

General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.
- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.
- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.
- This document is paginated as submitted by the original source.
- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Memorandum 33-466

Volume I

*VISCEL—A General-Purpose Computer Program
for Analysis of Linear Viscoelastic Structures*

User's Manual

F. A. Akyuz

E. Heer

3
9

FACILITY FORM 602

N71-18931
(ACCESSION NUMBER)
27
(PAGES)
CR-116823
(NASA CR OR TMX OR AD NUMBER)

63
(THRU)
(CODE)
08
(CATEGORY)



**JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA**

February 15, 1971

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Memorandum 33-466

Volume 1

*VISCEL—A General-Purpose Computer Program
for Analysis of Linear Viscoelastic Structures*

User's Manual

F. A. Akyuz

E. Heer

**JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA**

February 15, 1971

PREFACE

The work described in this report was performed by the Engineering Mechanics Division of the Jet Propulsion Laboratory.

ACKNOWLEDGMENT

The authors wish to thank Dr. K. K. Gupta for reviewing this report and for numerous suggestions relative to implementation of the VISCEL program. Thanks are due Mr. Douglas R. Maple for his assistance in editing this work.

CONTENTS

I.	Introduction	1
II.	Problem Definition	1
	A. Recursive Equations for Incremental Deformations and Accumulative Residual Forces and Stresses	1
	B. Synchronized Material Property Concept for Thermorheologically Simple Material	3
	C. Explicit Expressions for Incremental Deformations, Residual Forces, and Stresses	4
III.	Description of the Program	11
	A. Method of Solution and the Computer Program ELAS	11
	B. Capabilities and Limitations of the Program	11
	C. Programming Language and Physical Description of the Program	12
	D. Computer Hardware and Operational System and Functions of Various System Units	13
IV.	Preparation of Input	14
	A. General Information	14
	B. Description of Input Data for Linear Viscoelastic Problems and for Multiple Runs	15
V.	Description of the Output	18
	A. General Information	18
	B. Output for Linear Viscoelastic Problems and Multiple Runs	18
	References	20

TABLES

1.	Explicit recursive relations for deflections	8
2.	Explicit recursive relations for residual forces	9
3.	Explicit recursive relations for stresses	10
4.	The functions of the FORTRAN units as used in VISCEL	14

CONTENTS (contd)

TABLES (contd)

5.	Permanent and modifiable input items	17
6.	Summary of output items	19

FIGURES

1.	Schematic representation of the material properties and external disturbances at ξ_t	5
2.	Typical ξ interval setup	7
3.	Physical deck arrangement of the VISCEL program	12
4.	Schematic overlay configuration of the VISCEL program	13
5.	Physical arrangement of data deck for the VISCEL program	16

ABSTRACT

A general-purpose computer program (VISCEL) for the solution of linear viscoelastic structures is described herein. VISCEL is an extension of the program ELAS, which has been developed for the in-core solution of linear equilibrium problems of structural mechanics of limited size (approximately 500-600 unknowns) and with a minimum of computer time (3-4 min in the IBM 7094, Model I). The standard features and capabilities of ELAS have been preserved. The solution is obtained by means of the displacement method and the finite element technique together with the use of incremental time steps and a synchronized material property concept. The solution is obtained at the end of each time step as incremental and accumulative displacements and stresses. The recursive equations expressing the property of the material with memory have been incorporated in the computations. Also, a scheme which employs constant time steps in the logarithmic scale can be used to minimize the number of computations resulting from the accumulative effects of the material memory.

Almost any structure with linear elastic material properties and continuous structures with linear viscoelastic material properties can be handled by VISCEL. The program is written in FORTRAN IV language for use in 32K IBM 7094 machines with a standard hardware configuration having a minimum of 15 system units. The source deck consists of about 8600 cards; the object deck contains about 1500 cards. The physical program VISCEL is available from the Computer Software Management and Information Center (COSMIC), the NASA agency for the distribution of computer programs.

VISCEL - A GENERAL-PURPOSE COMPUTER PROGRAM
FOR ANALYSIS OF LINEAR VISCOELASTIC STRUCTURES

USER'S MANUAL

I. INTRODUCTION

This two-volume report describes a general-purpose computer program for the solution of linear viscoelastic continuous structures. The program is named VISCEL. It is an extension of the program ELAS, which has been developed for the in-core solution of linear equilibrium problems of structural mechanics (Ref. 1). Volume I of this report details the use of this program. The problem is defined with explicit forms of the expressions used in the development of the present solutions scheme, and the step-by-step analysis technique in the time domain is explained. The basic equations, which have been derived and presented previously in Ref. 2, are reviewed briefly. Descriptions of the program and input procedures are presented. Reference to ELAS is made for basic input information in the linear elastic case corresponding to the initial time and whenever necessary in the succeeding time steps.

Volume II of this report presents the complete list of programs as they appear in ELAS, the function of each subroutine, the list of important symbols and their meanings and locations, the flow charts, the complete listing of programs from ELAS which have been modified in VISCEL, and listings of additional programs.

II. PROBLEM DEFINITION

A. Recursive Equations for Incremental Deformations and Accumulative Residual Forces and Stresses

The equations for finite element analysis of linear thermoviscoelastic material have been derived in Ref. 2. For reference purposes, the explicit form of the recursive equations is presented herein to make clear the definition of the input data explained in the following sections.

