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FINAL REPORT
(PHASE-I)

FOR

AUTOMATIC RENDEZVOUS AND DOCKING

CONTRACT NAS8-23973

TO

NASA
MARSHALL SPACE FLIGHT CENTER

HUNTSVILLE, ALABAMA 35812

PREPARED BY

S.A.R. AYYAR

T.E. FLOM

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AEROSPACE/OPTICAL DIVISION ITT

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FORWARD

This document is the final report for Phase I of contract NAS8-23973, "Automatic Rendezvous and Docking." The program was sponsored by the Marshall Space Flight Center of the National Aeronautics and Space Administration, Huntsville, Alabama. The technical representative for NASA was J.D. (Dennis) Ellsworth.

This study contract was performed by the Aerospace/Optical Division of ITT, San Fernando. The program was performed in the Electro-Optical Laboratory under Thomas Dixon (Lab Director) and Leo Cardone (Associate Lab Director).

ITT's principal investigator for the study program was S.A.R. (Rama) Ayyar (Senior Scientist). Other contributors to the program were I. Barlia, L. Cardone, T. Flom and J. Steinhilber. Acknowledgement is made of the helpful discussions Mr. Ayyar has had with his fellow doctoral students at UCLA, System Science Department, and particularly with Mr. Yuan-shih Hsu.

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1.0 INTRODUCTION

The primary objective of Phase I of this study program (NAS8-23973, "Automatic Rendezvous & Docking") was to develop a digital computer program that simulated the two earth orbiting satellites while they were performing station keeping and docking closure maneuvers. The maneuvers were to be performed automatically using the Scanning Laser Radar (SLR) system that was developed on contract NAS8-20833 to close the loop. Automatic rendezvous and docking capability depends on the use of the on-board digital computer to produce steering commands to the autopilot generated from the sensed state parameters. For this study, the main sensor is the ITT Scanning Laser Radar which provides mutual range, range-rate, angle and angle-rate data to the vehicle computer. Since measurement parameters are corrupted by noise and the system state (vehicular positions and velocities) is defined by a dynamical system, a recursive filter was designed to estimate the vehicle state from the noisy measurement vector.

Section 2.0 of this report defines the general rendezvous and docking problem studied here and section 3.0 discusses the overall control problem. Section 4.0 and section 5.0 go into the detailed analysis and equation of motions developed for station keeping and docking closure respectively. Section 6.0 and section 7.0 discuss the Kalman filter and the combined filtering and control. Appendices A, B, and C are computer plots of the results for station keeping, docking closure, and the recursive filter simulations.

2.0 PROBLEM DEFINITION

Performance of rendezvous and docking requires knowledge of relative positions and velocities of the spacecraft involved. Generally a rendezvous and docking mission has a target vehicle in orbit with its orbital parameters known from ground tracking. The chaser vehicle is launched into orbit, and when the relative range is short enough (i.e., < 75 miles) for the chaser vehicle on-board sensors (i.e., Laser Radar) to acquire target vehicle relative position and velocity data, thrusting is applied to cause the chaser vehicle to come within close proximity (1000 feet) of the target vehicle. A station keeping mode is used for final checkout prior to closure for docking.

Considering the Scanning Laser Radar as the primary sensor on the chaser vehicle, digital computer programs were developed to simulate the target and chaser vehicle motions during certain phases of automatic rendezvous and docking. The two vehicles were put into earth orbit with an altitude of 200 nautical miles. Phase I, reported here, only considers the station keeping and docking closure phases. Station keeping is performed at a range of approximately 1,000 feet and docking closure is from 1,000 feet to 1 foot short of actual docking contact. Filtering of the Scanning Laser Radar observations and generation of minimum fuel-minimum end point error thrusting programs were developed. The digital computer programs, developed to study primarily the laser radar system for automatic docking and rendezvous, accept noisy measurements of range, elevation and azimuth angles. From this, the dynamic filter gives the estimate of state (i.e., position and velocity of each vehicle), assuming the computer has knowledge of the position and velocity of the target vehicle at all times. This estimated state is used to generate thrusts in accordance with station keeping and closure guidance laws. One closure technique involves the maintenance of the chaser within pre-determined range/range-rate criterion and line-of-sight (LOS) rate limits. An altenate technique, involving frequent solution on-board the spacecraft of a minimization equation to obtain the thrust program for specific initial conditions with given minimum energy criterion, has given us excellent results and is fully explained with results in Section 5. The station-keeping criterion is based on thrust-free movements of the chaser under two thrusts; one being the maintenance of the target within the view constraints of the laser radar, the second being the station-keeping tolerance. Thrusting will only occur if either or both constraints are violated.

3.0 OVERALL CONTROL PROBLEM DESCRIPTION

Figure 3.1 is a simplified block diagram of control loop simulation. The Scanning Laser Radar System (SLR) with its range and angle measurements corrupted by noise feeds into a vehicle guidance and control computer. This computer has two sub sections. The recursive filter sub section estimates the State (Position and Velocity) of the Chaser vehicle based on observations by the SLR systems. The coordinate system chosen for this program is a target centered rotating coordinate system. The filter output enters the thrust program and, based upon a typical station-keeping and closure guidance law, ΔV Commands are generated. Through the jet select logic, these commands are turned into a sequence of burn times for specific jets. This data is sent to the Reaction Control System of the Chaser vehicle.

Sections 4, 5, and 6 discuss the station-keeping, closure control and recursive filtering programs generated and make analysis of Simulation results. Detailed outputs of the computer are in appendix A, B and C.

Section 7 discusses the combined filters and control scheme and Figure 3-2 is representative of this scheme.

Section 6 explains inputs to the filter as shown in the Figure 3-2.

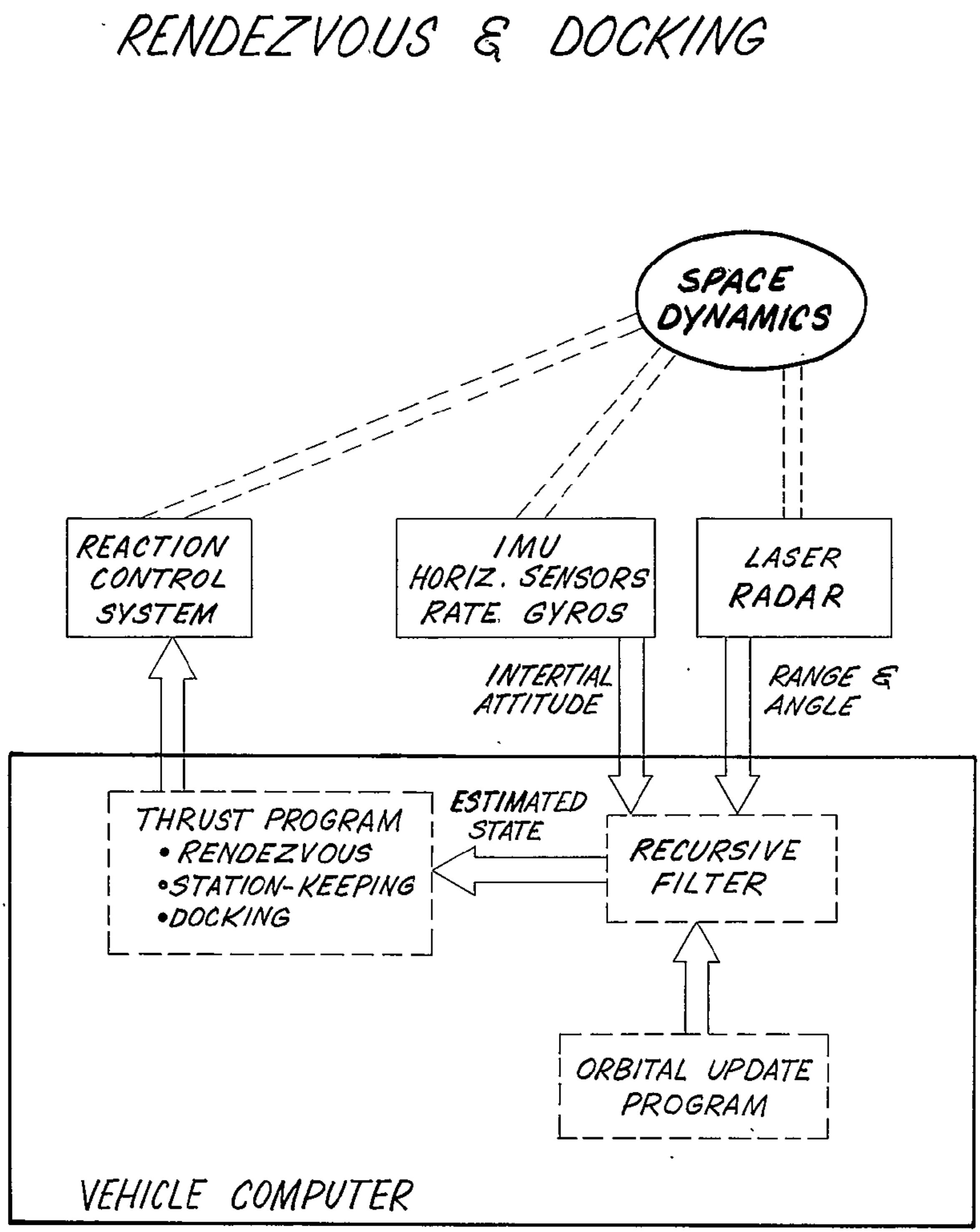


Figure 3-1. Simplified Block Diagram of Control Loop Simulation

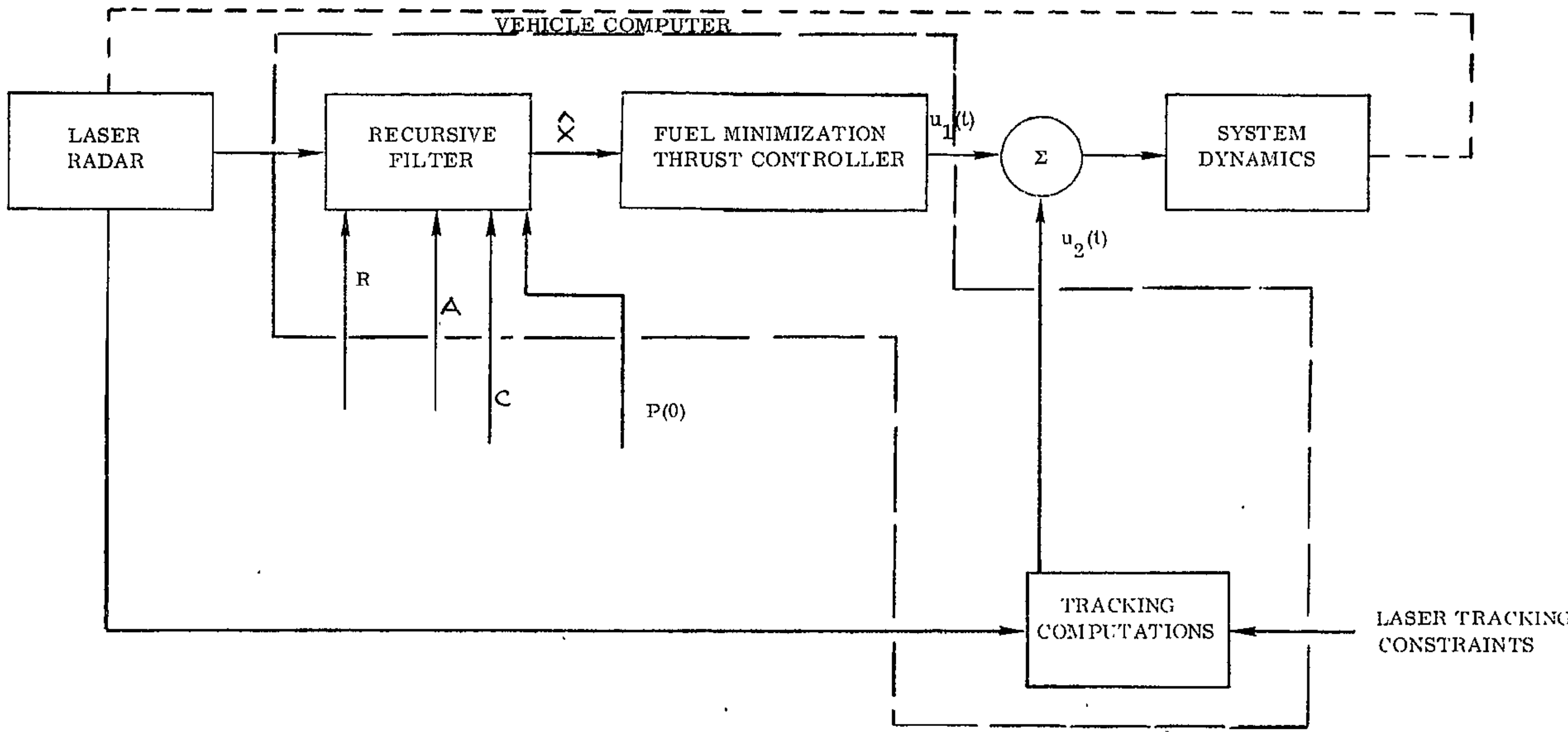


Figure 3-2. System Block Diagram

4.0 STATION KEEPING

4.1 ANALYSIS AND SIMULATION DETAILS

We use the x, y, z rotating frame of reference fixed to target for analysis of station keeping. The target is assumed at 200 n mile circular orbit. The relative equations of motion as shown in Section 5 are

$$\ddot{x} - 2\omega\dot{y} = \frac{T_x}{m} \tag{1}$$

$$\ddot{y} + 2\omega\dot{x} - 3\omega^2 y = \frac{T_y}{m} \tag{2}$$

If the applied thrusts are zero, the equations may be solved to yield $x, y, \dot{x}, \dot{y}$ for given values of $x(o)$, $y(o)$, $\dot{x}(o)$ and $\dot{y}(o)$ and fixed time. Using state notations as shown in Section 5 the state vector of chaser can be written as

$$X(t) = e^{A(t)}\underline{x}(0) + \int_0^t e^{A(t-\tau)} B\underline{u}(\tau)\, d\tau \tag{3}$$

where

$$e^{A(t)}\underline{X}(0) = \begin{bmatrix} 1 & \frac{4}{\omega}\sin\omega t - 3t & -6\sin\omega t + 6\omega t & -\frac{2}{\omega}\cos\omega t + \frac{2}{\omega} \\ 0 & 4\cos\omega t - 3 & -6\omega\cos\omega t + 6\omega & 2\sin\omega t \\ 0 & \frac{2}{\omega}\cos\omega t - \frac{2}{\omega} & -3\cos\omega t + 4 & \frac{\sin\omega t}{\omega} \\ 0 & -2\sin\omega t & 3\omega\sin\omega t & \cos\omega t \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_4(0) \end{bmatrix} \quad (4$$

The objective is to obtain a thrust program for automatic station keeping using as a measure of performance ΔV requirement. Equation (4) was programmed on the computer which gave the chaser state for any time duration for given initial conditions.

From physics considerations, for minimum ΔV, the chaser position should be in front or behind the target on the same orbit.

Due to the laser scanning radar field-of-view limitations, proper thrusting was done to maintain the chaser within a $\pm 15^{\circ}$ cone from the target during the station keeping period of 6000 seconds. Analysis and simulation results show that the laser radar measurement accuracy is the important parameter to minimize ΔV during station keeping. The L. S. R. with its range rate accuracy (3σ value) of $\pm$ .016 ft/sec. can reduce significantly station keeping fuel as discussed in the next subsection. It is noted that in order to maintain the L. S. R. element in the chaser within its own $\pm 15^{\circ}$ field-of-view some fuel will have to be expended for rotational control of the chaser vehicle. The extent of this addition will be looked into during Phase 2 studies of this contract.

4.2 DISCUSSION AND SUMMARY OF RESULTS

Two positions of chaser vehicle both in front and behind the target vehicle were simulated. Charts A-1 to A-6 in appendix cover the results of chaser being 1000 feet in front of target on the x axis.

Results are summarized as follows

a. Chaser initial state

$x = 1000$ feet

$y = 0$ feet

$\dot{x} = .064$ Ft/Sec

$\dot{y} = .064$ Ft/Sec

Δv applied at TR (Running time) $1500 = 2 \times .1356$ ft/sec

Δv applied at TR $3500 = 2 \times .1229$ ft/sec

Δv applied at TR $5300 = 2 \times .1218$ ft/sec

Total Δv for 6000 second period $= 0.760$ ft/sec

B-2 shows that the chaser vehicle is within $\pm 15^{\circ}$ F.O.V. of Target Segment of S. L. R.

b. Chaser initial state

$x = 1000$ feet

$y = 0$ feet

$\dot{x} = .032$ ft/sec

$\dot{y} = .032$ ft/sec

Total Δv for 6000 second period $= .38$ ft/sec

c. Chaser initial state

$x = 1000$ feet

$y = 0$ feet

$\dot{x} = .016$ ft/sec

$\dot{y} = .016$ ft/sec

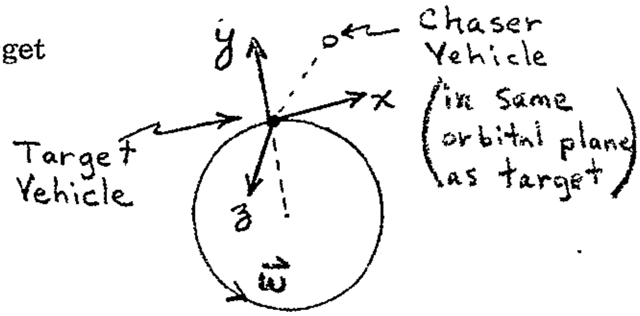
Total Δv for 6000 second period = .19 ft/sec

Charts A-7 to A-12 appendix show similar results for chaser having an initial position of 1000 feet in front of target. Since the S. L. R. has an accurate low range rate measurement capability, station keeping Δv's can be significantly reduced by incorporation of this type S. L. R. Of course for long time station keeping, the savings in fuel are more significant.

5.0 CLOSURE CONTROL

5.1 ANALYSIS

x, y, z is rotating frame of reference fixed to target



$$\vec{\omega} = \sqrt{\frac{g_E r_E^2}{r^3}}\ \vec{z}$$

= The angular rate of x, y, z rotating frame.

x axis - in orbital plane
z axis - normal to the orbital plane
y axis - in orbital plane and along local vertical

g_E = gravitational constant at earth surface

= 32.17 ft/sec^2 (mean value)

r_E = Earth radius = 20.89 x 10^6 feet (mean radius of earth)

r = distance to target from earth center

Assuming target at 200 n. mile circular orbit

r = 22.1052 x 10^6 feet

w = 1.14 x 10^{-3} rad/sec.

The differential equations of motion considering only in-plane motion, are

$$\ddot{x} - 2 w \dot{y} = \frac{T_x}{m} \qquad (1)$$

$$\ddot{y} + 2 w \dot{x} - 3 w^2 y = \frac{T_y}{m} \qquad (2)$$

Here T_x, T_y are applied thrusts along x and y.

If the applied thrust is zero, the right sides of (1) and (2) are zero and the equations may be solved to yield expressions for the relative rate and distance subject to initial values of rate $\dot{x}_o$, $\dot{y}_o$ and distances x_o, y_o. When this is done the rate and position at some later time is

$$\dot{x} = (4\dot{x}_o - 6 y_o w)\cos wt + 2\dot{y}_o \sin wt + 6 w y_o - 3\dot{x}_o$$

$$\dot{y} = (3 y_o w - 2\dot{x}_o)\sin wt + \dot{y}_o \cos wt$$

$$x = 2\left(\frac{2\dot{x}_o}{w} - 3 y_o\right)\sin wt - \frac{2\dot{y}_o}{w}\cos wt + (6 w y_o - 3\dot{x}_o)t + x_o + 2\frac{\dot{y}_o}{w}$$

$$y = \left(\frac{2\dot{x}_o}{w} - 3 y_o\right)\cos wt + \frac{\dot{y}_o}{w}\sin wt - 2\frac{\dot{x}_o}{w} + 4 y_o$$

Let
$$x = x_1 \; ; \quad \dot{x}_1 = \dot{x} = x_2$$
$$\dot{x} = x_2 \qquad \dot{x}_2 = \ddot{x}$$
$$y = x_3 \qquad \dot{x}_3 = x_4$$
$$\dot{y} = x_4 \qquad \dot{x}_4 = \ddot{y}$$

Then the differential equations of motion are

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \ddot{x} = \frac{T x_1}{m} + 2 w x_4$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = \ddot{y} = \frac{T x_3}{m} + 3 w^2 x_3 - 2 w x_2$$

<u>Note</u> "m" is the mass of the chaser vehicle

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2\omega \\ 0 & 0 & 0 & 1 \\ 0 & -2\omega & 3\omega^2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{Tx_1}{m} \\ 0 \\ \frac{Tx_3}{m} \end{bmatrix}$$

$$\underline{\dot{x}} = A\underline{x} + B\underline{u} \qquad (3)$$

where

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \qquad \underline{u} = \begin{bmatrix} 0 \\ \frac{Tx_1}{m} \\ 0 \\ \frac{Tx_3}{m} \end{bmatrix}$$

and

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2\omega \\ 0 & 0 & 0 & 1 \\ 0 & -2\omega & 3\omega^2 & 0 \end{bmatrix}$$

$B = I$ (the identity matrix)

The solution is

$$x_2 = \left(4x_2(0) - 6x_3(0)\omega\right)\cos\omega t + 2x_4(0)\sin\omega t + 6\omega x_3(0) - 3x_2(0)$$

$$x_4 = \left(3x_3(0)\omega - 2x_2(0)\right)\sin\omega t + x_4(0)\cos\omega t$$

$$x_1 = 2\left(\frac{2x_2(0)}{\omega} - 3x_3(0)\right)\sin\omega t - 2\frac{x_4(0)}{\omega}\cos\omega t + \left(6\omega x_3(0) - 3x_2(0)\right)t + x_1(0) + \frac{2x_4(0)}{\omega}$$

$$x_3 = \left(\frac{2x_2(0)}{\omega} - 3x_3(0)\right)\cos\omega t + \frac{x_4(0)}{\omega}\sin\omega t - 2\frac{x_2(0)}{\omega} + 4x_3(0)$$

In matrix notation the solution is

$$\underline{x}(t) = e^{A(t)}\underline{x}(0) + \int_0^t e^{A(t-\tau)} B\,\underline{u}(\tau)\,d\tau \qquad (4)$$

where

$$e^{A(t)}\underline{x}(0) = \begin{bmatrix} 1 & \frac{4}{\omega}\sin\omega t - 3t & -6\sin\omega t + 6\omega t & -\frac{2}{\omega}\cos\omega t + \frac{2}{\omega} \\ 0 & 4\cos\omega t - 3 & -6\omega\cos\omega t + 6\omega & 2\sin\omega t \\ 0 & \frac{2}{\omega}\cos\omega t - \frac{2}{\omega} & -3\cos\omega t + 4 & \frac{\sin\omega t}{\omega} \\ 0 & -2\sin\omega t & 3\omega\sin\omega t & \cos\omega t \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_4(0) \end{bmatrix}$$

The objective is to obtain a thrust program for automatic closure using as a measure of performance ΔV requirements and closure end point errors.

This is equivalent to minimization of the absolute value of the components of $\underline{u}$ (t).

A mathematically convenient performance measure can be the following. We seek to find the u(t) which will minimize

$$\int_0^T \| u(t) \|^2 dt + \| x(T) \|^2 \tag{5}$$

5.2 THE EPSILON TECHNIQUE

Instead of minimizing Equation (5), we use a so-called epsilon technique to solve the optimal control problem. That is, for each $\epsilon > 0$, we seek a $u(\epsilon, t)$ such that

$$J_\epsilon = \frac{1}{2\epsilon} \int_0^T \| \dot{x}(t) - A\,x(t) - B\,u(t) \|^2 dt + \int_0^T \| u(t) \|^2 dt + \| x(T) \|^2 \tag{6}$$

is minimized.

The optimal $u(\epsilon, t)$ must satisfy

$$\nabla_u \left[\| \dot{x}(t) - A\,x(t) - B\,u(t) \|^2 + \| u(t) \|^2 \right] = 0$$

or equivalently,

$$\tilde{u} = (B^* B + 2\epsilon I)^{-1} B^* Y \tag{7}$$

where $Y = \dot{x}(t) - A\,x(t)$

where * denotes the transpose.

With the relation of (7), we want to minimize Equation (6). Substituting $\tilde{u}$ of (7) to (6), we get

$$J_\epsilon = \frac{1}{2\epsilon}\int_0^T \left\{ \| \dot{x}(t) - Ax(t)\|^2 - \left[(\dot{x}(t) - Ax(t)), B\tilde{u}\right]\right\} dt + \|x(T)\|^2 \tag{8}$$

This is because

$$\begin{aligned} J_\epsilon &= \frac{1}{2\epsilon}\int_0^T \|\dot{x} - Ax(t) - B\tilde{u}(t)\|^2 + \int_0^T \|\tilde{u}(t)\|^2 dt + \|x(T)\|^2 \\ &= \frac{1}{2\epsilon}\int_0^T \left\{ \|\dot{x} - Ax\|^2 - 2[\dot{x} - Ax, B\tilde{u}] + [B\tilde{u}, B\tilde{u}] + 2\epsilon[\tilde{u}, \tilde{u}]\right\} dt + \|x(T)\|^2 \end{aligned}$$

notice that

$$\begin{aligned} [B\tilde{u}, B\tilde{u}] + 2\epsilon[\tilde{u}, \tilde{u}] &= [(B^*B + 2\epsilon)\tilde{u}, \tilde{u}] \\ &= [(B^*B + 2\epsilon)\tilde{u}, (B^*B + 2\epsilon)^{-1}B^*(\dot{x} - Ax)] \\ &= [\tilde{u}, B^*(\dot{x} - Ax)] = [\dot{x} - Ax, B\tilde{u}] \end{aligned}$$

$$\Rightarrow J_\epsilon = \frac{1}{2\epsilon}\int_0^T \left\{ \|\dot{x} - Ax\|^2 - [\dot{x} - Ax, B\tilde{u}]\right\} dt + \|x(T)\|^2$$

so our problem is as follows. With

$$\tilde{u} = (B^*B + 2\epsilon I)^{-1} B^*(\dot{x} - Ax)$$

find x (ϵ, t) such that (8) is minimized.

Notice that

$$\begin{aligned} x(T) &= e^{AT}x_0 + \int_0^T e^{A(T-s)} B\tilde{u}(s)\, ds \\ &= e^{AT}x_0 + \int_0^T e^{A(T-s)} B(B^*B + 2\epsilon I)^{-1} B^*(\dot{x} - Ax)\, ds \end{aligned}$$

$$\Rightarrow \nabla_x(\|x(T)\|^2) = -2A^*\int_0^T B(B^*B + 2\epsilon I)^{-1} \cancel{B^*} e^{A^*(T-s)} x(T)\, ds$$

so $$\nabla_x J_\epsilon = \frac{1}{2\epsilon}\left\{ \int_0^T \left[-2A^*(\dot{x} - Ax) + 2A^*B(B^*B + 2\epsilon I)^{-1}B^*(\dot{x} - Ax)\right] dt \right. - 2A^*\int_0^T B(B^*B + 2\epsilon I)^{-1} B^* e^{A^*(T-s)} x(T)\, ds$$

ζ = little t

Notice that this is true for each $\epsilon > 0$

$$\Rightarrow \quad (\dot{x} - Ax) + B(B^*B + 2\epsilon I)^{-1}B^*(\dot{x} - Ax)$$
$$= -2\epsilon B(B^*B + 2\epsilon I)^{-1}B^* e^{A^*(T-t)} x(T)$$

Letting $R_\epsilon = I - B(B^*B + 2\epsilon I)^{-1}B^*$

$$\Rightarrow \quad R_\epsilon(\dot{x} - Ax) = (-2\epsilon) B(B^*B + 2\epsilon I)^{-1}B^* e^{A^*(t-s)} x(T)$$

$$\Rightarrow \quad (\dot{x} - Ax) = (-2\epsilon) R_\epsilon^{-1} B(B^*B + 2\epsilon I)^{-1}B^* e^{A^*(T-s)} x(T) \qquad (9)$$

Notice that

$$e^{A(T-t)}(\dot{x} - Ax) = \frac{d}{dt} e^{A(T-t)} x(t)$$

$$\Rightarrow \int_0^t e^{A(T-s)}(\dot{x} - Ax)\,dt = \int_0^t e^{A(T-s)} \left(\frac{R_\epsilon}{-2\epsilon}\right)^{-1} B(B^*B + 2\epsilon I) B^* e^{A^*(t-s)} x(T)\,ds$$

i.e.

$$e^{A(T-t)} x(t) = e^{AT} x_0 + P(t)\, x(T) \qquad (10)$$

where $P(t) = \int_0^t e^{A(T-s)} \left(\frac{R_\epsilon}{-2\epsilon}\right)^{-1} B(B^*B + 2\epsilon I)^{-1} B^* e^{A^*(t-s)}\, ds \qquad (11)$

for $t = T$ equation (10) becomes

$$x(T) = e^{AT} x_0 + P(T)\, x(T) \qquad (12)$$

Eliminating x_0 from (12) and (10) we obtain

$$\dot{x}(t) - A\,x(t) = \left(\frac{R\epsilon}{-2\epsilon}\right)^{-1} B(B^*B + 2\epsilon I)^{-1} B^* e^{A^*(T-s)} \cdot \left(P(t) - P(T)\right)^{-1} e^{A(T-s)} x(t)$$

Hence finally we get

$$u(t) = (B^*B + 2\epsilon I)^{-1} B^* \left(\frac{R\epsilon}{-2\epsilon}\right)^{-1} B(B^*B + 2\epsilon I)^{-1} B^* \cdot e^{A^*(T-S)} \left(P(t) - P(T)\right)^{-1} e^{A(T-S)} x(t) \tag{13}$$

This is the synthesis form, u(t), and x(t) depend on ϵ. Also in Equation (13)

as $\epsilon \to 0$

$$(B^*B + 2\epsilon)^{-1} B^* \left(\frac{R\epsilon}{-2\epsilon}\right)^{-1} \longrightarrow B^*$$

and

$$\left(\frac{R\epsilon}{-2\epsilon}\right) \longrightarrow BB^*$$

and u(t) converges to

$$u(t) = B^* e^{A^*(T-t)} \left[\int_T^t e^{A(T-s)} BB^* e^{A^*(T-s)} e^{A^*(T-s)}\, ds\right]^{-1} \cdot e^{A(T-s)} x(t) \tag{14}$$

This is the optimal solution for our system. Since B = I in our case, (14) reduces to

$$u(t) = e^{A^*(T-t)}\left[\int_T^t e^{A(T-S)} e^{A^*(T-S)} ds\right]^{-1} e^{A(T-S)} x(t) \qquad (15)$$

so we have the solution for minimizing total energy and end position. We are not minmizing fuel, which is equivalent to min. $\int_0^T |u(t)| dt$. This is because of the difficulty in mathematics to handle absolute value. But we expect the optimal solution of min $\int_0^T |u(t)| dt$ will be very close to what we have got here.

5.3 Implementation

We now consider computer simulation of the closure problem. The procedure is as follows:

We estimate x(o) and compute u(o) by

$$u(0) = e^{-A^*T}\left[\int_0^T e^{A(T-S)} e^{A^*(T-S)} ds\right]^{-1} e^{AT} x(0)$$

Because w = 0.00114, a very small number e^{At} is almost linear within a period of 1 second.

So the integration can be approximated by trapozoidal rule with step size of 1 second. We also can maintain u to be constant within the period of 1 second.

So we have

$$\begin{aligned} u(t) &= u(0) && 0 \le t < 1 \\ u(t) &= u(1) && 1 \le t < 2 \\ &\vdots \\ u(t) &= u(T-1) && T-1 \le t < T \end{aligned}$$

Here we assume T is an integer. So in general, we calculate

$$u(K) = -e^{A^*(T-K)}\left[\int_K^T e^{A(T-S)} e^{A^*(T-S)} ds\right]^{-1} e^{A(T-K)} x(K)$$

and $x(K) = e^{A\cdot 1} x(K-1) + \left[\int_{K-1}^K e^{A(K-\tau)} d\tau\right] u(K-1)$

For $K = 0, 1, 2, \dots, T$.

An APL program is written for this optimal control problem. Several runs have been made on IBM 360/50 computer with different starting positions and closure times. The results are shown in Appendix B.

In the next section, we analyze the excellent results we have obtained.

5.4 EVALUATION AND SUMMARY OF RESULTS

Closure control simulations were performed for closure times (CT) of 900 seconds, 300 seconds and 150 seconds.

1. For the 900 second closure time, the chaser state at time "0" was

   $x = 500$ feet

   $y = -500$ feet

   $\dot{x} = -.6$ ft/sec

   $\dot{y} = .6$ ft/sec

   The first eight charts (Appendix B-1 to B-8) are the results for this case.

   Final value

   $$x = .000376 \text{ feet}$$

   $$\dot{x} = -.000623 \text{ ft/sec}$$

   $$y = -.000289 \text{ feet}$$

   $$\dot{y} = .00374 \text{ ft/sec}$$

   $\Delta V < 1.9$

   B-1 shows x vs TR where TR stands for running time

   B-2 shows y vs TR

   B-3 shows y vs x

   B-4 shows $\dot{x}$ vs TR

   B-5 shows $\dot{y}$ vs TR

   B-6 shows $\frac{Tx}{m}$ vs TR

B-7 shows $\frac{Ty}{m}$ vs TR

B-8 shows DELV (ft/sec) for the closure operation

Some comments are in order. Of all the runs this run is perhaps the best and indicative of the effectiveness of the optimal control program. As B-1 (x vs TR) and B-2 (y vs TR) indicate the last hundred seconds, the approach trajectory is a very desirable one. Also as B-3 indicates, the approach is almost within the $\pm 15^{o}$ cone from the target. Perhaps one or two rotational adjustments by the chaser may only be necessary. Further as evident from our Kalman filter simulations results, the combined filter and sub optimal control program will degrade the end point error to the extent of the 3σ values of the range and range rate accuracy of the Scanning Laser Radar. Additional simulation results in this area will come as part of Phase 2 studies.

As B-8 shows, we achieve a Δv in the area of 2 ft/sec for this 15 minute optimal control run.

2. For the 300 second closure time, two starting points were considered. Curves B-9 to B-24 in the Appendix tell all the story. However, the following data is a summary.

a. $x = -500$ feet

$y = -600$ feet

$\dot{x} = -.032$ ft/sec

$\dot{y} = -1.2$ ft/sec

Final value

$x = -.0067$ feet

$\dot{x} = 0.0434$ ft/sec

$y = -.0084$ feet

$\dot{y} = .0381$ ft/sec

$\Delta v \approx 14$

b. $x = 1000$ feet

$y = -1000$ feet

$\dot{x} = -.6$ ft/sec

$\dot{y} = .6$ ft/sec

Final value

$x = .0088$ feet

$\dot{x} = -.046$ ft/sec

$y = -.010$ feet

$\dot{y} = .0738$ ft/sec

$\Delta v \approx 18.7$

3. For the 150 second closure time, 14 starting points were considered. Curves B-25 to B-136 contain a wealth of data. The following summary is in order

a. $x = 500$ feet

$y = 0$ feet

$\dot{x} = 0.6$ ft/sec

$\dot{y} = -.032$ ft/sec

Final value

$x = .0213$ feet

$y = -.133$ ft/sec

$\dot{x} = -.0009$ feet

$\dot{y} = .0162$ ft/sec

$\Delta v \approx 12$

b. $x = 500$ feet

$y = 500$ feet

$\dot{x} = .6$ ft/sec

$\dot{y} = -.6$ ft/sec

Final value

$x = .0218$ feet

$y = -.1467$ ft/sec

$x = .0173$ feet

$y = -.1032$ ft/sec

$\Delta v \approx 20$

c. $x = -500$ feet

$y = 0$ feet

$\dot{x} = 1.2$ ft/sec

$\dot{y} = 1.2$ ft/sec

Final value

$x = -.0156$ feet

$y = .1086$ ft/sec

$\dot{x} = .0036$ feet

$\dot{y} = -.0267$ ft/sec

$\Delta v \approx 10$

d. $x = -500$ feet

$y = -500$ feet

$\dot{x} = -.6$ ft/sec

$\dot{y} = -.6$ ft/sec

Final value

$x = -.0223$ feet

$\dot{x} = .1493$ ft/sec

$y = -.0209$ feet

$\dot{y} = .1179$ ft/sec

$\Delta v \approx 22.5$

e. $x = -500$ feet

$y = +500$ feet

$\dot{x} = .6$ ft/sec

$\dot{y} = -.6$ ft/sec

Final value

$x = -.0174$ feet

$\dot{x} = .1053$ ft/sec

$y = .0184$ feet

$\dot{y} = -.1322$ ft/sec

$\Delta v \approx 18.7$

f. $x = 500$ feet

$y = -500$ feet

$\dot{x} = -.6$ ft/sec

$\dot{y} = .6$ ft/sec

Final value

$x = .0174$ feet

$\dot{x} = -.0153$ ft/sec

$y = -.0184$ feet

$\dot{y} = .1322$ ft/sec

$\Delta v \approx 18.7$

g. $x = 0$ feet

$y = -1000$ feet

$\dot{x} = 0$ ft/sec

$\dot{y} = 0$ ft/sec

Final value

$x = .0014$ feet

$\dot{x} = -.0291$ ft/sec

$y = .0398$ feet

$\dot{y} = -.2528$ ft/sec

$\Delta V \approx 22$

h. $x = -1000$ feet

$y = 1000$ feet

$\dot{x} = 1.2$ ft/sec

$\dot{y} = -1.2$ ft/sec

Final value

$x = -.0349$ feet

$\dot{x} = .2107$ ft/sec

$y = .0369$ feet

$\dot{y} = -.2645$ ft/sec

$\Delta V \approx 37.5$

i. $x = -1000$ feet

$y = -1000$ feet

$\dot{x} = .032$ ft/sec

$\dot{y} = -1.2$ ft/sec

Final value

$x = -.0410$ feet

$\dot{x} = .2834$ ft/sec

$y = -.0423$ feet

$\dot{y} = .2386$ ft/sec

$\Delta v \approx 42$

j. $x = -1000$ feet

$y = 0$ feet

$\dot{x} = 0$ ft/sec

$\dot{y} = .032$ ft/sec

Final value

$x = -.0392$ feet

$\dot{x} = .2520$ ft/sec

$y = .0012$ feet

$\dot{y} = -.0294$ ft/sec

$\Delta v \approx 22$

k. $x = 1000$ feet

$y = -1000$ feet

$\dot{x} = -.6$ ft/sec

$\dot{y} = +.6$ ft/sec

Final value $x = .0364$ ft.

$\dot{x} = -.2168$ ft/sec

$y = -.0389$ ft.

$\dot{y} = .2732$ ft/sec

$\Delta v \approx 38$

l. $x = 1000$ feet

$y = 1000$ feet

$\dot{x} = -.6$ ft/sec

$\dot{y} = -.6$ ft/sec

Final value $x = .038$ ft

$\dot{x} = -.2725$ ft/sec

$y = .0372$ ft.

$\dot{y} = -.2177$ ft. sec

$\Delta v \approx 38$

6.0 KALMAN FILTER

6.1 ANALYSIS AND SIMULATION DETAILS

The laser scanning radar measures range, azimuth and elevation angles between the chaser and the target. In terms of the coordinates chosen in Section 5.0 they are $(x_1^2 + x_3^2 + x_5^2)^{\frac{1}{2}}$, $\tan^{-1} \frac{-x_3}{-x_1}$ and $\sin^{-1} \frac{-x_5}{\left(x_1^2 + x_3^2 + x_5^2\right)^{\frac{1}{2}}}$

respectively. We recall that the coordinate system was target centered with the y or x_3 direction along the target local vertical, the x_5 or z direction is aligned with the orbit normal and the x or x_1 direction is in the orbit plane. Since we assume both chaser and target are on the same plane the laser scanning radar measures only range and azimuth angles. For analysis purposes the nonlinear measurement (in terms of x_1 and x_3) is rather cumbersome. Hence assume for each measurement, we can make a transformation to obtain the observation values of x_1 and x_3. Then we have the following observation equation

$$\begin{bmatrix} Z_1(t) \\ Z_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} N_1(t) \\ N_2(t) \end{bmatrix}$$

Our system model is then

$$\dot{x}(t) = A\, x(t) \tag{1}$$

$$\underline{z}(t) = C\; \underline{x}(t) + \mathbf{N}(t) \tag{2}$$

where A is specified in Section 5 and

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Since laser scanning radar takes discrete observations, our observation model is then

$$z\,(n\tau) = c\; x(n\tau) + N\,(n\tau)$$

and

$$\left[N(n\tau)\; N^{T}(K\tau)\right] = \begin{bmatrix} 0.011959\,\delta_n^K & 0 \\ 0 & 0.011959\,\delta_n^K \end{bmatrix}$$

The above noise convariance matrix is obtained using the value of 10 cm's as the three sigma position accuracy of laser scanning radar. Of course, once the transformation box referred to previously has been built, the convariance matrix of the noise N vector is obtained from experiments.

The laser statistics for the Kalman recursive filter is as follows

Range	0 - 120 km (75 miles)
Range accuracy (3σ)	$\pm$ 0.02% or $\pm$ 10 cm (whichever is greater)
Range rate	0-5 km/sec. (11,200 mph)
Range rate accuracy	$\pm$1.0% or $\pm$0.5 cm/sec. (whichever is greater)

σ here denotes the standard deviation and σ^2 is the variance.

The well-known Kalman-Bucy filter is utilized in the estimation of chaser position and velocities (i.e., coordinates x_1, x_2, x_3 and x_4) from noisy measurements azimuth and range by the laser scanning radar. The equations are

$$\dot{\hat{x}} = A\,\hat{x}(t) + K(t)\left[z(t) - c\,\hat{x}(t)\right]$$

$$K(t) = P(t)\,C^T R^{-1}(t)$$

$$\dot{P}(t) = A\,P(t) + P(t)\,A^T - K(t)\,C\,P(t)$$

with $\hat{x}(0)$ known, which is a small variation of the true value $x(0)$.

For Kalman filter mechanization we choose the following

$P(0) = P_0$ matrix

$$P_0 = \begin{bmatrix} .012 & 0 & 0 & 0 \\ 0 & .012\times10^{-2} & 0 & 0 \\ 0 & 0 & .012 & 0 \\ 0 & 0 & 0 & .012\times10^{-2} \end{bmatrix}$$

Since observation is discrete, we can select the observation period τ to be the integration step time in the Kalman recursive filter, which is taken as 0.1 sec.

Our Algorithm for Kalman filter is then

1. Calculate R^{-1}, where $R = E\left[N(K\tau)\,N^T(K\tau)\right]$

$$R^{-1} = \begin{bmatrix} 83.6127 & 0 \\ 0 & 83.6127 \end{bmatrix}$$

2. Calculate

$$z(o) = C\,x(o) + N(o)$$

$$K(0) = P(0)\,C^T R^{-1}$$

$$\dot{\hat{x}}(0) = A\,\hat{x}(0) + K(0)\left[z(0) - C\,\hat{x}(0)\right]$$

$$\dot{P}(0) = A\,P(0) + P(0)\,A^T - K(0)\,C\,P(0)$$

3. Calculate

$$\hat{x}(\tau) = \hat{x}(0) + \dot{\hat{x}}(0) + [\Delta T]$$

where $\Delta T = \tau = 0.1$ seconds

$$P(\tau) = P(0) + \dot{P}(0)\,[\Delta T]$$

$$K(\tau) = P(\tau)\,C^T R^{-1}$$

$$\dot{\hat{x}}(\tau) = A\,\hat{x}(\tau) + K(\tau)\left[z(\tau) - C\,\hat{x}(\tau)\right]$$

$$\dot{P}(\tau) = A\,P(\tau) + P(\tau)\,A^T - K(\tau)\,C\,P(\tau)$$

We proceed for $t = 2\tau, 3\tau, \ldots K\tau$ where $K\tau = T$, the final time for observation.

The simulation results are given in Appendix C. Discussions of the results follow.

6.2 DISCUSSION OF FILTER SIMULATION RESULTS

Charts C-1 to C-10 in the appendix show the effectiveness of the filter for various initial conditions of the Chaser vehicle .

a. Estimated State at time o

$$\hat{x}_1 = 1000.4 \text{ feet}$$

$$\hat{x}_2 = -1.02 \text{ ft/sec}$$

$$\hat{x}_3 = 1000.4 \text{ feet}$$

$$\hat{x}_4 = -1.02 \text{ feet}$$

True state at time "0" which was fed into the laser error model

$$x_1 = 1000 \text{ feet}$$

$$x_2 = -1 \text{ ft/sec}$$

$$x_3 = 1000 \text{ feet}$$

$$x_4 = -1 \text{ ft/sec}$$

Curve C_2 and C_3 shows the convergence of $\hat{x}$ and $\hat{y}$ to the true x and y in about 5 observations. Curves C_4 and C_5 show plots of $\hat{\dot{x}}$ and $\hat{\dot{y}}$ in relation to true values of $\dot{x}$ and $\dot{y}$. The convergence is slower than the range convergence since $\dot{x}$ and $\dot{y}$ are not observed variables.

b. Estimated State at time " 0 "

$$\hat{x}_1 = 500.4 \text{ feet}$$

$$\hat{x}_2 = +1.02 \text{ feet/sec}$$

$$\hat{x}_3 = 500.4 \text{ feet}$$

$$\hat{x}_4 = +1.02 \text{ feet/sec}$$

True state at time "0" which was fed into the laser error model

$$x_1 = 500 \text{ feet}$$

$$x_2 = 1 \text{ feet/sec}$$

$$x_3 = 500 \text{ feet}$$

$$x_4 = 1 \text{ feet/sec}$$

Curves C-6 to C-10 show similar results as case "a". In this case more observations are needed since initial estimate is further apart from true values than in the case "a".

7.0 COMBINED FILTERING AND CONTROL

7.1 ANALYSIS

The optimal control as carried out in section 5 is a continous thrust control which was used to obtain theoretical limit valves of ΔV for various closure times. When we use constant magnitude thrusters, we design a sub optimal control in the following manner. The change we make is to fix the amplitude of thrusts according to the design valve and to vary the thrust time length.

Let $\tilde{u}$ be the constant thrust force. From section 5, we know the optimal $u(t)$, $k \leq t \leq k+1$, is

$$u(t) = -e^{A^*(T-K)}\left[\int_K^T e^{A(T-s)} e^{A^*(T-s)}\, ds\right]^{-1} e^{A(T-K)} x(K$$

which is a 4 x 1 vector. Let

$$u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \end{bmatrix}$$

The sub optimal design involves the selection of the following:

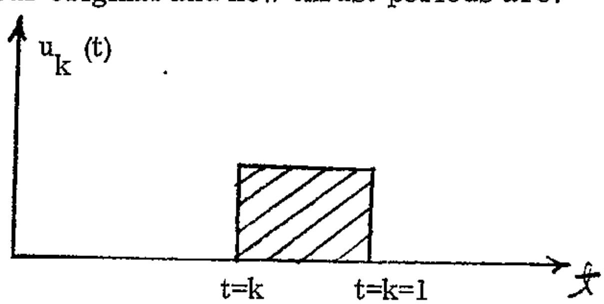
$$t_1(K) = \frac{|u_1|}{\tilde{u}}$$

$$t_2(K) = \frac{|u_2|}{\tilde{u}}$$

$$t_3(K) = \frac{|u_3|}{\tilde{u}}$$

$$t_4(K) = \frac{|u_4|}{\tilde{u}} \qquad \text{where } K = 0,1,2,\ldots(T-1$$

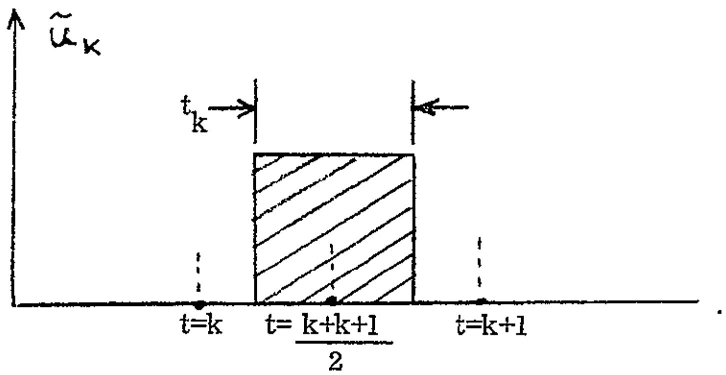
Then our original and new thrust periods are:



$k = 1,2,3,4$

$k = 0,1,2.\ldots,T-1$

optimal thrust in coordinate X_k



k = 1,2,3,4

k = 0,1,2,....,T-1

Sub optimal thrust in coordinate X_k

It should be noted that u_1 (*) and u_3 (*) correspond to impulses of velocity in X_1 and X_3 directions, where as u_2 (*) and u_3 (*) correspond to $\frac{Tx_1}{m}$ and $\frac{Tx_3}{m}$ which have units of acceleration (Ft/sec^2). Since the optimal control program, calls for thrust input to be constant for one second, acceleration integrated over one second gives us velocity change. Simulation results show for ~~clusre~~ closure time of 900 seconds, the u_1 (*) and u_3 (*) values are negligible compared to u_2 (*) and u_4 (*).

The sub optimal program is being tested. The results will be included in the Phase 2, [redacted].

7.2 Implementation

The following steps explain how sub optimal control program and Kalman filtering program are combined.

1. At t = 0, we calculate the optimal control u (0). Using this and $\tilde{u}$ we calculate t_k (0), k = 1,2,3,4.

2. $$\tilde{u}(t) = \begin{bmatrix} \tilde{u}_1(t) \\ \tilde{u}_2(t) \\ \tilde{u}_3(t) \\ \tilde{u}_4(t) \end{bmatrix} = 0 \quad \text{for} \begin{cases} 0 \le t \le \frac{1-t_1}{2}, & \frac{1+t_1}{2} \le t \le 1 \\ 0 \le t \le \frac{1-t_2}{2}, & \frac{1+t_2}{2} \le t \le 1 \\ 0 \le t \le \frac{1-t_3}{2}, & \frac{1+t_3}{2} \le t \le 1 \\ 0 \le t \le \frac{1-t_4}{2}, & \frac{1+t_4}{2} \le t \le 1 \end{cases}$$

and

$$\tilde{u}(t) = \begin{bmatrix} \tilde{u} \cdot \mathrm{sign}(u_1) \\ \tilde{u} \cdot \mathrm{sign}(u_2) \\ \tilde{u} \cdot \mathrm{sign}(u_3) \\ \tilde{u} \cdot \mathrm{sign}(u_4) \end{bmatrix} \quad \text{for} \begin{cases} \frac{1-t_1}{2} \le t \le \frac{1+t_1}{2} \\ \frac{1-t_2}{2} \le t \le \frac{1+t_2}{2} \\ \frac{1-t_3}{2} \le t \le \frac{1+t_3}{2} \\ \frac{1-t_4}{2} \le t \le \frac{1+t_4}{2} \end{cases}$$

where sign (u_k) =+1 if u_k is positive and sign (u_k) = −1 if u_k is negative.

3. When thrust is ended, Kalman filter is on till $t=1$ second. Based on estimated value of $\underline{X}$ (1), we do 1,2 for $1 \le t \le 2$ again.

4. Do 1,2,3 for $k \le t \le k+1$ until $k = T-1$ seconds. An important remark should be made, that in this type of sub optimal combined filter and thrust control program, for t=k, we treat the problem as if it were started at t = k seconds so that the accumulated error won't propagate (ie, in other words we determine the control u, based on latest information). Errors such as thrust end point etc., can conveniently be compensated by this technique.

The complete combined program will come out in the Phase 2 report.

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APPENDIX A

RESULTS OF STATION KEEPING SIMULATIONS

STATION-KEEPING

```
      SK1
INPUT X:
⎕:
       x     ẋ    y   ẏ
      1000 .064 0 .064
```

[Chaser behind Target]

Chaser State at TR = 1500

```
1062.249335
 ¯0.10075933
¯72.26286404
 ¯0.1356421212
```

Chaser State at TR = 3500

```
_1.131927732E3
¯4.554625260E¯2
¯4.804660202E1
¯1.229545646E¯1
```

Chaser State at TR = 5300

```
_1.215757153E3
_¯4.180229614E¯2
_¯4.640451585E1
_¯1.218202054E¯1
```

Chaser State at TR = 1500
after Δv in the y direction

```
1062.249335
 ¯0.10075933
¯72.26286404
  0.1356421212
```

Chaser State at TR = 3500
after Δv in the y direction

```
_1.131927732E3
¯4.554625260E¯2
¯4.804660202E1
 1.229545646E¯1
```

Chaser State at TR = 5300
after Δv in the y direction

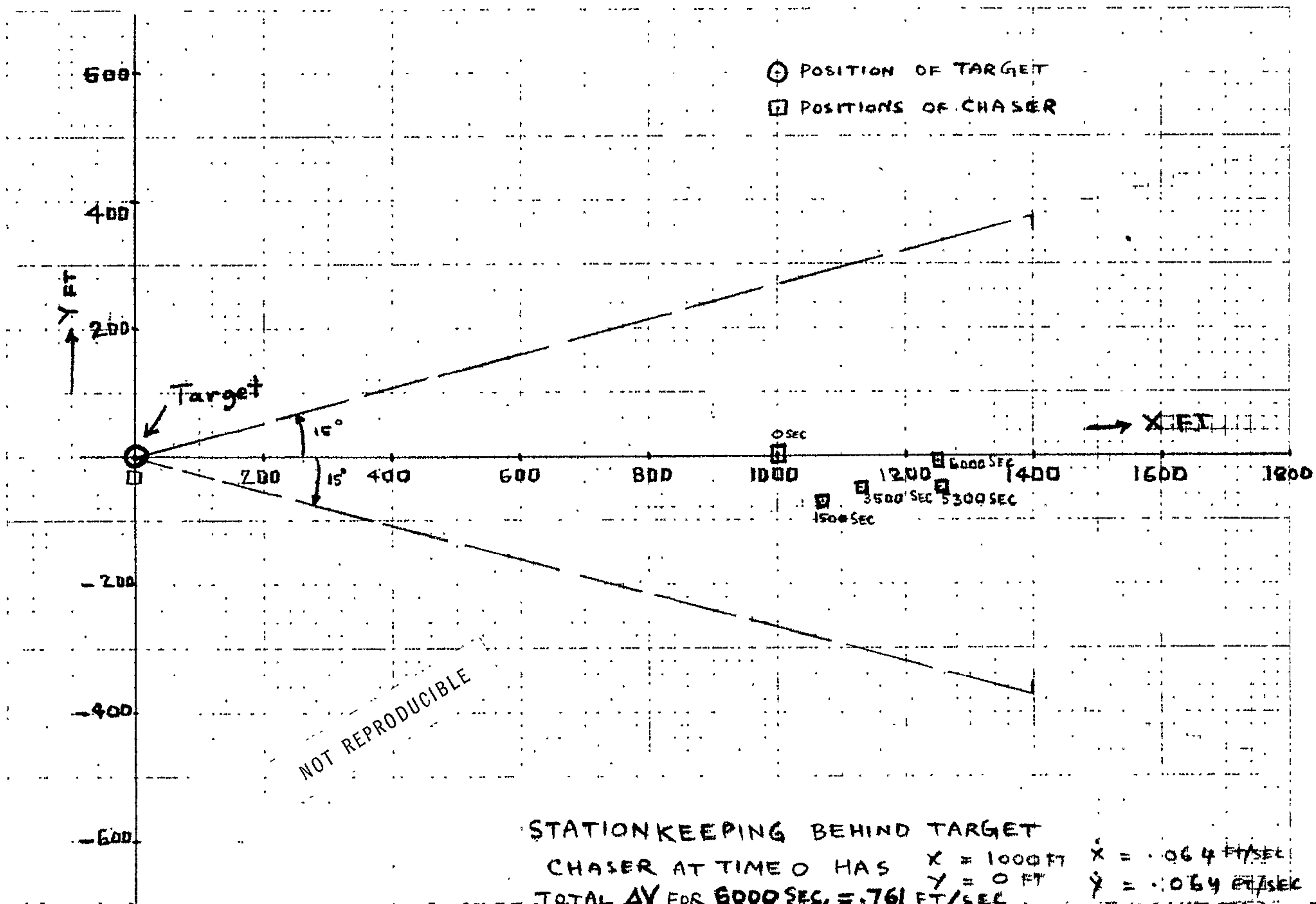
```
_1.215757153E3
¯4.180229614E¯2
¯4.640451585E1
 1.218202054E¯1
```

Chaser State at TR = 6000

```
1.240200155E3
8.729612056E¯2
1.021759673E1
3.127969406E¯2
```

Δ v for Period 6000 Seconds (Ft/sec)

```
0.7608337824
```



STATIONKEEPING BEHIND TARGET

CHASER AT TIME 0 HAS $X = 1000$ FT, $Y = 0$ FT, $\dot{X} = .064$ FT/SEC, $\dot{Y} = .064$ FT/SEC

TOTAL ΔV FOR 6000 SEC. = .761 FT/SEC

```
      SK1
INPUT X:
⎕:
        X     Ẋ   Y   Ẏ
      1000 .032 0 .032
```

[Chaser behind Target]

Chaser State at TR = 1500

```
1.031124668E3
¯5.037966501E¯2
¯3.613143202E1
¯6.782106058E¯2
```

Chaser State at TR = 3500

```
1.065963866E3
¯2.277312630E¯2
¯2.402330101E1
¯6.147728232E¯2
```

Chaser State at TR = 5300

```
1.107878576E3
¯2.090114807E¯2
¯2.320225792E1
¯6.091010269E¯2
```

Chaser State at TR = 1500 after Δv in the y direction

```
1.031124668E3
¯5.037966501E¯2
¯3.613143202E1
6.782106058E¯2
```

Chaser State at TR = 3500 after Δv in the y direction

```
1.065963866E3
¯2.277312630E¯2
¯2.402330101E1
6.147728232E¯2
```

Chaser State at TR = 5300 after Δv in the y direction

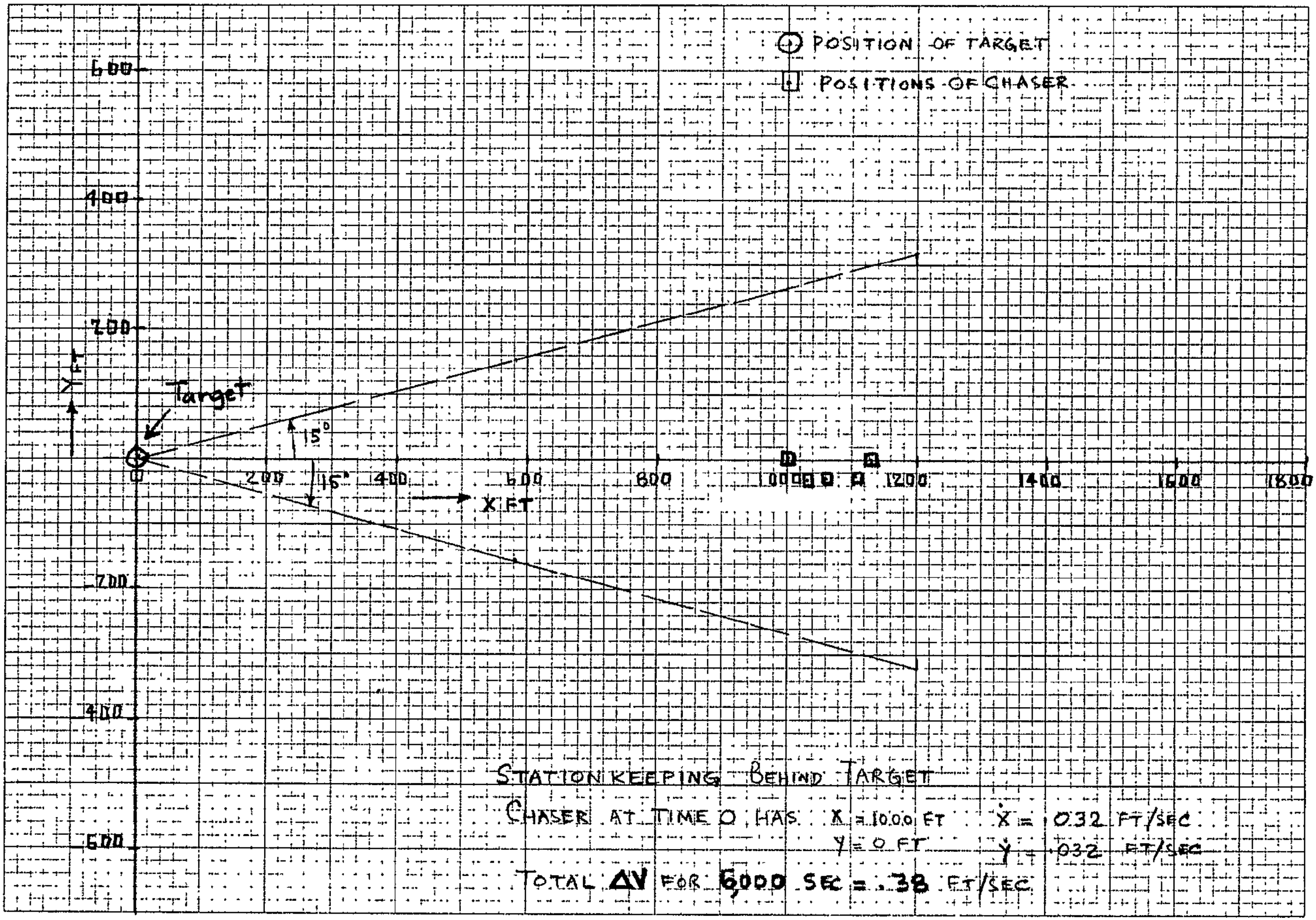
```
1.107878576E3
¯2.090114807E¯2
¯2.320225792E1
6.091010269E¯2
```

Chaser State at TR = 6000

```
1.120100077E3
4.364806028E¯2
5.108798367E0
1.563984703E¯2
```

Δv for Period 6000 Seconds (Ft/sec)

```
0.3804168912
```


POSITION OF TARGET
POSITIONS OF CHASER
600
400
200
Y FT
Target
15°
15°
0
200
400
600
800
1000
1200
1400
1600
1800
X FT
200
400
600
STATIONKEEPING BEHIND TARGET
CHASER AT TIME 0 HAS X = 1000 FT Ẋ = .032 FT/SEC
Y = 0 FT Ẏ = .032 FT/SEC
TOTAL ΔV FOR 6,000 SEC = .38 FT/SEC

SK1
INPUT X:
⎕:

x	ẋ	y	ẏ
1000	.016	0	.016

[Chaser behind Target]

Chaser State at TR = 1500

```
 1.015562334E3
¯2.518983250E¯2
¯1.806571601E1
¯3.391053029E¯2
```

Chaser State at TR = 3500

```
 1.032981933E3
¯1.138656315E¯2
¯1.201165050E1
¯3.073864116E¯2
```

Chaser State at TR = 5300

```
 1.053939288E3
¯1.045057403E¯2
¯1.160112896E1
¯3.045505134E¯2
```

Chaser State at TR = 1500 after Δv in the y direction

```
 1.015562334E3
¯2.518983250E¯2
¯1.806571601E1
 3.391053029E¯2
```

Chaser State at TR = 3500 after Δv in the y direction

```
 1.032981933E3
¯1.138656315E¯2
¯1.201165050E1
 3.073864116E¯2
```

Chaser State at TR = 5300 after Δv in the y direction

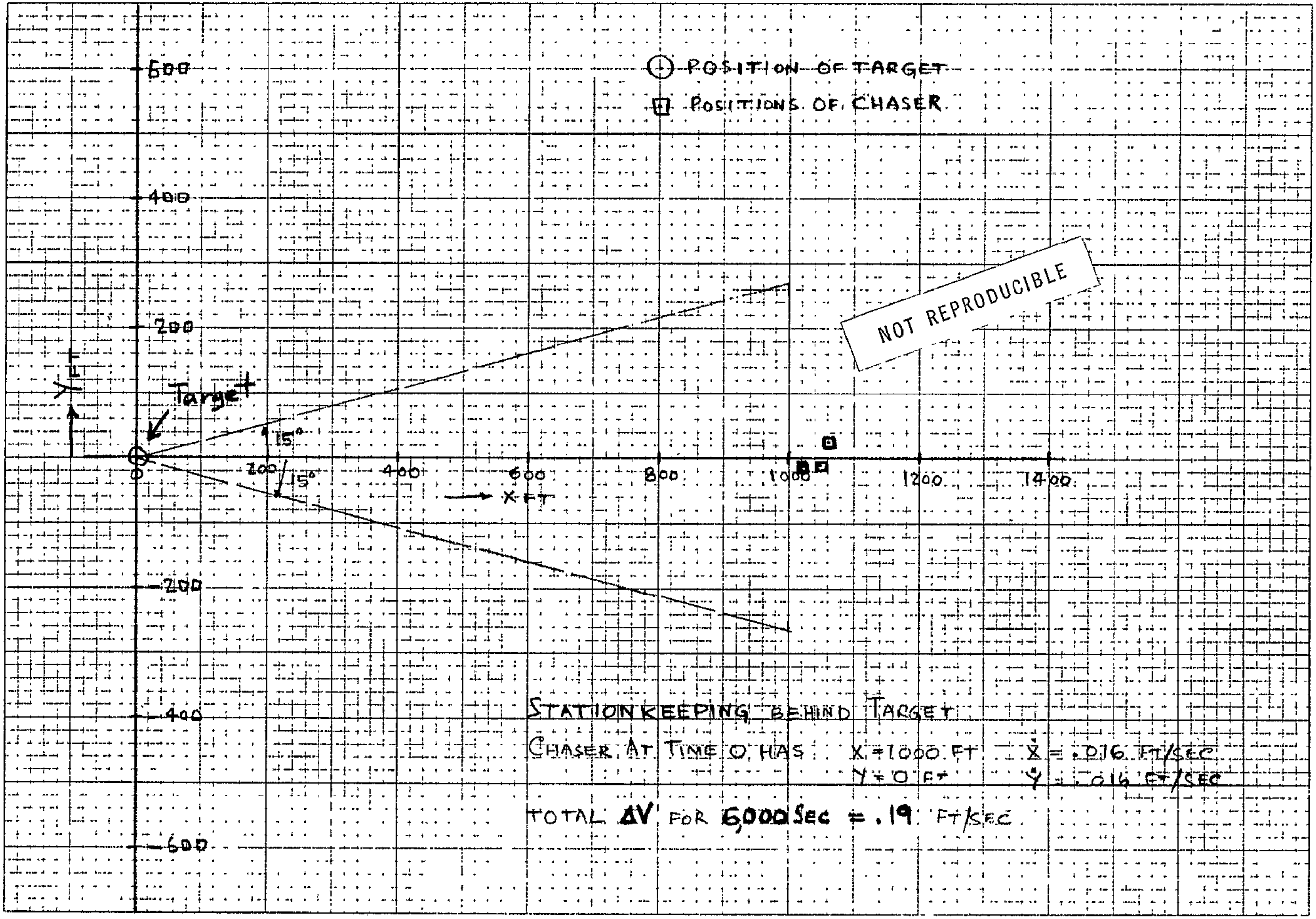
```
 1.053939288E3
¯1.045057403E¯2
¯1.160112896E1
 3.045505134E¯2
```

Chaser State at TR = 6000

```
1.060050039E3
2.182403014E¯2
2.554399184E0
7.819923516E¯3
```

Δv for Period 6000 Seconds (Ft/sec)

0.1902084456


POSITION OF TARGET
POSITIONS OF CHASER
600
400
200
-200
-400
-600
Y FT
Target
15°
15°
0
200
400
600
800
1000
1200
1400
X FT
NOT REPRODUCIBLE
STATIONKEEPING BEHIND TARGET
CHASER AT TIME 0 HAS X = 1000 FT Ẋ = .016 FT/SEC
Y = 0 FT Ẏ = .016 FT/SEC
TOTAL ΔV FOR 6,000 SEC = .19 FT/SEC

```
      SK1
INPUT X:
⎕:      X      Ẋ   Y  Ẏ
      ¯1000 .064 0 .064
```

[Chaser in front of Target]

Chaser State at TR = 1500

```
¯937.7506646
  ¯0.10075933
 ¯72.26286404
  ¯0.1356421212
```

Chaser State at TR = 3500

```
¯868.0722684
  ¯0.0455462526
 ¯48.04660202
  ¯0.1229545646
```

Chaser State at TR = 5300

```
¯784.2428473
  ¯0.04180229614
 ¯46.40451585
   0.1218202054
```

Chaser State at TR = 1500
after Δv in the y direction

```
¯937.7506646
  ¯0.10075933
 ¯72.26286404
   0.1356421212
```

Chaser State at TR = 3500
after Δv in the y direction

```
¯868.0722684
  ¯0.0455462526
 ¯48.04660202
   0.1229545646
```

Chaser State at TR = 5300
after Δv in the y direction

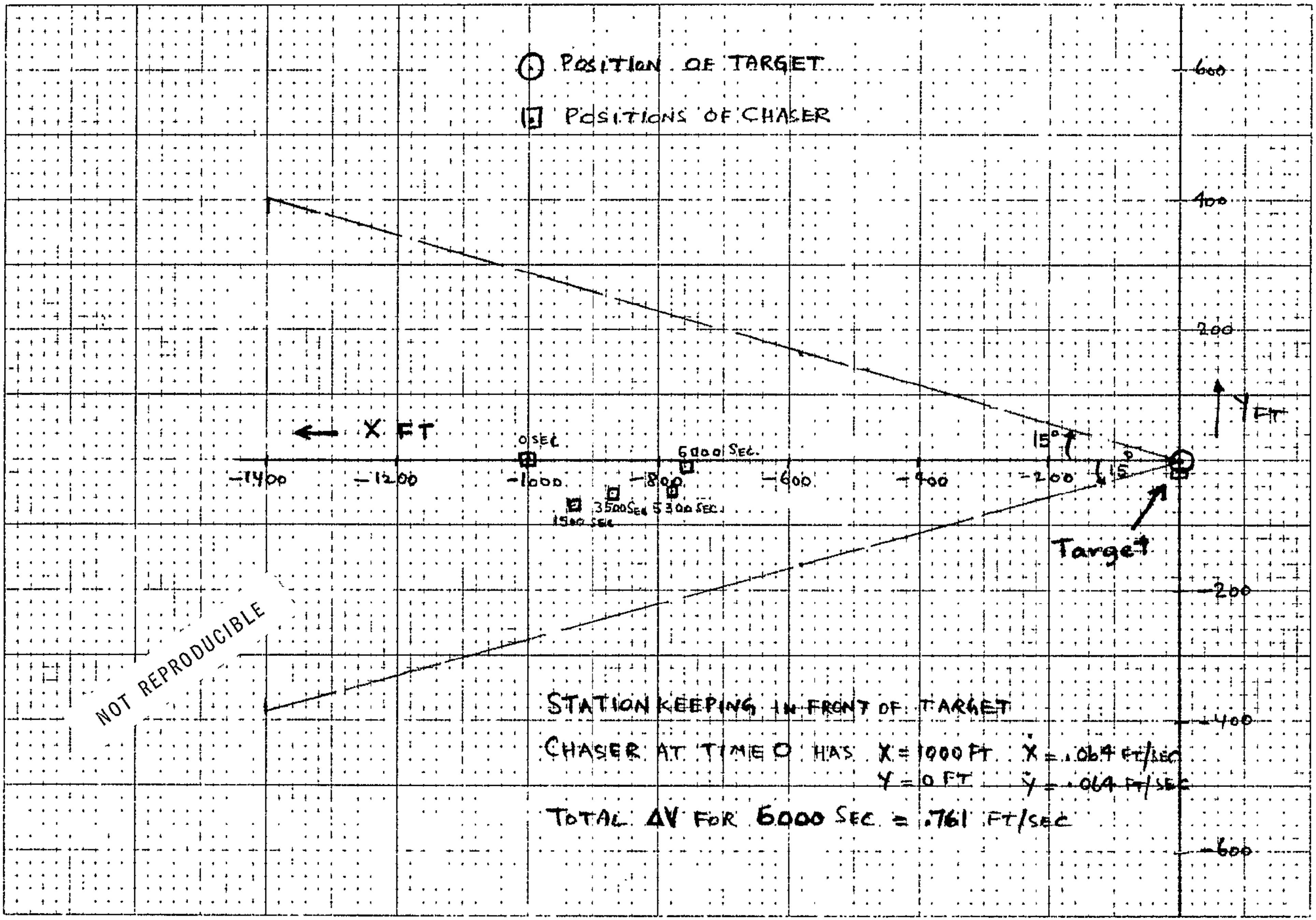
```
¯784.2428473
  ¯0.04180229614
 ¯46.40451585
  ¯0.1218202054
```

Chaser State at TR = 6000

```
¯759.799845
   0.08729612056
  10.21759673
   0.03127969406
```

Δ v for Period 6000 Seconds (Ft/sec)

```
0.7608337824
```


POSITION OF TARGET
POSITIONS OF CHASER
600
400
200
-200
-400
-600
← X FT
Y FT
-1400
-1200
-1000
-800
-600
-400
-200
0 SEC
6000 SEC.
3500 SEC
5300 SEC.
1500 SEC
15°
15°
Target
NOT REPRODUCIBLE
STATION KEEPING IN FRONT OF TARGET
CHASER AT TIME 0 HAS X = 1000 FT   Ẋ = .064 FT/SEC
Y = 0 FT   Ẏ = .014 FT/SEC
TOTAL ΔV FOR 6000 SEC = .761 FT/SEC

ITT

```
      SK1
INPUT X:
⎕:
        x     ẋ    y   ẏ
      ¯1000  .032  0  .032
```

[Chaser in front of Target]

Chaser State at TR = 1500

```
¯968.8753323
  ¯0.05037966501
 ¯36.13143202
  ¯0.06782106058
```

Chaser State at TR = 3500

```
¯934.0361342
  ¯0.0227731263
 ¯24.02330101
  ¯0.06147728232
```

Chaser State at TR = 5300

```
¯892.1214237
  ¯0.02090114807
 ¯23.20225792
  ¯0.06091010269
```

Chaser State at TR = 1500
after Δv in the y direction

```
¯968.8753323
  ¯0.05037966501
 ¯36.13143202
   0.06782106058
```

Chaser State at TR = 3500
after Δv in the y direction

```
¯934.0361342
  ¯0.0227731263
 ¯24.02330101
   0.06147728232
```

Chaser State at TR = 5300
after Δv in the y direction

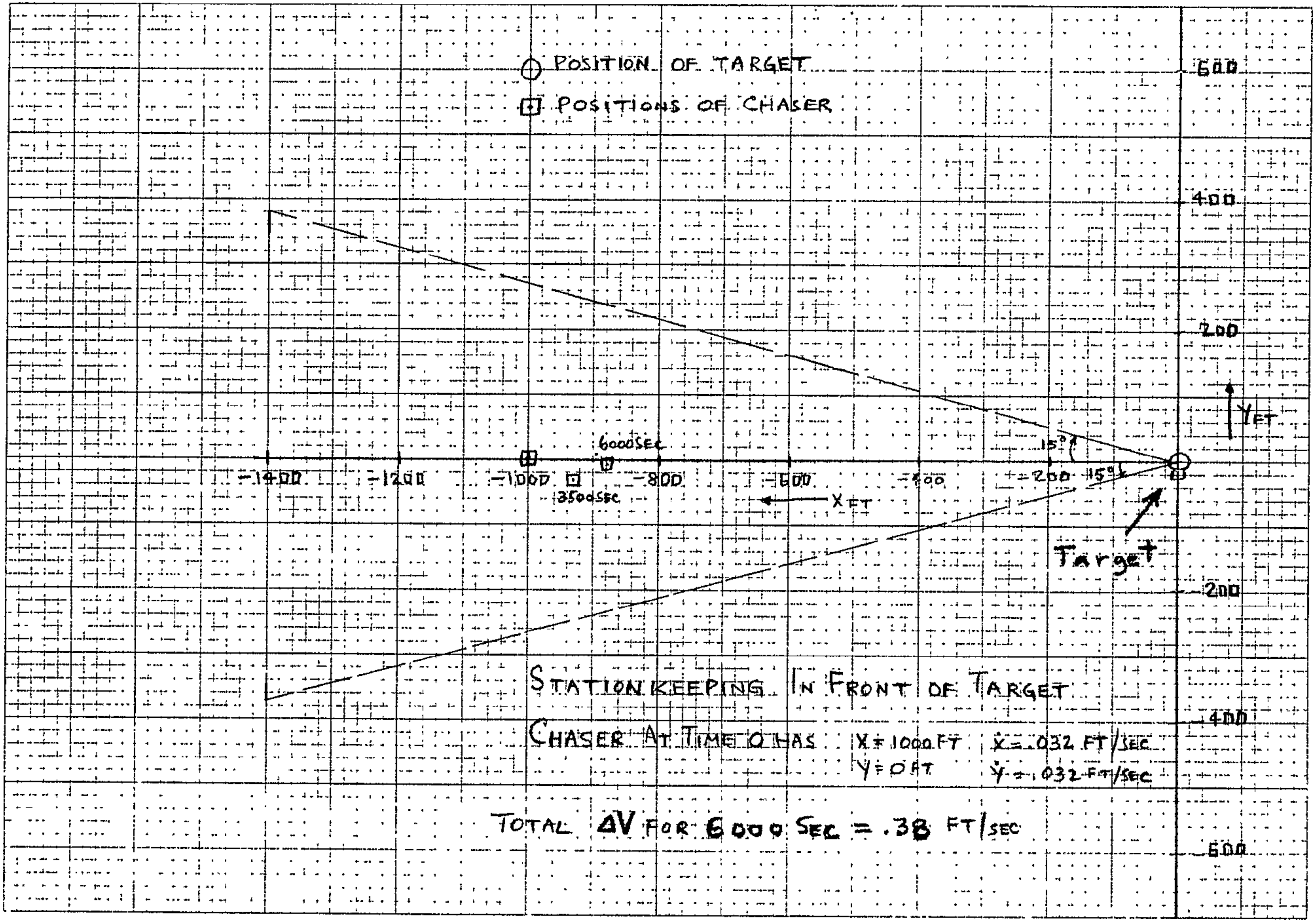
```
¯892.1214237
  ¯0.02090114807
 ¯23.20225792
   0.06091010269
```

Chaser State at TR = 6000

```
¯879.8999225
   0.04364806028
   5.108798367
   0.01563984703
```

Δv for Period 6000 Seconds (Ft/sec)

```
0.3804168912
```


POSITION OF TARGET
POSITIONS OF CHASER
600
400
200
200
400
600
-1400
-1200
-1000
-800
-600
-400
-200
6000SEC
3500SEC
15°
15°
X FT
Y FT
Target
STATION KEEPING IN FRONT OF TARGET
CHASER AT TIME 0 HAS X = 1000 FT Ẋ = .032 FT/SEC
Y = 0 FT Ẏ = .032 FT/SEC
TOTAL ΔV FOR 6000 SEC = .38 FT/SEC

```
      SK1
INPUT X:
⎕:
       X      Ẋ     Y   Ẏ
      ¯1000 .016 0 .016
```

[Chaser in front of Target]

Chaser State at TR = 1500

```
¯984.4376662
  ¯0.0251898325
 ¯18.06571601
  ¯0.03391053029
```

Chaser State at TR = 3500

```
¯967.0180671
  ¯0.01138656315
 ¯12.0116505
  ¯0.03073864116
```

Chaser State at TR = 5300

```
¯946.0607118
  ¯0.01045057403
 ¯11.60112896
  ¯0.03045505134
```

Chaser State at TR = 1500
after Δv in the y direction

```
¯984.4376662
  ¯0.0251898325
 ¯18.06571601
   0.03391053029
```

Chaser State at TR = 3500
after Δv in the y direction

```
¯967.0180671
  ¯0.01138656315
 ¯12.0116505
   0.03073864116
```

Chaser State at TR = 5300
after Δv in the y direction

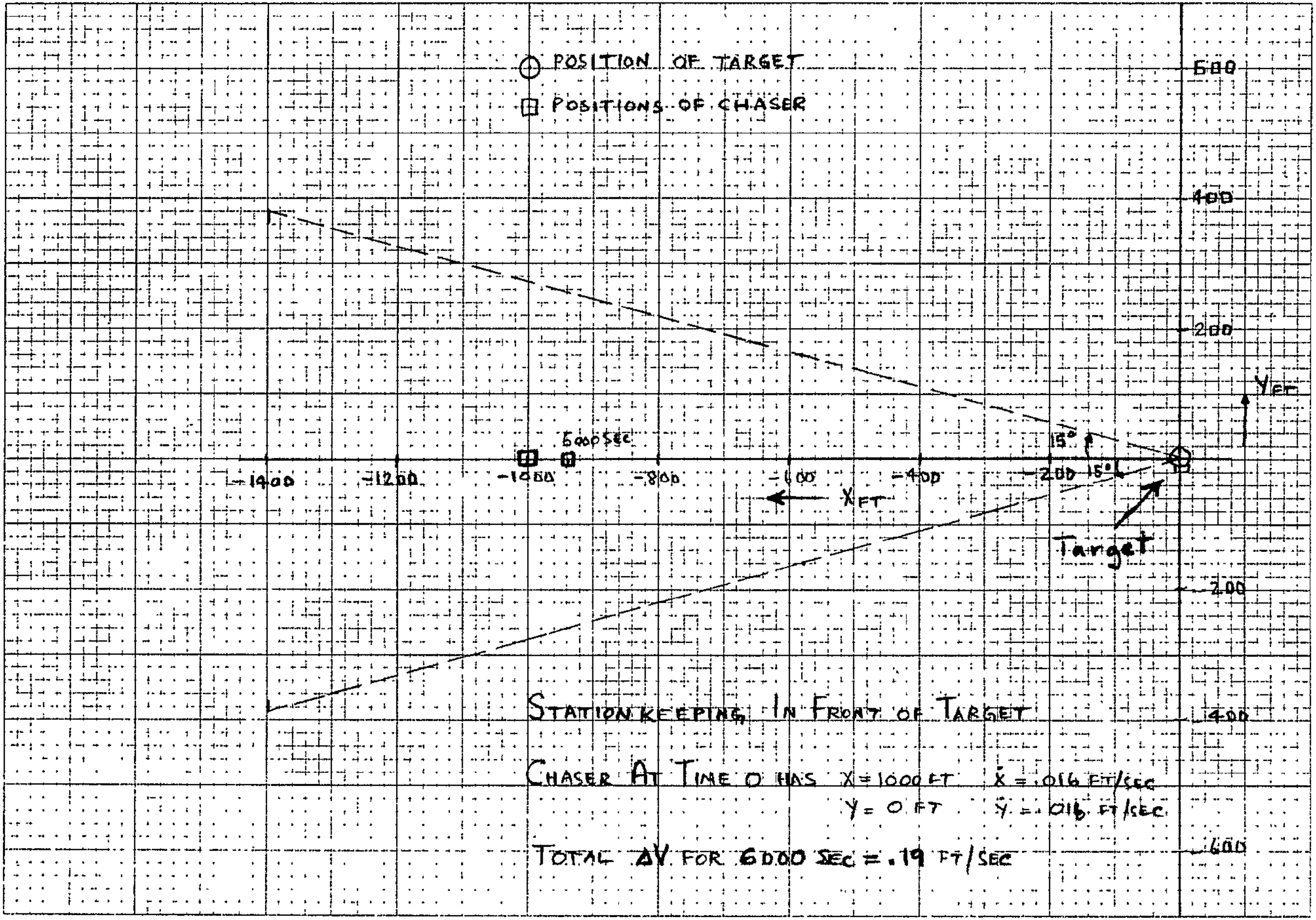
```
¯946.0607118
  ¯0.01045057403
 ¯11.60112896
   0.03045505134
```

