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M. L. Williams

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College of Engineering
University of Utah
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CUMULATIVE DAMAGE EFFECTS OF VISCOELASTIC MATERIALS

M. L. Williams
K. L. DeVriės

INTRODUCTION

For the past year a series of investigations relating to the energy concept of adhesive failure have been conducted. This proposal to continue our work will summarize our most recent investigations including: a simple pressurized disk test for measuring the adhesive value for a bonded elastomer and an application to a debonding problem in engineering design debonding of a rubber cylinder from a glass case, debonding between two rubber plates and the effect of an adhesive interlayer between elastic materials.

The following discussion will examine means by which such information can be used by engineers to predict failure of adhesive joints. In addition, we will discuss simple tests by which adhesive energy or "strength" might be measured until such time as it is possible to predict it reliably from basic material properties. This quantity can then be used as a design parameter to predict failure for other more comple ashesive geometries and loading configurations. This discussion is based in part on work published elsewhere.[1-5].

At the Fifth U. S. National Congress of Applied Mechanics, Williams discussed[6] an essential similarity between certain problems of adhesion and fracture. Considering, for example, the elastic analysis of a thin sheet in the neighborhood of a sharp geometric discontinuity such as a wedge point or crack tip, it is well known that a singularity in stress exists at the point of discontinuity and depends upon the local boundary conditions, loading, and properties of the material.[7-10] In the case of a central finite length crack in an infinite sheet subjected to tension, the classic Griffith problem gives a local stress variation which is proportional to the inverse square root of the distance from the crack tip.

Inasmuch as this (mathematically) infinite stress exists here for even the smallest loading, it appears that instantaneous fracture would occur and that stress analysis would not be useful for predicting a finite stress which the sheet could withstand before fracture. The essential contribution of Griffith,[11] however, was to develop an overall energy balance, which incorporated the integrable stress singularity, by equating the reduction in strain energy to the energy required to create new surface. The result was the prediction of a finite applied tensile stress, σ_{cr}, needed to initiate fracture, namely $\sigma_{cr} = \sqrt{2E\gamma_c/\pi a}$ in which E and γ_c are the Young's modulus and energy to create new fracture surface respectively, and 2a is the finite length of the crack in the thin sheet. It is apparent, therefore that the use of the integrated energy balance neatly circumvented the question of how infinite the infinite stress need become before fracture. It furthermore suggests the way in which other problems in stress analysis having stress singularities can be attacked in order to predict a finite stress at failure notwithstanding an infinite stress at the origin of the fracture initiation.

The character of elastic stress singularities to be expected for various geometric discontinuities was investigated by Williams,[7,8] and later applied to the specific situation of the interface between dissimilar media.[12] In this case too, when a crack existed along the line of demarcation of the two materials, the stress singularity was likewise singular, although not necessarily solely of the $r^{-\frac{1}{2}}$ type. It subsequently became attractive to several workers in the field to inquire whether the same approach as Griffith used could be applied to predict the stress required to further separate or fracture the (adhesively bonded) interface between two different media, again notwithstanding the predicted existence of an infinite stress at the crack point for even small applied loads.

The phenomenological similarity in the two cases becomes clear. In the Griffith problem the finite length central crack 2a, lies, say, along the x-axis with the upper and lower half planes occupied by the same material; in the second problem, the materials above and below the x-axis are different. For the purposes of discussion, we shall assume the material in the lower half plane to be infinitely rigid (e.g. glass) with respect to that in the upper half plane (e.g. rubber), and assume perfect adhesion over $|x|>a$. The stresses at the crack ends, $|x|=a$, are both singular. In the first case the Griffith critical stress is the classic example of cohesive fracture and well-known; in the second, the example of perfect adhesive failure is not.*

Before looking into the second problem in more detail, it is pertinent to comment upon the distinction between the mechanics and chemistry viewpoints. As structured above, the mechanics approach is straightforward and consists of two parts: (1) conduct the stress analysis for an edge-bonded specimen having a central finite crack at the interface with a rigid boundary, and (2) express the incremental new surface energy generated as the crack extends. This latter part however requires interpretation. In the cohesive fracture problem with the same material on both sides of the extending crack, Griffith used $\Delta S=4\gamma_c\Delta c$ as the incremental energy per unit thickness. The factor four arises because both ends of the crack are assumed to extend equally, and each end creates two new surfaces, one above and one below the crack.

* It should be clear that a continuum mechanics analysis does not, of itself, differentiate between a cohesive or adhesive mechanism of failure. The distinction lies in the behavior implied by using a particular one of the respective energies to create the new surface, namely γ_c (cohesive) or γ_a (adhesive). Furthermore there appears to be no direct association between the critical surface tension and the continuum mechanics analysis of the unstable infinitesimal deformation of a solid, although for special cases the critical surface stress to cause a spherical flaw to become unstable has been deduced by Williams and Schapery (see Intl. J. Frac. Mech., Vol. 1, No. 1 (1965) and Vol. 1, No. 4 (1965).

The specific energy, γ_c has been subscripted to denote the value associated with cohesive failure. For adhesive failure, it would be appropriate, although not necessarily unique, to write $\Delta S=2\gamma_a \Delta a$ denoting that only two new free surfaces are formed in the elastic material. While this leaves open to surface chemists the question of any quantitative relation between γ_a and γ_c, as long as γ_a is a fundamental material constant, it can be used subsequently for predicting adhesive failure in a different geometric or loading configuration. Suppressing further comment upon this point until later, let us proceed to a detailed consideration of perfect adhesion between elastic and rigid materials.

THE ELASTIC RIGID-ADHESIVE PROBLEM

The first distinction to be drawn is that between problems for which exact or approximate solutions are available. In the first category we find two useful geometric configurations: (1) the end-bonded half plane with an interface finite crack of length 2a, and (2) the end-bonded circular rod containing an interface penny-shaped crack of radius 2a. In both cases the discontinuity is located along the bond to the rigid boundary. The width or external rod diameter is assumed infinitely large. For the geometries of immediate concern (Figure 1), analytical solutions for the case of uniform internal pressure in the crack are already available.[13,14] Without reproducing the analysis* but merely equating the change in internal strain energy with respect to the increment in flaw enlargement, Δa, to the change in energy to create new adhesive surface, $\Delta S_1=(2\gamma_a b)\Delta a$ or $\Delta S_2=(2\pi a\gamma_a)\Delta a$ for problems one or two respectively, one easily deduces the results in Table 1.

In principle then, either of the configurations in Table 1 could be used to determine γ_a, and then verified by checking the results obtained in the other. As a practical matter, there are the usual

* The referenced results have been specialized to the case of an incompressible elastic material ($\nu=\frac{1}{2}$) bonded to a rigid support in which case the oscillating character of the stress singularity[12] disappears, and actually gives essentially the same behavior as the homogeneous problem.

Table 1--CRITICAL IMPOSED PRESSURE

Configuration	P_{crit}
End-bonded half-plane with line crack	$\sqrt{\frac{2}{\pi(1-\nu^2)}}\ \sqrt{\frac{E\gamma_a}{a}}$
End-bonded rod with penny-shaped crack	$\sqrt{\frac{2}{2(1-\nu^2)}}\ \sqrt{\frac{E\gamma_a}{a}}$

experimental difficulties, such as how one introduces uniform pressure into the crack without leakage, or how to control, repeatedly, the thickness of the adhesive coating in the various configurations. For these reasons, and to ascertain whether indeed there is a unique value of γ_a independent of configuration, it is useful to have as wide a range of test conditions as possible.* In the earlier paper(6) it was indicated that modern computational techniques are sufficiently accurate, to determine the energy gradient with crack size with sufficient accuracy, even in reasonably complicated three-dimensional problems. One could proceed to construct therefore almost any experimentally desired test configuration, although at some expense in complicating the stress analysis.

