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Optimal Time Invariant Output Feedback Controllers

by Derek E. McBrinn

Submitted on behalf of

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Rob Roy Professor Systems Engineering Division

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LIST OF SYMBOLS

Symbol	LIST OF SYMBOLS <u>Meaning</u>	Page First Used
A	System matrix	. 2
۵	Equivalent system matrix	. 36
В	Control matrix	. 2
С	Output/state matrix	. 8
ĉ	Equivalent output/state matrix	• 9
D	Output/control matrix	. 8
D	Arbitrary square matrix	, 14
D(t)	Riccati matrix	. 2
F	Terminal state cost matrix	. 2
G	Gradient matrix	. 15
GF	Gain function	. 4 <u>1</u>
I	Identity matrix	. 8
Ip	Permutation matrix	• 9
J	Cost functional	. 2
K	Matrix of feedback gains	• 3
к _о	Base gain matrix	. 47
ĸ	Perturbed gain matrix	. 47
М	Modal matrix	• 51
NC	Number of controls	. 2
NF	Number of feedback states	. 2
NS	Number of states	. 2
P	Equivalent cost matrix	• 37
ର୍	State cost matrix	. 2
Q, R, W	Cost matrices used in Cassidy's SOC technique	• 7
R	Control cost matrix	2

Symbol	Meaning	Page First Used
Т	Terminal time for optimization problem	. 2
v	Covariance matrix	• 39
W	Matrix of necessary conditions	. 44
t	Time	. 2
u	Scalar control	. 56
<u>u</u> (t)	Control vector	. 2
<u>v</u> r, v _i r	Row eigenvector of a matrix	. 1 ⁴
<u>w</u> , <u>w</u> i	Column eigenvector of a matrix	. 14
$\underline{\mathbf{x}}(t)$	State vector	. 2
$\underline{\hat{x}}(t)$	Subset of state vector	• 9
<u>y</u> (t)	Output vector	. 8
<u>y</u> (t)	Alternative state vector	. 40
<u>z</u> (t)	Alternative state vector	• 9
α	Parameter of a matrix	<u>.</u> 14
Δ	Small perturbation	. 16
ક	Small perturbation	<u>.</u> 47
e	Element of the set	. 2
Ø(t)	State transition matrix	• 36
Л	Diagonal matrix of eigenvalues	. 51
λ,λ _i	Eigenvalue of a matrix	. 14
μ	Step size (convergence) parameter	. 16
∇	Gradient matrix	. 16
(t)	Function of time	. 2
• as in $\underline{x}(t)$	Differentiation with respect to time	. 2
$^{\mathrm{T}}$ as in $^{\mathrm{T}}$	Transform of matrix or vector	. 2
-l as in R ^{-l}	Inverse of matrix	. 2

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Symbol	Meaning	Page First <u>Used</u>
ooas in ^o K.	Vector arrangement of a matrix by column ordering	. 15
[0, T]	Closed time interval from 0 to T	. 2
ij as in k _{ij} or []ij	Element in row i, column j of the matrix	. 10
i as in x _.	i component of vector	. 15
g as in <u>b</u> q	q th column of the matrix B	. 14
T	Evaluation at given point	• 33
	Euclidian norm	. 16
Re[Real part	. 15
e[]	Expected value	. 38
exp []	Exponentiation	• 37
tr[Trace of matrix	• 33

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ABSTRACT

The problem of the determination of optimal time-invariant output feedback controllers for linear dynamic systems with quadratic cost functionals is considered. Two distinct cases arise, depending on whether optimization is over a finite or a semi-infinite time interval.

For the semi-infinite time interval problem a gain initialization technique is derived to complement existing optimization techniques. The gain initialization technique determines the feedback gains required to (locally) maximize the system stability. A computational algorithm for the technique is incorporated in a digital computer program, and is used to stabilize a seven state model of a Saturn V booster rocket.

For the finite time interval problem a technique is derived to (locally) minimize the expected value of the cost functional. The technique uses the concept of Initial State Averaging. A computational algorithm is provided and incorporated in a digital computer program. The technique is illustrated by three examples.

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CHAPTER I

INTRODUCTION

I.1 Motivation

During the past decade much excellent work has been done in the field of Optimal Control Theory. Many books and a great number of technical papers have been published - the Bibliography of this report cites only a small portion of the total. Not all of this work has been purely theoretical. A substantial background has been developed in the important practical area of computation and it has for several years been feasible to determine optimal controllers for a wide variety of physical systems.

It would be expected that practical applications of this theory would have appeared in abundance. Optimization is the essence of all good engineering design, and practicing engineers should seize upon any techniques which might aid them in their work. Application of the new techniques has, however, been disappointingly slow. With a few notable exceptions the practical design of control systems has remained based on the transferfunction techniques developed prior to the 1960's.

The explanation of the above paradox is widely recognized. It lies in the nature of the word "optimal", which is meaningless without a criterion of optimality. By and large, control theorists have used criteria of optimality dependent only on the performance of the control system. Design engineers, on the other hand, interpret "optimal" as embracing both system performance and system cost. The difference may be best illustrated by a simple example.

Consider the problem of designing a speed control device for a cheap movie camera. The objective is to ensure that sixteen frames of film are exposed per second regardless of film tension, state of charge of the motor batteries, etc. The could be interpreted as comprising an example of the state-regulator problem in Optimal Control Theory, with the following . method of analysis. The equations governing the film transportation are determined and linearized to the form

$$\underline{\dot{x}}(t) = A \underline{x}(t) + B \underline{u}(t)$$
(I,l_1)

where $\underline{x}(t)$ is an NS-vector describing the deviation of the system . from its desired state at time t

- $\underline{u}(t)$ is an NC-vector defining the control inputs at time t
- A and B are matrices whose elements depend on the characterr. istics of the system.

A cost functional is formulated having the form

$$J = \underline{x}^{T}(T) = \underline{x}(T) + \int_{0}^{T} \left[\underline{x}^{T}(\boldsymbol{\tau}) \quad Q \; \underline{x}(\boldsymbol{\tau}) + \underline{u}^{T}(\boldsymbol{\tau}) \quad R \; \underline{u}(\boldsymbol{\tau}) \right] d\boldsymbol{\tau} \qquad (I, 1-2)$$

where T is the duration of the scene to be filmed. The objective is to determine the control input $\underline{u}(t)$ for $t \in [0, T]$ which minimizes the cost functional J. The positive semi-definite matrices F and Q are chosen to penalize any given control system for the deviations it allows from the desired state. The positive definite matrix R is a penalty for improvident usage of control power (i.e., battery energy). The solution to the problem defined by (I.1-1) and (I.1-2) can be shown² to be

$$\underline{u}(t) = -R^{-1} B^{T} D(t) \underline{x}(t)$$
 (1.1-3)

where D(t) satisfies the matrix Riccati equation

$$\dot{D}(t) = -D(t) A - A^{T}D(t) + D(t) BR^{-1} B^{T}D(t) - Q \qquad (1,1-4)$$

with

$$D(T) = F$$
(I.1-5)

Defining

$$K^{T}(t) = R^{-1} B^{T} D(t)$$
 (I.1-6)

and re-writing (I.1-3) as

$$\underline{\mathbf{u}}(t) = -\mathbf{K}^{\mathrm{T}}(t) \underline{\mathbf{x}}(t) \qquad (\underline{\mathbf{I}},\underline{\mathbf{I}},7)$$

we see that the optimal control input can be generated as a time-vanying linear combination of all of the states of the system. We note in passing that the less-fastidious photographer, willing to edit a few frames from the beginning and end of each scene and settle for good steady state performance, would be rewarded by simplification of the controller to the form²

$$\underline{u}(t) = -K^{\mathrm{T}} \underline{x}(t)$$
(T.1-8)

This time-invariant controller would be much simpler to mechanize than . that described by (I.1-7).

Now consider that speed control of the typical cheap movie camera is achieved by means of a simple flyball governor driving an on/off switch between the batteries and the motor. The flyball governor, invented by James Watt in 1788, was the first widely used automatic feedback controller.⁵ In its simpler form it suffers from "hunting" about the set point. In its application to the movie camera it is driven by the single system state, motor speed. The single feedback gain can be considered as constant in the vicinity of the set point. It is clear that the flyball governor does not satisfy the requirements for an optimal controller as defined by (1,1-7). Yet the flyball governor is indeed an optimal controller in the best engineering sense. It does an acceptable job at minimum cost. The mechanization of the "optimal" controller defined by (I.1-7) on the other hand, would be prohibitively expensive. It would require measurement and feedback of all system states. It would also require a digital computer to calculate and store the time-varying feedback gains for each scene to be filmed.

It should be clear from the above that the allowable degree of complexity of a control system is often constrained by considerations of cost. Weight, reliability and common engineering sense similarly often dictate simplicity. What the control system designer wants then is not that system which performs in the best possible manner. Rather he wishes the best possible performance for a given degree of complexity. This problem is considerably more difficult than that defined by (I.1-1) and (I.1-2). It must however, be solved if the benefits of optimal control are ever to be realized for the majority of potential applications.

This report considers a portion of the general problem of optimization within a given degree of control system complexity. Only linear systems with quadratic cost functionals are considered. Such combinations may be described by (I.1-1) and (I.1-2). The allowable degree of complexity of the controller is assumed to be time-invariant feedback of the system outputs only. The system outputs are those quantities which can be measured, and in most cases provide less than a full description of the system state at any time. The above limitations are those which are normally applied in classical control system analysis techniques. Thus this report seeks to optimize the classical controller, without increasing its complexity.

I.2 Historical Review

It is generally conceded that control system design prior to the Second World War was primarily an art. The techniques used were often empirical and thus confined to particular classes of problems. This situation was largely eliminated by the contributions of Nyquist,⁶ Bode⁷ and Evans⁸ who established the theory and techniques of control system design which predominate in practical work even today. These techniques, however, are most suited to the design of relatively simple systems. They work best for systems having a single input and a single output related by a transfer function. Also these techniques are purely analytical and cannot be used to directly synthesize a control system.

The problem of synthesis was first tackled by Wiener⁹, who considered the optimization of linear filters. This work was extended to control systems by Newton, Gould and Kaiser¹⁰, who, however, still retained the transfer-function approach; their technique was to determine the values of system parameters required to minimize a cost functional for a specific perturbation of the system. It is significant that they made use of the Calculus of Variations in their approach to this problem. The extension to the classical Calculus of Variations provided by Pontryagin's Maximum Principle,¹¹ and the control problem framework provided by Kalman¹² finally brought Optimal Control Theory to maturity. This maturity, combined with the state-variable formulation presented by DeRusso, Roy and Close¹ and the increasing capability of digital computers, finally allowed the determination of optimal controllers for a wide variety of systems.

The early excitment at the disclosure of the ability to compute optimal controllers faded when it was realized that such controllers were very difficult to mechanize. The reasons for this difficulty are twofold. First, an optimal controller requires knowledge of the complete state of the system. It may not, however, be feasible to measure all system states. Secondly, for finite time duration problems an optimal controller requires time-varying feedback.

There are two fundamental approaches to elimination of the first difficulty. The approach taken by Kalman and Bucy¹³ was to estimate the unmeasurable states from the (noise corrupted) system outputs or measurable states. This approach was extended by Luenberger¹⁴ and again by Ash^{15} . Estimation of the unmeasurable states allows the mechanization of a regular optimal controller. Note, however, that it addes the complexities of a state estimator to those of the optimal controller.

The second approach to the problem of unmeasurable states is to design a controller which uses only the available information. Such a controller will be "sub-optimal" in the mathematical sense of minimizing a cost functional such as that described by (I.1-2). It may well, however, be optimal in the engineering sense. This approach has been tried by a number of people. Newton, Gould and Kaiser essentially advocated such controllers, but optimized them for specific system disturbances. Consideration of only a specific disturbance is equivalent to imposing a specific initial condition on the system to be controlled. Max-min techniques were developed in an attempt to eliminate the dependence of the solution on the chosen initial condition.^{16,17} In the max-min procedure the maximum system cost with respect to a set of initial conditions is minimized with respect to the system feedback gains. It still remains to choose an appropriate set of initial conditions. Rekasius circumvented this problem by considering the initial condition giving the maximum ratio of output feedback cost to allstate feedback cost but was still limited to design for this specific initial condition. Levine¹⁹ determined the

controller which was optimal in an average sense and computed time varying feedback gains for the case of the finite time-interval problem. For the semi-infinite time interval problem he required his feedback gains to be time-invariant. Cassidy²⁰ considered the minimization of a modified cost functional having the form

$$J = \frac{1}{2} \int_{0}^{T} (\underline{x}^{T} Q \underline{x} + \underline{x}^{T} Q \underline{x} + \underline{x}^{T} Q \underline{x} + \underline{x}^{T} W \underline{u} + \underline{x}^{T} W \underline{y} + \underline{u}^{T} R \underline{u}) d\boldsymbol{\tau} \qquad (I.2-1)$$

where the matrices \hat{Q} and \hat{W} were chosen to ensure that the feedback gains for the unavailable states were zero. The resultant feedback gains were again time-varying for a finite time interval, time-invariant for the semi-infinite time interval.

It is considered that the approaches used by Levine and Cassidy are the most satisfactory of those considered, since they are both completely independent of initial conditions. They both, however, result in timevarying gains for the finite time-interval case. For the semi-infinite time interval both give constant feedback gains, but use computational algorithms which require initialization by a set of stabilizing feedback gains.

The problem of determining stabilizing output-feedback gains for complex systems has not, to the author's knowledge, been satisfactorily solved. Koenigsberg²¹ has considered local stability maxima, but his computational algorithm is felt to be inefficient. Jameson²² and Davison²³ have shown that as many system roots as there are feedback states can be arbitrarily determined, but the remaining roots are then unconstrained.

A solution to the problem of determining optimal constant or piecewise constant feedback gains for a finite time interval has been derived by Kleinman, Fortmann and Athans.²⁵ As with Levine,¹⁹ they considered optimality in an average sense, but confined attention to the allstate feedback case.

I.3 Scope and Contribution of This Work

This report considers the determination of optimal time-invariant output feedback control gains for systems which can be described by the equation

$$\dot{\mathbf{x}}(t) = \mathbf{A} \, \mathbf{x}(t) + \mathbf{B} \, \mathbf{u}(t) \tag{I.3-1}$$

where $\underline{x}(t)$ is an NS-vector describing the state of the system at time t

<u>u(t)</u> is an NC-vector describing the control inputs at time t A,B are time invariant matrices of appropriate order.

It is assumed, without loss of generality, that the system outputs are the first NF states. If this is not the case then a new set of state variables may be obtained as follows to satisfy the assumption. Suppose that the system described by (I.3-1) has NF independent outputs $\underline{y}(t)$ given by

$$y(t) = C x(t) + D u(t)$$
 (I.3-2)

We shall require that

$$\underline{u}(t) = -K^{\mathrm{T}} \underline{y}(t) \qquad (\underline{T}.\underline{3}-3)$$

where K is the time-invariant matrix of feedback gains. Thus

$$\begin{bmatrix} I + DK^{T} \end{bmatrix} \underline{y}(t) = C \underline{x}(t)$$
(I.3-4)

where I is the identity matrix of order NF. Assuming $\begin{bmatrix} I + DK^T \end{bmatrix}$ to be non-singular gives

$$\underline{\mathbf{y}}(t) = \widehat{\mathbf{C}} \underline{\mathbf{x}}(t) \tag{I.3-5}$$

where

$$\widehat{\mathbf{C}} = \left[\mathbf{I} + \mathbf{D}\mathbf{K}^{\mathrm{T}}\right]^{-1} \quad \mathbf{C}$$
(1.3-6)

We note that in most cases D will be zero, leaving \widehat{C} equal to C. We now form the vector $\underline{\widehat{x}}(t)$ comprising any (NS - NF) of the $\underline{x}(t)$ state variables which are independent of the $\underline{y}(t)$ variables. We note that such variables must exist since $\underline{x}(t)$ spans NS space while $\underline{y}(t)$ only spans NF space. Appending $\underline{\widehat{x}}(t)$ to $\underline{y}(t)$ gives

$$\underline{z}(t) = \begin{bmatrix} \underline{y}(t) \\ \underline{\hat{x}}(t) \end{bmatrix} = \begin{bmatrix} \hat{c} \\ Ip \end{bmatrix} \underline{x}(t)$$
(I.3-7)

where Ip is a permutation matrix describing how $\underline{\hat{x}}(t)$ was chosen from $\underline{x}(t)$.

• It follows that

$$\underline{\underline{z}}(t) = \begin{bmatrix} \hat{c} \\ Ip \end{bmatrix} \underline{\underline{x}}(t)$$
 (I.3-8)

i.e.,

.

$$\underline{\underline{\dot{z}}}(t) = \begin{bmatrix} C \\ Ip \end{bmatrix} A \underline{x}(t) + \begin{bmatrix} C \\ Ip \end{bmatrix} B \underline{u}(t)$$
(I.3-9)

Thus, from (I.3-7)

$$\underline{\dot{z}}(t) = \begin{bmatrix} C \\ Ip \end{bmatrix} A \begin{bmatrix} C \\ Ip \end{bmatrix} -1 \begin{bmatrix} C \\ Ip \end{bmatrix} = \underline{z}(t) + \begin{bmatrix} C \\ Ip \end{bmatrix} B \underline{u}(t)$$
(I.3-10)

and $\underline{z}(t)$ is the desired state-vector having its first NF states as the system outputs.

Thus the control inputs $\underline{u}(t)$ for the system described by (I.3-1) are to be optimized over the set

$$\underline{u}(t) = -K^{T} \underline{x}(t)$$
 (1.3-11)

where K, the time invariant matrix of feedback gains, is constrained to the form NC

$$K = \begin{bmatrix} k_{ij} \\ 0 \end{bmatrix}$$
 NF
$$k = \begin{bmatrix} k_{ij} \\ 0 \end{bmatrix}$$
 NS (1,3-12)

The cost functional is assumed to be of the form

$$J = \underline{x}^{T}(T) F \underline{x}(T) + \int_{0}^{T} \left[\underline{x}^{T}(T) Q \underline{x}(T) + \underline{u}^{T}(T) R \underline{u}(T) \right] dT \qquad (1,3-13)$$

where F and Q are positive semi-definite

R is positive definite

and optimization is over the time interval [0, T].

Two separate cases are investigated. For the case where the terminal time T is infinite it is considered that the techniques presented by either Levine¹⁹ or Cassidy²⁰ are adequate. Both these techniques, however, require initialization by stabilizing feedback gains. In practice, indeed the author has found that feedback gains giving marginal stability may be inadequate, due to numerical considerations. Chapter II of this report therefore presents a computational algorithm designed to find feedback gains of the form given by (1.3-12) such that the system stability is driven to a local maximum. Note that maximum stability is here defined to mean that the least stable of the system roots is made as stable as possible. The algorithm presented is felt to be superior to that given by Koenigsberg²¹ in that it is computationally faster.

For the case where the terminal time T is finite, an approach is taken similar to that used by Levine¹⁹, except that the feedback gains are required to be time-invariant. The resulting theory is presented in Chapter III, together with a computational algorithm to allow the determination of the optimal gains.

It is felt that this report will help to fill some of the gaps in the existing ability to determine practical feedback controllers which are optimal in the engineering sense.

CHAPTER II

THE GAIN INITIALIZATION PROBLEM

II.1 Introduction

The problem of determination of the output feedback gains required to stabilize a dynamic system is an interesting one in its own right. In this report, however, we are concerned with the determination of such gains as a pre-requisite to the application of optimization techniques.

Both Levine¹⁹ and Cassidy²⁰ have considered the determination of optimal output feedback controllers for linear systems. Both have derived iterative techniques resulting in time invariant feedback gains when the system is optimized over the semi-infinite time interval $[0, \infty]$. Each technique, however, requires initialization with a set of stabilizing feedback gains in order to ensure convergence of the computational algorithm.

Based on the author's practical experience²⁶ with Cassidy's technique it appears that an extra requirement on initial system stability may be added when the optimization algorithms are mechanized on a digital computer. It was found that with low order systems (five states or less) marginal stability was sufficient to ensure convergence of the optimization algorithm. When a twenty-one state model of the Saturn V booster rocket was considered, however, initialization by feedback gains giving marginal stability was insufficient to cause convergence. This was judged to be due to numerical truncation resulting from the finite word length in the digital computer. The problem was solved by computing a new set of feedback gains giving better than marginal stability. Re-initialization of the optimization algorithm with the new feedback gains resulted in convergence. It is considered likely that Levine's algorithm would have similar characteristics. It is clear that practical application of Levine's and Cassidy's techniques requires a method for determination of stabilizing output feedback gains. It would be desirable that the gains give more than marginal stability. A method is described in this Chapter for determination of such gains, when they exist.

II.2 Theory of the Gain Initialization Technique

The problem under consideration is as follows. Given the system

$$\dot{\underline{x}}(t) = A \underline{x}(t) + B \underline{u}(t) \qquad (11.2-1)$$

where $\underline{x}(t)$ is an NS-vector describing the state of the system at time t

- u(t) is an NC-vector describing the control inputs at time t
- A,B are matrices whose elements depend on the characteristics of the system

with

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$$\underline{u}(t) = -K^{\mathrm{T}} \underline{x}(t) \qquad (\mathrm{II}.2-2)$$

where K is a matrix of feedback gains such that

$$\underline{\underline{x}}(t) = \begin{bmatrix} A - BK^T \end{bmatrix} \underline{x}(t)$$
(II.2-3)

where K is constrained to the form NC

$$K = \left[\begin{array}{c} k_{ij} \\ \vdots \\ 0 \end{array}\right] \qquad \left.\right\} \qquad NF \qquad NS \qquad (II.2-4)$$

how should the feedback gain matrix K be modified, within the constraints of (II.2-4) to increase the stability of the least stable eigenvalue of (II.2-3)

The solution hinges on an expression given by Fadeev and Fadeeva²⁷ for the sensitivity of an eigenvalue of a matrix to a parameter of the matrix. Given any eigenvalue λ and corresponding row and column eigenvectors \underline{v}^{T} and \underline{w} of a square matrix D, the sensitivity of the eigenvalue to a parameter α of the matrix is given by

$$\frac{d\lambda}{d\alpha} = \frac{\underline{v}^{\mathrm{T}} \cdot \underline{\mathbf{\delta}} \cdot \underline{D}}{\underline{v}^{\mathrm{T}} \cdot \underline{\mathbf{w}}}$$
(II.2-5)

We note that <u>v</u> and <u>w</u> satisfy

$$\mathbf{D}^{\mathrm{T}}\underline{\mathbf{v}} = \lambda \, \underline{\mathbf{v}} \tag{II.2-6}$$

$$D\underline{w} = \lambda \underline{w} \tag{II.2-7}$$

To apply (II.2-5) to the system described by (II.2-3) and (II.2-4) we note that

$$\frac{\partial \left[A - BK^{T}\right]}{\partial k_{pq}} = \begin{bmatrix} 0 & b_{q} & 0 \end{bmatrix}$$
(II.2-8)
$$\sum_{p^{\text{th}} \text{ column}}$$

where \underline{b}_{q} is the q^{th} column of the matrix B. Thus if λ_{i} is any eigenvalue of (II.2-3) and if \underline{v}_{i}^{T} and \underline{w}_{i} are corresponding row and column eigenvectors we see that

$$\frac{\partial \lambda_{i}}{\partial k_{pq}} = \frac{\underline{v_{i}}^{T} \begin{bmatrix} 0 | \underline{b}_{q} | 0 \end{bmatrix} \underline{w}_{i}}{\underline{v_{i}}^{T} \underline{w}_{i}}$$
(II.2-9)

The sensitivity of the real part of λ_{i} to the real feedback gain k is simply the real part of the right hand side of (II.2-9).

Suppose now that it is desired to decrease the real part of λ_i . The direction of the steepest descent in feedback gain space is defined by the gradient matrix G where

$$g_{pq} = \operatorname{Re} \left\{ \frac{\partial \lambda_{i}}{\partial k_{pq}} \right\}$$
(II.2-10)

$$p = 1, \dots, NF$$

$$q = 1, \dots, NC$$

II.3 The Gain Initialization Algorithm

A gain initialization algorithm has been derived from the expressions given in (II.2-9) and (II.2-10), and has been incorporated in the digital computer program GRADGN. A listing of the program GRADGN is given in Appendix I. A description of the algorithm/program follows.

The algorithm is iterative. It increases the stability of the least stable eigenvalue of the system at each step. If, during a given interation a new eigenvalue should become less stable than the one being operated on, this fact is taken into account in subsequent iterations.

Consider the rearrangement of the variable elements of the feedback gain matrix K into an NF x NC vector ${}^{o}K{}^{a}$. The rearrangement is performed by column ordering of K so that

$$K_{pq} = {}^{\boldsymbol{\rho}} K_{(q-1)NF + p}^{\boldsymbol{\sigma}}$$
(II.3-1)

The gradient matrix G is similarly rearranged giving

$$G_{pq} = {}^{\boldsymbol{a}} G_{(q-1)NF}^{\boldsymbol{a}} + p \qquad (II.3-2)$$

The steepest descent method requires that variations in the gain vector ${}^{a}K{}^{a}$ must lie along the negative of the gradient vector ${}^{a}G{}^{a}$. Thus

$${}^{a}\Delta K^{a} = -\mu^{a}G^{a}$$
(II.3-3)

where μ is a positive constant determined by the step size to be taken and ${}^{B}\Delta K^{B}$ is a variation in the gain vector. Suppose now that we wish to determine the variation ${}^{B}\Delta K^{B}$ in the gain vector ${}^{B}K^{B}$ required to cause a small negative change $\Delta \operatorname{Re} \{\lambda_{i}\}$ in the real part of the eigenvalue λ_{i} . The variation ${}^{B}\Delta K^{B}$ may be computed from the relationship

$${}^{\mathfrak{g}} \mathcal{G}^{\mathfrak{g}} \mathcal{T} \mathcal{A} \mathcal{K}^{\mathfrak{g}} = \Delta \operatorname{Re} \{ \lambda_{j} \}$$
(II.3-4)

When the variation ${}^{B}\Delta K^{B}$ lies in the direction of steepest descent then, from (II.3-3) and (II.3-4)

$$-{}^{\boldsymbol{\mu}}\boldsymbol{G}^{\boldsymbol{\mu}}\boldsymbol{T} \quad \boldsymbol{\mu}^{\boldsymbol{\mu}}\boldsymbol{G}^{\boldsymbol{\mu}} = \boldsymbol{\Delta}\operatorname{Re}\left\{\boldsymbol{\lambda}_{\mathbf{i}}\right\} \tag{II.3-5}$$

whence

$$\mu = \frac{-\Delta \operatorname{Re} \{\lambda_{j}\}}{\left\| {}^{a} \operatorname{G}^{a} \right\|^{2}}$$
(II.3-6)

and so

$${}^{\boldsymbol{\mu}} \Delta \boldsymbol{K}^{\boldsymbol{\mu}} = \frac{-\Delta \operatorname{Re} \left\{ \lambda_{\underline{i}} \right\}}{\left\| {}^{\boldsymbol{\mu}} {}_{\mathrm{G}} {}^{\boldsymbol{\mu}} \right\|^{2}} {}^{\boldsymbol{\mu}} \boldsymbol{G}^{\boldsymbol{\mu}} \qquad (\text{II.3-7})$$

When the variation $\Delta \operatorname{Re} \{\lambda_{i}\}$ is sufficiently small the system corresponding to the feedback gains $({}^{0}K^{0} + {}^{0}\Delta K^{0})$ should have an eigenvalue whose real part approximates $\operatorname{Re} \{\lambda_{i}\} + \Delta \operatorname{Re} \{\lambda_{i}\}$. It remains to determine suitable values for the step size $\Delta \operatorname{Re} \{\lambda_{j}\}$. It is assumed that in most cases a user of GRADGN will have little idea of what comprises a suitable step size. The step size is therefore varied adaptively based on experience with previous iterations. The initial step size may either be assigned by the user or given a default value of -0.1. In either case the initial step size is used as a program criterion for termination due to diminishing returns. Should two successive iterations of the program fail to improve the stability by the initial step size, the program is terminated. It should be noted that apart from its use as a termination control the program is quite insensitive to the initial setp size, due to the rapidity of the step size adaptation.

The adaptation control for a given iterative step is described by Figure II.3-1. One of two basic step size adjustments is utilized, depending on the convexity of the trajectory of the least stable eigenvalue. If the trajectory is sufficiently convex then a quadratic curve is fitted to two points on the trajectory and the trajectory gradient at one of the points. This procedure is illustrated by Figure II.3-2. The gain vector corresponding to the minimum point of the quadratic curve is computed and tested for stability. If the trajectory is insufficiently convex then the step size is doubled. One or both of these procedures may be used repeatedly during a single iteration. Note that only one gradient vector is computed per iteration.

The initiation point for the next iteration is always the most stable gain vector found. The starting step size for the next iteration is set to one half of the current improvement in the least stable eigenvalue. This allows the step size to be changed by several orders of the magnitude in either direction during a single iteration, while allowing for the decreasing rate of improvement as a stability maximum is approached.



 G_{γ} = gradient vector at computation point 1

$$\Delta \lambda_{i}$$
 = step size from computation point 1 to computation point i

FIGURE II.3-1 Step Size Adaptation Control for the Gain Initialization Program GRADGN



- $\Delta \lambda_{i}$ = step size from computation point 1 to computation point i
 - λ_{i} = real part of least stable eigenvalue at computation point i

Quadratic curve fit to variation of λ with $\Delta\,\lambda$ gives

$$\lambda = a(\Delta \lambda)^2 + b\Delta \lambda + c$$

and

.

$$c = \lambda_{1}$$

$$b = \frac{\lambda_{\lambda}}{\lambda_{\Delta}}\Big|_{1} = 1$$

$$a = \frac{\lambda_{2} - b\Delta\lambda_{2} - c}{(\Delta\lambda_{2})^{2}}$$

Curve has minimum point at $\Delta \lambda_3$ where $\frac{\delta \lambda}{\delta \Delta \lambda} \Big|_3 = 0$ $\therefore \Delta \lambda_3 = -\frac{b}{2a} = -\frac{(\Delta \lambda_2)^2}{2(\lambda_2 - \lambda_1 - \Delta \lambda_2)}$

The computer program GRADGN incorporates an option for the degree of stability required for termination. Setting the input parameter ISTOP to zero instructs the program to maximize stability. Termination is then initiated by reaching a point of diminishing returns, as mentioned above. If ISTOP is set of 1 a minimum desired degree of stability, STOPR, may be input to the program. The program then terminates when the real part of the least stable eigenvalue becomes less than STOPR.

II.4 Example of the Use of the Gain Initialization Program GRADGN

The use of the gain initialization program GRADGN is illustrated in this Section by output feedback stabilization of a seven state model of the Saturn V booster rocket. The model represents the rocket dynamics at a point occuring 80 seconds after lift-off.

The seven states considered are

$$x_1 =$$
 measured pitch attitude angle ϕ_D
 $x_2 =$ measured pitch rate $\dot{\phi}_R$
 $x_3 =$ aerodynamic angle of attack α
 $x_4 =$ first bending mode deflection η_1
 $x_5 =$ first bending mode deflection rate $\dot{\eta}_1$
 $x_6 =$ engine gimbal angle β
 $x_7 =$ engine gimbal angle rate $\dot{\beta}$

It is assumed that only the first two states x_1 and x_2 are to be measured. Thus only these two states are available for feedback. Control is achieved via an actuator driving the engine gimbal angle at a rate proportional to the actuator input. The model may be represented by

$$\underline{\underline{x}}(t) = A \underline{x}(t) + \underline{b} \underline{u}(t) \qquad (II.4-1)$$

with

$$\underline{u}(t) = -k_1 x_1(t) - k_2 x_2(t)$$
 (II.4-2)

where

$$A = \begin{bmatrix} 0. & 1. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & .2030 & -.6535 & -.0020 & 2.558 & 0. \\ -.0137 & 1. & -.0407 & .0002 & -.0146 & -.0334 & 0. \\ 0. & 0. & 0. & 0. & 1. & 0. & 0. \\ 0. & 0. & 0. & 0. & 1. & 0. & 0. \\ 0. & 0. & 0. & -.44.67 & -.1337 & 254.6 & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 1. \\ 0. & 0. & 0. & 0. & 0. & -.50. & -10. \end{bmatrix}$$

$$b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$
 (II.4-5)

The derivation of this model is described by Cassidy.²⁰

The program GRADGN was used to determine the feedback gains k_1 and k_2 corresponding to a local stability maximum of the above system. The program output is given in Figure II.4-1.



