{\mathrm{a}}\right|_{1}, \eta_{\mathrm{b}}\right)
$$

where the calculation procedure is
(1) $\left(\frac{\mathrm{f}}{\mathrm{a}}\right)_{2 \mathrm{~T}}=0.02$
(2) $\left(\frac{\mathrm{f}}{\mathrm{a}}\right)_{3}=\left(\frac{\mathrm{f}}{\mathrm{a}}\right)_{1}+\left(\frac{\mathrm{f}}{\mathrm{a}}\right)_{2 \mathrm{~T}}$
(3) $\mathrm{h}_{2}=\operatorname{ENTHAL}\left[\mathrm{T}_{2},\left(\frac{\mathrm{f}}{\mathrm{a}}\right)_{3}, \eta_{\mathrm{b}}\right]$
(4) $\mathrm{h}_{1}=\operatorname{ENTHAL}\left[\mathrm{T}_{1},\left(\frac{\mathrm{f}}{\mathrm{a}}\right)_{1}, \eta_{\mathrm{b}}\right]$
(5) $\left(\frac{\mathrm{f}}{\mathrm{a}}\right)_{2}=\frac{\left(\mathrm{h}_{2}-\mathrm{h}_{1}\right)\left[1+\left(\frac{\mathrm{f}}{\mathrm{a}}\right)_{1}\right]}{\left[\eta_{\mathrm{b}} \mathrm{HV}\left(\mathrm{h}_{2}-\mathrm{H}^{\mathrm{o}}\right)\right]}$
(6) If $\left[\left(\frac{f}{a}\right)_{2}-\left(\frac{f}{a}\right)_{2 T}-1.0 \times 10^{-6}\right]>0$
let $\left(\frac{\mathrm{f}}{\mathrm{a}}\right)_{2 \mathrm{~T}}=\left(\frac{\mathrm{f}}{\mathrm{a}}\right)_{2}$ and return to step (2). If the difference is $\leq 0$ then do the following
(7) $\operatorname{FULAIR}=\left(\frac{f}{a}\right)_{2}$
(8) Return to main program

## Function TFROMS

This function evaluates temperature from entropy function using an iterative procedure. The function TFROMS is given as

$$
\text { Function TFROMS }\left(\varphi_{\mathbf{M}}, \frac{f}{a}, \eta_{\mathbf{b}}\right)
$$

The calculation procedure for this function follows:
(1) $G=\frac{\frac{f}{a} \eta_{b}}{\left(1+\frac{f}{a}\right)}$
(2) $\mathrm{T}_{\mathrm{a}}=\operatorname{POLY}\left(\varphi_{\mathrm{M}}, 8\right)$
(3) $\varphi_{a}=\operatorname{POLY}\left(\mathrm{T}_{\mathrm{a}}, 2\right)\left\{\frac{\varphi_{\mathrm{M}}}{\left[\operatorname{POLY}\left(\mathrm{T}_{\mathrm{a}}, 2\right)+(\mathrm{G}) \operatorname{POLY}\left(\mathrm{T}_{\mathrm{a}}, 5\right)\right]}\right\}^{1.2}$
(4) $\mathrm{T}_{\mathrm{a}}=\operatorname{POLY}\left(\varphi_{\mathrm{a}}, 8\right)$

At this point in the program, solve for $\mathrm{T}_{\mathrm{a}}$ by the false position method which obtains approximations for $T_{a}$. Thus, for the mixture, $\operatorname{TFROMS}=T_{a}$.

## Function GASCON

This function evaluates the gas constant as a function of fuel-to-air ratio.

$$
\text { Function GASCON }\left(\frac{\mathrm{f}}{\mathrm{a}}\right)
$$

$$
R=\frac{\left(53.349+55.088 \frac{\mathrm{f}}{\mathrm{a}}\right)}{\left(1+\frac{\mathrm{f}}{\mathrm{a}}\right)}
$$

and the GASCON $=R$.

## Function TFROMH

This function evaluates temperature from enthalpy and the fuel-to-air ratio of the mixture. This function also requires the burner efficiency and the function POLY. An iterative process is required to evaluate the mixture temperature. The function TFROMH is given as

$$
\text { Function TFROMH }\left(\mathrm{h}_{\mathrm{M}}, \frac{\mathrm{f}}{\mathrm{a}}, \eta_{\mathrm{b}}\right)
$$

where the calculation procedure is
(1) $G=\frac{\frac{f}{a} \eta_{b}}{\left(1+\frac{f}{a}\right)}$
(2) $h_{a}=h_{M}$
(3) $\mathrm{T}_{\mathrm{a}}=\operatorname{POLY}\left(\mathrm{h}_{\mathrm{a}}, 7\right)$
(4) $\psi_{M}(\mathrm{~h})=\operatorname{POLY}\left(\mathrm{T}_{\mathrm{a}}, 4\right)$
(5) $\left.h_{a}=h_{a}\left\{\frac{h_{M}}{\left[h_{a}+G \psi_{M}(h)\right.}\right]\right\} 1.2$
(6) $\mathrm{T}_{\mathrm{a}}=\operatorname{POLY}\left(\mathrm{h}_{\mathrm{a}}, 7\right)$

Now, solve for $\mathrm{T}_{\mathrm{a}}$ using the false position method, which obtains approximations for $\mathrm{T}_{\mathrm{a}}$. Then, for the mixture, $\mathrm{TFROMH}=\mathrm{T}_{\mathrm{a}}$.

## Function ENTHAL

This function gives the enthalpy as a function of temperature, burner efficiency, and fuel-to-air ratio. Function POLY is used in the procedure:

$$
\text { Function ENTHAL }\left(T, \eta_{b}, \frac{f}{a}\right)
$$

(1) $h_{a}=\operatorname{POLY}(T, 1)$
(2) $\psi_{M}(\mathrm{~h})=\operatorname{POLY}(\mathrm{T}, 4)$
(3) $\mathrm{G}=\frac{\frac{\mathrm{f}}{\mathrm{a}} \eta_{\mathrm{b}}}{\left(1+\frac{f}{a}\right)}$
(4) $h_{M}=h_{a}+G \psi_{M}(h)$
(5) ENTHAL $=h_{M}$.

## Function SPHTCP

This function gives specific heat at constant pressure as a function of temperature, fuel-to-air ratio, and burner efficiency.

$$
\text { Function } \operatorname{SPHTCP}\left(\mathrm{T}, \frac{\mathrm{f}}{\mathrm{a}}, \eta_{\mathrm{b}}\right)
$$

(1) $\mathrm{C}_{\mathrm{p}, \mathrm{a}}=\operatorname{POLY}(\mathrm{T}, 3)$
(2) $G=\frac{\frac{f}{a} \eta_{b}}{\left(1+\frac{f}{a}\right)}$
(3) $\psi\left(\mathrm{C}_{\mathrm{p}}\right)=\operatorname{POLY}(\mathrm{T}, 6)$
(4) $C_{p, M}=C_{p, a}+G \psi\left(C_{p}\right)$
(5) SPHTCP $=C_{p, M}$.

## Subroutine ADIABT

This subroutine evaluates an expansion or compression of a gas taking into account initial temperature and pressure, fuel-to-air ratio, adiabatic efficiency, and final pressure. Outputs of the subroutine are final temperature and enthalpy change. The enthalpy change for an expansion process will be a negative value.
(1) $\varphi_{1}=\operatorname{ENTROP}\left(T_{1}, \frac{f}{a}, \eta_{b}\right)$
(2) $\mathrm{R}=\operatorname{GASCON}\left(\frac{\mathrm{f}}{\mathrm{a}}\right)$
(3) $\varphi_{2}=\varphi_{1}+\left(\frac{\mathrm{R}}{778.16}\right) \log \left(\frac{\mathrm{p}_{2}}{\mathrm{p}_{1}}\right)$
(4) $\mathrm{T}_{2, \mathrm{id}}=\operatorname{TFROMS}\left(\varphi_{2}, \frac{\mathrm{f}}{\mathrm{a}}, \eta_{\mathrm{b}}\right)$
(5) $h_{1}=\operatorname{ENTHAL}\left(T_{1}, \frac{f}{a}, \eta_{b}\right)$
(6) $\mathrm{h}_{2, \mathrm{id}}=\operatorname{ENTHAL}\left(\mathrm{T}_{2, \mathrm{id}}, \frac{\mathrm{f}}{\mathrm{a}}, \eta_{\mathrm{b}}\right)$

For a compression process $\left(p_{2}>p_{1}\right)$
(7) $\Delta \mathrm{h}=\frac{\left(\mathrm{h}_{2, \mathrm{id}}-\mathrm{h}_{1}\right)}{\eta}$

For an expansion process $\left(p_{2}<p_{1}\right)$
(8) $\Delta \mathrm{h}=\eta\left(\mathrm{h}_{2, \mathrm{id}}-\mathrm{h}_{1}\right)$

The final enthalpy is
(9) $h_{2}=h_{1}+\Delta h$
and the exit temperature
(10) $\mathrm{T}_{2}=\operatorname{TFROMH}\left(\mathrm{h}_{2}, \frac{f}{\mathrm{a}}, \eta_{\mathrm{b}}\right)$

## Function GAMMA

This function evaluates the ratio of specific heats according to the perfect gas definitions.

$$
\text { Function GAMMA }\left(T, \frac{f}{\mathrm{a}}, \eta_{\mathrm{b}}\right)
$$

(1) $\mathrm{C}_{\mathrm{p}}=\operatorname{SPHTCP}\left(\mathrm{T}, \frac{\mathrm{f}}{\mathrm{a}}, \eta_{\mathrm{b}}\right)$
(2) GAMMA $=\frac{C_{p}}{\left(C_{p}-[\text { GASCON }] / 778.16\right)}$

## Function ENTROP

This function yields entropy function as a function of temperature, burner efficiency, and fuel-to-air ratio.

$$
\text { Function ENTROP }\left(T, \eta_{b}, \frac{f}{a}\right)
$$

(1) $\varphi_{a}=\operatorname{POLY}(T, 2)$
(2) $G=\frac{\frac{\mathrm{f}}{\mathrm{a}} \eta_{\mathrm{b}}}{\left(1+\frac{\mathrm{f}}{\mathrm{a}}\right)}$
(3) $\psi(\varphi)=\operatorname{POLY}(\mathrm{T}, 5)$
(4) $\varphi_{M}=\varphi_{a}+G \psi(\varphi)$
(5) $\mathrm{ENTROP}=\varphi_{\mathrm{M}}$

## Function VISCON

This function determines the viscosity of air from a table of viscosity against temperature. The function VISCON is an interpolation procedure (see appendix H for program listing).

## Function FMM

This function solves for Mach number for an input value of flow factor. The equation is solved by an iterative procedure. This expression is

$$
\mathrm{FF}=\frac{\mathrm{M} \sqrt{\gamma \mathrm{~g}}}{\left(1+\frac{\gamma-1}{2} \mathrm{M}^{2}\right)^{(\gamma+1) / 2(\gamma-1)}}
$$

Inputs are $\mathrm{FF}, \gamma, \mathrm{g}$ and the output is M .

## APPENDIX H

## PROGRAM LISTING

```
$IBFTC VTOLI
```

C

C

```
    DIMENSIDN FN3J(50),P2OP1J(50),W1J(50),FMSJ(100)
    DIMENSION ALPH5J(100),DTTJ(100), FMR5J(100),TBHJ(100),
l VLJJJ(100),
1 W1CMP(100),W1TST(5),FMSTST(5),ALPTST(5)
                    IF IOUT IS READ IN AS 1, COMP WILL BE REAPEADETLY
                        CALLED AND OUTPUT OBTAINED
```

            IF IOUT IS READ IN AS O.
                MINIMUMS WILL BE OBTAINED, AND MOST OF THE
                    OUTPUT WILL NOT BE PRINTED - IE. ONLY
                        MINIMUM CASES WILL BE PRINTED GUT. IN THIS CASE
                        IOUT WILL BE USED TO SUPPRESS OR NOT SUPPRESS
                        PRINTING
                    IN THE CALLING VECTOR, IF IOUT \(=0\), DO NOT WRITE
                OUTPUT
                    IN THE CALLING VECTOR. IF IOUT \(=1\), WRITE OUTPUT
                    JP7 IS THE INDEX IN THE FMSJ PROFILE
                        CORRESPONDING TO FMS \(=.7\)
                    THE VALUES IN THE FMSJ INPUT VECTOR MUST BE IN
                INCREASING OROER
                    SUB. COMP INTERATES ON FTAT, AND INITIAL GUESSES
                        FDR ETAT AND ETAN ARE VEEDED. FDR ANY TWO
                        RUNS USING IDENTICAL INPUT, EXCEPT FOR
                DIFFERENT ETAT AND ETAN INPUTS, THE RESULTS
                SHOULD BE IDENTICAL, BUT INFACT, COMPARE TO
                OVLY FOUR OR FIVE DECIMAL PLACES
                    IF IPLOTT \(=0\), AND IOUT \(=0\). THEN PLOTS WILL BE
                    DRAWN
                    IF IPLOTT \(=1\), PLOTS WILL NOT BE DRAWN
            1 READ(5,51) ITURB,IETAT,IWRITE ,INUT,JP7,IWGHT, IPLOTT
    IWRTSV = IWRITE
    IF(IETAT) \(160,160,57\)
    160 GO TO(2,161,3,2),ITURB

C
3 READ(5,50) FK, TRM, XIAL6,FLLC
WRITF (6,5)
WRIIE(6,555) ITURB, IETAT, IWRITE,IOUT, JPT, IWGHT, IPLOTT
WRITE (6,60) FK, TRM,XIAL6,FLLC
C
GO 10 88
57 WRITE(6.58)
WRITE(5,555) ITURB,IETAT,IWRITE,IOUT,JP7, IWGHT GOTO 38
161 WRITE(6,162)
WRITE(6,555) ITURB,IETAT,IWRITE,IOUT,JP7, IWGHT
GO TO 88
2 WRITE 5,54 )
WRITE(5,555) ITURB,IETAT,IWRITE,IDUT,JP7.IWGHT
C
88 IF (IWRITE) $56,56,87$
87 REAll 5,50 ) W1S2
WRITE(5.89) W1S2
C
C

| 56 READ (5,50) | TLH, OMEGAS, |  |
| :---: | :---: | :---: |
| 1 | OTLOSS. | ETABR, ETAF, ETAN, ETAT,FKV, |
| 2 | FM11, FM2, FM2E, FM3S, | FNB, FNS, FPR, H2P, HHV,P1, |
| 3 | P1413, PAMB, | PP2B,R,RHRT,RING, TAMB, |
| 4 | TL,T4,UFT, WIOBR, | WAl, WLRAR,CT, |
| 5 | CM,FKSB, FKSN, XIALS. | M, AFXP, |
| 6 THARC, CO | R,F14, ZIL |  |

