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PROBABILITY CONCEPTS APPLIED TO SEQUENTIAL EQUIPMENT OPERATION

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16. Abstract

Probability concepts are applied to an operation in which a given number of items of equipment are sequentially activated. Only one item is operating at any time. When it fails, another item is activated. The probability that the system will fail prior to a specified time is discussed, and some examples are presented.

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PROBABILITY CONCEPTS APPLIED TO SEQUENTIAL EQUIPMENT OPERATION

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INTRODUCTION

The problem under consideration is the calculation of the probability of failure for a system consisting of a number of sequentially activated components. An item, a lamp for example, is activated. If it fails before a specified time interval has elapsed, another item is activated as a replacement. This procedure is continued with a preassigned number of items until the supply is exhausted or until the specified time interval has elapsed. Assuming that information concerning the performance of the individual items is available, what is the probability that the sequence of operations terminates before the specified time interval has expired?

EXAMPLES

Assume that three lamps are available for a particular operation. One lamp is turned on. If it burns out within a year, another lamp is turned on and similarly with the third lamp. What is the probability that all three lamps will fail before the end of the year? Let the failure probability density as a function of time for lamp i be

$$p_i = p_i(t) . \quad i = 1, 2, 3 .$$

Let X_i be a point in time such that

$$p_i = 0 , \quad \text{for } 0 \leqslant t_i < X_i .$$

In other words, the lamp is considered certain not to fail between the time it is turned on, $t = 0$, and the time X_i. In many applications, X_i will be zero, but we include the capability of providing for nonzero values.

The probability that the system, that is, the tandem operation of the three lamps, will fail before the year is completed is

$$P = \int_{X_3}^{1-X_1-X_2} \int_{X_2}^{1-X_1-t_3} \int_{X_1}^{1-t_2-t_3} p_1\,dt_1\,p_2\,dt_2\,p_3\,dt_3\,,$$

where we assume that $X_1 + X_2 + X_3 < 1$.

If the probability densities, p_i, $i = 1, 2, 3$, are constant, the result is

$$P = \frac{p_1 p_2 p_3 [1 - (X_1 + X_2 + X_3)]^3}{3!}\,.$$

As an illustration, suppose that each of the three lamps may be represented by Figure 1.

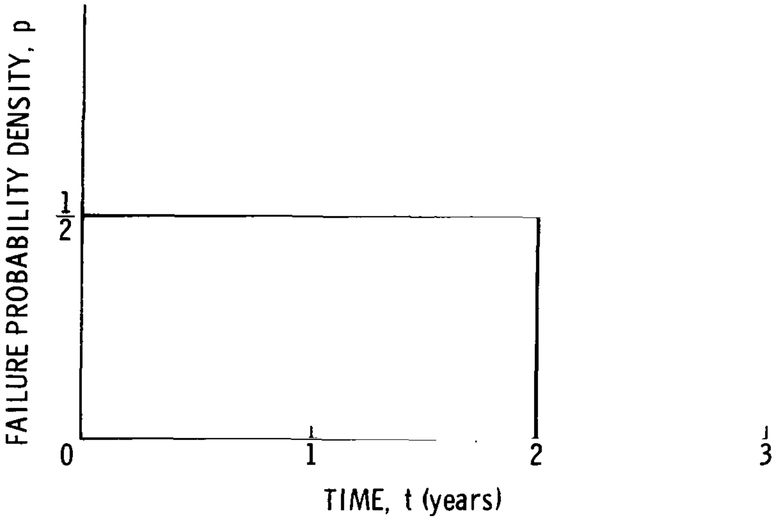

Figure 1—Failure probability density for the first example.

This means that each lamp may last from zero to 2 years and that the probability of failure is the same for all equal subintervals. Then

$$p_1 = p_2 = p_3 = 1/2$$

and

$$X_1 = X_2 = X_3 = 0\,.$$

The probability that the system will fail within a year is

$$P = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1^3}{3!} = \frac{1}{48}\,.$$

As another example, consider the probability density to be an exponential decay function. Assume for each lamp the failure performance indicated in Figure 2. Then