Figure II.4-1 Computer Print-Out For Example Problem

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FIGURE II.4-1 (Cont'd)

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FIGURE II.4-1 (Cont'd)

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-0.169915E 00 -0.113802E 00 -0.113802E 00		01223355 002655594 00			
	0.000000E	00 15 15			
MAXINUM REAL PART OF 0.6210702E-01	(ROOTS)				

FIGURE II.4-1 (Cont'd)

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GAIN SEARCH PROCEEDS ALONG A NEW GRADIENT ITERATION NUMBER 2
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SW1=0+1207E-02 ITER= 5 DIF=0.000DE 00
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LEIGENVELIUKS LURKES TU MRP EIGENVALUE
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D_{1} D_{2} D_{2
-0-1416449E-01 0.000000E 00 0.98801935 00 0.0000000E 00
-0.1529490E-02 0.0000000E 00 -0.6137960E-01 0.0000000E 00
-0.1885111E-01 0.0000000E 00 0.1733184E 00 0.0000000E 00
-0.1896896E-02. 0.0000000E 00 -0.1076721E-01 0.0000000E 00
GRADIENT MATRIX
A DIDELAN AND AND AND AND AND AND AND AND AND A
Verterit increase of the size and the size of the size
GATN MATRIX K
0.4054773E 03 ~0.4449835E -02
en e
ROOTS REAL PART IMAG PART
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-0.679102E 01
9.13/404E 01 0.644 [84E 0]
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BOOTS REAL PART -0.483643E 01 <
BOOTS REAL PART -0.483643E 01 -0.549917E 01 -0.307465E-01 0.612206E -0.612206E 01 -0.457808E.00 00.512206E -0.307465E-01 -0.612206E -0.307465E-01 -0.612206E -0.307465E-01 -0.612206E -0.512206E 01 -0.512206E 01
ROOTS REAL PART JMAG. PART -0.483643E 01 0.549917E 01 -0.483643E 01 -0.549917E 01 -0.307465E-01 0.612206E 01 -0.307465E-01 0.612206E 01 -0.307465E-01 0.612206E 01 -0.307465E-01 0.612206E 01 -0.307465E-01 -0.612206E 01 -0.307465E-01 -0.395643E 00
ROOTS REAL PART JMAG. PART -0.483643E 01 0.549917E 01 -0.483643E 01 -0.549917E 01 -0.307465E-01 0.612206E 01 -0.307465E-01 -0.612206E 01 -0.307465E-01 -0.612206E 01 -0.157808E 00 -0.395643E 00 -0.124568E 00 -0.395643E 00 -0.124568E 00 -0.395643E 00
ROOTS REAL PART IMAG. PART -0.483643E 01 0.549917E 01 -0.483643E 01 -0.549917E 01 -0.307465E-01 0.612206E 01 -0.307465E-01 0.395643E 00 -0.157808E 00 -0.395643E 00 -0.124568E 00 -0.395643E 00 -0.124568E 00 0.900000E 00

FIGURE II.4-1 (Cont'd)


FIGURE II.4-1 (Cont'd)



FIGURE II.4-1 (Cont'd)

STEP SIZE TOO LARGE RETURN TO PREVIOUS BEST GAINS AINEMATRIX K SIL 1011772309010231
 DTS
 REAL PART
 IMAG. PART

 0.480912E.014
 0.0564784E.01
 0.460912E.01

 0.80912E.01
 0.564784E.01
 0.460912E.01

 0.80912E.01
 0.612440E.01
 0.4602440E.01

 0.870506E-01
 -0.612440E.01
 0.4658613E.00

 -0.153032E.00
 -0.1658613E.00
 -0.1538613E.00

 -0.153032E.00
 -0.1658613E.00
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STABILLITY IMPROVEMENT RATE TOOTSUON. PROGRAM TERMINATED. PLAST RESULTS SHOW MOST STARLES CONDITION/ FRAMES COMPLIE TIMES 22.97 SIGTEX CONTONE TIMES AT 19 CEACHIES

2

FIGURE HI 4-1 G (Cont'd)

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In order to verify that GRADGN did indeed attain a stability maximum, the eigenvalues of the system described by (II.4-1) to (II.4-4) were obtained for a matrix of feedback gains. The values of the real parts of the least stable eigenvalues were cross-plotted against the feedback gains, resulting in the equi-stability contours shown in Figure II.4-2. The gain trajectory produced by GRADGN is superimposed on Figure II.4-2. The steepest descent nature of GRADGN, and the fact that a stability maximum was achieved (within the limits of the diminishing returns termination) are evident from Figure II.4-2.

II.5 Comments on Gain Initialization

The digital computer program GRADGN clearly provides a way to improve the stability of a system by output feedback. As such it is a useful adjunct to the output feedback optimization techniques of Cassidy and Levine. It must be remembered, however, that GRADGN is designed to find purely local stability maxima. Thus it may on occasion fail to find feedback gains giving sufficient stability for initialization of the optimization algorithms, even though such gains actually exist. In an attempt to ameliorate this problem, provision has been included in GRADGN for initializing its gain search at any desired location. Thus any desired volume of gain space may be searched by initializing GRADGN at a suitable number of discrete points.

Since GRADGN is designed to perform a similar function to Koenigsberg's²¹ algorithm, a comparison of the two technique's may be pertinent. Both are gradient techniques and both use adaptive step size variation, although Koenigsberg's adaptation procedure is much simpler than that used in GRADGN. A major difference occurs in the computation of the gradient. Koenigsberg's

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gradient computation is based on Reddy's 30 expression

$$\frac{\partial \lambda_{k}}{\partial \alpha_{p}} = \frac{\operatorname{Tr} \left(\operatorname{conjoint} \left[A\right] \left[\frac{\partial A}{\partial \alpha_{p}}\right]\right)}{\operatorname{Tr} \left(\operatorname{conjoint} \left[A\right]\right)} \right|_{\lambda = \lambda_{k}}$$
(II.5-1)

for the sensitivity of the eigenvalue λ_k of the matrix A to the matrix parameter α_p . The conjoint is defined by

conjoint
$$[A] = adjoint [A - \lambda I]$$
 (II.5-2)

The equivalent expression used by GRADGN is (from (II.2-5))

$$\frac{\partial \lambda_{k}}{\partial \alpha_{p}} = \frac{\underline{v}^{\mathrm{T}} \frac{\partial A}{\partial \alpha_{p}} \underline{w}}{\underline{v}^{\mathrm{T}} \underline{w}}$$
(II.5-3)

where <u>v</u> and <u>w</u> respectively are the row and column eigenvectors of the matrix A corresponding to the eigenvalue λ_k . Equation (II.5-3) may be expanded to

$$\frac{\partial \lambda_{k}}{\partial \alpha_{p}} = \frac{\sum_{i=1}^{NS} \sum_{j=1}^{NS} v_{i} \left[\frac{\partial A}{\partial \alpha_{p}}\right] w_{j}}{\sum_{i=1}^{NS} \sum_{j=1}^{NS} v_{i} w_{j}}$$
(II.5-4)

which may be reordered to

$$\frac{\partial \lambda_{k}}{\partial \alpha_{p}} = \frac{\sum_{j=1, \dots, i=1}^{NS} \sum_{i=1, \dots, i=1}^{NS} w_{j} v_{i} \left[\frac{\partial A}{\partial \alpha_{p}}\right]_{i,j}}{\sum_{j=1, \dots, i=1}^{NS} \sum_{i=1, \dots, i=1}^{NS} w_{j} v_{i}}$$
(II.5-5)

Inspection of (II.5-5) shows it to be the same as

$$\frac{\boldsymbol{\delta} \boldsymbol{\lambda}_{k}}{\boldsymbol{\delta} \boldsymbol{\alpha}_{p}} = \frac{\operatorname{tr} \left[\underline{\boldsymbol{w}} \ \underline{\boldsymbol{v}}^{\mathrm{T}} \ \frac{\boldsymbol{\delta} \boldsymbol{A}}{\boldsymbol{\delta} \boldsymbol{\alpha}_{p}} \right]}{\operatorname{tr} \left[\underline{\boldsymbol{w}} \ \underline{\boldsymbol{v}}^{\mathrm{T}} \right]} \tag{II,5-6}$$

Now the determination of a column eigenvector of a matrix may be achieved by the computation of only one column of the conjoint. Van Ness'²⁹ EIGVEC, used for eigenvector computation in GRADGN, gives both row and column eigenvectors with about the same computational effort as is required for only a column eigenvector. Thus (II.5-6) is computationally more efficient than (II.5-1). However (II.5-6) is simply (II.5-3) with each vector inner product replaced by the trace of a vector outer product, so that (II.5-3) is clearly preferable to (II.5-6) especially for systems of high order.

CHAPTER III

THE FINITE TIME INTERVAL PROBLEM

III.l Introduction

In this Chapter we are concerned with the determination of the optimal time invariant output feedback controller for the system

$$\underline{\underline{x}}(t) = A \underline{x}(t) + B \underline{u}(t)$$
 (III.1.1)

where $\underline{x}(t)$ is a NS vector

 $\underline{u}(t)$ is a NC vector

with the cost functional

$$J = \underline{x}^{T}(T) F \underline{x}(T) + \int_{0}^{T} \underline{x}^{T}(\tau) Q \underline{x}(\tau) + \underline{u}^{T}(\tau) R u(\tau) d\tau \qquad (III.1_{\tau}2)$$

where T is a fixed finite time and F, Q, R are suitably positive (semi) definite. Thus we require

$$\underline{u}(t) = -\underline{K}^{T} \underline{x}(t)$$
 (III,1-3)

and constrain K to be of the form

$$K = \begin{bmatrix} k_{ij} \\ 0 \end{bmatrix}$$
 NF
$$MF$$
 (III.1-4)

Two items set this problem apart from the normal linear-quadratic state regulator problem. These are the requirement for output feedback control and the requirement for the time-invariant feedback gains. It is remarkable that each of these requirements has been individually satisfied by different investigators, but by the use of similar techniques. Levine¹⁹ used the Initial State Averaging (I.S.A.) technique to determine the optimal time-variable output feedback controller for the system (III.1-1) with the cost functional (III.1-2). On the other hand Kleinman, Fortmann and Athans²⁵ used I.S.A. to determine the optimal time-invariant allstate feedback controller. This report considers the use of I.S.A. to solve both requirements simultaneously.

The theory underlying the I.S.A. approach is presented in Section III.2

III.2 Theory of the Initial State Averaging Technique

The solution to the problem posed by (III.1-1) to (III.1-3) is underdefined in the sense that the optimal controller is a function of the undefined initial condition $\underline{x}(0)$. In general each different initial condition requires a different set of feedback gains. Since no single controller can minimize the cost functional for all initial conditions it seems wise to seek that controller which minimizes the expected value of the cost functional. This is the rationale behind I.S.A.

From (III.1-1) and (III.1-3) we see that

$$\dot{\mathbf{x}}(t) = \hat{\mathbf{A}} \mathbf{x}(t)$$
 (III.2-1)

where

$$\mathbf{\hat{A}} = \begin{bmatrix} \mathbf{A} - \mathbf{B}\mathbf{K}^{\mathrm{T}} \end{bmatrix}$$
(III,2-1)

Thus for a given initial condition $\underline{x}(0)$

$$\underline{\mathbf{x}}(t) = \phi(t) \underline{\mathbf{x}}(0) \tag{III.2-3}$$

where $\phi(t)$ is the state transition matrix of \hat{A} and is defined by

$$\dot{\phi}(t) = \hat{A} \phi(t) \qquad (III, 2-4)$$

$$\phi(0) = \mathbf{I} \tag{III.2-5}$$

Since we are dealing only with linear time-invariant systems we have

$$\phi(t) = \exp \left[At \right]$$
 (III.2-6)

Repeating (III.1-2)

$$J = \underline{x}^{T}(T) F \underline{x}(T) + \int_{0}^{T} \underline{x}^{T}(\tau) Q \underline{x}(\tau) + \underline{u}^{T}(\tau) R \underline{u}(\tau) d\tau \qquad (III.2-7)$$

Using (III.1-3) reduces this to

$$J = \underline{x}^{T}(T) F \underline{x}(T) + \int_{0}^{T} x^{T}(\tau) \left[Q + KRK^{T} \right] \underline{x}(\tau) d\tau \qquad (III.2-8)$$

whence, by (III.2-3)

.

$$J = \underline{x}(0)^{\mathrm{T}} \not \otimes^{\mathrm{T}}(\mathrm{T}) F \not \otimes(\mathrm{T}) \underline{x}(0) + \int_{0}^{\mathrm{T}} \underline{x}^{\mathrm{T}}(0) \not \otimes^{\mathrm{T}}(\boldsymbol{\tau}) \left[Q + \mathrm{KRK}^{\mathrm{T}} \right]$$
$$\not \otimes(\boldsymbol{\tau}) \underline{x}(0) d\boldsymbol{\tau} \qquad (\mathrm{III.2-9})$$

Moving the time invariant $\underline{x}(0)$ outside the integration reduces (III.2-9) to

$$J = \underline{\mathbf{x}}(0)^{\mathrm{T}} \left[\phi^{\mathrm{T}}(\mathrm{T}) + \phi(\mathrm{T}) + \int_{0}^{\mathrm{T}} \phi^{\mathrm{T}}(\boldsymbol{\tau}) \left[\mathrm{Q} + \mathrm{KRK}^{\mathrm{T}} \right] \phi(\boldsymbol{\tau}) \, \mathrm{d}\boldsymbol{\tau} \right] \underline{\mathbf{x}}(0) \quad (\mathrm{III.2-10})$$

which may be rewritten as

$$J = \underline{x}^{\mathrm{T}}(0) P \underline{x}(0) \qquad (\text{III.2-11})$$

where

$$P = \phi^{T}(T) F \phi(T) + \int_{0}^{T} \phi^{T}(\tau) \left[Q + KRK^{T} \right] \phi(\tau) d\tau \qquad (III.2-12)$$

Thus

$$\mathbf{E}[\mathbf{J}] = \mathbf{E}\left[\mathbf{x}^{\mathrm{T}}(\mathbf{0}) \ \mathbf{P} \ \mathbf{x} \ (\mathbf{0})\right]$$
(III,2-13)

i.e.,

$$\mathbb{E}[J] = \mathbb{E}\left[\sum_{i=1}^{NS} \sum_{j=1}^{NS} x_i(0) \mathbb{P}_{ij} x_j(0)\right]$$
(III.2-14)

Now consider the $x_i(0)$, i = 1, ..., NS to be random variable whose distributions are dependent on the distribution of the initial conditions in state space. Then E[J] is given by (III.2-14) as the expected value of a finite sum of random variables. But the expected value of a finite sum of random variables is equal to the sum of the expected values. Thus

$$\mathbb{E}[J] = \sum_{i=1}^{NS} \sum_{j=1}^{NS} \mathbb{E}\left[\mathbb{x}_{i}(0) \mathbb{P}_{ij} \mathbb{x}_{j}(0)\right] \qquad (\text{III.2-15})$$

Rearranging (III.2-15) gives

$$\mathbb{E}[\mathbf{J}] = \sum_{i=1}^{NS} \sum_{j=1}^{NS} \mathbb{E}\left[\mathbb{P}_{ij} \mathbf{x}_{i}(0) \mathbf{x}_{j}(0)\right] \qquad (III.2-16)$$

It is clear from (III.2-12) that the P_{ij} in (III.2-16) do not depend on the $x_i(0)$ or the $x_j(0)$. Thus (III.2-16) can be rewritten as

$$E[J] = \sum_{i=1}^{NS} \sum_{j=1}^{NS} P_{ij} E[x_j(0) x_i(0)] \qquad (III.2-17)$$

Suppose now that each of the $x_i(0)$ has zero mean and that we can estimate the covariance matrix V of the initial conditions in state space, i.e.,

$$V_{ij} = E\left[x_i(0) \ x_j(0)\right]$$
(III.2-18)

Then, from (III.2-17) and (III.2-18)

$$E[J] = \sum_{i=1}^{NS} \sum_{j=1}^{NS} P_{ij} V_{ij} \qquad (IEI.2-19)$$

whence

$$E[J] = Tr [PV]$$
(III.2-20)

Now consider that the system described by (III.1-1), (III.1-3) and (III.1-4) is linear. Thus superposition holds and the system response can be scaled up or down with the magnitude of the initial conditions. This implies that the response of the system to an initial condition lying on the surface of the unit hypersphere in state space characterizes the response to any colinear initial condition state vector. Furthermore it is clear from (III.1-2) that if any initial condition is ratioed by an amount r then the cost is ratioed by r^2 . These two facts imply that the costs associated with the set of initial conditions lying on the surface of the unit hypersphere is state space can be used to conveniently characterize the costs for all initial conditions. It is shown below that with little loss of generality attention may be confined to a uniform distribution of the initial conditions on the surface of the unit hypersphere.

Suppose that the covariance matrix ∇ is non-singular with inverse v^{-1} . Suppose also that the square root of v^{-1} exists, and let it be denoted by $v^{-1/2}$. Consider the description of the system (III.2-1) in terms of a new set of state variables $\underline{y}(t)$ defined by

$$\underline{y}(t) = (NS)^{-1/2} v^{-1/2} \underline{x}(t)$$
 (III.2-21)

Then (III.2-1) becomes

$$(NS)^{1/2} y^{1/2} \underline{y}(t) = A(NS)^{1/2} y^{1/2} \underline{y}(t)$$
 (III.2-22)

i.e.,

$$\dot{\mathbf{y}}(t) = \mathbf{v}^{-1/2} \quad \mathbf{\hat{A}} \quad \mathbf{v}^{1/2} \quad \underline{\mathbf{y}}(t) \tag{III.2-23}$$

which is similar in form to (III.2-1).

Also we find that

$$\mathbb{E}\left[\underline{y}(0) \ \underline{y}^{\mathrm{T}}(0)\right] = \mathbb{E}\left[\operatorname{NS}^{-1/2} \ \mathbb{V}^{-1/2} \ \underline{x}(0) \ \underline{x}^{\mathrm{T}}(0) \ (\operatorname{NS})^{-1/2} \ \mathbb{V}^{-1/2}\right] (\operatorname{III}_{i}.2-24)$$

Thus

$$\mathbb{E}\left[\underline{y}(0) \ \underline{y}^{\mathrm{T}}(0)\right]_{ij} = (\mathbb{N}S)^{-1} \mathbb{E}\left[\sum_{p=1}^{\mathbb{N}S} \sum_{q=1}^{\mathbb{N}S} \left[\mathbb{V}^{-1/2}\right]_{ip} \mathbb{x}_{p}(0) \mathbb{x}_{q}(0) \left[\mathbb{V}^{-1/2}\right]_{qj}\right]$$
(III.2-25)

Interchanging finite sum and expectation gives

$$\mathbb{E}\left[\underline{y}(0) \ \underline{y}^{\mathrm{T}}(0)\right]_{ij} = (\mathrm{NS})^{-1} \sum_{p=1}^{\mathrm{NS}} \sum_{q=1}^{\mathrm{NS}} \mathbb{E}\left[\left[\mathrm{V}^{-1/2}\right]_{ip} x_{p}(0) x_{q}(0) \left[\mathrm{V}^{-1/2}\right]_{qj}\right]$$
(III.2-26)

$$= (NS)^{-1} \sum_{p=1}^{NS} \sum_{q=1}^{NS} \left[v^{-1/2} \right]_{ip} v_{pq} \left[v^{-1/2} \right]_{qj} (III.2-27)$$

.

$$=\frac{1}{NS}\left[V^{-1/2} \quad V \quad V^{-1/2}\right]_{ij}$$
(III.2-28)

$$= \frac{1}{NS} \left[I \right]_{ij}$$
(III.2-29)

where I is the identity matrix of order NS.

Thus the covariance matrix of the initial state vector $\underline{y}(0)$ is compatible with a uniform distribution of the $\underline{y}(0)$ vector on the surface of the unit hypersphere. Since such a distribution is convenient and since we can, within the limitations of the non-stringent assumptions made above, transform our initial state vector so that it is compatible with this distribution, we shall hereafter assume that

$$V = \frac{1}{NS} I \qquad (III.2-30)$$

Thus, from (III.2-20) we see that

$$E \left[J\right] = \frac{1}{NS} tr \left[P\right]$$
(III.2-3L)

and our I.S.A. optimal control is that which minimizes $\frac{1}{NS}$ tr [P] where P is given by (III.2-12).

III.3 The Initial State Averaging Algorithm

The objective of the I.S.A. algorithm is to determine that gain matrix K which minimizes the average cost of control as given by (III.2-31) under the constraint that K must be of the form given by (ITI.1-4). Thus we wish to minimize the gain function GF given by

$$GF = \frac{1}{NS} \operatorname{tr} \left[\phi^{\mathrm{T}}(\mathrm{T}) F \phi(\mathrm{T}) + \int_{0}^{\mathrm{T}} \phi^{\mathrm{T}}(\boldsymbol{\mathcal{T}}) \left[Q + KRK^{\mathrm{T}} \right] \phi(\boldsymbol{\mathcal{T}}) \, \mathrm{d}\boldsymbol{\mathcal{T}} \right] \quad (III.3-1)$$

Now for any two matrices A and B of suitable dimensions we have the trace identity

$$tr [AB] = tr [BA]$$
(III.3-2)

Using (III.3-2) and the fact that the integral of a matrix is the matrix of the integrals of the elements reduces (III.3-1) to

$$GF = \frac{1}{NS} \operatorname{tr} \left[F \ \phi(\mathbf{T}) \ \phi^{\mathrm{T}}(\mathbf{T}) + (Q + KRK^{\mathrm{T}}) \int_{0}^{\mathrm{T}} \phi(\mathbf{\mathcal{I}}) \ \phi^{\mathrm{T}}(\mathbf{\mathcal{I}}) \ d\mathbf{\mathcal{I}} \right] \quad (III.3-3)$$

Since the variable elements of the gain matrix K are unconstrained the necessary conditions for K to minimize GF are given by

$$\frac{\partial GF}{\partial k_{ij}} = 0$$
 (III.3-4)
 $i = 1, ..., NF$
 $j = 1, ..., NC$

Expanding (III.3-4) gives the necessary conditions as

$$\operatorname{tr}\left[\operatorname{F}\left[\frac{\partial \phi}{\partial k_{i,j}} \phi^{\mathrm{T}} + \phi \frac{\partial \phi^{\mathrm{T}}}{\partial k_{i,j}}\right]_{\mathrm{T}} + \left[\frac{\partial \kappa}{\partial k_{i,j}} \operatorname{R} \kappa^{\mathrm{T}} + \kappa \operatorname{R} \frac{\partial \kappa^{\mathrm{T}}}{\partial k_{i,j}}\right] \int_{\mathrm{O}}^{\mathrm{T}} \phi \phi^{\mathrm{T}} \, \mathrm{d} \, \boldsymbol{\gamma} + \left[\operatorname{Q} + \kappa \kappa \kappa^{\mathrm{T}}\right] \int_{\mathrm{O}}^{\mathrm{T}} \frac{\partial \phi}{\partial k_{i,j}} \phi^{\mathrm{T}} + \phi \frac{\partial \phi^{\mathrm{T}}}{\partial k_{i,j}} \, \mathrm{d} \, \boldsymbol{\gamma}\right] = 0 \quad (\text{III.3-5})$$

where the arguments of the ϕ functions have been dropped for clarity.

Now for any matrices A and B

$$tr \left[A\right] = tr \left[A^{T}\right]$$
(III.3-6)

anđ

$$tr \left[A + B\right] = tr \left[A\right] + tr \left[B\right]$$
(III.3-7)

Thus in (III.3-5)

$$\operatorname{tr}\left[\operatorname{F}\left[\frac{\partial \phi}{\partial k_{i,j}} \phi^{\mathrm{T}} + \phi \frac{\partial \phi^{\mathrm{T}}}{\partial k_{i,j}}\right]\right] = \operatorname{tr}\left[\operatorname{F}\frac{\partial \phi}{\partial k_{i,j}} \phi^{\mathrm{T}}\right] + \operatorname{tr}\left[\operatorname{F}\phi\frac{\partial \phi^{\mathrm{T}}}{\partial k_{i,j}}\right]$$

$$\operatorname{by}\left(\operatorname{III.3-7}\right)$$

$$= \operatorname{tr}\left[\operatorname{F}\frac{\partial \phi}{\partial k_{i,j}} \phi^{\mathrm{T}}\right] + \operatorname{tr}\left[\frac{\partial \phi}{\partial k_{i,j}} \phi^{\mathrm{T}}\right]$$

$$\operatorname{by}\left(\operatorname{III.3-6}\right)$$

$$= \operatorname{tr} \left[\operatorname{F} \frac{\partial \phi}{\partial k_{ij}} \phi^{\mathrm{T}} \right] + \operatorname{tr} \left[\operatorname{F} \frac{\partial \phi}{\partial k_{ij}} \phi^{\mathrm{T}} \right]$$

by (III.3-2)

$$= 2 \operatorname{tr} \left[\mathbb{F} \frac{\partial \phi}{\partial k_{ij}} \phi^{\mathrm{T}} \right]$$
 (III.3-8)

Similarly

$$\operatorname{tr}\left[\left[\frac{\boldsymbol{\partial}_{K}}{\boldsymbol{\partial}_{k}_{\mathbf{i}\mathbf{j}}} \operatorname{RK}^{\mathrm{T}} + \operatorname{KR}\frac{\boldsymbol{\partial}_{K}^{\mathrm{T}}}{\boldsymbol{\partial}_{k}_{\mathbf{i}\mathbf{j}}}\right]\int_{0}^{\mathrm{T}} \phi \phi^{\mathrm{T}} \,\mathrm{d}\boldsymbol{\mathcal{Z}}\right] = 2 \operatorname{tr}\left[\frac{\boldsymbol{\partial}_{K}}{\boldsymbol{\partial}_{k}_{\mathbf{i}\mathbf{j}}} \operatorname{RK}^{\mathrm{T}}\int_{0}^{\mathrm{T}} \phi \phi^{\mathrm{T}} \,\mathrm{d}\boldsymbol{\mathcal{Z}}\right]$$
(III.39)

and

•

$$\operatorname{tr}\left[\left[\mathbf{Q} + \operatorname{KRK}^{\mathrm{T}}\right] \int_{0}^{\mathrm{T}} \frac{\partial \phi}{\partial k_{\mathbf{i}\mathbf{j}}} \phi^{\mathrm{T}} + \phi \frac{\partial \phi^{\mathrm{T}}}{\partial k_{\mathbf{i}\mathbf{j}}} \,\mathrm{d}\boldsymbol{\tau}\right] = 2 \operatorname{tr}\left[\left[\mathbf{Q} + \operatorname{KRK}^{\mathrm{T}}\right] \int_{0}^{\mathrm{T}} \phi \frac{\partial \phi^{\mathrm{T}}}{\partial k_{\mathbf{i}\mathbf{j}}} \,\mathrm{d}\boldsymbol{\tau}\right]$$
(III.3-10)

Thus the necessary conditions of (III.3-5) reduce to

$$W = 0 \tag{III.3-11}$$

where the NF x NC matrix W has its elements defined by

$$w_{ij} = \operatorname{tr} \left[F \phi \frac{\partial \phi^{T}}{\partial k_{ij}} \right|_{T} + \left[Q + KRK^{T} \right] \int_{O}^{T} \phi \frac{\partial \phi^{T}}{\partial k_{ij}} d\mathcal{T} + \frac{\partial K}{\partial k_{ij}} RK^{T} \int_{O}^{T} \phi \phi^{T} d\mathcal{T} \right]$$
(III.3-12)

Equations (III.3-11) and (III.3-12) represent a set of NF x NC simultaneous equations in the NF x NC variable elements of the gain matrix
K. Solution of these equations gives candidates for the otpimal controller.
The solution is performed by Newton-Raphson iteration.

Consider the rearrangement of W into an NF x NC vector ${}^{B}W^{a}$ by column ordering

i.e.,
$$w_{ij} = {}^{a} W^{a}_{NF(j-1)+i}$$
 (III.3-13)

The variable elements of K may be similarly ordered, giving the gain vector ${}^{B}K{}^{B}$ where

$$K_{ij} = {}^{n} K^{n}_{NF(j-1)+i}$$
(III.3-14)

Suppose that ${}^{B}K(n)^{B}$ is the gain vector at the n^{th} iteration of the Newton-Raphson procedure, and that ${}^{B}W(n)^{B}$ is the corresponding

vector of necessary conditions. The Newton-Raphson technique computes ${}^{\boldsymbol{a}}_{K(n+1)}{}^{\boldsymbol{a}}$, an improved estimate of a gain vector satisfying the necessary conditions, by

$${}^{\boldsymbol{a}}_{K(n+1)} = {}^{\boldsymbol{a}}_{K(n)} - \mu \left[\nabla(n) \right]^{-1} {}^{\boldsymbol{a}}_{W(n)}$$
(III.3-15)

where $\nabla(n)$ is the (NF x NC) x (NF x NC) gradient matrix given by

$$\nabla(n)_{ij} = \frac{\partial^{\mathbf{n}} W(n)^{\mathbf{n}}_{j}}{\partial^{\mathbf{n}} K(n)^{\mathbf{n}}_{j}}$$
(III.3-16)

and $\boldsymbol{\mu}$ is a convergence factor .

