Reports of the Department of Geodetic Science
Report No. 140

SECOR OBSERVATIONS IN THE PACIFIC

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Ivan I. Mueller
James P. Reilly, Charles R. Schwarz, Georges Blaha

Prepared for

National Aeronautics and Space Administration
Washington, D.C.

Contract No. NGL 36-008-093
OSURF Project No. 2514

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PREFACE

This project is under the supervision of Ivan I. Mueller, Professor of the Department of Geodetic Science at The Ohio State University, and it is under the technical direction of Jerome D. Rosenberg, Project Manager, Geodetic Satellites Program, NASA Headquarters, Washington, D. C. The contract is administered by the Office of University Affairs, NASA, Washington, D. C. 20546.

TABLE OF CONTENTS

1. INTRODUCTION

SECOR, an acronym for Sequential Collation of Range, is an electronic distance measuring system in which four ground stations sequentially interrogate a satellite-borne transponder. The system was developed by the Cubic Corporation for the U.S. Army Corps of Engineers (Army Map Service). The geodetic applications of the SECOR system are numerous. For the purpose of this report we are only concerned about its use for inter-island and inter-datum geodetic ties.

The SECOR system, its operation and data characteristics are explained in great detail in [Cordova, 1965; Gross, 1968]. However, a short description may be necessary in order to more fully understand the discussion that is to follow.

The SECOR system is composed of four or more ground tracking stations and a satellite-borne transponder. The transponder in the satellite is in a stand-by condition when it is not being interrogated by the ground stations. Whenever the satellite appears above the horizon, the first station that detects it (referred to as the master station) sends a select signal to the satellite which activates the transponder. When activated, the transponder receives and retransmits the signals from each of four ground stations, one after the other. The transponder receives and retransmits to each ground station every 50 ms, or 20 times per second.

After the satellite moves out of range of the tracking network, the transponder again goes on stand-by, and if the satellite is in the sunlight the solar cells begin to charge the batteries in preparation for the next interrogation. The number of interrogations per day depends on the amount of time the satellite is exposed to the sun's rays. The power budget, as it is called, is watched very closely because if too much power is drained from the batteries the transponder may never be able to operate again.

The maximum number of stations that can observe the satellite at one time is four. However, there can be less than four stations observing, and this happens often when one or more of the stations looses "lock" on the satellite. If there are only one or two stations tracking, the satellite begins to oscillate (electronically, not physically), and it is very difficult for these stations to hold lock.

When the SECOR network is tracking the satellite using four ground stations, it is referred to as the Simultaneous Mode. Three of the tracking stations are observing from known positions, and the fourth tracking station is located at an unknown position. The positions of the satellite in space can be determined by the measurements from the three known stations, and when three or more space positions have been determined it is then possible to solve for the position of the unknown station. The station coordinates computed from observations on only one pass of the satellite will give very poor results because of the lack of good geometry. For this reason it is necessary to observe at least two passes, and preferably more than two passes. After the station coordinates of the unknown station have been determined, it is then referred to as a known station, and it observes with two other known stations for the purpose of determining the location of another tracking station at an unknown location.

The procedure described above is referred to as "leap-frogging." However, since the present reports on SECOR have been written, much more sophisticated computer programs have been developed which can adjust a complete SECOR network simultaneously [Krakiwsky and Pope, 1967; Brown, 1966]. For a simultaneous network adjustment enough ground station information must be known to constrain the translation and rotation of the network. The scale, of course, is determined from the measured ranges.

Another technique of using SECOR data is in the orbital mode in which it is not necessary to have four stations observing. The unknowns are not the satellite positions, but the six orbital elements for each pass of the satellite. These six elements will completely describe the orbit if the observations are limited to short arcs. A short arc has been defined as the maximum length of arc over which modeling errors are not greater than a few meters. For geodetic satellites, and using the orbital adjustment program developed at The Ohio State University [Schwarz, 1969], this is about 1/15 revolution, or about eight minutes for GEOS-I. If it is desired to have satellite passes longer in the orbital mode, it is necessary to include a more complex orbital model into the reduction, e.g., [Brown and Trotter, 1969].

In the fall of 1964, the operational mission of Secor began in Japan. The mission itself was to provide a tie between the Hawaiian Islands and Japan, with the ultimate goal of making a geodetic connection to North America. Part of this work was the determination of positions of certain islands in the Southwest Pacific.

When the operational mission began in 1964, the satellites used were special satellites of the EGRS (Engineer Geodetic Ranging Satellite) series designed especially for SECOR and launched "piggy-back" on rockets used to launch satellites for other missions.

In November, 1965, the National Aeronautics and Space Administration launched the first geodetic satellite of the GEOS series. This satellite was equipped with a flashing light, a Doppler transmitter, a range and range-rate transmitter, and a SECOR transponder. At that time the SECOR stations had already completed their observations on the stations in Japan and moved out onto the Pacific Islands. The SECOR stations then began to observe the GEOS satellite as well as their special EGRS satellites.

Since the Army Map Service was required to forward to the Space Science Data Center only the observations made on GEOS-I, it was impossible to tie the island stations to the Japanese Datum using the data available.

2. GEOMETRIC ADJUSTMENT

2.1 General Theory

The geometric adjustment technique used to perform the adjustments is based on the theory of least squares. All data must be observed from four stations simultaneously.

Figure 1 shows the three-dimensional terrestrial coordinate system XYZ, with a ground station i and a satellite position j. The XZ plane is parallel to the Mean Greenwich Astronomical Meridian as defined by the Bureau International de l'Heure (BIH), the Z axis is directed toward the Conventional International Origin (CIO) as defined by the International Polar Motion Service (IPMS), and the Y axis is in the plane of the equator forming a right-handed system. The observed quantity is the topocentric range r_{ij} from ground station i to satellite position j. The parameters X_i, Y_i, Z_i and X_j, Y_j, Z_j are the three dimensional rectangular coordinates of the ground station i and the satellite position j, respectively.

From Figure 1 it can easily be seen the the mathematical model can be written as

$$r_{ij} = [\, (X_j - X_i)^2 + (Y_j - Y_i)^2 + (Z_j - Z_i)^2 \,]^{\frac{1}{2}}$$

or

$$F_{ij} = [\, (X_j - X_i)^2 + (Y_j - Y_i)^2 + (Z_j - Z_i)^2 \,]^{\frac{1}{2}} - r_{ij} = 0 .$$