C
C
C
C
C
C
C
REAO (5,102) JFN3, (FN3J(J), J=1, JFN3)
REAil(5,102) JP20P1,(P20P1J(J), J=1,JP2OP1)
REAL(5,102) JWl,(W1J(J),J=1,JW1)
REAU(5,102) JFMS, (FMSJ(J),J=1,JFMS)
107 WRITE(S,101) (FN3J(J),J=1,JFN3)
WRITE(5.103) (P2OP1J(J), J=1,JP2OP1)
WRITE(S,104) (W1J(J),J=1,JWI)
WRITE(6,105) (FMSJ(J),J=1,JFMS)
C
WRITE (5,66)
WRITE $(5,66)$
C
WRITE(5,55) TLH,OMEGAS,
1 DTLOSS, ETABR,ETAF,ETAN,ETAT,FKV,
1 FM11,FM2,FM2B,FM3S, FNB,FNS,FPR,H2P,HHV,P1, 2 P1413, PAMB, PP2B,R,RHRT,RING, TAMB,
3 TI,T4,UFT, WIOER,WFWAI,WLEAR, CT,
4 EM,FKSE,FKSN,XIALS,CCM, AEXP,
5 THARC,COOS,AR,F14
C
C
$\mathrm{ADAS}=1 .+.1744 *$ TLH

$$
\begin{aligned}
& \text { DJ } 92 \text { I92 = 1, JP20P1 } \\
& \text { P2OP1 }=P 20 P 1 J(I 92)
\end{aligned}
$$

```
                DO93 [93 = 1,JWI
    WlSl = WlJ(I93)
            DO 91 191=1,JFV3
    FN3 = FN3J(I91)
    Wl = W1Sl
    IWRITE = IWRTSV
C
C
C
    7 1
                D) 94 I94=1,JFMS
            FMS = FMSJ(I94)
            Wl = WlSl
            IWRITE = IWRTSV
            94 CALL GJMP(IUUT, IWGHT,FMS,ALPH5,DTT,FMR5,TBH,WI,FN,VL)
            GO TO 91
    70 DEG75 = 75.*PIE/180.
                        rO OBTAIN VECTORS TO PLOT
                DO 25 J=1,JFMS
C
    IF(J-JP7) 21,22,21
    22 IOUT = 1
    21 FMS = FMSJ(J)
        W1 = W1S1
        IWRITE = IWRTSV
        CALL COMP(IOUT,IWGHT,FMS,ALPH5,DTT,FMR5,TBH,WI,FN,VL)
        ALPH5J(J) = ALPH5
        DTTJ(J) = DTT
        FMR5J(J) = FMR5
        TBHJ(J) = TBH
        WICMP(J) = Wl
        VLJJJ(J) = VL
    25 IOUT = O
    JMIN = I
C
                DO 27 J=2,JFMS
            IF(W1CMP(JMIN) - WICMP(J)) 27.27.26
    26 JMIN = J
    27 CONTINUE
                TO SEE IF WICMP(JMIN) IS AN END POINT
                        OR AN INTERMEDIATE POINT
                TO FIND WI MINIMUM, TWO POINTS
                                ARE TAKEN ON EITHER SIDE OF WICMP{JMIN)
    IF(JMIV - 2) 30,31,31
30 FMS = FMSJ(1)
```

```
    WITSTW = WICMP(1)
    ALPTST(1) = ALPH5J(1)
    JATST = 1
    GO TO 34
    31 IF(JMIN - JFMS) 32,33,33
    33 FMS = FMSJ(JFMS)
    WITSTW = WICMP(JFMS)
    ALPTST(JFMS) = ALPHSJ(JFMS)
    JATST = JFMS
    GO TO }3
    32 DEL13 = (FMSJ(JMIN) - FMSJ(JMIN - 1))/3.
    DEL23 = (FMSJ(JMIN * 1) - FMSJ(JMIN))/3.
    JJ. = 0
    SIGN = 1.
    DEL = DEL13
        DJ 35 I= 1,2
    SIG't = - SIGN
    IOUT = 0
C
C
C
C
    JMIN5 = 1
        DJ 37 J=2.5
    IF(W1TST(JMIN5) - WITST(J)) 37,3.7,38
38 JMIN5 = J
37 CONTIVUE
    JATST = JMIN5
    FMS = FMSTST(JMIN5)
    W1TSTW = W1TST(JMIN5)
34 IF(ALPTST(JATST) - DEG75) 28,28,29
28 IOUT = 1
    WRITE(6,9) FMS
    Wl=WISI
    IWRITE = IWRTSV
    CALL EJMP(IOUT,IWGHT,FMS,ALPH5,UTT,FMR5,TBH,Wl,FN,VLI
    IOUT = 0
    GO TO }3
```

alptstijatst) is greater than 75 deg.
TD FIND THE INTERVAL IN WHICH ALPH5J IS 75. DEG., USING THE READ IN, COARSE INTERVALS

```
    29 IF(JMIN - 1) 74,74,73
    73 JMINI = JMIN - 1
        JJ = JMIN - J
        IF(ALPH5J(JJ) - DEG75) 40,40,399
    40 J751= JJ
        J752 = JJ + 1
        GO TO 41
    399 CONTIVJE
    74 IF(JMIV - JFMS) 75,76,76
    75 JMIN2 = JFMS - JMIV
        JJ = JMIN
            DJ 42 J =1,JMIN2
        JJ=JJ+1
        IF(ALPH5J(JJ) - DEG75) 43,43,42
        43 J751 = JJ - 1
        J752 = JJ
        GO TO 4l
    42 CONTIVUE
    76 WRITE(5,44) FMSJ(JMIN),WICMP(JMIN), ALPHSJIJMIN)
        GO TO }3
    41 FMSI1 = {DEG75 - ALPH5J|J751))/(ALPH5J(J752) - ALPH5J(J751)|
        FMS = FMS11*(FMSJ(J752) - FMSJ(J751)) + FMSJ(J751)
        IOUT = 1
        WRITE(5,45) FMS
        Wl = WISI
        IWRITE = IWRTSV
        CALL CJMP\IUUT, IWGHT,FMS,ALPH5,DTT,FMR5,TBH,W1,FN,VLI
    39 DO 46 J=1,JFMS
        ALPH5J(J) = ALPH5J(J)/ALPH5J(JP7)
        OTTJ(J) = DTTJ(J)/DTTJ(JP7)
        FMR5J(J)= FMR5J(J)/FMR5J(JP7)
        TBHJ(J) = TBHJ(J)/TBHJ(JP7)
        W1CMP(J) = W1CMP(J)/W1CMP(JP7)
    4 6 ~ V L J J J ( J ) ~ = ~ V L J J J ( J ) / V L J J J ( J P 7 )
        IF(IPLOTT) 200,200,91
    200 CALL HALPLT(JFMS,FMSJ,ALPH5J,OTTJ,FMR5J,TBHJ,WICMP,
        1
                        VLJJJ,FPR,T1,T4,UFT,P1,ITURB)
    91 CONTINJE
    93 CONTIVUE
    9 2 ~ C O N T I N U E
                    TO MAKE CALCOMP PLGT
                            TO NON-DIMENSIONALIZE THE POINTS TO BE PLOTTED
                                    BY DIVIDING BY THE VALUES AT FMS = . }
```

C
C
C
C
C
C
C
C
C
C
C
C
C
C
C

IF(IWRITE) 56,1,1
C
50 FORMAT (9E8.5)
51 FORMAT (1814)
555 FORMAT $11 H 0,8 H$ ITURB $=I 4,5 \mathrm{X}, 7 \mathrm{HIETAT}=I 4,5 \mathrm{X}, 8 \mathrm{HIWRITE}=I 4,5 \mathrm{X}$,
$1 \quad 5 \mathrm{IIOUT}=[4,5 \mathrm{X}, 6 \mathrm{H}$ JP7 $=[4,5 \mathrm{X}, 8 \mathrm{H} \quad \mathrm{IWGHT}=\mathrm{I} 4)$
5 FORMAT(IHI, 27 H TWO STAGE TURBINE WITH OGV)
54 FORMAT(IHI, $30 H$ SINGLE STAGE TRUHINE WITH OGV)
58 FORMAT (IHI, $82 H$ INPJT FOLLOWS - TURBINE ROUTINE IS SKIPPED SINCE 1 THERE IS NO INTERATIOV ON ETAT)
55 FORソAT(1H, 7H TLH =E12.5.5X,7HOMEGAS=E12.5,5X,7HDTLOSS=E12.5, /1H ,
7HETABR =E12.5.5X,7H ETAF =E12.5.5X,
$\begin{array}{ll}2 \\ 3\end{array} \quad 7 \mathrm{HHETABR}=\mathrm{E} 12.5,5 \mathrm{X}, 7 \mathrm{H}$,
7H ETAN $=E 12.5,5 \mathrm{X}, 7 \mathrm{H}$ ETAT $=E 12.5 / 1 \mathrm{H}$,
7 H FKV $=\mathrm{E} 12.5,5 \mathrm{x}, 7 \mathrm{H}$ FMII $=\mathrm{E} 12.5,5 \mathrm{X}, 7 \mathrm{H}$
FM2 $=\mathrm{E} 12.5,5 \mathrm{X}$,
$7 H$ FM2B $=E 12.5,5 \mathrm{X}, 7 \mathrm{HFM} 3 \mathrm{~S}=\mathrm{E} 12.5 / 1 \mathrm{H}$,
7H FNB $=E 12.5,5 \mathrm{X}, 7 \mathrm{H}$ FNS $=\mathrm{E} 12.5 .5 \mathrm{X}$,
7H $F P R=E 12.5,5 X, 7 H \quad H 2 P=E 12.5 / 1 H$,
7H $\quad H H V=E 12.5,5 X, 7 H \quad P 1=E 12.5,5 X, 7 H P 1413=E 12.5,5 X$,
7 H PAMB $=E 12.5 / 1 \mathrm{H}$,
19X, $\quad 5 \mathrm{X}, 7 \mathrm{H}$ PP2R $=E 12.5 .5 \mathrm{X}, 7 \mathrm{H} \quad \mathrm{R}=\mathrm{E} 12.5 .5 \mathrm{X}$, 7H RHRT $=E 12.5,5 \mathrm{X} .7 \mathrm{H}$ RING $=\mathrm{E} 12.5 / 1 \mathrm{H}$.

7H TAME $=E 12.5 .5 \mathrm{X}, 7 \mathrm{H} \quad \mathrm{T} 1=E 12.5 .5 \mathrm{X}$,
$7 H \quad T 4=E 12.5 .5 \mathrm{X}, 7 \mathrm{H} \quad$ UFT $=E 12.5 / 1 \mathrm{H}$,
THW1OBR $=E 12.5,5 X, 7 H W F W A 1=E 12.5,5 X$,
$\begin{array}{lll}6 & 7 H W L B A R=E 12.5, ~ 5 X, 7 H \quad C T=E 12.5 / 1 H O \\ 7 & 74 & C M=E 12.5,5 X, 7 H F K S H=E 12.5,5 X, 7 H F K S N=E 12.5,5 X,\end{array}$
$8 \quad 7$ HXIALS $=[12.5,5 X, 7 H \quad C C M=E 12.5 / 1 H .7 H \quad A E X P=E 12.5$,
$1 \quad 5 \mathrm{X}, 7 \mathrm{HTHARC}=\mathrm{E} 12.5 .5 \mathrm{X}, 7 \mathrm{H} \operatorname{COOS}=\mathrm{El} 2.5,5 \mathrm{X}, 7 \mathrm{H} \quad \mathrm{AR}=\mathrm{E} 12.5$,
$25 \mathrm{~F}, 7 \mathrm{H}$ F14 =E12.51
60 FORMAT $11 H, 7 H \quad F K=E 12.5,5 X, 7 H \quad$ TRM $=E 12.5,5 X, 7 H X I A L 6=E 12.5$,
$1 \quad 5 \mathrm{X}, 7 \mathrm{H}$ FLLC $=\mathrm{E} 12.51$
89 FORMAT ( 1 HO, 7 H W1S2 $=$ E12.5)
102 FORMAT(I4, (9E8.5))
101 FORMAT (1H0,2OH FN3 PROFILE FOLLOWS/(8E15.5))
103 FORMAT (1HO, $2 H$ P2OPI PROFILE FOLLOWS/(8E15.5))

105 FORMAT (1HO, 2UH FMS PROFILE FOLLOWS/(8E15.5))
162 FORMAT(IHI, $36 H$ SINGLE STAGE TURBIME - STATORLESS)
66 FOR:AT (IHK)
44 FORUAT (lHO, $104 H$ FOR THE GIVEN FMS RANGF, ALPHS IS NEVER LESS THAN 175 UESREES - FOR THIS RANGE. THE MINIMUM WI FOLLOWS/
$2 \quad 7 \mathrm{H} \quad \mathrm{FMS}=\mathrm{E} 12.5,5 \mathrm{X}, 7 \mathrm{H} \quad \mathrm{WI}=\mathrm{E} 12.5,5 \mathrm{X}, 7 \mathrm{HALPH} 5=\mathrm{E}$ 12.5)
45 FORMAT(IHO, 5OH THE CASE CORRESPDNDING TG ALPHS = 75 DEG. FQLLUWS/
$11 \mathrm{H}, 7 \mathrm{H}$ FMS =E12.5)
9 FOKMAT $11 H 0$, 39H DUTPUT FUR THE MINIMUM W1 CASE FOLLOWS/IH,
1 6H FMS =E12.5)
ENO

## \$IBFTC TUßBI

```
C YAY 1968
    SUBROJTINE TURBIIT4,W4,P4,PSS, DFT,DT, FMS,ETAN,
        F3,FM10,P10,PAMR,
                        TS 5,V5,FM5, ALPH5,VZ5,VU5,P5,U,BETA5,
                        R5,FMR5,R6,ETAB,TS6,VZ6,FMZ6,FMRG,RETA6,
                        RU6,OMEGA,ALPHG,PG,PS,TG,FMSCAL, WLEAK,
                            PHI,PS 10,SPR,ZLEAK,ETATLI, ETAT,
                        FLAM,FIUUBR,ETABR,
                        CM,FKSF,FKSN,XIALS,CCM,
                        AZ5,FMG,FM7.5,PSFAN,RE,RUS,TS,TSS,VO,VU6,XIM,
                        XIN,XIP,XIS,ZETAAS,ZETAB,ZETAM, LETAP,LETASS,
                        ZETAS,TSP,DHTTI, CPG,DH,AEXP,DTR,
            TBH,THARC,COOS,AR,F141
            WRITTEN FOR HANK HALLER MAY 1968
                SINGlE StAGE TURBINE WITH OGV
                    MAIN REFERS TO THE CALLING PROGRAM
                    WFWA3P IV MAIN IS F3 IN TURBDP
                FMII IV MAIN IS FMIO IV TURHUP
                PlI IN MAIN IS PIO IN TURBDP
                PSCHK IN MAIN IS PS IN TURROP
                PSIL IV MAIN IS PSIO IN TURBDP
G=32.174
FJ = 778.
PIE = 3.1415926
PIE180 = 180./PIE
```

```
RE = 24.*W4/0T/VISC(T4)
```

RE = 24.*W4/0T/VISC(T4)
REXP2 = (RE/1.5E6)**.2
REXP2 = (RE/1.5E6)**.2
XIP = XIALS/REXPZ
XIP = XIALS/REXPZ
XIS = FKSN*XIALS
XIS = FKSN*XIALS
PS5 = PSS
PS5 = PSS
PS6 = PS5
PS6 = PS5
C
GASKON = GASCON(F3)
R = GASKON
FM5C = 2.*FJ/R
DLHBC = 2.*G*FJ
RHO6C = 144.*PS6/R
C
C
C
C
FM5 = 1.
KCT = 0
C
11 IF(FM5 - 1.)6,7,7
6 XIM = 0.
GO TO 8

```
```

7 XIM = 0.0075*(FM5*FM5-FM5)
8 XIN = XIP + XIS + XIM
ETAN = 1.-XIN

```

C
    14 CP6 \(=\) SPHTCP(TS5,F3,ETABR)
    \(F M Z 6 C=G A M * G * R\)
    GAMI = GAM-1.
    GAM11 = GAM1/GAM
    GAM2 \(=\) GAM/GAM1
    GAM3 \(=2 . /\) GAMI
    GAM4 \(=(1 .-G A M) / G A M\)
    GAM5 \(=\)-GAM4
    \(V 5=S\) SRT (2.*G*FJ*ABS (OH))
    P5 = PS 5*(1.*0.5*(GAM-1.)*FM5*FM5)**GAM2
    RHO \(5=144 . * \mathrm{PS} 5 / \mathrm{TS} 5 / \mathrm{R}\)
    FMR5C \(=S\) QRT(GAM*G*R*TS5)
    DELB \(=2 . *\) BETAS
    ZETABC = PIEI8O*DELB/100.
    CALL ADIABT(T4,P4,ETAN, F3,TS5,PS5,DH,ETABR)
    GAM \(=\) 乞̂AMMA(TS5,F3,ETABR)
    FM5 \(=\) SQRT(FM5C*ABS(OH)/TS5/GAM)

        IF(KCT-1)9,10,10
        \(9 \mathrm{KCT}=1\)
        FM5SV = FM5
        GO TO 11
    \(10 \operatorname{IF}(A B S(F M 5-F M 5 S V)-.002) 14,14,12\)
    12 FM5SV \(=\) FM5
    \(K C T=K C T+1\)
    IF(KCT-100)11,11,15
    15 WRITE(S,16) FM5,FM5SV,ETAN
    16 FORMATIIH, \(36 H\) IN SUB TURBDP, FM5 CAN NOT BE FOUNDIIH.
        16 FM5 =E12.5,5X,8H FM5SV =E12.5,5X,7H ETAN =E12.5)
C
BEGIN FMZ5 COMPUTATION
    \(K M S=0\)
    FMZS \(=\) FMS*CCM
    34 ALPH5S = FMZ5/FM5
    ALPH5 = ARCOS (ALPH5C)
    VZ5 \(=V 5 * A L P H 5 C\)
    VU5 = V5*SIN(ALPH5)
    \(A Z 5=144 . * W 4 / V Z 5 /\) RHO5
    RU5 = VU5-U
    BETA5 = ATAN2(RU5,VZ5)
    \(R 5=S Q R T(V Z 5 * V 25+R J 5 * R U 5)\)
    FMRS = 25/FMR5C
    to compute the bucket efficiency, etab
```

    ZETABS = 0.025 + 0.034*ZETABC*ZETABC*SORT(ZETABC)
    ZETAP = ZETABS/REXP2
    ZETAS = FKSB*ZETABS
    IF(FMR5-1.) 42,43,43
    4 2 ~ Z E T A M ~ = ~ 0 . 0 ~
GO TO 44
43 ZETAM = EM*(FMR5*FMR5 - FMR5)
44 ZETAB = ZETAP + ZETAS + ZETAM
ETAB = 1.- ZETAB
IF(THARC - (360. - .01)) 60.61.61
60 SPACE ■ TBH/COOS/AR
ARCL = PIE*OT*THARC/350.
FKPA = 1. - SPACE/3./ARCL
R6 = R5*SQRT(ETAB)*FKPA
GO TO 62
C
61 R6 = R5*SQRT(ETAB)
62 DLHB = (R5*R5-R6*R6)/DLHBC
TS6 = TS5 +DLHE/CP6
RHOl6 = RHO6C/TS6
VZ6 = 144.*W4/RHO6/AZ5
FMZ6S = SQRT(FMZ6C*TS6)
FMZ6 = VZ6/FML6S
FMRG = R6/FMZ6S
RU6 = SQRT(R6*R6-V26*VZ6)
BETAG= ATAN2(RUG,VZG)
VU6 = RUG-U
C
C
DELALP= PIE18O*ABS(ALPH6)
DLALPC = DELALP/100.
ZETAAS = 0.025 + 0.034*DLALPC*DLALPC*SQRT(OLALPC)
ZETASS = ZETAAS*(1./REXP2 + FKSB)
C
51 OMEGA = ZETASS*(1. + 0.5*GAM*FMS*FMS)
C
52 FMG = FMZ6/COS(ALPH6)
P6 = PS6*(1.+0.5*(GAM-1.)*FM6*FM6)**GAM2
PS = PG-OMEGA*(PG-PS6)
T6 = TS6*(1.*0.5*GAM1*FM6*FM6)
FMSCAL = SQRT(GAM3*((PS/PSS)**GAMI1- 1.))
C
IF(ABS(FMSCAL-FMS) -.001) 31.31,30
30 KMS = KMS +1 <
IF(KMS-25) 32,32,33
32 FMZ5 = FMZ5*FMS/FMSCAL
GO TO 34
33 WRITE(6,35) FMS,FMSCAL,FMZ5
35 FORMATIIH , 9OH IN SUB. TURB68,FMZ5 CAN NOT BE FOUND - PROGRAM
ICONTIVUES USING THE LAST VALUE OBTAINED/IH.
2
6H FMS =E12.5.5x.7HFMSCAL=E12.5.5X,7H FMZ5 =E12.5)
C
C
FMZ5 IS FOUND
31 TS = TS
TSS = TS/(1.*.5*GAM1*FMS*FMS)

```
```