It can be seen from (III.3-13) and (III.3-14) that computation of ∇ (n) is equivalent to computation of $\frac{\partial w_{gh}}{\partial k_{pq}}$ for g = 1, ..., NF, h = 1, ..., NC; p = 1, ..., NF; q = 1, ..., NC. From (III.3-12) we have

$$\frac{\partial w_{gh}}{\partial k_{pq}} = tr \left[F \left[\frac{\partial \phi}{\partial k_{pq}} - \frac{\partial \phi^{T}}{\partial k_{gh}} + \phi \frac{\partial^{2} \phi^{T}}{\partial k_{pq} \partial k_{gh}} \right]_{T} + \left[Q + KRK^{T} \right] \int_{0}^{T} \frac{\partial \phi}{\partial k_{pq}} - \frac{\partial \phi^{T}}{\partial k_{pq}} + \phi \frac{\partial^{2} \phi^{T}}{\partial k_{pq} \partial k_{gh}} - d\gamma + \left[\frac{\partial K}{\partial k_{pq}} - RK^{T} + KR \frac{\partial K^{T}}{\partial k_{pq}} \right] \int_{0}^{T} \phi \frac{\partial \phi^{T}}{\partial k_{gh}} d\gamma + \frac{\partial^{2} K}{\partial k_{pq} \partial k_{gh}} RK^{T} - \int_{0}^{T} \phi \phi^{T} d\gamma + \frac{\partial K}{\partial k_{pq} \partial k_{gh}} RK^{T} - \int_{0}^{T} \phi \phi^{T} d\gamma \right]$$

(III.3-17)

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Clearly

$$\frac{\partial^2 k}{\partial k_{pq} \partial k_{gh}} = 0 \qquad (III.3-18)$$

Also, by (III.3-6) and (III.3-2)

$$\operatorname{tr}\left[\operatorname{KR}\frac{\partial \operatorname{K}^{\mathrm{T}}}{\partial \operatorname{k}_{\mathrm{pq}}}\int_{0}^{\mathrm{T}} \phi \frac{\partial \phi^{\mathrm{T}}}{\partial \operatorname{k}_{\mathrm{gh}}} \,\mathrm{d}\tau\right] = \operatorname{tr}\left[\int_{0}^{\mathrm{T}}\frac{\partial \phi}{\partial \operatorname{k}_{\mathrm{gh}}} \phi^{\mathrm{T}} \,\mathrm{d}\tau \frac{\partial \operatorname{K}}{\partial \operatorname{k}_{\mathrm{pq}}} \,\operatorname{RK}^{\mathrm{T}}\right]$$
$$= \operatorname{tr}\left[\frac{\partial \operatorname{K}}{\partial \operatorname{k}_{\mathrm{pq}}} \,\operatorname{RK}^{\mathrm{T}} \int_{0}^{\mathrm{T}}\frac{\partial \phi}{\partial \operatorname{k}_{\mathrm{gh}}} \phi^{\mathrm{T}} \,\mathrm{d}\tau\right] (\operatorname{III.3-19})$$

so that (III.3-17) reduces to

$$\frac{\partial W_{gh}}{\partial K_{pq}} = \operatorname{tr} \left[\operatorname{F} \left[\frac{\partial \phi}{\partial k_{pq}} - \frac{\partial \phi^{\mathrm{T}}}{\partial k_{gh}} + \phi - \frac{\partial^{2} \phi^{\mathrm{T}}}{\partial k_{pq} \partial k_{gh}} \right]_{\mathrm{T}} \right] + \left[\operatorname{Q} + \operatorname{KRK}^{\mathrm{T}} \right] \int_{0}^{\mathrm{T}} \frac{\partial \phi}{\partial k_{pq}} - \frac{\partial \phi^{\mathrm{T}}}{\partial k_{gh}} + \phi - \frac{\partial^{2} \phi^{\mathrm{T}}}{\partial k_{pq} \partial k_{gh}} - \operatorname{d} \mathcal{T} \right] + \frac{\partial K}{\partial k_{pq}} \operatorname{RK}^{\mathrm{T}} \int_{0}^{\mathrm{T}} \frac{\partial \phi}{\partial k_{gh}} \phi^{\mathrm{T}} + \phi - \frac{\partial \phi^{\mathrm{T}}}{\partial k_{gh}} - \operatorname{d} \mathcal{T} \right] + \frac{\partial K}{\partial k_{gh}} \operatorname{RK}^{\mathrm{T}} \int_{0}^{\mathrm{T}} \frac{\partial \phi}{\partial k_{gh}} \phi^{\mathrm{T}} + \phi - \frac{\partial \phi^{\mathrm{T}}}{\partial k_{gh}} - \operatorname{d} \mathcal{T} \right] + \frac{\partial K}{\partial k_{gh}} \operatorname{RK}^{\mathrm{T}} \int_{0}^{\mathrm{T}} \frac{\partial \phi}{\partial k_{pq}} \phi^{\mathrm{T}} + \phi - \frac{\partial \phi^{\mathrm{T}}}{\partial k_{pq}} - \operatorname{d} \mathcal{T} + \frac{\partial K}{\partial k_{gh}} - \operatorname{RK}^{\mathrm{T}} - \frac{\int_{0}^{\mathrm{T}} \partial \phi}{\partial k_{pq}} - \frac{\partial \phi^{\mathrm{T}}}{\partial k_{pq}} - \operatorname{d} \mathcal{T} + \frac{\partial K}{\partial k_{gh}} - \operatorname{RK}^{\mathrm{T}} - \frac{\partial \phi}{\partial k_{pq}} - \frac{\partial \phi^{\mathrm{T}}}{\partial k_{pq}} - \frac{\partial \phi^{\mathrm{T}}}{\partial k_{pq}} - \operatorname{d} \mathcal{T} + \frac{\partial K}{\partial k_{gh}} - \operatorname{RK}^{\mathrm{T}} - \frac{\partial \phi}{\partial k_{pq}} - \frac{\partial \phi}{\partial k_{pq}} - \frac{\partial \phi^{\mathrm{T}}}{\partial k_{pq}} - \frac{\partial$$

From (III.3-3), (III.3-12) and (III.3-20) it can be seen that computation of the cost GF, the necessary conditions vector ${}^{a}W{}^{a}$ and the gradient matrix ∇ for a given matrix K is dependent on the computation

of a number of functions of the state transition matrix $\phi(t)$. Basic to evaluation of these functions is evaluation of $\frac{\partial \phi(t)}{\partial k_{pq}}$ and $\frac{\partial^2 \phi(t)}{\partial k_{pq}}$ for p=1,...,NF; q=1,...,NC; g=1,...,NF; h=1,...,NC.

Consider the state transition matrix $\mathscr{P}_{\widehat{K}}(t)$ corresponding to a gain matrix \widehat{K} given by

$$\hat{\mathbf{K}} = \mathbf{K}_{0} + \frac{\mathbf{\delta}\mathbf{K}}{\mathbf{\delta}\mathbf{k}_{pq}} \quad \mathbf{\delta}\mathbf{k}_{pq} + \frac{\mathbf{\delta}\mathbf{K}}{\mathbf{\delta}\mathbf{k}_{gh}} \quad \mathbf{\delta}\mathbf{k}_{gh}$$
(III.3-21)

where δk_{pq} and δk_{gh} are small. By (III.2-4) $\phi_{\tilde{K}}(t)$ can be computed as the solution to

$$\frac{d \phi_{\tilde{K}}(t)}{dt} = \left[\tilde{A} - BK_{0}^{T} - B(\frac{\partial K^{T}}{\partial k_{pq}} \delta k_{pq} + \frac{\partial K^{T}}{\partial k_{gh}} \delta k_{gh}) \right] \phi_{\tilde{K}}(t) \quad (III.3-22)$$

with

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$$\phi_{\hat{K}}(0) = I$$
 (III.3-23)

Thus

$$\frac{d \phi_{\hat{K}}(t)}{dt} = \left[A - BK_0^T\right] \phi_{\hat{K}}(t) - B\left[\frac{\delta K^T}{\delta k_{pq}} \quad \delta k_{pq} + \frac{\delta K^T}{\delta k_{gh}} \quad \delta k_{gh}\right] \phi_{\hat{K}}(t)$$
(III.3-24)

Treating the second term in (III.3-24) as an external input and using (III.2-6)

$$\phi_{\tilde{K}}(t) = \exp\left[\left[A - BK_{0}^{T}\right]t\right] - \int_{0}^{T} \exp\left[\left[A - BK_{0}^{T}\right](t-\tau)\right]B$$

$$\left[\frac{\partial K^{T}}{\partial k_{pq}} \delta_{k_{pq}} + \frac{\partial K^{T}}{\partial k_{gh}} \delta_{k_{gh}}\right] \phi_{\tilde{K}}(\tau) d\tau \qquad (III.3-25)$$

We now substitute the function described by (III.3-25) for $\oint_{\tilde{K}}(\boldsymbol{\gamma})$ in (III.3-25). This is similar to the method for computation of the matizant given in DeRusso, Roy and Close¹, and results in

$$\begin{split} \phi_{\tilde{K}}(t) &= \phi_{K_{O}}(t) - \int_{0}^{t} \phi_{K_{O}}(t-\tilde{\tau}) B \left[\frac{\delta K^{T}}{\delta k_{pq}} - \delta k_{pq} + \frac{\delta K^{T}}{\delta k_{gh}} \delta k_{gh} \right] \\ & \left[\phi_{K_{O}}(\tilde{\tau}) - \int_{0}^{\tau} \phi_{K_{O}}(\tilde{\tau}-s) B \left[\frac{\delta K^{T}}{\delta k_{pq}} - \delta k_{pq} + \frac{\delta K^{T}}{\delta k_{gh}} - \delta k_{gh} \right] \phi_{\tilde{K}}(s) ds \right] d\tau \\ & (III.3-26) \end{split}$$

where $\phi_{K_0}(t)$ is the state transition matrix corresponding to the gain matrix K_0 . Thus, to second order in δk_{pq} and δk_{gh} ,

$$\begin{split} \phi_{\widehat{K}}(t) &= \phi_{K_{0}}(t) - \int_{0}^{t} \phi_{K_{0}}(t-\tau) \mathbb{B} \left[\frac{\Im K^{T}}{\Im k_{pq}} \int k_{pq} + \frac{\Im K^{T}}{\Im k_{gh}} \int k_{gh} \right] \\ & \left[\phi_{K_{0}}(\tau) - \int_{0}^{\tau} \phi_{K_{0}}(\tau-s) \mathbb{B} \left[-\frac{\Im K^{T}}{\Im k_{pq}} \int k_{pq} + \frac{\Im K^{T}}{\Im k_{gh}} \int k_{gh} \right] \phi_{K_{0}}(s) \mathbb{d}s \right] \mathbb{d}\tau \\ & (\text{III.3-27}) \end{split}$$

Expansion of (III.3-27) gives

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$$\phi_{\widehat{K}}(t) = \phi_{K_{O}}(t) - \left[\int_{0}^{t} \phi_{K_{O}}(t-\tau) B \frac{\Im K^{T}}{\Im k_{pq}} \phi_{K_{O}}(\tau) d\tau \right] \delta_{k_{pq}}$$

$$- \left[\int_{0}^{t} \phi_{K_{O}}(t-\tau) B \frac{\Im K^{T}}{\Im k_{gh}} \phi_{K_{O}}(\tau) d\tau \right] \delta_{k_{gh}}$$

$$+ \left[\int_{0}^{t} \phi_{K_{0}}(t-\tau) B \frac{\partial \kappa^{T}}{\partial k_{pq}} \int_{0}^{\tau} \phi_{K_{0}}(\tau-s) B \frac{\partial \kappa^{T}}{\partial k_{pq}} \phi_{K_{0}}(s) ds d\tau \right] \delta^{2}k_{pq}$$

$$+ \left[\int_{0}^{t} \phi_{K_{0}}(t-\tau) B \frac{\partial \kappa^{T}}{\partial k_{pq}} \int_{0}^{\tau} \phi_{K_{0}}(\tau-s) B \frac{\partial \kappa^{T}}{\partial k_{gh}} \phi_{K_{0}}(s) ds d\tau \right] \delta k_{pq} \delta k_{gh}$$

$$+ \left[\int_{0}^{t} \phi_{K_{0}}(t-\tau) B \frac{\partial \kappa^{T}}{\partial k_{gh}} \int_{0}^{\tau} \phi_{K_{0}}(\tau-s) B \frac{\partial \kappa^{T}}{\partial k_{pq}} \phi_{K_{0}}(s) ds d\tau \right] \delta k_{pq} \delta k_{gh}$$

$$+ \left[\int_{0}^{t} \phi_{K_{0}}(t-\tau) B \frac{\partial \kappa^{T}}{\partial k_{gh}} \int_{0}^{\tau} \phi_{K_{0}}(\tau-s) B \frac{\partial \kappa^{T}}{\partial k_{pq}} \phi_{K_{0}}(s) ds d\tau \right] \delta k_{pq} \delta k_{gh}$$

$$+ \left[\int_{0}^{t} \phi_{K_{0}}(t-\tau) B \frac{\partial \kappa^{T}}{\partial k_{gh}} \int_{0}^{\tau} \phi_{K_{0}}(\tau-s) B \frac{\partial \kappa^{T}}{\partial k_{gh}} \phi_{K_{0}}(s) ds d\tau \right] \delta^{2}k_{gh}$$

$$(III.3-28)$$

Now consider that $\not 0(t)$ can be expanded in a Taylor's series about K_{\bigcup} giving

$$\begin{split} \phi_{\tilde{K}}(t) &= \phi_{K_{0}}(t) + \frac{\partial \phi(t)}{\partial k_{pq}} \bigg|_{K_{0}} \qquad \delta \quad k_{pq} + \frac{\partial \phi(t)}{\partial k_{gh}} \bigg|_{K_{0}} \qquad \delta \quad k_{gh} \\ &+ \frac{1}{2!} \left[\left. \frac{\partial^{2} \phi(t)}{\partial k_{pq}^{2}} \right|_{K_{0}} \qquad \delta^{2} k_{pq} + \frac{2}{\partial k_{pq}} \frac{\partial^{2} \phi(t)}{\partial k_{pq}} \right|_{K_{0}} \qquad \delta \quad k_{gh} \\ &+ \frac{\partial^{2} \phi(t)}{\partial k_{gh}^{2}} \bigg|_{K_{0}} \qquad \delta^{2} k_{gh} \bigg]. \end{split}$$

+ terms of higher order

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(111.3-29)

Comparison of (III.3-28) and (III.3-29) gives

$$\frac{\partial \phi(\mathbf{t})}{\partial \mathbf{k}_{pq}} = - \int_{0}^{\mathbf{t}} \phi_{K_{0}}(\mathbf{t}-\boldsymbol{\tau}) B \frac{\partial \kappa^{T}}{\partial \mathbf{k}_{pq}} \phi_{K_{0}}(\boldsymbol{\tau}) d\boldsymbol{\tau} \qquad (\text{III.3-30})$$

and

$$\frac{\partial^{2} \hat{\varphi}(t)}{\partial k_{pq} k_{gh}} = \int_{0}^{t} \phi_{K_{0}}(t-\tau) B \frac{\partial K^{T}}{\partial k_{pq}} \int_{0}^{\tau} \phi_{K_{0}}(\tau-s) \frac{\partial K^{T}}{\partial k_{gh}} \phi_{K_{0}}(s) ds d\tau$$

$$+ \int_{0}^{t} \phi_{K_{0}}(t-\tau) B \frac{\partial K^{T}}{\partial k_{gh}} \int_{0}^{\tau} \phi_{K_{0}}(\tau-s) \frac{\partial K^{T}}{\partial k_{pq}} \phi_{K_{0}}(s) ds d\tau$$

$$(III.3-3L)$$

The result given by (III.3-30) agrees with a similar expression presented by Levine and Athans³¹ for the semi-infinite time interval problem.

We note from (III.3-30) and (III.3-31) that

$$\frac{\partial^{2} \phi(t)}{\partial k_{pq} \partial k_{gh}}_{K_{0}} = - \int_{0}^{t} \phi_{K_{0}}(t-\tau) B \left[\frac{\partial K^{T}}{\partial k_{pq}} \frac{\partial \phi(t)}{\partial k_{gh}} \Big|_{K_{0}} + \frac{\partial K^{T}}{\partial k_{gh}} \frac{\partial \phi(t)}{\partial k_{pq}} \Big|_{K_{0}} \right] d\tau$$
(III.3-32)

The functions of the state transition matrix to be evaluated are now seen to be given by

$$\phi \left. \frac{\partial \phi^{\mathrm{T}}}{\partial k_{\mathrm{pq}}} \right|_{\mathrm{T}} = - \phi(\mathrm{T}) \int_{0}^{\mathrm{T}} \phi^{\mathrm{T}}(\boldsymbol{\tau}) \frac{\partial K}{\partial k_{\mathrm{pq}}} B^{\mathrm{T}} \phi^{\mathrm{T}}(\mathrm{T} - \boldsymbol{\tau}) d\boldsymbol{\tau} \qquad (\mathrm{III}.3 - 33)$$

$$\frac{\partial \phi}{\partial k}_{pq} \frac{\partial \phi^{T}}{\partial k}_{gh} \bigg|_{T} = \int_{0}^{T} \phi(T-\tau) B \frac{\partial \kappa^{T}}{\partial k}_{pq} \phi(\tau) d\tau \int_{0}^{T} \phi^{T}(s) \frac{\partial \kappa}{\partial k}_{gh} B^{T} \phi^{T}(T-s) ds$$
(III.3-34)

$$\phi \frac{\mathbf{a}^{2} \phi^{\mathrm{T}}}{\mathbf{a}_{\mathrm{pq}}^{\mathrm{p}} \mathbf{b}_{\mathrm{gh}}^{\mathrm{p}}} \bigg|_{\mathrm{T}} = -\phi(\mathrm{T}) \int_{\mathrm{O}}^{\mathrm{T}} \left[\frac{\mathbf{a} \phi^{\mathrm{T}}(\boldsymbol{\tau})}{\mathbf{a}_{\mathrm{p}}^{\mathrm{k}} \mathbf{p}_{\mathrm{q}}} \frac{\mathbf{a}_{\mathrm{K}}}{\mathbf{a}_{\mathrm{gh}}^{\mathrm{p}}} + \frac{\mathbf{a} \phi^{\mathrm{T}}(\boldsymbol{\tau})}{\mathbf{a}_{\mathrm{k}}^{\mathrm{k}} \mathbf{p}_{\mathrm{q}}} \frac{\mathbf{a}_{\mathrm{K}}}{\mathbf{a}_{\mathrm{p}}^{\mathrm{k}} \mathbf{p}_{\mathrm{q}}} \right] \mathbf{B}^{\mathrm{T}} \phi(\mathrm{T} \cdot \boldsymbol{\tau}) \mathrm{d} \boldsymbol{\tau}$$

$$(\mathrm{III.3-35})$$

$$\int_{0}^{T} \phi \frac{\partial \phi^{T}}{\partial k_{pq}} d\tau = - \int_{0}^{T} \phi(\tau) \int_{0}^{\tau} \phi^{T}(s) \frac{\partial K}{\partial k_{pq}} B^{T} \phi^{T}(\tau-s) ds d\tau \qquad (III.3-36)$$

$$\int_{0}^{T} \frac{\partial \phi}{\partial k_{pq}} \frac{\partial \phi^{T}}{\partial k_{gh}} d\tau = \int_{0}^{T} \int_{0}^{\tau} \phi(\tau - \nu) B \frac{\partial K^{T}}{\partial k_{pq}} \phi(\nu) d\nu$$

$$\int_{0}^{\tau} \phi^{T}(s) \frac{\partial K}{\partial k_{gh}} B^{T} \phi^{T}(\tau - s) ds d\tau$$
(III.3-37)

and

$$\int_{0}^{T} \phi \frac{\boldsymbol{\delta}^{2} \boldsymbol{\phi}^{T}}{\boldsymbol{\delta}_{k}^{k} \boldsymbol{p}_{q}^{k} \boldsymbol{g}_{h}^{k}} d\boldsymbol{\mathcal{T}} = - \int_{0}^{T} \phi(\boldsymbol{\mathcal{T}}) \int_{0}^{\boldsymbol{\mathcal{T}}} \left[\frac{\boldsymbol{\delta} \phi(s)}{\boldsymbol{\delta}_{k}^{k} \boldsymbol{p}_{q}} \frac{\boldsymbol{\delta}_{k}}{\boldsymbol{\delta}_{k}^{k} \boldsymbol{p}_{q}} + \frac{\boldsymbol{\delta} \phi^{T}(s)}{\boldsymbol{\delta}_{k}^{k} \boldsymbol{g}_{h}} \cdot \frac{\boldsymbol{\delta}_{k}}{\boldsymbol{\delta}_{k}^{k} \boldsymbol{p}_{q}} \right] \mathbb{B}^{T} \phi(\boldsymbol{\mathcal{T}} - s) ds d\boldsymbol{\mathcal{T}}$$

$$(III.3-38)$$

Consider (III.3-33) as being representative of (III.3-33) through (III.3-38). Supposing \hat{A} to have distinct eigenvalues we can evaluate $\phi(t)$ by

$$\phi(t) = M \exp \left[\Lambda t \right] M^{-1}$$
 (III.3-39)

where M is the modal matrix of eigenvectors of \hat{A}

 Λ is the diagonal matrix of eigenvalues of \hat{A} Substitution of (III.3-39) in (III.3-33) yields

$$\phi \frac{\partial \phi^{\mathrm{T}}}{\partial k_{\mathrm{pq}}} \bigg|_{\mathrm{T}} = -Me^{A_{\mathrm{T}}} M^{-1} \int_{0}^{\mathrm{T}} \left[M^{-1} \right]^{\mathrm{T}} e^{A_{\mathrm{T}}} M^{\mathrm{T}} \frac{\partial K}{\partial k_{\mathrm{pq}}} B^{\mathrm{T}} \left[M^{-1} \right]^{\mathrm{T}} e^{(\mathrm{T}-2)} M^{\mathrm{T}} d_{\mathrm{T}} \chi$$
(III.3-40)

Now $\frac{\partial K}{\partial k_{pq}}$ is simply an NS x NC matrix having a 1.0 in it pq position and zeros elsewhere. Thus the ij element of $\oint \frac{\partial \phi^{T}}{\partial k_{pq}} \Big|_{T}$ is given by

$$\begin{bmatrix} \phi \cdot \frac{\delta \phi^{\mathrm{T}}}{\delta k_{\mathrm{pq}}} \Big|_{\mathrm{T}} \end{bmatrix}_{\mathrm{ij}} = -\sum_{\substack{n_{1}, n_{2}, n_{3} \\ n_{4}, n_{5} = 1}}^{\mathrm{NS}} M_{\mathrm{in}_{1}} e^{\lambda n_{1} \mathrm{T}} M_{n_{1}n_{2}}^{-1} \int_{0}^{\mathrm{T}} M_{n_{3}n_{2}}^{-1} e^{\lambda n_{3}} M_{\mathrm{pn}_{3}} B_{\mathrm{n}_{4}} q$$

$$M_{n_5 n_4}^{-1} e^{\lambda n_5 (\mathbf{I}-\boldsymbol{\tau})} M_{jn_5} d\boldsymbol{\tau}$$
(III.3-41)

where λ is the nth eigenvalue of A. n_1 Performing the integration yields

$$\left[\phi \frac{\delta \phi^{T}}{\delta k_{pq}} \Big|_{T}\right]_{ij} = -\sum_{\substack{n_{1}, n_{2}, n_{3} \\ n_{1}, n_{5}=1}}^{NS} M_{in_{1}} M_{n_{1}n_{2}}^{-1} M_{n_{3}n_{2}}^{-1} M_{pn_{3}} B_{n_{4}q} M_{n_{5}n_{4}}^{-1} M_{jn_{5}}$$

$$\begin{bmatrix} \inf_{n_{5}=n_{3}}^{\text{iff}} T e^{(\lambda n_{1}+\lambda n_{5})^{T}} + \inf_{n_{5}\neq n_{3}}^{\text{iff}} \frac{e^{(\lambda n_{1}+\lambda n_{3})^{T}} (\lambda n_{1}+\lambda n_{5})^{T}}{\lambda n_{3}-\lambda n_{5}} \end{bmatrix} (\text{III.3-42})$$

Thus (III.3-33) can be evaluated in terms of the eigenvalues and eigenvectors of A. Equations (III.3-34) through (III.3-38) can be similarly evaluated. The terms can then be combined by (III.3-3), (III.3-12) and (III.3-20) to give the average cost GF, the necessary conditions vector ${}^{a}W^{a}$ and the gradient matrix ∇ respectively.

The only other item required for application of the Newton-Raphson iteration technique described by (III.3-15) is a value for the convergence factor μ . Determination of a suitable value of the convergence factor is described in Section III.4.

III.4 Mechanization of the Initial State Averaging Algorithm

The algorithm described above for computation of optimal constant output feedback gains has been mechanized in the digital computer program ISAFT. The program was written in Fortran IV for an IBM 360/50 computer. A listing is given in Appendix II. The program comprises a MAIN and several subprograms. The functions of the major subprograms are described briefly below.

MAIN: performs most input-output functions and acts as the controller for the other subprograms. The MAIN subprogram also computes a suitable value for the Newton-Raphson convergence factor μ (see (III.3-15) and controls program termination. The operation is illustrated by the data flowchart given in Figure III.4-1.

Computation of the convergence factor μ is performed adaptively. A trial step is made with μ set to 1.0. The value of the gain function GF for the new gains is compared with its former value. If a decrease has occurred the program proceeds to the next iteration. Otherwise the value of μ is halved. This process is repeated up to five times. If no improvement in the gain function has resulted by the fifth cycle, i.e., $\mu = 2^{-5}$, the program is terminated due to poor convergence.

Assuming no termination due to poor convergence, normal program termination will occur when the feedback gains approach their final values. The criterion for this termination is that the proportional change in each individual gain during a single iteration should be less than an amount GNSTOP. The parameter GNSTOP is input by the user.





STRAM: computes the eigenvalues and the model matrix of the matrix A. The subprograms VECT, HSBG and ATEIG are used in the eigenvalue determination. The subprograms MSQ and EIGVEC are used to determine the model matrix and its inverse.

FEFN: computes the functions of the state transition matrix described by (III.3-33) through (III.3-38).

GAINFN: computes the value of the gain function GF at the beginning of each iteration - see (III.3-3).

NESCON: computes the necessary conditions vector ${}^{D}W{}^{D}$ - see (III.3-12).

GRADNT: computes the gradient matrix ∇ - see (III.3-20). INVERT: inverts the gradient matrix

NEWRIT: performs one iteration of the Newton-Raphson algorithm see (III.3-15).

GAIN2: computes the value of the gain function GF at the end of each iteration for use in the determination of a suitable convergence factor μ - see (III.3-15).

III.5 Examples of the Use of the Optimization Program ISAFT

Three examples are given to illustrate the use of the digital computer program ISAFT. In keeping with the expository nature of this Chapter the examples are all short and are designed to illustrate different aspects of the computations.

Example 1

This example considered the case where all system states were available for feedback. The system equations were

$$\begin{bmatrix} \dot{\mathbf{x}}_{1}^{(t)} \\ \dot{\mathbf{x}}_{2}^{(t)} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} \mathbf{x}_{1}^{(t)} \\ \mathbf{x}_{2}^{(t)} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \mathbf{u}(t) \quad (\text{III.5-1})$$

and the cost functional was

$$J = \underline{x}^{\mathrm{T}}(1) \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \underline{x}(1) + \int_{0}^{1} \underline{x}^{\mathrm{T}}(\boldsymbol{\tau}) \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \underline{x}(\boldsymbol{\tau}) + 0.1 u^{2}(\boldsymbol{\tau}) d\boldsymbol{\tau} (\text{III.5-2})$$

The control was constrained to be of the form

$$u(t) = -k_1 x_1(t) - k_2 x_2(t)$$
 (III.5-3)

It was assumed that there was no prior knowledge about the system, and the initial feedback gains were set to the default value of zero. The computer print out is given in Figure III.5-1. The optimal feedback gains were computed to be

$$k_1 = 2.30$$
 (III.5-4)
 $k_2 = 3.42$ (III.5-5)

To verify that the values given by (III.5-4) and (III.5-5) were indeed optimal the results were checked by independent means. A matrix of feedback gains in the vicinity of the solution gains was chosen. At each set of gains the system described by (III.5-1) and (III.5-3) was simulated over the optimization time interval, starting from a number of different initial conditions, and the costs were computed numerically using (III.5-2). The average cost was then computed for each set of feedback gains. The initial conditions were chosen to be equally spaced around the unit circle in state space. Such an independent check will hereafter be referred to as an I.C. Cross-plotting the I.C. data resulted in the equal-cost contours shown in Figure III.5-2. The gain trajectory produced by ISAFT is superimposed

TATES CONTROLS	FEEDBACKS	TGATHS Q		ů
SYSTEM MATRIX A 0-0000000	1.0000000	-2.0000000	-3.0000000	
CONTROL NATRIX B	1.000000			
TERMINAL TIME Y =	1-000000			
TERFINAL COST MATRIX 2.0000000	F 0.000000	0.000000	1.0000000	`
STATE WEIGHTING MATRI 1.0000000	X Q Q.0000000	0.0000000	2.0000000	
CONTROL WEIGHTING MAT	RIX R			
	••			
		· · · · · · · · · · · · · · · · · · ·		
		- 	<u></u>	<u> </u>
			·····	
		· · · · · · · · · · · · · · · · · · ·		

FIGURE III.5-1 Computer Print-Out For Example 1

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TTERATION NUMBER 1 GAIN MATRIX 0.000000 0.000000 ----
 SYSTEM EIGENVALUES

 -C.200CC00D 01
 -0.0C00000D 00

 -C.10CCCC0D 01
 -0.0C0000D 00
 TAVERAGE CUST # 0.1227681D 01 . "NECESSARY CONDITIONS VECTOR -C.6159624D-01 -C.9331765D-01 GRACIENT MATRIX C.2239452D 00 -0.9323973D-01 -0.9323973D-01 0.1338320D 00 INVERSE GRADIENT MATRIX 6.2898689 4.3821046 4.3821046 10.5250361 ---- . • · -- · · · · . . . -- -. - -- -. -----• • • • • • • 4. - · . . -- •- -- ------ ----, . . -------

FIGURE III.5-1 (Cont'd)

ITERATION NUMBER 2 NEW GAINS 0.7963600 1.2520928 SYSTEM EIGENVALUES . -C.3438549D 01 -C.0C0C0C0D 00 -C.8131438D C0 -0.0CC00C0D 00 GAIN TOLERANCE ACHIEVED -1.0000000 REQUIRED STOPPING TELERANCE = 0.0100000 AVERAGE COST = 0.1121653D 01 NECESSARY CONDITIONS VECTOR -C.2673182D-01 -C.3301758D-01 GRACIENT MATRIX G.1144905D CO -0.5721144D-01 -0.5721144D-01 0.66735890-01 . INVERSE GRADIENT MATRIX 15-2801933 13-0994265 13-0994265 26-2143374 г, · · · • ------. . · ··· · **.** . -- -. . . . ----------- - - - * ----------. . . -----.

FIGURE III.5-1 (Cont'd)

ITERATION NUMBER	3			
NEW GAINS 1.6373387	2.4677983			
SYSTEM EIGENVALUES -C.4692691D 01 -C.7751C72D CO	-C.OCCOCOD 00 -C.OCCCOCOD 00			
GAIN TOLERANCE A	CHIEVED =	0.5136254		
RECLIRED STOPPING TO	LERANCE =	0.0100000		
AVERAGE COST *	C.1083638D 01			
NECESSARY CONDITIONS -C.8372973D-C2	VEC TOR -0.9586460D-02			
GRACIENT NATRIX C.69093130-01	-0.3735754D-01	-0.3735754D-01	0.3910556D-01	
INVERSÉ GRADIENT MAT 29.5352104	RIX 28.5971025	28,5971025	52.8906197	

FIGURE III.5-1 (Cont'd)

•

ITERATION NUMBER NEW CAINS 2.1621304 3.2142749 SYSTEM EIGENVALUES -C.5450676D C1 -C.0C0C000D 00 -C.7635585D C0, -C.CCCC0C0D 00 GAIN TOLERANCE ACHIEVED = 0.2427197 RECLIRED STOPPING TELERANCE = 0.0100000 AVERAGE COST = C.1C771720 01 NECESSARY CONDITIONS VECTOR -C.13328130-C2 -C.1736675D-02 GRACIENT MATRIX C.5362835D-C1 -0.25760C7D-01 -0.2976007D-01 0.2943540D-01 INVERSE GRADIENT MATRIX 42.4808458 42.9494143 42.9494143 77.3958539 -

FIGURE III.5-1 (Cont'd)

-ITERATION NUMBER **、** • ~ ~ 5 -NEW GAINS • •---_____ 2.2933385 3.4059298 -SYSTEM EIGENVALUES -C.5645432D G1 -C.7604978D C0 -C.OCCCCODD 00 ---C.CCCCCCCD 00 ---------GAIN TOLERANCE ACHIEVED = 0.0572127 REQUIRED STOPPING TOLERANCE = ----- ----- , 0.0100000 -• • • --. . AVERAGE COST = " 0.10769100 01 - -_ _ * NECESSARY CONDITIONS VECTOR -C.4792496D-C4 -0.86927270-04 -----GRACIENT MATRIX C.5059286D-C1 -0.28172380-01 -0.28172380-01 0.27458210-01 ----. INVERSE GRADIENT MATRIX 47-3089873 47-3082380 47.3082380 84.9576547 ------ - - -- - - -----------. -. -----·· · ٠ . -. -.. - -• • --•-----------. . ------. --. - -- -- -- - - ----1.

FIGURE III.5-1 (Cont'd)

62
ITERATION NUMBER
 6

 "NEW CAINS 2.2996607
 3.4155822

 SYSTEM EIGENVALUES --C.5655292D C1
 -0.000000D G0

 -C.5655292D C1
 -0.000000D G0

 -C.7602297D C0
 -C.0C0C000D G0

 EAIN TOLERANCE ACHIEVED =
 0.0028260

 REQUIRED STOPPING TOLERANCE =
 0.0100000

 SOLUTION IS COMPLETE. ABOVE GAINS ARE OPTIMAL

 "AVERAGE CDST =
 0.1076909D 01

 COMPILE TIME=
 28.40 SEC.EXECUTION TIME=
 193.64 SEC.OBJECT CODE=
 64192 BYTES,

•

ī.



FIGURE III.5-2 Verification of the ISAFT results for Example 1

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on Figure III.5-2. It is apparent that ISAFT determined feedback gains giving a minimum average cost.

To provide a comparison for the results produced by ISAFT, the problem defined by (III.5-1) and (III.5-2) was solved for the optimal time varying feedback gains by the normal procedure of Ricatti matrix backwards integration.² The resulting feedback gains are compared in Figure III.5-3. with those produced by ISAFT. The average cost for the optimal time varying gains was computed by numerical integration from a number of initial conditions. The average cost for the time-varying gains was 1.030 compared with 1.077 for the time invariant gains.

Example 2

This example used the same system equations and cost functional as Example 1 - see (III.5-1) and (III.5-2) respectively. The feedback structure however, was constrained to feedback of only the first state i.e.,

$$u(t) = -k_1 x_1(t)$$
 (III.5-6)

A rather inaccurate guess at the optimal feedback gain resulted in the computer print out given in Figure III.5-4. The use of the convergence factor is evident.

