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STABILITY CHARACTERIZATION OF ADVANCED INJECTORS

Design Guide, Volume 2, Revision A
Operation of the Computer Program

Report 20672-P3D

February 1971



AEROJET LIQUID ROCKET COMPANY

A DIVISION OF AEROJET-GENERAL

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ERRATA

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Report 20672-P2D dtd. May 1970
Volume I: Design Application

Revise values in Table 1 -- "Chamber Response as a Function of Chamber Type" (page 23) to read:

<u>Type</u>	<u>(N chambers) (Fundamental and Higher Order Modes)</u>
Cylindrical	0.70 to 1.50
Annular	1.80 to 2.60
Conical	0.90 to 1.50
Rectangular	2.0 to 2.80

ERRATA

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DESIGN GUIDE

Volume 2, Operation of the Computer Program

Revision A

Report 20672-P3D

Prepared by:

Aerojet-General Corporation
Liquid Rocket Division

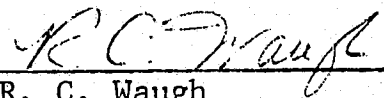
Contract NAS 8-20672

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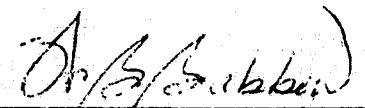
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Marshall Space Flight Center
Huntsville, Alabama

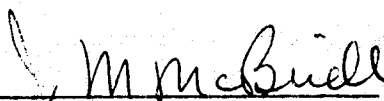
Prepared by


R. C. Waugh

Approved by


W. B. Bakken

Approved by


J. M. McBride

Approved by


N. E. VanHuff

Report 20672-P3D

FOREWORD

This volume is submitted as partial fulfillment of contract provisions of Contract NAS 8-20672 and meets the requirements of Supplemental Agreement No. 13 of that contract. The NASA project engineer was Mr. R. J. Richmond.

Volume I of this report is written specifically for the designer with the intent of providing combustion stability information which may be applied directly during a program's design phase. This volume (Volume II), on the other hand, is to provide the tools by which the analyst may characterize the combustion stability of a system. The combustion model which provided the basis for this analytical technique is the Sensitive Time Lag theory first developed by Dr. L. H. Crocco and later extended by Dr. F. H. Reardon. This volume provides description of a computer program, written in FORTRAN V, which may be utilized in characterizing a variety of combustion systems. The program appearing in this edition has been updated to include a high Mach number analysis, hence the original edition of Vol. II is superceded by this revision.

The work on this edition was conducted by the Engine Components Department, Aerojet Liquid Rocket Company, with Dr. N. E. Van Huff, program manager; Mr. J. M. McBride project manager; and Mr. W. W. Howard and Mr. W. B. Bakken, project engineers.

Special acknowledgement is given to Mr. R. C. Waugh for his contribution in the development of analytical models, Mr. D. P. Dudley for programing and conversion of the computer program, Mr. R. K. Turner for the reduction of combustion stability theory into design criteria and analysis and correlations of test data, and Mr. R. J. Wondra and Mr. N. B. Davidson for preparation and editing the material in this report.

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N71-20416

I. PROBLEM

A. ABSTRACT

This program solves the combustion instability problem using the sensitive time lag theory. This amounts basically to solving the non-homogeneous Helmholtz equation for the condition of neutral stability. The non-homogeneous terms in the Helmholtz equation account for the mean flow and combustion effects. The program considers longitudinal and transverse modes of oscillation. It includes effects of non-uniform injection, velocity effects, and non-linear combustion response.

The program is divided into several subprograms which can either be run together or separately. These subprograms include analyses of nozzle admittance, concentrated combustion, and distributed combustion with a transverse mode -- as well as those which calculate describing functions for combustion response and nonuniform injection parameters.

B. TECHNICAL DESCRIPTION

1. The Nature of Combustion Instability

It is well known that the processes occurring within a liquid rocket combustion chamber are never entirely smooth. Even when the mean operating conditions are constant, fluctuations around these mean values occur in all of the quantities that characterize the flow. The nature of the fluctuations can vary widely from one combustor to another and in a single combustor for different operating conditions. If the fluctuations are random and of small amplitude, this unsteadiness is referred to as "combustion noise". With random fluctuations of large amplitude, the operation of the rocket is said to be "rough", and the functioning of the system of which the rocket is a part may be impaired.

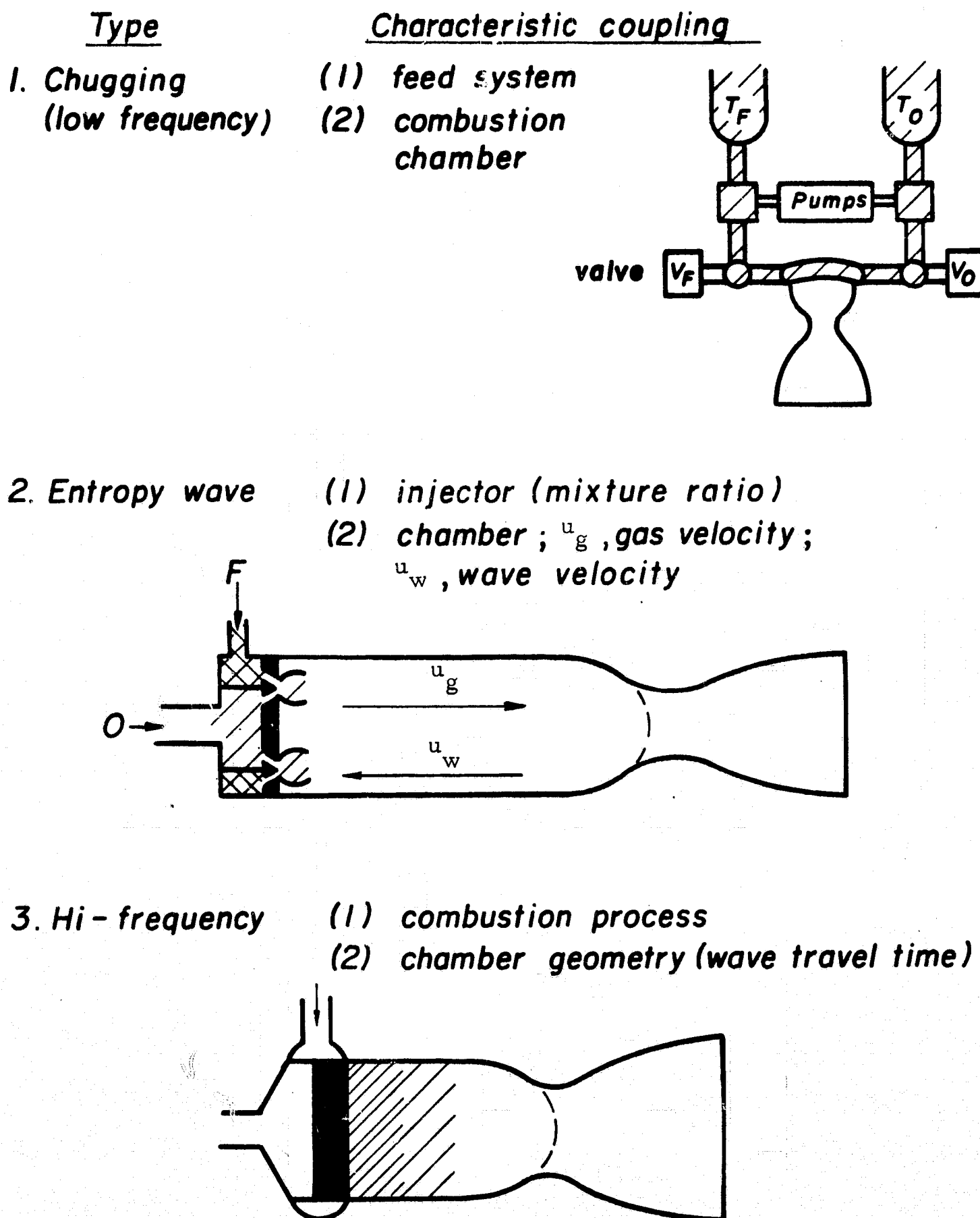


Figure 1 -- Liquid Propellant Combustion Instability

I, B, Technical Description (cont.)

Much more serious than rough operation is the problem of combustion instability, also termed unstable combustion, oscillatory combustion, or resonant combustion. Whereas rough combustion refers to random fluctuations, combustion instability consists of organized oscillations that are maintained and amplified by the combustion process itself. The various types of combustion instability can be classified roughly into three categories: low frequency, intermediate frequency, and high frequency. However, the classification is not based simply on frequency alone. Just as electrical and mechanical systems respond to specific frequencies depending on the type of coupling, so also liquid rocket systems exhibit representative frequency and amplitude patterns.

The basic coupling mechanisms for the three general types of combustion instability found in liquid propellant rocket engines are illustrated in Figure 1. For the low frequency ("chugging") type, interaction between the propellant feed system and the combustion chamber places the frequency generally less than 200 Hertz. The coupling is effected by the oscillating propellant feed rates. In the case of intermediate frequency combustion instability (sometimes referred to as entropy wave instability), the injector characteristics (especially the internal injector manifolding and orifice impedances) account for part of the interaction, with the mean gas flow and pressure wave propagation in the combustion chamber completing the process. Typical frequencies are in the several hundred Hertz range.

In this report, attention will be focused on the third type of combustion instability, namely, high frequency instability. This type depends upon a coupling between the combustion processes and flow oscillations in the combustion chamber. Such coupling requires no input from the feed system, although it is possible for the feed system to have an effect when the combustion chamber is large and acoustic frequencies are reduced to several hundred cycles per second. Normally, the frequencies to be expected are in the thousand Hertz range for most current engines.

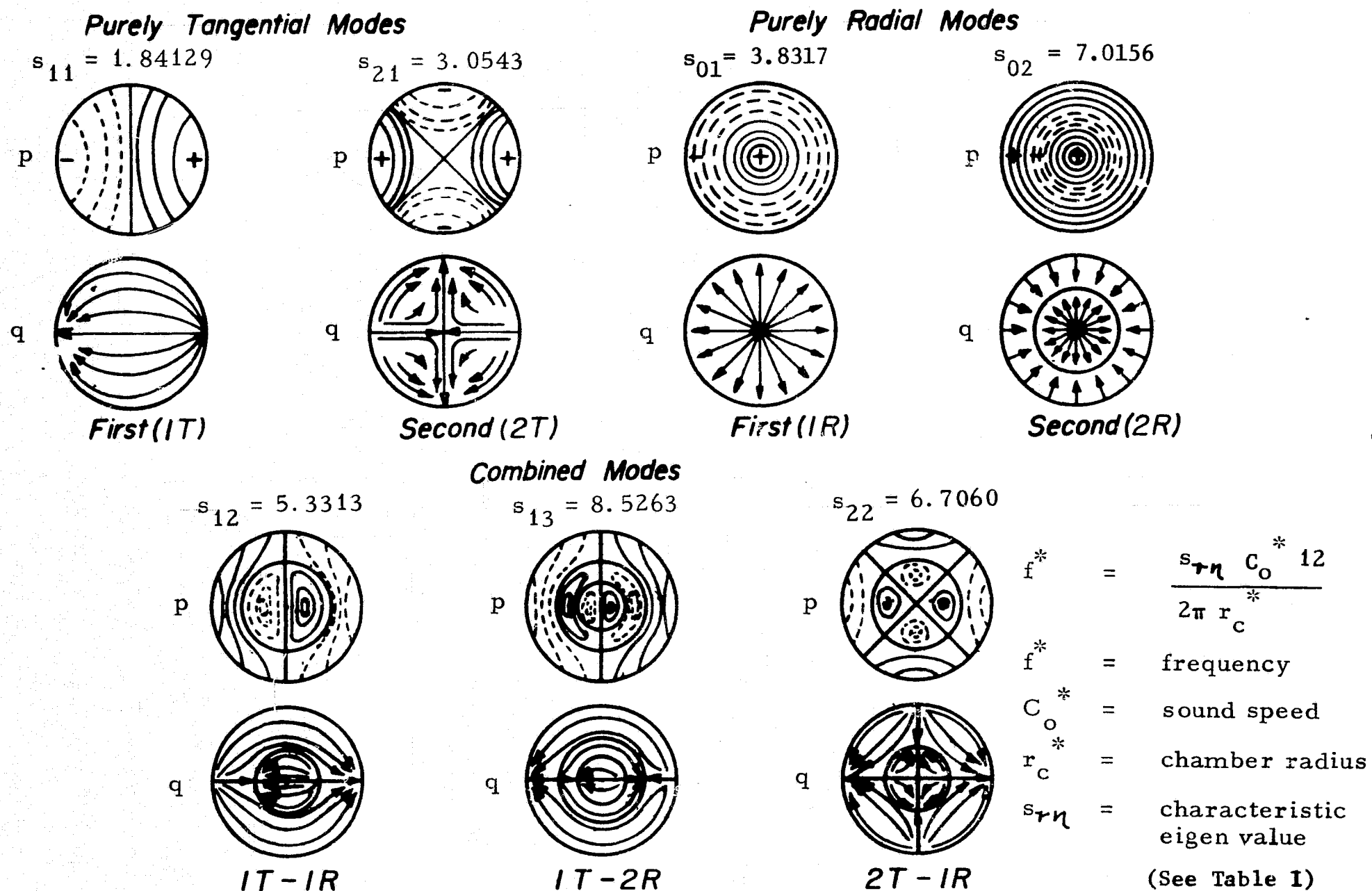


Figure 2 -- Pressure, p, and Velocity, q, Characteristics of Several Transverse Modes

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TABLE I

$s_{v\eta}$ FOR A CYLINDRICAL CHAMBER

For transverse modes the frequency of oscillation is given by:

$$f^* = \frac{s_{v\eta} C_o^* \times 12}{2 \pi r_c^*}$$

<u>Mode</u>	<u>v</u>	<u>n</u>	<u>$s_{v\eta}$</u>
1st radial	0	1	3.8317
2nd radial	0	2	7.0156
3rd radial	0	3	10.1734
1st tangential	1	1	1.8413
2nd tangential	2	1	3.0543
3rd tangential	3	1	4.2012
4th tangential	4	1	5.3175
5th tangential	5	1	6.4154
6th tangential	6	1	7.5013
1T, 1R combined	1	2	5.3313
2T, 1R combined	2	2	6.7060
3T, 1R combined	3	2	8.0151
4T, 1R combined	4	2	9.2825
1T, 2R combined	1	3	8.5263

For a longitudinal mode the frequency can be estimated by:

$$f^* = \frac{C_o^* \times 12}{2 (L_c^* + 2/3 L_N^*)} (1 - M_m^2)$$

where: L_N^* = length of nozzle in inches.

M_m = average Mach number in nozzle-chamber combination

I, B, Technical Description (cont.)

The combustion chamber geometry is an important factor in high frequency combustion instability because the possible frequencies depend on the internal geometry. Since it is effectively closed at one end by the injector and has a choked-flow exhaust nozzle at the other end, the chamber acts acoustically much as a double closed-end cavity. For instance, for purely longitudinal modes, an approximate point of effective reflection in the nozzle can be determined theoretically; between that point and the injector face standing patterns of acoustic waves can be established. Similarly in the transverse plane, tangential modes of the spinning or standing types, as well as radial modes, may be established. Frequencies correspond approximately to those of the acoustic modes, either the fundamental or higher harmonics. Modes containing combinations of tangential, radial and longitudinal oscillations may also exist, each characterized by its own frequency. Pressure and velocity patterns for several transverse modes are illustrated in Figure 2. However, despite the similarity of the modes and the closeness of frequencies, the continuous generation of gases produces effects that do not exist in a closed chamber. In a closed chamber, the only source of damping originates from the friction on the walls. This source of damping is active of course also in the combustion chamber, but it plays a very modest role compared to other, more powerful sources of damping. Indeed, the very existence of the nozzle produces damping in the case of pure or combined longitudinal modes because the reflection of waves from the convergent (subsonic) portion of the nozzle departs from that of the ideal closed end. For purely transverse waves this source of damping is missing, and actually is replaced by a slight source of amplification.

The most important source of damping, however, is related to the process of gas generation itself, and consists of two parts of approximately equal importance. The most obvious comes from the fact that, from the conditions of the steady propellant injection flow (steady, of course, only if the feed system perturbations are disregarded) the combustion gases must have

I, B, Technical Description (cont.)

acquired, at the moment of generation, the perturbed momentum corresponding to the oscillatory flow. The acquisition of this momentum demands a certain work which must be absorbed from the system. The fact that momentum exchanges due to the drag of the droplets can take place prior to the moment of generation can only add additional damping.

The other, more subtle source of damping comes from the fact that at the moment of generation the volume of the propellant must change from its practically negligible liquid volume, to the full volume of the burned gases. To this change of volume corresponds a certain "pumping work" proportional to the local instantaneous pressure. Hence, more work must be absorbed from the system when the pressure is higher, and less when it is lower, thus providing an effective damping mechanism.

The significance of the preceding discussion is that, from the point of view of instability, each combustion system is characterized by certain well-defined proper frequencies at which the gases can oscillate in well-defined modes, and by certain damping mechanisms which absorb energy from the oscillating system. It is clear that self-sustained oscillations can exist (and instability appear) only if the combustion process is able to generate, at any one of the proper frequencies, enough feedback combustion energy (in excess of the steady-state conditions) to restore continually the amount which is being lost.

Suppose, for a moment, that the amount of combustion feedback energy is independent of frequency. Then the only important factor in the balance would be the energy damping corresponding to each mode, that (or those) mode(s) becoming unstable which correspond(s) to the lowest level of damping. Experience shows that this is not the case, and that, for a given injector, the selection of the unstable model is governed by frequency. This has been shown to be the case both

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I, B, Technical Description (cont.)

longitudinal and for transverse forms of instability. It has been demonstrated experimentally that the frequency at which combustion oscillations become organized is more closely related to the combustion process (the driving mechanism) than it is to the chamber geometry (the resonating chamber). When a chamber that became unstable in the fundamental mode was lengthened so that the frequency of the fundamental mode now became a harmonic mode, the combustion oscillations continued to organize at the same frequency. Since the physical length of the harmonic wall length provides considerably more viscous damping than that of the fundamental mode, it would seem logical that the oscillations would organize in the fundamental mode. It is not our purpose to discuss here the details of the transition from one mode to the other (it does not take place suddenly), but rather to point out that the transition occurs in such a way as to maintain the oscillation frequency within a well-defined narrow range.

The only logical interpretation of this observation is that combustion energy must be fed into the chamber in a narrow range around one, well defined, frequency. The reason for the energy being confined to this narrow range is complicated -- it depends on several geometrical, physical, and chemical conditions -- but its existence is the only logical explanation for the driving frequency being of sufficient magnitude to overcome the viscous damping associated with the increased wave length of the harmonic frequencies.

The empirically observed existence, for a given combustion system, of one narrow frequency range in which instability can appear can be interpreted by stating that, among the features characterizing the response of a given combustion system to oscillatory conditions, one can single out a "characteristic time" simply proportional to the reciprocal of frequency, such that only when its ratio to the oscillation period is around a certain value, maximum feedback can be generated and, possibly, instability produced.

This behavior has similarities to that of a resonant system, which is able to oscillate at certain natural frequencies with amplitudes that depend on the value of the frequency of the exciting forces compared to those of the natural frequencies. In the combustion instability problem also the exciting

I, B, Technical Description (cont.)

force due to the unsteady combustion processes is characterized by its own frequency (or its characteristic time), and maximum amplitudes are to be expected where there is coincidence with one of the proper frequencies. However, there is an important difference, residing in the fact that the exciting force is not independently applied from the outside, but is produced (in a sort of feedback loop) by the oscillations themselves, with the result that considerations of stability appear and take fundamental importance, and that the eventual oscillatory situation is determined by nonlinear effects. Nevertheless the terms "resonance" and "resonant" are often applied also to this case.

It appears that the knowledge of the characteristic time associated with a particular combustion system would be of primary importance in the design of new rockets, since it determines in which mode instability is able to appear. Such a knowledge could, for instance, allow the choice of the chamber and injector geometry in such a way that all of the proper frequencies would be too high to become unstable. Unfortunately, even assuming the characteristic time to be known, the choice of the propellant combination, as well as the chamber and basic injection system geometry, has always been fixed in the past during the early stages of development programs based on other stringent requirements of size, weight and performance, and there is very little chance that even in the future the designers may base their designs only on stability requirements. Therefore, a more sophisticated approach to the problem of stability is necessary, in which the second condition for appearance of instability, that of the energy balance, is also taken into consideration. In other words, even if it is impossible to avoid having some of the proper frequencies fall in the range where they may become unstable for the given combustion system, one should make sure that for the corresponding modes the combustion feedback is not sufficient to balance the damping.