        VS = FMS*SORT(GAM*R*G*TSS)
        PS10= P10/(1.+0.2*FM10*FM10)**3.5
        DELPS = 1.20*PAME*(P10-PS10)/P10
        PSFAN = PAMB-UELPS
        SPR = PSFAN/PSS
    C
PHI = 0.725*SURT(1.-SPR**AEXP)
DELLIT = 0.00025*UFT
WLEIK = PIF*DTR*DELLIT*SWRT(G/R)*PS5*0.615*PHI/SQRT(TS5)
C
41 ZLEAK = WLEAK/W4
CALL ADIABT(T4,P4,1,0,F3,TSP,PS,DHTTI,ETABR)
C
C CHANGES MADE 3-7-69
IF(THARC - (360. - .01)) 63,64.64
63 ETATL2 = RUJ + RUG - . 5*F14*U*U*COOS*SPACE/V5/ARCL/ALPH5C
ETATLI=U*ETATL2/ABS(DHTTI)/G/FJ
GO TO S5
64 ETATLL = U*(RU5 + RU6)/AHS(DHTTI)/G/FJ
C
65 ETAT = ETATLI*(1.-ZLEAK)
FLAM = U/(RU5+RUG)
VO = V5/SQRT(ETAN)
40 FNUSR = U/VU
C
C
RETURV
END
\$[BFTC TURBZ
C
FERRUARY 1969
SURROJTINE TURBZIT4.W4.P4,PSS, DFT,DT, FMS,ETAN,
2
3
4
5
6
7
8
9
1-CM,FKSR,FKSN,XIALS,
1 AL5,FMG,FMZ5,PSFAN,RE,RU5,TS,TSS,VD,VUG,XIM,
2 XIN,XIP,XIS, LETAB,ZETAM,ZETAP,
3
1
C
c
C
c
c
C
C
C
MAIN zEFERS TI THE CALLING pROGRAM
WFWABP IN MAIV IS F3 IN TURBDP
FMII IV MAINIS FMIO IN TURBDP

```

PII IN MAIN IS Plo IN TURBDP PSCHK IN MAIN IS PS IN TURBDP PSII IV MAIN IS PSIO IN TURBDP
        FM5 \(=1\).
    \(K C T=0\)
C
11 IF(FM5-1.)6,7,7
    \(6 \times I M=0\).
    GO ro 8
    \(7 \mathrm{XIM}=0.0075 *(F M 5 *\) FM5-FM5)
    8 XIN \(=X I P+X I S+X I M\)
    ETAN \(=1 .-X I N\)
C
C
    CALL \(\triangle\) DIABT(T4,P4,ETAN, F3,TS5,PS5,DH,ETABR)
    GAM \(=\) GAMMA(TS5,F3,ETABR)
    \(F M 5=S Q R T(F M 5 C * A B S(D H) / I S 5 / G A M)\)

IF(KCT-1)9,10,10
\(9 \mathrm{KCT}=1\)
FM5SV = FM5
GO TO 11
10 IF(ABS (FM5-FMSSV)-.002)14,14,12
12 FM5SV = FM5
GASKOV = GASCON(F3)
R \(=\) GASKON
FM5C \(=2 . * F J / R\)
DLHAC \(=2 . * G * F J\)
RHO6C \(=144 . * P S 6 / R\)

THE INITIAL MS VALUE FOLLOWS

FM5 \(=1\).
\(K C T=0\)
\(c\)
11 LF(FM5-1.)6,7,7
\(6 \times I M=0\).
GO 10 8
\(7 \mathrm{XIM}=0.0075 *(F M 5 *\) FM5-FM5)
8 XIN \(=X I P+X I S+X I M\)
ETAN \(=1 .-X I N\)

C
CALL \(\triangle\) DIABT(T4,P4,ETAN, F3,TS5,PS5,DH,ETABR)
GAM \(=\) GAMMA(TS5,F3,ETABR)
FM5 \(=\) SQRT(FMJC*ABS(DH)/IS5/GAM)
\(K C T=K C T+1\)
IF(KCT-100)11,11,15
15 WRITE (5,16) FM5,FM5SV,ETAN
16 FORMAT IIH, 36H IN SUB TURBDP, FMS CAN NOT BE FDUND/IH,
1 6H FM5 =E12.5.5X,8H FM5SV =E12.5.5X.7H ETAN =E12.5)
BEGIN STATOR EXIT MACH NUMBER AXIAL COMPONENT FLOW FMZ5 CALCULATION
```

    14 CP6= SPHTCP(TS5,F3,ETABR)
    ```


    \(F M Z 6 C=G A M * G * R\)
    GAMI = GAM-1.
    GAMLI = GAMI/GAM
    GAM2 \(=\) GAM/GAM1
    GAM3 \(=2 . /\) GAMI
    GAM4 \(=\) (1.-GAM)/GAM
    GAM5 \(=\)-GAM4
    \(V 5=S 2 R T(2 . * G * F J * A B S(D H))\)
    P5 = PS5*(1.*0.5*(GAM-1.)*FM5*FM5)**GAM2
    RHO5 \(=144 . * P S 5 / T S 5 / R\)
    FMR5C \(=S\) QRT(GAM*G*R*TS5)

BEGIN FMZ5 COMPUTATION
\(K M S=0\)
FMZ5 \(=F M S\)
34 ALPH5C = FMZ5/FM5
\(A L P H 5=A R C O S(A L P H 5 C)\)
VZ5 = V5*ALPH5C
VU5 = V5*SIN(ALPH5)
\(A Z 5=144 . * W 4 / V Z 5 /\) RHO 5
RU5 = VU5-U
\(B E T A 5=A T A N 2(R \cup 5, V Z 5)\)
\(R 5=S Q R T(V 25 * V Z 5+R U 5 * R U 5)\)
FMRっ= \(25 /\) FMR5C
TO COMPUTE THE BUCKET EFFICIENCY, ETAB
DEL \(\delta=2\). RETAS
ZETABC = PIE180*DELB/100.
\(\angle E T A B S=0.025+0.034 * 2 E T A B C * Z E T A B C * S Q R T(Z E T A B C)\)
ZETAP \(=\) ZETABS/REXPZ
ZETAS = FKSB*ZETABS
IF(FMR5-1.) 42,43,43
42 ZETAM \(=0.7\)
GO TO 44
43 ZETAM = CM* (FMR5*FMR5 - FMR5)
44 ZETAB \(=\) ZETAP + ZETAS + ZETAM
ETAB = 1.- ZETAB

IF(THARC - (360. - .01)) 60,61,61
60 SPACE \(=T B H / C O O S / A R\)
ARCL \(=P I E * D T * T H A R C / 360\).
FKPA \(=1\). - SPACE/3./ARCL
\(R 6=R 5 * S Q R T(E T A B) * F K P A\)
GO TO 62
61 R6 \(=\) R5*SQRT(ETAB)
62 DLH: \(=(R 5 * R 5-R 6 * R 6) /\) DLHBC
TS6 = TS5 + DLHB/CP6
RHOG \(=\) RHO6C/TS6
VZ6 \(=144 . * W 4 /\) RHO6/AZ5
FMZGS \(=\) SQRT(FMZ6C*TS6)
FMZ6 \(=\) VZ6/FMZ6S
```

    FMR6 = R6/FMZ6S
    RU6 = SQRT(RG*RG-VZ6*VZ6)
    BETAG= ATAN2(RUG,VZ6)
    VU6 = RUG-U
    C
C
C
C
52 FMG = FMZ6/COS(ALPH6)
P6 = PS6*(1.*0.5*(GAM-1.)*FM6*FM6)**GAM2
PS = P6
T6 = TS6*(1.+0.5*GAM1*FM6*FM6)
FMSCAL = FMZ6
C
PS=PSS*(1.+.5*(GAM-1.)*FMSCAL**2)**GAM2
IF(ABS(FMSCAL-FMS) -.001) 31,31,30
30 KMS = <MS +1
IF(KMS-100) 32,32,33
32 FMZ5 = FMZ5*FMS/FMSCAL
GO TO 34
33 WRITE(B,35) FMS,FMSCAL,FMZ5
35 FORMATIIH, 9OH IN SUB. TURBS8,FMZ5 CAN NOT BE FOUND - PROGRAM
ICONTIVJES USING THE LAST VALUE OBTAINED/IH,
2 6H FMS =E12.5,5X,7HFMSCAL=E12.5.5X,7H FMZ5 =E12.5)
C
C
C
31 TS = TS6*(1. + 0.5*(GAM - 1.)*FMZ6*FMZ6)
TSS = TSG
VS = FMS*SQRT(GAM*R*G*TSS)
PS10= P10/(1.+0.2*FM10*FM10)**3.5
DELPS = 1.20*PAMB*(P10-PS10)/P10
PSFAN = PAMB-DELPS
SPR = PSFAN/PS5
C
C
C
C
C
IFITHARC - (360. - .01)) 63,64,64
63 ETATL2 = RU5 + RU6 - .5*F14*U*U*COOS*SPACE/V5/ARCL/ALPH5C
ETATLI=U*ETATL2/ABS(DHTTI)/G/FJ
GO TO 65
64 ETATLI = U*(RU5 + RU6)/ABS(DHTTI)/G/FJ
C
65 ETAT = ETATLI*(1.-ZLEAK)
FLAM = U/(RU5+RU6)
VO = V5/SQRT(ETAN)
40 FNUBR = U/VO
C
C
RETURN
END

```

IBFTC TURB3
```

    SUBZDJTINE TURB3 (T4,W4,P4,PSS, DFT, FMS,ETAN,
                        F3,FMIC,P1O, PAMB,FKS,SWID,FNSML,
                        ETABR,FK,
                                ALPH5,ALPH6,ALPH7,ALPH8,AZ5,AZ7,AZ8,AZS,
                                RETA5, EETAG,BETA7,BETA8,CPG,CP8,DELBT1,
                                DELBT2,DHN1,DHN2,DLHB1,DLHB2,DT,ETAB1,
                                ETAH2,ETAN1,ETAN2,ETAT,ETATLI,
                                FKM,FKM1, FKM2,FM5,FM6,FM7,FMB,FMR5,
                        FMR6,FMR7,FMR B,FMZ5,FMLSSV,FMZ6,FMZ7,
                        FMZ7SV,FMZ8,GAM5,GAM6,GAM8,P5,P6,
                                P7,P8,PHI,PS,PS5,PS6,PS7,PSO,PS1O,R5,R6,R7,
                                R8,RHO5,RHO6,RHO7,RH08,RU5,RU6,RU7,
                        RU8,SHP,SPR,
                        T6,T8,TS,TSS,TSP, TS5,TS6,TS7,TS8,U,
                                V5,V7,VS, VU5,VU6,VU7,VU8,VZ8,VZ5.
                                VZ6,VZ7,VZS,WIDEAL,WLEAK, XIBETI,
                                    XIBETZ.ZLEAK,
    OM&GA,CM,FKSB,FKSN,XIALS,TRM,XIALG,CCM,FLLC,ZIL)
    G=32.174
    FJ = 778.
    PIE = 3.1415926
    PIEI8J = 180./PIE
    FKS1 = FKS
    FKS? = FKSI
    R = GASCON(F3)
    RGG= S*R
    ETA|C1 = (1. +FKSS)* XIALP
    PS5 = FLLC*SQRT(P4*PAMB)
    RHODC = 144.*PS5/R
    DLH:3C = 2.*G*FJ
    W414 = 144.*W4
    AZSC = R#W4/PSS
    FMS = FMS
                            STARTING VALUES FOR FMZ5 AND FMZ7
    RE4 = 24.*W4/DT/VISC(T4)
    REXP2 = (RE4/1.5E6)**.2
    XIP4 = XIALS/REXP2
    XIS4 = FKSN*XIALS
    KCT =0
    C
C
C
ETANl = ETAN
15 CALL ADIABT(T4,P4,ETANI,F3,TS5,PS5,DHNI,ETABR)
DHNI = ABS(DHN1)
V5 = SORT(OLHBC*OHV1)
GAMS = GAMMA (TS5,F3,ETABR)
FM5SV = FM5
FM5 = V5/SQRT(GAM5*2GG*TS5)
IF(ABS(FMSSV-FM5)-.002) 12.12.13

```
```

    13 KCT = KCT+1
    IF(KCT - 100)14,14,16
    16 WRITE(S,17) FM5,FM5SV,ETAN,ETAVI
    17 FORMATIIH,5UHIN TURB2 AT STATEMENT NO. 13, FM5 CAN NOT BE FOUND/
        l 5H FM5=El2.5,5X,7H FM5SV=E12.5,5X,6H ETAN=EI2.5,
        2 5X,8H ETAN1 =E12.5)
    GO TO 12
    14 IF(FM5-1.)6,7,7
    6 XIM4 = 0.
    GO TO 8
    7 XIM4 = .0075*(FM5*FM5 - FM5)
    8 XIN4 = XIP4 + XIS4+ XIM4
    ETANI = 1. - XIN4
    C
GO 10 15
12 GAM51 = GAM5-1.
GAM52 = GAMS/GAM51
C
C
C
C
COMPUTE STARTING VALUES
FOR FMZ5 AND FMZT COMPUTATIONS
FMZ5 = FMS*CCM
FMZ7 = FMZ5
FMSCAL = 0
KMS =0
C
C BEGINNING OF THE FMZ5 AND
C FMZT こJMPUTATIONS
C
42 FMZ505 = FMZ5/FM5
ALPH5 = ARCOS(FMZ505)
VZ5= V5 *FMZ505
VU5 = V5 * SORT(1.-FM2505*FM2505)
P5 = PS5 *(1.*.5*GAM5I*FM5*FM5I**GAM52
RH05 = RH05C/TS5
AZ5 = W414/VZ5/RHO5
RU5 = VU5 -U
BETA5= ATAN2(RU5,VZ5)
R5 = S2RT(VZ5*VZ5+RU5\#RU5)
FMR;j = R5/SQRT(GAM5*RGG*TS5)
DELBTI = 2.*BETA5
C
ZETABC = PIE18O*DELBT1/100.
ZETABS = 0.025 + .034*ZETABC*ZETABC*SQRT(ZETABC)
ZETAP = ZETABS/REXP2
ZETAS = FKSB*ZETABS
IF(FMR5 -1.) 52.53.53
52 ZETAM = 0.0
GO TO 54
53 ZETAM = CM*(FMR5*FMR5 - FMR5)
54 ZETAB1 = ZETAP + ZETAS + ZETAM
ETABl = 1. - ZETABI
R6 = R5*SQRT(ETAB1)
DLHB1 =(R5*R5-R6*R6)/DLHEC
CP6 = SPHTCP(TS5,F3,ETABR)
TS6 = TS5 + DLHB1/CP6
PS6 =PS5
AZ6 = Al5
RHO6 = 144.*PS6/R/TS6

