THE TRANSMISSION OF SONIC BOOM SIGNALS INTO ROOMS THROUGH OPEN WINDOWS

PART 1: THE STEADY STATE SOLUTION

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SUMMARY

As a first step in calculating transient pressure time-histories in rooms, due to sonic booms, a solution is sought for the pressure field generated inside a room due to an incoming harmonic wave, incident onto a window.

The basic problems of sound radiation and diffraction, related to this problem, are first discussed. These are made use of to obtain a solution in the case of a room with hard walls and normal incidence, first by viewing the room as a terminated duct and later by the Green's function method.

The concept of the mode excitation distribution function is introduced and is used to match the boundary conditions. This concept has been extended for oblique incidence. Some general properties of the distribution function have been derived.

Extension of these results to transient pressure fields is being presented in a subsequent report (Part 2).

INTRODUCTION

The interest in the transmission of sonic booms through open windows inside rooms is mainly from the subjective point of view. In a general sense, by the term open window is meant any opening left in the wall facing the boom. Such openings, even if they are small, can transmit an appreciable amount of sound. The characteristics of the sound field inside the room depend on the geometry of the window, the location of the window, dimensions of the room and the characteristics of the boom.

The main task of this and the next report is to describe the methods of computing the characteristics of the pressure field inside the rooms once the characteristics of the boom are known. There are several methods available. Usually the boom is described in terms of its pressure time-history. One method of analysis is to obtain, firstly, the spectrum of the sonic boom in the frequency domain. This can readily be done analytically (1) if the time history is given by an analytic expression (e.g., an 'N' or modified 'N' wave with prescribed durations, rise times, etc.), or this can be done experimentally by analysing the signals by a spectral analyser. Then the steady state response of each frequency component is computed to arrive at the frequency spectrum of the response inside the room.

However, it appears that for the subjective judgement of the loudness of the boom, it is the earlier part of the signal (first 70 milliseconds, say) which is most crucial (2). Lochner and Burger (3) have devised criteria based on such assumptions to predict the relative loudness of various transient sound signals once their time domain description is known. Therefore there is a considerable interest in obtaining the final response in the time domain.

The procedure that in practice leads to fairly accurate results consists of obtaining the steady state response of the system in the frequency domain and then obtaining the Laplace transform of the solution in the time domain. This part of the report mainly describes the first part of this method: i.e., the steady state solution.

Another practical method is to work throughout in the time domain. This is achieved by considering the transient diffraction pattern generated due to the passage of the boom through a single aperture first and then to sum up the contributions due to various apertures situated at the various image positions of the window, created due to the presence of walls. (4, see p.344, 5) This method will be briefly described in the third report.

SYMBOLS

a	width of the room
A	amplitude of the incident wave
b	height of the room
c	speed of sound
d	depth of the room
e	base of the natural logarithm
E	defined by equation (3.3.7)
F	force

g	Green's function inside the room
G	Green's function for a half infinite space
$G_{m,n}$	defined by equation (2.4.13)
i	square root of minus one
I	defined by equation (1.2.8)
k	wave number $(= \omega/c)$
k^*	cut-off wave number, see equation (2.3.4)
m	modal parameter
M	mobility of an orifice
n	modal parameter
p	acoustic pressure phasor
q	see equation (2.2.1)
Q'	see equation (2.2.3)
$Q_{m,n}$	parameter for the $(m, n)^{th}$ mode, see equation (2.3.7)
r	radial distance
R_{rad}	radiational resistance
S	cross sectional area of an orifice
S'	cross sectional area of a duct
t	time
T	kinetic energy
U	uniform velocity
v	velocity in general
V	velocity in general
$\left.\begin{matrix} x \\ y \\ z \end{matrix}\right\}$	coordinates

$\delta_{i,j}$	Kronecker delta, has a value of one if $i = j$ and of zero if $i \neq j$
ε_j	a parameter which is equal to two if $j = 0$ and is equal to $\sqrt{\text{two}}$ otherwise
ϕ	velocity potential (velocity being the positive gradient of velocity potential)
λ_1	ratio of the window width to the room width
λ_2	ratio of the window height to the room height
ρ	density
ρ_o	mean density
ψ	normalised eigenfunction
ω	circular frequency
Ω	generalised frequency (see (2.2.10))
τ	characteristic impedance

1.1 RADIATION IMPEDANCE OF ORIFICES IN PLANE INFINITE SCREENS

One of the most important characteristics of an orifice is its radiation impedance. For 'plane waves', it is defined as the force F required to move air in the plane of the opening as a rigid piston, with normal velocity V:

$$Z_{rad} = F/V. \tag{1.1.1}$$

In general the impedance is a complex quantity, and it is customary to express it as

$$Z_{rad} = R_{rad} - i\omega m' ; \tag{1.1.2}$$

where the real part R_{rad} is termed as the radiational resistance and the term m' is called, by an analogy with a simple oscillator problem, the effective mass of the fluid outside the opening. This mass can thus be regarded to be attached to the piston.

For a piston in an infinite rigid baffle with a velocity at a point on the piston equal to V(x', y', 0) the velocity potential ϕ at a point in space is given by (see, for example, reference 18, p. 807)

$$\phi(x,y,z) = \frac{1}{4\pi} \int\int_S V(x', y', 0)\, G(x, y, z/x', y', 0)dx'dy', \quad (1.1.3)$$

where the Green's function for the half space is given by

$$G(x,y,z/x',z',0) = 2\,\frac{e^{ik\sqrt{(x-x')^2 + (y-y')^2 + z^2}}}{\sqrt{(x-x')^2 + (y-y')^2 + z^2}}\,. \quad (1.1.4)$$

When the piston velocity is uniform, i.e. $V(x',y',0) = U$, a constant, the velocity potential is given by

$$\phi(x,y,z) = \frac{-U}{4\pi} \int_S\int G(x,y,z/x',y',0)dx'dy' \quad (1.1.5)$$

and the pressure is given by $-\rho_o \frac{\partial\phi}{\partial t}$: i.e.,

$$p(x,y,z) = \frac{-i\omega\rho_o U}{4\pi} \int\int_S G(x,y,z/x',y',0)dx'dy'. \quad (1.1.6)$$

The pressure at $z = 0$ (i.e., along the plane of the aperture) can be written as

$$p(x,y,0) = \frac{-i\omega\rho_o U}{4\pi}\, I(x, y), \quad (1.1.7)$$

if we define

$$I(x, y) \equiv \int\int_S G(x,y,0/x',y',0)dx'dy'. \quad (1.1.8)$$

The total force F on the piston moving with a uniform velocity U over the orifice is thus

$$F = \frac{-i\omega\rho_o U}{4\pi} \int\int_S I(x, y)dxdy. \quad (1.1.9)$$

At low frequencies the Green's function $G(x,y,0/x',y',0)$ is approximated by the first two terms of its expansion in $k\sqrt{(x-x')^2+(y-y')^2}$:

$$G(x,y,0/x',y',0) = \frac{2}{\sqrt{(x-x')^2 + (y-y')^2}} + 2ik. \quad (1.1.10)$$

A substitution in (1.2.8) and then in (1.2.9) followed by a comparison with (1.2.2) leads to the following results

$$R_{rad} = \frac{\rho_o \omega^2 S^2}{2\pi c} \tag{1.1.11}$$

and

$$m' = \frac{\rho_o}{4\pi} \iint_S \mathrm{Re}\,(I(x,\ y))\ dxdy. \tag{1.1.12}$$

Further the kinetic energy is given by

$$T_1 = \tfrac{1}{2} m' U^2. \tag{1.1.13}$$

1.2 THE TRANSMISSION OF SOUND THROUGH APERTURES IN PLANE INFINITE SCREENS

Attempts at mathematical solutions of this diffraction problem go back to the days of Rayleigh (6). Only comparatively recently (7, 8, 9) results have been obtained which are valid over a large frequency range. Experimental investigations have been reported for circular and rectangular plates (10, 11, 12).

Following the argument by Rayleigh an incident wave ϕ is considered to impinge upon a plane rigid infinite screen containing an orifice of cross sectional area S, as shown in Figure 1. If the screen had continued across the orifice a block reflected wave ϕ_2 would have been obtained.

Thus if ϕ_1 be given by

$$\phi_1 = Ae^{ikz}e^{-i\omega t}, \tag{1.2.1}$$

then

$$\phi_2 = Ae^{-ikz}e^{-i\omega t}. \tag{1.2.2}$$

However, the passage of sound through the orifice gives rise to a velocity distribution $V(x',y',0)$ across the surface S, which in turn gives rise to two further, partial velocity potentials: ϕ_3 for $z < 0$ and ϕ_4 for $z > 0$:

$$\phi_3(x,y,z) = \frac{-1}{4\pi} \iint_S V(x',y',0)\ G(x,y,z/x',y',0)dx'dy', \quad z < 0 \tag{1.2.3}$$