Chaser State at TR = 6000

```
¯9.399499613E2
 2.182403014E¯2
 2.554399184E0
 7.819923516E¯3
```

Δv for Period 6000 Seconds (Ft/sec)

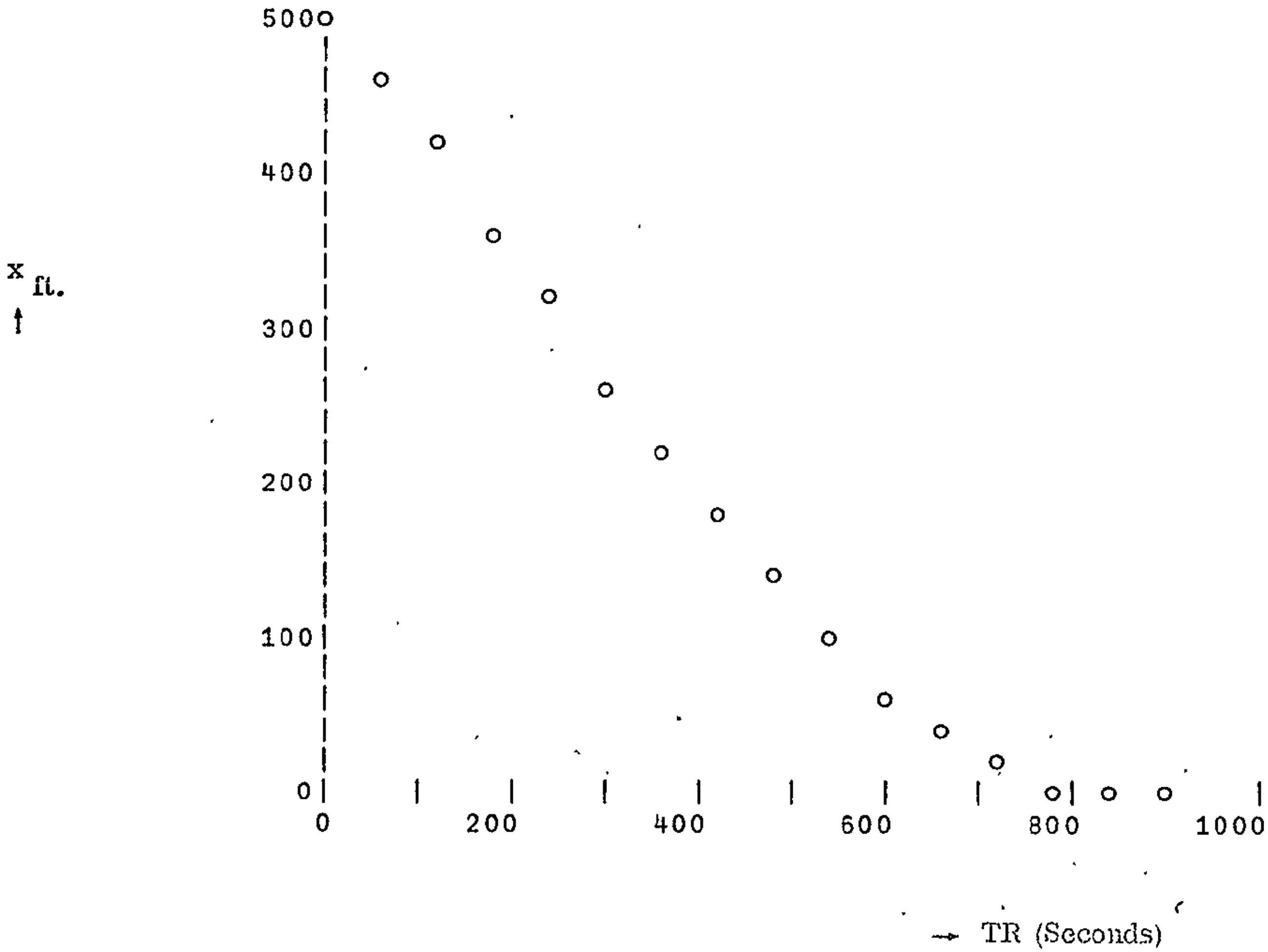
```
0.1902084456
```


POSITION OF TARGET
POSITIONS OF CHASER
600
400
200
Y_{FT}
6000 SEC
15°
15°
-1400
-1200
-1000
-800
-600
-400
-200
X_{FT}
Target
-200
-400
-600
STATIONKEEPING IN FRONT OF TARGET
CHASER AT TIME 0 HAS X = 1000 FT $\dot{X}$ = .016 FT/SEC
Y = 0 FT $\dot{Y}$ = -.016 FT/SEC
TOTAL ΔV FOR 6000 SEC = .19 FT/SEC

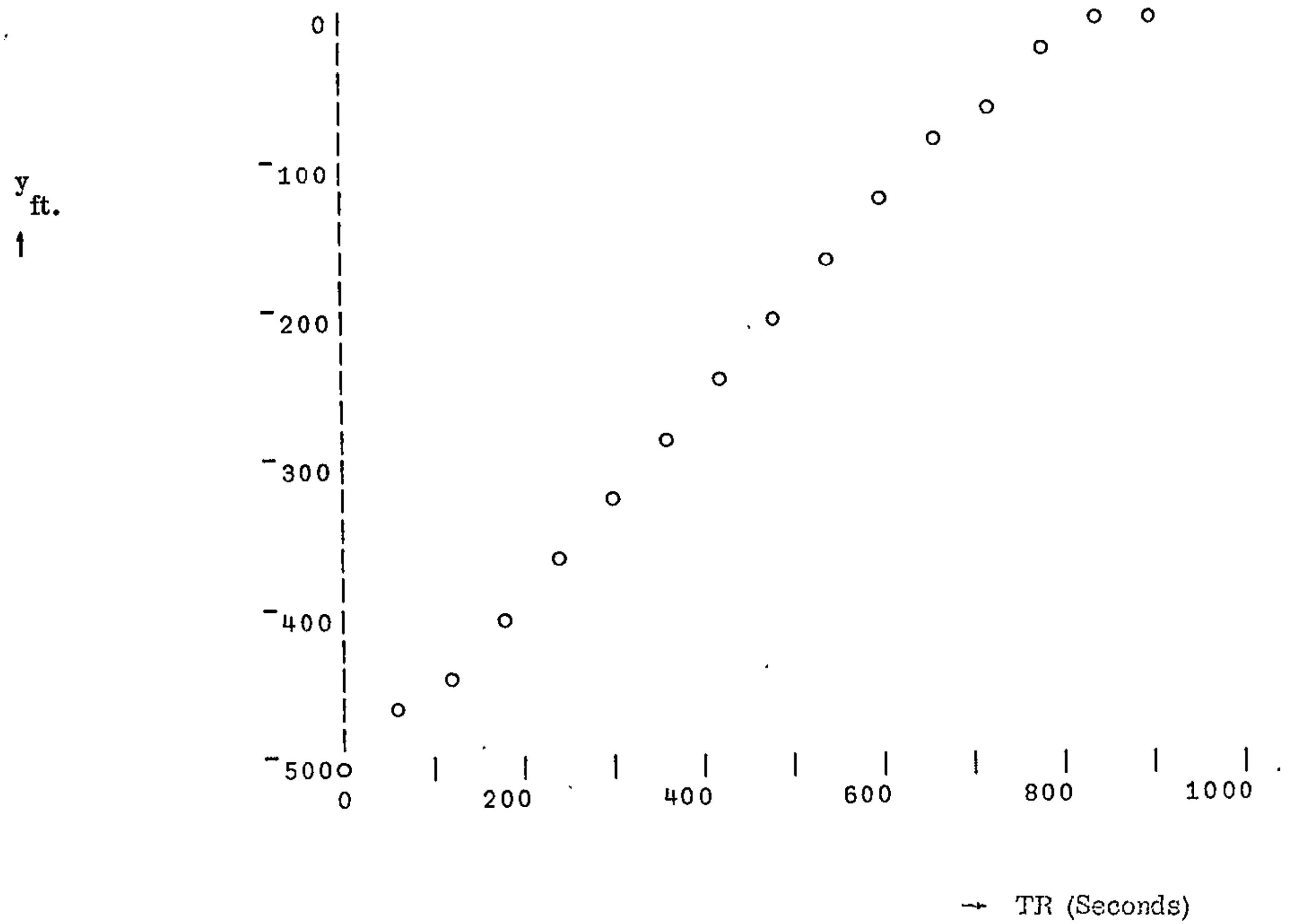
APPENDIX B

RESULTS OF CLOSURE CONTROL SIMULATIONS

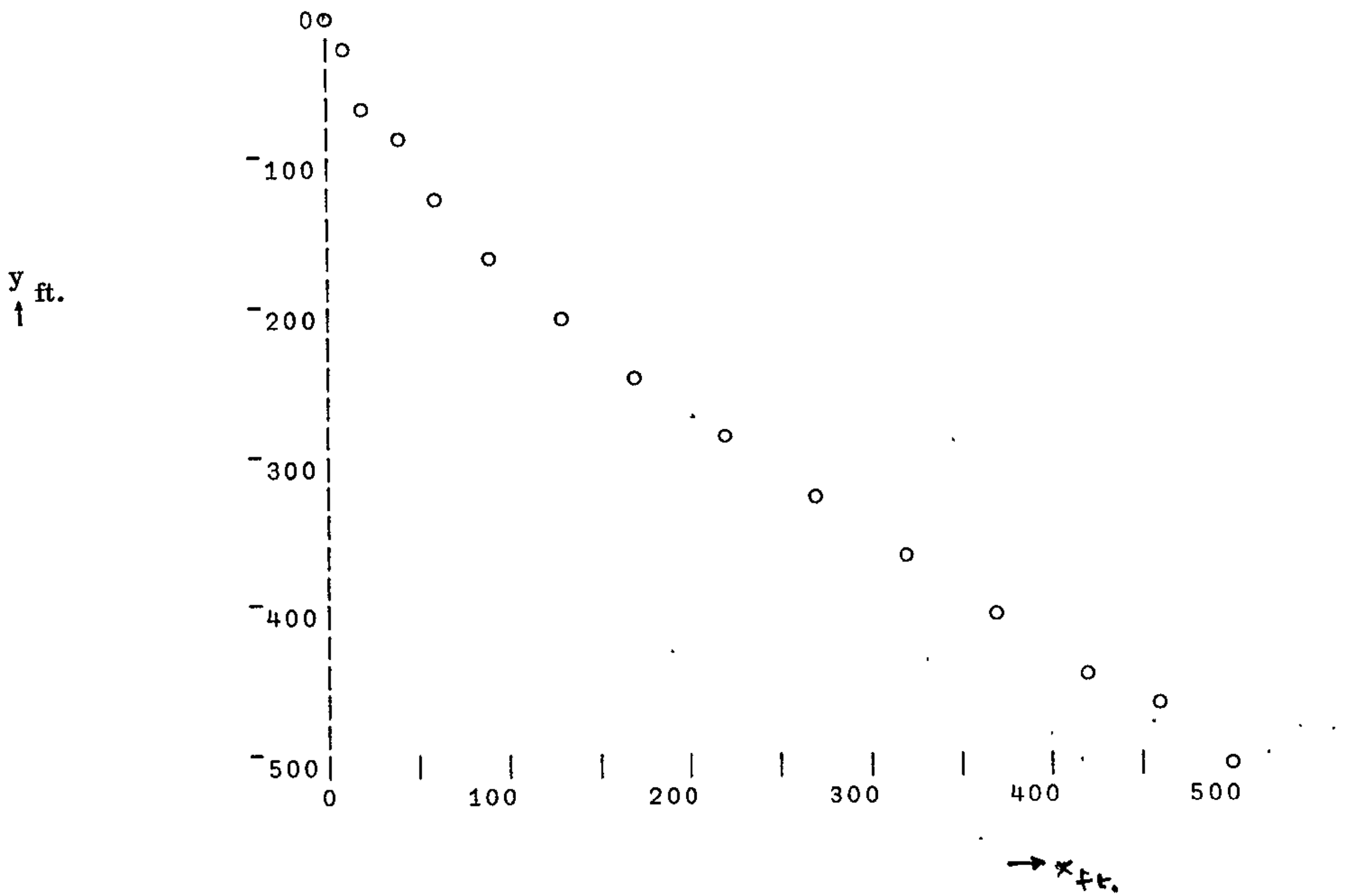
CT = 900 Sec.
x = 500 ft.
y = -500 ft.
$\dot{x}$ = -.6 ft/sec.
$\dot{y}$ = .6 ft/sec.



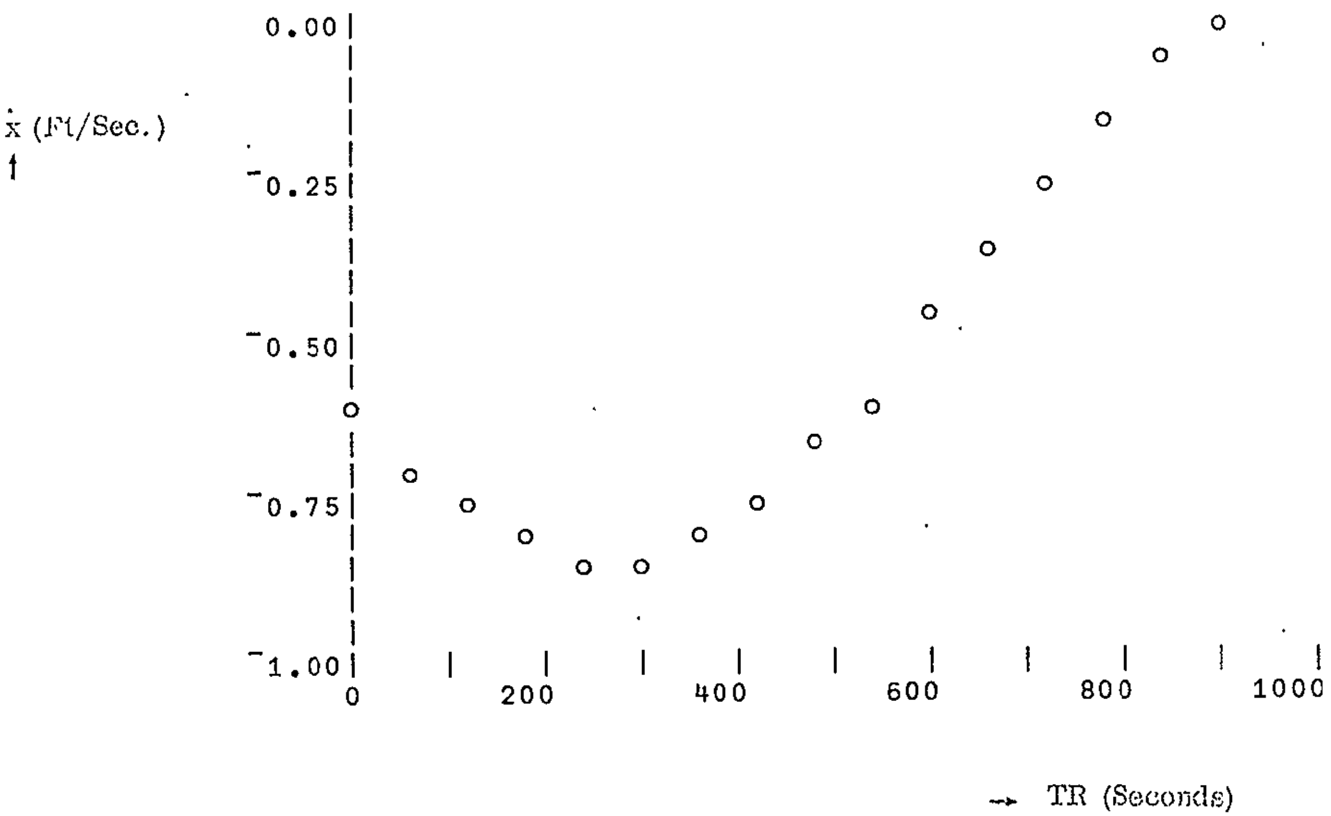
CT = 900 Sec.
x = 500 ft.
y = -500 ft.
$\dot{x}$ = -.6 ft/sec.
$\dot{y}$ = .6 ft/sec.

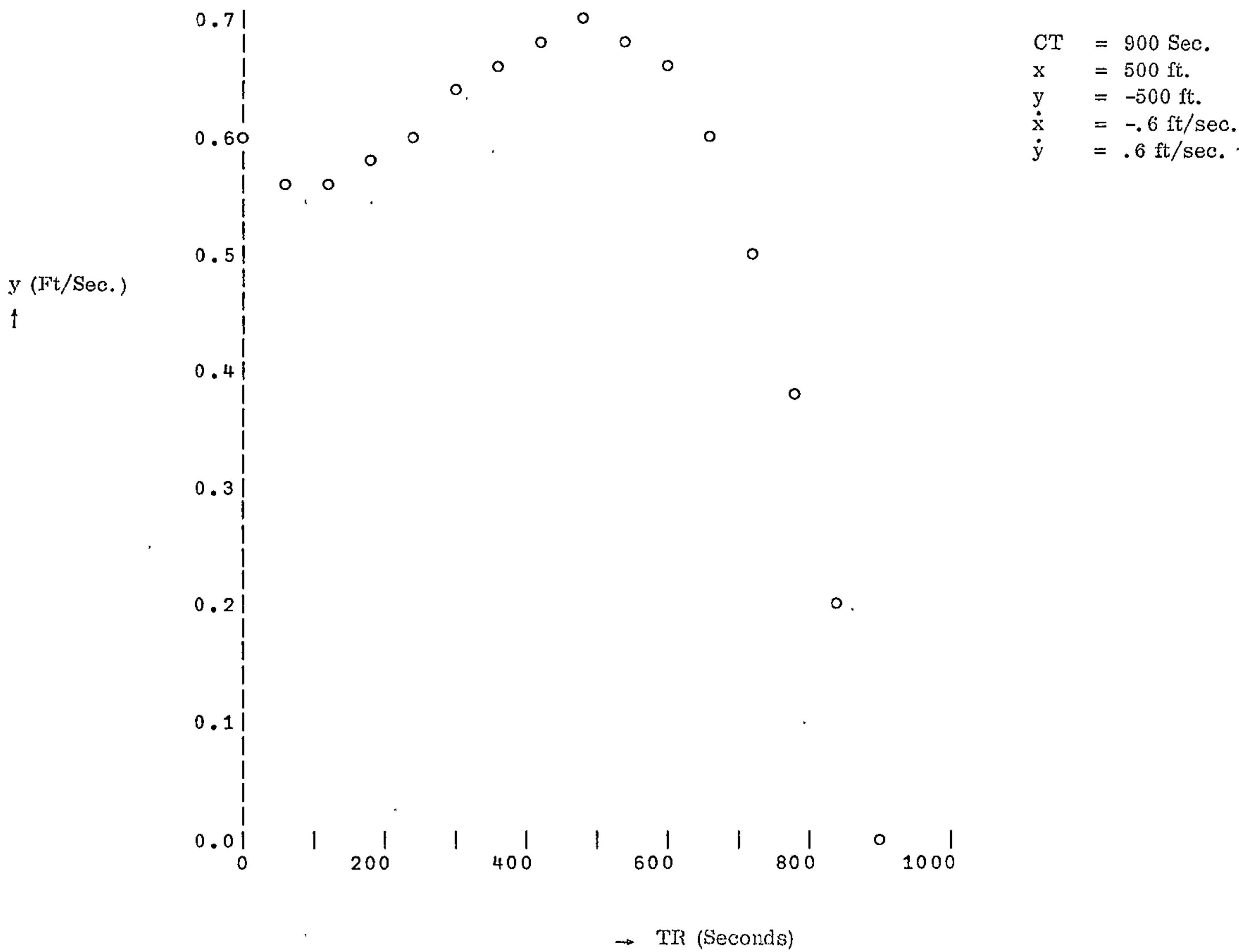


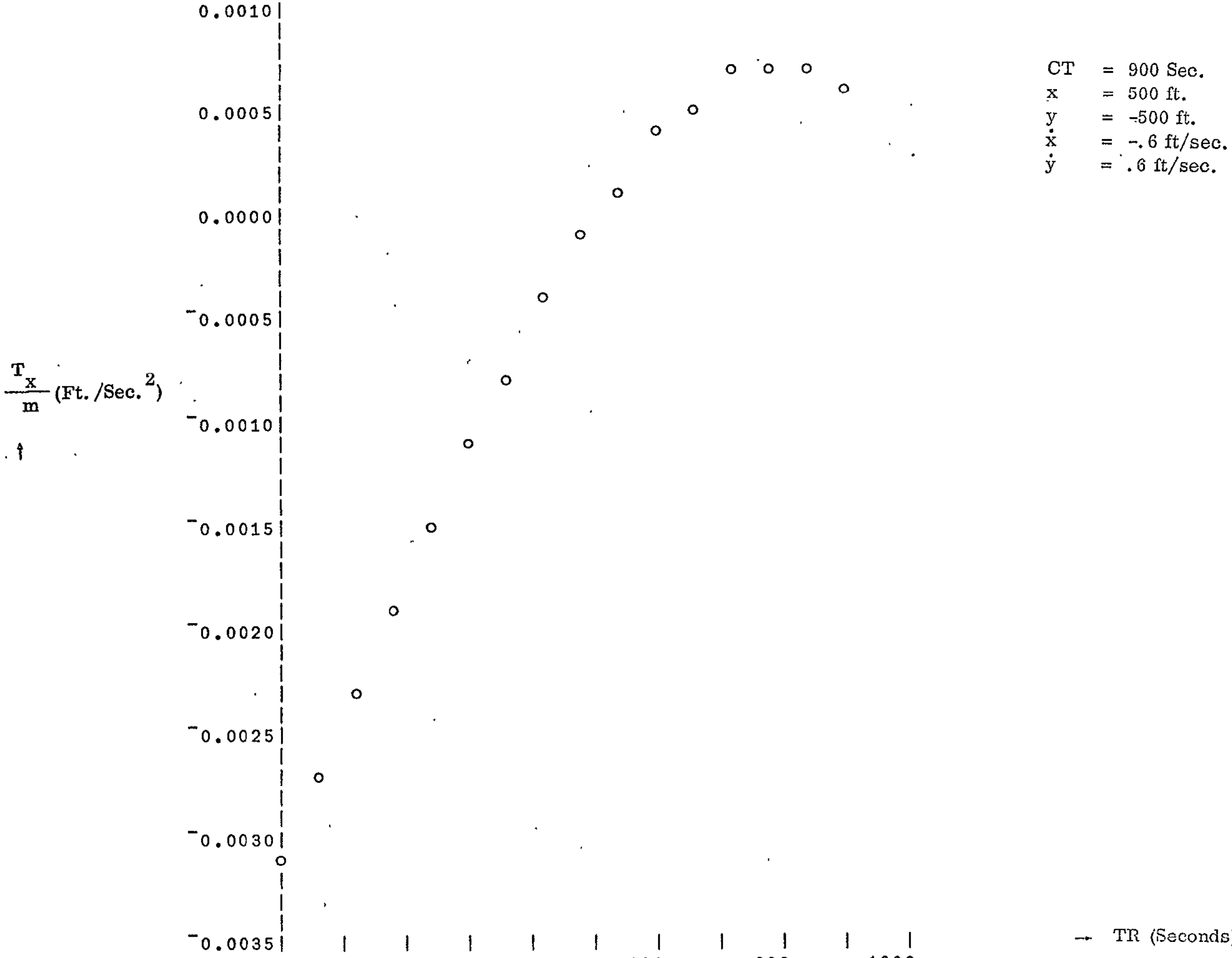
CT	=	900 Sec.
x	=	500 ft.
y	=	-500 ft.
$\dot{x}$	=	-.6 ft/sec.
$\dot{y}$	=	.6 ft/sec.



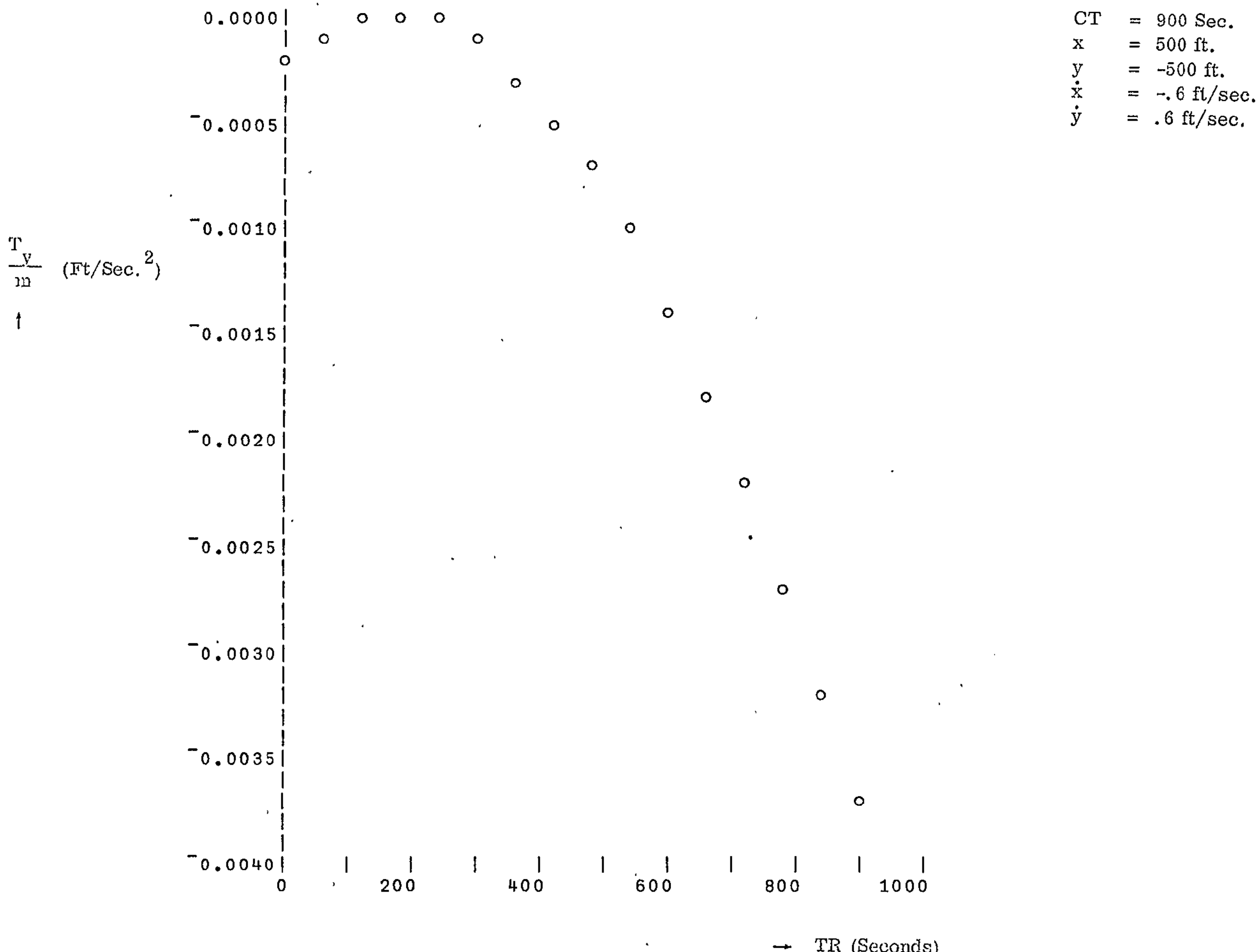
CT = 900 Sec.
x = 500 ft.
y = -500 ft.
$\dot{x}$ = -.6 ft/sec.
$\dot{y}$ = .6 ft/sec.



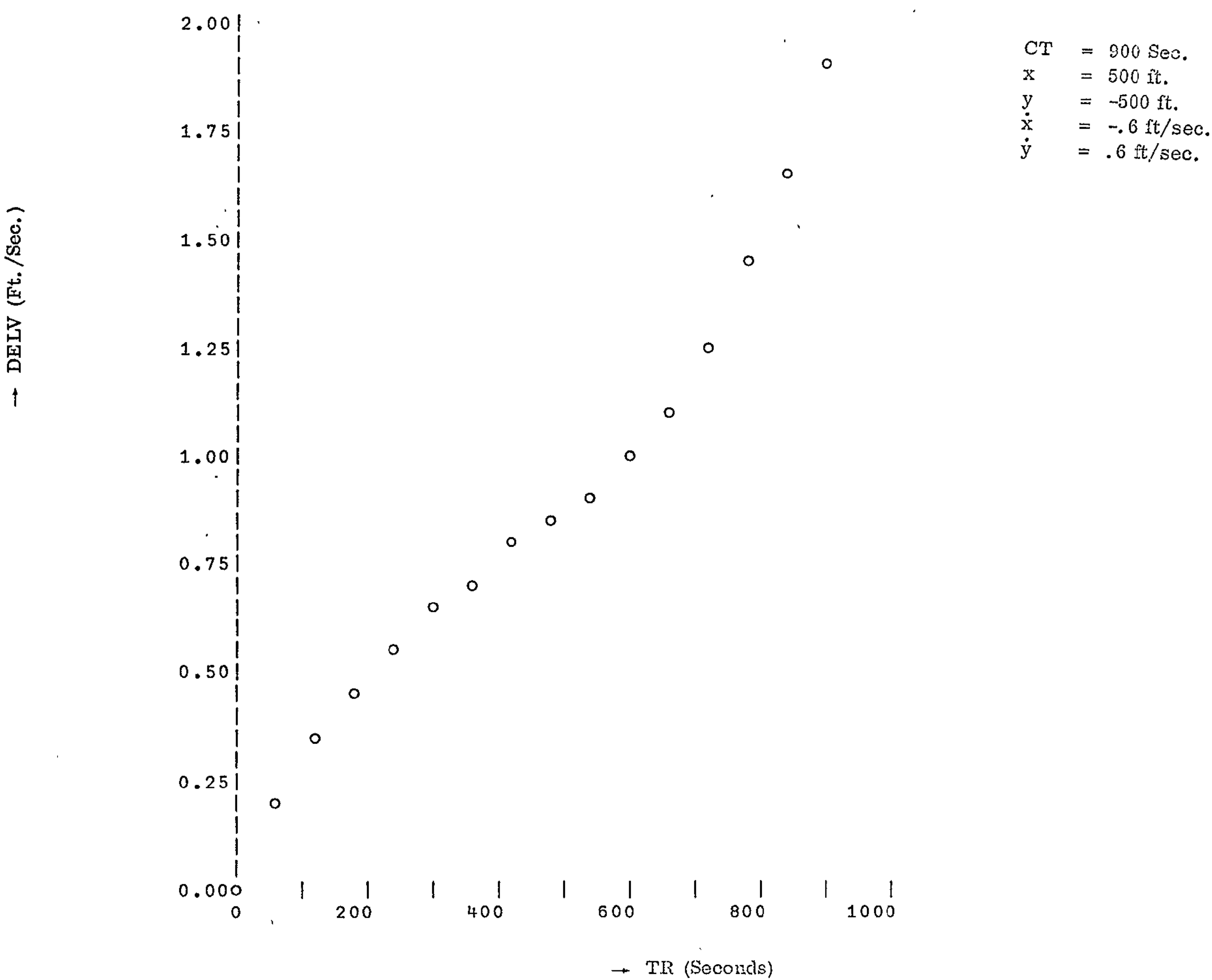

0.7
0.60
0.5
0.4
0.3
0.2
0.1
0.0
y (Ft/Sec.)
0
200
400
600
800
1000
TR (Seconds)
CT = 900 Sec.
x = 500 ft.
y = -500 ft.
ẋ = -.6 ft/sec.
ẏ = .6 ft/sec.



CT = 900 Sec.
x = 500 ft.
y = -500 ft.
ẋ = -.6 ft/sec.
ẏ = .6 ft/sec.
$\frac{T_x}{m}$ (Ft./Sec.^{2})
0.0010
0.0005
0.0000
-0.0005
-0.0010
-0.0015
-0.0020
-0.0025
-0.0030
-0.0035
0
200
400
600
800
1000
TR (Seconds)

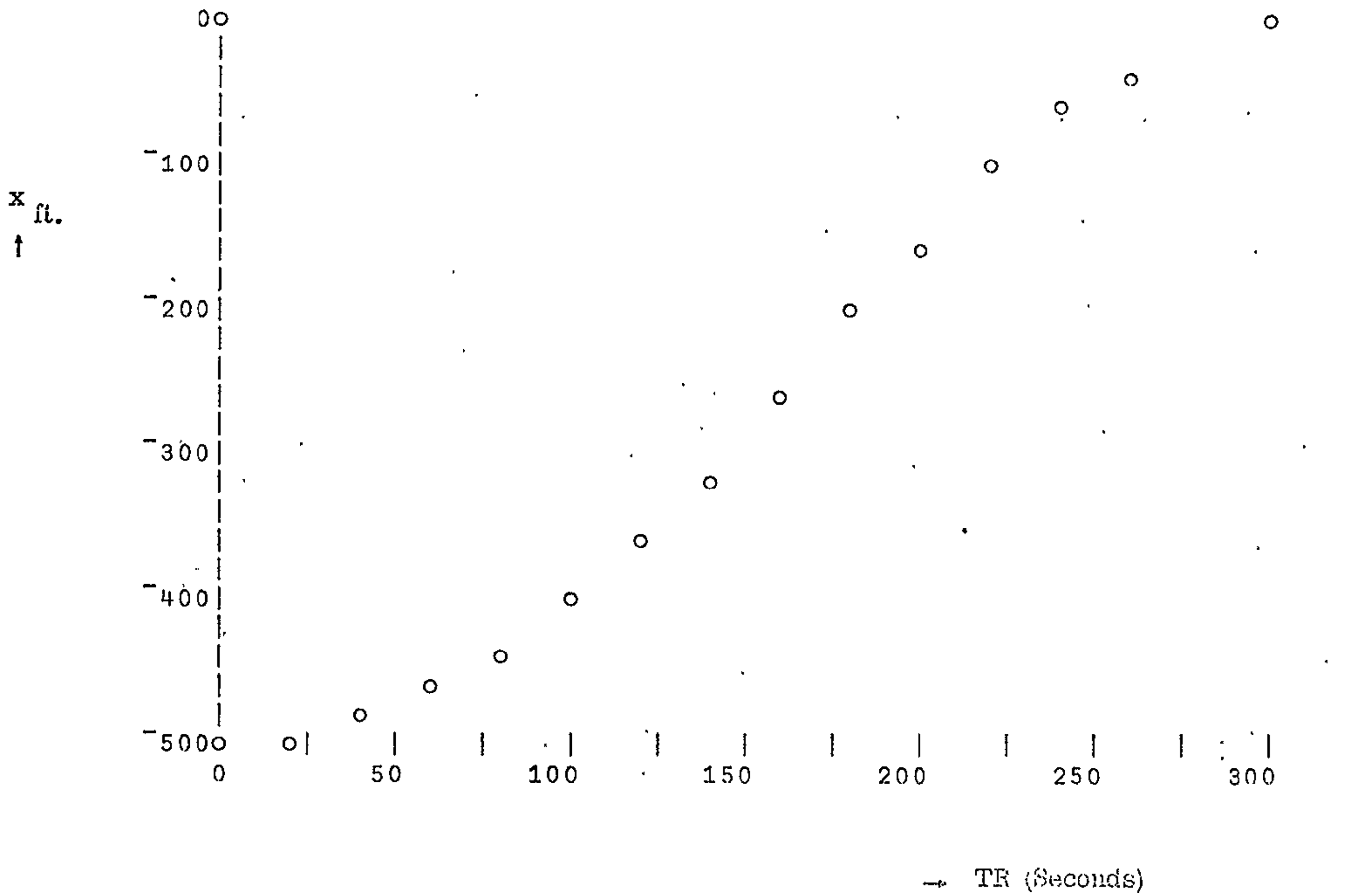


0.0000
-0.0005
-0.0010
-0.0015
-0.0020
-0.0025
-0.0030
-0.0035
-0.0040
0
200
400
600
800
1000
$\frac{T_y}{m}$ (Ft/Sec.^{2})
→ TR (Seconds)
CT = 900 Sec.
x = 500 ft.
y = -500 ft.
$\dot{x}$ = -.6 ft/sec.
$\dot{y}$ = .6 ft/sec.

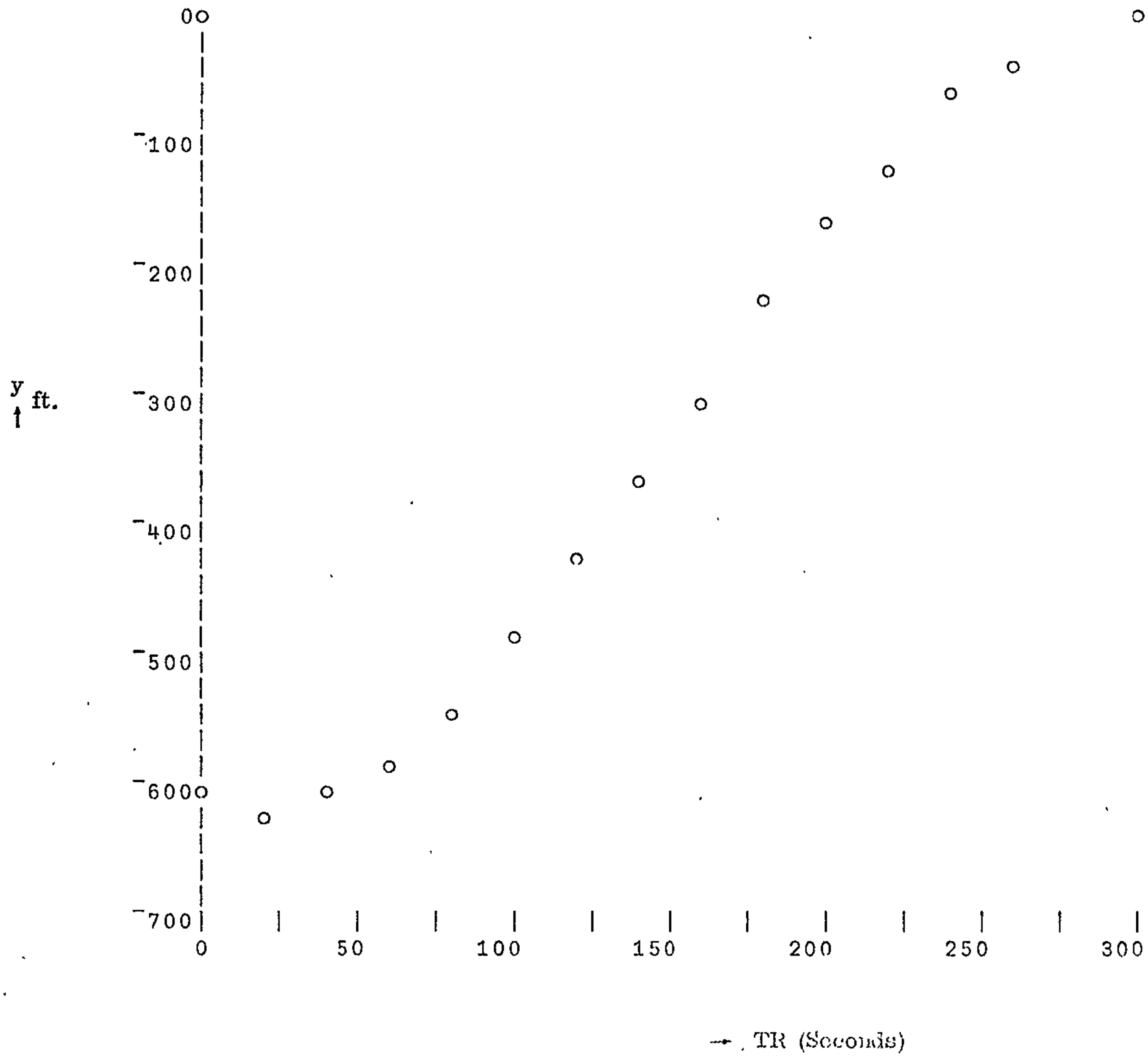


CT = 900 Sec.
x = 500 ft.
y = -500 ft.
ẋ = -.6 ft/sec.
ẏ = .6 ft/sec.
→ DELV (Ft./Sec.)
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0
200
400
600
800
1000
→ TR (Seconds)

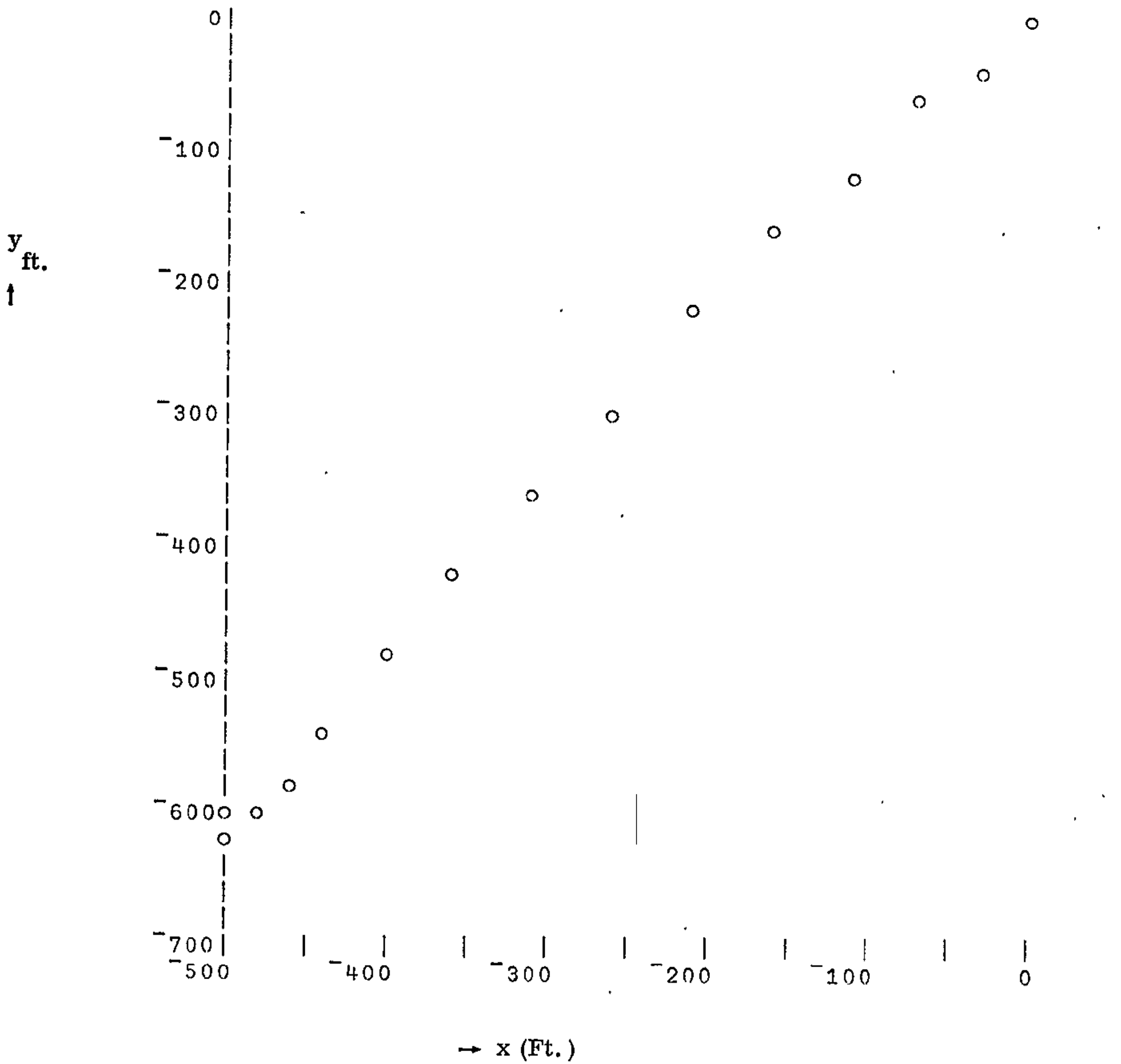
CT = 300 Sec.
x = -500 ft.
y = -600 ft.
$\dot{x}$ = -.032 ft/sec.
$\dot{y}$ = -1.2 ft/sec.

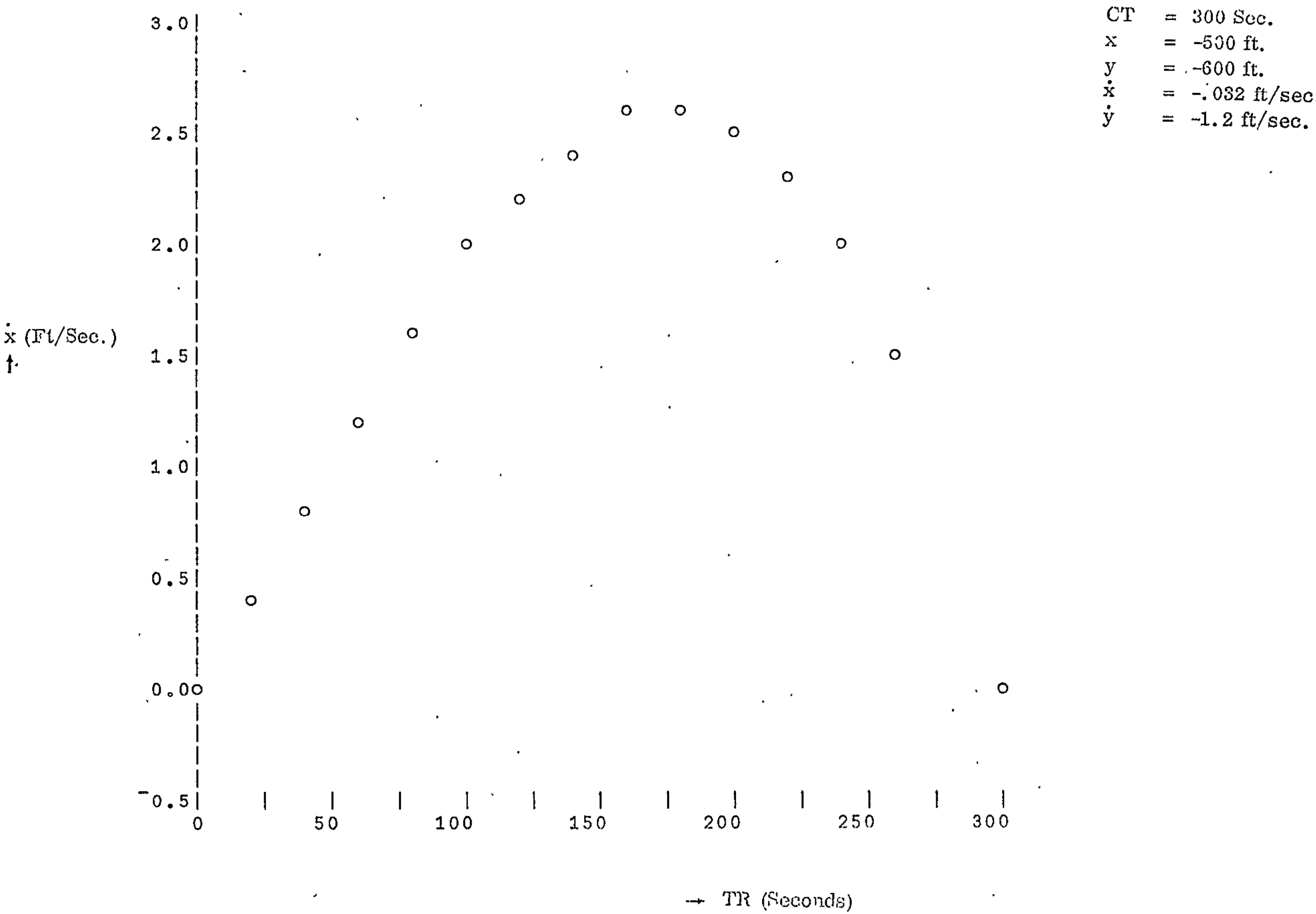


CT = 300 Sec.
x = -500 ft.
y = -600 ft.
$\dot{x}$ = -.032 ft/sec.
$\dot{y}$ = -1.2 ft/sec.



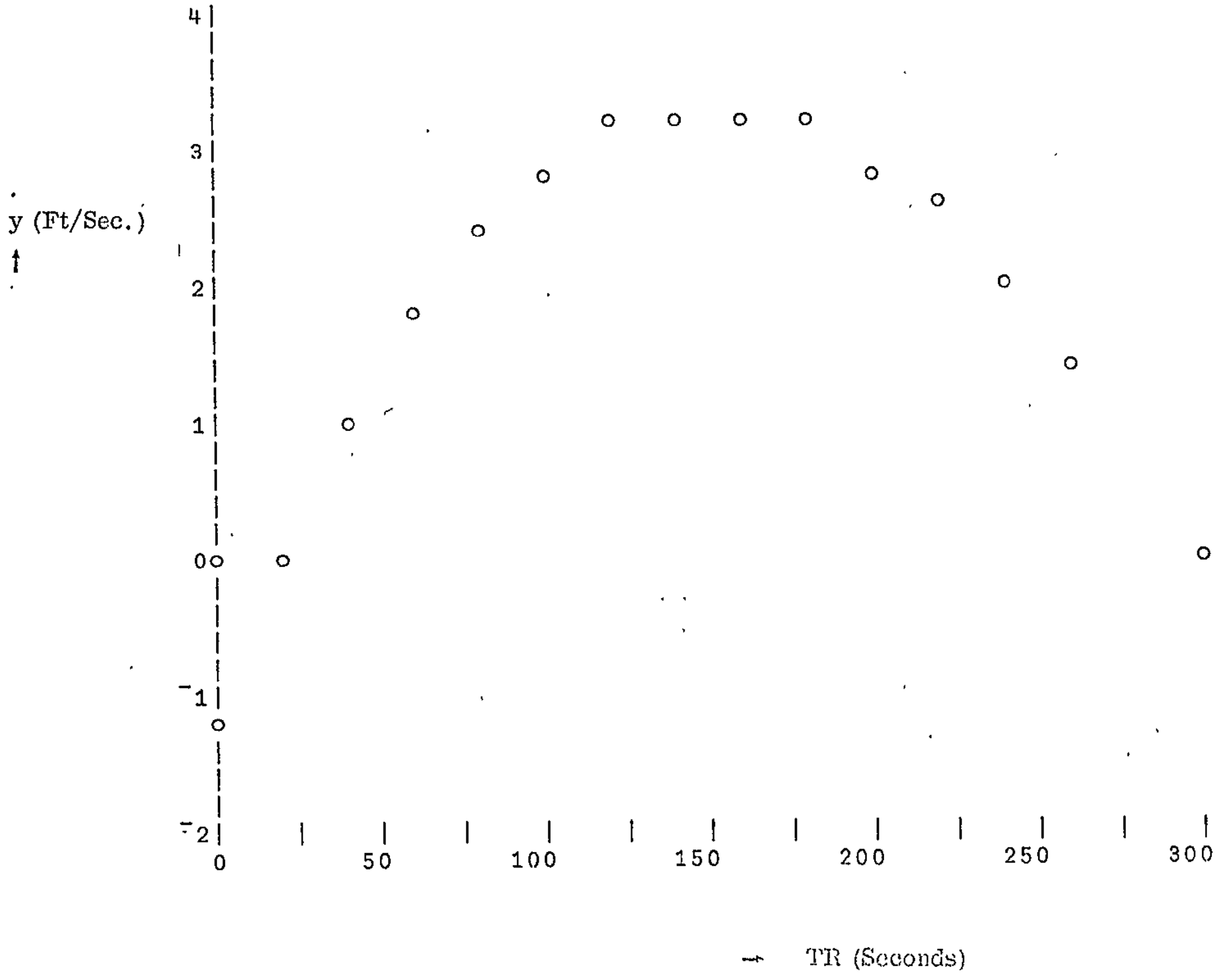
$CT = 300$ Sec.
$x = -500$ ft.
$y = -600$ ft.
$\dot{x} = -.032$ ft/sec.
$\dot{y} = -1.2$ ft/sec.

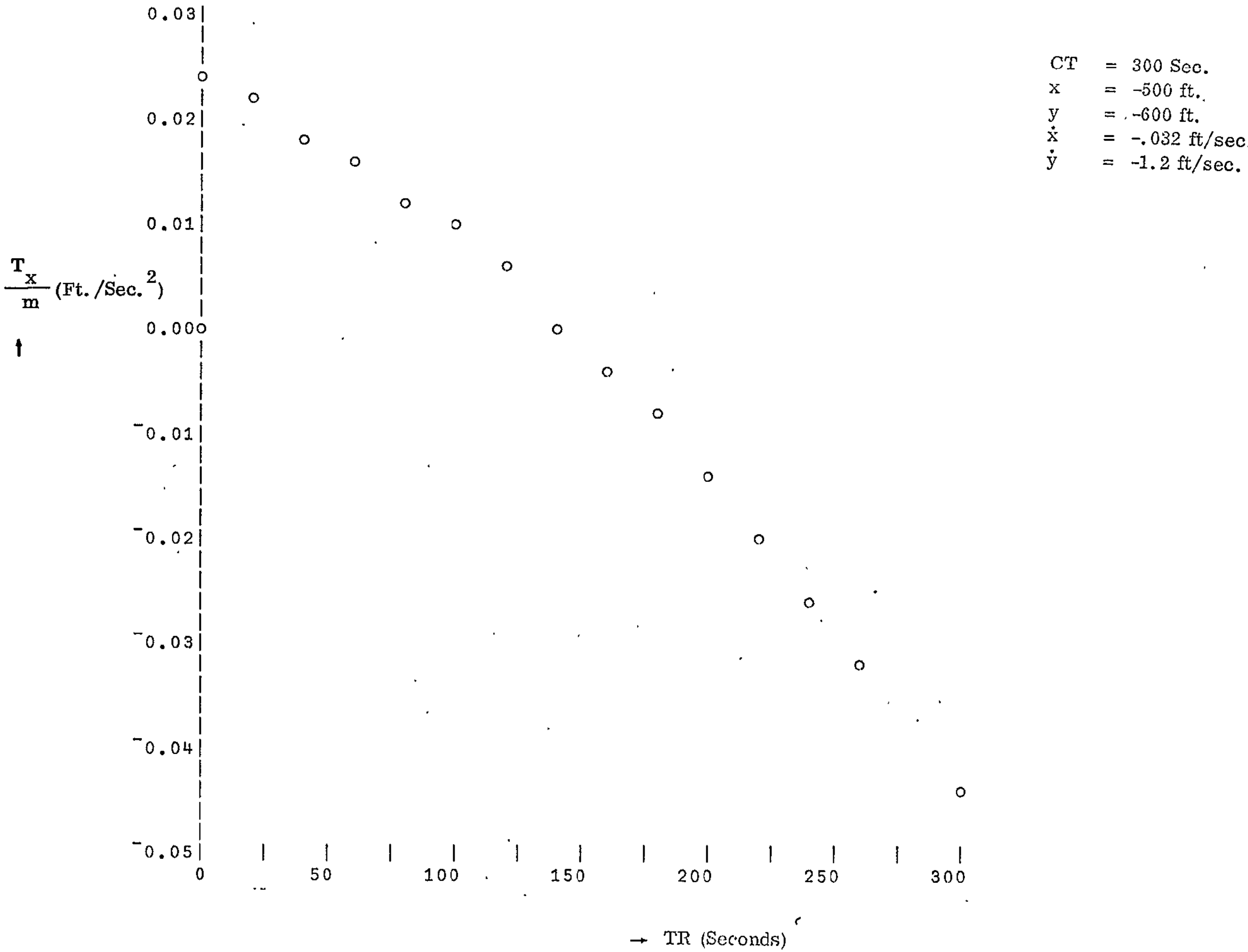




CT = 300 Sec.
x = -500 ft.
y = -600 ft.
ẋ = -.032 ft/sec.
ẏ = -1.2 ft/sec.
ẋ (Ft/Sec.)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
0
50
100
150
200
250
300
TR (Seconds)

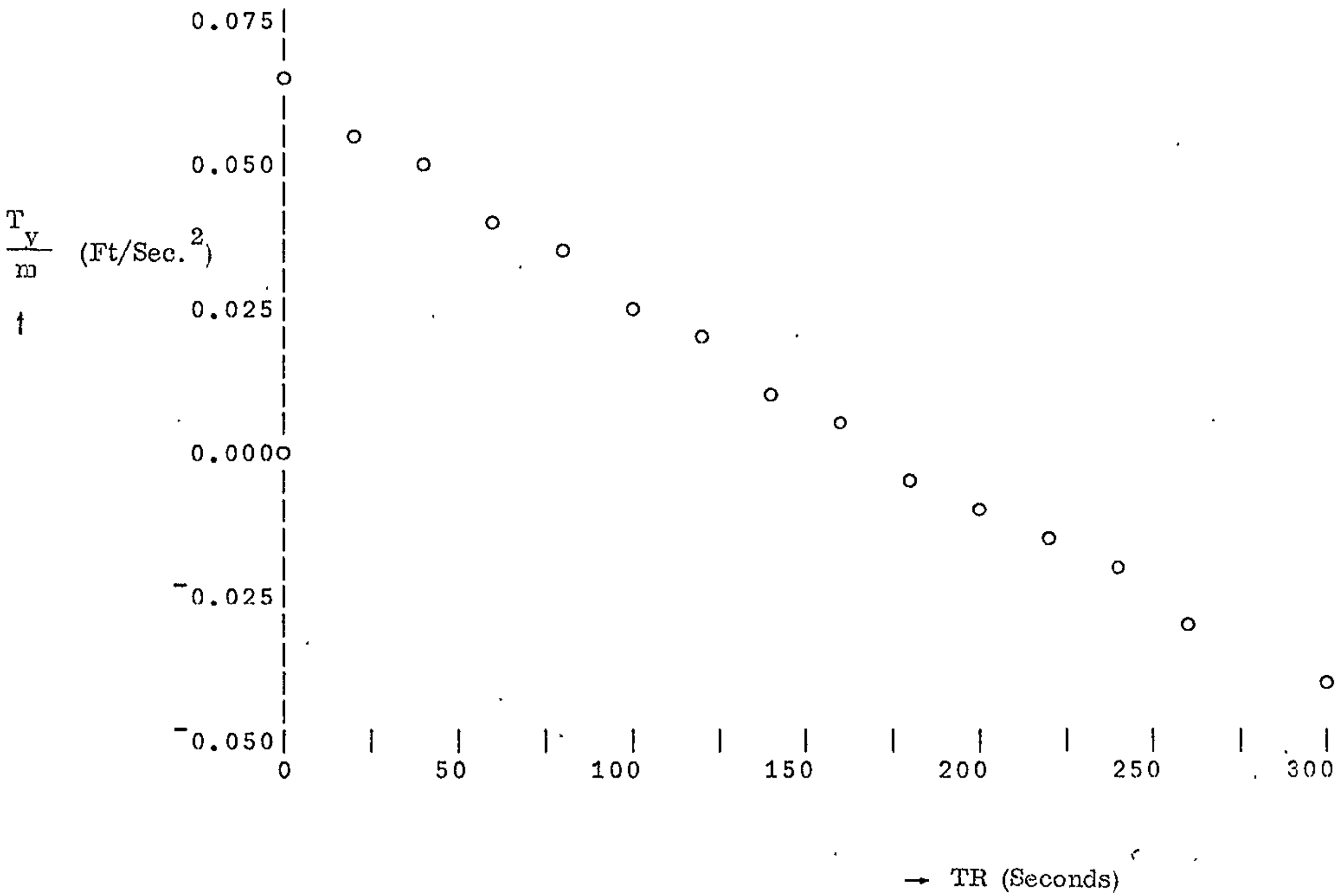
CT	=	300 Sec.
x	=	-500 ft.
y	=	-600 ft.
$\dot{x}$	=	-.032 ft/sec.
$\dot{y}$	=	-1.2 ft/sec.

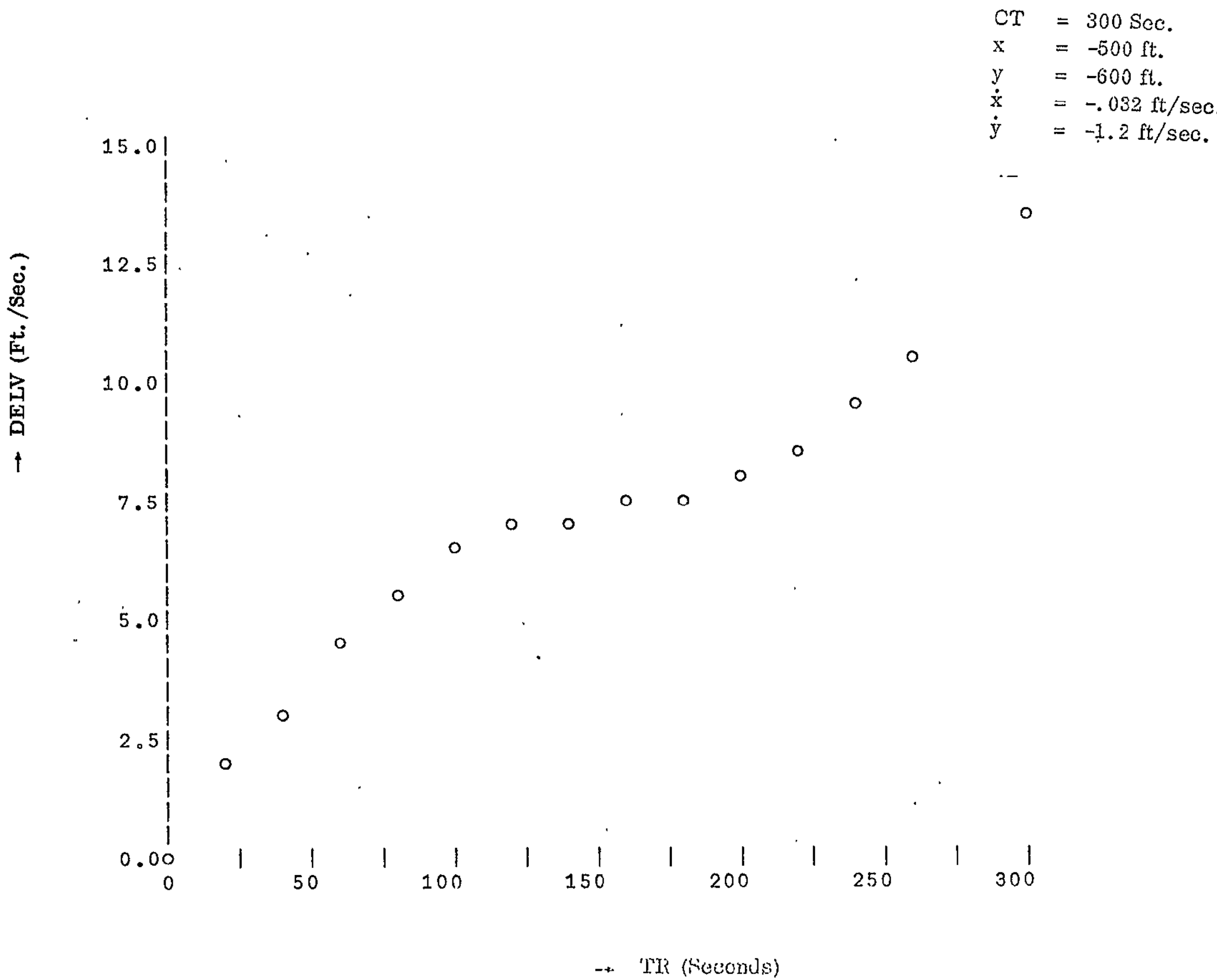




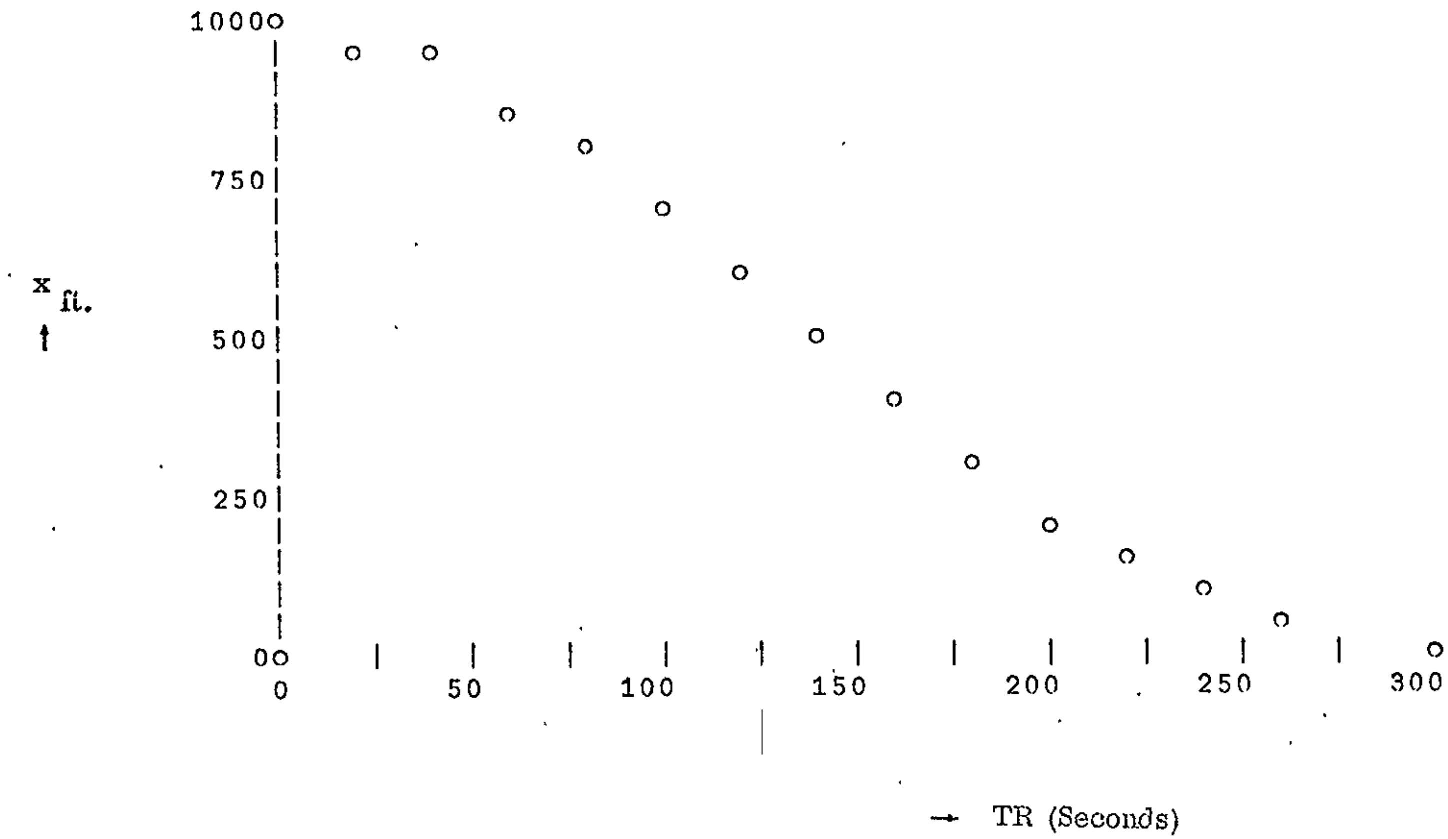
CT = 300 Sec.
x = -500 ft.
y = -600 ft.
ẋ = -.032 ft/sec.
ẏ = -1.2 ft/sec.
Tx/m (Ft./Sec.2)
0.03
0.02
0.01
0.000
-0.01
-0.02
-0.03
-0.04
-0.05
0
50
100
150
200
250
300
TR (Seconds)

CT = 300 Sec.
x = -500 ft.
y = -600 ft.
$\dot{x}$ = -.032 ft/sec.
$\dot{y}$ = -1.2 ft/sec.

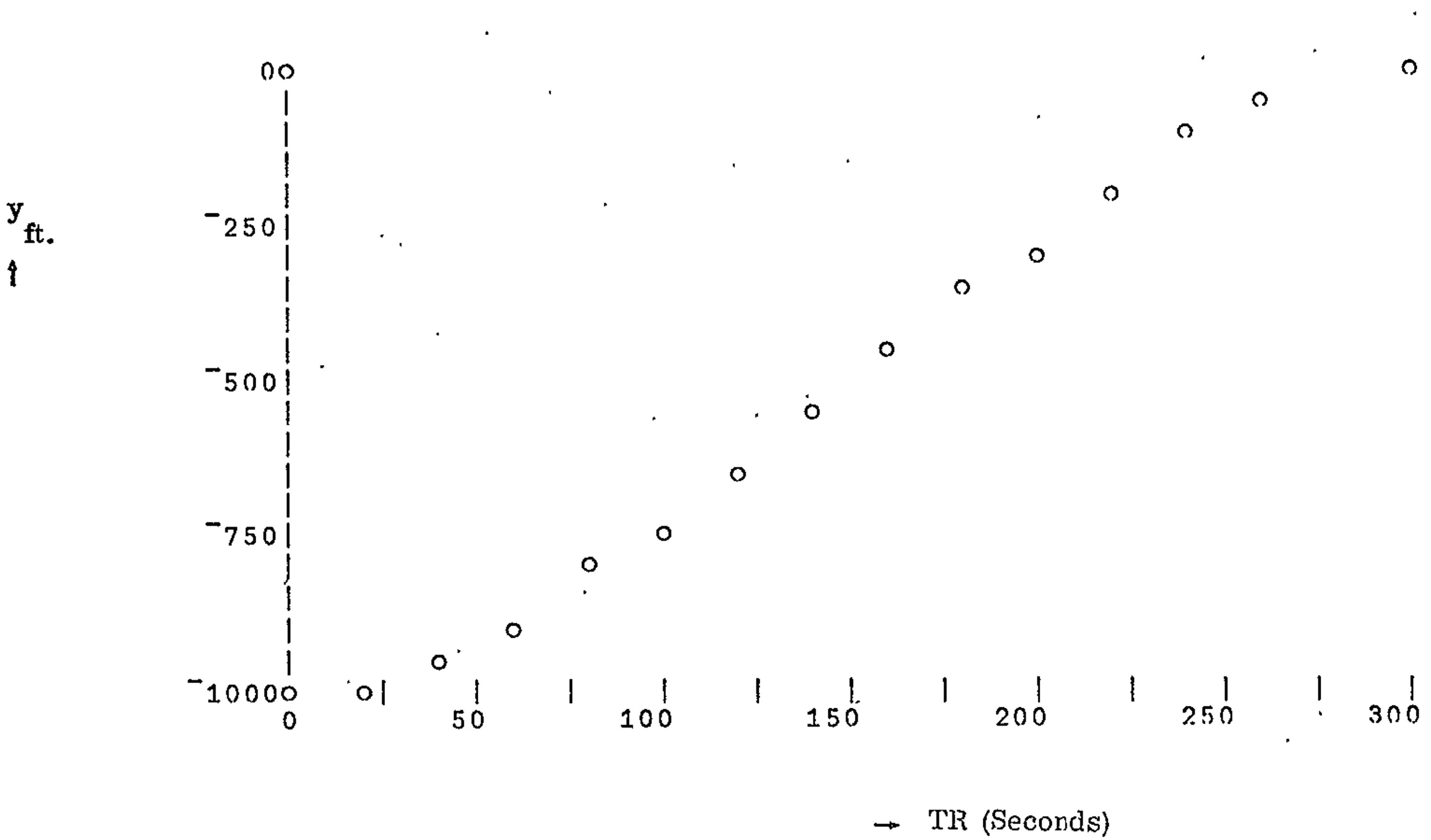



CT = 300 Sec.
x = -500 ft.
y = -600 ft.
ẋ = -.032 ft/sec.
ẏ = -1.2 ft/sec.
DELV (Ft./Sec.)
15.0
12.5
10.0
7.5
5.0
2.5
0.00
0
50
100
150
200
250
300
TR (Seconds)

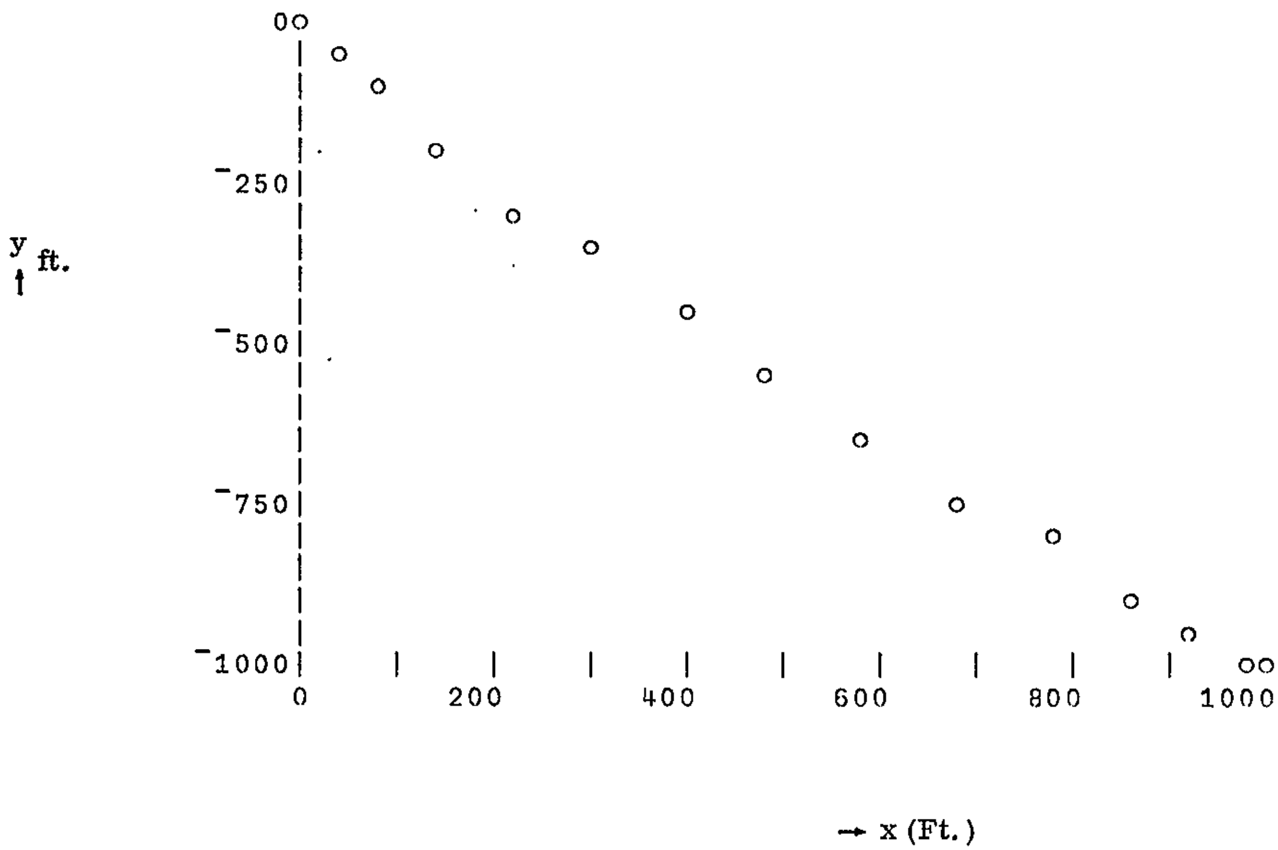
CT = 300 sec.
x = 1000 ft.
y = -1000 ft.
$\dot{x}$ = -.6 ft/sec.
$\dot{y}$ = .6 ft/sec.



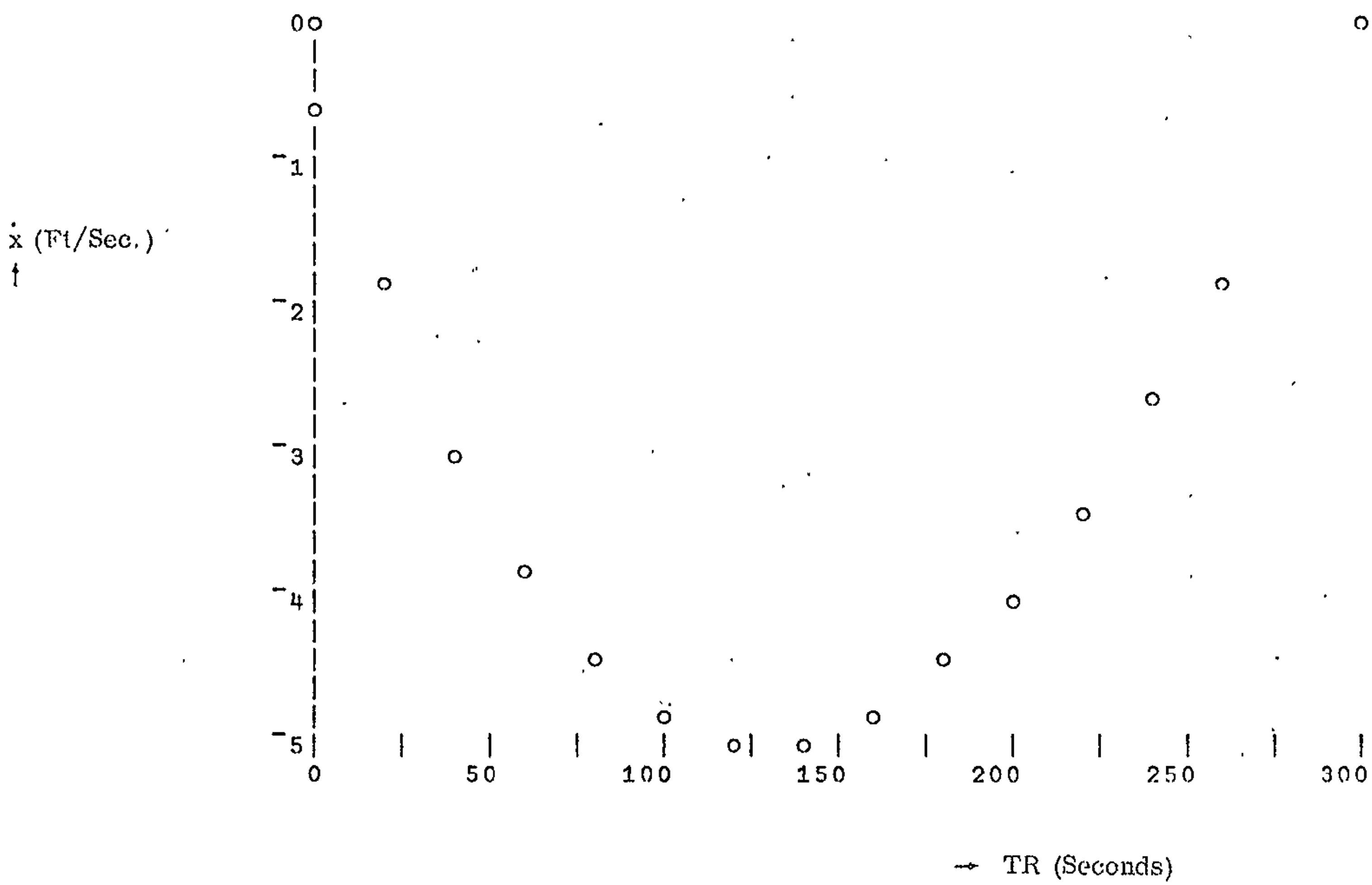
CT = 300 sec.
x = 1000 ft.
y = -1000 ft.
$\dot{x}$ = -.6 ft/sec.
$\dot{y}$ = .6 ft/sec.