```
```

    VZ6 = W414/RHO6/AZ6
    GAMG = GAMMA(TS6,F3,ETABR)
    GAMO1 = GAMG-1.
    GAM62 = 0.5#GAM6 1
    GAM63= GAM6/GAM61
    FMZOSQ=SQRT(GAM6*RGG*TS6)
    FMZS= VZ6/FMZ6SQ
    FMRG = R6/FMZ6SQ
    RU6 = SQRT(R6*R6-VZ6*VZ6)
    BETAG = ATAN2(RUG,VZG)
    VU6 = RUG-U
    ALPH6 = ATAN2(VUG,VZ6)
    FM6 = FMZ6*SQRT(VUG*VU6+VZ6*VZ6)/VZ6
    T6 = TS6*(1.+GAMO2*FN6*FM6)
    P6 = PS6*(1.+GAM62*FM6*FM6)**GAM63
    PS7 =PSS
    FM7 = FMS
    ETAY2 = ETAN
    C
VISCT6 = VISC(TG)
RE6 = 24.*W4/(VISCT6*DT*TRM)
REXPG = (REG/1.5E6)**.2
XIPh = XIALG/REXPG
XISO = FKSN*XIALG
C
2 6
CALL ADIABT(T6,Pb,ETAN2,F3,TS7,PS7,DHN2,ETABR)
DHN2 = ABS(DHN2)
GAM7 = GAMMA(TS7,F3,ETABR)
V7 = S2RT(DLHBC*DHN2)
FM7SV = FM7
FM7 = V7/SQRT(RGG*GAM7*TST)
IF(ABS(FM7-FM7SV) -.002) 21,21,22
22 IF(KCT-100) 23,23,24
23 IF(FM7-1.) 55,56,56
55 XIMG = 0.0
GO TO 57
56 XIM6 = .0075*(FM7*FM7 - FM7)
57 XIN2 = XIP6+ XIS6 + XIM6
ETAIN2 = 1.- XIN2
C
GO TO 26
24 WRITE (6,25) FM7,FM7SV,ETAN2,GAM7
25 FORMATIIH, 5HFM7 =E12.5,5X,8H FM7SV =E12.5,5X,8H ETAN2 =E12.5,
1 5X,6H GAM7=E12.5)
21 GAM71=うАМ7-1.
GAM72=3.5*GAM71
FK=「稆
GAM73= SAM7/GAM71
FMZIC = FMZ7/FM7
ALPHT = ARCOS(FMZ7C)
VZ7 = V7*FMZ7C
VU7=V7*SQRT(1.-FMLTC**2)
P7 = PS7*(1.+GAM72*FM7*FM7) **GAM73
RHO7= 144.*PS7/R/TS7
AZ7 = W414/VZ7/RHO7
U2 = TZM*U
RUT = VUT-UZ
BETAT = ATAN2(RUT,VZ7)

```
```

    R7=SQRT(VZ7*VZ7+RU7*RU7)
    FMR7 = R7/SQRT(GAM7*RGG*TS7)
    DELBT2 = 2.*BETAT
    C
C
5% ZETAM = 0.0
GO TO 50
59 ZETAM = CM*(FMR7*FMR7 - FMR7)
60 ZETABZ = ZETAP + ZETAS + ZETAM
ETAB2 = 1.- ZETAB2
R8 = R7*SQRT(ETAB2)
DLHB2 = (R7*R7 -R8*R8)/DLHBC
CP8 = SPHTCP(TS7,F3,ETABR)
TS8 = TS7+DLHB2/CP8
PS8 = PS7
RHOB = 144.*PS8/R/TS8
AZ8 = AL7
VZ8 = W414/RHO\&/AZ8
GAM8 = GAMMA(TS8,F3,ETABK)
GAM81 = GAM8-1.
GAMB2 = 0.5*GAM81
GAM83 = GAM8/GAM81
GAM84 = GAM81/GAM8
FMZBC = SQRT(GAM8*RGG*TS8)
FMZ8 = VZ8/FMZBC
FMR8 = R8/FMZ8C
RU8= SQRT(R8*R8-VZ8*VZ8)
BETA8 = ATAN2(RU8,VZ8)
VU8=RUB-U2
T8=TS8*(1.+GAM82*FM8*FM8)
ALPH8 = ATAN2(VU8,VZ8)
FM8 = FMZ8*SQRT(VU8*VU8+VZ8*VZ8)/VZ8
P8 = PS8*(1.+ GAM82*FM8*FM8)**GAM83
C
C
C}
DELALP = PIE18O*ABS(ALPH8)
DLALPC = DELALP/100.
ZETAAS = 0.025 + 0.034*DLALPC*DLALPC*SQRT(OLALPC)
ZETASS = ZETAAS*(1./REXP6 + FKSB)
OMEGA = ZETASS*{1.0+0.5*GAM8*FMS*FMS)
PS = PB - OMEGAF(P8-PS8)
T8 = TS8*(1.+GAM82*FM8*FM8)
TS=T8
FMSCAL=SQRT(2./GAM81*((PS/PSS)**GAM84 -1.))
DEBUG AZ8,VZ8,GAMB,GAM84,FMZ8C,FMZ8,RU8,BETA8,VU8,T8,ALPH8,FM8,P8,
XIP,WSBAR,PS,T8,TS,FMSCAL
IF(ABS(FMS -FMSCAL) -.001) 40,40,41
41 KMS = KMS+1
IF(KMS-100) 45,45,43
45 FMZ7SV = FMZ7
FMZ5SV = FMZ5
FMZ5 = FMZ5*FMS/FMSCAL
FMZ7 = FMZ5
FMSCAL = FMS
GO TO 42

```
```

    4 3 \text { WRITE (6,44) FMS,FMSCAL,FML5}
    4 4 \text { FORMAT(IH,48H IN TURH2 AT STATE. NO. 4I, FMS CAN NOT BE FOUND/}
    l lH ,jH FMS=E1:.5:5X,9H FMSCAL =&12.j,5x,
    2 GH FMZ5=E12.5)
    C
C
C
40 TSS=TS/(1.+GAM82*FMS*FMS)
VS= FMS*SQRT{GAM\&*RG(F*TSS)
VZS=VS
AZS = AZSC*TSS/VS
DTR=xIBET2
P10=PlJ
FMl )=FM10
PS1J= P10/(1.*0.2*FM10*FM10)**3.5
PSIil=PS10
GELPS = 1.2U*PAMB*(P10-PS10)/P10
PSFAN = PAMB-OFLPS
SPR = PSFAN/PS5
AEXP=XIBETI
PHI = 3.925*SQRT(1.-SPR*\#AFXP)
DELLIT = 0.00025*OFT
WLEAK = PIE*DTR*OELLIT*SORT(G/R)*PS5*0.615*PHI/SQRT(TS5)
C
ZLEAK = WLEAK/W4
CALL ADIABT(T4,P4,1.0,F3,TSP, PS ,DH4S,ETABR)
OH4S = ABS(DH4S)
WIOLAL = W4\#DH4S
SHP=U*(RU5+RU6+TRM*(1.-ZIL)*(RU7+RU8))*W4:(G*FJ)
ETATLI= SHP/WIDENL
ETAT = ETATLI*(1.-ZLEAK)
4 6 ~ R E T U R V ~
END
SIBFTC TU2B́
SUB<NUTINE TURB4IT4,W4,P4,PSS, DFT,DT, FMS,FTAN,
2
3
4 TS5,V5,FM5,ALPH5,VZ5,VU5,P5,U,BETAS,
5 R5,FMRS,R6,FTAB,TS6,VZ6,FMZG,FMRG,BETAG,
6 RJ6,TMEGA,ALPHG,PG,PS,T6,FMSCAL,WLEAK,
7 PHI,PS10,SPR,ZLEAK,ETATLI, ETAT,
8 FLAM,FNUBR,ETABR,
9 CM,FKS?,FKSN,XIALS,CCM,
1 AL5,FMO,FML5,PSFAV,RE,RU5,TS,TSS,VO,VUG,XIM,
2 XIN,XIP,XIS,ZETAAS,ZETAB,ZETAM,ZETAP,ZETASS,
3 ZETAS,TSP,DHTTI, CPG,DH,AEXP,DTR,
1 TBH,THARC,CCOS,AR,F14)
C
C
C
C MAIV REFERS TO THE CALLING PROGRAM
C
C
WFWABP IN MAIN IS F3 IV TURBUD
HMII IV MAIV IS FNIO IV TURBDP
PII IN MAIN IS PIO IN TURBDP

```
```

C PSCHK IN MAIN IS PS IN TURBOP
C
PSLI IV MAIA IS PSIO IN TURBDP
C
C
C
C
G = 32.174
FJ = 778.
SIGS=1.2
PIE = 3.1415926
PIEI8J = 180./PIE
C
RE = 24.*W4/0T/VISC(T4)
REXP2 = (RE/I.5E6)**.2
XIP = XIALS/REXP2
XIS = FKSN*XIALS
PS5 = PSS
PS6 = PS5
C
GASKOV = GASCUN(F3)
R = GASKON
FMSC = 2.*FJ/R
DLHBC = 2.*G*FJ
RHO6C = 144.*PSG/R
C
C
C
C
FMS = l.
KSD=1
SD=. 3
95 KMSS=0
101 FMZ5 = FMS*CCM
KCT = 0
C
11 IF(FM5 - 1.16,7,7
6 XIM = 0.
GO TO B
7 XIM = 3.0075*(FM5*FM5-FM5)
8 XIN = XIP + XIS + XIM
ETAY = 1.- XIN
C
C
CALL ADIABT(T4,P4,ETAN, F3,TS5,PS5,DH,ETABR)
GAM = GAMMA(TS5,F3,ETABR)
FM5 = SQRT(FM5C*ABS(DH)/TS5/GAM)
C
C
IF(KCT-1)9,10,10
9 KCT = 1
FM5SV = FM5
GO TO 11
10 IF(ABS(FM5-FMSSV)-.002)14,14,12
12 FM5SV = FM5
C
KCT = KCT+1
IF(KCT-100)11,11,15
15 WRITE(5,16) FM5,FM5SV,ETAN
16 FORMATIIH, 36H IN SUB TURBDP, FM5 CAN NOT BE FOUND/IH,

```
```

6H FM5 =E12.5,5X,8H FM5SV =E12.5,5X,7H ETAN =E12.51

```
```

FMRIC SQRT(GAM*G*R*TS5)

```
C BEGIN FMZ5 COMPUTATION
C
    34 ALPH5C \(=\) FML5/FM5
    \(A L P H 5=A R C O S(A L P H 5 C)\)
    VZ5 = V5*ALPH5C
    VU5 = V5*SIN(ALPH5)
    AL5 \(=144 . * W 4 /\) VZ5/R405
    RUS = VU5-U
    BETA5 = ATAN2(RU5,VZ5)
    \(R 5=S Q R T(V 25 * V 25+R U 5 * R U 5)\)
    FMRS \(=\) R5/FMR5C
                            to compute the bucket efficiency, etab
    DELB \(=2 . *\) BETAS
C
    ZETABC = PIE180*DELB/100.
    \(Z E T A B S=0.025+0.034 * Z F T A B C * Z E T A B C * S Q R T(Z E T A B C)\)
    ZETAP \(=\) ZETABS/REXP2
    ZETIS = FKSB\#ZETABS
    IF(FMR5-1.) 42,43,43
    \(42 \mathrm{ZETAM}=0.0\)
    GO TO 44
    43 ZETAM = CM* (FMR5*FMR5 - FMR5)
    44 ZETAB = ZETAP + ZETAS + ZETAM
        ETAB \(=1 .-Z E T A B\)
    C
C
        [F(THARC - (360. - .01)) 60,61,61
    60 SPACE \(=T B H / C O O S / A R\)
        ARCL \(=\) PIE*DT*THARC/360.
        FKPA \(=1 .-\mathrm{SPACE} / 3 . / A R C L\)
        \(R 6=R 5 * S Q R T(E T A B) * F K P A\)
        GOTO 52
C
    61 R6 \(=\) R5*SQRT(ETAB)
    62 DLHB \(=(R 5 * R 5-R 6 * R 6) / D L H B C\)
        TS6 = TS5 + DLHB/CP6
        RHOG \(=\) RHO6C/TS6
        VZ6 \(=144 . *\) W4/RH06/AZ5
        FMZ6S \(=\) SORT(FMZ6C*TS6)

FMZO \(=\) VZ6/FMZ6S
FMRB \(=\) RG/FMZGS
\(R \cup 6=د Q R T(R 6 * R 6-V Z 6 * V Z 6)\)
BETAG = ATAN2 (RUG,VZ6)
VU6 \(=\) RU6-U
ALPH6 \(=\) ATAN2(VU6,VZ6)
51 OMEGA \(=.008 * S I G S * E X P(3.7 * S D) / C O S(A L P H 6)\)
52 FMG \(=\) FMZ6/COS (ALPHS)
P6 \(=\) PS6*(1.+0.5*(GAM-1.)*FM6*FM6)**GAM2
PS \(=\) P6-OMEGA*(P6-PS6)
T6 \(=\) TS6*(1.+0.5*GAM1*FM6*FM6)
V6 \(=5 Q P T(V U 6 * * 2+V Z 6 * * 2)\)
VS = V6* (1.-SD+SIN(ALPH6)/2./SIGS)
IF(KMS.EQ.0) TSS=TS6
FMSCAL \(=V S / S Q R T(G A M * 32.17 * R * T S S)\)
\(T S=T 6\)
TSS=TS/(1.+GAM1/2.*FMS**2)
PSSN=PS/(1.+(GAM-1.)/2.*FMS**2)**GAM2
PS5 =PS5*14.7/PSSN
PS6=PS5
IF(ABS(PSSN-14.7).LT..O1) GO TO 31
PSS=PSSN
\(30 \mathrm{KMS}=K M S+1\)
IF (KMS-100) 32,32,33
32 GO TO 101
33 WRITE(S,35) FMS,FMSCAL,FMZ5
35 FORMATIIH, \(90 H\) IN SUB. TURB68,FMZ5 CAN NOT BE FOUND - PROGRAM lCONTIVUES USING THE LAST VALUE DBTAINED/IH,
\(2 \quad 6 \mathrm{H}\) FMS \(=\mathrm{E} 12.5,5 \mathrm{X}\). 7HFMSCAL=E12.5.5X,7H FMZ5 =E12.51

FMZ5 IS FDUND
31 GO ro (80.81,90), KSD
80 FMS 1=FMSCAL
SDI \(=\) SD
\(S D=.6\)
\(K S O=2\)
GO TO 75
81 FMS2=FMSCAL
SD2 \(=\) SD
\(K S D=3\)
\(S D=(S D 2-S D 1) /(F M S 2-F M S 1) *(F M S-F M S 1)+S D 1\)
GO TO 95
90 CONTINJE
DEBUG FMS,FMSCAL,SD,PSS,PS5,VS
\(T S=T 6\)
PS10= P10/(1.+0.2*FM10*FM10)**3.5
DELPS \(=1.20 * P A M B *(P 10-P S 10) / P 10\)
PSFAN \(=\) PAMB-DELPS
SPR \(=\) PSFAN/PS5
IF (SPR.GT.1.) SPR=1.
PHI \(=3.925 * S Q R T(1 .-S P R * * A E X P)\)
DELLIT \(=0.00025 *\) DFT
WLEAK \(=\) PIE*DTR*DELLIT*SQRT(G/R)*PS5*0.615*PHI/SQRT(TS5)
41 ZLEAK = WLEAK/W4
CALL ADIABT(T4,P4,1.0,F3,TSP,PS,DHTTI,ETABR)
C
IF (THARC - (360. - .01)) 63,64,64
```

    63 ETATL2 = RU% + RUS - .5*F14*U*U*COOS*SPACE/V5/ARCL/ALPH5C
        ETAILI=U*ETATL2/ABS(DHTTI)/G/FJ
        GO 10 55
    64 ETATLI = U*(RU5 + RU6)/AHS(DHTTI)/G/FJ
    C
65 ETAT = ETATLI*(1.-ZLEAK)
FLAA. = U/(RU5+{U6)
VO = V5/SORT(ETAN)
40 FNU:R = :J/Vu
C
C
RETUPV
END

```

\section*{sIBFTC HAPLT}
```

C
SUBROJTINE HALPLT(JFMS,FMSJ, ALPH5J,DTTJ,FMR5J,TBHJ,WICMP, 1 VLJJJ,FPR, 「I,T4,UFT,P1,ITURRI
C
C
COMMOV/CLPLUT/XPEN,YPEN, MX,NY,IPEN,XLABEL(10), YLABEL(10)
DATA BEDX/GHSTATOR, $6 H$ EXIT, GHMACH V, GHD., MS/,
1 SCDY/OHPARA., GHIRATIO,.6H BASE, $6 H P T . M S, 6 H=0.5 /$,
2 LABEL/6HALPH5, GHDTT , 6HMR5, 6HTBH , 6HWI.
3 SHVL /
C
DATA VAM1/SHFPR $=/$,NAM $2 / 6 \mathrm{H} \mathrm{Tl}=/$,NAM3/6HT4 $=/$, NAM4/6HUFT $=1$,
1 VAM5/6HP1 = 1 , NAM7/6HITURB=/
C
DIMFNSION ALPHSJ(100), OTTJ(100),FMR5J(100), TBHJ(100),
1 VLJJJ(100), WICMP(100),FMSJ(100)
DIM:-NSION YJ(100),BCDX(4), BCDY(5), LABEL(6)
c
CALL IDENT
CALL PLOT (0.,.5,-3)
C
FITURB $=$ ITURB
$D X=.35$
XMI ; = FMSJ(1) - DX
FLOMGX $=$ AES(FMSJ(JFMS) - FMSJ(1))/DX +1.
YMIV $=.8$
$D Y=.1$
FLO:AGY $=8$.
$\mathrm{YHISH}=.1$
TO DRAW THE X AXIS
CALL. AXIS(9.,0., BCDX, -24, FLONGX,0.0.XMIN,DX,IU.)
C
C
TO DRAW Y AXIS
CALL AXIS(0.,O., BCDY, 30, FLONGY,90.,YMIN,OY,10.)
C
C
TO DRAW GRID LINES

```
```

        63 ETATL2 = RU5 + RU6 - .5*F14*U*U*COOS*SPACE/V5/ARCL/ALPH5C
        ETAILI=U*ETATL2/ABS(DHTTI)/G/FJ
        GO TO 65
    64 ETATLI = U*(RU5 + RU6)/ABS(DHTTI)/G/FJ
    C
        65 ETAT = ETATL1*(1.-ZLEAK)
        FLAM = U/(RU5+RU6)
        VO = V5/SQRT(ETAN)
    40 FNUBR = U/VU
    C
C
RETUPV
END
sIBFTC HAPLT
C