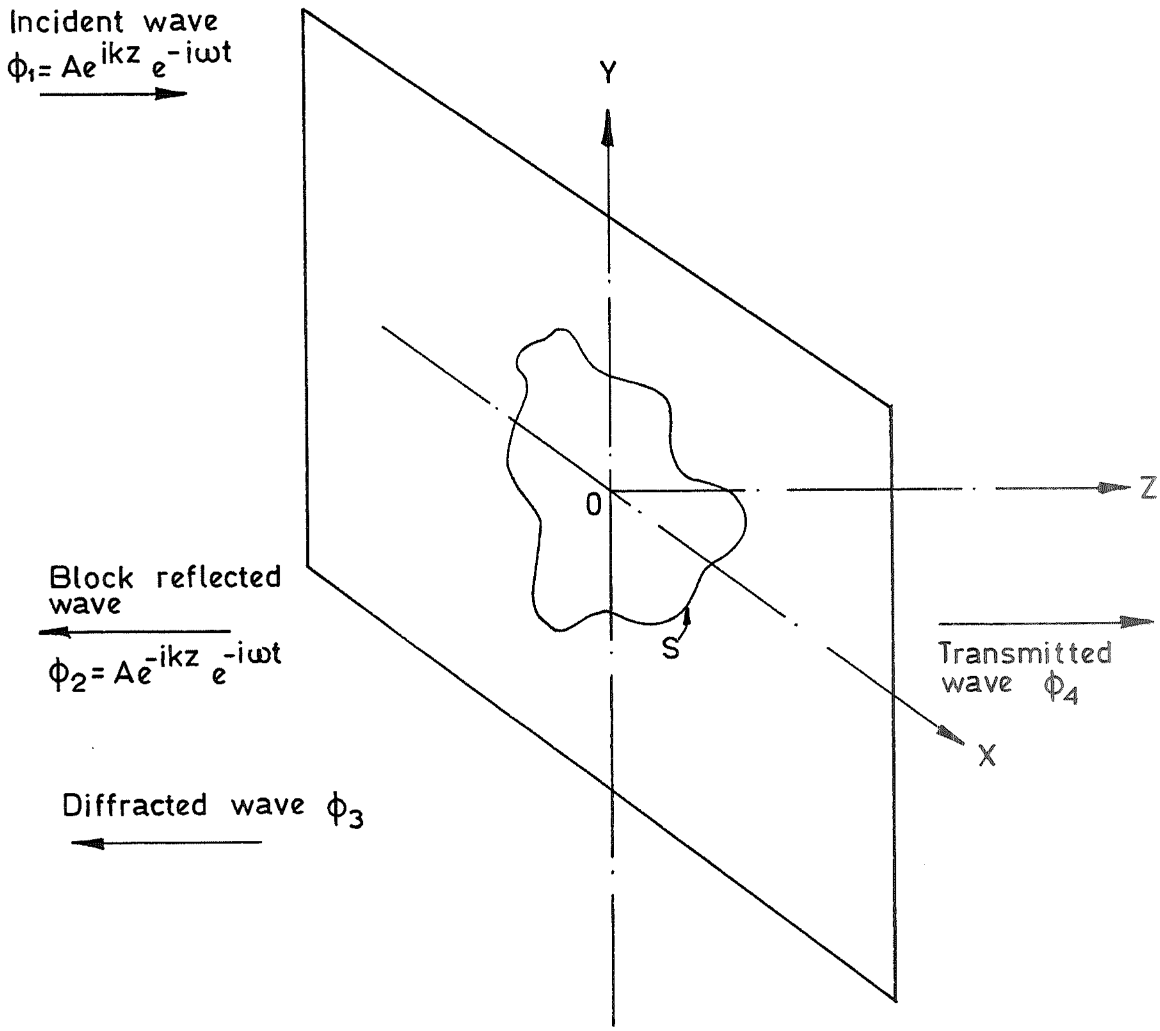

FIGURE 1

The transmission of sound through apertures in infinite screens; the figure shows how a harmonic wave, incident normally onto the screen gives rise to various wave systems.

and

$$\phi_4(x,y,z) = \frac{+1}{4\pi} \int\int_S V(x',y',0)\; G(x,y,z/x',y',0)dx'dy', \qquad z > 0, \tag{1.2.4}$$

the outward normal convention being followed.

The continuity of velocity potential condition across the surface S gives

$$2A + \phi_3(x,y,0) = \phi_4(x,y,0), \quad \text{for} \quad (x, y) \quad \text{in} \quad S:$$

i.e.,

$$2A = 2\phi_4(x, y, 0)$$

and therefore

$$\phi_4(x, y, 0) = -\phi_3(x, y, 0) = A. \tag{1.2.5}$$

This leads to the well known result that, for extremely thin orifices, the velocity potential across the orifice is the same as that which would have been there due to the incoming wave in the absence of the screen.

A similar assumption, which is quite satisfactory in optics, namely that the normal velocity at the orifice is the same as the incoming wave velocity, leads to serious errors (6, p.140).

The normal velocity at the aperture, in fact, rises very steeply near the edges. For an inviscid fluid and infinitesimal thickness of the screen it goes to infinity at the edges. In practice, however, the viscous losses at the edge and finite thickness of the screen make this rise considerably smaller.

The velocity is appreciably uniform near the centre of the aperture and the sharp rise is restricted to such a small area that the extra contribution to the integral (1.3.4), for example, is comparatively small. Rschevkin (11, see p.220) has estimated that an error due to assuming that the velocity is uniform (yet distinct from free field) is about 7.5%.

Once a uniform velocity assumption is made results of the previous section become applicable. Ingerslev and Nielsen (12, p.7) have shown how this method can be used with some modifications to arrive at fairly good approximations.

The parameter of most interest here, as will be explained in the next chapter, is the acoustic mobility, M, (also known as the conductivity of an orifice. Strictly the concept of mobility is applicable in the case of an incompressible flow. However, for low frequency waves, fairly good results can be obtained by using the concept. Van Bladel (13) has shown the method to extend the static results to higher frequencies.

When a fluid passes through an aperture, its streamlines are constrained to pass through a restricted region. This leads to an increase in the kinetic energy of the fluid. Let us denote this kinetic energy by T_2. Now the mobility of the flux is defined by the following equation:

$$T_2 = \tfrac{1}{2}\rho_o \frac{D^2}{M} \tag{1.2.6}$$

where D is the total instantaneous volume flow through the orifice.

If the volume flow were uniformly redistributed over the cross sectional area, S, and if the kinetic energy were found, we would have obtained a value which, in accordance with Rayleigh's principle, would have been greater in magnitude than T_2. However, it would not greatly differ from T_2 due to its stationary nature. A comparison with (1.1.13) and (1.1.12) leads to the following approximate relations:

$$M = \frac{\rho_o S^2}{m'} \tag{1.2.7}$$

and

$$\iint_S \mathrm{Re}\ I(x,y)dxdy = \frac{4\pi S^2}{M}, \tag{1.2.8}$$

and from (1.2.7) and (1.1.10)

$$\iint_S I(x,y)dxdy = 4\pi S^2 \left[\frac{1}{M} + \frac{ik}{2\pi}\right]. \tag{1.2.9}$$

Relations (1.1.2), (1.1.11) and (1.2.7) lead to the following expression for Z_{rad}:

$$Z_{rad} = S^2 \left[\frac{\rho_o \omega^2}{2\pi c} - \frac{i\omega\rho_o}{M}\right]. \tag{1.2.10}$$

These results are used in the next chapter. It has been shown by Rayleigh (6, see p.178) that the mobility of a circular orifice is equal to its diameter. He also showed that the mobility of an ellipse was given by

$$M = 2\sqrt{\frac{S}{\pi}}\left(1 + \frac{e^4}{64} + \frac{e^6}{64} + \text{higher terms}\right), \tag{1.2.11}$$

where S is the cross sectional area and e is the eccentricity.

For orifices of not too elongated shapes, an approximate value of the mobility is given by that of a circle with the same area as that of the orifice: i.e., $M = 2\sqrt{S/\pi}$. For a rectangle, it has been suggested in reference (12) that an ellipse, with the same area as that of the rectangle and same ratio of major and minor axes as the ratio of the sides of the rectangle would give an even more accurate result.

The relationship between the imaginary part of the radiation impedance and the mobility has been used by Morfey (14) to provide formulae for various shapes.

When the screen has an appreciable thickness the expressions for mobility and for the sound transmission in general, must be modified. These modifications have been dealt with by Nielsen (15) and Gomperts (16).

Transmission of sound through an aperture at an oblique incidence has been discussed by Miles (17). A common feature to be noted is that the dependence of the transmitted intensity, etc., on the cosine of the angle from the normal is only in coefficients of factors of order $(ka)^2$ or higher. It thus appears that at low frequencies we can use the same values for mobility for sound fields incident at an angle not too oblique, as in the case of normal incidence. This result is again made use of in the next chapter.

2.1 A SIMPLIFIED MODEL

In this chapter a steady state solution, of a simplified model of the room, is considered. The model is sketched in Figure 2a.

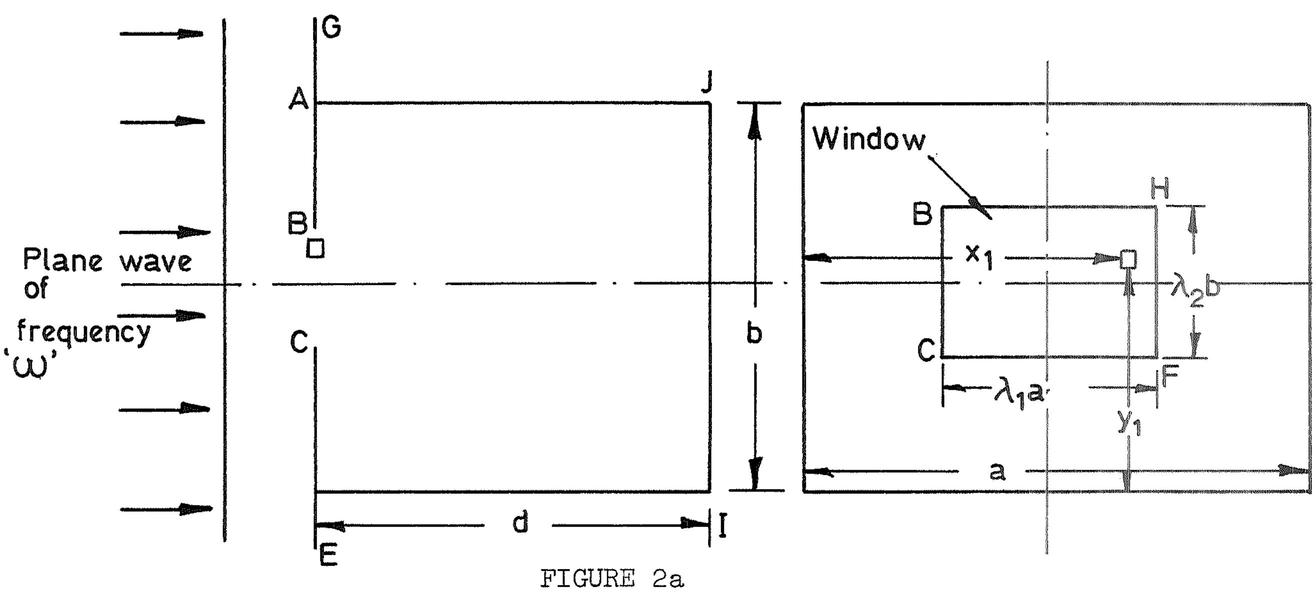

FIGURE 2a
A sectional end view of the room and the front view is shown. The plane harmonic wave is normally incident onto the face GE.