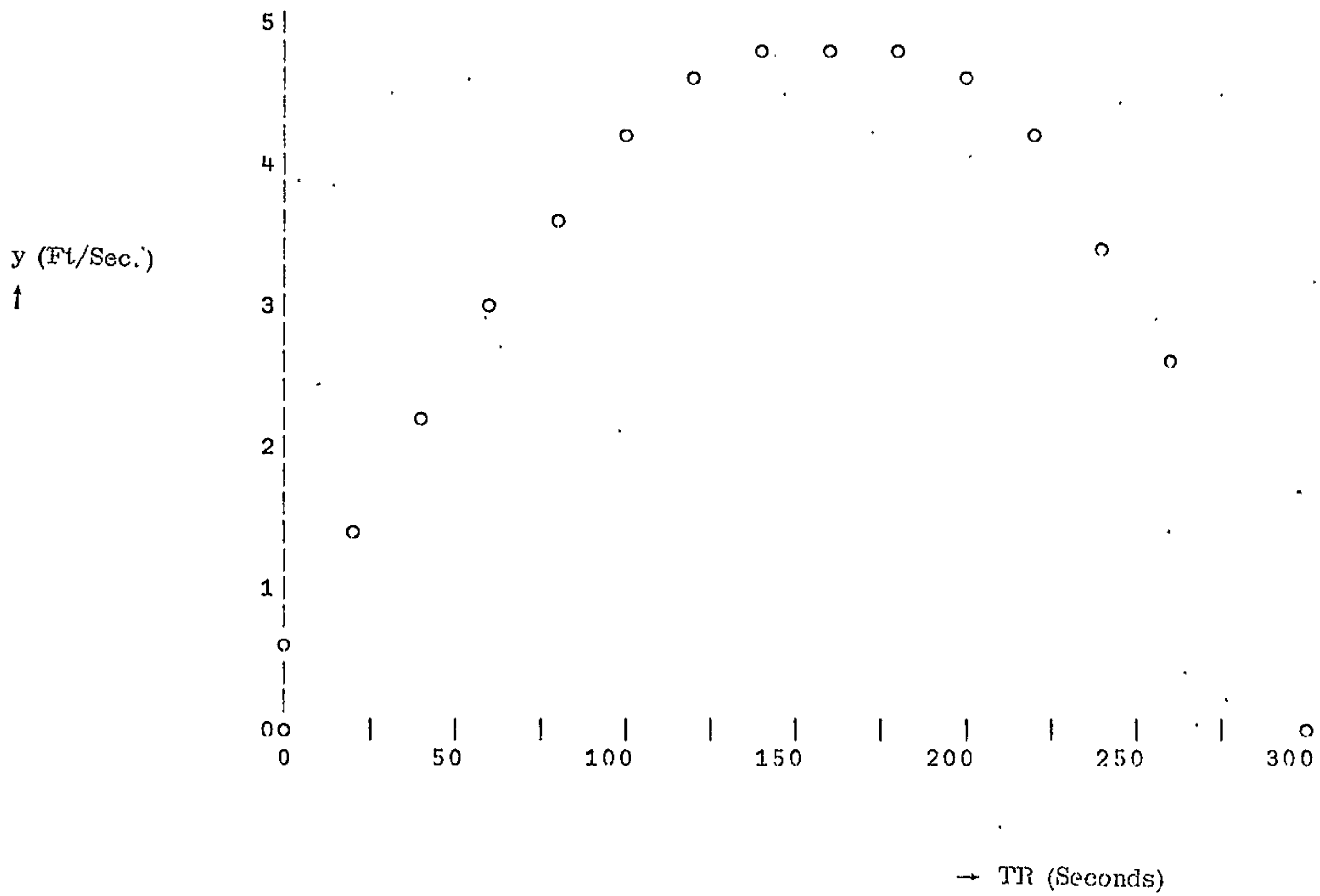
$CT = 300$ sec.
$x = 1000$ ft.
$y = -1000$ ft.
$\dot{x} = -.6$ ft/sec.
$\dot{y} = .6$ ft/sec.

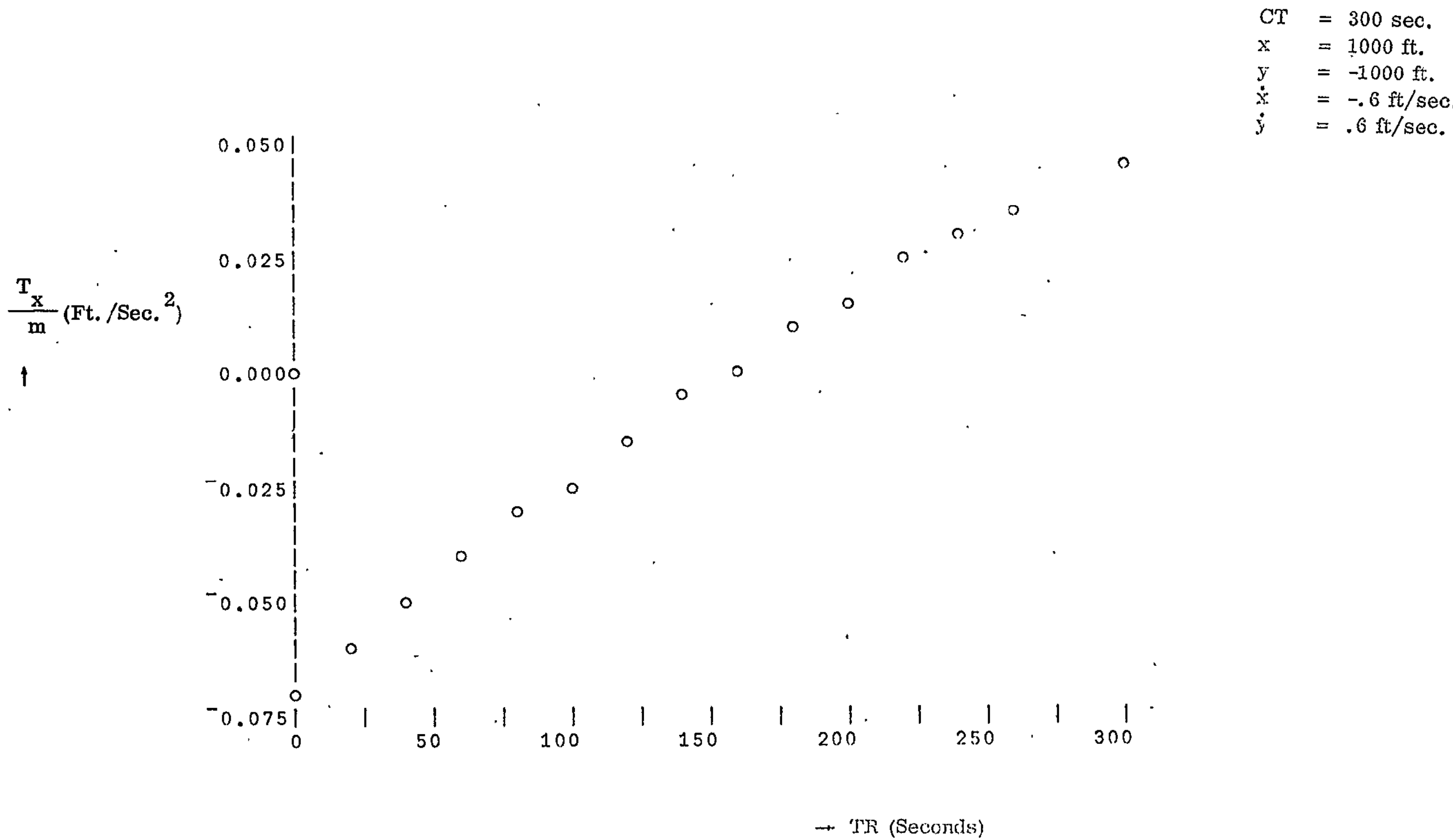


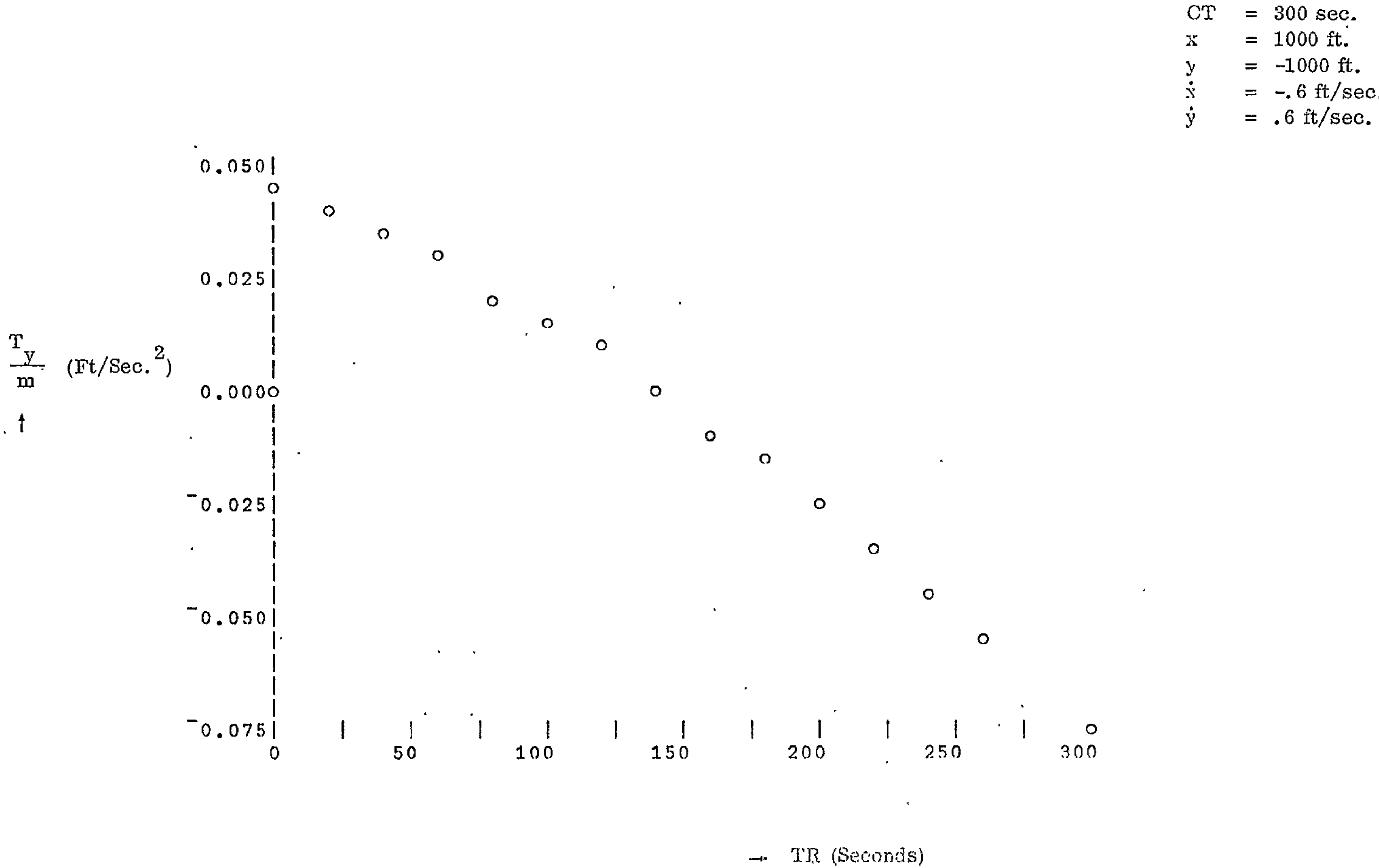
CT	=	300 sec.
x	=	1000 ft.
y	=	-1000 ft.
$\dot{x}$	=	-.6 ft/sec.
$\dot{y}$	=	.6 ft/sec.



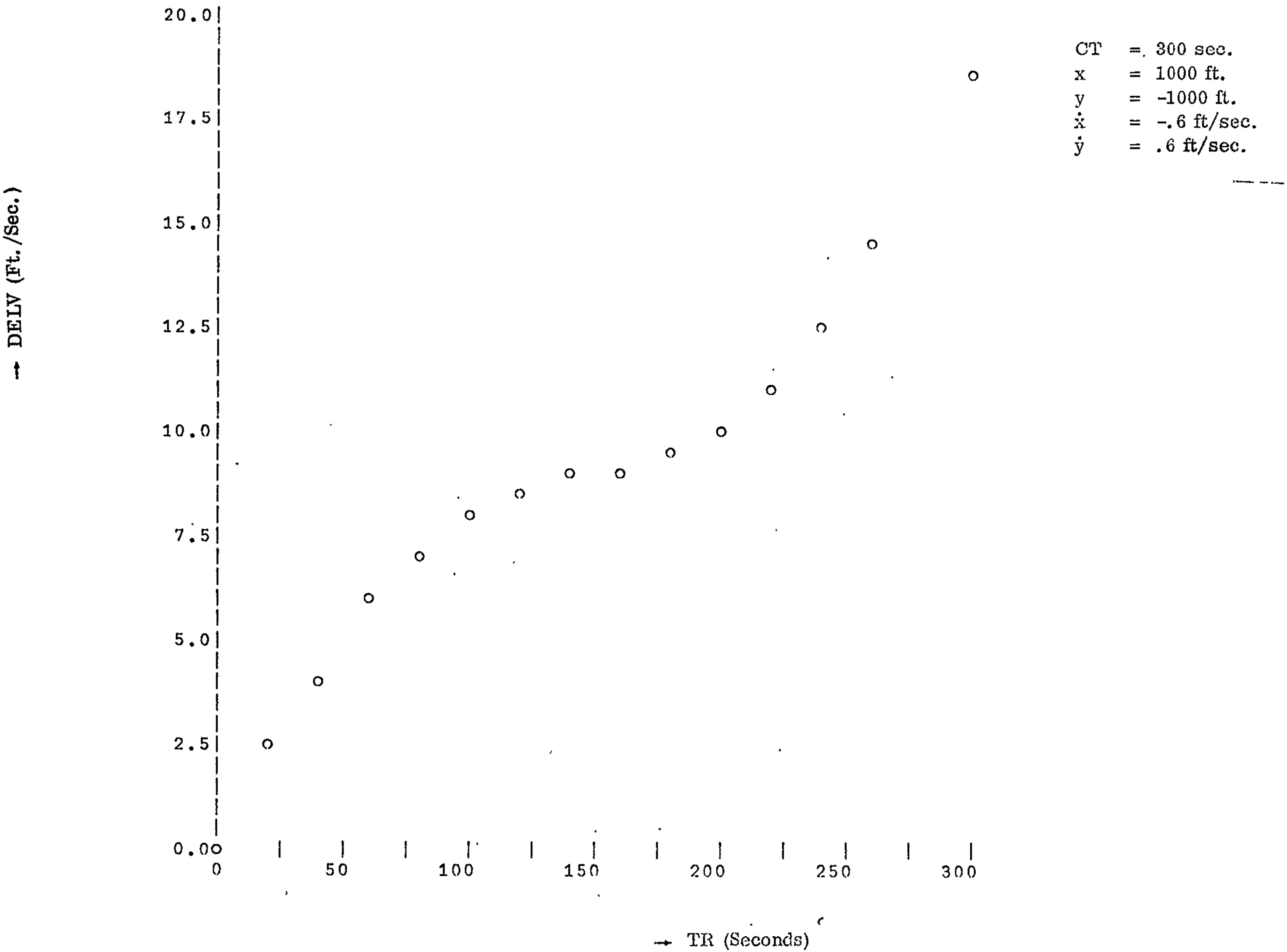
CT = 300 sec.
x = 1000 ft.
y = -1000 ft.
$\dot{x}$ = -.6 ft/sec.
$\dot{y}$ = .6 ft/sec.



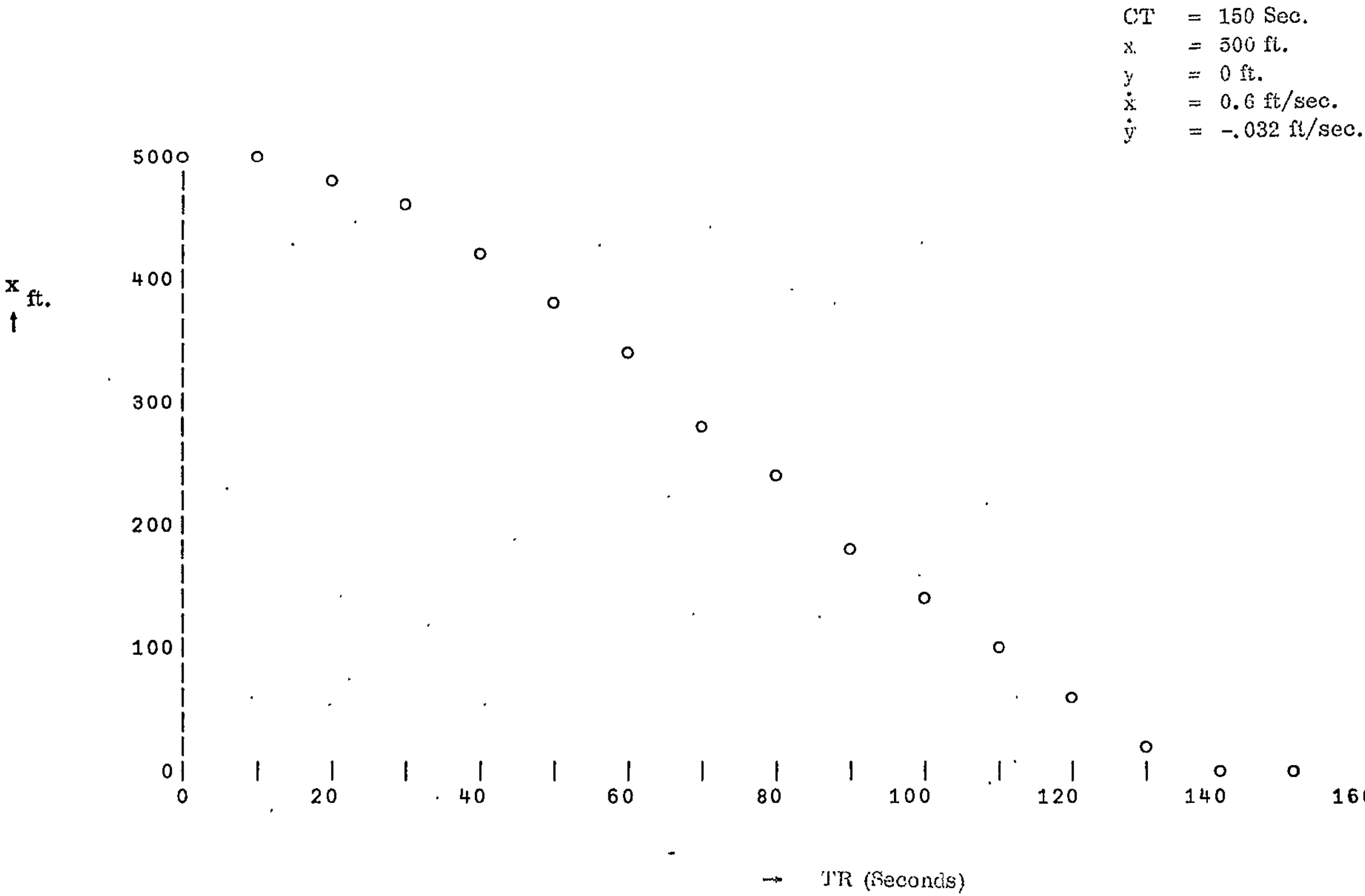

CT = 300 sec.
x = 1000 ft.
y = -1000 ft.
ẋ = -.6 ft/sec.
ẏ = .6 ft/sec.
$\frac{T_x}{m}$ (Ft./Sec.^{2})
0.050
0.025
0.0000
-0.025
-0.050
-0.075
0
50
100
150
200
250
300
→ TR (Seconds)



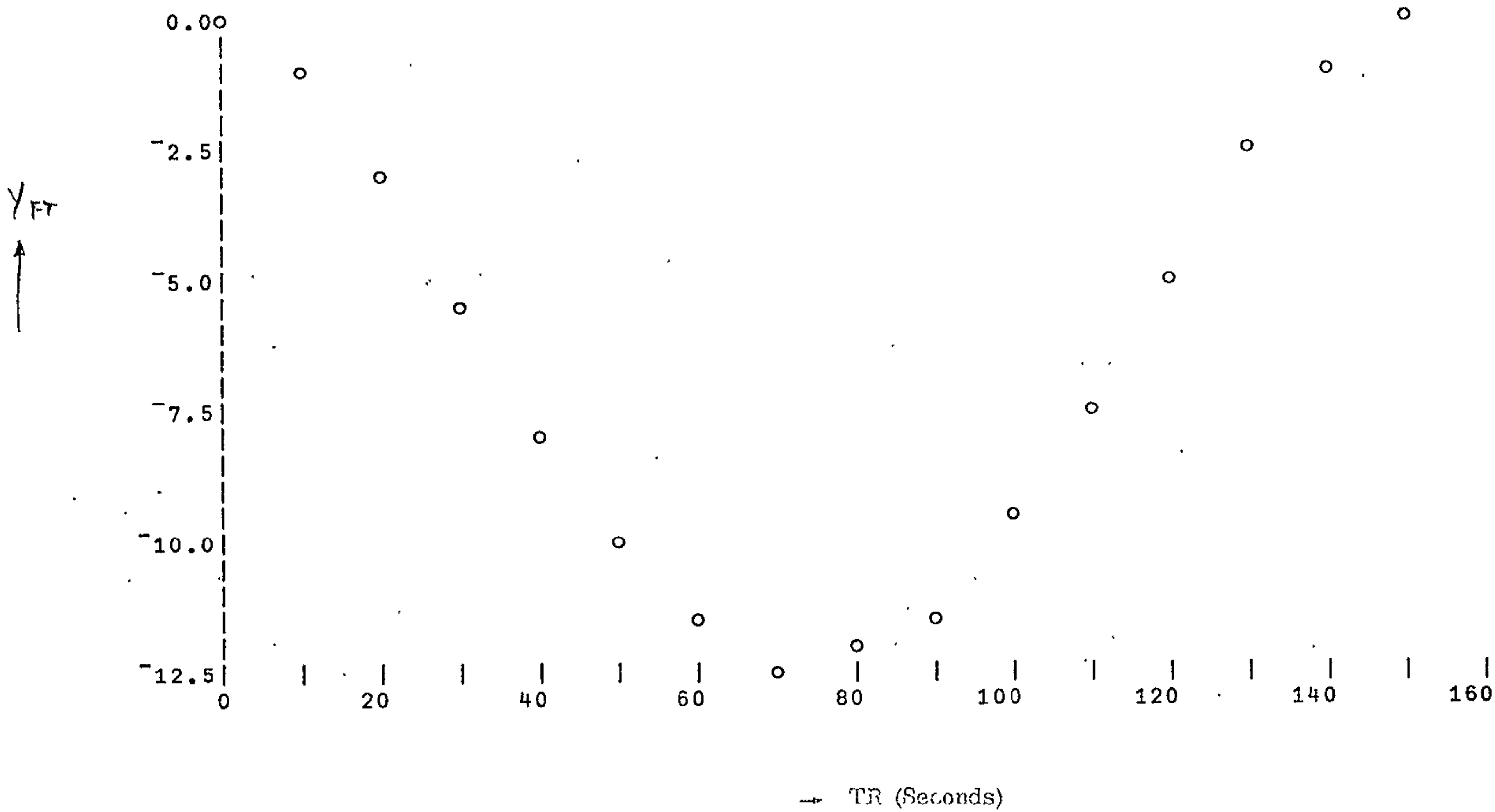
CT = 300 sec.
x = 1000 ft.
y = -1000 ft.
ẋ = -.6 ft/sec.
ẏ = .6 ft/sec.
$\frac{T_y}{m}$ (Ft/Sec.^{2})
0.050
0.025
0.000
-0.025
-0.050
-0.075
0
50
100
150
200
250
300
TR (Seconds)



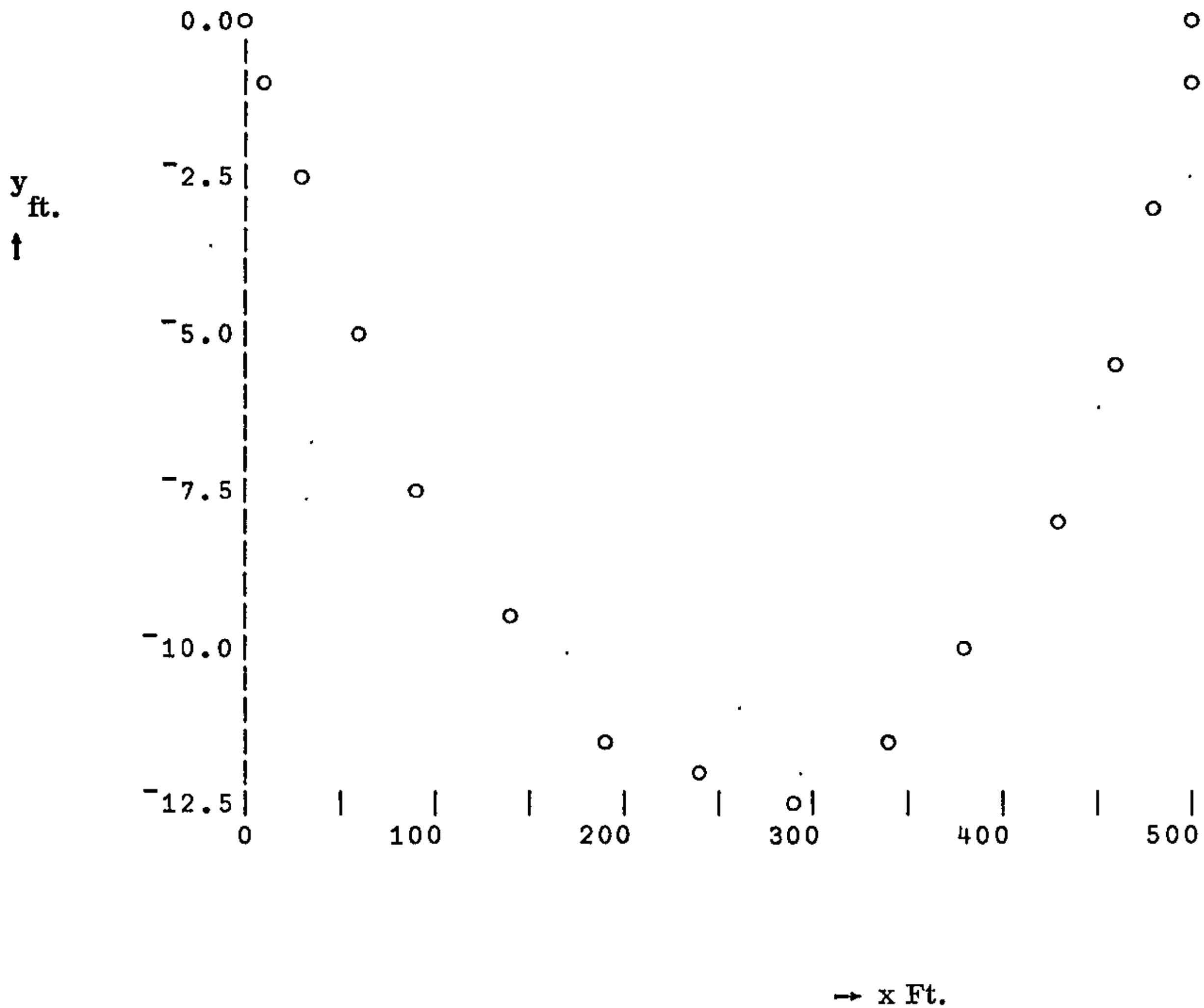
CT = 300 sec.
x = 1000 ft.
y = -1000 ft.
ẋ = -.6 ft/sec.
ẏ = .6 ft/sec.
DELV (Ft./Sec.)
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.00
0
50
100
150
200
250
300
TR (Seconds)

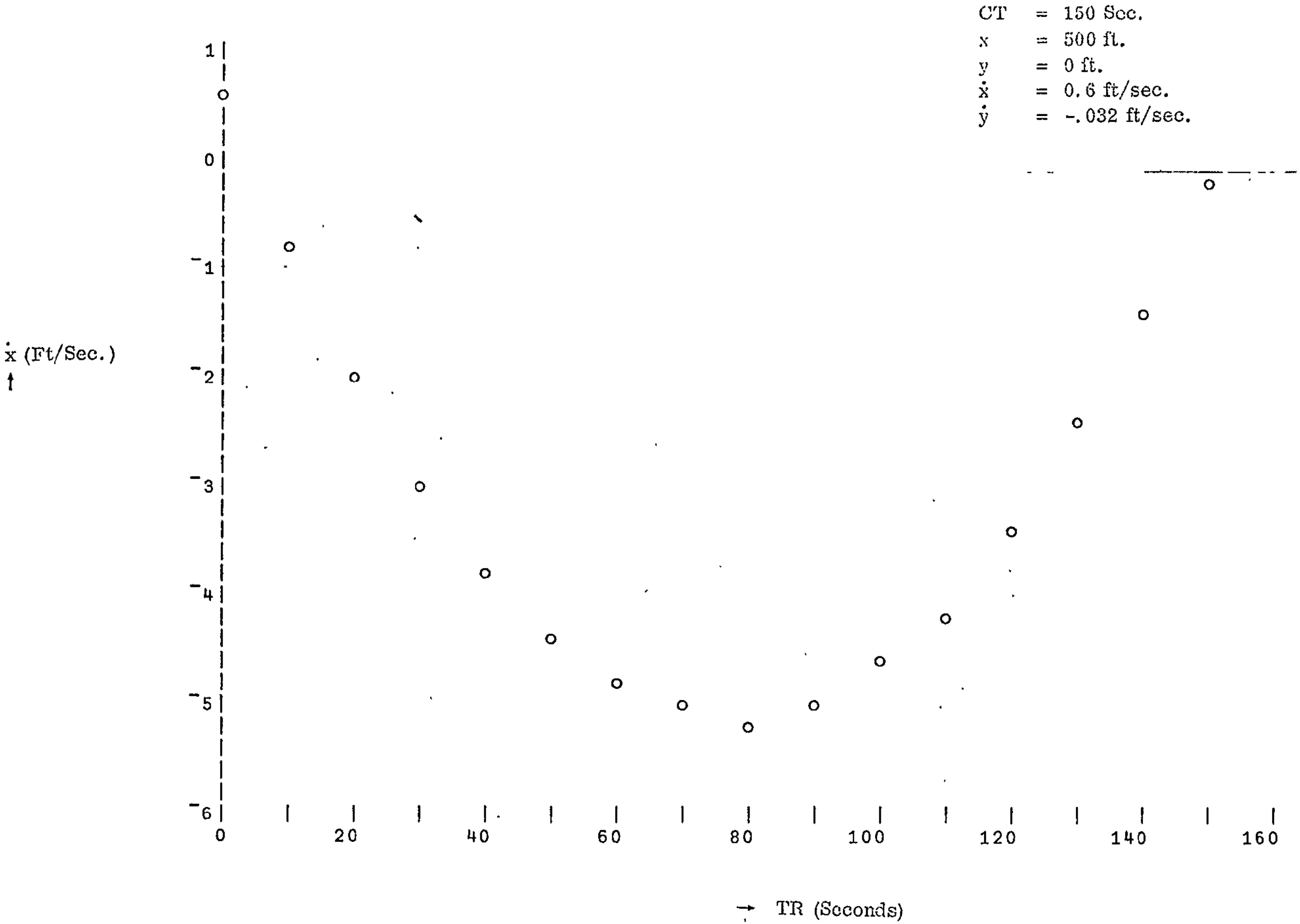

CT = 150 Sec.
x = 500 ft.
y = 0 ft.
ẋ = 0.6 ft/sec.
ẏ = -.032 ft/sec.
x ft.
500
400
300
200
100
0
0
20
40
60
80
100
120
140
160
TR (Seconds)

CT = 150 Sec.
x = 500 ft.
y = 0 ft.
$\dot{x}$ = 0.6 ft/sec.
$\dot{y}$ = -.032 ft/sec.

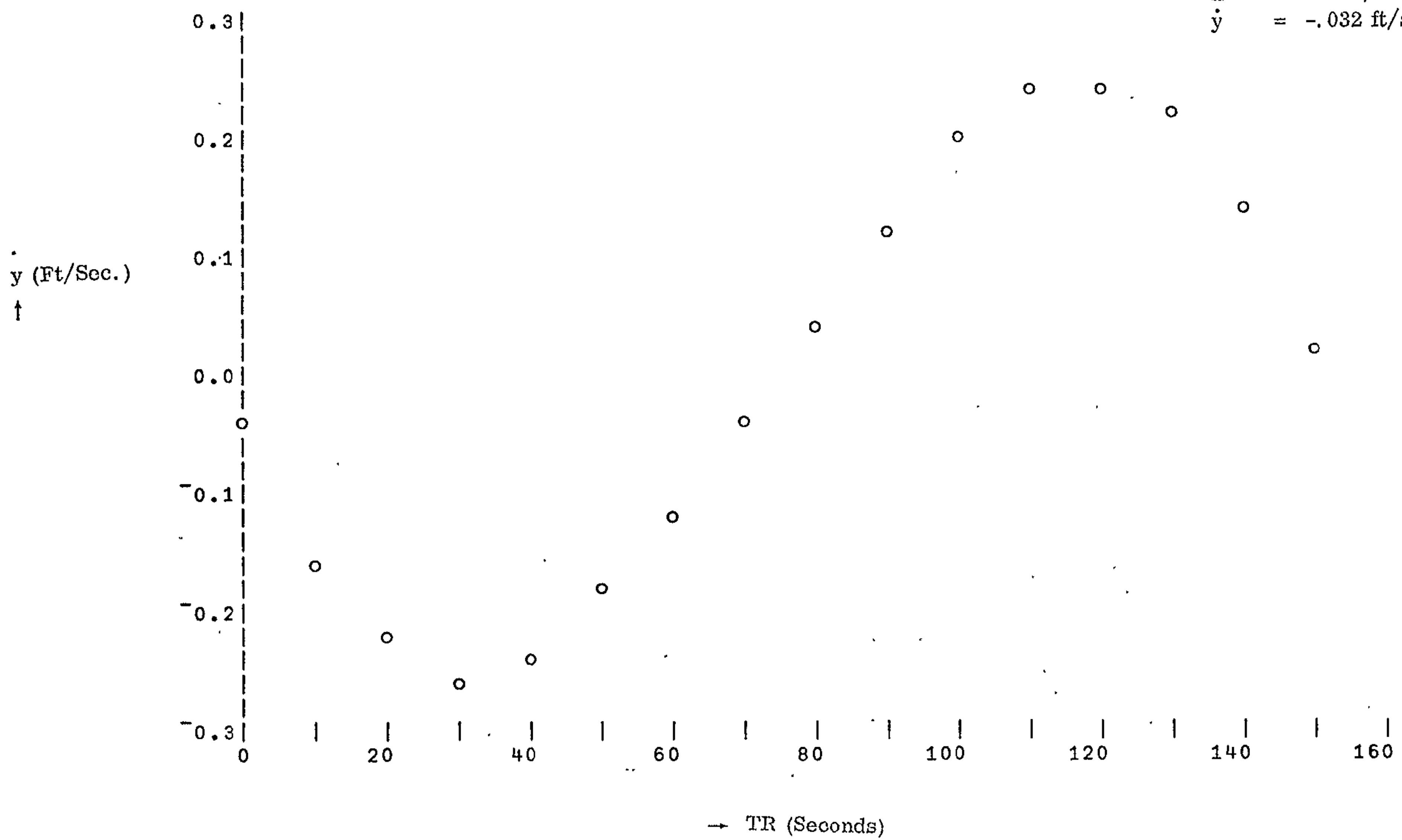


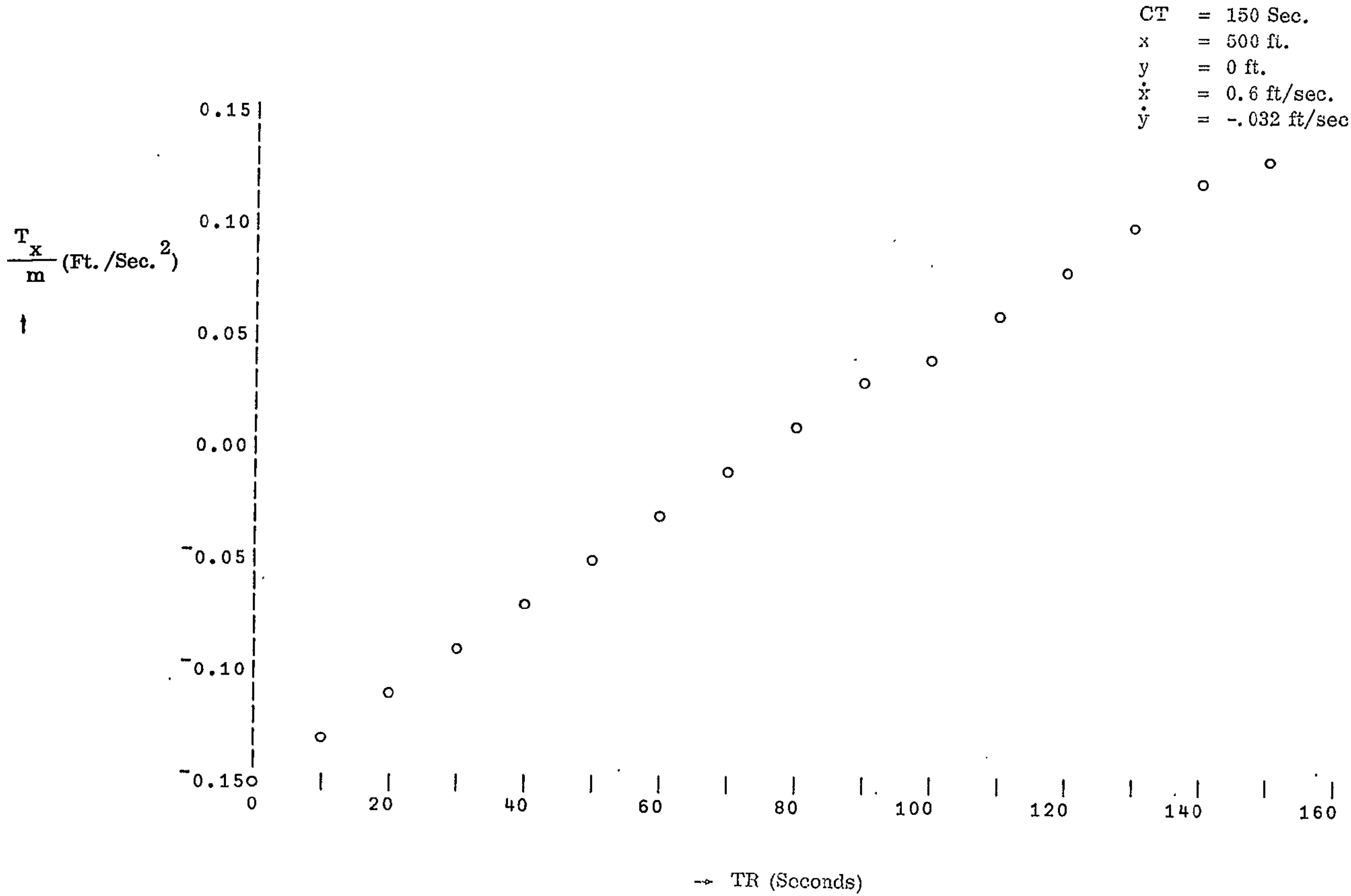
CT = 150 Sec.
x = 500 ft.
y = 0 ft.
$\dot{x}$ = 0.6 ft/sec.
$\dot{y}$ = -.032 ft/sec.



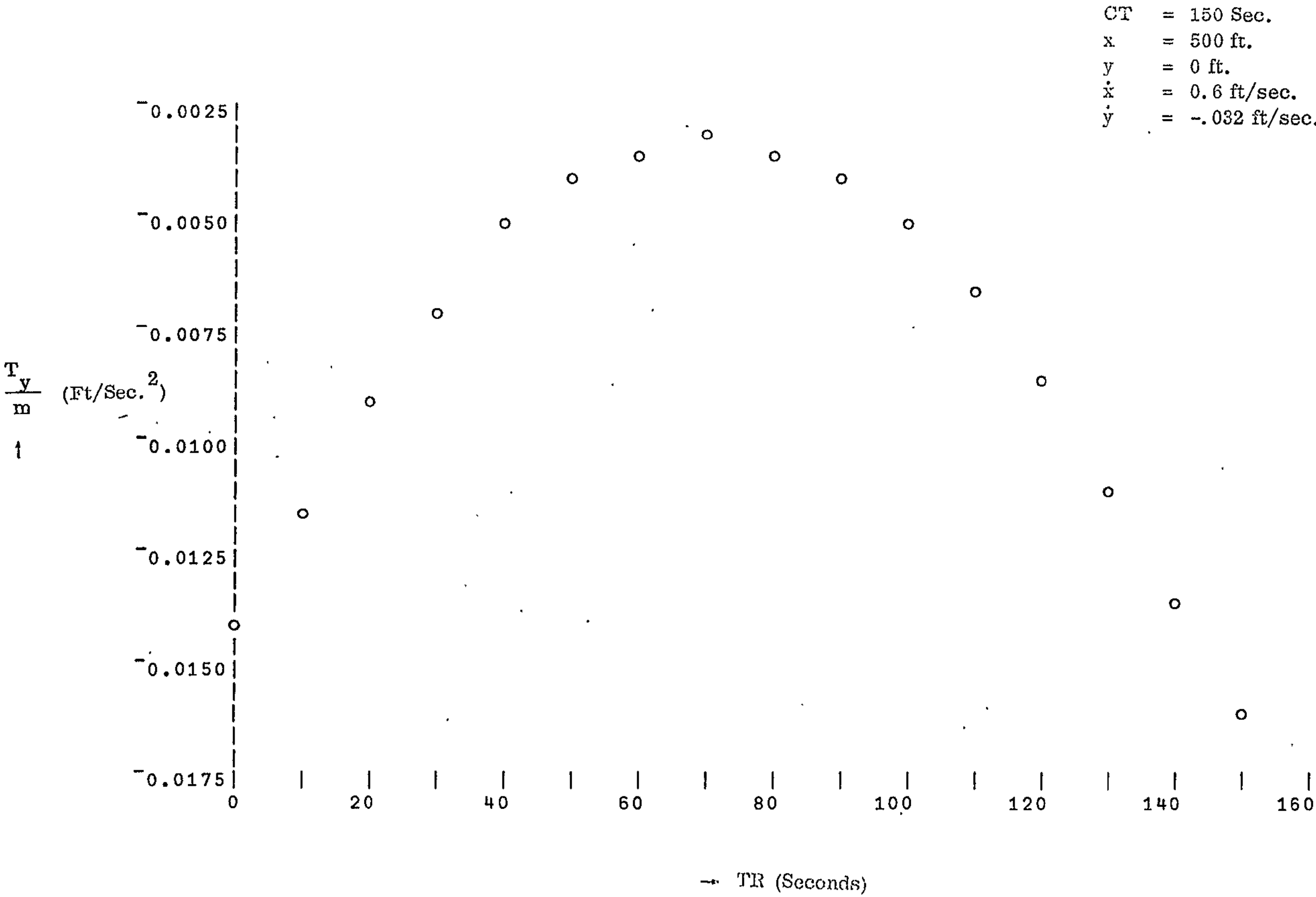

CT = 150 Sec.
x = 500 ft.
y = 0 ft.
ẋ = 0.6 ft/sec.
ẏ = -.032 ft/sec.
ẋ (Ft/Sec.)
1
0
-1
-2
-3
-4
-5
-6
0
20
40
60
80
100
120
140
160
TR (Seconds)

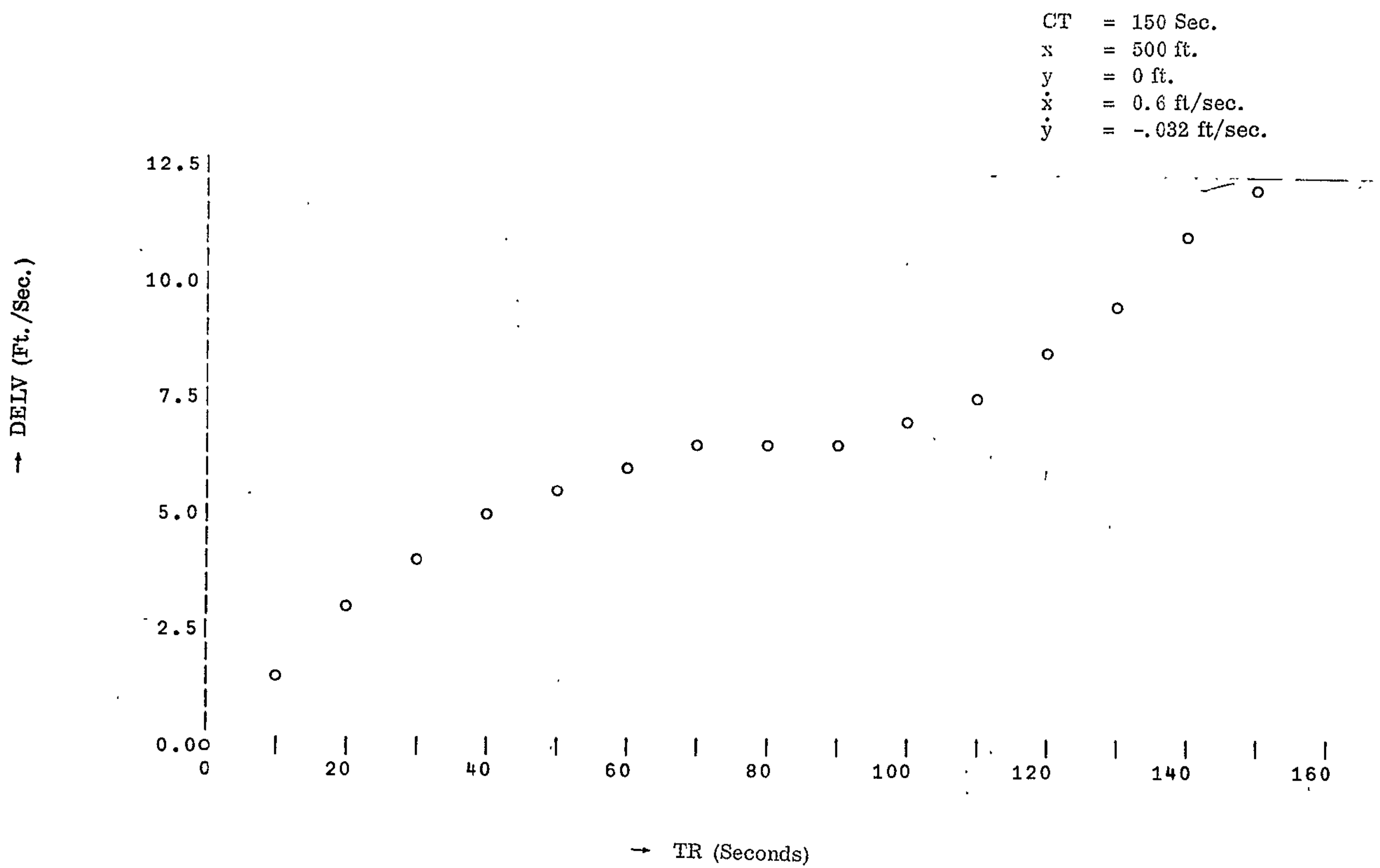
CT	=	150 Sec.
x	=	500 ft.
y	=	0 ft.
$\dot{x}$	=	0.6 ft/sec.
$\dot{y}$	=	-.032 ft/sec.

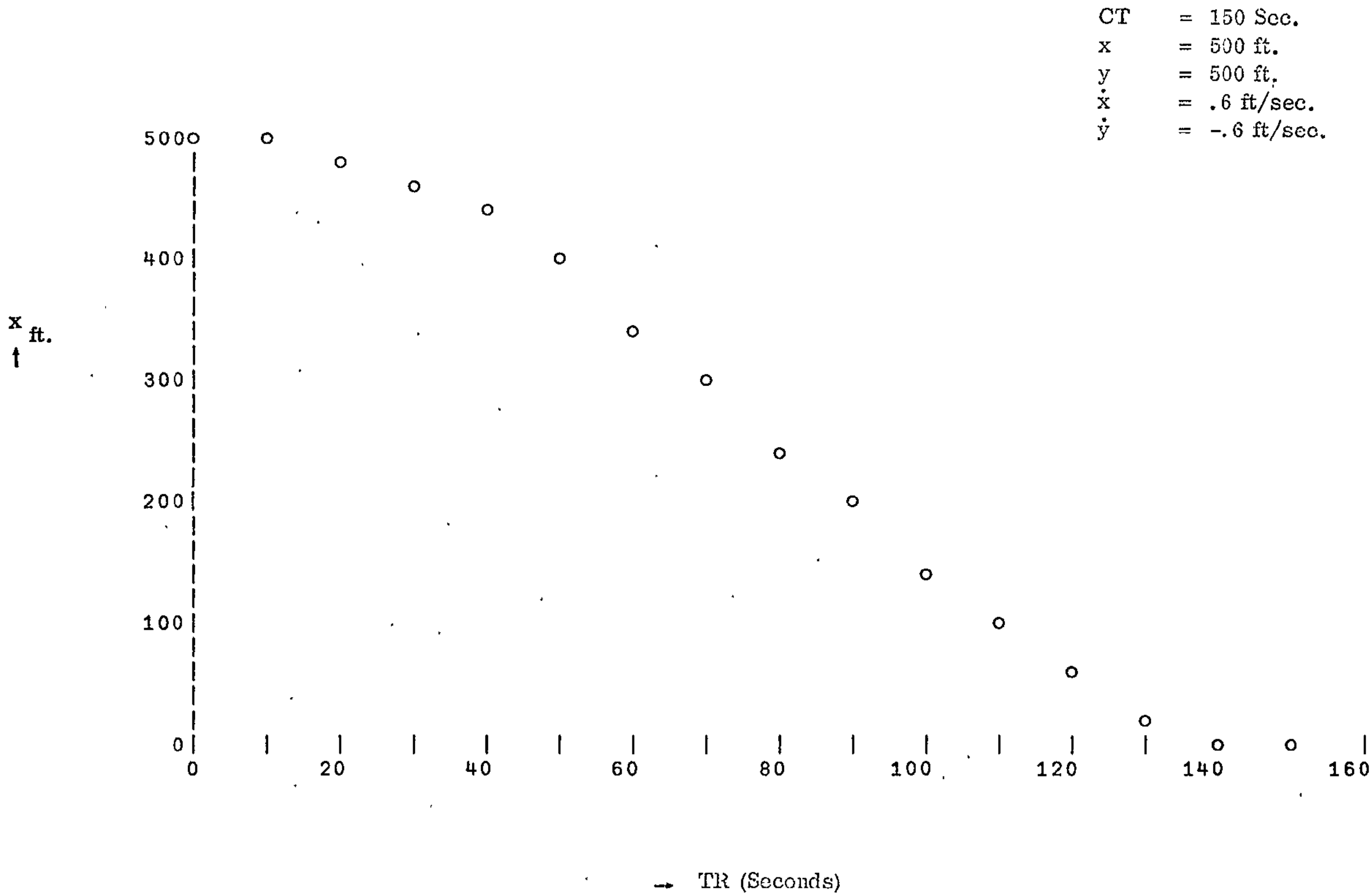




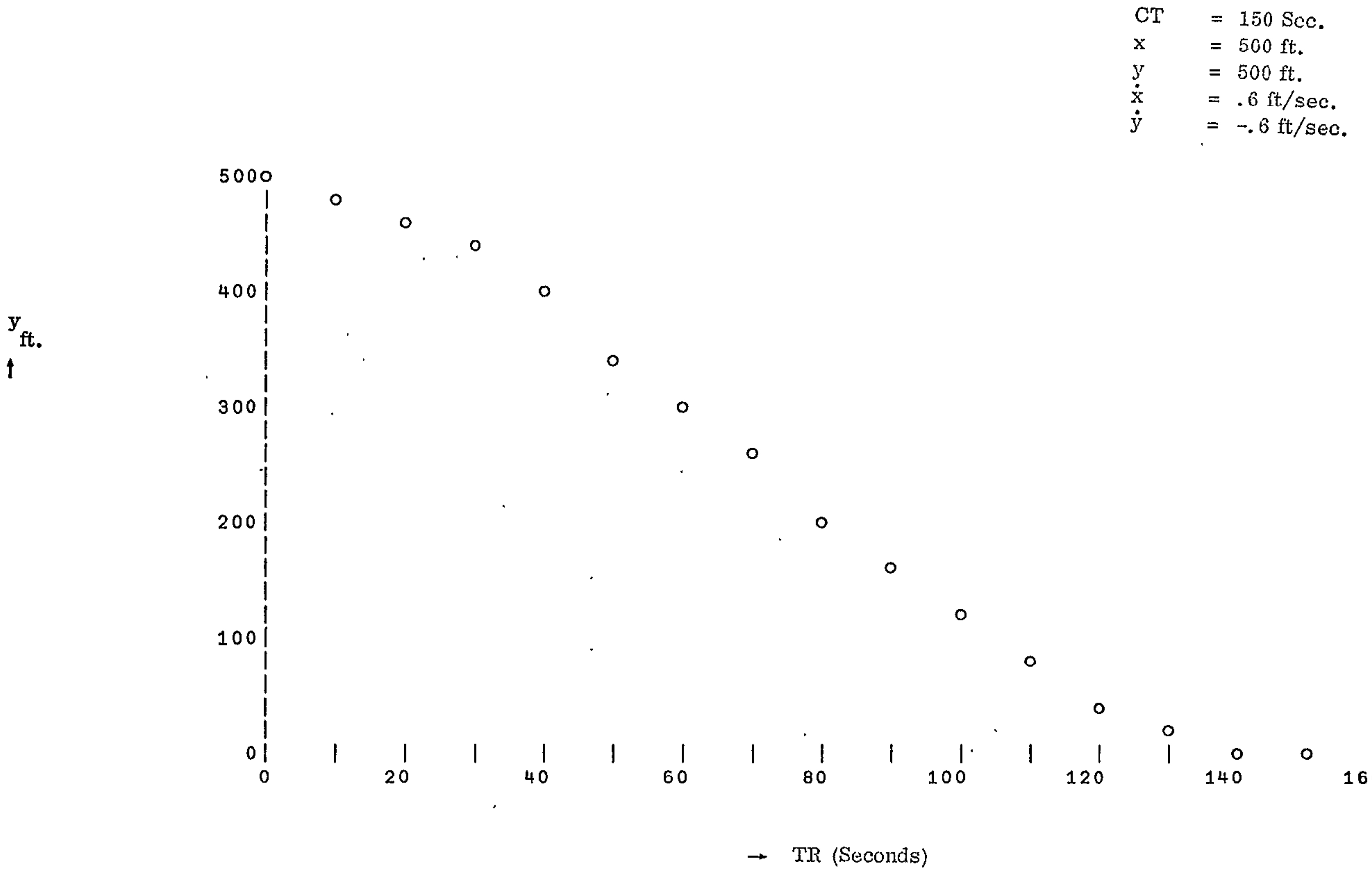
CT = 150 Sec.
x = 500 ft.
y = 0 ft.
$\dot{x}$ = 0.6 ft/sec.
$\dot{y}$ = -.032 ft/sec.
$\frac{T_x}{m}$ (Ft./Sec.^{2})
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
0
20
40
60
80
100
120
140
160
TR (Seconds)

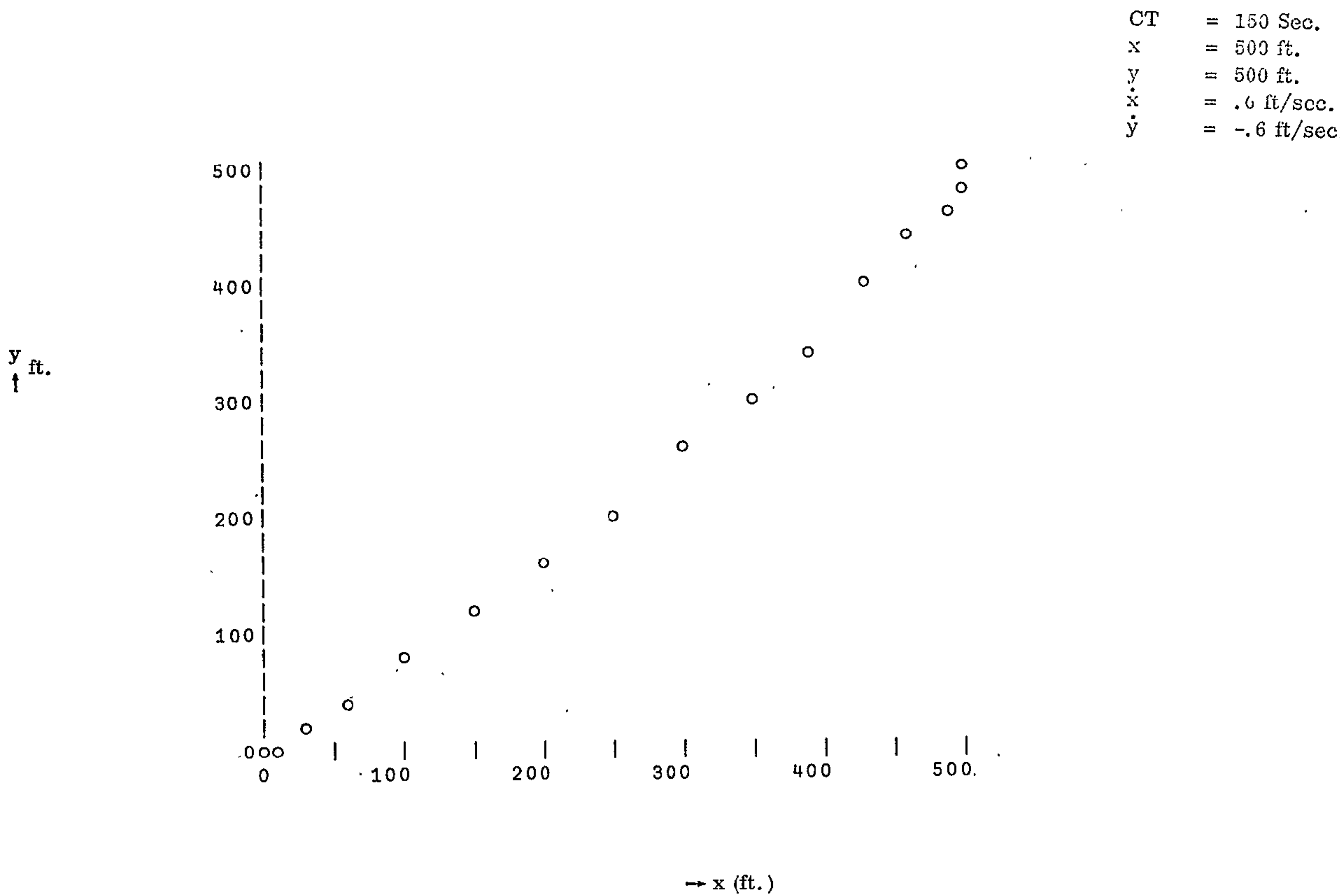

CT = 150 Sec.
x = 500 ft.
y = 0 ft.
ẋ = 0.6 ft/sec.
ẏ = -.032 ft/sec.
-0.0025
-0.0050
-0.0075
-0.0100
-0.0125
-0.0150
-0.0175
Ty/m (Ft/Sec.2)
0
20
40
60
80
100
120
140
160
TR (Seconds)

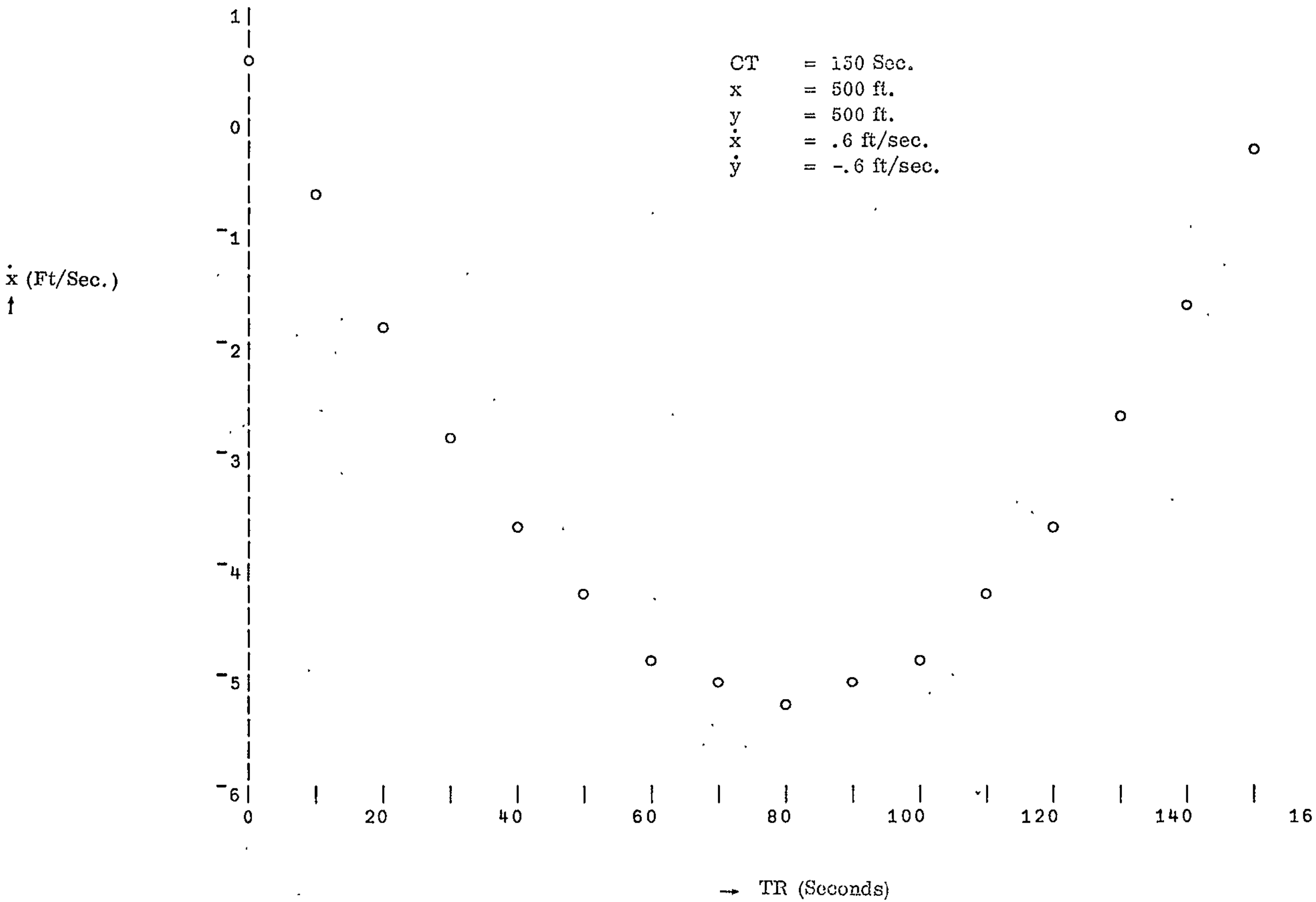

CT = 150 Sec.
x = 500 ft.
y = 0 ft.
ẋ = 0.6 ft/sec.
ẏ = -.032 ft/sec.
12.5
10.0
7.5
5.0
2.5
0.00
0
20
40
60
80
100
120
140
160
DELV (Ft./Sec.)
TR (Seconds)



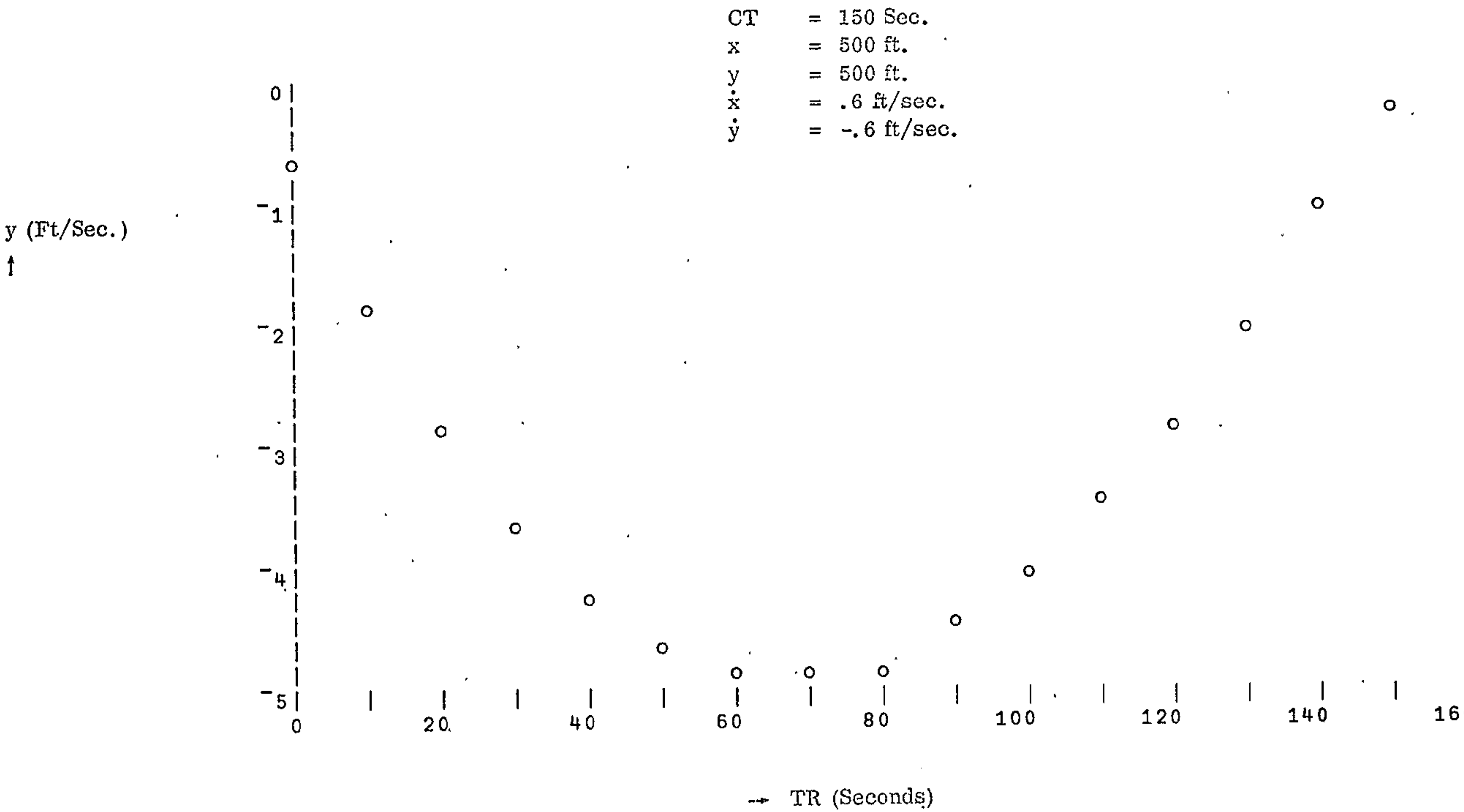
CT = 150 Sec.
x = 500 ft.
y = 500 ft.
ẋ = .6 ft/sec.
ẏ = -.6 ft/sec.
x ft.
500
400
300
200
100
0
0
20
40
60
80
100
120
140
160
TR (Seconds)


CT = 150 Sec.
x = 500 ft.
y = 500 ft.
$\dot{x}$ = .6 ft/sec.
$\dot{y}$ = -.6 ft/sec.
y ft.
500
400
300
200
100
0
0
20
40
60
80
100
120
140
160
TR (Seconds)

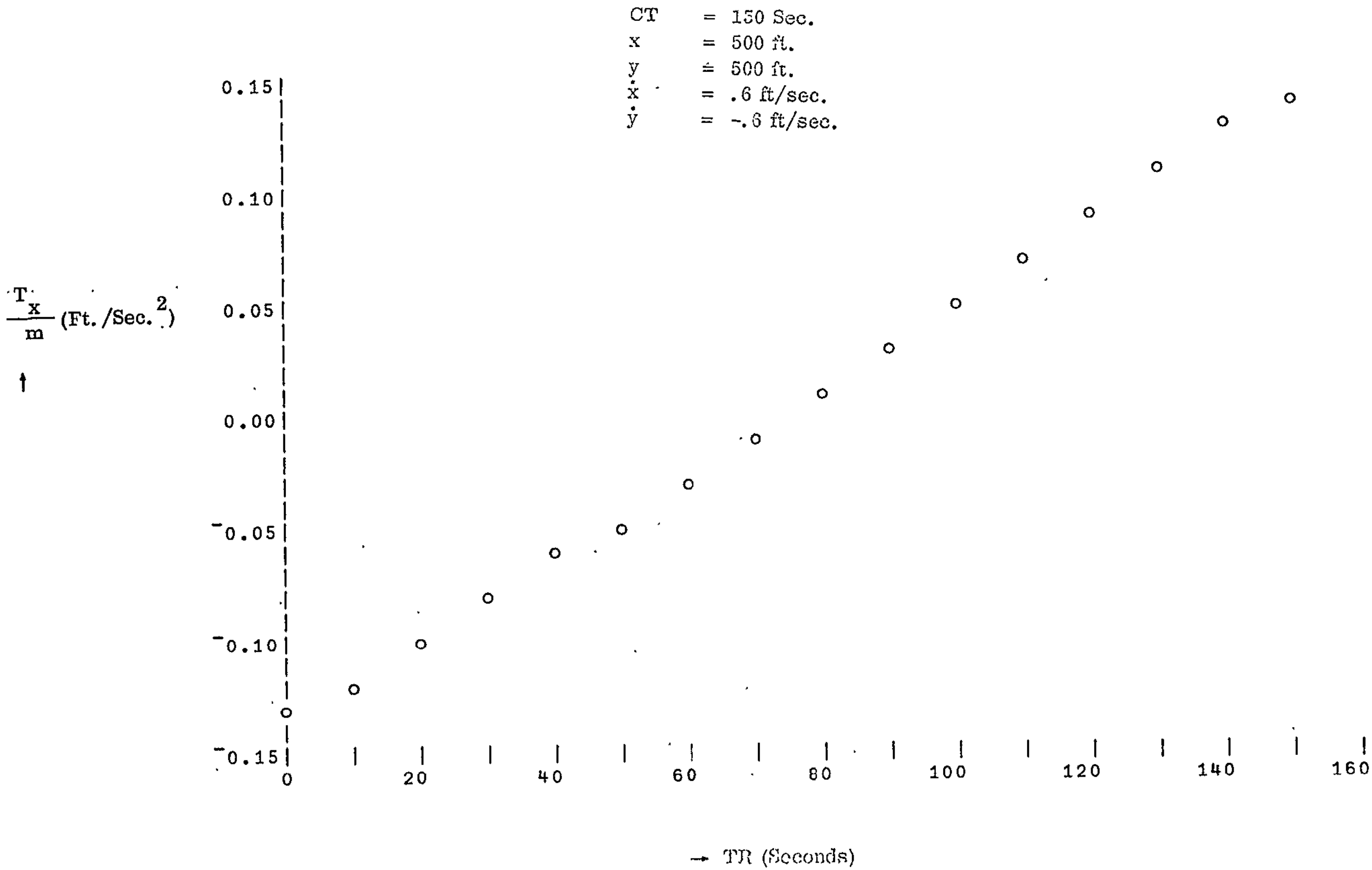

CT = 150 Sec.
x = 500 ft.
y = 500 ft.
$\dot{x}$ = .6 ft/sec.
$\dot{y}$ = -.6 ft/sec.
500
400
300
200
100
y ft.
0
100
200
300
400
500
x (ft.)



