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TWO-PHASE HYDROGEN DENSITY MEASUREMENTS USING AN OPEN-ENDED MICROWAVE CAVITY

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16. Abstract

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SUMMARY

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INTRODUCTION

In many applications where hydrogen density measurements are required, the hydrogen exists as a mixture of two phases: either solid-liquid (slush) or liquid-gas. The liquid-gas condition frequently occurs in liquid hydrogen transfer systems where small heat leaks cause some of the liquid to vaporize.

The density of single-phase hydrogen is normally determined by measuring its pressure and temperature and then computing its density using the equation of state for hydrogen. This procedure is unuseable when two phases are present. The pressure and temperature of a two-phase mixture do not lead to a unique density.

A second technique for determining the density of hydrogen is to measure its dielectric constant and then to compute the density using an equation such as the Clausius-Mossotti equation which relates dielectric constant to density. The dielectric constant is not directly related to the phase since it depends only on the polarizability, orientation, and spacing of the molecules.

A device suitable for measuring the dielectric constant of hydrogen is an open-ended microwave cavity (refs. 1 to 3). The cavity technique is particularly well suited for flow line applications since the body of the cavity can form a segment of the flow line and the open ends allow the hydrogen to freely pass through the cavity without an appreciable pressure drop.

Experimental results on single-phase hydrogen density measurements using an open-ended cavity have been reported previously (ref. 1). In this report, an extension of that work to two-phase hydrogen is presented to demonstrate the effectiveness of the cavity technique under limited liquid-gas, two-phase conditions. The theory of two-phase hydrogen density measurements using a microwave cavity is presented first. An experimental apparatus for producing a two-phase mixture of liquid hydrogen with small, uniformly distributed hydrogen gas bubbles is then described followed by a discussion of the associated instrumentation. Particular attention is given to insuring that all assumptions inherent in the theory are satisfied in the experiment. Finally, measurements of two-phase density using the cavity are presented and discussed.

THEORY OF TWO-PHASE DENSITY MEASUREMENTS

The measurement of hydrogen density using a microwave cavity is based on two important relationships: (1) the relationship between the resonant frequency of the cavity and the dielectric constant of hydrogen in the cavity, and (2) the relationship between the dielectric constant and the density of hydrogen.

In practice, the Clausius-Mossotti equation is commonly used to relate the density of hydrogen to its relative dielectric constant. The Clausius-Mossotti equation is a derived equation that is based on several assumptions that are not entirely valid for hydrogen. It generally fits experimental data only to within ±0.3 percent.

An improved equation that relates density to relative dielectric constant has been derived by Böttcher (ref. 4). This equation is given by

$$\rho = \frac{m\epsilon_0}{3\alpha} \frac{(K - 1)(2K + 1)}{K} - \frac{m}{6\pi a_0^3} \frac{(K - 1)^2}{K} \tag{1}$$

where ρ is the mass density of hydrogen, K the relative dielectric constant, m the mass of a hydrogen molecule, α the average polarizability of a hydrogen molecule, a_0 the effective molecular radius of hydrogen, and ϵ_0 the electric permittivity of free space. (Symbols are defined in the appendix.) A rearrangement of Böttcher's equation reveals that a plot of $3K\rho/[(K - 1)(2K + 1)]$ against $(K - 1)/(2K + 1)$ is a straight line with a slope of $-m/2\pi a_0^3$ and an intercept of $m\epsilon_0/\alpha$. Numerical values for the slope and intercept can be found by fitting equation (1) to experimental data.

The most accurate and complete experimental data on the density-dielectric constant relationship for hydrogen were taken by Stewart (ref. 5). These experimental data consist of 205 points of dielectric constant as a function of density for parahydrogen over the density range 2 to 80 kilograms per cubic meter, the pressure range of 2.026×10^5 to 239.1×10^5 newtons per square meter (2 to 236 atm), and the temperature range 24 to 100 kelvin. The error in these data in density is estimated to be less than 0.1 percent.

Fitting Böttcher's equation to Stewart's data with a least error squared fit of $3K\rho/[(K - 1)(2K + 1)]$ against $(K - 1)/(2K + 1)$ gives a slope of -55.31 kilograms per cubic meter and an intercept of 332.2 kilograms per cubic meter. These values for slope and intercept differ slightly from those previously reported in reference 1 because the present results include a density correction term given by Stewart. This correction, given for each data point, is necessary because of a temperature gradient in the hydrogen sample holder. In addition, three data points were considered to be invalid points and were excluded since they deviated from the least error squared fit line by 0.32 to 1.75 percent. Of the 202 remaining points 118 points were within 0.05 percent of the line, 189 points were within 0.1 percent of the line, 11 points were 0.1 to 0.15 percent from the line, and 2 points were 0.15 to 0.20 percent from the line. On the basis of these results, equation (1) can be used to relate density to relative dielectric constant over the range 2 to 80 kilograms per cubic meter with a probable error in density less than 0.1 percent.

If a microwave cavity is completely filled with single-phase, homogeneous hydrogen of relative dielectric constant K, the resonant frequency f of the cavity is given by

$$f = f_0/\sqrt{K} \qquad (2)$$

where f_0 is the resonant frequency of the cavity when it is evacuated. Thus, for single-phase hydrogen equations (1) and (2) can be combined to relate density to resonant frequency.

In using equation (2) to determine K, care must be taken to insure that f_0 is measured at approximately the same temperature as f so that the effect of dimensional changes in the cavity can be eliminated. This requirement is not very severe for liquid hydrogen applications, however, since the thermal expansion coefficient for possible cavity materials such as stainless steel is of the order $10^{-6}\ K^{-1}$ at 20 kelvin. Unless the temperature varies considerably (i.e., more than 30 kelvin) it is generally adequate to use one value for f_0.

If the cavity is filled with inhomogeneous hydrogen, such as a mixture of liquid and gas, equation (2) yields an effective dielectric constant of the mixture K_{eff}. If this value of K_{eff} is substituted into equation (1) an effective density of the mixture can be computed. The effective density is, of course, not necessarily equal to the actual density of the mixture which is what is desired. The difference between the two densities can only be determined if the relative amounts, location, and orientation of the liquid and gaseous regions in the cavity are completely specified.