The outer wall, FE, extends indefinitely on both the sides and contains an open, centrally situated, rectangular window. A plane wave, of radian frequency ω is normally incident on it. An expression for the sound field generated inside the room is to be obtained.

This problem leads to several analytical problems of interest. It is worth pursuing some of them, in order to retain generality, until a clear physical picture of the system is obtained. Thereafter any simplifying assumptions of interest, for example that ω is very small (or to be specific that the corresponding wavelength is many times greater than the room dimensions a, b and d and/or many times greater than the window dimension $\lambda_1 a$ and $\lambda_2 b$), can be made. In particular, the assumption that the window perimeter is relatively small compared with representative wavelengths is especially relevant, since the predominant frequencies encountered in the sonic boom spectrum are, in general, low in this sense.

The problem is that of seeking a solution to the wave equation for the given boundary conditions. These conditions would be some sort of impedance conditions on all the walls and continuity conditions across the window.

A brief account was given in the last chapter of the result that in the case of a thin infinite screen (in the absence of a backing cavity, etc.) the potential at the window due to an incoming, normally incident, wave is the same as the incoming potential. This result is no longer valid in the particular problem of interest here because the room is capable of reradiating back into the open, and the assumption of symmetry across GE is not valid.

Looking from another point of view the problem is analogous to that of a Helmholtz resonator which is externally excited. This analogy shows that there would be a significant frequency dependence of the system. A behaviour similar to the cavity resonance may well be expected at the right sort of frequencies. This simplified model is modified by the fact that higher order modes will be set up in the room to a varying degree and the usual formulae for obtaining the lumped circuit elements of the resonator will not be satisfactory, except at very low frequencies.

In what follows it has been tacitly assumed that the magnitude of the velocity across the window, as shown in Figure 2, is already known. A simplified calculation based on this assumption is first made. The room is assumed to be closed and the window is replaced by a piston possessing the same velocity as the actual velocity in the initial problem.

In section (2.2), the field generated inside a room due to a rigid piston, moving with a uniform velocity, is computed. Rigourously, this piston should have been a flexible piston. The method used here can be extended to a flexible piston problem. However, the rigid piston approach itself is sufficient to bring out clearly the relevant parameters of the system.

The procedure followed in (2.2) has one advantage. The final expression obtained takes an account of the absorption in the room. However, the initially assumed velocity is left indeterminate. Even if this velocity were to be known it is difficult in practice to compute accurately the time domain behaviour from the expressions such as (2.2.12). Therefore a different method is followed in (2.3) which regards the room as a terminated duct.

2.2 PRESSURE FIELD DUE TO A PISTON, SET IN A WALL, IN A ROOM

The room shown in Figure 2 is considered. However, the window is replaced by a uniform rigid piston executing a simple harmonic motion with velocity given by $Ue^{-i\omega t}$. Let us choose the origin at one of the corners. Also let the piston cover the area enclosed by $\frac{a}{2}(1 - \lambda_1) \leqslant x \leqslant \frac{a}{2}(1 + \lambda_1)$ and $\frac{b}{2}(1 - \lambda_2) \leqslant y \leqslant \frac{b}{2}(1 + \lambda_2)$. Consider now a small elemental area, on the piston, $dx'dy'$, centered at (x',y'). This area acts as a source whose strength $q(x,y,z)e^{-i\omega t}$ is given by $q(x,y,z) = Sp\, U\delta(x-x')\delta(y-y')\delta(z)$, where

$$Sp = dx'dy'. \tag{2.2.1}$$

The acoustic pressure $p(x,y,z)e^{-i\omega t}$ satisfies the following equation (4, see p.313):

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right] p(x,y,z)e^{-i\omega t} = -\rho_o \frac{\partial}{\partial t}\left[q(x,y,z)e^{-i\omega t}\right]. \tag{2.2.2}$$

In general both p and q can be expressed as series expansions in the normal functions $\psi_{\ell mn}(x,y,z)$:

$$q(x,y,z) = \sum_{\ell,m,n = 0} Q'_{\ell mn}\psi_{\ell mn}(x,y,z). \tag{2.2.3}$$

The functions $\psi_{\ell mn}$ are normalised orthogonal functions so that

$$Q'_{\ell mn} = \iiint_V q(x,y,z)\psi_{\ell mn}(x,y,z)dxdydz.$$

For the present case

$$Q'_{\ell mn} = USp\ \psi_{\ell mn}(x',y',0). \qquad (2.2.4)$$

Also one can express the pressure as

$$p(x,y,z) = \sum_{\ell,m,n,=0}^{\infty} a_{\ell mn}\psi_{\ell mn}(x,y,z). \qquad (2.2.5)$$

The $\psi_{\ell mn}$'s, in fact, satisfy the homogeneous equation. It follows that

$$c^2\nabla^2\psi_{\ell mn}(x,y,z) = -\Omega^2_{\ell mn}\psi_{\ell mn}(x,y,z). \qquad (2.2.6)$$

Substituting this equation in (2.2.2) leads to

$$a_{\ell mn} = -i\rho c^2\omega\ Q_{\ell mn}\ \left[\omega^2 - \Omega^2_{\ell mn}\right], \qquad (2.2.7)$$

and therefore

$$dp(x,y,z) = -i\rho c^2\omega\ dx'dy'\ Ue^{-i\omega t}\sum_{\ell mn}\frac{\psi_{\ell mn}(x',y',0)\psi_{\ell mn}(x,y,z)}{\omega^2 - \Omega^2_{\ell mn}} \qquad (2.2.8)$$

When the walls are perfectly rigid, it can be shown that

$$\psi_{\ell mn} = \left[\frac{\varepsilon_\ell\varepsilon_m\varepsilon_n}{\sqrt{abd}}\right]\cos\frac{\ell\pi x}{a}\ \cos\frac{m\pi y}{b}\ \cos\frac{n\pi z}{d},$$

and

$$\Omega_{\ell mn} = c\left[\left(\frac{\ell\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2 + \left(\frac{n\pi}{d}\right)^2\right],$$

where

$$\varepsilon_j = 2 \text{ if } j = 0 \quad\text{and}\quad \varepsilon_j = \sqrt{2} \text{ if } j = 1, 2, \ldots \qquad (2.2.9)$$

If, however, the walls are not absolutely rigid and yet not too absorbent it can be shown (reference 3) that the $\Omega_{\ell mn}$'s above become complex. Thus

$$\Omega_{\ell mn} = \omega_{\ell mn} - i\mu_{\ell mn}$$

$$\omega_{\ell mn} \simeq c \left[(\frac{\ell\pi}{a})^2 + (\frac{m\pi}{b})^2 + (\frac{n\pi}{d})^2\right],$$

$$\mu_{\ell mn} \simeq \frac{c}{8abd}\left[\frac{\varepsilon_\ell a_x}{2} + \frac{\varepsilon_m d_y}{2} + \frac{\varepsilon_n a_z}{2}\right], \qquad (2.2.10)$$

where a_x represents the total absorption of the x-walls, etc.

Thus neglecting $\mu^2_{\ell mn}$ on the assumption of small damping,

$$dp(x,',y',z') = \frac{\rho c^2\omega}{abd}\, dx'dy'\, Ue^{-i\omega t}$$

$$\times \sum_{\ell,m,n=0}^{\infty} \frac{\cos\frac{\ell\pi x'}{a}\cos\frac{m\pi x'}{a}\cos\frac{n\pi y}{b}\cos\frac{n\pi z}{d}}{\left[2\omega_{\ell mn}\mu_{\ell mn} - i(\omega^2 - \omega^2_{\ell mn})\right]}\,\varepsilon^2_\ell\varepsilon^2_m\varepsilon^2_n \qquad (2.2.11)$$

The desired result is obtained when this expression is integrated throughout the area of the piston:

$$P(x,y,z) = \int_{\frac{a}{2}(1-\lambda_1)}^{\frac{a}{2}(1+\lambda_1)}\int_{\frac{b}{2}(1-\lambda_2)}^{\frac{b}{2}(1+\lambda_2)} dp(x,',y',z')$$

$$= \frac{\rho c^2\omega}{d}\, Ue^{-i\omega t}\sum_{\ell,m,n=0}^{\infty}\frac{\cos\frac{\ell\pi x}{a}\cos\frac{m\pi y}{b}\cos\frac{n\pi z}{d}}{2\omega_{\ell mn}\mu_{\ell mn} - i(\omega^2 - \omega^2_{\ell mn})}\,\varepsilon^2_n\iint_S\frac{\varepsilon^2_m\varepsilon^2_\ell}{ab}\cos\frac{m\pi x'}{a}\cos\frac{n\pi y'}{b}dx'dy'$$

$$= \frac{\rho c^2\omega}{d}\, Ue^{-i\omega t}\sum_{\ell,m,n=0}^{\infty}\frac{\varepsilon^2\sigma_{\ell,m}\cos\frac{\ell\pi x}{a}\cos\frac{m\pi y}{b}\cos\frac{n\pi z}{d}}{\left[2\omega_{\ell mn}\mu_{\ell mn} - i(\omega^2 - \omega^2_{\ell mn})\right]} \qquad (2.2.12)$$

where

$$\sigma_{\ell,m} = \varepsilon_\ell^2 \varepsilon_m^2 (-1)^{m/2}(-1)^{n/2} \lambda_1 \lambda_2 \frac{\sin\theta_m}{\theta_m}\frac{\sin\theta_n}{\theta_n} \qquad (m,n \quad \text{even})$$

$$= 0 \quad (m \text{ or } n \quad \text{odd}) \tag{2.2.13}$$

and

$$\theta_m = \frac{m\pi\lambda_1}{2}, \qquad \theta_n = \frac{n\pi\lambda_2}{2}. \tag{2.2.14}$$

Each term in the series (2.2.12) shows a resonance behaviour when $\omega = \omega_{\ell mn}$ and thus peak amplitudes are reached for that particular mode, subject, of course, to the spatial variation of the numerators with (x,y,z).