The expression for observation equations in matrix form would be

$$A_{ij} X_{ij} + L_{ij} = V_{ij} \qquad (2.1)$$

(i and j designate ground and satellite points, they are not the dimensions

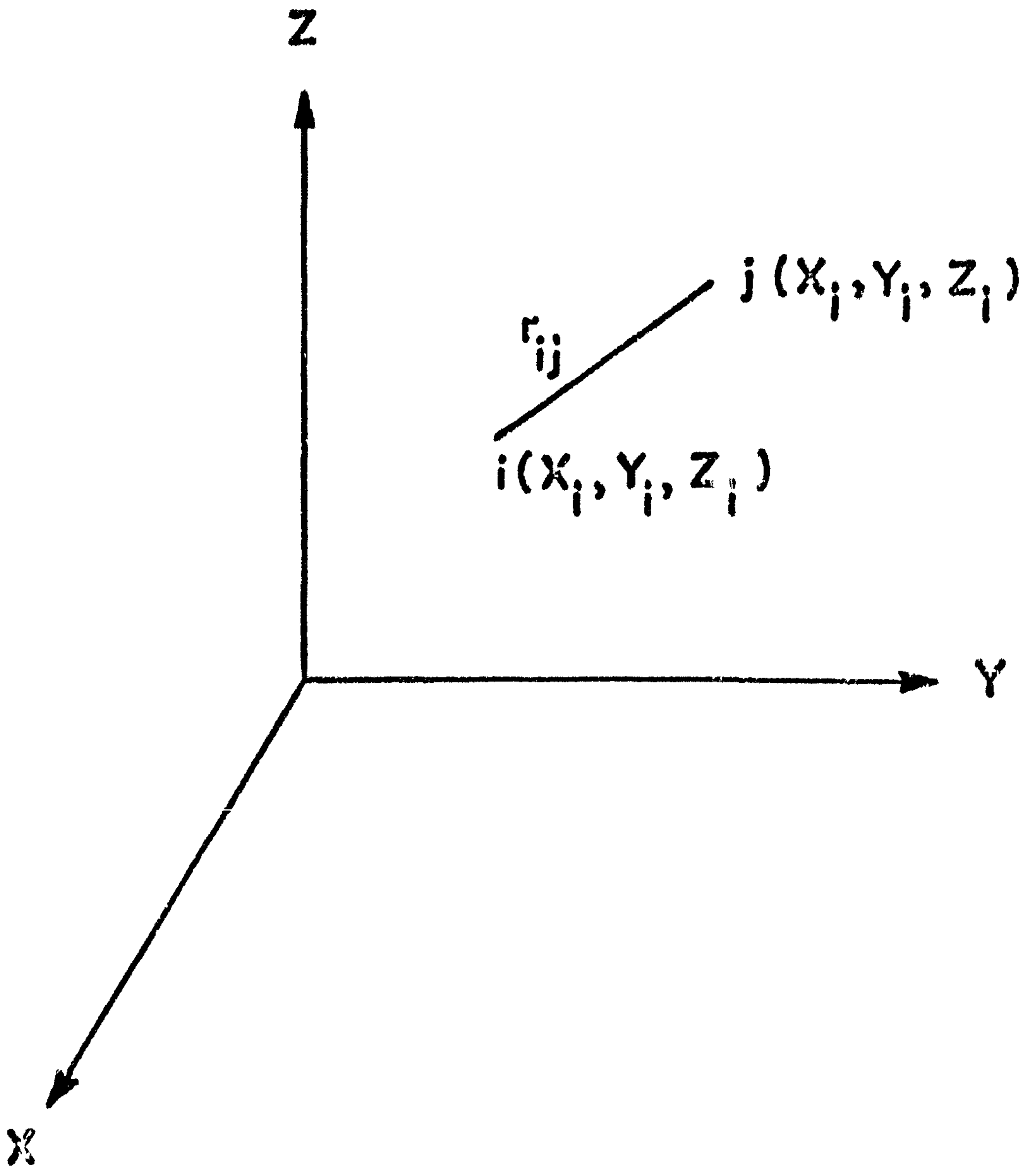

Fig. 1 The XYZ coordinate system

on the arrays)

where

$$A_{ij} = \frac{\partial F_{ij}}{\partial(\text{unknown parameters})} \qquad \frac{\partial F_{ij}}{\partial X, \partial Y, \partial Z}$$

$$X_{ij} = \begin{bmatrix} X_i \\ X_j \end{bmatrix}$$

where

$$X_i = \begin{bmatrix} dX_i \\ dY_i \\ dZ_i \end{bmatrix}$$

and

$$X_j = \begin{bmatrix} dX_j \\ dY_j \\ dZ_j \end{bmatrix}$$

$$L_{ij} = r^{o}_{ij}\,(\text{computed}) - r^{b}_{ij}\,(\text{observed}) .$$

V_{ij} is the residual of the adjustment (in meters) corresponding to the observed ranges r^{b}_{ij}. Using matrix notation and eliminating the i and j ground point and satellite subscripts, the principal of least squares is

$$V'PV \quad \text{minimum}. \tag{2.2}$$

The quantity P is the weight matrix of independent observed topocentric ranges.

By performing the matrix manipulation required [Uotila, 1967, pp. 37-38], the expression for normal equations will be

$$NX + U \quad 0 . \tag{2.3}$$

The solution vector X can now be solved by

$$X = -\dot{N}^{-1} U . \tag{2.4}$$

The correction vector X is made up of the corrections to both satellite and station coordinates. The satellite positions are really nuisance parameters and can be eliminated from the solution by using the partitioned normal equations

$$\left[\begin{array}{c|c} \dot{N} + \dot{P} & \overline{N} \\ \hline \overline{N}' & \ddot{N} + \ddot{P} \end{array}\right] \left[\begin{array}{c} X_i \\ \hline X_j \end{array}\right] + \left[\begin{array}{c} U_i \\ \hline U_j \end{array}\right] = 0 \tag{2.5}$$

where as before the subscripts i and j refer to the ground and satellite points. The notation in (2.5) has been popularized by Duane Brown (1968]. The normal equations can be accumulated and the satellite positions eliminated from the final solution in the following manner: The expression for observation equations should be written

$$AX_i + BX_j + L = V \tag{2.6}$$

where

$$A = \frac{\partial F}{\partial X_i, \partial Y_i, \partial Z_i}$$

$$B = \frac{\partial F}{\partial X_j, \partial Y_j, \partial Z_j} .$$

Let the weight matrix for the observations be designated W. Using the notation of (2.5),

$$\dot{N} = \Sigma\ A'WA$$

$$\ddot{N} = \Sigma\ B'WB$$

$$\overline{N} = \Sigma\ A'WB$$

$$U_i = \Sigma\ A'WL$$

$$U_j = \Sigma\ B'WL .$$

The terms $\dot{P}$ and $\ddot{P}$ are the weights that are applied to the station coordinates and satellite coordinates respectively.

Equation (2.5) can now be rearranged to eliminate X_j, and the resulting equation is

$$[\dot{N} + \dot{P} - \overline{N}(\ddot{N} + \ddot{P})^{-1}\overline{N}'] X_i = \overline{N}(\ddot{N} + \ddot{P})^{-1} U_j - U_i . \qquad (2.7)$$

Because the correction vector X_j can be eliminated from the solution, the computer programs developed at OSU were designed to form the normal equations one event at a time and accumulate the normals from each observed event. An event is a set of four observations from four ground stations to a single satellite position.