The incremental deflection vector of the complete structure at the nth step is obtained from the following recursive equation:

$$\begin{aligned} \{\Delta \bar{U}_n\} = & \left[[B][A][K_{n,n-1}][A]^T[B]^T \right]^{-1} \left\{ \{\bar{P}\} - \sum_{m=1}^{m=n-1} [B][A][K_{n,m-1}][A]^T[B]^T \{\Delta \bar{U}_m\} \right. \\ & \left. - [B][A][K_n][A]^T[B]^T \{\bar{U}_0\} + \sum_{m=1}^{m=n} [B][A] \{E_{n,m-1}\} [\Delta(\alpha\theta)_m] - [B][A] \{F_n\} \right\} \quad (1) \end{aligned}$$

where $\{\Delta \bar{U}_n\}$ is the compact incremental deflection vector at the step n, $[B], [A]$ express boundary conditions and coordinate transformation matrices, $[K_{i,j}]$ is the stiffness matrix obtained from the material matrix corresponding to the reduced time difference $\Delta \xi_{i,j} = \xi_j - \xi_i$, and $\{\bar{P}_n\}$ is the compact external load vector at the step n. The summation expresses the memory of the material. In the second summation, the expression $E_{n,m-1}$ represents the internal forces due to temperature changes obtained from the material matrix computed from $\Delta \xi$ as above, $\Delta(\alpha\theta)_m$ is the temperature and coefficient of expansion change, and F_n is the body force vector.

The residual force vector of an element i at the nth step is obtained from the following recursive equation:

$$\{R_n^i\} = \sum_{m=1}^{m=n-1} [K_{n,m-1}^i] \{\Delta U_m^i\} + [K_{n,0}^i] \{\Delta U_0\} \quad (2)$$

where $\{R_n^i\}$ is the residual forces vector on the nodes of the ith element, $[K_{n,m-1}^i]$ are the elemental stiffness matrices of the ith element corresponding to the time $\Delta \xi_{n,m-1} = \xi_n - \xi_{m-1}$, and $\{\Delta U_m^i\}$ are the nodal deflection vector of the ith element. The residual forces are defined as the total external force vector acting at the nodal points of the structures, including the reactions at the supports and roundoff errors at the free nodes. The residual force vector of the complete structure can be obtained by suitable combinations and summations of the elemental residual vectors.

The stresses in the i th element at the n th step can be computed from

$$\begin{aligned} \{Z_n^i\} &= [E_n^i][\psi^i][\phi^i]^{-1}[I^i][A]^T[B]^T\{U_0^i\} \\ &+ \sum_{m=1}^{m=n} [E_{n, m-1}^i] \left\{ [\psi^i][\phi^i]^{-1}[I^i][A]^T[B]^T \{ \Delta U_m^i \} - \{ \Delta(\alpha\theta)_m^i \} \right\} \end{aligned} \quad (3)$$

where $\{Z_n^i\}$ is the stress vector of the i th element, $[E_n^i]$ is the material matrix corresponding to the time ξ_n , $[\psi^i]$ is the differentiation matrix, $[\phi^i]^{-1}$ is the transformation matrix from the generalized coordinate system to the overall coordinates, and $[I^i]$ is the location matrix which generates the nodal point deflection vector from the overall deflection vector. The definitions of the remaining terms are equivalent to those described above.

Equations (1)-(3) indicate several major computations during a typical n th step, involving, for example, stiffness matrix assembly ($n + 1$ times), matrix multiplication by a vector ($2n$ times), and one matrix inversion. The triple products can be included in the assembly time by suitable operations on indices. The computation time for these matrix operations will become excessive after a few steps. By suitable choice of the time steps, the computation time can be minimized.

B. Synchronized Material Property Concept for Thermorheologically Simple Material

In the following computational scheme it is assumed that, starting from time zero, the material properties and the external disturbances can be expressed as a function of one parameter ξ . This parameter might represent time, reduced time, or any other suitable variable. For each value of ξ , the material properties and external disturbances are assumed defined from the analytical considerations or from the experimental results. Furthermore, the functional relations are given in the form of tables for each material property or external disturbance, and these values are functions of ξ only; i. e., any interaction between them is ignored or assumed included in the predetermined tabulated values of these functions. These values are assumed to be synchronized with respect to the parameter ξ .

The material can be isotropic, orthotropic, or general if the proper definition of the material matrix is available from experimental results in which rheological properties of the material are considered.

The deflection boundary conditions are assigned at the initial loading and assumed to be unchanged during the rheological process. The force boundary conditions can be input as a function of the parameter ξ . The basic information concerning the geometry of the structure, the material, and the boundary conditions corresponding to the initial loading can be found in Ref. 1, Section III.

The schematic representation of the material property and external disturbances are given in Fig. 1. Two basic assumptions and simplifications are included in the computations:

- (1) From the definition of the material properties, it is assumed that

$$K_{i,j} = K_{l,k} \quad \text{if } i - j = l - k \quad (4)$$

where i, j and l, k refer to the arbitrarily chosen values of the parameter ξ .

- (2) The expressions corresponding to the temperature change of Eq. (1) are equivalent to

$$\Delta(\alpha\theta) = \Delta\alpha \cdot \theta + \alpha \cdot \Delta\theta \quad (5)$$

In order to simplify the input procedure, the first term which contains the differential value of the material expansion coefficient in the interval considered is neglected and $\Delta(\alpha\theta)$ is approximated by $\Delta\theta\alpha$. This permits the input of only the temperature difference and the average value of the material expansion coefficient at the step considered.

C. Explicit Expressions for Incremental Deformations, Residual Forces, and Stresses

Let the parameter ξ be expressed as the summation of incremental value $\Delta\xi$ in the form

$$\xi_j^i = \sum_{i=1}^M \sum_{j=1}^{N(i)} \left(\frac{i-1}{N(i)} \Delta\xi \right)_j \quad (6a)$$

