N71-19048
NASA CR-116860

A THREE-LAYER DISTRIBUTED RC NETWORK WITH TWO TRANSMISSION ZEROS

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Prepared under Grant NGL-03-002-136
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Abstract: This report describes the properties of a three-layer distributed RC network consisting of two resistive layers separated by a dielectric which may be used to realize two zeros of transmission on the $j\omega$ axis of the complex frequency plane. The relative location of the two zeros is controlled by the location of a contact placed on one of the resistive layers.

October 1970

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I. Introduction

This is one of a series of reports describing the use of digital computational techniques in the analysis and synthesis of DLA (Distributed-Lumped-Active) networks. This class of networks consists of three distinct types of elements, namely distributed elements (modeled by partial differential equations), lumped elements (modeled by algebraic relations and ordinary differential equations), and active elements (modeled by algebraic relations). Such a characterization is especially applicable to a broad class of circuits, especially including those usually referred to as linear integrated circuits, since the fabrication techniques for such circuits readily produce elements which may be modeled as distributed, as well as the more conventional lumped and active ones. The network functions which describe distributed elements, however, involve hyperbolic irrational functions of the complex frequency variable. The complexity of such functions make the use of digital computational techniques most desirable in analyzing and synthesizing these networks.

In this report we shall consider the application of such digital computational techniques to the analysis and synthesis of a three-layer distributed network element. It will be shown that such an element can be designed to have a pair of transmission zeros, and that the location of the values of these zeros may be adjusted by properly choosing the network parameters. Thus, in many applications, a single three-layer distributed RC network may be used to replace two twin-tee lumped element circuits which would normally require a total of twelve lumped elements! This makes the three-layer distributed RC network especially valuable for the case where a low-pass network with high-attenuation of multiple harmonic frequencies is required.

II. The Three-Layer Distributed RC Network

A three-layer uniform distributed-RC network as analyzed in this report, consists of two layers of resistive material separated by a layer of dielectric material. As shown in Fig. 1, such a network may be considered as a three-terminal network element by adding three conducting terminal strips, two across the ends of one of the resistive layers, and one across the mid-region of the other resistive layer. The network may be characterized schematically as shown in Fig. 2, in which R is the total resistance of the resistive layer