Returning to one of the initial points, however, it may also prove useful to inquire into the potential accuracy of approximate solutions for easily tested experimental configurations. One of the commonest ones used in adhesion evaluation is the strip peel test (Figure 2), which has several variations as discussed and reviewed, for example, by Bikerman(15) and Kaelble.(16) While variations of this test are attractive especially for ranking purposes, its analysis is not thought to be completely satisfactory for our present purposes, although in principle it could be made so even if one resorted to numerical computations. Another example developed for cohesive failure is the

* For example, one can easily obtain similar results for the related case of a concentrated central splitting force normal to the interface.

cantilever split beam proposed by Obreimoff[17] and subsequently modified by Gilman[18] and Berry[19] for evaluating the cohesive fracture energy in thin sheets like mica (Figure 3). In addition, the cantilever beam and the point loaded circular plate (Figure 4) tests proposed by Malyshev and Salganik,[20] seem very attractive to include in a family of practical tests for determining the adhesive energy. As a final configuration forming the series of possible geometries which can be studied, it should be noted that the previous <u>exact</u> solutions for the edge-bonded plane strain specimen and the rod specimen may be viewed as a limit situation as the beam or plate thickness, h, becomes infinite.

Viewing all the solutions in this context, both exact and approximate, and with due consideration of experimental convenience, it is proposed that Malyshev and Salganik's analysis can be supplemented by a pressurized bubble test (Figure 4, except using a uniform pressure instead of their point loading). Conceptually this type of test is also not new, although perhaps some improvements in the analytical expression of the results can be achieved. Dannenberg[21] for example has discussed the measurement of adhesion by a "blister method," which is essentially a pressurized bubble. In that case the work of adhesion was deduced from measurements of the work input, pdV, of the pressurizing fluid for application to the adhesion of paint. It may be noted, incidentally, that the consecutive detachment of the coating which he notes experimentally, is possibly related to the same "stick-slip" phenomenon frequently observed in the cohesive fracture of polymers.

From the principle of energy conservation, one may write that the work done by the applied pressure moving through the virtual displacement must be balanced by the change in internal strain energy plus the change in the energy to create any new surface. Inasmuch as the change in internal energy is one-half the applied work for a linear load-deflection relation by Clapeyron's theorem,[22] one has

$$(\tfrac{1}{2})\left\{ 2\pi \int_0^a p_0 \delta w(r)\, r\, dr \right\} = \delta(\gamma_a \pi a^2) \qquad (1)$$

From plate theory, one finds that the deflection of a uniformly loaded clamped plate of radius a is given by [23]

$$w(r) = (1/64)(p_0/D)(a^2-r^2)^2 \tag{2}$$

where $D=Eh^3/12(1-\nu^2)$ is the plate flexural rigidity, so that the energy balance yields

$$p_a = \left[\frac{32}{3(1-\nu^2)} \left(\frac{h}{a}\right)^3\right]^{1/2} \sqrt{\frac{E\gamma_a}{a}} \tag{3}$$

which can be compared to the rod result (Table 1) of

$$p_{cr} = \left[\frac{\pi}{2(1-\nu^2)}\right]^{1/2} \sqrt{\frac{E\gamma_a}{a}} \tag{4}$$

The comparative dependency upon (h/a) is to be noted.

Several of the pertinent cases have been collected in Table 2 for use in experimental studies. Using the pressurized crack configuration for example, one can construct the curve in Figure 5 which shows the predicted limit results for bonded rods or plane stress) plates. By establishing these limits it is possible to bracket, with engineering accuracy, the adhesive energy value, γ_a, for a wide range of experimental configurations without having to compute, explicitly, the difficult transition cases.*

As a concluding point before discussing some representative test results, it should be emphasized that the analyses of all the thin disk configurations are approximate because only beam and plate theory has been used. Actually (mathematically) infinite stresses exist at the bonded end of the beam or clamped edge of the plate at the specimen-bond interface. These are not included in the approximate analysis, and thus contribute to the potential error in adhesive energy determinations. Its degree must be ascertained. In the meantime, it is reasonable to inquire as to the degree of accuracy obtained with the

* If warranted, these results can be extended to very thin plates or membranes for which the load-deflection relations are nonlinear.[25,26]

Table 2--SELECTED CRITICAL LOADINGS FOR ADHESIVE FAILURE TO A RIGID MEDIUM

AXIALLY SYMMETRIC

	Geometry	Reference	Critical Loading
1.	Bonded rod with small penny-shaped crack; uniform pressure, p	14	$p^2_{cr} = \frac{\pi}{2(1-\nu^2)} \frac{E\gamma_a}{a}$
2.	Bonded disk with small penny-shaped crack; uniform pressure, p	Eqn. 4	$p^2_{cr} = \frac{32}{3(1-\nu^2)} \left(\frac{h}{a}\right)^3 \frac{E\gamma_a}{a}$
3.	Bonded disk with small penny-shaped crack; central conc. load, P	20	$P^2_{cr} = \frac{8\pi^2}{3(1-\nu^2)} Eh^3\gamma_a$

THIN SHEETS (PLANE STRESS*)

	Geometry	Reference	Critical Loading
1.	End-loaded cantilever, P (length, a; width, b)	20	$P^2_{cr} = \frac{Eb^2h^3\gamma_a}{6a^2}$
2.	Uniformly loaded cantilever (length, a; width, b)	--	$p^2_{cr} = \frac{2Eh^3\gamma_a}{3a^4}$
3.	Centrally loaded, cantilever on both ends (length, 2a; width, b) P	--	$P^2_{cr} = \frac{8Eb^2h^3\gamma_a}{3a^2}$
4.	Uniformly loaded, cantilever on both ends (length, 2a; width, b)	--	$p^2_{cr} = \frac{3Eh^3\gamma_a}{2a^4}$
5.	Edge-bonded thin sheet (crack length, 2a); central concentrated load P(lbs/in)**	13	$P^2_{cr} = 2\pi E\gamma_a a$
6.	Edge-bonded sheet (crack length, 2a); uniform pressure loading	13	$^2_{cr} = \frac{2}{\pi} \frac{E\gamma_a}{a}$

* Plane strain obtained by replacing E by $E/(1-\nu^2)$.

** Exact for plane strain and incompressible elastic medium. Estimate only for plane stress due to oscillating type singularity; exact analysis follows from reference (13).

with the simple analyses.

One may note in passing, however, that the experimental solution achieved by Broutman and McGarry[24] for adhesive work measurements might be adopted. The double cantilever beam test, which they used, leads theoretically to a load-deflection relation containing the beam (crack) length raised to the third power when elementary beam theory is used. By actually plotting deflection per unit force versus crack length they obtained an exponent from their tests which was a constant, somewhat less than three, over a substantial range of the crack length. If this approach was applied to the pressurized disk by plotting experimental results of deflection/pressure as a function of debonding radius, perhaps some of the uncertainty due to using Kirchoff-Lowe plate theory could be eliminated. For the time being however, it is reasonable to inquire first into the degree of accuracy obtained with the simple analysis.

EXPERIMENTAL RESULTS

Malyshev and Salganik[20] have conducted experiments using a split beam, end-loaded cantilever, and a point-loaded circular plate (Figure 4) using a steel-plexiglas combination. For the circular plate configuration, they obtained the criticality relation as (Table 2)

$$\gamma_a = \frac{P^2}{32\pi^2 D} = (8Dw_{max}^2/a^4) \tag{5}$$

This result indicates that if γ_a is a material constant, the applied force to debond the disk would remain a constant while the central deflection increased proportional to the square of the disk radius. As is seen in Figure 6, reproduced from their work, there is a substantial portion of the deflection over which the applied force does remain constant. They point out that the best correlation would be expected if the deflection is smaller than the disk thickness due to limitations of the basic plate theory. This method for determining the adhesive energy, γ_a, seems very attractive and reasonably simple to obtain. It furthermore has the advantage, in contrast to the beam-like specimens, that bonding control need only be attained at the initial fracture front because there are no "sides" to a circular disk specimen as there are when bonding two beam strips

With the thought that improved sensitivity could be obtained using this same specimen but with pressurization rather than point loading, plus the fact that there results a smaller maximum deflection for the same total load, W. B. Jones has conducted some experiments using a glass-rubber combination.[27] Using $\nu=\frac{1}{2}$ for the Poisson's ratio of the rubber, one expects

$$\gamma_a = \frac{9}{128}\;\frac{P^2a^4}{Eh^3} = \tfrac{1}{8}Pw_{max} = \left[\frac{32}{9}\;\frac{Eh^3}{a^4}\right]\; w^2_{max} \tag{6}$$

It may also be noted that, in contrast to the concentrated loading, there is no necessity for the hole through which the load is introduced to remain concentric with the debonding disk for the analytic result to apply. In this respect the pressure loading is also experimentally simpler, although the fracture once started will be unstable while the concentrated load configuration is neutrally stable.