One of the most unique properties of the program used to generate normal equations is the ability to test the observations made at each event and to either reject the event or include it in the formation of the normal equations. After four simultaneous range observations have been detected by the computer program, the approximate position of the satellite is computed by taking the mean value of the latitude and longitude of the four observing stations and using a height of 1.6 million meters (1.6 million meters is satisfactory for GEOS-I. Another satellite may need a different height.). These approximate coordinates are then converted to rectangular coordinates. The mathematical model is

$$\Delta X_i = XS - X_i$$
$$\Delta Y_i = YS - Y_i$$
$$\Delta Z_i + ZS - Z_i$$

where

XS, YS, ZS is the space position of the satellite

X_i, Y_i, Z_i is the position of station i,

$R = [\Delta X_i^2 + \Delta Y_i^2 + \Delta Z_i^2]^{\frac{1}{2}}$

where

R the computed range .

The least squares adjustment is as described in [Uotila, 1967, pp. 37-38]. As can be expected, there will be very large corrections on the first iteration. The computer program iterates until the change in the correction to any one of the parameters is less than 0.01 m. After the solution converges, equation (2.1) is solved for the residual vector V. The program then computes the unit variance of the event

$$\sigma_0^2 = \frac{V'PV}{d.o.f.} \; .$$

Since the degrees of freedom will always be one for a four station track,

$$\sigma_0^2 = V'PV \; .$$

At this point the entire event can be rejected, or it can be included in the formation of normal equations. This is determined by a test value which is input to the program with the observational data. The test value is compared to the value of σ_0^2 for each event, and if σ_0^2 is less than the test value the event is accepted and its contribution to the normal equations is computed. The coordinates of all four stations are held fixed during the above computations, and if the coordinates are only very rough approximations (the first iteration of an adjustment), the test value must be of sufficient magnitude so as not to reject good observations.

2.2 External Constraints

In order to perform a network adjustment there has to be established a working coordinate system. In the general case this is done by defining

(a) three parameters (coordinates) for the origin of the system,

(b) three parameters (coordinates) for determining the orientation of the system, and

(c) one parameter to establish the scale.

In the case of range observations since the scale is determined by the observations themselves, only (a) and (b) must be defined. The normal equations that are generated by the geometric adjustment program are singular with a nullity of six. Therefore, it is necessary to apply a minimum of six constraints in order to satisfy conditions (a) and (b). In the preliminary work (data screening) it is necessary to impose only this minimum set of constraints, because any larger set of constraints would only increase the residuals and mask the internal consistency of the data set (see Section 2.3). After the quality of the data is determined and the final set of data arrived at, the network is to be readjusted using all available external information (directions, heights, etc.) as additional constraints. In the adjustment reported here four types of external constraints were used. These were constraints on

(a) directions between two stations,
(b) relative positions of two stations,
(c) station positions, and
(d) geodetic heights.

The conventional method of handling constraints among stations is to add their contribution to the $\dot{N}$ matrix. In the case of the adjustment described above, it is more convenient to add the contributions to the reduced normal equations (Equation (2.7)).

2.21 Directional Constraints.

The directional constraint between two stations i and j is accomplished by applying weights to two angles α and β defining the direction between them.

From the approximate (X°, Y°, Z°) coordinates of the two stations, values are computed for the two angles, referred to as α° and β°, in the following manner:

$$\alpha^\circ \qquad \tan^{-1}\frac{\Delta Y^\circ}{\Delta X^\circ}$$

$$\beta^\circ \qquad \tan^{-1}\frac{\Delta Z^\circ}{R^\circ}$$

where

$$\begin{aligned} \Delta X^\circ &= X_i^\circ - X_j^\circ \\ \Delta Y^\circ &\quad Y_i^\circ - Y_j^\circ \qquad (2.8) \\ \Delta Z^\circ &\quad Z_i^\circ - Z_j^\circ \end{aligned}$$

and

$$R^\circ = (\Delta X^{\circ 2} + \Delta Y^{\circ 2})^{\frac{1}{2}}$$

A matrix of partial derivatives G, is then formed [Uotila, 1967, p. 33],

$$G = \begin{bmatrix} \frac{\partial \alpha^\circ}{\partial \Delta X^\circ} & \frac{\partial \alpha^\circ}{\partial \Delta Y^\circ} & \frac{\partial \alpha^\circ}{\partial \Delta Z^\circ} \\ \frac{\partial \beta^\circ}{\partial \Delta X^\circ} & \frac{\partial \beta^\circ}{\partial \Delta Y^\circ} & \frac{\partial \beta^\circ}{\partial \Delta Z^\circ} \end{bmatrix}$$

where

$$\frac{\partial \alpha^\circ}{\partial \Delta X^\circ} = \cos^2 \alpha^\circ \tan \alpha^\circ / \Delta X^\circ$$

$$\frac{\partial \alpha^\circ}{\partial \Delta Y^\circ} = -\cos^2 \alpha^\circ / \Delta X^\circ$$

$$\frac{\partial \alpha^\circ}{\partial \Delta Z^\circ} = 0$$

$$\frac{\partial \beta^\circ}{\partial \Delta X^\circ} \qquad \Delta X^\circ \cos^2 \beta^\circ \tan^2 \beta^\circ / R^{\circ 2}$$

$$\frac{\partial \beta^\circ}{\partial \Delta Y^\circ} \qquad \frac{\partial \beta^\circ}{\partial \Delta X^\circ} \tan \alpha^\circ$$

$$\frac{\partial \beta^\circ}{\partial \Delta Z^\circ} \qquad - \cos^2 \beta^\circ / R^\circ \quad .$$

The procedure then is to form a matrix

$$\tilde{N} \quad G' W G \tag{2.9}$$

where W is the weight matrix estimated from the statistics for α° and β° the customary way as follows:

$$W = \begin{bmatrix} \sigma_{\alpha^\circ}{}^2 & \sigma_{\alpha^\circ \beta^\circ} \\ \sigma_{\alpha^\circ \beta^\circ} & \sigma_{\beta^\circ}{}^2 \end{bmatrix} \quad . \tag{2.10}$$

The matrix $\tilde{N}$ is then added to the diagonal elements of the reduced normal equations (2.7) that correspond to each of the ground stations, i.e., $\dot{N}_{ii}$ and $\dot{N}_{jj}$ and is subtracted from the off-diagonal elements $\dot{N}_{ij}$ and $\dot{N}_{ji}$.

If the directional constraints are computed from a priori information and not the approximate station coordinates used in the adjustment, the correction below must be applied to the right hand side of the reduced normal equations

$$\tilde{U} = G' W S \tag{2.11}$$

where

$$\delta = \begin{bmatrix} \alpha - \alpha^\circ \\ \beta - \beta^\circ \end{bmatrix} \quad .$$

In this case the a priori direction angles are α and β, and the weight matrix W (2.10) is computed using the variance and covariance of α and β, not α° and β°. This weight matrix is then used in Equations (2.9) and (2.11). The vector $\tilde{U}$ is then applied to the right hand side of the equations (2.7), added to the part that corresponds to station i, and subtracted from station j.