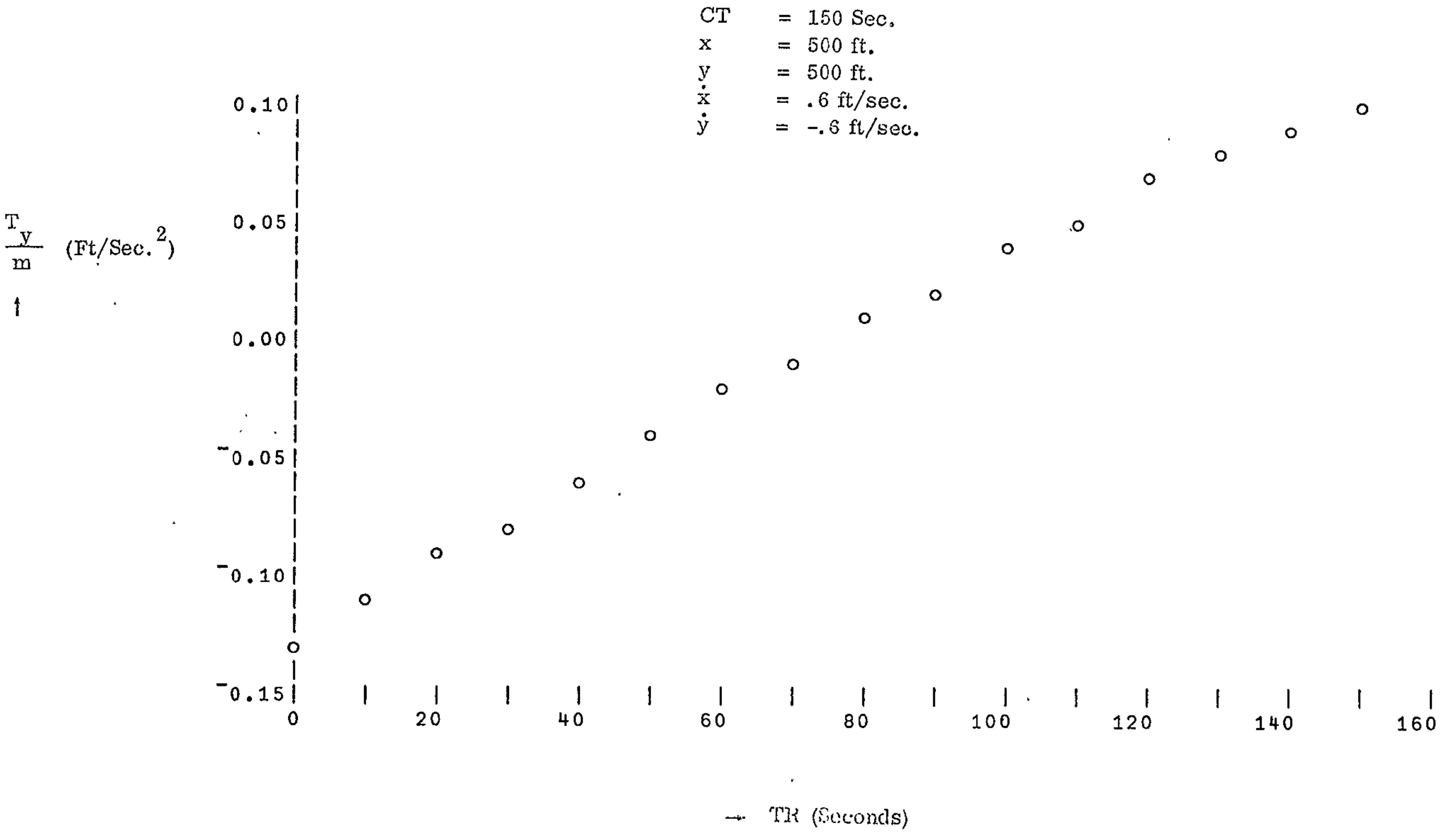
CT = 150 Sec.
x = 500 ft.
y = 500 ft.
ẋ = .6 ft/sec.
ẏ = -.6 ft/sec.
ẋ (Ft/Sec.)
1
0
-1
-2
-3
-4
-5
-6
0
20
40
60
80
100
120
140
160
TR (Seconds)



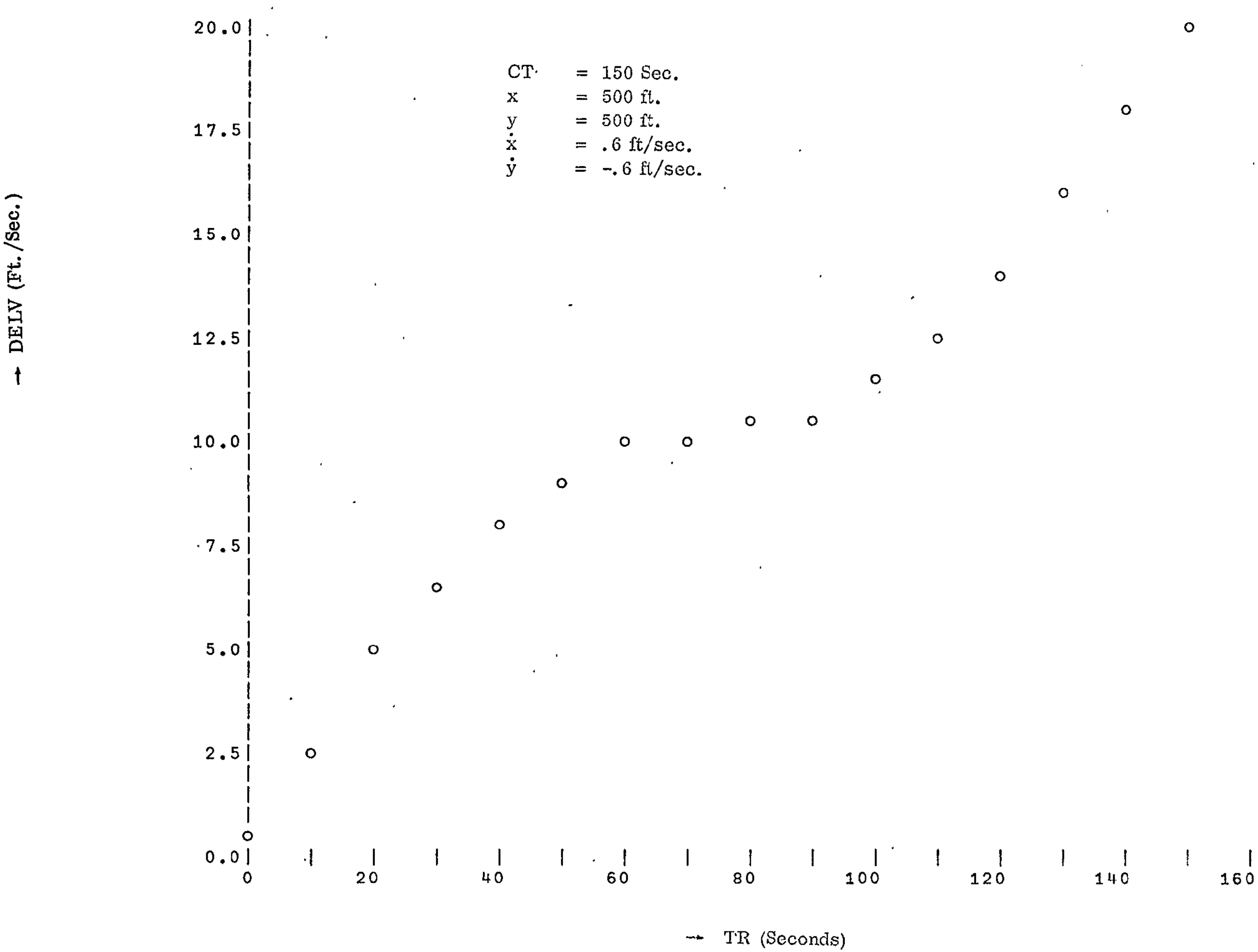
CT = 150 Sec.
x = 500 ft.
y = 500 ft.
$\dot{x}$ = .6 ft/sec.
$\dot{y}$ = -.6 ft/sec.
y (Ft/Sec.)
0
-1
-2
-3
-4
-5
0
20
40
60
80
100
120
140
160
TR (Seconds)

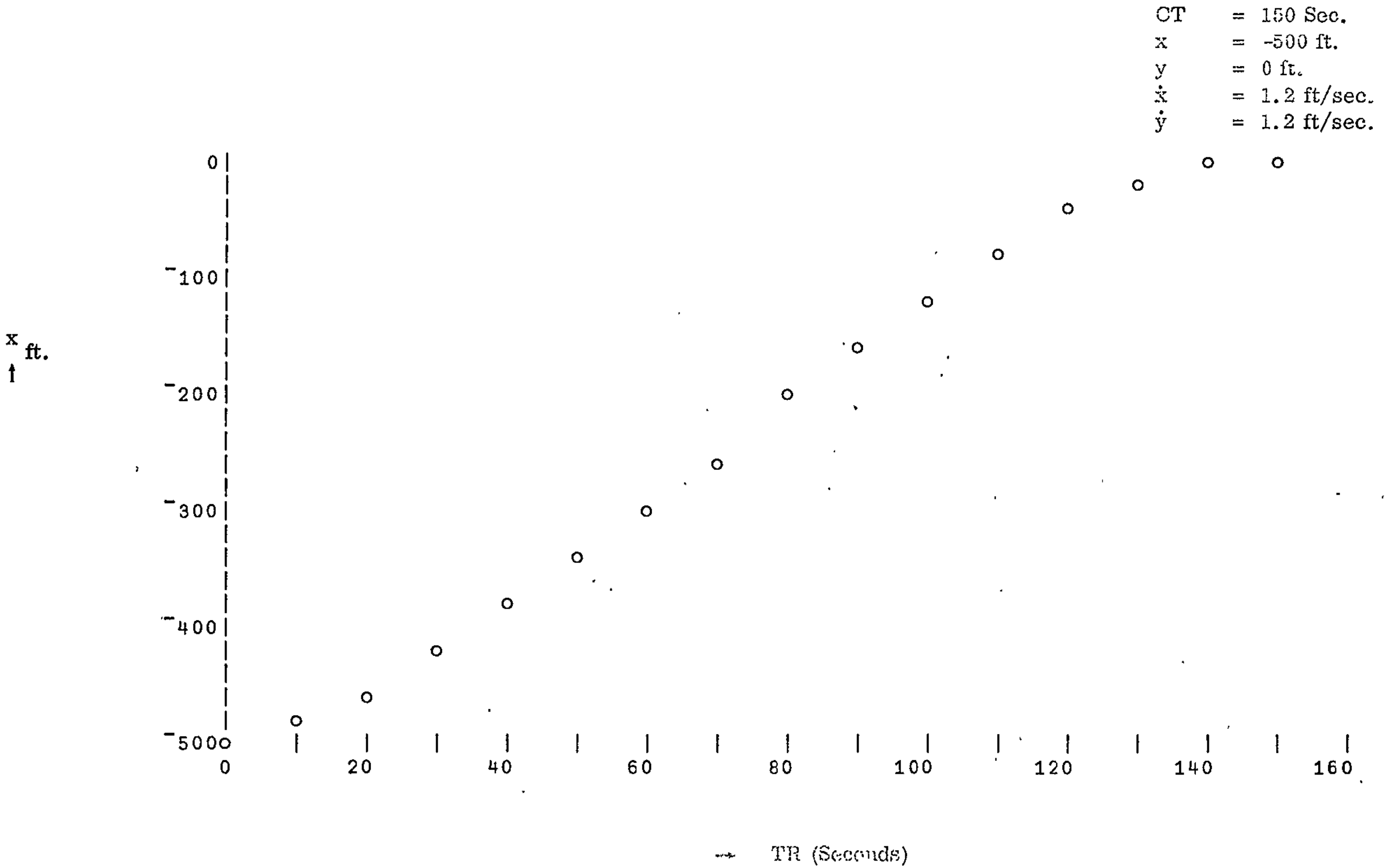


CT = 150 Sec.
x = 500 ft.
y = 500 ft.
ẋ = .6 ft/sec.
ẏ = -.6 ft/sec.
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
Tx/m (Ft./Sec.²)
0
20
40
60
80
100
120
140
160
TR (Seconds)

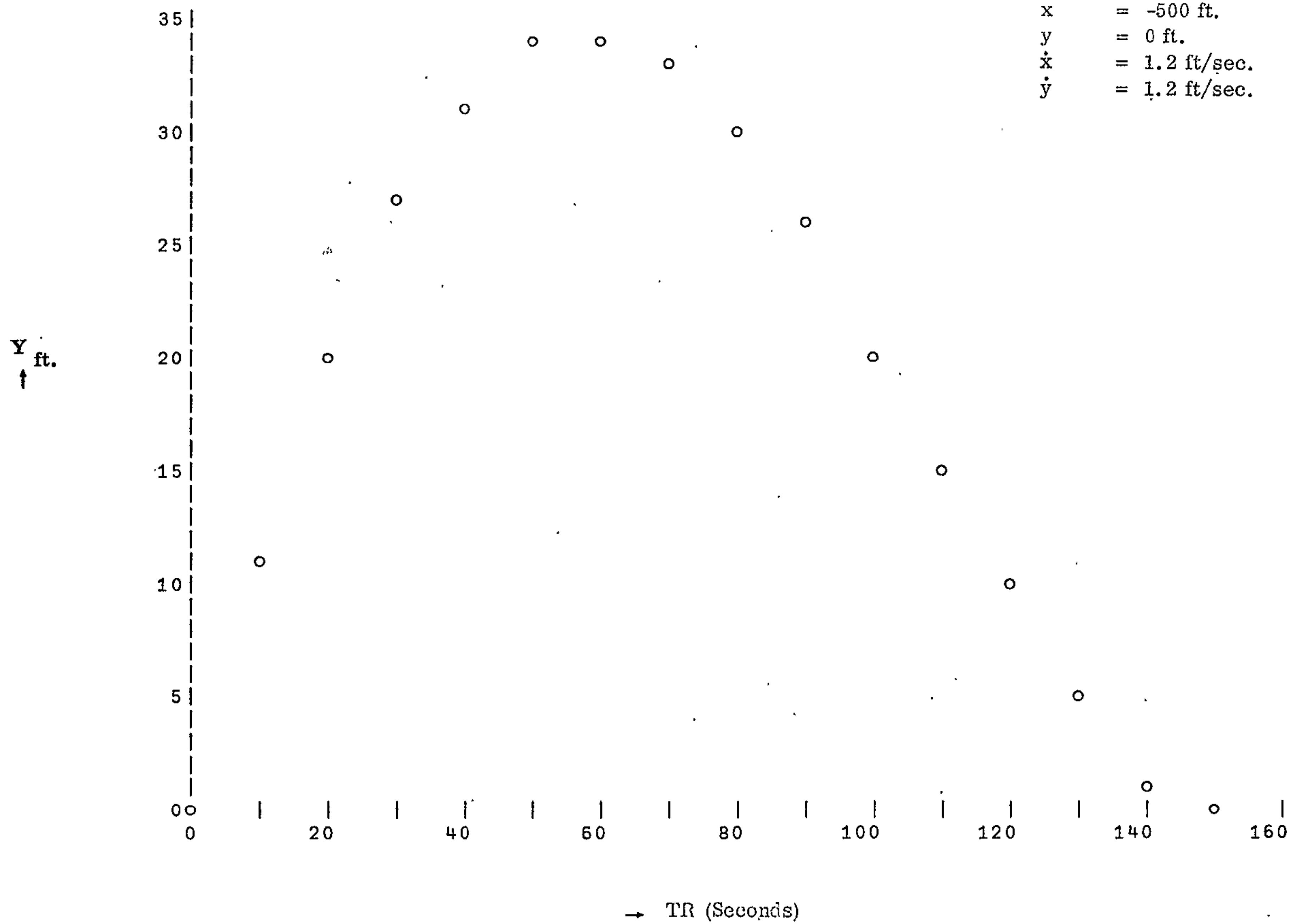


CT = 150 Sec.
x = 500 ft.
y = 500 ft.
$\dot{x}$ = .6 ft/sec.
$\dot{y}$ = -.6 ft/sec.
$\frac{T_y}{m}$ (Ft/Sec.^{2})
0.10
0.05
0.00
-0.05
-0.10
-0.15
0
20
40
60
80
100
120
140
160
→ TR (Seconds)

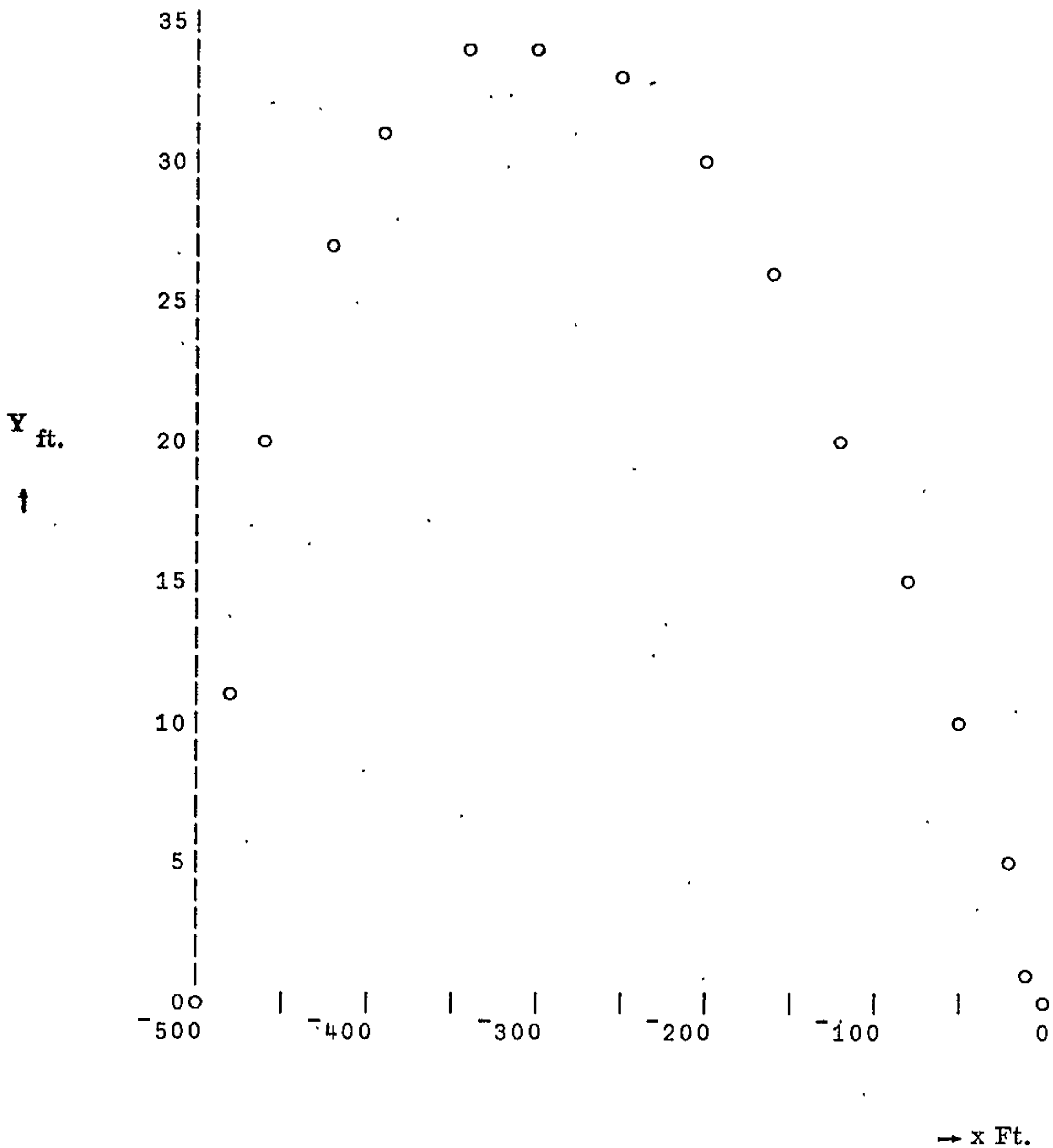

CT = 150 Sec.
x = 500 ft.
y = 500 ft.
ẋ = .6 ft/sec.
ẏ = -.6 ft/sec.
DELV (Ft./Sec.)
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
0
20
40
60
80
100
120
140
160
TR (Seconds)


CT = 150 Sec.
x = -500 ft.
y = 0 ft.
ẋ = 1.2 ft/sec.
ẏ = 1.2 ft/sec.
x ft.
0
-100
-200
-300
-400
-500
0
20
40
60
80
100
120
140
160
TR (Seconds)

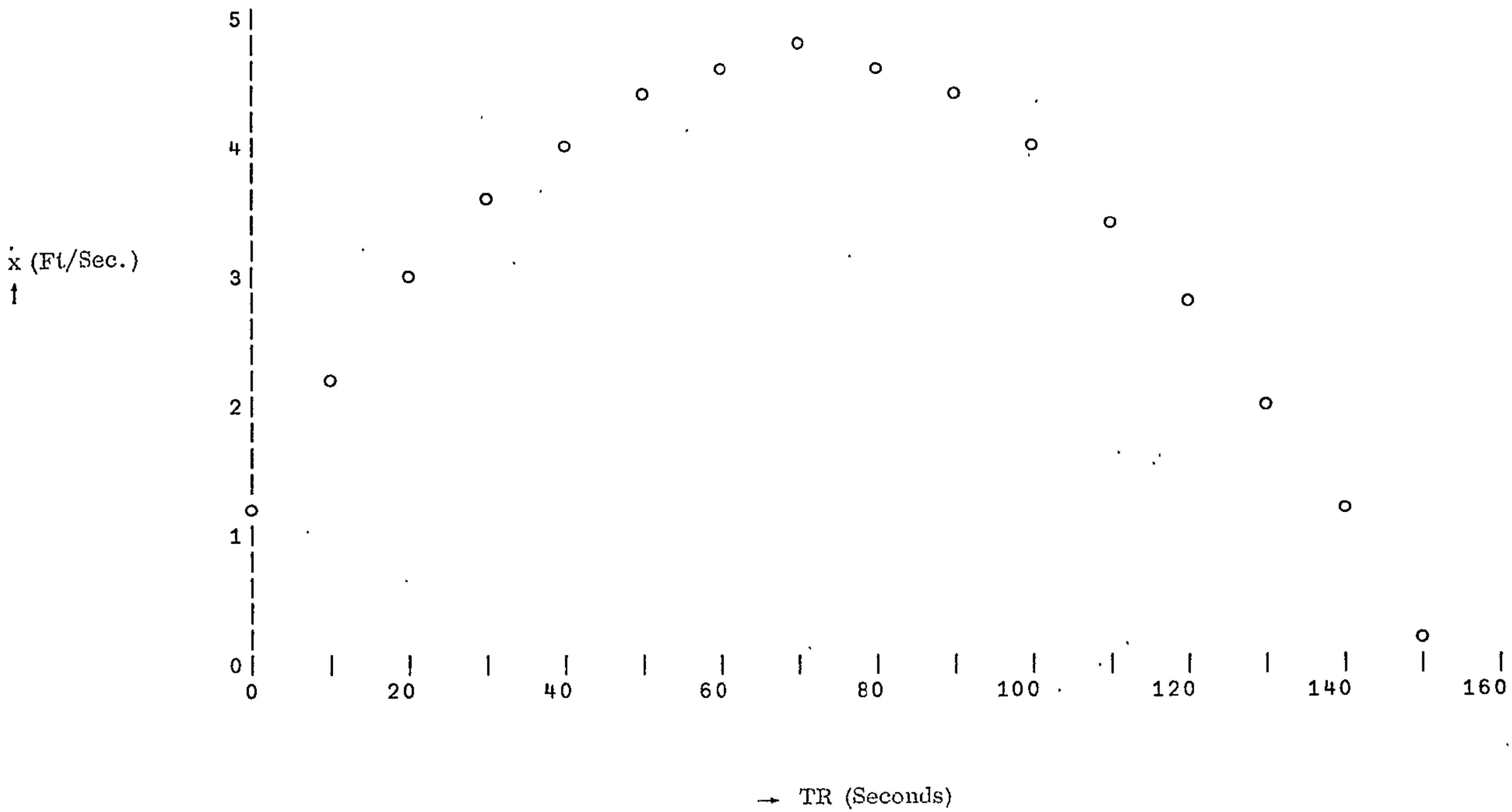
CT	=	150 Sec.
x	=	-500 ft.
y	=	0 ft.
$\dot{x}$	=	1.2 ft/sec.
$\dot{y}$	=	1.2 ft/sec.

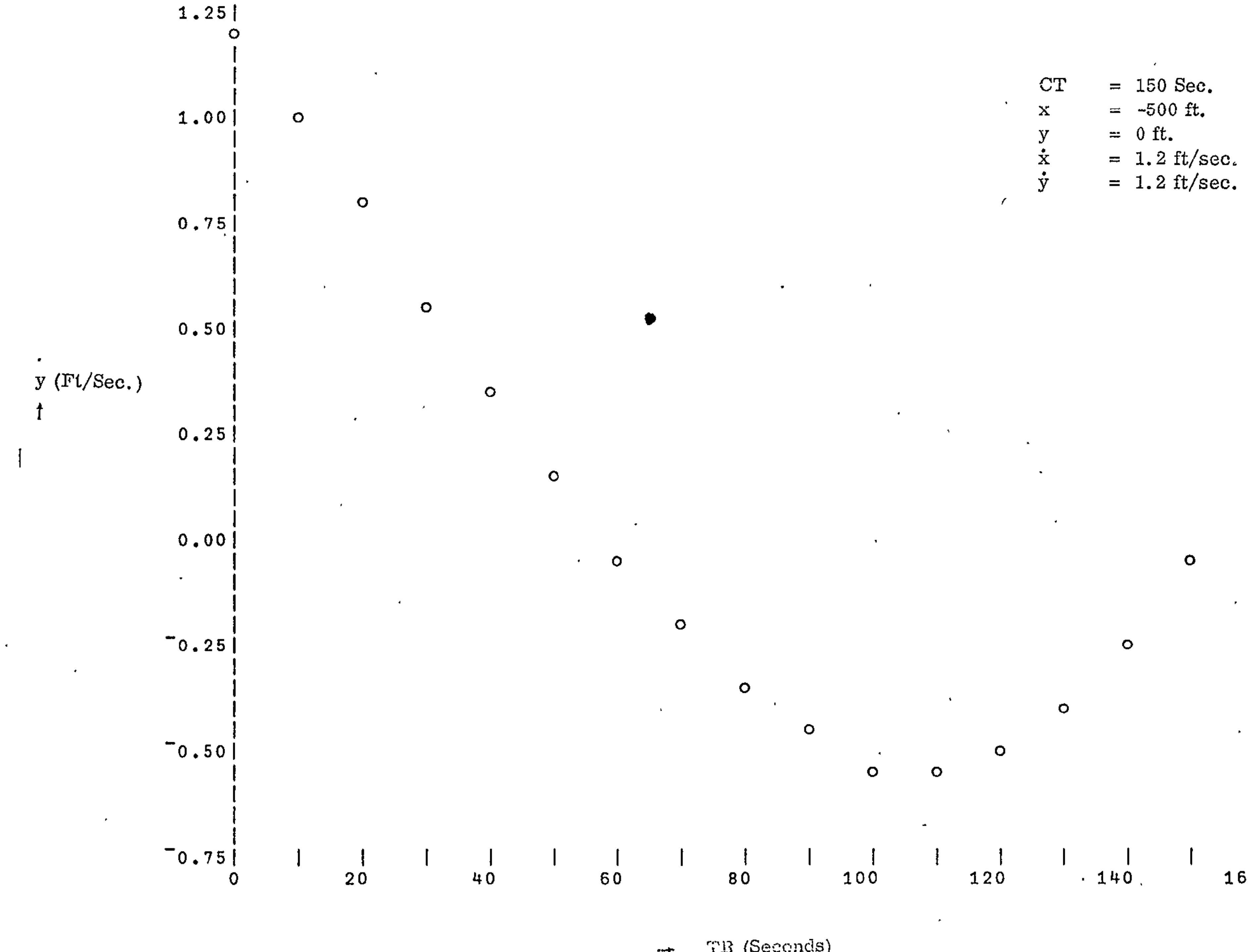


CT	=	150 Sec.
x	=	-500 ft.
y	=	0 ft.
$\dot{x}$	=	1.2 ft/sec.
$\dot{y}$	=	1.2 ft/sec.



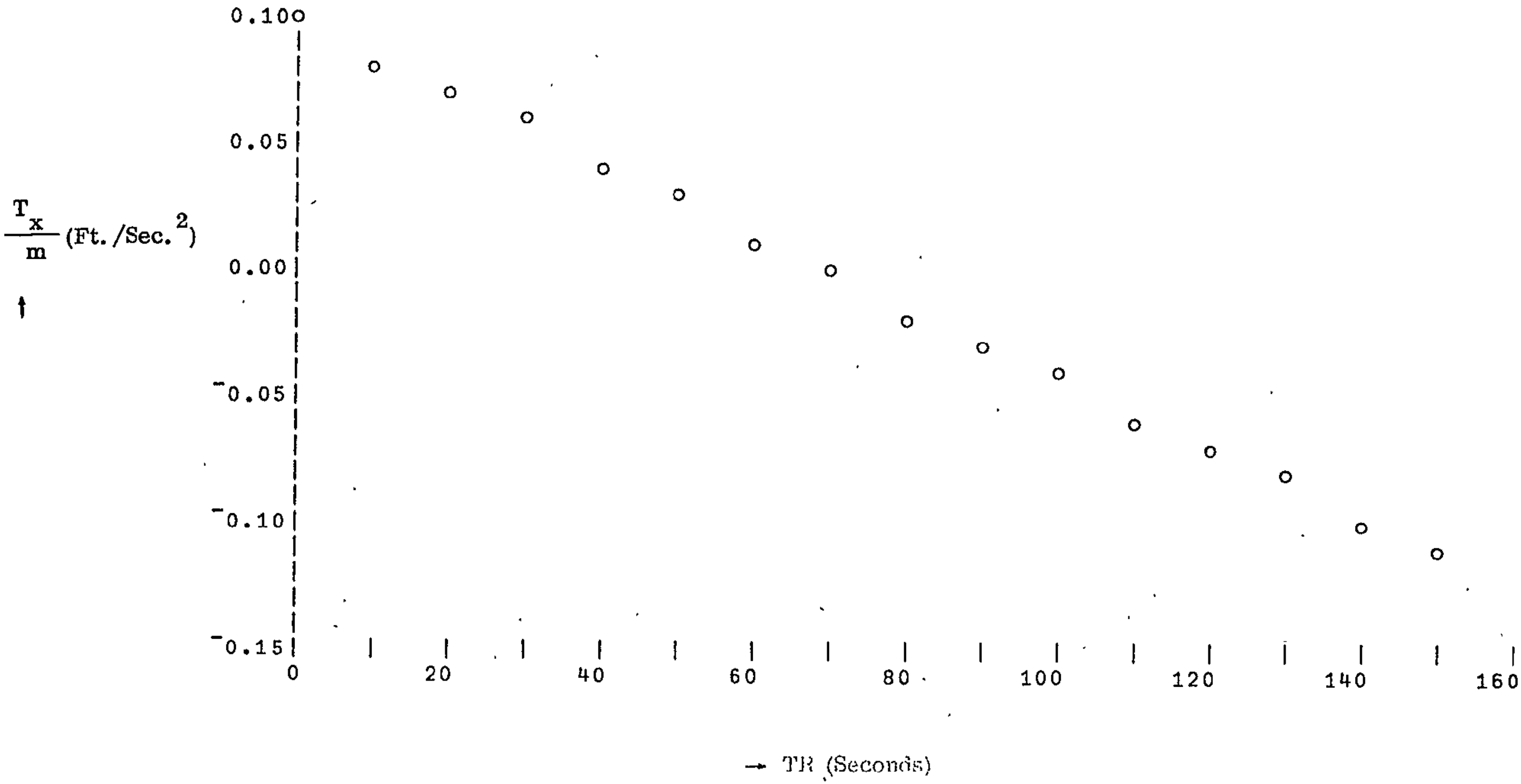
CT	=	150 Sec.
x	=	-500 ft.
$\dot{y}$	=	0 ft.
$\dot{x}$	=	1.2 ft/sec.
$\dot{y}$	=	1.2 ft/sec.



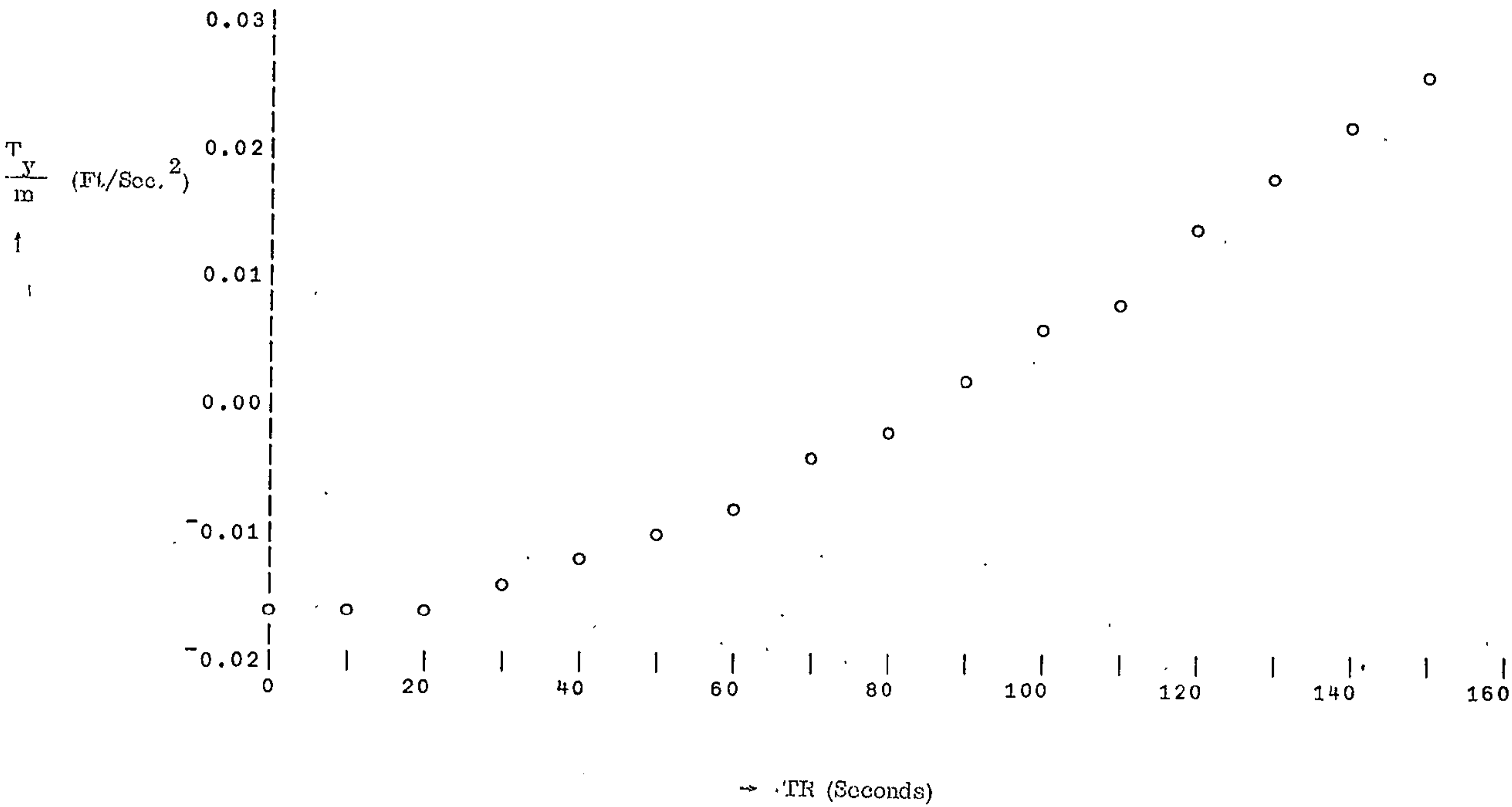


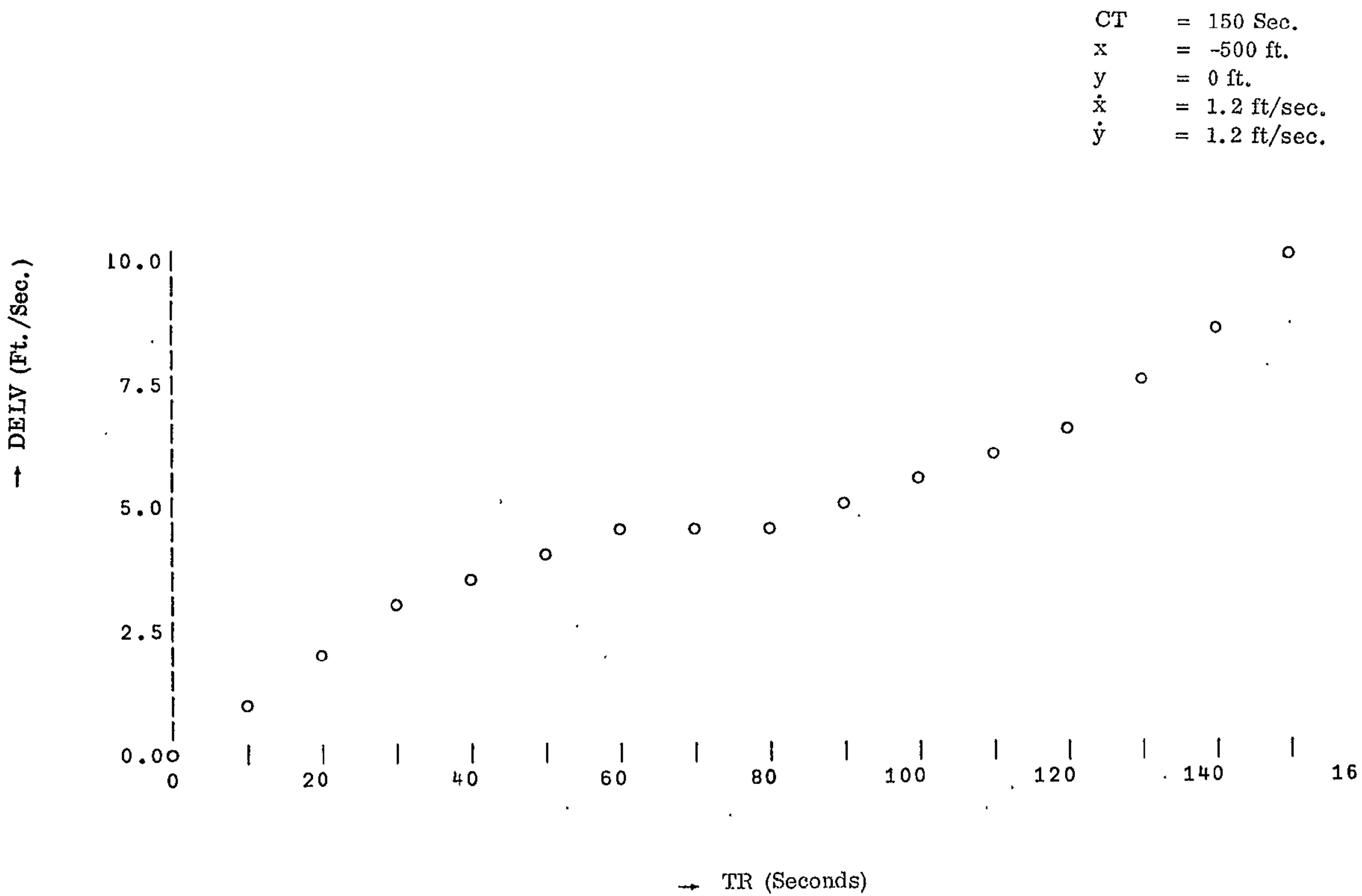
CT = 150 Sec.
x = -500 ft.
y = 0 ft.
$\dot{x}$ = 1.2 ft/sec.
$\dot{y}$ = 1.2 ft/sec.
$\dot{y}$ (Ft/Sec.)
1.25
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
0
20
40
60
80
100
120
140
160
TR (Seconds)

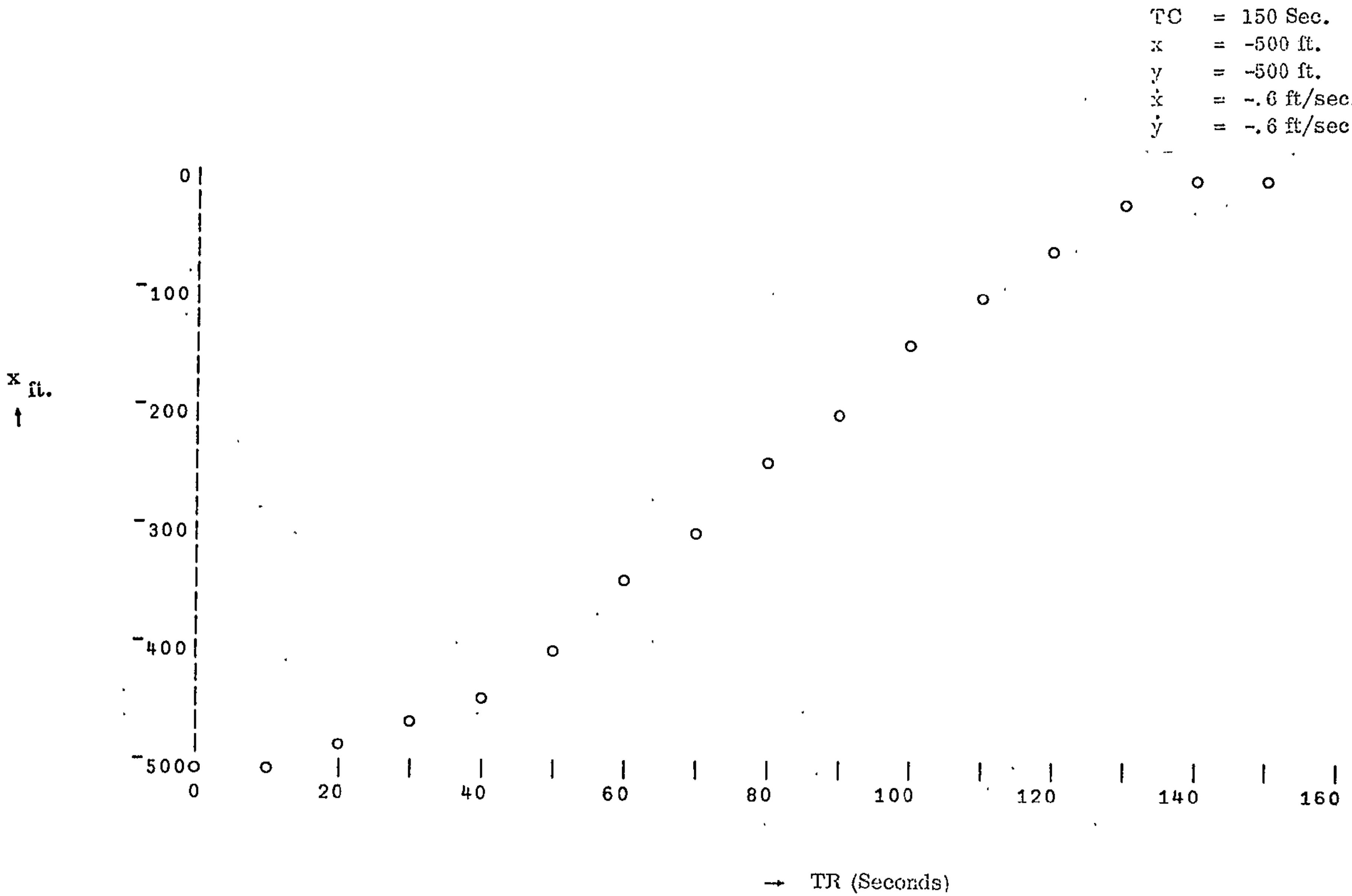
CT	=	150 Sec.
x	=	-500 ft.
y	=	0 ft.
$\dot{x}$	=	1.2 ft/sec.
$\dot{y}$	=	1.2 ft/sec.



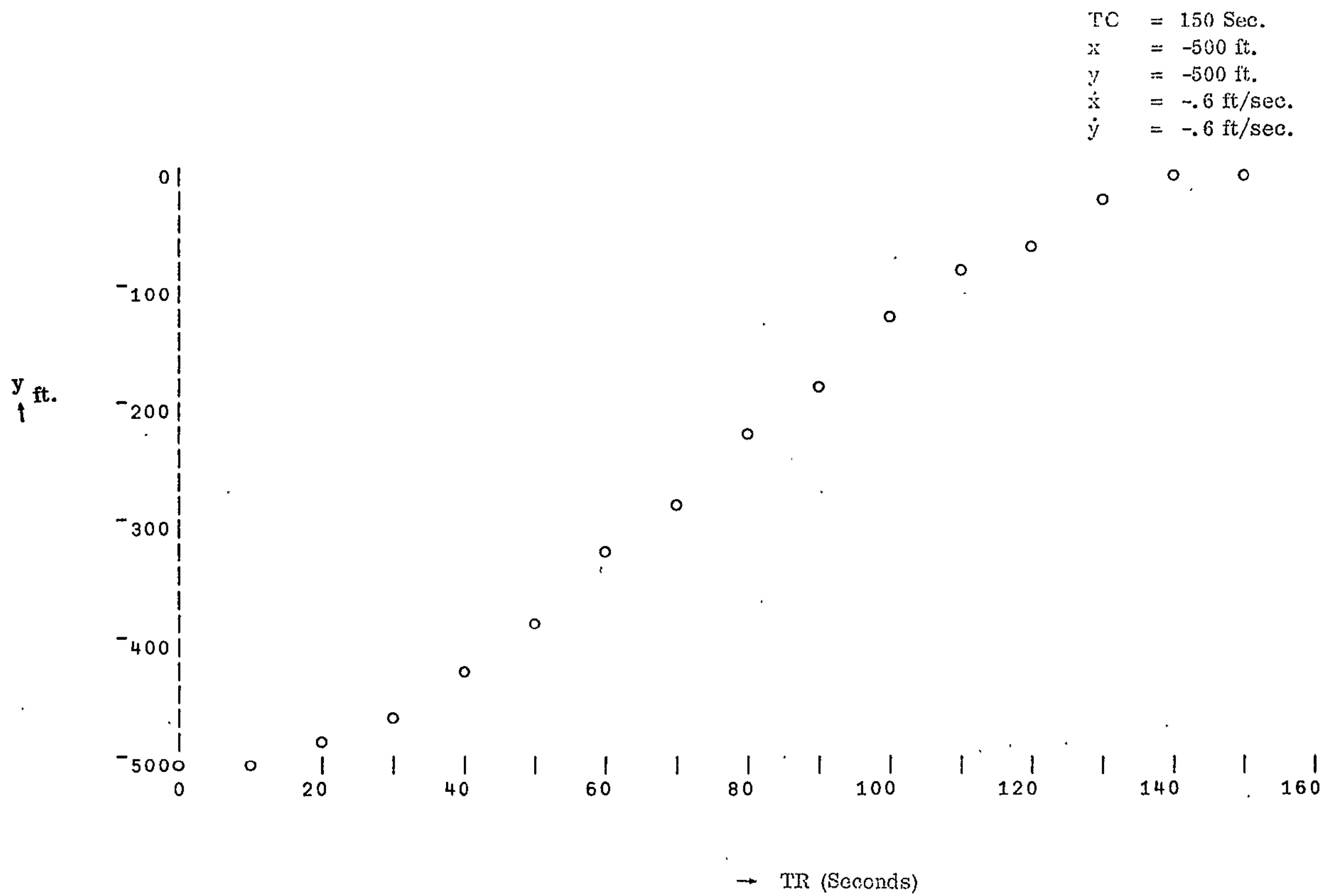
CT = 150 Sec.
x = -500 ft.
y = 0 ft.
$\dot{x}$ = 1.2 ft/sec.
$\dot{y}$ = 1.2 ft/sec.




CT = 150 Sec.
x = -500 ft.
y = 0 ft.
$\dot{x}$ = 1.2 ft/sec.
$\dot{y}$ = 1.2 ft/sec.
DELV (Ft./Sec.)
10.0
7.5
5.0
2.5
0.00
0
20
40
60
80
100
120
140
160
TR (Seconds)

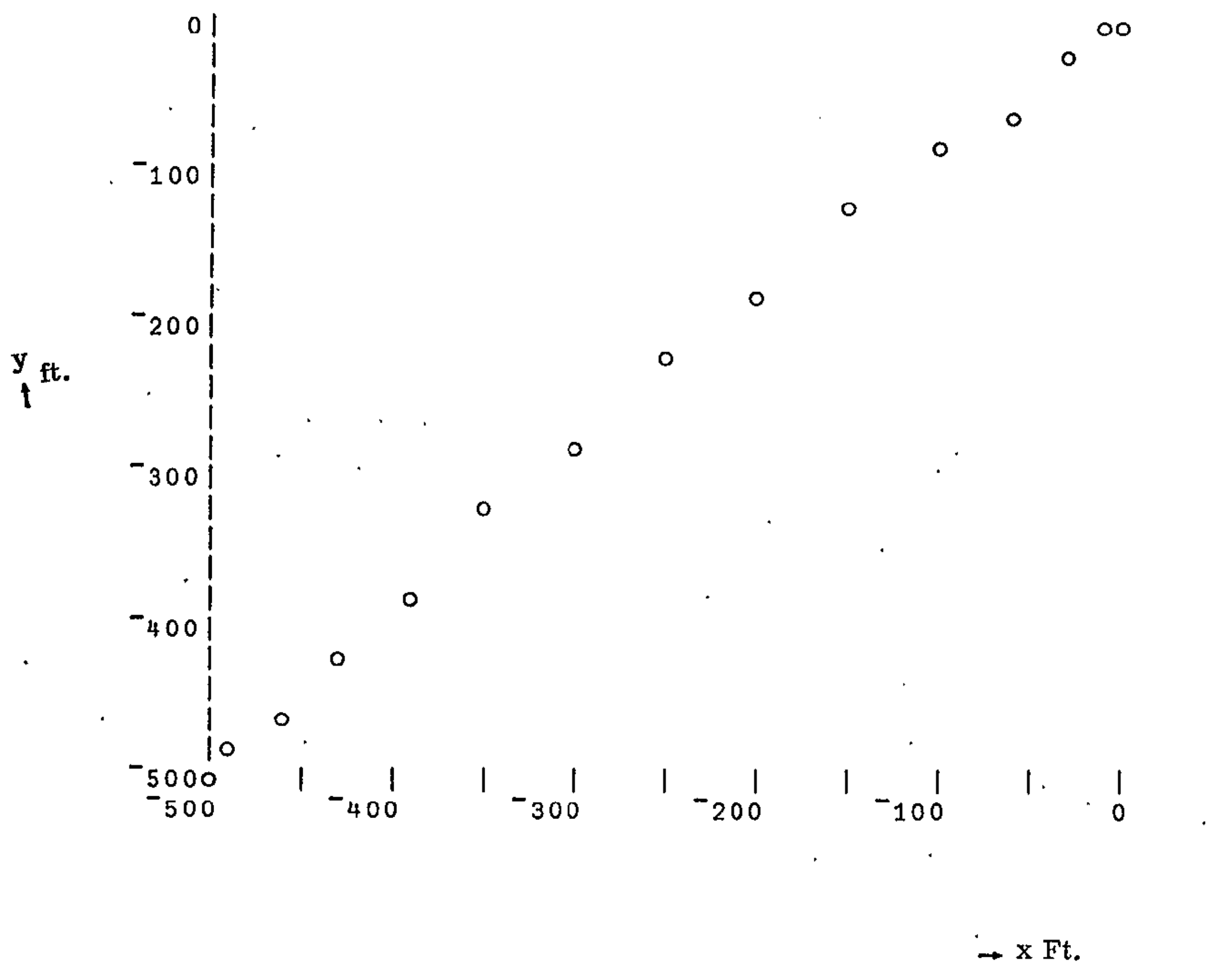


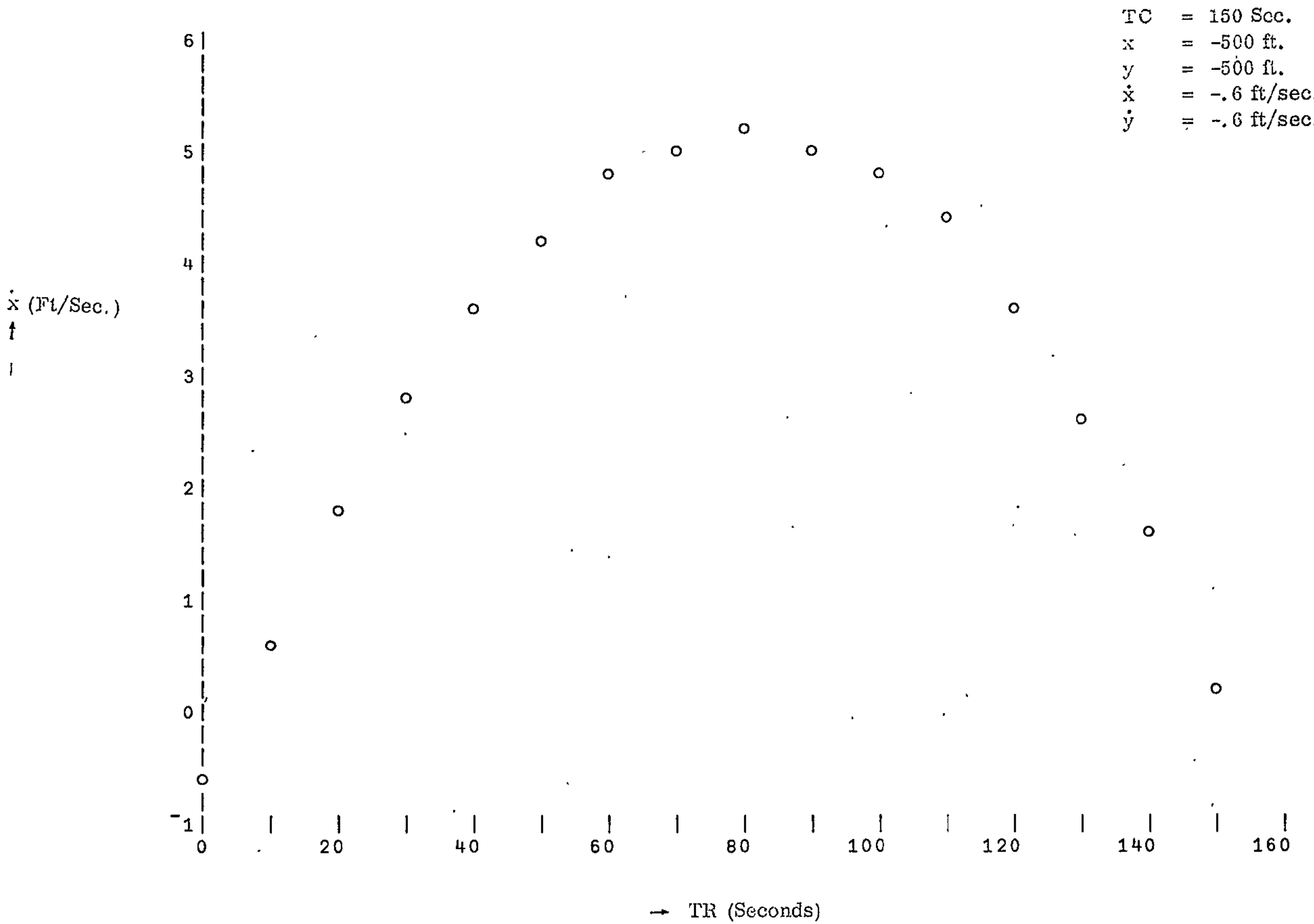
TC = 150 Sec.
x = -500 ft.
y = -500 ft.
ẋ = -.6 ft/sec.
ẏ = -.6 ft/sec.
x ft.
0
-100
-200
-300
-400
-500
0
20
40
60
80
100
120
140
160
TR (Seconds)



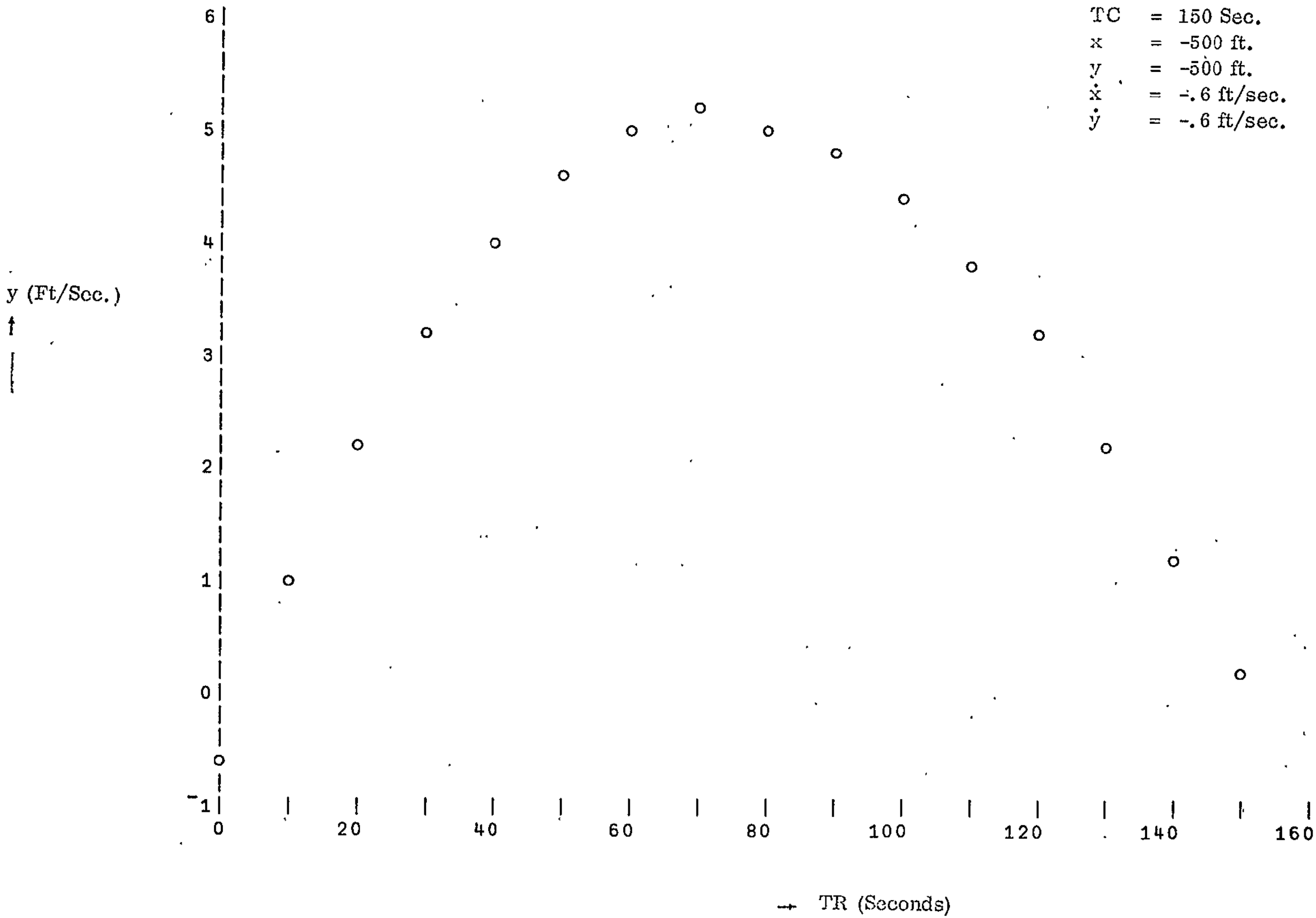
TC = 150 Sec.
x = -500 ft.
y = -500 ft.
ẋ = -.6 ft/sec.
ẏ = -.6 ft/sec.
y ft.
0
-100
-200
-300
-400
-500
0
20
40
60
80
100
120
140
160
TR (Seconds)

TC	=	150 Sec.
x	=	-500 ft.
y	=	-500 ft.
$\dot{x}$	=	-.6 ft/sec.
$\dot{y}$	=	-.6 ft/sec.

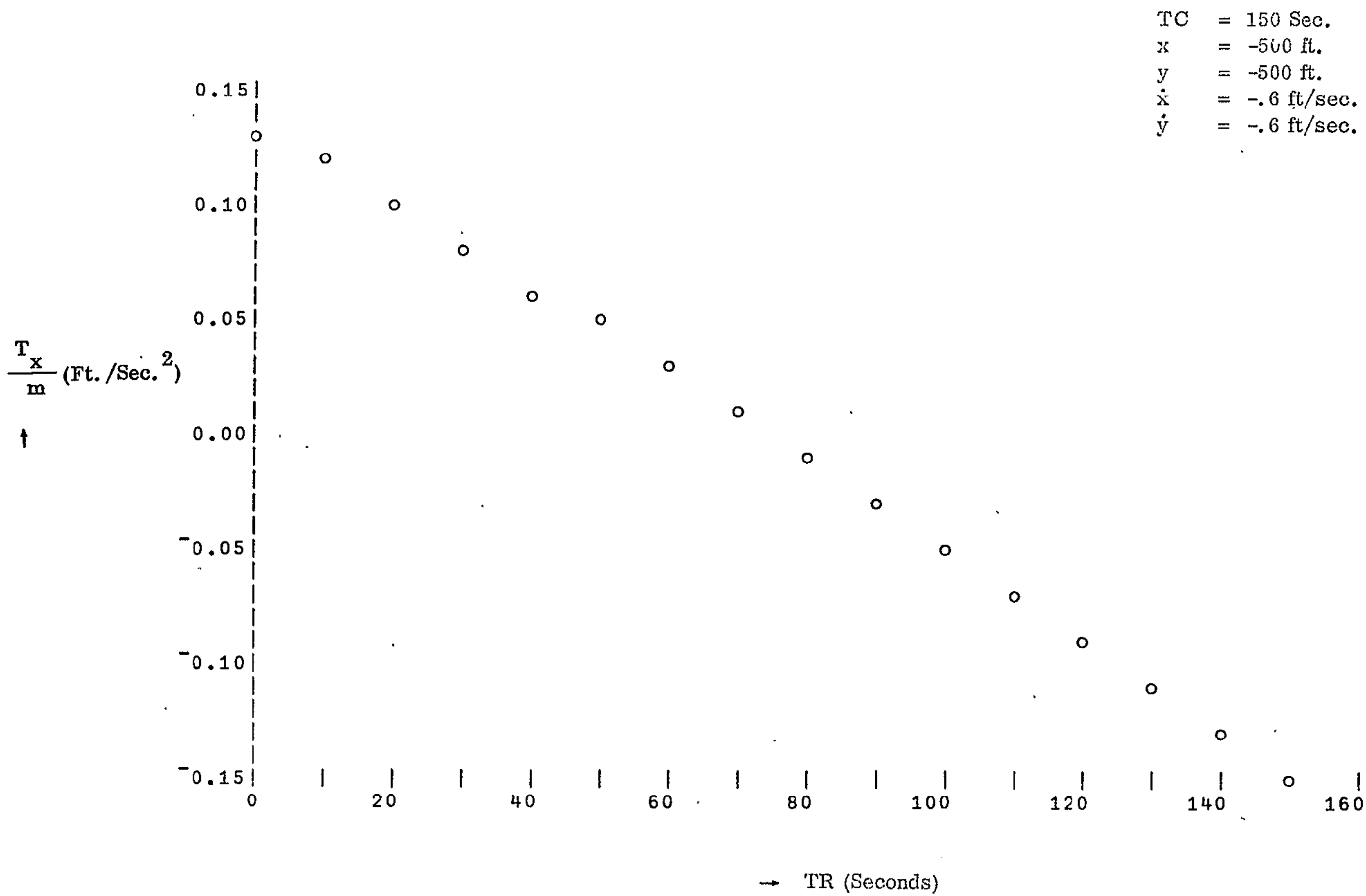




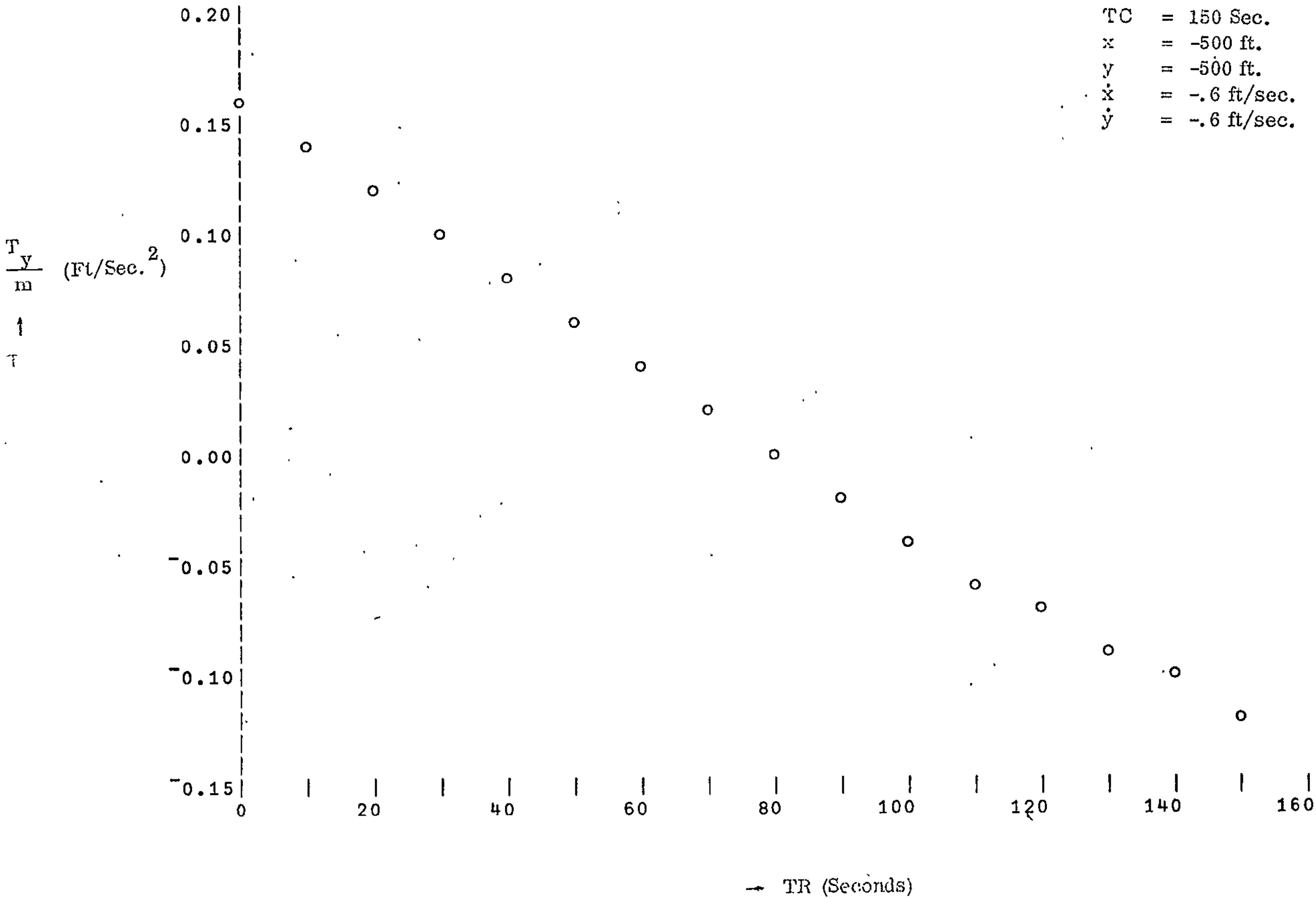
TC = 150 Sec.
x = -500 ft.
y = -500 ft.
$\dot{x}$ = -.6 ft/sec.
$\dot{y}$ = -.6 ft/sec.
$\dot{x}$ (Ft/Sec.)
-1
0
1
2
3
4
5
6
0
20
40
60
80
100
120
140
160
TR (Seconds)

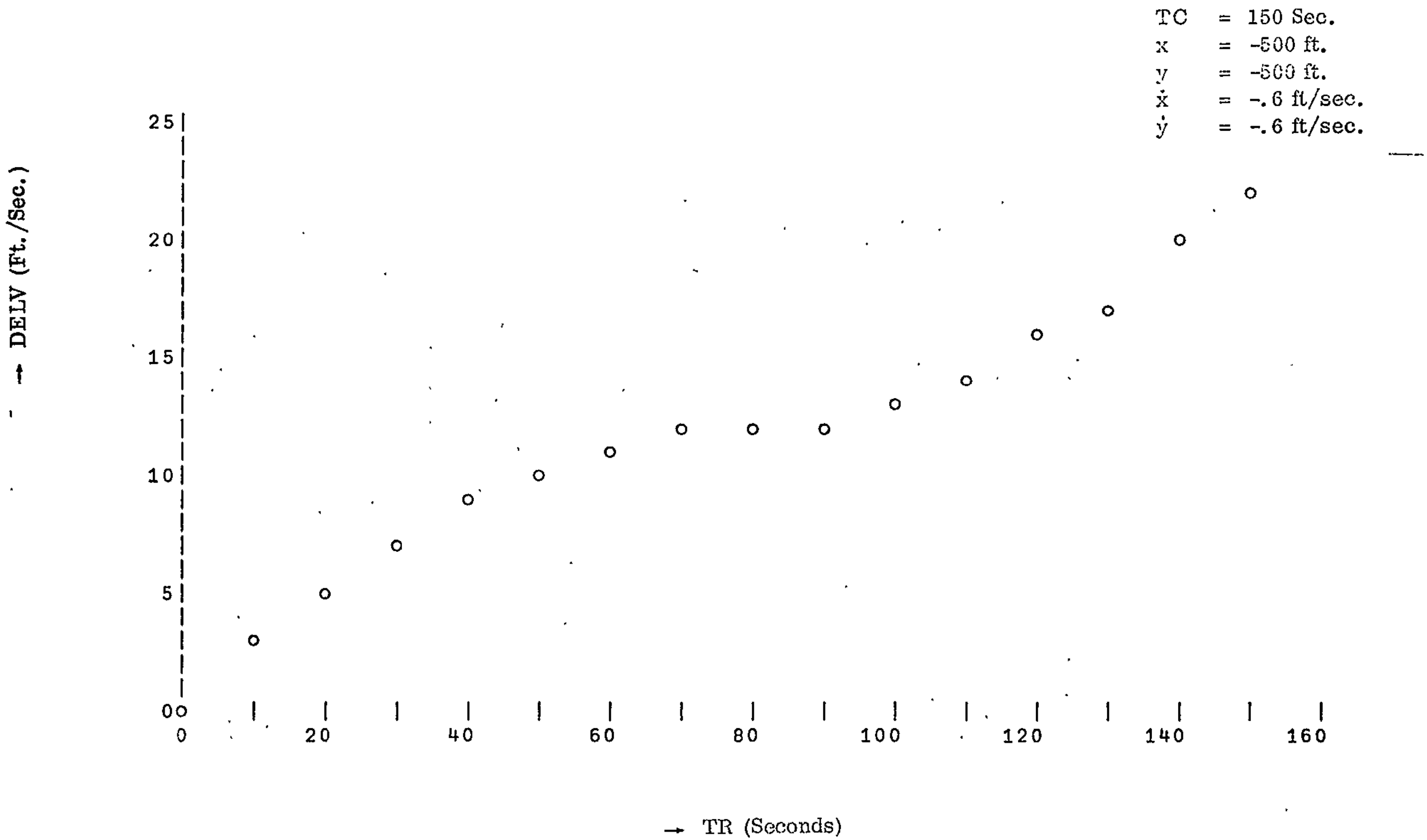


TC = 150 Sec.
x = -500 ft.
y = -500 ft.
ẋ = -.6 ft/sec.
ẏ = -.6 ft/sec.
y (Ft/Sec.)
6
5
4
3
2
1
0
-1
0
20
40
60
80
100
120
140
160
TR (Seconds)



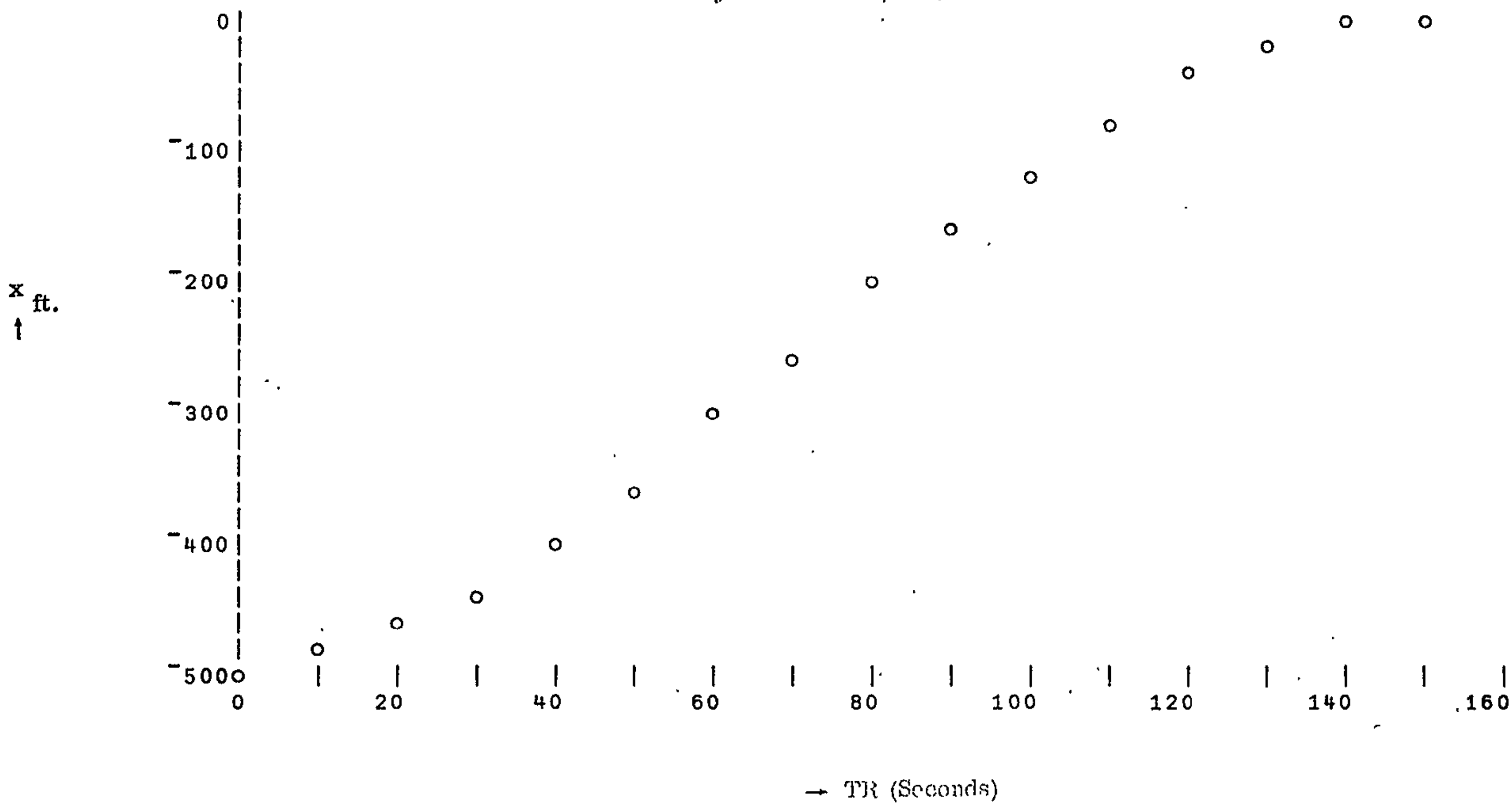
TC = 150 Sec.
x = -500 ft.
y = -500 ft.
ẋ = -.6 ft/sec.
ẏ = -.6 ft/sec.
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
Tx/m (Ft./Sec.2)
0
20
40
60
80
100
120
140
160
TR (Seconds)

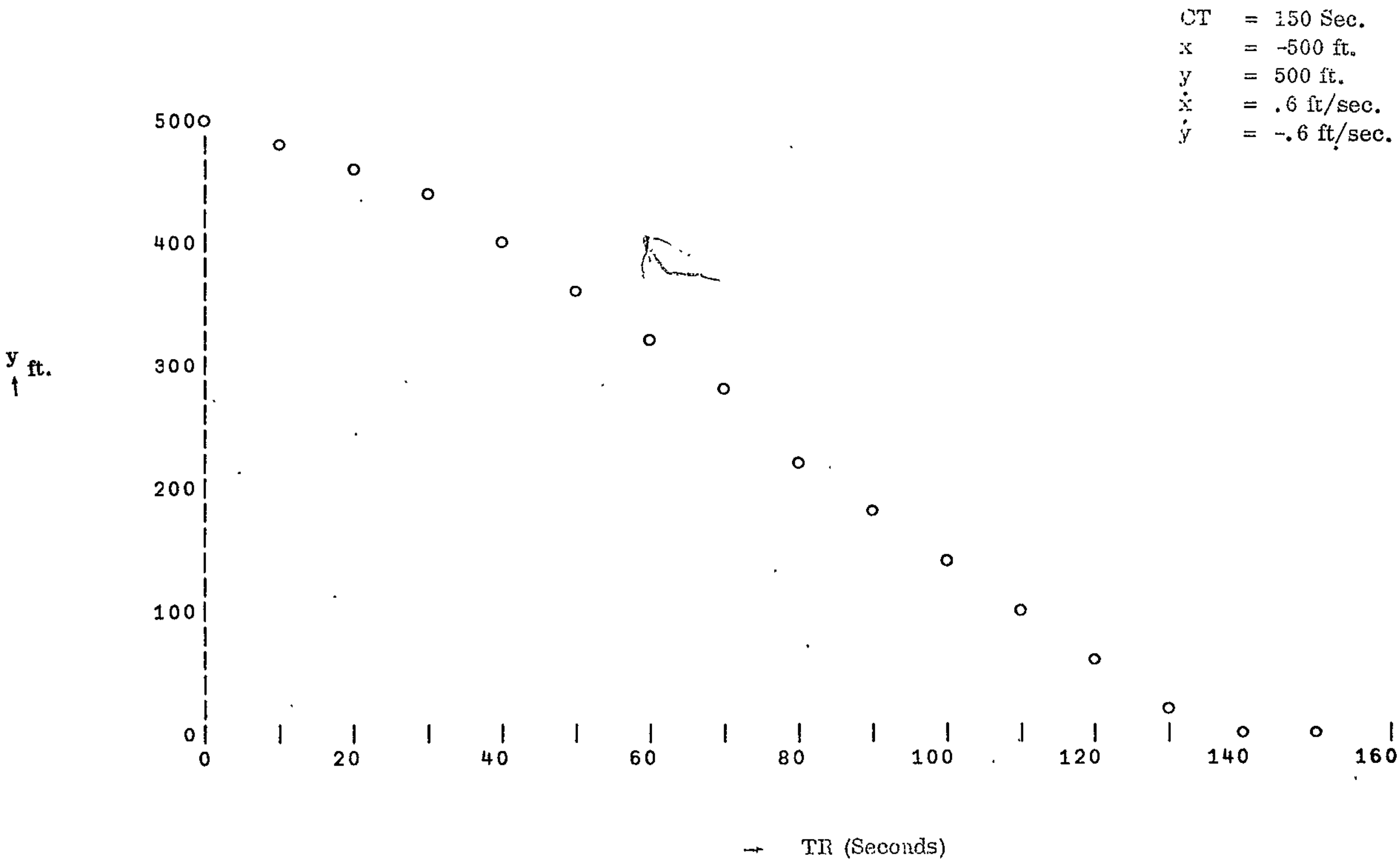

TC = 150 Sec.
x = -500 ft.
y = -500 ft.
ẋ = -.6 ft/sec.
ẏ = -.6 ft/sec.
$\frac{T_y}{m}$ (Ft/Sec.^{2})
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
0
20
40
60
80
100
120
140
160
→ TR (Seconds)



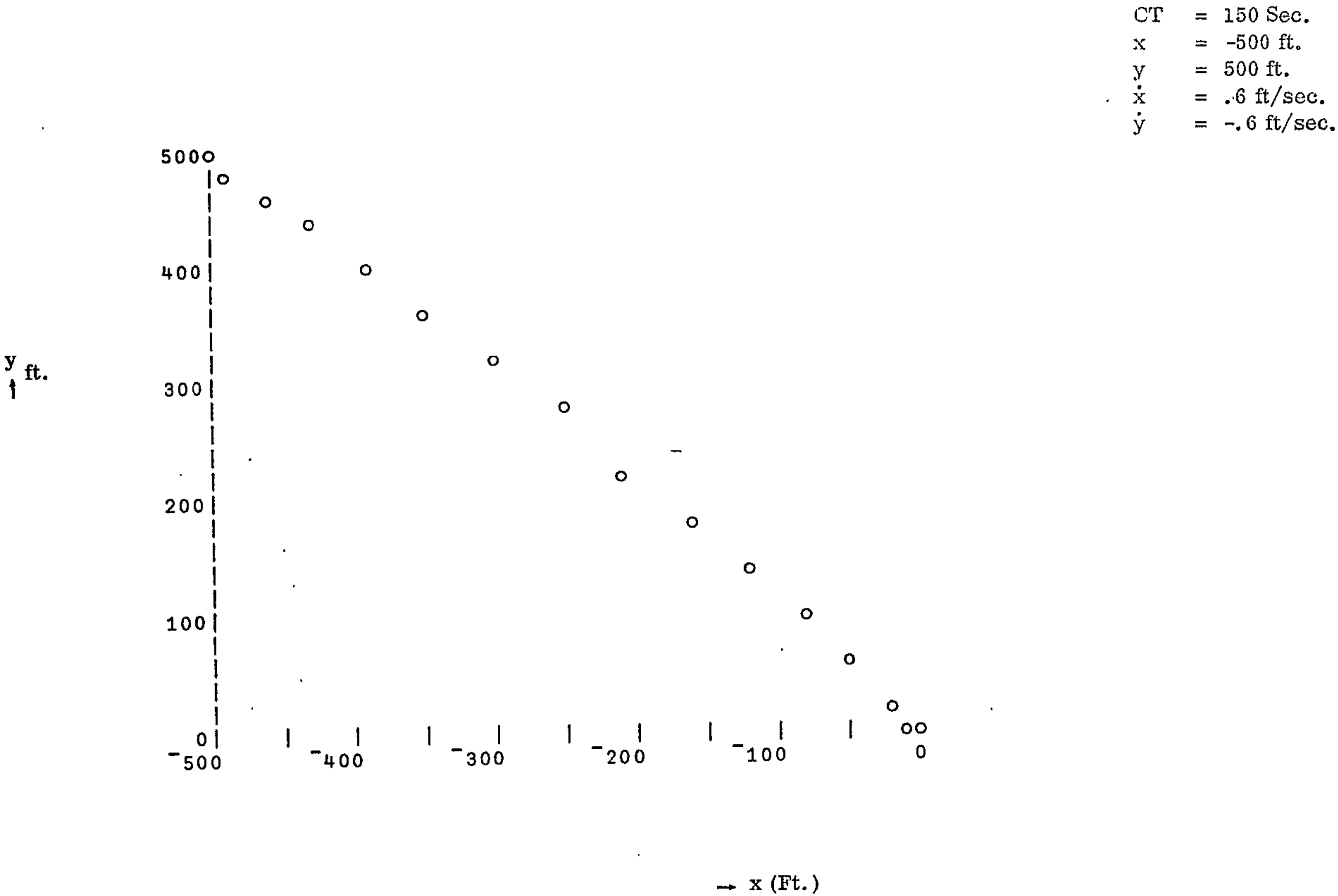
TC = 150 Sec.
x = -500 ft.
y = -500 ft.
ẋ = -.6 ft/sec.
ẏ = -.6 ft/sec.
→ DELV (Ft./Sec.)
0
5
10
15
20
25
0
20
40
60
80
100
120
140
160
→ TR (Seconds)

CT = 150 Sec.
x = -500 ft.
y = 500 ft.
$\dot{x}$ = .6 ft/sec.
$\dot{y}$ = -.6 ft/sec.

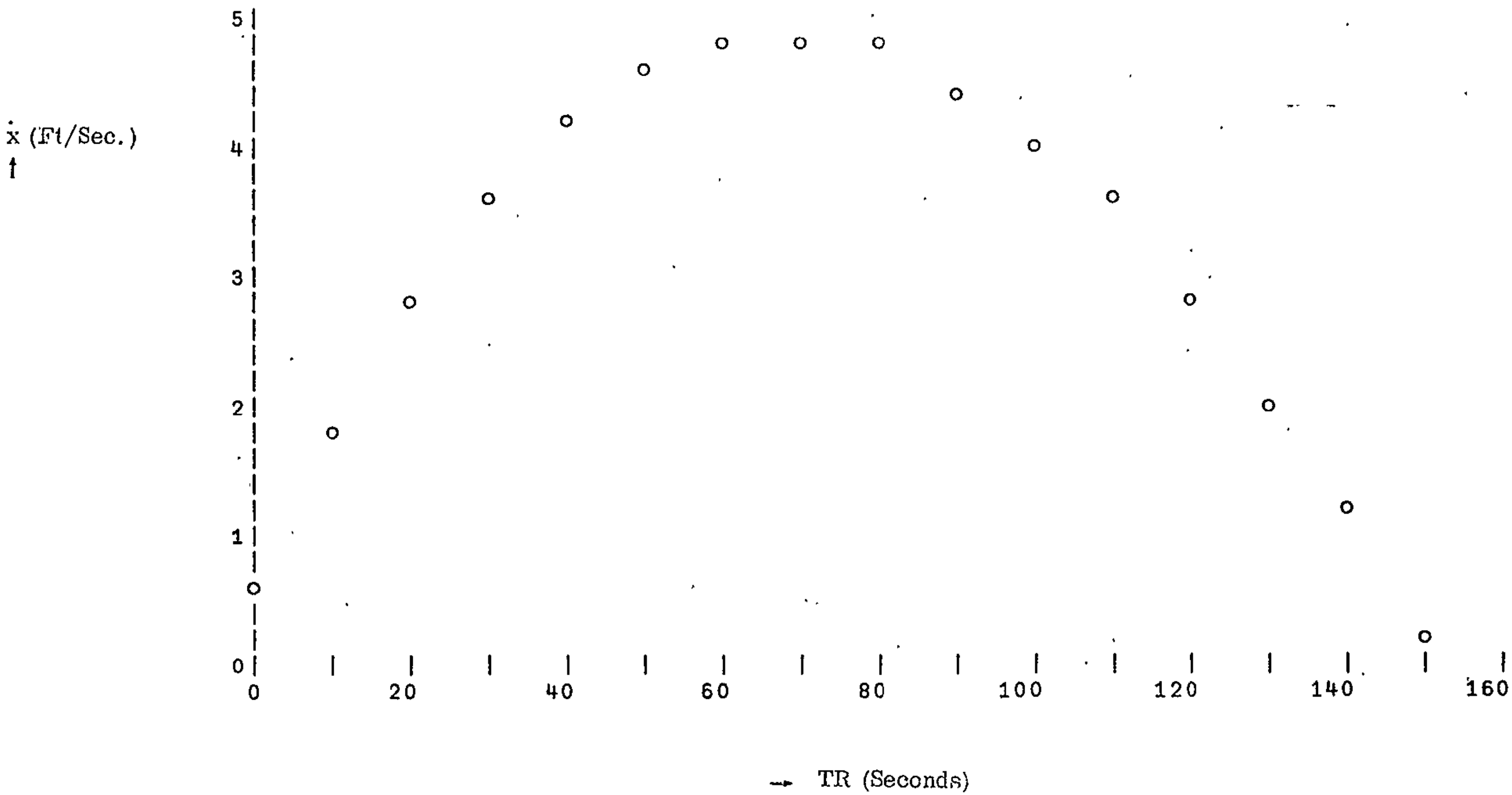


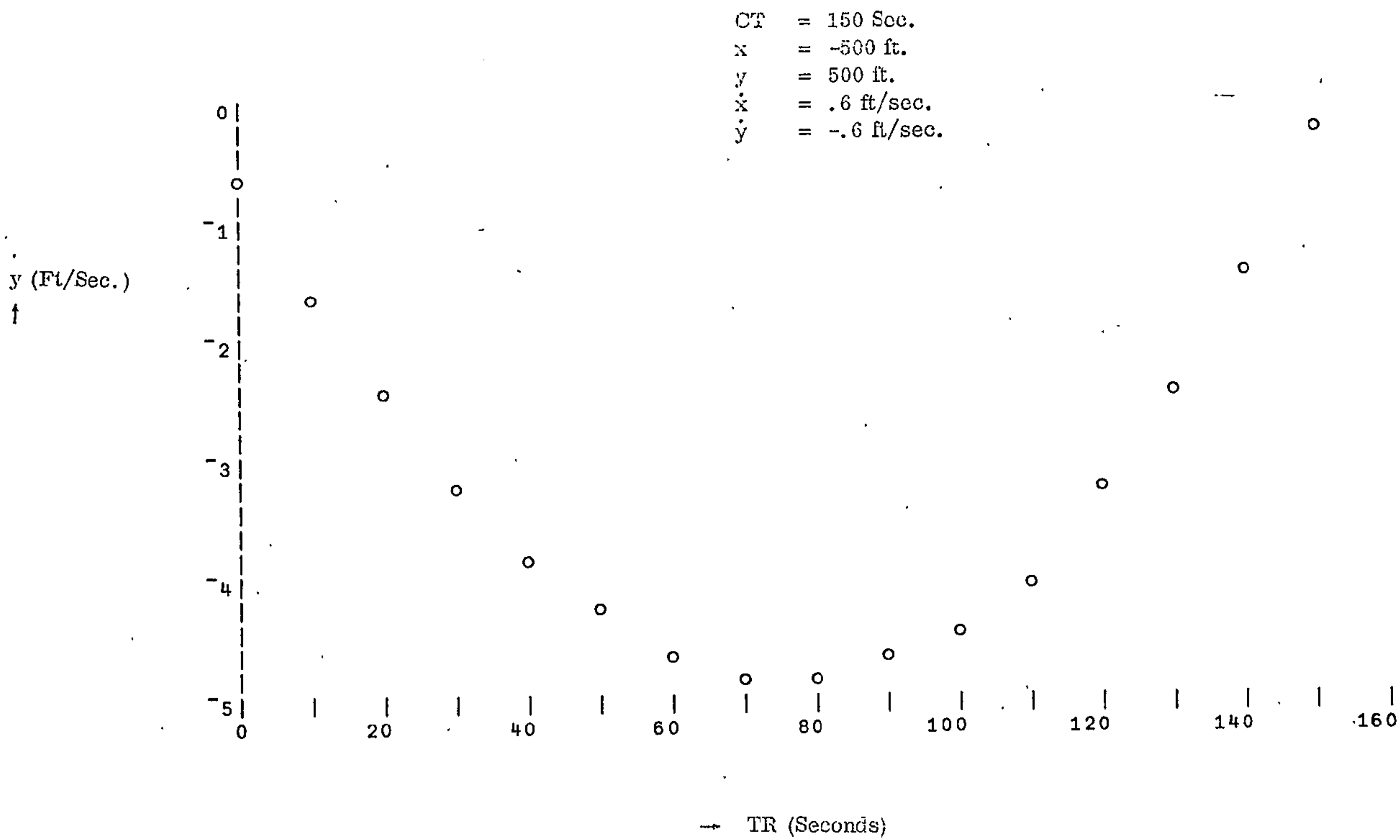


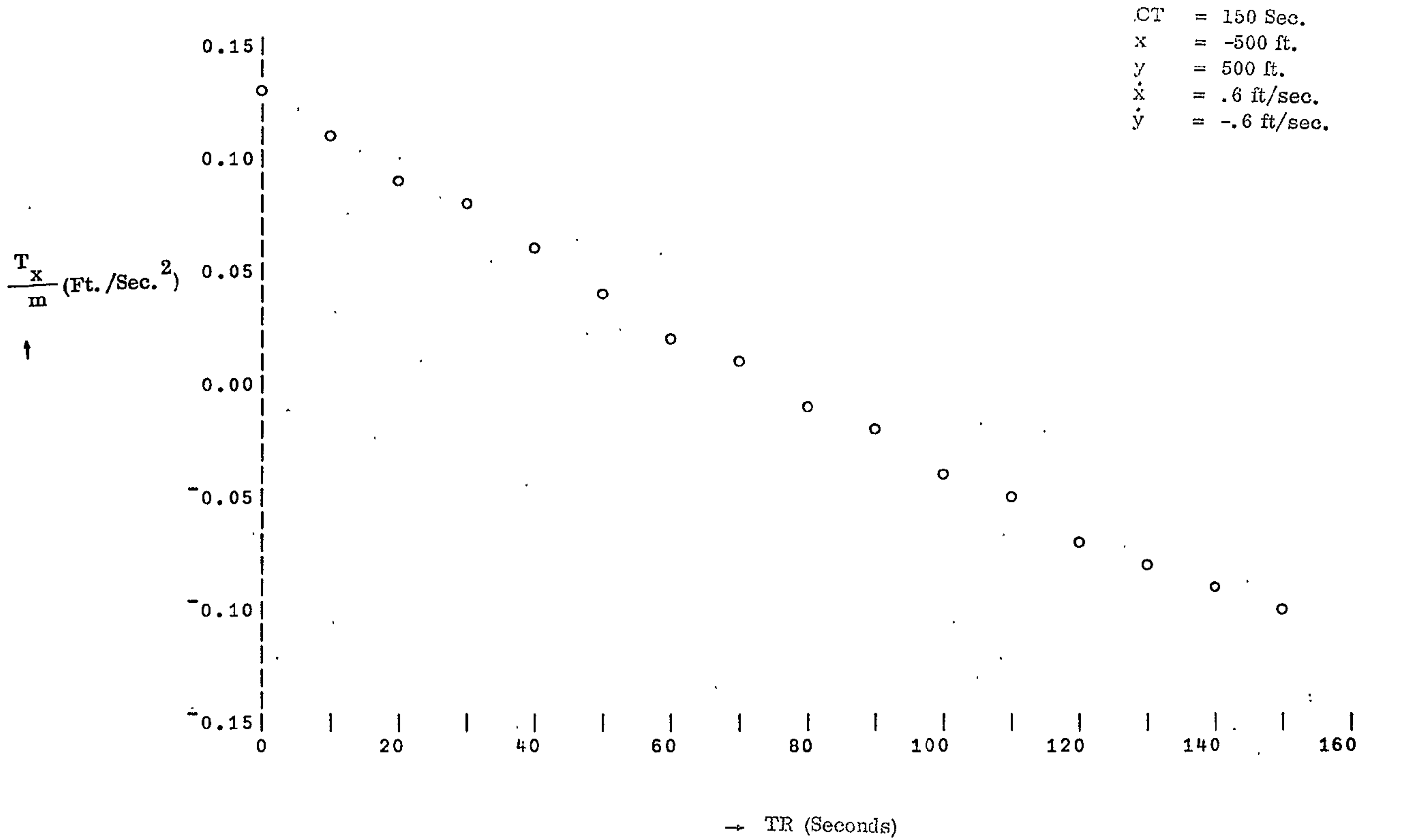
CT = 150 Sec.
x = -500 ft.
y = 500 ft.
$\dot{x}$ = .6 ft/sec.
$\dot{y}$ = -.6 ft/sec.
y ft.
500
400
300
200
100
0
0
20
40
60
80
100
120
140
160
TR (Seconds)

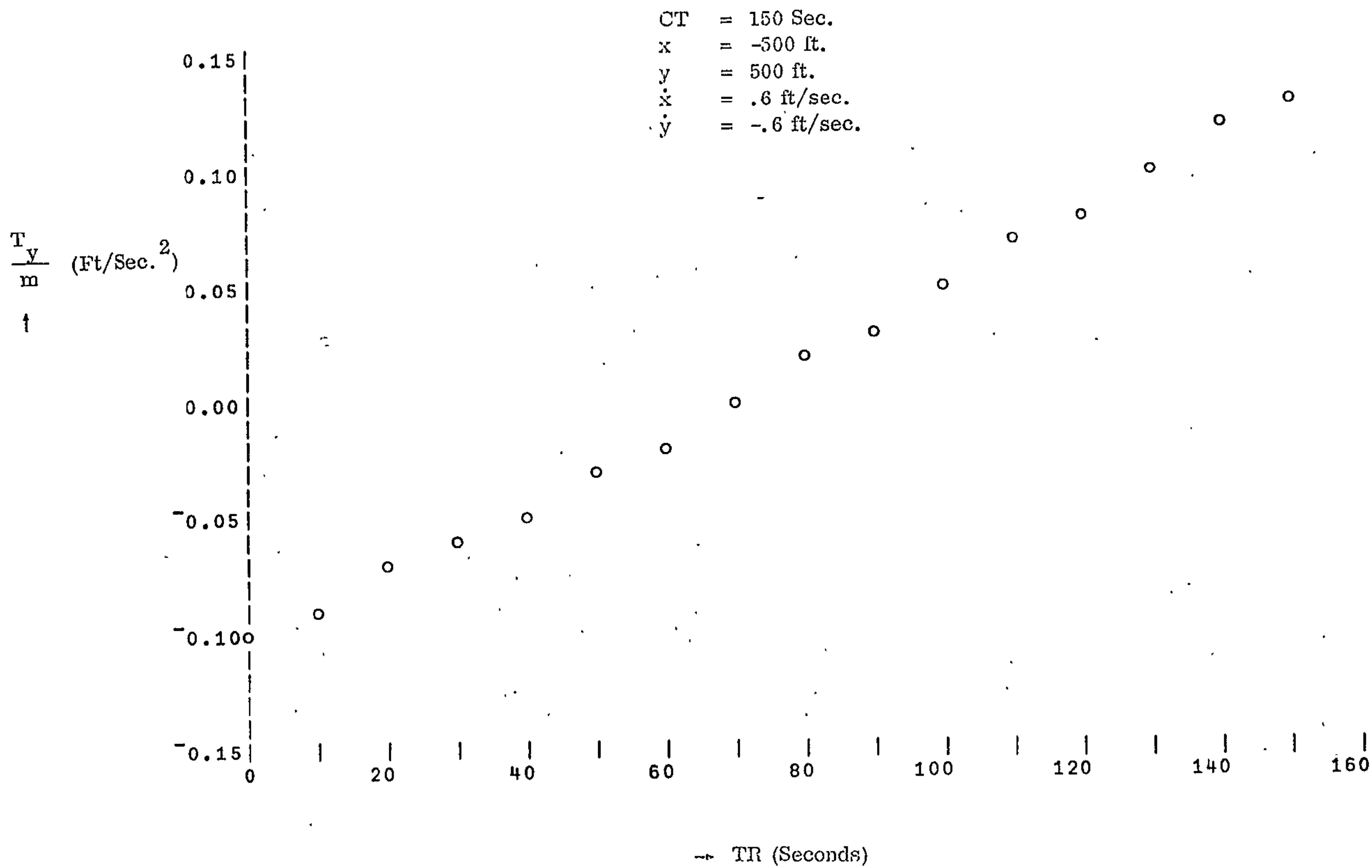

CT = 150 Sec.
x = -500 ft.
y = 500 ft.
$\dot{x}$ = .6 ft/sec.
$\dot{y}$ = -.6 ft/sec.
y ft.
500
400
300
200
100
0
-500
-400
-300
-200
-100
0
x (Ft.)