Figure III.5-5 compares the results given by ISAFT with those determined by an I.C. It is again clear that ISAFT determined a feedback gain giving a minimum average cost.

The minimum average cost for the single time-invariant feedback was 1.219 compared with 1.077 for allstate time invariant feedback and 1.030 for allstate time varying feedback.

Example 3

This example was chosen to check the effect of complex eigenvalues







and and the second of the seco

ITERATION NUMBER	1
GAIN MATRIX 5.0000000	- • • • • • • • • • • •
SYSTEM EIGENVALUES -0.15000000 01 -0.15000000 01	G.2179449C 01 -C.2179449C 01
_AVERAGE CDST =	0.21604570.01
NECESSARY CONDITION 0.2729108D CO	S VECTOR
GRADIENT MATRIX 0.98928970-02	· · · · · · · · · · · · · · · · · · ·
INVERSE GRADIENT MA 101-0826292	TRIX
مىرىيىيىتىكەر مەرىمە « مەرىپىيى بەرىپىيىيى بەرىپىيى بەرىپىي	
	· · · · · · · · · · · · · · · · · · ·
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FIGURE III.5-4 (Cont'd)

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ITERATION NUMBER 2 -NEH GAINS--22.5865428 SYSTEM EIGENVALUES -0.6278760D 01 -0.00000000 00 0.3278760D 01 -0.00000000 00 STEP SIZE TOO LARGE NEW AVERAGE COST HOULD HAVE BEEN 0.37338830 04 GAIN ADJUSTMENT IS HALVED · · · S. NEW GAINS -8.7932714 SYSTEM EIGENVALUES -0.45072030 01 -0.45072030 01 1641 5 11.65 -0.45072030 01 -0.00000000 00 4.9.6 80.10 1.14 STEP SIZE TOO LARGE NEW AVERAGE COST HOULD HAVE BEEN 0.5075050D 02 GAIN ADJUSTMENT IS HALVED • $\mathcal{L}_{\mathcal{L}}$ 1.51 1.1.53 NEW GAINS -1.8966357.+ ALCENS. Con Contact 27. SYSTEM EIGENVALUES GAIN TOLERANCE ACHIEVED = 3.6362469 0.0100000 - 14 C AVERAGE_COST - 0.19246070 01 testine rong the NECESSARY CONDITIONS VECTOR 1. 17 GRADIENT MATRIX 0.58702700 00

FIGURE III.5-4 (Cont'd)

INVERSE GRADIENT MATRIX 1+7034993

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2.	
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FIGURE III.5-4 (Cont'd)

ITERATION NUMBER 3
NEW GAINS
SYSTEM EIGENVALUES -0.2397460D 01 -C.0CC0:00D 00 -0.6025401D 00 -0.0000u00C 00
GAIN TOLERANCE ACHIEVED = 2.4146897
REQUIRED STOPPING TOLERANCE = 0+0100000
AVERAGE COST = C.13002640 C1
GRADIENT MATRIX 0.30303690 CO
INVERSE GRADIENT MATRIX 3-2999278
· · · · · · · · · · · · · · · · · · ·
•
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FIGURE III.5-4 (Cont'd)

71

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ITERATION NUMBER				
NEW GAINS				
0+1278936				
SYSTEM EIGENVALUES -0.1849437D 01 -0.1150563D 01			• *	
GAIN TOLERANCE	ACHIEVED =	5.3429401		
REQUIRED STOPPING	TOLERANCE =	0.0100000		
AVERAGE COST =	0.12215920 01			
NECESSARY CONDITIO	NS VECTOR	• • • • • • • • • • • • • • • • • • •	-	
-0.3396536D-01				
GRADIENT HATRIX 0.20829570 00	· · · · · · · · · · · · · · · · · · ·	•	· ·	-
INVERSE GRADIENT N 4.8008683	ATRIX			

FIGURE III.5-4 (Cont'd)

ITERATION NUMBER 5 . . . 1 0.2909588 SYSTEM EIGENVALUES 0.2023779D 00 -0.2023779D 00 -0.15000000 01 -0.15000000 01 GAIN TOLERANCE ACHIEVED - 035604378 0.0100000 REQUIRED STOPPING TOLERANCE = 「いったい」と <u>ka</u>r AVERAGE COST.= 0.12187380 01 . . Ì., (. - . NECESSARY CONDITIONS VECTOR . . 970 A. V. GRADIENT MATRIX . ` ٤ 0.18960870 00 ٠., ~ -INVERSE GRADIENT MATRIX 5.2740210

٠.

FIGURE III.5-4 (Cont'd)

ITERATION NUMBER 6 ---³/* - _____ 11 . •• , NEW GAINS 0.2990916 ۰, · • ÷. _ -2 . . . ۰, . 2 SYSTEM EIGENVALUES `;÷ ţ --0.1500000D 01 -0.2215661D 00 -0.1500000D 01 ŧf -0.22156610 00 <u>.</u> GAIN TOLERANCE ACHIEVED = 0.0271982 3 . the P REQUIRED STOPPING TOLERANCE = 0.0100000 · * _ - - -_ < - -2.1 5 -AVERAGE COST - 0.12187320 01 • `; • ` • • • • • · • • • • 5. , " . NECESSARY CONDITIONS VECTOR -0.36478570-05 في والعرب ألحاله الم الجه 1. A. ۹, 1 GRADIENT MATRIX - 0.1887124D 00 С <u>يې د او</u> -7 P INVERSE GRADIENT MATRIX 4 ٢, · '+ 7 5.2990702 j.S ٩, . ۰. • -.-2 ۲, • , •,• 1 ۰. . ħ, . ĺ., ٦, <u>: 1</u> ٠., . 1 - .. 25 :: . -÷, ۰. ي مان المراجع ð X-. . . . -¢ . ۲<u>۰</u>۰ <u>_</u>--

FIGURE III.5-4 (Cont'd)

74

ITERATION NUMBER NO. 2 CONTRACTOR OF A CONTRACT
NEW GAINS
0;2991109
SYSTEM EIGENVALUES -0.15000000 01 0.22160980 00 -0.15000000 01 -0.22160980 00
GAIN TOLERANCE ACHIEVED = 0.0000646
REQUIRED STOPPING TOLERANCE = 0.0100000
SOLUTION IS COMPLETE, ABOVE GAINS ARE OPTIPAL
AVERAGE COST = 0.1218732D 01
CONPILE TIME= 27.14 SEC.EXECUTION TIME= 69.23 SEC.OBJECT CODE= 64216 BYTES

FIGURE III.5-4 (Cont'd)



on the computations. The system equations were

$$\begin{bmatrix} \dot{x}_{1}(t) \\ \dot{x}_{2}(t) \\ \dot{x}_{3}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & 7 & 0 \end{bmatrix} \begin{bmatrix} x_{1}(t) \\ x_{2}(t) \\ x_{3}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$
 (III.5-7)

with the cost functional

$$J = \underline{x}^{\mathrm{T}}(1) \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \underline{x}(1) \div \int_{0}^{1} \underline{x}^{\mathrm{T}}(\tau) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \underline{x}(\tau) + 0.1 u^{2}(\tau) d\tau$$
(III.5-8)

It was assumed that only the first state was available for feedback so that

$$u(t) = -k_1 x_1(t)$$
 (III.5-9)

The computer print-out for a starting gain of zero is given in Figure III.5-6 and is compared with an I.C. in Figure III.5-7. There is an offset in Figure III.5-7 between the average costs produced by ISAFT and those produced by the I.C. This is due to the difficulty in approximating a uniform distribution on the surface of a sphere by a finite number of discrete points. (This difficulty increases rapidly with the order of the system). It is nevertheless clear that ISAFT achieved a minimum average cost.

III.6 Comments on the Finite Time Problem

The objective of the I.S.A. approach is to determine the feedback gains giving the minimum value of the gain function GF, which is the expected value of the cost functional. The I.S.A. algorithm presented in Section III.3, however, is designed only to determine feedback gains

STATES 3	CONTROLS	FEEDBACKS	TGAINS '
	ŀ · · :		· · · ·
SYSTEM NA O.	TRIX A TODOCOO DOCODO	1.0000000	0-0000000 6-0000000 7-0000000
0.1	000000	· ·	
CONTROL M	TRIX B	0.0000000	1-0000000
TERMINAL	FIME T	1.0000000	
TERMINAL 3.1	OST HATRIX F	0.0000000	9-0000000
2. 1. 	1000000	0-0000000	0.0000000 0.000000000000000000000000000
STATE WEI	SHTING MATRIX	q 0.0000000	0+0000000
2.0	000000	0-000000	0.0000000
CONTROL U	IGHTING MATR	IX R	

FIGURE III.5F6 Computer Print-Out For Example 3

FIGURE III.5-6 (Cont'd)

 ITERATION NUMBER
 2

 NEH GAINS
 6.1790200

 \$YSTEM EIGENVALUES
 0.265984470 01

 0.26598470 01
 0.00006000 00

 0.26538700° 01
 0.00006000 00

 0.26538700° 01
 0.00006000 00

 0.2653766880-01
 0.00006000 00

 CAINATOLERANCE ACHIEVED 1.00000000

 REQUIRED STOPPING TOLERANCE 0.005000003

 AVERAGE COST 0.25636430 03

 VECESSARY_CONDITIONS_VECTOR
 3.00000003

 SRADIENT_MATRIX 0.16112070.02
 3.00000003

0.0620653

INVERSE GRADIENT NATRIX

FIGURE III.5-6 (Cont'd)

TTERATION NUMBER	3	· ·		المستقدمين المراجع الم المستقدمين المراجع المرا
ANEW GAINS 8.7598845	The second s			Calify States Strates
SYSTEM EIGENVALUE 	S. -0.00000000.00 0.00000000 00			
0.40366570.00	0.000000D 00			
GAIN TOLERANC	E ACHIEVED =	0.2946231	•	
	ا المواجع المو المواجع المواجع		· · · ·	
REQUIRED STOPPING	TOLERANCE	0.0500000		
AVERAGE COST =	0.2174369D 03	<u> </u>		
* * * *				
NECESSARY CONDITI -0.54271280_01	ONS VECTOR		· · · · · ·	
GRADIENT MATRIX 0.12017290 02	· · · · · · · · · · · · · · · · · · ·		-	· · · · · · · · · · · · · · · · · · ·
		н. Н. Н. Н.		
INVERSE GRADIENT	MATRIX			
0.0002125				
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بر ب	74 40 x	- t, -		

FIGURE III.5-6 (Cont'd)

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FIGURE III.5-7 Verification of the ISAFT results for Example 3 satisfying certain necessary conditions. The necessary conditions are that the partial derivatives of the gain function with respect to each feedback gain should be zero. These necessary conditions may be satisfied by local maxima, minima or points of inflection. With a multidimensional problem points of inflection seldom occur in practice. The convergence factor μ built into the digital computer program ISAFT prevents convergence to a local maximum. There still remains the problem of convergence to a local rather than a global minimum. No solution to this problem was found. It is felt, however, that it should be feasible to guarantee the existence of a single (global) minimum for suitable system and cost matrices, and useful work could be done in this area.

The digital computer program ISAFT computes the various required functions of the state transition matrix (see (III.3-33)) through (III.3-38)) analytically. It was felt that this best illustrated the procedure. For high order systems however, it should be more economical to evaluate the state transition matrix at a number of discrete time instants and then evaluate the required functions by numerical integration.

CHAPTER IV

SUMMARY AND CONCLUSIONS

This report has considered the determination of optimal time-invariant output-feedback controllers for linear dynamic systems with quadratic cost functionals. The need for such controllers in practical engineering was illustrated, and the contributions of earlier researchers in the field were reviewed.

It was determined that deficiencies existed in two areas. For the case where optimization was to take place over the semi-infinite time interval Levine¹⁹ and Cassidy²⁰ had each derived a suitable computational algorithm. Both algorithms, however, required initialization by suitably stabilizing feedback gains and neither author gave a method for determination of such gains. For the case of optimization over a finite time interval, no satisfactory existing techniques were uncovered.

The problem of the determination of stabilizing feedback gains was approached via a gradient technique. The technique evolved from an eigenvalue sensitivity relationship given in Fadeev and Fadeeva²⁷. It was mechanized in the digital computer program GRADGN (Appendix 1) and provides a practical method for determination of local stability maxima in feedback gain space. It is felt that this gradient technique should complement the existing optimization algorithms for the semi-infinite time interval problem.

A new technique was derived for the finite time interval problem. The technique is based on the Initial State Averaging concept, previously used for somewhat different problems by Levine¹⁹ and by Kleinman, Fortmann, and Athans²⁵. A computational algorithm was derived and is incorporated in the digital computer program ISAFT (Appendix II). The algorithm satisfies a

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set of necessary conditions by Newton-Raphson iteration, using their gradient with respect to the feedback gains. This is equivalent to minizization of the expected value of the cost functional by the method of second variations.

The contributions made by this report should aid in the search for practical optimal controllers.

Several outstanding problems remain. Foremost among these is the question of the sufficiency of the solutions obtained by the optimization technique described above. The technique was designed to determine local minima of the expected value of the cost functional. An examination of the convexity of this quantity in feedback gain space might uncover conditions ensuring a single (global) minimum. The gain initialization technique is similarly local and would also benefit by extension to a global technique.

The computational methods used in the digital computer program ISAFT were designed to illustrate the theory. They are satisfactory for low order systems, but not for high order systems. It is felt that relatively simple modifications to ISAFT should eliminate this deficiency.

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APPENDIX I

This Appendix comprises a listing of the gain initialization digital computer program GRADGN.

	/Ju8	4257 MCRKINN,LINES=55
1		DT4=NSION W(3),4),XR(3)),XI(3,1),VR(3,0),VI(3/1),IROW(3),2),
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2		2 6130,30) Common Amatia 201 Bhatiad 201 Anato201 Anato201 Asudiad 201
2		$\frac{1}{28} = \frac{1}{23} + \frac{1}{23} $
3		KFALK(3, 13.)
м	10	FORMAT (711)
ب	2.2.	FDRMAJ (4E13./)
υ	25	FORMAT (2044)
7	30	FORMAT (7F10,4)
<i>c</i> ,	35	- FURMAL STATESTIC ALL STATESTIC AND ROLS AND FEEDBACKS AND IGAINST.
)	3.6	1 4 (,'1)%2 (',4,,'15,00') FR2W1T (//13, (51ARI) ITV INCREASE INCREMENT =),623 ()
1	37	FORMAT (//T3, MINIPUM STABLITY REGULARED FOR TERMINATION', 3X_F20_8
11	40	FORMAT (//T3, SYSTEM MATRIX AMAT()
. 12		FORMAT (//I3, (CONTROL MATRIX BMAI)
13	יוב י	FORMAT (//T2, MAXIMUM REAL PART UF ROOTS')
14	6.1	FORMAL (7/13,*EIGVEC ERROR MESSAGES*) FORMAL (7/13, 15, 1-5, 510 (100 HITED-1 TE 100 HELC) + 515 ()
1 A -		ENERAT (//T3.16 GENVETTERS CODDEC TO MOD ELCENTATION
17	75	FORMAT (7/1-) EIGENVECTORS CORRES TO MAY EIGENVELDE")
•••	• •	1 SX, COL IMAG PARTI)
13	. <u>8</u> .,	FORMAT (//T3, GRADIENT MATRIX')
19	82	FORMAT ('1',T), PROGRAM TERMINATION DUE TO EXCESSIVE NUMBER OF ITE
		IRAIICYS')
2.	84	FORMAT ('1',T3,'SPECIFIED STABLITY HAS BEEN ATTAINED')
21	80	FURMAL (11,13, GAIN SEARCH PROCEEDS ALONG A NEW GRADIENT. ITERATI
25	87	TUN NOMOGE ', 22 (12)
C		INTEL
23	8.6	FORMAT (//T3, STABILITY INCREASE STEP SIZE = +, F2J.8)
24	70	FORMAT (//T3, STEP SIZE IS DOUBLED')
25	92	FORMAT (//T3,+CONVEX CURVE FIT TO STABILITY LOCUS+),
<u>26</u>	93	FORMAT 1//T3, STEP SIZE TOO LARGE. RETURN TO PREVIOUS BEST GAINS!)
27	94	FORMAT (//T3, CURVE FIT NOT SUCCESSFUL. RETURN TO PREVIOUS BEST GA
÷.	25	LINST) . Eosmat (7/13) (STED 5175 TOO 13205 COMDUTE NEW CONDIENT AT DESUDU
£.2	7.7	IS ABST GAINSTI
27	96	FORMAT (11, T3, STABILITY IMPROVEMENT RATE TOO SLOW. PROGRAM TERMI
		1NATED. ()
3	97	FORMAT (//T3, LAST RESULTS SHOW MOST STABLE CONDITION FOUND .)
3±	98	FORMAT (//T3, STEP SIZE TOO LARGE, PROGRAM CONTINUES WITH REDUCED
	·· <u> </u>	
	с Г	NS # NIMRED OF SYSTEM STATES
	č	
-	Č.	NF = NUMBER OF FEEDBACK STATES
	C	IGAINS = γ - INITIAL FEEDBACK GAINS ARE ZERO
	Ç	= 1 - INITIAL ÉEEDBACK GAINS ARE READ IN
	ç	IDELR = 0 - STABILITY INCREMENT = 3.1
	C C	= 1 - STABILITY INCREMENT IS READ IN AS DELR
••	 C	= 1 - PROGRAM TERMINATES AT START ITY SPECIFIED BY STODE
	č	······································
		DEPRODUCIBLE
		NOI RELINC
		a service a

.

	C	
32		READ (1,17) NS,NC,NF,IGAINS,IDELR,ISTOP
33		WRITE (3.32)
34		WRITE (3,1J) NS,NC,NF,IGAINS,IDELR,ISTOP
	Ç	
	<u> </u>	DELR CONTROLS STABILITY INCREASE INCREMENT
,	C	
35		IF (IDELR) 120,120,110
<u>36</u>	<u>110</u>	READ (1,30) DELR -
37		GO TO 130
38	120	DELR=0.1
	130	DELR1=DELR
4-1	_	WRITE (3,36) UELR
	C	· · · · · · · · · · · · · · · · · · ·
	<u> </u>	STOPR CONTROLS STABILITY REQUIRED FOR PROGRAM TERMINATION.
	Ç.	
41		IF (ISTUP) 153,150,140
42	140	READ (1,30) STOPR
43		GO TO 160
44	150	S10PR=-1.0E6
45	100	<u>WRITF (3,37) STOPE</u>
46		D0 170 I=1.NS
41		001170 J=1,NC
48	170	K(I, j) =0,
	L,	
	L C	SYSTEM DATA MATRICES ARE READ IN RY RUWS
	<u> </u>	ANAL IS SYSTEM MAIKIX
	5	BRAT 15 CUNIKUL MAIKIX
40	6	NEAD 21 201 (ZANATZI I) 1-1 NEA AND THE NEA
<u> </u>		KEAU (1130) (IAMAIII.JJ.J=LENSIEIELS)
51		MALLE (2949) Malte (293) //ANAT/1 18 (2930) (21) NOS
52		MRIIE (3423) ((AMAI(143)454440)1) MRIIE (3424) ((AMAI(143)454440)1)
52		LOTT (1) CLOMAINT, JI, J=1, NCI 1, J=1, NCI
5/		RKLIE (2949)
24	r	WILL 194201 (IBMAIL140/40-1946/41-1445)
	r	י איז איז איז איז איז איז איז איז איז אי
55	U	TE (TRATAS) 101.105 (R)
56	180	$R_{FA} = 0.11331 + 1.0112310 + 1.121 + N_{FA} + 1.012 + 1.01$
57	100	
58	. 170	
	C.	
•	č	SUBROUTINE STAR COMPUTES THE SYSTEM STARTITY
	c	
59	195	CALL STAB (NS-NC-NF-K)
60	198	IF(RR(1)-STOPR) 999.200.200
61	200	KOUNT=KCUNT+1
62		IF (KOUNT-1.) 210-210-11-0
	с	
	č	GAIN SEARCH PROCEEDS ALONG A NEW GRADIENT
	Č	
63	210	ROOTR=RR(1)
64		ROOTI=RI(1)
65		hRITE (3.5.3)
~ •		
		م ماد به بالا بالا بالا بالا بالا بالا بالا ب
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66		WRITE (3.2)) 200TR
67		
69		
00		
69		KI=KUUNI-I
70		WRITE (3,86) K1
	<u>_C</u>	· · · · · · · · · · · · · · · · · · ·
	C	COMPUTATION OF EIGENVECTORS
	Ċ.	
71	v	CALL MSQUAS)
7 7 7		CALL STORECTS ANAL ACON H TOOL YO ME NO DE DOOTS DOOTS DE 33 C
• 2		CALL EIGVECTS AMAISASARSWITKUWARRALIYYRYVI, KUUIK, KUUIII, NS, SJ, U,
	_ 1	SW1 +1 (EK + 01F + 2)
	<u>C</u>	· · · · · · · · · · · · · · · · · · ·
	C	SW1 = J FOR AN EXACT EIGENVALUE AND NO RCUND-OFF ERROR
	С	ITER = NUMBER OF ITERATIONS USED TO FIND EIGENVECTORS.
	С	IF TOLERANCE IS NOT ACHIEVED. PROGRAM ACCEPTS VALUES AT ITER = 15.
	C	DIE = LARGEST CHANGE IN ANY SIGENVECTOR COMPONENT AT EINAL LIER
	ñ	and any contract of the state of the source of the state of the
7,	v	
		MR[]E 13;01]
74		WRITE (3,65) SWI, LIER, DIF
73		WRI(E (3,77)
70		WRITE (3+75)
77		WRITE $(3,20)$ $(VR(E),VI(I),XR(I),XI(I),I=1.NS)$
	С	1.BL
	с	NORMALISE EIGENVECTORS INNER PRODUCT
	- <u></u>	
. 7 .	C	CPRC CPRC
10		RE'
19		VECMULE J.
80		DO 24J I=1,NS
81		VECMGR=VECMGR+VR(I)=XR(I)=VI(I)=XI(I)
	240	VECMGI=VECMGI+VR(I) *XI(I) *XI(I) *XR(I)
83		VECMGS=VECMGR+VECMGI+VECMGI
84		DO 253 I=1-NS
55		VRN(T) = (VR(T) * VECMGR * VT(T) * VECMGT) / VECMGS
N 4	250	$\mathbf{U} = \mathbf{U} + $
00	~ 2.70	TIN(1)- TTI(1)- VECHOR-V(1)- VECHOI) VECHOS
	<u>ь</u>	
	<u> </u>	CUMPULE GRADIENT MAIRIX
	C	
87		DO 3CO J=1,4F
88		DO 300 L=1+NC
89		GRADR(J+L)=0.
96		GRADI(.t.) = 0.
01		
		$\frac{1}{2}$
92	0.0.5	GRAUK(J)LJ=GRAUK(J)LJ=VRN(LJ)=BRAI(LJ)LJ
93	290	GRAUI(J+L)=GRAUI(J+L)+VIN(I)*BMA1(I+L)
94	300	GRAD(J,L)=GRAUR(J,L)=XR(J)=GRADI(J,L)=XI(J)
95		WRITE (3,80)
96		WRITE (3,20) {[GRAD(1,J),J÷1,NC),I=1,NF]
97		WRITE (3-88) DELR
01		
70		
00		
99		D0 329 J=1,NC
99 199		
99 <u>179</u> 101	320	GRDSQ=GRDSQ+GRAD(I,J)*GRAD(I,J)
99 <u>179</u> 101 102	320	GRDSQ=GRDSQ+GRAD(I,J)*GRAD(I,J) DELK1=+DELR/GRDSQ
99 <u>179</u> 101 102 103	320	GRDSQ=GRDSQ+GRAD(I,J)*GRAD(I,J) DELK1=+DELR/GRDSQ DO 340 I=1.NF
99 <u>179</u> 101 102 103 104	320	GRDSQ=GRDSQ+GRAD(I,J)*GRAD(I,J) DELK1=+DELR/GRDSQ DO 340 I=1.NF DO 340 J=1.NC