2.22 Relative Position Constraints.

If the relative position (ΔX°, ΔY°, ΔZ°) of two stations is known, along with the standard deviation of these relative positions, the constraint can be formed. If the constraint is to be computed based on the approximate coordinates of stations i and j used in the adjustment, then

$$\tilde{N} = W$$

and

$$\tilde{U} = 0$$

where

$$W = \begin{bmatrix} \frac{1}{\sigma^2_{\Delta X^\circ}} & 0 & 0 \\ 0 & \frac{1}{\sigma^2_{\Delta Y^\circ}} & 0 \\ 0 & 0 & \frac{1}{\sigma^2_{\Delta Z^\circ}} \end{bmatrix} . \qquad (2.12)$$

$\tilde{N}$ is then applied to the reduced normal equations (2.7) in the same manner as described in Section 2.21.

If the constraint is based on relative coordinates (ΔX, ΔY, ΔZ) that are different from the approximate coordinates (ΔX°, ΔY°, ΔZ°) used in the adjustment, then the variances in the weight matrix (2.12) are replaced by the variances $\sigma^2_{\Delta X}$, $\sigma^2_{\Delta Y}$, and $\sigma^2_{\Delta Z}$, where ΔX, ΔY, ΔZ are the a priori relative positions. Because of the discrepancy on the right hand side of the reduced normal equations, the vector

$$\tilde{U} = W\,\delta ,$$

with

$$\delta = \begin{bmatrix} \Delta X - \Delta X^\circ \\ \Delta Y - \Delta Y^\circ \\ \Delta Z - \Delta Z^\circ \end{bmatrix} ,$$

must be computed. $\tilde{N}$ and δ are then applied to the reduced normals (2.7) in the same manner as described in Section 2.21.

2.23 Station Position Constraints.

If the approximate station coordinates of station i used in the adjustment are to be constrained, then

$$\tilde{N} = W$$

and

$$\tilde{U} = 0 ,$$

where

$$W = \begin{bmatrix} \frac{1}{\sigma^2_{X_i^\circ}} & 0 & 0 \\ 0 & \frac{1}{\sigma^2_{Y_i^\circ}} & 0 \\ 0 & 0 & \frac{1}{\sigma^2_{Z_i^\circ}} \end{bmatrix} . \qquad (2.13)$$

The values $\sigma^2_{X_i^\circ}$, $\sigma^2_{Y_i^\circ}$, $\sigma^2_{Z_i^\circ}$ are the variances of the approximate coordinates of station i.

In the event that the station coordinates to be constrained (X, Y, Z) are different from the approximate coordinates (X°, Y°, Z°), the vector

$$\tilde{U} \quad W\delta$$

with

$$\delta \quad \begin{bmatrix} X_i - X_i^\circ \\ Y_i - Y_i^\circ \\ Z_i - Z_i^\circ \end{bmatrix} ,$$

must be computed again. In this case the weight matrix W is as above but with the variances $\sigma^2_{X^i}$, $\sigma^2_{Y^i}$, $\sigma^2_{Z^i}$. The W matrix is also used to form $\tilde{N}$.

Since only one station is involved in this constraint, $\tilde{N}$ is added only to the diagonal of the reduced normal Equations (2.7), and $\tilde{U}$, if needed, is added at the appropriate location on the right hand side.

2.24 Height Constraints.

If the height of the station (h_i°) to be constrained is that of the approximate height used in the adjustment,

$$\tilde{N} = \alpha W \alpha'$$

and

$$\tilde{U} = 0$$

where

$$W = 1/\sigma^2_{h_i^\circ} , \qquad (2.14)$$

and

$$\alpha = \begin{bmatrix} \cos\varphi_i^\circ \cos\lambda_i^\circ \\ \cos\varphi_i^\circ \sin\lambda_i^\circ \\ \sin\varphi_i^\circ \end{bmatrix} ,$$

where the values of φ°_i and λ°_i are the approximate (geodetic) coordinates of the station, and $\sigma^2_{h^\circ_i}$ is the variance of the station height.

If the height (h_i) is different from the approximate height (h°_i), then

$$\tilde{U} \qquad \alpha W \delta$$

where

$$\delta = h_i - h^\circ_i ,$$

and the variance in the weight matrix (2.14) is replaced by the variance $\sigma^2_{h_i}$. As before, this same weight matrix W is used in the formation of $\tilde{N}$. The quantities $\tilde{N}$ and $\tilde{U}$ are applied as described at the end of Section 2.23.

It should be mentioned that when a constraint of any kind is added to the normal equations, its contribution to $\Sigma V' PV$ must also be considered. In addition, the degrees of freedom will also change. These must be accounted for so that the proper standard deviation of unit weight may be computed.

2.3 Inner Constraints

Even though the selection of a coordinate system is arbitrary in the case of a minimum constraint adjustment, the selection of the six coordinates (at more than two stations) to be constrained is very critical, since one set of constraints would give a different solution than another set. The best solution is arrived at in a coordinate system defined through the use of a set of constraint equations called "inner" constraints [Rinner, et. al., 1969]. In this sense, "best" means resulting in the smallest covariance matrix for the unknowns. Covariance matrices may be compared by means of their traces, and the inner constraint equations are characterized by the property that the trace of the covariance matrix obtained with their use is a minimum among those obtained by adjusting a given set of range observations augmented by a minimal set of six constraint equations. This property also implies that the mean square uncertainty of the unknowns is smaller when the inner adjustment equations are used. From this property it follows that in the preliminary minimum constraint adjustment leading to the selection of the final data set these inner constraints should be used

The inner constraint equations are written in the form

$$CX = 0$$

where

$$C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} ,$$

X is the set of corrections of the approximate coordinates of the unknown points, and

$$C_1 = \left[\begin{array}{ccc|c} 1\,0\,0 & 1\,0\,0 & \cdots \\ 0\,1\,0 & 0\,1\,0 & \cdots \\ 0\,0\,1 & 0\,0\,1 & \cdots \end{array}\right] ,$$

$$C_2 = \left[\begin{array}{ccc|ccc|c} 0 & -Z_1^\circ & Y_1^\circ & 0 & -Z_2^\circ & Y_2^\circ & \cdots \\ Z_1^\circ & 0 & -X_1^\circ & Z_2^\circ & 0 & -X_2^\circ & \cdots \\ -Y_1^\circ & X_1^\circ & 0 & -Y_2^\circ & X_2^\circ & 0 & \cdots \end{array}\right] .$$

Both the C_1 and C_2 matrices are made up of similar 3 × 3 blocks, where each block contains the coefficients of the unknown coordinates of a point. The symbols (X_i°, Y_i°, Z_i°) denote the approximate coordinates of the ith unknown point, where both the ground points and the satellite positions are considered.