Clearly there are two ways in which this balance can be improved in favor of stability: one is the depression of the combustion feedback, the other the increase of the damping. Of the two solutions, the

I, B, Technical Description (cont.)

second has been favored in recent times because it is better understood and hence easily controlled. Such damping devices as baffles or acoustic liners have been introduced, to the cost of more or less profound design complications, and have been very effective in producing substantial levels of damping, due to the dissipation caused by the devices. At the same time these devices also entail a change of the proper frequencies, and, in view of the previous discussion, this change can also have an effect on the balance.

On the other hand, the solution of depressing the combustion feedback has not been used in any consistent and systematic fashion, but only by looking for injector designs which, fortuitously or nearly so, result in the most stable operation on the test stand.

It may be added that a third, and the most effective solution, clearly consists in applying simultaneously the two solutions above, by (1) using an injector as stable as possible, and (2) adding a certain amount of extra damping to attain whatever safety margin is required.

The difficulty with a systematic application of the second and third solutions has been that knowledge of the behavior of combustion systems under oscillatory conditions is still very limited. Thus, it is impossible, starting from the fundamental physico-chemical processes involved, to predict the value of the characteristic time, the magnitude of the energy feedback, or -- with any certainty -- even the direction in which these quantities are affected by a change in design.

However, a less fundamental approach exists, which may still help the designer considerably, that is based upon the idea of establishing empirical correlations between the characteristic time and the energy feedback on one side and, on the other side, a certain number of appropriate combinations of parameters characterizing the injector geometry, the propellant mixture and the

I, B, Technical Description (cont.)

operating conditions. The feasibility of such an approach is offered by the sensitive time lag concept, introduced by Crocco in 1951 (Ref. 1). Although originally formulated with the sole intent of gaining an insight into the essential features of the high frequency instability phenomenon, the sensitive time lag combustion model was later found to predict accurately the quantitative behavior of the system in quite a few cases. In fact, this was beyond the expectations for such a heuristic approach.

This is indeed the primary advantage of the sensitive time lag concept, that the complexity of the actual combustion process can be avoided through the use of a very small number of lumped parameters. This is not to say that phenomena, such as droplet breakup, vaporization, mixing, chemical reactions, etc., are not important in determining stability and performance characteristics, but rather that, lacking such specific knowledge, the general nature of the coupling between the chamber conditions and the combustion process may still be described.

According to the Crocco model, the dynamic aspects of the injection-combustion process that are of significance in high frequency instability are characterized by a time lag which is sensitive to the local, instantaneous, values of pressure, temperature, gas velocity, etc. The degree of sensitivity is measured by one or more interaction indices. The sensitive time lag, then, plays the role of the above discussed characteristic time, while the interaction indices, properly combined, hold the key to the magnitude of the energy feedback. Thus, the occurrence of high frequency combustion instability is seen to result from the matching of the sensitive combustion time lag with one of the proper frequencies of the combustion chamber, provided that the degree of sensitivity of the combustion is sufficiently large to offset the damping effects present in the chamber. The stability conditions can then be expressed only in terms of the time lag and the interaction indices, and the ways to stabilization are easily discussed.

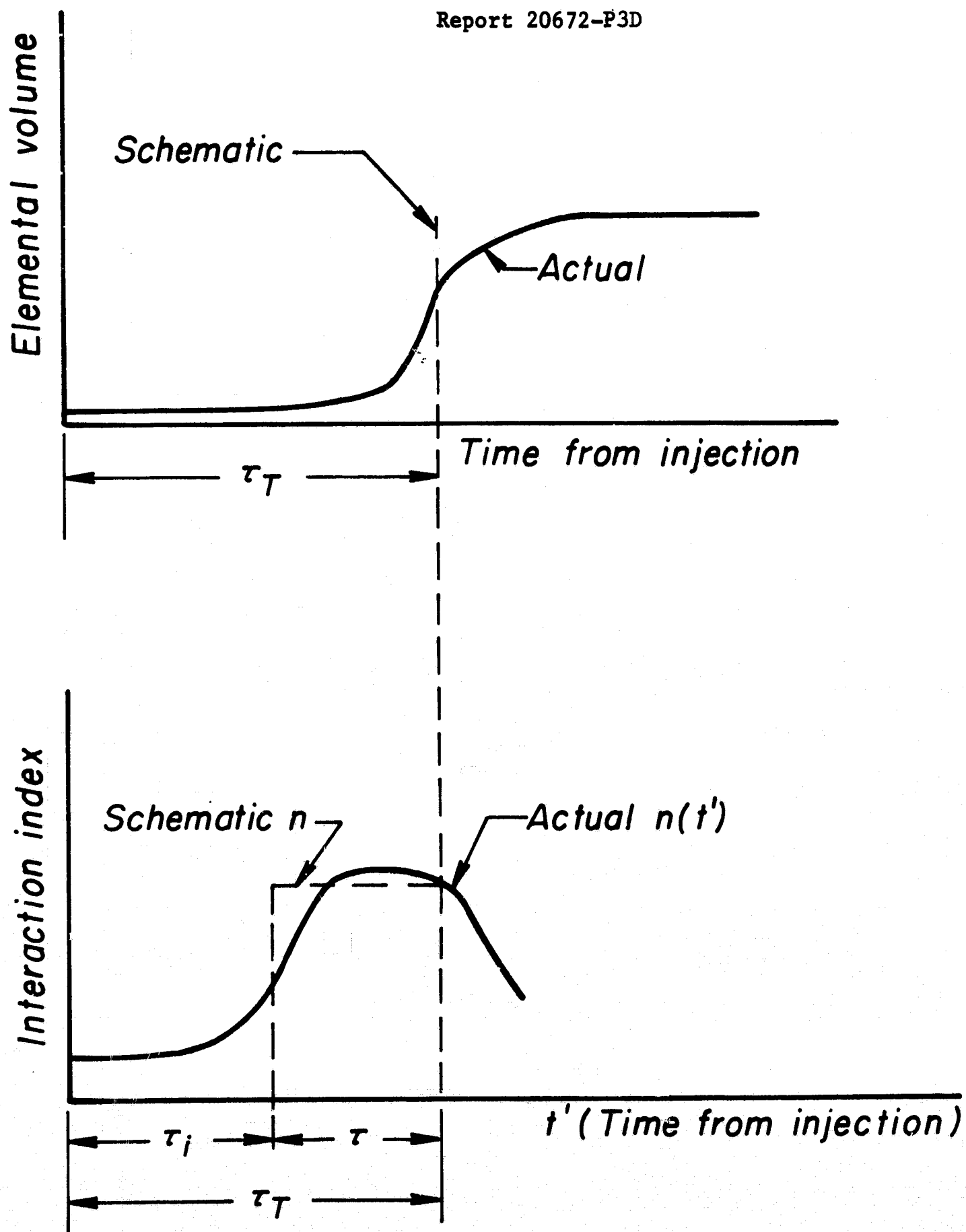


Figure 3 -- Time Lag Schematization

I, B, Technical Description (cont.)

2. The Sensitive Time Lag Concept

The liquid propellant combustion process may be represented schematically as in Figure 3, which shows the volume of a propellant element from the instant of injection until conversion to final combustion products is accomplished. Initially, in the liquid state, the volume is small. The conversion to gaseous products takes place more or less gradually, depending on the degree of atomization, the intensity of mixing, the chemical nature of the reactants, and so on. This gradual change is replaced, for analytical simplicity, by a step function, as shown in Figure 3. This is not to say that the combustion process is thought to occur instantaneously, but that such simplification may reasonably describe the essential nature of the combustion process from a dynamic point of view. The step function approximation of the overall combustion process defines a characteristic time, called the "total time lag" for the propellant element considered and denoted by τ_T . This time, of the order of a few milliseconds, is not representative of the characteristic times of high frequency instability. Rather, the total time lag is basic to the low frequency type of combustion instability. However, the total time lag, together with the velocity history of the injected propellants, determines the space lag, that is, the location in the chamber at which combustion of the particular propellant element is taking place. It is this aspect of the total time lag that cannot be disregarded when considering high frequency instability. This point will be discussed further in a later section. It suffices to say here that the step function approximation is compatible with any combustion distribution in the chamber, since τ_T can be different for different propellant elements.

The lower diagram of Figure 3 illustrates the important concept that not all of the processes that occur during the combustion of liquid propellants are equally affected by the combustion environment. Consider first only the effect of pressure (and correlated temperature) oscillations.

I, B, Technical Description (cont.)

Initially, only liquid streams, ligaments, or relatively large droplets are available in conditions unfavorable to combustion. It is to be expected that pressure perturbations interact only slightly with propellants in that degree of preparation. In the later portion of the preparation phase (as the nominal τ_T value is approached), droplets are small and interspersed; moreover fuel and oxidizer species are mixed. Hence, burning rates are readily influenced by instantaneous pressure changes. This increased sensitivity to disturbances in pressure and temperature is represented on the figure by a higher instantaneous "pressure interaction index", n . The portion of the total time lag that is associated with this high sensitivity can be assigned an approximately constant mean value of the interaction index and is called the "sensitive time lag", denoted by τ . The early portion is referred to as the "insensitive time lag", τ_i , and the corresponding interaction index is taken to be zero. Again, the step-function approximation is useful to reach a simple analytical description. The sensitive time lag is of the order of a few tenths of a millisecond (an order of magnitude smaller than the total time lag). It plays the role in the frequency combustion instability of the "characteristic time" discussed previously. The energy feedback resulting from pressure oscillations can be calculated in terms of τ and the interaction index, n . Similarly the energy feedback resulting from gas velocity oscillations can be calculated in terms of a velocity-sensitive time lag (which may coincide with the one already defined) and of an appropriate "velocity interaction index", and the same would apply to the interaction with other possible physical quantities.

In the schematic representation just discussed, two (or more) combustion parameters have been defined that are of key importance to further discussions of high frequency instability. The first is the sensitive time lag, τ , which places the combustion process in proper perspective with the times corresponding to the various acoustic modes associated with the chamber geometry. The second parameter, the interaction index, n (and, possibly, other parameters, representing the additional interaction indices of relevance), must exceed a certain minimum level if the damping is to be balanced and self amplifying or sustained oscillations are to be generated.

I, B, Technical Description (cont.)

These points are further clarified (where only one interaction index is relevant) by a stability diagram using τ as the abscissa and n as the ordinate (Figure 4), relative to the first tangential (1T) mode. As already observed, the resonant behavior of the system is such that the maximum amount of energy feedback is obtained when the ratio of the characteristic time to the oscillation period (in this case of the 1T mode) is around a certain value (which for the present simple combustion model is $1/2$). At the corresponding value of τ , evidently, the damping processes can be balanced with the minimum value of $n_{\min}$. At other values of τ the effectiveness of the feedback process decreases, and hence larger values of n are required to reach the balance, the values increasing with increasing deviations of τ from the value of maximum effectiveness. The curve of Figure 4 represents, for the given mode, the n - τ combinations for which a balance between feedback and damping is obtained. If, for given τ , n is smaller than the value which produces the balance, perturbations will decay as a result of the excessive damping; if on the contrary n is larger, perturbations will amplify as a result of the excessive feedback. Hence the curve provides the stability boundary between the stable region of Figure 4 (under the curve) and the unstable region (above the curve).

A given combustion system is characterized by a certain value of the interaction index. If this value is less than $n_{\min}$ no instability at all is possible. If the interaction index is just equal to $n_{\min}$, then instability is possible, but only if the matching of the proper time of the chamber and the characteristic combustion time is perfect. For $n > n_{\min}$ the time-matching requirement is less stringent, i.e., a certain mismatch of the times will still result in instability. As the value of n increases, the amount of allowable mismatching also increases.

For other modes than the 1T mode, the stability boundary can be represented in a similar way, the scale of τ properly shifted. Putting the stability boundaries corresponding to different modes on the same plot results

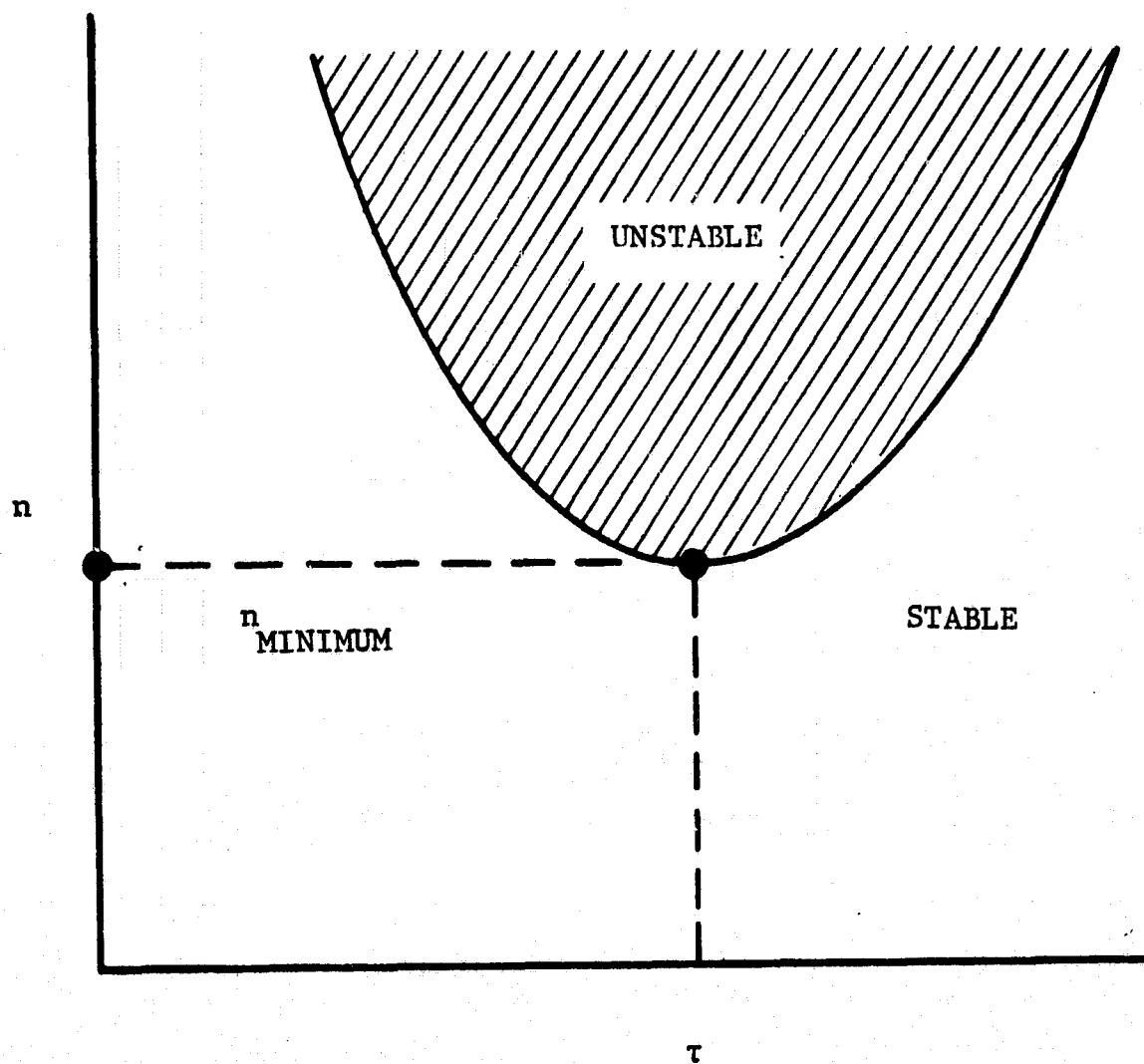


Figure 4 -- Stability Diagram, n, τ , - Plane

uniformly distributed combustion,
 $\bar{u}_e = 0.10$

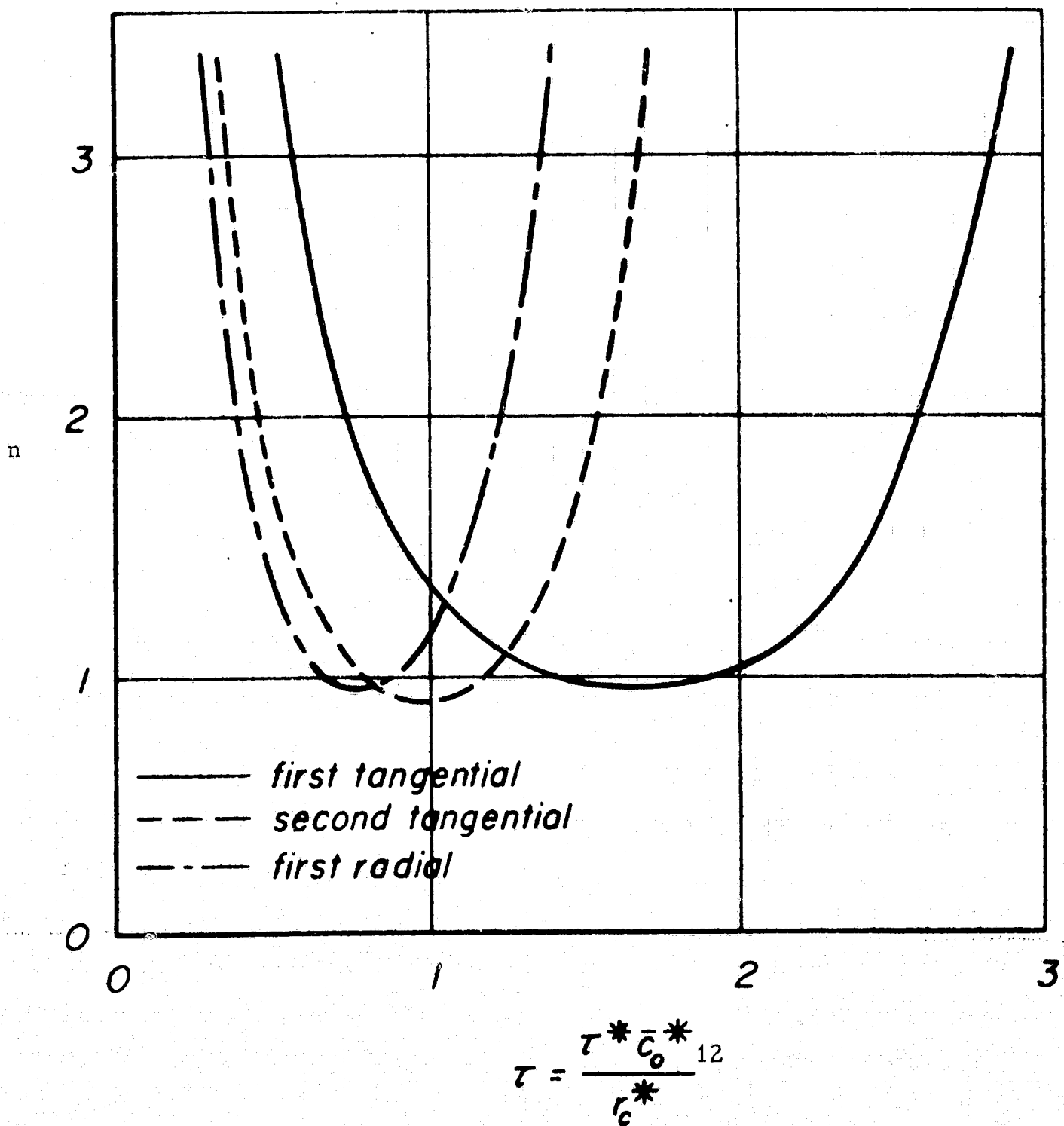


Figure 5 -- Theoretical Stability Limits for Several Purely Transverse Modes

I, B, Technical Description (cont.)

in Figure 5, where an overall stability region, regardless of mode, is now evident. Plots of this kind are fundamental to the methods to be discussed extensively in this report. However, two remarks, to be substantiated later, should be made.

The first remark is that such a clearcut boundary between stability and instability is obtained only if the perturbations are assumed to be of small amplitude. This is indeed the assumption on which most of the theoretical developments on combustion instability have been based. Within this assumption all effects of the perturbations can be assumed to vary linearly with the perturbation amplitudes, the mathematical treatment is accordingly substantially simpler, and the (linear) stability boundary unequivocally defined, the linearly unstable region being practically interpreted as that in which oscillations grow spontaneously out of the random combustion noise. However, in real practice the perturbations are not necessarily limited to the range where all of their effects vary linearly with the amplitude, and when the contrary happens important nonlinear effects may appear. Some effects of nonlinearity can be derived theoretically, at the cost of substantial mathematical complexities. But what is important to the present qualitative discussion is that, while the linearly unstable region always remains a region of instability, the corresponding statement for the linearly stable region is not true. In other words, a system corresponding in the $n-\tau$ plane to a point of the linearly stable region may be triggered into amplifying or self-sustaining oscillations by a perturbation (for instance, a pulse) of sufficiently large amplitude. Only if the pulse remains under a certain critical level do the resulting oscillations decay - in agreement with the predictions of the linear theory. This nonlinear behavior plays an important role in rockets, and has to be taken into account when interpreting the experimental results.