2.3 TERMINATED DUCT APPROACH

For the original problem stated in section 2.1, assume that the velocity potential of the incoming wave is given by the equation

$$\phi_1 = Ae^{ikz}e^{-i\omega t} \tag{2.3.1}$$

where k is the wave number $(= \omega/c)$. The solution is obtained by a combination of the solutions of the two problems described below:

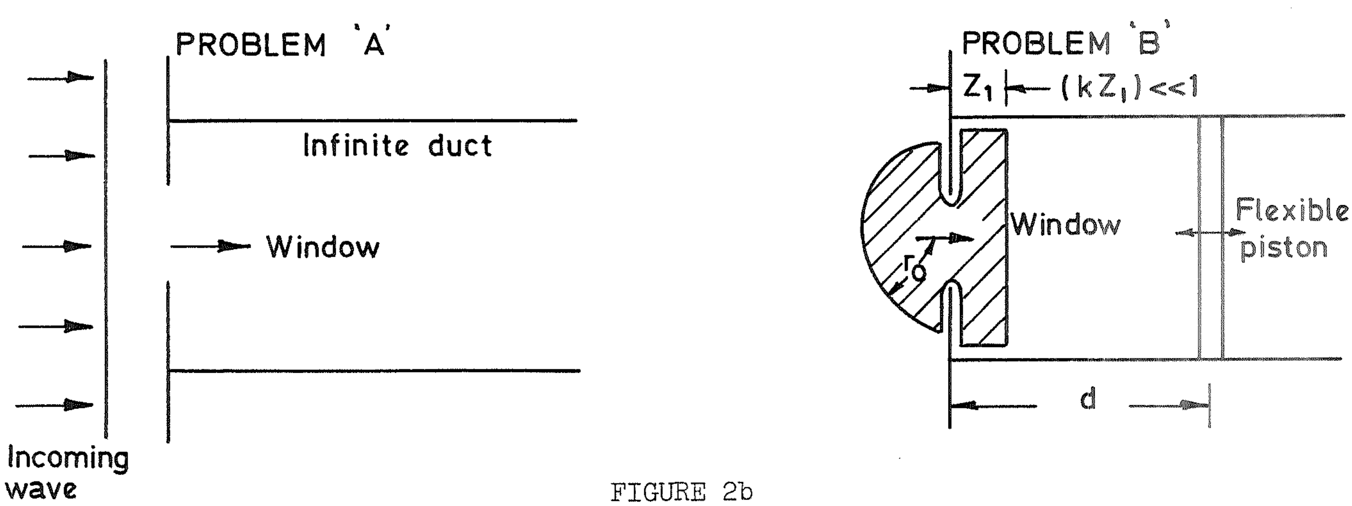

FIGURE 2b

The figure shows two simplified problems whose individual solutions can be combined to obtain the solution of the problem presented in Figure 2a.

Problem A is that of an infinite duct with a flanged opening on which is incident a plane wave, and problem B is that of a duct excited by a flexible piston, giving rise to waves which pass down the duct on one side and pass through the window out into the open on the other side. The solution of these problems in turn is dependent on two classical problems: (i) radiation from a source of sound in a duct; (ii) diffraction of sound through an aperture in a screen.

The analytical task is considerably reduced if the long wavelength assumption be made. This assumption may give rise to considerable errors in the region very near the edges of the window. However, far from these regions, the analysis is expected to be reasonably accurate. Also, if desired, this assumption may be removed at a later stage, and results recalculated for higher frequencies, along the lines indicated in section 2.4.

If the outer wall had continued in the region occupied by the window, a block reflected potential, ϕ_2, would be generated:

$$\phi_2 = Ae^{-ikz}e^{-i\omega t}.$$

This would give rise to a potential of amplitude 2A across the surface of the baffle facing the wave. It is then argued that, as far as the duct is concerned, the effect of the external perturbation can be represented by a potential perturbation, of amplitude 2A, applied at the window. This perturbation would give rise to a system of acoustic waves represented by

$$\phi_3 = 2A \sum_{m,n} \sigma_{mn} \cos \frac{m\pi x}{a} . \cos \frac{n\pi y}{b} \cos k^*_{m,n} z e^{-i\omega t}, \qquad (2.3.3)$$

$$k^*_{m,n} = \sqrt{k^2 - (\frac{m\pi}{a})^2 - (\frac{n\pi}{b})^2} . \qquad (2.3.4)$$

It has been assumed that the walls are rigid and the origin is situated at the corner. The normalizing parameters, $\sigma_{m,n}$, satisfy the following condition:

$$\sum_{m,n} \sigma_{m,n} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} = 1, \quad (\text{if } \frac{a}{2}(1 - \lambda_1) < x < \frac{a}{2}(1 + \lambda_1) \text{ and } \frac{b}{2}(1 - \lambda_2) < y < \frac{b}{2}(1 + \lambda_2)),$$

and

$$= 0 \quad (\text{if otherwise}). \qquad (2.3.5)$$

Thus at $z = 0$, ϕ_3 adds up to 2A over the window and to zero elsewhere. This condition is used to evaluate $\sigma_{m,n}$. Multiplying both sides of (2.3.5) by $\cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b}$ and integrating, leads to

$$\int_0^b \int_0^a \sigma_{m,n} \cos^2 \frac{m\pi x}{a} \cos^2 \frac{n\pi y}{b} \, dxdy = \int_{\frac{a}{2}(1-\lambda_1)}^{\frac{a}{2}(1+\lambda_1)} \int_{\frac{b}{2}(1-\lambda_2)}^{\frac{b}{2}(1+\lambda_2)} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \, dxdy, \tag{2.3.6a}$$

and therefore

$$\sigma_{m,n} = (2 - \delta_{o,m})(2 - \delta_{o,n})(-1)^{m/2}(-1)^{b/2} \lambda_1 \lambda_2 \frac{\sin \theta_m}{\theta_m} \frac{\sin \theta_n}{\theta_n}, \quad \text{(m,n both even)}$$

$$= 0 \text{ if either } m \text{ or } n \text{ is odd,} \tag{2.3.6b}$$

where

$$\theta_m = \frac{m\pi\lambda_1}{2}, \qquad \theta_n = \frac{n\pi\lambda_2}{2},$$

and

$$\delta_{o,m} = 1 \text{ if } m = 0,$$
$$= 0 \text{ if } m \neq 0, \text{ etc.}$$

In the main problem, equation (2.3.3) by itself will suffice as a solution for the wave equation inside the room if it so happens that $\partial\phi_3/\partial z$ equals zero. In general this is not the case and hence a balancing solution ϕ_4, which represents waves radiated by a flexible piston, is required. These waves in turn create an additional potential ϕ_5 outside the window: i.e., for $z < 0$. ϕ_4 once again satisfies the boundary conditions at the side walls. Thus it can be expressed as

$$\phi_4 = \sum_{m,n} R_{m,n} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \left[\sin k^*_{m,n} z + Q_{m,n} \cos k^*_{m,n} z\right]. \tag{2.3.7}$$

$Q_{m,n}$ in general is complex and has a value which takes into account the attached mass and dissipation at the window. Its value can be estimated reasonably well by making use of the formulae for radiation impedance derived in the last chapter.

From (1.3.9) the radiation impedance at the window is

$$Z_{rad} = S^2 \left[\frac{\rho_o \omega^2}{2\pi c} - \frac{i\rho_o \omega}{M} \right] .$$

The radiation impedance as viewed by the duct with a cross sectional area S_1 (= ab) is $Z_{rad}(S_1/S_2)^2$ because of the 'transformer' effect of the area (11, p.220). Briefly, this effect is due to the fact that while going from the window to the duct the velocity, following the continuity equation, reduces in magnitude, while the pressure remains relatively unchanged, consequently increasing the force. The specific acoustic impedance (p/v) at the entrance of the duct is given by $(1/S_1)$ times the value of the radiation impedance: i.e.,

$$\zeta_{z\to 0} = S_1 \left[\frac{\rho_o \omega^2}{2\pi c} - \frac{i\rho_o \omega}{M} \right] . \tag{2.3.8}$$

The specific impedance can also be computed from (2.3.8).

In the $(m, n)^{th}$ mode the pressure amplitude along a plane very near the entrance of the duct is given by

$$p_{m,n} = i\omega\rho_o [\phi]_{z\to 0}$$

$$= i\omega\rho_o R_{m,n} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} [Q_{m,n}] ; \tag{2.3.9}$$

and the velocity by

$$v_{m,n} = \left[\frac{\partial\phi}{\partial z}\right]_{z\to 0}$$

$$= k^*_{m,n} R_{m,n} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} . \tag{2.3.10}$$

Equating the two expressions for the specific modal impedance gives rise to

$$S_1 \left[\frac{\rho_o \omega^2}{2\pi c} - \frac{i\rho_o \omega}{M} \right] = \frac{i\omega\rho_o Q_{m,n}}{k^*_{m,n}}$$

and therefore

$$Q_{m,n} = -k^*_{m,n}\ ab \left[\frac{1}{M} + \frac{ik}{2}\right] . \tag{2.3.11}$$

A more accurate estimate of $Q_{m,n}$ can be made, and this is done in the next section.