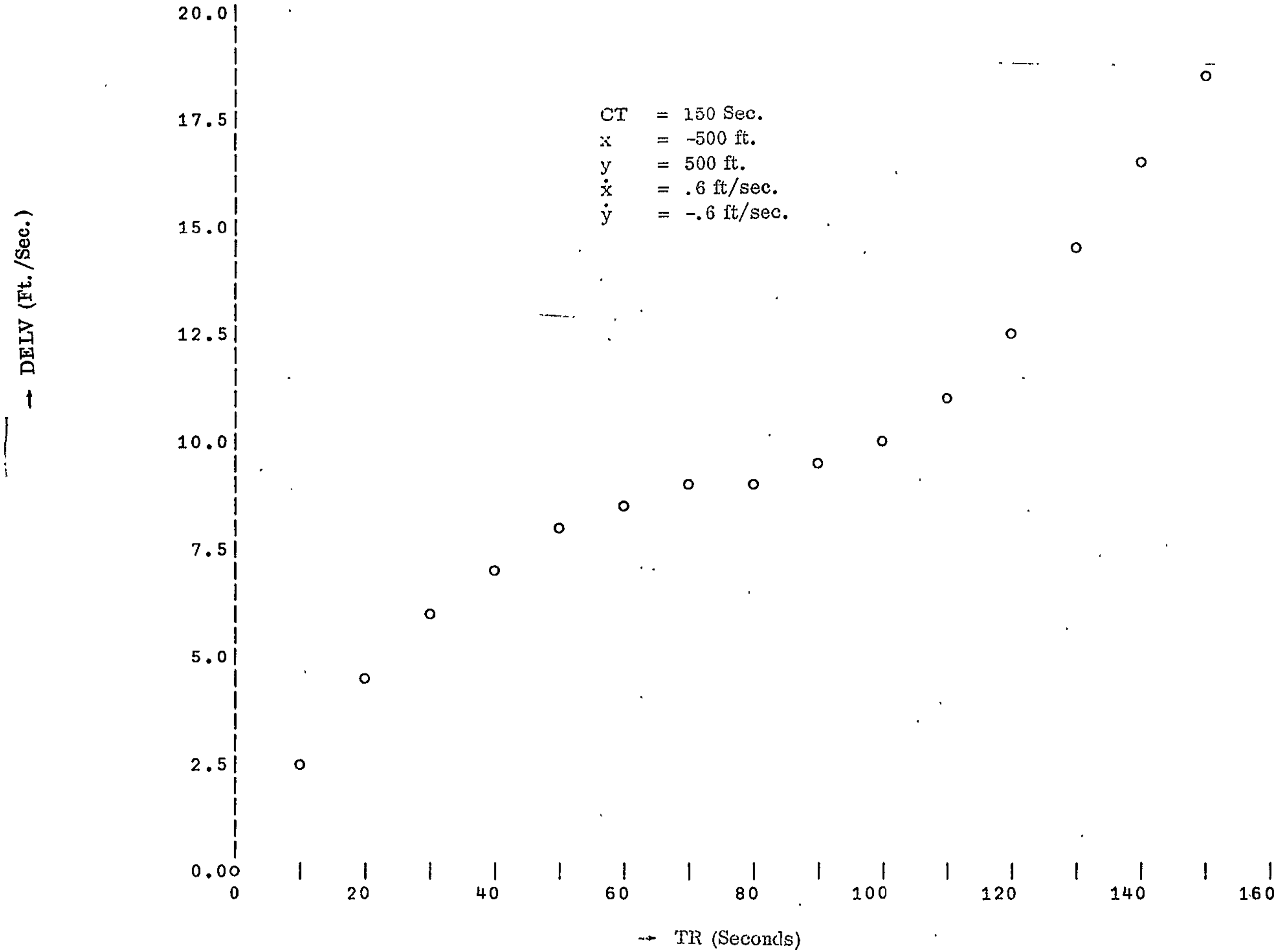
CT = 150 Sec.
x = -500 ft.
y = 500 ft.
$\dot{x}$ = .6 ft/sec.
$\dot{y}$ = -.6 ft/sec.

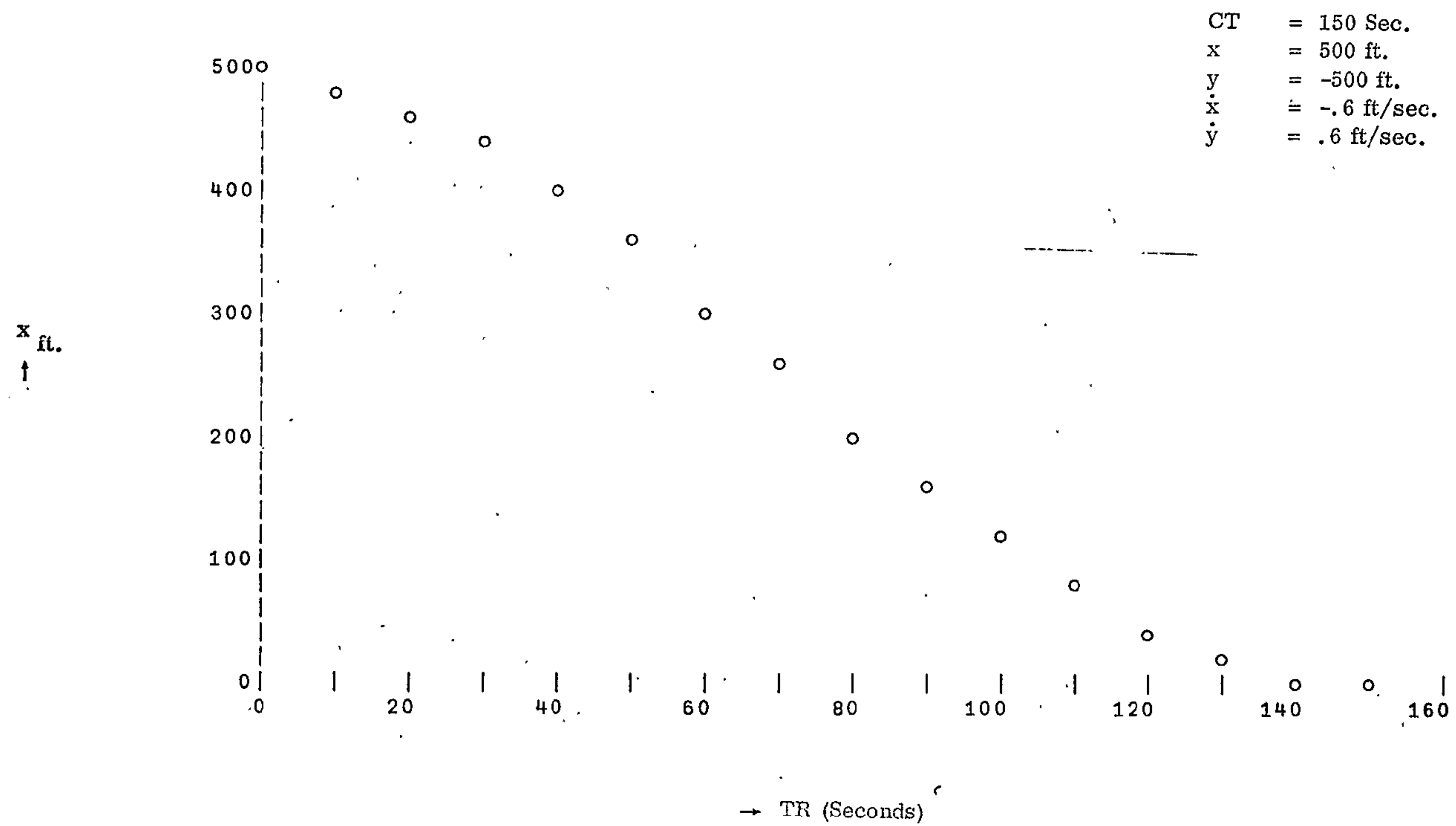


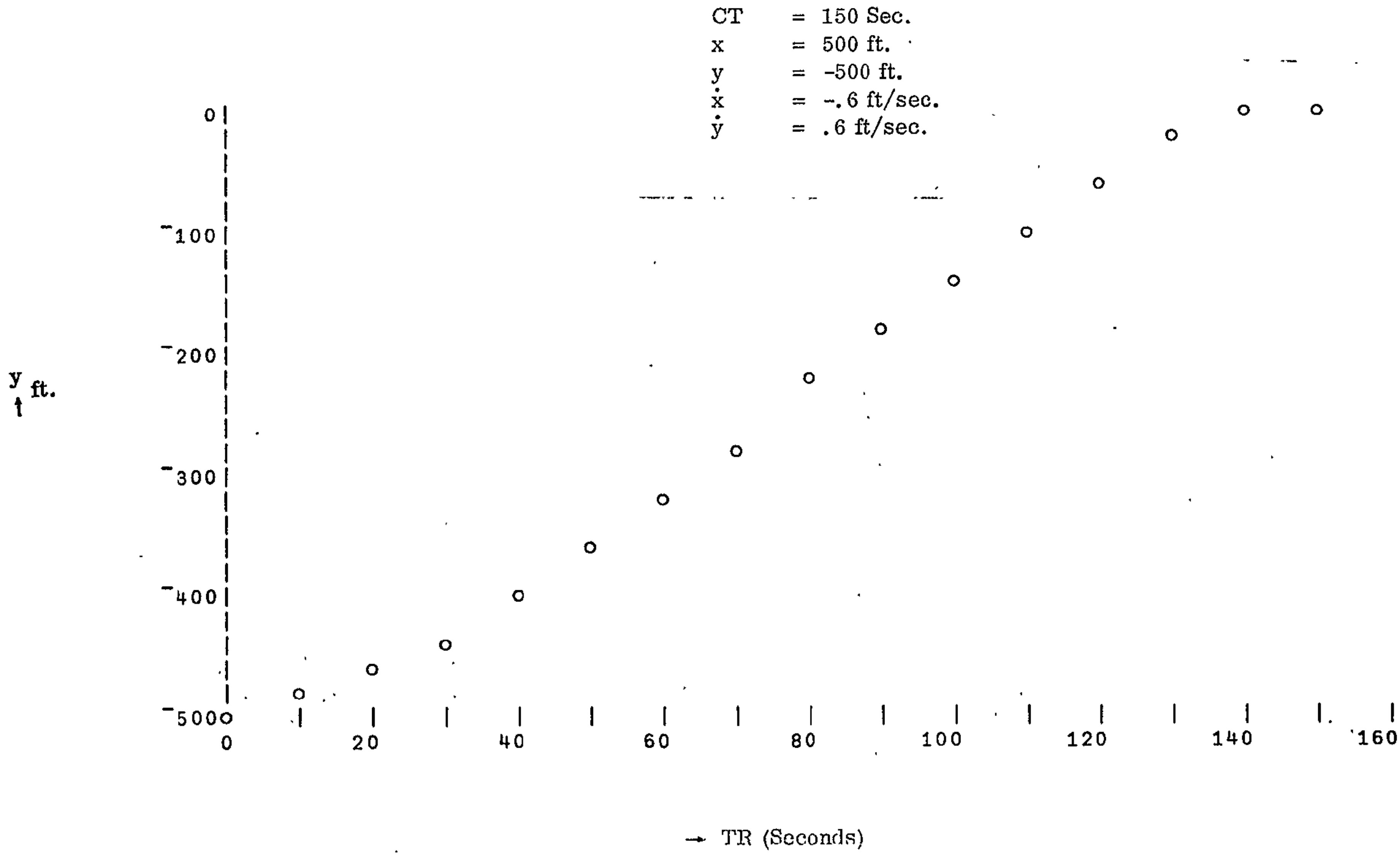

CT = 150 Sec.
x = -500 ft.
y = 500 ft.
$\dot{x}$ = .6 ft/sec.
$\dot{y}$ = -.6 ft/sec.
$\dot{y}$ (Ft/Sec.)
0
-1
-2
-3
-4
-5
0
20
40
60
80
100
120
140
160
TR (Seconds)

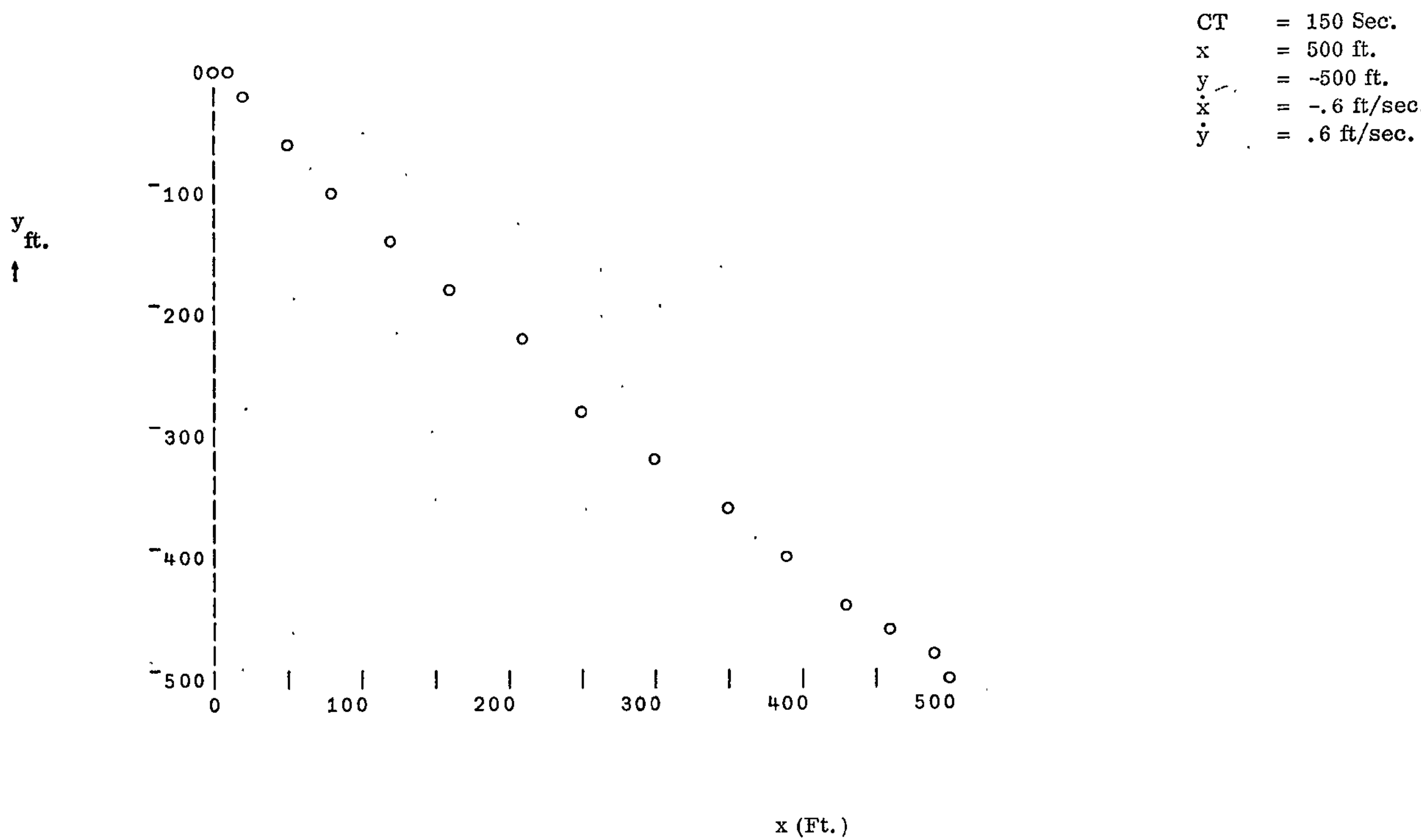

CT = 150 Sec.
x = -500 ft.
y = 500 ft.
ẋ = .6 ft/sec.
ẏ = -.6 ft/sec.
$\frac{T_x}{m}$ (Ft./Sec.^{2})
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
0
20
40
60
80
100
120
140
160
TR (Seconds)


CT = 150 Sec.
x = -500 ft.
y = 500 ft.
ẋ = .6 ft/sec.
ẏ = -.6 ft/sec.
Ty/m (Ft/Sec.²)
0.15
0.10
0.05
0.00
-0.05
-0.100
-0.15
0
20
40
60
80
100
120
140
160
TR (Seconds)

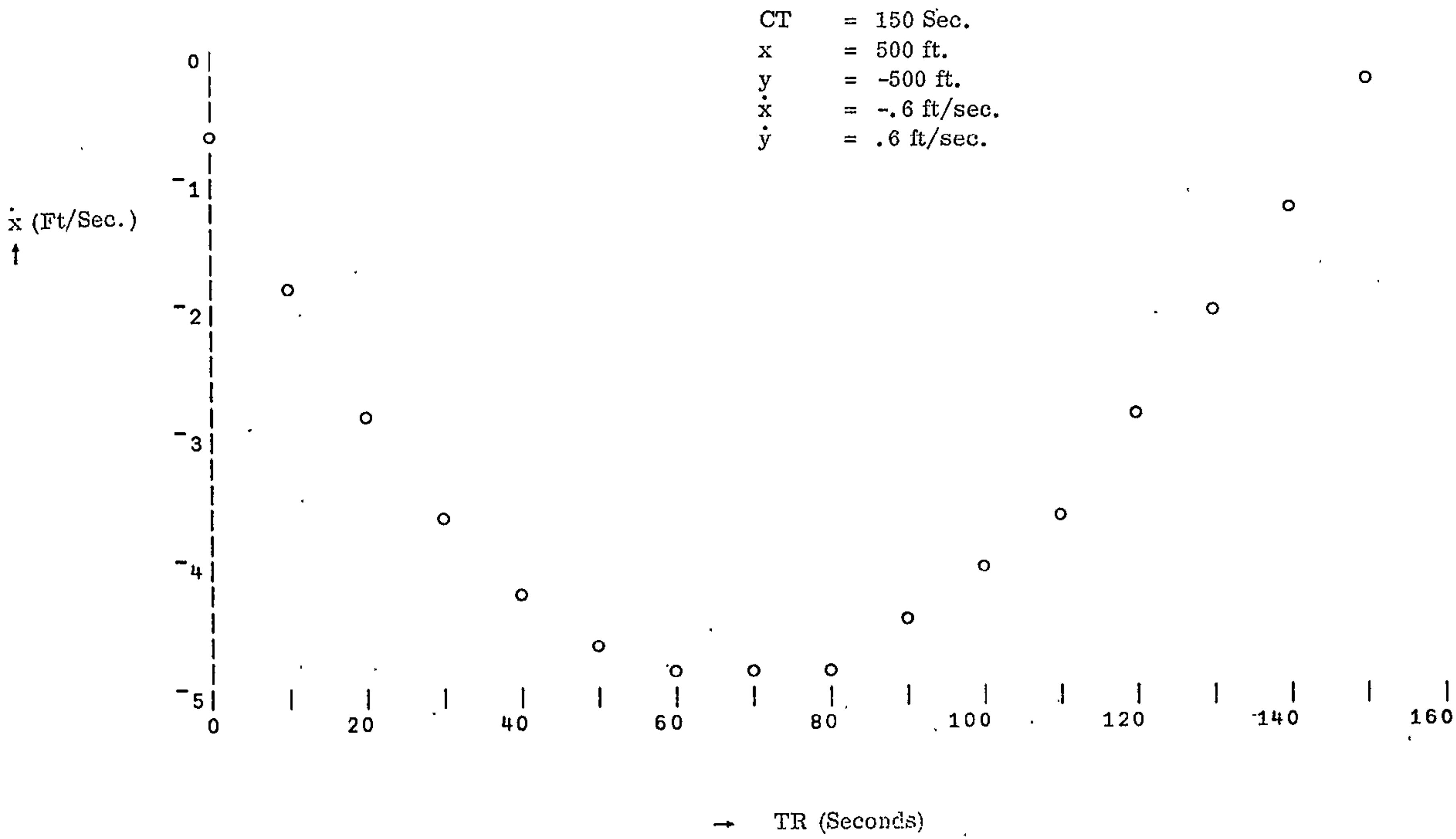

CT = 150 Sec.
x = -500 ft.
y = 500 ft.
ẋ = .6 ft/sec.
ẏ = -.6 ft/sec.
DELV (Ft./Sec.)
TR (Seconds)
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.00
0
20
40
60
80
100
120
140
160


CT = 150 Sec.
x = 500 ft.
y = -500 ft.
ẋ = -.6 ft/sec.
ẏ = .6 ft/sec.
x ft.
500
400
300
200
100
0
0
20
40
60
80
100
120
140
160
TR (Seconds)

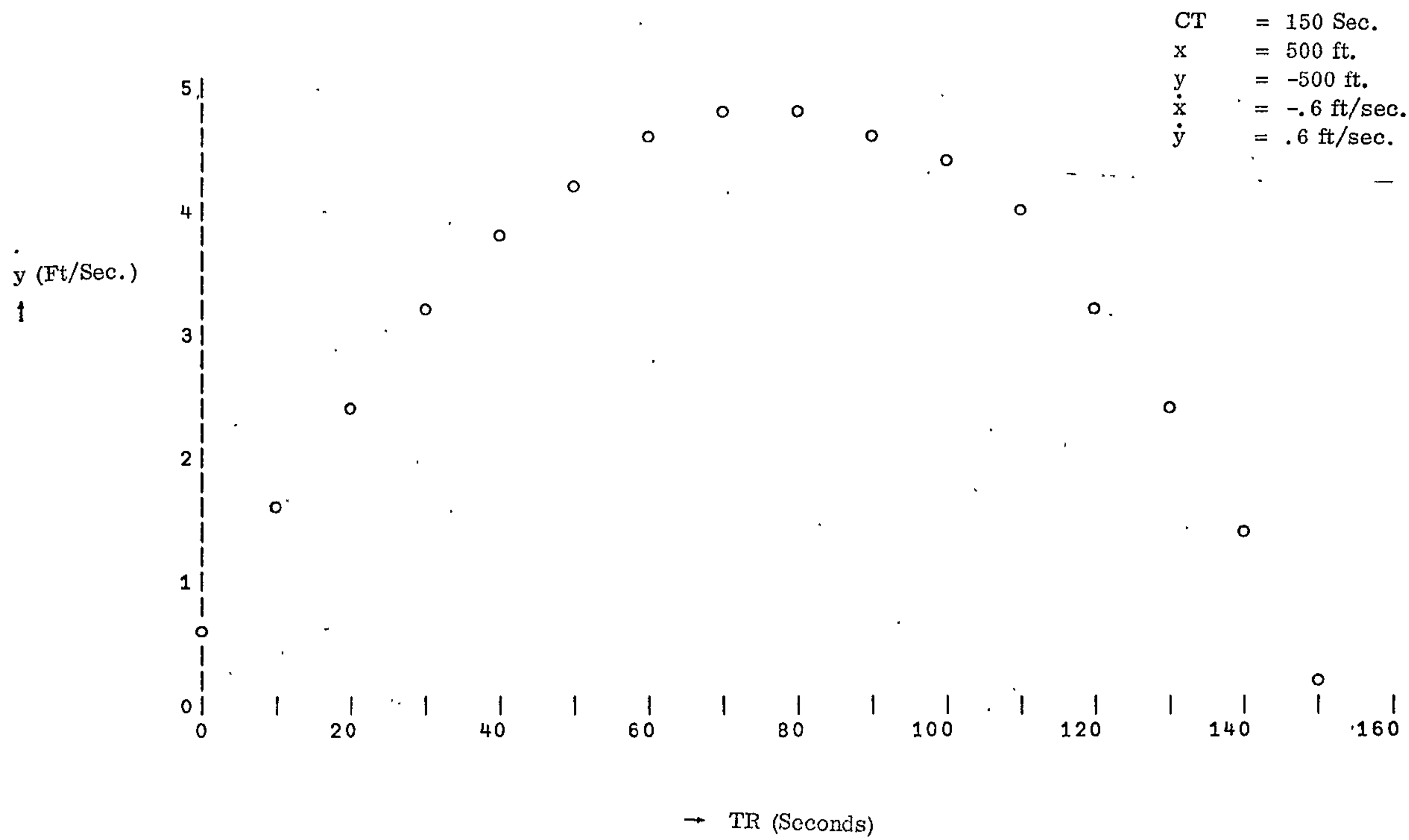

CT = 150 Sec.
x = 500 ft.
y = -500 ft.
$\dot{x}$ = -.6 ft/sec.
$\dot{y}$ = .6 ft/sec.
y ft.
0
-100
-200
-300
-400
-500
0
20
40
60
80
100
120
140
160
→ TR (Seconds)

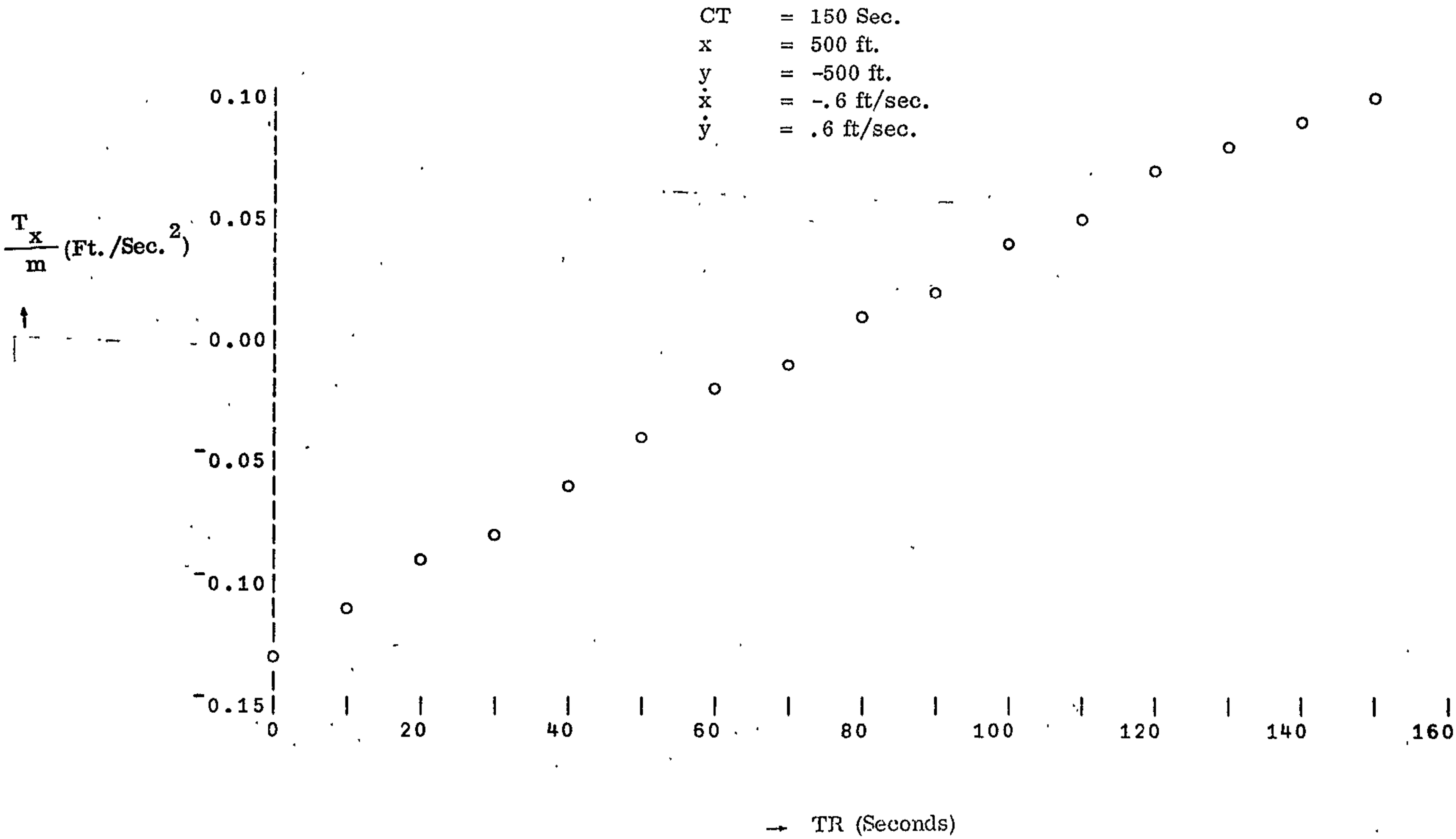


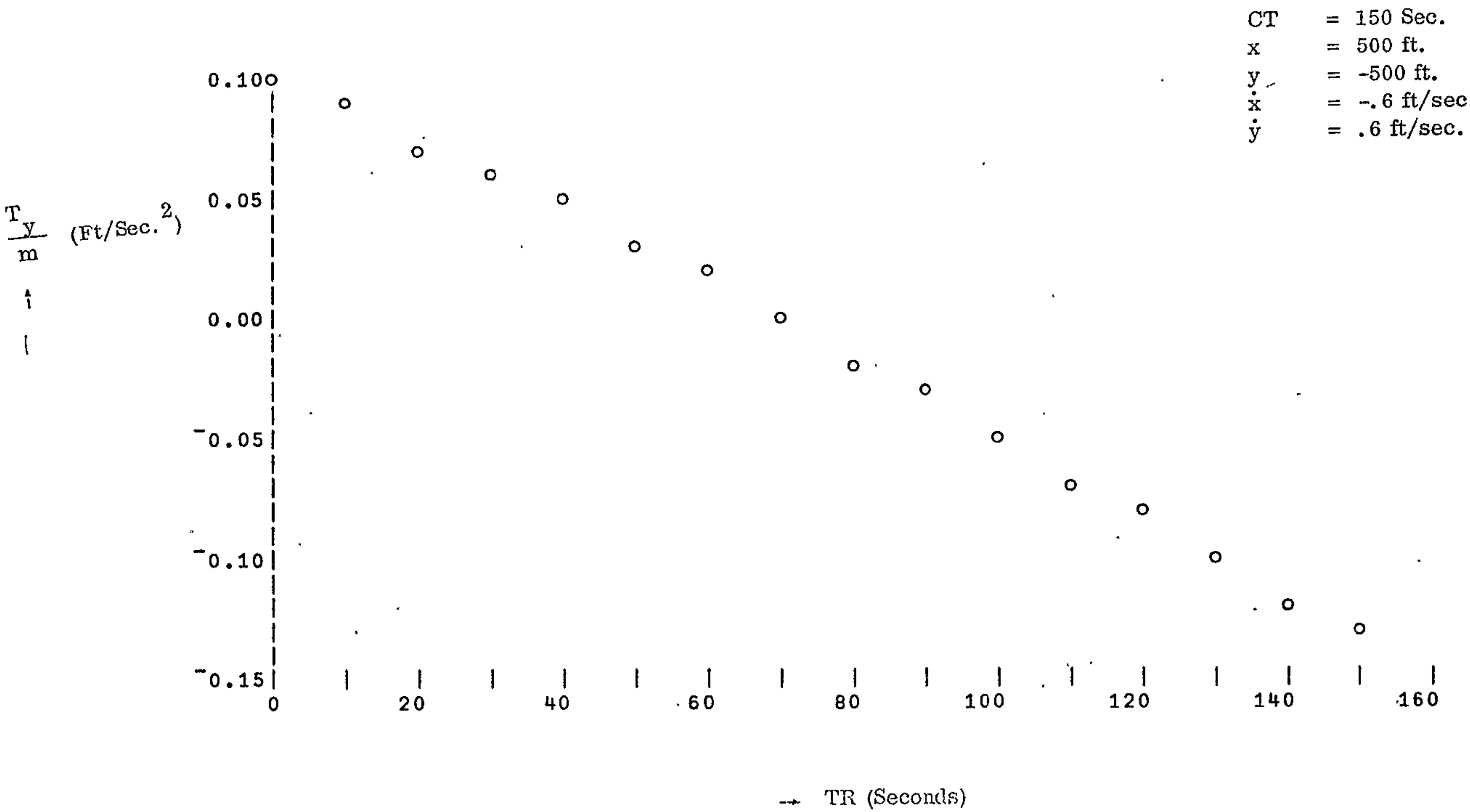
CT = 150 Sec.
x = 500 ft.
y = -500 ft.
$\dot{x}$ = -.6 ft/sec.
$\dot{y}$ = .6 ft/sec.
y ft.
000
-100
-200
-300
-400
-500
0
100
200
300
400
500
x (Ft.)

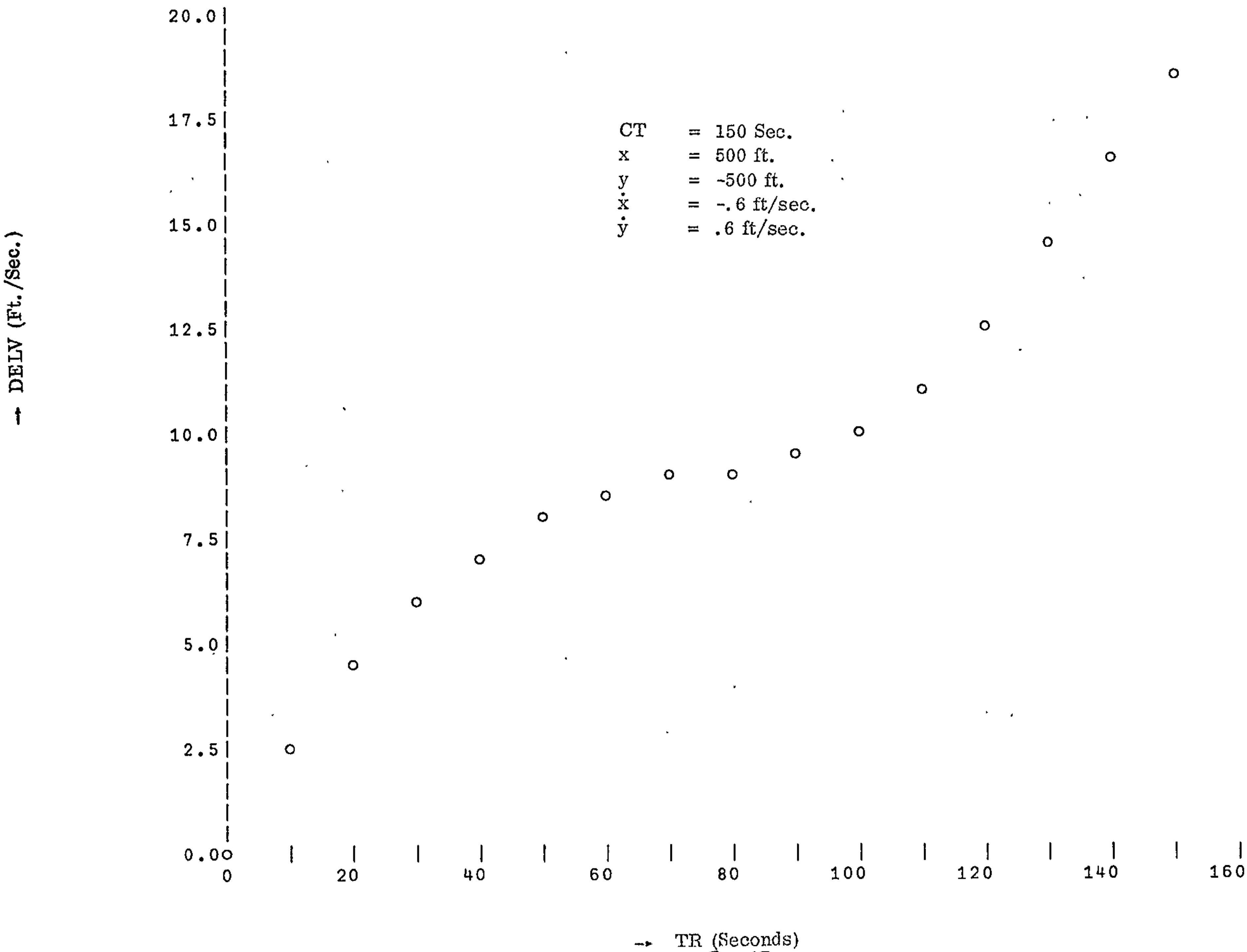


CT = 150 Sec.
x = 500 ft.
y = -500 ft.
ẋ = -.6 ft/sec.
ẏ = .6 ft/sec.
ẋ (Ft/Sec.)
0
-1
-2
-3
-4
-5
0
20
40
60
80
100
120
140
160
TR (Seconds)

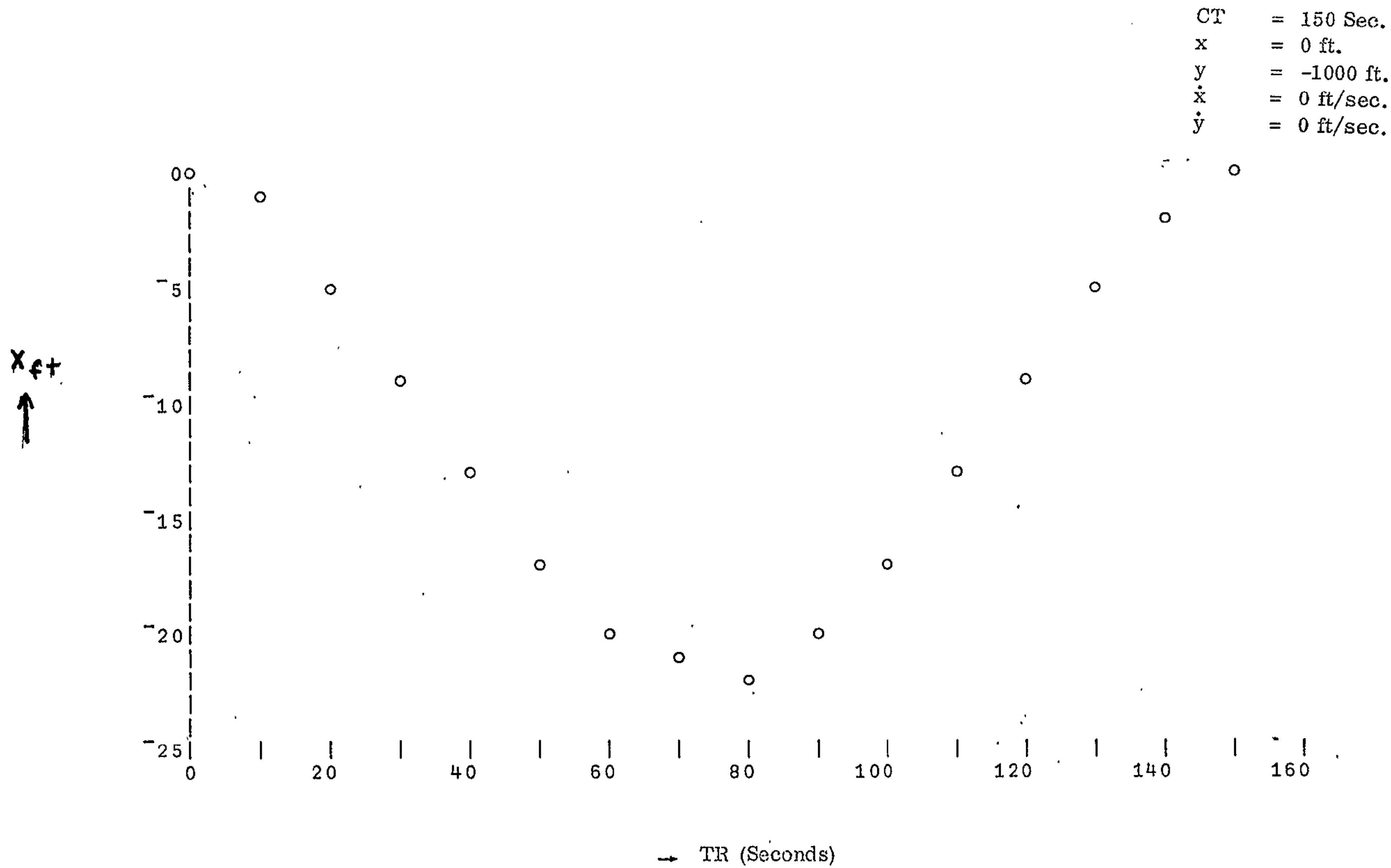

CT = 150 Sec.
x = 500 ft.
y = -500 ft.
$\dot{x}$ = -.6 ft/sec.
$\dot{y}$ = .6 ft/sec.
$\dot{y}$ (Ft/Sec.)
0
1
2
3
4
5
0
20
40
60
80
100
120
140
160
→ TR (Seconds)


CT = 150 Sec.
x = 500 ft.
y = -500 ft.
ẋ = -.6 ft/sec.
ẏ = .6 ft/sec.
Tx/m (Ft./Sec.²)
0.10
0.05
0.00
-0.05
-0.10
-0.15
0
20
40
60
80
100
120
140
160
TR (Seconds)

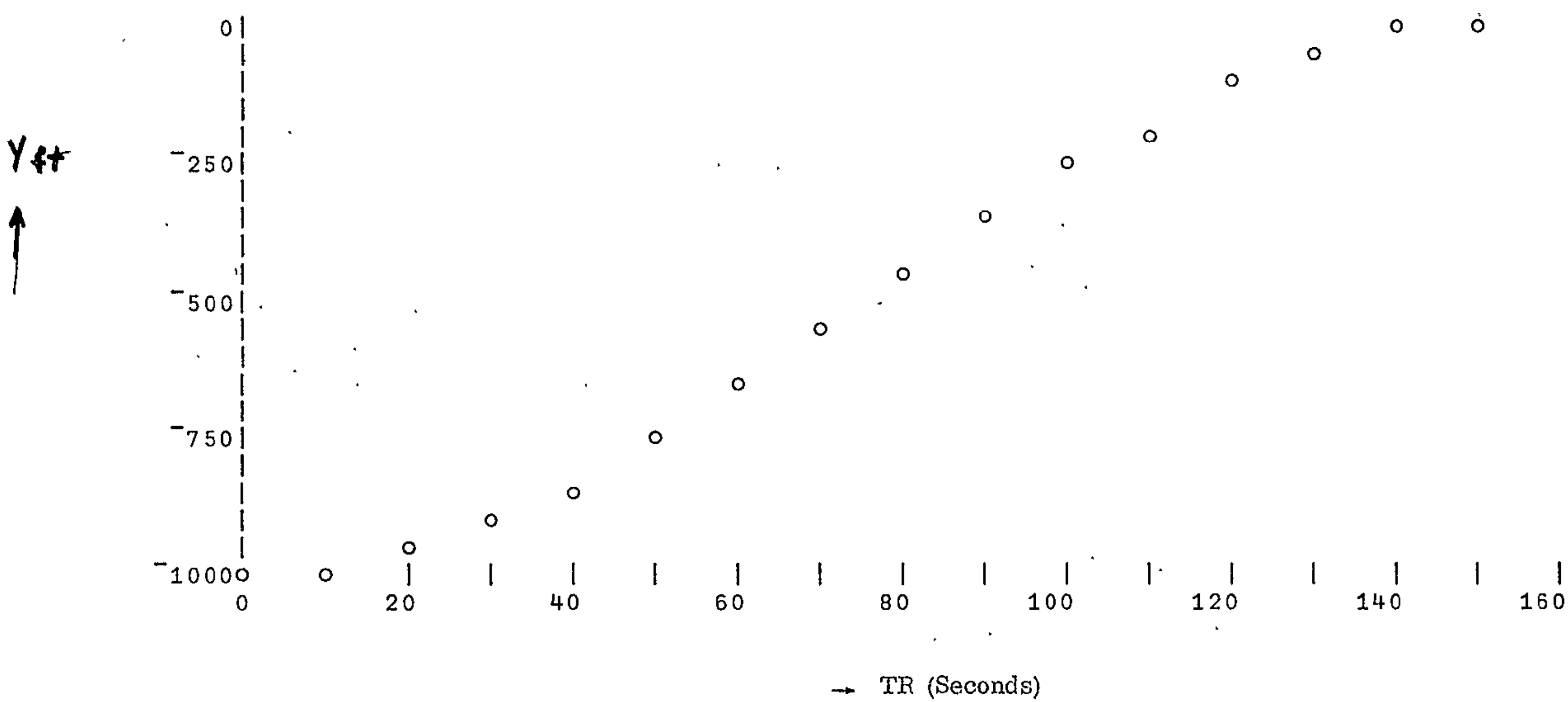

CT = 150 Sec.
x = 500 ft.
y = -500 ft.
$\dot{x}$ = -.6 ft/sec.
$\dot{y}$ = .6 ft/sec.
$\frac{T_y}{m}$ (Ft/Sec.^{2})
0.100
0.05
0.00
-0.05
-0.10
-0.15
0
20
40
60
80
100
120
140
160
TR (Seconds)



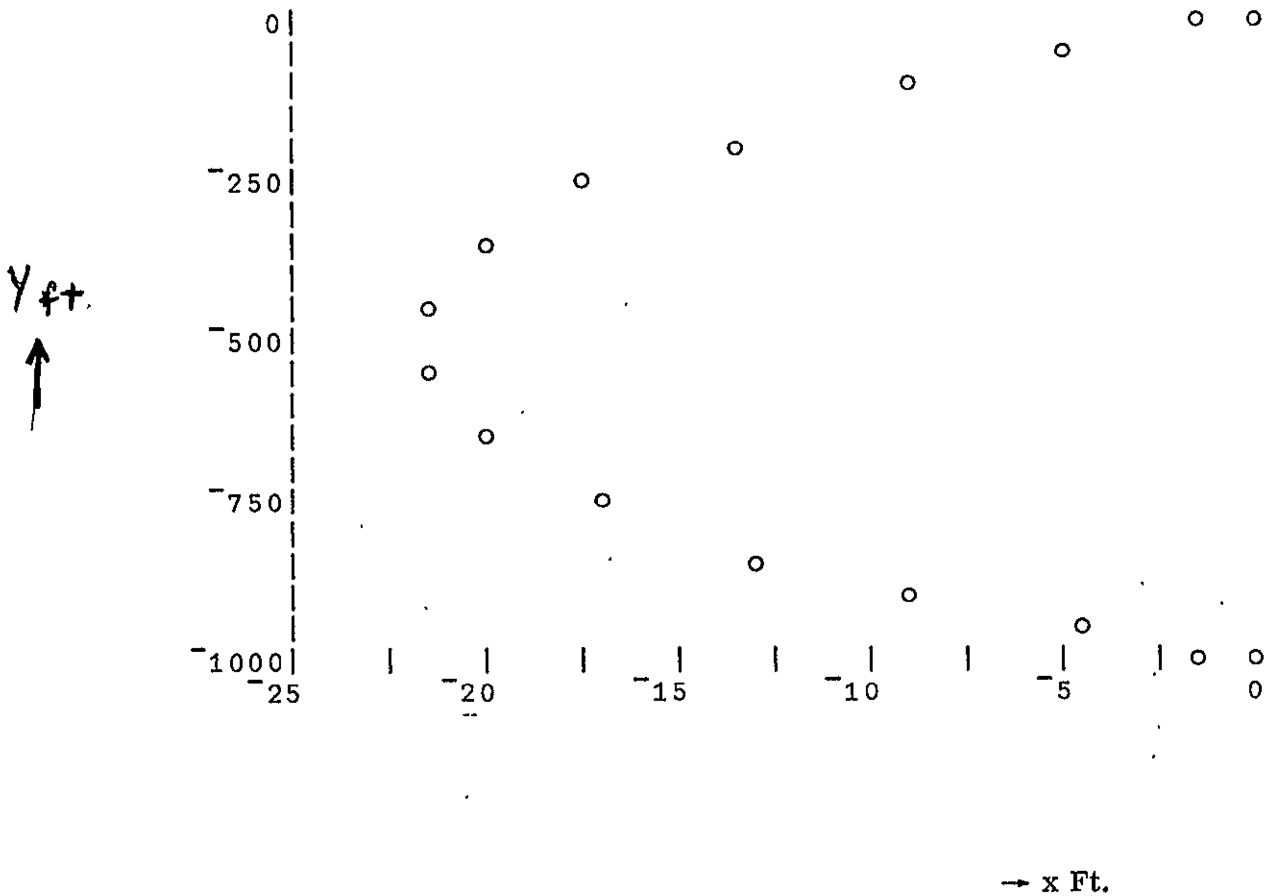
CT = 150 Sec.
x = 500 ft.
y = -500 ft.
ẋ = -.6 ft/sec.
ẏ = .6 ft/sec.
DELV (Ft./Sec.)
TR (Seconds)
0.0 2.5 5.0 7.5 10.0 12.5 15.0 17.5 20.0
0 20 40 60 80 100 120 140 160

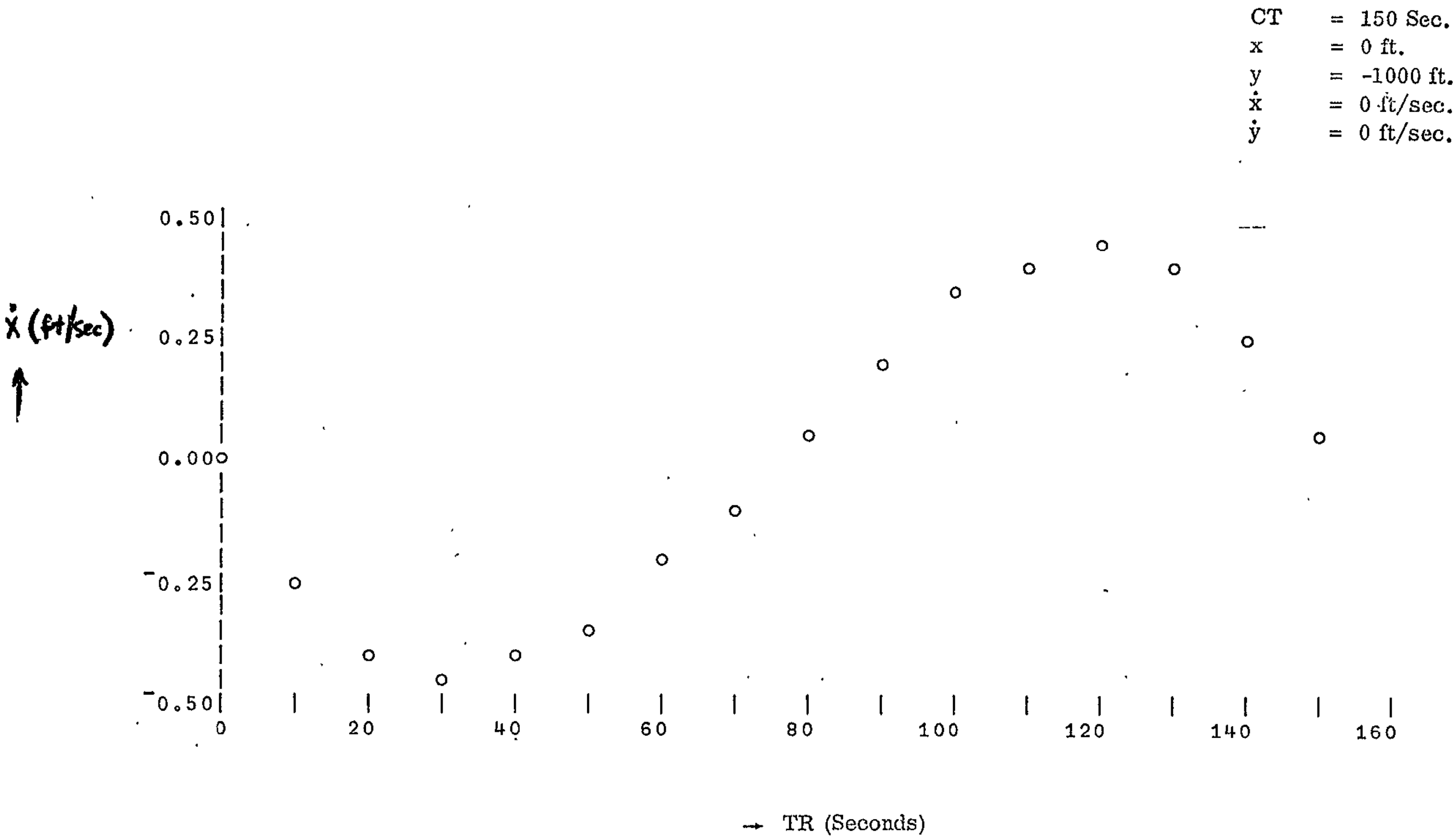

CT = 150 Sec.
x = 0 ft.
y = -1000 ft.
ẋ = 0 ft/sec.
ẏ = 0 ft/sec.
X ft
00
-5
-10
-15
-20
-25
0
20
40
60
80
100
120
140
160
TR (Seconds)

CT = 150 Sec.
x = 0 ft.
y = -1000 ft.
$\dot{x}$ = 0 ft/sec.
$\dot{y}$ = 0 ft/sec.

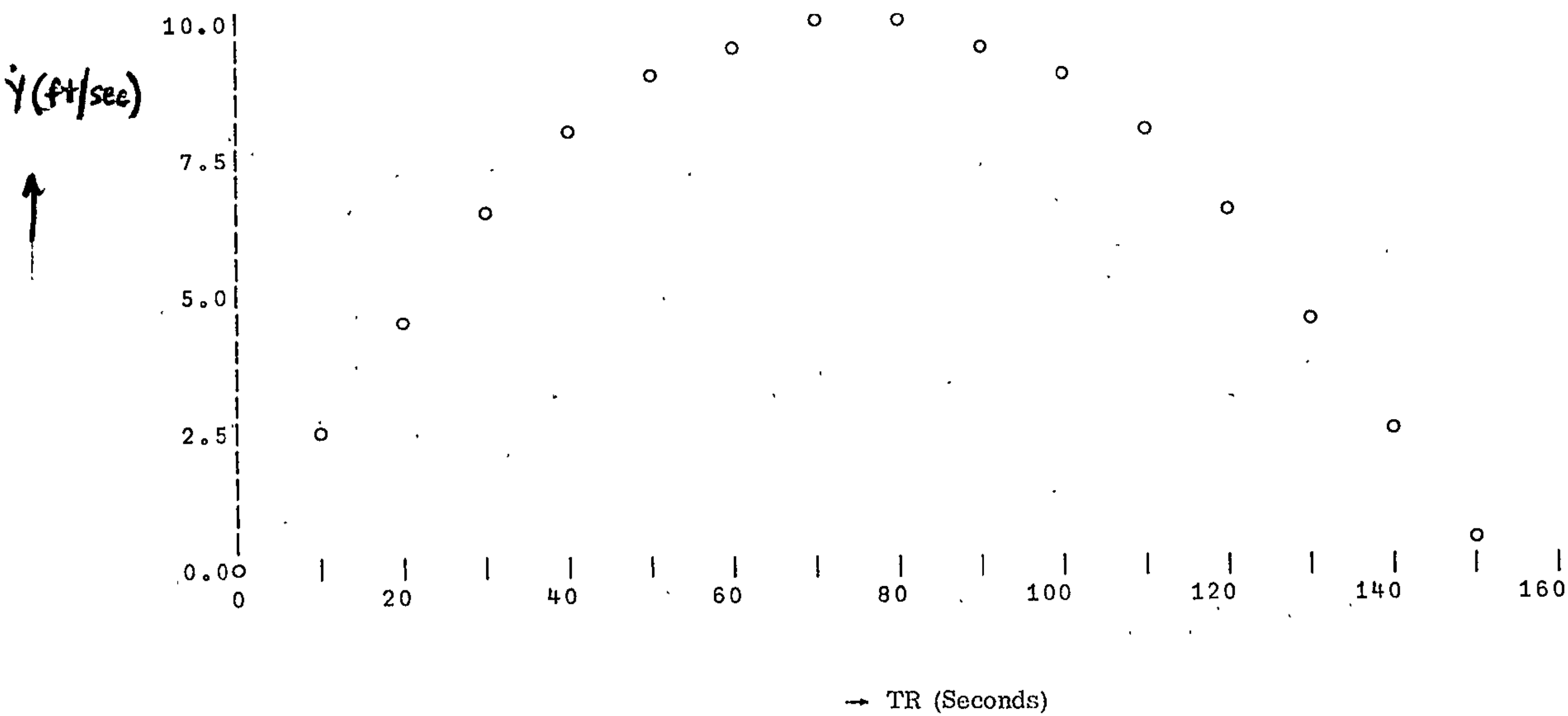


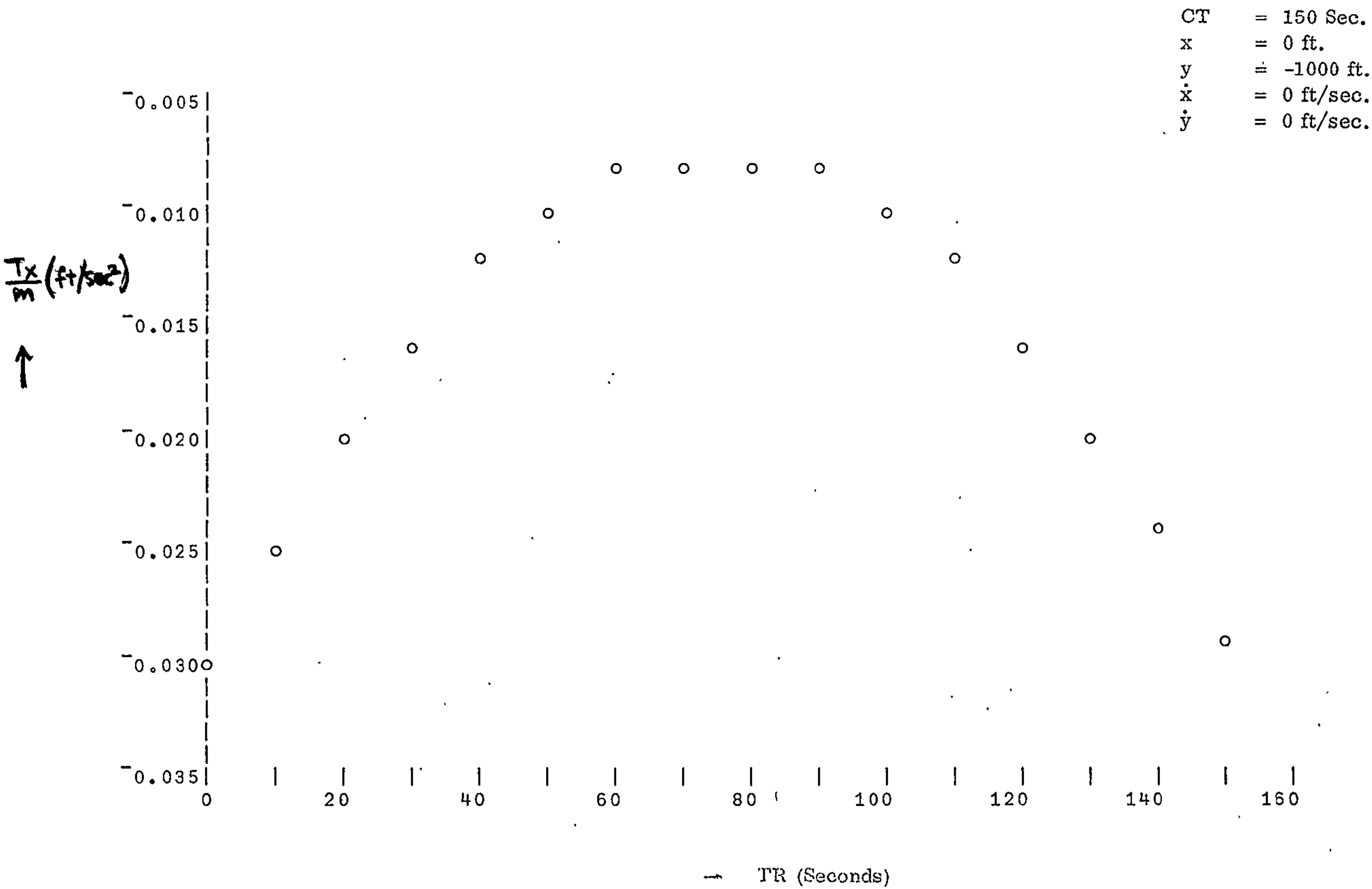
CT	=	150 Sec.
x	=	0 ft.
y	=	-1000 ft.
$\dot{x}$	=	0 ft/sec.
$\dot{y}$	=	0 ft/sec.

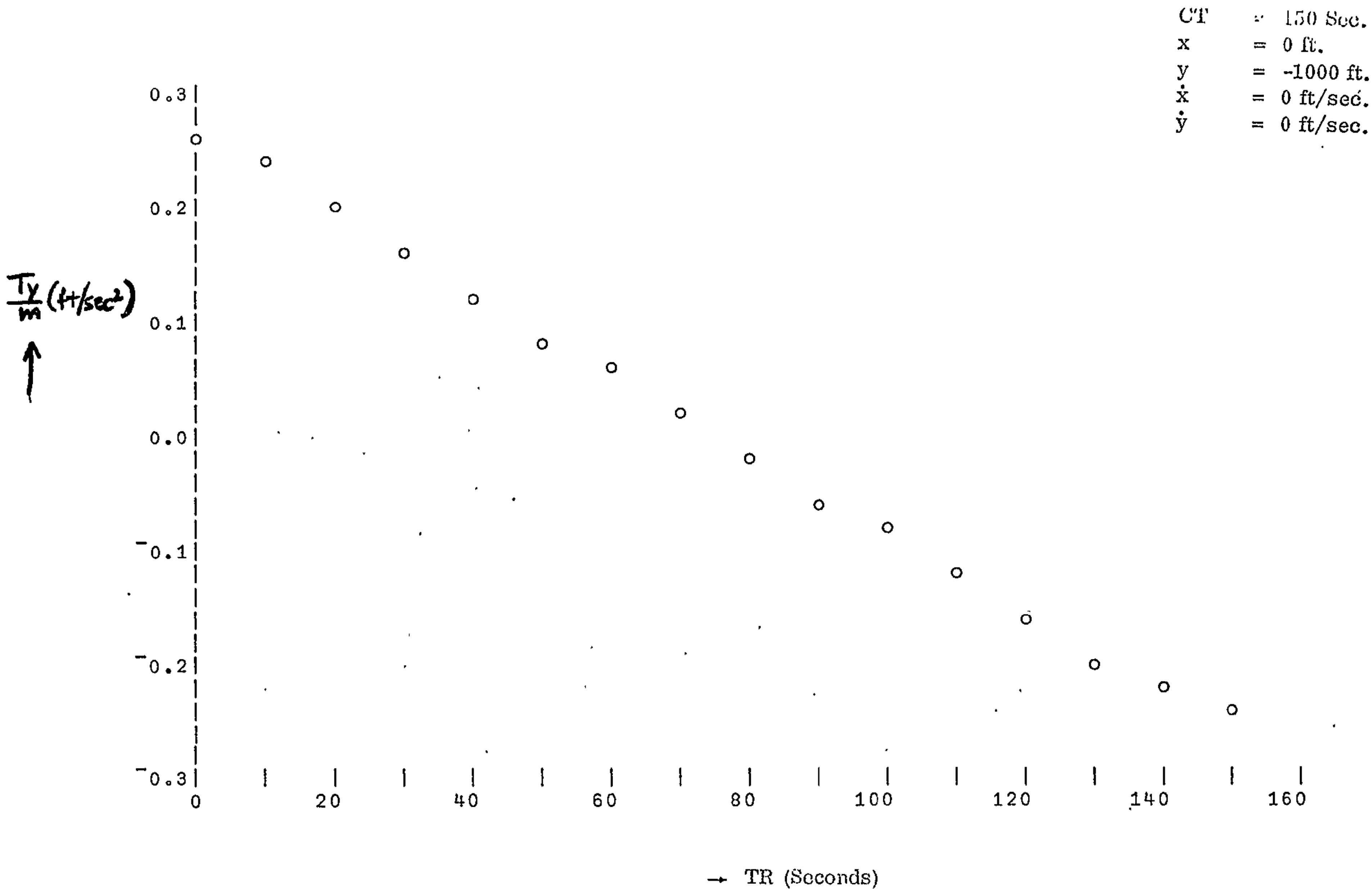



CT = 150 Sec.
x = 0 ft.
y = -1000 ft.
$\dot{x}$ = 0 ft/sec.
$\dot{y}$ = 0 ft/sec.
$\dot{x}$ (ft/sec)
0.50
0.25
0.00
-0.25
-0.50
0
20
40
60
80
100
120
140
160
TR (Seconds)

CT	=	150 Sec.
x	=	0 ft.
y	=	-1000 ft.
$\dot{x}$	=	0 ft/sec.
$\dot{y}$	=	0 ft/sec.

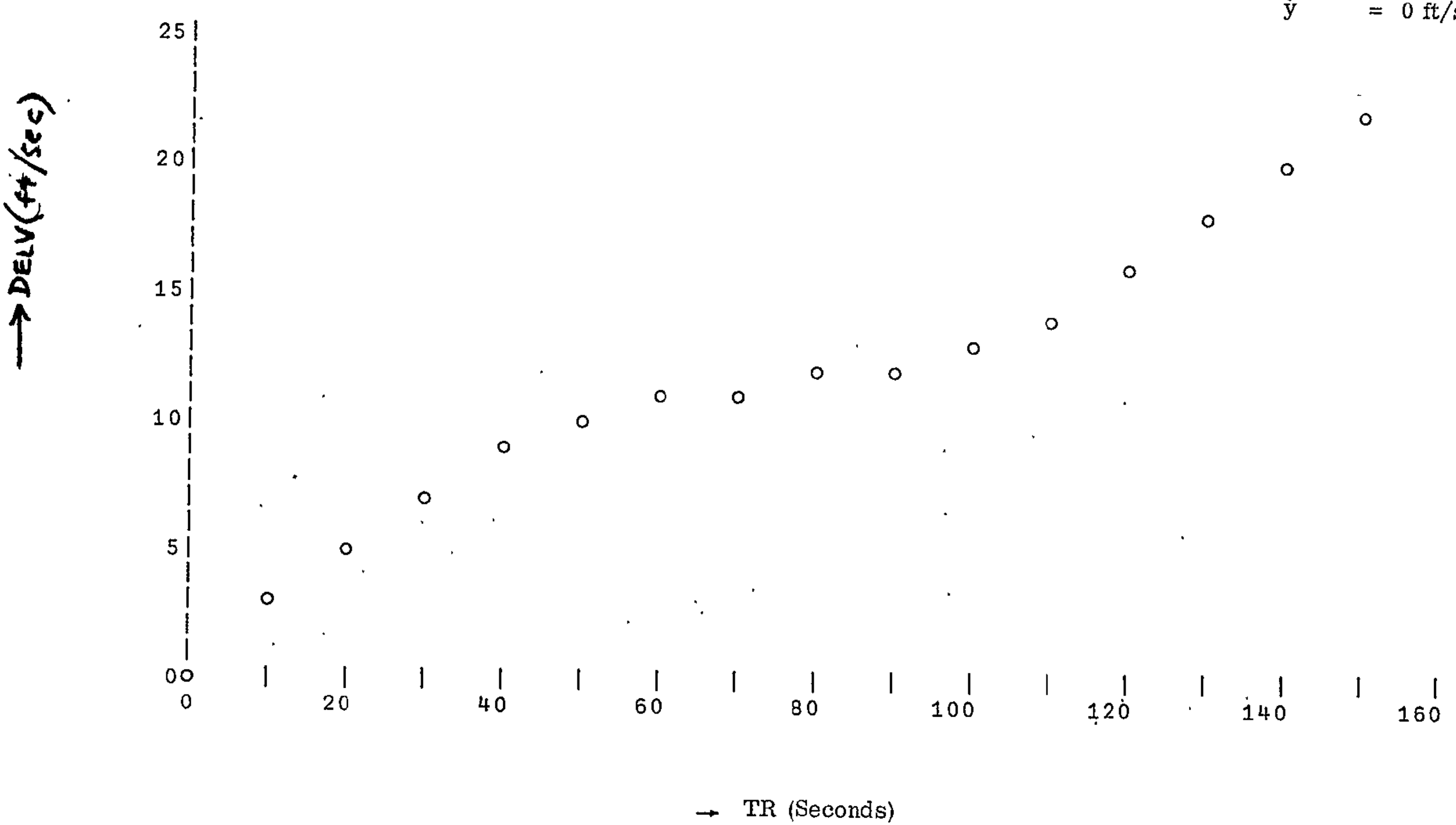


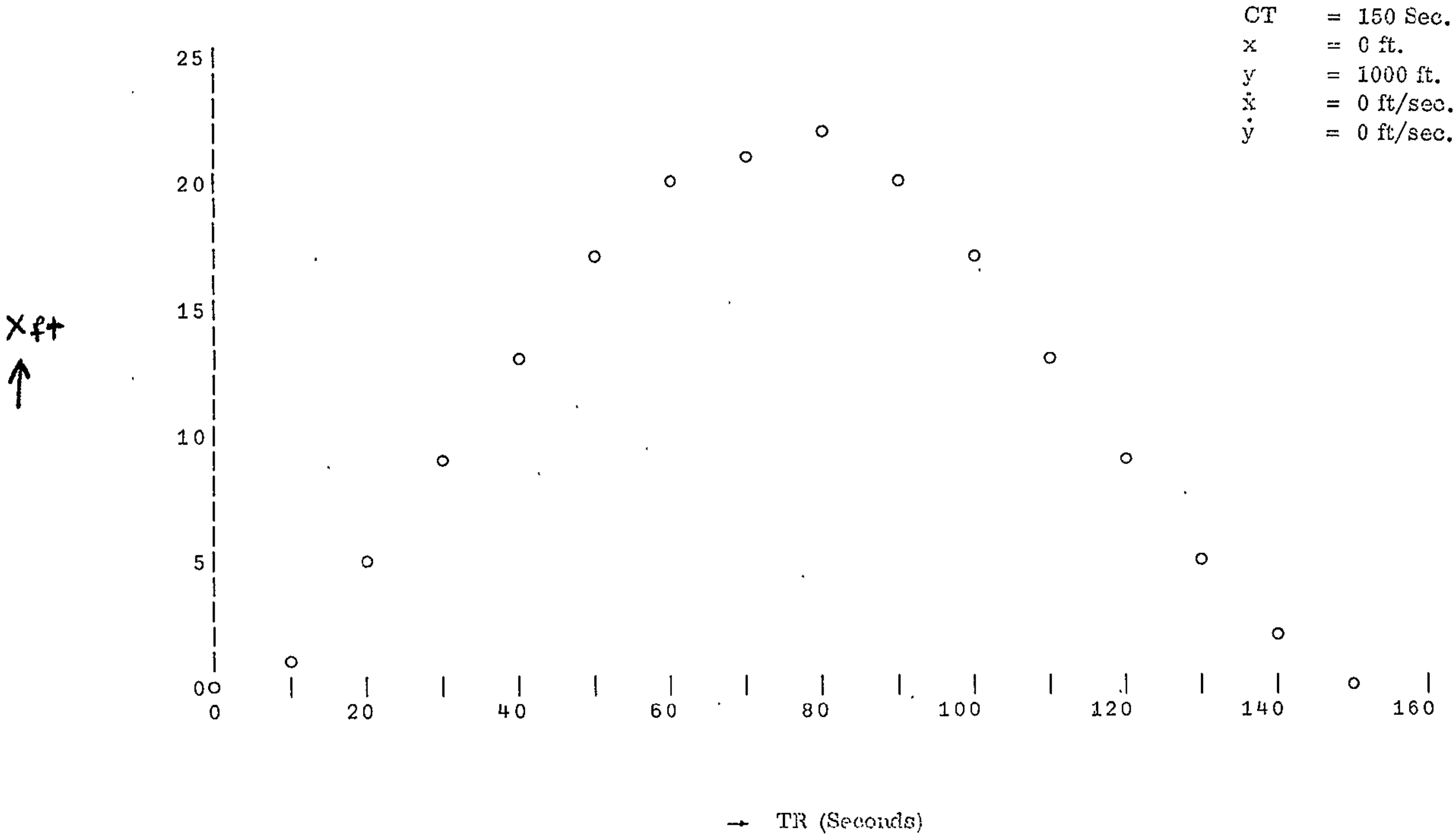

CT = 150 Sec.
x = 0 ft.
y = -1000 ft.
$\dot{x}$ = 0 ft/sec.
$\dot{y}$ = 0 ft/sec.
$\frac{T_x}{m}$ (ft/sec²)
-0.005
-0.010
-0.015
-0.020
-0.025
-0.030
-0.035
0
20
40
60
80
100
120
140
160
TR (Seconds)



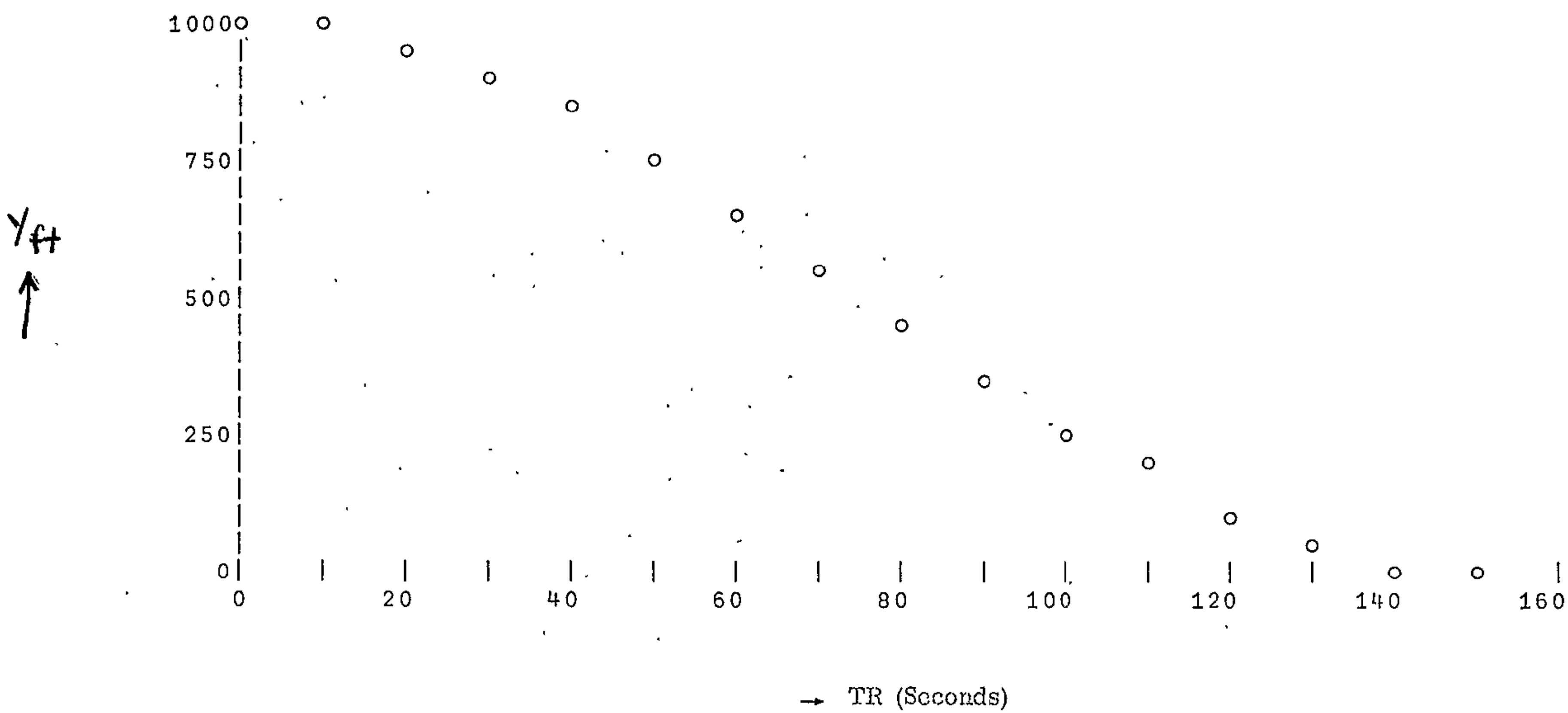
CT = 150 Sec.
x = 0 ft.
y = -1000 ft.
ẋ = 0 ft/sec.
ẏ = 0 ft/sec.
Ty/m (ft/sec²)
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
0
20
40
60
80
100
120
140
160
→ TR (Seconds)

CT = 150 Sec.
x = 0 ft.
y = -1000 ft.
$\dot{x}$ = 0 ft/sec.
$\dot{y}$ = 0 ft/sec.