DESCRIPTION OF TEST RESULTS

The test vehicle for the surface energy-bond tests consisted of a glass disk having a central hole and a thin sheet of polyurethane rubber cast-bond to it. The glass disk was of lens quality crown glass, 3.8 inches in diameter and 0.128 inches thick. A brass pressure fitting was bonded to the plate. Through it pressure was introduced, thereby causing progressive debonding (Figure 7). Simultaneous measurements were made of the diameter of the debonded area and the imposed pressure.

The disk was prepared for casting the polyurethane by first wiping with commercial grade acetone, then with chloroform. A metal pin was then wiped with silicone vacuum grease and inserted into the hole through the brass fitting so that the end was flush with the top surface of the disk. A circular area about 1/2 inch in diameter was then thinly covered with vacuum grease to initiate unbonding. The assembly was then placed in the oven to preheat before casting.

The polyurethane rubber was made of equal volumes of Solithane* 113 and castor oil. The components were preheated to 140°F and mixed. The mixture was degassed in a vacuum for about 10 minutes, then poured to the desired thickness on the glass disks. The rubber was subsequently cured for about two hours at 280°F. The oven was then cut-off and allowed to cool with the samples inside. After about two hours, the samples were removed and readied for testing. A metal wire was pushed through the hole to clear the flashing and to initiate unbond. The samples were attached to a regulated air supply.

Procedures usually were to pressurize the sample so that debonding initiated and propagated beyond the edges of the brass fitting so that it could be more easily observed. On some tests, however, measurements were also taken near the center of the sample. The pressure was slowly increased until debonding initiated, then was slowly decreased until the flaw line stopped propagating. Pressure values from a mercury manometer were recorded as a function of the diameter of the debond. Some typical data are shown in Figure 8.

In some tests crossed polaroids were used to enhance the contrast at the edge of the unbond.

APPLICATION TO A MORE COMPLICATED GEOMETRY

With the foregoing reasonable results obtained from simple experiments it appears possible to utilize the known value of the adhesive energy in investigating adhesive failure in more complicated geometries. As an illustration, a rather sophisticated numerical analysis was used to obtain the unbonding threshold for a hollow thick-walled cylinder of polyurethane rubber cast into a glass tube and subjected to a temperature drop.[6]

The thermal stresses in the cylinder were computed numerically, normalized upon $E\alpha\Delta T$, and the strain energy integrated over the cylinder. Then assuming only one new interface, the change of potential energy, V, with crack length, ℓ, was equated for rigid boundary displacements to

* A commercial polyurethane rubber available from the Thiokol Chemical Corporation.

the change in energy to form new surface, i.e.

$$2\pi[Ea\Delta T]^2 \delta V(\bar{\sigma}_{ij})/\delta\ell\Big|_{\substack{\ell=\ell_{cr}\\ \sigma = \text{const.}}} = 2\pi b\gamma_a \tag{7}$$

leading, at criticality, to

$$Ea\,T_{cr} = \sqrt{\frac{E\gamma_a}{\ell_{cr}\,\delta I[(\ell/b)^2]/\delta[(\ell/b)^2]\big|_{\ell=\ell_{cr}}}} \tag{8}$$

where $I[(\ell/b)^2]$ is the non-dimensional energy. This result for the special geometry computed is given in Figure 9.

The implications of this result appear rather interesting. Note first that for small debonding lengths, the basic variation is consistent with the usual Griffith inverse dependence upon crack length, and qualitatively that a lower temperature difference is required to propagate a larger crack than a smaller one. On the other hand, note that for $\ell/b \gtrsim 0.17$ the functional dependence upon the fracture length is reversed, namely that an increasing temperature is required to make a crack longer. The key to this behavior predicted by the numerical analysis is found in evaluating the rate of energy release with respect to crack length. For the shorter values of ℓ_{cr}, the strain energy being released is so great that it cannot be completely absorbed by the creation of new surface, and crack instability results, as in the Griffith centrally-cracked sheet. On the other hand, due to the three-dimensionality of the geometry and the existence of hoop stress, once the crack is sufficiently large, in this case $\ell/b \approx 0.17$, the energy release is just that to create the new surface--a stable growth proportional to the amount of driving force (temperature difference) present.

One therefore predicts in an elastic medium that a small crack, once started propagating by a temperature difference greater than critical for that particular length, will extend or jump to its other stable position across the "well" in Figure 9, path ABC, and proceed toward D if the temperature difference is further increased. On the other hand, if the crack begins at a larger length, path A'B', as the temperature increases it will merely grow larger toward D, and grow in a stable fashion.

It is thus seen that the debonding characteristics could be predicted, as well as the value A' for a critical stable crack length ℓ^*/b, by using the value of adhesive energy γ_a from Figure 8, although the assumed independence of γ_a upon temperature should be verified. It is implicit, of course, that this fracture analysis also assumes the same quality of bond in both the laboratory test of the disk and the filled cylinder. While the example chosen has been deliberately more complicated than other typical engineering design problems with, say, mechanical loading, the proposed analysis technique is thought to be direct and simple.

THE ADHESIVE BONDING OF A DOUBLE-SIDED BLISTER

To this point, we have treated cases where one material was considered as rigid in comparison to the other. In some cases, however, it may be desirable to adhesively bond two disks of different elastic properties and thicknesses. Thus, instead of the pressure raising a blister off a rigid, flat surface, and, at the critical pressure, extending the radius of the blister, it may be of interest to separate two layers like a diaphragm.

If each is circular at the bond line, the deflection function under uniform pressure, will be of the form[23]

$$w(r) = c_1 + c_2 r^2 + pr^4/(64D) \tag{9}$$

where D is the flexural rigidity of the plate, $D=Eh^3/[12(1-\nu^2)]$. Requirements for matching the deflection and negative slope at r=a, plus conditions for zero moment and shear at the edge of the bonded region, r=a, lead to

$$\underset{(\ell)}{w_u} = \frac{\underset{(\ell)}{p_u}\, a^4}{64\underset{(\ell)}{D_u}} \left[1 - \left(\frac{r}{a}\right)^2\right]^2 \tag{10}$$

where u and ℓ stand for the upper and lower plates respectively.

By Clapeyron's Theorem, the strain energy stored in the plate is

$$U_{\underset{(\ell)}{u}} = \frac{2\pi p_{\underset{(\ell)}{u}}^2 a^6}{(128)(6)D_{\underset{(\ell)}{u}}}$$

upon noting that $p_u = -p_\ell \equiv p$, the total strain energy in the plate is

$$U = \frac{2\pi p^2 a^6}{6(128)D_u}\left[1 + \frac{D_u}{D_\ell}\right] \tag{11}$$

The adhesive energy to create new surface is

$$S = \pi a^2 \gamma_a \tag{12}$$

The threshold for adhesive fracture is found by equating $\partial U/\partial a$ to give the critical pressure as

$$p_{cr} = \sqrt{\frac{32}{3(1-\nu_u^2)}\left(\frac{h_u}{a}\right)^3}\ \sqrt{\frac{E_u\ \gamma_a/a}{1 + (D_u/D_\ell)}} \tag{13}$$

where the original result for a rigid lower plate, $D_\ell \to \infty$ is recovered in the limit. The effect of an elastic lower plate is given by the factor $1+(D_u/D_\ell)$. Note that a resilient base tends to reduce the stress required to fracture the adhesive bond, and that it can result from either a change in tensile modulus, E_ℓ, or thickness, h_ℓ.