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105		उ(1,J)≖K(1,J)
106	340	$K\{I,J\}=G[I,J]+DELKI*GRAD(I,J)$
157		POUT1=800T3
108		CALL STAB(NS.NC.NF.K)
109		IF (RR(1)-STOPR) 999-390-390
110	390	RC0T2=RR(1)
111		
112		
113		
114	40.5	$\frac{11}{12} \frac{1}{12} $
115	400	
112	420	DELK2=+3+DELK+DELKI/(RUG12-RUG11+DELK)
117		
118		DU 440 J=I, NC
	440	$\frac{K(1,J) = G(1,J) + UELKZ * GRAD(1,J)}{2}$
120		CALL STAB(NS,NC,NF,K)
121		IF(RR(1)-STOPR) 999,999,460
122	_460	ROOT3=RR(1) -
123		IF(ROUT3-RCOT2) 475,471,480
124	470	IF(KOUNT-2) 475,475,472
125	472	IF(RTEST-RCOT3-DELR1) 1200,475,475
· 126	475	DELR=.5+(ROOTL-RCOT3)
127		GO TO 200
128	480	IF(KOUNT-2) 485,485,482
129	482	1F(RTEST-RCOT2-DELR1) 483,485,485
135	483	DO 484 I=1.NF
131		DD 484 J=1.NC
132	484	K(I,J) = G(I,J) + DELK1 + GRAD(I,J)
133		WRITE (3,93)
134.		CALL STAB (NS_NC_NF_K)
135		60 TO 1200
136	485	DE18=.5+(800T,-800T2)
137		D(EE = 1) - (AO(1) + AO(1))
139		
130	•	
140	400	
- 1 4 1		
141	000	
142	200	
<u> </u>	<i>.</i>	
144		1+(KUON11-11) 205,505,502
145	502	DELR=.5+(ROOT1-ROOT2)
		<u>MRITE (3,87)</u>
147		GO TO 2 JU
148	505	WRITE (3,9)
149	<u> </u>	DD 510 I=1,NF
. L50 · /		• 00 510 J=1,NC
451 -	-510	K(I,J)=G(I,J)+DELK2+GRAD(I,J)
152 - 2		CALL STAB(NS.NC.NF.K)
153	_	IF(RR(1)-STOPK) 999,999,520
154	520	R00T3=RR(1)
155	-	IF (RU013-R0012) 530-531-58J
156	530	80012=80013
157 %		DELK1=DELK2
-15R'	·	60 TO 400
150	580	TE(KOUNT-2) 585-585-582
- <i>2</i> 7	200	······································
	•	· · · ·
` ;		5
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160	582	IF(RTEST-ROUT2-DELR1) 583,585,685
161	583	DN 584 I=1, NF
162	E94	$DU > 84 = 1 = 1 \cdot NL$
164 -	204	N(1;J)=U(1;J)+UELNI#GKAU(1;J) LOITE /2.03)
165		
166		
167	585.	DELR = 5*(ROOT - RCOT2)
168	_	WRITE (3,95)
169		D0 590 I=1,NF
170		DO 590 J=1,NC
171	590	K(I,J) = G(I,J) + DELK1 + GRAD(I,J)
172		GN TO 195
173	630	KOUNT2=KOUNT2+1
<u>174</u>		IF(KOUNI2-6) 0J5,605,1133
175	605	DELK2=,5=DELR=DELK1/(ROOT2-ROOT1+DELR)
176		WRITE (3,92)
····· <u>L [[</u>		
170	42.1	
18-1	020	$ \begin{array}{c} K(1) G(1) G(1$
181		
182	640	
183		IE(R0013-R0011) 680-680-660
184	660	
185		WRITE (3,98)
186		R00T2≍R00T3
187		GO TO 600
188	680	IF(KOUNT-2) 685,685,682
189	682 .	IF(RTEST-R00T3-0ELR1) 1200.685.685
19.	685.,	DELR=.5*(ROOTRCOT3)
191	000	
102	444	WRITE (3:64)
192	1100	
195	1100	
196	1200	WRITE (3.96)
197		WRITE- (3.97)
198	200ບ	CONTINUE
199		STUP
20 ت		END
		,
		```````````````````````````````````````
		·
		)
		· · · · · · · · · · · · · · · · · · ·

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201 202	SUBROUTIVE STAB(NS,NC,NF,K) CONMON AMAT(3_,30),EMAT(3+,3_),AHAT(3,3),AAAA(9)),ASUR(3,3), BRI3 1.201.31.1000/311
203 204 10 205 20	REAL K(30,30) FORMAT (//T3,'GAIN MATRIX K') FORMAT (4E18.7)
206 30 207 40 208	FORMAT (//T2, 'ROOTS',5X, 'REAL PART',11X, 'IMAG. PART') FORMAT (2E2).0) . IA=NS
209 210 211	WRITE (3,1) WRITE (3,2) ((K(I,J),J=1,NC),I=1,NF) CALL AMHT(NS+NC+K)
212 213 	CALL VECT(NS) CALL HSBG(NS,AAAA,IA) <u>CALL ATEIG(NS,AAAA,RB,SI,LANA,L</u> AL
215 216 	WRITE (3,30) WRITE (3,40) (RR(I),RI(I),I=1,NS) <u>CALL MAXRI(NS)</u>
218 219	
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	· · · · · · · · · · · · · · · · · · ·
<u> </u>	

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22:	SUBRUUTINE AMUT(NS,NC,K)
	C COMPUTES AHAT = AMAT - BMAT+K(TRANSPOSE)
221	CUMMON AMAT[30,30],BMAT[30,30],AHAT[30,30],AAAA[900],ASQR[30,30], 1 RR[30],RT[30],IANA[30]
222	REAL K(3/,3/)
223	DO 100 I=1,VS
225	AHAT(I,J) #AMAT(I,J)
226	DC 100 L=1,NC
227	<u>100 AHAT(I+J)=AHAT(I+J)=BHAT(I+L)=K(J+L)</u>
229	END
	<b>,</b>
<u></u> .	
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<u></u>	, , , , , , , , , , , , , , , , , , ,
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	and the second

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23,	SUBROUTINE VELTINS)
c	CONVERTS AHAT TU SINGLE SUBSCRIPT FORM APAA
۲۰ 231, ۲	COMMON AMAT(3,30), BMAT(3,3), AHAT(30,30), AAAA(900), ASQR(30,30),
232	DO 145 J=1, NS
233	DO 111 / I=1+NS
234	
236	RETURN
237	END
	NOIBLE
	DEPRODUCIÓ
	NOT REI
	στοστημα που τ τη προγογηματική του
	· · ·
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	, <u>,</u> , .
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12 12 1 2 2 2 2	· · · · · · · · · · · · · · · · · · ·

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238	3	SUBROUTINE HSQ(NS)
	C	
	C	COMPUTES ASOR .= AHAT=AHAT
	С	
239	2	COMMON AMAT(30,30),BMAT(30,30),AHAT(30,30),AAAA(900),ASQR(30,30),
		1 RR (30) , RT (30) , TANA (30)
240	)	DG 100 I=1,NS
241		DO 100 J=1,NS
	1	ASOR(1.J)30.
243	3	DO 100 K=1,NS
244	100	ASQR(I,J)=ASQR(I,J)+AHAT(I,K)=AHAT(K,J)
.24:	5	RETURN
246	5	END
•		

249		SUBROUTINE HSBG(N.A.TA)
	С	
	č.	CONVERTS & TO UPPER HESSENDERG FORM
	C	
254	-	DOUBLE PRECISION DARS DELOAT DISTON DRIE DEXP. DI DG. DI DGID. DATAN
	. 1	LOSIN DE S. DSORT DIADH-DHOD-DHAYI DHINI
251		
252		
253		
254		
255		NIA~L*IA   TA~L*IA
255	20	
220	20_	
257	40	
255	-	
253	· · · · · · · · · · · · · · · · · · ·	
200		1508=114+L .
201		IPIV=ISUB-IA
262		PIV=ABS(A(IPIV))
263		IF(L-3) 90,91,50
264	50	M=IPIV-IA
265		DN 87 I=L,M,IA
266		T=ABS(A(I)) / · ·
267		IF(T-PIV)' 80,80,60
268	<u> </u>	
269		PIV=T
27J	8J	CONTINUE
271	<u>93</u>	IF(PIV) 100,320,100
272	100	IF(PIV-ABS(A(ISUB))) 180,180,120
273	120	H=IPIV-L
274		D0 140 I=1.L
275		J=M+1
276		(L) A=1
277		KILIA+I
278		A(J) = A(K)
279	140	A(K) = T
280		M=12-M/IA REP.
281		DO 16.1 = 11.014.14
282		
283		
284	······································	
285	169	
284	180	
287	2.1.3	
101	203	
200		
207		
290		
291	•	
293		
294		
295	221	A(KJ) = A(KJ) - A(KL)
295	240	CONTINUE -
297		K=-IA
<u>298</u>		PQ 3.00 I=1,N
299		K=K+IA

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101

	LK=K+L L
301	S=A(LK)
332	
205	DA SRA Partires
204	
366 200	
307 200	A11×1−C 9=9±81C91×419×1×1°(A);
308 309	AILNI~0 DD 310 [~1 14.14
309 310	-00-517.1×L12124124 •δ(1)±0.0
310 320	1 a 1
311	G0 T0 20
312 369	RETURN
313	END
	······································
	· · ·
	· · · · · · · · · · · · · · · · · · ·
····=··=	······································
	· <u>····································</u>
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	<u></u>
<u> </u>	

31 [°] 4	SUBROUTINE ATEIG(M, A, RR, RI, IANA, IA)
č -	COMPUTES ROOTS OF UPPER HESSENBERG MATRIX A
515 AG	DOUBLE PRECISION DABS, OFLOAT, DS TGN, DBLE, DEXP, DLOG, DLOGIO, DATAN P.DSIN, DCDS, DSQRT, DTANH, DMOD, DMAX1, DMINI
316	DIMENSION A(900),RR(30),RI(30),PRR(2),PRI(2),IANA(30)
317	INTEGER P,PI,Q
318	B7=1-0E-8
319 320 321	E6=170E-6
322	MAXLT=30
323	N=M
324 2	0 N1=N-1
125	TN=N1+TA
-326	NN=TN+N
-327	TIE (N1) 30-1300-30
328, 3	0 NP=N+1
329	IT=0
<u>330</u>	00 40 1=1.2
331	PRR(1)=0.0
332	0 PRI(1)=0.0
333	- PAN=0.0
334	PAN1=0.0
-335	R=0.0
336	S=0.0
338 339	NZ=NI=I INI=IN-TA NNI=NI+N
340	N1N=IN+N1
341	N1N1=IN1+N1
342 6	O_T=A (N1N1)-A (NN)
343	U=T+T
1344 10 10 10	V=4.0*A(NIN)*A(NN1)
345 10 10	IF(ABS(V)=U*E7)*100.100.65
346 6	5 T=U+V
347	IF(ABS(T)-AMAX1(U+ABS(V))*E6) 67+67+68
<u>348 6</u>	7 T=O+0
349	8°U=(A1N1N1/3A(NN1)/2:0
350	V=SQRT(ABS(T1))/2:0
351	AIF(T1)140:70:70
352-7	U 1F(U) 80,75,75
353 7	5 RR(N1)=L+V
354	
356	0 RR(NL)=U=V
357	<u>CONTENTION</u>
359 10 360 11	$\frac{0.110 + 100}{0.110 + 100}$ $\frac{0.110 + 100}{0.110 + 100}$
3623 363 12	G0 T0 T30 O <u>R (NL) = A (NN)</u>
204 8 53 9 11	CL (5)1=4 (NINT)

365	13) RI(N)=+,)
360	$RI(i 1) = 1_{+}$
367	<u> </u>
368	140 RR(N1)=L
369	RR(N)=U
370	RI(N1)=V
371	RI(N) = -V
372	16) IF(N2)1280,1233,180
373	180 N1N2=N1N1-IA
374	RMOD=RR(N1)*RK(N1)+RI(N1)*RI(N1)
375	- EPS=E10+SQRT(KMOD)
376	IF (ABS(A(N1N2))-EPS)1280-1280-240
377	24J IF(ABS(A(NY1))-E1J*ABS(A(NN))) 133)+1330+253
375	25y IF(ABS(PAN1-A(N1N2))-ABS(A(N1N2))*E6) 1240,1241,265
377	26U IF(ABS(PAN-A(NN1))-ABS(A(NN1))+E6)1240-1240-300
38,	300 IF(IT-MAXIT) 323-1240-1240
381	320 J≠1
382	DO 36D 1=1+2
383	K=NP-I
384	IF(ABS(RR(K) - PRR(I)) + ABS(RI(K) - PRI(I)) - DELTA + (ABS(RR(K)))
	$1 + ABS(RI(K)) + 343 \cdot 362 \cdot 363$
385	340 .1=.1+1
386	360 CONTINUE
387	G0 T0 (440-460-480)-J
198	44'1 B=0.0
384	$S=J_{\bullet}$
395	GD TO 513
391	460 J=N+2-J
392	R=RR(J)+RR(J)
393	S=RR(J)+RR(J)
394	GO TO 5)·J
395	483 R=RR(N)*RR(N1)-RI(N)*RI(N1)
396	S=R3(N)+RR(N))
397	500 PAN=A(NN1)
398	PAN1=A[N1N2]
399	DO-520 [=1.2 ²
، ( 4	K=NP-I ····
4 51	PRR([)=RR(K)
4.12	520 PRI(I)=RI(K)
403	P ≥N2
404	IF(N-3)630,630,525
415	525 [P]=N142
400	DO 58J J=2,N2
437	$I P I = I P I - I \Delta - 1$
4 (•8	IF(ABS(A(IPI))-EPS) 61).600.530
409	530 IPIP≠IPI+IA
410	IPIP2=IPIP+IA
411	D = A(IPIP) = (A(IPIP) - S) + A(IPIP2) + A(IPIP+1) + R
412	IF(D)54),56J,24J
413	540 [F(ABS(A(IPI)*A(IPIP+1))*(ABS(A(IPIP)+A(IPIP2+1)-S)+ABS(A([PIP2+2)
	1 )) -ABS(D)*EPS) 62J.620.560
414	560 P=N1-J
	580 CONTINUE
415	
415 416	600 Q=P

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418	62J P1=P-1 . ·
419	<b>U</b> =P1
420	IF(P1-1)68J.65J.65J
421	650 00 660 I=2.Pl
422	$IPI \neq IPI = IA = 1$
423	IF(ABS(A(TPI))-EPS)680.680.660
474	660 (1) (2) (3)
425	
423	
420	
421	
428	
429	<u>IF(1-P)720+73-723</u>
43	/J) [P]=1[+]
431	1P1P=1IP+1
432	$G_{1}=A(II)+(A(II)-S)+A(IIP)+A(IPI)+R$
433	G2=A(IPI)*(A(IPIP)+A(II)-S)
434	G3=A(IPI)*A(IPIP+1)
435	<u>A(IPI+1)≈3.2</u>
436	GO TO 78J
437	720.61=A(111)
438	G2=A(III+1)
439	IF(1-N2)74).74).76
440	740, 63=4(11)+2
441	
447	
443	$781(\Delta P = 50PT(G) + G) + G2 + G2 + G3 + G3$
444	
445	800 15(C1)820, 840, 840
445	
447	
<u>-7771</u>	
440	
449	
45:	<u>AEPHA=7, 3731, 1+PS11+PS12+PS12</u>
451	
452	860 ALPHA=2.0
453	PSI1=0.3
454	PSI2=0.0
455	88J IF(I-Q)93J,96J,930
<u>   456                                 </u>	90J 1F(I-P)92,1,94,,92
457	92ú A(III)=-CAP
458	GO TO 960 .
459	940 A(III) = -A(III)
46.1	96 J [J=II
46í	DO 134) J=I+N
462	T = PS(1) + A(1) + 1
463	1F(1-N1)980.1.40.1000
464	980 1921=1.1+2
465	
466	1.006 $FTA=AI$ PHA*(T+ $_{A}$ (T,1))
467	$\Delta(T_{i}) = \Delta(T_{i}) = -\overline{T} \Delta$
469	MIAU/-MIAU/ EIM
440	
ייד א עסיי	4) 11-9112 / 64924994949 1036 4 / 10211-4 / 10219-6 / 67 / 67 / 67 / 67 / 67 / 67 / 67 /
470	LOCU ALIFCJI=ALIFCJI+FSIZ#EIA
4/1	
472	TECT-NT) TAQAT 1903 1390

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473	1J6J K=N	
474	GO TO 1100	
	108J K=I+2	
476	1100 IP=IIP-I	
477	00 1180 J=Q+K	
478	JIP≂IP+J	
479	JI=JIP-IA	
48_	$T=PSII * \Delta(JIP)$	
481	TE(1-N1)1120-1140-1140	
482	1120 HD2= HD+1A	
483		
484	1/40 ETA-ALDWA#(T+A(11))	
404		
405		
400	ALUINI-ALUINI-CIA#POIL TCLL NINILLS - NOG INDO	
401		
488	1100  A(JIPZ) = A(JIPZ) - E(A + PS) Z	
489	1180 CUNTINUE	
491	IF(I-N2)1200,1220,1220	
491	1200 JI=11+3	
492	JIP=JI+IA	
493	JIP2=JIP+IA	······································
494	ETA=ALPHA+PSI2+A(JIP2)	
495	A(JI)=-ETA	
496	A(JIP)=-ETA*PSI1	······································
497	A(JIP2)=A(JIP2)-ETA*PSI2	•
498	1220 II=IIP+1	4
494	IT=IT+1	
500	GO TO 60	•
501	1240 IF(ABS(A(NN1))-ABS(A(N1N2))) 1300-1280-1280	
·502	1280 IANA(N)=0	
503	IANA (NI)=2	······································
504	N=N2	
535	TE(N2114)1.14.1.20	
50.6	$1300 \text{ RP}(N) = \Lambda(NN)$	
507		
509	- NI 1997-940 TANA / MIN - T	
5779	170/ M-HT	
510	1520 N=41	
512	1400 RETURN	
513	END	
	•	
		*
		•
	······································	,
	· ·	•
	,	
	· · ·	
		······

514	6	SUBROUTINE MAXRT(NS)
	C	COMPUTES ROOT HAVING MAXIMUM REAL PART
515	C	COMMON AMAT(3J,30),BMAT(30,30),AHAT(30,30),AAAA(900),ASOR(30,30), RR(30),R(30),TANA(30)
516 517		00 100 1=2,NS IF (RR(1)-RR(1))-50,107,102
519 520	100	RR(1)=RR(1) CONTINUE
<u>521</u>		RETURN .
		· · · · · · · · · · · · · · · · · · ·
		•.
		· · · · · · · · · · · · · · · · · · ·
<u></u>		
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<b>5</b> 23	SUBROUTINE EIGVECLIVC, A, B, M, IRON, XR, XI, VR, VI, ROOTRE, E	ESV 1.
	1 RODTIE, NE, NMAX, T2, SW1, COUNTE, ERR, MMM)	SV1
	C SUBRUUTINE TO FIND THE EIGENVECTORS OF A NON-SYMMETRIC MARKIN I	بيي
		-014 -041
	C FIND ONLY THE REGULAR FIGENVECTORS FALLS I ANDON N.	1511
<u>بيب متنظنات 196 (199</u>	C · " 2 FIND ONLY THE TRANSPOSED EIGENVECTORS (AT V = LAMBDA V)	ESY1
, .	C 3 FIND BOTH TYPES OF EIGENVECTORS.	SY1
524	DIMENSION A130, 301 . B 30, 301 . W 30. 4 . XR (30) . XI (30) . VR (30) . VI (30) .	
	1 IROW(30,2) / 2015 2016 2016 2016 2016 2016 2016 2016 2016	2 5
525	INTEGER COUNT: COUNTE: T2	-\$71
- 526		0724 (SPA)
528		cui
520	ROOTH = ROOTLE	C G V 1.
53.	N = NE	SVE
531	MM = NMM - 1	SVR
<u>. 332</u>	N1 = N - 1	SYL
533	NP1 = N + 1	ESVL
534	IVC1 = IVC - 1	esv1
, 235		<u>e sañ</u>
220 527 \	CUUNI = 1	SV1
538		
239	XR (1=0-0FA)	
540	400 CONTINUE	·
541	CLIM = 1.0E-4	ESY1
542	15(ROOTI)) 60, D	ESVI
		ES∀L
	COMPLEX EIGENVALUE	ESYI
647		ESY1
544		237L
545	TENR2-POINTPAROTTAPOOTTAPOOTT	492 YZ 4
546	300	£57
5473	200-808-12=1. N.	ESYL
548	IF (T2), 600, 603, 600	ESY1
549	600 00 602 J = 1, N	ÉSY1
550		<u>esyi</u>
551	1E139-25118602 6018 601	ESV1
- 202 F		ESAT
554	$602 \text{ R(I_1)} = 4(I_1) \text{ BEEVE} + 1(I_1)$	<u>5671</u> 5911
555		6211 6311
556	603 DD 694 J = 1 • N	ESY1
557	604 8(1,J) = A(15J) TEMP ( BLT J)	ESV1
.558	5 605 BITTE BITTE TENP2	ESY1
.559.33	ST 506 A (1-,17) A (131) A (131) A (131) A (131)	EŚŶ1
560	IF(T2 • NE• 0) REWIND T2	ESY1
561	50 IU 709	ESY1
202	DUL UTILL, 022, 008, 622	
12 A A	MATE TY CTACITAD	2011
		COTL COVI
563	622 1F(1VC2) 623, 625, 623	ESVI

564	623	00.624  LL = 1.000  M	ESY:
565. 566	624	Witt(2)=0.0	
567	AN PERCH	1F,(1VC1)2625, 514, 625	ESY:
568	625	D0.626 $LL = 1, N$	ESY:
570	626	VI(LL)=0.0	<u>, , , , , , , , , , , , , , , , , , , </u>
571	<b>.</b> .	GO TO 511	ESY:
88-54A	<u>C</u>	MATRIX NOTSCINCHTAD	ESY ESY
14.53	V C (23)		EST.
572	ACC2 608		ESY.
574		W(LL,2)=1.0	
575		W(LL.3)=1.0	
516	SS 609		+ e u [‡]
578	610	DO STINI - LANSA	857 257
579		12 = IROW(1,2)	÷SY
581	1	X15121:4"9(1+1)*R0011 D0:611 1 = 1 N	T ESY
582	<b>於</b> [在611]	XI(12) = XI(12) + A(T-J) × H(J-2)	ESY
583		LF (IVCI)1.612, 500, 612.2	ESY
585	210 Sealer	VICTOR = WILSINGOTI	ESY
586		DO 613 J = 1.N	257 257
587	613 When Jacobs	$\frac{V(I)}{V(I)} = V(I) + A(J_{+}I) + W(J_{+}4)$	£SY
589	615	CERR = 0.0	ESY ESY
590	Chi. She Chi.	1F(1VC2) 616, 619, 616	ESY
591	616	$\frac{100618 \text{ F} = 1 \text{ N}}{100000000000000000000000000000000000$	ESY
593		$\frac{1}{10}$ $\frac{1}{17}$ $\frac{1}{3}$ $\frac{1}{10}$ N	≡S¥ ≓sγ
594	as in 617	XR TIDE XR HDLA ACTAINXT (UN	ËSY
295	618	XR(1) = XR(1)/ROOTI	ESY
597	619	DO 621 I $=$ 1, N	, £51 FSY
598		VR(1) = -H(1,4)	ËSY
600%	÷ 620	$\frac{DU}{SVR} = \frac{1}{1 + 1} + \frac{1}{2} + \frac{1}{2}$	≟SY
601*	<b>新学学621</b>	VR LIJY = VR LLP/ROOTL	EST
12.01	VECTINE:		ESY
•	ւ. Ըր	SCARCH VEGIURS FUR LARGEST ELEMENT AND NORMALIZE.	ESY
602	627	AMAX = 0.0	EST
6034	1527	DD 629 L TELEN	ESY
605		IE(TEMP2- ANAX) 629. 629. 628	ESY ESY
606	628	AMAX = TEMP	ESY
607 638	. 629	12 = L CONTINUS	ESY
609		CI'= VR(T2)/AMAX	ESY Ecv
610		G2 = -VI(12)/AMAX	ESY
612		$DU = 639'' = 1, N_{-}$	ESY
		- Myre - Packar	= S Y
		•	

	9.8							
613	V,I ( L	= VR(L) *C2	+ TEMP*C1				E	S¥
614	630 VR (L	} = VR(L)+C1	- TEMP+C2			e.	΄ E	SY
<u>615</u>	<u>IF(C)</u>	<u>1UNT ED. 1)</u>	<u>GO TO 632</u>				E	SY
610	275 278° UU-26	SILLS TISN					E	SY
5210 A		AMAXITUERI	ABSTVRIL	L12=2H(LL,3))	ABS(VI(LL')	Se Marri	4)))) E	SY
619	422 AMAY	- 0 0	16 10 3 3 4 TY SU ()	Real and the second of the second	· · · · · · · · · · · · · · · · · · ·		<u>)- · · E</u>	SY.
620		= (J+U 25 E m 1 N	•	-			E	<u>sy</u>
621	TEMD	JJ L H IJ N - YDII 1447 J					· E:	ŞY
622333	SPREATE IT		35 4625 22	28.22 A. 2				27
5623	634 ANAX	TEMP					S	21
624	Sec. 12.52	120					S. J. S. C.	31 67
625	635 CONT	INUE	·····				E.	<u> </u>
626	G1 =	XR(12)/AMAX		· .				5 Y
627.	<u> </u>	-X1(12)/AMA)		•			F	ŝv
628	THE DO/6	16 LEBEL, NA				7 44 44 18	E SE	ŚÝ
11629%FILE	TEMP	=XL(U)					E	ŝŶ
<u>CEG302455-</u>	83	N=RXR ([_] +C2*	+TEMP+C1			1		\$Ý.
631	636 XR(L)	= XR(L)+C1	- TEMP=C2	· ·	-		E	S¥.
632	1111	JUNT + EQ. 1)	GO TO 646	•			E	\$¥
233 VA2 601	00 00 762720E00	A LL = 1. N			And a water to water . Set	e. Michard and an	E	s <u>x</u>
	PLOSE CENT	AMAALIUERN	ABSIARIC	しりぶきんり(しじうし)」	ABSIXILLI		[2]]]·]型E	SY.
		STAFORACINA	PCENES	OF STREET		Serve Process	IN SOLUTION	SX
								λŢ.
635	638 IF(CC	IUNT LÉD. IN	GOTTO AAA'	1.1			23	91 97
636	IF(CE	RR GE IOF	-4) GO TO	839			5	31 6V
637参》第	的2%此IFICE	RREGENCLIN	GO TO 64	BUSTANS	the second	1031-4	382 3 Set 2	ац. сv.
638	CUIM	E CERR	Martin Sparting					S V.
2639.7	IFICI	TH TELLOF	-8)-60-TO	548 ····				ςγ.
`640,.` <i>`</i>	-639 IF(CC	IUNT _ GE 15)	GO TO 68	·美丽1491 / / /	-	``	É	SY.
641.	647 COUNT	= COUNT 4 1			<b>`</b>		E.	S¥.
-642- ···	IFIRO	071) 642 . 67	3. 642					SY.
6043 100	2042 1711	U21.840 544	£ 640				FE:	ŚΥ
446	1040-100504	AND LEWEN LEWIS		and the second	the second s	The share	E.	S¥ĝ
646	SAT ULLY	TIME XKULED	1. 15 4 15 X 10 1 1 1 1 1 1 1 1	1	TEST TEST AND A DESCRIPTION OF THE		Statistics E	<u>SY</u>
6478	TEIT	(1) $(444)$ $(1)$	666 × 1.184			-j	ES ES	SY,
648	5644 NO 64	5 415 E-1-2N3		die ander			<u>-</u>	SY;
649	Sales Milete	31H-SVRLLITE	erns Heres	Station and the state	Service and in the service	Analy all the series	<u></u>	3 Y.
650	645 HILLS	4) == VI(LE)		a the second		1.00	S MARTIN	2 T : C V
651 87	Section Sector	6992 44	EAST AND				The second s	بر د ن
652	646 CERR-	يَرْبُرُ بِنَبْ 10 0 €		1	Les Date on the	and a start of the second	1. E. C. C.	¢ v
_653发行	SP 注意 IE(IIC	C) 648, 647;	,698-25					S¥.
654	648 ERR =	CERR			公理部的代表。使得	1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1	6	SÝ
6555	COUNT	E = COUNT	and a second		Section & Frank Section 4	3. A. 1. 155	on the second	ŠΥ.
656	Sector IF(RO	DT112667.966	854667					5 V
1021-32	100 FUU 64	<u>9 1 7 1 N 1</u>		NOT THE OWNER OF THE OWNER OF THE	1. 17. 5 - 17 18 - 18 - 18 - 18 - 18 - 18 - 18 - 1	Co Stre Bach	12:5:00 F	ςΫ́
1020	209778 (:171 865778 (:171	IF = ATTFILE	RUDTR				E.	SY
66020	ARCIUK	MAL DATE OF THE					E:	ŝΥ,
6612	A SCENATO	SAARTON CONTRA	STATES AND	A sing was a set of a set	2	منابعة المرية معنى ترجع مترجعة رجعه تطريق جوالي	5%	ςγ;
SAL C						4. <b>5</b> 3 3	e se E	Ľ.
a start		ANTELOENVECT			WALK STATES		3.77	1
SATE TO	R. S. C. S. TRAFF.	and a state of the		2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2	and the second	an property with	Contraction Contraction	÷,
2. 1 C	they they want	Sarta Manual Strate						1

662 663	ESY ESY
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	ESY ESY ESY ESY
668 652 IF (ICC) 680, 685, 680	ESY ESY ESY
669)/c 680 [F(IVC2] 681, 683, 681 c 942 c 40 c	ESY ESY ESY
672, $682$ XI(L) = 0.0 672, $683$ IF(IVCI), $603$ , $514$ , $683673$ , $683$ DO $684$ L = 1, N 674, $684$ VI(1), $-0.0$	ESY ESY ESY ESY
675 GO TO 511 C MATRIX NOT SINGULAR.	ESY ESY ESY
<u>676 685 IF(1VC2) 653, 656, 653</u> 677 653 D0 654 U 51 N	ESY ESY ESY ESY
679 679 680 680 656 00 657 L = 1 , N	ESY ESY
682 GOLTO 499 C C ANORMALIZE REAL VECTORS:	EST ESY ESY ESY
L. 1	ESY ESY ESY
686 687 687 687 687 197 197 197 197 197 197 197 19	ESY
689 1F(TEMP - C1) 660, 660, 659 690 5 659 C1 = TEMP 691 659 62 = MI(L)	ESY ESY ESY
(692), 660 CONTINUE (693), -> DO 661 (C =>1.> N (694) + XI((L)→CXI((L)/C2)) + (C =>1.2 (C =>	ESY ESY ESY
696 (62 CZ=010) (62, 638, 662) (698 CZ=010) (62, 638, 662)	ESY ESY ESY
699 C1=0.0 700 D0 664 L= 7/1.1 N 701 TEMPT = ABS (VI(L))	ESY ESY
702 703 663 <u>C1 C11</u> 664, 663 704 704 C2 VII(U)	ESY ESY ESY
$\frac{706}{4}$	ESY

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ZO7 ESY

	-	· · · · · · · · · · · · · · · · · · ·	
708	-1 1	CERR = AMAXI (CERR, ABS (VI(LL) - W(LL, 1).)	SYI
709		W(LL,1)=VI(LL)	
710	665	VR(LL)=W(LL,L)	_1
§ 711 '	•	GO TO 638	SY[]
17.12 -	668	-IF(1VC2) 669 671 669	SY
<u>{-7.13</u>	669	00 670 L = 1 + N	<u>SY</u> ?
1.14	ໍ່ຄາກ	XI(L) = 0.00	21
2-716	671		ζVI?
717	-672	$VI(L) = D_{2}O$	SY:
718	· 70	RETURN	SY.:
7.19	<u>     673</u>	IF(1VC2) 674, 502, 674	SY.
720	.674	DO 675 I = 1, N	sy¦'
721		12 = 1ROW(1,2)	SY 🕻
722	675	XI(I2) = XR(T)	SX.,
123			211
	r	BACK SUBSTITUTION SECTION	31. CV:
· · · · · · · · · · · · · · · · · · ·	C	SHOR SOBSTITUTION SECTIONS	S Y
\$ 724	- 499	IF(IVC2) 500, 502, 500	SY
<u>725</u>	500	DD 501 T = 2. N	SY.
726	· • •	<b>14</b> = <b>1</b> - <b>1</b>	SΎ
1727	₹ <b>₩</b> *	DD 501 J = 1, 11	SY;
728	501	XF(I) = XF(I) - B(I,J) * XF(J)	S¥:
723	511	IF(1VU1) 502, 514, 502	SY.
731	202	DU DIV I = I I W S I + I + I + I + I + I + I + I + I + I	31.
732	· ·	16(18) 503. 505. 503	<u>ς</u> Υ:
733	503	90° 504×11° = Ro. 11	SY.
734	504	VI(I) = VI(I) - B(J) I (J) + VI(J) + S - S - S - S - S - S - S - S - S - S	SY.
735		IF(ICC) 505, 506, 505	SY
736, /	505	IF(B(I,1)) 506, 507, 506 E	SY?
737	<u> </u>	$\frac{VI(I) = VI(I)/B(I,I)}{E}$	SY.
-;1/28 ⊶ -,71210	·		SY:
1746	· <u>502</u>	-116-11-2089-2099 208 -11613 - VECTARD-DEALK - 200 / 200 / 200 / 200 / 200 / 200 / 200 / 200 / 200 / 200 / 200 / 200 / 200 / 200 / 20	SY:
741			<u>cv.</u>
742	. 509	VI(I) = 1.0	SY.
743	510	CONTINUE ).	SY.
744		IF(INC2) 514, 5256 514	SY;
745	, 514	DO 522 I = 1. N	SY:
1.746			SY
747	61 E	17(1 - 1) 515, 517, 515	5¥3
. 740	515	14 = 10 + 1	31
750	- 516	$X1(IR) = X1(IR) - B(IR, I) \times I(II)$	T.
751 ·	ماييس ري	IF(ICC) 517.518. 517	SY1
752	517	IF(B (IR, IR)) 518, 519, 518	sŸ۱
753	518	XI(IR) = XI(IR)/B(IR)IR	SYI
754	•	GD TO 522	SYI
755	519	IF(XI.(IR)) 520, 521, 520	<u>SY</u> :
756	520	XI(IR) = XI(IR)*1.0E+15	SYY
1-151.7	ະ 	-60 RU-622 Contraction of the second state of	SY)
.759	522	CONTINUE	AL V
		الله المراجع الأن المحمد بين المركب التي المراجع المحمون المراجع المراجع مراجع المراجع ال مراجع المراجع ا	

and the second second in the second	:.
[760" [16] [16] [16] [16] [16] [16] [16] [16]	ESY!
761 - 525 D0 526 I = 2, N	ĘSY.
762 IR = NP1 - 1	ESY.