It will be assumed that the hydrogen in the cavity is principally liquid with some bubbles. If the bubbles are spheriodal in shape and uniformly distributed in the liquid, Taylor (ref. 6) has shown that

$$K_{eff} = K_{\ell} - \frac{\delta K_{eff}(K_{\ell} - K_g)}{(1 - N)K_{eff} + NK_g} \tag{3}$$

where K_{ℓ} and K_g are the dielectric constants of the liquid and gaseous hydrogen, respectively, δ is the fraction of the total cavity volume that is occupied by the gas, and N is the depolarization factor for the bubbles. The depolarization factor is a function of the bubble shape and orientation with respect to the electric field, but it is independent of the absolute size of the bubble. Equation (3) is a derived equation based on a quasi-static approximation for the electromagnetic fields. In addition, the derivation neglects the interaction effects between the polarization fields of the bubbles in determining K_{eff}. To satisfy these approximations the following inequalities must be true:

$$r_b << \lambda \tag{4}$$

$$\delta^2 << 1 \tag{5}$$

where r_b is a characteristic dimension of the bubbles such as their radius and λ is the wavelength at which the effective dielectric constant is determined. Equation (4) is a result of the quasi-static approximation, and equation (5) is a result of neglecting the interaction effects between the bubbles. Equation (3) can also be used where the hydrogen in the cavity is principally liquid with small solid particles or principally gas with small liquid droplets by simply using the appropriate dielectric constants.

The difference between the effective and actual density can be determined for specific cases. Consider the case where hydrogen is boiling at a pressure of 1.013×10^5 newtons per square meter (1 atm). This gives a liquid density ρ_{ℓ} of 70.78 kilograms per cubic meter and a gas density ρ_g of 1.34 kilograms per cubic meter. The corresponding dielectric constants K_{ℓ} and K_g are 1.2297 and 1.0040, respectively. Figure 1 shows the percentage difference between the effective and actual density as a function of the fraction of the volume occupied by the gas. The bubbles are assumed to be evenly distributed in the liquid and either spherical or oblate spheroidal in shape, which are the common shapes found in practice. For spherical bubbles the difference between effective and actual density is approximately 0.01 percent for δ of 0.1, as shown. As the eccentricity of the bubbles increases, the difference increases going positive or negative depending on the orientation of the bubbles with respect to the electric field in the cavity. If the bubbles are randomly oriented, which often occurs when turbulence is present, these differences will tend to cancel giving a net difference much less than the extremes shown. Thus, for the case of small uniformly distributed bubbles it is generally adequate to assume that the effective and actual densities are equal if the concentration of gas bubbles is low so that equation (5) is satisfied.

Some work has been done (ref. 1) to test the range of the assumption given by equation (4) concerning the size of the bubbles compared with a wavelength. The liquid hydrogen was simulated by polystyrene based plastic-foam particles of relative dielectric constant 1.2, and the gas bubbles were simulated by voids between the particles. It was found that a value of r_b/λ of the order of 0.1 was too large, giving differences between effective and actual density of up to 25 percent for $\delta = 0.4$. A value of r_b/λ of the order of 0.01, which gave a difference of 0.22 percent for $\delta = 0.4$, appeared to be small enough to satisfy the quasi-static approximation.

The difference between the effective and actual density can also be computed for other configurations of liquid and gas in the cavity. The main difficulty, however, is not in calculating these differences, but in having the liquid and gaseous hydrogen assume a known configuration in the cavity so that these calculations will apply. The initial configuration analyzed, liquid hydrogen with small uniformly distributed gas bubbles, appears to be a reasonable configuration to enforce. If the cavity were located in a flow line, for example, a screen or mixing device could be placed upstream from the cavity to break up the large bubbles and to generate turbulence to uniformly distribute and randomly orient the bubbles.

THE EXPERIMENT

The purpose of this experiment was to demonstrate that two-phase hydrogen density measurements can be made using an open-ended microwave cavity and the theory presented in the previous section. The general procedure used was to generate a two-phase hydrogen sample and determine its density by measuring the pressure, volume, and temperature (PVT) of each phase, to measure the density of the sample using the microwave cavity, and to compare the results.

The experiment was performed under static or zero liquid flow conditions. The cavity, which is in the form of a right circular cylinder, was mounted vertically in a liquid hydrogen bath. Bubbles of hydrogen gas having uniform size were continuously generated at 30 evenly spaced points across the base of the cavity. These bubbles, after reaching terminal velocity, rose through the cavity giving a relatively homogeneous two-phase mixture within the cavity.

The density of the two-phase mixture of liquid and gas ρ_t is given by

$$\rho_t = \frac{\rho_\ell V_\ell + \rho_g V_g}{V} = \rho_\ell - \frac{V_g}{V}(\rho_\ell - \rho_g) \tag{6}$$

where ρ_ℓ and ρ_g are the densities of the liquid and gas, respectively, and V_ℓ and V_g are the volumes occupied by the liquid and gas, respectively, within the total active volume V of the cavity. The active volume V is given by

$$V = AL \tag{7}$$

where A (1190 mm^2) is the cross sectional area of the cavity and L (34.6 mm) is the spacing between the partitions within the cavity (refs. 2 and 3). The volume occupied by the gas in the cavity can be expressed as

$$V_g = A_g L \tag{8}$$

where A_g in the effective cross sectional area for gas flow. The mass flow rate of the hydrogen gas transported by the bubbles $\dot{M}$ is given by

$$\dot{M} = \rho_g v A_g \tag{9}$$

where v is the terminal velocity of the bubbles. Combining equations (6) to (9) to eliminate V_g and V then give

$$\rho_t = \rho_\ell - \frac{\dot{M}}{vA}\left(\frac{\rho_\ell}{\rho_g} - 1\right) \tag{10}$$

This is the basic equation that is used to determine the density of the two-phase mixture for comparison with the microwave cavity measurements.

The requirement imposed by equation (5) on the fraction of the total volume occupied by the gas bubbles can be expressed as

$$\delta^2 = \left(\dot{M}/\rho_g vA\right)^2 \ll 1 \tag{11}$$

This equation will be used to determine an upper limit for $\dot{M}$. The following sections give a detailed description of the experimental apparatus and the instrumentation used to determine the parameters in equation (10).

Experimental Apparatus

The experiment was performed in a vacuum jacketed test dewar having inner dimensions of 0.15 meter in diameter and 0.75 meter in height. Two open-ended cavities were used initially. They were connected end to end and suspended by their rigid coaxial feed lines in approximately the center of the dewar (see fig. 2).

A helium diffuser was located at the bottom of the dewar. The hydrogen bath could be supercooled by blowing helium gas through the porous plate in the diffuser which formed a cloud of bubbles in the bath. The presence of the helium bubbles caused a portion of the hydrogen bath to vaporize and thus lose some of its thermal energy. The bath could be cooled several degrees below its normal boiling point by this procedure.