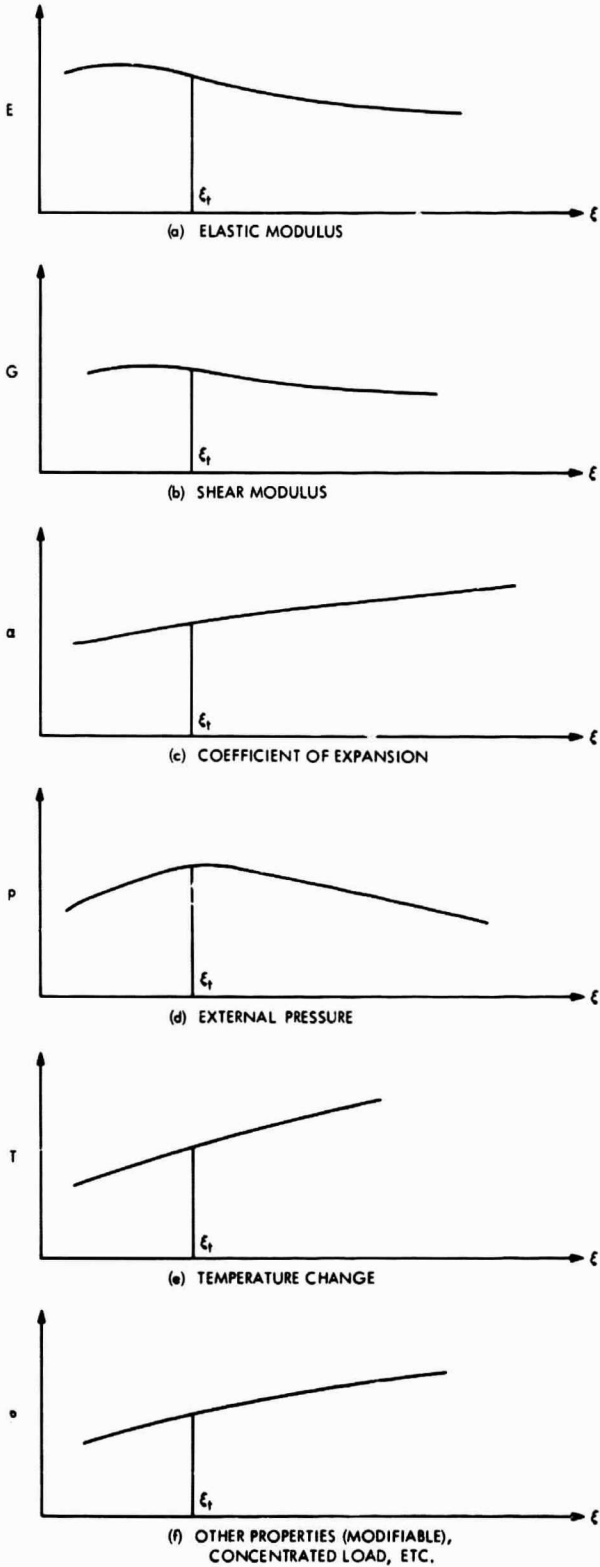


Fig. 1. Schematic representation of the material properties and external disturbances at ξ_t

where ξ_j^i is the value of the relevant parameter at the end of $n = i \times j$ steps, M, N are integer values which can be chosen optionally by the user, and $\Delta\xi$ is a basic interval value of the parameter ξ . The summation order in Eq. (6a) is not interchangeable. One should notice that for $M = 1$ the expression (6a) amounts to a simple summation of $\Delta\xi$; $N(i)$ refers to the fact that N might be assigned various values for different values of i . The subscript j is used to denote the number of steps in any particular group identified by the superscript i . The values of i, j determine, therefore, the value of the parameter ξ_j^i once the M, N , and $\Delta\xi$ are defined. If for each ξ_j^i the material property values and external disturbances are evaluated, then it is possible to use the recurrence formulas presented in Section II-B.

Figure 2 illustrates a typical plot of such values with respect to the parameter ξ . In this example, $M = 3$, $N(1) = 2$, $N(2) = 3$, and $N(3) = 2$, and the corresponding material property values, stiffness matrices, and external disturbances are given with the corresponding superscript and subscript. The input data are evaluated at the points on the curve C_M as follows:

A, B, C, D, \dots , etc.

corresponding to the values of ξ :

$$\begin{aligned}
 \xi_1^1 &= \Delta\xi \\
 \xi_2^1 &= 2\Delta\xi & \xi_1^2 &= 2\Delta\xi \\
 & & \xi_2^2 &= 4\Delta\xi & (6b) \\
 & & \xi_3^2 &= 6\Delta\xi & \xi_1^3 &= 6\Delta\xi \\
 & & & & \xi_2^3 &= 12\Delta\xi
 \end{aligned}$$