1263 The II and I have been a strategy of the	ESA3.
¥764> < -> 00 526	ESX:
1765 526 VI (JR) = VI (JR) - B(J) TR) = VI (J)	ESY.
(66, 00, 527 L = 1, N,	ESY:
767   12 = 1  ROW(L, 1)	ESY:
(708 + 727) VR(12) = VI(12)	-E:21
7 7692 DU 528 L + 19 No of the set of states to be a set of the se	EST.
TATU TO 228 VICEJ = VR(L):	EST.
L L.C.L	E.3.I_
CACTOR MATOLY	-231 CCV
G PALTUK MAIREA	EST.
772 700-100 - 0	- CV:
(1773 SWI-1,0172) SWI-1,0172	C014
	'c cv-?
-775 701 (1900) (1 - 11 - 11)	É CY
776 - 00708 k = 1.01	ECV'
$777$ $\Delta MAY = ABS(B(K, NY))$	έςλ.
778	ESY"
779. metha = 1 K + 1 + 1 K + 1	ESY
780 D0 702 F = K1 . N	FSY
781 IF(AMAX -GT- ABS(B(I+K))) GO TO 702	ESY
782 AMAX = $ABS(B(1+K))$	ESY
783 IMAX = I	ESY
784 702 CONTINUE	E SY
785 IF (AMAX .LT. SW1) SW1 = AMAX	ESY
.786	ESY
787 $B(K,K) = 0.0$	ESY
1.26 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 + 1.25 +	ESY
789 <u>60 TO 708</u>	ESY
790	ESY
791. > (DD 703 J = 1, N	₩ESY
- 792 · · · · · · · · · · · · · · · · · · ·	ESY
793 $B(K,J) = B(IMAX,J)$	ESY
794 703 $B(IMAX,J) = AMAX$	ESY
1.795 12' = IROW(K,1)	ESY
796 MARTINE IROW(K)1) = IROW(KNAX,1)	÷E\$Y-
(7.97,,, 1ROW(1MAX+1)) = 12	ESY
198 104 DO 707 I = KI N	_ESY,
199 IF(B(I,K)) 705, 707, 705	ESY
1 800 - (05'8(L5K) = 8(L5K)/B(K,K)	ેESX
801 UU (Ub J 3: KL, N,	_ESY
BOAL THE B(I,J) = B(I,J) = B(K,J)+B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I,K)-B(I	ESY;
1002 TOTOLOGIANUE	ESY
1004 JUS-CUNTINUE	_ESX
1002 ARAA = ABS(B(N,N)); $100$	ESY
1000 IF(AMAX = 1.00E-25) (12, (13,	ESY
<u>LOW() IIC DININI EU-U</u>	
1. ONO - 2 × 1. SMI≖U+U - × × × 1. SM * × × 1. SM * × × × × × × × × × × × × × × × × × ×	
	ESY.
1 - 0 + 0 = 0 $1 = 1 + 0 + 0 = 0$ $1 = 1 + 0 + 0 = 0 = 0$	_ESX
JOIL SE SERVICE ANALIS DELL SEL SEL SEL SE ANAL	. ESY
general sector and the sector of the sector	

312	7J9 IF(ICC .LE. Ism) GC TU 71:	ESY
613	IF(MM) 1.05 (, 1.05), 1:51	
		V
816	CDUNIE 2 J DETION	EQY
817	1050 WRITE(103-1052) ICC .	231
312	(1) 00 711 LL = 1, N	ESY
813	I2 = [ROW (LL, 1)]	ESY
32,	711 IRUW(12,2) = LL	ESY
821	IF(ROOTI) 697, 652, 607	ESY
822	1052 FORMAT(///23H ****** WARNING ****** , SUBROUTINE EIGVEC HAS	ESY
	IFOUND AN EIGENVALUE OF APPARENT HULTIPLICITY.	FZA -
	1 1477238," CUMPUTATION OF EL 2/ENVECTORIST CONTINUES AT USED S OPTIONICAT	52Y
323	LI FORMATION MONE THAN IS LODES FOR FIGENVERTOR OF 2612-4.	EST EST
	2 14H DIFFERENCE OF E12.4)	ESY
824	102 FORMAT(16HO+***WARNING**** , 14, 71H ZERCS ON DIAGONAL OF FACTORED	ESY
	1 MATRIX. CHELK FOR MULTIPLE EIGENVALUES./20X.	ESY
	2. SUBRUUTINE LIGVEC WILL NOT PERFORM COMPUTATION FOR THIS EIGENVEC	ESY
	3TOR 1//)	ESY
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## APPENDIX II

This Appendix comprises a listing of the I.S.A. optimization digital computer program ISAFT.

	/J08	4257	MCPRINN.LINES=55
	C		
	ι c	PREGRAM ISAF	
	د م	DETERMINES UN	THE CLARK DECITATES BALBLERS
	с с	FUR FINITE II	ACCIDED TO DE ETAST AC STATES
	r r	ULIPUIS ERA /	SOUND IN DE FINSI NE STRICS
1	Ý	COMMON M(3,3),MI	((3,3), C(3), AHAT(3,3), T, K(3,3), 5(3,3), 4 ((2,3),
		1 FT(?,3),f	FTI(3,3),DFT(3,3,3,3),FCFTI(3,3,3),
		2 D2FT(3,3	,3,3,3,3),DFUFTI(3,3,3,3,3,3,3),FC2FTI(3,2,3,3,3,3),
		3 VW(9),VG	RAC(9+9), GRADI(9,9), 8(3,3), F(3,3), R(3,3), NS, NC+'\F
2		DIMENSION A(3,3)	1,6(3,3)
3		COMPLEX#16 M.MI	RC
4		COUBLE PRECISIC	<pre>A AHAT, T,K,G,CHAT, FT, FFTI, CFT, FCFFI, CFFI, CFCFII,</pre>
		1	FC2FTI,VN,VGKAD,GRADI
5		COUBLE PRECISIC	N GE,GE2,TEST:TEST:GASTCE
6		DCUBLE PRECISIO	V CMAX1,DAES
7	1.	FORMAT (4F14.7)	
8	11	FORMAT (4D17)	
9	15	FCRMAT (711.)	
10	20	FORMAT (1415)	
11	25	FORMAI (*1*,14,	'STATES',4X, 'CUAIRCUS',4X, 'FEEDMACKS',4X, 'ESAINS')
12	30	FORMAT (//13, 1	VVERSE GRADIENI MATRIX")
13	35	FURMAI (7713, 5	YSIEP MAIRIX P')
14	40	- FURMA1 (7713, "N	28 (AINS') (ATOCI - AITOTY - 41)
15	45	- FORMAT 1//13, C	JATA ANDIAN ALE ANTOINN
17	55	EPRMAT (//13. T	$\frac{1}{1} \frac{1}{1} \frac{1}$
	60	ECRMAT (//17,10	$\Delta T = T = T = T = T = T = T = T = T = T $
19	62	FORMAT L//T3. A	VERAGE COST ='+C2 .7)
20	65	FORMAT (//13. T	ERMINAL COST MATRIX F"}
21	70	FCRMAT (//T3. 48	ECUIRED STOPPING TOLERANCE =*,F13.7}
22	75	FCRMAT (//13, 15	TATE WEIGHTING MATRIX (")
23	មជ	FCRMAT (7773, C	CNTRCL WEIGHTING MATRIX R*)
24	85	FORMAT ('1','IT	ERATION NUMBER', 11-)
25	90	FORMAT (//T3,'S	CLUTION IS COMPLETS. ABOVE GAINS ARE (PTIMAL!)
. 26	92	FORMAT (//T3, 'S	TEP SIZE TOO LARGE. NEW AVERAGE COST HOULD HAVE THE
		1N, D16.7)	
27	93	FORMAT LI3, CAL	N ACJUSIMENT IS HALVED')
28	50	FLRMA1 1//13, "	UNVERGENCE ILL SLLW. PRIGRAM TERMINATHUMI
29	100	READ (1.12) N21	NC (NE ) IGAINS
3U 21		PEAD (1+1-) (14	{ 1 } J = J = 1 → N ⊂ J = 1 → N ⊂ J { 1 → J → J → N ⊂ J → I → N ⊂ J
_ JL _ 32		READ (1.11) T	(190)40-14(0)41-14(0)
32		READ (1,17) ((E	(1,.),.=!=!.NS).[=!.NS)
34		READ (1.10) ((0	$(I \cdot J) \cdot J = I \cdot NS \cdot I = I \cdot NS$
35		READ (1.1.) ({3	$\{1, J\}, J=1, NC\}, I=1, NC\}$
36		READ (1.11) GAS	TCP
37		WRITE (3,25)	
38		WRITE (3,15) NS	+NC +NF + IGAINS
39		hRITE (3,35)	
4.)		WRITE (3,1 ) ((	4(I,J),J=1,NS),[=1,NS)
4 <u>1</u> '		WRITE (3,45)	
42		WRITE (3,11) ((	B(I+J)+J=1+NC)+I=1+NS)
43		wRITE (3,55) T	

44 WRITE (3:65) hRITE (3,1_) ((F(I,J),J=1,KS),I=1,NS)
WRITE (3,75) 45 46 47 WRITE (3,11) ((Q(I,J),J=1,NS),I=1,NS) WRITE (3,82) WRITE (3,12) ((R(I,J),J=1,NC),I=L,NC) 48 49 5υ NFP1=NF+1 51 NFC=NF+NC CC 1000 I=1.NS CC 1000 J=1.NS C(I,J)=0.000 52 53 54 55 1666 K(I,J)=5.000 IF(IGAINS) 1200,1200,1100 1103 READ (1,11) ((<(I,J),J=1,NC),I=1,NF) 56 57 12CC CENTINUE 58 59 CG 1223 I=1,NS 60 CC 1220 J=NF+NS QHAT(1,J)=C(1,J) 61 1220 AHAT(1, J)=A(1, J) 62 63 IT=1 64 WRITE (3,85) IT WRITE (3,50) 65 WRITE (3,10) ((K(I,J),J=1,NC),I=1,NF) 66 67 1230 CONTINUE 68 CC 1250 I=1,NS CG 125J J=1,NF 69 7 J  $A \vdash AT(I,J) = A(I,J)$ 71 {L,I) 2= (L,I) TAH2 CO 1250 N1=1+NC 72 AHAT(I,J) = AHAT(I,J) - P(I,N1) + K(J,N1)73 74 CC 1250 N2=1,NC 75 1250 CHAT(I,J)=CHAT(I,J)+K(I,N1)+R(N1,N2)=K(J,N2) 76 CALL STRAM 1260 CALL FEFN 77 78 IT=IT+1 79 CALL GAINFN(GF) 80 WRITE (3,62) GF CALL NESCON 81 CALL GRADNT CALL INVERTIVGRAD, GRADI, NFC) 82 83 WRITE (3,31) 84 WRITE (3,12) ((GRADI(I,J),J=1,NFC),I=1,NFC) 85 86 CALL NEWRIT WRITE (3,85) IT 87 kRITE (3,42)
kRITE (3,1^{*}) ((G(I,J),J=1,\C),I=1,\F) 88 89 90 IT1=C 91 1285 CONTINUE 92 DO 1300 I=1+NS 93 DC 1300 J=1,NF 94 AHAT(I,J) = A(I,J)95 CHAT(I,J)=C(I,J)DO 1300 N1=1.NC 96 97 AHAT(I,J)=AFAT(I,J)-B(I+N1)*G(J,N1)98 CO 13CO N2=1,NC

99	1330	CHAT(1,J)=L+AT(1,J)+G(1,N1)+F(N1,N2)+G(J,A7)
105		CALL STRAM
101		CALL GAIN2(GF2)
1.)2		IF(GF~GF2.GT.L.SC1) GC TC 13P)
103		WRITE (3,92) GF2
104		WRITE (3,93)
105		CO 135) I=1+NF
106		CO 1353 J=1,NC
107	1350	G(I,J)=G(I,J)/2.+K(I,J)/2.
108		WRITE (3,41)
109		WRITE (3,10) ((G(1,J),J=1,NC),I=1,AF)
115		IT1=IT1+1
111		[F(IT1.LE.5) GC TC 128h
112		G0 TC 3000
113	1380	TEST1=P.CO.
114		CC 2CCJ I=1+NF
115		CC 2000 J=1+Nu
116		IF(G(I+J)+EC+L+CCC) GC TC 140J
117		TEST≭DABS((G(I,J)-K(I,J))/G(I,J))
118		TEST1=DMAX1(TEST,TEST1)
119		GC TC 2001
120	146-	IF(G(I,J)-K(I,J).EC.6.3DJ) GC TC 2532
121		TEST1=1CCG++GNSTCP
122	2000	K(I,J)≭G(I,J)
123		WRITE (3,60) HESTI
124		WRITE (3,7.) GNSTEP
125		IF(TEST1-GNSTLP+GT+C+rC2) GC TC 126C
126		WRITE (3,97)
127		WRITE (3,62) GF2
128		GC TE BCCC
129	3000	WRITE (3,95)
13J	805.	CONTINUE
131	5000	CONTINUE
132		STOP DUCIBLE
133		END
		NOT KEI
		10.

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134		SUBRELTINE STRAM
	C	
	C C	COMPLTES THE STATE TRANSITION MATRIX
1 2 5	L	
132		$COMMEN \ \ M[3,3], M[(3,3], KC(3], AHA[(3,3], ], [K(3,3], G(3,3], QFA[(3,3], ])]$
		1 F1(3,3),FF11(3,3),DF1(3,3,3,3),FDF11(3,3,3,3),
		$2 \qquad D2F1(3_13_13_23_13_1) DEDF11(3_13_13_13_13_13_15_1) FU2F11(3_13_13_13_13_13_1) FU2F1(3_13_13_13_13_13_13_13_1) FU2F1(3_13_13_13_13_13_13_13_13_1) FU2F1(3_13_13_13_13_13_13_13_13_13_13_1) FU2F1(3_13_13_13_13_13_13_13_13_13_13_1) FU2F1(3_13_13_13_13_13_13_13_13_13_13_13_1) FU2F1(3_13_13_13_13_13_13_13_13_13_13_13_1) FU2F1(3_13_13_13_13_13_13_13_13_13_13_13_13_1) FU2F1(3_13_13_13_13_13_13_13_13_13_13_13_1) FU2F1(3_13_13_13_13_13_13_13_13_13_1) FU2F1(3_13_13_13_13_13_13_13_1) FU2F1(3_13_13_13_13_13_13_13_13_1) FU2F1(3_13_13_13_13_13_13_1) FU2F1(3_13_13_13_13_13_13_1) FU2F1(3_13_13_13_13_13_1) FU2F1(3_13_13_13_13_13_1) FU2F1(3_13_13_13_13_13_1) FU2F1(3_13_13_13_13_13_1) FU2F1(3_13_13_13_13_1) FU2F1(3_13_13_13_1) FU2F1(3_13_13_1) FU2F1(3_13_13_13_1) FU2F1(3_13_13_1) FU2F1(3_13_1) FU2F1(3_13_$
		3 Vk(9), VGRAD(9,9), GRAD[(9,9], B(3,3), F(3,3), R(3,3), NS, NC, NF
136		CIMENSION AAAA(9), RR (3), RI (3), ASCR (3, 3), ASC2 (3, 3), XR (3), XI (3), VR (3
·	-	1 1, VI(3), IANA(3), IRCW(3,2), VRN(3), VIN(3), W(3,4)
137		COMPLEX*16 M,MI,RC
138		COMPLEX#16 DCMPLX,CCCNJG
139		DOUBLE PRECISION APAT, T, K, G, CHAT, FT, FFTI, CFT, FCFTI, D2FT, DF0ETI,
		L FC2FTI,VW,VGRAD,GRADI
140		DOUBLE PRECISION AAAA, RR, RI, ASQR, ASQ2, XR, XI, VR, VI, VR, VI, W, W, VECMGR
		1 .vec/gi,vec/gs, <u>swi</u>
141	10	FORMAT (2018.7)
142	30	FORMAT (//T3, SYSTEP EIGENVALUES)
143		CALL VECT (AHAT, AAAA, NS)
144		CALL HSBG(NS,AAAA,NS)
145		CALL ATEIG(NS,AAAA,RR,RI,IANA,NS)
146		WRITE (3,32)
147		WRITE (3,10) (RR(I),RI(I),I=1,NS)
148		CALL MSC(AAAA,NS,ASQR)
149		
150		<pre>%C(I)=DCMPLX(RR(I),RI(I))</pre>
151		00 1C0 J=1,NS
152	100	ASQ2(1,J)=ASCK(1,J)
153		CALL EIGVEC(3,AHAT,ASGR,W,IRCW,XR,XI,VR,VI,RR(1),RI(1),NS,NS,O,
		1 Shl,ITER,DIF,2)
154		VECMGR=C.000
155		VECMGI=C.DD.
156		CO 11C I=1,NS
157		VECMGR=VECMGR+VR(I)*XR(I)-VI(I)*XI(I)
158	110	VECMGI=VECMGI+VR(I)*XI(I)+VI(I)*XR(I)
159		VECMGS=VECMGR+VECMGR+VECMGI
160		CO 123 I=1,NS
161		VRN(I)=(VR(I)*VECMGR+VI(I)*VECMGI)/VECMGS
162		VIN(I)=(VI(I)*VECMGR-VR(I)*VECMGI)/VECMGS
163		P(1,1)=DCPPLX(XR(1),XI(1))
164	120	<pre>MI(1,1)=DCMPLX(VRN(I),VIN(I))</pre>
165		CC 100) KOUNT=2+NS
166		KOUNT1=KOLNT-1
167		IF(RR(KCUNT)-++R(KCUNT1)) 2,144,22)
168	14,2	CONTINUE
169		CC 15_ I=1+\S
17)		W(I,KCUNT)=CCLNJG(W(I,KCUNT1))
171	155	MI(KCUNT,I)=DCCNJG(MI(KCUNT1,I))
172		60 TC 1005
173	200	CO 21_ I=1,NS
174		CG 21: J=1,NS
175	213	△\$QQ(I,J)=4\$C2(I,J)
176		CALL EIGVEC(3,AHAT,ASGR,W,IRCW,XR,X[,VR,VI,RR(K(UNT),RI(KOUNT),NS,
		I NS,G,SW1,ITER,GIF,2)
177		VECMGR=J.JCJ

179 DO 22G I=1,NS 180 VECMGR=VECMGR+VR(I)*XR(I)-VI(I)*XI(I)
180 VECMGR=VECMGR+VR(I)*XR(I)-VI(I)*XI(I)
181 _ 220 VECMGI=VECMGI+VR(I)*XI(I)+VI(I)*XR(I)
182 VECHGS=VECHGR+VECHGR+VECHGI+VECHGI
183 DC 230 I=1,NS .
184 VRN(I)=(VR(I)*VECMGR+VI(I)*VECMGI)/VECM
185 VIN(I)=(VI(I)*VECHGR-VR(I)*VECHGI)/VECH
186 M(I,KOUNT)=CCMPLX(XR(I),XI(I))
187 230 MI(KOUNT, I)=CCMPLX(VRN(I), VIN(I))
188 1000 CONTINUE
189 RETURN
190 END

191	•	SUBROUTINE FEFN
	С	
	С	COMPUTES REQUIRED FUNCTIONS OF THE STATE TRANSITION MATVIX
	С	
192		CGMMCN M(3,3),MI(3,3),AC(3),AHAT(3,3),T,K(3,3),G(3,3),(HAT(3,3),
		1 FT(3,3),FFTI(3,3),CFT(3,3,3,3),FCFT1(3,3,3,3),
		2 C2FT(3,3,3,3,3,3),CFDFT1(3,3,3,3,3,3),FC2FT1(3,3,3,3,3),
		3 VW(3),VGRAD(9,9),GRAD[(3,5),E(3,3),F(3,3),A(3,3),VC,K
193		CIMENSION C1(3,3,3,3,3),C2(3,3,3,3,3,3)
194		COMPLEX*16 EX1(3),EX2(3,3),R1(3,3),R2(3,3),CUM1,DUM2,CUM3,UUM4,
		1 CL#5,CL#6,DU#7
195		COMPLEX*16 M.M.I.RC
196		CCMPLEX*16 CCcXP
197		DOUBLE PRECISION AHAT,T,K,G,CHAT,FF,FFTI,EFT,FCFTI,U2ff,U46FTI,
		1 FC2FTI,VW,VCRAL,GRACI
198		DCUBLE PRECISION D1,D2
199		∧ S1 = NS−1
200		
201		
202		
200		$\{X_1(1)\}=\bigcup \{X_1(R_1), \{X_1(1)\}\}$
204		
205		
200	1.5.3	
207	160	
200		CONCLETING PURCHELL
203		ET (1) - C (C)
213		
212		
213		CO 3CQ K2=1,NC
214		CFT(I,J,K1,K2)=2.900
215		FDFTI(I,J,K1,K2)=J.CU
216		DG 3C3 K3=1,NF
217		DO 3¢9 K4=1,NC .
218		DZFT([,J,K1,K2,K3,K4)=3.003
219		DFDFTI(I,J,K1,K2,K3,K4)=0.000
220	300	FD2FTI(I,J,K1,K2,K3,K4)=、. U.
221		CC 1000 N1=1,NS
222		FT(I, J) = FT(I, J) + P(I, NI) + P((N1, J) + EX1(N1)
ZZ3		DD ICCJ NZ=1, NS
224		
225		UMI = P(1, NI) = P(1, NI, NZ) = P(J, AJ)
220		$FFII(1, J) = FFII(1, J) + UMI * MI(N3, N2) \circ (EX2(N1, N3) - 1, )/RI(31, 13)$
221		
228		UU DUU KZEIAN Na na
229		UCM2-F(1;)11/#F1()1;)2/*D()2;N2;#F(N1;N3) TE(N1,N2) 32/ 314 32/
233	210	17(NI-N3) 22493109324 Petti (VI V3)-Detfi (VI V3)-DH024V1(N2 (V4T4EV3/N))
232	510	C = C + C + C + C + C + C + C + C + C +
233	320	0 1 1
	520	13.NI
234	330	CO 600 N4=1+NS
235		CO 6CO N5=1,NS
236		CUM3=DUM1*MI{N3,N4}*B(Ň4,K2)*M(K1,N5)*MI(N5,N2)

•

237		IF(N3-N5) 352,342,352
238	340	FCFTI(I,J,K1,K2)=FCFTI(I,J,K1,K2)-CUM3*(EX2(N1,N3)*(K1(x1,N3)*T-1.
		1)+1.)/(R1(N1,N3)*R1(N1,N3))
239		GC TC 360
24)	35C	FDFTI(1,J,K1,K2)=FCFTI(1,J,K1,K2)-DUM3+((EX2(N1,N5)-1.)/X1(N1,N5)-
	-	1(EX2(N1,N3)-1.)/R1(N1,N3))/K2(N5,N3)
241	360	CO 6CC K3≠1,NF
242		00 600 K4=1.NC
243		DLM4=2.*DUM2*MI(N3,N4)*H(N4,K4)*M(K3,N3)*MI(N5,J)
244		IF(N3-N5) 411.370.444
245	370	IF(N1-N3) 392.380.39C
246	380	$C2FT(1, J_{*}K_{1}, K_{2}, K_{3}, K_{4}) = C2FT(1, J_{*}K_{1}, K_{2}, K_{3}, K_{4}) + CUF(4+T+T+E(1, N+1)/2)$
247		GC IC 455
248	390	C2FT(1, J + K1 + K2 + K3 + K4) = C2FT(1 + J + K1 + K2 + K3 + K4) + CUV4 + ((R2(+3++)) + T-1))
		$1 + F \times 1 (N_3) + F \times 1 (N_1) / (R_2(N_3, N_1) + R_2(N_3, N_1))$
249		66 16 455
251	4E G	CONTINUE
251		IE(N1-N3) 421-410-420
252	410	C2FT(1,1,K), K2,K3,K4)=C2FT(1,1,K1,K2,K3,K4)=C104+T+FX1(N1)/R2(N5,N
672	710	
253		
256	4.2.1	
2 / 4	720	UZTICIJUTEINZINZINZINZINZINZINZINZINZINZINZINZINZI
365	435	
277 752	430	1F(N1=N3) 44,9430,944 Pactor 3 V1 V2
290	447	027111; J;K1;K2;K3;K4;=02711; J;K1;K2;K3;K4;F0094*(CA1(3)/=CA1(41))
957		1/(K2(N),N1)=K2(N),N3))
237	100	
208	450	62F1{1+J+K1+K2+K2+K4}=62F1{1+J+K1+K2+K3+K4}+60F4*1*2×1(N31/K2(N3+N
259	455	$U$ but $Nb \neq 1$ as
263		60 655 N/=1,NS
261		GUM5=PI(N4, N5)+E(N5, K4)+P(K3, N6)+PI(N6, N7)
262		CUM6=CUM2+MI(A3,N7)+M(J,N4)+CUM5
263		CUM7=2.**(I;N1)**I(N1;N7)**(J;N2)**I(N2;N3)*E(N3;K2)**(K1;N4)*CUM5
264		IF(N1-N3) 490,460,49C
265	460	1F(N4-N6) 480,470,480
266	470	CFDFTI(I;J;K1;K2;K3;K4)=CFCFTI(I;J;K1;K2;K3;K4)+CUM6*((T*T*R1(N1;N
		14)*R1(N1,N4)-2.*T*R1(N1,N4)+2.)*EX2(N1,N4)-2.)/(R1(N1,N4)+31(N1,N4)
		2)*R1(N1,N4))
267		GC TC 520
268	48ů	CFDFTI(I,J,K1,K2,K3,K4)=CFCFTI(I,J,K1,K2,K3,K4)+CUM6+(((R1(N1,N6)+
		1T-1.)*EX2(N1.N6)+1.)/(R1(N1.N6)*R1(N1.N6))-((R1(N1.N4)*T-L.)*EX2(N
		21,N4)+1.)/(R1(N1,N4)*R1(N1,N4))/R2(N6,N4)
269		GO TC 52L
275	496	IF(N4-N6) 510,500,510
271	5CC	CFDFTI(1, J, K1, K2, K3, K4)=CFDFTI(I, J, K1, K2, K3, K4)+DUM6*(((K1(N3, N4)*
		$1T-1_{*} = X^{(N)} = X^{(N)} + 1_{*} / (R^{(N)} + N^{(N)} + R^{(N)} + R^{(N)} + (R^{(N)} + N^{(N)} + T^{(N)} + 1_{*}) = EX^{(N)}$
		21.N4)+1.)/(R1(N1.N4)*R1(N1.N4))/R2(N3.N1)
272		60 TC 520
273	510	CEDETI(1.J.K).K2.K3.K4)=CECETI(1.J.K).K2.K3.K4)+DUM6*((5x2)N3.N6)-
		$1_{1}$ /R1(N ₃ , N6)-(FX2(N ₃ , N4)-), /R1(N ₃ , N4)-(FX2(N ₁ , N6)-), /R1(N ₁ , N6)-)
		$2(EX2(N), K_{0}) = 1.201(N), K_{0} = 1.202(N), K_{0} = 0.202(N), K_{0} = 0.202(N),$
274	520	TEINGANA) 560 SEC. Sec. Sec.
275	520	IF(N2-N4) 552 566 557
276	540	
210	240	, THEFTELESTATESTERESTERESTERESTERESTERESTERESTER

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		1) = (T * T * RI (\1, N2) * RI (\1, N2) - 2 • * F * RI (N1, N2) + 2 • ) - 2 • ) / (RI (N1, N2) * KI (NI
		2,N2)*R1(N1,N2))
277		GD TC 652
278	55J	FD2FTI(I, J, K1, K2, K3, K4)=FD2FTI(1, J, K1, K2, K3, K4)+CUM7*((EX2(N1, N4)*
		1(R1(N1,N4)*T-1.)+1.)/(R1(N1,N4)*R1(N1,N4))-(EX2(N1,N4)-1.)/(R1(N1,N4))
		2N4 + R2(N4, N2) + (EX2(N1, N2) - 1, )/(R1(N1, N2) + R2(N4, N2)))/R2(N4, N2)
279		GC TC 625
280	56C	IF(N2-N4) 581-570-580
281	570	FD2FTI(I, J, K1, K2, K3, K4)=FD2FTI(I, J, K1, K2, K3, K4)-DUM7*(=X2(\1, \2)*(
		1R1(N1+N2) = T-1+ + 1+ + 1 + (R1(N1+N2) = R1(N1+N2) = R2(N6+N2))
282		GO TC 59C
283	580	FD2FTI(1, J,K1,K2,K3,K4)=FD2FTI(1,J,K1,K2,K3,K4)-CUM7*((EX2(N1,N4)
		1-1, $1/R1(N1,N4)-(EX2(N1,N2)-1)/R1(N1,N2))/(R2(N4,N2))R2(N6,N4))$
284	59C	IF(N2-N6) 594,592,594
285	592	FC2FTI(I,J,K1,K2,K3,K4)=FC2FTI(I,J,K1,K2,K3,K4)+DUK7*(EX2(N1,VC)*
		1(R1(N1,N6) = T - 1.) + 1.) / (R1(N1,N6) = R2(N6,N4) = R1(N1,N6))
286		GO TO 600
287	594	FD2FT1(I,J,K1,K2,K3,K4)=FC2FT1(I,J,K1,K2,K3,K4)+DUM7*((¿X2(N1,N6)-
		11.)/R1(N1,N6)-(EX2(N1,N2)-1.)/R1(N1,N2))/(R2(N6,N2)*R2(N6,N4))
288	663	CONTINUE
289		CO 7C3 K1=1,NF
290		CC 7CQ K2=1,NC
291		DC 700 K3=1,NF
292		CO 760 K4=1,NC
293		D1(I,J,K1,K2,K3,K4)=C2FT(I,J,K1,K2,K3,K4)
294	730	D2(I,J,K1,K2,K3,K4)=FD2FTI(I,J,K1,K2,K3,K4)
295		CC 800 K1=1,NF.
296		DO 800 K2=1,NČ
297		DD 800 K3=1,NF
298		CC 800 K4=1,NC
299		$D2FT(I_{1}J_{1}K1_{1}K2_{1}K3_{1}K4) = (D1(I_{1}J_{1}K1_{1}K2_{1}K3_{1}K4) + C1(I_{1}J_{1}K3_{1}K4_{1}K1_{1}K2))/2.$
300	800	FD2FTI(I,J,K1,K2,K3,K4)=(D2(I,J,K1,K2,K3,K4)+C2(I,J,K3,K4,K1,K2))/
		12.
301	1000	CCNTINUE
302		RETURN
363		END

+4T(3,3), ,3,3,2,3),
******
,LF0F <b>F1</b> ,
1

318 SUBRGUTINE NESCON	
C CEMPCIES RECESSART CONDITIONS	
313 CUMMUR F(3)3)F1(3)3)TCD1914DA(3)3)T1F(3)3)T0A(3)3)	
3 VM(9), VGRAD (9,9), GRADI (9,9), P(3,3), F(3,3), R(3,3), NS, NC, NE	
320 COMPLEX#16 #-ML+RC	
321 DOUBLE PRECISICN AHAT, T, K, G, CHAT, FF, FFTI, CFT, FCFTI, D2FT, CFDFTI,	
1 FD2FTI,VW,VGRAD,GRADI	
322 10 FORMAT (//T3, NECESSARY CONDITIONS VECTOR)	
323 20 FORMAT (4D18.7)	
324 NFC=NF+NC	
325 00 1143 I=1,NF	
32600 1100 J=1,KC	
$\frac{32i}{220} = \frac{11 + 1}{220} + \frac{11 + 1}{200}$	
320 YW(IN)-0-0000 320 NO 1100 NI-1-NS	
33, $CP$ 100 N2=1.NS	
331 VW(IN)=VW(IN)+OHAT(N1.N2)+FDFTI(N2.N1.J.J)	
332 00 1CC0 N3=1.NS	
333 1000 VW(IN)=VW(IN)+F(NI,N2)=FT(N2,N3)*CFT(N1,N3,I,J)	
334 CO 1100 N4=1,NC	
<u>335 1100, VW(IN)=VW(IN)+R(J,N4)+K(N1,N4)+FFTI(N1,1)</u>	
336 WRITE (3,13)	
337 WRITE (3,22) (VW(I),I=L,NFC)	
338 RETURN	
339 END	
	^
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·	
······································	

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340	SUBROUTIN	E GRADNT
	C CCMPLT	ES GRADIENT MATRIX
241	С	2.21.WT/3.21.0C/31.AWAT/2.31.T.K/3.31.C(3.3).LHAT/3.31.
341	1 57	292797123939800379800113937979797900293790079577 73 31 861772 31 861713 3 3 3 31 6651173 3 3 3 31
	2 02	CT12 2 2.2.2.2.2 DEPETI(2.3.2.2.2.3.3). EP2ETI(3.3.3.3.3.3).
	2 UZ1 2 Wh	/di ucoso/o oi.coso//di.gi.gi.gi.gi.gi.gi.gi.gi.gi.gi.gi.gi.gi
347	CCN93 5741	(у) фаски су у) факмал су ууу фака факт са факт су у у на булат. К. М. МТ. ОС
343		SCISION ARATITAKIGICHATISTIERTIICETIECETTIOPETIA
747	1	
344	10 FORMAT (/	/T3_*GRACIENT #ATRIX*)
345	20 FORMAT (4)	
346	NEC=NE+NC	
347	CO 1160 I	=1•NF
348	CC IIC) J	=1.NC
349	IN=NF+{J-	1)+I
350	DO 1103 K	1=1,NF
351	DO 1103 K	2=1,NC
352	$ID = NF \neq IK2$	-1)+K1
353	VGRAD(IN,	IC)=R(J,K2)+FFTI(K1,1)
354	CG 1100 N	1=1,05
355	CC 1000 N	2=1,15
356	VGRAD(IN,	IC)=VGRAD(IN,ID)+CHAT(N1,N2)*(DFEFTI(N2,N1,K1,K2,I,J)+FD2
	1FTI(N2+N1	+K1+KZ+I+J}}
357	DO 1063 N	3=1, NS
358	1000 VGRAD(IN.	1D)=VGRAC(IN,1D)+F(N1,N2)*(DFT(N2,N3,K1,K2)+OFT(N1,N3,I,J
	2)+FT(N2,N	3)#D2FT{N1,N3,K1,K2,1,J}}
359	DO 1103 N	14=1,NC
360	11CC VGRAC(IN,	IC) = VGRAC(IN, ID) + R(K2, N4) + K(N1, N4) + (FCFTI(N1, K1, I, J) + IC) = VGRAC(IN, ID) + R(K2, N4) + K(N1, N4) + (FCFTI(N1, K1, I, J) + IC) = VGRAC(IN, ID) + R(K2, N4) + K(N1, N4) + (FCFTI(N1, K1, I, J) + IC) = VGRAC(IN, ID) + R(K2, N4) + K(N1, N4) + (FCFTI(N1, K1, I, J) + IC) = VGRAC(IN, ID) + R(K2, N4) + K(N1, N4) + (FCFTI(N1, K1, I, J) + IC) = VGRAC(IN, ID) + R(K2, N4) + K(N1, N4) + (FCFTI(N1, K1, I, J) + IC) = VGRAC(IN, ID) + R(K2, N4) + K(N1, N4) + (FCFTI(N1, K1, I, J) + IC) = VGRAC(IN, ID) + R(K2, N4) + K(N1, N4) + (FCFTI(N1, K1, I, J) + IC) = VGRAC(IN, ID) + R(K2, N4) + K(N1, N4) + (FCFTI(N1, K1, I, J) + IC) = VGRAC(IN, ID) + R(K2, ID)
	1	$FOFTI(K1_N1_FI_FJ)) + R(J_FNA) * K(N1_FNA) *$
	2	(FDFTI(N1,I,K1,K2)+FDFTI(I,N1,K1,K2))
361	WRITE (3,	10)
362	WRITE (3,	20) ((VGRAD(I,J),J=1,NFC),T=1,NFC)
363	RETURN	
364	END	

			•	
365	c	SUBROUTINE INVERTIA, P.A.	)	
	č	INVERTS A TO GIVE B		
34.6	C	DINESSIEN ALO 01 040 01		-
500		DIMENSION A177719017971	- v	
106		LOUDLE PRECISION AND A	744	
368		COUBLE PRECISION DABS		
369		IF(N-1) 103,150,101		×
37.,	100	B(1+1)=1+COC/A(1+1)		
371		RETURN.	w -	
372	101	CO 1C2 I=1+N		
373		DO 1C2 J=1,N		,
374	102	8(I,J)=0.0D0		
375		DO 103 I=1.K		
376	193	9(I+I)=1+7DC		
	c	PICK UP PIVOT ELEMENT		
377	-	CO 114 K=1.N		
378		1 =K		
379		TE(N=K) 111-110-104		
281	104	T-V-1		
101 102	104			
381		DU LOO JJ=len		
382		TEINVR2IV(11*K1)-DVR2IV	(F*K))))(0*1)	-0, £0, ; , , , , , , , , , , , , , , , , , ,
383	105	L ≖J j	•	
- 384	166	CONTINUE		•
385		IF(L-K) 107,110,107		
	С	PERFORM ROW INTERCHANGE		
386	167	CG 168 J=K,N		
387		C=A(K,J)		
388		A(K,J) = A(L,J)		
389	168	$A\{1,J\}=C$		·
390	200	EC 109 J=1.N		• •
301		C-B(K. 1)	•	
302		818.11-911.11		
202	100			
375	C 103		• 、、	واستنبين يستنبرني والالالا بالم منيو
	ь 	CULUPN ELIFINATION		
394	115	DC 114 L=1+N		
395		$IF(K-I) 111_{1}I_{4}I_{1}I_{1}$		· · · · · · · · · · · · · · · · · · ·
396	111	D=A(I,K)/A(K,K)		
397		CO 112 J=K,N		
398	112	∆(I,J)=A{[,J}-C+A{K,J}	,	
399		A(I,K)=C.CDG		· · · · · · · · · · · · · · · · · · ·
400		CO 113 J=1.N		•
401	113	$B(I_{J})=B(I_{J})-C+B(K_{J})$		
402	114	CONTINUE		
	с	SOLVE FOR INVERSE		
£ 14	~	CC 115 J=1-N		
404		DO 115 1-1.N	••	
404		UC 112 171911 V-D(1 1)/8/5 1)		
400		A-0114J/A(141) D/T 1)_V		•
406	112		·····	
407		KEIUKN		•
408		ENU		

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469	SUBRCUTINE NE*RI1
	C
	C PERFORMS NEWTON RAPHSON ITERATION
	C .
41ú	COMMON #(3.3).WI(3.3).RC(3).AHAT(3.3).T.K(3.3).G(3.3).SHAT(3.3),
	1 FT(3,3).FFTI(3,3).EFT(3,3,3,3).FCFTI(3,3,3,3).
	2 C2FT(3,3,3,3,3,3) DEDET1(3,3,3,3,3,3) FC2FTI(3,3,4,3,3,3)
	3 WE (91, WERAF (9,9), GRADI (9,9), B(3,3), F(3,3), R(3,3), NS, KC, NE
411	
711	CONFLUXATE FULLY AND AN AT THE COLATER FOR SET SET SET SET SET
412	DUBLE PRECISION ARAIETERS BALERIERIERIERIERIERIERIERIERIERIERIERIERIE
	I FCZFTI#VW#VGKAD#GKADI
413	CC 1COU 1=1,NF
414	DC 1000 J=1,NC
415	G[I,J]=K(I,J)
416	IN=NF+(J~1)+I