I, B, Technical Description (cont.)

The second remark is that the curves of Figures 4 and 5 depend not only on the chamber and nozzle geometries, but also on the transverse and longitudinal distributions of combustion, the first being substantially determined by the distribution of the injection flux across the injector, the second by the details of the individual combustion processes (such as atomization, evaporation, mixing and chemical reactions, recirculation flow, etc.). The significance of the total time lag τ_T in this connection has been mentioned already. Large values of τ_T , other factors being equal, will spread the combustion toward the nozzle end of the chamber. If the total time lag is too large for the chamber in question, performance will suffer because of incomplete burning prior to the nozzle entrance. Since the region of maximum combustion is closely associated with the region where propellants have reached the sensitive state in the preparation process, it is in this general location that interaction between the combustion process and the acoustic modes reaches a maximum level.

For the fundamental longitudinal mode, pressure antinodes are found at the injector and the nozzle end of the chamber, as shown in Figure 6. Higher harmonics will have additional pressure antinode locations. If combustion is uniformly distributed from one end of the chamber to the other, maximum instability coupling cannot take place since there is a region (or regions, in the harmonics) in which a pressure nodal environment is approached. If the wave is sinusoidal, a true node is found; otherwise, only an oscillation with reduced amplitude will be observed. The limiting case of a non-sinusoidal wave is that of a shock wave, the amplitude variation of which is shown in Figure 6. In the nodal region, even with proper time phasing, too little energy is available from the increased burning rate to cause the pressure oscillations to be amplified. However, if combustion is concentrated at the injector end, the best environment for energy transfer to the pressure oscillations is provided.

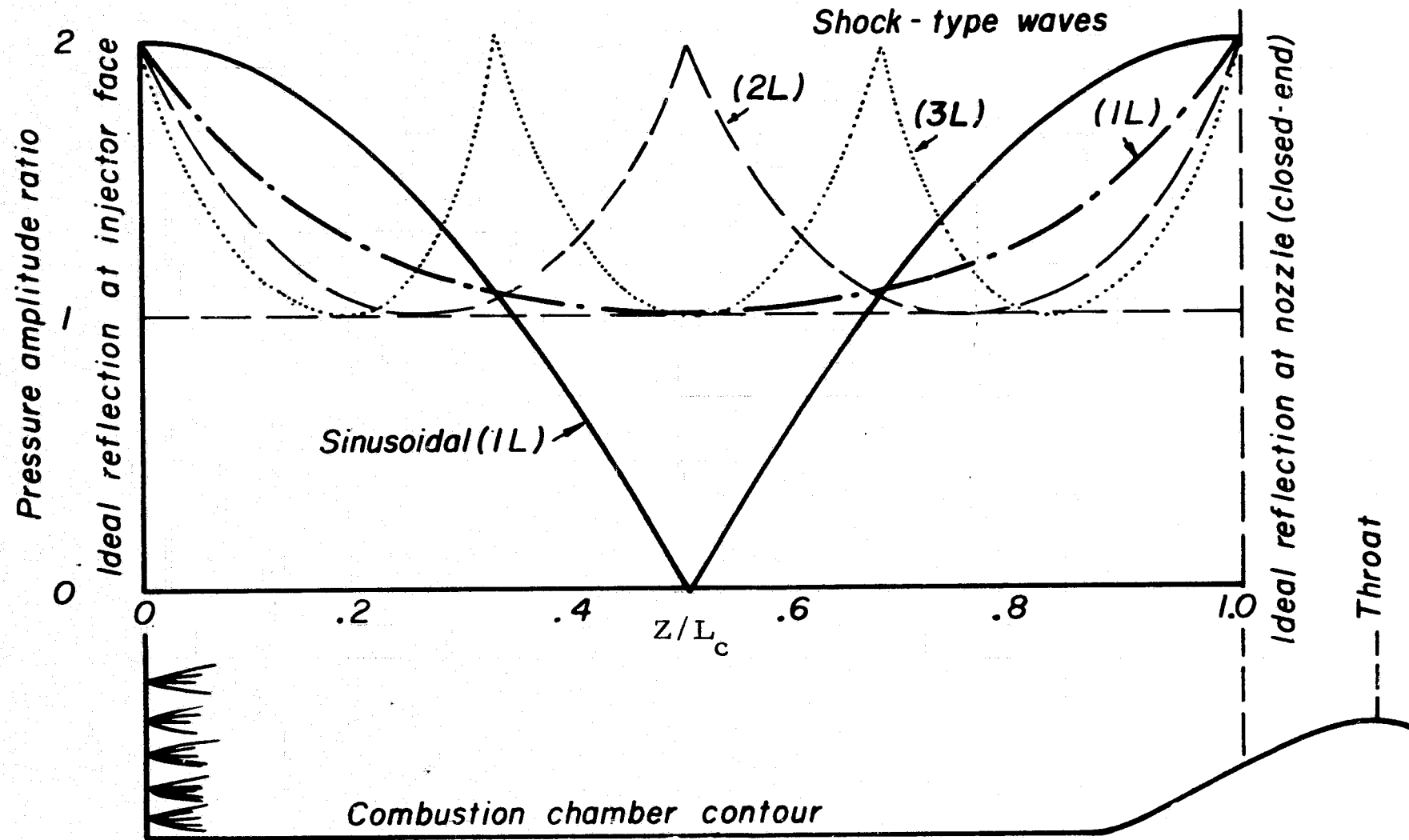


Figure 6 -- Axial Pressure Variations for Several Longitudinal Modes

I, B, Technical Description (cont.)

The steady state combustion rate is approximately proportional to the rate of change of axial gas velocity with distance from the injector. Typical velocity profiles are shown in Figure 7. Two extremes are apparent: (1) the concentrated combustion case in which near-maximum gas velocities are produced within a short distance from the injector, as shown by curve "A", and (2) the nearly uniform combustion case in which the axial velocity curve is nearly linear, as represented by curve "B". Rocket experience has shown that the actual velocity distributions fall within these extremes. Thus, combustion concentrated near the nodal point and combustion concentrated near the antinode at the nozzle end are both unrealistic situations.

The transverse modes may also be discussed on a somewhat similar basis. Experimental measurements at a number of laboratories have shown that maximum amplitudes for the tangential modes are always found at the injector end. Thus, to reduce coupling, axially distributed combustion can offer considerable improvement as compared to the concentrated combustion case, just as was found for the longitudinal mode. In the radial direction the combustion distribution picture is more complicated. As shown in Figure 8, pressure oscillation amplitudes vary radially, and the vibration is strongly dependent on the mode. The first and second tangential modes are similar to each other but quite different from the first radial mode. To promote instability in the first tangential mode, assuming that the n , τ values are suitable, one would choose an injector design that would provide rapid burning (i.e., small τ_T , to keep the combustion near the injector face) and would have the injection orifices concentrated at the outer radii. Either spreading the combustion axially or moving the injection toward the center of the chamber would reduce the degree of coupling. If the first radial mode is also considered, then a compromise location near the half-radius point would prove to be the best injection point. This principle has been verified in an investigation of a number of injection distributions by the Aerojet-General Corporation using high-thrust hardware (Ref 2).

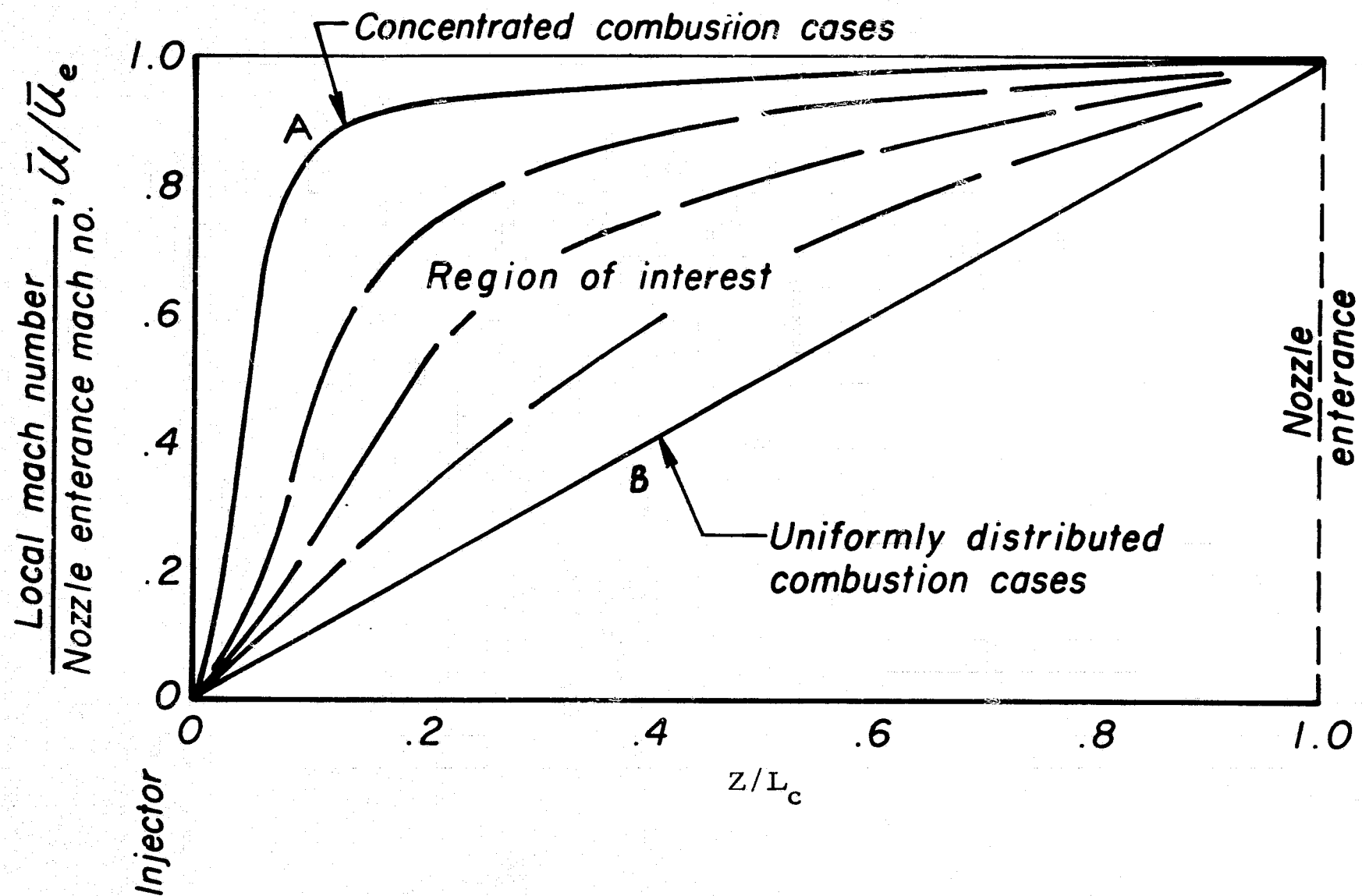


Figure 7 -- Typical Combustion Distributions

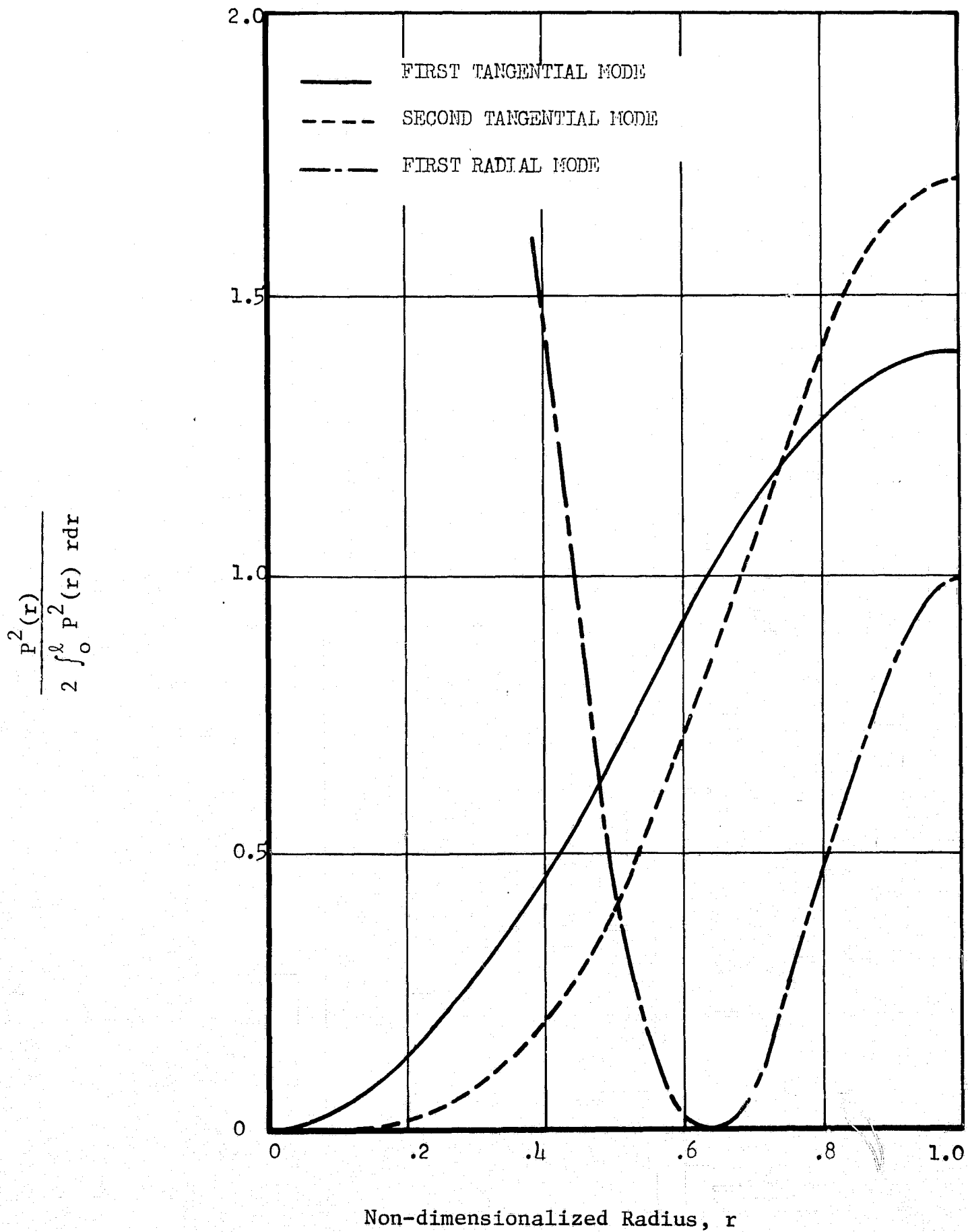


Figure 8 -- Pressure Non-Uniformity Coefficient for Concentrated Combustion

I, B, Technical Description (cont.)

The qualitative discussion just given is based on the assumption that there is only one relevant interaction index. When more than one has to be taken into account, the effect of each additional interaction index produces a displacement of the stability boundaries in the $n-\tau$ plane, and the interpretation becomes correspondingly more difficult. Effects of this kind are found, for instance, when the combustion rates are sensitive to the transverse velocity perturbations characteristic of transverse oscillations. One can visualize important coupling mechanisms related to the inequities in the displacements of liquid drops and propellant vapors that are produced by such oscillations. The local mixture ratio can be altered, with resulting changes in the burning rate, if one of the propellants is displaced with respect to the other.

It is interesting in this connection to observe that this effect may be particularly intense in the region immediately adjacent to the injector where one can expect droplets and vapors to be present in abundance, and in very unmixed conditions. Thus, it is clear that in the use of baffles and liners there must be a consideration of the combustion distribution in the chamber. In general, it has been found that for tangential modes the oscillation amplitudes are greatest at the injection end of the chamber, combustion rates are maximum a few inches from the injector, and coupling between the combustion process and tangential waves is greatest in the outer regions of the injector face. If the chamber volume is subdivided by baffles, the dimensions of the cavities between baffles determine the period of oscillation. Combining these principles, it is concluded that if baffles are to be used to control tangential modes they must be placed at the injector end, they must

I, B, Technical Description (cont.)

protect the preparation zone found in the first few inches downstream of the injector, they must extend as close as possible to the chamber wall, and the circumferential blade spacing must be small enough to prevent cavity modes from existing. Similarly, acoustic liner orifices are most effective near the injector end of the chamber, and liner absorption characteristics must be designed to match the resonance properties of the chamber and the combustion process, as described by the sensitive time lag theory.

3. Development of the Sensitive Time Lag Theory

The historical background of the development of the sensitive time lag theory is important to the understanding of the concepts described in the preceding section. The earliest published paper on combustion instability theory was that of Gunder and Friant (Ref 3) in 1950, with a subsequent discussion by Yachter (Ref 4). Probably the most important contribution of these early treatments was the introduction of the concept of a combustion time lag (conceived independently, but not published, by other groups) between the instant of injection of a propellant element and the succeeding instant of burning, in which the propellant element is transformed into hot gas capable of contributing to the chamber pressure.

Interest at Princeton University in the problem of combustion instability in liquid propellant rocket motors was given impetus by a Bureau of Aeronautics Symposium held at the Naval Research Laboratory in December, 1950. This interest resulted in theoretical analyses by Professors Summerfield and Crocco.

Professor Summerfield's work (Ref 5) considered the effects of inertia in the propellant feed lines and the capacitance of the combustion chamber, assuming a constant combustion time lag. His analysis treated the case of low frequency oscillations for frequencies up to about 200 cps.

I, B, Technical Description (cont.)

Professor Crocco advanced the concept of the pressure dependence of the combustion time lag. His paper (Ref 1) presented the fundamentals resulting from this concept. In his paper, Crocco treated the case of low frequency instability in a bipropellant rocket, and also the case of high frequency instability with combustion concentrated at the injector end of the combustion chamber.

The analytical work on the high frequency case was continued by S. I. Cheng under the direction of Professor Crocco. His studies of the effects of the axial distribution of combustion (on the longitudinal modes) were published as his Ph.D. thesis. A thorough discussion of these and other aspects of the theory was published by Crocco and Cheng in 1956 as an AGARD monograph (Ref 6). The general theories of low and longitudinal high frequency instability, the effects of the combustion distribution, the influence of the exhaust nozzle, as well as the (scarce) experimental evidence substantiating the analyses were all discussed at length. A brief discussion of the transverse modes of combustion instability was included, and general adherence to the sensitive time lag model was predicted.

The extension to the transverse modes was initiated by S. M. Scala (Ref 7). Following Crocco's pressure dependence model, he determined the fundamental behavior of the transverse modes, including the influence of the nozzle. In addition, Scala treated the case of intermediate frequency instability, in which the coupling mechanism consists of entropy perturbations, generated by off-design mixture ratio combustion, which reflect from the nozzle as pressure waves and propagate back to the injector to cause perturbations in the injection rates.

I, B, Technical Description (cont.)

The study of transverse instability was continued by F. H. Reardon (Ref 8), who developed several extensions to the basic theory to explain certain experimental results (which are discussed in the following section). The sensitive time lag concept was extended to include sensitivity to the transverse components of the oscillating gas velocity. The effect on the combustion rate was visualized in the oscillatory displacement of the vapors of one propellant with respect to the liquid droplets of the other. In addition, Reardon introduced an approximate treatment of the effects of nonuniform distribution of propellant injection on the transverse modes, and applied the modified theory to a sector-shaped combustor, which simulates the "pocket-mode" behavior of a baffled chamber.

I, B, Technical Description (cont.)

4. Theory

a. General Approach

The simplifying assumptions on which the mathematical treatment of combustion instability is based are the following:

(1) The substance contained in the combustion chamber is either in the form of liquid propellants, of practically zero volume, or in the form of complete combustion gases. It will be noticed that this assumption disregards the contribution of the propellants in vapor form, and of the intermediate products of combustion. Hence the assumption is equivalent to saying that not only the liquids, but also the vapors and the intermediates occupy a negligible volume compared to the final products of combustion. This assumption (first used by Crocco in Ref. 1) is actually in agreement with the step-function combustion model discussed in Section I,B,2. Even more important, it represents quite well the actual conditions in rockets where indeed, with the exception of the region immediately adjacent to the injector, practically the same gas temperature is observed throughout the chamber. Of course, it is clear that the above is true when using liquid propellants, and not for the combustion of gaseous propellants. The difference is that while in the latter case there is a constant mass flux with energy addition, in the case of liquid propellants the gas flow has a variable mass flux with energy addition being produced through an addition of mass to the gaseous flow.