```
```

    SUBQUJTINE HALPLT(JFMS,FMSJ,ALPH5J,DTTJ,FMR5J,TBHJ,WLCMP,
    ```
    SUBQUJTINE HALPLT(JFMS,FMSJ,ALPH5J,DTTJ,FMR5J,TBHJ,WLCMP,
    l
C
C
    COMMON/CLPLUT/XPEN,YPEN,NX,NY,IPFN,XLABEL(10), YLABEL(10)
    DATA BEDX/GHSTATOR,GH EXIT, GHMACH N,GHO., MS/,
    1 SCDY/GHPARA. ,GHRATIO,,GH BASE,GHPT. MS,GH=0.5%%
    2 LABEL/GHALPH5,6HDTT ,6HMR5 ,6HTBH ,GHW1,
    3 SHVL /
C
    DATA VAM1/GHFPR = /,NAM2/6H T1 = /,NAM3/6HT4 = /,NAM4/GHUFT = /,
l VAM5/6HPl=1, NAM7/6HITURB=/
C
    DIMENSION ALPHSJ(103),OTTJ(100),FMR5J(100),TBHJ(100),
1 VLJJJ(100),W1CMP(100),FMSJ(100)
    DIMIZNSION YJ(100),BCDX(4),BCDY(5),LABEL(6)
C
    CALL IDENT
    CALL PLOT(0.,.5,-3)
C
    FITURB = ITURB
    DX=.05
    XMIN = FMSJ(1) - DX
    FLONGX = ABS(FMSJ(JFMS) - FMSJ(1))/DX + 1.
    YMIN = .8
    DY = . 1
    FLONGY = 8.
    YHICH}=.
C
C TO DRAW THE X AXIS
    CALL AXIS(O.,O.,BCOX,-24,FLONGX,O.0.XMIN,DX,1U.)
C
    CALL AXIS(0.,O.,BCDY,30,FLONGY,90.,YMIN,DY,10.)
C
C
                                    TO DRAW GRID LINES
```

```
    FLGXX = FLGNGX + .1*FLONGX
    JLONGX = IFIX(FLGXX)
C
            31 DO 36 J=1,JFMS
            36 YJ(J) = ALPH5J(J)
        GO In 30
        32DO 37 J=1,JFMS
        37 YJ(J) = DTTJ(J)
        GO TO 30
        33 DO 38 J=1,JFMS
        38 YJ(J) = FMRSJ(J)
        GO TO 30
        34 DO 39 J=1,JFMS
        39 YJ(J) = TBHJ(J)
        GO TO 30
        35DO 40 J=1,JFMS
        40 YJ(J) = W1CMP(J)
        GO TO 30
        30 DUMY = DUMY
    300 CALL LINE(FMSJ,YJ,JFMS,1,0,1,XMIN,DX,YMIN,DY)
                                    LINE PLDTS ARE DRAWN
                    TO LABEL CURVES
    X=(FMSJ(1) - XMIN)/DX/2.
    DO 45 JJ=1,5
    GO TO (46,47,48,49,50),JJ
46 Y = ALPH5J(1)
    GO TO 51
47Y= DTTJ(1)
    GO IO 51
48 Y = FMR5J(1)
    GO TO 51
49 Y = TBHJ(1)
    GO rO 51
```

$50 \mathrm{Y}=\mathrm{WLCMP}(1)$
GO TO 51
$72 Y=$ VLJJJ(1)
51 DUMY = DUMY
$Y=(Y \quad-Y M I N) / D Y \quad+.1$
45 CALL SYMBOL $(X, Y, Y H I G H, L A B E L(J J), 0.0,6)$

C
$c$

C
RETURN
END
\$IBFTC SFALSE
SUBROUTINE FALSE(X,Y,COVTST, J, KCOUNT)
C
C
C
C
C
C
CCC
C
CCC
C
C
c
c
c
c
C
c
CCC
C
c
c
C
TO DRAW LEGEND PLOT
TO MOVE TO NEW PLOT TO WRITE LEGEND

CALL PLOT (10., 0., -3)
CALL SYMBOL (0.0.9.0., 15, NAM1.0.0.6)
CALL VUMBER(1.05,9.0,. 15,FPR,0.0,3)
CALL SYMBOL (0.0,8.75,.15,NAM2,0.0.6)
CALL VJMBER(1.05,8.75,.15, T1,0.0.3)
CALL SYMBOL (0.0.8.50,. 15, NAM 3.0.0.6)
CALL NJMBER(1.05,8.50,.15,T4,0.0.3)
CALL SYMBOL(0.0,8.25,.15,NAM4,0.0,6)
CALL NJMBER(1.05,8.25..15,UFT,0.0.3)
CALL SYMBOL (0.0,8.00,. 15, NAM $5,0.0,6)$
CALL NUMBER(1.05,8.00,.15, P1,0.0.6)
CALL SYMBOL $(0.0,7.75, .15$, NAM $7,0.0,6)$
CALL NJMBER(1.05,7.75,.15,FITURB,0.0.6)
CALL PLOT(10.,0.,-3) $F(X)=Y=0$

ECC REFERS TO WHAT MUST BE DONE IN CALLING PROG.
COVTST IS THE CONVERGENCE CRITERION AND IS
COMPUTED IN THE CALLING PROGRAM MUST BE MET

Y IS ALWAYS COMPUTED IN THE CALLING PROG. TO BE USED IN FALSE

J IS AN INDICATOR CALLING PROGRAM
$J=0,1,2$ ARE SET IN FALSE

TO FIND $x$ BY THE METHOD DF FALSE POSITION SUCH THAT THE CONDITION ABS(Y) LESS THAN COVTST J MUST bE INITIALLY SET EQUAL TO - 1 IN THE

```
J = - M MEANS }x=XFIRST IS BEING USED IN FALSE
J}=0\mathrm{ MEANS }x=XSECOND IS BEING USED IN FALSE
J}=1\mathrm{ MEANS }X\mathrm{ IS TO BE COMPUTED IN FALSE AND
                                    RETURNED TO MAIN
J = 2 IS A FLAG SENT TO THE CALLING PROGRAM SAYING
        wE have Convergence
    IN THE CALLING PROCRAM.TWO READ IN X INITIAL GUESS ARE
        NEEDED, XFIRST AND XSECOND. THESE TWO GUESSES
                ARE USED ONLY IN THE CALLING PROG.
IN THE CALLING PROG.
        IF J = - 1 SET X = XFIRST
        IF }J=0\mathrm{ , SET }X=XSECOND
        IF J = 1 , USE THE X FROM FALSE
        IF J = 2 , STOP. X IS FOUND
KCOUNT IS GNLY USEO IN FALSE. IT IS IN THE CALLING VECTIR TO TELL THE CALLING PROG. HOW MANY INTERATIONS WERE NEEDED
IF(J)10,15,20
C
\(10 \times 1=X\)
\(Y 1=Y\)
\(J=0\)
KCOUNT \(=0\)
GO TO 40
C
\(15 \times 2=X\)
\(Y 2=Y\)
\(J=1\)
GO TO 30
C
C
20 IF(ABS(Y) - COVTST) 21,21,22
\(21 \mathrm{~J}=2\)
GO TO 40
C
\(22 \mathrm{KCOUNT}=\mathrm{KCOUNT}+1\)
IF(KCOUNT- 200)18,18,23
23 WRITE (5,24) \(X, Y\), COVTST
\(J=2\)
GO TO 40
\(C\)
C
18 IF(ABS(Y1) -ABS(Y2))16.16.17
\(16 \mathrm{Y} 2=Y\)
\(\mathrm{X} 2=\mathrm{X}\)
GO TO 30
C
\(17 \mathrm{Yl}=\mathrm{Y}\)
\(X 1=X\)
C
C
\(30 X=X 2-Y 2 *(X 2-X 1) /(Y 2-Y 1)\)
GO TO 40
```

C
C

```
24 FORMAT(IH,5OH IN THE FALSE POSITION ROUTINE, KCOUNT IS EXCEEDED/
    1 4H X =E12.5.5X,4H Y =E12.5.5X.9H COVTST =E12.5)
```

C
40 RETURV
END
\$IBFTC TFQOS

FUNCTITN TFROMS(S,F,ETABR)
C
C TO SOLVE FCR TA BY THE FALSE POSITION METHOD TO GET A FIRST APPROXIMATION FOR TA
C
$S F=F * E T A B R /(1 .+F)$
FTATST $=.0001 * A B S(S)$
TA $=\operatorname{POLY}(S, 8)$
PHIMS $=$ POLY(TA,2) $+S F * P O L Y(T A, 5)$
PHIA $=$ PDLY(TA,2)*(S/PHIMS)**I. 2
$T A I=P J L Y(P H I A, \dot{U})$
C
c. bEGIN FALSE POSITION METHOD

C
$T A 2=T A 1+.02 * T A 1$
JFALSE $=-1$
7 IF (JFALSE) 11,L2,9
$11 \mathrm{TA}=\mathrm{TAI}$
GO TO 9
$12 \mathrm{TA}=\mathrm{TA} 2$
9 FTA $=\operatorname{POLY}(T A, 2)+S F \# P O L Y(T A, 5)-S$
CALI FALSE(TA,FTA,FTATST,JFALSE,K)
IF (JFALSE-1)7,9,30
30 TFRONS = TA
RETURV
END

SIBFTC TFQOH
FUNCTIJN TFROMHI(HM,F,ETABR)

C
C
C
C

TO SOLVE FOR TA BY THF FALSE POSITION METHOD TO GET A FIRST APPRDXIMATION FOR TA

```
    SF=F*ETAEBR/(1.+F)
    FTATST =.0001*AES(HM)
    HA=HM
    TA= POLY(HA,T)
    PSIH = POLY(TA,4)
    HMSTR = HA + SF*PSIH
    HA =HA*(HM/HMSTR)**1.2
```

TAl $=\operatorname{POLY}(H A, 7)$

## BEGIN FALSE POSITION METHOD

C
TA2 $=$ TA1+.02*TA1
JFALSE $=-1$
7 IF(JFALSE) 11,12,9
$11 \mathrm{TA}=$ TAl
GO 「0 9
$12 \mathrm{TA}=\mathrm{TA} 2$
9 FTA $=P O L Y(T A, 1)+S F * P O L Y(T A, 4)-H M$
CALL FALSE(TA,FTA,FTATST,JFALSE,K)
IF(JFALSE-1)7,9,30
30 TFROMH = TA
RETURV
END
\$IBFTC POLYA

```
    FUVCTIJN PMLY(X,M)
    CIMFNSION ICC(8),IORD(8),C(300)
    DATA(ICC(I),I= 1,8)/133,146,161,174,187,200,209,226/
    DATA(I JRD(I), I =1,8)/4,5,4,4,4,2,6,6/
    DATA(に(I),I = 133.242)/100..107.2616.2.4975E-1,-2.2658E-5,
    11.96075E-8,-3.675E-12,2000.,116.484,.209610,2.j54713E-5,-3.338588E
    2-9,1.8943E-13,6000..100..1.14506.1.49413E-3.-1.79831E-6,1.39476E-9
    3,-5.8514E-13,1.0156E-16,1400.,1.42938,4.437122E-4,-1.48918E-7.
    43.4527E-11,-4.513E-15,2.47E-19,6000.,100.,.25232,-5.44152E-5,
    57.0682E-8,-2.U171E-11,-5.1E-16,1400.,.1861,8.0148E-5,-2.3278E-8,
    63.41635E-12.-1.98YE-16.6000.,10U..957.028.-1.397247E-2.2.728083E-4
    7,-7.874997E-8,1.131184E-11,2000..989.0299,2.421058E-2.1.719862E-4,
    8-2.1315lE-8,1.027056E-12,6000.,100.,-. 224336,4.84768E-4,-7.184546E
    9-8,-4.315008E-12,3.134744E-15,2000.,-4.089761,5.504657E-3,-2.44669
    AGE-i,4.8993, 1E-10.-3.617383E-14.6000..100.,3.986078E-2,3.562965E-4
    B,-6.512821E-8,2000.,.2063979,1.764343E-4.-1.633137E-8,6000.,100.,
    C-62.685516,-6.9221196F-1,1.9220060E-2,-1.1345011E-5,-9.8360870E-8,
    D2.2458196E-10,-1.4509678E-13,614.,1946.1462,-8.5195086,2.6225174E-
    E2,-2.5543759E-5,1.1479667E-8,-6.4275539E-13,-0.082041E-16,2000.,
    F1.4,415.37102,58.865753,-312.50006,16.528755,-29.769562.5.6792908.
    G53.206712,1.93344,553.84976,123.43567,-174.32129,22.184326,-104.70
    H823,-43.322374,82.938049,2.3/
    J=I^ES(M)
    K=ICC(J)
10 L=IURD(J)+2
11 [F(X-C(K))12,13,13
12 K=K-L
    GO TO 1l
13 K=K+L
    IF(X-C(K)) 15,15,13
15 IF(M) 20,16,16
16 L=L-1
    POLY=E(K-1)
    0O 17 V=2.L
    KN=K-V
```

17 POLY=PJLY*X+C(KN) GO TO 22
$20 \mathrm{~L}=\mathrm{L}-2$
POLY=FLOAT(L)*C(K-1)
DO $21 \mathrm{~V}=2, \mathrm{~L}$
$K N=K-N$
$L N=L-N+1$
21 POLY=POLY*X+FLOAT(LN)*C(KN)
22 RETURN
END
\$IBFTC FM3S

C

C
C
C TO SOLVE AN FQUA. IN FM3 BY NEWTON'S METHOD
C

C
$S=2.0 *(G 3-1) /.(G 3+1$.
$S 1=1 . / S$
$F 1=\{F 3 / S Q R T(G * G 3)) * * S$
$F 2=3.5 *(G 3-1) * F$.
$X=(F 1+.25 * F 2) * * S 1$
C
$10 \times S=X * * S$
$F=F 1+F 2 * x * x-x S$
$D F=2 * * F 2 * x-S * X S / X$
$X S V=x$
$11 X=X-F / D F$
C
IF(.JJ1*ASS(XS) - ABS(F)) 13,13,12
12 IF (. $05005 * A B S(X+X S V)-A B S(X-X S V) 113,13,15$
C
$13 \mathrm{~J}=\mathrm{J}+1$
IF (J-1 20) 10, 1U,14
14 WRITE 5,17 ) X,XSV,F
C
15 FMM $=X$
C
16 RETURV
17 FORMAT (1H , 6 H FM3 $=\mathrm{E} 12.5,5 \mathrm{X}, 8 \mathrm{H}$ FM35V $=\mathrm{E} 12.5,5 \mathrm{X}, 4 \mathrm{HF}=\mathrm{F} 12.51$ END

## \$IBFTC ENTRJ

FUNCTITN EVTRUP (T,F,ETABR)
PHIA $=\operatorname{POLY}(T, 2)$
$S F=F * E T A B R /(1 .+F)$

PSI $=\operatorname{POLY}(T, 5)$
PHIM $=$ PHIA + SF*PSI ENTROP = PHIM
7 RETURV
END
\$IBFTC ENTHA

FUNCTIJN ENTHAL (T,ETABR,F)
C
C
HA = POLY(T, 1)
PSIH $=\operatorname{POLY}(T, 4)$
$4 \mathrm{SF}=\mathrm{F}$ *ETABR/(1-+F)
$\mathrm{HF}=\mathrm{SF}$ ? PSIH
$H M=H A+H F$
ENTHAL $=H M$
6 RETURV END
\$IBFTC FULAR

C
FUNCTIJN FULAIR(T1, T2,F1,ETARR)
C
$K C T=3$
$F 2 T=.02$
$3 \mathrm{KCT}=\mathrm{KCT}+1$
$F 3=F 1+F 2 T$
$\mathrm{H}_{2}=$ CNTHAL (T2,F3,ETABR)
$H 1=E N T H A L(T 1, F 3, E T A B R)$
$F 2=(1 .+F 1) *(H 2-H 1) /(18630 . * E T A B R+H 1-H 2)$
6 IF (ABS(F2 - F2T) -1.OE-6) 9,9,7
$7 \mathrm{~F} 2 \mathrm{~T}=\mathrm{F} 2$
8 IF(KCT -100) 3,3,11
11 WRITE(S,12) F2,F2T,H1,H2
12 FORMATIIH, 36 H IN SUB. FULAIR, F2 CAN NDT BE FUUND/LH.
$1 \quad 4 \mathrm{HF} 2=\mathrm{E} 12.5,5 \mathrm{X}, 4 \mathrm{HF} 2 \mathrm{~T}=\mathrm{E} 12.5,5 \mathrm{X}, 4 \mathrm{HH} 1=\mathrm{E} 12.5,5 \mathrm{X}, 4 \mathrm{HH} 2=\mathrm{E} 12.51$
9 FULAIR = F2
10 RETURV
END