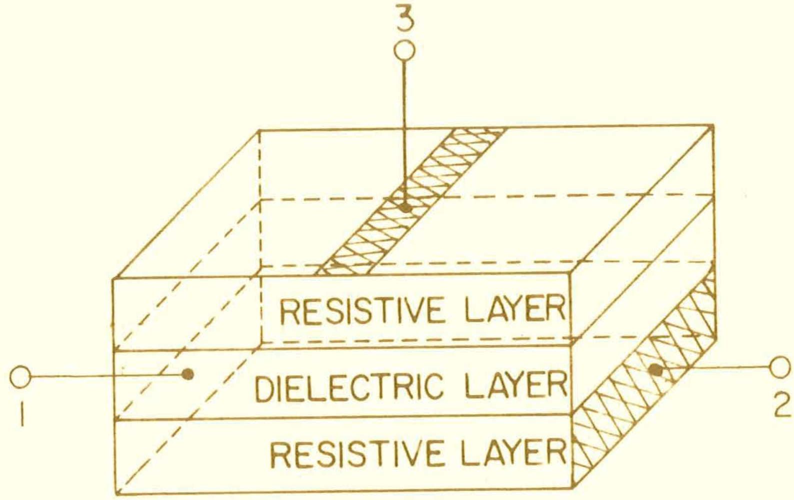


FIG. 1

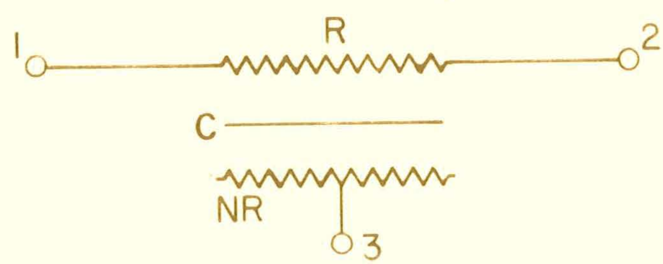


FIG. 2

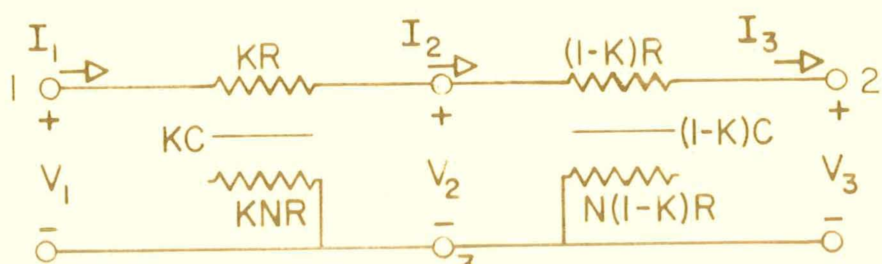


FIG. 3

with two terminals, NR (where N is a positive constant of proportionality) is the total resistance that is measured between the ends of the resistive layer with the single terminal (if terminal strips were provided at the ends), and C is the total capacitance that exists between the two resistive layers. Because of the assumption that the distributed resistance and capacitance are uniform, i.e., R and C are constant rather than being functions of position, the network shown in Fig. 2 may be modeled as a cascade connection of two uniform distributed RC networks connected in cascade as shown in Fig. 3. In this figure, the constant K has been defined to indicate the relative position of the terminal strip on the single-terminal resistive layer. For example, for K equals zero, the terminal strip is at the left end of the network shown in Fig. 1, and for K equals unity, it is at the right end. The z-parameters of the separate distributed networks shown in Fig. 3 are specified by the following equations[1]

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{R}{\theta} \begin{bmatrix} \frac{N}{N+1}\left(K\theta - 2\tanh\frac{K\theta}{2}\right) + \frac{N+1}{\tanh K\theta} & \frac{N}{\tanh K\theta} + \frac{1}{\sinh K\theta} \\ \frac{N}{\tanh K\theta} + \frac{1}{\sinh K\theta} & \frac{N+1}{\tanh K\theta} \end{bmatrix} \begin{bmatrix} I_1 \\ -I_2 \end{bmatrix} \quad (1)$$

and

$$\left.\begin{matrix} V_3 \\ \\ V_2 \end{matrix}\right] = \frac{R}{\theta}\begin{bmatrix} \frac{N}{N+1}(M\theta-2\tanh\frac{M\theta}{2}) + \frac{N+1}{\tanh M\theta} & \frac{N}{\tanh M\theta} + \frac{1}{\sinh M\theta} \\ \\ \frac{N}{\tanh M\theta} + \frac{1}{\sinh M\theta} & \frac{N+1}{\tanh M\theta} \end{bmatrix}\left[\begin{matrix} I_3 \\ \\ I_4 \end{matrix}\right. \tag{2}$$

where the quantity M is defined as

$$M = 1-K \tag{3}$$

and the quantity θ has the value

$$\theta = \sqrt{pRC(N+1)}$$

From the parameters given in (1) and (2), the open-circuit voltage transfer function $T(p) = V_3(p)/V_1(p)$ (for $I_3(p) = 0$) for the three-layer distributed RC network is readily shown to be

$$T(p) = \frac{(N\coth K\theta + \frac{1}{\sinh K\theta})\ (N\coth M\theta + \frac{1}{\sinh M\theta})}{(NK\theta-2N\tanh\frac{K\theta}{2} + \frac{(N+1)^2}{\tanh K\theta})\ (\coth K\theta + \coth M\theta)-(\frac{N}{\tanh K\theta}+\frac{1}{\sinh K\theta})^2} \tag{5}$$

This relation may be put in a form more amenable to computation by making manipulations that will make each term in the equation single-valued. To do this we first multiply the numerator and denominator by the quantity

$$\frac{\sinh^2 K\theta\ \sinh M\theta}{\theta} \tag{6}$$

and apply the identities

$$\tanh\frac{x}{2} = \frac{\cosh x - 1}{\sinh x} \tag{7}$$

$$\sinh x \cosh y + \cosh x \sinh y = \sinh(x+y) \tag{8}$$

The resulting voltage-transfer function is

$$T(p) = \frac{\frac{\sinh\theta}{\theta}(N\cosh K\theta + 1)\ (N\cosh M\theta + 1)}{\frac{\sinh\theta}{\theta}[NK\theta^2\ \frac{\sinh K\theta}{\theta} + (N^2+1)\cosh K\theta + 2N] - \frac{\sinh M\theta}{\theta}(N\cosh K\theta + 1)^2} \tag{9}$$

This function will form the basis for the analysis of the three-layer distributed RC network which is described in the following sections. It is readily shown that for varying values of K (except K equal to 0.5) the two networks shown in Fig. 2 will produce transmission zeros at different frequencies, since varying K effectively constitutes different frequency normalizations for each network. In addition, second network has a loading effect on the first. Thus, the exact determination of the location of the two transmission zeros is most conveniently performed by a digital computer optimization process as described in the following section.