(2) The combustion gases are of constant composition, they obey the perfect gas law and have constant specific heats.

(3) Frictional effects on the walls are neglected, and only those are taken into account which result in the liquid droplet drag.

I, B, Technical Description (cont.)

Also, Reynolds stresses associated with the high turbulence level caused by combustion are neglected, in spite of the fact that they may play an important role with respect to the uniformity assumption (Assumption 5).

(4) The flow of injected propellants is unaffected by oscillations in the chamber, and hence the injection flux and velocity are always the same as in steady conditions. This assumption is not necessarily verified in actual rockets where, especially for large rockets, the possibility of matching the wave propagation times in the feed lines and in the chamber may lead to interactions. However, the assumption is rather good if matching is avoided, and substantially simplifies the treatment by making the chamber behavior independent of the feed system varieties and complexities. The resulting instability problem has been termed "intrinsic instability" of the combustion chamber.

(5) The steady-state gas flow is uniform across any chamber section. This is possible partly because the previous assumption allows the boundary layer formation on the walls to be disregarded. However, it involves more. For instance, it would require the injected propellants to be uniformly distributed so as to produce no recirculation. Of course, this is not the actual situation, and the uniform flow considered in the theory should be interpreted as an average flow from which the actual flow can depart substantially if the injection is far from being uniform. For large rockets the injection systems are generally rather uniform, and hence the uniformity assumption can be quite accurate. However, it will be seen in the following that transverse stability conditions can be improved by using particular non-uniform injection systems. It is felt that the contradiction that results in these cases is not very important for not too large flow Mach numbers because of the strongly equalizing effects of the high turbulence due to combustion.

I, B, Technical Description (cont.)

(6) In steady-state, the total energy (internal and kinetic) of the droplets remains constant. Obviously this is not exactly true, because of the heat exchanges affecting the internal energy and the droplet drag affecting the kinetic energy in ways that are not so simply related. The assumption has, however, the advantage of providing a substantial simplification of the treatment, and it is believed not to hide any of the fundamental effects.

(7) The steady-state flow in the nozzle is one-dimensional. Although not true, this assumption is known to result in very accurate predictions concerning the steady flow itself. Here, however, the same assumption is extended to the treatment of the oscillatory nozzle flow obtained when the steady flow is perturbed. It should be observed that this assumption is consistent with that of uniform steady flow in the chamber (Assumption 5).

(8) The unsteady, oscillatory quantities in the chamber and in the nozzle can be obtained by superposing small perturbations to the steady-state quantities. By "small" is meant as usual, that only "linear" (first order) terms in the perturbations are to be retained, while terms containing products or powers of perturbations (second and higher order) are to be neglected. The great advantage of this assumption is of a mathematical nature, since the resulting equations, being linear, can be treated in a much simpler way. One of the simplifications is that a harmonic time dependence can be chosen, as discussed under Assumption 10. But, of course, the disadvantage is that only the linear effects can be accounted for, all the nonlinear effects being left out. Under the small perturbation assumption only the "linear" stability problem can be attacked.

(9) In the distributed combustion analysis the gas flow Mach number is always sufficiently small so that the square can be neglected compared to unity. Because of this assumption, the analysis cannot

I, B, Technical Description (cont.)

be applied to thrust chambers in which the Mach number exceeds 0.3. Such is not an essential assumption; its relaxation leads only to complexity. This assumption does not apply to the concentrated combustion analysis.

(10) The time dependence of the perturbations can be expressed in complex form as $\exp(\sigma t)$ where $\sigma = \lambda + i\omega$ is the same complex quantity for all perturbations. Here ω is the angular frequency and λ the amplification coefficient. As usual when using the complex representation each perturbation is obtained by multiplying the above exponential by a corresponding complex amplitude factor which is a function of the space coordinates only, and by taking the real part of the result. The assumption of exponential time dependence is common in problems of linear stability. It does not affect the generality of the result, since within the linear framework the development of the most general perturbation can be obtained from the superposition of independent components each behaving exponentially, and since, from the point of view of stability, the only thing that matters is that no single component should show amplification. Hence all the information required is the behavior of exponentially varying perturbations when the frequency is made to change over all possible values.

b. Governing Equations

(1) General Equations

The equations governing the unsteady, two-phase flow in the combustion chamber are derived from the principles of conservation of mass, momentum, and energy, and the equation of state, according to the assumptions discussed in the previous section. It is convenient to work with the equations in nondimensional form. The reference quantities for the nondimensionalization are taken as the stagnation gas properties at the injector face (pressure, temperature, density, and speed of sound) together with a

I, B, Technical Description (cont.)

reference length, which is the chamber radius for transverse modes. (The chamber length is generally used for longitudinal modes). The governing equations take the following forms:

Conservation of mass,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{q}) = Q = - \frac{\partial \rho_L}{\partial t} - \nabla \cdot (\rho_L \vec{q}_L) \quad (1)$$

Conservation of momentum,

$$\rho \frac{\partial \vec{q}}{\partial t} + \rho \vec{q} \cdot \nabla \vec{q} + \frac{1}{\gamma} \nabla p = (Q + \kappa \rho_L) (\vec{q}_L - \vec{q}) \quad (2)$$

Conservation of energy,

$$\rho \frac{\partial h_s}{\partial t} + \rho \vec{q} \cdot \nabla h_s - \frac{\gamma-1}{\gamma} \frac{\partial p}{\partial t} = Q (h_{Ls} - h_s) \quad (3)$$

Equation of state for the gas,

$$p = \rho T \quad (4)$$

Droplet drag (gas/liquid momentum interchange) using Stokes law,

$$\frac{\partial \vec{q}_L}{\partial t} + \vec{q}_L \cdot \nabla \vec{q}_L = \kappa (\vec{q} - \vec{q}_L) \quad (5)$$

And droplet energy,

$$\frac{\partial h_{Ls}}{\partial t} + \vec{q}_L \cdot \nabla h_{Ls} = 0 \quad (6)$$

I, B, Technical Description (cont.)

The symbols used in these equations are defined in Section I.D.

It is necessary to express the governing Equations, (1) to (6), in steady state form and perturbed form. The separation is accomplished by writing all the dependent variables, as the sum of a steady state solution and an unsteady solution. For example,

$$p = \bar{p} + p' e^{\sigma t} \quad (7)$$

$$u = \bar{u} + u' e^{\sigma t}$$

It is noted that the perturbed portion of each parameter is harmonic with respect to time.

(2) Steady State Equations

The steady state solution is assumed to be one dimensional, although the perturbations are considered in three dimensions. Therefore, the steady state vectorial components of gas velocity, $\vec{q}$, and liquid velocity, $\vec{q}_L$, are given by the following relationships:

$$\begin{aligned} \vec{q} &= \bar{u} , \quad \vec{q}_L = \bar{u}_L \\ \bar{v} &= \bar{w} = \bar{v}_L = \bar{w}_L = 0 \end{aligned} \quad (8)$$

where u, u_L are the axial components, v, v_L are the radial components, and w, w_L are the tangential components of $\vec{q}, \vec{q}_L$.

I, B, Technical Description (cont.)

Substituting the relationships given by (7) and (8) into (1) to (6) and neglecting the time dependence yields the following system of steady state equations:

Conservation of mass

$$\frac{d}{dz} (\bar{\rho} \bar{u}) = \bar{Q} = - \frac{d}{dz} (\bar{\rho}_L \bar{u}_L) \quad (9a)$$

Conservation of momentum

$$\frac{d}{dz} (\bar{\rho} \bar{u}^2) + \frac{d}{dz} (\bar{\rho}_L \bar{u}_L^2) = - \frac{1}{\gamma} \frac{d\bar{p}}{dz} \quad (9b)$$

Conservation of energy

$$\bar{\rho} \bar{u} \frac{d\bar{h}_s}{dz} = - \bar{Q} (\bar{h}_s - \bar{h}_{Ls}) \quad (9c)$$

Equation of state (gas)

$$\bar{p} = \bar{\rho} \bar{T} \quad (9d)$$

Droplet drag

$$\bar{u}_L \frac{d\bar{u}_L}{dz} = \kappa (\bar{u} - \bar{u}_L) \quad (9e)$$

Droplet energy

$$\frac{d\bar{h}_{Ls}}{dz} = 0 \quad (9f)$$

I, B, Technical Description (cont.)

It should be noted that at $Z = 0$, $u = 0$ and $\bar{p} = \bar{\rho} = 1$ because of the nondimensionalizing scheme. At $Z = L_c$, combustion is assumed to be complete so that $\bar{\rho}_L = 0$ and $\bar{Q} = 0$.

(3) The Perturbation Equations: Wave Equations

The perturbed equations are considered in three dimensional form in cylindrical coordinates and the resulting system of equations is linearized. That is, the perturbed quantities are considered to be small, and therefore, all products of two or more perturbed terms are considered to be zero. Substituting Equations (7) and (8) into Equations (1) to (6) yields the following system of equations.

$$\begin{aligned} \left(\sigma + \frac{d\bar{u}}{dZ} \right) \rho' + \bar{u} \frac{\partial \rho'}{\partial Z} + \frac{d\bar{\rho}}{dZ} u' + \bar{\rho} \left(\frac{\partial v'}{\partial r} + \frac{v'}{r} \right. \\ \left. + \frac{1}{r} \frac{\partial w'}{\partial \theta} + \frac{\partial u'}{\partial Z} \right) = \bar{Q} (Pp' + Rv' + Tw') \end{aligned} \quad (10a)$$

where

$$P = n (1 - e^{-\sigma \tau}), \quad R = \frac{1}{r} (1 - e^{-\sigma \tau}) = \frac{1}{n} P, \quad \text{and} \quad T = \frac{1}{\theta} (1 - e^{-\sigma \tau}) = \frac{1}{n} P$$

Conservation of axial momentum

$$\begin{aligned} \left(\sigma \bar{u} + 2\bar{u} \frac{d\bar{u}}{dZ} \right) \rho' + \left(\sigma \bar{u}_L + 2\bar{u}_L \frac{d\bar{u}_L}{dZ} \right) \rho'_L + \bar{u}^2 \frac{\partial \rho'}{\partial Z} + \bar{u}_L^2 \frac{\partial \rho'_L}{\partial Z} \\ + \left(\sigma \bar{\rho} + 2\bar{\rho} \frac{d\bar{u}}{dZ} + 2\bar{u} \frac{d\bar{\rho}}{dZ} \right) u' + \left(\sigma \bar{\rho}_L + 2\bar{\rho}_L \frac{d\bar{u}_L}{dZ} + 2\bar{u}_L \frac{d\bar{\rho}_L}{dZ} \right) u'_L \\ + 2\bar{\rho} \bar{u} \frac{\partial u'}{\partial Z} + 2\bar{\rho}_L \bar{u}_L \frac{\partial u'_L}{\partial Z} + \bar{\rho} \bar{u} \left(\frac{\partial v'}{\partial r} + \frac{v'}{r} + \frac{1}{r} \frac{\partial w'}{\partial \theta} \right) \\ + \bar{\rho}_L \bar{u}_L \left(\frac{\partial v'_L}{\partial r} + \frac{v'_L}{r} + \frac{1}{r} \frac{\partial w'_L}{\partial \theta} \right) = - \frac{1}{\gamma} \frac{\partial p'}{\partial Z} \end{aligned} \quad (10b)$$

I, B, Technical Description (cont.)

Conservation of radial momentum

$$\begin{aligned} & (\sigma \bar{\rho} + \bar{\rho} \frac{d\bar{u}}{dZ} + \bar{u} \frac{d\bar{\rho}}{dZ}) v' + (\sigma \bar{\rho}_L + \bar{\rho}_L \frac{d\bar{u}}{dZ} + \bar{u}_L \frac{d\bar{\rho}_L}{dZ}) v'_L \\ & + \bar{\rho} \bar{u} \frac{\partial v'}{\partial Z} + \bar{\rho}_L \bar{u}_L \frac{\partial v'_L}{\partial Z} = - \frac{1}{\gamma} \frac{\partial p'}{\partial r} \end{aligned} \quad (10c)$$

Conservation of tangential momentum

$$\begin{aligned} & (\sigma \bar{\rho} + \bar{\rho} \frac{d\bar{u}}{dZ} + \bar{u} \frac{d\bar{\rho}}{dZ}) w' + (\sigma \bar{\rho}_L + \bar{\rho}_L \frac{d\bar{u}_L}{dZ} + \bar{u}_L \frac{d\bar{\rho}_L}{dZ}) w'_L \\ & + \bar{\rho} \bar{u} \frac{\partial w'}{\partial Z} + \bar{\rho}_L \bar{u}_L \frac{\partial w'_L}{\partial Z} = - \frac{1}{\gamma r} \frac{\partial p'}{\partial \theta} \end{aligned} \quad (10d)$$

Conservation of energy and equation of state, combined

$$\begin{aligned} & (\sigma \bar{\rho} + \bar{Q} - \bar{u} \frac{d\bar{\rho}}{dZ}) \rho' + \bar{\rho} \bar{u} \frac{\partial \rho'}{\partial Z} - (\gamma-1) \bar{\rho} \bar{u} (\sigma \bar{\rho} + \bar{Q} + \bar{\rho} \frac{d\bar{u}}{dZ}) u' \\ & - (\gamma-1) (\bar{\rho} \bar{u})^2 \frac{\partial u'}{\partial Z} = (\sigma \frac{\bar{\rho}}{\gamma} + \bar{Q} - \bar{u} \frac{d\bar{\rho}}{dZ}) p' + \bar{\rho} \bar{u} \frac{\partial p'}{\partial Z} \end{aligned} \quad (10e)$$

Droplet dynamics

defining

$$K = \frac{\kappa}{\kappa + \sigma}$$

and

$$\xi = (\kappa + \sigma) \int_0^Z \frac{dZ}{\bar{u}_L}$$

then

$$u'_L = Ku' - \frac{K}{\bar{u}_L} e^{-\xi} \left[\int_0^Z \frac{d\bar{u}_L}{dZ} e^{\xi} u' dZ + \int_0^Z \bar{u}_L e^{\xi} \frac{\partial u'}{\partial Z} dZ \right] \quad (10f)$$

I, B, Technical Description (cont.)

$$v'_L = K (1-e^{-\xi}) v' - K e^{-\xi} \int_0^Z e^{\xi} \frac{\partial v'}{\partial Z} dZ \quad (10g)$$

$$w'_L = K (1-e^{-\xi}) w' - K e^{-\xi} \int_0^Z e^{\xi} \frac{\partial w'}{\partial Z} dZ \quad (10h)$$

$$\begin{aligned} & (\sigma + \frac{d\bar{u}}{dZ}) \rho' + (\sigma + \frac{d\bar{u}_L}{dZ}) \rho'_L + \bar{u} \frac{\partial \rho'}{\partial Z} + \bar{u}_L \frac{\partial \rho'_L}{\partial Z} \\ & + \bar{\rho} \left[\frac{\partial v'}{\partial r} + \frac{v'}{r} + \frac{1}{r} \frac{\partial w'}{\partial \theta} + \frac{\partial u'}{\partial Z} \right] + \bar{\rho}_L \left[\frac{\partial v'_L}{\partial r} + \frac{v'_L}{r} \right. \\ & \quad \left. + \frac{1}{r} \frac{\partial w'_L}{\partial \theta} + \frac{\partial v'_L}{\partial Z} \right] + \frac{d\bar{\rho}}{dZ} u' + \frac{d\bar{\rho}_L}{dZ} u'_L = 0 \end{aligned} \quad (10i)$$

In general, separation of variables is not possible with these equations. A solution can be obtained by writing each quantity in a series such that each successive term in the series is less than its predecessor. Therefore, the pressure perturbation is written in the form

$$p' = p_0 + p_1 + p_2 + p_3 + \dots p_n + \dots \quad (11)$$

where p_0 can assume any magnitude within the restrictions of the analysis. Then $p_1 = O(\bar{u}_e \cdot p_0)^*$, $p_2 = O(\bar{u}_e^2 \cdot p_0)$ and $p_n = O(\bar{u}_e^n \cdot p_0)$ where $\bar{u}_e$ is essentially the chamber Mach number. Applying this approach to Equations (10a) through (10i), collecting terms of like order, and solving for the pressure results in wave equations for p_0 and p_1 :

$$\sigma^2 p_0 - \nabla^2 p_0 = 0 \quad (12a)$$

* This notation is used to indicate the order of magnitude of a given parameter.

I, B, Technical Description (cont.)

$$\sigma^2 p_1 - \nabla^2 p_1 = \sigma \gamma \bar{Q} (P p_0 - R \frac{1}{\sigma \gamma \rho} \frac{\partial p_0}{\partial r} - T \frac{1}{\sigma \gamma \rho \theta} \frac{\partial p_0}{\partial \theta}) - \sigma \left[2 \frac{d\bar{u}}{dz} + (\gamma-1) \frac{\bar{Q}}{\rho} + \sigma \frac{\kappa}{\kappa+\sigma} \frac{\rho_L}{\rho} \right] p_0 \quad (12b)$$

For a cylindrical chamber the solutions of (12a) and (12b) are given by

$$p_0(z, r, \theta) = P_{00} \cosh(\Omega Z) \psi(r) \theta(\theta) \quad (13a)$$

$$p_1(z, r, \theta) = P_{00} (\Gamma_{11} P + \Gamma_{10}) \psi(r) \theta(\theta) \quad (13b)$$

where

$$\psi(r) = J_\nu(s_{\nu\eta} \cdot r) \quad (13c)$$

$$\theta(\theta) = \begin{cases} \cos \nu \theta & \text{STANDING MODE} \\ e^{-\nu \theta} & \text{SPINNING MODE} \end{cases} \quad (13d)$$

and J_ν represents the Bessel function of the first kind of order ν (see Table I)

Also,

$$\Gamma_{11} = \frac{1}{\Omega} (-\sigma \gamma A_{\nu\eta} + \frac{1}{n} B_{\nu\eta} + \frac{1}{n} C_{\nu\eta}) \int_0^Z \bar{Q}(\zeta) \sinh[\Omega(Z-\zeta)] d\zeta \quad (13e)$$

where

$$\Omega^2 = \sigma^2 + s_{\nu\eta}^2$$

the factors $A_{\nu\eta}$, $B_{\nu\eta}$ and $C_{\nu\eta}$ are discussed in Section I.B.4.b.(5), p. 61.

I, B, Technical Description (cont.)

$$\begin{aligned}
 \Gamma_{10} = \frac{\sigma}{\Omega} \left\{ (\gamma-1) \int_0^Z \bar{Q}(\zeta) \sinh [\Omega (Z-\zeta)] d\zeta \right. \\
 + 2 \int_0^Z \frac{d\bar{u}}{dZ}(\zeta) \sinh [\Omega (Z-\zeta)] d\zeta \\
 \left. + \frac{\kappa\sigma}{\kappa+\sigma} \int_0^Z \frac{\bar{\rho}_L}{\rho}(\zeta) \sinh [\Omega (Z-\zeta)] d\zeta \right\} \quad (13f)
 \end{aligned}$$

These solutions depend on the following boundary conditions:

$$Z = 0, \quad u' = 0$$

$$r = 1, \quad v' = 0$$

$$r = 0, \quad v' < \infty$$

To complete the analysis, the combustion terms must be considered in detail, and the boundary condition at the nozzle entrance (viz., the nozzle admittance condition) must be specified.

The preceding analysis has been for distributed combustion and has been limited to low Mach numbers. The extension of this analysis of high Mach numbers is so difficult it has not been successfully completed. An analysis at high Mach number has been performed using the simplifying assumption that the combustion is concentrated at a plane. Using this assumption, the conservation equations across the combustion front can be written as follows: (The + indicates before combustion and the - indicates after combustion)

I, B, Technical Description (cont.)

Conservation of mass

$$u'_+ + \dot{m}'_b = \rho'_- \bar{u}_- + \bar{\rho}_- u'_- \quad (14a)$$

Conservation of momentum

$$\frac{p'_+ - p'_-}{\gamma} = \bar{u}_-^2 \rho'_- + 2\bar{\rho}_- \bar{u}_- u'_- - \dot{m}'_b \bar{u}_L \quad (14b)$$

Conservation of energy

$$\begin{aligned} \bar{\rho}_+ \bar{h}_+ u'_+ + \dot{m}'_b (\bar{h}_L + (\gamma-1) \frac{\bar{u}_L^2}{2}) &= (\gamma-1) \left(\frac{\bar{u}_-^3}{2} \rho'_- + \bar{\rho}_- \frac{\bar{u}_-^2}{2} u'_- + \bar{\rho}_- \bar{u}_-^2 u'_- \right) \\ + \bar{h}_- \bar{u}_- \rho'_- + \bar{h}_- \bar{\rho}_- u'_- + \bar{\rho}_- \bar{u}_- h'_- &\quad (14c) \end{aligned}$$

Equation of state

$$p'_- = \rho'_- + T'_- \quad (14d)$$

Definition of enthalpy

$$h' = T' \quad (14e)$$

Upstream of the combustion front the mean velocity is zero. If the interaction between the gas and drops is neglected, the dynamics of the region upstream of the combustion is governed by the homogeneous Helmholtz equation:

$$\nabla^2 p' - \sigma^2 p' = 0 \quad (14f)$$

Downstream of the combustion front a nozzle admittance condition exists.