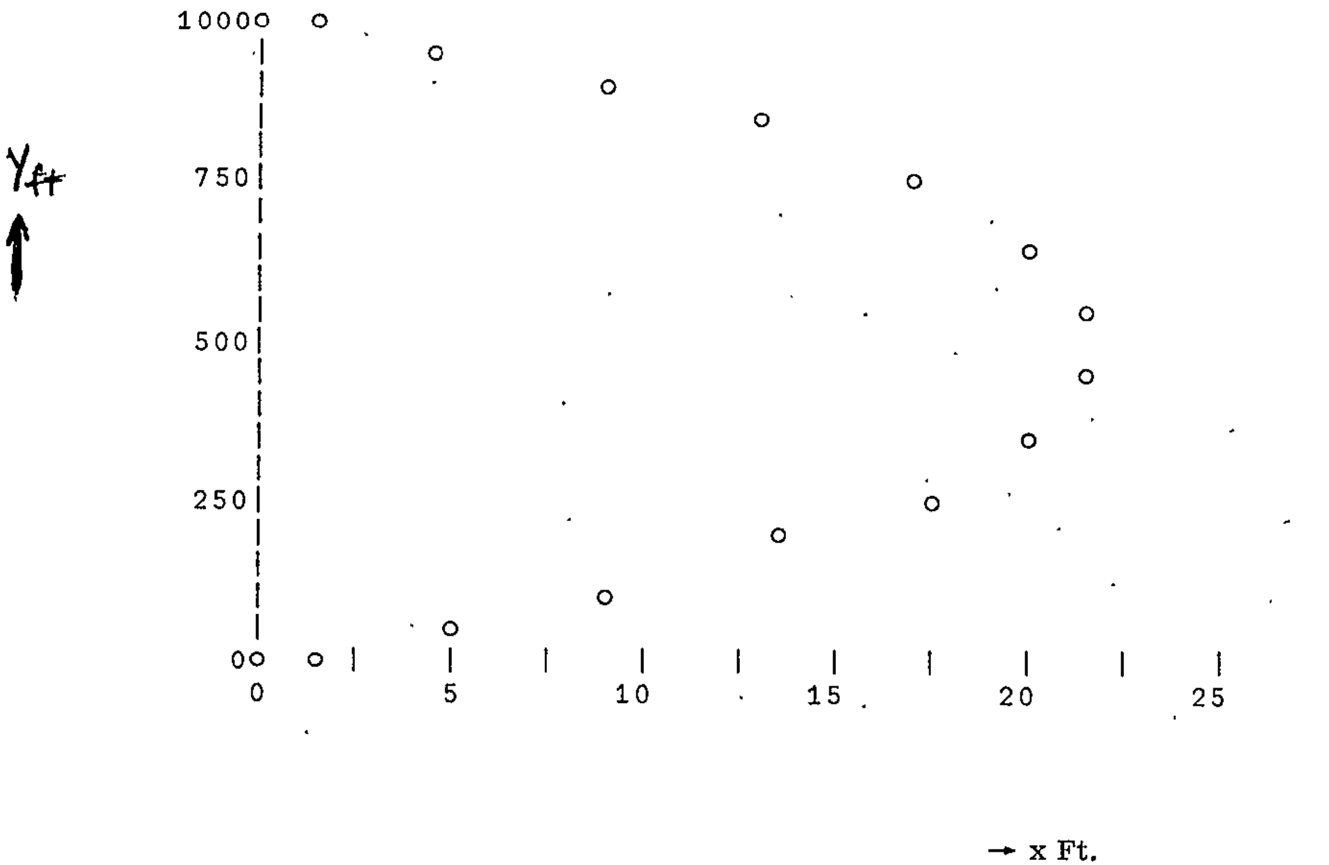


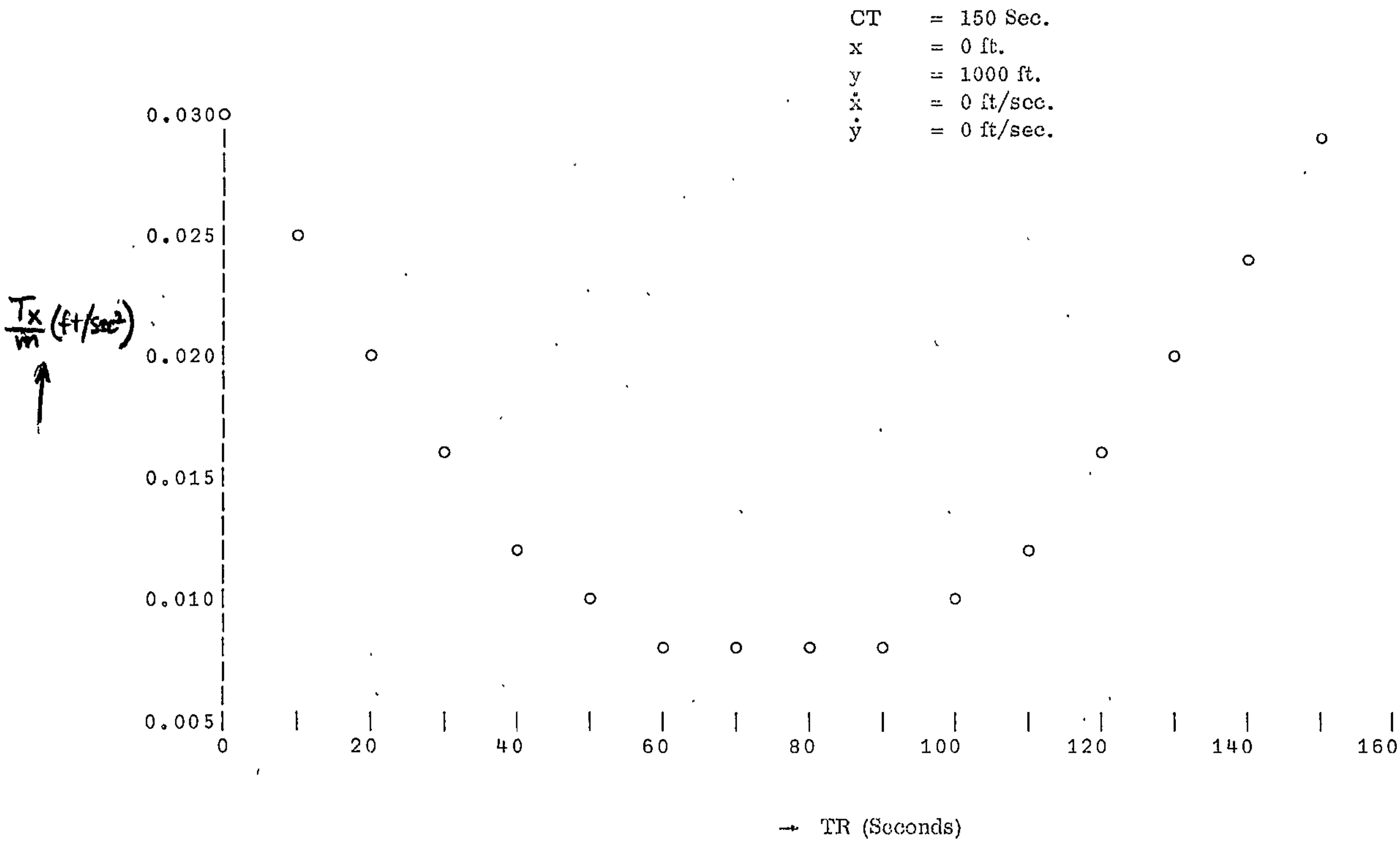

CT = 150 Sec.
x = 0 ft.
y = 1000 ft.
$\dot{x}$ = 0 ft/sec.
$\dot{y}$ = 0 ft/sec.
X ft
25
20
15
10
5
0
0
20
40
60
80
100
120
140
160
TR (Seconds)

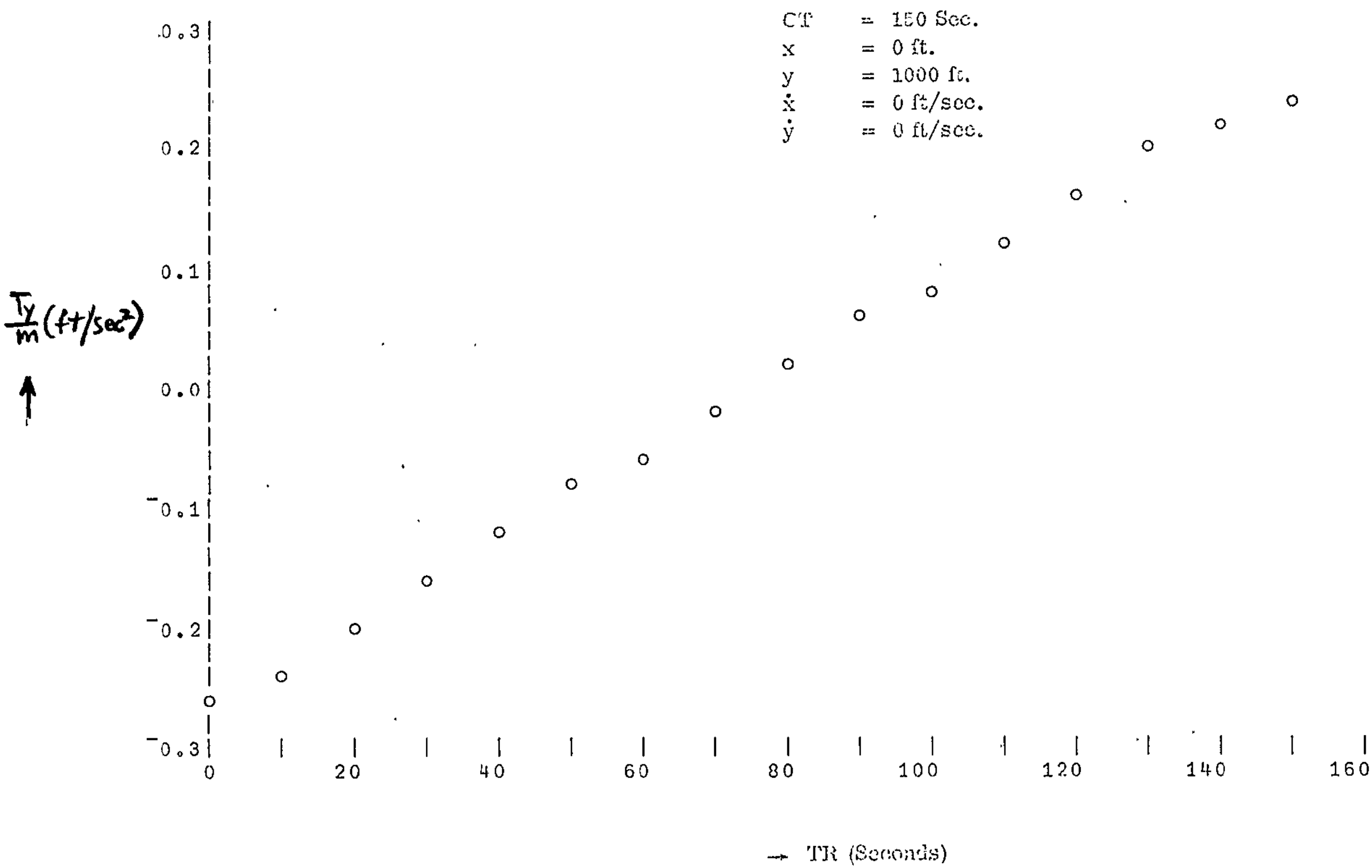
CT = 150 Sec.
x = 0 ft.
y = 1000 ft.
$\dot{x}$ = 0 ft/sec.
$\dot{y}$ = 0 ft./sec

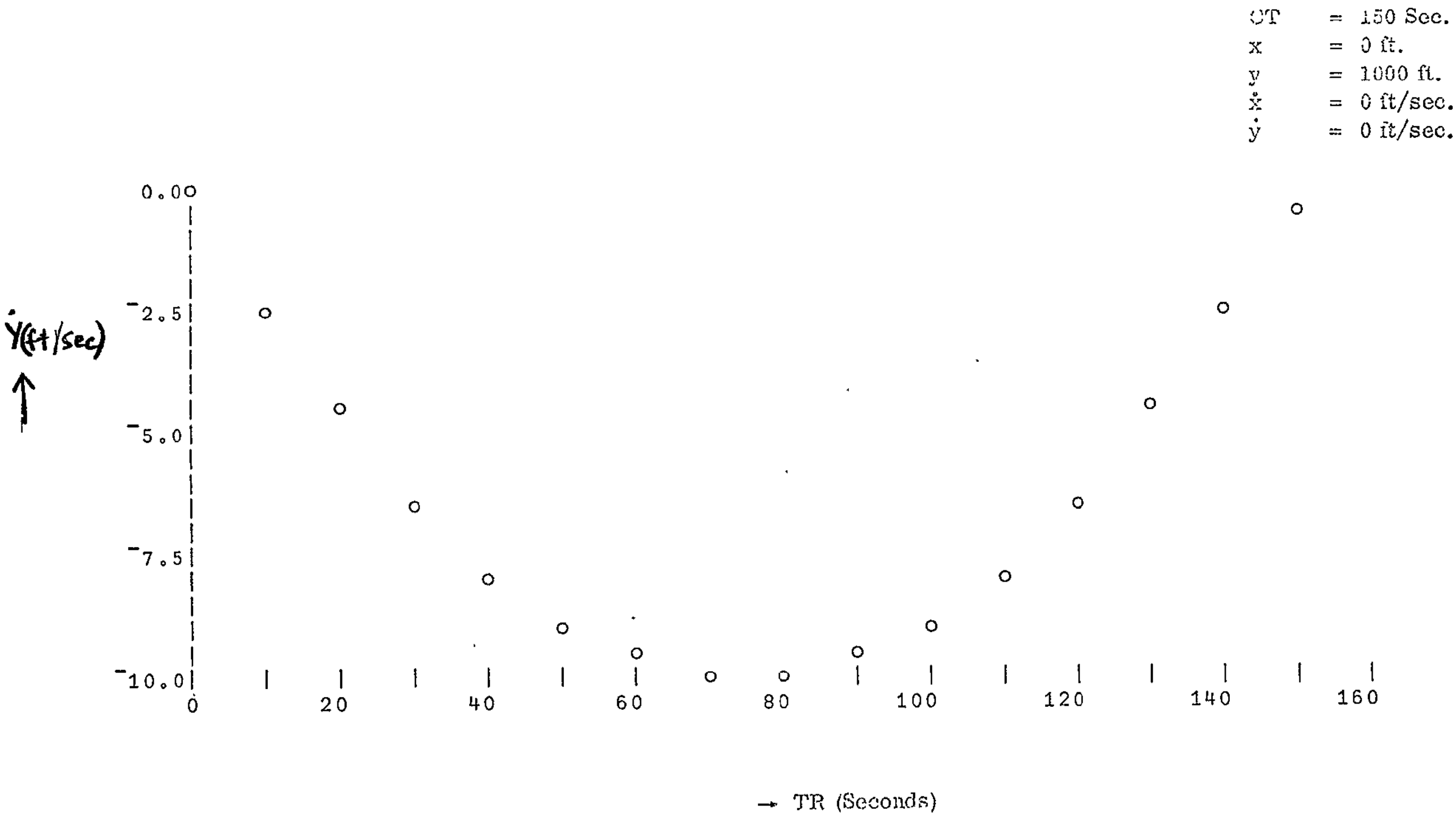


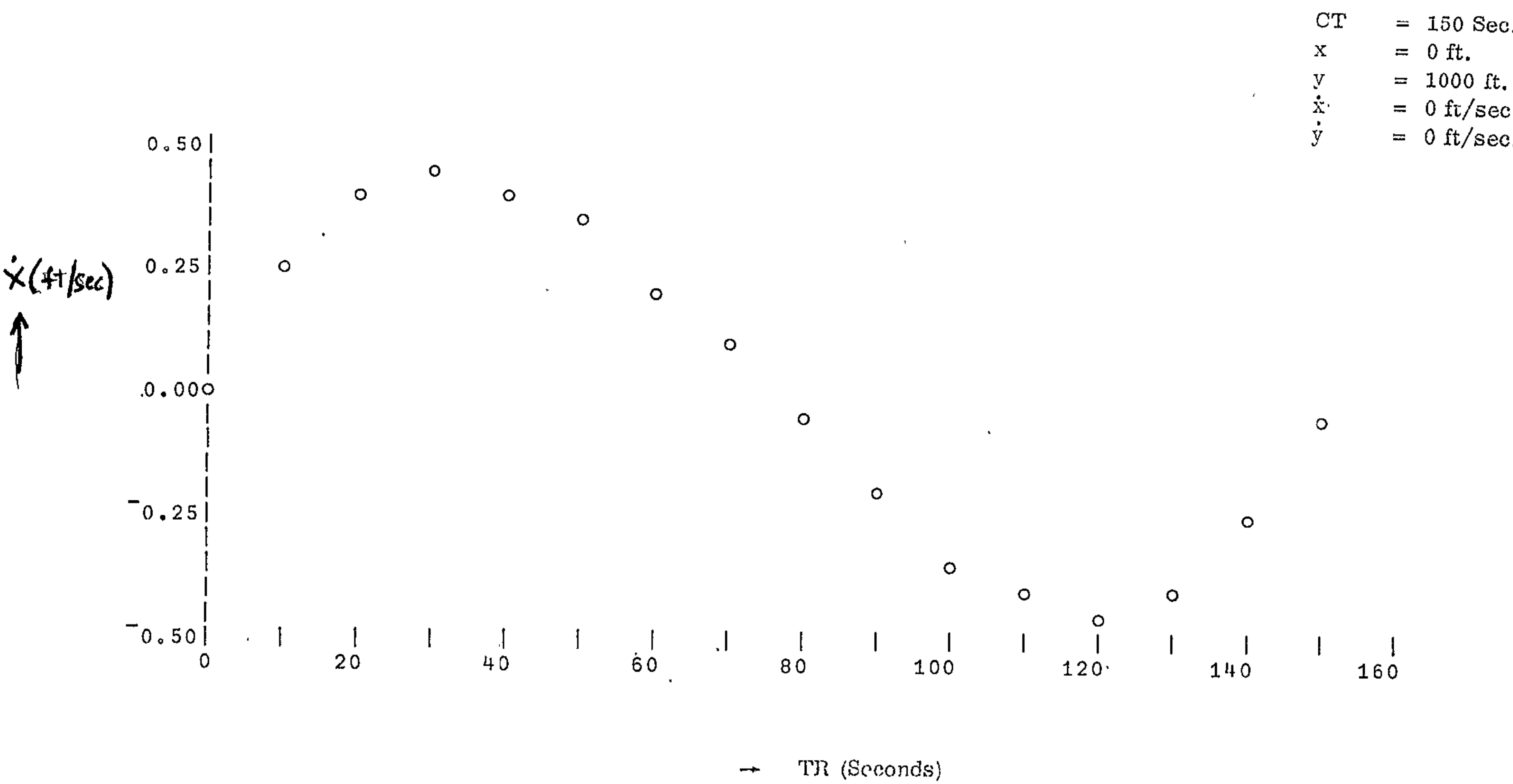
CT	=	150 Sec.
x	=	0 ft.
y	=	1000 ft.
$\dot{x}$	=	0 ft/sec.
$\dot{y}$	=	0 ft/sec.

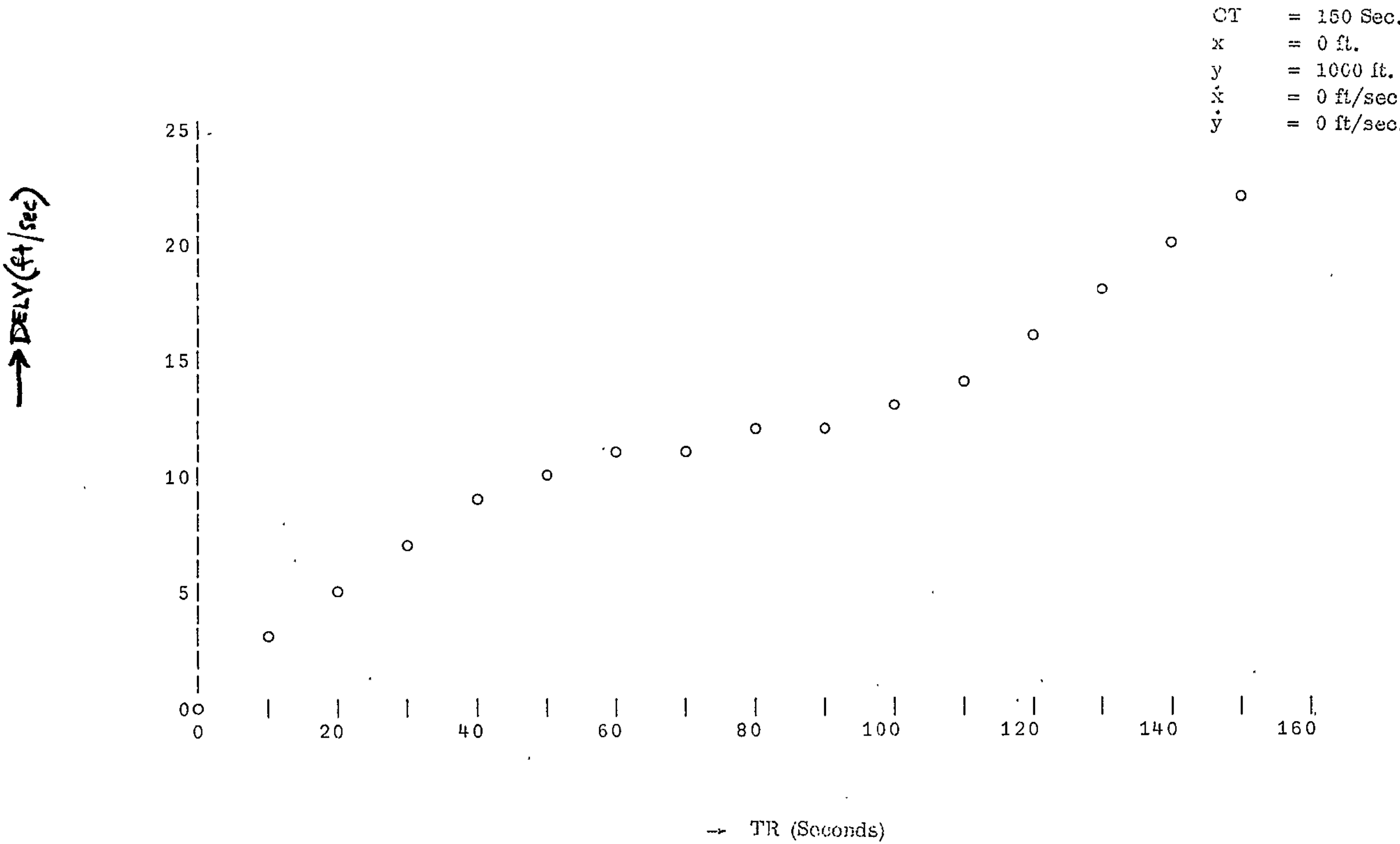



CT = 150 Sec.
x = 0 ft.
y = 1000 ft.
ẍ = 0 ft/sec.
ẏ = 0 ft/sec.
0.030
0.025
0.020
0.015
0.010
0.005
$\frac{T_x}{m}$ (ft/sec²)
0
20
40
60
80
100
120
140
160
TR (Seconds)

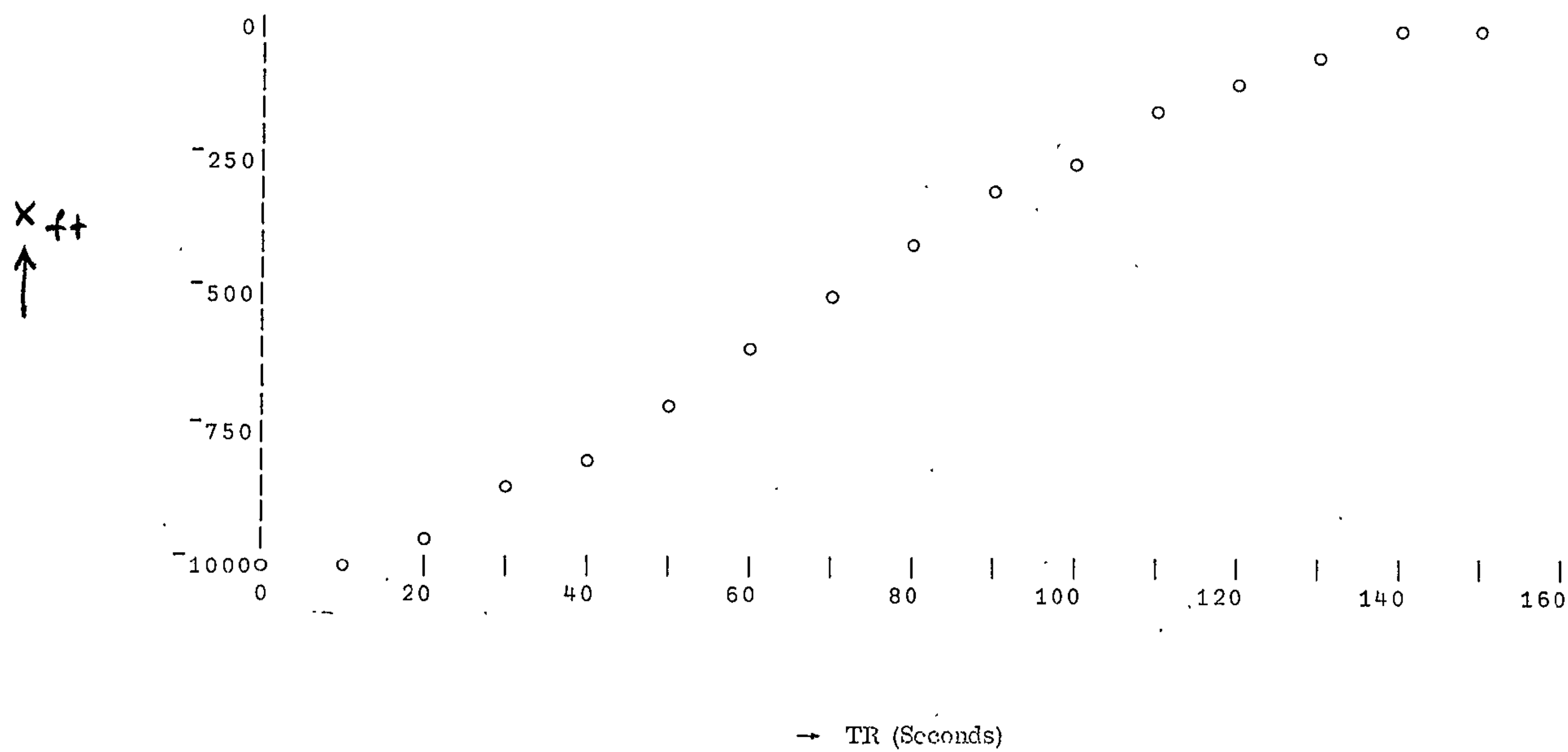

CT = 150 Sec.
x = 0 ft.
y = 1000 ft.
ẋ = 0 ft/sec.
ẏ = 0 ft/sec.
Ty/m (ft/sec²)
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
0
20
40
60
80
100
120
140
160
TR (Seconds)


CT = 150 Sec.
x = 0 ft.
y = 1000 ft.
ẋ = 0 ft/sec.
ẏ = 0 ft/sec.
0.00
-2.5
-5.0
-7.5
-10.0
Ẏ(ft/sec)
0
20
40
60
80
100
120
140
160
→ TR (Seconds)

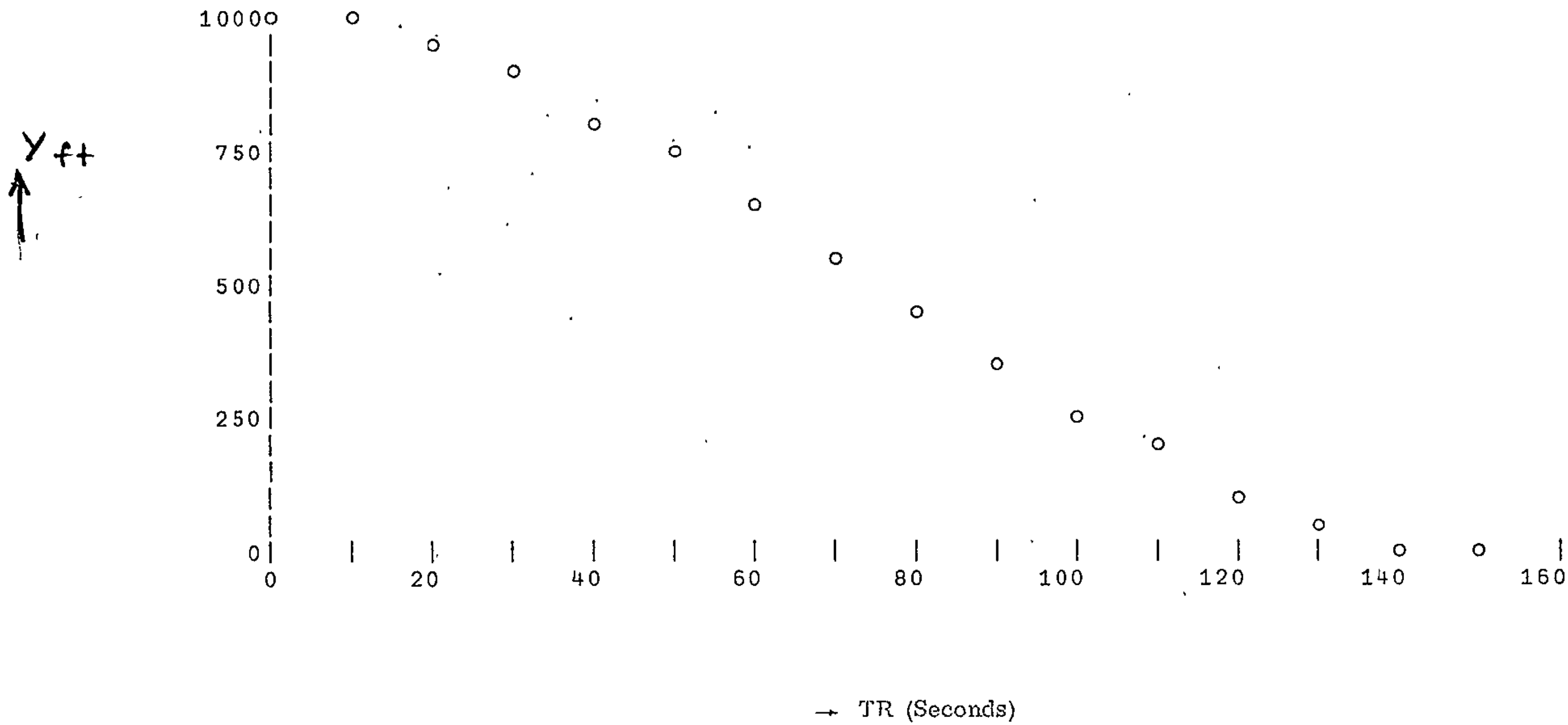

CT = 150 Sec.
x = 0 ft.
y = 1000 ft.
ẋ = 0 ft/sec.
ẏ = 0 ft/sec.
0.50
0.25
0.000
-0.25
-0.50
ẋ (ft/sec)
0
20
40
60
80
100
120
140
160
TR (Seconds)


CT = 150 Sec.
x = 0 ft.
y = 1000 ft.
ẋ = 0 ft/sec.
ẏ = 0 ft/sec.
DELY (ft/sec)
25
20
15
10
5
0
0
20
40
60
80
100
120
140
160
TR (Seconds)

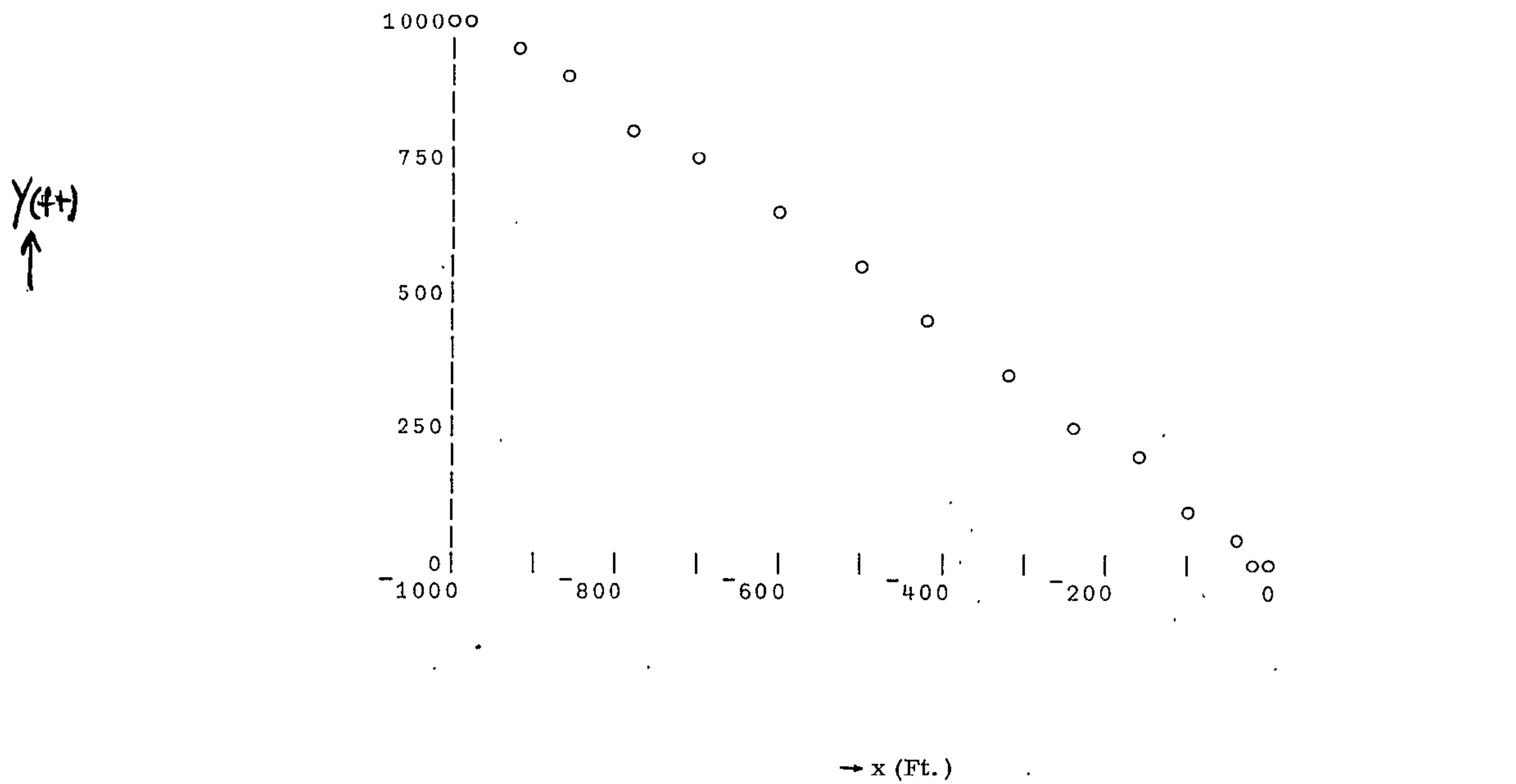
CT = 150 Sec.
x = -1000 ft.
y = 1000 ft.
$\dot{x}$ = 1.2 ft/sec.
$\dot{y}$ = -1.2 ft/sec.



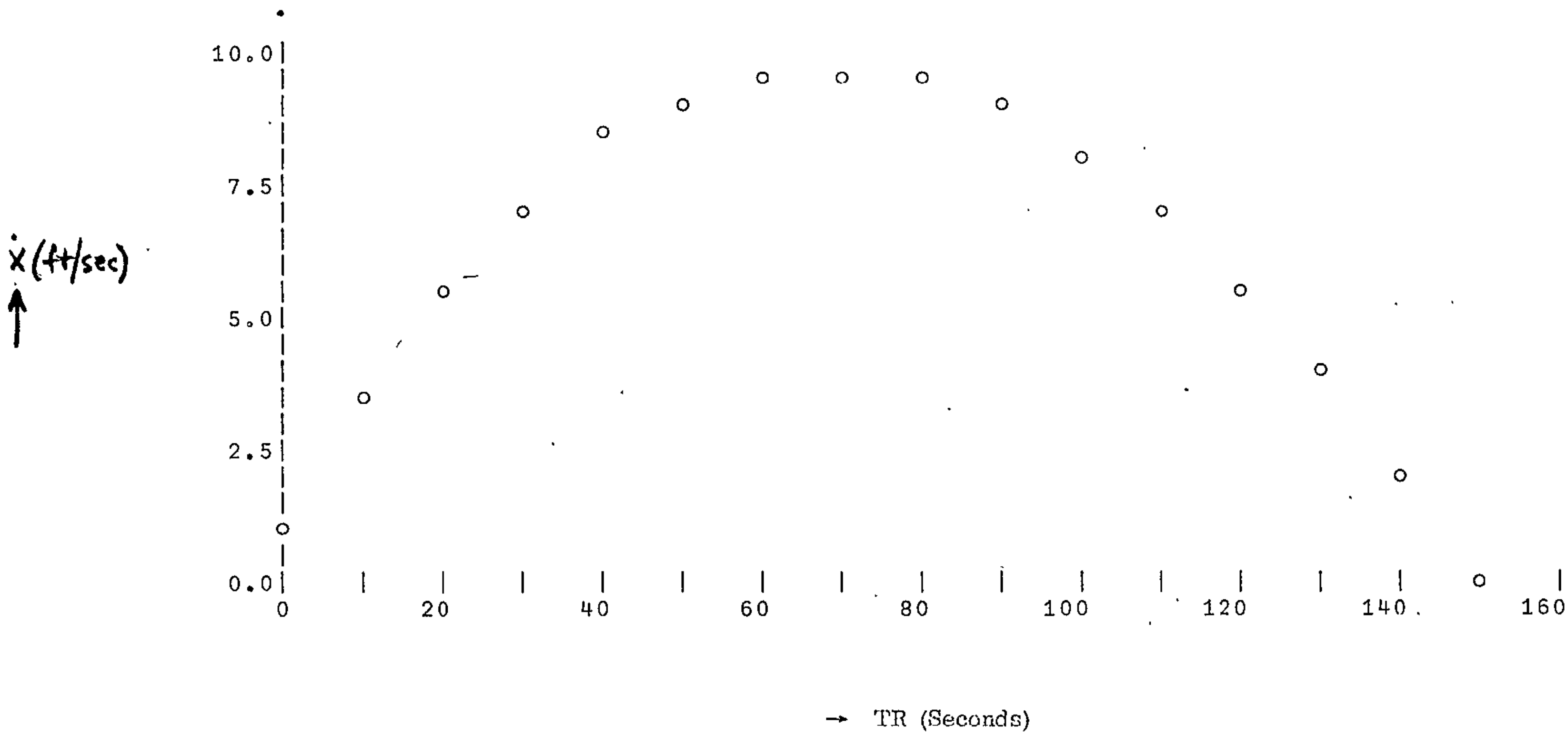
CT	=	150 Sec.
x	=	-1000 ft.
y	=	1000 ft.
$\dot{x}$	=	1.2 ft/sec.
$\dot{y}$	=	-1.2 ft/sec.



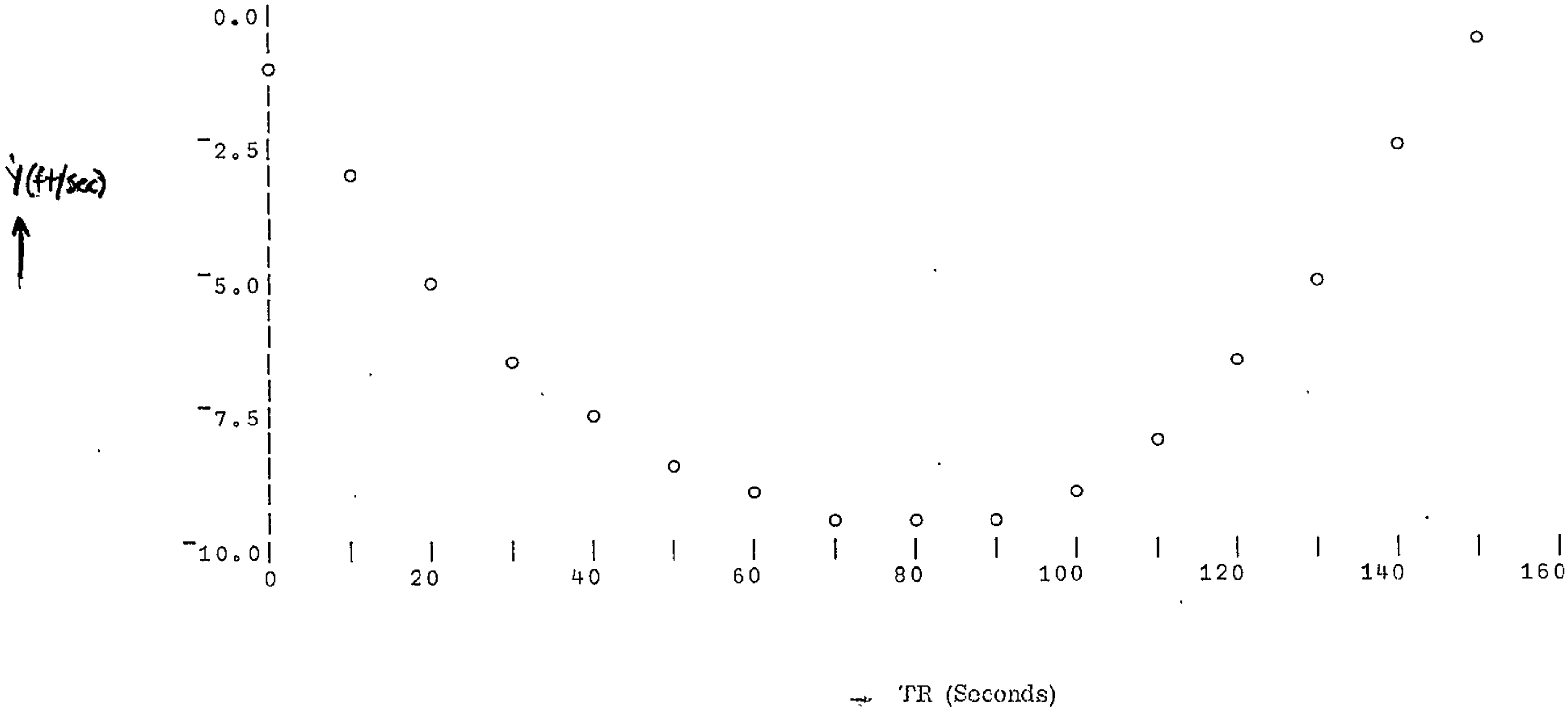
CT	=	150 Sec.
x	=	-1000 ft.
y	=	1000 ft.
$\dot{x}$	=	1.2 ft/sec.
$\dot{y}$	=	-1.2 ft/sec.

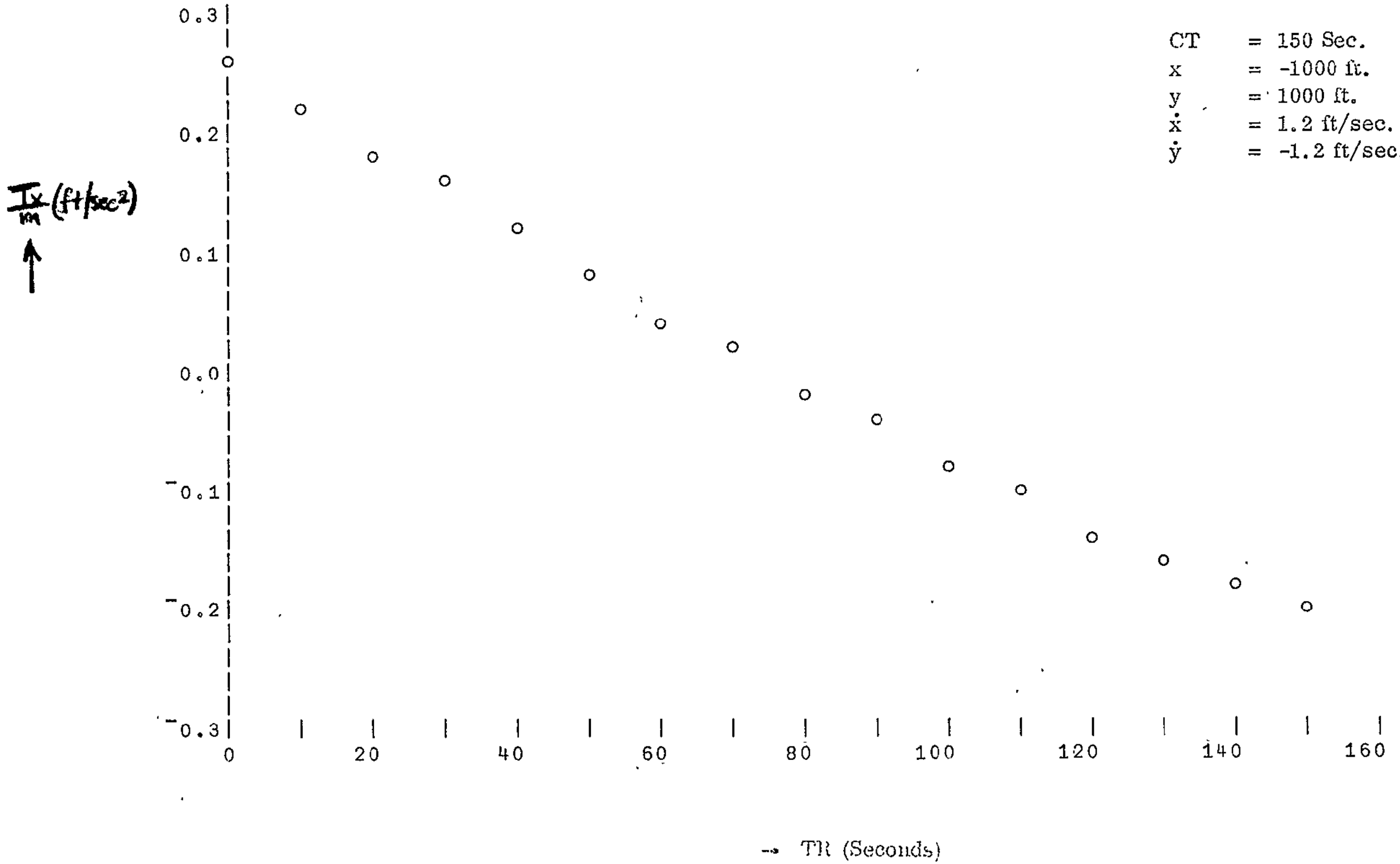


CT = 150 Sec.
x = -1000 ft.
y = 1000 ft.
$\dot{x}$ = 1.2 ft/sec.
$\dot{y}$ = -1.2 ft/sec.

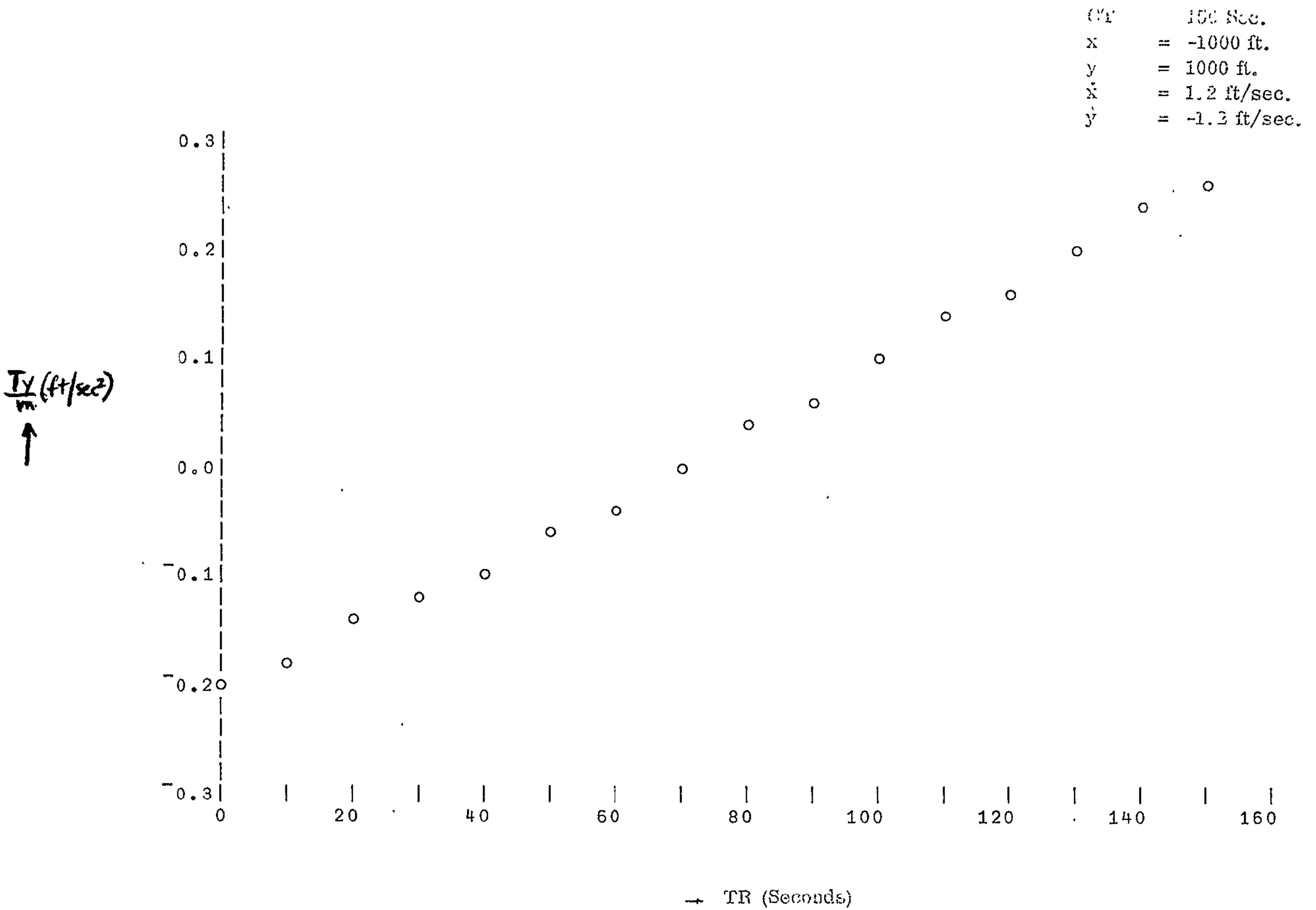


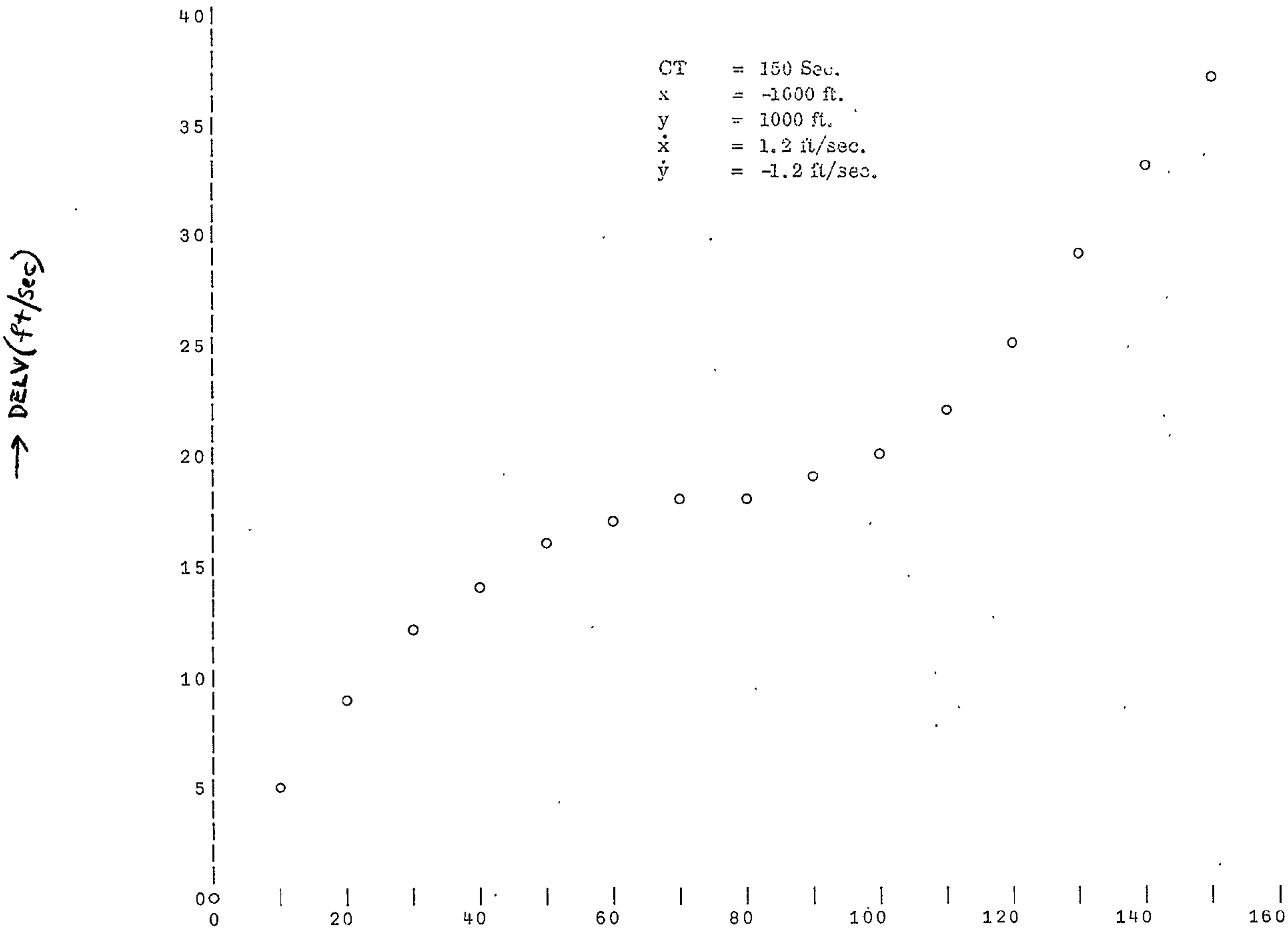
CT = 150 Sec.
x = -1000 ft.
y = 1000 ft.
$\dot{x}$ = 1.2 ft/sec.
$\dot{y}$ = -1.2 ft/sec.



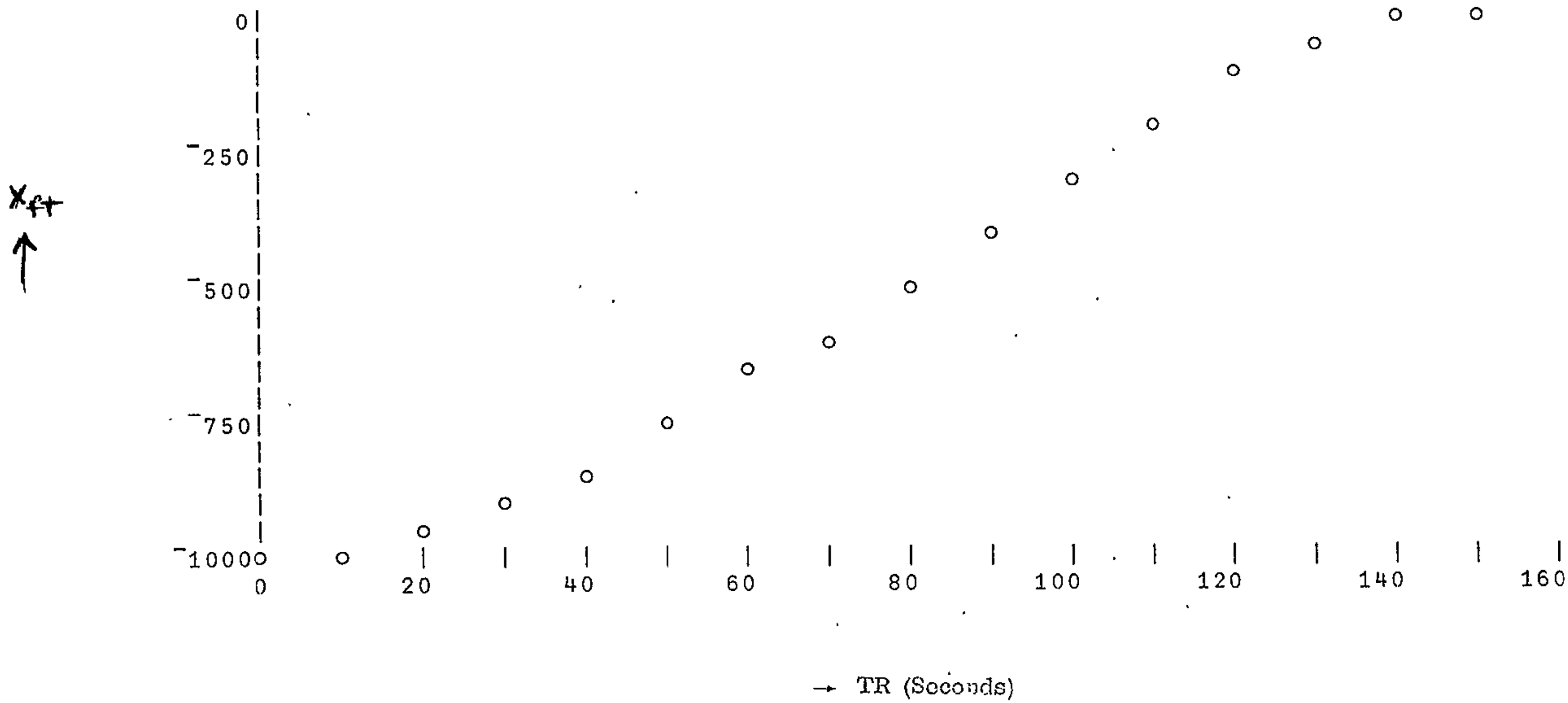


CT = 150 Sec.
x = -1000 ft.
y = 1000 ft.
$\dot{x}$ = 1.2 ft/sec.
$\dot{y}$ = -1.2 ft/sec.
$\frac{I_x}{m}$ (ft/sec²)
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
0
20
40
60
80
100
120
140
160
TR (Seconds)

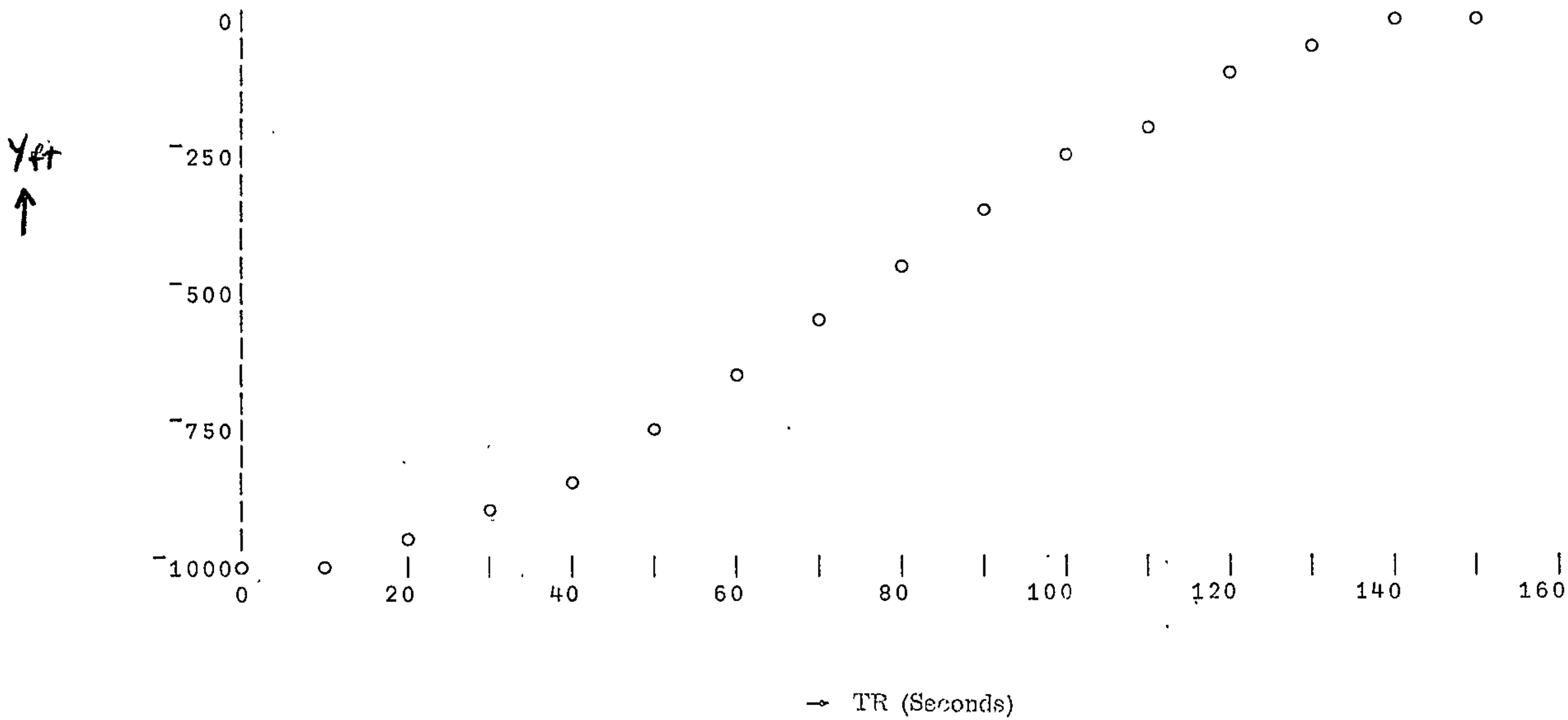

CT 150 Sec.
x = -1000 ft.
y = 1000 ft.
$\dot{x}$ = 1.2 ft/sec.
$\dot{y}$ = -1.2 ft/sec.
0.3
0.2
0.1
0.0
-0.1
-0.20
-0.3
$\frac{T_y}{m}$ (ft/sec^2)
0
20
40
60
80
100
120
140
160
TR (Seconds)


CT = 150 Sec.
x = -1000 ft.
y = 1000 ft.
ẋ = 1.2 ft/sec.
ẏ = -1.2 ft/sec.
→ DELV (ft/sec)
→ TR (Seconds)
0
5
10
15
20
25
30
35
40
20
40
60
80
100
120
140
160

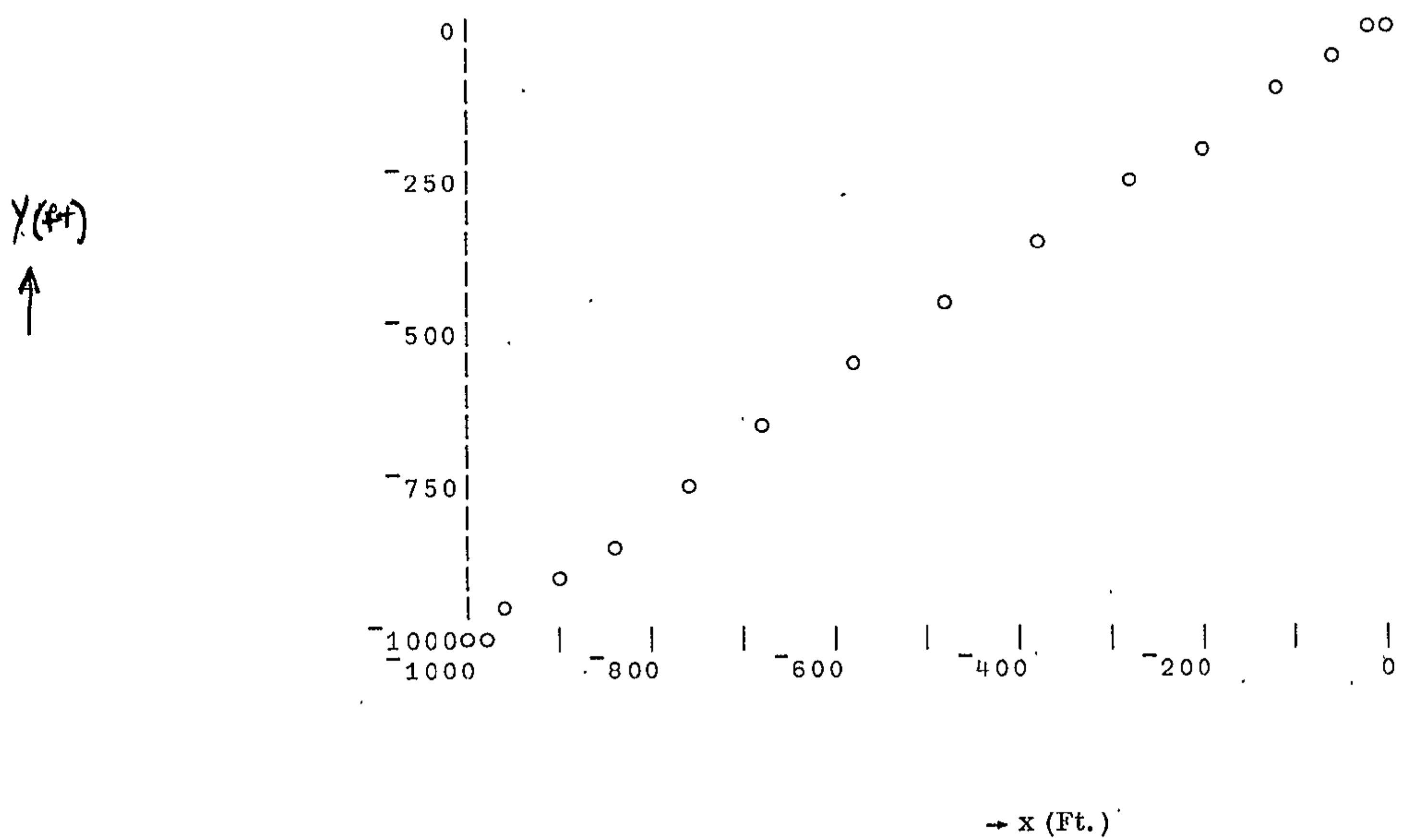
CT	=	150 Sec.
x	=	-1000 ft.
y	=	-1000 ft.
$\dot{x}$	=	.032 ft/sec.
$\dot{y}$	=	-1.2 ft/sec.



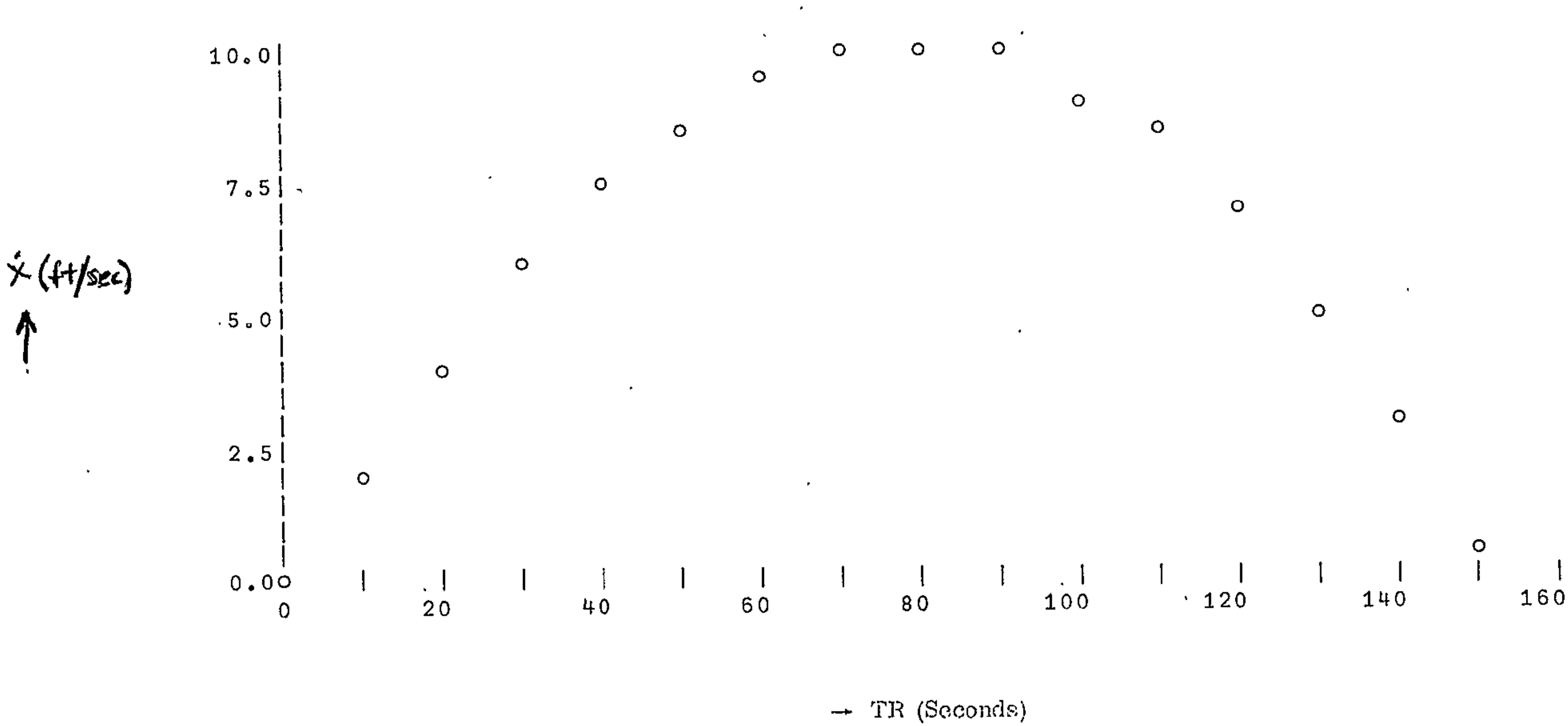
CT = 150 Sec.
x = -1000 ft.
y = -1000 ft.
$\dot{x}$ = .032 ft/sec.
$\dot{y}$ = -1.2 ft/sec.

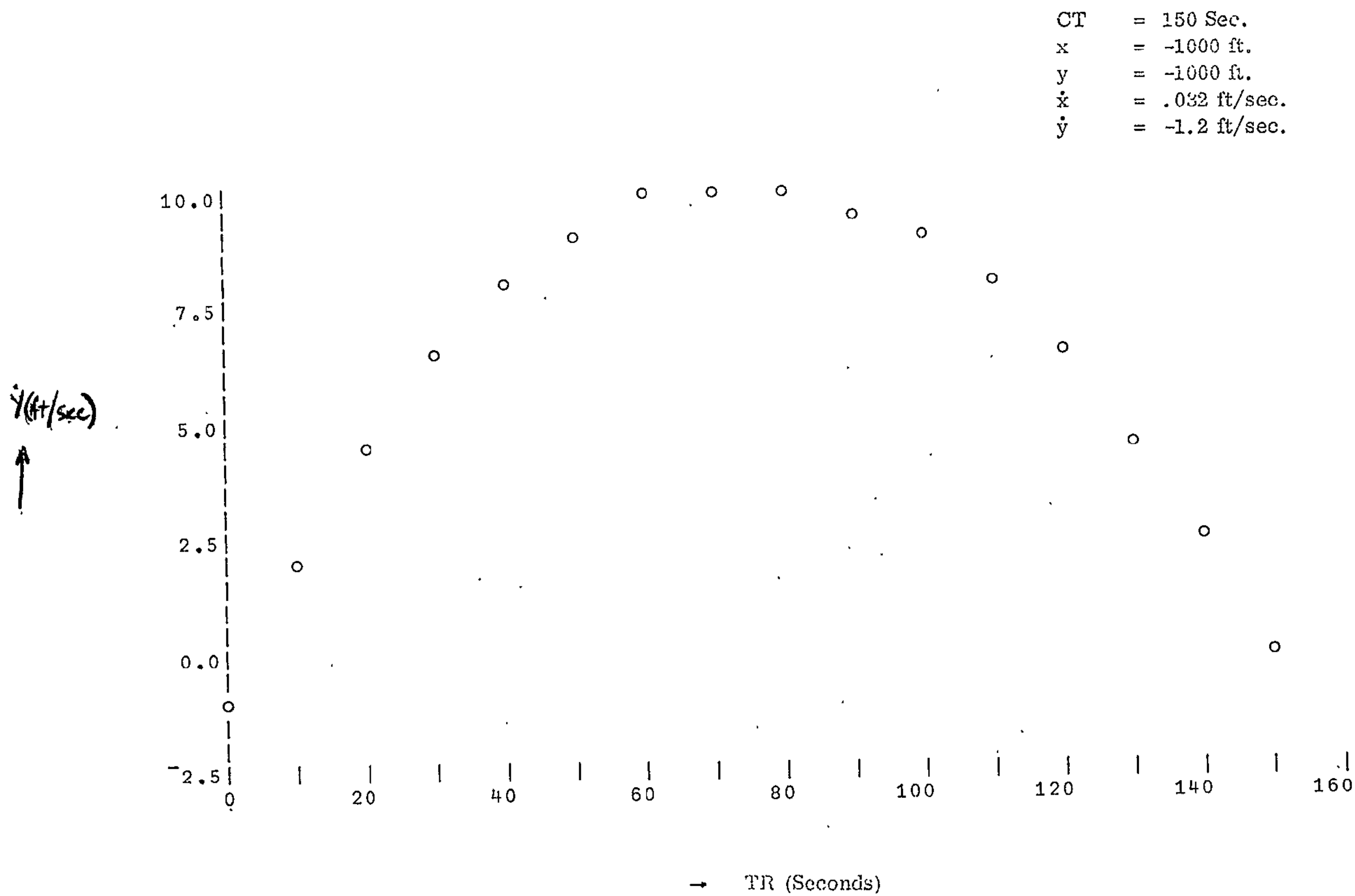


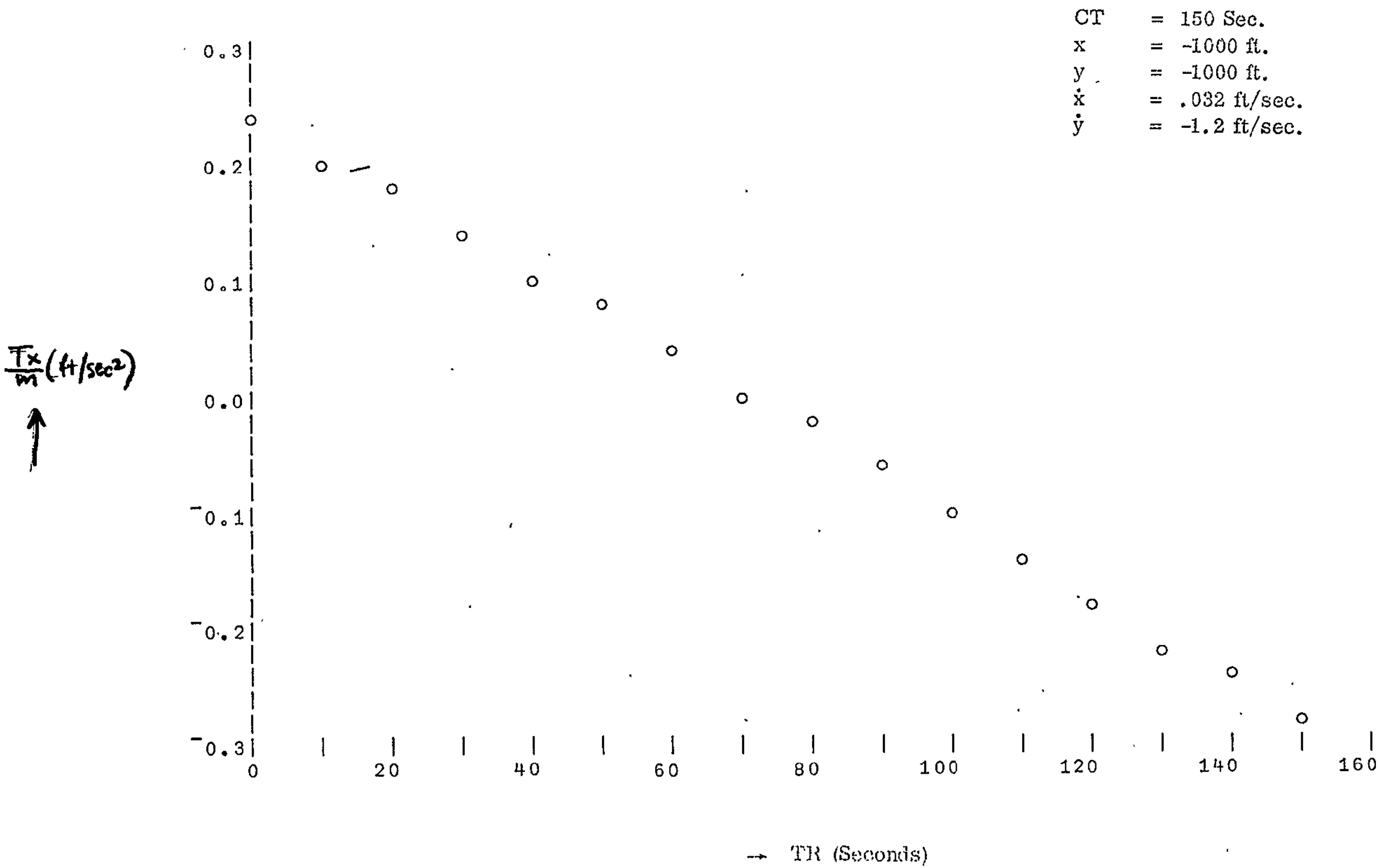
CT	=	150 Sec.
x	=	-1000 ft.
y	=	-1000 ft.
$\dot{x}$	=	.032 ft/sec.
$\dot{y}$	=	-1.2 ft/sec.

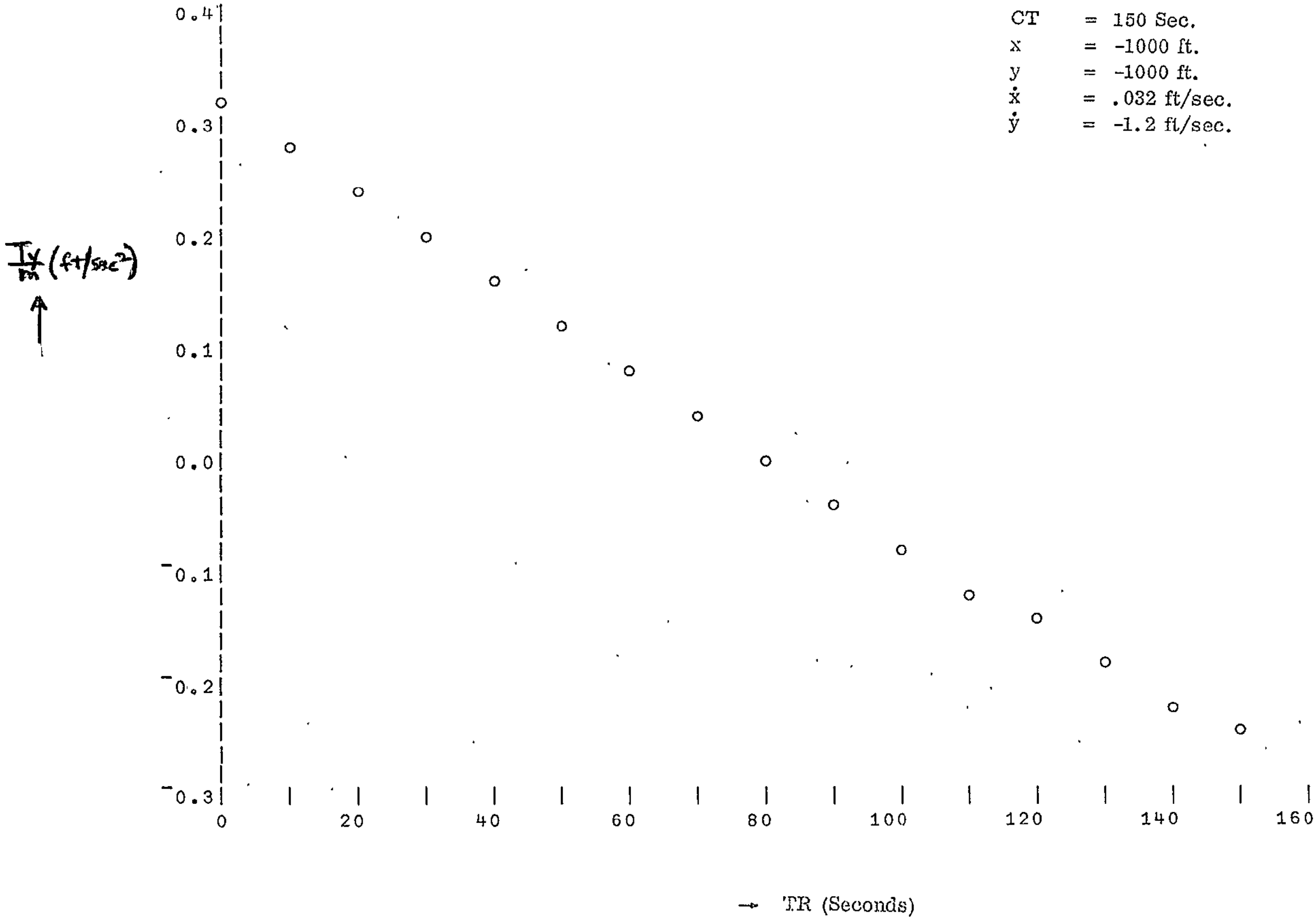


CT = 150 Sec.
x = -1000 ft.
y = -1000 ft.
$\dot{x}$ = .032 ft/sec.
$\dot{y}$ = -1.2 ft/sec.