Bubble generator. - Ultrapure hydrogen gas was injected into the cavity through thirty 0.375-millimeter-inside-diameter by 0.500 millimeter-outside-diameter tubes which protrude through the bottom of a cup that is attached to the lower end of cavity 1. The cup was made deep enough to allow the bubbles to reach terminal velocity before entering the cavities.

The 100-millimeter long, 0.375-millimeter-inside-diameter tubes are connected to 100-millimeter long, 1.0-millimeter-inside-diameter tubes. These are connected to 1.0-meter long, 1.5-millimeter-inside-diameter tubes which are in turn connected to the manifold.

The tube diameters were kept as large as practicable before necking them down at the exit in order to keep the flow velocity of the gas relatively small. The low flow allowed the gas sufficient time to cool to the bath temperature before leaving the tubes.

The valve manifold was made circular to facilitate the connection of the 30 tubes while keeping their lengths approximately equal. Each of the 30 lines has a needle valve (VB-1 to VB-30) to adjust the flow rate and an on-off valve (VC-1 to VC-30) to facilitate calibration. The manifold supplies gas to each of the lines at the same pressure. The pressure regulator PR-2 sets the pressure in the manifold and keeps it constant. The nitrogen trap was used to filter out impurities in the hydrogen gas that could freeze at liquid hydrogen temperatures and plug the flow lines.

A bank of five rotameters, calibrated with an inlet pressure of 6.90×10^5 newtons per square meter (100 psi), was used to measure the mass flow rate of the gas. Pressure gage P-1 was used to monitor the inlet pressure to the rotameters, and regulator PR-1 was used to set it. The probable error in the mass flow rate measurement of hydrogen gas was estimated to be 5 percent.

Using the most sensitive rotameter, the needle valves on the valve manifold were set one at a time with constant pressure in the manifold to give equal flow rates in each of the tubes. The object of maintaining equal flow rates was to generate a homogeneous distribution of bubbles in the liquid. The total mass flow rate of hydrogen $\dot{M}$ was adjusted by varying the pressure in the manifold using pressure regulator PR-2. It is assumed that the mass flow rate of hydrogen gas as measured by the rotameter is equal to the mass flow rate of hydrogen gas transported by the bubbles in the cavity and, hence, is the proper value of $\dot{M}$ to use in equation (10). This assumption will be discussed in detail in the section Experimental Procedure.

Temperature and pressure measurement. - Temperature measurements were made using platinum resistance temperature sensors. It was not possible to put a sensor inside the cavity where the hydrogen density is actually measured because the presence of the sensor would interfere with the operation of the cavity. Consequently, three surface sensors were soldered to the outer wall of the lower cavity; one at the top, one at the center, and one at the bottom. A fourth sensor was suspended in the liquid over the lower cavity. (The upper cavity was only used during the initial phases of the experiment when temperature measurements were not needed and was removed before the temperature sensors were installed.)

The resistances of the sensors were determined by passing a constant current through the four sensors which were connected in series with a standard resistance and measuring the voltage drop across each sensor and the standard. Measurements were made with the current in both the forward and reverse directions to determine the thermoelectric potentials. The voltage measurements were made with an integrating digital voltmeter and recorded on a digital recorder. It was estimated that the sensor temperatures could be resolved to within 0.002 kelvin and were accurate to within 0.02 kelvin. The temperature of the hydrogen in the cavity could not be determined to this accuracy, however, because the temperature differences between the sensor locations and the hydrogen in the cavity were unknown.

The pressure of the hydrogen in the cavity was equal to the barometric pressure at the test area plus the differential pressure between the dewar and the test area. A barometer with a calibration corrected to the altitude at the test area was used to measure the atmospheric pressure. The differential pressure between the dewar and the test area was measured with a gage in the top cover of the dewar. This pressure never exceeded 1.2×10^3 newtons per square meter (0.012 atm). The differential pressure in the dewar between the surface of the liquid and the cavity level is less than 1×10^2 newtons per square meter (0.001 atm) and was neglected.

Knowing the pressure and temperature of the hydrogen in the cavity allows the liquid and gas densities ρ_ℓ and ρ_g in equation (10) to be calculated using the equation of state for hydrogen (ref. 7).

Microwave instrumentation. - The resonant frequency of the open-ended cavity was determined by constructing an oscillator whose frequency was controlled by the cavity. The oscillator frequency, which ranges from 9.2 to 10.2 gigahertz depending on the density in the cavity, was measured using an automatic heterodyne frequency meter and recorded on a digital recorder.

A block diagram of the system is shown in figure 3. The oscillator consists of a loop formed by the travelling wave tube microwave amplifiers, a delay line, the cavity, and the attenuators. A directional coupler is also inserted in the loop to supply a sample of the signal to the frequency meter. The system will oscillate when the open-loop gain is 0 decibel (dB) or greater and when the phase shift is some integral multiple of 2π radians.

The loop gain was set by adjusting the attenuators to give a total open-loop gain of 3 dB at the center frequency of the cavity. This restricted the frequency of oscillation to within the 3 dB "down" points of the cavity which are separated by f/Q. The Q of the cavity was set at approximately 8000 by adjusting the coupling loops in the cavity at the ends of the coaxial lines (see fig. 2(b)). Thus, if the loop oscillates, the frequency will be bounded by the cavity center frequency ±0.006 percent.

To insure that oscillation occurs, the phase shift around the loop must be an integral multiple of 2π radians within ±0.006 percent of the cavity center frequency for all center frequencies in the range from 9.2 to 10.2 gigahertz. Since a signal through the cavity shifts in phase approximately $\pi/2$ radians between the 3 dB down points, it is necessary to insert a delay line in the loop that will provide a phase shift of at least $3\pi/2$ radians for a 0.012-percent frequency change. This can be achieved with a minimum time delay of 0.68 microsecond. In the experiment a delay of 1.0 microsecond was used.

It was found that the loop gain need not be precisely set to 3 dB at the cavity center frequency to insure that the frequency of oscillation is within the 3 dB down points of the cavity. If the gain exceeds 3 dB there are two or more possible frequencies of oscillation. However, the loop oscillated at the frequency that had the maximum loop gain which is always the one nearest the cavity resonant frequency.

Experimental Procedure

The last parameter in equation (10) to be determined is v, the terminal velocity of the bubbles as they rise through the hydrogen bath. In addition, it is necessary to determine the bubble size to verify that r_b/λ is of the order 0.01 or less. These two requirements were accomplished by a direct measurement of the bubble velocity and then by calculating the bubble size using several theories relating bubble size to velocity.