I, B, Technical Description (cont.)

For an annular chamber the modification to the cylindrical chamber analysis imposed by the annular geometry enters through the solution for $\psi(r)$. The differential equation for $\psi(r)$ is the classic Bessel equation which results in the following general solution:

$$\psi_{v\eta}(r) = C_1 J_v(s_{v\eta} \cdot r) + C_2 Y_v(s_{v\eta} \cdot r) \quad (15)$$

where J_v is the Bessel function of the first kind, Y_v is the Bessel function of the second kind, and $s_{v\eta}$ (where v specifies the order of the Bessel equation) is the transverse acoustic mode number. Specifically, this means that at all chamber walls

$$\frac{d\psi_v}{dr} = 0 \quad (16)$$

Applying this condition to (16) yields

$$\psi'_v = 0 = J'_v(s_{v\eta} \cdot r_c) + \frac{C_2}{C_1} Y'_v(s_{v\eta} \cdot r_c) \quad (17)$$

at the outer wall, $r_c = 1$, equation (17) reduces to

$$J'_v(s_{v\eta}) + \frac{C_2}{C_1} Y'_v(s_{v\eta}) = 0 \quad (18)$$

For the cylindrical chamber case, equation (18) reduces to

$$J'_v(s_{v\eta}) = 0 \quad (19)$$

since B must be zero because Y_v becomes infinite at $r = 0$. The solution of equation (19) serves to define the transverse acoustic mode number, $s_{v\eta}$, for cylindrical combustion chambers.

I, B, Technical Description (cont.)

The annular chamber has its inner wall located in the range $0 < r_i < 1$. Therefore, for the inner wall, equation (17) is written:

$$J'_v(s_{v\eta} \cdot r) + \frac{C_2}{C_1} Y'_v(s_{v\eta} \cdot r) = 0, @ r = r_i \quad (20)$$

whereas at the outer wall, $r = 1$ and equation (18) is still applicable. Solution of (18) and (20) simultaneously yields

$$J'_v(s_{v\eta}) Y'_v(s_{v\eta} \cdot R) - J'_v(s_{v\eta} \cdot R) Y'_v(s_{v\eta}) = 0 \quad (21)$$

The solution of equation (21) defines the transverse acoustic mode number, $s_{v\eta}$, for annular chambers. Fortunately, equation (21) has been solved in Reference (9) and the values of $s_{v\eta}$ are listed as a function of R , where $R = r_i/r_o$, on Table II.

Thus the use of an annular chamber will alter the frequencies of the transverse modes. The extent of the alteration is illustrated in Figure 9. In addition, radial injection distribution effects will be minimized by the use of the annular chamber so that in most cases the distribution coefficients $A_{v\eta}$, $B_{v\eta}$, and $C_{v\eta}$ can be assumed to be unity.

TABLE II

BESSEL FUNCTION VALUE $(s_{v\eta})_{\text{ann}}$ FOR TANGENTIAL MODES IN ANNULAR CHAMBERS *

Annular Chamber Geometry		Tangential Modes					
(Radius) Inner	(Radius) Outer	1T	2T	3T	4T	5T	6T
0.910		$(s_{v\eta})_{\text{ann}} = 1.04802$	2.09602	3.14401	4.19197	5.23989	6.28778
0.833		1.092	2.1846	3.27672	4.368	5.4588	6.5496
0.667		1.209	2.412	3.61	4.80	5.98	7.14
0.500		1.35	2.68	3.98	5.18	6.34	7.45
0.400		1.41	2.85	4.10	5.27	6.40	7.5
0.333		1.54	2.91	4.17	5.31	6.42	7.5
0.286		1.60	2.99	4.18	5.31	6.42	7.5
0.250		1.64	3.00	4.18	5.31	6.42	7.5
0.222		1.67	3.02	4.18	5.31	6.42	7.5
0.200		1.70	3.025	4.18	5.31	6.42	7.5

- NOTE: 1. Figure 9 gives the frequency ratio viz. $\frac{(s_{v\eta})_{\text{annular}}}{(s_{v\eta})_{\text{cylindrical}}}$ for different chamber geometries.
2. Calculation of annular chamber tangential frequencies

$$f^* = \frac{C_o^* (s_{v\eta})_{\text{annular}}^{12}}{2\pi r_c^*}$$

* Bridge and Angrist, "Math. of Computation, 16, 78," April 1962.

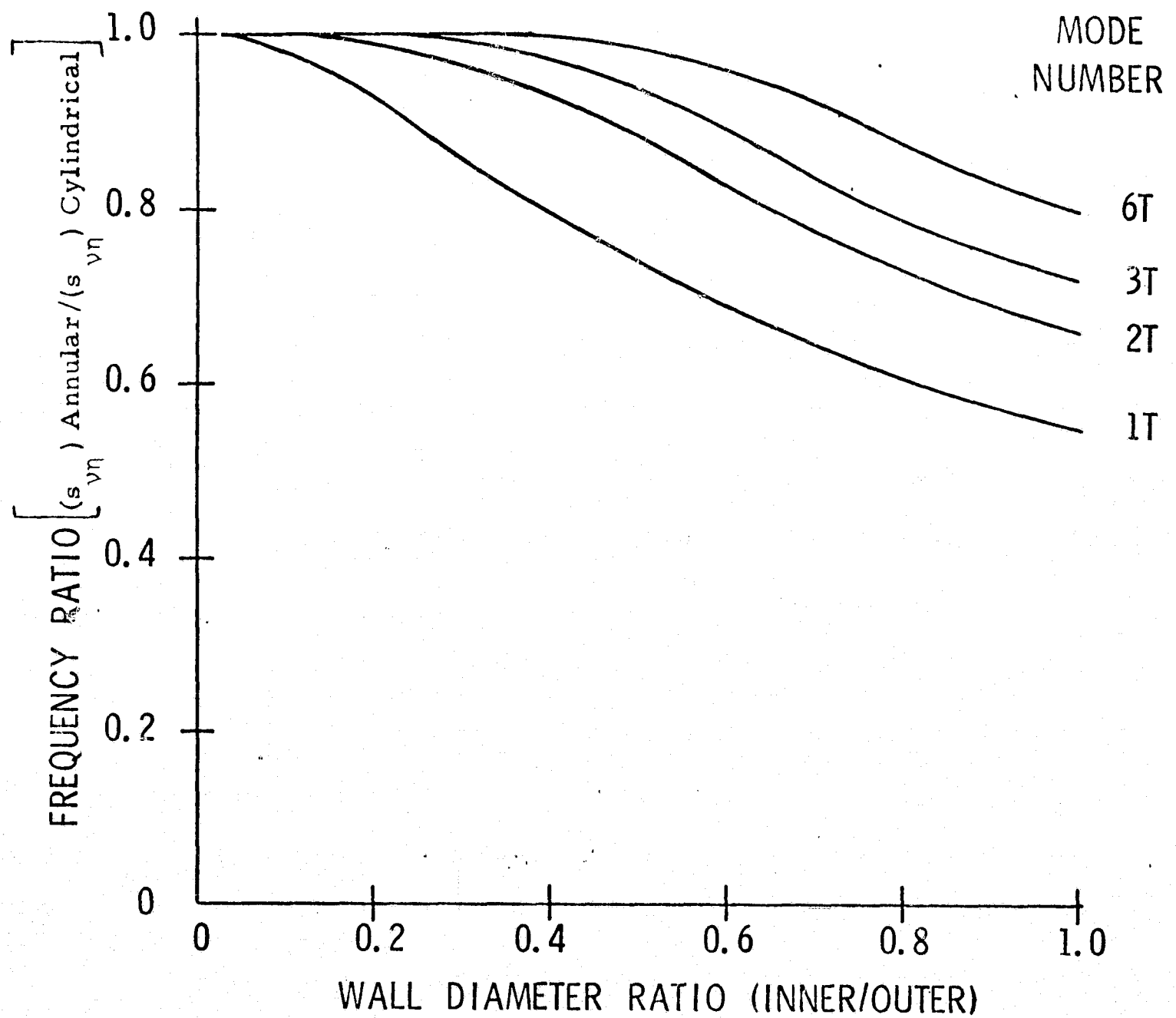


Figure 9 -- Tangential Mode Acoustic Frequencies for Annular Chambers

I, B, Technical Description (cont.)

(4) Combustion Response

(a) General Formulation

In Equation 10a combustion terms are written. It is necessary now to provide a mathematical formulation for the combustion terms in the flow equations in terms of the chamber conditions. As discussed previously our quantitative understanding of the actual combustion processes is not sufficient to provide a mathematical model. Fortunately the heuristic formulation based on the sensitive time lag concept seems to provide a good representation of the actual combustion response. The relation between time lag and burning rate is immediately found by considering that the fraction of propellant burning at a certain station in a time interval dt must have been injected during the interval $d(t - \tau_T)$. If, then $\dot{m}_b$ and $\dot{m}_i$ are the corresponding burning and injection rates we must have

$$\dot{m}_b dt = \dot{m}_i d(t - \tau_T)$$

In steady-state τ_T does not vary with time, and hence, if the injection rate is unaffected by the oscillations

$$\bar{\dot{m}}_b = \bar{\dot{m}}_i$$

From these two relations we obtain the fractional perturbation of the burning rate in the form

$$\frac{\dot{m}_b'}{\bar{\dot{m}}_b} = \frac{\dot{m}_b - \bar{\dot{m}}_b}{\bar{\dot{m}}_b} = - \frac{\partial \tau_T}{\partial t} = \frac{-d\tau}{dt} \quad (22)$$

I, B, Technical Description (cont.)

where, in accordance with the definition of sensitive time lag, the variation of τ_T with time is entirely due to the variation of τ , and where τ has been assumed to be the same for all propellant elements, and hence independent of the space coordinates.

Turning to the evaluation of $d\tau/dt$, a satisfactory mathematical description is obtained if one imagines that during the sensitive time lag certain preparatory processes, which need not be more precisely defined, take place at a rate depending on the local, instantaneous values of quantities representing the state and the motion of the gas and droplets. When these preparatory processes, integrated over the duration of the sensitive time lag, reach a certain fixed level, the conversion into hot gases takes place abruptly. It is clear, then, that when the state and the motion conditions vary, also the duration of the sensitive (and hence, the total) time lag will vary, resulting in a variable rate of gas production.

The quantitative formulation follows at once. If the rate of the preparatory processes is given by a function $f(p, T, v, \dots)$ of the pertinent values of the pressure, temperature, any representative velocity v (for instance the radial gas velocity), and possibly other quantities representing the conditions in the chamber, the sensitive time lag τ for an element burning at time t will be given by the equation

$$\int_{t-\tau}^t f(p, T, v, \dots) dt_1 = \text{const.}$$

where t_1 represents the burning time. Here the values of $p, T, v, \dots$ must be evaluated not only at time t_1 , but also at the position where the particular propellant element finds itself at that time. Since the above relation must be satisfied also in steady operation, indicating the steady-state quantities with a superimposed bar, we must have

I, B, Technical Description (cont.)

$$\int_{t-\tau}^t f(p, T, v, \dots) dt_1 = \int_{t-\tau}^t f(\bar{p}, \bar{T}, \bar{v}, \dots) dt_1 \quad (23)$$

Now we introduce the perturbations, such that

$$p = \bar{p} + p', \quad T = \bar{T} + T', \quad v = \bar{v} + v' \quad \dots$$

and expand the rate function in a Taylor series

$$f(p, T, v, \dots) = \bar{f} + \bar{f}_p p' + \bar{f}_T T' + \bar{f}_v v' + \dots$$

where $\bar{f} = f(\bar{p}, \bar{T}, \bar{v}, \dots)$ and similarly for the partial derivatives $\bar{f}_p$, $\bar{f}_T$, $\bar{f}_v$ of f with respect to the subscripts. Observe that, under the small perturbations assumption, the Taylor series must be stopped after the first order terms.

If it is assumed that the temperature is a function only of the pressure (for instance, through the isentropic defining relation), $T' = p' (dT/dp)$. Then, defining the nondimensional interaction indices $n, l, \dots$ as

$$n = \frac{\bar{f}_p + \bar{f}_T (dT/dp)}{\bar{f}}, \quad l = \frac{\bar{f}_v}{\bar{f}}, \quad \dots \quad (24)$$

$$f(p, T, v, \dots) = \bar{f} (1 + n p' + l v' + \dots)$$

and Equation (23) can be written in the form

I, B, Technical Description (cont.)

$$\int_{t-\tau}^{t-\bar{\tau}} \bar{f} (1 + np' + lv' + \dots) dt_1 + \int_{t-\bar{\tau}}^t \bar{f} (1 + np' + lv' + \dots) dt_1$$

$$= \int_{t-\tau}^t \bar{f} dt_1$$

Here the integration interval at the L.H.S. of Equation (23) has been split into two parts. The first interval, from $t - \tau$ to $t - \bar{\tau}$ is of duration $\bar{\tau} - \tau$ and hence of the order of the perturbation of the time lag. Hence, compared with the other two integrals, the first integral is of the order of a perturbation. As a result in its evaluation one can disregard in the integrand the terms containing the perturbations which, in view of the small perturbation assumption, would result in a negligible second order contribution. Then, simplifying, the above equation becomes

$$\int_{t-\tau}^t \bar{f} dt_1 = \int_{t-\bar{\tau}}^t \bar{f} (np' + lv' + \dots) dt_1$$

In the combustion zone $\bar{p}$ is approximately constant, and if v is the radial gas velocity, so that $\bar{v} = 0$, then $\bar{f}(\bar{p}, \bar{v})$ is a constant, and so are n and l .

Then we obtain simply

$$\bar{\tau} - \tau = n \int_{t-\bar{\tau}}^t p' dt_1 + l \int_{t-\bar{\tau}}^t v' dt_1 + \dots$$

or, differentiating with respect to t

I, B, Technical Description (cont.)

$$-\frac{d\tau}{dt} = n \left[p'(t) - p'(t - \bar{\tau}) \right] + 1 \left[v'(t) - v'(t - \bar{\tau}) \right] + \dots \quad (25)$$

Again, it must be specified that while $p'(t)$ is evaluated at the conversion instant t at the location where the conversion takes place, $p'(t - \bar{\tau})$ must be evaluated not only at time $t - \bar{\tau}$, but also at the location where the propellant was at that time. However, the displacement of the propellant during the time $\bar{\tau}$ produces an effect of second order in Equation (25). As a reasonable approximation, therefore, both $p'(t)$ and $p'(t - \bar{\tau})$ can be evaluated at the station when the conversion into burned gases takes place. And, of course, the same applies to the velocity effect in Equation (25), and to other possible effects.

In Equation (25) only the pressure sensitivity and the radial velocity sensitivity are explicitly considered. Concerning the last it must be added that other components of the gas velocity can be treated in exactly the same fashion. If one is interested, for instance, in the effect of the transverse non-uniformity of the gas composition, then also the tangential velocity component is relevant, and correspondingly another velocity sensitive term must appear in Equation (25), which becomes, in the absence of other interactions

$$-\frac{d\tau}{dt} = n \left[p'(t) - p'(t - \bar{\tau}) \right] + 1_r \left[v'(t) - v'(t - \bar{\tau}) \right] + 1_\theta \left[w'(t) - w'(t - \bar{\tau}) \right] \quad (26)$$

It must be observed that, actually, when only the effects of the nonuniform gas composition on the burning rate are sought, what counts is the displacement of the gases with respect to the droplets,

I, B, Technical Description (cont.)

rather than the relative velocity. This can be formalized by writing, for instance, instead of Equation (26).

$$-\frac{d\tau}{dt} = n \left[p'(t) - p'(t - \bar{\tau}) \right] + m_r \left[\delta'_r(t) - \delta'_r(t - \bar{\tau}) \right] + m_\theta \left[\delta'_\theta(t) - \delta'_\theta(t - \bar{\tau}) \right] \quad (27)$$

where m_r and m_θ are two displacement indices relative to the radial and tangential displacements δ_r , δ_θ respectively. The two formulations (26) and (27) are closely correlated, because of the relations existing between velocities and displacements.

If the time dependence of the perturbations in Equations (26) and (27) is taken to be $\exp(st)$, the combined effect is:

$$-\frac{d\tau}{dt} = - (P p' + R_v v' + T_v w') \text{ or } - (P p' + R_\delta \delta'_r + T_\delta \delta'_\theta) \quad (28)$$

where the quantities defined by

$$\frac{P}{n} = \frac{R}{l_r} = \frac{T}{l_\theta} = \frac{R_\delta}{m_r} = \frac{T_\delta}{m_\theta} = 1 - e^{-\sigma \bar{\tau}}, \quad (\sigma = \lambda + i\omega) \quad (29)$$

are to be interpreted as feedback factors. In this case the relation between velocities and displacements is simply $v' = \sigma \delta'_r$, $w' = \sigma \delta'_\theta$ so that the same expression can be used for $-\frac{d\tau}{dt}$ for velocity or displacement effects if R_δ is equivalent to σR and T_δ is equivalent to σT .

(b) Velocity Sensitive Combustion

It is desirable to take a closer look at the dynamic aspects of velocity sensitive combustion. The most significant

I, B, Technical Description (cont.)

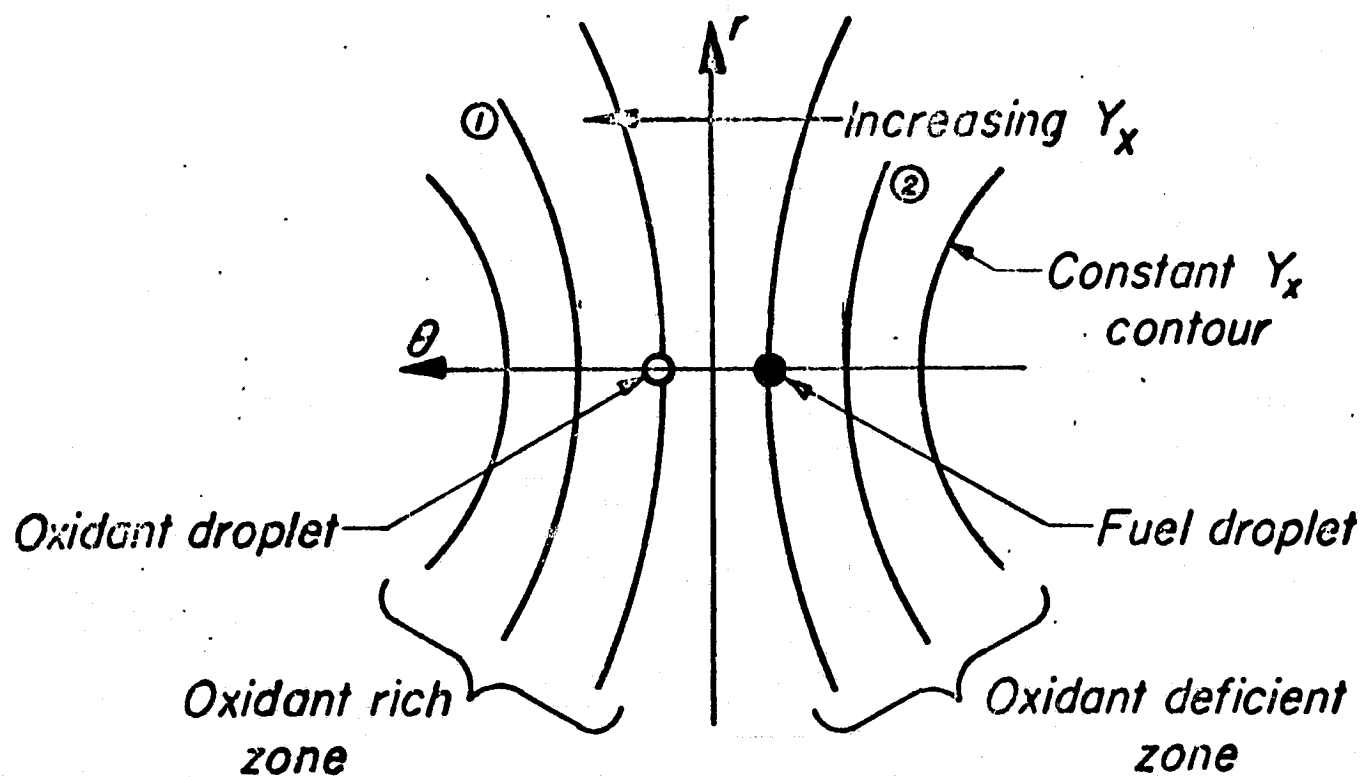
velocity effects are those resulting from the radial and tangential components of the gas velocity perturbation. In the case of purely transverse modes, the longitudinal velocity perturbation component is always much smaller than the transverse components. In addition, the longitudinal component vanishes at the injector face and has its smallest magnitude in the early combustion zone, the region which appears to have the greatest significance for transverse modes. It is possible that in certain combustion chambers the axial spreading of the combustion will result in sizable longitudinal velocity oscillations for higher order longitudinal or combined transverse-longitudinal modes. However, in such cases, the pressure perturbation will become correspondingly small in that region, thus, the decreased pressure effect will cancel the increased velocity effect. In the present analysis, therefore, only the effects of the transverse velocity oscillations will be considered in the combustion response.