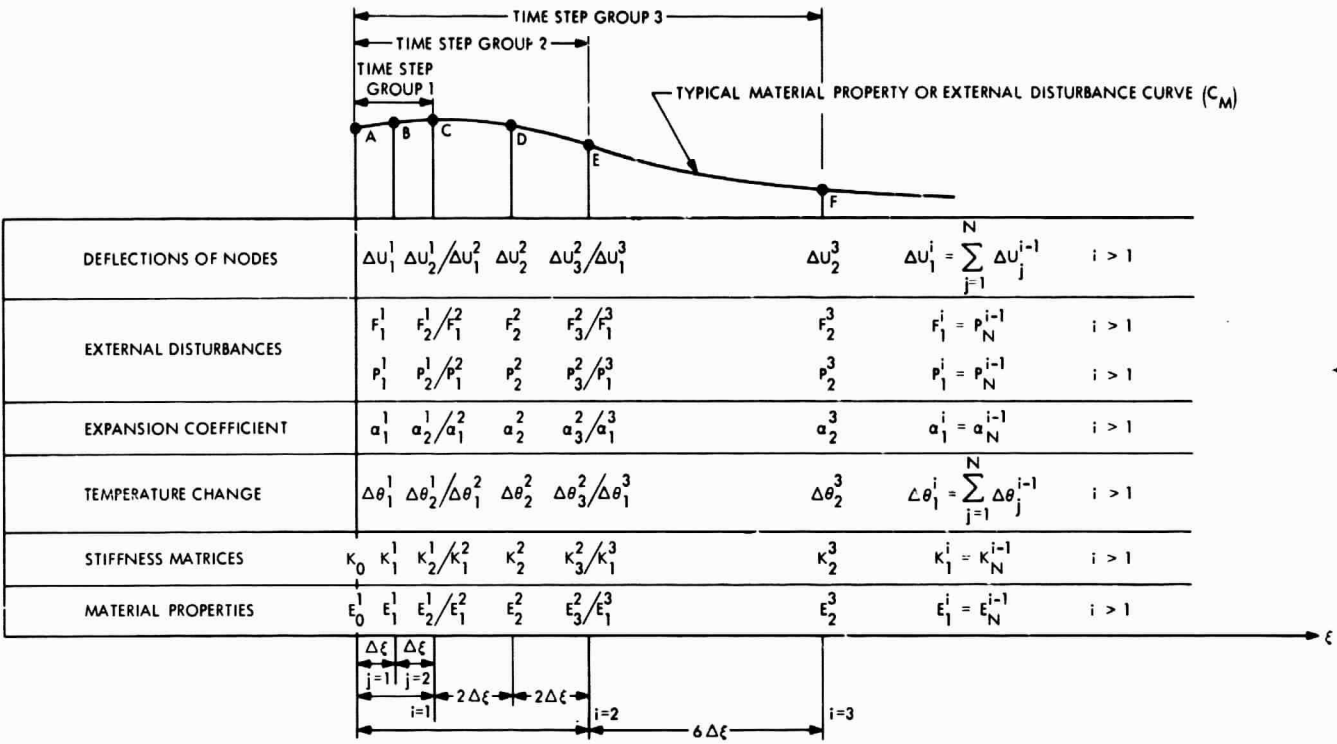


Fig. 2. Typical ξ interval setup

By using the intervals evaluated in accordance with the expressions (6), one can use the recursive Eqs. (1)-(3) with reduced amounts of accumulative recursive computations; i. e., at the end of each time step group, the first step of the next group starts with the values of the input data and deflection values already computed and stored from the recursive relations applied to this group. The time intervals become constant, in the general sense, in the logarithmic scale, and the amount of recursive computations becomes limited with the value of $N(i)$ in each group. The explicit recursive relations for deflections and for residual forces and stresses are given in Tables 1, 2, and 3, respectively, and correspond to the setup of ξ intervals in Fig. 2. Tables 1, 2, and 3 are self-explanatory; definitions of terms are given in Section II-A. The computer program is based on these expressions and described in the following section.

Table 1. Explicit recursive relations for deflections^a

Value of ξ	Incremental deflections
0(initial time)	$U_0 = [K_0]^{-1}[P_0 - E_0(\alpha\Delta\theta)_0 + F_0]$
$\Delta\xi^1$	$\Delta U_1^1 = [K_1^1]^{-1}[P_1^1 - K_1^1 U_0 - E_1^1(\alpha\Delta\theta)_1^1 + F_1^1]$
$2\Delta\xi^1$	$\Delta U_2^1 = [K_1^1]^{-1}[P_2^1 - K_2^1(\Delta U_1^1 + U_0) - E_1^1(\alpha\Delta\theta)_2^1 - E_2^1(\alpha\Delta\theta)_1^1 + F_2^1]$
$\Delta\xi^2 = 2\Delta\xi^1$	$\Delta U_1^2 = \Delta U_1^1 + \Delta U_2^1 \quad K_1^2 = K_2^1 \quad E_1^2 = E_2^1 \quad \alpha_1^2 = \frac{\alpha_1^1 + \alpha_2^1}{2} \quad \Delta\theta_1^2 = \Delta\theta_1^1 + \Delta\theta_2^1$ $P_1^2 = P_2^1 \quad F_1^2 = F_2^1$
$2\Delta\xi^2$	$\Delta U_2^2 = [K_1^2]^{-1}[P_2^2 - K_2^2(\Delta U_1^2 + U_0) - E_1^2(\alpha\Delta\theta)_2^2 - E_2^2(\alpha\Delta\theta)_1^2 + F_2^2]$
$3\Delta\xi^2$	$\Delta U_3^2 = [K_1^2]^{-1}[P_3^2 - K_2^2\Delta U_2^2 - K_3^2(\Delta U_1^2 + U_0) - E_1^2(\alpha\Delta\theta)_3^2 - E_2^2(\alpha\Delta\theta)_2^2 - E_3^2(\alpha\Delta\theta)_1^2 + F_3^2]$
$\Delta\xi^3 = 3\Delta\xi^2$	$\Delta U_1^3 = \Delta U_1^2 + \Delta U_2^2 + \Delta U_3^2 \quad K_1^3 = K_3^2 \quad E_1^3 = E_3^2 \quad \alpha_1^3 = \frac{\alpha_1^2 + \alpha_2^2 + \alpha_3^2}{3}$ $\Delta\theta_1^3 = \Delta\theta_1^2 + \Delta\theta_2^2 + \Delta\theta_3^2 \quad P_1^3 = P_3^2 \quad F_1^3 = F_3^2$
$2\Delta\xi^3$	$\Delta U_2^3 = [K_1^3]^{-1}[P_2^3 - K_2^3(\Delta U_1^3 + U_0) - E_1^3(\alpha\Delta\theta)_2^3 - E_2^3(\alpha\Delta\theta)_1^3 + F_2^3]$

^aCapital letters represent matrices.

Table 2. Explicit recursive relations for residual forces^a

Value of ξ	Accumulative residual forces of the i th element
0 (initial time)	$R_0^i = K_0^i U_0^i$
$\Delta \xi^1$	$R_1^{i1} = K_1^{i1} \Delta U_1^{i1} + K_1^{i1} U_0^i$
$2\Delta \xi^1$	$R_2^{i1} = K_1^{i1} \Delta U_2^{i1} + K_2^{i1} (\Delta U_1^{i1} + \Delta U_0^{i1})$
$\Delta \xi^2 = 2\Delta \xi^1$	$K_1^{i2} = K_2^{i1} \quad R_1^{i2} = K_1^{i2} (\Delta U_1^{i2} + U_0^i)$
$2\Delta \xi^2$	$R_2^{i2} = K_1^{i2} \Delta U_2^{i2} + K_2^{i2} (\Delta U_1^{i2} + U_0^i)$
$3\Delta \xi^2$	$R_3^{i2} = K_1^{i2} \Delta U_3^{i2} + K_2^{i2} \Delta U_2^{i2} + K_3^{i2} (\Delta U_1^{i2} + U_0^i)$
$\Delta \xi^3 = 3\Delta \xi^2$	$K_1^{i3} = K_3^{i2} \quad R_1^{i3} = K_1^{i3} (\Delta U_1^{i3} + U_0^i)$
$2\Delta \xi^3$	$R_2^{i3} = K_1^{i3} \Delta U_2^{i3} + K_2^{i3} (\Delta U_1^{i3} + U_0^i)$

^a K^i are elemental stiffness matrices.