Bubble terminal velocity. - The bubble velocity was measured using the two open-ended cavities suspended in the hydrogen bath as shown in figure 2. The two identical cavities with equal resonant frequencies were mounted vertically one over the other so that the bubbles would rise first through one and then the other. Different microwave instrumentation from that previously discussed was used for the bubble velocity measurement only since it was not necessary to measure the cavity resonant frequencies precisely. The entire system is illustrated in figure 4.

The mass flow rate of the hydrogen gas and the frequency of the microwave generator were set to produce the following conditions:

(1) When the valve V-A was closed, there was only liquid in the cavities, and their resonant frequencies were just below the frequency of the generator.

(2) When the valve V-A was open, there was a mixture of liquid and gas bubbles in the cavities, and their resonant frequencies were much higher than the frequency of the generator.

When valve V-A was first opened, a cloud of bubbles rose through the liquid. As the leading edge of the bubble cloud approached the upper end of the lower partition in each cavity, the resonant frequency increased passing through the frequency of the microwave generator. This generated a voltage pulse at the output of the detectors. The pulse from cavity 1 preceded the pulse from cavity 2 by the time it took the leading edge of the bubble cloud to rise from one cavity to the other. The bubble velocity is, therefore, the spacing between the upper ends of the lower partition in each cavity divided by the time interval between pulses. The terminal velocity of the bubbles equaled 250 millimeters per second in this experiment.

It is important that the bubbles all have the same size for this measurement to be valid. If the bubbles were of various sizes, some would rise faster than others because the terminal velocity is dependent on the bubble size. The measurement just described would then give the velocity of the fastest bubble. To insure uniformity in bubble size the ends of the tubes on which the bubbles are produced were carefully finished by squaring the ends, reaming the inside diameter, and burring the edges.

Additional tests were made with the bubble generator using water and alcohol as the fluid and air and helium as the gas. By observing the bubbles from each tube with a stroboscope, the uniformity of the bubble size and velocity could be assured. It should be emphasized that the cavity performance does not depend on all bubbles having the same size. The uniformity in bubble size is only necessary to make the bubble velocity measurement meaningful.

Bubble size. - Since the bubbles could not be observed in the liquid hydrogen, their size was calculated using several theories relating bubble size and velocity. Figure 5 shows how the terminal velocity of a bubble is related to its size. The values shown are for hydrogen at the boiling point at a pressure of 1.013×10^5 newtons per square meter (1 atm). Very small bubbles are spherical in shape and Stokes' law applies. As the bubbles become larger, their shape changes to an oblate spheroid with the minor diameter in the direction of motion. This case is considered by Moore (refs. 8 and 9). Very large bubbles have a spherically shaped cap followed by a long trailing turbulent wake. Davies and Taylor (ref. 10) considered the case where the cap is exactly spherical. Collins (ref. 11) noted that the cap of a large bubble is not truly spherical and developed equations using a more accurate representation for the cap. The dashed curve is our interpolation between the three different cases.

The abscissa in figure 5 is labeled equivalent radius r_e to take into account the nonspherical shapes. The smaller bubbles have a volume equal to that of a sphere of radius r_e. The caps on the larger bubbles have a radius of curvature at the stagnation point equal to r_e.

There are obviously three different values of r_e (i.e., 0.23, 0.75, and 15 mm) that could produce the measured terminal velocity of 250 millimeters per second. To resolve which of the three sizes actually occurred, consider the static forces acting on a bubble as it grows at the end of the tube near the time of separation. There are two forces upward, the bouyant force and the force due to the difference in pressure between the liquid and gas, and there is one restraining force downward due to the surface tension. The maximum value of the restraining force is limited to the product of the tubes outer diameter, the surface tension of liquid hydrogen, and the sine of the contact angle between the bubble and the tube. As the bubble grows in size the forces tending to lift the bubble increase until a critical bubble radius is reached where the lifting force is equal to the maximum restraining force. At this point separation occurs. Evaluating these forces for the three possible bubble radii using 1.93×10^{-3} newton per meter for the surface tension of liquid hydrogen and 0.5-millimeter outside diameter for the tube radius reveal that the largest bubble radius of 15 millimeters can be excluded. The maximum value of the restraining force is much less than that required to hold this bubble to the tube. The two smaller radii are possible choices, however. Because the two smaller radii yield values for r_b/λ of 0.008 and 0.025 (λ = 30 mm), it is not necessary to determine which of the two possibilities actually occurred. Both values of r_b/λ are of the order 0.01 or less as required.

Two-phase density measurements. - The two-phase density measurements were performed with cavity 2 removed to permit the installation of a temperature sensor in the bath directly over cavity 1. The most difficult part of the experiment was to determine M, the mass flow rate of the gas transported by the bubbles. As discussed in the section Bubble generator, this measurement was made using a rotameter in the hydrogen gas supply line. However, the rotameter reading may differ considerably from the actual mass flow rate of gas in the cavity. If the hydrogen bath is supercooled, most if not all of the gas supplied to the test dewar will condense to liquid in the tubes. A stream of liquid rather than bubbles will be emitted from the 30 tubes at the base of the cavity. Conversely, if the hydrogen bath is boiling, the gas in the tubes will not condense. However, gas bubbles due to the boiling bath will be present in the cavity in addition to those emitted from the tubes. Only when the bath temperature is just slightly below the boiling point will the rotameter reading be indicative of the true mass flow rate of gas transported by the bubbles.

Rather than to try to control the bath temperature at a point just below boiling, it was simpler to first supercool the bath and then allow it to warm up to the boiling point. Specifically, the following procedure was used:

(1) The hydrogen gas flow rate $\dot{M}$ was set to a desired rate as indicated by the rotameter and held constant for one set of data.

(2) Helium gas was supplied to the diffuser until the bath cooled approximately 1 degree.

(3) After the helium source was turned off, the bath warmed up to the boiling point. Temperature and resonant frequency data were taken continuously during this warmup period which ranged from 1 to 15 minutes.

(4) After the bath reached the boiling point the hydrogen gas flow rate was set to a new value and the entire process was repeated.

Although all data were of interest, a comparison between the cavity measurements and the density as determined from PVT data was only meaningful at the boiling point temperature.

Results

Four temperature sensors were used to determine the variation in temperature around the cavity assembly. All sensors read the same temperature immediately after the bath was supercooled. Then their temperatures increased at slightly different rates reaching the boiling point temperature in an order depending on their respective vertical positions in the dewar. The upper most sensor reached the boiling point first, then the second highest, and so on. This was expected because most of the heat input came from the top of the test dewar and the warmest hydrogen being the least dense would tend to remain near the top. The maximum temperature difference between the highest and lowest sensors was less than 0.5 kelvin. The temperature of the sample of hydrogen being measured was assumed to be the temperature indicated by the sensor on the outer wall at the center of cavity 1. Sensors could not be located in the cavity, as mentioned previously, because they would interfere with its operation.