It is also possible to design a set of constraints that will result in the best solution for only a subset of the points. In our adjustments we were only interested in the ground station unknowns, so that we wanted to obtain the best possible solution for those unknowns while using only six constraint equations. This implies that the trace of only that portion of the covariance matrix corresponding to the ground station unknowns is minimized, while the variances of the satellite position unknowns are not included in the minimum sum. The constraint equations that will produce a solution which is best for only some of the unknown points have the same form as those producing the best solution for all the points; however, 3 × 3 blocks of zeros are inserted into those positions of C_1 and C_2 which correspond to unknowns whose variances are not to be included in the minimum sum.

The inner adjustment constraint equations can be given a geometrical interpretation that appeals to intuition. Let X_i^o denote the set of approximate coordinates of the ith unknown point, dX_i denote the corrections to these coordinates, and X_i denote the adjusted coordinates; i.e.,

$$X_i = X_i^o + dX_i \ .$$

The first set of constraint equations, $C_1 X = 0$, is then equivalent to the set of conditions

$$\sum_i dX_i = 0 \, .$$

The geometrical interpretation of these conditions is that the center of gravity of all the points will not change after adjustment; i.e.,

$$\sum_i X_i = \sum_i X_i^o \, .$$

The second set of constraint equations, $C_2 X = 0$, correspond to the conditions

$$\sum_i X_i^o \times dX_i = 0 \, .$$

If the center of the system remains fixed, then the cross products $X_i^0 \cdot dX_i$ reflect rotations of the points around the fixed center. These constraint equations insure that the sums of the rotations around all three coordinate axes are zero. The corresponding geometrical interpretation is that the mean orientation of the system of points will not change after adjustment either.

Thus, the respective equations $C_1X = 0$ and $C_2X = 0$ effectively specify the origin and the orientation of the adjustment coordinate system. A seventh "inner adjustment" equation is also available to specify the scale of the system. However, this scale equation is only used when the observations themselves do not determine the scale. This would be the case, for instance, if a set of optical satellite observations were to be adjusted.

A more complete description of the inner adjustment is described in [Blaha, 1971].

3. DATA

3.1 SECOR Observations

The data used in the adjustments was selected from the 78000+ observations available in the Space Science Data Center, plus several passes of data received directly from the U.S. Army Topographic Command. The data that is available from the Space Science Data Center is listed in [NASA, 1969]. The observing stations and their coordinates as given in the Geodetic Satellites Observation Station Directory [Geonautics, 1969] are listed in Table 1. The network configuration is shown in Figure 2.

The first set of data received was all of the SECOR data from the Pacific Network that was available at the Data Center as of July 1, 1968. The data received were 56258 range measurements contained on three magnetic tapes, the greater percentage being simultaneous from four stations. However, there was an adequate amount of data from only the following seven quadrangles:

5401-2-3-4
5402-3-4-5
5402-3-5-6
5402-5-6-7
5404-5-6-7
5403-6-7-8
5403-7-8-10.

There was not enough data to extend the network to Maui, Hawaii. For this reason, all of the preliminary work was on the network that ended at Midway Island.

In order to perform an adjustment, it is normally expected that one or

Table 1

Station Coordinates and Datum Information Given for the Ten SECOR Stations Used in the Network Adjustment

GOCC No.	Name	Latitude	Longitude	Height (above MSL) (meters)	Datum
5401	Truk Island	7°27'39".307	151°50'31".282	5.95	1947 Navy Iben Astro
5402	Swallow	-10 18 21 .42	166 17 56 .79	9.52	1966 SECOR Astro
5403	Kusaie	5 17 44 .432	163 01 29 .881	7.5	Astro 1962, 65 (Allen Sodano Light)
5404	Gizo	- 8 05 40 .580	156 49 24 .825	49.53	Provisional DOS
5405	Tarawa	1 21 42 .130	172 55 47 .268	7.36	1966 SECOR Astro
5406	Nandi	-17 45 31 .012	177 27 02 .833	17.65	Viti Levu 1916
5407	Canton	- 2 46 28 .99	188 1[illegible] 43 .47	6.11	1966 Canton Astro
5408	Johnson	16 43 51 .681	190 28 41 .555	6.3	Johnson Island (1961)
5410	Midway	28 12 32 .061	182 37 49 .531	6.097	Midway Astro 1961
5411	Maui	20 49 37.004	203 31 52.772	32.33	Old Hawaiian

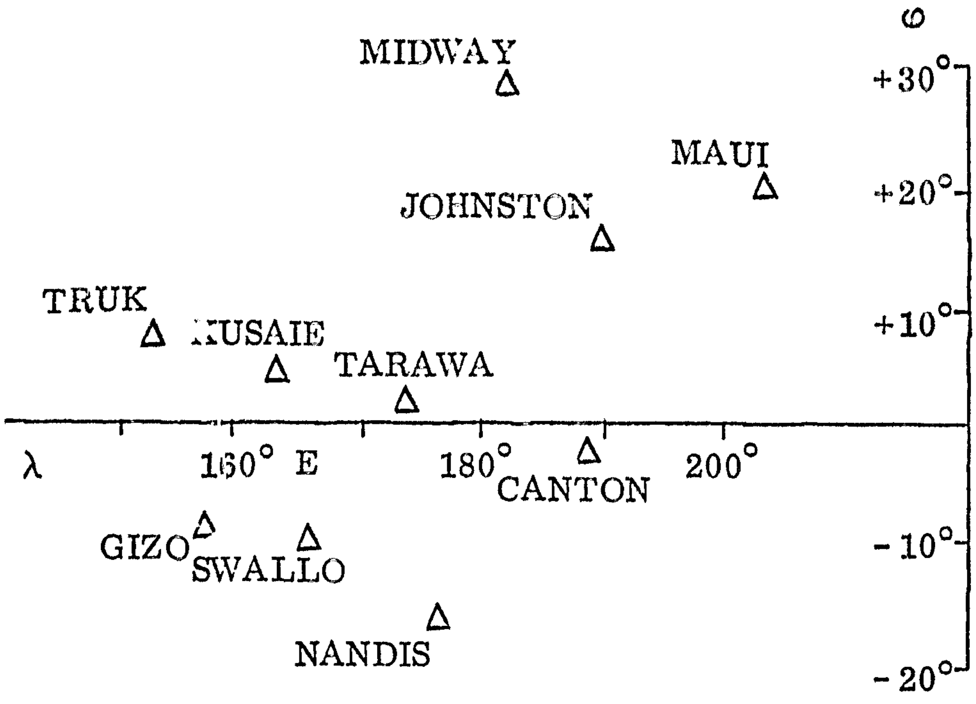

Figure 2. Location of the SECOR Stations in the Pacific

more of the observing stations have known coordinates, and the majority of the observations are reliable. If one has fairly good station positions, it is not difficult to find and remove bad data; conversely, if one has good data a rigid geodetic network can easily be constructed. In the case of the Pacific SECOR network there were no station coordinates given on any major datum, the only coordinates available were those listed in Table 1. As a further complication, the quality of a large amount of the data is questionable, since in its early days the SECOR system was plagued by ambiguities, calibration errors, and possibly unreliable determination of ionospheric refraction. Therefore as a first step it was necessary to find a set of data that was at least internally consistant. This was done by performing network adjustments using only the six minimum constraints, which were weights applied to the x, y, z coordinates of station 5401, the y coordinates of station 5402, and the x and z coordinates of station 5407. These were later verified by the "inner adjustment" technique to be a reasonable set of constraints.