III. The Transmission Zeros of the Three-Layer Distributed RC Network

The determination of the properties of the three-layer distributed RC network is accomplished by the use of the GOSPEL optimization program described in a previous report.[2] Since we are concerned with the transmission zeros of this network, the technique of complex optimization also described in a previous report will be used to facilitate the analysis.[3] As documented in the above references, the first step in such an application is the preparation of a subroutine named ANLYZ which determines the magnitude of the network function at a given complex value of the frequency variable p. Such a subroutine providing an algorithm for the relation of (9) is shown in Fig. 4. In this subroutine, the constant K (CAY) is treated as a functional parameter H(1), and the imaginary part of the complex frequency variable p is treated as the variable X(1) whose value is to be found. Thus, since the real part of p is specified as zero, the parameter X(1) is the value of sinusoidal frequency (in rad/sec) at which the zero of transmission occurs, i.e., at which the value of the magnitude of the network transfer function as computed as the variable G(1) is minimized by the optimization strategy. It should be noted that a value of 0.086266738 is used for the value of N (the program variable is EN) which is used to ensure that a zero occurs on the $j\omega$ axis. This value is appreciably dif-

```
      SUBROUTINE ANLYZ                                                  HUELS
      COMMON /OPT/X(20),XL(20),XU(20),RX(20),XO(20),H(20),R(20),S(20)   HUELS
     1,G(20),PARAM(10,7),HCPT(10,10),N,M,YEMX,ITER,ERMIN,ITMAX,ALFA(3)  HUELS
C     CENTER-TAPPED NOTCH FILTER (3-LAYER)                              HUELS
      COMPLEX P,TH,EPK,ENK,SK,CK,EPM,ENM,SM,CM,SH,VN,VD                 HUELS
      P=CMPLX(0.,X(1))                                                  HUELS
      EN=1.0862674                                                      HUELS
      CAY=H(1)                                                          HUELS
      RO=1.                                                             HUELS
      CO=1.0                                                            HUELS
      EM=1.-CAY                                                         HUELS
      TH=CSQRT(P*RO*CO*(EN+1.))                                         HUELS
      EPK=CEXP(CAY*TH)                                                  HUELS
      ENK=CEXP(-CAY*TH)                                                 HUELS
      SK=(EPK-ENK)/2.                                                   HUELS
      CK=(EPK+ENK)/2.                                                   HUELS
      EPM=CEXP(EM*TH)                                                   HUELS
      ENM=CEXP(-EM*TH)                                                  HUELS
      SM=(EPM-ENM)/2.                                                   HUELS
      CM=(EPM+ENM)/2.                                                   HUELS
      SH=(CEXP(TH)-CEXP(-TH))/2.                                        HUELS
      VN=SK*(EN*CK+1.)*(EN*CM+1.)/TH                                    HUELS
      VD=SH*(EN*CAY*TH**2*SK/TH+(EN**2+1.)*CH+2.*EN)/TH                 HUELS
     1-SM*(EN*CK+1.)**2/TH                                              HUELS
      G(1)=CABS(VN/VD)                                                  HUELS
      G(19)=REAL(VN/VD)                                                 HUELS
      G(20)=AIMAG(VN/VD)                                                HUELS
      RETURN                                                            HUELS
      END                                                               HUELS
```

ferent than the value of 0.0866 which is reported in the literature.[1] For a value of K equal to 0.5, the two networks shown in Fig. 3 are identical. Such a cascade produces a second-order transmission zero in the voltage-transfer function T(p) of (9). The location of this zero, for normalized values of unity for the total resistance R (of the two lines), and for the total capacitance C (of the two lines) is 72.687558 rad/sec. This value was determined using a Fletcher-Powell optimization algorithm to minimize the magnitude of T(p). At the stated value of the zero, the magnitude of T(p) is approximately 3.5×10^{-12}.