We are presently experimentally investigating these relations. A photograph of the experimental arrangement designed by W. B. Jones and G. D. Hone is shown in Figure 10. As shown in Figure 11, the preliminary results of these experiments indicate that the critical pressure is directly proportional to $1/a^2$ in agreement with (13). The transition in slope in this figure is attributed to the pressure connections on the bottom plate increasing the effective stiffness of this plate.

THE EFFECT OF AN ADHESIVE INTERLAYER

In many cases, one material is not cast and cured against the second material, but rather it is bonded to it by an intermediate, usually thin, layer of a different material. The previously mentioned analysis does not specifically treat this important class of problems but we shall illustrate the modifications which can be adopted to do so.

As a model of the phenomenon involved, consider instead of a circular blister, an elastic plate strip of material properties E and ν, and thickness h, infinite in the lateral y-direction (plane strain) and appearing in cross section essentially as a beam (Figure 12). The plate specimen is mounted upon an interface elastic adhesive layer of material properties E' and ν' and thickness h'. The substrate underneath the adhesive layer is considered infinitely rigid, as before. From the standpoint of linear plate theory, the problem can be formulated by assuming an equivalent Winkler elastic foundation[(23)] modulus, k, which leads then to a simple analysis of the classic beam supported by an elastic foundation.* The deflection of the central portion of the strip, $|x|<a$, loaded by pressure only, is then determined from standard beam theory, which for the outer portions, $|x|>a$--that supported by the elastic foundation, is found from the Winkler equation. The deflection, slope, moment, and shear are matched at $|x|=a$ to complete the solution. The essential details follow.

The inner solution, $|x|<a$. For this region the governing differential equation is

$$D(d^4w/dx^4) = p \tag{14}$$

with the solution

$$Dw(x) = px^4/24 + C_2x^2/2 + C_0 \tag{15}$$

* It should be noted that the essential assumption introduced by Winkler allows for vertical motion and dilatation stress only. Shear effects in the foundation can be incorporated however, e. g., K. S. Pister and M. L. Williams, "Bending of Plates on a Viscoelastic Foundation," Proceedings of the Amer. Soc. of Civil Engineers, October 1960, pp. 992-1005.

where the flexural rigidity is $D=Eh^3/[12(1-\nu^2)]$.

The outer solution, $|x|>a$. In this region, the Winkler equation is

$$D(d^4w/dx^4) + kw = 0 \tag{16}$$

with the solution,

$$w(x) = (C_3 \cos \lambda x + C_4 \sin \lambda x)\exp(-\lambda x) \tag{17}$$

where that part of the solution corresponding to $\exp(+\lambda x)$ has been neglected due to the assumed lateral infinite extent of the adhesive layer. The constant λ is defined through

$$k \equiv 4D\lambda^4 \tag{18a}$$

where it is also useful to define for later use the dimensionless parameters representing the (damped) wave length of the deformation in the foundation

$$\mu \equiv \lambda a \tag{18b}$$

The constants C_0, C_2, C_3 and C_4 are determined by requiring that the deflection, slope, moment and shear-$w(a)$, $dw(a)/dx$, $D(d^2w/dx^2)$, and $D(d^3w/dx^3)$ respectively, for the inner and outer solutions match at $x=a$. An application of these conditions leads to

$$\begin{aligned}
C_0 &= pa^4\left[\frac{1}{24} + \frac{2\mu^3 + 5\mu^2 + 6\mu + 3}{12\mu^3(1+\mu)}\right] \\
C_2 &= -pa^2\left[\frac{1}{6} + \frac{2\mu + 3}{6\mu(1 + \mu)}\right] \\
C_3 &= \frac{pa^4}{D\exp(-\lambda a)}\left[\frac{(2\mu^2 + 6\mu + 3)\cos\mu + (2\mu^2 - 3)\sin\mu}{12\mu^3(1 + \mu)}\right] \\
C_4 &= \frac{pa^4}{D\exp(-\lambda a)}\left[\frac{(2\mu^2 + 6\mu + 3)\sin\mu - (2\mu^2 - 3)\cos\mu}{12\mu^3(1 + \mu)}\right]
\end{aligned} \tag{19}$$

With the constants now determined, the strain energy per unit lateral length stored in the various parts of the specimen can be calculated.

Using Clapeyron's theorem,[29] one finds that the energy in the plate is

$$U = \frac{1}{2} \int_{-a}^{a} pw(x)\, dx \tag{20}$$

whereupon using (15), along with (19)

$$\frac{p^2a^5}{45D}\left\{1 + \frac{5}{4}\ \frac{4\mu^3 + 12\mu^2 + 18\mu + 9}{\mu^3(1 + \mu)}\right\} \tag{21}$$

where the first term in the bracket may be identified as the elementary solution contributed by the central portion of the plate strip alone with clamped edges at $|x|=a$, i.e. $\mu\to\infty$.

It is necessary next to calculate the increment in adhesive energy per unit lateral length to create new surface at the ends of the strip, S. It is assumed that the debonding will take place along the interface between the plate and the adhesive layer, and that the specific adhesive energy associated with the debonding is γ_a. Thus one has

$$S=2a\gamma_a \tag{22}$$

The energy balance at criticality, $\partial U/\partial a = \partial S/\partial a$, upon noting (18b), leads easily to

$$p_{cr} = \frac{\frac{3}{2}\left(\frac{h}{a}\right)^3\left(\frac{E\gamma_a}{a}\right)}{1 + \frac{16\mu^4 + 56\mu^3 + 84\mu^2 + 63\mu + 18}{4\mu^3(1 + \mu)^2}} \tag{23}$$

where the numerator is the clamped-clamped strip solution, i.e., $\mu\to\infty$, given earlier (Table 1, Formula 4). The second term in the denominator therefore represents the effect and presence of an elastic foundation, within the Winkler approximation, and is the desired correction factor for a finite thickness adhesive bonding layer.

It is useful to note that in many cases the parameter $\mu=\lambda a$ will be sufficiently large that the correction factor may be adequately represented by

$$p_{cr} \simeq \frac{3}{2}\left(\frac{h}{a}\right)^3 \frac{E\gamma_a}{a}\left[1 - \frac{4}{\mu} + \cdots\right] \qquad ;\mu\to\infty \tag{24}$$

$$= \frac{3}{2}\left(\frac{h}{a}\right)^3 \frac{E\gamma_a}{a}\left[1 - 4\sqrt[4]{\frac{1}{3}\left(\frac{h}{a}\right)^3 \frac{E}{ka}} + \cdots\right] \qquad ;\mu\to\infty \tag{25}$$

Also, the foundation modulus k may be estimated from the characteristics of the adhesive layer. Assuming for the thin layer that its lateral strain is inhibited ($\epsilon_x=\epsilon_y=0$) and that only normal strain is permitted, which is consistent with the Winkler hypothesis, one finds that the foundation constant relating applied normal pressure and deflection, w, is*

$$k = \frac{1 - \nu'}{(1 - 2\nu')(1 + \nu')} \frac{E'}{h'} \tag{26}$$

which permits a direct evaluation of $\mu=\lambda a$, viz

$$\frac{1}{\mu} = \frac{h}{a}\sqrt[4]{\frac{(1 + \nu')(1 - 2\nu')}{3(1 - \nu')(1 - \nu^2)} \cdot \frac{E/h}{E'/h'}} \tag{27}$$

For the case when the foundation is "stiff" $k\sim\mu^{\frac{1}{4}}\to\infty$,

$$p_{cr}^2(\mu) \simeq p_{cr}^2(\infty)\left[1 - 4\frac{h}{a}\sqrt[4]{\frac{(1 + \nu')(1 - 2\nu')}{3(1 - \nu')(1 - \nu^2)} \frac{E/h}{E'/h'}}\right] \tag{28}$$

and the stiffer the adhesive layer, i.e., higher modulus or thinner layer, the higher the fracture strength. As a matter of fact, directly from (25) one can find that

$$\left.\frac{\Delta p_{cr}}{p_{cr}}\right|_{\mu} \simeq \frac{1}{2\mu}\frac{\Delta k}{k} \tag{29}$$