The station coordinates, as given in Table 1, are all based on local datums. Because the majority of these local datums use the International Ellipsoid (see Table 2) this ellipsoid was used in all of the preliminary solutions. Many adjustments were performed to try to find a set of coordinates that would fit the given data, and by using the rejection criteria feature of the adjustment program and performing several iterations a set of data was selected that appeared to be realistic. With this data and the constraints described above, it was possible to solve for the station coordinates of all the stations in the network except Maui. This solution was reported in [Reilly, 1969].

After the initial request for SECOR data, more GEOS-I data was placed in the Data Center. This data was requested, and received. There were several passes of data from quadrangles that contained Maui, but the solution showed very large residuals and as a result was not usable.

Table 2

Datum Parameters

Datum Name	Semi-Major Axis	1/f
1966 Canton Astro	6378388.0 m	297.0
Johnston Island 1961	6378388.0	297.0
Midway Astro 1961	6378388.0	297.0
Navy Iben Astro 1947	6378206.4	294.9787...
Provisional DOS	6378388.0	297.0
Astro 1962, 65 Allen Sodano Light	6378388.0	297.0
1966 SECOR Astro	6378388.0	297.0
Viti Levu 1916	6378249.1	293.4663
Old Hawaiian	6378206.4	294.9787...

A request was made to the U.S. Army Topographic Command (TOPOCOM) for some additional SECOR data that could tie the present network to Maui. TOPOCOM forwarded data from six passes of the EGRS-7 satellite observed from Kusaie, Johnston, Midway and Maui (5403-8-10-11). With this data, it was now possible to perform an adjustment of the network and arrive at a better solution. The time spans of data that was now available and that appeared to be realistic are listed in Table 3 (passes 1-53).

An additional subset of usable data was made available near the end of the investigations. The usefulness of this data set was due to the fact that the station coordinates of the SECOR stations had now been determined sufficiently that it was possible to perform short arc orbital mode adjustments for the purpose of recovering biases. The time spans of this data are also listed in Table 3 (passes 54-67).

3.2 Data for External Constraints

Although there was no direct data available on the absolute or relative position of any SECOR station, we were able to find several indirect sources of positional information which could be utilized as external constraints in the adjustment. This information consisted of the following (Tables 4 and 5):

(1) On Maui there were camera stations from the Coast and Geodetic Survey's Worldwide Geometric Satellite Network (BC-4), and from the Smithsonian Astrophysical Observatory's Network (Baker-Nunn). Both of these stations had been tied into the local survey system together with the Maui SECOR station. The relative positions of these three stations provided $3 \times 3 = 9$ constraint equations.

(2) On Johnston Island there was a PC-1000 camera, operated by the U.S. Air Force almost at the same location that had been occupied by the SECOR station. This PC-1000 camera had observed PAGEOS, ECHO I,

Table 3. Time-spans of Data Used

	Pass No.	Date		From			To		
		1966		h	m	s	h	m	s
GEOS - I	1	6	1	7	55	12			
	2	6	1	18	43	28	18	50	40
	3	6	5	18	50	36	18	50	40
	4	6	8	6	00	20	6	02	56
	5	6	17	15	30	04	15	33	16
	6	6	18	4	38	56	4	42	48
	7	6	19	4	45	04	4	46	44
	8	6	20	4	46	24	4	54	24
	9	6	26	14	08	00	14	09	44
	10	6	28	3	12	08	3	19	52
	11	7	3	1	31	40	1	37	24
	12	7	5	1	40	32	1	48	28
	13	7	7	23	53	16	23	58	32
	14	7	9	10	51	12	10	51	16
	15	7	9	21	59	28			
	16	7	10	0	01	28	00	09	48
	17	7	11	11	02	16			
	18	7	12	11	03	20			
	19	7	14	22	22	32	22	22	36
	20	7	15	22	19	16	22	29	48
	21	7	16	9	17	44	9	20	20
	22	7	16	22	24	00	22	29	16
	23	7	17	9	20	08	9	25	28
	24	7	17	22	30	04	22	35	24
	25	7	19	4	44	04	4	46	44
	26	7	21	20	43	24	20	48	44
	27	7	22	20	44	24	20	52	20
	28	7	23	7	40	44	7	40	48
	29	7	23	18	47	00			
	30	7	23	20	56	56	20	59	36
	31	7	24	7	46	44	7	49	24
	32	7	24	18	50	24	18	53	04
	33	7	24	20	58	44	21	05	28
	34	7	25	18	55	24			
GEOS - I	35	7	26	18	55	21	19	02	08
	36	8	6	17	40	48	17	46	32
	37	11	5	14	57	24	15	00	00
	38	11	5	21	06	08	21	12	12
	39	11	17	17	48	04	17	52	00
	40	11	19	17	57	20	18	01	24
	41	11	20	18	05	16	18	10	32
	42	11	22	18	10	56	18	16	12
	43	11	23	16	14	44			
	44	11	24	16	19	28	16	22	08
	45	11	25	16	22	24			
	46	11	26	16	28	04	16	33	20
	47	11	27	16	31	20	16	36	36
EGRS-7	48	12	18	16	07	40	16	12	40
	49	12	20	15	39	03	15	45	43
	50	12	21	14	05	47	14	06	27
	51	12	22	15	12	22	15	16	37
	52	12	24	14	39	00	14	43	45
	53	12	27	15	21	16	15	25	00
GEOS - I	54	5	24	22	04	07	22	07	07
	55	6	7	18	56	44	18	59	48
	56	6	11	6	13	48	6	16	04
	57	7	21	9	34	44	9	37	20
	58	7	27	19	01	56	19	05	44
	59	8	22	14	39	56	14	46	44
	60	9	5	11	30	12	11	37	44
	61	9	23	6	34	40	6	36	24
	62	10	20	0	09	12	0	10	20
	63	12	6	15	03	52	15	09	48
	64	12	9	13	10	04	13	15	04
	65	12	11	13	19	08	13	23	04
	66	12	16	11	39	36	11	41	48
	67	* 1	8	6	55	44	7	03	36

* 1967

Table 4. Relative Positions from Local Ground Surveys

From To	Name	Type	Δx(m)	Δy(m)	Δz(m)	Estimated σ in each coordinate (m)
5408 3475	Johnston Johnston	SECOR PC-1000	3.8	0.8	-1.2	0.5
5411 6011	Maui Maui	SECOR BC-4	2001.2	-22992.3	-10965.0	0.5
5411 9012	Maui Maui	SECOR BN	1951.7	-22873.4	-11000.9	0.5
6011 9012	Maui Maui	BC-4 BN	-49.5	118.9	-35.9	0.5
5410 2724	Midway Midway	SECOR Doppler	-882.6	1911.2	-1481.4	0.5

and ECHO II simultaneously with BC-4 cameras on Maui, Wake, and Christmas Islands [Huber, 1969]. Since the three BC-4 stations were part of the Coast and Geodetic Survey's world net, coordinates of the Johnston PC-1000 on the North American Datum, together with the direction Johnston (PC-1000) - Maui (BC-4) could be determined. The relative positions of the Johnston stations and the direction Johnston-Maui provided 3 + 2 = 5 additional constraint equations.