Of the various intermediate processes occurring during the combustion of liquid bipropellants, those most sensitive to velocity are the vaporization of the liquid droplets and the mixing of the vaporized propellants that must precede chemical reaction. The theoretical study of unsteady vaporization by Wieber and Mickelsen (Ref. 10) indicates that the evaporation rate is dependent on the absolute magnitude of the relative velocity between droplet and gas; therefore, the vaporization velocity effect is seen to be essentially nonlinear, and cannot be treated within the framework of a linearized theory. On the other hand, the mixing of the propellants by the oscillating velocities may be linearized, and gives rise to important modifications of the stability behavior of a combustor. Although no detailed description of such a complex phenomenon is now possible, the following discussion illustrates one process by which the burning rate may be caused to oscillate by an oscillating transverse gas velocity.

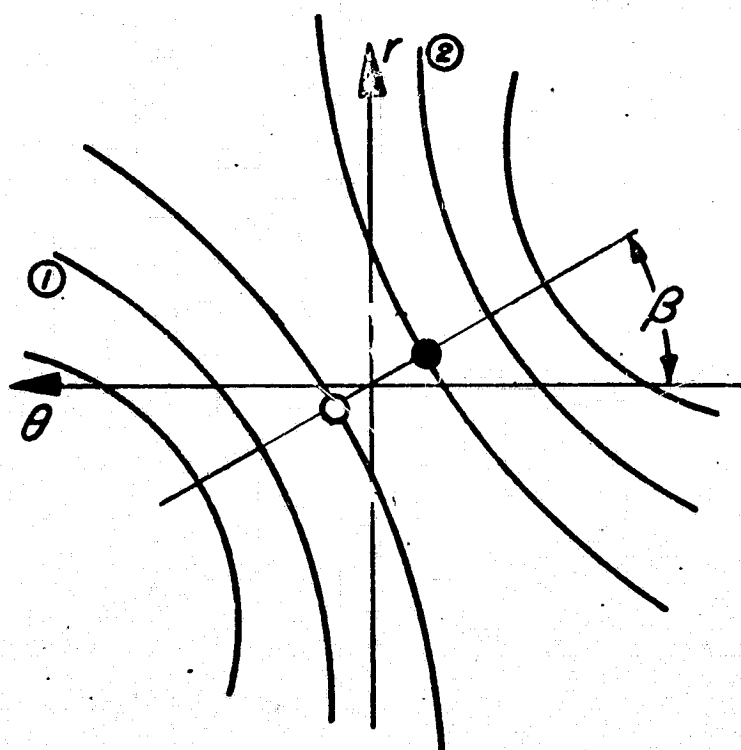
I, B, Technical Description (cont.)

Consider first the mixture of gaseous combustion products, vaporized propellants (oxidizer and fuel), and liquid propellant droplets at some axial station downstream from a fuel-on-oxidizer impinging doublet injector element. Since liquid mixing is imperfect, some stratification will exist in the mixture. For concreteness, assume that the line of centers of the doublet is aligned tangentially (i.e., normal to a radius). Then the stratification is almost entirely in the tangential direction, as shown schematically in Figure 10 by the lines of constant mass fraction of vaporized oxidizer in the mixture Y_x . The exact shape of the constant Y_x contours will be dependent on the injector design, operating conditions, and propellant characteristics. Because of the turbulence in the combustion chamber, the stratification pattern shown represents only a mean condition.

As a droplet evaporates, the vapor diffuses away and must mix with the other vaporized propellant in the propellant proportions for chemical reaction. In a rocket combustor, the transport and mixing are most likely to be carried out by turbulence rather than by molecular diffusion. The overall burning rate of a fuel-rich droplet will, therefore, be a function of the amount of oxidizer vapor near the droplet. In the presence of small, periodic, tangential gas velocity oscillations ($w'_e e^{i\omega t}$) the gaseous mixture will be displaced relative to the droplets, causing oscillations of the local mass fractions of both oxidizer and fuel. Since a fuel droplet is in an oxidant-deficient region, a velocity perturbation which increases the oxidizer fraction in the vicinity of the droplet will increase the contribution of that droplet to the overall burning rate. The opposite is true for an oxidizer-rich droplet subject to the same perturbation, since an oxidizer fraction increase corresponds to a fuel fraction decrease. Thus, the effects of the same velocity perturbation on the two droplets will tend to cancel, unless the propellants have significantly different vaporization rates. In the latter case, at any axial station, there will be a greater number of droplets of the less-volatile propellant, and summation



(a) Spray produced by tangentially oriented injector spud



(b) Spray produced by rotated spud

Figure 10 -- Sprays Produced by Various Orientations of the Injector Spud

I, B, Technical Description (cont.)

of the velocity effect over all of the droplets in the spray will result in a net contribution to the burning rate. This contribution will clearly depend on the amplitude of the velocity as well as its direction. For small perturbations, and for the doublet spray shown in Figure 10a, the burning rate contribution can be written in the form

$$f' = l w' e^{i\omega t}$$

where l is a velocity interaction index analogous to the pressure interaction index defined by Crocco. In the case of an arbitrarily oriented spray, such as shown in Figure 10b, the burning rate perturbation due to velocity effects becomes

$$f' = (l_r v' + l_\theta w') e^{i\omega t} \quad (30)$$

so that, in general, two velocity indices are necessary.

It is clear that this linearized expression will not be valid for all types of injection patterns. For example, approximately linear effects can be expected with a fuel-on-oxidizer doublet and for a like-on-like pattern if the spacing between unlike fans is sufficiently small. However, for large spacings, nonlinear velocity effects must be taken into consideration.

At present, the magnitudes of the velocity indices cannot be calculated because of the lack of quantitative knowledge of the processes involved in liquid propellant combustion under turbulent conditions.

I, B, Technical Description (cont.)

It is also possible to formulate the above discussion in terms of displacement interaction indices. Letting the radial and transverse components of the displacements be $\delta'_r e^{i\omega t}$ and $\delta'_\theta e^{i\omega t}$, the net combustion process rate perturbation can be written

$$f' = (m_r \delta'_r + m_\theta \delta'_\theta) e^{i\omega t} \quad (31)$$

It is clear that $m_r = i\omega l_r$ $m_\theta = i\omega l_\theta$. Thus, the displacement indices present at 90° phase shift with respect to the velocity indices.

The analysis of the effects of velocity (or displacement) sensitivity on the stability of a combustor is considerably simplified by assuming that the velocity effects occur during the same time interval (the sensitive time lag) as the pressure effects. In this case, the burning rate perturbation becomes

$$Q' = \bar{Q} (P_p' + R v' + T w') \quad (32)$$

where

$$\begin{aligned} P &= n (1 - e^{-\sigma \tau}) \\ R &= l_r (1 - e^{-\sigma \tau}) \\ T &= l_\theta (1 - e^{-\sigma \tau}) \end{aligned}$$

In Equation (32), additional simplifications have been introduced by assuming that all propellant elements have equal mean sensitive time lags, and that the space lag associated with the sensitive time lag is a negligible fraction of the wave length. In general, of course, the mean sensitive time lag varies from one propellant element to another. Crocco and Cheng have shown that this nonuniformity of the sensitive time lag leads to increased stability of the combustor. Therefore, the assumption of a uniform time lag produces a conservative stability prediction.

I, B, Technical Description (cont.)

It would be possible to generalize the burning rate expression to allow for different time lags for pressure and velocity effects. However, both mathematical and physical considerations indicate the desirability of the simpler formulation.

(c) Approximate Treatment of Nonlinear Combustion Response

Nonlinearities associated with oscillatory combustion chamber operation can derive from two sources: (1) the fluid mechanical behavior of the gases in the chamber, and (2) the dynamics of the combustion process. It is clear that significant interactions between the two kinds can also occur. The studies of Priem and Guentert have shown that combustion process nonlinearities can be important even for oscillation amplitudes less than 20% of the mean chamber pressure. Thus, it is worthwhile to consider nonlinearity of the combustion response while retaining the linearized fluid mechanical analysis with its attendant simplification.

To insert the nonlinear combustion dynamics into the framework of the linear theory, some method of equivalent linearization must be used. The method selected in this analysis is the "describing function" method.

When a sinusoidal signal is input to a nonlinear element, the output will not, in general, be sinusoidal. Fourier analysis of the output will reveal many frequency components, among which is one (the fundamental) that corresponds to the frequency of the input signal.

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I, B, Technical Description (cont.)

For the analysis of stability, only the fundamental frequency component is required, as shown by Reardon (Ref 8). An equivalent linear transfer function for the nonlinear element can be defined as the ratio of the fundamental component of the output to the input. Thus, if

$$I(t) = I e^{i\omega_f t}$$

is the input, and

$$O(t) = O_1(\omega_f) e^{i\omega_f t} + \sum_{j \neq 1} O_j e^{i j \omega_f t} \quad (33)$$

is the output, the equivalent linear transfer function is

$$TF = \frac{O_1(\omega_f)}{I}$$

For nonlinearities that can be treated by this method, linear behavior is obtained for limiting values of the input (e.g., $I \rightarrow 0$ or $I \rightarrow \infty$). It is convenient to define a "describing function" F as

$$F(\omega) = TF / (TF)_{LIM} \quad (34)$$

where $(TF)_{LIM}$ is the limiting linear transfer function. Thus, a linear analysis can be extended to include isolated nonlinear effects by replacing the linear transfer function of the nonlinear element by $F(\omega) \cdot (TF)_{LIM}$.

In applying this approach to the combustion instability problem, it is assumed that the only significant nonlinearities

I, B, Technical Description (cont.)

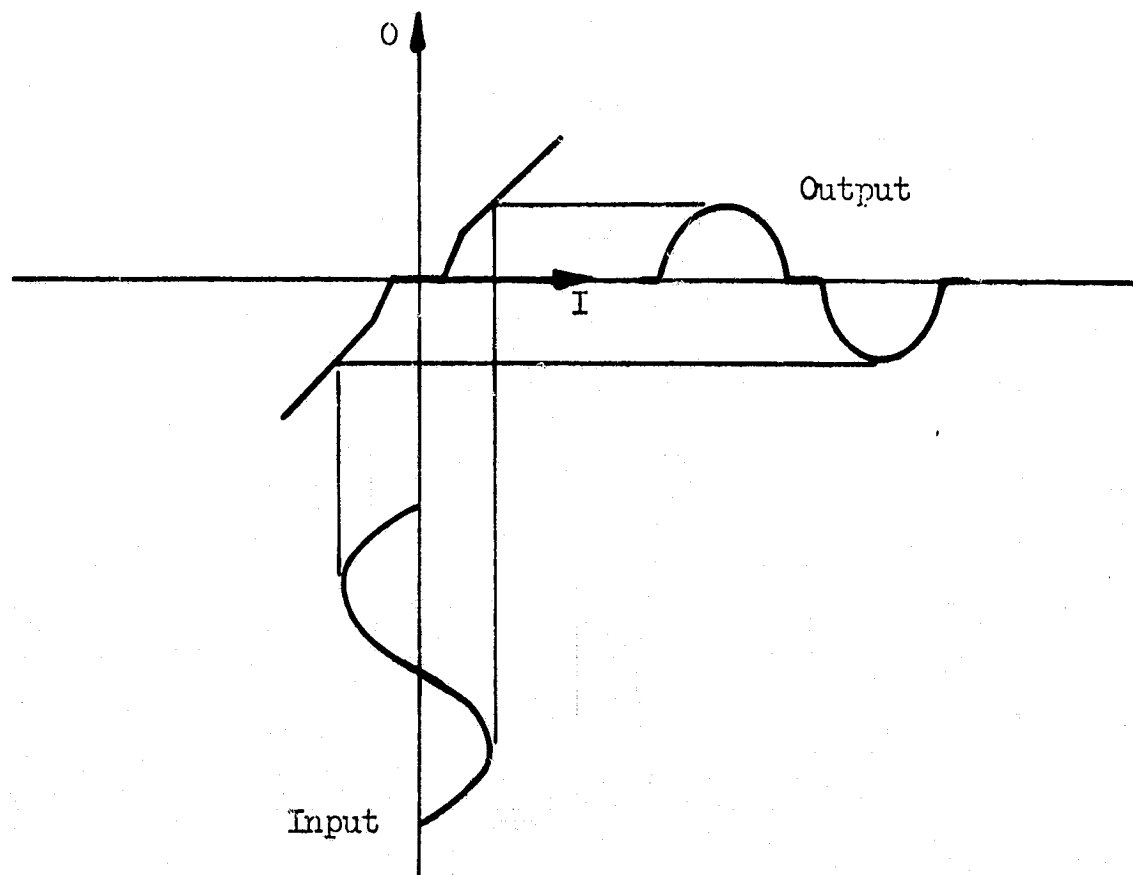
are those associated with the response of the combustion process to pressure and velocity perturbations. Three describing functions are required, corresponding to the combustion response to pressure, radial velocity, and tangential velocity perturbations. Thus, the combustion rate perturbation becomes

$$Q' = \bar{Q} \left[F_P(p') P_p' + F_R(v') Rv' + F_T(w') Tw' \right] \quad (35)$$

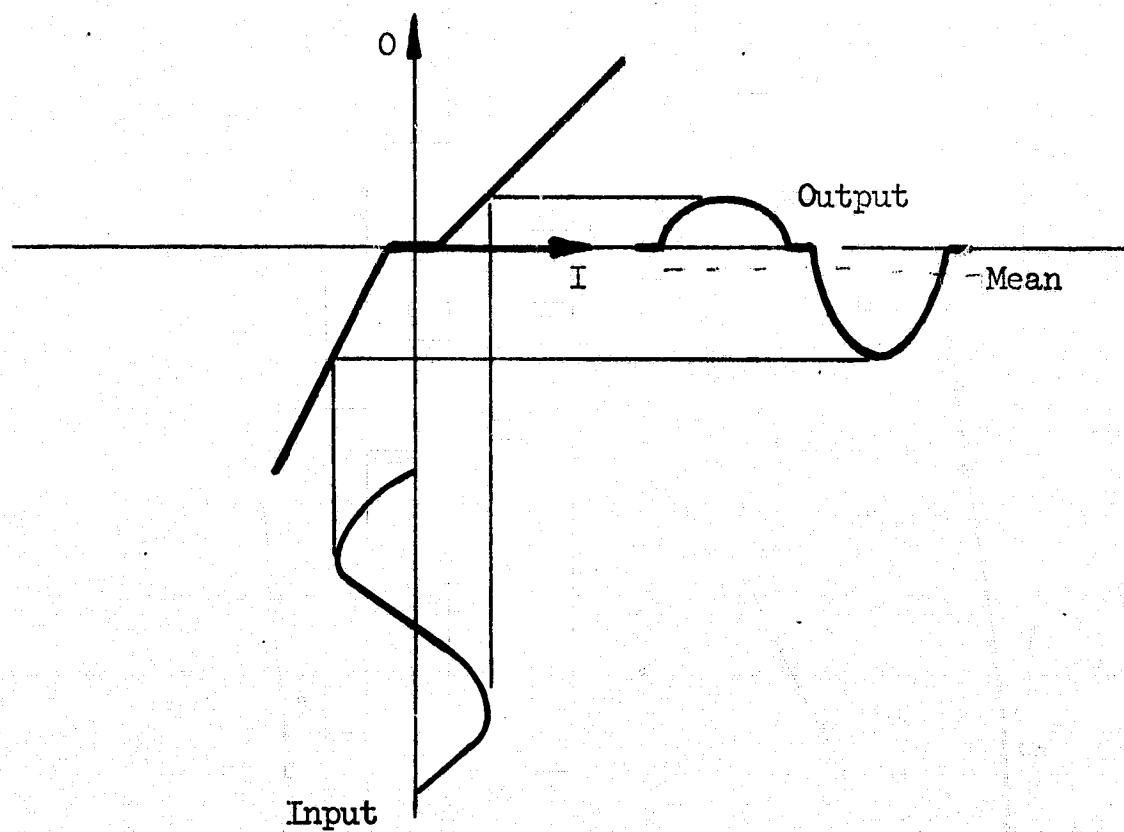
In the above expression, the dependence of the describing functions on the perturbation (input) amplitude is shown explicitly. This dependence on amplitude introduces complications into the solution of the perturbation equations. In general, the input amplitude is a function of the axial, as well as the transverse, space coordinate. To obtain a solution, it is necessary to introduce the additional simplification of neglecting the axial variation of perturbation amplitude in the evaluation of the describing function. For purely transverse modes, this is not an unreasonable approximation, and it breaks down significantly only for higher-order longitudinal modes. The error made by using the perturbation amplitudes at the injector face will be small.

To calculate the describing functions, it is necessary to know the shape of the combustion response to each perturbation. Since it is assumed that the effects are independent of each other, the burning rate perturbation can be written

$$\frac{Q'}{\bar{Q}} = \phi_P (P') + \phi_R (v') + \phi_T (w')$$



(a) Response Function with Odd-Symmetry



(b) Asymmetric Response Function

Figure 11 -- Examples of Nonlinear Response Functions

I, B, Technical Description (cont.)

The procedure for calculating each describing function is the same; therefore, it is necessary only to discuss one, say, the pressure-effect describing function.

The input perturbation is $p' = P_o \cos \chi$ where $\chi = \omega t$ and, from Fourier analysis the fundamental term of the output series is

$$\phi_P(P_o) = \frac{1}{\pi} \int_{C-\pi}^{C+\pi} \phi_P(P_o \cos \chi) e^{-i\chi} d\chi$$

Thus, the describing function is given by

$$F_P = \frac{1}{\pi P_o (TF)_{LIM}} \int_{C-\pi}^{C+\pi} \phi_P(P_o \cos \chi) e^{-i\chi} d\chi \quad (36)$$

In the expressions above, C is an arbitrary constant, and $(TF)_{LIM}$ is a suitable normalizing factor, such that $F \rightarrow 1$ for equivalent linear operation. The choice of $(TF)_{LIM}$ depends on the characteristics of each nonlinear response function, and a general rule does not appear feasible.

The describing function method applies well to nonlinearities with odd symmetry (Figure 11a). It is not applicable to response functions with even symmetry, such as the velocity effect on vaporization, since there is no contribution to the fundamental term of the Fourier series. An intermediate case is that of asymmetric response function (Figure 11b). In this case, there will be a significant contribution to the fundamental oscillation, but a change in the mean burning rate as well. This change in the mean burning rate occurs only during the sensitive portion of the total time lag and so will have a negligible influence on the steady state solution.

I, B, Technical Description (cont.)

In general, the describing function is complex; that is, the nonlinear combustion response introduces a phase shift as well as an amplitude change. However, for response functions with odd symmetry, the fundamental component of the output is in phase with the input, so that the describing function is real.

(5) Effect of Non-Uniform Injection

The non-uniformity of mass injection in the r and θ direction can be characterized by a burning rate distribution function $\mu(r, \theta)$ which is the ratio of the local injection density to the mean injection density so that

$$\pi = \int_0^1 \int_0^{2\pi} \mu r dr d\theta.$$

When $\bar{Q}$ is multiplied by μ in Equations 12b, terms on the right hand side are known functions of r and θ , as well as Z . The solution of this type of inhomogeneous differential equation can be expressed as a Bessel-Fourier series as follows:

$$p_1 = (A_{v\eta} P_{a v\eta} + B_{v\eta} P_{b v\eta} + C_{v\eta} P_{c v\eta} + P_d) \psi_{v\eta} \theta_v +$$

$$\sum_{p \neq v} \sum_{q \neq \eta} (A_{pq} P_{apq} + B_{pq} P_{bpq} + C_{pq} P_{cpq}) \psi_{pq} \theta_p$$

where the subscript a indicates pressure effects (associated with P) b radial velocity effects (associated with R), c tangential velocity effects (associated with T) and d damping effects (which are independent of r and θ).