417	CD 1CD) KI=I.NF
418	$BG_{1}(G) K_{2=1-NG}$
410	
717	10-01-10-01-01-00401714 1014047101
427	
421	RETURN
422	END

• 423 SUBROUTINE VECT (AHAT, AAAA, NS) C C Ç . CONVERTS AHAT TO SINGLE SUBSCRIPT FORM ###A ----------DIMENSION AHAT(3,3), AAAA(9) Double Precision Ahat, AAAA 424 -425 . . 426 427 00 100 J=1.NS -----. 428 K=(J-1)*NS+I 429 LCC AAAA(K) = AHAT(I,J) -----RETURN 431 END . . .... • • -- -د. د میلیونسو در ۲۰ تر بر بر ۲۰ تابی است. این ۲۰ ^{میر} بیرون در ۲۰ ^{میر} بیرون در ۲۰ ماله است. ----------- --. ... -----. -. -..... --------. . ------للماجا فالمرابية فستستحدث المتحج فليساح بالتما سرار بالتيات فت . ___ ..... ----•--_ . .. . . ----_____ . . ------, -------------..... . ---------.

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432	c	SUBRCUTINE GAIN2(GF2) .
		COMPLTES AVERAGE COST FCR CONVERGENCE CHECK
<b>43</b> 3	 -	COMMGN #(3,3), #I(3,3), RG(3), AHAT(3,3), T,K(3,3), G(3,3), QHAT(3,3), FT(3,3), FTI(3,3), DFT(3,3,3), FDFTI(3,3,3,3), D2FT(3,3,3,3,3), DFDFTI(3,3,3,3,3,3), FCFTI(3,3,3,3,3,3), VK(9), VGRAD(9,9), GRADI(9,9), B(3,3), F(3,3), R(3,3), NS, NC, NF
434		COMPLEX*16 EX1(3).EX2(3.3).R1(3.3).CUM1
435		COMPLEX*16 M.#I.RC
436		COMPLEX*16 CDEXP
437		DOUBLE PRECISION AHAT.T.K.G.CHAT.FT.FETI.CET.FCFTI.D2FT.CEDETI.
	1	FC2FTI-VW-VGRAD-GRADI
438		DOUBLE PRECISION GF2
439		DO 5CC $I=1.NS$
440		EX1(I) = CDEXP(RC(I) + T)
441		DG 500 J=1+NS
442		EX2(I,J)=CDEXP((RC(I)+RC(J))=T)
443	500	R1(I+J)=RC(I)+RC(J)
444		CO 1CCO I=1+NS
445		C0 1C60 J=1+NS
446		FT([+J]=3.303
447		FFTI(1.J)=G.0C3
448		$CO \ 1COO \ N1=1.NS$
449		FT(I,J) = FT(I,J) + W(I,NI) + WI(NI,J) = EXI(NI)
450		$CO \ 1CGJ \ N2=1,NS$
451		DO 1000 N3=1,NS
452		DUM1=#(1,N1)*/I(N1,N2)*/(J,N3)
453	1609	FFTI(I,J) = FFTI(I,J) + CUM1 * MI(N3, N2) * (EX2(N1, N3) - 1) / R1(N1, N3)
454		GF2=C.0DG
455		D0 2000 N1=1+NS
456		DC 2CG3 N2=1.NS
457		GF2=GF2+QHAT(N1,N2)*FFTI(N2,N1)
458		CC 2007 N3=1,NS
459	2000	GF2=GF2+F(N1,N2)*FT(N2,N3)*FT(N1,N3)
460		GF2=GF2/NS
461		RETURN
462		END

463		SUBROUTINE HSBG (N, A, IA)
	С	
	С	CONVERTS A TO UPPER HESSENBERG FORM
	C	
464		DIMENSION A(9)
465		DOUBLE PRECISION A.PIV, T.S
466	<u> </u>	DOUBLE PRECISION DABS
467		
408		NIA≠L+LA /
409		
470	20	IF(L=3) 300990940 ITA-ITA-ITA
472	40	LIA*LIA ⁻ IA
473		
474		
475		TPTV=TS19-TA
476		PIV=DABS(A(IPIV))
477		IF(1-3) 90.90.50
478	50	M=1PIV-IA
479		DO 8C I=L, F, IA
480		T=DABS(A(I))
481		IF(T-PIV) 80,80,60
482	60	IPIV=I
483		PIV=T
484	60	CONTINUE
485	90	IF(PIV) 100,320,10C
486	100	IF(PIV-DABS(A(ISUB))) 180,180,120
487	120	M=IPIV-L
488		DG 140 I≖1,L
489		J=M+I
490		<u></u>
491		
492	1/0	
493		
494		
472		DO 104 T=CT\$MIN\$IN DO 104 T=CT\$MIN\$IN
407		
409		
400	160	
500	180	
501	200	
502		
503		D0 240 [=1.L2
504		J=J+IA
505		L-L+J
506		DO 220 K=1,L1
507		K ≠ J
508		KL=K+LIA
509	22G	A(KJ)=A(KJ)-A(LJ)*A(KL)
510	240	CENTINUE
<u>511</u>		<u>K=-IA</u>
512		DC 300 I=1,N
513	-	K=K+IA
514		<u>LK=K+L1</u>
•		·
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515 516 S=A(LK) LJ=L-IA 517 518 CO 280 J=1,L2 - - --.. . 519 LJ=LJ+IA 520 280 S=S+A(LJ)*A(JK)*1.CDC . 300_A(LK)=S DO 310_I=L,LIA,IA 310_A(I)=0.GDC 521 522 523 320 L=L1 GO TC 20 360 RETURN 524 525 526 .... - 1 -. . ----. ... 527 END, . - ---. _ . . . . . . _____ . . .. ---------ww. . ............ .... ----------. . . . • . . . . . . _ .... . .. . . ----------. ------ --- --------ي ويدير دروريد فيجيع الد متداريوند د . . • -•• ... ··· -_____ -. . . . - , **. ...** .. . . . . . . . . . -- - --------. . . . ------······ . .. . . ---------. --------- - -------. . . . .

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528	SUBROUTINE ATEIG (M.A.RR.RI, TANA, TALES
. C	COMPUTES ROOTS OF UPPER HESSENBERG HATRIX A
C	
529	DIMENSION A(9), RR(3), RI(3), PRR(2), PRI(2), TANA(3)
1	The RI, RHOD, EPS, D, GI, G2, G3, CAP, PS 11, PS12, ALPHA, ETA
531	DOUBLE PRECISION DABS, DSQRT, DHAX1
532 ·. 533	INTEGER P.P1.4
534	E6=1.00-6
535	E10=1.00-10
537	DELTA=04500
538	N=M
539 20	N1=N-1
540	IN=NI+IA NN=IN+N
542 2 5 5	-1F(N1) 30,1300,30
543 30	NP=N+1
544 . 545	IT=0 00 40 L=1.2
546. 200	PRR(1)=0.000
547 40	PRT(1)=0.0D0
548 540	PAN=0.000
550	R=0.000
551	S=0.0D0
552	N2=N1-1
554	NNI=TNI+N
555	N1N=IN+N1
556 5.	N1N1=IN1+N1
558 Hereicher	
559	V=0.00004(N1N)00(G01)
<u>560</u>	IF(DABS(V)-U+E7) 100,100,65
562	IFIDABS(T)-ENAX1(U-DABS(V))#E6) 67-67-68
563 67."	T=0.000
564 85 68	ZU=(A(NIN1)+A(NN))/2:0D0
566	TF(T)140,70,70
567. 70	IF(U) 80,75,75
568 75	RR(N1)=U+V
569	
571 80	RRINIJ=L-V
2572-99933	RRINDUW
574 100	GU 10 15057 34 10 10 10 10 10 10 10 10 10 10 10 10 10
575 110	RRINIJ=AININIJ
576	RRINIEAINNIGTOCOCCUS
-21 (2) (2) (2) (2) (2) (2) (2) (2) (2) (2)	RRUNISALINN

579	$R(N) = \Delta(N1N1)$	
580	136 RI(N)=2.00C	
581	$RI(N1)=C_{\bullet}GEC$	
582	RI(N1)=C.	
583	GO TC 16G	
584	145 RR(N1)=L	
585	BB(N)=II	
586		
597		
588	167 TEINO1280.1280.186	
590		
500		
501		
502	TEINACHININAN LECCU 1286 1280 260	
502	10/10/2014/11/2//=E/3/ 12/0/12/0/2/0/2/0/2/0/2/0/2/0/2/0/2/0/2/	
273	290 IF (DAD3)A(NKL)/TELC*DAD3)A(NKY)/ 1300/1300/1300/200	
574	220 IF(DAD3(FANTAINING))-CAD3(A(NINI)-CAD3(A(N)))-COT 12(3)(12(1))(12(1))	
595	200 IF(UAD3)PAN-2(RA1)J-LAD3(A(RA1)J=C0) 1240(1240,500	
570	200 ITTITTAATII 260112401640	
391		
398	UL 2011 (*1)2 Kanal	
233	N = KF = I TE (N ABE ( BO ( K ) _ BED ( T ) \ LFARE ( D T ( K )_ DD T ( T ) \ _ FE ( T A + / FARE ( DD ( K ) )	
067	$\frac{1}{1} = \frac{1}{1} = \frac{1}$	
401	1 TUADIRI(NIII) 94313001300	
601	249 J=341	
CU2		
613		
CC 4	446 KEJSOUJ E-C 400	
605		
600		
607		
608		
609		
615		
611	40C R = RR(IN) = RI(IN) = RI	
612		
615		
014		
615	EU 22 = 1 = 1 + 2	
c10		
D1/		
618		
619		
02-	171N+3/050,000,000 2011 101-0100	
621	223 191-51142 DD 264 1-2 12	
622	しい うだい ジェイトバイ コート・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・	
623	IF 1910-14-1	
624	1010-1011-111-1-223 (C+)-2011-22	
625	DJG IFIFEIFIA TRIDACIONAL	
626		
-627	C=V(1)1(0(1)1)+(0(1)1)-2)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)1(5)+0(1)(1)(1)(1)(1)(1)(1)(1)(1)(1)(1)(1)(1)(	
628	11(0)243,555,243	
629	540 IF(DABS(A(IFI)*A(IFIP+1))*(CABS(A(IPIP)+A(IPIP2+1)-S)*CARS(A(IPIP2	
	- 1 +2))]-CARS(C)*EPS) 62:+62.+56.	
63J	560 P=N1-J	
631	58J CONTINUE	
632	6C 🗘	Q = P
---------------	----------	-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------
633		GO TO 680
634	624	P1=P-1
635		
636		
030		1F(F[-1]000;000;C3C
637	650	DU 66G 1#2,P1
638		
639		IF(DABS(A(IPI))-EPS) 680,680,660 -
640	660	C=O-1
641	680	II = (P-1) + IA + P
642		EC 1220 T#P.N1
643		
644		
645		IF(1+P)720-701-720
646	760	
640		
440	-	
040		$G_{1}$
049		$G_2 = A(IPI) * (A(IPIP) + A(II) = S)$
. 650		G3=A(1P1)+A(1P1P+1)
651		A(IPI+1) ≒C.3CO
652		GO TC 780
653	72C	G1=A(III)
654		G2=A(II1+1)
655		IF(I-N2)74C.740.76C
656	74 C	G3=A(III+2)
657		60 10 786
658	760	63=0-000
650	784	C A D = C S A D T ( C 1 = C 1 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2 + C 2
4413	100.	
660	000	
C01	000	171613020404040
664	820	
663	646	
664		PS11=G271
665		PSI2=G3/T
666		ALPHA=2.GDJ/(1.GCO+PSI1+PSI1+PSI2+PSI2)
667		GO TC 880
668	860	ALPHA=2.303
669		PSI1=0.dDQ
679		PS12=0.40C
671	880	IF(I-0)900-96v-900
672	95.0	IF(1+P)927 - 64 - 920
472	020	
474	724	
C (4 / 7 C	<u>.</u>	
610	94_	
010	20~	
677		U0 154) J=1+N
678		T=PSI1#A(IJ+1)
679		IF(I-N1)980,1000,100C
680	98J	IP2J=IJ+2
681		T=T+PSI2*A(IP2J)
682	1.00	ETA=ALPHA+(T+A(IJ))
683		A ( I J ) = A ( I J ) = - E A
684		Δ([]+1)=Δ([]+1)-PST1=FTΔ
685		1E(1-N1)1926-1240-1040
686	1626	
400	****	
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68	87	1040	IJ=IJ+II
68	38		IF(I-N1)1980,1060,1060
68	39	1060	K =N
	90		GO TC 11CC
69	91 [–]	1080	K=I+2
69	92	1100	IP=IIP-I
69	93		00 1180 J=0+K
69	94		JIP=IP+J
69	35		JI=JIP-IA
69	36		T=PST1#A/.ITP)
69	97		IE(I-N1)1120-1140-1140
Ā	18	1120	JID2=JID4TA
Ă	a o	****	T=T+DST2#AfitD21 *
~ 74	).) ).)	3140	
70	,, ,,	1140	AIII)-AIII)_AIIIA
70	12		A(1101-A(01)-CTA=CCT)
	12		16/1_0111140 1100 1200
70	10	1140	1///TENT////OV/1100
70	15	1100	HIJIFCI-AIJIFCI-EIFFFDIC
	12	1100	
70	10		1F(I=N2)1200,1220,1220
10	) ( 	1200	J1=11+3
	28		JIP=JI+IA
/0	<u> </u>		JIP2=JIP+IA
71	10		ETA=ALPHA+PSI2+A(JIP2)
1	11.,.		A(JI)=-ETA
71	12		A(JIP)=-ETA*PSI1 .
71	13		A(JIP2)=A(JIP2)-ETA=PSI2
71	14	1220	II=IIP+1
71	15		IT=IT+1
71	L6		GO TO 60
71	17.	1240	IF(DABS(A(NN1))-DABS(A(N1N2))) 1300,1280,1280
	18	1280	IANA(N)=0
71	18 19	1280	IANA(N)≠G IANA(N1)=2
71	18 19 20	1280	IANA(N)≠G IANA(N1)=2 N=N2
71 7 7	18 19 <u>20 -</u> 21	1280	IANA(N) = G IANA(N1) = 2 N = N2 IF(N2) 14G0 + 14G0 + 20
71 	18 19 2 <u>0</u> 21 22	1280	IANA(N)=C IANA(N1)=2 <u>N=N2</u> IF(N2)14CO,14CO,2C RR(N)=A(NN)
71 	18 19 2 <u>0</u> 21 22 23	1280	IANA(N)=C IANA(N1)=2 <u>N=N2</u> IF(N2)14CO,14CO,20 RR(N)=A(NN) RI(N)=0.CDC
71 71 72 72 72 72 72 72	18 19 20 21 22 23 23	1280	IANA(N) = G IANA(N1) = 2 N=N2 IF(N2)14CO,14CO,20 RR(N) = A(NN) RI(N) = 0,CDC IANA(N) = 1
71 	18 19 20 21 22 23 24 25	1280	IANA(N) = G $IANA(N1) = 2$ $N = N2$ $IF(N2) I 4 GO + 14 GO + 20$ $RR(N) = A(NN)$ $RI(N) = 0, GOC$ $IANA(N) = 1$ $IF(N1) I 4 GO + 14 GO + 13 20$
71 71 72 72 72 72 72 72 72	18 19 20 21 22 23 24 25 26	1280 1300	IANA(N)=C IANA(N1)=2 <u>N=N2</u> IF(N2)14CO,14CO,2C RR(N)=A(NN) <u>RI(N)=0,CDC</u> IANA(N)=1 IF(N1)14CO,14CO,1320 N=N1
71 71 72 72 72 72 72 72 72 72 72 72 72	18 19 20 21 22 23 24 25 26 27	1280 1300 <u>1326</u>	IANA(N) = G $IANA(N1) = 2$ $N=N2$ $IF(N2) 14G0, 14G0, 20$ $RR(N) = A(NN)$ $RI(N) = 0$ $IANA(N) = 1$ $IF(N1) 14G0, 14G0, 1320$ $N=N1$ $GD IC 2C$
71 71 72 72 72 72 72 72 72 72 72 72 72 72 72	18 19 20 21 22 23 24 25 26 26 27 28	1280 1300 <u>1326</u>	IANA(N)=C IANA(N1)=2 N=N2 IF(N2)14CO,14CO,2C RR(N)=A(NN) RI(N)=0,CDC IANA(N)=1 IF(N1)14CO,14CO,1320 N=N1 GO TC 2C PETURN
71 71 72 72 72 72 72 72 72 72 72 72 72 72 72	18 19 20 21 22 23 24 25 26 26 27 28 28	1280 1300 <u>1326</u> 1400	IANA(N)=C IANA(N1)=2 N=N2 IF(N2)14CO,14CO,20 RR(N)=A(NN) RI(N)=0,COC IANA(N)=1 IF(N1)14CO,14CO,1320 N=N1 GO TC 2C RETURN END
71 71 72 72 72 72 72 72 72 72 72 72 72 72 72	18 19 20 21 22 23 24 25 26 27 28 29 	1280 1300 132 <u>6</u> 1400	IANA(N)=C IANA(N1)=2 N=N2 IF(N2)14C0,14C0,20 RR(N)=A(NN) RI(N)=0,CDC IANA(N)=1 IF(N1)14C0,14C0,1320 N=N1 GO TC 2C RETURN END
71 71 72 72 72 72 72 72 72 72 72 72 72 72 72	18 19 20 21 22 23 24 25 26 27 28 29 29	1280 1300 132 <u>6</u> 1400	IANA(N)=C IANA(N1)=2 N=N2 IF(N2)14C0,14C0,20 RR(N)=A(NN) RI(N)=0,CDC IANA(N)=1 IF(N1)14C0,14C0,1320 N=N1 GO TC 2C RETURN END
71 71 72 72 72 72 72 72 72 72 72 72 72 72 72	18 19 20 21 22 23 24 25 26 27 28 29	1280 1300 132 <u>6</u> 1400	IANA(N)=C IANA(N1)=2 N=N2 IF(N2)14C0,14C0,20 RR(N)=A(NN) RI(N)=0,CDC IANA(N)=1 IF(N1)14C0,14C0,1320 N=N1 GO TC 2C RETURN END
	18 19 20 21 22 23 24 25 26 27 28 29	1280 1300 <u>1326</u> 1400	IANA(N)=C IANA(N1)=2 N=N2 IF(N2)14CO,14CO,2C RR(N)=A(NN) RI(N)=A(NN) RI(N)=0,CDC IANA(N)=1 IF(N1)14CO,14CO,1320 N=N1 GO TC 2C RETURN END
71 71 72 72 72 72 72 72 72 72 72 72 72 72 72	18 19 20 21 22 23 24 25 26 27 28 29	1280 1300 132 <u>6</u> 1400	IANA(N)=C IANA(N1)=2 N=N2 IF(N2)14CO,14CO,2CO RR(N)=A(NN) RI(N)=0,CDC IANA(N)=1 IF(N1)14CO,14CO,1320 N=N1 GO TC 2C RETURN END
	18 19 20 21 22 23 24 25 26 27 28 29 	1280 1300 132 <u>6</u> 1400	IANA(N)=C IANA(N1)=2 N=N2 IF(N2)14CO.14CO.20 RR(N)=A(NN) RI(N)=0.COC IANA(N)=1 IF(N1)14CO.14CO.1320 N=N1 GO TC 2C RETURN END
	18 19 20 21 22 23 24 25 26 27 28 29 	1280 1300 132 <u>6</u> 1400	IANA(N)=C IANA(N)=C N=N2 IF(N2)14C0,14C0,20 RR(N)=A(NN) RI(N)=0,CD0 IANA(N)=1 IF(N1)14C0,14C0,1320 N=N1 GO TC 2C RETURN END
71 71 72 72 72 72 72 72 72 72 72 72 72 72 72	18 19 20 21 22 23 24 25 26 27 28 29 	1280 1300 <u>1326</u> 1400	IANA(N)=C IANA(N1)=2 N=N2 IF(N2)14C0,14C0,20 RR(N)=A(NN) RI(N)=0,CDC IANA(N)=1 IF(N1)14C0,14C0,1320 N=N1 GO TC 2C RETURN END
	18 19 20 21 22 23 24 25 26 27 28 29 	1280 1300 <u>1326</u> 1400	IANA(N)=C IANA(N1)=2 N=N2 IF(N2)14CO,14CO,2C RR(N)=A(NN) RI(N)=0,CDC IANA(N)=1 IF(N1)14CO,14CO,1320 N=N1 GO TC 2C RETURN END
	18 19 20 21 22 23 24 25 26 27 28 29 28 29	1280 1300 132 <u>6</u> 1400	IANA(N)=C IANA(N1)=2 N=N2 IF(N2)14CO,14CO,2C R(N)=A(NN) R(N)=A(NN) RI(N)=0,GOC IANA(N)=1 IF(N1)14CO,14CO,1320 N=N1 GO TC 2C RETURN END
	18 19 20 21 22 23 24 25 26 27 28 29 	1280 1300 132 <u>6</u> 1400	IANA(N)=C IANA(N1)=2 N=N2 IF(N2)14CO,14CO,20 RR(N)=A(NN) RI(N)=O,COC IANA(N)=1 IF(N1)14CO,14CO,1320 N=N1 GO TC 2C RETURN END
	18 19 20 21 22 23 24 25 26 27 28 29 	1280 1300 132 <u>6</u> 1400	IANA(N)=C IANA(N)=C N=N2 IF(N2)14C0,14C0,20 RR(N)=A(NN) RI(N)=0,CD0 IANA(N)=1 IF(N1)14C0,14C0,1320 N=N1 GO TC 2C RETURN END
	18 19 20 21 22 23 24 25 26 27 28 29 	1280 1300 1326 1400	IANA(N)=C IANA(N)=C N=N2 IF(N2)14C0,14C0,20 RR(N)=A(NN) RI(N)=0,CDC IANA(N)=1 IF(N1)14C0,14C0,1320 N=N1 GO TC 2C RETURN END
	18 19 20 21 22 23 24 25 26 27 28 29 	1280 1300 132 <u>6</u> 1400	IANA(N)=C IANA(N1)=2 N=N2 IF(N2)14C0,14C0,20 RR(N)=A(NN) RI(N)=0,CDC IANA(N)=1 IF(N1)14C0,14C0,1320 N=N1 GO TC 2C RETURN END
	18 19 20 21 22 23 24 25 26 27 28 29 	1280 1300 132 <u>6</u> 1400	IANA(N)=C IANA(N1)=2 N=N2 IF(N2)14CO,14CO,2C R(N)=A(NN) R(N)=0,CDC IANA(N)=1 IF(N1)14CO,14CO,1320 N=N1 GO TC 2C RETURN END
	18 19 20 21 22 23 24 25 26 27 28 29 	1280 1300 132 <u>6</u> 1400	IANA(N)=C IANA(N1)=2 N=N2 IF(N2)14CO,14CO,2C R(N)=A(NN) RI(N)=0,GOC IANA(N)=1 IF(N1)14CO,14CO,1320 N=N1 GO TC 2C RETURN END
	18 19 20 21 22 23 24 25 26 27 28 29 	1280 1300 132 <u>6</u> 1400	IANA(N)=C IANA(N)=C N=N2 IF(N2)14C0,14C0,20 R(N)=A(NN) R(N)=0,CD0 IANA(N)=1 IF(N1)14C0,14C0,1320 N=N1 GO TC 2C RETURN END
	18 19 20 21 22 23 24 25 26 27 28 29 	1280 1300 1326 1400	IANA(N)=C IANA(N)=C N=N2 IF(N2)14C0,14C0,20 R(N)=A(NN) RI(N)=0,CDC IANA(N)=1 IF(N1)14C0,14C0,1320 N=N1 GO TC 2C RETURN END
	18 19 20 21 22 23 24 25 26 27 28 29 	1280 1300 132 <u>6</u> 1400	IANA(N)=C IANA(N1)=2 N=N2 IF(N2)14CO,14CO,2C R(N)=A(NN) RI(N)=0,CDC IANA(N)=1 IF(N1)14CO,14CO,1320 N=N1 GO TC 2C RETURN END

.

730		SUBREUTINE MSC(AAAA,NS,ASQR)				
	C					
	с	COMPLTES ASCR = AAAA+AAAA				
	C	·				
731	·	DIMENSION AAAA(9),ASCR(3,3)		`		and the second
732		DOUBLE PRECISION AAAA, ASCR	•	•		
733		DO 100 I=1.NS				
734		00 100 J=1,NS				
735		ASOR(1,J)=C.GC0			•	
736		DO 100 K=1.NS				
737		K1=NS+(J-1)+K			****	
738		K2=NS+(K-1)+I				
739	100	ASOR(I,J) = ASOR(I,J) + AAAA(K1) + AAAA(K2)				•
740		RETURN				
741		END				
				+		

747		SUBDRITING STONECTIVE. A R. L. TOEL, YE. YI. VP. VI. REALES	
142		SUBRELING EIGTEGUITE, AJ DJ AJ INCAJ ANJ ALT ANJ ALT AGOMET	
	c	CLODEL, RET REAT IZ SHIT CORRECTED CONTRACTOR AND	- s
	r r	SUBRELING TE TING THE ELECTICS OF A CONSTRUCTION AND AND AND AND AND AND AND AND AND AN	
	5	COLTON THE CONTRACTOR STRACTS THERAFTER FEILURA	
	<u>د</u>	CONTROL IVE CLEE IS	F 2 F 5 1
	ι.	I FIND UNLY THE REGULAR EIGENVELTURS (A A - LAMDUA A)	53
	C	2 FIND UNLY THE TRANSPESED FIGENVECTORS (AT V = LAMBUA V	162
_	C	3 FINC BOTH TYPES OF EIGENVECTORS.	55.
743		CIMENSION A(3,3), E(3,3), K(3,2), XR(3), XI(3), VR(3), VI(3), IRON(3,2)	
744		DOUBLE PRECISION RCOTR, RCCTI, RCCTR, RCCTIE, TEMP, TEMP2, AMAX, C1, C2,	
	-	1 SW1+W+XR+XI+VR+V[+B+ZERC+ECERR+A	
745		DOUBLE PRECISION CABS,CSIGN,CSGRT,EMAX1	
746		INTEGER COUNT, COUNTE, T2	5.5
747		101=1	
748		103=3	
749		BCOTR = BCCTRF	= S '
750		Prost = Proste	ŝ
751			
752			
752		FE - FOF - L	÷.
754			24
734		$\mathbf{N}\mathbf{P}\mathbf{I} = \mathbf{N} + \mathbf{I}$	1.2
. (22)	•	IVCI = IVC - I	12.
756		1VC2 = 1VC1 - 1	
757		COUNT = 1	r, S
_758	_	DD 4CG I=1+N	
759		w([,])=G.CBC	
760		XR(I)=0.CDC	
761	400	CONTINUE	
762		$CLIM = 1 \cdot CE - 4$	Ξ2.
763		IF(RQOTI) 1, 63, 1	£5'
	C		E۵
	C	CCMPLEX EIGENVALUE.	нS1
	C		15
764	-	1  TEMP = -  RCCTR -  RCCTR	= S
765			=S1
766		TEMP2=RUGTR #RECTR #RECTI#RECTI	
767			= S
768		DD = 666 I = 1. N	÷ 5*
769		IE(T2) 600, 673, 600	
770	60		- c
771			
772			e.
772	40		20
774			
775	60	NEAD VIEL VNACLIII LE A 1922/1	
772	60	2 0(1, j) = A(1, j) = 1644 + M(1, j) =	- < +
	<u> </u>		22
111		2 80 804 J 7 19 N	13 20
118	60	4 B(I}J) → A(I)J)#IEPP + B(I)J	53
		D B(1+1) = B(1+1) + 1EPP2	->
780	60	6  A(I,I) = A(I,I) - RCCTR	25
781		IF(T2 .NE. G) REWIND T2	ΞΞ,
_782_		GC TC 7CO	- 5
783	60	17 IF(ICC) 622, 608, 622	ES
	C		ĽS
	Ç	MATRIX SINGULAR.	£S

	C			٤S
	784	622	IF(IVC2) 623, 625, 623	ΕS
	785	623	BC 624 LL = 1 + N	ES.
	786		W(LLy2)=0+CDC	
•	787	624	XI(LL)=C-ODC	-
	788		IF(IVC1) 625. 514. 625	ES'
	789	625	$DG 626 11 = 1 \cdot N$	ES.
	790			
	791	626	VI(11)=0-CD3	
	792		6C TC 511	ES
-	<u> </u>	- •		55.
	č		MATRIX NOT SINGULAR.	ES.
	· C			ĒŜ,
	793	808	CO 609 LL = 1, N	ES'
	794		$w(LL_1) = 1.500$	
	795		h(LL,2) = 1.00	
-	796		W(LL,3)=1.CDC	
	797 0	609	h(LL,4)=1.000	
	798	699	IF(IVC2) 61C, 612, 610	ESI
	799	610	CO 611 I = 1, N	`ΕS`
	800		$I2 = IRCw(I_{2})$	ē\$'
	801		$XI(I2) = W(I_{+1}) * RCCTI$	ēS1
•	802		$CC \ 611 \ J = 1, N$	ESI
	803	611	X1(12) = X1(12) + A(1.3) + W(3.2)	ES'
	804		IF(IVC1) 612. 500. 612	ES
	805	612	DC 613 I = 1 • N	ÉŚ
	806		$VI(I) = W(I \cdot 3) \cdot RCGTI$	ES
	807		$D0 613 J = 1 \cdot N$	ES
	808	613	$VI(I) = VI(I) + A(J_{1}) + W(J_{2})$	ES'
	809		GO TO 499	ES
	810	615	CFRR = 0.0	ES
•	811 ~		DCERR=0_000	
	812		IF(INC2) 616. 619. 616	esi
	813	616	$RO 618 T \approx 1. N$	651
	814		XR(1) = -W(1,2)	ĒŚ
	815		DR 617 J $\Rightarrow$ 1. N	ES'
	816	617	XR(T) = XR(T) + A(T,I) = XI(J)	FS
-	817	618	XR(I) = XR(I)/RGCII	251
	818		IF(IVC1) 615. 633. 619	ES
	819	619	CD = 623 T = 1 - N	= 51
	820	- # T	$VR(\mathbf{I}) = -W(\mathbf{I}, 4)$	ĒŠ
	821		DC 620 J = 1. N	ESI
	822	620	VR(T) = VR(T) + A(I,T) * VT(I)	ĒĊ
	823	621	VR(1) = VR(1)/RGC11	= = 5 \
	с. С			551
	ř		SEARCH VECTORS FOR LARGEST FLEMENT AND NORMALIZE.	
	č		Contraction and the following the second second	= 51
	824	27		
	825		B0.629.1 = 1.N	= 51
-	826		TEMP = VR(1) + 2 + VT(1) + 2	201
	827		F(TFMD = AWAX) 625 629 628	FCI
	828	628	$\Delta M \Delta X = TEMP$	201
-	829	JĘJ		Ēč
	830	679	CONTINUE	- gev
	831	927		500
			NA TOTAN (COMPA	

832		C2 = -VI(I2)/AMAX	121
833		CC 630 L = 1 + N	251
834		TEMP = VI(L)	: 51
835		VI(1) = VR(1) + C2 + TEMP + C1	25
836	630	VR(1) = VR(1) + C1 - TEMP + C2	- 5'
837	00.3		20
020			
020	(3)	b = 0 = 0 = 1 + R	7.3
839	631	CCERR=DPAXI(ECERR+DAPS(VRILL)-W(LL+3))+LAES(VI(LL)-W(LL+4))	
840	632	IF(IVC2) 633, 638, 633	55
84I	633	AMAX=0.000	
842		CO 635 L = 1, N	2 S '
843		$TEMP = XR\{L\} + + 2 + XI\{L\} + + 2$	2S'
844		IF(TEMP - AMAX) 635, 635, 634	15,
845	634	AMAX = TEMP	551
846			151
847	635	CENTINUE	יצב -
848		C1 = XR(12)/AMAX	C \$1
849		$C_2 = -x_1(T_2)/\Delta M \Delta x$	Ξ¢.
850			Ę (,
050			
021		TMP  = XI(L)	- 2
852		$X_1(L) = X_1(L) + (L) $	23
853	636	XK(L) = XK(L) + C1 - 1EMP + C2	= 2
854		1F(CCUNT •ER• 1) GC TC 646	÷5,
855		CO 637 LL = 1, N	a 5 '
856	637	DCERR=DMAX1(DCERR,DA0S(XR(LL)-W(LL,I)),CAPS(XI(LL)-W(LL,2)))	
	С		15
	С	TEST FOR CONVERGENCE.	ЕS
	C		151
857	638	IF(CCUAT .EC. 1) GC TC 646	35
858		CERR=DCER3	
859		LE(CERR .GE. 1.)E-4) GC TG 639	5
860		LE(CER2 .GE. CLIM) GC TC 648	551
861			
942			
002	620		
002	717	$\frac{1}{2} \left( \frac{1}{2} \right) = \frac{1}{2} \left( \frac{1}{2} \right) = \frac{1}$	
864	647		: 2
865		IF(RLUII) 642, 673, 642	= 2
866	642	1+(1V(2) 64., 644, 64.	= 5
867	649	UU 641 LL = 1, N	- 51
868		h(LL,1) = X(LL)	- ÷
869	641	w(LL,2) = xI(LL) DEPRO	÷ 5
870		IF(IVC1) 644, 613, 644	- 5 -
871	644	$CC 645 LL = 1, N \setminus NV'$	- 5
872		W(LL,3) = VR(LL)	221
873	645	h(LL,4) = VI(4L)	- 5
874	•••		
875	646	CEBR = 1.5	÷.
876	0.0		, -
010			
011	22.0	1) (100) 0404 0474 040 CDD - CCDD	
8/8	048		
879			- 5
68.)		IF(REGII) 667, 668, 667	. 5
881	667	CC 649 I = 1, N	۳S
882	649	A(I,I) = A(I,I) + RCCTR	<del></del>
000			⊢ <b>S</b>
683		NETONA	·

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$ \begin{array}{c} C \\ C $	884 885	68 C	PRINT 1C1. RCCTR, ROCTI, CERR GO TC 648	85' 85'
886       60 [5% = 1       [55]         887		Č.	REAL EIGENVECTORS.	<u>ES</u> ,
888       C0 65C J = 1, N       E3         889       65G 81(J, J) = A(I, J)       E5         890       651 8(J, I) = B(I, I) = RCTR       E5         891       GO TO 700       E5         892       652 IF(ICC) 680, 665, 680       E5         C       SINGULÄR #ATRIX.       E5         C       SINGULÄR #ATRIX.       E5         893       680 IF(IVC2) 681, 683, 681       E5         894       661 00 682 L = 1, N       E5         895       662 XI(L)=0.00C       E5         896       664 VI(L)=0.00C       E5         897       683 D0 654 L = 1, N       E5         898       664 VI(L)=0.00C       E5         899       GO TC 51L       E5         C       MATRIX NOT SINGULAR.       E5         900       653 IF(IVC2) 653, 656, 653       E5         901       653 00 654 L = 1, N       E5         902       654 XI(L)=1.000       E5         903       IF(IVC1) 655, 50C, 656       E5         904       656 00 657 L = 1, N       E5         905       657 VI(L)=1.000       E5         906       GO TC 459       E5         907       655 CERR = C.C	886 887	00	ISW = 1 $DC 651 I = 1 \cdot N$	ES'
8901       691       691       601       700       652         892       652       IF(ICC) 680, 685, 680       ES         C       SINGULAR MATRIX.       SS         893       680       IF(IVC2) 681, 683, 681       ES'         894       661       06 682       1.1.1       863       681         894       661       06 682       1.1.1       863       681       ES'         895       682       XI(L)=0.000       ES'       ES'       ES'         896       IF(IVC1) 683, 514, 683       ES'       ES'       ES'         897       683       D6 684       L = 1, N       ES'         898       664       VI(L)=0.00C       ES'       ES'         900       665       IF(IVC1) 652, 656, 653       ES'       ES'         901       653       D0 654       L = 1, N       ES'         902       654       XI(L)=1.4000       ES'       ES'         903       IF(IVC1) 656, 500, 656       ES'       ES'       ES'         904       656       D06 671       = 1, N       ES'       ES'         905       657       VI(L)=1.4000       ES'       ES'       E	888 889	650	$\begin{array}{l} UU \ 65C \ J = I_{F} \ N \\ B(I_{F}J) = A(I_{F}J) \\ B(I_{F}J) = DCCTD \\ D(I_{F}J) = DCCTD \\ D(I_{F}J$	52,