The next task is to determine $R_{m,n}$. Since the wall at $z = d$ is rigid $\left[\frac{\partial\phi_3}{\partial z} + \frac{\partial\phi_4}{\partial z}\right] = 0$ at all x, y. Hence, from (2.3.3) and (2.3.7),

$$2A\ \sigma_{m,n} \sin k^*_{m,n}\ d = R_{m,n}\ (\cos k^*_{m,n}\ d - Q_{m,n} \sin k^*_{m,n}\ d).$$

Therefore

$$R_{m,n} = \frac{2A\ \sigma_{m,n} \sin k^*_{m,n} d}{(\cos k^*_{m,n} d - Q_{m,n} \sin k^*_{m,n} d)} \tag{2.3.12}$$

and hence

$$\phi = \phi_3 + \phi_4,$$

$$= 2A \sum_{m,n} \sigma_{m,n} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \frac{\cos k^*_{m,n}(d - z)}{(\cos k^*_{m,n} d - Q_{m,n} \sin k^*_{m,n} d)}.$$

Replacing the time factor, we see that an incoming wave, $Ae^{-i\omega t}$, gives rise to a velocity potential inside the room of

$$Qe^{-i\omega t} = 2A \sum \sigma_{m,n} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \frac{\cos k^*_{m,n}(d - z)}{\cos k^*_{m,n} d - Q_{m,n} \sin k^*_{m,n} d}. \tag{2.3.13}$$

The incoming pressure is

$$p_1 = i\omega\rho_o Ae^{-i\omega t},$$

and the velocity is

$$v_i = ikAe^{-i\omega t}.$$

Inside the room the pressure is

$$p = (i\omega\rho_o)2A \sum \sigma_{m,n} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \frac{\cos k^*_{m,n}(d - z)}{\cos k^*_{m,n} d - Q_{m,n} \sin k^*_{m,n} d} e^{-i\omega t}, \tag{2.3.14}$$

the axial velocity is

$$v = 2A \sum \sigma_{m,n} k^*_{m,n} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \frac{\sin k^*_{m,n}(d - z)}{\cos k^*_{m,n} d - Q_{m,n} \sin k^*_{m,n} d} d^{-i\omega t}, \tag{2.3.15}$$

and the specific impedance for the (m, n)th mode is

$$\zeta_{m,n} = (\rho_o c) \frac{ik}{k^*_{m,n}} \cot k^*_{m,n}(d - z). \tag{2.3.16}$$

The ratio of the pressure in the (m, n)th mode, at the centre of the window, to the incoming pressure p_i is given by

$$\frac{(p_{m,n})}{(p_i)} = \frac{2\sigma_{m,n}}{(1 - Q_{m,n} \tan k^*_{m,n} d)} \tag{2.3.17}$$

This equation shows a highly modified form of the quarter-wavelength effect.

2.4 GREEN'S FUNCTION FORMULATION

The approach in (2.3) was essentially based on a physical interpretation of the problem. Its success was based on the fact that in the time domain it led to a transform (to be described in the next report) which was easier to interpret in terms of a multiple-reflection description.

It was felt necessary to formulate the same problem on a more exact basis to understand the implications of the approximations more thoroughly. An obvious method was the integral equation approach based on Green's functions. This method was also felt to be useful when it came to include room-absorption.

However, the usual formula for rooms

$$g(x,y,z/x',y',z') = \sum_{\ell mn} \frac{4\pi c^2}{abd} \varepsilon_\ell^2 \varepsilon_m^2 \varepsilon_n^2 \cos \frac{\ell\pi x}{a} \cos \frac{n\pi y}{b} \cos \frac{m\pi z}{d} \times$$

$$\frac{\cos \frac{\ell\pi x'}{a} \cos \frac{m\pi y'}{b} \cos \frac{n\pi z'}{d} e^{-i\omega t}}{\Omega^2_{\ell mn} - \omega^2} \tag{2.4.1}$$

led only to a modified form of the approach in 2.2.

Luckily, a discontinuous form was found in Morse and Feshbach (reference 18, p.1434) which leads to a result similar to 2.3.

In Figure 2a let $(x', y', 0)$ be an arbitrary point on the window surface. Let its axial velocity be described by $\frac{\partial\phi(x',y',0)}{\partial z}$. By Green's theorem, the potential ϕ_5 generated by the window in the infinite half-space, $z < 0$, due to the velocity distribution in the window is

$$\phi_5(x,y,z) = \frac{1}{4\pi} \iint \frac{\partial\phi(x',y',0)}{\partial z} G(x,y,z/x',y',0)dx'dy' \quad (z < 0) \qquad (2.4.2)$$

where G is the Green's function for the half-space, given by

$$G(x,y,z/x',y',z') = \frac{2e^{ik\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}}}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}}, \qquad (2.4.3)$$

and the Green's function for interior of the room to be used is

$$g(x,y,z/x',y',z') = \frac{-4\pi}{ab} \sum_{m,n} \frac{(2 - \delta_{o,m})(2 - \delta_{o,n})}{k^*_{m,n} \sin k^*_{m,n}}$$

$$\cos\frac{m\pi x'}{a} \cos\frac{n\pi y'}{b} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \left\{\begin{bmatrix} \cos k^*_{m,n} z' \cos k^*_{m,n}(d - z) \\ \cos k^*_{m,n} z \cos k^*_{m,n}(d - z') \end{bmatrix}\right\} \left\{\begin{bmatrix} z < z' \\ z' < z \end{bmatrix}\right\}. \qquad (2.4.4)$$

Therefore the velocity potential, ϕ_1, inside the room is given by

$$\phi(x,y,z) = \frac{-1}{4\pi} \iint \frac{\partial\phi}{\partial z}(x',y',0)\, g(x,y,z/x',y',0)dx'dy'. \qquad (2.4.5)$$

It follows from (2.4.4) that

$$g(x,y,z/x',y',0) = \frac{4\pi}{ab} \sum_{m,n} (2 - \delta_{o,n})(2 - \delta_{o,m}) f_{m,n}(d,z) \cos\frac{m\pi x'}{a} \cos\frac{n\pi y'}{b} \times$$

$$\cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b}, \qquad (z > 0), \qquad (2.4.6)$$

where

$$f_{m,n}(d - z) \equiv \frac{\cos k^*_{m,n}(d - z)}{k^*_{m,n} \sin k^*_{m,n} d} \qquad (2.4.7)$$

If the region that the window occupies is denoted by s_w and if the magnitude of the window area is given by S (= $ab\lambda_1\lambda_2$), it follows from (1.2.9) that

$$\iint_{s_w} \iint_{s_w} G(x,y,0/x',y',0)dx'dy'dxdy = 4\pi S^2 \left(\frac{1}{M} + \frac{ik}{2\pi}\right) \qquad (2.4.8)$$

For the region to the left of the window, given by $z < 0$, an incoming wave of potential ϕ_1, $= Ae^{ikz}$, gives rise to a block reflected potential ϕ_2 (= Ae^{-ikz}), due to the presence of the baffle. The velocity set up in the space occupied by the window gives rise to an additional potential, ϕ_5. For the region occupied by the room, i.e. for $z > 0$, the velocity distribution at the window creates the potential ϕ_1. The condition for the continuity of velocity, across the window, has already been used. The condition that the potentials are continuous across the window is expressed by equating the limiting values of the potentials on both the sides; i.e.,

$$2A + \phi_5(x,y,0) = \phi_1(x,y,0) \quad \text{(for } (x,y) \text{ lying in } s_w). \qquad (2.4.9)$$

Therefore from equation (2.4.2) and (2.4.5),

$$2A = \frac{-1}{4\pi} \iint \frac{\partial\phi}{\partial z}(x',y',0) \left[G(x,y,0/x',y',0) + g(x,y,0/x',y',0)\right] dx'dy'$$

$$\text{(for all } x,y, \text{ in } s_w). \qquad (2.4.10)$$

g is already expressed in the form of a sum over the indices m and n. Let $\frac{\partial\phi}{\partial z}(x',y',0)2A$ and $G(x,y,0/x',y',0)$ be also expressed in a similar fashion:

$$\frac{\partial\phi}{\partial z}(x',y',0) = \sum_{m,n} V_{m,n} \cos\frac{m\pi x'}{a} \cos\frac{n\pi y'}{b}, \qquad (2.4.11)$$

$$2A = \sum_{m,n} (2A\sigma_{m,n}) \cos\frac{m\pi x'}{a} \cos\frac{n\pi y'}{b}, \qquad (2.4.12)$$

$$G(x,y,0/x',y',0) = \sum_{m,n} G_{m,n} \cos\frac{m\pi x'}{a} \cos\frac{n\pi y'}{b} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b}. \qquad (2.4.13)$$

In (2.4.12) a property of $\sigma_{m,n}$ has been used. This expansion would lead to the correct value of 2A over the region s_w. $G_{m,n}$ is also found by choosing the true value of G over the region s_w. For the rest of the region the choice is arbitrary and it is most convenient to take it to be zero. $G_{m,n}$ can be evaluated by multiplying both the sides of (2.4.13) by $\cos\frac{m\pi x'}{a} \cos\frac{n\pi y'}{b} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b}$ and integrating over the area twice. Thus

$$G_{m,n} = \frac{(2-\delta_{o,m})^2\,(2-\delta_{o,n})^2}{S^2} \int_{s_w}\!\!\int \int_{s_w}\!\!\int G(x,y,0/x',y',0)$$

$$\cos\frac{m\pi x'}{a} \cos\frac{n\pi y'}{b} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b}\, dx'dy'dxdy \qquad (2.4.14)$$