---

* For a beam, rather than a plate strip,

$$k = \frac{E'/h'}{1 - \nu'^2}$$

There are several points to be emphasized. First, the energy calculations have been simplified by using linear elastic beam theory; large deflections are not admissible in this solution. Second, a Winkler type elastic foundation, neglecting shear deformation and shear strain energy, has been used. Third, and most important, separation along the line between the plate and adhesive layer has been assumed. It is known that in many cases the fracture progresses <u>into</u> either the adhesive or parent material, in which case an alternative calculation is required. Furthermore, in the case that the adhesive layer thickness vanishes, $k=\infty$, a uniform solution results as far as the continuum mechanics solution is concerned, i.e. (27) with $\mu=\infty$. It is not clear from the microscopic point of view, however, that the adhesive fracture energy γ_a is the same for separation of the plate from the rigid base ($\mu=\infty$) as for separation of the plate from the adhesive layer ($\mu\to\infty$); indeed it is probably not. Finally, it appears that the primary engineering design quantity is not either the layer modulus, E', or layer thickness, h', separately, but, from (20), is the effective spring constant, or foundation modulus, $k\sim E'/h'$, of the layer. The relative change of the fracture stress with spring constant is inversely proportional quantitatively to the dimensionless wave length $\mu=\lambda a$.

Nevertheless, it is believed that the nature of the results reported herein are sufficiently quantitative to assess with fair accuracy the importance and trade-off between modulus and thickness of a vanishingly thin adhesive interlayer, having as its upper limit in strength a value determined by the characteristic specific adhesive fracture energy of the mating surfaces.

As a practical matter, the strip geometry just treated is difficult to test because of the necessity of confirming the pressure at the edges of the specimen. The blister geometry is in this respect much simpler and hence its analysis would be advantageous. Recently Burton, Jones and Williams[4] successfully performed such an analysis. The analysis while being much more complex than that of the strip parallels the analysis just presented and will not be given here. Suffice it to say that the equation obtained was similar in form to (13) and experimental confirmation of the results has been presented.[4]

ADHESIONS IN METALS

When making the thermal dynamic energy balance for crack growth non-elastic materials, care must be taken to include all terms. In metals one might expect the energy associated with plastic deformation to be significant. Such effects can be included in the γ term and at times increase it significantly. Indeed for cohesive failure in ductile metals γ_c is determined largely by the plastic dissipation.[28] (γ_c can be 10^5 and 10^7 ergs/cm^2 for ductile metals while for brittle metals it may be as low as 10^2 ergs/cm^2 or less.) Very high strength adhesive bonds (welds) might be expected to exhibit similar effects. Preliminary experiments indicate such is the case.[29]

Professor Arthur Ezra, University of Denver, supplied us with 1/8 inch thick steel plate explosively welded to a 1/2 inch steel plate of similar material by techniques developed in their laboratory. Holes of various radius (a in our analysis) were carefully milled through the 1/2 inch plate to the interface between the two plates. This hole was then tapped for attachment of high pressure fittings and the sample subsequently loaded by internal pressure to failure. The pressure was supplied by an air-driven Sprague pump capable of approximately 30,000 psi.

Figure 13 shows sectioned samples after failure. The failure surface can be readily seen in the photograph. The critical pressure observed qualitatively exhibited the correct dependence on blister diameter.

Experimentally γ_a was found to be approximately 10^5 ergs/cm^2. This is within an order of magnitude of the cohesive fracture energy and an indication of the very high strength joint produced by explosive welding techniques.

Analytical work is presently underway by Williams to incorporate effects of plasticity (through limit analysis) in our model.

CONCLUSIONS

Notwithstanding the promising success in using the continuum mechanics approach to adhesives, it is well to emphasize some of the qualifications. First, it is not necessary from the engineering standpoint that the adhesive and cohesive energy values be associated. It suffices that they can be measured. To the extent that the bonding process in any design situation satisfactorily duplicates the process used in the specimen preparation, e.g. bond thickness, cure temperatures and pressure, and that the failure mode is the same, i.e. interfacial adhesive separation (or else cohesive failure analysis would be used) then a direct and reliable prediction can be made. Second, if subsequent experience proves the approach is practical, additional analytical refinements can be made, either by way of numerical calculations in a given configuration, or additional analytical sophistication in the way of allowing for elastic deformations in all of the bonding components. Third, from the chemical standpoint it should be interesting to inquire into the possible molecular associations of the cohesive and adhesive energy values, as well as the non-isotropic boundaries, or effects of adhesive migration into the base polymer. Finally a serious attempt should be made to reduce the peel test data, by appropriate analysis, to a form from which the specific energy values can be extracted and thus increase its quantitative value by widening the range of application of peel test data. If this could be accomplished, it would be very helpful and provide a great deal of data and parameter since so many tests of the this type have been conducted in the past.

More information on the nature and cause of γ_a would be helpful. The ultimate goal, of course, would be to predict reliably γ_a from basic principles. There is evidence that γ_a can be time dependent.[4] This is analogous to the time dependence of γ_c in solid rocket propellant observed by Bennett, Anderson and Williams.[30] Experiments presently being conducted should provide answers to some of these questions.

PROPOSED INVESTIGATIONS

Recently, we have undertaken an analytical study of the effects of cyclic loading on material with a spherical flow. Previous work by other researchers has attempted to analyze more complex flow geometries and generally they have resorted to numerical or other approximate techniques to obtain their solutions.

We believe the following outlined investigations will yield valuable information on failure of materials.

1) Time dependence of γ_a. A study of the time dependence of γ_a is essential if we are to predict cumulative damage at adhesive interfaces. We propose using a modification of the Bennett method for measuring the time dependence of γ_c.

2) The measurement of adhesive fracture energy of binder to A.P. crystals. Large single A.P. crystals are available to us and we propose applying the blister technique to measure γ_a between rubber binders and the crystals. The technique can be used to investigate the effect of surface preparation and modification techniques on γ_a.

3) The measurement of γ_a between highly filled rubbers and rigid substrates. We question whether the same basic equations apply to composite systems (with the composite E) as apply for the simpler systems previously studied.

4) Analyzation of the Peel test results. We are currently conducting tests to analyze the peel test including the effect of plasticity in terms of the fundamental thermodynamic balance. The analytical section of this study is progressing and would propose to undertake an experimental study to verify our results.

5) Analytical analysis of the blister test. We have been engaged in studies on the analytical analysis of the blister test (2-D) including the effect of plasticity. We have undertaken a study of explosively welded plates and propose to continue these tests.

6) Spherical flow in a viscoelastic material. We propose to continue our studies of the spherical flow in a viscoelastic material under cyclic loading. We believe this problem can benanalytically solved.

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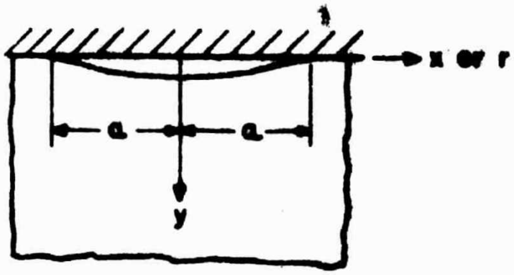

Figure 1. Cross section geometry of a rigid-elastic bond in a sheet of rod specimen. The region $|x|<a$ may be subjected to uniform pressure p_0.

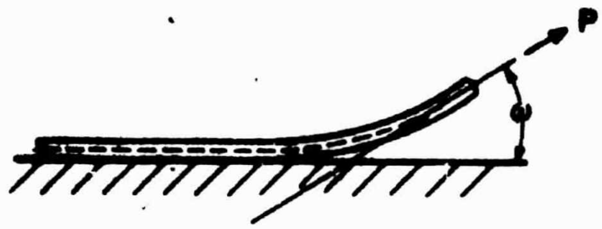

Figure 2. External forces acting on the flexible member.