SIBFTC ADIAB
SUBROUTINE ADIABT(T1,P1,ETAT,F,T2,P2,DH,ETABR)
C

C
$1 S 1=E N T R \cap P(T 1, F, E T A B R)$
$R=3 A S C O V(F)$
$S 2=S 1+(R / 778.161) * A L O G(P 2 / P 1)$
T2J = TFROMS(52,F.ETAHR)
Hl $=$ ENTHAL (Tl,F,ETABR)
$2 \mathrm{H} 2 \mathrm{~J}=\mathrm{ENTHAL}(T 2 J, F, E T A B R)$
IF(P2-P1) 4,5,3
$3 \mathrm{DH}=\left(\mathrm{H} 2 \mathrm{~J}-\mathrm{H}_{1}\right) / E T A T$
GO 105
$5 \mathrm{DH}=0.0$
GO TO 6
$4 \mathrm{DH}=\mathrm{ETAT} *(H 2 \mathrm{~J}-H 1)$
$6 \mathrm{H} 2=\mathrm{Hl}+\mathrm{DH}$ T2 = TFROMH(H2,F,FTARR)
10 RETURN
END
\$IBFTC IDFNT
SUBUOJTINE IDENT
COMMOV/XCPIDX/NELONO, IRUNO, CPID(B)
DATA RJNMES, BLKMES/GHRUN NO, 6HBLK NO/
CALL PLOT(ก..O., - 3)
FBLONO = MBL OivO
CALL SYMBOL(0.0,1.0,.2,CPID(1),90.,24)
CALL SYMBOL (0.5,1.0,.2,RUNMES,90.,6)
CALI SYMBOL (.5,2.5,.2,IRUNO,90.,61
CALL SYMBOL (.5,4.,.2, BLKMES.90.,6)
CALI. VUMBER(.5,6.,.2,FBLCNO, 90.,-1)
CALL PLOT (5.,0., -3)
RETURV
END
\$IBFTC GAMMAF
FUNETIJN GAMMA(T,F,ETABR)
$C P=\operatorname{SPHTCP}(T, F, E T A S R)$
GAMMA $=C P /(C P-G A S C O V(F) / 778.16)$
10 RETURV
END
\$IBFTC SPHTC
C
FUNCTIJN SPHTCP(T,F,ETABR)

C
$C P A=P O L Y(T, 3)$
$S F=F * E T A B R /(l-+F)$
PSICP $=\operatorname{POLY}(T, G)$
CPM $=$ CPA + SFFPSICP
SPHTCP = CPM
10 RETURV
END

## SIBFTC GASCN

FUNCTION GASCON(F)
$R=(53.349+55.088 * F) /(1 .+F)$
GASCOV $=R$
15 RETURN
END

## \$IBFTC VISCOS

FUNCTIJN VISC(T)
DIMENSION A(50), V(50)
DATA (V(J), J=1,50)/73.8,136.0,185.2.227.2.264.7.299.2.331.3, A $361.4,389.8,417.1,443.5,469.5,495.1,519.7,543.6,567.0,589.8,612.1$, B633.9,555.3.676.3.697.0.717.3.737.3,757.0,776.5,795.6,814.5,833.2, C851.6, $969.8,887.8,905.6,923.2,940.6,957.9,974.9,991.8,1008.6$, D1025.2,1041.6,1058.0.1074.1,1090.2.1106.1.1121.9.1137.5.1153.1, E1168.5,1183.81
TKELVV=T*5.19.
$A(1)=100$.
DO $10 \mathrm{~V}=2,50$
$A(N)=A(N-1)+100$.
70 CONTIVUE
IF (TKFLVN.LE.A(1)) GO TC 71
IF (TくELVN.GF.A(50)) GO TO 72
$K=2$
75 IF (TKELVN.LT.A(K)) GO TO 74
$K=K+1$
GO TO 75
74 TOP = A (K)-TKELVN
DELTT= TOP/100.
$D V=V(K)-V(K-1)$
DIFF = DELTT*DV
VISM= V(K)-DIFF
VISC=VISM/14882000.
GO TO 80
71 VISC=V(1)/14882000.
WRITE(6.76)
76 FORMAT (32H TKELVN IS LESS THAN 100 DEGREFS)
GO TO 50
72 VISC=V(50)/14882000.
WRITE 5,77 )
77 FORMATI $36 H$ TKELVN IS GREATER THAN 5000 DEGREES)
80 RETURV
END

```
$IBFTC COMP
    LISTING UF ALTER DECK.
        *ALTER 143,144
        *ENDAL
        SUBROUTINE COMP(IOUT, IWGHT,FMS,ALPH5,DTT,FMR5,TBH,WI,FN,VLI
C
            IF IWIRTE = 2, THEN COMPUTE WITH THREE VALUES OF WI
            THE THIRD VALUE OF WI (WlS3) IS COMPUTED. THE FIRST AND
                        SECOND VALUES ARE KEAD IN
            THE THIRD VALUE OF FN (FN3) IS READ IN. THE FIRST ANO
                        SECOND VALUES ARE COMPUTED.
            IF IDUT = O. DO NOT WRITE DUTPUT
            IF IOUT = 1, WRITE OUTPUT
        IF(IOUT) 81,81,73
        73 WRITE(6,106) FMS,P2OP1,W1,FN3
        81 PIE4=4./PIE
        G = 32.16
        GAM = 1.4
        XIBETl=AEXP
C
C
C
C
                TO CONVERT ALPH7 TO RADIANS
```

```
    GAMC1 = GAM-1.
```

    GAMC1 = GAM-1.
    PDPS = 1. - .1*(1. - 1./ADAS/ADAS)*(1. - 1./11. + 0.5*
    PDPS = 1. - .1*(1. - 1./ADAS/ADAS)*(1. - 1./11. + 0.5*
    1 GAMC1*FMS*FMS)**(GAM/GAMC1))
    1 GAMC1*FMS*FMS)**(GAM/GAMC1))
    GAMC2 = GAMCI/GAM
    GAMC2 = GAMCI/GAM
    GAMC3 = 2./GAMCI
    ```
    GAMC3 = 2./GAMCI
```

```
    ZLEAK = 0.
```

    \(W 2=W 1\)
    \(T 3=T 4\)
    \(T 10=T A M B\)
    T11 = 110
    P2 \(=P 1 * P 20 P 1\)
    T2 \(=\) T1 \({ }^{\circ}\) DTLOSS
    R2 \(=\) GASCON(WFWA1)
    CP2 \(=\) SPHTCP(T2,WFWA1,ETABR)
    GAM2 \(=\) CP2/(CP2-R2/778.16)
    GAM2C1 \(=\) GAM2 +1 .
    GAM2C2 \(=\) GAM2 - 1 .
    GAM2C3 \(=0.5 *\) GAM2C1/GAM2C2
    GAM2C4 \(=0.5 *\) GAM2C2
    FF2Cl \(=\) FM2*SQRT(GAM2*G)
    FF2 \(=\) FF2C1/(1. +.5*GAM2C2*FM2*FM2)**GAM2C3
    \(A 2=W 2 * S Q R T(T 2 * R 2) / P 2 / F F 2\)
    C
DEBUG P2,T2,R2,CP2,GAM2C3,FF2C1,FF2,A2
$\mathrm{A} 2 \mathrm{Bl}=(1 .+G A M 2 C 4 * F M 2 B * F M 2 B) /(1.0+G A M 2 C 4 * F M 2 * F M 2)$
$A 2 B=A 2 * F M 2 / F M 2 B * A 2 B 1 * * G A M 2 C 3 / F N B$
D2B $=\operatorname{SQRT}(P I E 4 * A 2 B)$
WFWA3 = FULAIR(T2,T3.WFWA1,ETABR)
WFWA3P = WFWA1 + WFWA3
$W 3=W 2 *(1 .+W F W A 3)$
$W_{4}=W 3$
R3 $=$ GASCON(WFWA3P)
CP3 $=$ SPHTCP(T3,WFWA3P,ETABR)
GAM3 $=$ CP3/(CP3-R3/778.1 $)$
GAM3C1 = GAM3+1.
GAM3C2 $=$ GAM3-1.
GAM 3C $3=0.5 *$ GAM $3 C 1 /$ GAM $3 C 2$
$P 3 P 2 B=1.0-P P 2 B$
$P 3=P 2 * P 3 P 2 B$
C
C
C
C
C
Q3S $=P 3 *(1 .-1 . /(1 .+($ GAM3-1.) *FM3S*FM3S/GAM3)**(GAM/(GAM-1.) ))
$P 4=P 3-$ OMEGAS*Q3S
FF3 $=W 3 * S Q R T(T 3 * R 3) / P 3 / A 2 B$
FM3 $=$ FYM(FF3,GAM3,G)

```
A3S1 = (1. + 0.5*GAM3C2*FM3S*FM3S)/(1.+GAM3C2*FM3*FM3)
A3S = A2E *FM3/FM3S*A3S1**GAM3C3/FNS
D3S = SQRT(PIE4*A3S)
```

GAMS $=1.35$
GAMS $1=$ GAMS +1 .
GAMS2 $=$ GAMS -1 .
GAMS $3=0.5 *$ GAMS $1 /$ GAMS2
GAMS4 $=$ GAMS/GAMS 1
GAMS5 $=$ GAMS $2 /$ GAMS
GAMS6 $=$ GAMS/GAMS 2
FFS $1=1 .+0.5 * G A M S 2$ *FMS*FMS
FFS $=F M S * S Q R T(G A M S * G) / F F S 1 * * G A M S 3$
$P S D=P A M B$
FFD $=F F S / P D P S / A D A S$
C
FMD $=$ FMM (FFD,GAMS,G)

C
$23 \mathrm{PD}=\mathrm{PAMB} *(1 .+0.5 * G A M S 2$ - $\mathrm{FMD} * F M D) * * G A M S 6$ $P S=P D * P D P S$
c
c
C
43 CALL AJIABT(T4, P4, ETAT,WFWA3P,TS,PS, DHACT, ETABK)
DHACT $=A B S(D H A C T)$
$R S=$ GASCON (WFiNA3P)
C

C
C
33 PD $=$ PAMB*(1. + $0.5 * G A M S 2$-FMD*FMD)**GAMS6
$P S=P D * P D P S$
PSS = PS/FFS 1**GAMS6
CALL ADIABT(T4,P4,ETAT,WFWA3P,TS,PS, DHACT,ETABR)
DHACT $=\triangle B S(D H A C T)$
$30 W T=W 4 * D H A C T$
PLI $=1 .+0.5 * G A M S 2 *$ FMD*FMD
$P L=P D *(1 .-W L B A R *(1 .-1 . / P L 1 * * G A M S 6))$
FML $=$ SORT(2./OAMS2*((PL/PAMR)**GAMS5-1.))
$T L=T S$
$T S L=T L /(1 .+.5 * G A M S 2 * F M L * F M L)$
$V L=F Y L * S Q R T(G A M S * G * R S * T S L)$
$F N T=(W 4 *(1 .-L L E A K) * V L / G) * C T$
$T D=T S$
$A S=W 4 *(1 .-Z L E A K) * S Q R T(T S * R S) / P S / F F S$
$A D=W 4 *(1 .-2 L E A K) * S Q R T(T D * R S) / P D / F F D$
$P 10=P A M B$
P11O = 1.-WIOBR*\{1. -1./(1. + . 2*FM11*FM11)**3.5)
P11 = P110-P10
P13 = FPR*P11
CALL ADIABT(T11,P11,ETAF,0.,T13,P13,DHFACT,0.)
$W F=W T$
DHFACT $=A B S(D H F A C T)$
W1O = NF/DHFACT
PS11 = P11/(1. + 0.2*FM11*FM11)**3.5
TSIL = T11/(1. + 2*FMII*FM11)
V11 = 49.1*SQRT(TS11)*FM11
P14 = P1413*P13
FM14 = SQRT(GAMC 3*(P14/PAMB)**GAMC2 - GAMC3)

```
    P1514=1. -WLBAR*(1. -1./(1. +.2*FM14*FM14)**3.5)
    PS14= PAMB
    PS15 = PAMB
    P15S15 = P1514*P14/PS 14
    FM15 = SQRT(GAMC3*(P15S15)**GAMC2 -GAMC3)
    FM13 = FM11 + FKV*(FM15-FM11)
    PS13C = 1. +.2*FM13*FM13
    PS13=P13/PS13C**3.5
    TS13=T13/PS13C
    V13=49.1*FM13*SQRT(TS13)
    T14= 「13
    T15= T13
    TS14=T14/(1.4.2*FM14*FM14)
    V14 = 49.1*FM14*SGRT(TS14)
    TS15 = T15/(1.*.2*FM15*FM15)
    V15=49.1*FM15*SQRT(TS15)
    FNF =(W10 *V15/G)*CT
    FN=FNT+FNF
C
C
C
C
C
    127 IF(KETA-1) 120,121,121
    120 AS55 = AS
        GO TO 124
    121 IF(ITURB - 2) 122,122,123
    122 AS55 = AZ5
        GO TO 124
    123 AS55 = AS
    124 IF(THAZC - (360. - .01)) 126,129,129
C
C
C
    126 DTT = SQRT(360.*PIF4*AS55/THARC + DTR*DTR)
        GO TO 128
C
C
    129 DTT =1.1284*SQRT(UTR*OTR/PIE4 +AS55)
    128 TRR = DTR/DIT
        TBH = 0.5* (DTT -DTR)
        DT = . 5*(DTR + DTT)
        U = UFT*DT/DFT
        ETATSV = ETAT
        IF(IETAT) 133,133,40
    133 GO TO (41,132,52.998), ITURB
C
    132 CALL TURB2IT4,W4,P4,PSS, DFT.DT, FMS.ETAN.
    2
                                F3,FM11,P11,PAMB,
```

```
        3
4 TS5,V5,FM5,ALPH5,V15,VU5,P5,U,BETA5,
5
6
7
8
9
1
2
3
l
GO TO 53
C
C
    5 2 ~ C A L L ~ T U R B 3 ~ ( T 4 , W 4 . P 4 , P S S , ~ D F T , ~ F M S , E T A N ,
    1 WFWA3P,
    1 FMIL,P11, PAMB,FKS,SWID,FNSML,
    2
    3
    4
5 DELBT2,DHN1,DHN2,DLHBI,DLHB2,DT,ETAB1,
6 ETAB2,ETANI,ETAN2,ETAT,ETATLI,
7 FKM,FKM1,FKM2,FM5,FM6,FM7,FMB,FMR5,
8 FMRG,FMR7,FMR8,FMZ5,FMZ5SV,FMZ6,FMZ7.
9 FMZ7SV,FMZ8,GAM5,GAM6,GAM8,P5,P6,
1 P7,PG,PHI,PS,PS5,PS6,PS7,PS8,PS1O,R5,R6,R7.
2 R8,RHO5,RHO16,RHO7,RHOB,RU5,RU6,RU7,
3 RUB,SHP,SPR,
4
5
6
7
8
TBH,THARC,COOS,AR,F14)
                                ETABR,FK,
    ALPH5,ALPH6,ALPH7,ALPH8,AZ5,AZ7,AL8,AZS,
    BETA5,BETA6,BETA7,BETA8,CP6,CP8,DELBT1.
    T6,T8,TS,TSS,TSP, TS5,TS6,TS7,TS8,U,
    V5,V7,VS, VU5,VU6,VU7,VU8,VZ8,VZ5,
    VZG,VZ7,VZS,WIDEAL,WLEAK, XIBET1,
    XIRET2,ZLEAK.
        OMEGA,CM,FKSB,FKSN,XIALS,TRM,XIAL6,CCM,FLLCI
C
    GO TO 53
C
    41 CALL
        2
        3
        4
        5
        6
        7
        8
        9
        1
        2
        3
        TURB1(T4,W4,P4,PSS, DFT,OT, FMS,ETAN,
        WFNA 3P,FM11,P11,PAMB,
    TS5,V5,FM5,ALPH5,VZ5,VU5,P5,U,BETA5,
    R5,FMR5,R6,ETAR,TS6,VZ6,FMZ6,FMR6,BETA6,
    RJG,OMEGA,ALPHG,PG,PSCHK,TG,FMSCAL,WLEAK,
    PHI,PS11,SPR,ZLEAK,ETATLI, ETAT,
    FLAM,FNUBR,ETABR,
    CM,FKSF,FKSN,XIALS,CCM,
    AZ5,FM6,FMZ5,PSFAN,RE,RU5,TS,TSS,VO,VU6,XIM,
    XIN,XIP,XIS,ZETAAS,ZETAB,ZETAM,ZETAP,ZETASS,
    ZETAS,TSP,OHTTI, CPG,DH,AEXP,DTR,
    TBH, THARC,COOS,AR,F14)
    GO TO 53
998 CALL TURB4I
        T4,W4,P4,PSS, DFT,DT, FMS,ETAN,
        2
        F3,FM11,P11,PAMB,
        TS5,V5,FM5,ALPH5,VZ5,VU5,P5,U,BETA5,
        R5,FMR5,R6,ETAB,TS6,VZ6,FMZ6,FMR6,BETAG,
        RUG,OMEGA,ALPHG,P6,PS,TG,FMSCAL,WLEAK,
        PHI,PS10,SPR,LLEAK,ETATLI, ETAT,
        FLAM,FNUBR,ETABR,
        CM,FKSB,FKSN, XIALS,CCM,
```

TBH, THARC, COOS,AR,F14)
53 IF (ABS(ETAT-ETATSV) -. 001 ) $40.40,42$
42 KETA $=$ KETA +1
IF(KETA -100$) 43,43,44$
44 WRITE( 6,45 ) ETAT, ETATSV
45 FORMAT (1H, 52 H IN MAIN PROGRAM, ETAT CAN NOT BE FOUND AT STATE. 41
$1 / 1 H, 7 H$ ETAT =E12.5,5X,7HETASV =E12.5)
40 WFSTR $=14.696 * W 11 * S Q R T(T 11 / 518.7) / P 11$
IF(IETAT) 61,61,63
61 GD TD $(63,63,64,63)$, ITURB
63 IFIIOUTI 75,75,74
74 WRITE(S.67) A11, A13, A14, A15.A2,A2B,A3S,AD,AS, D2B,D3S,
DFR, DFT, DTR, DT, DTT, ETATSV,FM13,FM14,FM15,
2 FM3,FMD,FML,FN,FNF,FNT,P11,P13,P14,P2,P3,
$3 \quad \mathrm{P} 4, \mathrm{PL}, \mathrm{PD}, \mathrm{PS}, \mathrm{PSS}, \mathrm{PSIL}, \mathrm{PS} 13, \mathrm{Tl4,T2,TBH}$,
4 TRR,TSL
WRITE(5,68) TSP,TS11,TS13,TS14,TS15,U,V11,V13,V14,V15.
1 VL,W3, WFWA3,WFWA3P,WT,W10
V6 $=$ SQRT (VU6**2+VZ6**2)
WRITE(6,69) ALPH5,ALPH6,AZ5,BETA5,BETAG,ETAB,ETAN,ETAT,
1 ETATLI, FLAM,FM5,FM6,FMR5,FMRG,
2 FMSCAL,FMZ5,FMZ6,FNUBR, P5,PG,PHI,
3 PSIL,V6 ,PSFAN,R5,R6,RE,RU5,RU6,SPR,T6,
4 TSO,TS,TSS,V河,VO,VU5,VUG,VZ5,VIG,WLEAK,XIM, XIN
WRITE(S,70) XIP, XIS, ZETAR, ZETAM, ZETAP, ZETAS,
1 ZLEAK,TSP:DHITI,TS5,CPG,DH
75 GO TO $(47,47,65,47)$, ITURB
47 IF (IOJT) 65,65,89
89 WRITE 6,48 ) OMEGA, ZETAAS,ZETASS
GO TO 65
64 IF(IDUT) 65,65,76
76 WRITE(5,67) A11,A13,A14, A15, A2, A $28, A 35, A D, A S, D 2 B, D 3 S$,
1DFR, DFT, DTR, DT, DTT, ETATSV,FM13,FM14, FM15,
2FM3, FMD,FML,FN,FNF,FNT,P11,P13,P14,P2,P3,
3P4, PL, PD, PS, PSS, PS $11, P S 13, T 14, T 2, T B H, T R R, T S 1$
C

$$
G A M 7=F K
$$

WRITE(S,68) TSP,TS11,TS13,TS14,TS15,U,V11,V13,V14,V15,
1VL, W3, WFWA3, WFWA3P,WT,WIO