A substitution of (2.4.12), (2.4.11), (2.4.13) and (2.4.6) in (2.4.7) leads to

$$\sum_{m,n} 2A\sigma_{m,n} + \left[\frac{V_{m,n}G_{m,n}\,(ab)}{4\,(2-\delta_{o,m})(2-\delta_{o,n})}\right] \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} = \sum_{m,n} V_{m,n} f_{m,n}(d,0)$$

$$\cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b}. \qquad (2.4.15)$$

This expression is valid for all (x, y) in s_w. It follows therefore that

$$V_{m,n} = \frac{2A\sigma_{m,n}}{f_{m,n}(d,0) - \dfrac{G_{m,n}(ab)}{4(2-\delta_{o,m})(2-\delta_{o,n})}}. \qquad (2.4.16)$$

A substitution of (2.4.11) and (2.4.6) in (2.4.5) leads to an expression for ϕ:

$$\phi = \sum_{m,n} V_{m,n}\, f_{m,n}(d,\, z) \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \qquad (2.4.17)$$

or

$$\phi = \sum_{m,n} 2A\sigma_{m,n} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \left[\frac{f_{m,n}(d,\, z)}{f_{m,n}(d,0) - \dfrac{G_{m,n}(d,b)}{4\pi(2-\delta_{o,m})(2-\delta_{o,n})}}\right]. \tag{2.4.18}$$

It follows, from the definition of $f_{m,n}(d,\, z)$ ((2.4.7)), that ϕ can also be expressed as

$$\phi = \sum 2A\sigma_{m,n} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \frac{\cos k^*_{m,n}(d - z)}{\cos k^*_{m,n} d - Q_{m,n} \sin k^*_{m,n} d} \tag{2.4.19}$$

where

$$Q_{m,n} = \frac{k^*_{m,n} G_{m,n}(ab)}{4\pi(2-\delta_{o,m})(2-\delta_{o,n})}. \tag{2.4.20}$$

The correspondence of the equations (2.4.19) and (2.3.15) is striking. Equation (2.4.20) can be used to obtain a more accurate value of $Q_{m,n}$. From equations (2.4.14) and (2.4.20) it follows that

$$Q_{m,n} = \frac{k^*_{m,n}(ab)}{S^2} (2 - \delta_{o,m})(2 - \delta_{o,n}) \iint_{S_W} \iint_{S_W} G(x,y,0/x',y',0)$$
$$\cos\frac{m\pi x'}{a} \cos\frac{n\pi y'}{b} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b}\, dx'dy'dxdy. \tag{2.4.21}$$

If in this equation, the averaged value of G be substituted and if the integration be carried out,

$$Q_{m,n} = k^*_{m,n}(ab)\left(\frac{1}{M} + \frac{ik}{2\pi}\right) \frac{(ab)^2}{S^2} \left\{\frac{\sigma^2_{m,n}}{(2-\delta_{o,m})(2-\delta_{o,n})}\right\} \tag{2.4.22}$$

or

$$Q_{m,n} = k^*_{m,n}(ab)\left(\frac{1}{M} + \frac{ik}{2\pi}\right) \left\{\frac{1}{(\lambda_1\lambda_2)} \frac{\sigma^2_{m,n}}{(2-\delta_{o,m})(2-\delta_{o,n})}\right\}. \tag{2.4.23}$$

It can be readily verified that this agrees with the value obtained in section (2.3) for m =0 and n = 0. A better value for $Q_{m,n}$ can be obtained by carrying out the integration in equation (2.4.21). For the purpose of the present work, equation (2.4.23) is quite satisfactory. The expression (2.4.19) will be made use of to obtain the time domain response to a sonic boom signal.

3.1 MODE DISTRIBUTION FUNCTIONS: INTRODUCTION

The necessity for the distribution functions arises when the boundary conditions of a duct with a cross sectional area, S', are to be matched along an orifice with a cross sectional area S. The distribution functions represent a suitable distribution of the disturbance along S'. When the disturbance incorporates a phase variation, the distribution functions become complex to account for the phase variation. This will be observed in (3.3). Certain properties of these functions are derived in (3.4).

3.2 DISTRIBUTION FUNCTION FOR A WINDOW AT A CORNER

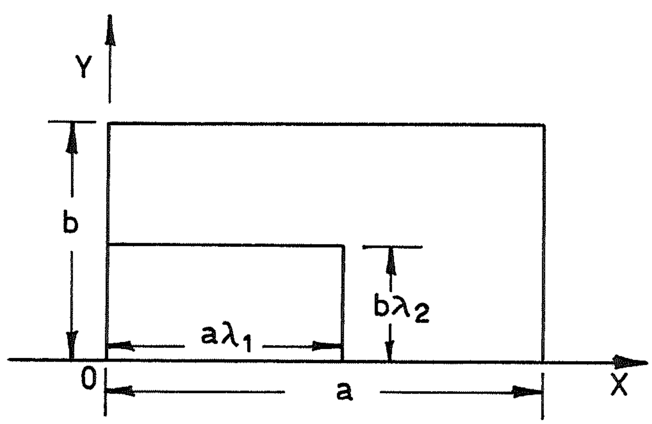

FIGURE 3

Front view of a room having a window in a corner.

When the window is not centrally situated, we can apply exactly the same procedure as in 2.3 to obtain $\sigma_{m,n}$'s. Once these are obtained, the rest of the procedure remains unaffected.

To illustrate the nature of the results to be expected, consider the window shown in Figure 3. Its dimensions along x is $a\lambda_1$ and along y is $b\lambda_2$; one of its corners is at 0. It is required that

$$\sum_{m,n} \sigma_{m,n} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} = 1, \quad \text{for } x,y \text{ in the window,} \qquad (3.2.1)$$
$$= 0, \quad \text{for } x, y \text{ outside.}$$

Therefore, integrating over the whole area, after multiplying through by $\cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b}$, gives

$$\frac{ab\ \sigma_{m,n}}{(2-\delta_{o,m})(2-\delta_{o,n})} = ab\lambda_1\lambda_2 \ \frac{\sin m\pi\lambda_1}{m\pi\lambda_1} \ \frac{\sin n\pi\lambda_2}{n\pi\lambda_2},$$

and

$$\sigma_{m,n} = \lambda_1\lambda_2 \left[(2-\delta_{o,m})(2-\delta_{o,n}) \left(\frac{\sin m\pi\lambda_1}{m\pi\lambda_1}\right) \left(\frac{\sin n\pi\lambda_2}{n\pi\lambda_2}\right) \right]. \qquad (3.2.2)$$

It can be noted that, in general, both m and n can be both odd and even.

3.3 DISTRIBUTION FUNCTION FOR OBLIQUE INCIDENCE

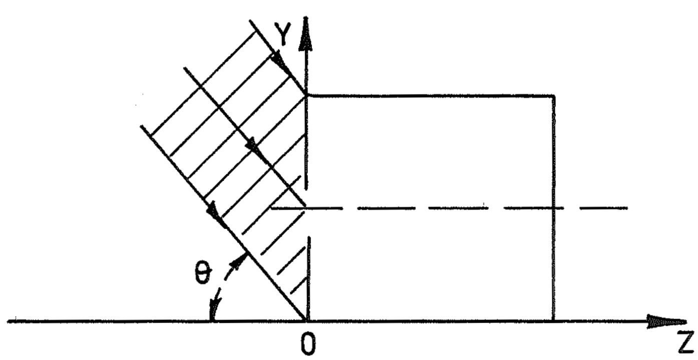

FIGURE 4
Case of an obliquely incident wave

Figure 4 shows an obliquely incident wave on a window which is centrally situated in a wall of a room which has the same dimensions as the room shown in Figure 2. Let the direction of wave propagation make an angle θ to the horizontal, as shown.

The 'terminated duct' approach of 2.3 is found useful here. Thus the incoming potential can be written as

$$\phi_1 = Ae^{ikz\cos\theta}\, e^{-iky\sin\theta} \tag{3.3.1}$$

and the 'block-reflected' potential as

$$\phi_2 = Ae^{-ikz\cos\theta}\, e^{-iky\sin\theta}. \tag{3.3.2}$$

This produces a driving potential (see 2.3), at $z = 0$, given by

$$(\phi_1 + \phi_2)_{z=0} = 2Ae^{-iky\sin\theta}. \tag{3.3.3}$$

Now the $\sigma_{m,n}$'s are required to satisfy

$$\begin{aligned} e^{-iky\sin\theta} &= \sum \sigma_{m,n} \cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}, \quad (x,y) \text{ in the window,} \\ &= 0, \quad (x,y) \text{ outside the window.} \end{aligned} \tag{3.3.4}$$

Thus

$$\begin{aligned} \frac{ab\,\sigma_{m,n}}{(2-\delta_{o,m})(2-\delta_{o,n})} &= \int_{\frac{b}{2}(1-\lambda_2)}^{\frac{b}{2}(1+\lambda_2)} \int_{\frac{a}{2}(1-\lambda_1)}^{\frac{a}{2}(1+\lambda_1)} e^{-iky\sin\theta}\cos\frac{m\pi y}{a}\cos\frac{n\pi y}{b}\,dxdy \\ &= (-1)^{m/2}\frac{\sin\theta_m}{\theta_m}(a_1)\int_{\frac{b}{2}(1-\lambda_2)}^{\frac{b}{2}(1+\lambda_2)} e^{-iky\sin\theta}\cos\frac{n\pi y}{b}\,dy \\ &\qquad\qquad \text{m even;} \\ &= 0, \quad \text{m odd.} \end{aligned} \tag{3.3.5}$$