To determine the effect of varying K on the position of the zeros of T(p), a series of 96 optimization problems were run using the same Fletcher-Powell optimization algorithm to determine the values of the zero locations for a sequence of values of K at 0.01 intervals from 0.03 to 0.5. The tabulated results are shown in Fig. 5. This figure also shows the ratio of the zero-locations for the various values of K. The total computer time required for the 96 optimization runs was less than 10 sec. on a CDC 6400 computer. It should be noted that these values are for normalized values of unity for R and C, the total distributed resistance and capacitance. A plot of the zero locations

K	LOWER ZERO	UPPER ZERO	RATIO
.03	1.4313022E+01	2.0167020E+04	1.4090421E+03
.04	1.9717453E+01	1.1343650E+04	5.7531036E+02
.05	2.0134742E+01	7.2528541E+03	3.6021614E+02
.06	2.0565419E+01	5.0397657E+03	2.4506020E+02
.07	2.1010064E+01	3.7040446E+03	1.7629959E+02
.08	2.1469286E+01	2.8365920E+03	1.3212326E+02
.09	2.1943731E+01	2.2416332E+03	1.0215359E+02
.10	2.2434080E+01	1.8159262E+03	8.0944714E+01
.11	2.2941050E+01	1.4974287E+03	6.5272093E+01
.12	2.3465399E+01	1.2587094E+03	5.3641340E+01
.13	2.4007933E+01	1.0728163E+03	4.4685309E+01
.14	2.4565502E+01	9.2524575E+02	3.7664303E+01
.15	2.5151010E+01	8.0614656E+02	3.2052270E+01
.16	2.5753410E+01	7.0864159E+02	2.7516429E+01
.17	2.6377713E+01	6.2781022E+02	2.3800783E+01
.18	2.7024994E+01	5.6005603E+02	2.0723655E+01
.19	2.7696395E+01	5.0270616E+02	1.8150535E+01
.20	2.8393137E+01	4.5373261E+02	1.5980365E+01
.21	2.9116500E+01	4.1150127E+02	1.4135071E+01
.22	2.9867363E+01	3.7504152E+02	1.2556691E+01
.23	3.0648627E+01	3.4315973E+02	1.1196556E+01
.24	3.1460534E+01	3.1517649E+02	1.0018154E+01
.25	3.2305081E+01	2.9048102E+02	8.9918360E+00
.26	3.3184096E+01	2.6657763E+02	8.0335346E+00
.27	3.4099472E+01	2.4903673E+02	7.3031077E+00
.28	3.5053258E+01	2.3154597E+02	6.6059737E+00
.29	3.6047637E+01	2.1398551E+02	5.9362536E+00
.30	3.7064927E+01	2.0175714E+02	5.4440832E+00
.31	3.8167630E+01	1.8895539E+02	4.9506713E+00
.32	3.9292451E+01	1.7733492E+02	4.5125112E+00
.33	4.0480297E+01	1.6675454E+02	4.1193999E+00
.34	4.1716262E+01	1.5709483E+02	3.7657744E+00
.35	4.3009715E+01	1.4824963E+02	3.4468371E+00
.36	4.4364274E+01	1.4012789E+02	3.1585751E+00
.37	4.5783951E+01	1.3235946E+02	2.8909583E+00
.38	4.7272672E+01	1.2559470E+02	2.6568197E+00
.39	4.8835315E+01	1.1916987E+02	2.4402394E+00
.40	5.0476699E+01	1.1330393E+02	2.2446779E+00
.41	5.2202269E+01	1.0786198E+02	2.0662238E+00
.42	5.4017940E+01	1.0280421E+02	1.9031494E+00
.43	5.5930013E+01	9.8096125E+01	1.7539085E+00
.44	5.7945382E+01	9.3707116E+01	1.6171528E+00
.45	6.0071595E+01	8.9619723E+01	1.4917319E+00
.46	6.2317569E+01	8.5784628E+01	1.3765721E+00
.47	6.4692230E+01	8.2090625E+01	1.2689410E+00
.48	6.7205439E+01	7.8528104E+01	1.1684734E+00
.49	6.9871171E+01	7.4000000E+01	1.0590920E+00

Fig. [illegible]

as a function of K is shown in Fig. 6. A plot of the ratio of zero locations as a function of K is shown in Fig. 7. This latter figure is particularly appropriate as a design chart for specifying the parameters of the network when harmonic frequency components are to be filtered out.

To investigate the frequency response of the three-layer distributed RC network, a series of evaluations of the subroutine ANLYZ given in Fig. 4 were made for a logarithmically spaced set of values from 10 to 1000 rad/sec of the frequency parameter X(1) using this parameter as an independent variable, and specifying values of 0.2, 0.3, 0.4, and 0.5 for the functional parameter H(1) which represents the constant K. The results are shown in Fig. 8. The manner in which the single zero (for K equal to 0.5) separates into two zeros, and diverges is readily apparent in the figure. The low-pass nature of the resulting characteristic is also easily seen.