(3) On Midway Island there was a TRANET Doppler station which had been tied to the local survey system, as had the Midway SECOR station. The coordinates of the Doppler station on the Mercury Ellipsoid had been published as part of the NWL-8D solution [Anderle and Smith, 1967]. Performing a datum transformation, we were able to infer NAD coordinates for the Midway Doppler, and thus determine the direction Midway (Doppler) - Johnston (PC-1000). The relative positions of the stations on Midway and the direction Midway-Johnston provided again 3 + 2 = 5 constraint equations.

Table 5. Station Coordinates Used in the Network Orientations.

GOCC #	Name	Type	Datum	Latitude	σ	Longitude (+E)	σ	h (m)	σ(m)	Note
6011	Maui	BC-4	NAD	20°42′26″.139	0″.351	203°44′42″.886	0″.396	3001.4	12.0	1
9012	Maui	Baker-Nunn	SAO 1969	20 42 25.66	0.250	203 44 33.48	0.25	3029.0	7.0	5
6012	Wake	BC-4	NAD	19 17 28.247	0.470	166 36 43.564	0.515	-159.2	17.2	1
6059	Christmas	BC-4	NAD	2 0 13.185	0.487	202 35 20.508	0.380	-22.4	13.3	1
3475	Johnston	PC-1000	NAD	16 43 44.209	0.254	190 28 49.931	0.313	-90.2	7.9	2
2724	Midway	Doppler	Mercury	28 11 48.79		182 36 40.13		-14		3
2724	Midway	Doppler	NAD	28 11 50.47		182 36 46.16		-117		4

Notes:

1. Coast and Geodetic Survey preliminary coordinate.
2. Obtained at OSU by adjusting the ACIC optical data, weighting the coordinates of Maui, Wake, and Christmas according to their uncertainties.
3. NWL-8D Solution [Anderle and Smith, 1967]. Uncertainty is 25 m in each Cartesian coordinate.
4. Obtained from NWL-8D Mercury Datum Coordinate, using translation parameters of $\Delta x = -40$ m, $\Delta y = 163$ m, $\Delta z = 186$ m [Anderle and Smith, 1967].
5. Computed from Cartesian coordinates, with $\sigma = 7$ m for each coordinate, and referred to the SAO ellipsoid of the following parameters: $a = 6378155$ m, $1/f = 298.255$ [Gaposchkin and Lambeck, 1969].

(4) With the above information, the relative and/or the SAO 1969 positions of the stations on Maui, Johnston and Midway could be determined except for the scale, which was available only in the NAD system as propagated through the BC-4 net. These islands being at the eastern end of the network, the station positions at the western end were quite weak, especially in the vertical components. This phenomena was attributed to the cantilever effect of error propagation. In order to reduce this effect more external information was brought into the adjustment in the form of the geodetic heights of the ten SECOR stations. These heights were determined from the SAO 1969 Standard Earth geoid map by adding the Geoid undulations to the heights above sea level as determined from spirit leveling [Gaposchkin and Lambeck, 1969]. This procedure resulted in heights with respect to the SAO ellipsoid (a = 6378155 m; f = 1/298.255). Derived from a geoid map, these heights were quite uncertain. We estimated that 15-25 meters was a reasonable value for the standard deviation of a single height determination and derived weights for the height constraint equations from this figure. Even though they had relatively low weights, these constraint equations effectively nullified the cantilever effect and greatly improved the determination of station positions at the western end of the network.

(5) The Baker-Nunn station on Maui was selected as the origin of the system in the final (SP-7) solution. Its coordinates in the SAO 1969 Standard Earth System (see Table 5) were constrained with weights based on the standard deviations as given by SAO [Gaposchkin and Lambeck, 1969].

4. DESCRIPTION OF SOLUTIONS

4.1 The Different Solutions

Although many intermediate adjustments were performed, only three are worth mentioning. These are designated as SP-5, SP-6 and SP-7. The last one is our preferred solution.

4.11 The SP-5 Solution.

The GEOS-I data used for this adjustment was selected during the very early stages of the experiments, data that appeared to be free from ambiguities and calibration error (passes 1-47 in Table 3). Also included was the EGRS-7 data received from TOPOCOM (passes 48-53). There were 976 range observations, with the network extending from Truk Island to Maui. The NAD coordinates of Johnston Island SECOR was inferred from the position of the PC-1000 camera as described in Section 3.2. The directions from Johnston to both Maui and Midway were constrained by weighted constraint equations, so that the scale of the solution was determined from the SECOR observations alone.

Height constraints were applied to all stations except Johnston, Midway and Maui. For each of these height constraints, a standard deviation of 25 meters was used. The NAD coordinates of Johnston defined the origin of the system.

4.12 The SP-6 Solution

The data used in the SP-6 solution was the same used in the SP-5 solution. The height constraints were also identical. The difference was that the NAD coordinates of Johnston, Midway and Maui were all constrained, so that these coordinates also contributed to the scale determination. The results of the SP-5 and SP-6 solutions were presented at the GEOS-II Program Review Meeting in June, 1970.

4.13 The SP-7 Solution.

The SP-5 and SP-6 solutions gave a set of station coordinates that appeared to be reasonably consistent. With the adjusted coordinates of the preliminary solutions (specifically the SP-5) it was possible to perform short arc orbital mode adjustments for the purpose of recovering biases. It was suspected that much of the deleted data from earlier solutions was good, except that the observations contained constant biases. These constant biases are made up of ambiguities, which occur in multiples of 256 meters, and calibration errors, which are generally under 30-40 meters. By performing short arc orbital mode adjustments in which the station coordinates were all constrained, ambiguities and calibration corrections were determined for passes 54-67 in Table 3. The observations for which the biases had been recovered were corrected, and the corrected observations were added into the set of usable data. Since very few passes lasted over ten minutes and covered significant ranges in altitude, no attempt was made to solve for refraction or other error model terms.

It was also possible to make a reasonable estimate of the calibration error for some of the data that constituted only a very short segment of an arc. In many instances it was noted that residuals for a given station in the geometric mode solutions were fairly large, constant, and of the same sign. For these observations, the mean residuals served as estimates of the calibration errors. This data was removed, corrected for this calibration error, and added back into the usable set. These ambiguity and calibration corrections are listed in Table 6.