Now if $k_2 = -k\sin\theta$,

$$\int_{\frac{b}{2}(1+\lambda_2)}^{\frac{b}{2}(1-\lambda_2)} e^{-iky\sin\theta} \cos\frac{n\pi y}{b}\,dy = \int_{\frac{b}{2}(1-\lambda_2)}^{\frac{b}{2}(1+\lambda_2)} e^{ik_2 y} \cos\frac{n\pi y}{b}\,dy$$

$$= \left[\frac{e^{ik_2 y}}{(\frac{n\pi}{b})^2 - k_2^2} \left(\frac{n\pi}{b}\sin\frac{n\pi y}{b} + ik_2\cos\frac{n\pi y}{b}\right)\right]_{\frac{b}{2}(1- \ _2)}^{\frac{b}{2}(1+ \ _2)}$$

$$= \frac{2e^{\frac{ik_2 b}{2}}}{(\frac{n\pi}{b})^2 - k_2^2} \left[\cos\frac{n\pi}{2}\left\{\frac{n\pi}{b}\cos\frac{k_2 b\lambda_2}{2}\sin\frac{n\pi\lambda_2}{2} - k_2\sin\frac{k_2 b\lambda_2}{2}\cos\frac{n\pi\lambda_2}{2}\right\}\right.$$

$$\left. + i\sin\frac{n\pi}{2}\left\{\frac{n\pi}{b}\sin\frac{k_2 b\lambda_2}{2}\cos\frac{n\pi\lambda_2}{2} - k_2\cos\frac{k_2 b\lambda_2}{2}\sin\frac{n\pi\lambda_2}{2}\right\}\right]. \qquad (3.3.6)$$

From equations (3.3.5) and (3.3.6)

$$\sigma_{m,n} = 0, \quad \text{if } m \text{ be odd,}$$

$$= (2 - \delta_{o,m})\,\lambda_1(-1)^{m/2}\frac{\sin\theta_m}{\theta_m}E_n \text{ (say), if } m \text{ be even,} \qquad (3.3.7)$$

where

$$E_n = \frac{2e^{\frac{-ibk\sin\theta}{2}}(-1)^{n/2}}{(\frac{n\pi}{b})^2 - (k\sin\theta)^2}\left[\frac{n\pi}{b}\cos\left(\frac{k_1 b\lambda_2\sin\theta}{2}\right)\sin\frac{n\pi\lambda_2}{2}\right.$$

$$\left. - k\sin\theta\sin\left(\frac{kb\lambda_2\sin\theta}{2}\right)\cos\frac{n\pi\lambda_2}{2}\right], \quad n \text{ even}$$

$$= \frac{2i(-1)^{\frac{n+1}{2}} e^{\frac{-ibk \sin\theta}{2}}}{(\frac{n\pi}{b})^2 - (k \sin\theta)^2} \left[\frac{n\pi}{b} \sin\left(\frac{kb\lambda_2 \sin\theta}{2}\right) \sin\frac{n\pi\lambda_2}{2} - k \sin\theta \cos\left(\frac{kb\lambda_2 \sin\theta}{2}\right) \sin\frac{n\pi\lambda_2}{2}\right], \quad n \text{ odd.} \tag{3.3.8}$$

It can be seen that E_n, and consequently $\sigma_{m,n}$, is complex. For a centrally situated window $\sigma_{m,n}$ is zero for m and n odd in the case of normal incidence. However, for oblique incidence this ceases to be so and consequently more modes are generated.

If the direction of propagation is oblique with respect to both the axes, the result is

$$\sigma_{m,n} = E_m E_n, \tag{3.3.9}$$

where E_m is defined in a fashion similar to E_n.

3.4 SOME PROPERTIES OF DISTRIBUTION FUNCTIONS

A few general properties of the various $\sigma_{m,n}$'s encountered so far are derived here.

Consider firstly the central window case:

$$\sum \sigma_{m,n} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} = 1, \quad \text{if } \frac{a}{2}(1-\lambda_1) < x < \frac{a}{2}(1+\lambda_2) \text{ and } \frac{b}{2}(1-\lambda_2) < y < \frac{b}{2}(1+\lambda_2),$$
$$= 0 \text{ outside.} \tag{3.4.1}$$

Except when λ_1 or λ_2 is unity, if $x = 0$, $y = 0$, then

$$\sum \sigma_{m,n} = 0 \tag{3.4.2}$$

and if $x = \frac{a}{2}$, $y = \frac{b}{2}$, then

$$\sum (-1)^{m/2} (-1)^{n/2} \sigma_{m,n} = 1. \tag{3.4.3}$$

Choose also

$$\sum_{m,n} \sigma_{m,n} \cos\frac{\mu\pi x}{a} \cos\frac{\nu\pi y}{b} = \left\{\begin{matrix} 1, & \text{in window} \\ 0, & \text{outside window}\end{matrix}\right\} . \qquad (3.4.4)$$

Multiply both sides of equations (3.4.4) and (3.4.1) to get

$$\sum_{\mu,\nu} \sum_{m,n} \sigma_{m,n}\sigma_{\mu,\nu} \cos\frac{m\pi x}{a} \cos\frac{\mu\pi x}{a} \cos\frac{\nu\pi y}{b} \cos\frac{n\pi y}{b} = \{{}^{1}_{0}\}. \qquad (3.4.5)$$

Integrating over the area and using the orthogonality property results in

$$\sum \frac{\sigma_{m,n}}{(2-\delta_{o,m})(2-\delta_{o,n})} = \lambda_1\lambda_2. \qquad (3.4.6)$$

Consider now the distribution function for oblique incidence:

$$\sum \sigma_{m,n} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} = e^{ik_2 y} \quad \text{in window,}$$
$$= 0 \quad \text{outside.}$$

Therefore

$$\sum \sigma^*_{m,n} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} = e^{ik_2 y} \quad \text{in window,}$$
$$= 0 \quad \text{outside.}$$

Multiplying and integrating as before results in

$$\sum_{m,n} \frac{|\sigma_{m,n}|^2}{(2-\delta_{o,m})(2-\delta_{o,n})} = \lambda_1\lambda_2.$$

3.5 COMPUTATION OF $\sigma_{m,n}$

A computer programme has been developed to compute $\sigma_{m,n}$'s for various window sizes.

Some of the results obtained are illustrated in Figures 5, 6 and 7. Figure 5 shows some of the $\sigma_{m,n}$'s for window sizes given by $\lambda_1 = \lambda_2 = 0.1$.

Only two sets are shown for the sake of clarity.

In Figure 6, the absolute values are plotted for several values of n. The characteristic window ratios are 0.4 and 0.5. All (10, n) modes are seen to be zero because of the particular value of λ_1 chosen.

Figure 7 is a normalised plot of the absolute values of several window sizes. It can be seen that when the window occupies a substantial part of the cross sectional area, the lower modes are more predominant and values of the distribution function reduce rapidly as we go to the higher modes, whereas, for windows which occupy very small cross-sections, the values are relatively level. Thus a large number of higher order modes have to be taken into account to build up a realistic description.

CONCLUSIONS

The ultimate aim of the work is to formulate an analytical method which is capable of predicting the characteristics of the transient pressure field inside a room once the characteristics of the incident boom are known.

In this part of the report, attention was focused on the steady state response. A simple model of the room was used to obtain the pressure field inside a room with hard walls.

The sound field inside the room, shown in Figure 2a, due to an incoming harmonic wave of amplitude A is given by (2.3.15). The spatial variation is through the terms in x, y and z. The distribution functions $\sigma_{m,n}$ depend on the relative size of the window and the position of the window. It is shown in 3.3 that for an oblique incidence these functions become complex. The parameter $Q_{m,n}$, which represents the effect of both the inertia and dissipation associated with the window has been estimated in 2.3 and a better value for it is found in 2.4, as seen in 2.4.20. The contribution of each mode goes to its maximum when $\cot k^*d = \mathrm{Re}(Q_{m,n})$. This is the 'generalised' quarter wave-length effect, as can be seen by setting $m = n = \mathrm{Re}(Q_{m,n}) = 0$.

For every mode there is a frequency below which $k^*_{m,n}$ becomes imaginary. The contribution due to the mode, at a point, then alters considerably.

Detailed calculations in this case have not been presented because in further work the expressions have been converted into the time domain and explicit expressions have been obtained which enable the detailed calculations to be made directly. These calculations illustrating various representative cases are to be presented in Part II of this report.

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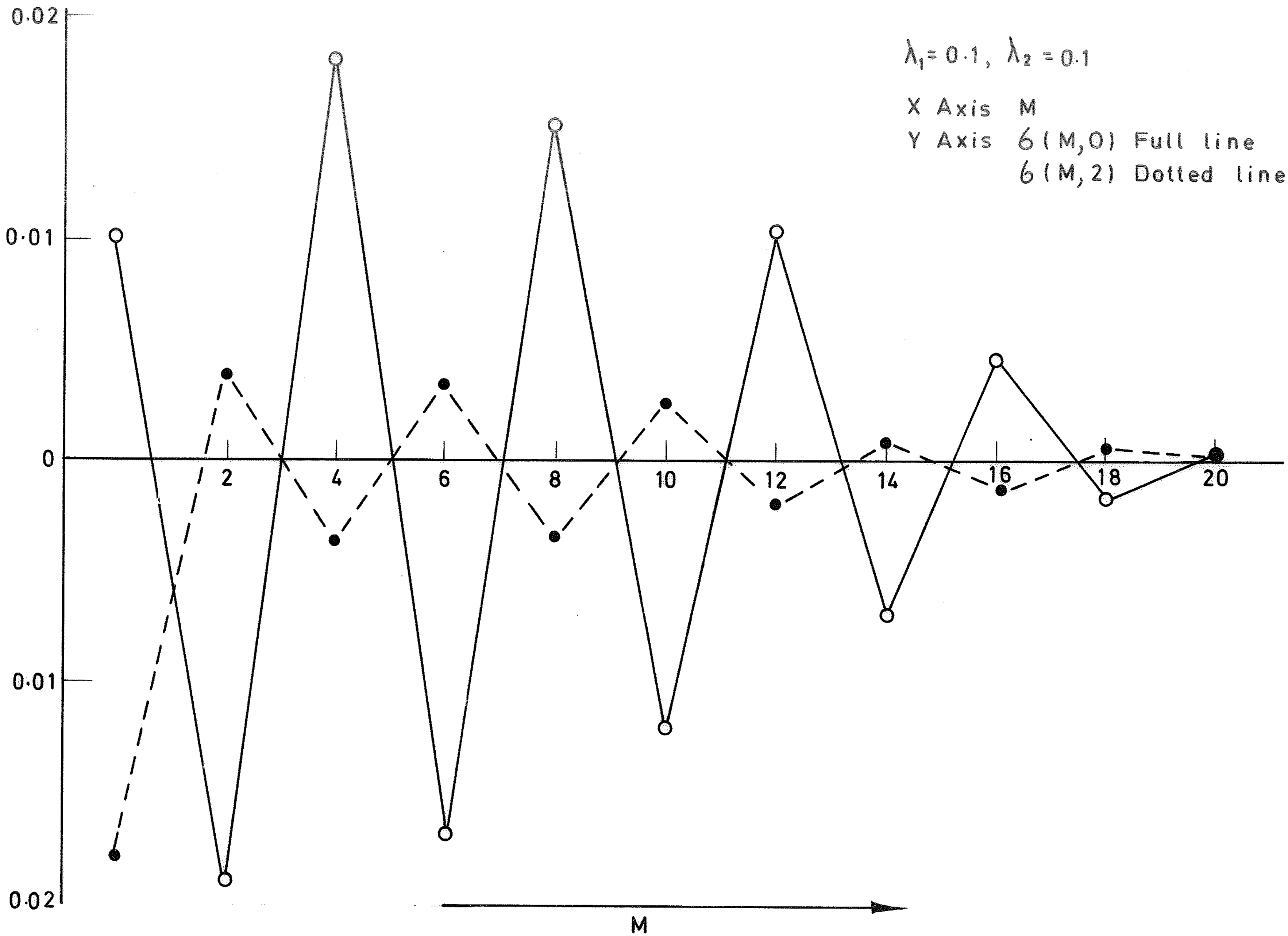