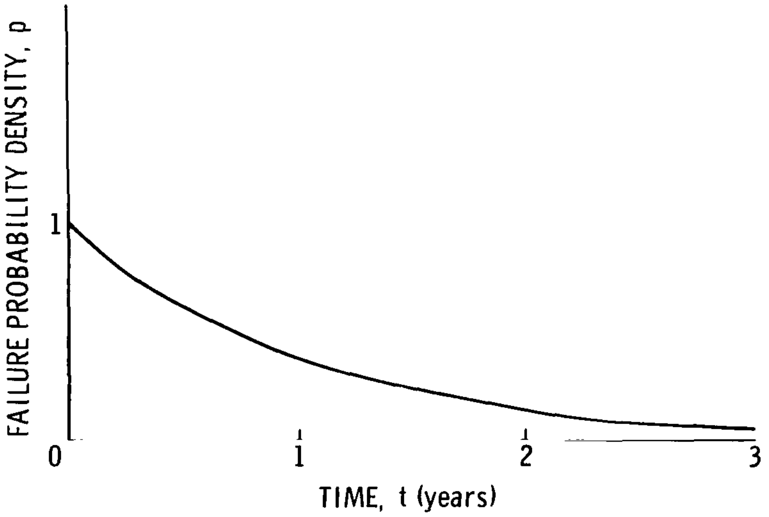

Figure 2—Failure probability density for the second example.

$$p_1 = p_2 = p_3 = e^{-t}$$

and

$$X_1 = X_2 = X_3 = 0\,.$$

The probability of system failure during the first year is

$$P = \int_0^1 \int_0^{1-t_3} \int_0^{1-t_3-t_2} e^{-t_1}\, dt_1\, e^{-t_2}\, dt_2\, e^{-t_3}\, dt_3$$

or

$$P = 1 - \frac{5}{2e} \approx 0.08\,.$$

A third example is provided by the situation where the probability that a lamp is still burning satisfactorily at time t, after being turned on at $t = 0$, is $e^{-\lambda t}$. Specifically, let

$$p_1 = p_2 = \lambda_1 e^{-\lambda_1 t}\,, \quad p_3 = \lambda_2 e^{-\lambda_2 t}\,, \quad \lambda_1 > 0\,, \quad \lambda_2 > 0\,, \quad \lambda_1 \neq \lambda_2\,,$$

where p_i, $i = 1, 2, 3$, are the failure probability density functions and t is assumed to be in years.

The probability that the system will fail during the first three years is

$$P = \int_0^3 \int_0^{3-t_3} \int_0^{3-t_3-t_2} \lambda_1 e^{-\lambda_1 t_1}\, dt_1\, \lambda_1 e^{-\lambda_1 t_2} dt_2\, \lambda_2 e^{-\lambda_2 t_3} dt_3$$

$$= 1 - e^{-3\lambda_2} + \frac{\lambda_2}{(\lambda_1 - \lambda_2)^2}\left[(\lambda_2 - 2\lambda_1)e^{-3\lambda_2} + (2\lambda_1 - \lambda_2 - 3\lambda_1\lambda_2 + 3\lambda_1^2)e^{-3\lambda_1}\right].$$

GENERAL CASE

In order to generalize the development, let n represent the number of items and T the specified time. The probability of system failure prior to time T is

$$P = \int_{X_n}^{T-X_1-X_2-\cdots-X_{n-1}} \int_{X_{n-1}}^{T-X_1-X_2-\cdots-X_{n-2}-t_n}$$

$$\cdots \int_{X_2}^{T-X_1-t_3-t_4-\cdots-t_n} \int_{X_1}^{T-t_2-t_3-\cdots-t_n} p_1\,dt_1\,p_2\,dt_2 \cdots p_n\,dt_n\,,$$

where

$$\sum_{i=1}^{n} X_i < T\,.$$

For $p_1 = p_2 = \ldots p_n = \lambda e^{-\lambda t}$, the probability of failure prior to time T is

$$P = e^{-\lambda X} - e^{-\lambda T}\left[\sum_{k=0}^{n-1} \frac{\lambda^k (T-X)^k}{k!}\right],$$

where

$$X = \sum_{i=1}^{n} X_i\,.$$

If each p_i, for $i = 1, 2, \ldots n$, is constant throughout a continuous time interval whose duration is at least $T - X$ and is zero elsewhere, the probability of failure prior to time T is

$$P = p_1 p_2 \ldots p_n \left[\frac{(T-X)^n}{n!}\right].$$

In spacecraft operations, where equipment failure is often irreparable, calculations such as these may be useful in determinations of the number of components to be included and in estimations of operating lifetime expectancies.

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Mr. E. L. Davis of the Manned Spacecraft Center, Houston, Texas, provided the sources listed in the bibliography. They contain material related to the subject of this paper.

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