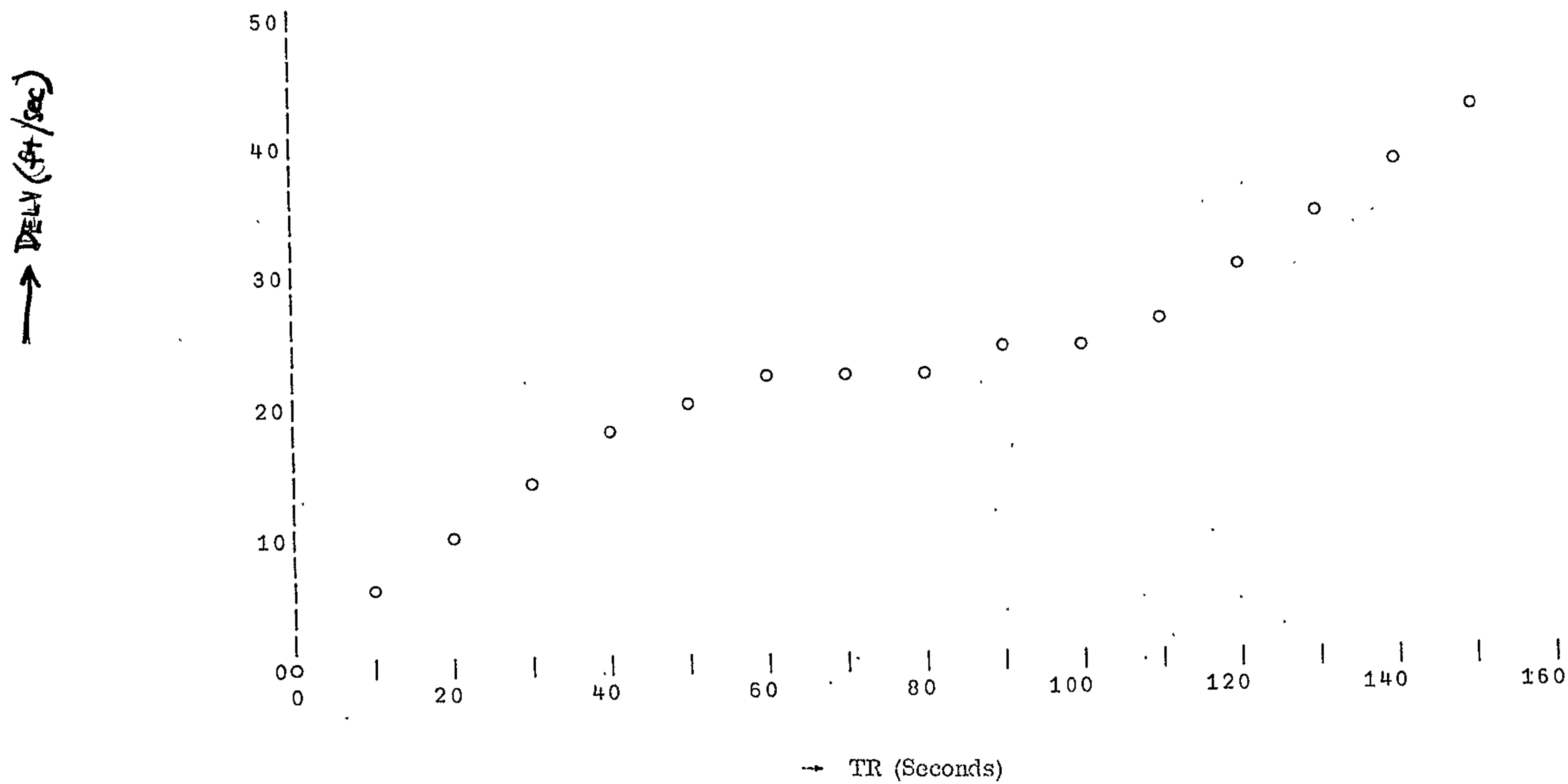



CT = 150 Sec.
x = -1000 ft.
y = -1000 ft.
ẋ = .032 ft/sec.
ẏ = -1.2 ft/sec.
10.0
7.5
5.0
2.5
0.0
-2.5
ẏ(ft/sec)
0
20
40
60
80
100
120
140
160
TR (Seconds)

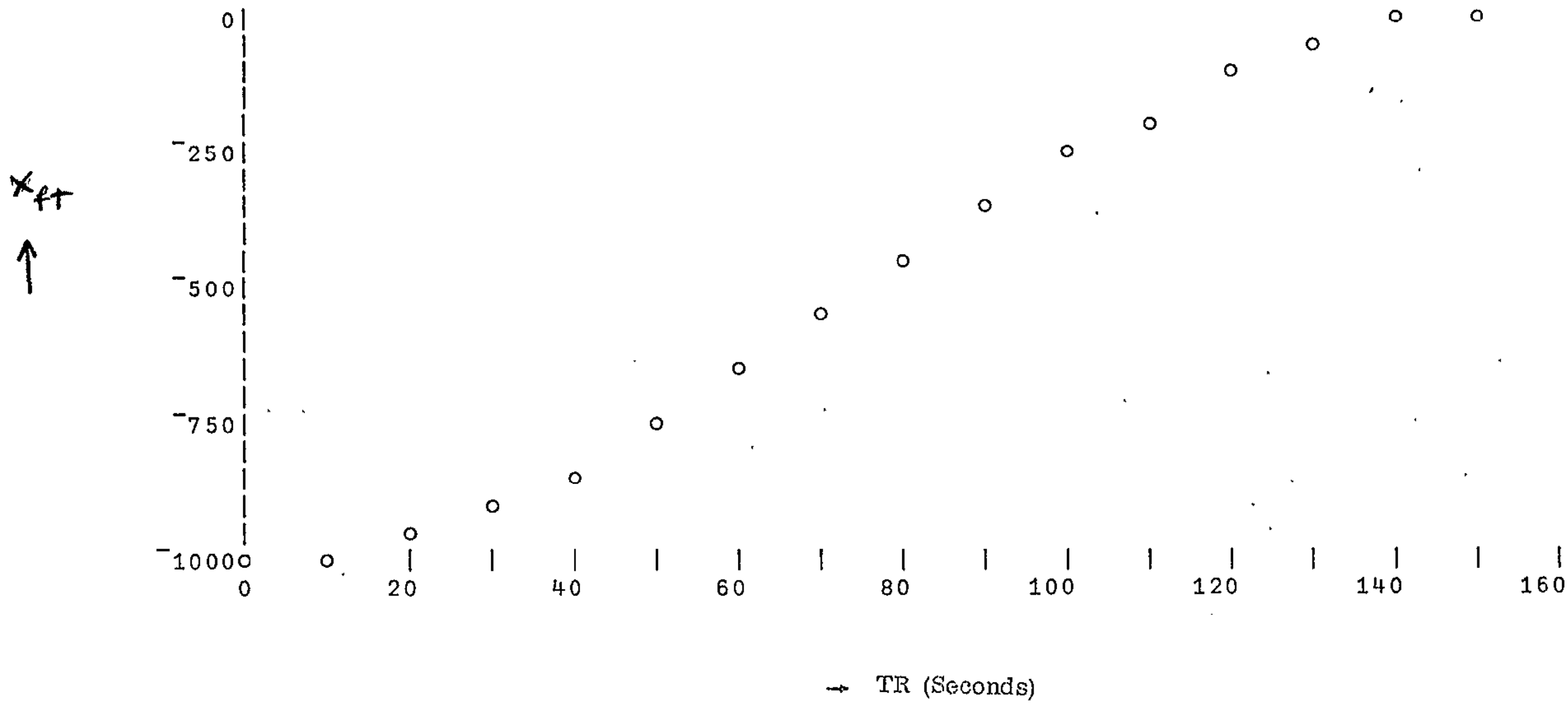

CT = 150 Sec.
x = -1000 ft.
y = -1000 ft.
ẋ = .032 ft/sec.
ẏ = -1.2 ft/sec.
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
Tx/m (ft/sec²)
0
20
40
60
80
100
120
140
160
TR (Seconds)

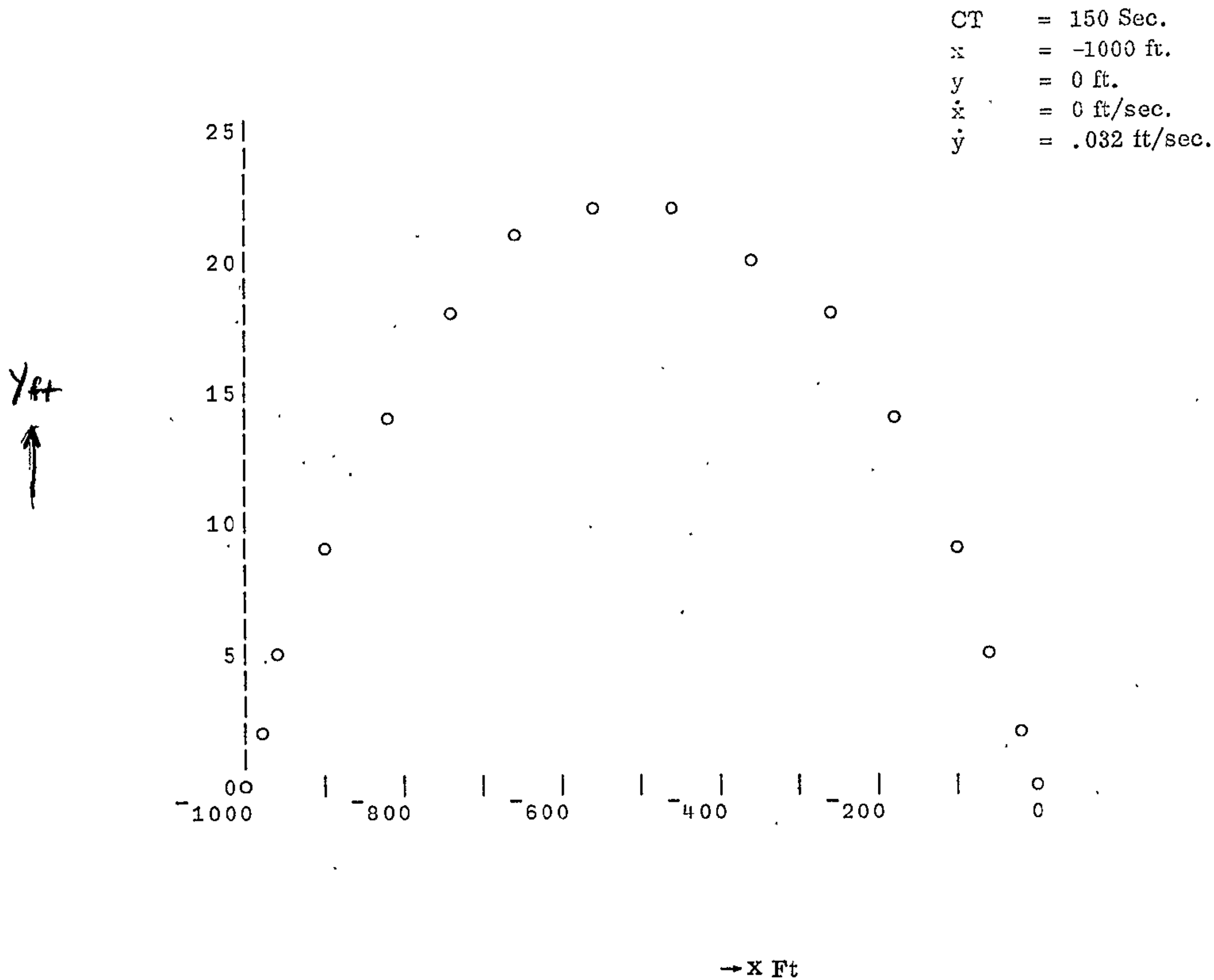

CT = 150 Sec.
x = -1000 ft.
y = -1000 ft.
ẋ = .032 ft/sec.
ẏ = -1.2 ft/sec.
Iy/m (ft/sec²)
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
0
20
40
60
80
100
120
140
160
TR (Seconds)

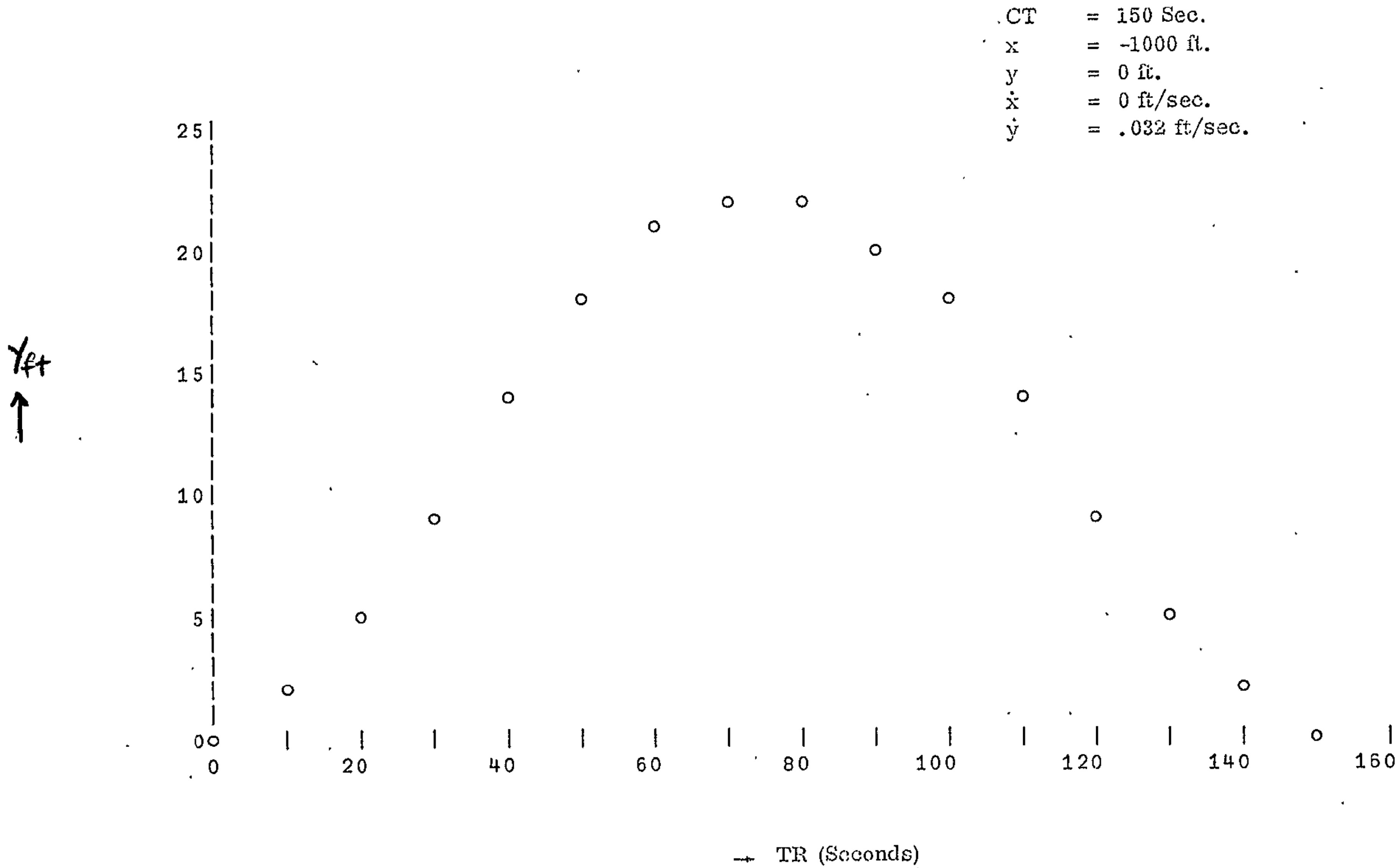
CT	=	150 Sec.
x	=	-1000 ft.
y	=	-1000 ft.
$\dot{x}$	=	.032 ft/sec.
$\dot{y}$	=	-1.2 ft/sec.



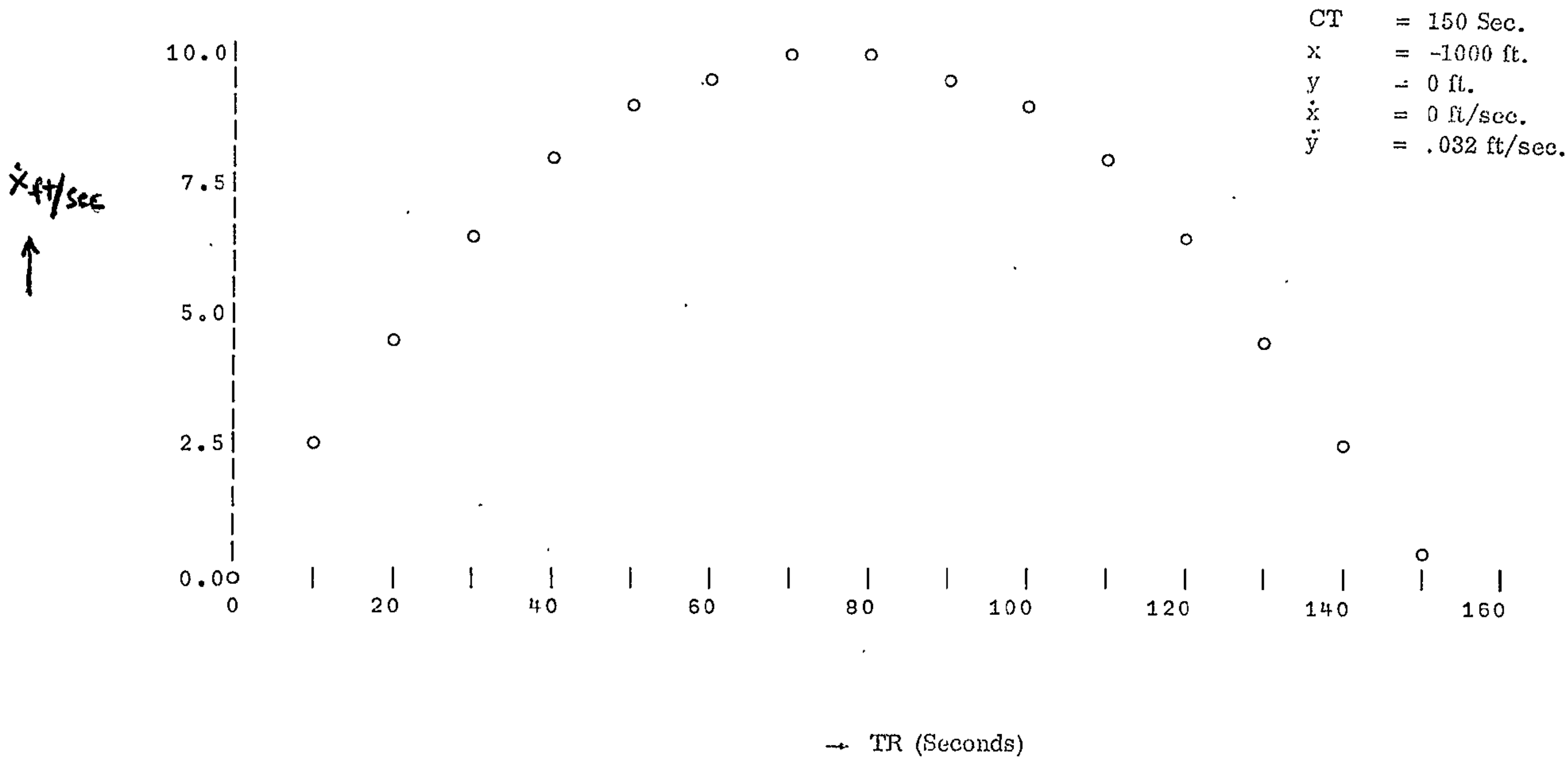
CT = 150 Sec.
x = -1000 ft.
y = 0 ft.
$\dot{x}$ = 0 ft/sec.
$\dot{y}$ = .032 ft/sec.



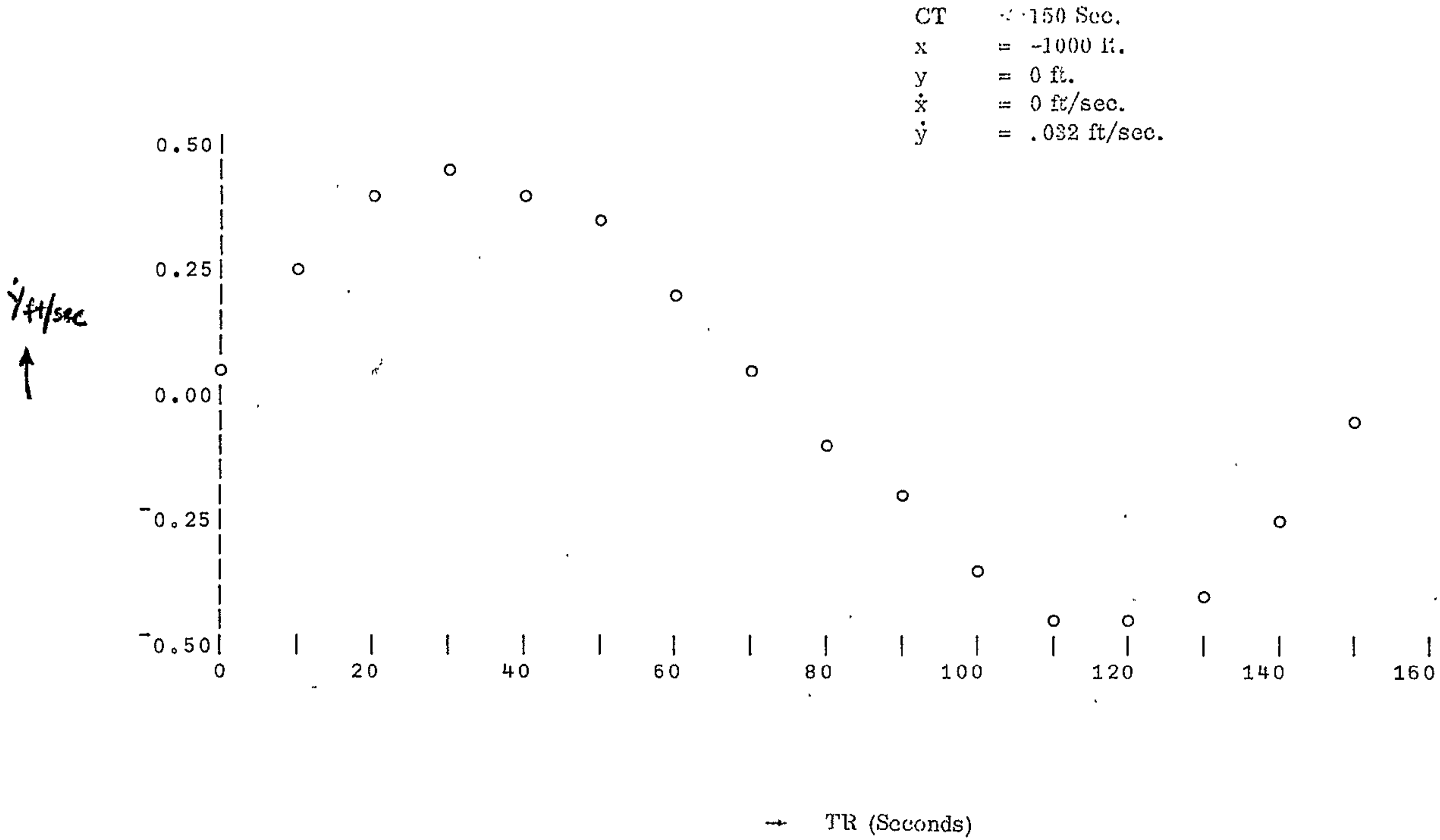

CT = 150 Sec.
x = -1000 ft.
y = 0 ft.
ẋ = 0 ft/sec.
ẏ = .032 ft/sec.
Y ft
25
20
15
10
5
0
0
-1000
-800
-600
-400
-200
0
→ X Ft



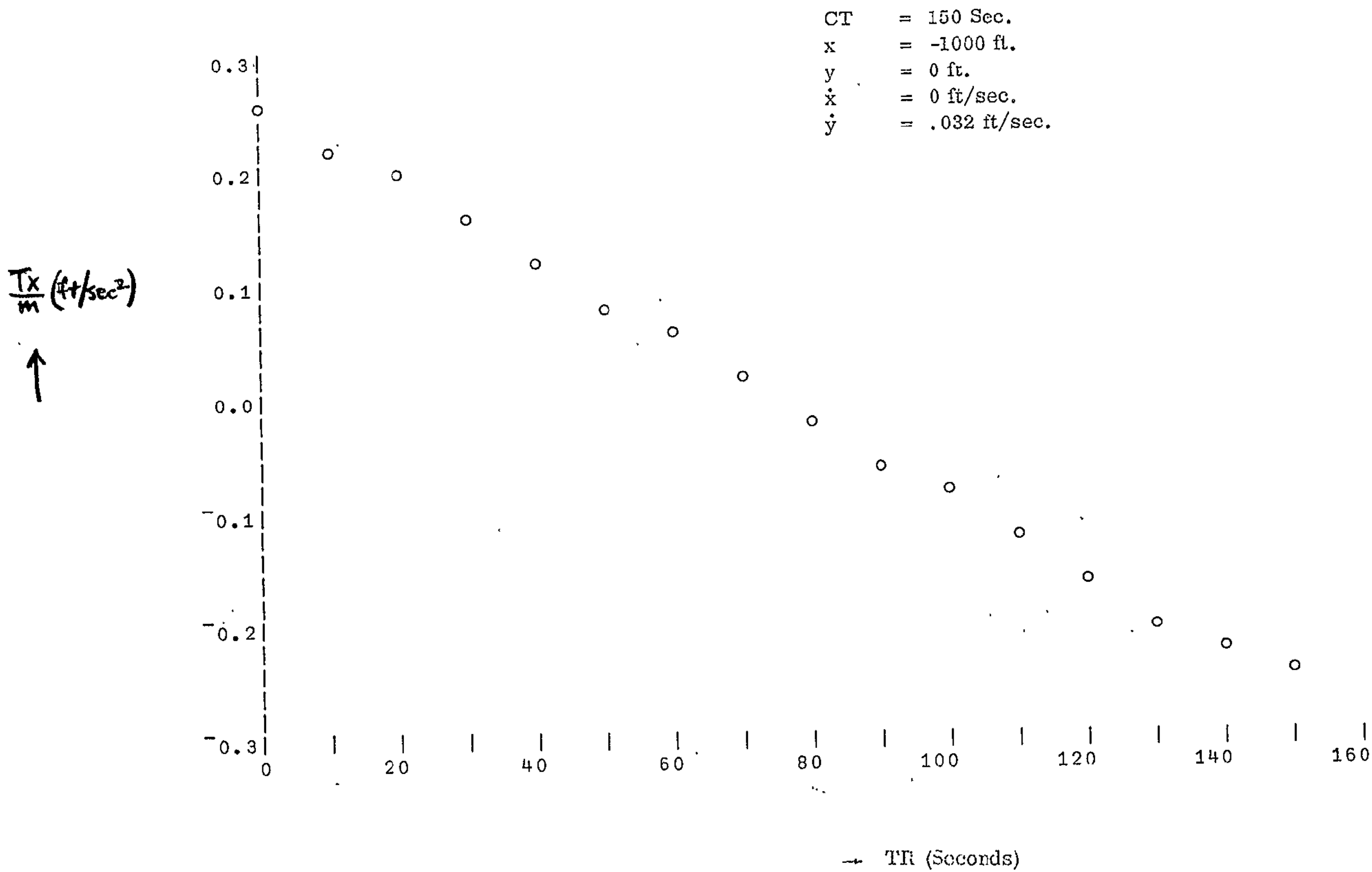
CT = 150 Sec.
x = -1000 ft.
y = 0 ft.
$\dot{x}$ = 0 ft/sec.
$\dot{y}$ = .032 ft/sec.
Y ft
25
20
15
10
5
0
0
20
40
60
80
100
120
140
160
TR (Seconds)

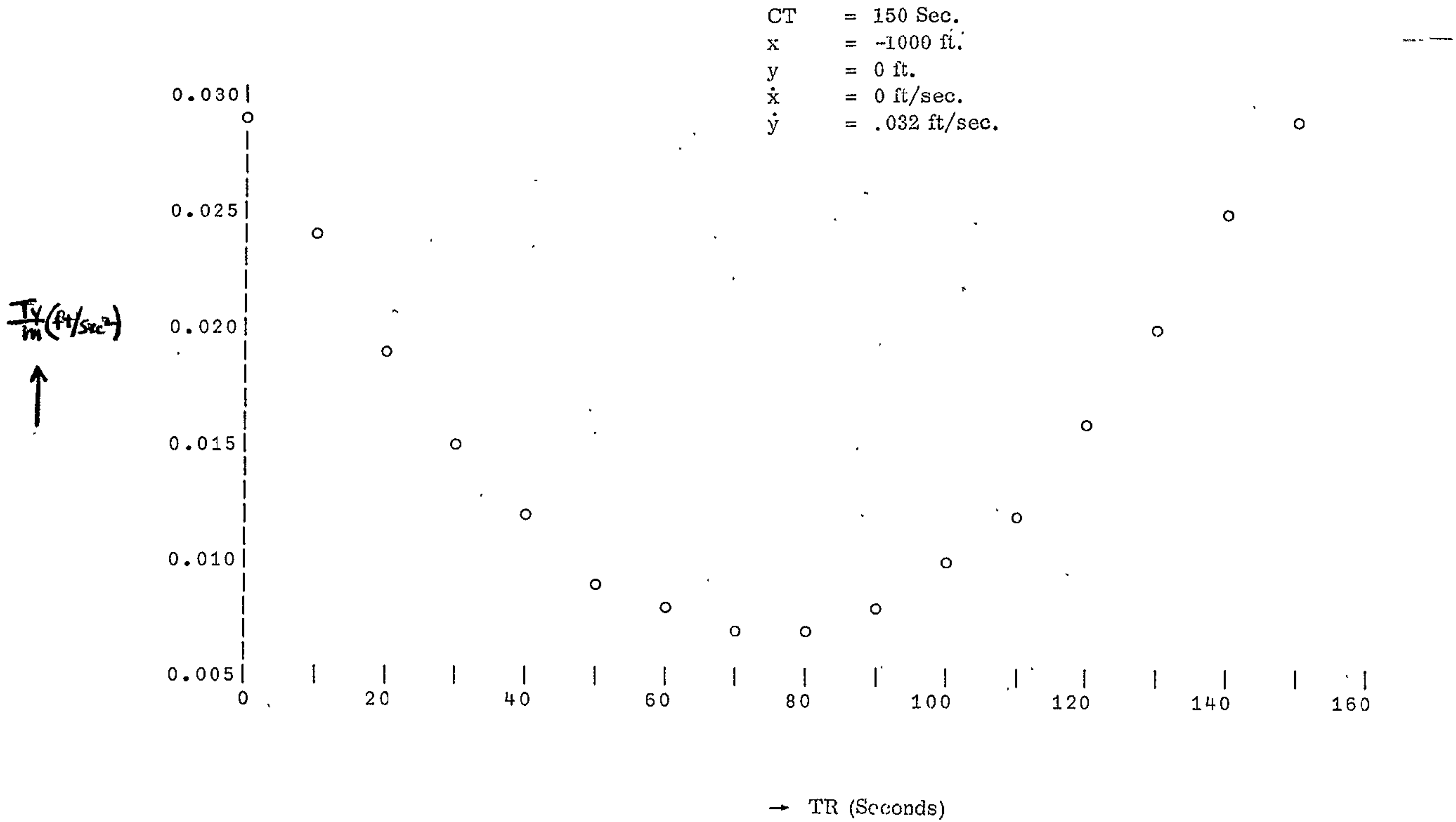


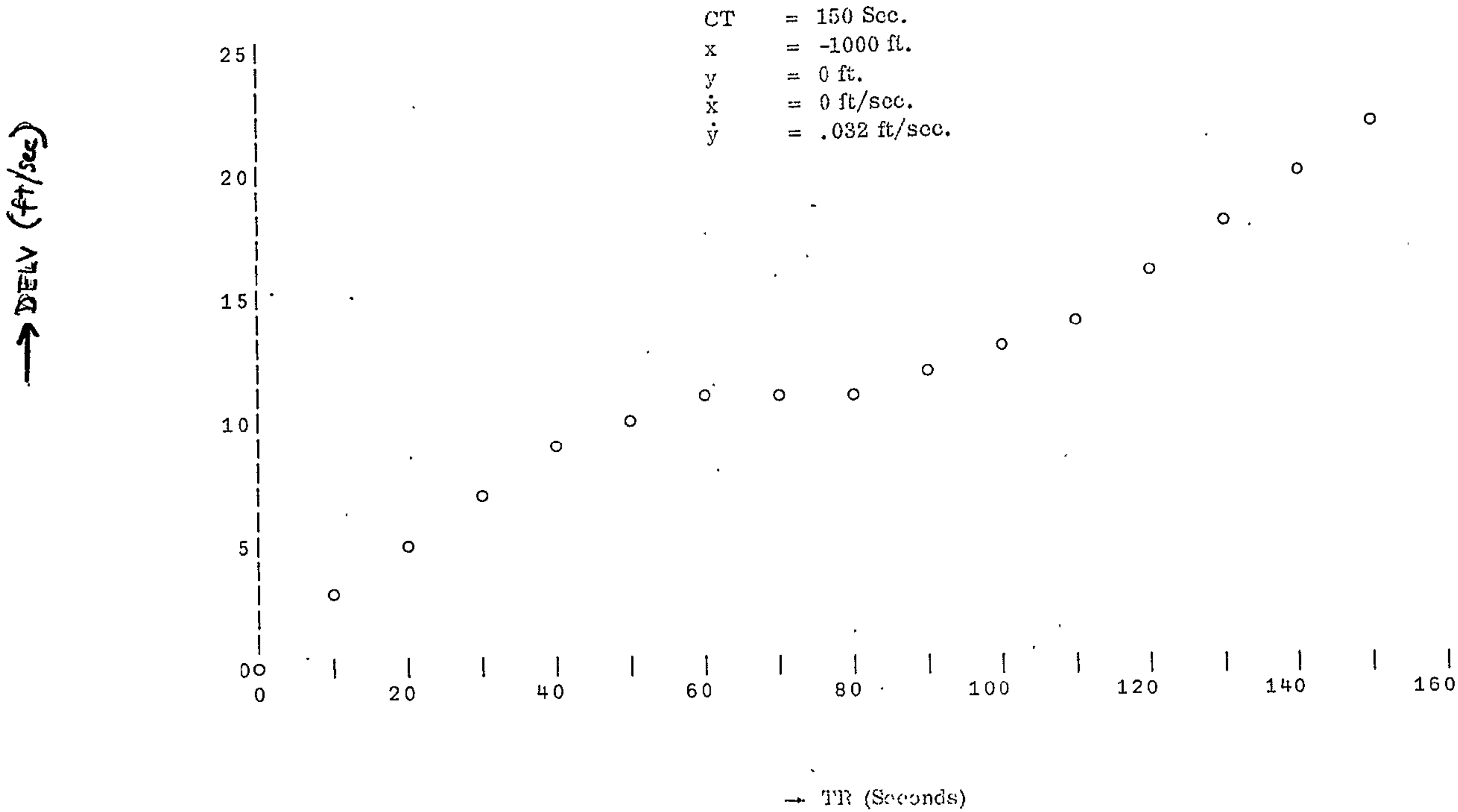
CT = 150 Sec.
x = -1000 ft.
y = 0 ft.
ẋ = 0 ft/sec.
ÿ = .032 ft/sec.
ẋ ft/sec
10.0
7.5
5.0
2.5
0.00
0
20
40
60
80
100
120
140
160
TR (Seconds)

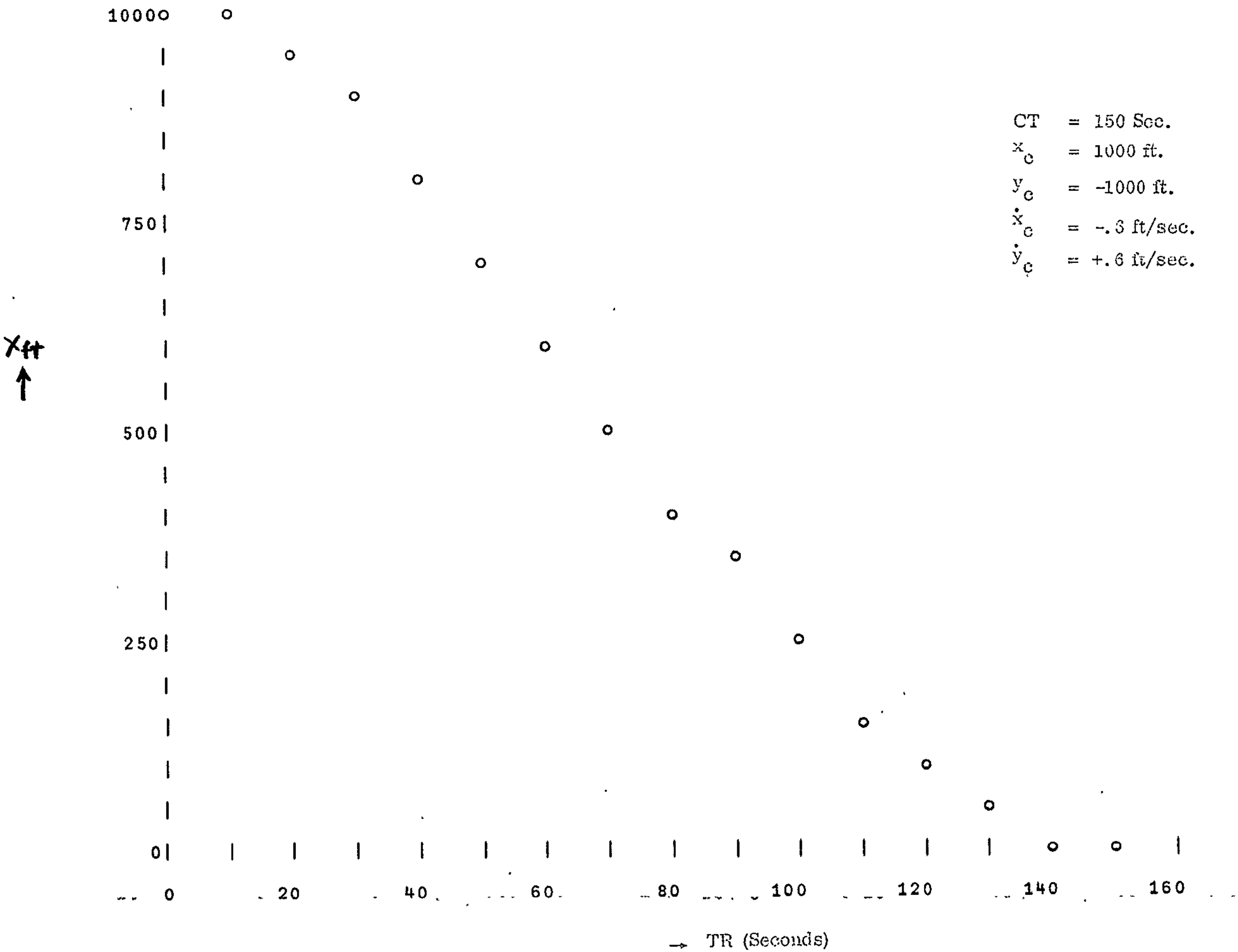


CT = 150 Sec.
x = -1000 ft.
y = 0 ft.
ẋ = 0 ft/sec.
ẏ = .032 ft/sec.
0.50
0.25
0.00
-0.25
-0.50
ẏ ft/sec
0
20
40
60
80
100
120
140
160
TR (Seconds)


CT = 150 Sec.
x = -1000 ft.
y = 0 ft.
ẋ = 0 ft/sec.
ẏ = .032 ft/sec.
Tx/m (ft/sec²)
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
0
20
40
60
80
100
120
140
160
TR (Seconds)


CT = 150 Sec.
x = -1000 ft.
y = 0 ft.
ẋ = 0 ft/sec.
ẏ = .032 ft/sec.
Ty/m (ft/sec²)
0.030
0.025
0.020
0.015
0.010
0.005
0
20
40
60
80
100
120
140
160
TR (Seconds)


CT = 150 Sec.
x = -1000 ft.
y = 0 ft.
ẋ = 0 ft/sec.
ẏ = .032 ft/sec.
→ DELV (ft/sec)
0
5
10
15
20
25
0
20
40
60
80
100
120
140
160
→ TR (Seconds)



1000
750
500
250
0
X ft
0
20
40
60
80
100
120
140
160
TR (Seconds)
CT = 150 Sec.
x_c = 1000 ft.
y_c = -1000 ft.
$\dot{x}_c$ = -.3 ft/sec.
$\dot{y}_c$ = +.6 ft/sec.

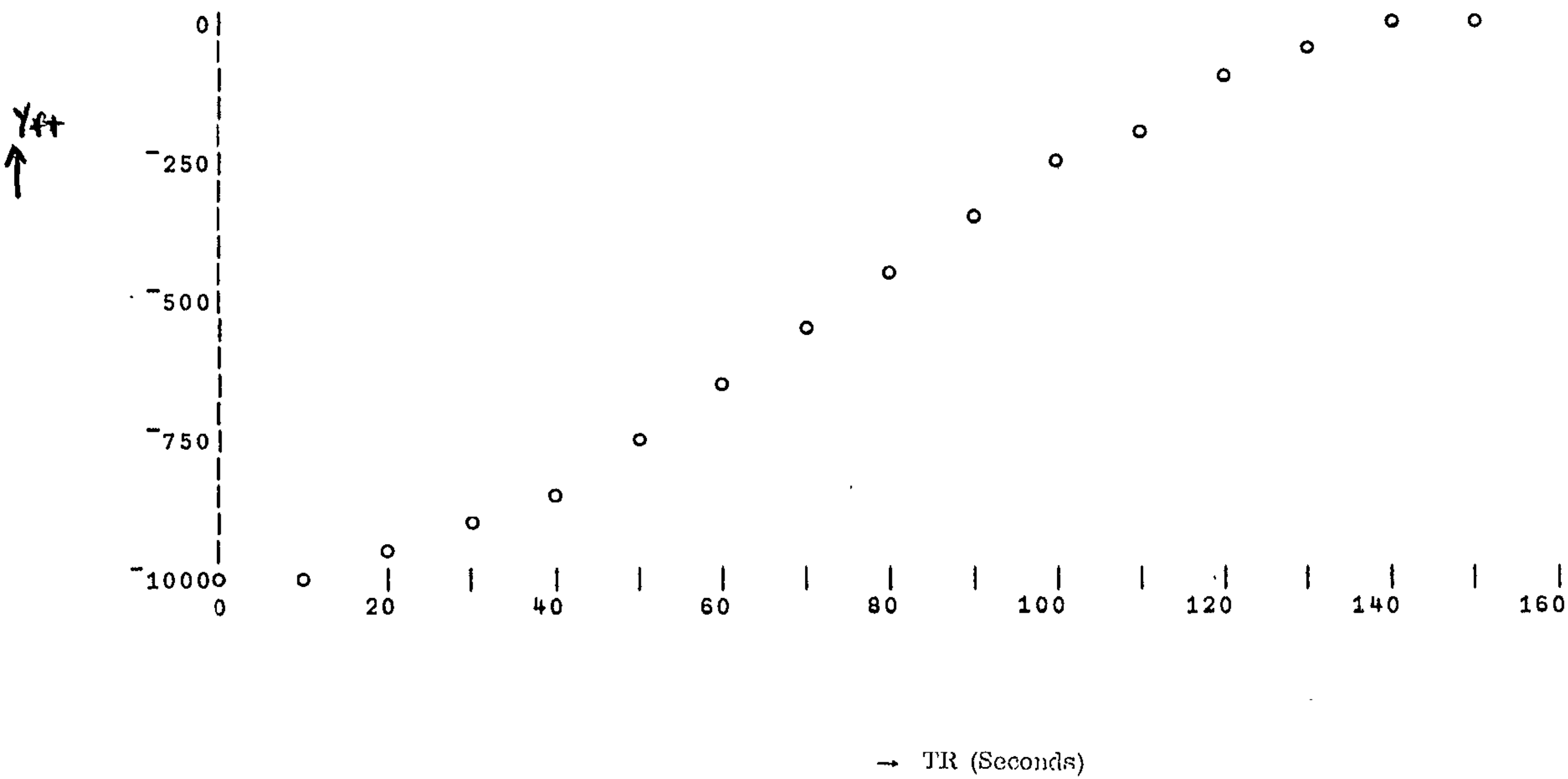
CT = 150 Sec.

x_c = 1000 ft.

y_c = -1000 ft.

$\dot{x}_c$ = -.6 ft/sec.

$\dot{y}_c'$ = +.6 ft/sec.



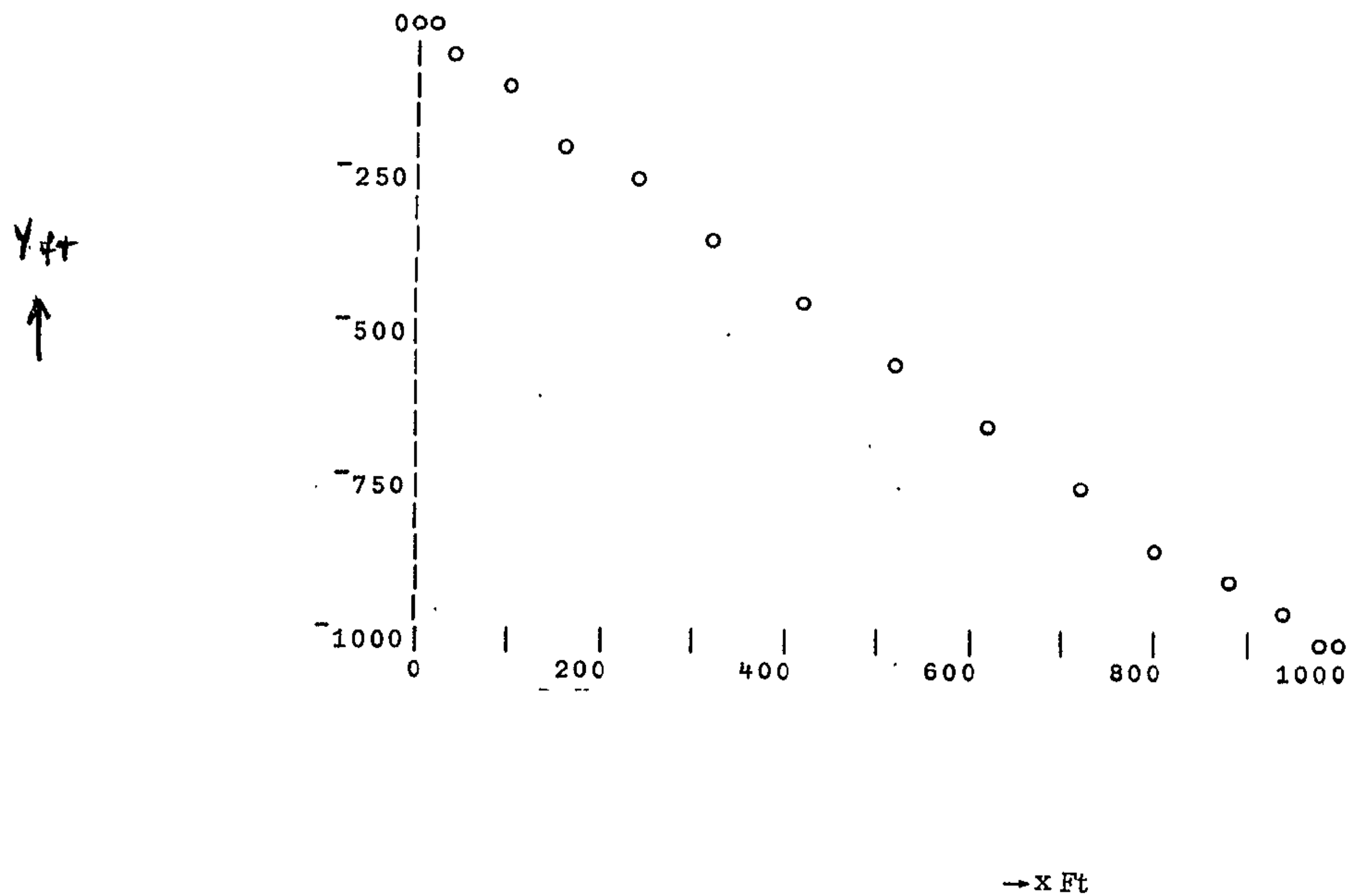
$CT = 150$ Sec.

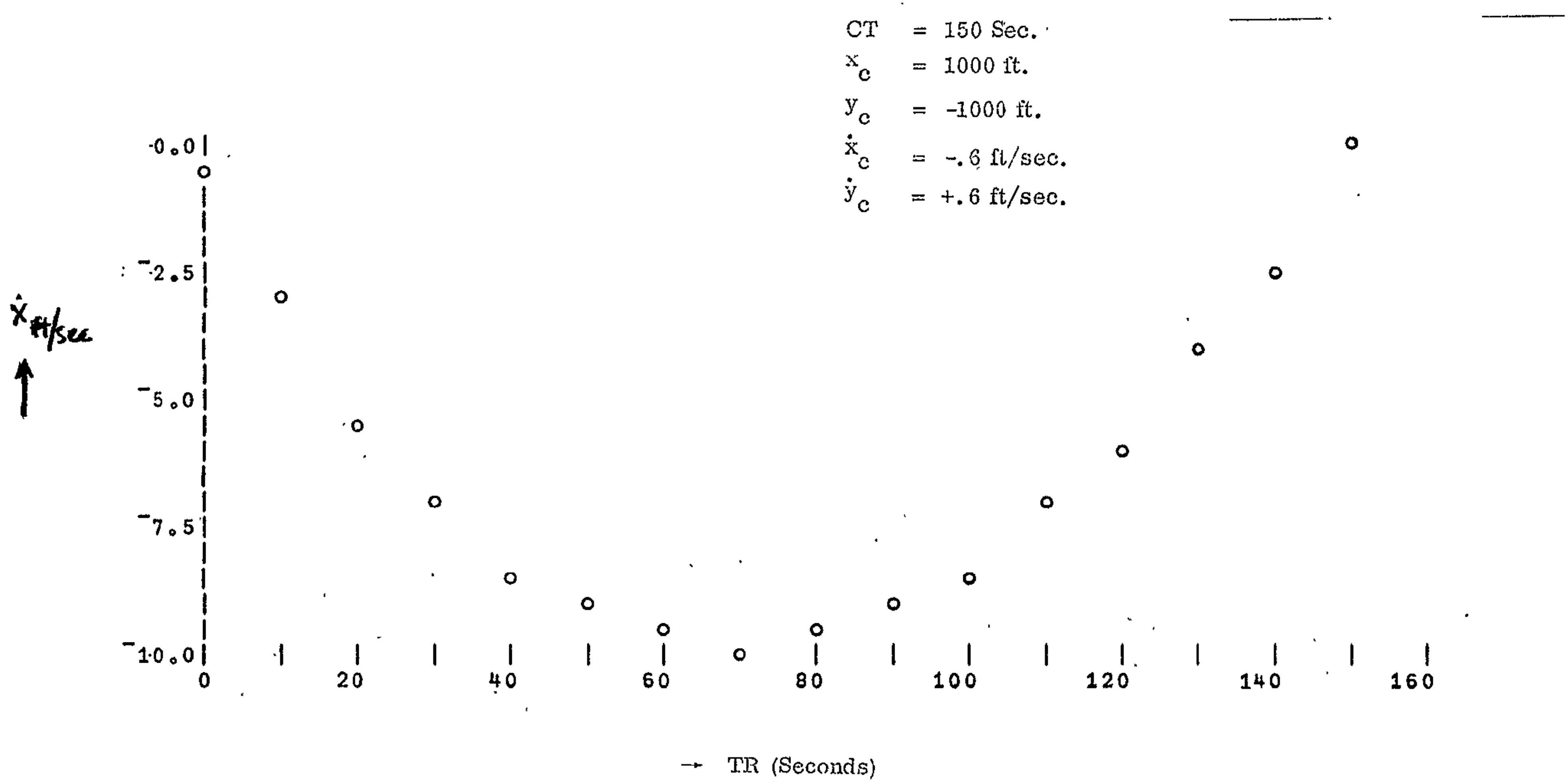
$x_c = 1000$ ft.

$y_c = -1000$ ft.

$\dot{x}_c = -.6$ ft/sec.

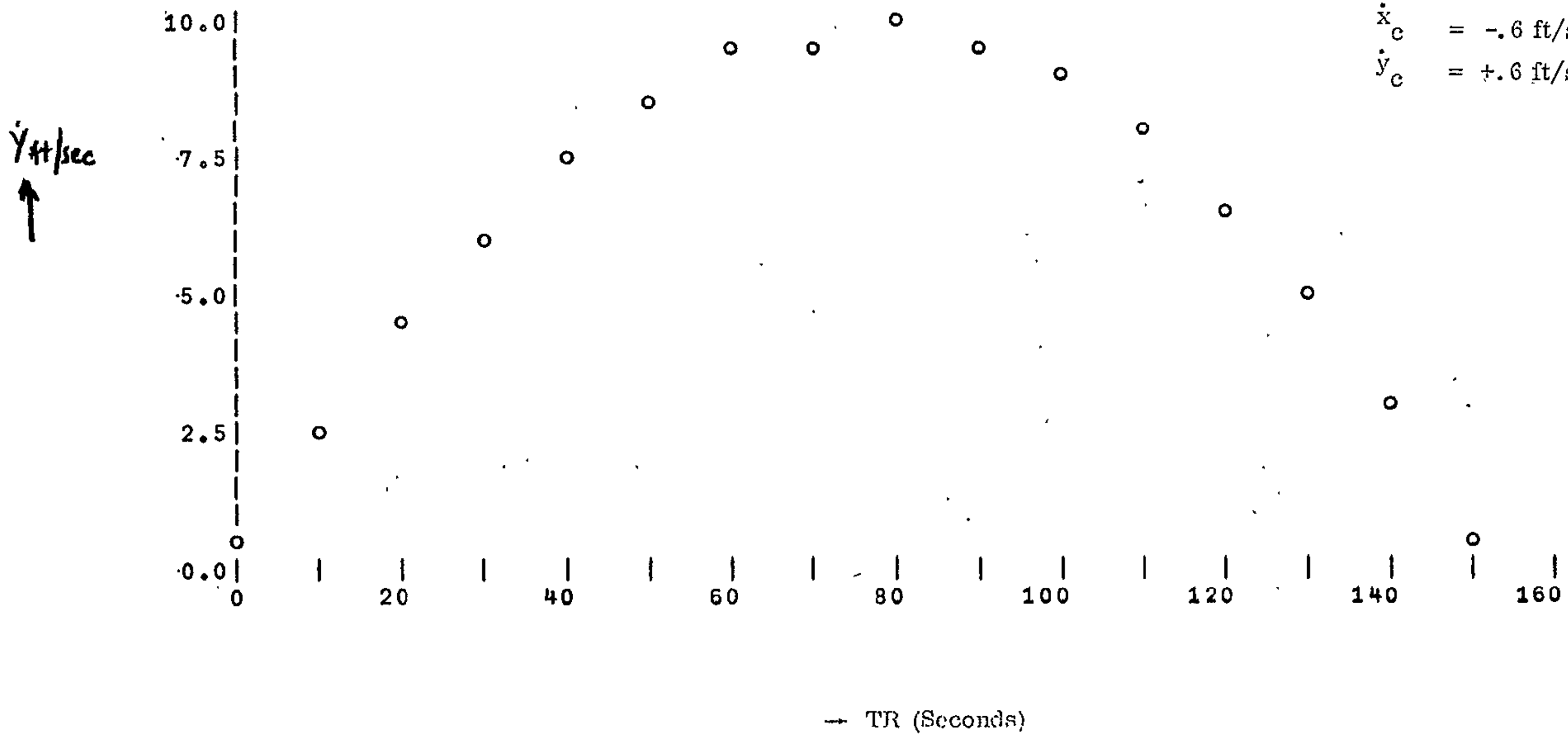
$\dot{y}_c = +.6$ ft/sec.

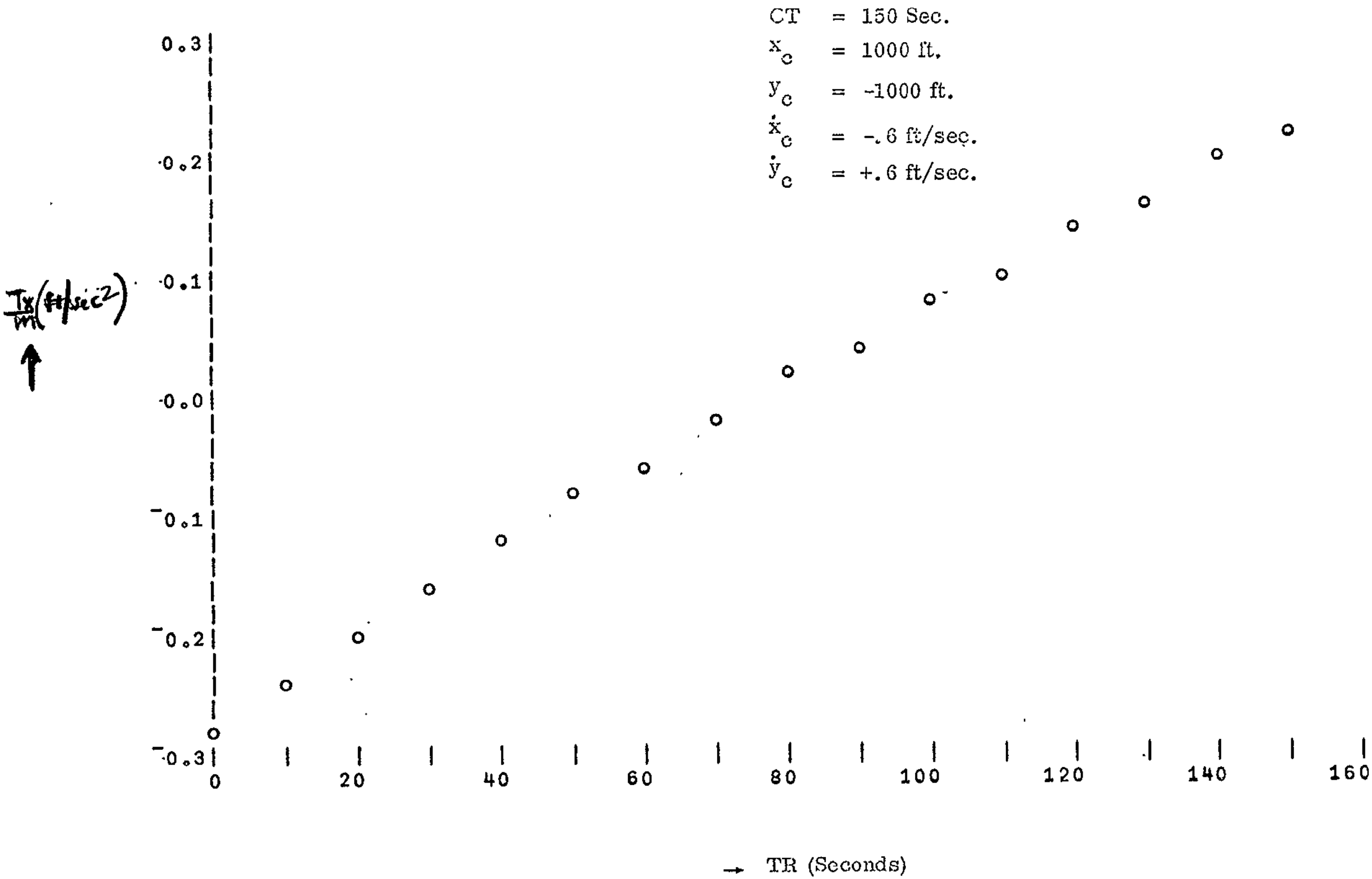


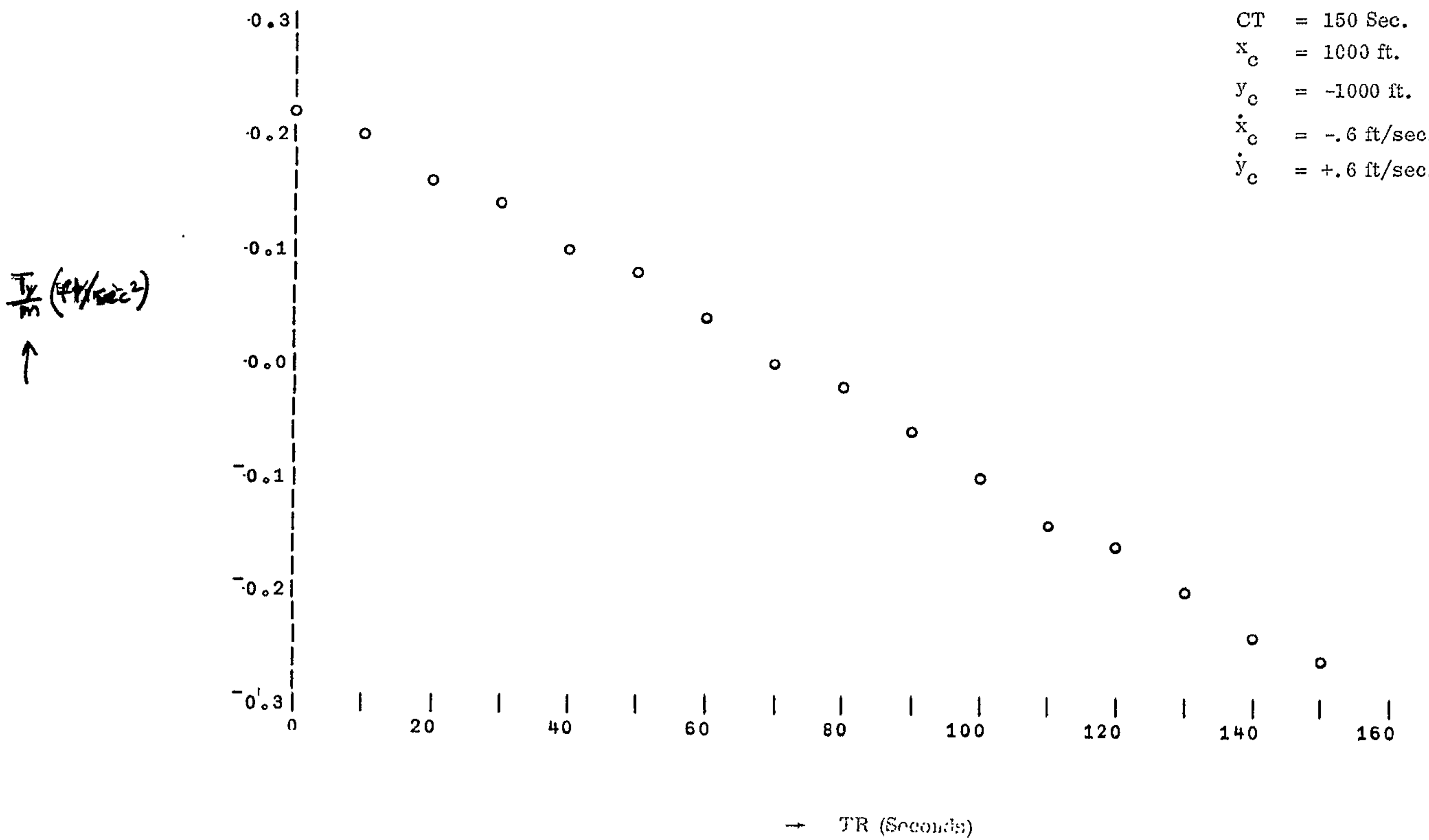


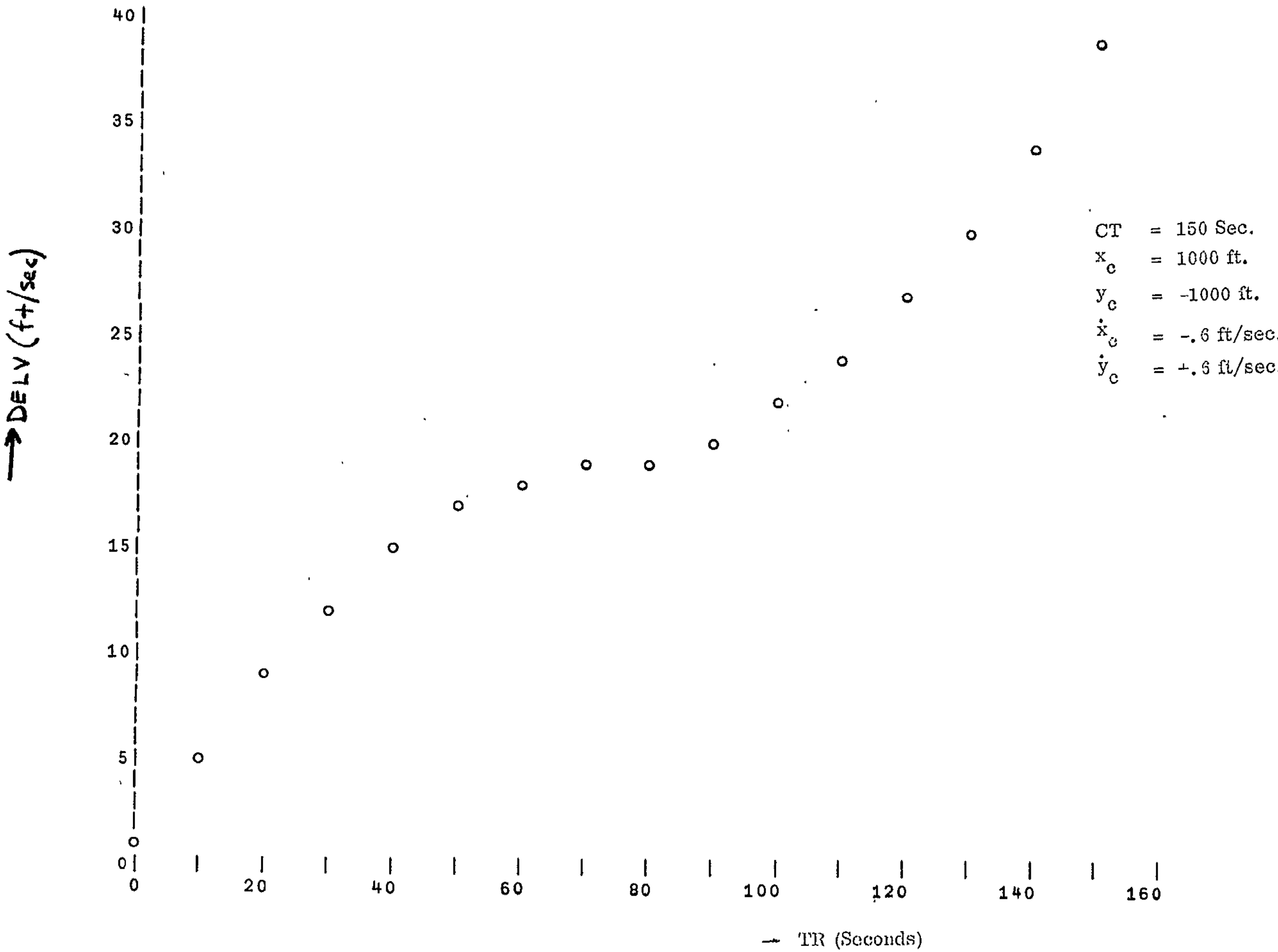
CT = 150 Sec.
x_c = 1000 ft.
y_c = -1000 ft.
$\dot{x}_c$ = -.6 ft/sec.
$\dot{y}_c$ = +.6 ft/sec.
$\dot{x}$ ft/sec
-0.0
-2.5
-5.0
-7.5
-10.0
0
20
40
60
80
100
120
140
160
→ TR (Seconds)

$CT = 150$ Sec.
$x_c = 1000$ ft.
$y_c = -1000$ ft.
$\dot{x}_c = -.6$ ft/sec.
$\dot{y}_c = +.6$ ft/sec.

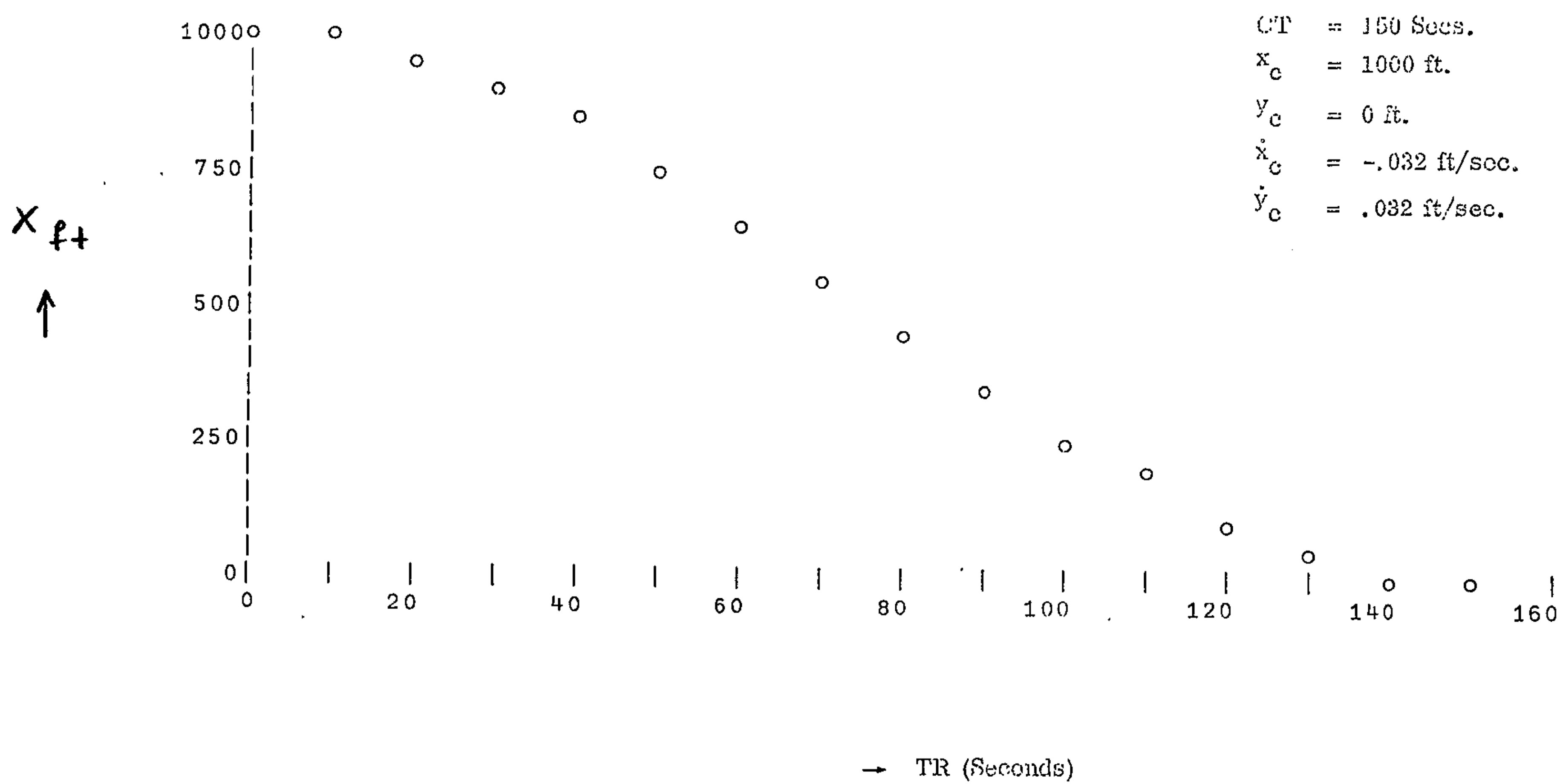



CT = 150 Sec.
x_c = 1000 ft.
y_c = -1000 ft.
$\dot{x}_c$ = -.6 ft/sec.
$\dot{y}_c$ = +.6 ft/sec.
$\frac{T_x}{m}$ (ft/sec²)
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
0
20
40
60
80
100
120
140
160
TR (Seconds)

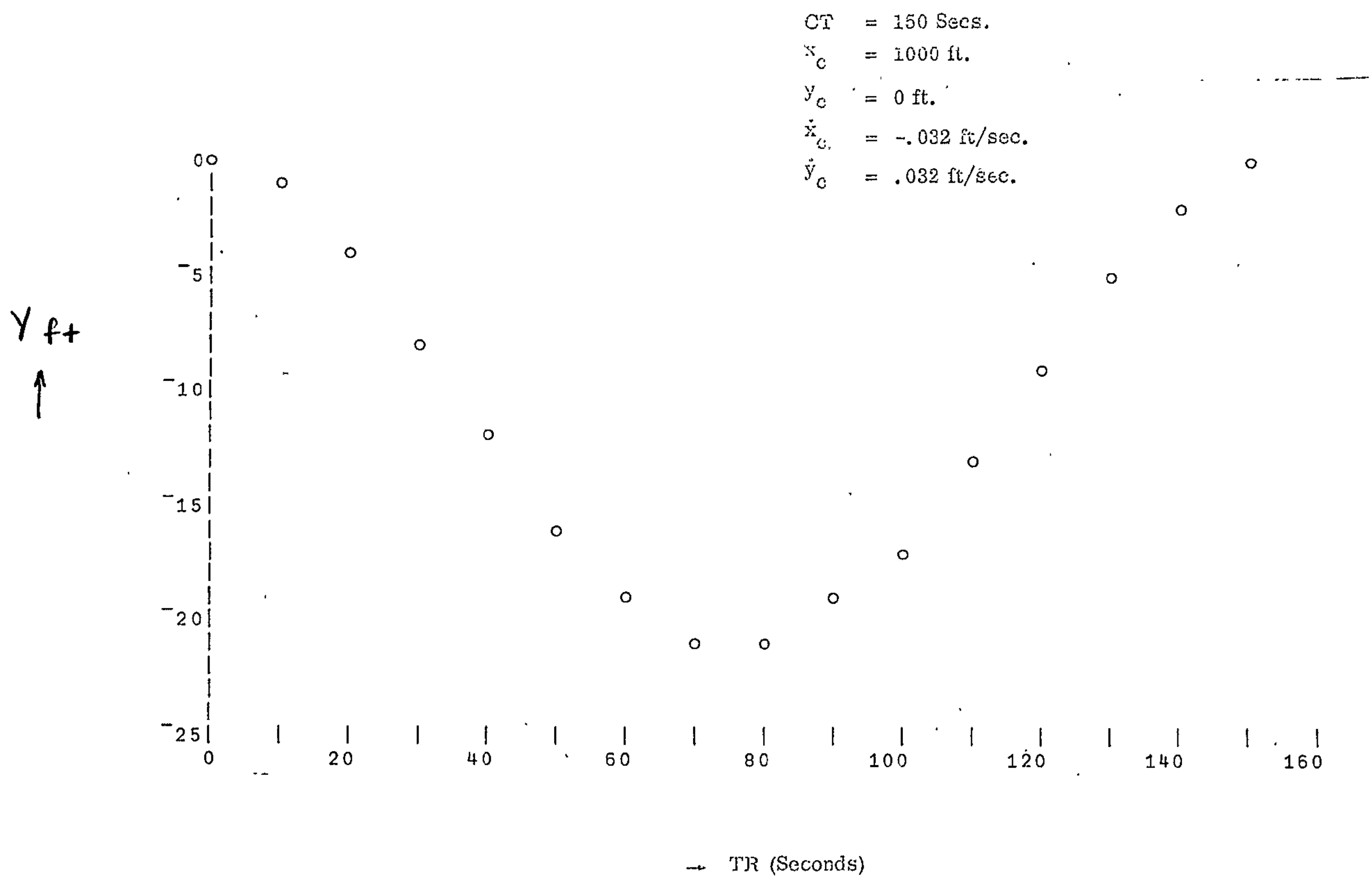

CT = 150 Sec.
x_c = 1000 ft.
y_c = -1000 ft.
$\dot{x}_c$ = -.6 ft/sec.
$\dot{y}_c$ = +.6 ft/sec.
$\frac{T_y}{m}$ (ft/sec²)
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
0
20
40
60
80
100
120
140
160
TR (Seconds)



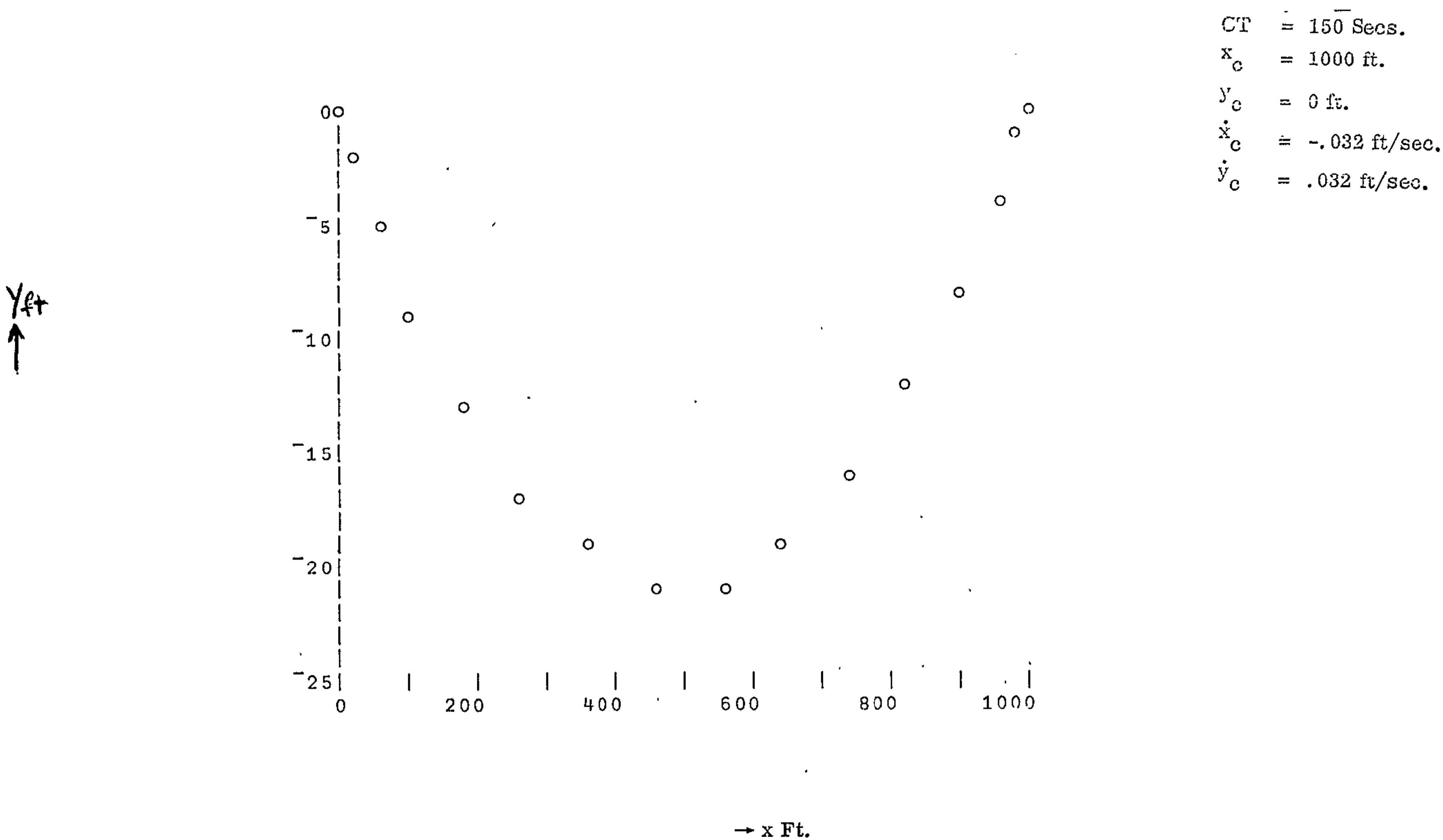
DELV (ft/sec)
TR (Seconds)
CT = 150 Sec.
x_c = 1000 ft.
y_c = -1000 ft.
$\dot{x}_c$ = -.6 ft/sec.
$\dot{y}_c$ = +.6 ft/sec.