The density of the two-phase sample in the cavity was determined using equation (10). The liquid density was determined from the PVT data using an equation of state for hydrogen at a pressure of 1.013×10^5 newtons per square meter (1 atm) (ref. 7). The correction for the change in liquid density with pressure, which is approximately 0.02 ppm per newton per square meter (0.2 percent/atm), was negligible. The barometric pressure at the test site was 0.99×10^5 newtons per square meter (0.98 atm), and the differential pressure between the dewar and test site averaged 0.01×10^5 newtons per square meter (0.01 atm). The pressure readings were necessary, however, to determine the boiling point temperature. For a pressure range from 0.99×10^5 to 1.01×10^5 newtons per square meter (0.98 to 1.00 atm), the boiling point varies from 20.20 to 20.27 kelvin.

The gas density used in equation (10) is its value at the boiling point at a pressure of 1.013×10^5 newtons per square meter (1 atm). Thus, the computed two-phase density is correct only at the boiling point. This step is reasonable, however, since $\dot{M}$ is equal to the rotameter reading only at the boiling point. To obtain more accurate values for the gas density and mass transported by the bubbles as a function of the bath temperature would require a detailed knowledge of the temperature distributions and heat transfer properties along the 30 tubes leading from the manifold to the cavity. The correction for the change in gas density with pressure was not made because it changed the total density by less than 0.05 percent for the highest value of $\dot{M}$ used.

The density as measured using the cavity was determined by first using equation (2) to compute the relative dielectric constant and then using equation (1) to obtain the density. The value used for resonant frequency in vacuum f_0 was calculated from data when the cavity was completely filled with liquid hydrogen since it was not possible to simultaneously cool the cavity and evacuate the test dewar. This procedure is valid since equations (1) and (2) have proven to give accurate results for the all liquid case in earlier tests when f_0 could be determined experimentally (ref. 1).

Experimental results for eight different values of $\dot{M}$ ranging from 6.0 to 18.4 milligrams per second are shown in figure 6. The largest value of $\dot{M}$ used corresponds to 4.6 percent of the total volume occupied by gas which easily satisfies the inequality given by equation (11). The top dashed curve for $\dot{M} = 0$ in figure 6 corresponds to the all liquid case and shows the variation in liquid density with temperature at a pressure of 1.013×10^5 newtons per square meter (1 atm). The lower dashed curve shows the variation in density with temperature for the two-phase mixture at the specified value for $\dot{M}$. This curve is hypothetical since the gas cannot exist at temperatures below the boiling point. The only realizable point on this curve is at the boiling point.

The measured density using the cavity is in good agreement with the $\dot{M} = 0$ curve at temperatures more than a few tenths of a degree below boiling. This is expected since all the hydrogen gas is being condensed to liquid and, consequently, no bubbles are being produced. As the temperature increases, the measured density begins to drop faster than the $\dot{M} = 0$ curve since all the gas is not condensing and some bubbles are being produced. At the boiling point none of the gas should condense in the tubes, and the measured density curve should intersect the calculated curve.

The boiling points corresponding to pressures of 0.99×10^5, 1.00×10^5, and 1.01×10^5 newtons per square meter (0.98, 0.99, and 1.00 atm) are shown in figure 6. Based on the barometric pressure and differential pressure measurements between the test dewar and the test area, it is estimated that the pressure in the cavity was $1.00\pm0.005\times10^5$ newtons per square meter (0.99 ± 0.005 atm). The intersections between the data curve and the hypothetical curve for the same value of $\dot{M}$ occur approximately at the boiling point corresponding to 1.00×10^5 newtons per square meter (0.99 atm). There is, however, a slight decrease in the temperature at the intersection point as the flow rate $\dot{M}$ increases. This decrease is probably due to the temperature difference between the cavity wall on which the sensors were mounted and the hydrogen within the cavity. When $\dot{M}$

was small the temperature increased slowly with time so that the wall and fluid temperatures were always nearly equal. For larger values of $\dot{M}$ the temperature increase was more rapid so that the measured wall temperatures lagged the fluid temperature in time and thus gave a temperature indication which was too low.

CONCLUDING REMARKS

The technique of measuring hydrogen density using an open-ended microwave cavity was demonstrated for the two-phase condition of liquid with small uniformly distributed gas bubbles. The most probable application of this instrument is its use in conjunction with a volume flowmeter to measure mass flow rate or total mass transfer. In a liquid hydrogen transfer system it is extremely difficult to create a flow without bubbles. The results of this experiment show that this technique for measuring hydrogen density is effective in the presence of limited quantities of bubbles. Some precautions must be taken, however, to insure accurate readings. For example, the cavity should not be located where gas pockets could form. In addition, means should be included for limiting the size of the bubbles and uniformly distributing them over the pipe cross section by using screens or a mixing device upstream from the cavity.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, November 20, 1970,
128-31.

APPENDIX - SYMBOLS

A	cross sectional area of cavity
A_g	effective cross sectional area for gas flow
a_o	molecular radius of hydrogen
f	resonant frequency of cavity when filled with hydrogen
f_o	resonant frequency of cavity when evacuated
K	relative dielectric constant of hydrogen
K_{eff}	effective relative dielectric constant of mixture
L	spacing between partitions in cavity
$\dot{M}$	mass flow rate of hydrogen gas
m	mass of hydrogen molecule
N	depolarization factor for ellipsoidal bubbles
Q	2π times ratio of energy stored in cavity to energy dissipated per cycle of oscillation
r_b	characteristic dimension of bubble
r_e	effective radius of nonspherical bubble
V	active volume of cavity
v	terminal velocity of bubble
α	average polarizability of hydrogen molecule
δ	fraction of cavity volume occupied by gas
ϵ_o	electrical permittivity of free space
λ	wavelength of electromagnetic waves
ρ	mass density of hydrogen