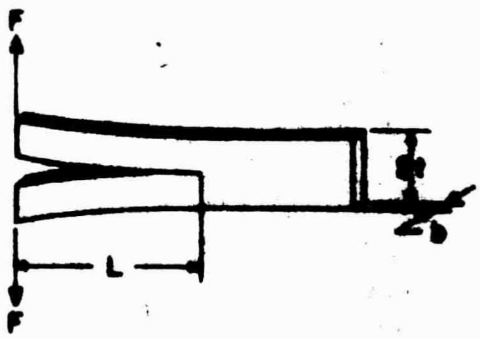

Figure 3. Double-cantilever cleavage specimen (Ref. 18)

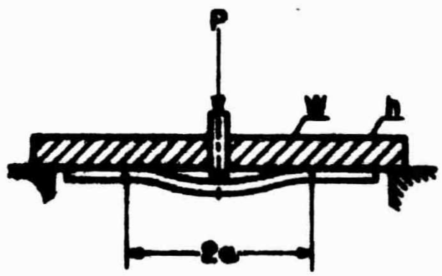

Figure 4. Circular bonded plate with a concentrated load (Ref. 20)

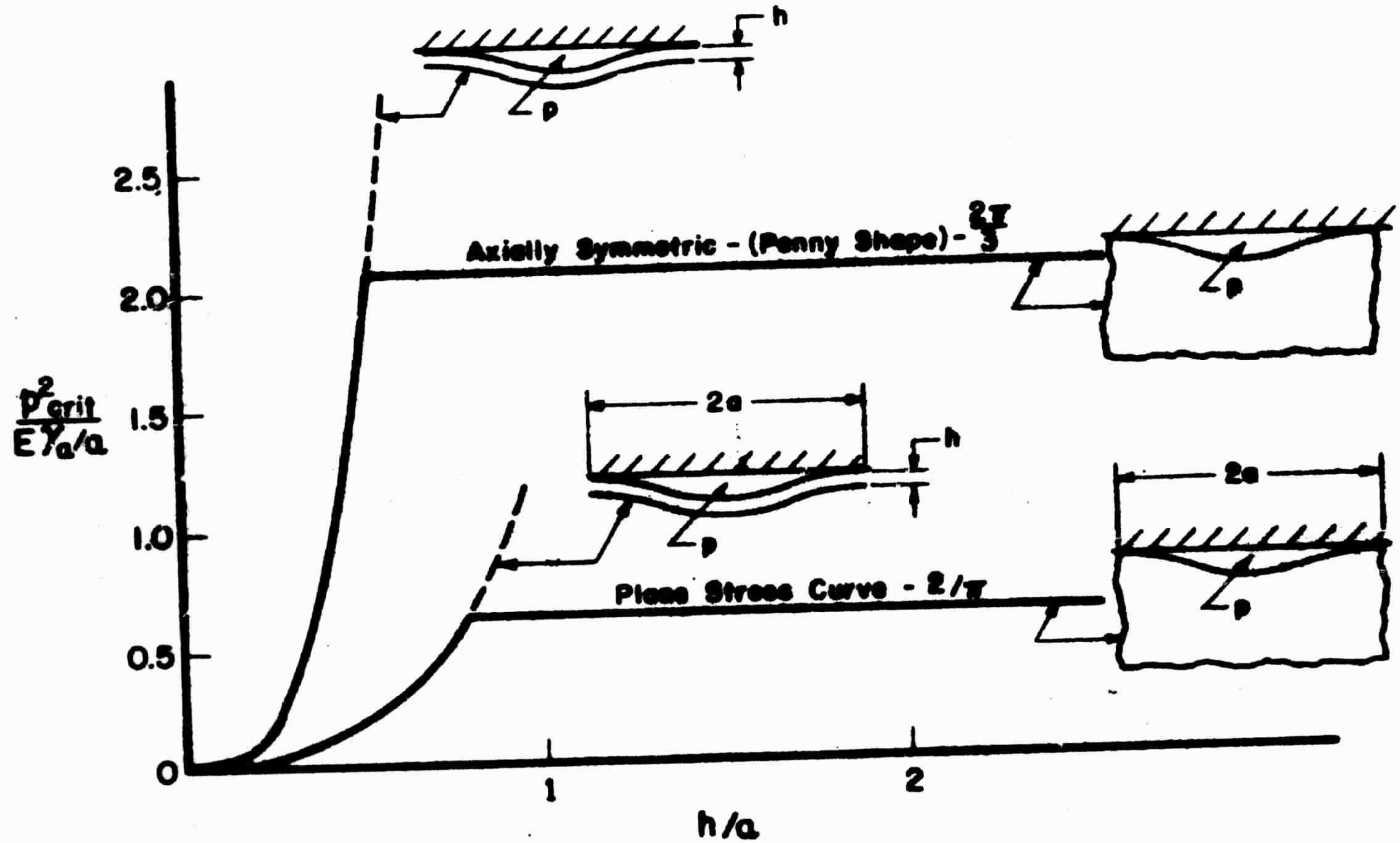

Figure 5. Asymptotic critical pressures for rigid-elastic incompressible adhesive end-bonds in thin sheets (Plane stress) or circular rods. The circular or slit flaw size, 2a, is small compared to the width or diameter of the specimen.

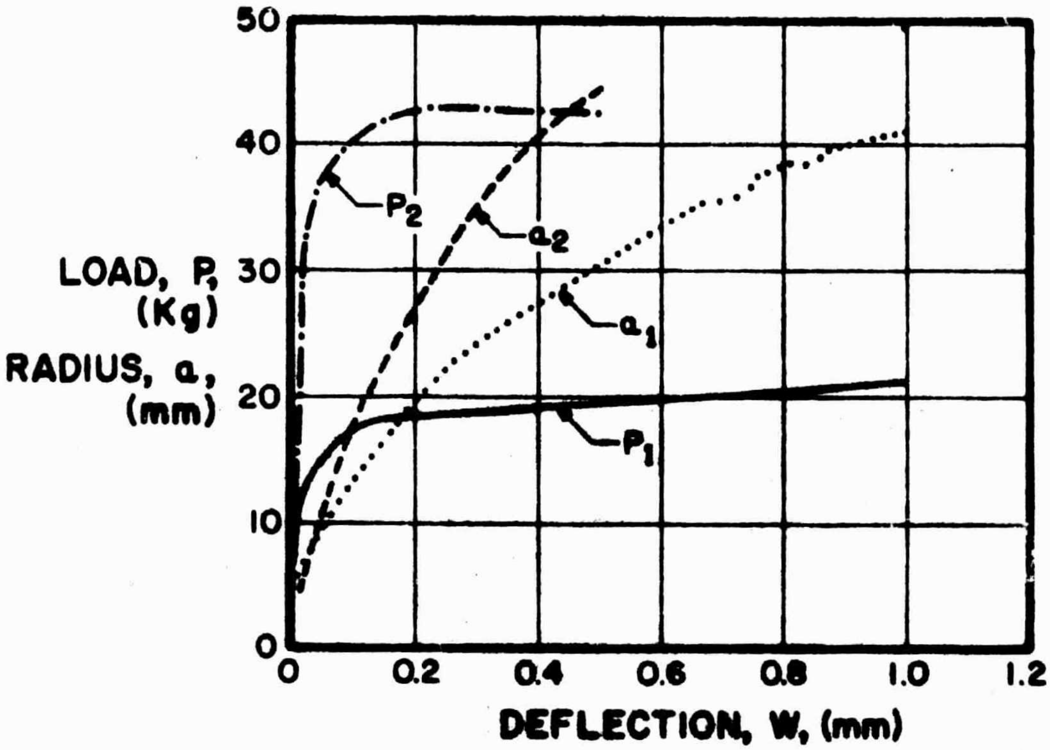

Figure 6. Tearing force, P, and flaw radius, a, versus maximum plate deflection. Experimental data (Ref. 20) for 3mm and 5mm thick plexiglass sheets bonded to steel.