The SP-7 solution was the final adjustment. The geocentric coordinates of the Maui Baker-Nunn station defined the origin, their weights were based on the standard deviations of 7 meters for each Cartesian component as given in SAO. The orientation was aided by constraining the directions

Table 6. Recovered Ambiguity and Calibration Corrections
(In Meters)

Station	5401		5402		5403		5404		5405		5406		5407		5408		5410		5411	
Pass No.	Amb	Cal	Amb	Cal	Amb	Cal	Amb	Cal	Amb	Cal	Amb	Cal	Amb	Cal	Amb	Cal	Amb	Cal	Amb	Cal
1				-3		+3				-2		+3								
3				+3		-2				+2		-3								
5				+5		-6				+4		-4								
6		-5		-5		+4		+5												
10		+5		+6		-5		-6												
11		+2		+3		-2		-3												
16				-5		+6				-5		+5								
16a				+5		-6				+4		-4								
20				+2		-2				+2		-3								
20a				+6		-8				+6		-3								
30				-5		+7				-5		+3								
35								+5		-5		-3		+4						
38						+1						-3		+4		-3				
46						+1						-2		+3		-3				
49						-2										+6		0		-5
50						-8										+15		-1		-8
51						-5										-15		0		+12
53						-3										+13		0		-12
54	-256		-512		-256		-512													
55			-256	+1	0	-1			0	+1	-256	-2								
55a			-256	+3	0	-2			0	+2	-256	-4								
56			-256	-2	-256	+1			0	-1	-256	+2								
57			-512		0				-256		-512									
58							256	-6	256	+6	-256	+5	-256	-6						
58a							256		256		-256		-256							
59							-256		-512		-768		-512							
60							-512	+3	-768	-5	-768	-3	-768	+6						
61					-256	-1					0	+2	0	-3	0	+2				
62					-768						-256		-256		-768					
63					-1280	0					-768	0	-768	+1	-768	-1				
64					-768								-256				-512		-256	
65					-768								-512				-512		-512	
66					-768								-256				-512		-512	
67					512	+2									256	-8	0	-1	0	+9
67a					512	-1									256	+3	0	0	0	-8

from Johnston to Maui and Johnston to Midway, the same orientation as in the SP-5 solution. A change from the previous solutions was that the geodetic heights of all stations were constrained, and they were constrained with weights corresponding to a standard deviation of 15 meters. There were a total of 1188 range observations (at 4^s - 60^s intervals) which, with the external constraint equations, resulted in 287 degrees of freedom.

4.2 Results

The adjusted coordinates from all three solutions are shown in Table 7. For ease of comparison, the coordinates of the SP-5 and SP-6 solutions have also been converted to the SAO-1969 system to be compatible with the SP-7 solution. In the solutions SP-5 and SP-6, the standard deviation of a single range estimated a posteriori from the solution was 8.6 meters. The SP-7 solution reduced this standard deviation to 3.2 meters. As can be seen by examining Table 7, the additional data, and the removal of the systematic errors from the existing data, made the SP-7 solution far superior to any of the earlier adjustments.

Table 8 gives the geodetic coordinates of the SP-7 solution on the North American Datum. To transfer the coordinates from the SAO system to the NAD, the following translation parameters were used [Badekas, 1969]:

$$\Delta x = 38\,\text{m}, \quad \Delta y = -164\,\text{m}, \quad \Delta z = -175\,\text{m}.$$

These parameters are in the sense NAD-SAO.

4.3 Conclusions

Our experiences with the SECOR observations of GEOS-I in the Pacific indicate that with a great deal of effort one can obtain satisfactory solutions. Since none of the observing stations are positioned on major

Table 7. SAO - 1969 Coordinates

GOCC #	NAME		SP-5	σ	SP-6	σ	SP-7	σ
5401	Truk	x	-5576046 m	20 m	-5576050 m	20 m	-5576050 m	12 m
		y	2984663	22	2984651	24	2984667	12
		z	822370	35	822391	41	822438	15
5402	Swallo	x	-6097439	14	-6097445	14	-6097450	8
		y	1486476	27	1486472	33	1486518	15
		z	-1133253	23	-1133237	27	-1133244	10
5403	Kusaie	x	-6074526	13	-6074532	14	-6074527	8
		y	1854349	17	1854340	20	1854359	10
		z	583794	23	583811	28	583838	11
5404	Gizo	x	-5805386	16	-5805390	16	-5805394	9
		y	2485301	27	2485295	32	2485342	14
		z	- 892938	29	- 892947	35	- 892882	12
5405	Tarawa	x	-6327917	11	-6327924	13	-6327924	7
		y	784564	18	784558	22	784583	11
		z	150802	17	150815	20	150834	9
5406	Nandis	x	-6070188	19	-6070195	19	-6070207	11
		y	270635	35	270636	41	270690	18
		z	-1932863	21	-1932851	23	-1932851	11
5407	Canton	x	-6304300	16	-6304305	16	-6304308	9
		y	- 917656	22	- 917657	26	- 917626	13
		z	- 307105	15	- 307097	15	- 307106	9
5408	Johnston	x	-6007969	8	-6007974	6	-6007981	5
		y	-1111233	9	-1111238	8	-1111240	8
		z	1824153	8	1824160	7	1824156	7
5410	Midway	x	-5618708	15	-5616715	13	-5618721	10
		y	- 258181	20	- 258193	13	- 258217	10
		z	2997221	21	2997228	13	2997241	10
5411	Maui	x	-5468005	11	-5468005	9	-5468010	6
		y	-2381408	12	-2381408	9	-2381410	7
		z	2253172	8	2253172	10	2253175	7

Table 8. North American Datum Coordinates

(Solution SP-7)

GOCC #	Name	Latitude	σ	Longitude (+E)	σ	Height	σ
5401	Truk	7°27′27″.1	0″.5	151°50′33″.2	0″.4	-127 m	12 m
5402	Swallo	-10 18 18.7	0.3	166 18 01.1	0.5	- 39	8
5403	Kusaie	5 17 10.4	0.3	163 01 32.0	0.4	- 84	7
5404	Gizo	- 8 06 12.7	0.4	156 49 30.1	0.5	- 21	10
5405	Tarawa	1 21 45.9	0.3	172 56 0.7	0.4	- 95	6
5406	Nandis	-17 45 36.8	0.3	177 26 53.6	0.6	53	11
5407	Canton	- 2 46 48.8	0.3	188 16 59.0	0.4	- 23	9
5408	Johnston	16 43 44.0	0.2	190 28 50.2	0.3	-105	5
5410	Midway	28 12 45.4	0.3	182 37 58.6	0.4	-121	10
5411	Maui	20 49 24.6	0.2	203 32 7.9	0.2	- 24	6

datums, external information must be used to tie the network into existing coordinate systems. Since ambiguity and calibration corrections can be extracted reliably only from those data subsets that constitute passes, and only a very few of the passes are long enough to allow the use of an error model more extensive than the single constant bias term, small systematic errors are still suspected to be present in some of the data.

The solutions for the station coordinates (Tables 7 and 8) appear to be completely valid. The standard deviations of the coordinates are all acceptable. There seems to be some rise in the standard deviations toward the western and southern parts of the network, probably because all direction control is in the northeastern part of the net. If ballistic camera data or other directional information were available from some of the stations on the western end, the whole network could be further strengthened.

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