A final investigation which is pertinent to the analysis and use of the three-layer distributed RC network is the relative selectivity of the transmission notches. A set of curves specifying the width of the notches of the lower and upper transmission zeros of the network for values of K of 0.2,

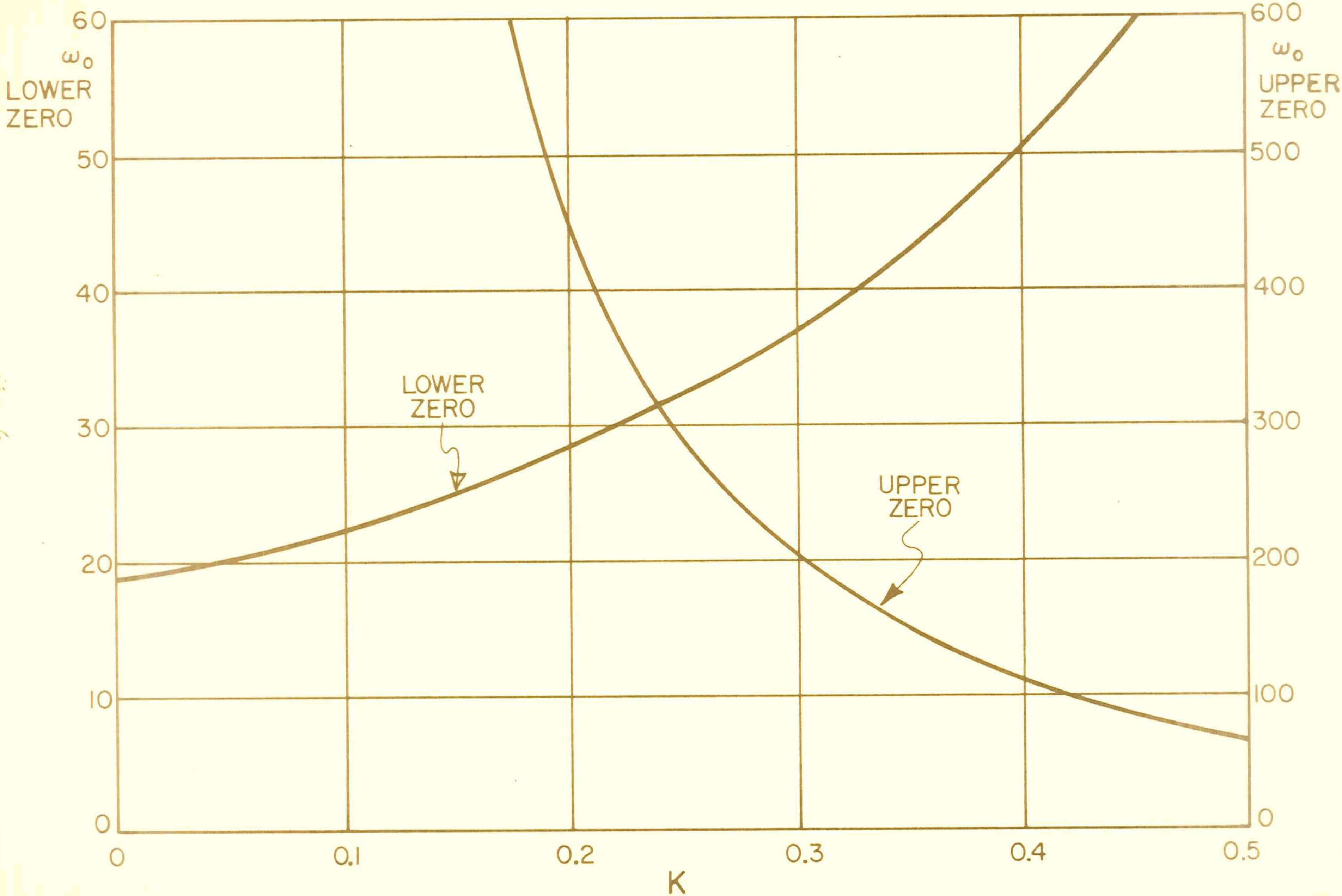
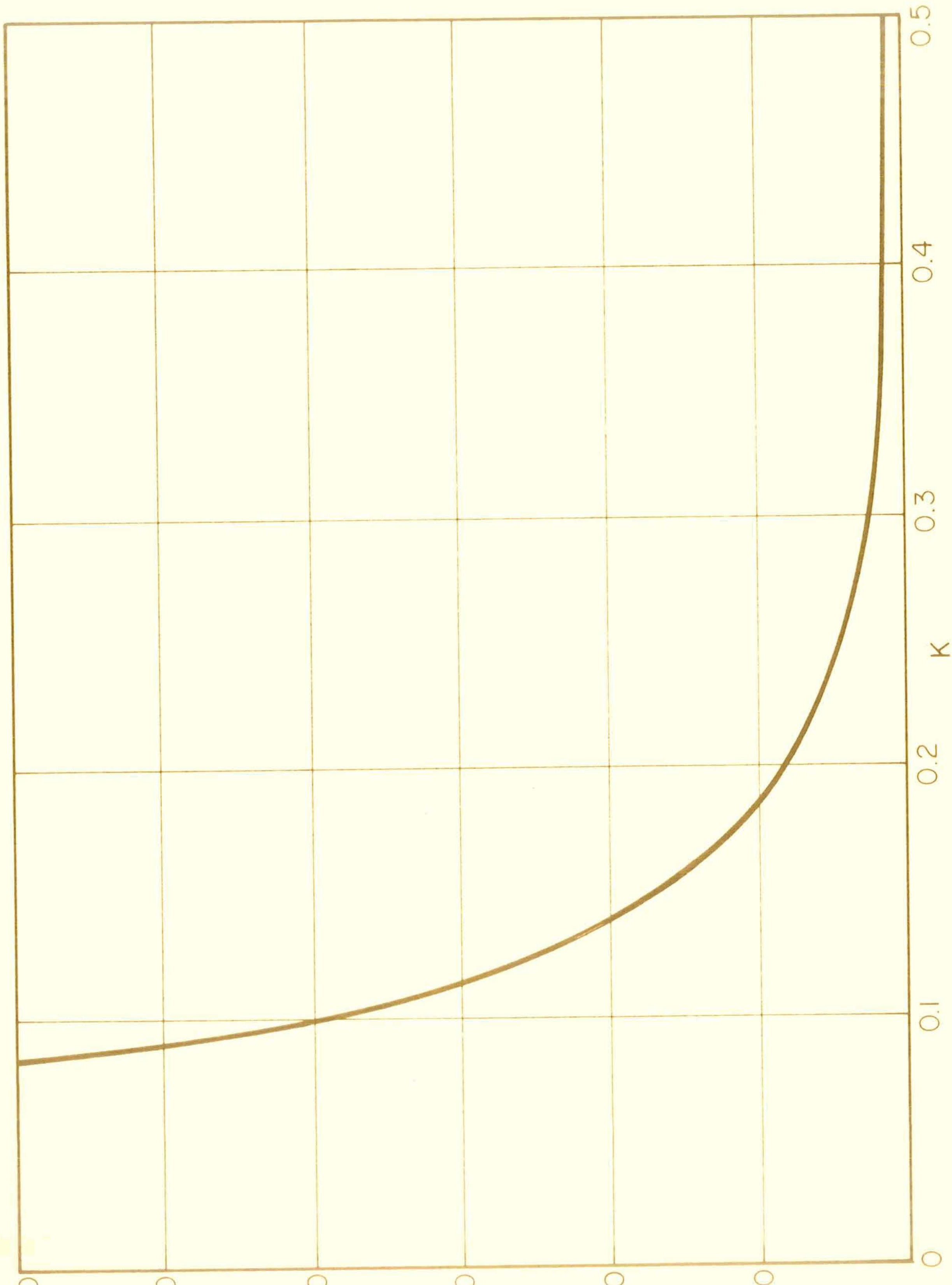
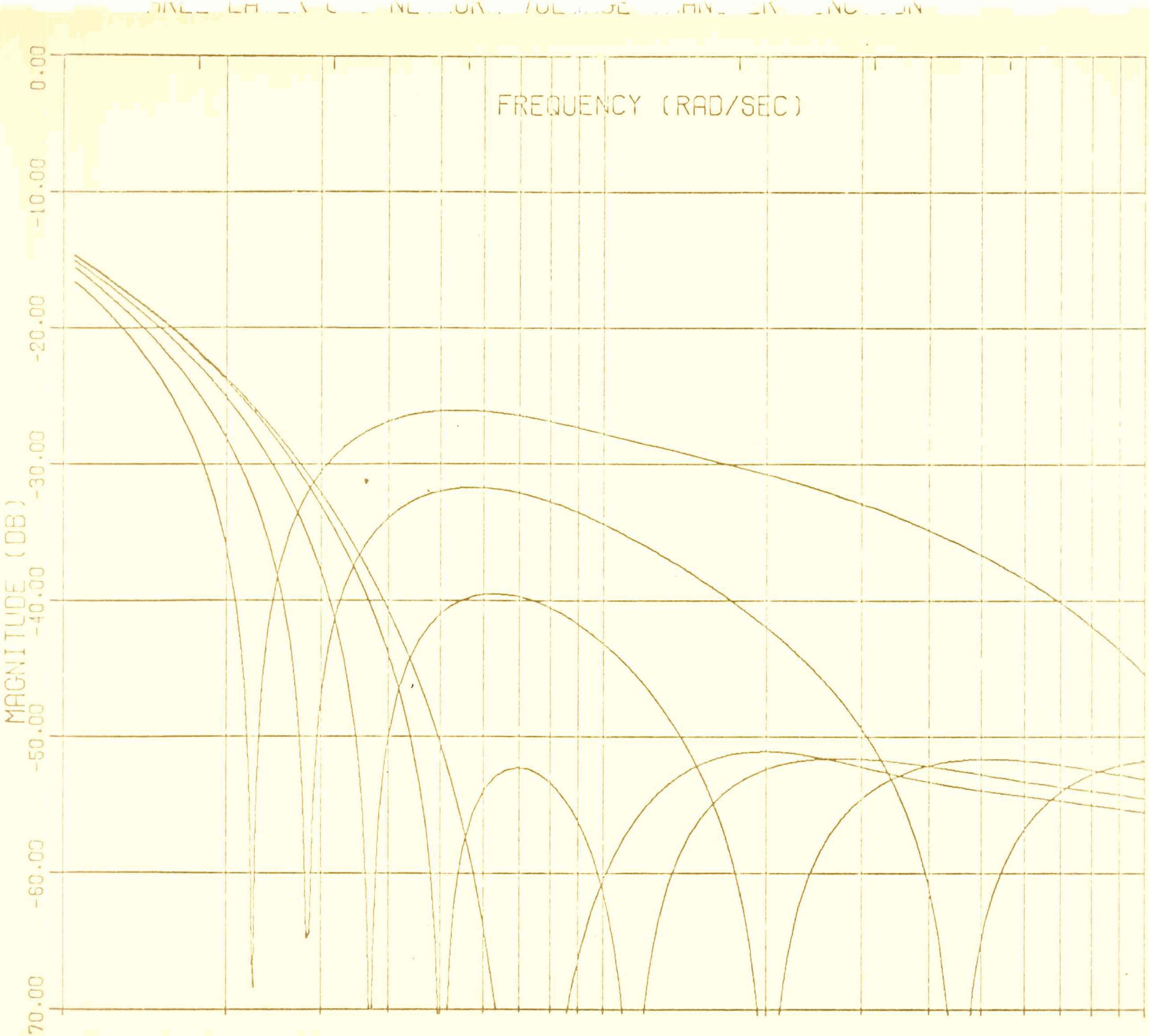