I, B, Technical Description (cont.)

When the orthogonality property of the functions ψ and θ is used on the system characteristic equation the coefficients $A_{v\eta}$, $B_{v\eta}$ and $C_{v\eta}$ are found to be:

$$\begin{aligned}
 A_{v\eta} &= \frac{\int_0^1 \int_0^{2\pi} \mu(r, \theta) \psi_{v\eta}^2 \theta_v \theta_v^* r dr d\theta}{\int_0^1 \int_0^{2\pi} \psi_{v\eta}^2 \theta_v \theta_v^* r dr d\theta} \\
 B_{v\eta} &= \frac{\int_0^1 \int_0^{2\pi} \mu(r, \theta) \frac{d\psi_{v\eta}}{dr} \psi_{v\eta} \theta_v \theta_v^* r dr d\theta}{\int_0^1 \int_0^{2\pi} \psi_{v\eta}^2 \theta_v \theta_v^* r dr d\theta} \\
 C_{v\eta} &= \frac{\int_0^1 \int_0^{2\pi} \mu(r, \theta) \psi_{v\eta}^2(r, \theta) \frac{d\theta_v}{d\theta} \theta_v^* dr d\theta}{\int_0^1 \int_0^{2\pi} \psi_{v\eta}^2 \theta_v \theta_v^* r dr d\theta}
 \end{aligned} \tag{37}$$

In section I,B,4,b (7) it is seen how these coefficients affect the characteristic equation. It should be noted that, if the energy addition follows a non-linear relationship, a describing function must also be included in the above coefficients.

I, B, Technical Description (cont.)

(6) Nozzle Admittance

In any rocket combustion instability analysis, it is desirable to apply a boundary condition at the nozzle entrance to describe the effect of the nozzle upon wave motion in the combustion chamber. In a linearized analysis, this boundary condition is written in the form of an admittance relation; that is, a linear relation between the perturbations of two thermodynamic properties and of the velocity components. The coefficients in this relation are termed admittance coefficients and are calculated by means of an analysis of the oscillatory flow in the nozzle. In this section, the analysis and numerical integration which lead to the determination of these coefficients are discussed.

The divergent portion of the supercritical nozzle need not be analyzed; all that is pertinent is the subsonic flow in the convergent portion since any disturbances to the supersonic flow cannot propagate upstream through the throat. Therefore, disturbances in the subsonic portion of the nozzle and in the chamber are neither affected nor caused by disturbances in the supersonic region. (The opposite, however, is not true.)

To date, two types of nozzles have been analyzed: axisymmetric designs and two-dimensional designs. The axisymmetric case is presently the one of the most practical significance and is the one to be discussed here. The two-dimensional case applies to thin annular chambers and to certain experimental configurations. The analyses of the two cases are similar; details of both are given in References 11 and 12.

The unperturbed, or steady-state, flow is considered to be one-dimensional in order to simplify the analysis. The perturbed flow, however, may be three-dimensional. The combustion process is

I, B, Technical Description (cont.)

assumed to be completed before the flow enters the nozzle so that there are no source terms in the differential equations of motion. The equations do allow for the occurrence of entropy waves and vorticity waves in the nozzle due to the combustion chamber.

The three-dimensional coordinate system (Figure 12) employs the values of the velocity potential ϕ and the stream function ψ of the unperturbed flow in addition to the azimuthal angle, with

$$\bar{q} = \frac{\partial \phi}{\partial s}$$

$$r \rho \bar{q} = \frac{\partial \psi}{\partial n}$$

where s is the streamline direction and n is the direction normal to the streamline. Since the value of the stream function is a constant at the nozzle walls where the boundary conditions are applied, separation of variables is allowed.

Under the usual assumption of small-amplitude oscillations, linear partial differential equations are obtained that govern the perturbations. These equations are separated under the assumption that the nozzle is sufficiently long that the cosine of the semi-angle of convergence may be approximated by unity. The time and azimuthal dependencies are given by sinusoidal functions. The radial dependencies are given in terms of Bessel functions of the first kind and their derivatives. The axial dependencies are related to the solution to a certain second-order linear ordinary differential equation with complex-variable coefficients which can only be obtained in exact form by numerical integration.

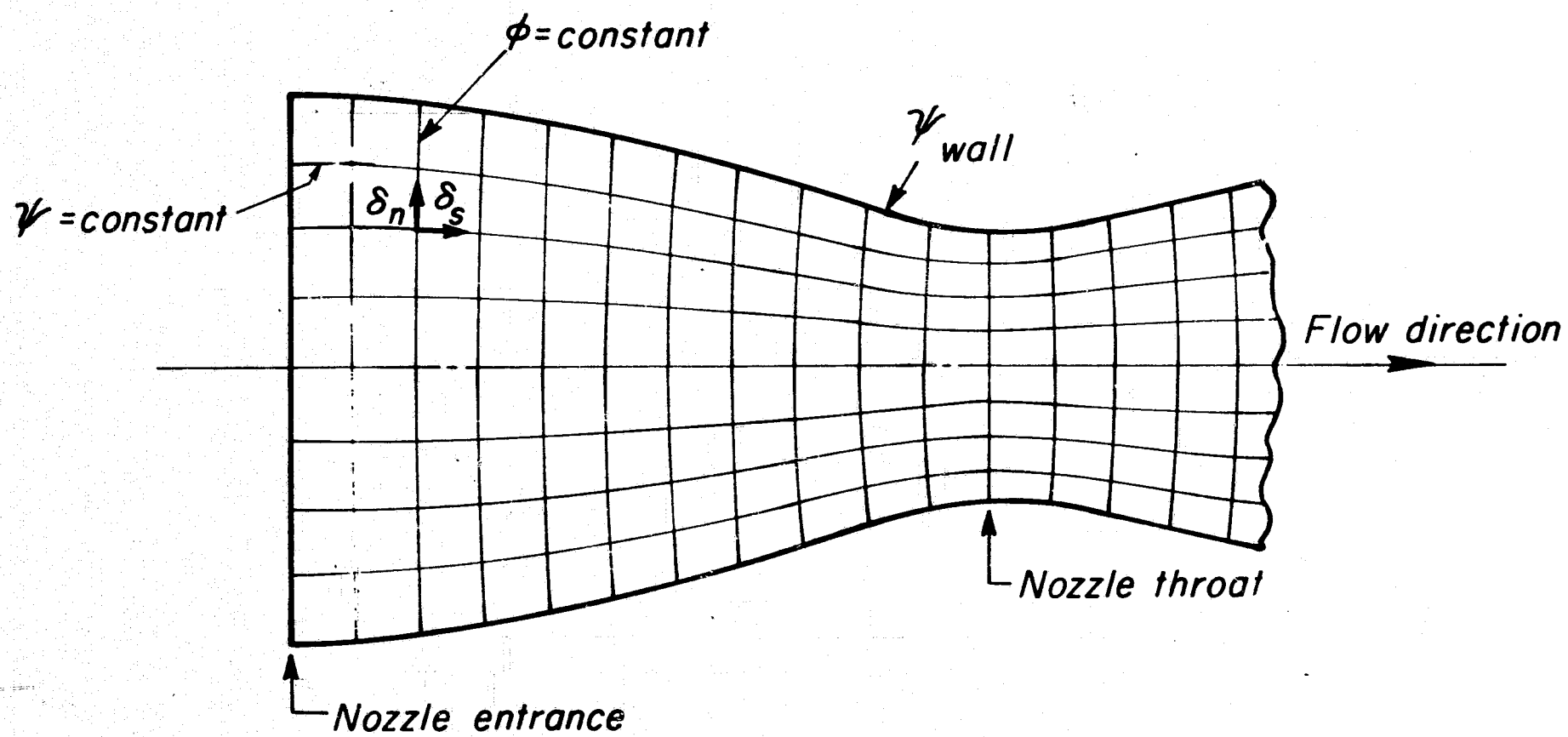


Figure 12 -- Exhaust Nozzle Coordinate System

I, B, Technical Description (cont.)

This second-order differential equation is singular at the throat; one of the homogeneous solutions will be regular there and the other one will be singular. The singular solution is cast away. This procedure has been demonstrated to be equivalent to disallowing perturbations to propagate upstream from the supersonic portion of the nozzle (Ref 6).

It has been shown that the admittance coefficients are functions of the solutions to certain first order equations that are obtained by reduction of the original second-order equation. So, while it would be necessary to integrate the second-order equation in order to determine the variation of the flow properties, it is not necessary for the purpose of determining the admittance coefficients. Since the interest lies in the prediction of global stability characteristics and not in the details of the flow itself, only the equations immediately needed to determine the admittance coefficient will be presented. The derivations and additional analyses may be found in Reference 11.

The admittance coefficients for a given geometry are determined as functions of the axial coordinate or, equivalently, of the local mean-flow Mach number. This implies that, when the admittance coefficient at the nozzle entrance is desired, the axial coordinate at the entrance or the entrance Mach number must be known before the admittance coefficients can be determined.

The linear admittance condition is given by

$$U + AP + B V + CS = 0 \quad (38)$$

I, B, Technical Description (cont.)

where U, P, V, and S are the axial dependencies of the nondimensional perturbations of axial velocity, pressure, radial velocity, and entropy, respectively.

The annular nozzle analysis used in this program is based on Reference 11. It contains certain inconsistencies which should be aired. The current analysis is an approximation to an annular nozzle analysis which is expected to be exact for a certain case.

The cylindrical nozzle analysis postulates the existence of streamlines and a velocity potential. The equations of motion are written in terms of stream functions and velocity potentials. The limits of integration in the radial direction must be written in terms of these streamlines. For a conventional nozzle these limits are the nozzle centerline and the outer wall, which are both streamlines. To make an annular analysis from the conventional nozzle analysis with a minimum of fuss, a different streamline can be chosen for the inner boundary. A new corresponding value of $s_{v\eta}$ can also be determined. It is clear that once the outer boundary is chosen the number of possible inner boundaries is severely limited to one which is a constant fraction of the outer radius.

The nozzle admittance analysis requires a table of the square of the velocity versus the velocity potential. For a general annular nozzle inner and outer contour this can be calculated by first calculating area ratios versus distance, then velocities versus distance, and the velocity potential from velocity and distances. As this program is written, the table of velocity squared versus velocity potential is calculated for a very general annular nozzle configuration. It should be clear from the discussion above that the configuration for which the admittance is obtained is not in general the configuration originally input.

I, B, Technical Description (cont.)

Basic changes to the nozzle admittance analysis are needed to rectify this limitation.

(7) Characteristic Equation

Solution of the perturbation equations gives the pressure perturbation in the form of a Bessel-Fourier series:

$$p' = P_{oo} [P_{v\eta}(Z) \psi_{v\eta}(r) \theta_v(\theta) + \sum_{\substack{p \neq v \\ q \neq \eta}} \sum P_{pq}(Z) \psi_{pq}(r) \theta_p(\theta)]$$

In this expression the indices v and η refer to the fundamental term, that is, to the oscillatory mode under consideration. The other terms in the series account for the distortion introduced by flow, injection distribution, velocity effects, and nonlinear combustion response. For values of the indices p, q different from v, η , each term in the series includes an integration constant. However, the integration constant for the fundamental term can be shown to be of order M^3 , and so can be neglected in the present analysis. The constant P_{oo} represents the perturbation amplitude level; in this linearized analysis, the amplitude has no effect on the stability solution.

Since the perturbation solution is obtained in the form of a series, the nozzle admittance boundary condition must be applied term by term. For each $p, q \neq v, \eta$, the nozzle boundary condition can be used to determine the integration constant. Application of the remaining condition,

$$U_{v\eta}(Z_e) + AP_{v\eta}(Z_e) + BV_{v\eta}(Z_e) + CS(Z_e) = 0 \quad (39)$$

results in an eigenvalue problem. That is, this equation is the characteristic equation for the eigenvalues $\sigma = \lambda + i\omega$. For a given combustor geometry

I, B, Technical Description (cont.)

and for a given value of the combustion parameters n , τ , 1_r and 1_θ , the characteristic equation can, in principle, be used to determine the frequency of oscillation ω and the growth rate λ of the perturbation amplitude.

However, since the coefficients of the characteristic equation are function of the variable σ , it is more convenient to regard σ as one of the independent variables. Considerable simplification results if $\lambda = 0$, that is, $\sigma = i\omega$, which is interpreted physically as an oscillation, the amplitude of which neither grows nor decays with time. The neutral condition is clearly the boundary between stable and unstable operation, and is sometimes referred to as the stability limit.

For neutral oscillations, regarding the frequency as an independent variable, the characteristic equation becomes a relation between the combustion parameters. Since the equation is complex, two of the combustion parameters can be determined in terms of the other two. It is natural to select the sensitive time lag τ and the pressure interaction index n as the dependent variables, because these parameters are significant to all modes of oscillation. Following this procedure, the characteristic equation can be written in the form:

$$n (1 - e^{-i\omega\tau}) = \frac{h(\omega)}{A_{v\eta} + \frac{1}{\gamma\omega} (B_{v\eta} \frac{1_r}{n} + C_{v\eta} \frac{1_\theta}{n})} = \tilde{h}_r + i \tilde{h}_i \quad (40)$$

The solution is

$$n(\omega, \frac{1_r}{n}, \frac{1_\theta}{n}) = \frac{\tilde{h}_r^2 + \tilde{h}_i^2}{2 \tilde{h}_r} \quad (41)$$

I, B, Technical Description (cont.)

$$\tau \left(\omega, \frac{1}{n}, \frac{1}{n} \right) = \tan^{-1} \left(\frac{\tilde{h}_i}{n - \tilde{h}_r} \right) \quad (42)$$

where τ is determined to within an additive constant $\frac{2\pi}{\omega}$.

A typical solution for $n(\omega)$ and $\tau(\omega)$ for assumed values of the velocity indices is shown in Figure 13. This solution applies at the stability limits, where $\lambda = 0$. It can be seen that for any value of τ only one value of n is consistent with neutral oscillations. For the same τ , a larger n corresponds to instability ($\lambda > 0$) and a smaller n to stability ($\lambda < 0$). In the case of transverse modes, for values of n in the range of interest (≤ 2), the frequency varies over a very narrow range, and is very nearly equal to the corresponding acoustic frequency. For longitudinal modes, both the frequency range and the departure from the acoustic-mode frequency are somewhat larger. The narrow frequency range result is related directly to the fact that high frequency instability involves the interaction of the combustion chamber. The chamber acoustics are somewhat modified by the presence of the exhaust nozzle and the mean gas flow, but a good approximation of the resonant frequencies can be made without reference to the combustion effects.

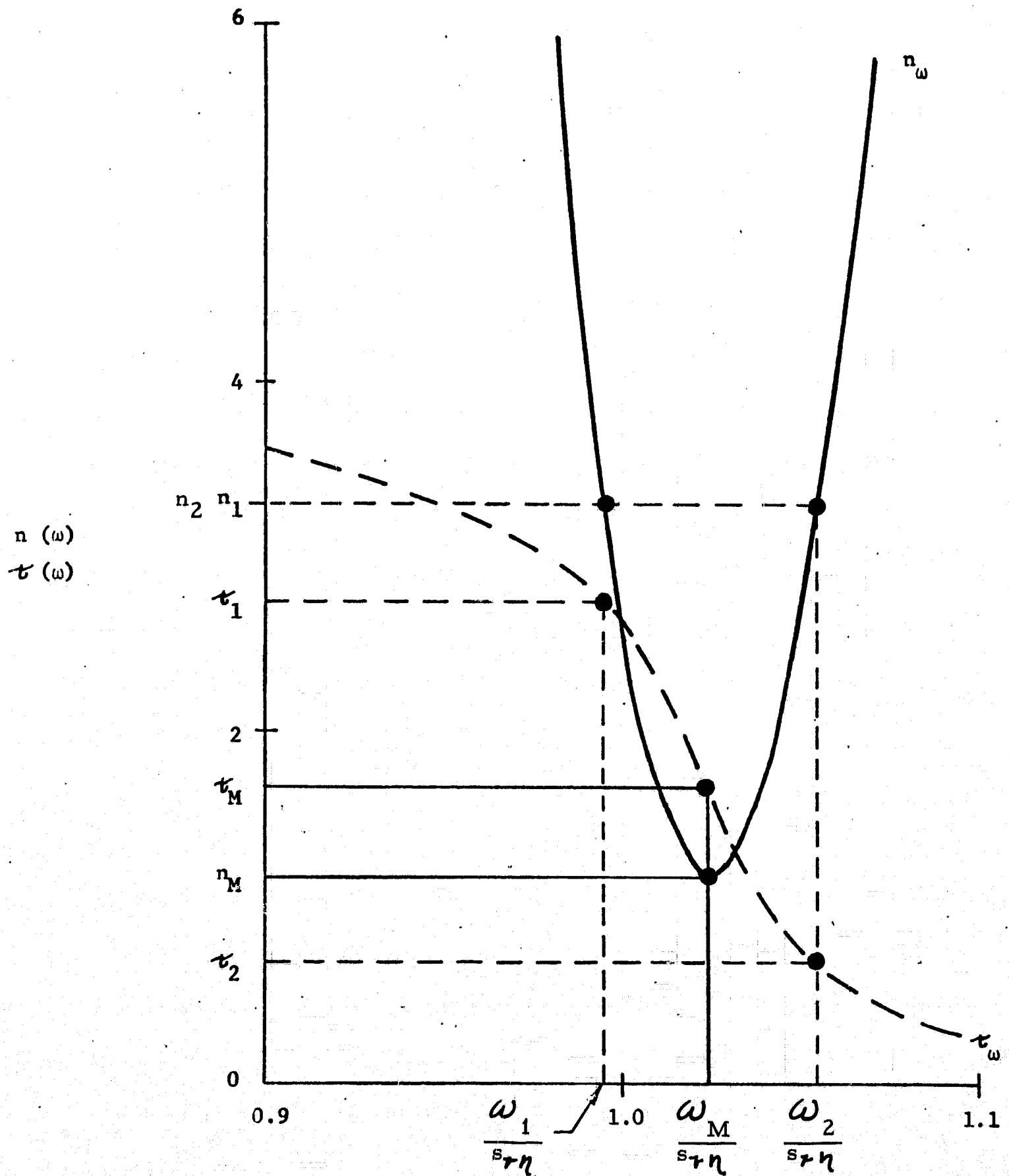


Figure 13 -- Typical Solutions of $\eta(\omega)$ and $\tau(\omega)$

I, Problem (cont.)

C. EQUATIONS

1. Program A-Distributed Combustion Analysis

$$\text{Input} = s_{v\eta}, \gamma, D_1(Z), r_c^*, L_c^*, C_o^*, \bar{u}_{Lo}^*, \kappa, \bar{u}_e, E$$

$$\bar{u} = \frac{D_1 \bar{u}_e}{D_1(Z=L_c)}$$

$$\text{Solve } \frac{d \bar{u}_L}{dZ} = \frac{\kappa[\bar{u} - \bar{u}_L]}{\bar{u}_L} \text{ for } \bar{u}_L$$

$$\bar{\rho} = [1 + \frac{\gamma-1}{2} \bar{u}^2]^{-\frac{1}{\gamma-1}}$$

$$\rho_L = [\bar{u}_e (1 + \frac{\gamma-1}{2} \bar{u}_e^2)]^{-\frac{1}{\gamma-1}} - \frac{\bar{\rho} \bar{u}}{\bar{u}_L}$$

$$\bar{Q} = \frac{[(1-\gamma\bar{u}^2)\bar{\rho} \frac{d\bar{u}}{dZ} - \gamma\bar{u} \bar{\rho}_L \kappa (\bar{u}-\bar{u}_L)]}{1 + \gamma\bar{u}(\bar{u} - \bar{u}_L)}$$

$$\Omega = (s_{v\eta}^2 - \omega^2)^{1/2}$$

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I, C, Equations

$$Y_1 = \Omega \sinh (\Omega Z_e) - i\omega E \cosh (\Omega Z_e)$$

$$Y_2 = - \int_0^{Z_e} Q_1(Z) \cosh [\Omega (Z_e - Z)] dZ + \frac{i\omega E}{\Omega} \int_0^{Z_e} Q_1(Z) \sinh [\Omega (Z_e - Z)] dZ$$

$$Y_3 = \int_0^{Z_e} Q_2(Z) \cosh [\Omega (Z_e - Z)] dZ - \frac{i\omega E}{\Omega} \int_0^{Z_e} Q_2(Z) \sinh [\Omega (Z_e - Z)] dZ$$

$$Q_1 = -\sigma \left[(\gamma+1) \bar{Q} + \frac{K\sigma}{K+\sigma} \rho_L \right]$$

$$Q_2 = \sigma \gamma \bar{Q}$$

$$h = \frac{Y_1 + Y_2}{Y_3}$$

I, C, Equations (cont.)