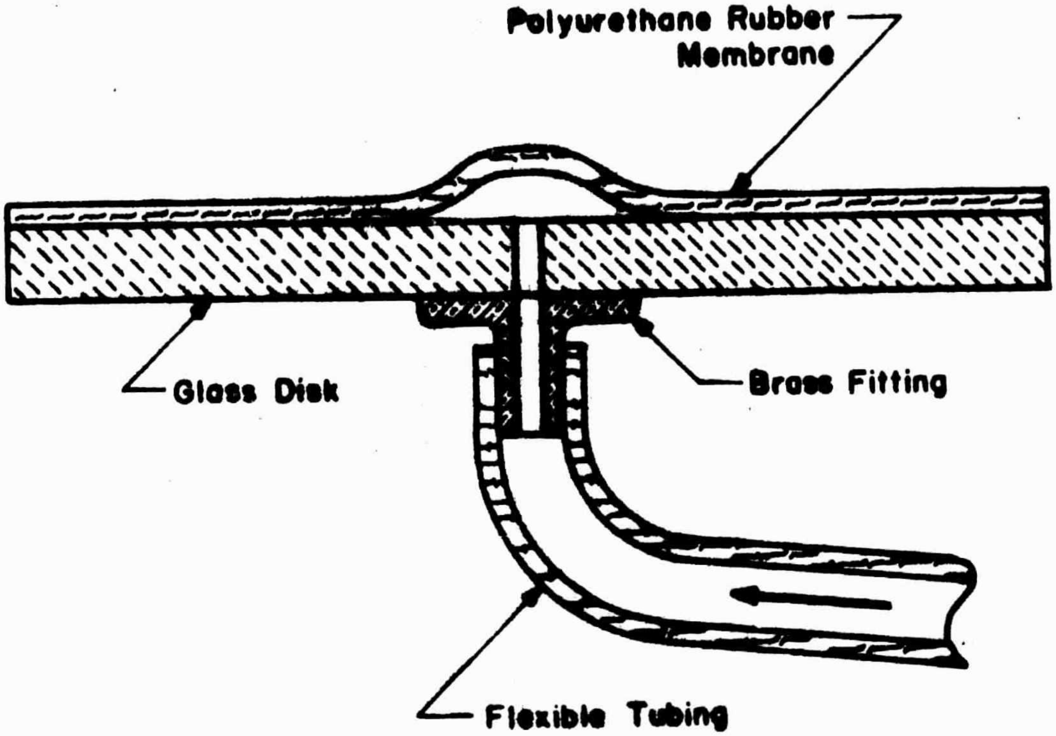

Figure 7. Sketch of disk test specimen

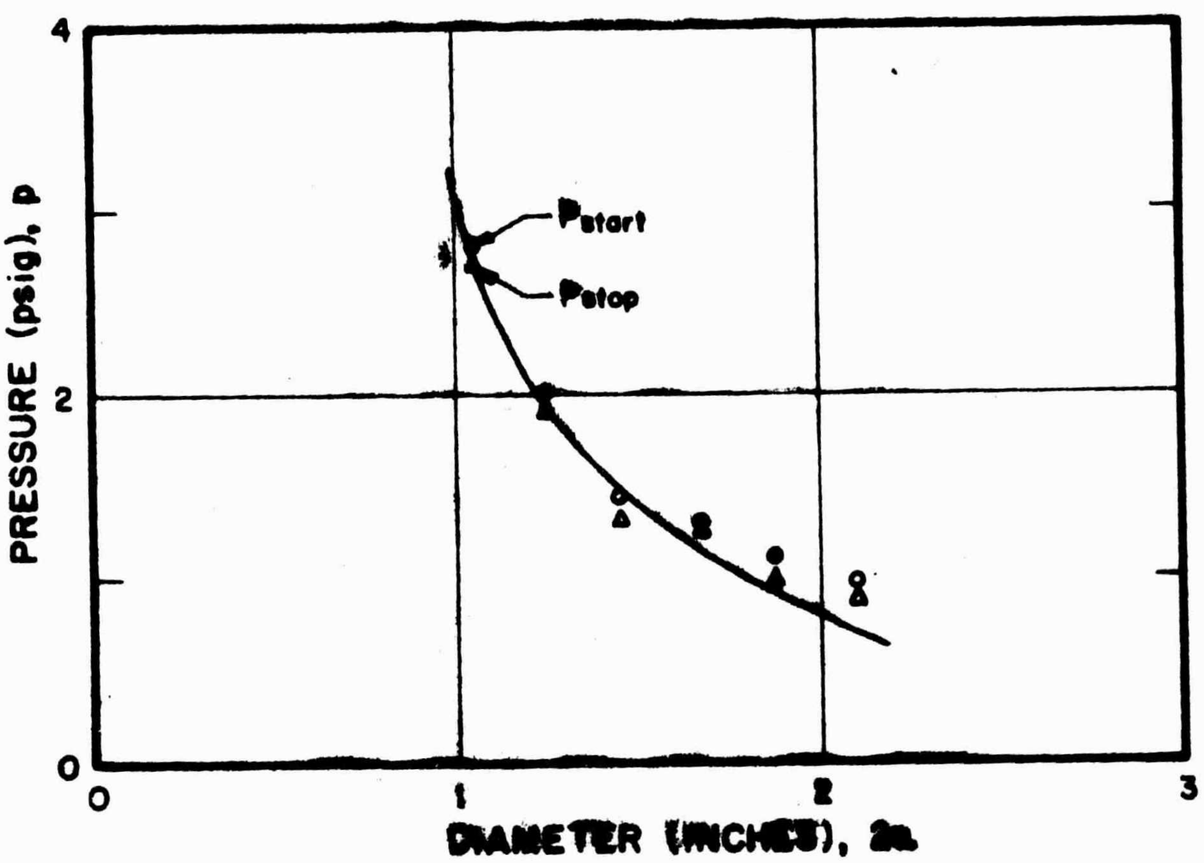

Figure 8. Typical pressure, p, versus diameter, 2a, data for the pressurized disk. E=400 psi, h=0.043 inch, giving γ_a=1.4 in-lbs/in^2.

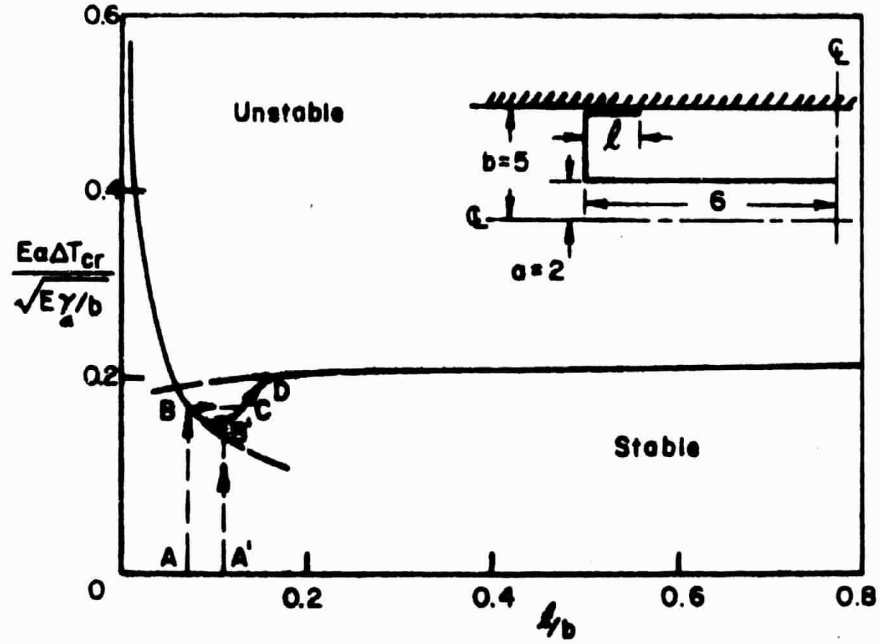

Figure 9. Critical temperature differential as function of crack length.