ω_o
LOWER
ZERO
ω_o
UPPER
ZERO
60
50
40
30
20
10
0
600
500
400
300
200
100
0
0
0.1
0.2
0.3
0.4
0.5
K
LOWER
ZERO
UPPER
ZERO

Fig. 6



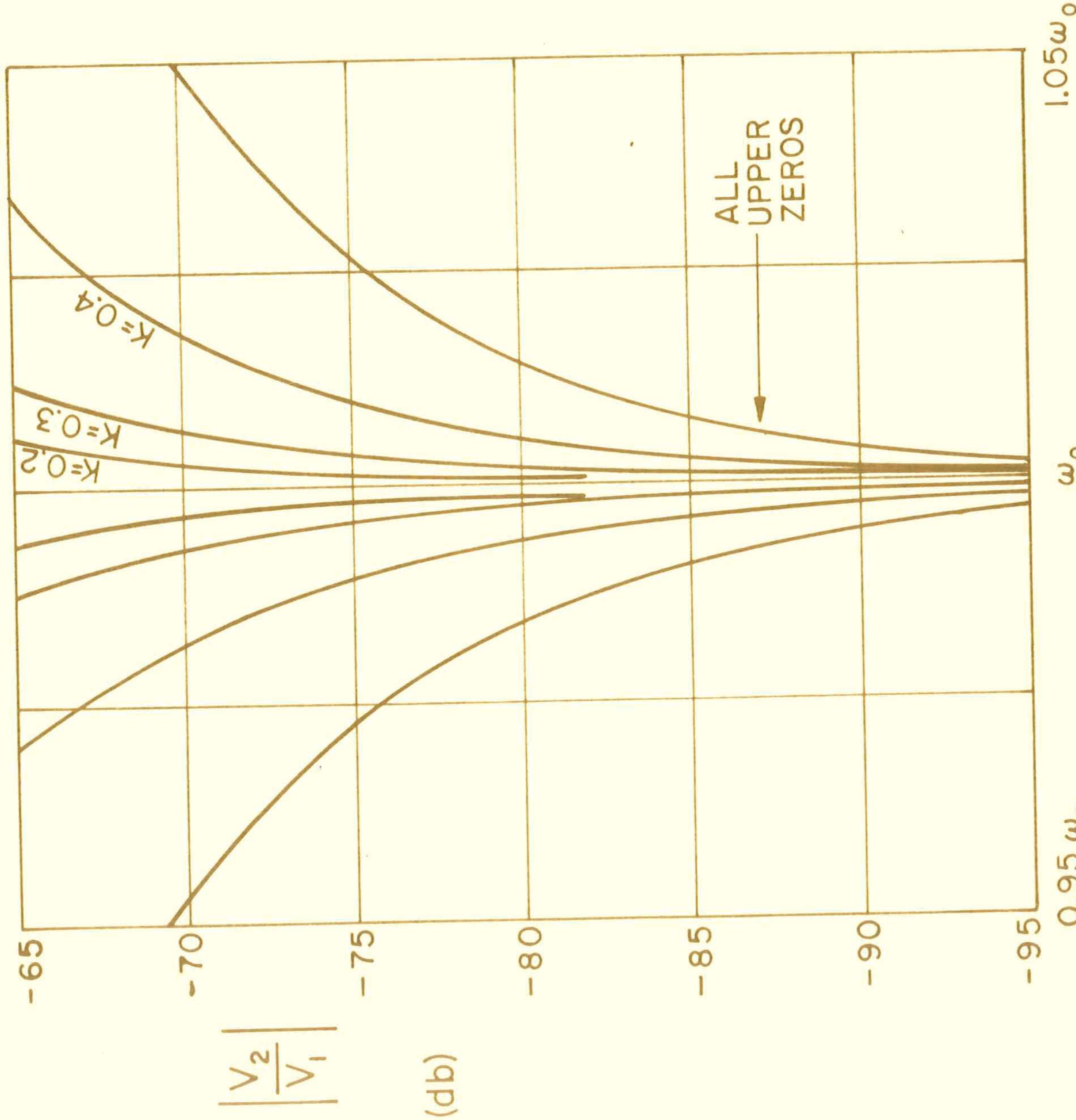
0.5
0.4
0.3
K
0.2
0.1
0
120
100
80
60
40
20
$\frac{\omega_H}{\omega_L}$


FREQUENCY (RAD/SEC)
MAGNITUDE (DB)
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
-60.00
-70.00

0.3, 0.4, and 0.5 are shown in Fig. 9. For convenience in comparing these results, all the curves are plotted over a linear frequency range from $0.95\omega_0$ to $1.05\omega_0$ (plus and minus five percent of ω_0) where ω_0 is the zero location. From these curves, the interesting result is observed that the normalized selectivity of the lower notch narrows as the value of K is decreased. The normalized selectivity of the upper notch, however, remains constant.

IV. Conclusion

The three-layer distributed RC network described in this report should have considerable application in a variety of filtering problems. As a low-pass network which may be adjusted to provide high attenuation of the harmonic components of a given periodic signal it has the advantages of circuit simplicity, ease of design, and minimization of space required. Considerable additional application of this network is possible as a feedback element in an active network. As such, it has the capability of generating a set of four complex-conjugate poles so as to provide wide-band bandpass performance. In such a usage, the ratio of the resistances of the two resistive layers as specified by the value of N, would have to be determined so as to move the transmission zeros of



-65
-70
-75
-80
-85
-90
-95
$\left|\frac{V_2}{V_1}\right|$
(db)
0.95 ω_o
ω_o
1.05ω_o
ALL UPPER ZEROS
K=0.4
K=0.3
K=0.2

Fig. 9

the distributed network into the left-half plane. Additional research is currently being conducted to determine the design specifications for such an application.

Acknowledgment

The support of this research by the Instrumentation Division of the Ames Research center, National Aeronautics and Space Administration, under Grant NGL 03-002-136 is most gratefully acknowledged. The author would also like to express his appreciation to Dr. R. H. Mattson, Head of the Department of Electrical Engineering, and Dr. W. J. Fahey, Dean of the College of Engineering, of the University of Arizona, for their support in providing university facilities. The network function and the frequency characteristics shown in Fig. 8 also appeared in a term paper entitled "A Center Tapped Three Layer Distributed R-C-NR Network" prepared by J. Hameenantila to satisfy an assignment in graduate course, EE 322 at the University of Arizona.

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