```
    V6=SQRT(VU6**2+VZ6**2)
    V8=SQRT(VU8**2+VZ8**2)
```

DEBUG T4,W4,ALPHS,ALPH6,ALPH8,AZ5,AZ7,AZ8,AZS,BETA5, 1 BETAG, BETAT, BETAB,CP6,CP8

DEBUG DHN1, DHN2, ETAB1, ETAB2, ETAN1, ETAN2,ETAT,ETATL1, 1FM5, FMS, FM7,FM8, FMR5, FMR6, FMR7,FMRB, FMZ5,FMZ6,FMZ7
C
DEBUG FMZ8,GAM5,GAM6,GAM8, P5,P6,P7,P8,PHI,PS5,PS6,
1PS7,PS8,PS10,R5,R6,R7,R8,RHO5,RH06,RHO7,RHO8,RU5,
2RU6,RJ7,RU8,SHP,SPR
DEBUG T6,T8,TS,TSS,TSP,TS5,TS6,TS7,TS8,V5,V6,V7,VB,VS. 1VU5,VU6,VUT,VUB,VZ8,VZ5,VZ6,VZ7,WIDEAL,WLEAK, 2ZLEAK, OMEGA, GAM 7 , ALPH7
65 WF3P $=$ WFWA3P*W2

```
        SFC = WF3P/FN
        BPR = WlO/W4
C
    CALL ADIABT(T1,P1,1.0,F1,TS1,PAMB,OHENG,ETABR)
    DHE|G = ABS(DHENG)
    VENG = SQRT(2.*G*778.16*DHENG)
    THE CT WHICH MULTIPLIED FNEAG WAS REMOVED 3-7-69
C
C
    FNE'NG = WI*VENG /G
    AUG = FN/FNENG
C
    IF(IOJT) 77,77,78
    78 WRITE(6,71) WFSTR,WF3P,SFC,BPR,AUG
C
C
    77 IF(IWRITE) 94,94,80
    80 IF(IWRITE - 1) 84,84,83
    83 Wl= WlS2
        FNl = FN
        IF(IOUT) 79,79,86
    86 WRITE(6,66)
    WRITE(5,82) WIS2
    79 IWRITE = 1
        GO TO 81
    84 FN2 = FN
    W1S3 = W1S1 + (W1S2 - W1S1)*(FN3 - FN1)/(FN2 - FN1)
    Wl = WlS3
    IWRITE = 0
    IF(IOUT) 81.81,87
87 WRITE(6,66)
    WRITE(5,85) W1S3
    GO TO 81
C
C
    94 IF(IOJT) 95,95,88
    8 8 \text { CONTIVUE}
C
C
```



```
        5X,7H V14 =E12.5,5X,7H V15 =E12.5/
        1H,7H VL =E12.5.5X,7H W3 =E12.5,
        5X,7HWFWA3 =E12.5,5X,7HWFWA 3P=E12.5,5X,
    7H WT =E12.5/1H, 7H W1O=E12.5)
69 FORMAT\1HO,7HALPH5 =E12.5,5X,7HALPHG =E12.5,5X,7H AL5 =E12.5,
        5X,7HBETA5 =E12.5,5X,7HBETAG =E12.5/
        1H,7H ETAB = E12.5,5X,7H ETAN =E12.5,5X,7H ETAT =E12.5,
        5X,7HETATLI=E12.5/
        1H,7H FLAM = E12.5,5X,7H FM5 =E12.5,5X,7H FM6 =E12.5.
        5X,7H FMR5 =E12.5,5X,7H FMRG =E12.5/
        1H,7HFMSCAL=E12.5,5X,7H FMZ5 =E12.5,5X,7H FNZ6 =E12.5,
        5X,7HFNUBR =E 12.5
        1H,7H P5 = E12.5.5X,7H P6 =E12.5,5X,7H PHI =E12.5,
        5X,7H PSII =E12.5,5X,7H V6 =E12.5/
        1H,7HPSFAN =E12.5,5X,7H
            R5 =E12.5,5x,7H R6 =E12.5.
            RUS =E12.5/
```