CT = 150 Secs.
x_c = 1000 ft.
y_c = 0 ft.
$\dot{x}_c$ = -.032 ft/sec.
$\dot{y}_c$ = .032 ft/sec.
X ft
1000
750
500
250
0
0
20
40
60
80
100
120
140
160
→ TR (Seconds)

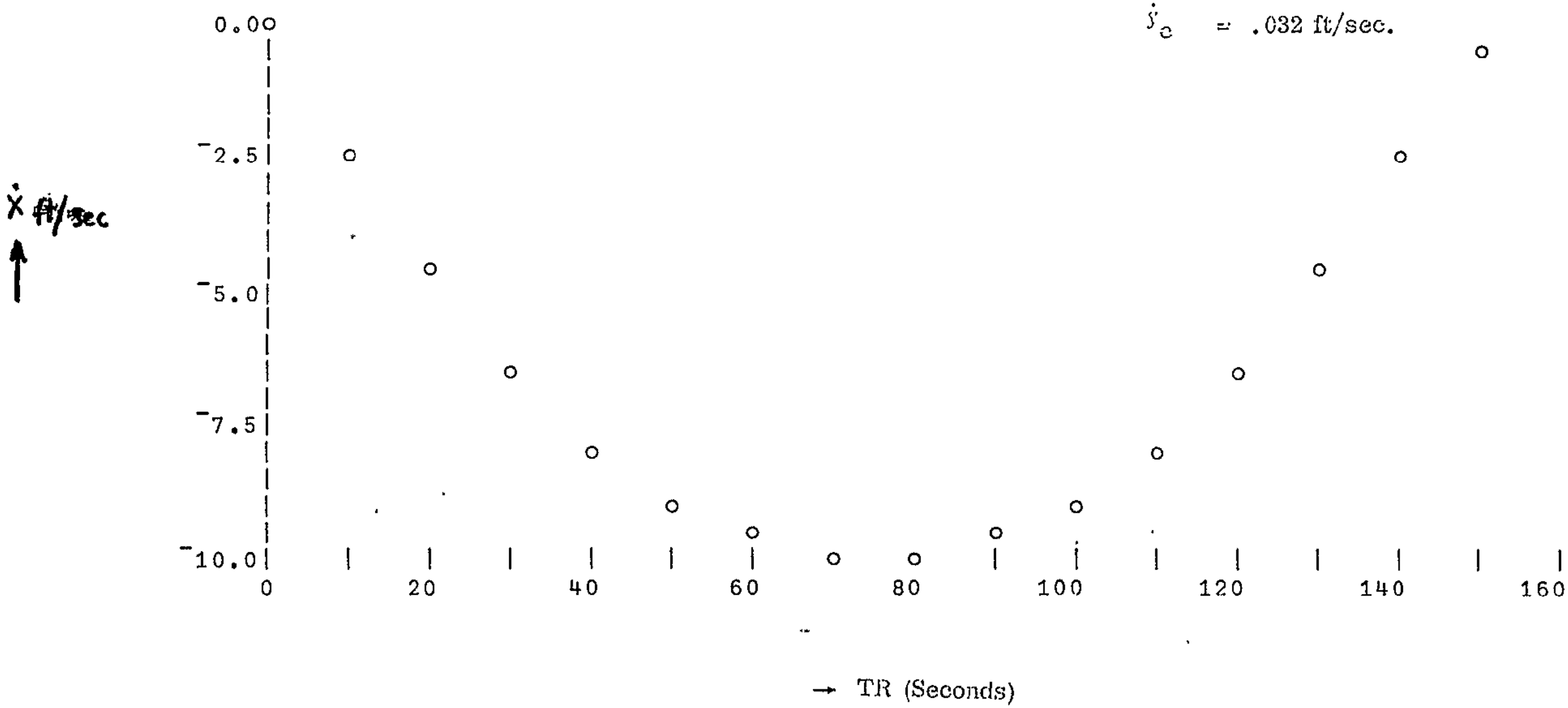


CT = 150 Secs.
x_c = 1000 ft.
y_c = 0 ft.
$\dot{x}_c$ = -.032 ft/sec.
$\dot{y}_c$ = .032 ft/sec.
Y ft
0
-5
-10
-15
-20
-25
0
20
40
60
80
100
120
140
160
TR (Seconds)



CT = 150 Secs.
x_c = 1000 ft.
y_c = 0 ft.
$\dot{x}_c$ = -.032 ft/sec.
$\dot{y}_c$ = .032 ft/sec.
Y ft
0
-5
-10
-15
-20
-25
0
200
400
600
800
1000
→ x Ft.

CT = 150 Secs.
x_c = 1000 ft.
y_c = 0 ft.
$\dot{x}_c$ = -.032 ft/sec.
$\dot{y}_c$ = .032 ft/sec.



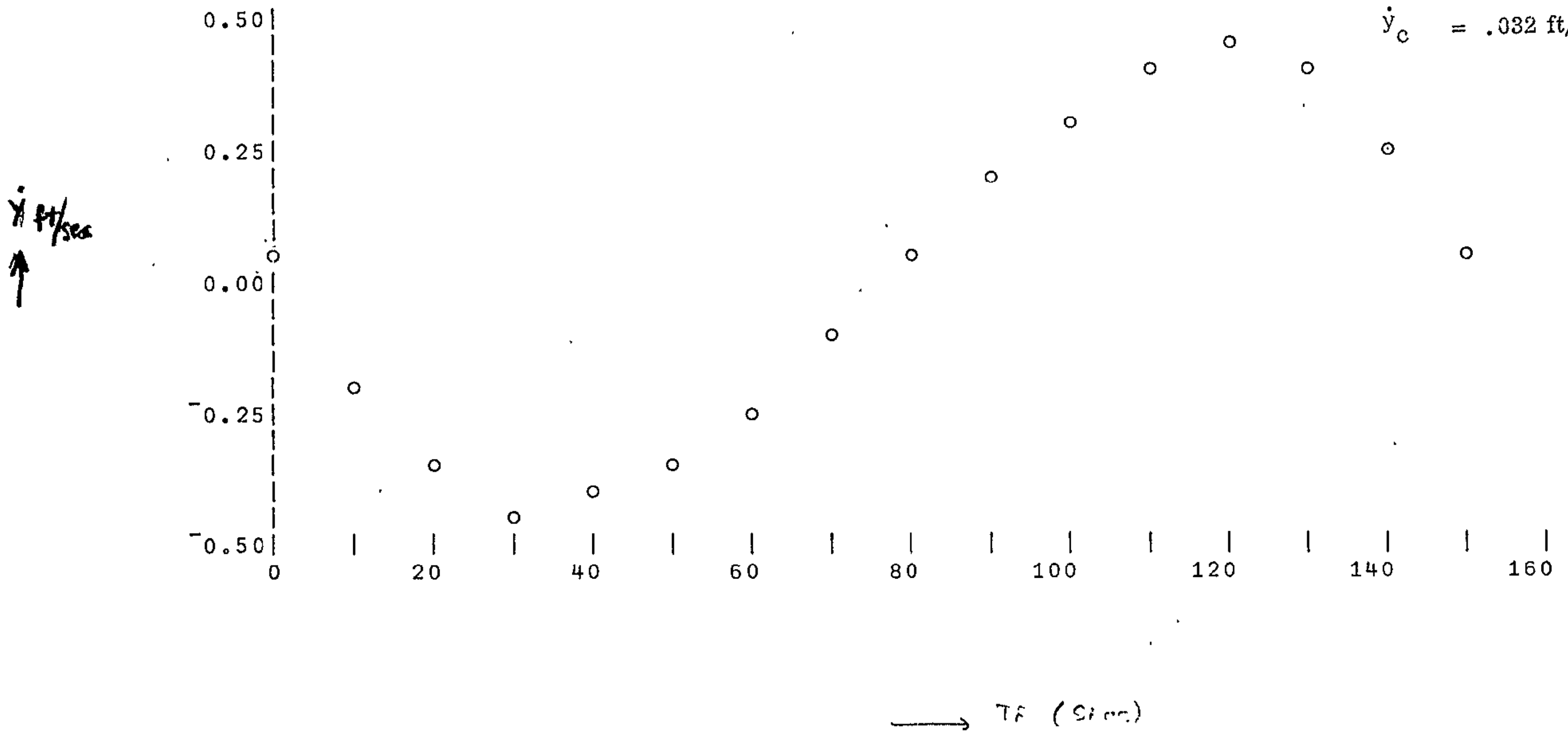
CT = 150 Secs.

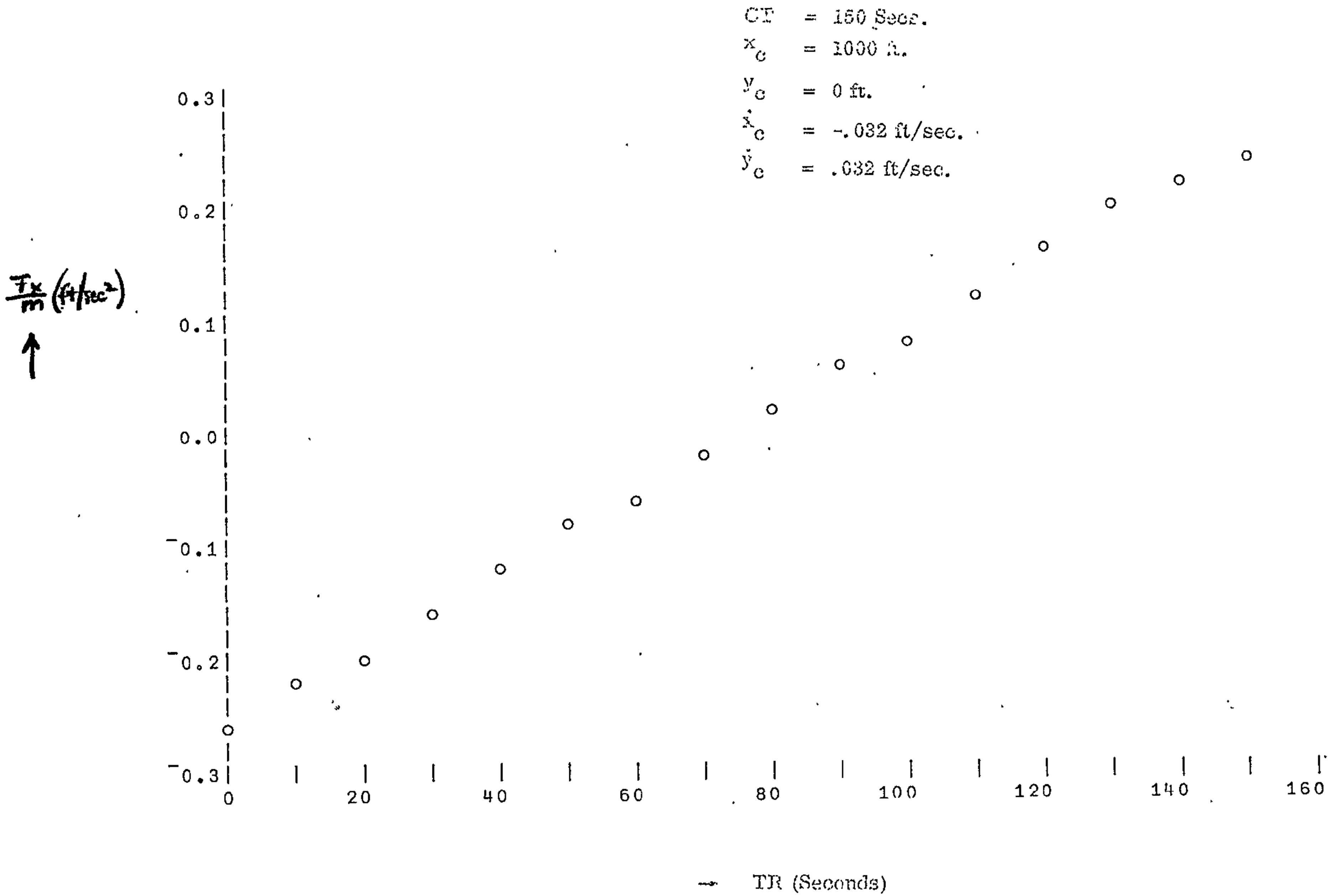
x_c = 1000 ft.

y_c = 0 ft.

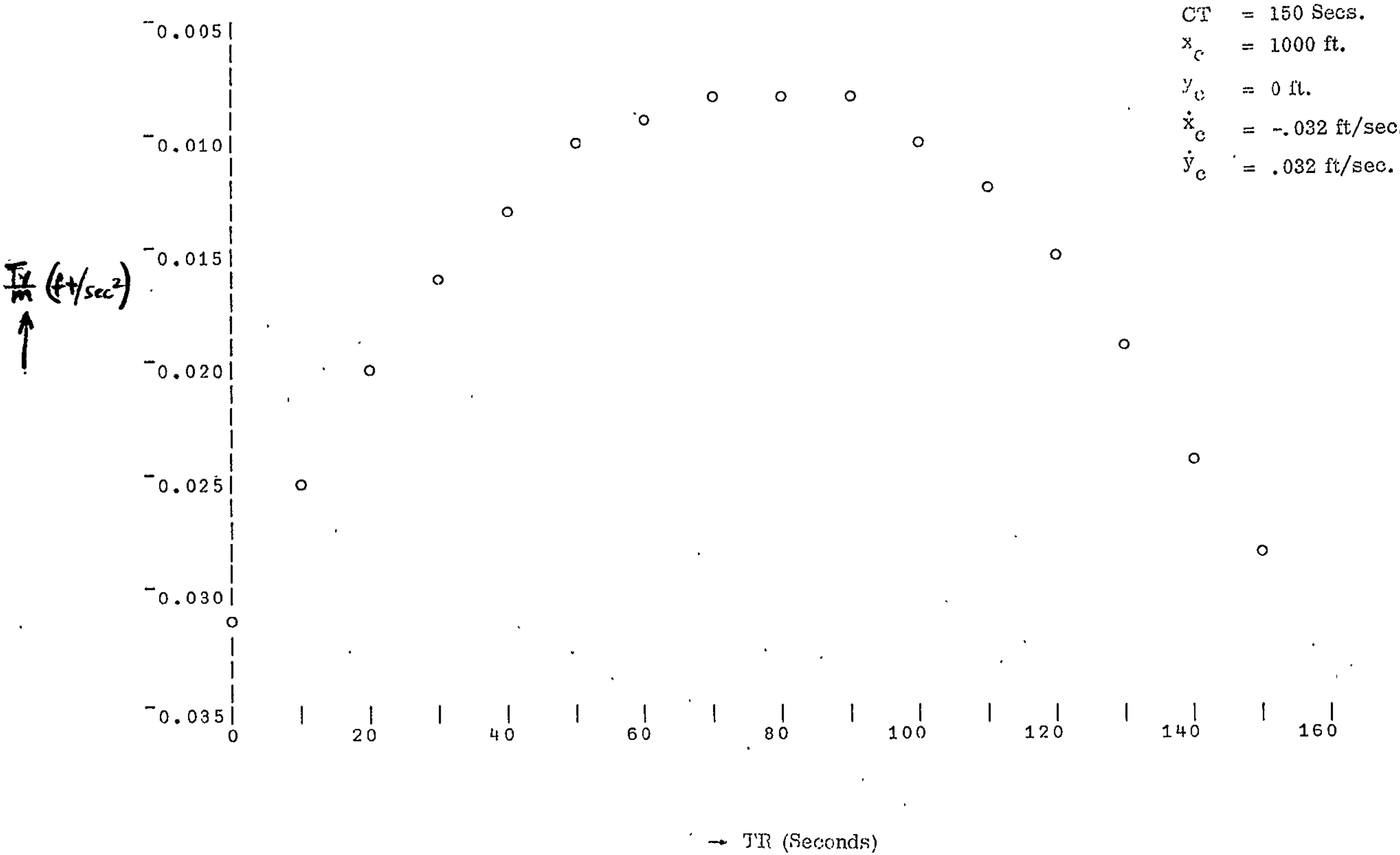
$\dot{x}_c$ = -.032 ft/sec.

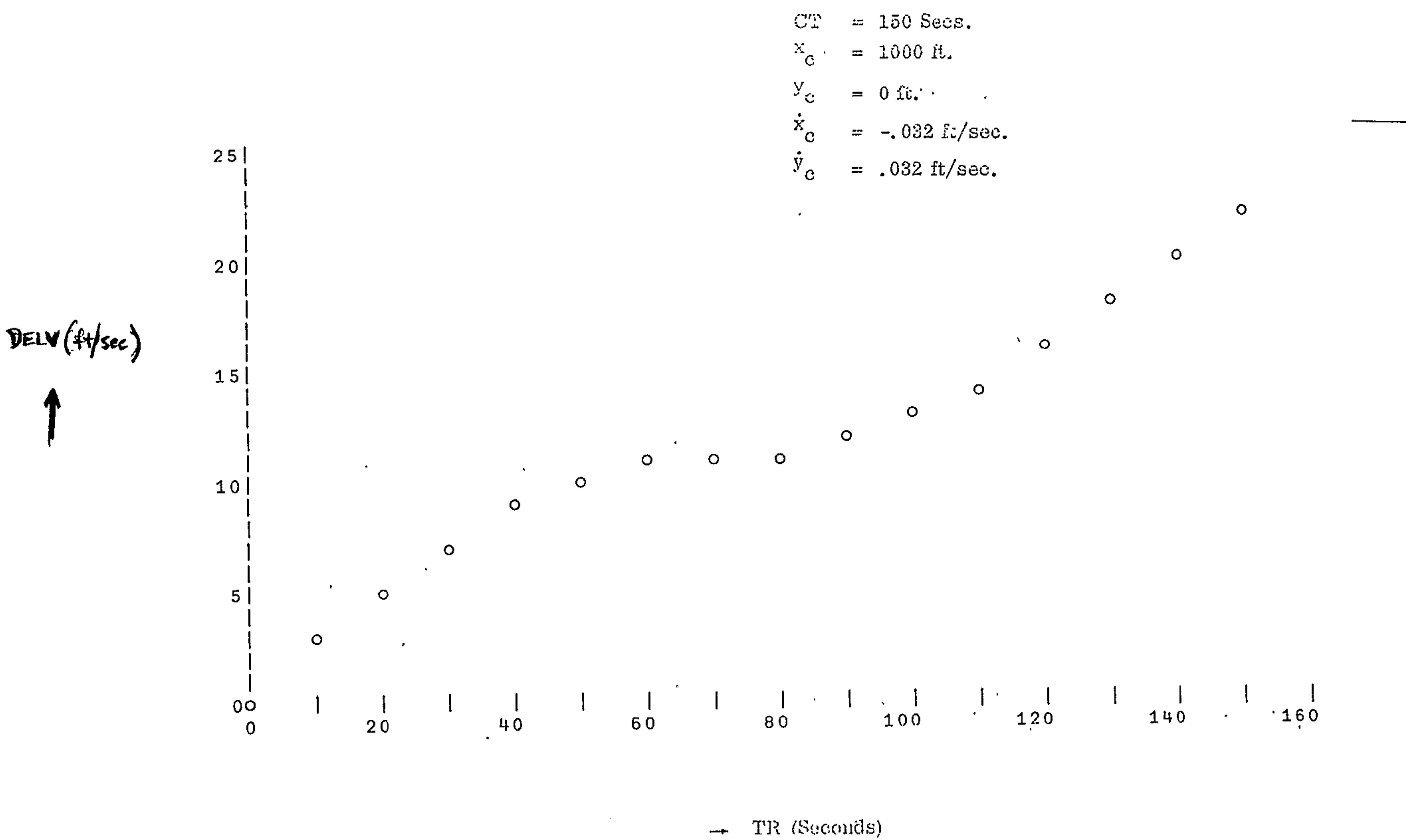
$\dot{y}_c$ = .032 ft/sec.



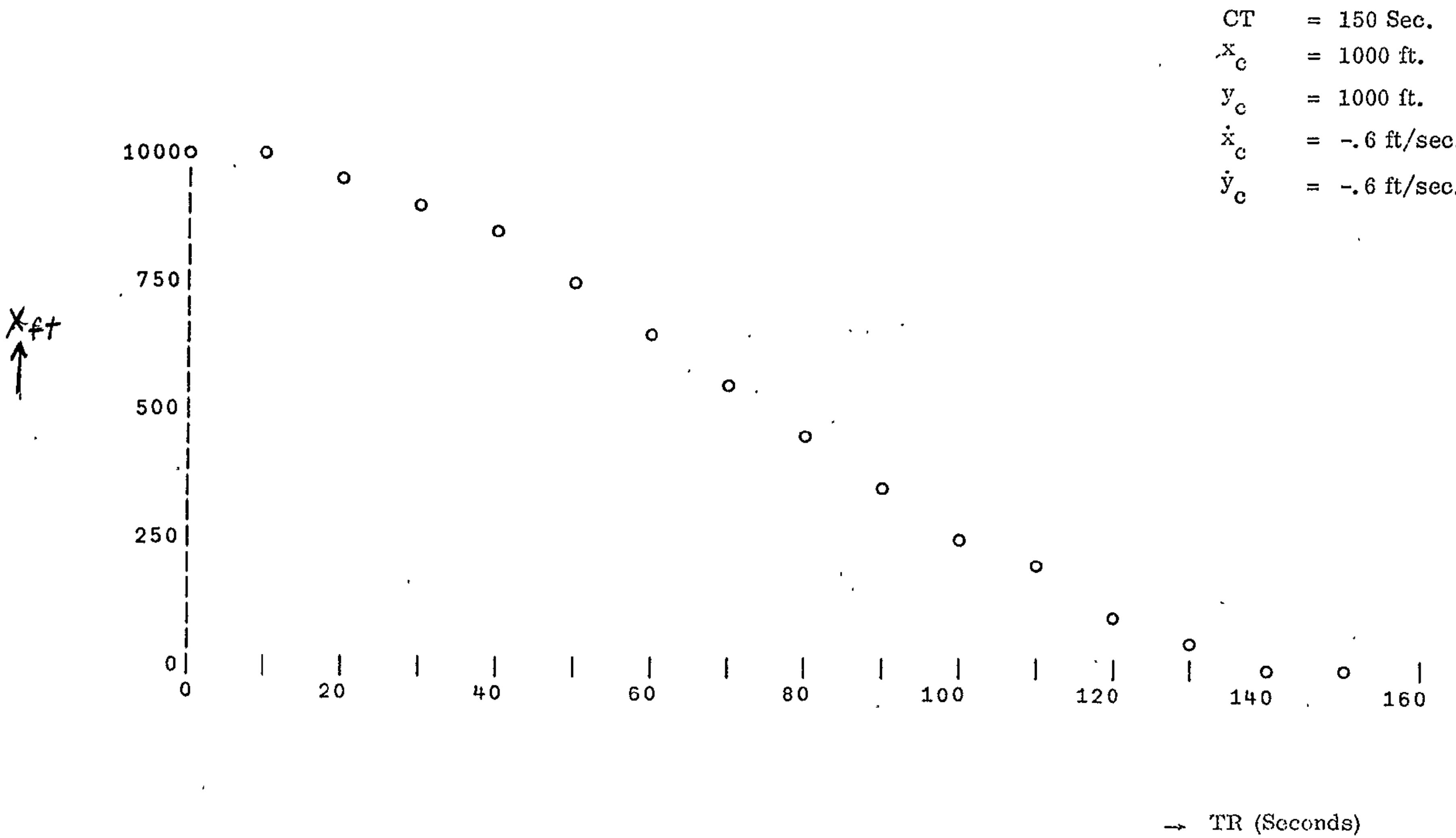


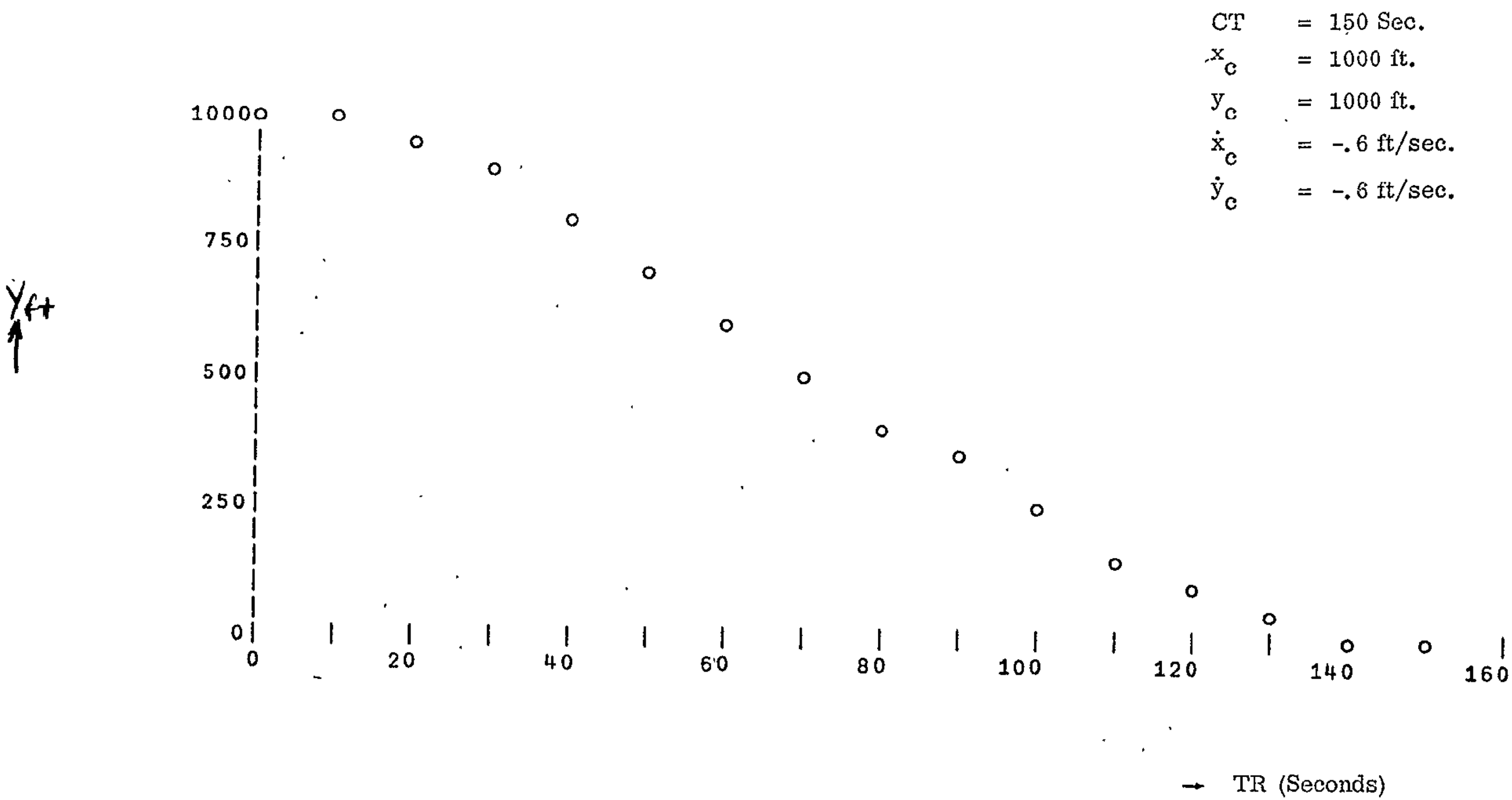
CT = 160 Secs.
x_c = 1000 ft.
y_c = 0 ft.
$\dot{x}_c$ = -.032 ft/sec.
$\dot{y}_c$ = .032 ft/sec.
$\frac{F_x}{m}$ (ft/sec²) ↑
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
0
20
40
60
80
100
120
140
160
→ TR (Seconds)

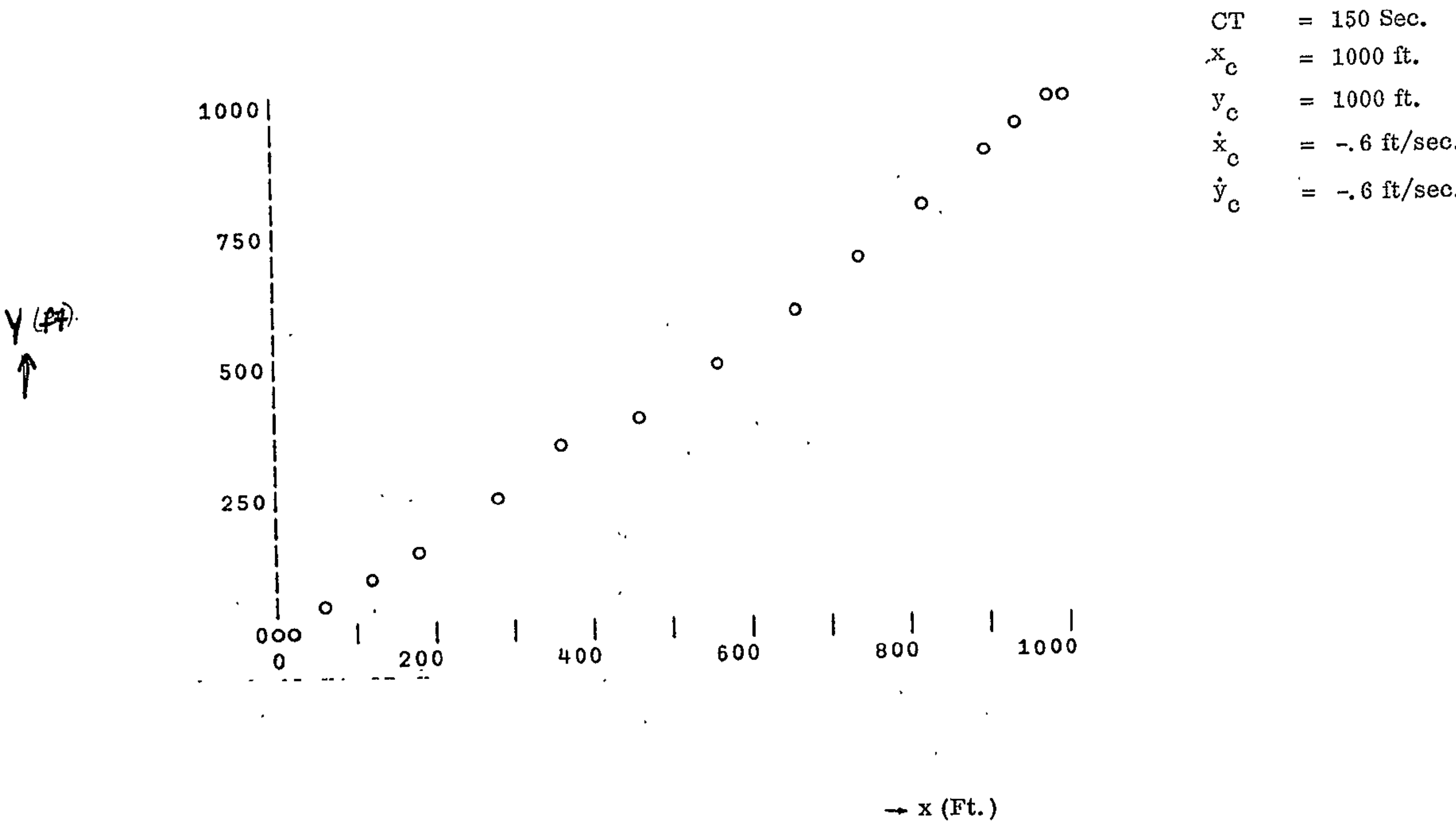

CT = 150 Secs.
x_c = 1000 ft.
y_c = 0 ft.
$\dot{x}_c$ = -.032 ft/sec.
$\dot{y}_c$ = .032 ft/sec.
-0.005
-0.010
-0.015
-0.020
-0.025
-0.030
-0.035
$\frac{T_y}{m}$ (ft/sec^2)
0
20
40
60
80
100
120
140
160
→ TR (Seconds)

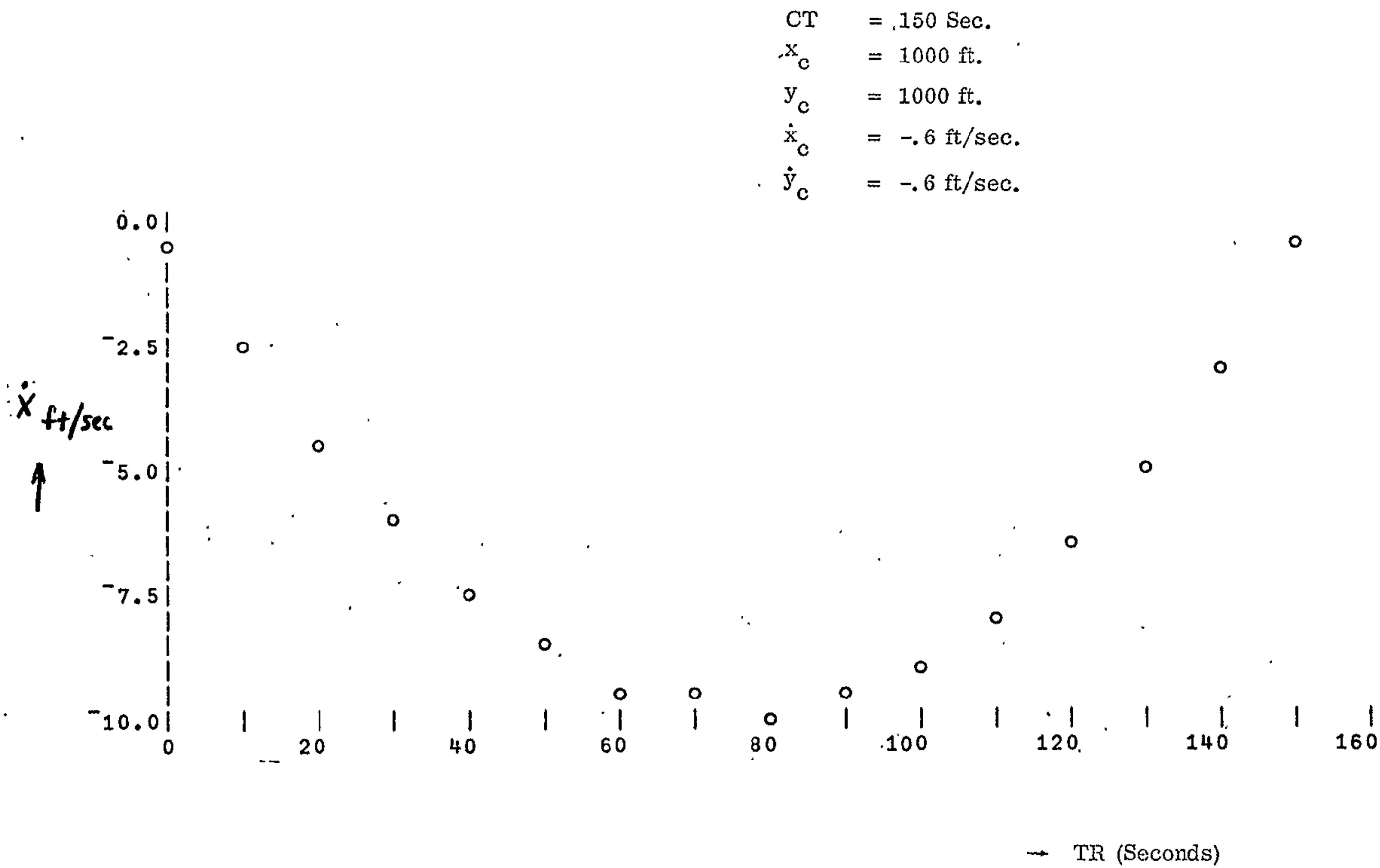


CT = 150 Secs.
x_c = 1000 ft.
y_c = 0 ft.
$\dot{x}_c$ = -.032 ft/sec.
$\dot{y}_c$ = .032 ft/sec.
DELV (ft/sec)
25
20
15
10
5
0
0
20
40
60
80
100
120
140
160
TR (Seconds)

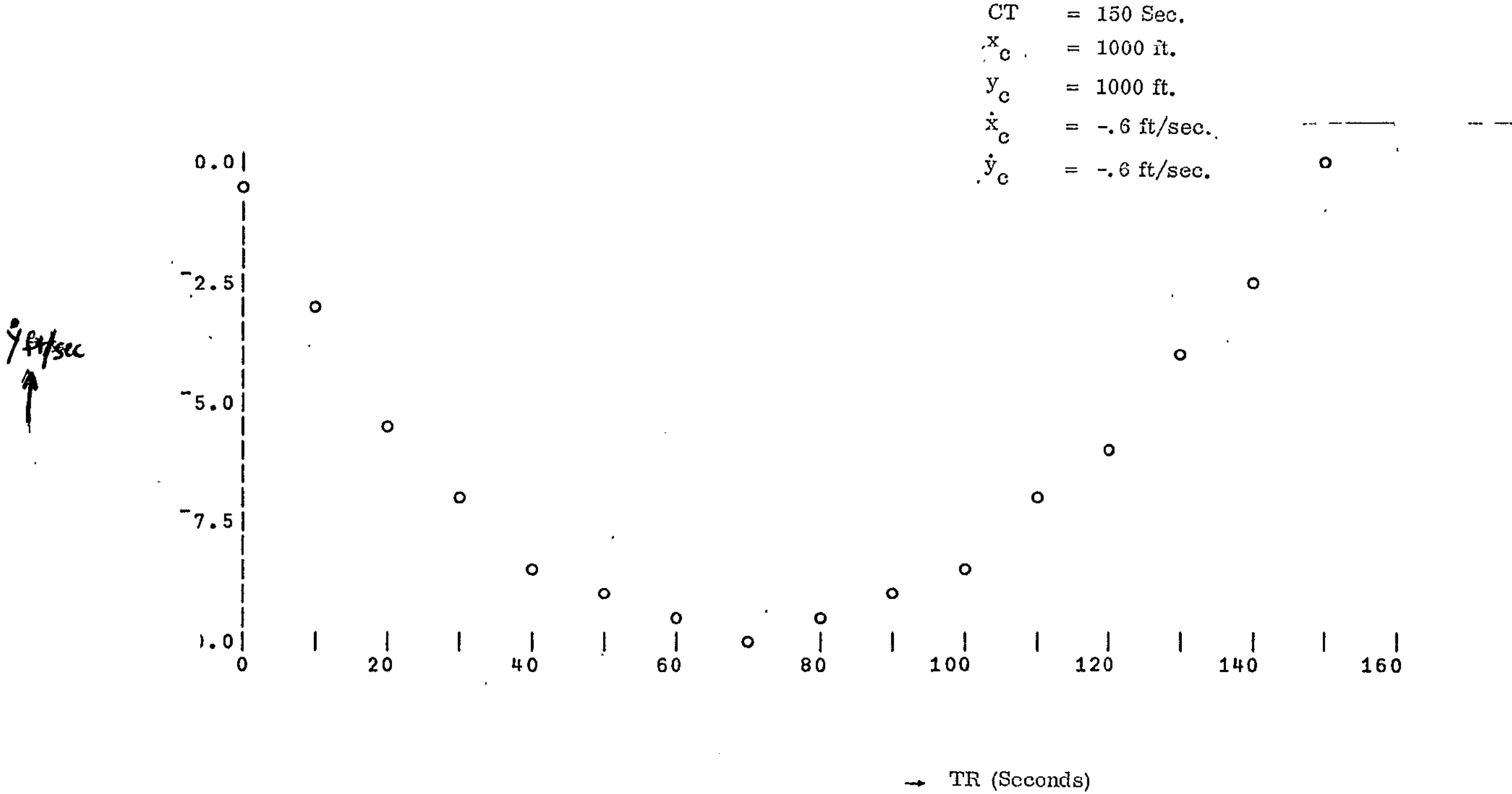

CT = 150 Sec.
x_c = 1000 ft.
y_c = 1000 ft.
$\dot{x}_c$ = -.6 ft/sec.
$\dot{y}_c$ = -.6 ft/sec.
1000
750
500
250
0
X_{ft}
0
20
40
60
80
100
120
140
160
→ TR (Seconds)

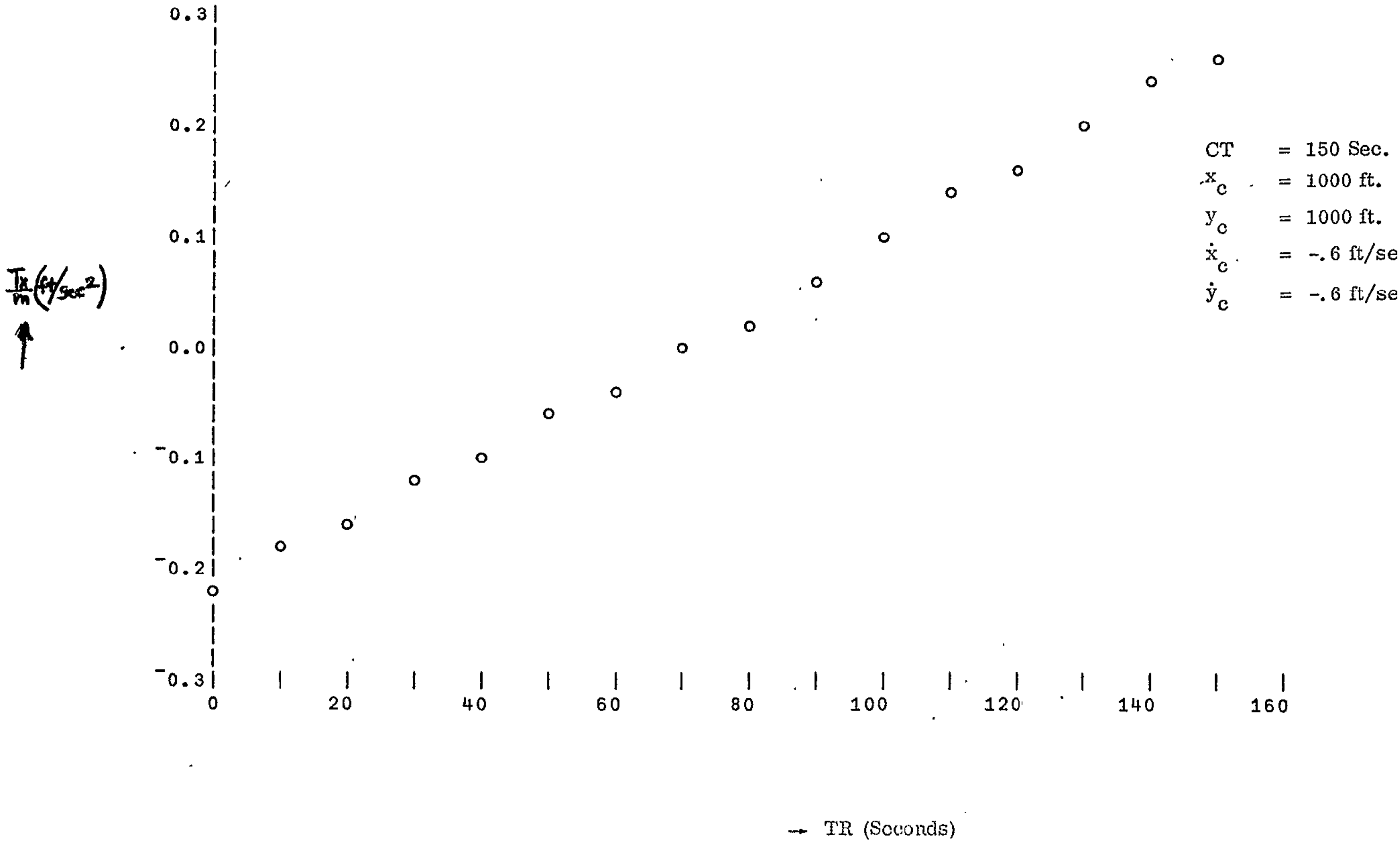

CT = 150 Sec.
x_c = 1000 ft.
y_c = 1000 ft.
$\dot{x}_c$ = -.6 ft/sec.
$\dot{y}_c$ = -.6 ft/sec.
Y ft
1000
750
500
250
0
0
20
40
60
80
100
120
140
160
TR (Seconds)

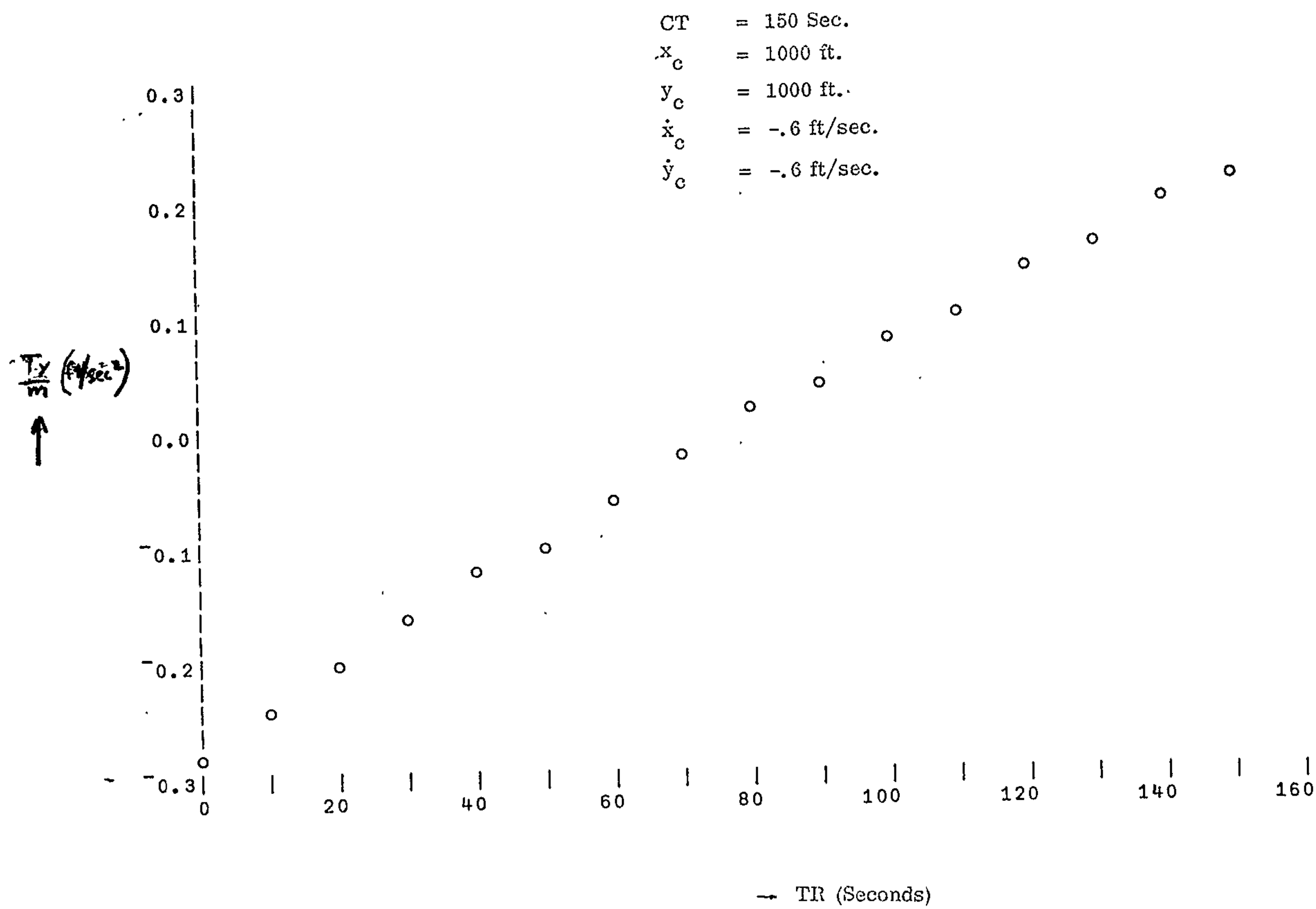

CT = 150 Sec.
x_c = 1000 ft.
y_c = 1000 ft.
$\dot{x}_c$ = -.6 ft/sec.
$\dot{y}_c$ = -.6 ft/sec.
Y (ft)
1000
750
500
250
0
200
400
600
800
1000
x (Ft.)



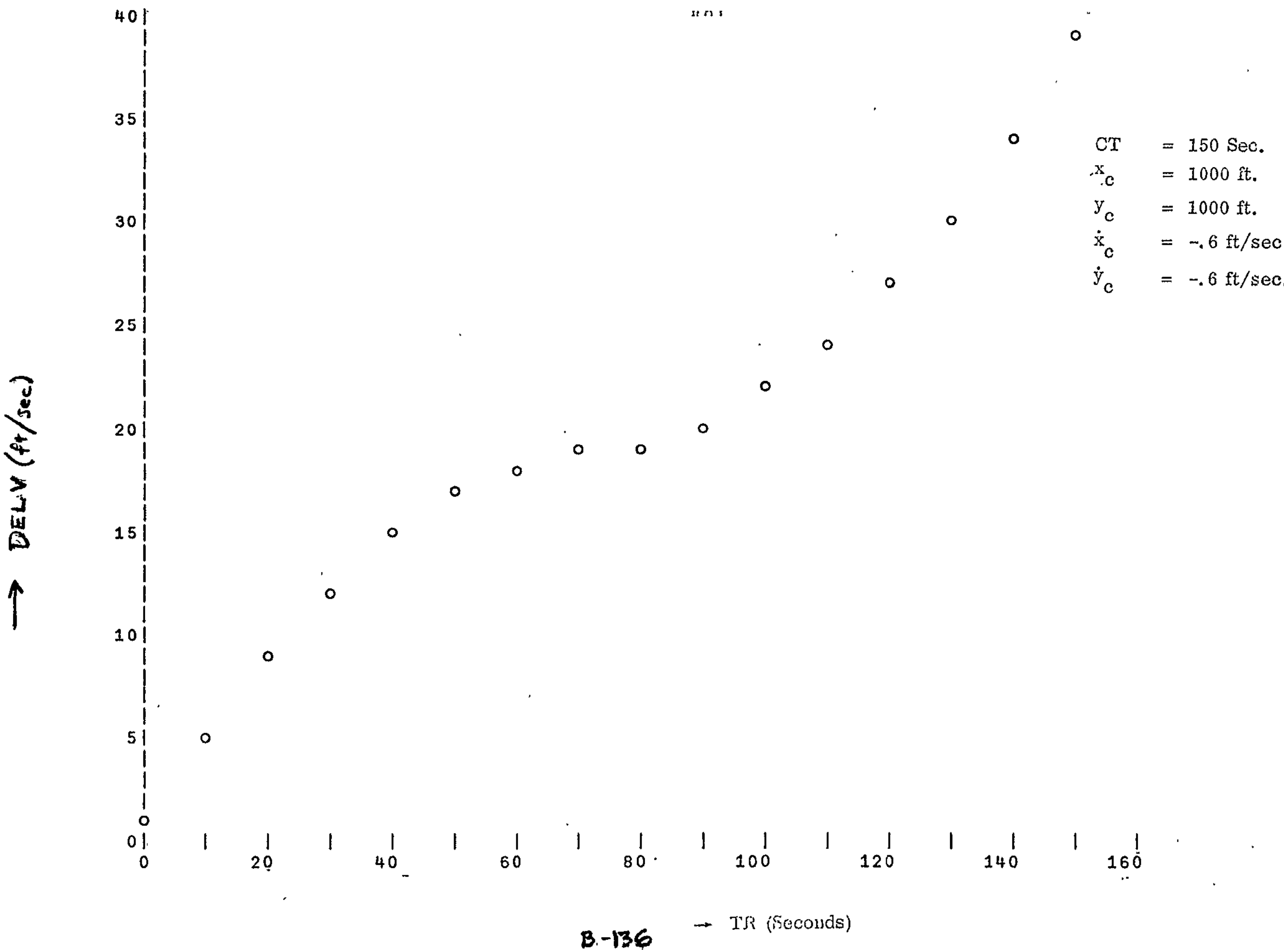
CT = 150 Sec.
x_c = 1000 ft.
y_c = 1000 ft.
$\dot{x}_c$ = -.6 ft/sec.
$\dot{y}_c$ = -.6 ft/sec.
0.0
-2.5
-5.0
-7.5
-10.0
$\dot{X}$ ft/sec
0
20
40
60
80
100
120
140
160
→ TR (Seconds)


CT = 150 Sec.
x_c = 1000 ft.
y_c = 1000 ft.
$\dot{x}_c$ = -.6 ft/sec.
$\dot{y}_c$ = -.6 ft/sec.
$\dot{y}$ ft/sec
0.0
-2.5
-5.0
-7.5
).0
0
20
40
60
80
100
120
140
160
TR (Seconds)


0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
Tx/m (ft/sec²)
0
20
40
60
80
100
120
140
160
CT = 150 Sec.
xc = 1000 ft.
yc = 1000 ft.
ẋc = -.6 ft/se
ẏc = -.6 ft/se
TR (Seconds)



CT = 150 Sec.
x_c = 1000 ft.
y_c = 1000 ft.
$\dot{x}_c$ = -.6 ft/sec.
$\dot{y}_c$ = -.6 ft/sec.
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
$\frac{T_y}{m}$ (ft/sec²)
0
20
40
60
80
100
120
140
160
TR (Seconds)



CT = 150 Sec.
x_c = 1000 ft.
y_c = 1000 ft.
$\dot{x}_c$ = -.6 ft/sec.
$\dot{y}_c$ = -.6 ft/sec.
DELV (ft/sec)
0
5
10
15
20
25
30
35
40
0
20
40
60
80
100
120
140
160
→ TR (Seconds)

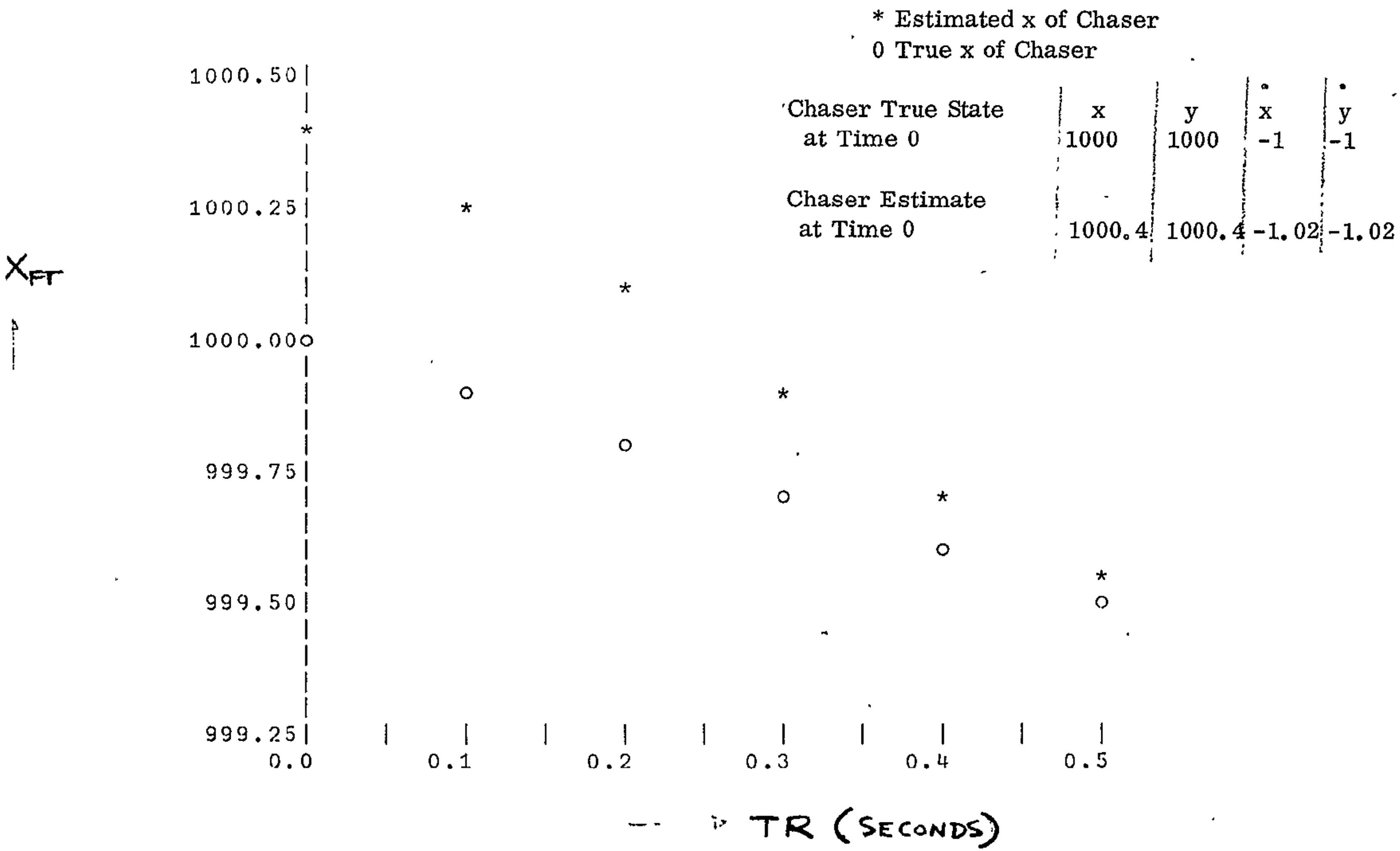
APPENDIX C

RESULTS OF RECURSIVE FILTER SIMULATION

```
      FILTER
THIS PROGRAM NEEDS MATRIX A4×4 AND ATRANSPOSE AND H2×4 AND HTRANSPOSE AND RINVERSE
INPUT T
⎕:
      0
INPUT X
⎕:
      1000 ¯1 1000 ¯1
INPUT XH
⎕:
      1000.4 ¯1.02 1000.4 ¯1.02
INPUT P
⎕:
      .012 0 0 0 0 .012E¯2 0 0 0 0 .012 0 0 0 0 .012E¯2
```

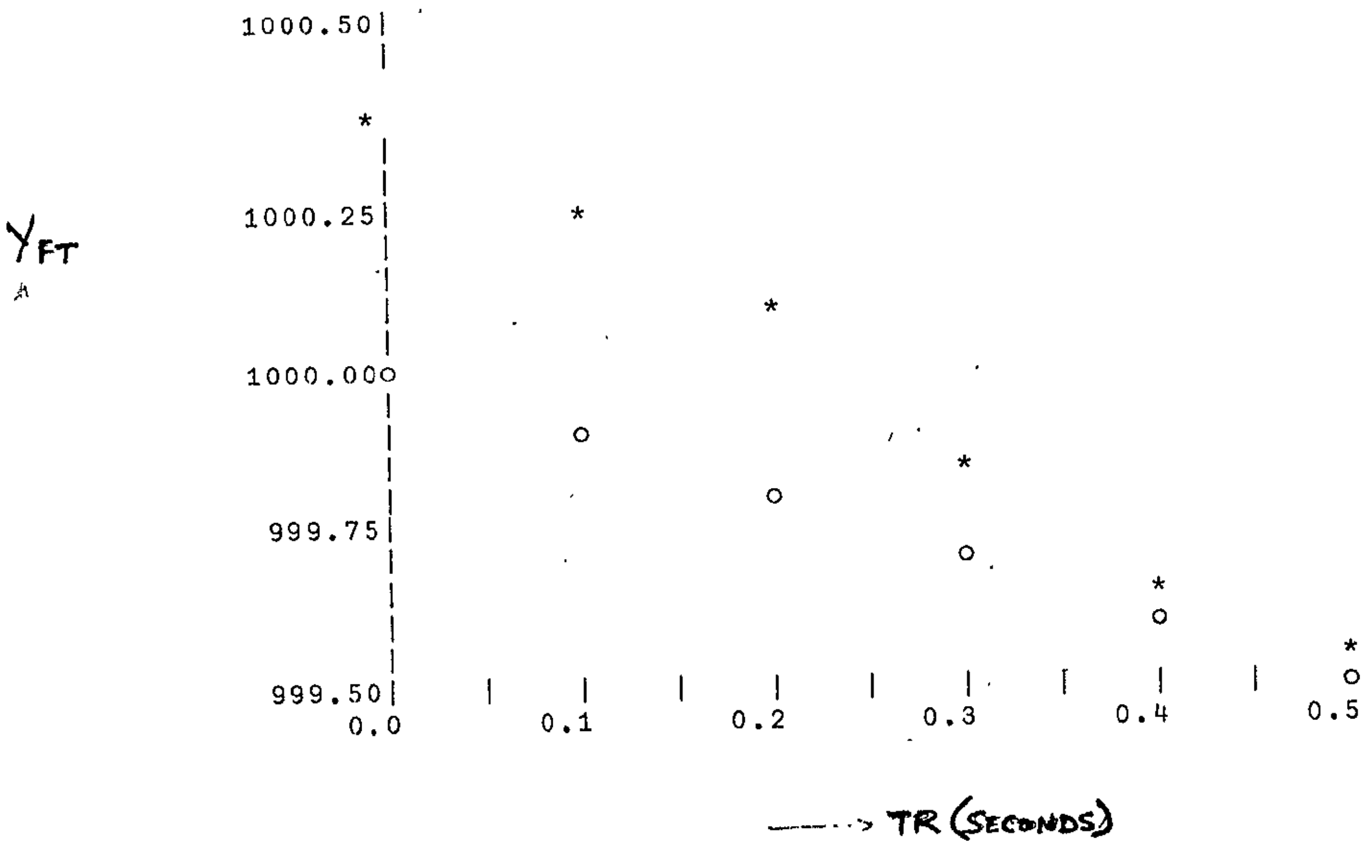
RECURSIVE FILTER

```
 0.1

   999.9
    ¯1.000228
   999.9
    ¯0.99938212

   1000.253181
     ¯1.02023256
   1000.250301
     ¯1.019377404
 0.2

   999.7999772
    ¯1.000455859
   999.8000618
    ¯0.998764227

   1000.09557
     ¯1.02049424
   1000.083859
     ¯1.018788782

 0.3

   999.6999316
    ¯1.000683577
   999.7001854
    ¯0.998146321

   999.8911503
    ¯1.020839878
   999.8427006
    ¯1.018320454
 0.4

   999.5998633
    ¯1.000911155
   999.6003707
    ¯0.9975284021

   999.7103287
    ¯1.021211574
   999.656554
    ¯1.017847291

 0.5

   999.4997721
    ¯1.001138591
   999.5006179
    ¯0.9969104701

   999.5540906
    ¯1.02158159
   999.5513122
    ¯1.017233492
```


* Estimated x of Chaser
0 True x of Chaser
Chaser True State at Time 0 | x 1000 | y 1000 | ẋ -1 | ẏ -1
Chaser Estimate at Time 0 | 1000.4 | 1000.4 | -1.02 | -1.02
XFT
1000.50
1000.25
1000.00
999.75
999.50
999.25
0.0
0.1
0.2
0.3
0.4
0.5
TR (SECONDS)

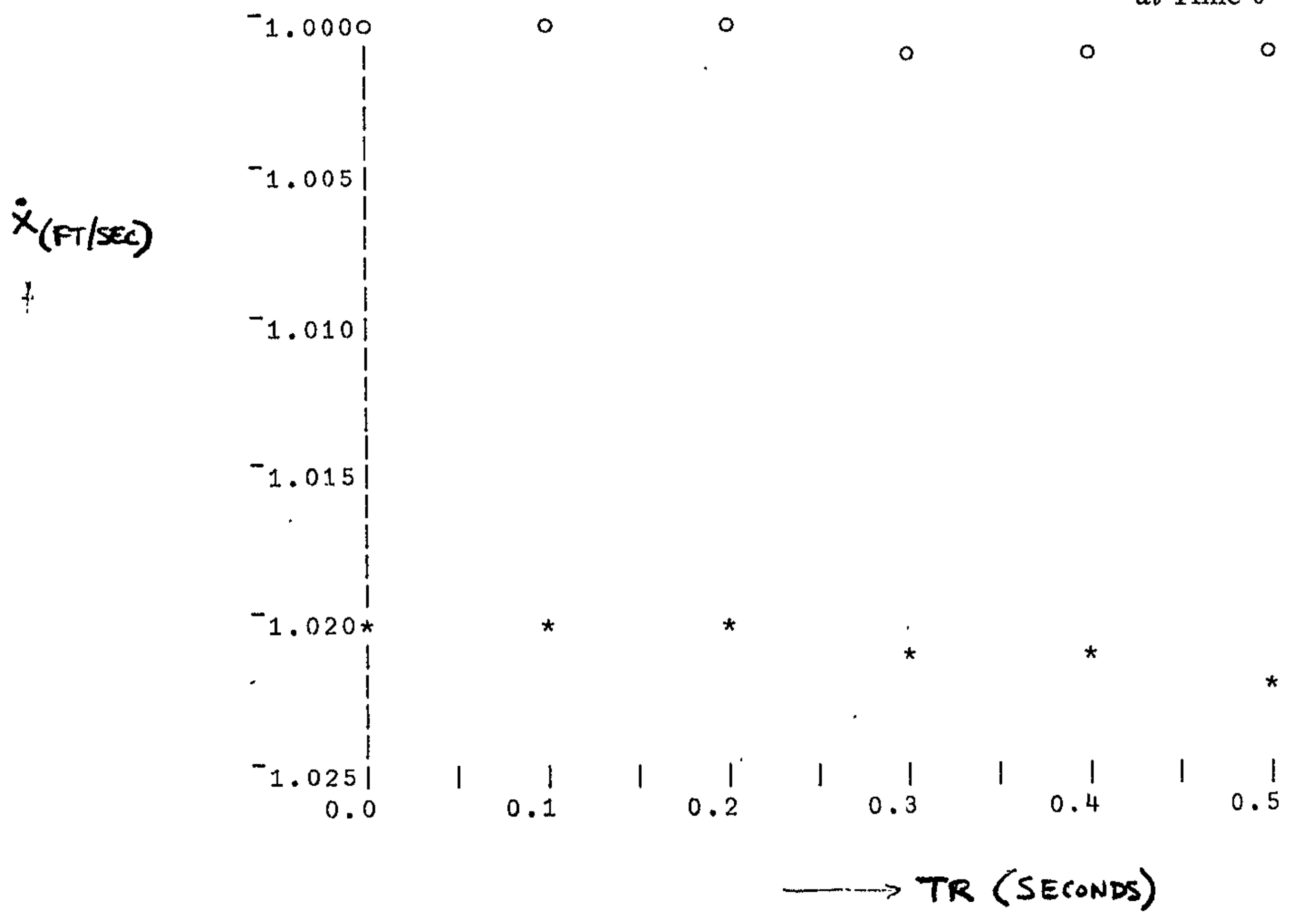
* Estimated y of Chaser
0 True y of Chaser

	x	y	$\dot{x}$	$\dot{y}$
Chaser True State at Time 0	1000	1000	-1	-1
Chaser Estimate at Time 0	1000.4	1000.4	-1.02	-1.02



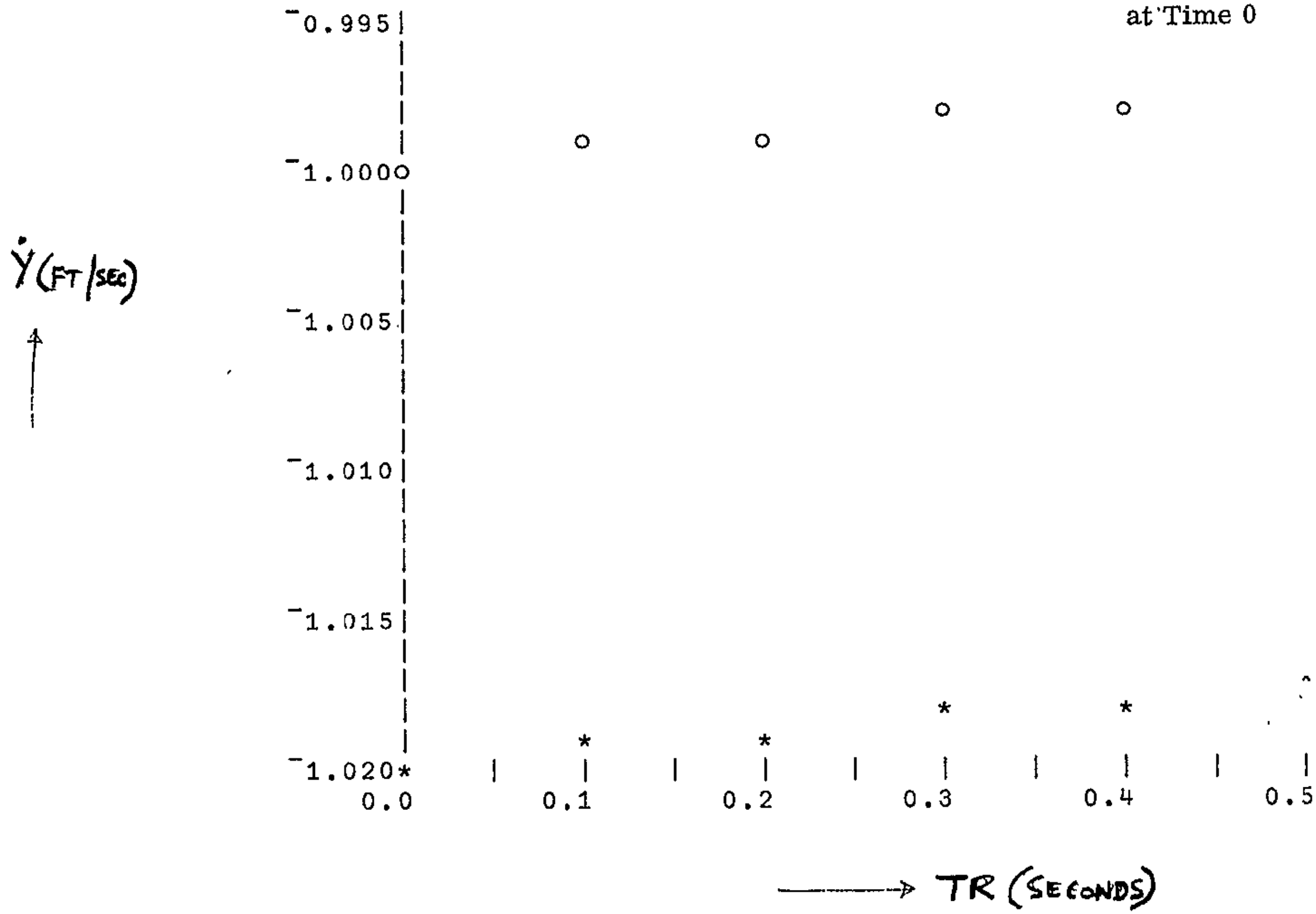
*Estimated x of Chaser
0 - True $\dot{x}$ of Chaser

	x	y	$\dot{x}$	$\dot{y}$
Chaser True State at Time 0	1000	1000	-1	-1
Chaser Estimate at Time 0	1000.4	1000.4	-1.02	-1.02



* Estimated $\dot{y}$ of Chaser
0 True $\dot{y}$ of Chaser

	x	y	$\dot{x}$	$\dot{y}$
Chaser True State at Time 0	1000	1000	-1	-1
Chaser Estimate at Time 0	1000.4	1000.4	-1.02	-1.02

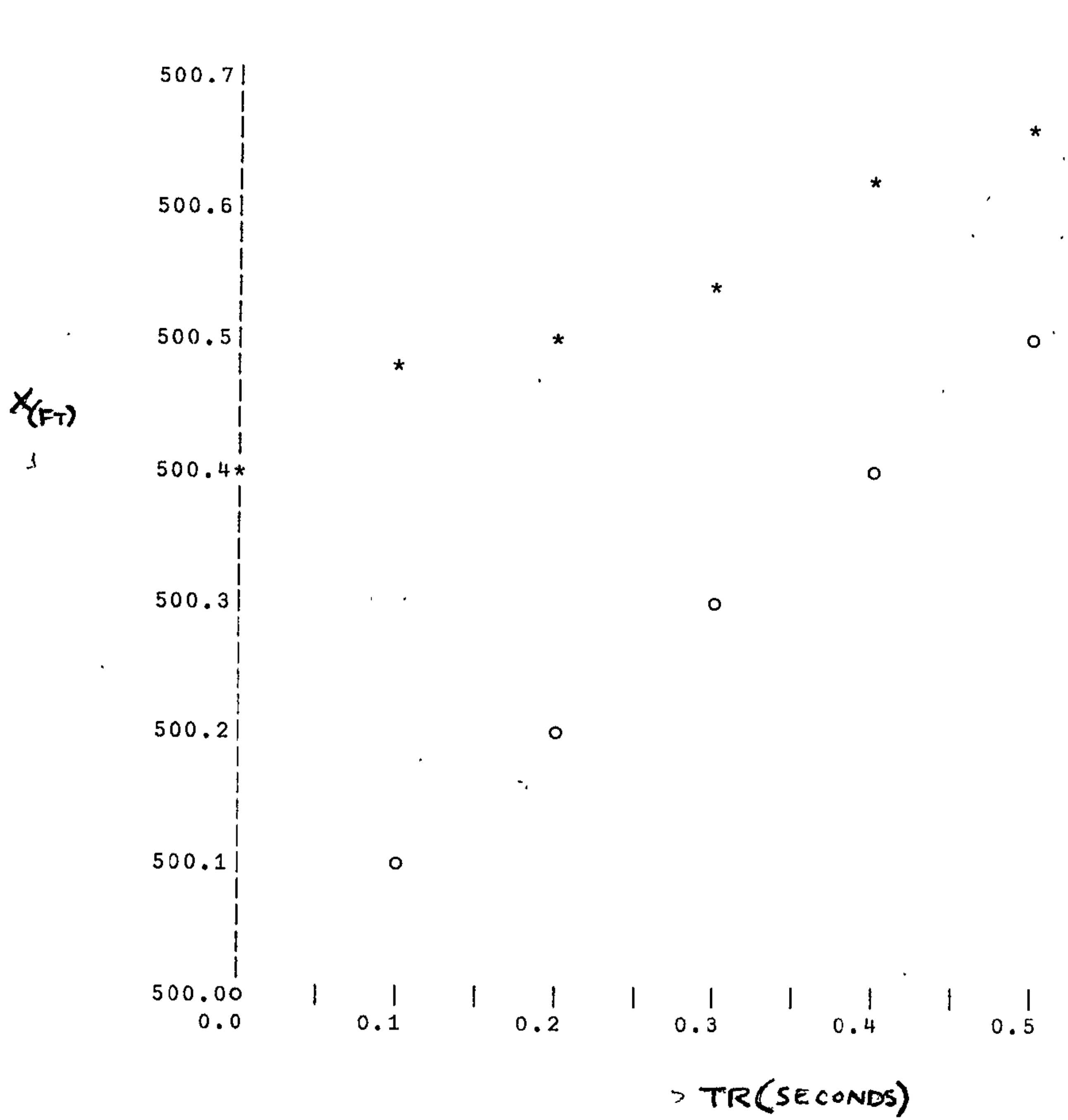


```
      FILTER
THIS PROGRAM NEEDS MATRIX A4×4 AND ATRANSPOSE AND H2×4 AND HTRANSPOSE AND RINVERSE
INPUT T
⎕:
      0
INPUT X
⎕:
      500 1 500 1
INPUT XH
⎕:
      500.4 1.02 500.4 1.02
INPUT P
⎕:
      .012 0 0 0 0 .012E¯2 0 0 0 0 .012 0 0 0 0 .012E¯2
```

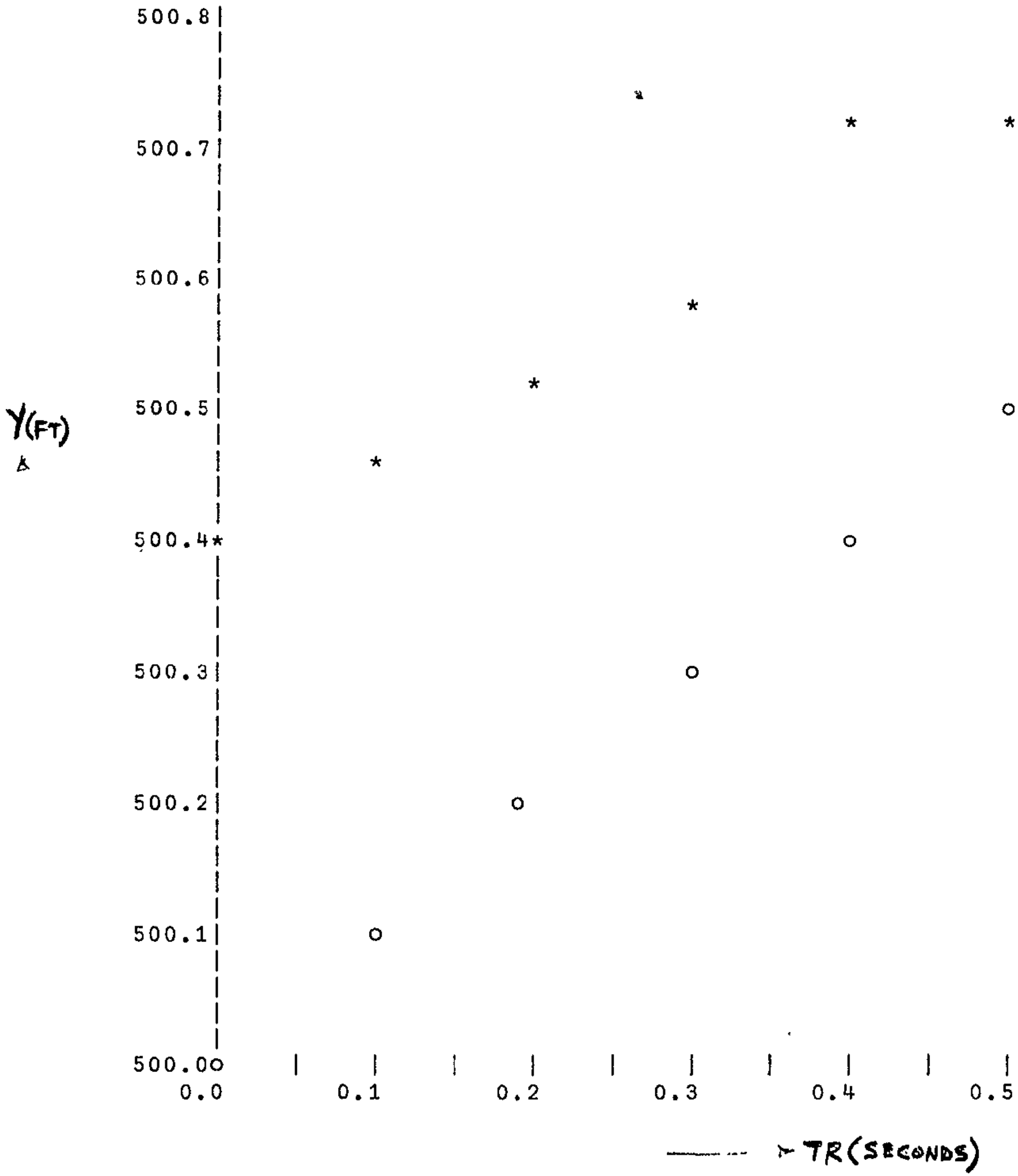
```
 0.1

   500.1
     1.000228
   500.1
     0.99996694

   500.4797361
     1.02023256
   500.4649254
     1.019962536
 0.2

   500.2000228
     1.000455992
   500.1999967
     0.999933867

   500.4928603
     1.020418314
   500.5296099
     1.019905395
```

```
 0.3

   500.3000684
     1.000683977
   500.2999901
     0.999900781

   500.531259
     1.020580384
   500.5808506
     1.019811681
 0.4

   500.4001368
     1.000911955
   500.3999802
     0.999867682

   500.6158131
     1.0207819
   500.7179543
     1.019836407
```

```
 0.5

   500.500228
     1.001139925
   500.4999669
     0.99983457

   500.6636147
     1.020876009
   500.7162605
     1.019534557
```



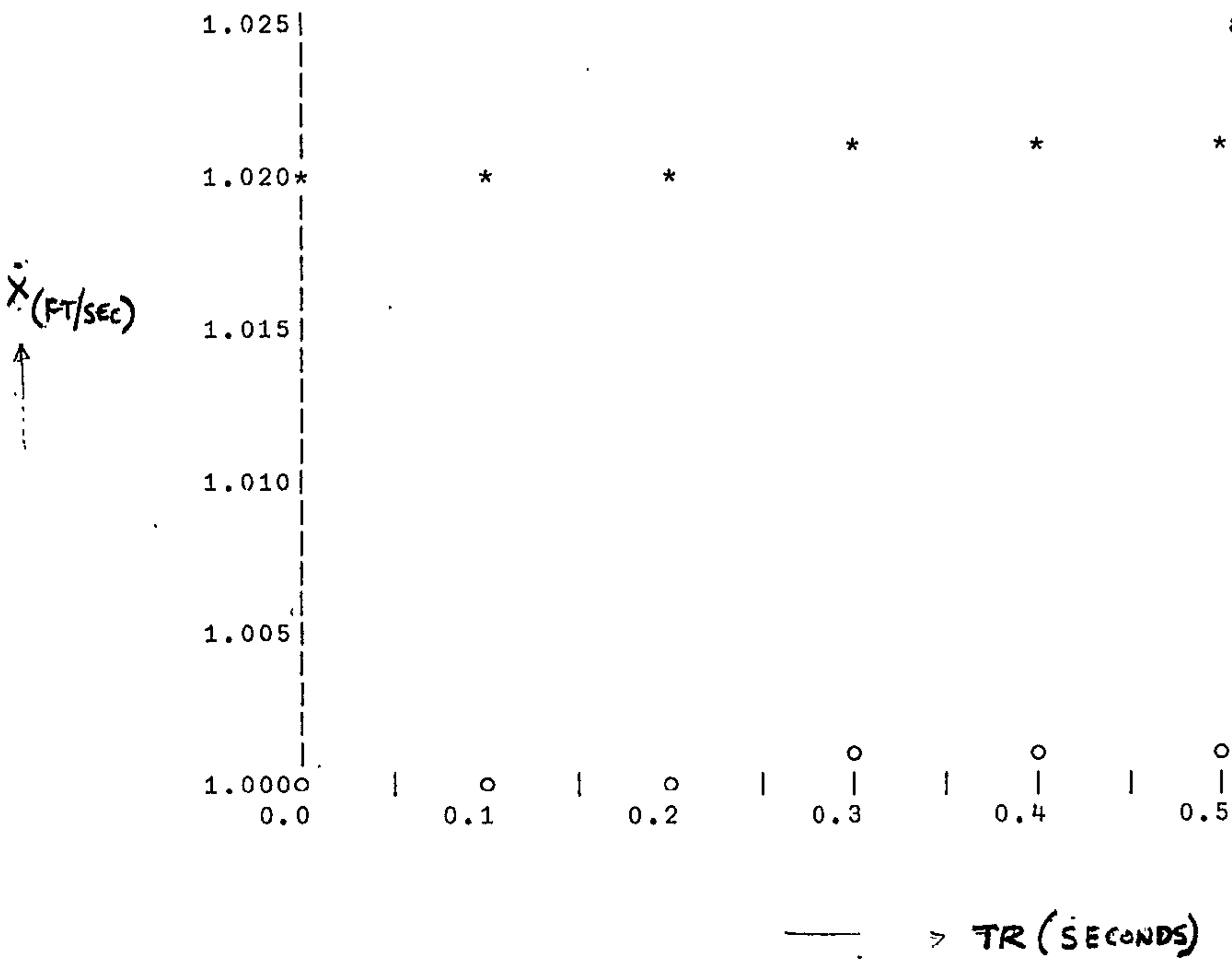
	x	y	$\dot{x}$	$\dot{y}$
Chaser True State at Time 0	500	500	1	1
Chaser Estimate at Time 0	500.4	500.4	1.02	1.02



* Estimated y of Chaser
o True y of Chaser

	x	y	$\dot{x}$	$\dot{y}$
Chaser True State at Time 0	500	500	1	1
Chaser Estimate at Time 0	500.4	500.4	1.02	1.02

Chaser True State at Time 0	x	y	$\dot{x}$	$\dot{y}$
	500	500	1	1
Chaser Estimate at Time 0	500.4	500.4	1.02	1.02



* Estimated $\dot{y}$ of Chaser
o True $\dot{y}$ of Chaser

	x	y	$\dot{x}$	$\dot{y}$
Chaser True State at Time 0	500	500	1	1
Chaser Estimate at Time 0	500.4	500.4	1.02	1.02