Table 3. Explicit recursive relations for stresses

Value of ξ	Accumulative stresses in the i th element
0 (initial value)	$Z_0^i = [E_0^i] \{U_0^i - (\alpha \Delta \theta)_0^i\}$
$\Delta \xi^1$	$Z_1^{i1} = [E_1^{i1}] \{U_0^i + \Delta U_1^{i1} - (\alpha \Delta \theta)_1^{i1}\}$
$2\Delta \xi^1$	$Z_2^{i1} = [E_2^{i1}] \{U_0^i + \Delta U_1^{i1} - (\alpha \Delta \theta)_1^{i1}\} + [E_1^{i1}] \{\Delta U_2^{i1} - (\alpha \Delta \theta)_2^{i1}\}$
$\Delta \xi^2 = 2\Delta \xi^1$	$\Delta U_1^{i2} = \Delta U_1^{i1} + \Delta U_2^{i1}$
$2\Delta \xi^2$	$Z_2^2 = [E_2^{i2}] \{U_0^i + \Delta U_1^{i2} - (\alpha \Delta \theta)_1^{i2}\}$ $+ [E_1^{i2}] \{\Delta U_2^{i2} - (\alpha \Delta \theta)_2^{i2}\}$
$3\Delta \xi^2$	$Z_3^2 = [E_3^{i2}] \{U_0^i + \Delta U_1^{i2} - (\alpha \Delta \theta)_1^{i2}\}$ $+ [E_2^{i2}] \{\Delta U_2^{i2} - (\alpha \Delta \theta)_2^{i2}\} + [E_1^{i2}] \{\Delta U_3^{i2} - (\alpha \Delta \theta)_3^{i2}\}$
$\Delta \xi^3 = 3\Delta \xi^2$	$\Delta U_1^{i3} = \Delta U_1^{i2} + \Delta U_2^{i2} + \Delta U_3^{i2}$
$2\Delta \xi^3$	$Z_2^3 = [E_2^{i3}] \{U_0^i + \Delta U_1^{i3} - (\alpha \Delta \theta)_1^{i3}\} + [E_1^{i3}] \{\Delta U_2^{i3} - (\alpha \Delta \theta)_2^{i3}\}$

III. DESCRIPTION OF THE PROGRAM

A. Method of Solution and the Computer Program ELAS

The purpose of VISCEL is to solve the equilibrium problems of one-, two-, and three-dimensional linear and linear viscoelastic structures. The basic data preparation procedures for VISCEL are identical to those of ELAS for linear elastic structures and for the initial time solution of the linear viscoelastic structures. Reference 1, therefore, is considered indispensable to this report.

The viscoelastic solution is obtained by using a step-by-step analysis procedure. At the beginning of each step the pertinent information concerning the change of data must be input. At the end of each step the accumulative nodal deflections and the residual forces acting on the nodes and the stresses are obtained. The computer time for each step is approximately equivalent to that of the elastic solution corresponding to the same structure. Information such as material properties and temperature changes and computed values such as stiffness matrices and deflections are stored in the tapes and updated at the end of each equal time step group.

B. Capabilities and Limitations of the Program

Any structure which can be handled by ELAS can be handled by VISCEL for linear problems, but with VISCEL the size of the problem might be slightly smaller. The linear viscoelastic solution is possible only for continuous structures with the actual version of the program. The program can handle up to 99 different types of pressures, material properties, thicknesses, temperature changes, and temperature gradients. Regional viscoelastic behavior of material can be accounted for owing to the inherent versatility of the finite element method. The force boundary conditions can be given as a function of the parameter ξ . The displacement boundary conditions assigned initially may not be changed at the subsequent steps. The coordinate of the nodes and mesh topology information remain identical to the initial input.

Up to about 600 unsuppressed degrees of freedom can be handled by the 32K machine. The solution time would be 3-5 min per step for such problems with the IBM 7094, Model I. By choosing, for example, $\Delta\xi = 1$ sec initial time increment and with $N = 3$, $M = 13$, one can reach, in 40 steps, a few years of elapsed time. Therefore, computation time would be of the order of 2-3 hr in the IBM 7094, Model I for problems having 300 or more unsuppressed degrees of freedom.

C. Programming Language and Physical Description of the Program

The VISCEL program is derived from the FORTRAN IV version of ELAS, with modifications and additional programs in FORTRAN IV. Less than 1% of the program is in FAP language. The subprograms in FAP are TICK*, BUFF., BUFF*, XLOC., and ILED. Figure 3 illustrates the physical arrangement of the program and is self-explanatory. Figure 4 illustrates the configuration of the program. More detailed information concerning the function and the length of the program is presented in Volume II of this report. The input decks for different jobs can be put successively at the end of the program. Linear elastic or linear viscoelastic problems of an indefinite number of different jobs can be solved successively one after the other.