Subscripts:

g	gas
ℓ	liquid
t	two-phase mixture

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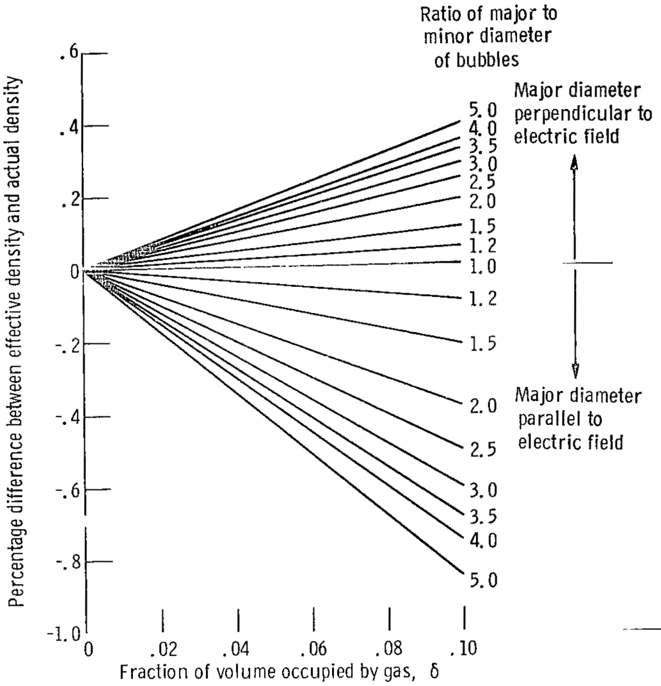
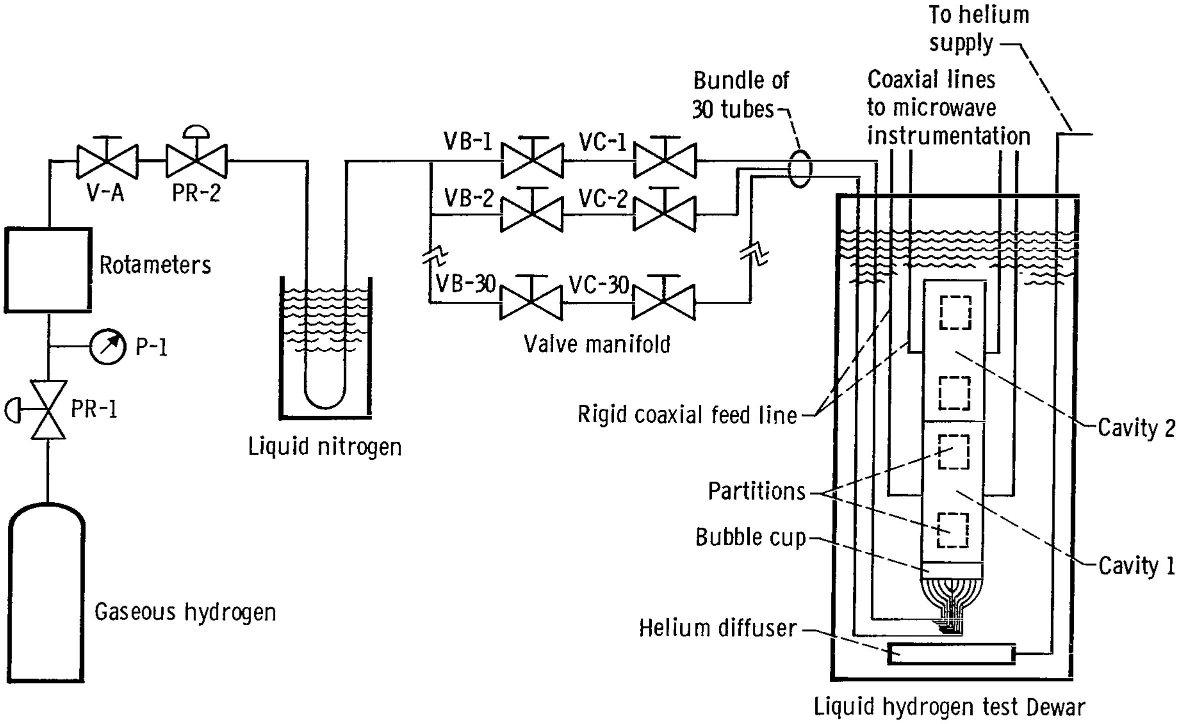
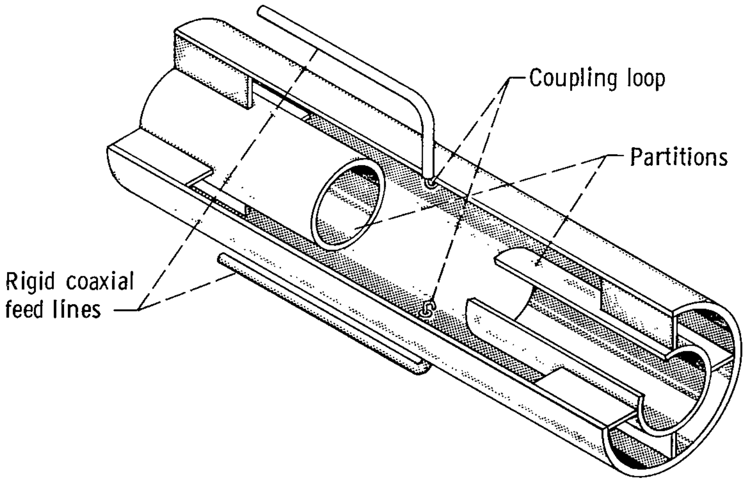


Figure 1. - Theoretical percentage difference between effective density and actual density against fraction of volume occupied by gas for oblate spheroidal bubbles. Relative dielectric constant of liquid, 1.2297; relative dielectric constant of gas, 1.0040; liquid density, 70.784 kilograms per cubic meter, gas density, 1.3378 kilograms per cubic meter.



(a) Experimental apparatus for generating two-phase hydrogen.



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(b) Detail of open-ended microwave cavity.

Figure 2. - Two-phase hydrogen experiment.

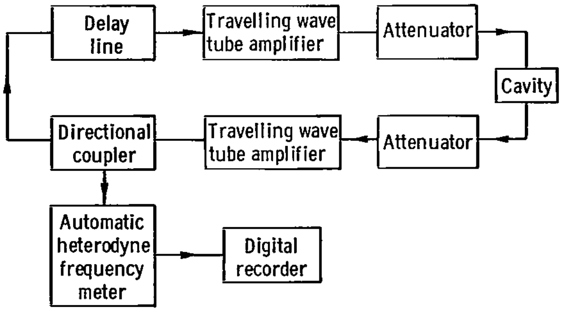


Figure 3. - Microwave instrumentation for determining resonant frequency of cavity.