Fig. 5.

The modal distribution parameters. These represent the ratios in which the various modes are generated, due to a uniform sourse distribution over the window area.

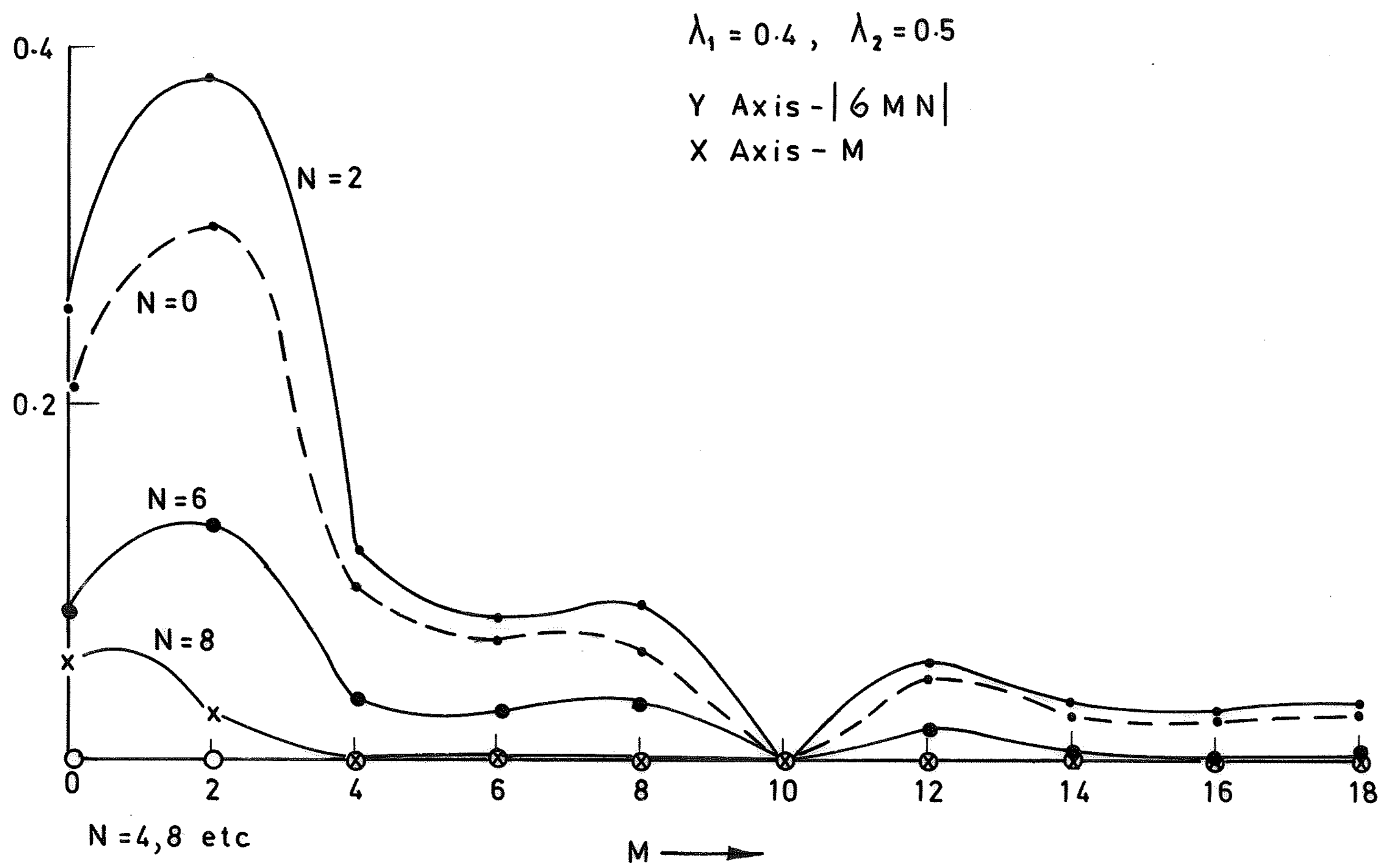

Fig. 6

The absolute value of $6_{m,n}$, for various 'm', and 'n'.

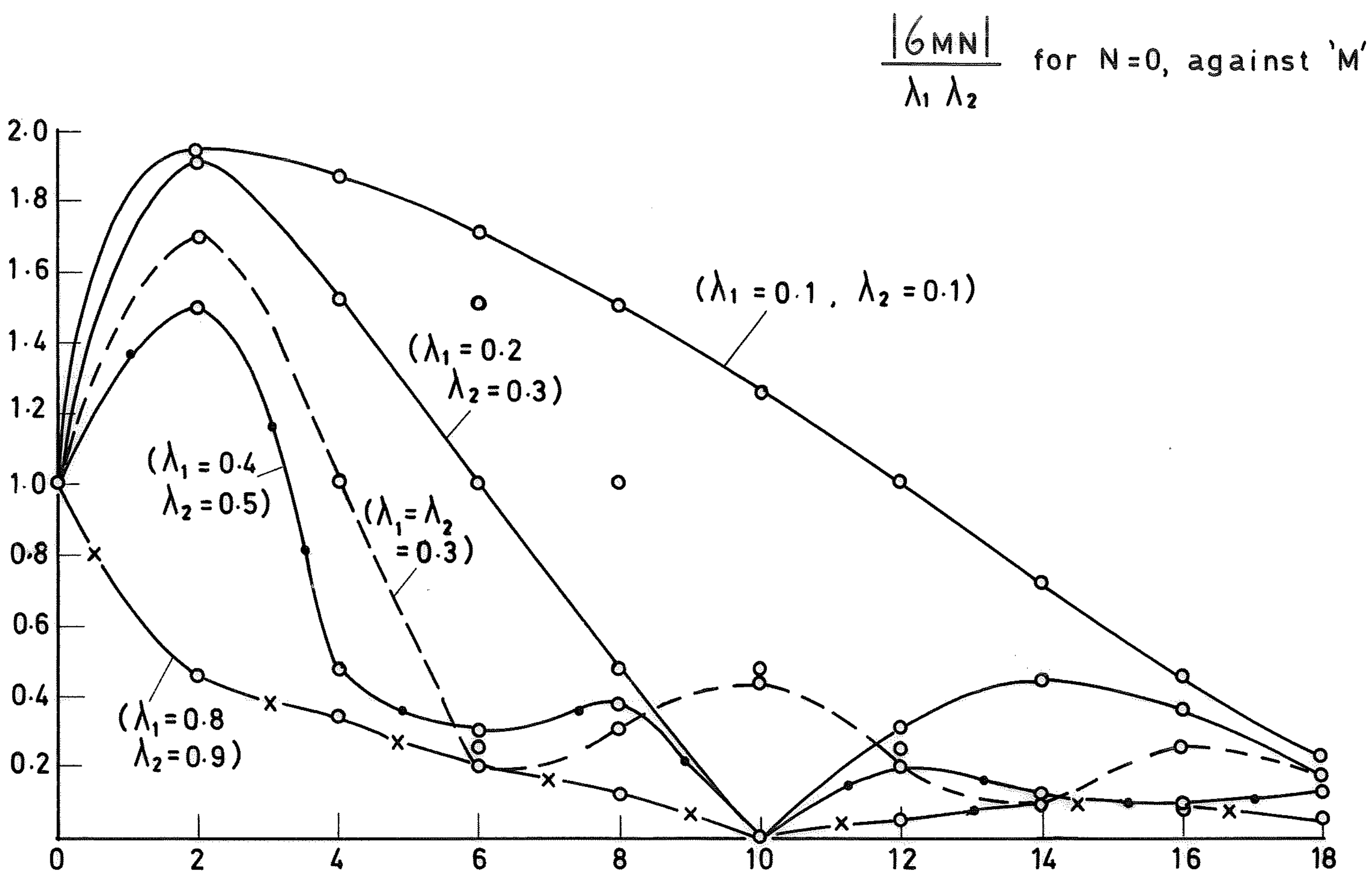

Fig. 7

Normalised modal distribution parameter $\frac{\sigma_{mn}}{\lambda_1 \lambda_2}$ for various values of 'm' and 'n'