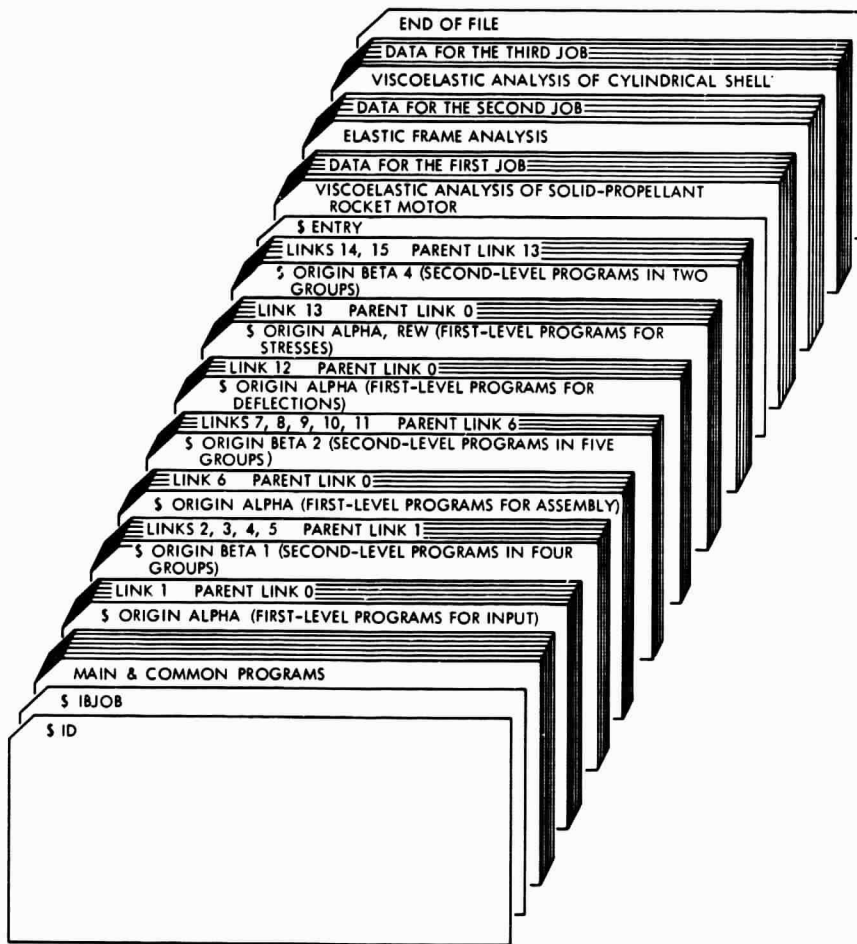


Fig. 3. Physical deck arrangement of the VISCEL program

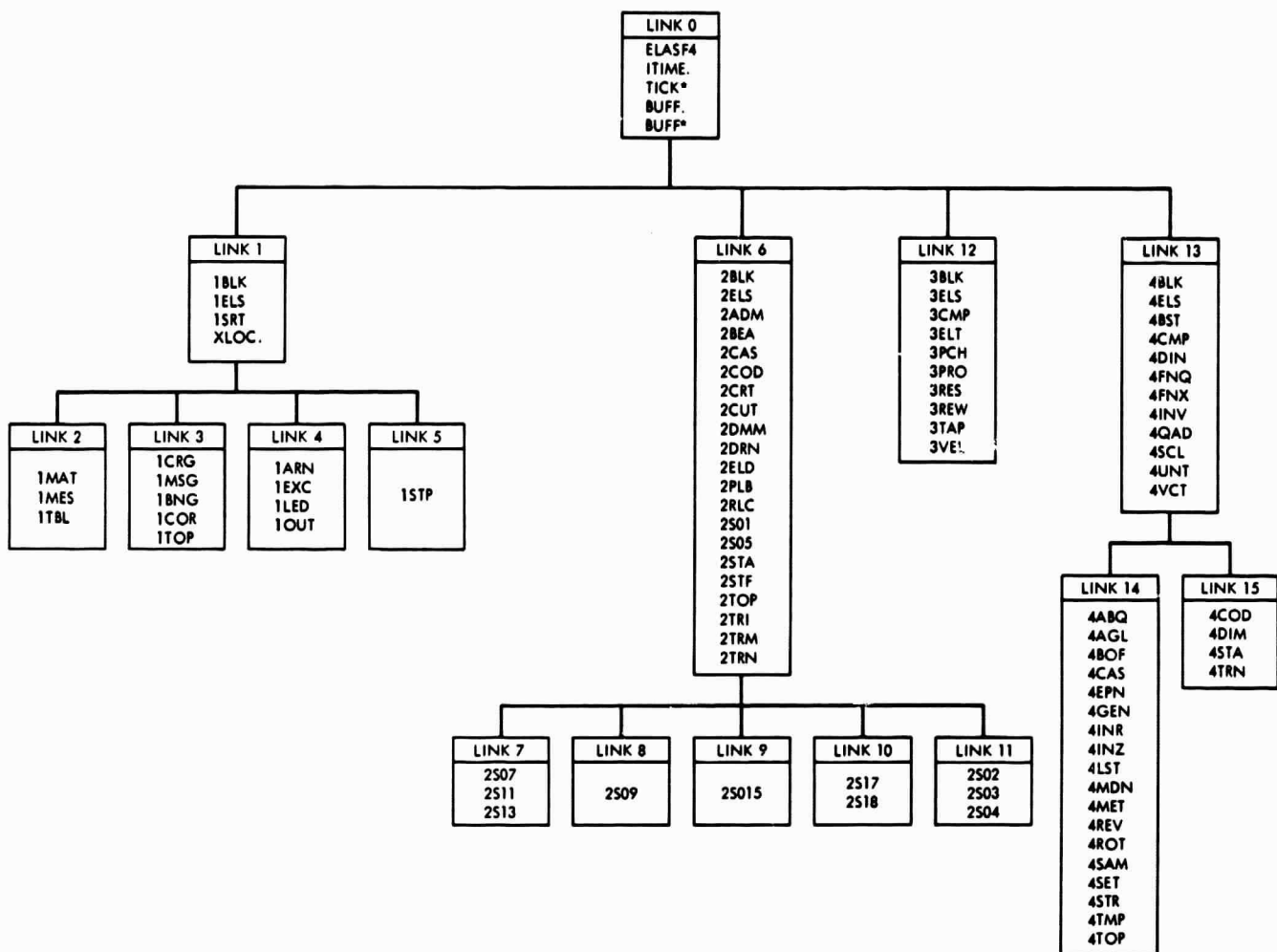


Fig. 4. Schematic overlay configuration of the VISCEL program

D. Computer Hardware and Operational System and Functions of Various System Units

The program has been written in FORTRAN IV language for the 32K IBM 7094, with a few additional subroutines in FAP language. The program uses 15 FORTRAN units, including standard system and input/output units. The function of each unit as used in VISCEL is described in Table 4. For systems and compilers where the functions of the units are not compatible with those in Table 4, the numbers of the units in the program must be reviewed and corrections must be made.