```
        5X,7H TS6 =E12.5,5X,7H TS =E12.5/
        1H,7H TSS = E12.5,5X,7H VS =E12.5,5X,7H VO =E12.5.
        5X,7H VUS =E12.5,5X,7H VUG =E12.5/
        IH,IH VZ5 =E12.5,5X,7H VZ6 = E12.5.5X,7HWLEAK =E12.5.
        5X,7H XIM =E12.5,5X,7H XIN =E12.5)
70 FORMATI1H, 7H XIP =E12.5,5X,7H XIS = E12.5,5X,7HLETAB =E12.5.
            5X,7HLETAM =E12.5
            1H,7HLETAP = E12.5.5X,7HZETAS =E12.5,
                        5X,7HZLEAK =E12.5,
                        5X,7H TSP = E12.5/
                                1H,7HOHTTI =F12.5,5X,7H TS5 =E12.5.5X,
                            7H CPG =E12.5,5X,7H OH =E12.5)
71 FORMAT(IH, 7HWFSTR =E12.5,5X,7H WF3P =E12.5,5X,7H SFC =E12.5,
    1 5X.7H BPR =E12.5.5X,7H AUG =E12.5)
66 FORMAT (1HK)
82 FORMAT(IHO,9H WI (2) =E12.5)
85 FORMAT (1HO,9H W1 (3) =E12.5)
106 FORMAT(IHI, 7H FMS =E12.5,5X,7HP2OP1 =E12.5.5X,7H W1 =EL2.5,
    l 5X,7H FN3 =E12.5)
4 8 \text { FORWAT(1H, THUMEGA =E12.b,4X, BHZETAAS =E12.5,4X,8HZETASS = E12.5)}
95 RETURV
    END
```


## APPENDIX I

## PROGRAM NOMENCLATURE

## Program Inputs

| FORTRAN symbol | Engineering symbol | Definition |
| :---: | :---: | :---: |
| AR | $\mathrm{AR}_{T}$ | turbine blade aspect ratio |
| AEXP | b | exponent in seal leakage equation |
| CCM | CC | turbine rotor axial velocity multiplier |
| CM | $\mathrm{K}_{\text {Sh }}$ | turbine blade Mach number correction coefficient |
| CT | $\mathrm{C}_{\mathrm{T}}$ | thrust coefficient (turbine and fan) |
| COOS | ${ }^{\sigma} \mathrm{T}$ | turbine blade solidity |
| F14 | $\mathrm{K}_{\mathrm{pa}}$ | partial-admission loss coefficient |
| HV | HV | lower heating value of fuel, Btu/lbm |
| H2P | $\mathrm{H}^{\text {O }}$ | liquid fuel inlet enthalpy, Btu/lbm |
| OMEGAS | $\mathrm{K}_{\mathrm{sc}}$ | scroll-loss coefficient |
| FKSB | $\left(\mathrm{K}_{\text {Se }}\right)_{\mathrm{B}}$ | rotor blade secondary-loss coefficient |
| FKSN | $\left(\mathrm{K}_{\text {Se }}\right)_{\mathrm{N}}$ | stator secondary-loss coefficient |
| FKV | $\mathrm{K}_{\mathrm{F}}$ | fan internal loss factor |
| TLH | ${ }^{(L / H)}{ }_{\text {D }}$ | turbine exit duct length to height ratio |
| FLLC | X | two-stage turbine work split factor |
| FM2 | $\mathrm{M}_{2}$ | inlet duct exit Mach number |
| FM2B | $\mathrm{M}_{2 \mathrm{~b}}$ | burner inlet flow Mach number |
| FM3S | $\mathrm{M}_{3 \mathrm{sc}}$ | scroll inlet flow Mach number |
| FMS | $\mathrm{M}_{\mathrm{S}}$ | turbine exit stator exit Mach number |
| FM11 | $\mathrm{M}_{10}$ | fan inlet axial Mach number |
| FNS | $\mathrm{n}_{\mathrm{A}}$ | number of scroll arms per inlet duct |
| FNB | $\mathrm{n}_{\mathrm{b}}$ | number of burners |
| P1 | $\mathrm{p}_{1}^{\prime}$ | inlet duct inlet total pressure, psia |


| FORTRAN symbol | Engineering symbol | Definition |
| :---: | :---: | :---: |
| P20P1 | $\mathrm{p}_{2}^{\prime} / \mathrm{p}_{1}^{\prime}$ | inlet duct pressure ratio |
| PAMB | $\mathrm{p}_{0}$ | ambient pressure, psia |
| FPR | $\mathrm{p}_{13}^{\prime} / \mathrm{p}_{11}^{\prime}$ | fan-stage pressure ratio |
| P1413 | $\mathrm{p}_{14} / \mathrm{p}_{13}^{\prime}$ | fan outlet duct pressure ratio |
| PP2B | $\Delta \mathrm{p}^{\prime} / \mathrm{p}_{2}^{\prime}$ | burner pressure drop fraction |
| R | R | air gas constant, $53.35 \mathrm{ft}-\mathrm{lbf} /(\mathrm{lbm})\left({ }^{( } \mathrm{R}\right)$ |
| RHRT | $\mathrm{Y}_{\mathrm{F}}$ | fan hub-tip ratio at rotor inlet |
| RING | $\Delta \mathrm{r}_{\mathrm{c}}$ | fan-shroud carrier height, in. |
| TAMB | $\mathrm{T}_{0}$ | ambient temperature, ${ }^{\circ} \mathrm{R}$ |
| TRM | TRM | two-stage turbine mean line radius change factor |
| T1 | T ${ }_{1}^{\prime}$ | inlet duct inlet total temperature, ${ }^{\circ} \mathrm{R}$ |
| T4 | T ${ }_{4}$ | turbine inlet total temperature, ${ }^{\circ} \mathrm{R}$ |
| DTLOSS | $\Delta \mathrm{T}_{\mathrm{d}}^{\prime}$ | inlet duct temperature loss, ${ }^{0} \mathrm{R}$ |
| UFT | $\mathrm{U}_{\mathrm{F}, \mathrm{t}}$ | fan tip speed, $\mathrm{ft} / \mathrm{sec}$ |
| W1 | ${ }^{W} 1$ | inlet duct weight flow, lbm/sec |
| W FWA 1 | $\left.\frac{\mathrm{f}}{\mathrm{a}}\right\|_{1}$ | fuel-to-air ratio at inlet duct |
| ZIL | $\mathrm{Z}_{6}$ | interstage leakage ratio |
| ETABR | $\eta_{\mathrm{b}}$ | burner efficiency |
| ETAF | $\eta_{\mathbf{F}}$ | fan-stage adiabatic efficiency |
| ETAN | $\eta_{\mathrm{N}}$ | initial value of inlet stator efficiency |
| ETAT | $\eta_{\mathbf{T}}$ | initial value of turbine total-to-total efficiency |
| W10BR | $\bar{\omega}_{10}$ | fan inlet loss coefficient |
| WLBAR | $\bar{\omega}_{L}$ | louver loss coefficient |
| THARC | $\theta$ | turbine admission arc, deg |
| XIALS | $\left(\xi_{p}^{*}\right)_{N}$ | inlet or interstage stator profile loss |

## Program Outputs - Main Program

| FORTRAN <br> symbol | Engineering symbol | Definition |
| :---: | :---: | :---: |
| A11 | ${ }^{\text {A }} 11$ | fan flow area at rotor inlet, in. ${ }^{2}$ |
| A13 | $\mathrm{A}_{13}$ | fan flow area at stator exit, in. ${ }^{2}$ |
| A14 | $\mathrm{A}_{14}$ | fan flow area at duct exit, in. ${ }^{2}$ |
| A15 | ${ }^{\text {A }} 15$ | fan flow area at louver exit, in. ${ }^{2}$ |
| A2 | $\mathrm{A}_{2}$ | turbine flow area at inlet duct exit, in. ${ }^{2}$ |
| A2B | $\mathrm{A}_{2 \mathrm{~b}}$ | turbine flow area at burner inlet, in. ${ }^{2}$ |
| A3S | $\mathrm{A}_{3 \mathrm{sc}}$ | turbine flow area at scroll inlet, in. ${ }^{2}$ |
| AD | ${ }^{A_{D}}$ | turbine flow area at exit duct, in. ${ }^{2}$ |
| AS | $\mathrm{A}_{S}$ | turbine flow area at exit stator exit, in. ${ }^{2}$ |
| D2B | $\mathrm{d}_{2 \mathrm{~b}}$ | burner diameter, in. |
| D3S | $\mathrm{d}_{3 \mathrm{sc}}$ | scroll inlet diameter, in. |
| DFR | $\mathrm{d}_{\mathrm{F}, \mathrm{h}}$ | fan rotor diameter at hub, in. |
| DFT | ${ }_{\text {d }}^{\text {F, }}$, | fan rotor diameter at tip, in. |
| DTR | $\mathrm{d}_{\mathrm{T}, \mathrm{h}}$ | rotor diameter at turbine hub, in. |
| DT | $\mathrm{d}_{\mathrm{T}, \mathrm{m}}$ | rotor diameter at midturbine, in. |
| DTT | ${ }^{\text {d }}$ T, t | rotor diameter at turbine tip, in. |
| FN | F | calculated total thrust, lbf |
| FNF | $\mathrm{F}_{\mathrm{F}}$ | calculated fan thrust, lbf |
| FNT | $\mathrm{F}_{\mathrm{T}}$ | calculated turbine thrust, lbf |
| FM13 | $\mathrm{M}_{13}$ | Mach number at fan stator exit |
| FM14 | $\mathrm{M}_{14}$ | Mach number at fan duct exit |
| F'M15 | $\mathrm{M}_{15}$ | Mach number at fan louver exit |
| FM3 | $\mathrm{M}_{3}$ | Mach number at burner exit |
| FMD | $M_{D}$ | Mach number at turbine duct exit |
| FML | $M_{L}$ | Mach number at turbine louver exit |
| P11 | $\mathrm{p}_{11}^{\prime}$ | total pressure at fan rotor inlet, psi |


| FORTRAN <br> symbol | Engineering symbol | Definition |
| :---: | :---: | :---: |
| P13 | $\mathrm{p}_{13}^{\prime}$ | total pressure at fan stator exit, psi |
| P14 | $\mathrm{p}_{14}$ | total pressure at fan duct exit, psi |
| P2 | p ${ }_{2}$ | total pressure at inlet duct exit, psi |
| P3 | $\mathrm{p}_{3}^{\prime}$ | total pressure at burner exit, psi |
| P4 | $\mathrm{p}_{4}^{\prime}$ | total pressure at turbine inlet, psi |
| PL | $p_{\text {L }}^{\prime}$ | total pressure at turbine louver exit, psi |
| PD | $\mathrm{p}_{\mathrm{D}}^{\prime}$ | total pressure at turbine duct exit, psi |
| PS | $\mathrm{p}_{\mathrm{S}}^{\prime}$ | total pressure at turbine exit stator exit, psi |
| PSS | $\mathrm{p}_{S}$ | static pressure at turbine exit stator exit, psi |
| PS11 | $\mathrm{p}_{11}$ | static pressure at fan rotor inlet, psi |
| PS13 | $\mathrm{p}_{13}$ | static pressure at fan stator exit, psi |
| T2 | T ${ }_{2}$ | total temperature at inlet duct exit, ${ }^{0} \mathrm{R}$ |
| T14 | T ${ }_{14}$ | total temperature at fan duct exit, ${ }^{0} \mathrm{R}$ |
| TBH | $\mathrm{H}_{\mathrm{B}}$ | turbine blade height, in. |
| TRR | $\mathbf{Y}_{\mathbf{T}}$ | turbine hub-tip ratio |
| TSL | $\mathrm{T}_{\mathrm{L}}$ | static temperature at turbine louver exit, ${ }^{0} \mathrm{R}$ |
| TS11 | $\mathrm{T}_{11}$ | static temperature at fan rotor inlet, ${ }^{\circ} \mathrm{R}$ |
| TS13 | $\mathrm{T}_{13}$ | static temperature at fan stator exit, ${ }^{0} \mathbf{R}$ |
| TS14 | $\mathrm{T}_{14}$ | static temperature at fan duct exit, ${ }^{0_{R}}$ |
| TS15 | $\mathrm{T}_{15}$ | static temperature at fan louver exit, ${ }^{0} \mathbf{R}$ |
| U | . $\mathrm{U}_{\mathbf{T}}$ | turbine midwheel speed, ft/sec |
| V11 | $\mathrm{V}_{11}$ | axial velocity at fan rotor inlet, $\mathrm{ft} / \mathrm{sec}$ |
| V13 | $\mathrm{V}_{13}$ | axial velocity at fan stator exit, $\mathrm{ft} / \mathrm{sec}$ |
| V14 | $\mathrm{V}_{14}$ | axial velocity at fan duct exit, $\mathrm{ft} / \mathrm{sec}$ |
| V15 | $\mathrm{V}_{15}$ | axial velocity at fan louver exit, $\mathrm{ft} / \mathrm{sec}$ |
| VL | $\mathrm{V}_{\mathrm{L}}$ | axial velocity at turbine louver exit, $\mathrm{ft} / \mathrm{sec}$ |
| W3 | ${ }^{\mathbf{w}} 3$ | mass flow at burner exit, lbm/sec |


| FORTRAN symbol | Engineering symbol | Definition |
| :---: | :---: | :---: |
| WFWA 3 | $\left.\frac{\mathrm{f}}{\mathrm{a}}\right\|_{\mathrm{b}}$ | fuel-to-air ratio added in burner |
| WFWA 3P | $\left.\frac{\mathrm{f}}{\mathrm{a}}\right\|_{4}$ | total fuel-to-air ratio at burner exit |
| WT | $\mathrm{P}_{\mathbf{T}}$ | turbine power output, Btu/sec |
| W10 | ${ }^{\mathbf{w}} 10$ | fan air flow rate, lbm/sec |
| Program Output - Turbine Subroutine |  |  |
| AUG | A UG | thrust augmentation ratio |
| AZ 5 | $\mathrm{A}_{5}$ | rotor inlet annular flow area, in. ${ }^{2}$ |
| BPR | BPR | bypass ratio |
| CP6 | $\mathrm{C}_{\mathrm{p}, 6}$ | specific heat at rotor exit, $\mathrm{Btu} /(\mathrm{lb})\left({ }^{\mathrm{O}} \mathrm{R}\right)$ |
| SD | D | exit stator diffusion factor |
| DH | $\Delta h_{N}$ | inlet stator actual enthalpy drop, Btu/lbm |
| DHTT1 | $\Delta h_{\text {T, id }}$ | isentropic total-to-total enthalpy drop, Btu/lbm |
| FM5 | $\mathrm{M}_{5}$ | absolute rotor inlet Mach number |
| FM6 | $\mathrm{M}_{6}$ | absolute rotor exit Mach number |
| FMR5 | $M_{\mathbf{r , 5}}$ | relative rotor inlet Mach number |
| FMR6 | $M_{r, 6}$ | relative rotor exit Mach number |
| FMSCAL | $\mathrm{M}_{\text {S(calculated) }}$ | stator exit Mach number (calculated) |
| FMZ 5 | $\mathrm{M}_{\mathrm{x}, 5}$ | axial rotor inlet Mach number |
| F'MZ6 | $\mathrm{M}_{\mathrm{x}, 6}$ | axial rotor exit Mach number |
| P5 | $\mathrm{p}_{5}^{\prime}$ | total pressure at rotor inlet, psi |
| P6 | $\mathrm{p}_{6}^{\prime}$ | total pressure at rotor exit, psi |
| PS11 | $\mathrm{p}_{11}$ | static pressure at fan rotor inlet, psi |
| PSFAN | $\mathrm{p}_{\mathrm{ex}}$ | static pressure at bellmouth (seal exit), psi |


| FORTRAN <br> symbol | Engineering symbol | Definition |
| :---: | :---: | :---: |
| R5 | $\mathrm{W}_{5}$ | relative velocity at rotor inlet, $\mathrm{ft} / \mathrm{sec}$ |
| R6 | $\mathrm{W}_{6}$ | relative velocity at rotor exit, $\mathrm{ft} / \mathrm{sec}$ |
| RE | Re | Reynolds number |
| RU5 | $\mathrm{W}_{\mathrm{u}, 5}$ | tangential component of relative velocity at rotor inlet, $\mathrm{ft} / \mathrm{sec}$ |
| SFC | SFC | specific fuel consumption of tip-turbine fan, $\frac{\mathrm{lb}-\text { fuel } / \mathrm{hr}}{\mathrm{lb} \text {-thrust }}$ |
| SPR | SPR | seal pressure ratio |
| T6 | $\mathrm{T}_{6}{ }^{\prime}$ | absolute total temperature at rotor exit, ${ }^{o_{R}}$ |
| TS6 | $\mathrm{T}_{6}$ | static temperature at rotor exit, ${ }^{0} \mathrm{R}$ |
| TS | T'S | total temperature at exit stator exit, ${ }^{\circ} \mathrm{R}$ |
| TSP | $\mathrm{T}_{\mathbf{S}}^{\mathbf{\prime}}$, id | isentropic total temperature at exit stator exit, ${ }^{\circ} \mathbf{R}$ |
| TSS | $\mathrm{T}_{S}$ | static temperature at exit stator exit, ${ }^{0} \mathbf{R}$ |
| TS5 | $\mathrm{T}_{5}$ | static temperature at rotor inlet, ${ }^{0} \mathrm{R}$ |
| V5 | $\mathrm{V}_{5}$ | absolute velocity at rotor inlet, $\mathrm{ft} / \mathrm{sec}$ |
| V6 | $\mathrm{V}_{6}$ | absolute velocity at rotor exit, ft/sec |
| VO | $\mathrm{V}_{\mathrm{j}}$ | ideal jet velocity, ft/sec |
| VU5 | $\mathrm{V}_{\mathrm{u}, 5}$ | tangential component of the absolute velocity at rotor inlet, ft/sec |
| VU6 | $\mathrm{V}_{\mathrm{u}, 6}$ | tangential component of the absolute velocity at rotor exit, ft/sec |
| VZ5 | $\mathrm{V}_{\mathrm{x}, 5}$ | axial component of velocity at rotor inlet, $\mathrm{ft} / \mathrm{sec}$ |
| VZ6 | $\mathrm{V}_{\mathrm{x}, 6}$ | axial component of velocity at rotor exit, $\mathrm{ft} / \mathrm{sec}$ |
| W FSTR | ${ }^{\mathbf{w}}$ eq | fan corrected mass flow rate, lbm/sec |
| WF3P | $\mathrm{w}_{\mathrm{f}, 4}$ | total fuel flow, lbm/sec |
| WLEAK | ${ }^{\text {w }}$ leak | seal leakage flow rate, $1 \mathrm{lbm} / \mathrm{sec}$ |
| ZLEAK | Z | ratio of inlet seal leakage to turbine flow rate |


| FORTRAN symbol | Engineering symbol | Definition |
| :---: | :---: | :---: |
| ALPH5 | $\alpha_{5}$ | rotor absolute inlet angle, rad |
| ALPH6 | $\alpha_{6}$ | rotor absolute exit angle, rad |
| BETA 5 | $\beta_{5}$ | rotor relative inlet angle, rad |
| BETA6 | $\beta_{6}$ | rotor relative exit angle, rad |
| ETAB | $\eta_{\mathrm{B}}$ | turbine blade row efficiency |
| ETAN | $\eta_{\mathrm{N}}$ | inlet stator blade row efficiency |
| ETAT | $\eta_{\mathrm{T}}$ | turbine total-to-total efficiency |
| ETATL1 | $\eta_{\mathrm{T}, \text { leak }}$ | turbine total-to-total efficiency (with leakage) |
| FLAM | $\lambda$ | speed-work parameter |
| FNUBR | $\nu$ | blade-jet speed ratio |
| OMEGA | $\mathrm{K}_{\mathrm{S}}$ | stator total pressure-loss coefficient |
| PHI | e | seal leakage expansion function |
| ZETASS | $\xi_{S}$ | stator total-loss coefficient |
| XIS | $\left.{ }^{\left(\xi_{\mathrm{se}}\right.}\right)_{\mathrm{N}}$ | inlet stator secondary-loss coefficient |
| ZETAB | $\xi^{\text {B }}$ | rotor total-loss coefficient |
| ZETAM | $\left.{ }^{\left(\xi \xi_{\text {sh }}\right.}\right)_{\text {B }}$ | rotor shock-loss coefficient |
| ZETAP | $\left(\xi_{\mathrm{p}}\right)_{\mathrm{B}}$ | rotor profile-loss coefficient |
| ZETAS | $\left.{ }^{\left(\xi \xi_{\text {se }}\right.}\right)_{\text {B }}$ | rotor secondary-loss coefficient |
| XIM | $\left.{ }^{(\xi}{ }_{\text {sh }}\right)_{N}$ | inlet stator shock-loss coefficient |
| XIN | $\xi_{\mathrm{N}}$ | inlet stator total-loss coefficient |
| XIP | $\left(\xi_{\mathrm{p}}\right)_{\mathrm{N}}$ | inlet stator profile-loss coefficient |

Additional Output for Two-Stage Turbine
$\mathrm{A}_{\mathrm{S}}$
second-stage rotor inlet annular flow area, in. ${ }^{2}$ second-stage rotor exit annular flow area, in. ${ }^{2}$ exit stator exit annular flow area, in. ${ }^{2}$

| FORTRAN symbol | Engineering symbol | Definition |
| :---: | :---: | :---: |
| CP8 | $C_{p, 8}$ | specific heat at second-stage rotor exit, $\mathrm{Btu} /(\mathrm{lbm})\left({ }^{\circ} \mathrm{R}\right)$ |
| DHN1 | $\Delta h_{N, 5}$ | actual enthalpy drop across first-stage stator, $\mathrm{Btu} / \mathrm{lbm}$ |
| DHN2 | $\Delta h_{N, 7}$ | actual enthalpy drop across second-stage stator, Btu/lbm |
| FM7 | $\mathrm{M}_{7}$ | absolute second-stage rotor inlet Mach number |
| FM8 | $\mathrm{M}_{8}$ | absolute second-stage rotor exit Mach number |
| FMR7 | $M_{r, 7}$ | second-stage relative rotor inlet Mach number |
| FMR8 | $\mathrm{M}_{\mathrm{r}, 8}$ | second-stage relative rotor exit Mach number |
| FMZ7 | $\mathrm{M}_{\mathrm{x}, 7}$ | second-stage rotor axial inlet Mach number |
| FMZ8 | $\mathrm{M}_{\mathrm{x}, 8}$ | second-stage rotor axial exit Mach number |
| P7 | $\mathrm{p}_{7}^{\prime}$ | total pressure at second rotor inlet, psi |
| P8 | $\mathrm{p}_{8}^{\prime}$ | total pressure at second rotor exit, psi |
| PS5 | $\mathrm{p}_{5}$ | static pressure at first rotor inlet, psi |
| PS6 | $\mathrm{p}_{6}$ | static pressure at first rotor exit, psi |
| PS7 | $\mathrm{p}_{7}$ | static pressure at second rotor inlet, psi |
| PS8 | $\mathrm{p}_{8}$ | static pressure at second rotor exit, psi |
| R7 | $\mathrm{W}_{7}$ | second-stage rotor inlet relative velocity, $\mathrm{ft} / \mathrm{sec}$ |
| R8 | $\mathrm{W}_{8}$ | second-stage rotor exit relative velocity, $\mathrm{ft} / \mathrm{sec}$ |
| RU7 | $W_{u, 7}$ | second-stage rotor inlet tangential component of relative velocity, ft/sec |
| RU8 | $\mathrm{W}_{\mathrm{u}, 8}$ | second-stage rotor exit tangential component of relative velocity, $\mathrm{ft} / \mathrm{sec}$ |
| T8 | $\mathrm{T}_{8}{ }^{\text {\% }}$ | absolute total temperature at second rotor exit, ${ }^{0} \mathbf{R}$ |
| TS7 | $\mathrm{T}_{7}$ | static temperature at second rotor inlet, ${ }^{0} \mathbf{R}$ |
| TS8 | $\mathrm{T}_{8}$ | static temperature at second rotor exit, ${ }^{0} \mathbf{R}$ |
| V7 | $\mathrm{V}_{7}$ | absolute velocity at second rotor inlet, $\mathrm{ft} / \mathrm{sec}$ |

FORTRAN
symbol
Engineering
symbol

Definition
absolute velocity at second rotor exit, $\mathrm{ft} / \mathrm{sec}$ stator exit velocity, $\mathrm{ft} / \mathrm{sec}$ second-stage rotor inlet tangential component of absolute velocity, ft/sec
second-stage rotor exit tangential component of absolute velocity, $\mathrm{ft} / \mathrm{sec}$
second-stage rotor inlet axial velocity, $\mathrm{ft} / \mathrm{sec}$ second-stage rotor exit axial velocity, ft/sec ideal stage power, Btu/sec actual power output, Btu/sec second-stage rotor absolute inlet angle, rad second-stage rotor absolute exit angle, rad second-stage rotor relative inlet angle, rad second-stage rotor relative exit angle, rad first-stage rotor blade row efficiency second-stage rotor blade row efficiency first-stage stator efficiency second-stage stator efficiency ratio of specific heats, first rotor inlet ratio of specific heats, first rotor exit ratio of specific heats, second rotor inlet ratio of specific heats, second rotor exit first-stage rotor inlet density, $\mathrm{lbm} / \mathrm{ft}^{3}$ first-stage rotor exit density, $\mathrm{lbm} / \mathrm{ft}^{3}$ second-stage rotor inlet density, $\mathrm{lbm} / \mathrm{ft}^{3}$ second-stage rotor exit density, $\mathrm{lbm} / \mathrm{ft}^{3}$

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Figure 1. - Single-stage tip-turbine fan model used in analysis showing major components and station designations.


Plan view
Figure 2 - Lift fan schematic.


Figure 3. - Tip-turbine concepts.


Figure 4. - Main program iteration flow chart.


Figure 5. - Typical turbine parametric data plot.


Figure 6. - Variation of inlet duct flow rate with exit stator exit axial Mach number. Total thrust, 10000 pounds; inlet duct inlet pressure, 117.6 psia; turbine inlet temperature, $1900^{\circ} \mathrm{R}$; inlet stator exit angle, $\mathrm{Z} 75^{\circ}$.


Figure 7. - Comparison of louver exit velocity for four turbine stage configurations. Total thrust, 10000 pounds; inlet duct inlet pressure, 117.6 psia; turbine inlet temperature, $1900^{\circ}$ R; inlet stator exit angle, $<75^{\circ}$.


Figure 8. - Variation of turbine tip diameter with exit stator exit axial Mach number. Total thrust, 10000 pounds; inlet duct inlet pressure, 117.6 psia; turbine inlet temperature, $1900^{\circ} \mathrm{R}$; inlet exit angle, $<75^{\circ}$.


Figure 9. - Two-stage tip-turbine station designations.


Figure 10. - Scroll configuration.


Figure 11. - Fan rotor schematic.


Figure 12. - Single-stage tip-turbine velocity diagrams.


Figure 14. - Profile loss coefficients for inlet and second-stage stator (kinetic energy basis). Data obtained from reference 12.


Figure 13. - Two-stage tip-turbine velocity diagrams.


Figure 15. - Estimated efficiency variation for inlet and interstage stators.


Figure 16. - Proposed shock loss coefficient variation for inlet and interstage stators.


Figure 17. - Profile loss coefficients for turbine rotor blades. Data obtained from reference 12.


Figure 18. - Proposed variation of turbine rotor blade turning loss coefficient. Reference Reynolds number, $1.5 \times 10^{6}$.


Figure 19. - Shock loss coefficient variation for turbine rotor.


Figure 20. - Enthalpy-entropy diagram for overall efficiency determination, single-stage turbine.


Figure 21. - Enthalpy-entropy diagram for overall efficiency determination, two-stage turbine.


Figure 22. - Variation of orifice function with seal pressure ratio for single-element seal.


Figure 23. - Variation of exponent in equation (F5) for expansion function, single element seal.


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