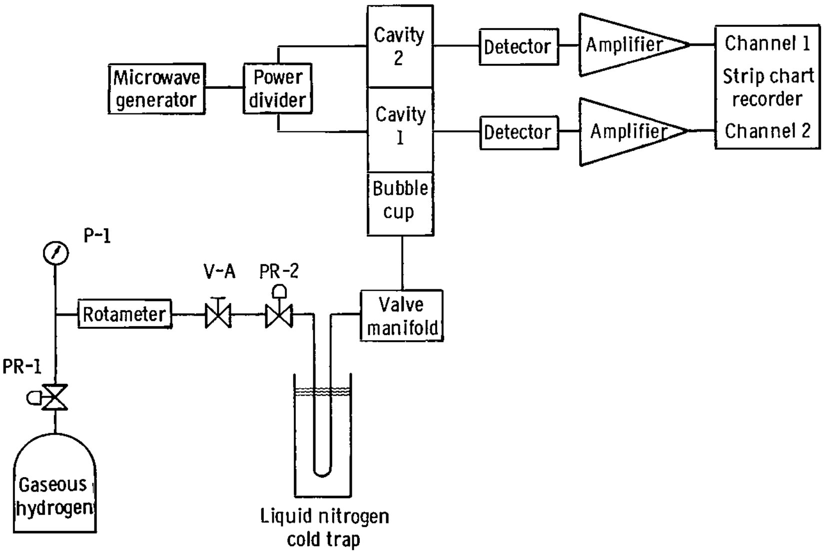


Figure 4. - Experimental apparatus for measuring terminal velocity of hydrogen bubbles.

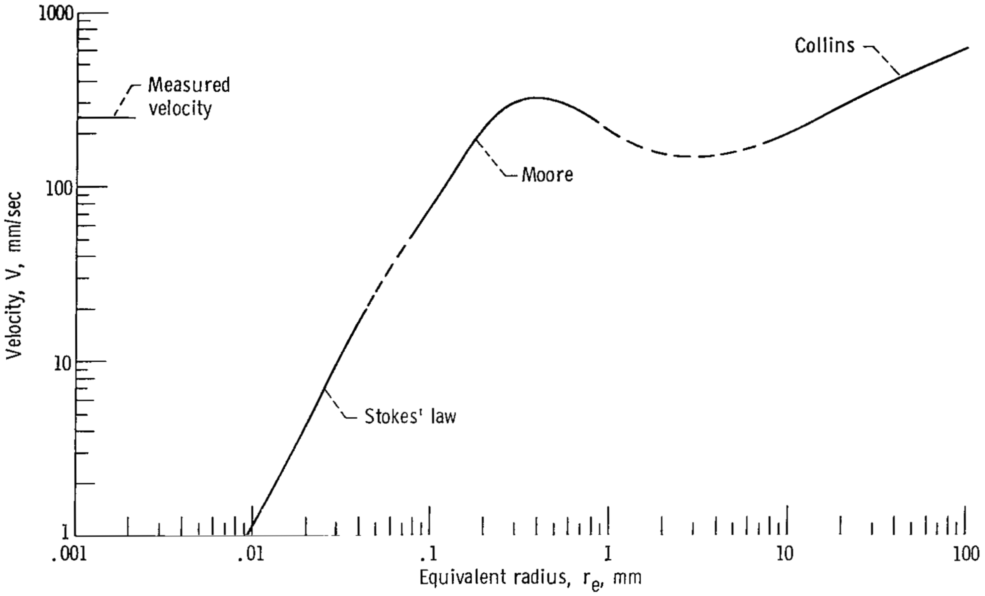
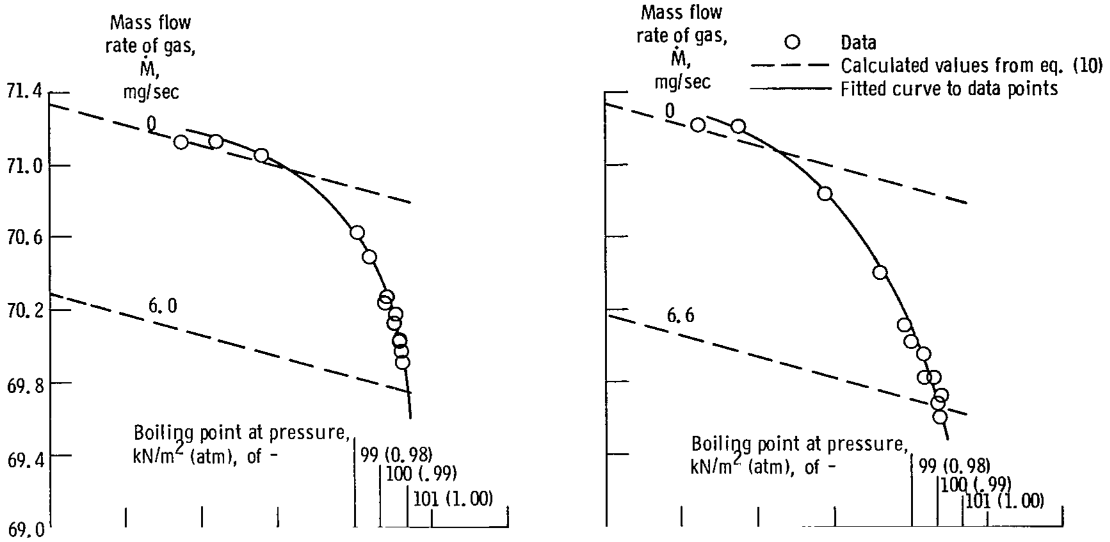
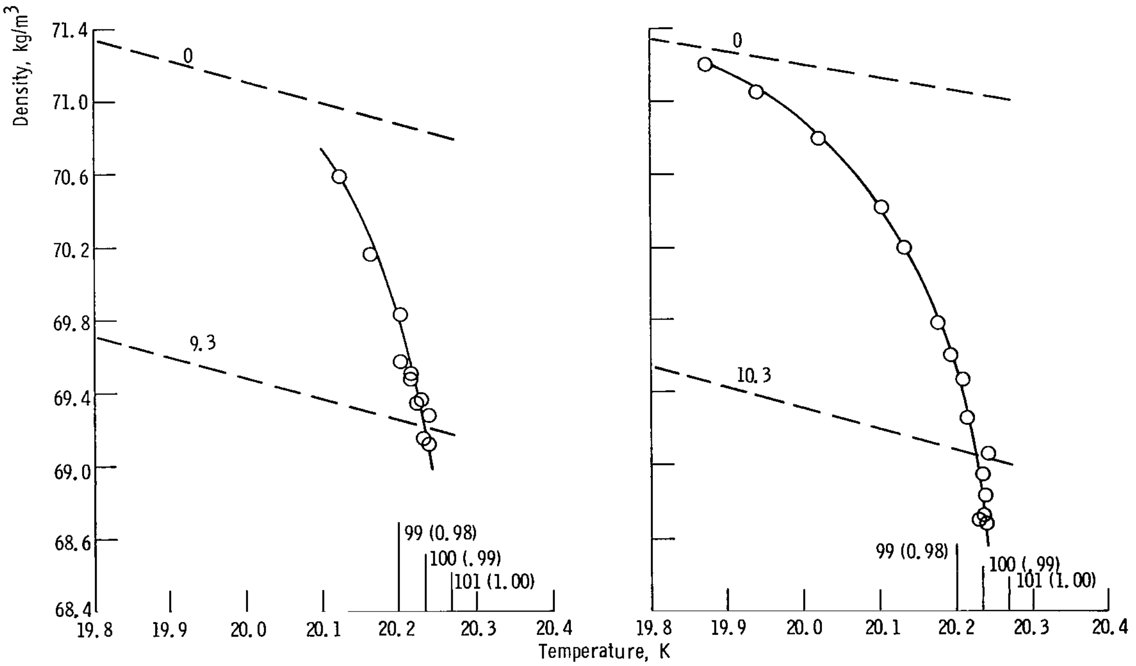