Table 4. The functions of the FORTRAN units as used in VISCEL

FORTRAN unit number	Function of the unit
1	System
2	Chain
3	Scratch for topological information generated in Link 3
4	Storage for deflections
5	Input
6	Output
7	Punch
8	Overlays for FORTRAN IV
9	Storage for material elastic constants
10	Storage for material expansion coefficients
11	Storage for temperature changes
12	Storage for elemental stiffness matrices
13	Storage for overall stiffness matrices
14	Scratch for elemental stiffness matrices in Link 1, 6
15	Scratch for incremental deflections for ISTEP = 1, Link 12
15	Storage for stiffness matrix decomposed by Choleski scheme

IV. PREPARATION OF INPUT

A. General Information

The input data for VISCEL can be classified and described in two groups:

- (1) The basic input data for elastic problems and for initial time solution of viscoelastic problems.
- (2) The additional input data for successive, multiple solutions of the elastic problems or for viscoelastic solutions corresponding to the positive incremental values of the parameter ξ . The portion of the input data which can be modified, optionally, in the additional input data deck, is

predetermined in the program VISCEL and governed by the related constants in the control cards.

Input procedure for (1) is identical to that described in Section IV of Ref. 1.

B. Description of Input Data for Linear Viscoelastic Problems and for Multiple Runs

Figure 5 illustrates the arrangement of the data deck for VISCEL for the solution of linear viscoelastic structures. The master deck is prepared in accordance with the definitions of Ref. 1. In accordance with these definitions, the input items are presented here in two groups: (1) permanent information, (2) modifiable information.

Table 5 presents the list of permanent and modifiable input items with reference to the input items of Ref. 1, Section IV. The field specification for the END card (input item 19) of the master deck and additional input data for VISCEL are in the following form:

70X, I7, 3HEND (7)

where I7 field corresponds to an integer ISUCA, which should be set as follows:

- (1) ISUCA = 0, for linear elastic problems.
- (2) ISUCA < 0, for multiple runs.

When the user wants to solve a problem for various cases by changing only modifiable constants, the END card of the master deck is prepared in accordance with Eq. (7), and the following deck consists of only a title card, a control card, a modifiable information deck, and an end card. By using the same format for the end card of this deck, the problem can be solved for different values of modifiable information, with additional input decks consisting of title card, control card, modifiable information deck, and END card.

- (3) ISUCA > 0, for linear viscoelastic problems.
ISUCA = 1, for the master deck and the following deck.
ISUCA > 1, repeated or in ascending sequence for the following additional decks.