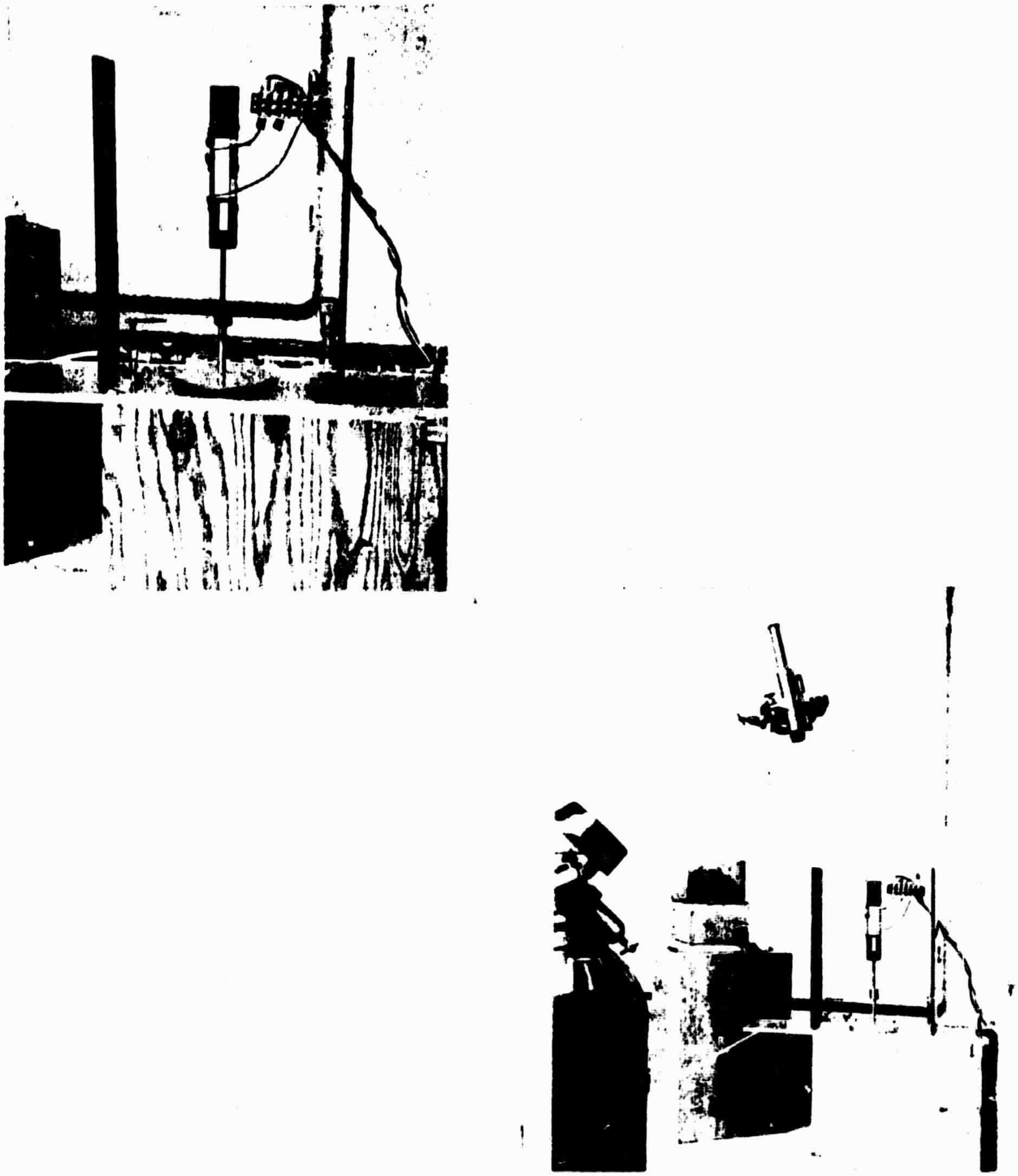

Figure 10. Experimental apparatus for the double blister test.

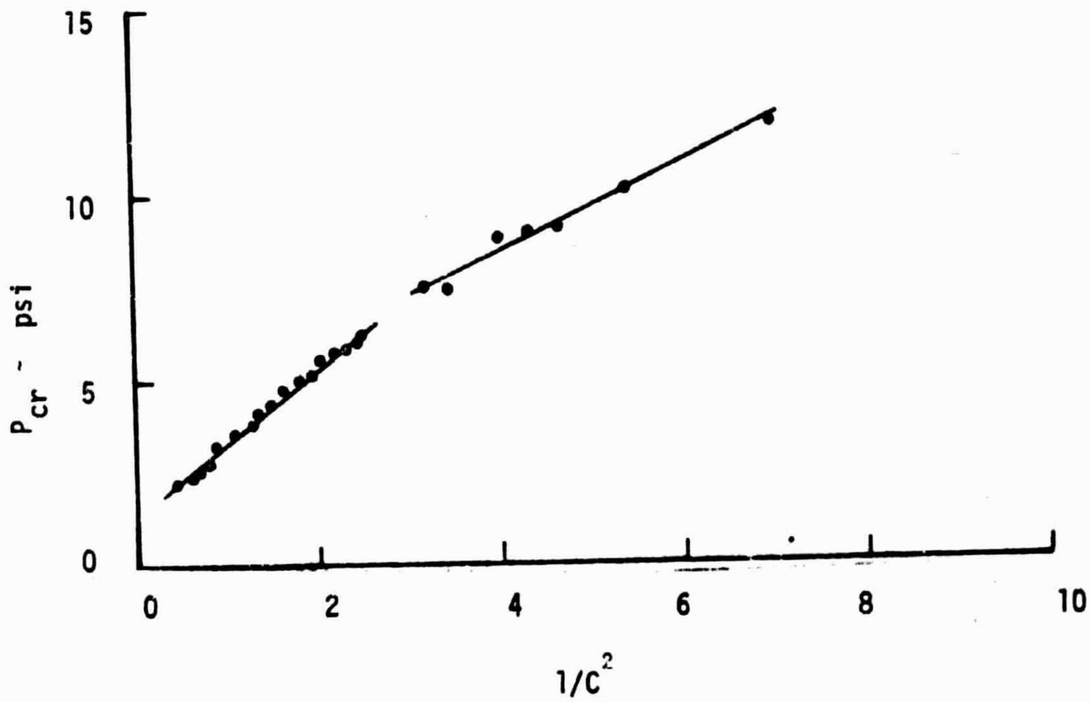

Figure 11. Critical pressure as a function of crack radius. For Solithane 113 (γ_a=0.81lbs/in)

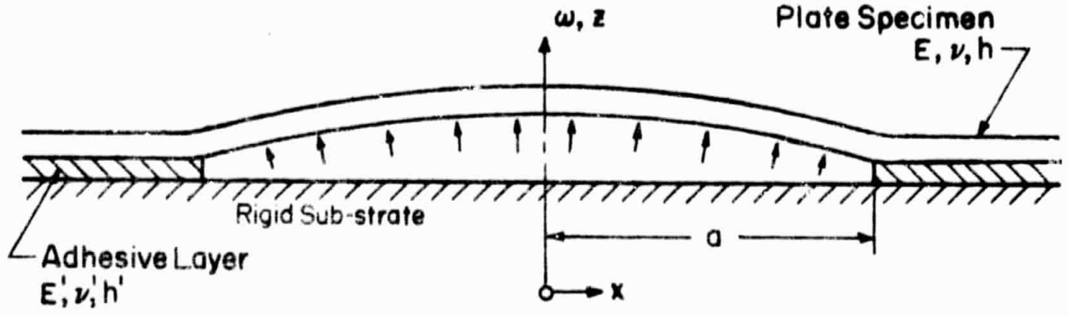

Figure 12. Cross-section of the configuration.

Figure 13. Photograph of explosively welded plates after blister testing sectioning and polishing.