Figure 5. - Velocity of hydrogen bubbles in liquid hydrogen from Stokes' law, Moore (refs. 8 and 9), and Collins (ref. 11).



(a) Mass flow rate of gas, 6.0 milligrams per second.

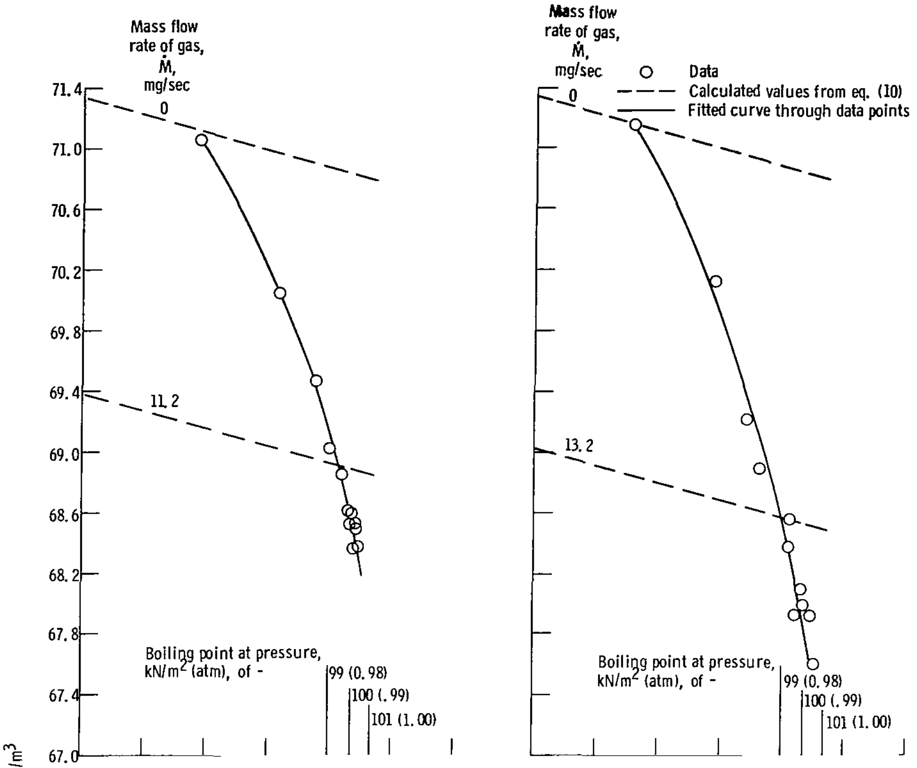
(b) Mass flow rate of gas, 6.6 milligrams per second.



(c) Mass flow rate of gas, 9.3 milligrams per second.

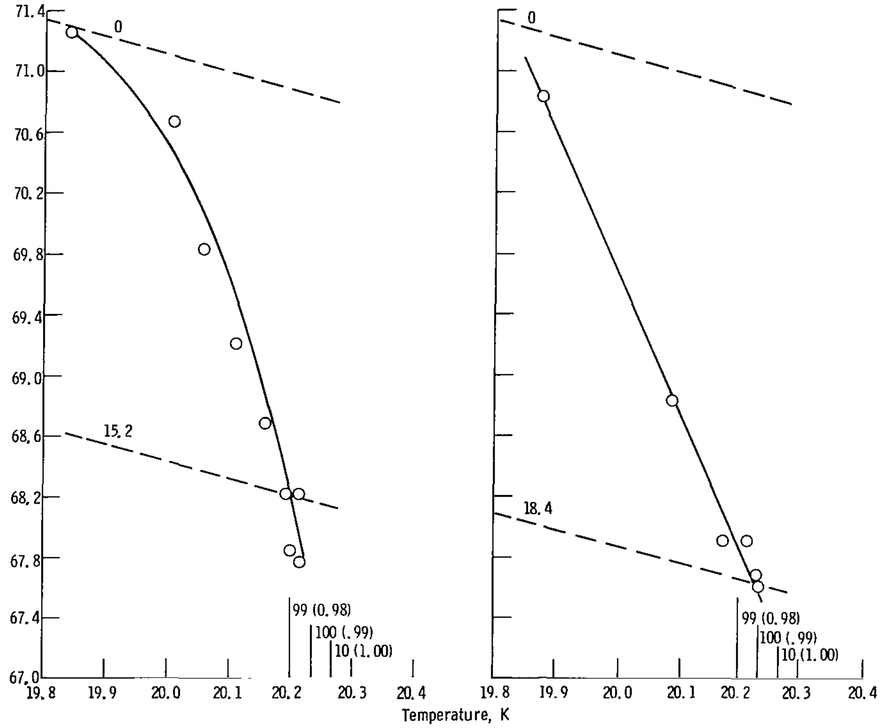
(d) Mass flow rate of gas, 10.3 milligrams per second.

Figure 6. - Two-phase hydrogen density measurements.



(e) Mass flow rate of gas, 11.2 milligrams per second.

(f) Mass flow rate of gas, 13.2 milligrams per second.



(g) Mass flow rate of gas, 15.2 milligrams per second.

(h) Mass flow rate of gas, 18.4 milligrams per second.

Figure 6. - Concluded.

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> — National Aeronautics and Space Act of 1958

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