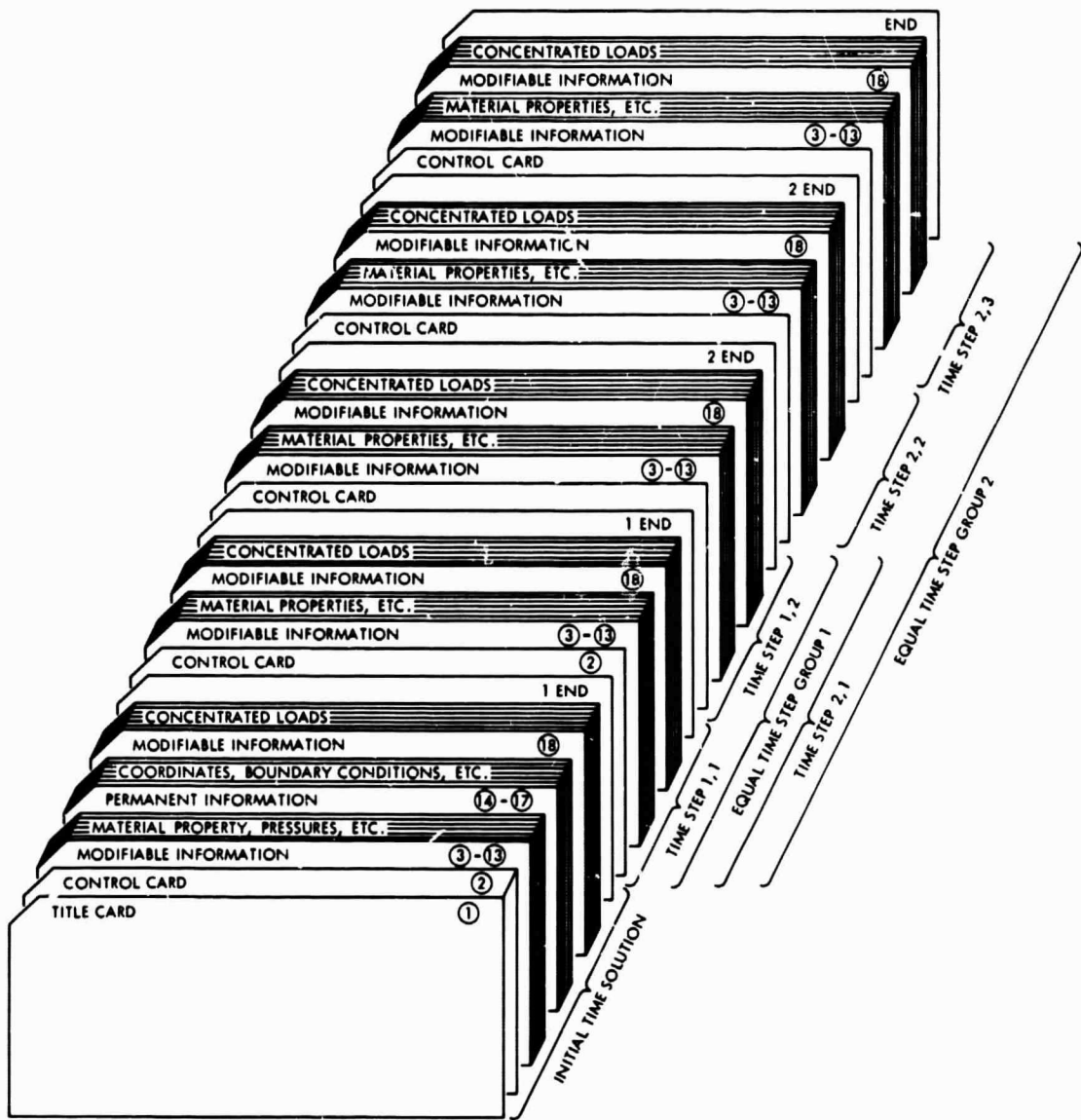


Fig. 5. Physical arrangement of data deck for the VISCEL program

The modifiable information in the decks following the master deck consists of the values of modifiable constants determined from the tables or diagrams as illustrated in Fig. 2. The value of ISUCA determines the equal time step group of the modifiable information. The number of repeated values of ISUCA in the successive decks determines the number of incremental steps in each equal time step group.

Each of the decks following the master deck contains a control card, a modifiable information deck, and an end card.

Table 5. Permanent and modifiable input items

Input item number	Description of input item	Qualifications	Existence in the master deck	Existence in the successive decks
1	Title card	For master deck only	*	-
2	Control card	For master and successive decks	*	*
Modifiable information				
3	Material types	For master and successive decks	*	o
4	Pressure types	For master and successive decks	o	o
5	Thickness types	For master and successive decks	o	o
6	Temperature increase types	For master and successive decks	o	o
7	Temperature gradient types -local y-axis direction	For master and successive decks	o	o
8	Temperature gradient types -local z-axis direction	For master and successive decks	o	o
9	Cross-sectional area types	For master and successive decks	o	o
10	Torsional constant types	For master and successive decks	o	o
11	y-moment-of-inertia types	For master and successive decks	o	o
12	z-moment-of-inertia types	For master and successive decks	o	o
13	Angle types-fixing local y and z axes	For master and successive decks	o	o
Permanent information				
14	Mesh point coordinates	For master deck only	o	-
15	Deflection boundary conditions	For master deck only	o	-
16	Element descriptions	For master deck only	o	-
17	Relabelling information	For master deck only	o	-
Modifiable information				
18	Concentrated loads	For master and successive decks	o	o
19	End card	For master and successive decks	*	*
<p>* = the card(s) must exist. o = the card(s) exist optionally depending on the control constant in the control card and relate to the modifiable information. - = the card(s) must not exist.</p>				

For categories (2) and (3) the following may be noted:

- (1) In viscoelastic problems ($ISUCA \geq 1$), only the master deck has a title card.
- (2) The values of the constants in the control card of the successive data groups should not exceed the corresponding values in the control card of the master deck. They may be less or zero.
- (3) The modifiable information must be read in the same sequence and order as the initial values are read in the master deck. The number can be reduced, if necessary, by reducing the corresponding number in the control card.
- (4) Together with the data prepared in accordance with Section IV of Ref. 1, the constant $ISUCA$ is the only additional constant to be read in order to determine the nature of the problem and the information about the steps in the viscoelastic solution.

V. DESCRIPTION OF THE OUTPUT

A. General Information

The output for VISCEL in linear elastic problems is described in Section VI of Ref. 1. For linear viscoelastic problems, the output corresponding to the initial time solution is identical to that of linear elastic problems except for the very first qualification title of the problem. The level of the output indicated by three different values (0, 1, 2) of INP can optionally be controlled for initial time solutions and viscoelastic or multiple run solutions. The highest level of the output is obtained by $INP = 2$ and yields an excessive amount of printout. Its use is not meaningful for users who are not concerned with the structure of the program.

B. Output for Linear Viscoelastic Problems and Multiple Runs

Table 6 presents a comprehensive summary of the output items for initial time solutions and viscoelastic solutions. The output titles which appear in the printout are as explicit as possible.

Column 1 of Table 6 refers to the output items number of Ref. 1, Section IV. Only those output items corresponding to $INP = 1$ and $ISHUF = 1$ have been listed.

Table 6. Summary of output items

Output item number	Output for linear elastic solution or initial time solution of linear viscoelastic problems	Output for linear viscoelastic solution
1	Linear elastic problem or linear viscoelastic problem (1) Title of the problem (2) Table for control constants	Number of equal time step group... Number of time steps in the group...
2	Modifiable information (material properties, pressure types, etc.)	Modifiable information (material properties, pressure types, etc.)
3	Nodal coordinates	
4	Mesh topology; element property types	
5	Relabelling message	
6	Topology of the reduced stiffness matrix	
7	Stiffness matrix requires... storage locations	
8	Total common length is (decimal)... storage locations	
9	Count of main diagonal elements of row listed stiffness matrix	
10	Force and displacement boundary conditions in directions 1 (2, 3, 4, 5, 6)	
13	Input link took... seconds	Input link took... seconds
17	Generation link took... seconds	Generation link took... seconds
19	Nodal deflections	Accumulative nodal deflections
20	Forces acting at the nodes	Forces acting at the nodes
21	Deflection link took... seconds	Deflection link took... seconds
22	Stresses at the nodes	Stresses at the nodes
25	Stress link took... seconds	Stress link took... seconds

The second column describes the output items of the linear elastic solution and initial time solution for linear viscoelastic problems as they appear in the title of a corresponding list or in the actual printout. The third column gives the output in the following steps of the viscoelastic solutions. Users should note that the title and the table do not appear in the viscoelastic solution, and the only indication of initial time solution of viscoelastic problems is the qualification title before the title of the problem.

REFERENCES

1. Utku, S., and Akyuz, F. A., ELAS - A General-Purpose Computer Program for the Equilibrium Problems of Linear Structures, Vol. I, User's Manual. Technical Report 32-1240, Jet Propulsion Laboratory, Pasadena, Calif., Feb. 1, 1968.
2. Heer, E., and Chen, J. C., Finite Element Formulation for Linear Thermo-viscoelastic Materials, Technical Report 32-1381. Jet Propulsion Laboratory, Pasadena, Calif., June 1, 1969.