G. 2       G. 2       SINGULÄR WATRIX.       ES.         C       SINGULÄR WATRIX.       ES.         893       680       IF(IVC21 681, 683, 681.       ES.         894       681 D0 682 L = 1, N       ES.         895       682 XI(L)=0.000       ES.         896       IF(IVC21 683, 514, 683.       ES.         897       683 DD 664 L = 1, N       ES.         898       664 VI(L)=0.00C       ES.         899       GG TC 511       ES.         C       MATRIX NOT SINGULAR.       ES.         C       MATRIX NOT SINGULAR.       ES.         900       665 IF(IVC21 653, 656, 653       ES.         901       653 DD 654 L = 1, N       ES.         902       654 XI(L)=1.600.       ES.         903       IF(IVC1) 656, 500, 656       ES.         904       656 DD 657 L = 1, N       ES.         905       657 VI(L)=1.600.       ES.         906       GO TD 459       ES.         907       655 CERR = C.C       ES.         908       DCEERR=0.000.       ES.         910       657 CI = TEMP       ES.         911       C2       C.G. 000       ES. <t< td=""><td>891 892</td><td><u>. 021</u>. 652</td><td>$\frac{0}{10} \frac{1}{10} = \frac{0}{10} \frac{1}{10} \frac{1}{10} = \frac{0}{10} \frac{1}{10} \frac{1}{10} = \frac{0}{10} \frac{1}{10} \frac{1}{10} \frac{1}{10} = \frac{0}{10} \frac{1}{10} \frac{1}{10} \frac{1}{10} = \frac{0}{10} \frac{1}{10} \frac{1}{$</td><td>ES' ES'</td></t<>	891 892	<u>. 021</u> . 652	$\frac{0}{10} \frac{1}{10} = \frac{0}{10} \frac{1}{10} \frac{1}{10} = \frac{0}{10} \frac{1}{10} \frac{1}{10} = \frac{0}{10} \frac{1}{10} \frac{1}{10} \frac{1}{10} = \frac{0}{10} \frac{1}{10} \frac{1}{10} \frac{1}{10} = \frac{0}{10} \frac{1}{10} \frac{1}{$	ES' ES'
c       ES         893       680 [F(1VC2] 681, 683, 681       ES         894       681 D0 682 L = 1, N       ES         895       682 X1(L)=0.000       ES         896       IF(1VC1) 683, 514, 683       ES         897       683 D0 684 L = 1, N       ES         898       684 V1(L)=0.000       ES         899       GG TC_511       ES         c       MATRIX NOT SINGULAR.       ES         900       653 D0 654 L = 1, N       ES         901       653 D0 654 L = 1, N       ES         902       654 X1(L)=1.4000       ES         903       IF(1VC1) 656, 500, 656       ES         904       656 D0 657 L = 1, N       ES         905       657 V1(L)=1.4000       ES         906       GO TO 459       ES         907       655 CERR = 0.0       ES         908       DCERR=0.000       ES         910       658 C1=0.000       ES         911       C2=0.000       ES         912		c	SINGÜLÄR MATRIX.	E\$' E\$'
894       681 D0 682 L = 1, N       63         895       682 XI(L)=0.0D0       683.514.683       655         897       683 D0 664 L = 1, N       655         898       684 VI(L)=0.0DC       655         899       GC TC.511       655         C       MATRIX NOT SINGLLAR.       655         901       653 D0 654 L = 1, N       655         902       654 XI(L)=L600       655         903       IF(IVC1) 655.665.653       655         904       650 D0 654 L = 1, N       655         905       657 VI(L)=La000       655         906       GO TO 459       655         C       NCRMALIZE REAL VECTORS.       655         907       655 CERR = C.C       655         908       DCER = 0.0DQ       557         909       IF(IVC2) 658.662.658       659         909       IF(IVC2) 659.660.659       651         914       IF(TEMP - C1).660.659       651         915       659 C1 = TEMP       651         918       DC 660 L = 1.N       851         918       DC C601 L = 1.N       851         919       XI(L) = XI(L)/C2       653         920       DCERRED	893	C 68 <u>0</u>	<u>IF(IVC2) 681, 683, 681</u>	ES' ES'
897       683 CO 664 L = 1, N       ES         898       664 VI(L)=0.00C       ES         C       MATRIX NOT SINGULAR.       ES         C       MATRIX NOT SINGULAR.       ES         900       6.65 IF(IVC2) 653, 656, 653       ES         901       653 DO 664 L = 1, N       ES         902       654 XI(L)=1.600       ES         903       IF(IVC1) 656, 500, 656       ES         904       656 DO 657 L = 1, N       ES         905       657 VI(L)=1.000       ES         906       GO TO 459       ES         907       655 CERR = 0.C       ES         908       DCERR=0.000       ES         909       IF(IVC2) 658, 662, 658       ES         907       655 CERR = 0.C       ES         908       DCERR=0.000       ES         910       658 D1=C.00C       ES         911       C2=C.00C       ES         912       E0 660 L = 1, N       ES         913       TEMP=DABS(XI(L))       ES         914       IF(IPP - C1) 660, 660, 659       ES         918       ET 661 L = 1, N       ES         919       XI(L) = XI(L)/C2       ES	895 895	682	DU 682 E = 1, N XI(L)=0.0DC IF(IVC)) 683. 514. 683	ES'
899       GG TC 511       ES         C       MATRIX NOT SINGULAR.       ES         000       645 IF(1VC2) 653, 656, 653       ES         901       653 D0 654 L = 1, N       ES         902       654 X1(L)=LCDQ       ES         903       IF(1VC1) 656, 500, 656       ES         903       IF(1VC1) 656, 500, 656       ES         903       IF(1VC1) 656, 500, 656       ES         904       656 D0 657 L = 1, N       ES         905       657 V1(L)=L0DQ       ES         906       GO TO 459       ES         C       NCRMALIZE REAL VECTORS.       ES         907       655 CERR = 0.0DQ       ES         908       DCERR=0.0DQ       ES         909       IF(1VC2) 658, 662, 658       ES         901       658 C1=0.0DC       ES         911       C2=0.0D2       ES         912       D0 660 L = 1, N       ES         913       TEMP=DABS(XI(L))       ES         918       DC 661 L = 1, N       ES         919       X1(L) = X1(L)/C2       ES         920       DCERR=0MAXI/DEER,DABS(XI(L)-XR(L))       ES         921       661 L = 1, N       ES </td <td>897 898</td> <td></td> <td>CC 664 L = 1, N •VI(L)=0.0DC</td> <td>EŚ</td>	897 898		CC 664 L = 1, N •VI(L)=0.0DC	EŚ
C       MATRIX NOT SINGLLAR.       ES         G00       685 IF(1VC2) 653, 656, 653       ES         901       653 D0 654 L = 1, N       ES         902       654 X1(L)=1.000       ES         903       IF(1VC1) 656, 500, 656       ES         904       656 D0 657 L = 1, N       ES         905       657 V1(L)=1.000       ES         906       G0 T0 459       ES         C       NCRMALIZE REAL VECTORS.       ES         C       NCRMALIZE REAL VECTORS.       ES         906       DCERR=0.000       ES         907       655 CERR = C.C       ES         908       DCERR=0.000       ES         911       C2 c.002       ES         912       D0 660 L = 1, N       ES         913       TEMP=DABS(XI(L))       ES         914       IF(1PP - C1) 660.600.659       ES         915       659 C1 = TEMP       ES         918       DC 661 L = 1, N       ES         919       X1(L) = XI(L)/C2       ES         920       DC 661 L = 1, N       ES         921       661 XR(L) = XI(L)/C2       ES         922       IF(1VC1) 662, 63E, 662       ES	899	Ċ	GG TC <u>511</u>	E <u>S</u> ` ES`
300       685       1F(1VC21 653, 656, 653       65         901       653 00 654 L = 1, N       55         903       1F(1VC1) 656, 500, 656       65         904       656 00 657 L = 1, N       55         905       657 V1(L)=1.000       55         906       60 TO 459       55         C       NCRMALIZE REAL VECTORS.       65         C       NCRMALIZE REAL VECTORS.       65         907       655 CERR = C.C       55'         908       DCERR=0.000       55'         909       IF(1VC2) 658, 662, 658       55'         907       655 CERR = C.C       55'         908       DCERR=0.000       55'         909       IF(1VC2) 658, 662, 658       55'         910       658 C1=C.00C       55'         911       C2=C.002       55'         912       DC 660 L = 1, N       55'         913       TEMP=DABS(XI(L))       55'         914       IF(TEMP - C1) 660, 660, 659       55'         915       659 C1 = TEMP       5'         918       DF 661 L = 1, N       5'         921       661 XR(L) = XI(L)/C2       5'         923       662 C2=0.00C </td <td>••• • •</td> <td>C C</td> <td>MATRIX NOT SINGLEAR.</td> <td>ES</td>	••• • •	C C	MATRIX NOT SINGLEAR.	ES
303       IF(IVCI) 656, 500, 656       ES         903       IF(IVCI) 656, 500, 656       ES         904       656 D0 657 L = 1, N       ES         905       657       VI(L)=1,000       ES         906       G0 TO 459       ES         C       ES       ES         C       NCRMALIZE REAL VECTORS.       ES         907       655 CERR = C.C       ES         908       DCERR=0.000       ES         909       IF(IVC2) 658, 662, 658       ES         910       658 C1=C.00C       ES         911       C2=C.002       ES         912       E0 660 L = 1, N       ES         913       TEMP=DABS(XI(L))       ES         914       IF(TEMP - C.1) 660, 660, 659       ES         915       659 C1 = TEMP       ES         916       C2 = XI(L)       ES         917       660 CNIINUE       ES         919       XI(L) = XI(L)/C2       ES         919       XI(L) = XI(L)/C2       ES         921       661 XR(L) = XI(L)       ES         922       G62 C2=0, CDC       ES         923       662 C2=0, CDC       ES         9	900 901 902	653 654	$\frac{1}{1000} = \frac{1}{1000} = \frac{1}{1000} = \frac{1}{1000} = \frac{1}{10000} = \frac{1}{10000000000000000000000000000000000$	ESV
905       457       VI(L)=1.0D0       FS         906       GO TO 459       FS         C       NCRMALIZE REAL VECTORS.       ES         907       655 CERR = C.C       ES         908       DCCERR=0.0D0       ES         909       IF(1VC2) 658, 662, 658       ES         910       658       C1=C.0DC         911       C2=C.0D2       ES         912       E00 660 L = 1, N       ES         913       TEMP=DABS(XI(L))       ES         914       IF(TEMP - C1) 660, 660, 659       ES         915       659 C1 = TEMP       ES         916       C2 = XI(L)       ES         917       660 CCNIINUE       ES         918       DF 661 L = 1, N       ES         919       XI(L) = XI(L)/C2       ES         921       661 L = XI(L)/C2       ES         922       IF(1VC1) 662, 638, 662       ES         923       662       C2=0.0DC         924       C1=C.0DC       ES         923       662       C2=0.0DC         924       C1=C.0DC       ES         925       CD 664 L = 1, N       ES         926       TEM	903 904		IF(IVC1) 656, 500, 656 DO 657 L ≠ 1, N	ES' ES'
C         NCRMALIZE REAL VECTORS.         ES           907         655 CERR = C.C         ES           908         DCERR=0.0DQ         ES           909         IF(1VC2) 658, 662, 658         ES           910         658 C1=C.0DC         ES           911         C2=C.0D2         ES           912         DD 660 L = 1, N         ES           913         TEMP=DABS(XI(L))         ES           914         IF(TEMP - C1) 660, 660, 659         ES           915         659 C1 = TEMP         ES           916         C2 = xI(L)         ES           917         660 CCNIINUE         ES           920         DCERR=DMAX1(DCERR, DABS(XI(L)-XR(L)))         ES           921         661 L = 1, N         ES           922         IF(1VC1) 662, 63E, 662         ES           923         662 C2=0.0CC         ES           924         C1=0.0DC         ES           925         CD 664 L = 1, N         ES           926         TEMP=DABS(VI(L))         ES	905 906	657	VI(L)=1.0D0 GO TŨ 459	ES
907       655 CERR = C.C       ES'         908       DCERR=0.0D0       ES'         909       IF(IVC2) 658, 662, 658       ES'         910       658 C1=C.0DC       ES'         911 $C2=C.0D2$ ES'         912       EO 660 L = T, N       ES'         913       TEMP=DABS(XI(L))       ES'         914       IF(TEMP - C1) 660, 660, 659       ES'         915       659 C1 = TEMP       ES'         916       C2 = XI(L)       ES'         917       660 CCNIINUE       ES'         918       EC 661 L = 1, N       ES'         920       DCERR=DMAX1(DCERR,DABS(XI(L)-XR(L)))       ES'         921       661 XR(L) = XI(L) /C2       ES'         922       IF(IVC1) 662, 63E, 662       ES'         923       662 C2=0.0DC       ES'         924       C1=0.0DC       ES'         925       CD 664 L = 1, N       ES'         926       TEMP=DABS(VI(L))       ES'	• 	с с	NCRMALIZE REAL VECTORS.	ES ES FS
909       IF(IVC2) 658, 662, 658       ESN         910       658       C1=C.0DC         911 $C2=C_{\bullet}$ 0D2       ESN         912       EO 660 L = 1, N       ESN         913       TEMP=DABS(XI(L))       ESN         914       IF(TEMP - C1) 660, 660, 659       ESN         915       659       ESN         916       C2 = XI(L)       ESN         917       660       CONTINUE         919       XI(L) = XI(L)/C2       ESN         919       XI(L) = XI(L)/C2       ESN         920       DCERR=DMAX1(DCERR, DABS(XI(L)-XR(L)))       ESN         921       661 XR(L) = XI(L)       ESN         922       IF(IVC1) 662, 63E, 662       ESN         923       662 C2=0, CDC       ESN         924       C1=C.0DC       ESN         925       CD 664 L = 1, N       ESN         926       TEMP=DABS(VI(L))       ESN	907 908	655	CERR = C.C DCERR=0.000	ES
911 $C2=C_0OC$ 912 $CO \ 660 \ L \neq 1$ , N       ESN         913 $TEMP=DABS(XI(L))$ ESN         914 $IF(TEMP - C1) \ 660, \ 660, \ 659$ 915 $659 \ C1 = TEMP$ ESN         916 $C2 = xI(L)$ ESN         917 $660 \ CCNTINUE$ ESN         918 $DC \ 661 \ L \neq 1$ , N       ESN         919 $XI(L) = XI(L)/C2$ ESN         920 $DCERR=DMAX1(DCERR, DABS(XI(L)-XR(L)))$ ESN         921 $661 \ XR(L) = XI(L)$ ESN         922 $IF(IVC1) \ 662, \ 63E, \ 662$ ESN         923 $662 \ C2=0, CDC$ ESN         924 $C1=C \ CDC$ ESN         925 $CD \ 664 \ L = 1, N$ ESN         926 $TEMP=DABS(VI(L))$ ESN	909 910	658	IF(1VC2) 658, 662, 658 C1=G.GDC	ESN
913 $IEMP=DABS(XI(LT))$ 914 $IF(TEMP - C_1) \ 660, \ 660, \ 659$ 915       659         916 $C2 = XI(L)$ 917       660         918 $DC \ 661 \ L = 1, \ N$ 919 $XI(L) = XI(L)/C2$ 920 $DCERR=DMAXI(DCERR, DABS(XI(L)-XR(L)))$ 921       661 $XR(L) = XI(L)$ 922 $IF(IVC1) \ 662, \ 63E, \ 662$ 923       662 $C2 = 0, CDC$ 924 $C1 = C, ODC$ 925 $CD \ 664 \ L = 1, \ N$ 926 $TEMP = DABS(VI(L))$	911 912		<u>C2=C.0D3</u> C0 660 L ≠ 1, N	ES
916 $C2 = xI(L)$ ES         917       660 $CCNTINUE$ ES         918 $BC$ 661 $x = 1$ , N       ES         919 $XI(L) = xI(L)/C2$ ES       ES         920 $DCERR=DMAX1(DCERR, DABS(XI(L)-XR(L)))$ ES       ES         921       661 $XR(L) = xI(L)$ ES         922 $IF(IVC1)$ 662, 638, 662       ES         923       662 $C2=0, CDC$ ES         924 $C1=C, CDC$ ES       ES         925 $CD$ 664 $L = 1, N$ ES         926 $TEMP=DABS(VI(L))$ ES       ES	913 914 015	- <u>65</u> 6	IF(TEMP - C1) = 60 = 60 = 659	ES)
918 $DC 661 L \neq 1, N$ ES1         919 $XI(L) = XI(L)/C2$ ES'         920 $DCERR=DMAX1(DCERR, DABS(XI(L)-XR(L)))$ ES'         921       661 XR(L) = XI(L)       ES'         922       IF(IVC1) 662, 638, 662       ES'         923       662 C2=0.0DC       ES'         924       C1=C.0DC       ES'         925       CD 664 L = 1, N       ES'         926       TEMP=DABS(VI(L))       ES'	916 917	660	C2 = XI(L) CCNTINUE	ES) 
920       DCERR=DMAX1(DCERR,DABS(X1(L)-XR(L)))         921       661 XR(L) = XI(L)         922       IF(IVC1) 662, 638, 662         923       662 C2=0.0CC         924       C1=0.0DC         925       DD 664 L = 1, N         926       TEMP=DABS(VI(L))	918 919		BC 661 L = 1, N XI(L) = XI(L)/C2	ES1 ES1
923 $662$ $C2=0.0DC$ $23$ $924$ $C1=0.0DC$ $25$ $925$ $CD.664L = 1.N$ $ESN$ $926$ TEMP=DABS(VI(L)) $ESN$	920 921	661	DCERR=DMAX1(DCERR,DABS(XI(L)-XR(L))) $XR(L) = XI(L)$ $Tr(L) = XI(L)$	ESI
925 CO 664 L = 1, N ESN 926 TEMP=DABS(VI(L))	922 923	662	1+(1VC1) 662, 632, 662 C2=0.600	<b></b>
	925 9 <u>2</u> 6		CO 664 L = 1, N TEMP=DABS(VI(L))	ESI

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0.07			
927		IF(1EMP - CI) 664, 664, 663	ES)
928	663	CT = TEMB	Ē٤١
929		C2 = VI(L)	ES۱
930_	664	CONTINUE	ESY
931		$CO \ 665 \ LL = 1. N$	ESI
932		VI(LL) = VI(LL)/C2	ESY
933		DCERP = CMAY1 (DCERP - CARS (VT (11) - W(11 - 1)))	
034	~ `	DELAN-DERATIOLAN DERESTATICLI-AGELLIZZ	•
025	445	M ( L ) L ) M ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N ( L ) N	
933	002		
430		<u>GU 1C 638</u>	E2)
937	668	IF(IVC2) 669, 671, 669	ES١
938	669	DC 670 L = 1, N	ES.
939	670	XI(L)=0.0DG	
940		IF(IVC1) 671, 79, 671	ÉSN.
941	671	$CC \ 672 \ L = 1, \ N$	ESY
942	672	VI(L)=0-CDC	
643	70	RETIRN	ES1
440	673	IE(1000) 474. 502. 474	601
777	675		C3.
342.	- 014		52.
946		12 = 1 K C H (1, 2)	521
947	675	XI(I2) = XR(I)	<u>₽</u> \$`
9 <u>48</u>			ē2`
	C		ESY
	С	BACK SUBSTITUTION SECTION.	ESY
	C.		ES
949	499	TE(1VC2) 50C, 502, 500	EST
950	567		20
051	200		601
.724 .			63
952			5
953	501	XI(1) = XI(1) - U(1+J) + XI(J)	ES.
954_	511	IF(IVCI) 532, 514, 502	FS,
955	502	CO 510 I = 1, N	ësi
956		I1 = I - 1	٤٤,
957		IF(I1) 5C3, 5L5, 5C3	ēS'
958	503	100504 J = 1, 11	EŜ
959	504	$VI(I) = VI(I) - 8(J \cdot I) * VI(J)$	'ES'
960		TE(ICC) 505- 506- 505	ES
961	565	IF(B(1,1)) 5/6, 5/7, 5/6	60
962	506	V(t) = V(t)/R(t, t)	6,00
042	200		54
	607		C 3 1
964	507	IF(VI(I)) 508, 509, 508	ES.
965	508	$V(1) = V(1) + 1 \cdot 0 + 15$	E2,
966 _	-	GO TO 510	E2,
967	509	$VI(I) = 1 \cdot C$	ESY
968	51C	CENTINUE	ES'
969		IF(IVC2) 514, 525, 514	ES'
'970	514	DC 522 I $\neq$ 1, N	651
971	•	IR = NP1 + I	Ē
972			E 6 1
972 973		171 - 10 + 1	ES.
972 973	515	$\frac{1}{12} = \frac{1}{12} + \frac{1}{12}$	ES
972 973 974	515	$   17(1 - 1) 515_{7} 517_{7} 515   12 = IR + 1   CR 516 J = 12_{7} N   17516 J = 12_{7} N $	ES ES ES
972 973 974 975	515 516	IF(I + 1) = 515, 517, 515 $I2 = IR + 1$ $CR = 516 J = 12, N$ $XI(IR) = XI(IR) - B(IR, J) + XI(J)$	ES' ES' ES'
972 973 974 975 976	515 516	IF(I - 1) 515, 517, 515 $I2 = IR + 1$ $CR 516 J = I2, N$ $XI(IR) = XI(IR) - B(IR,J) * XI(J)$ $IF(ICC) 517, 518, 517$	ES' ES' ES' ES'
972 973 974 975 976 977	515 516 517	IF(I - 1) 515, 517, 515 $I2 = IR + 1$ $CR 516 J = I2, N$ $XI(IR) = XI(IR) - B(IR,J) + XI(J)$ $IF(ICC) 517, 518, 517$ $IF(B(IR,IR)) 518, 519, 518$	ES' ES' ES' ES' ES'

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979		CO TC 522	5 C N
98.0	510	JE(VI(ID)) 527- 521, 520	ECV
900	520	$\frac{1}{100} = \frac{100}{100} = \frac{100}{100}$	E 21
992	226	$\mathbf{C} = \mathbf{C} + $	ECV
092	521	YTERDIAL ANA	· · · · · · · · · · · · · · · · · · ·
084	522	CONTINUE	
985	522	TE(1V(1) 525, 529, 525	
986	525		
997	222	10 - 101 - 1	= < \
988		10 = 10 + 1 12 = 10 + 1	ECI
999		$10.526.1 \pm 12.8$	μει. 
440	526	$vf(tR) = vf(tR) - f(t_rtR) + vf(t)$	E CI
991	200	00 527 1 = 1. N	FC.
992		12 = 1RGW(1,1)	
993	527	vR(12) = vI(L)	ESN
994		60 528 I = 1. N	541
995	528	VI(1) = VR(1)	ES\
996	529	IF(RCOTI) 615. 655. 615	EST
	с ⁷ С		ES
	č.	FACTOR FATRIX.	ESY
	č		ES
997	700	ICC = 0	ES
998		SW1=1-0072	
999		DO 701 LL = 1. N	EST
1000	701	IROW(L1.1) = LL	ES*
ີ້າດໍ່ຄຳ		DC 708 K = 1. N1	ES1
1002		AMAX=DABS(B(K-K))	201
1003		IMAX # K	ESV
1004		K1 = K + 1	ES
1005		00 702 I # KI. N	ESI
1006		TE(AMAX.GT.CARS(B(T.K))) GO TE 702	
1007		AMAX=DABS(B(T.K))	
1008		IMAX = I	EST
1009	702	CONTINUE	• ES1
1010		IF (AMAX .LT. SWL) SWL = AMAX	ESI
1011		IF(AMAX.GE.1.00-25) GO TC 723	
1012		8(K.K)=0.000	
1013		ICC = ICC + 1	ESI
1014		GO TO 7C8	ESI
1015	723	IF(IMAX .EC. K) GO TC 704	ESY
1016		DC 703 J = 1, N	ESI
1017		AHAX # B{K+J}	ESI
1018		$B\{K_{*}J\} = B\{IHAX_{*}J\}$	ESI
1019	703	B(IMAX,J) = AFAX	ESI
1020		$I2 = IROW(K_{*}1)$	ESY
1021		IROW(K,1) = IROW(INAX,1)	ESI
1022		$IRDW(IMAX_{+}1) = I2$	ESI
1023	764	DG 707 I + KL, N	ESI
1024		IF(B(I,K)) 705, 707, 705	ESI
1025	705	B(I,K) = B(I,K)/B(K,K)	ESI
1026		DO 706 J = K1, N	ESI
1027	706	$B(I,J) = B(I,J) \rightarrow B(K,J) \neq B(I,K)$	ESI
1028	707	CONTINUE	ESI
1029	708	CONTINUE States and a state of the state of	ESI
1030		AMAX=DA###################################	

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1031	IF(AMAX-1.0D-25) 712,712,713	
1032	712 B(N+N)=0.0D0	
1033	SWI=C.ODO	
1034	$\underline{ICC = ICC + 1}$	ESY
1035	GD TC 709	ESY
1036	713 [F(AMAX .LT. SW1) SW1 = AMAX	ESI
1037	709 IF(ICC .LE. ISW) GC TG 710	ESY
1038	IF(MM) 1050-1050	
1039	1051 WRITE(103.102) ICC	
1040		F۶۱
1041	R STIIGN	ECN
1042	1050 WEITE/102-10621 100	64
1043		6 C \
1044	12 - 100 / 14 / 10 / 10 / 10 / 10 / 10 / 10	- 5 \
1045	12 - INUN (LL)II 711 IOD/17 11 - 11	621
1045	$\begin{array}{cccc} 11 & 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 $	621
1040	1052 FORMATIA 224 BUT	221
TOAL	1032 FURMA 1///230 ***** RARING ***** , SUBRUUIINE EIGVEL HAS	53
	IFUURL AN EIGENVALUE OF APPARENT PULTIPLICITY,	E21
	14,/23X,* COMPUTATION OF EI	ESI
	2GENVECTOR(S) CONTINUES AT USER S OPTION'//)	ESY
1048	IDI FURMATI38HOMORE THAN 15 LCGPS FOR EIGENVECTOR OF,2E12.4,	ES,
	2 14H DIFFERENCE CF.E12.4)	551
1049	102 FORMAT(16H0++++WARKING++++) + 14+ 71H ZERCS ON DIAGONAL OF FACTORED	ES.
•	1 MATRIX. CHECK FOR FULTIPLE EIGENVALUES./20X,	٤S١
	2 SUBROUTINE EIGVEC WILL NOT PERFORM COMPUTATION FCR THIS EIGENVEC	EŞV
	3TOR •//)	٤Sı
1050	END ,	
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