2. Program B-Concentrated Combustion Analysis

The perturbation equations in this program are solved using matrices as shown here.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & & & & & \\ a_{31} & & & & & \\ a_{41} & & & & & \\ a_{51} & & & & & \\ a_{61} & & & & & \end{bmatrix} \begin{bmatrix} p_- \\ \rho_- \\ v_- \\ p_+ \\ h_- \\ s_- \end{bmatrix} + \dot{m}_b \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{bmatrix} = 0$$

where

$$\begin{aligned} a_{14} &= Y & a_{41} &= 1 \\ a_{12} &= -\bar{v}_- & a_{42} &= -1 \\ a_{13} &= -\bar{\rho}_- & a_{45} &= -1 \\ a_{21} &= -\frac{1}{\gamma} & a_{51} &= \frac{\gamma-1}{\gamma \bar{\rho}_-} \\ a_{22} &= -\frac{\bar{v}_-^2}{\gamma} & a_{55} &= -1 \\ a_{23} &= -\bar{\rho}_- \bar{v}_- & a_{56} &= \bar{T}_- \\ a_{24} &= +\frac{1}{\gamma} \frac{\bar{v}_-^3}{2} & a_{61} &= E \text{ (when } S_{v\eta} = 0 \text{ } A_{61} = A_Y) \\ a_{32} &= -(\gamma-1) \frac{\bar{v}_-^3}{2} - \bar{h}_- \bar{v}_- & a_{63} &= \gamma \\ a_{33} &= -(\gamma-1) \frac{3}{2} \bar{\rho}_- \bar{v}_-^2 - \bar{h}_- \bar{\rho}_- & a_{66} &= C_Y \\ a_{34} &= Y \bar{\rho}_+ \bar{h}_+ \\ a_{35} &= -\bar{\rho}_- \bar{v}_- \end{aligned}$$

I, C, Equations (cont.)

$$Y = - \frac{(\sigma^2 + S_{v\eta}^2)}{\sigma\gamma} \tanh \left[(\sigma^2 + S_{v\eta}^2)^{1/2} x \right] \quad \text{where } \sigma = j\omega$$

All other a's are zero.

Also,

$$b_1 = 1, b_2 = \bar{v}_L, b_3 = \bar{h}_L + (\gamma-1) \frac{\bar{v}_L^2}{2}, b_4 = 0, b_5 = 0, b_6 = 0.$$

The a's and b's must be evaluated from the following input parameters:

$$S_{v\eta}, \omega^*, x^*, E(\omega), C(\omega), \bar{v}_-, \bar{v}_L^*, \bar{c}_O^*, r_c^*, \bar{w}^*, \gamma, \bar{p}_O^*$$

as well as the conditions:

$$\bar{p}_+ = 1, \bar{\rho}_+ = 1, \bar{h}_+ = 1, \bar{h}_L = 1$$

The following equations can be used to determine other variables:

$$\bar{v}_L = \frac{\bar{v}_L^*}{\bar{c}_O^*}; \quad g^* R_O^* T_O^* = \frac{\bar{c}_O^{*2}}{\gamma}; \quad \bar{\rho}_O^* = \frac{\bar{p}_O^*}{g^* R_O^* T_O^*};$$

$$\bar{\rho}_- = \frac{\bar{w}^*}{\bar{\rho}_O^* g^* \pi r_c^{*2} \bar{c}_O^* \bar{v}_-}; \quad \bar{p}_- = 1 - \gamma(\bar{\rho}_- \bar{v}_-^2 - \bar{\rho}_- \bar{v}_- \bar{v}_L);$$

$$\bar{h}_- = 1 + \frac{\bar{v}_L^2}{2} - \frac{\bar{v}_-^2}{2}; \quad \omega = \frac{\omega^* r_c^*}{\bar{c}_O^*}; \quad x = \frac{x^*}{r_c^*}; \quad \bar{T}_- = \bar{h}_-.$$

L_c^* replaces r_c^* for longitudinal modes in the definition of x and ω .

I, C, Equations (cont.)

After the matrix equation is multiplied by the inverse of matrix (a), it represents a series of equations relating each perturbation variable to $\dot{m}'_b$. The first and fourth of these relate p'_+ and p'_- to $\dot{m}'_b$. A characteristic equation is formed by combining these equations.

$$\dot{m}'_b = \bar{\rho}_- \bar{v}_- \left[\frac{p'_+ + p'_-}{2 \bar{p}_-} \right] \eta (1 - e^{-i\omega\tau}) ,$$

$$p'_+ = K_1 \dot{m}'_b ,$$

$$p'_- = K_2 \dot{m}'_b ;$$

$$\frac{2 \bar{p}_-}{\bar{\rho}_- \bar{v}_- (K_1 + K_2)} = h$$

I, C, Equations (cont.)

3. Program C - Nozzle Admittance

Nozzle admittance is calculated for an axisymmetric conventional or annular nozzle with an inner wall radius r_i and outer wall radius r_o . These radii are functions of axial distance Z from the nozzle throat which are defined as follows:

$$r_o^*(Z^*) = R_{ATo}^* + R_{CTo}^* - \sqrt{R_{CTo}^{*2} - Z^{*2}}, \quad 0 \leq Z^* \leq Z_{1o}^*$$

$$\text{where } Z_{1o}^* = R_{CTo}^* \sin \alpha_o$$

$$r_o^*(Z^*) = R_{ST1o}^* + \frac{Z^* - Z_{1o}^*}{Z_{2o}^* - Z_{1o}^*} (R_{ST2o}^* - R_{ST1o}^*), \quad Z_{1o}^* < Z^* \leq Z_{2o}^*$$

$$\text{where } R_{ST1o}^* = R_{ATo}^* + R_{CTo}^* (1 - \cos \alpha_o)$$

$$Z_{2o}^* = Z_{1o}^* + (R_{ST2o}^* - R_{ST1o}^*) \cot \alpha_o$$

$$R_{ST2o}^* = R_{ACo}^* - R_{CCo}^* (1 - \cos \alpha_o)$$

$$r_o^*(Z^*) = R_{ACo}^* - R_{CCo}^* + \sqrt{R_{CCo}^{*2} - (Z_{3o}^* - Z^*)^2}, \quad Z_{2o}^* < Z^* \leq Z_{3o}^*$$

$$\text{where } Z_{3o}^* = Z_{2o}^* + R_{CCo}^* \sin \alpha_o$$

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I, C, Equations (cont.)

$$r_o^* (Z^*) = R_{ACo}^* , \quad Z_{3o}^* \leq Z^*$$

$$r_i^* (Z^*) = R_{ATi}^* + R_{CTi}^* - \sqrt{R_{CTi}^{*2} - Z^{*2}} , \quad 0 \leq Z^* \leq Z_{1i}^* ,$$

$$R_{CTi}^* > 0$$

$$r_i^* (Z) = R_{ATi}^* + R_{CTi}^* + \sqrt{R_{CTi}^{*2} - Z^{*2}} , \quad 0 \leq Z^* \leq Z_{2i}^*$$

$$R_{CTi}^* < 0$$

$$r_i^* (Z^*) = 0 , \quad R_{ATi}^* = 0$$

$$R_{ACi}^* = 0$$

$$\text{where } Z_{1i}^* = R_{CTi}^* \sin \alpha_i$$

$$r_i^* (Z^*) = R_{ST1i}^* + \frac{Z^* - Z_{1i}^*}{Z_{2i}^* - Z_{1i}^*} (R_{ST2i}^* - R_{ST1i}^*) \quad Z_{1i}^* < Z^* \leq Z_{2i}^*$$

$$\text{where } R_{ST1i}^* = R_{ATi}^* + R_{CTi}^* (1 - \cos \alpha_i)$$

$$Z_{2i}^* = Z_{1i}^* + (R_{ST2i}^* - R_{ST1i}^*) \cot \alpha_i$$

$$R_{ST2i}^* = R_{ACi}^* - R_{CCi}^* (1 - \cos \alpha_i)$$

I, C, Equations (cont.)

$$r_i^*(Z^*) = R_{ACi}^* - R_{CCi}^* + \sqrt{R_{CCi}^{*2} - (Z_{3i}^* - Z^*)^2}, \quad Z_{2i}^* < Z^* \leq Z_{3i}^*$$

$$R_{CCi}^* > 0$$

$$r_i^*(Z^*) = R_{ACi}^* - R_{CCi}^* - \sqrt{R_{CCi}^{*2} - (Z_{3i}^* - Z^*)^2}, \quad Z_{2i}^* < Z^* \leq Z_{3i}^*$$

$$R_{CCi}^* < 0$$

$$\text{where } Z_{3i}^* = Z_{2i}^* + R_{CCi}^* \sin \alpha_i$$

$R_{ATo}^*, R_{ATi}^*, R_{CTo}^*, R_{CTi}^*, R_{ACo}^*, R_{ACi}^*, R_{CCo}^*, R_{CCi}^*, \alpha_o$ and α_i are input data defined by Figures 14 and 15.

From the nozzle radius calculate the nozzle area a^* .

$$a^*(Z^*) = \pi [r_o^{*2}(Z^*) - r_i^{*2}(Z^*)]$$

Mach number, $M(Z)$, can be obtained by creating a table of M versus a^* from the equation.

$$\frac{a^*(Z)}{a^*(Z^*=0)} = \frac{1}{M(Z^*)} \left[\left(\frac{2}{\gamma+1} \right) \left(1 + \frac{\gamma-1}{2} M^2(Z^*) \right) \right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

The velocity divided by the speed of sound at the throat, $\bar{q}$, (commonly called M^*) is given by

$$\bar{q}(Z)^2 = \frac{\frac{\gamma+1}{2} M^2(Z^*)}{1 + \frac{\gamma-1}{2} M^2(Z^*)}$$

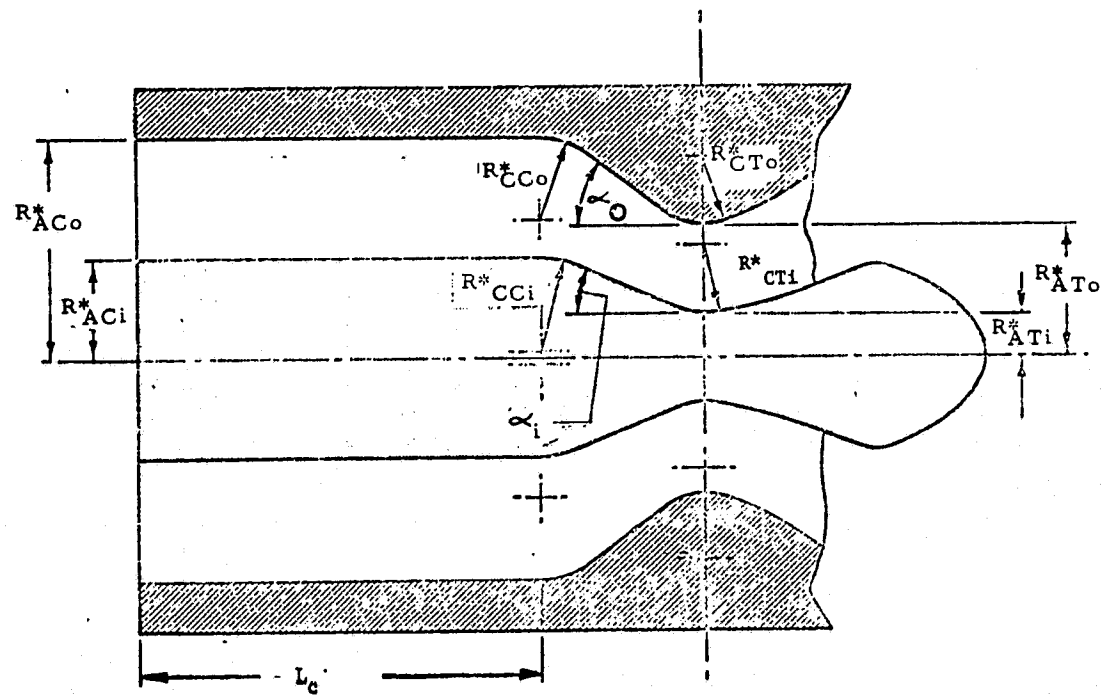


Figure 14 -- Computer Input Parameters -- Annular Chamber

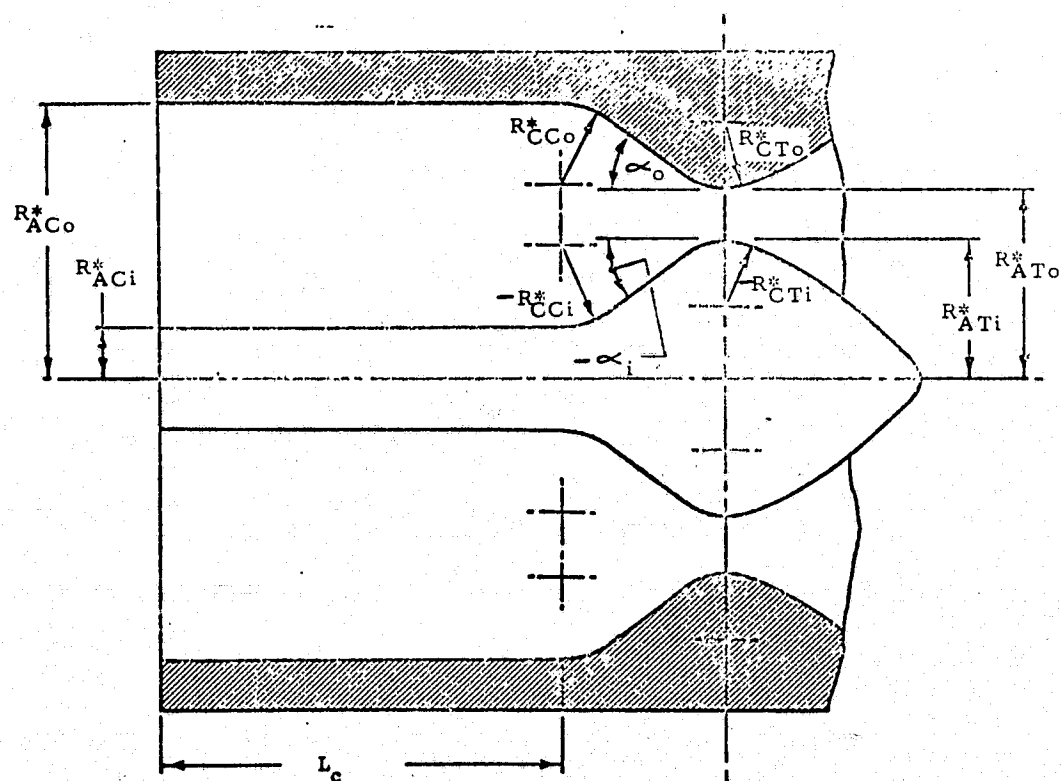


Figure 15 -- Computer Input Parameters -- Annular Chamber

I, C, Equations (cont.)

The velocity potential, which is the independent variable for the equations of motion in the nozzle, is given by

$$\hat{\phi}(Z^*) = \frac{2\hat{K}}{R_{AT}^*} \int_0^{Z^*} \bar{q}(Z^*) dZ^* \quad (43)$$

Auxiliary functions $\hat{\zeta}$, $\hat{\xi}^{(1)}$, $\hat{\xi}^{(2)}$, f_3 which are required to define the admittance coefficients in the nozzle are given by the following differential equations

$$b \left(\frac{d\hat{\zeta}}{d\hat{\phi}} + \hat{\zeta}^2 \right) = (g + ih) \hat{\zeta} - (j + ik)$$

$$\begin{aligned} \frac{d}{d\hat{\phi}} [(1 - \bar{q}^2) \hat{\xi}^{(1)}] = & - \left[\hat{\zeta} - i\hat{\omega} \left(\frac{1}{2\bar{q}^2} + \frac{2}{(\gamma+1)(1-\bar{q}^2)} \right) \right] (1-\bar{q}^2) \hat{\xi}^{(1)} \\ & + \frac{2}{\gamma+1} \frac{\hat{s}_{vn}^2}{4} \frac{\bar{c}^{\frac{2}{\gamma-1}}}{\bar{q}} \end{aligned}$$

$$\begin{aligned} \frac{d}{d\hat{\phi}} [(1-\bar{q})^2 \hat{\xi}^{(2)}] = & - \left[\hat{\zeta} - i\hat{\omega} \left(\frac{1}{2\bar{q}^2} + \frac{2}{(\gamma+1)(1-\bar{q}^2)} \right) \right] (1-\bar{q}^2) \hat{\xi}^{(2)} \\ & + \frac{2}{\gamma+1} \left[\frac{df_3}{d\hat{\phi}} + \frac{\hat{s}_{vn}^2 \bar{c}^{\frac{2}{\gamma-1}}}{2i\hat{\omega}} \bar{q} \left(\frac{1-\bar{q}^2}{2\bar{q}^2} + \frac{\hat{c}^2}{\bar{q}} f_3 \right) \right] \end{aligned}$$

$$\frac{d}{d\hat{\phi}} (\bar{c}^2 f_3) - \frac{i\hat{\omega}}{2\bar{q}^2} (\bar{c}^2 f_3) = \frac{1}{2} \frac{d\bar{q}^2}{d\hat{\phi}}$$

where $b = \bar{q}^2 (\bar{c}^2 - \bar{q}^2)$

I, C, Equations (cont.)

$$g = \frac{\gamma+1}{2} \frac{(\bar{q})^2}{\bar{c}^2} \frac{d(\bar{q})^2}{d\phi}$$

$$h = \hat{\omega} (\bar{q})^2$$

$$j = \frac{\hat{\omega}^2}{4} - \frac{\bar{q} (\bar{c})^{\frac{2}{\gamma-1}}}{4} (\hat{s}_{v\eta})^2$$

$$k = \frac{\gamma-1}{4} \frac{(\bar{q})^2}{\bar{c}^2} \frac{d(\bar{q})^2}{d\phi} \hat{\omega}$$

$$\hat{\omega} = \omega / \hat{K} \quad (44)$$

$$\hat{s}_{v\eta} = s_{v\eta} / \hat{K} \quad (45)$$

$$\hat{K} = \left(\frac{d\bar{q}}{dZ} \right)_{Z=0} = R_{AT0}^* \left[\frac{2}{\gamma+1} \left(\frac{R_{AT0}^*}{R_{CT0}^*} - \frac{R_{AT1}^*}{R_{CT1}^*} \right) / \left(R_{AT0}^{*2} - R_{AT1}^{*2} \right) \right]^{1/2}$$

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I, C, Equations (cont.)

The equations for $\hat{\xi}^{(1)}$, $\hat{\xi}^{(2)}$, and $\hat{\zeta}$ are singular at the throat, $\hat{\phi} = 0$. The solutions to these equations at the throat can be found using a series expansion about the throat. The numerical integration can be started at some small value of $\hat{\phi} = \hat{\phi}_i$. At $\hat{\phi}_i$ the initial values are:

$$\bar{c}^{-2} f_3 = 0.5 \hat{\phi}_i$$

$$(1-\bar{q}^{-2}) \hat{\xi}^{(1)} = \frac{\frac{2}{\gamma+1} \frac{\hat{s}_{v\eta}^2}{4} \hat{\phi}_i}{1 + \frac{i \hat{\omega}}{\gamma+1}}$$

$$(1-\bar{q}^{-2}) \hat{\xi}^{(2)} = \frac{\hat{\phi}_i}{\gamma+1 + i2\hat{\omega}}$$

$$\hat{\zeta} = \alpha_0 + \hat{\phi} \alpha_1$$

where

$$\alpha_0 = \frac{\hat{\omega}^2 - \hat{s}_{v\eta}^2 - i(\gamma-1)\hat{\omega}}{4(\frac{\gamma+1}{2} + i\hat{\omega})}$$

$$\alpha_1 = \frac{\frac{\gamma+1}{2} \alpha_0^2 + \frac{\gamma+1}{2} \alpha_0 (\frac{\gamma+1}{2} + 2\tilde{b}) + \alpha_0 i\hat{\omega} + \frac{\hat{s}_{v\eta}^2}{8} (1-\gamma) + \frac{i\omega(\gamma-1)}{4} (\frac{\gamma+1}{2} + 2\tilde{b})}{-(\gamma+1 + i\hat{\omega})}$$

and $\tilde{b}$ comes from the expansion $\bar{q}^{-2} = 1 + \hat{\phi} + \tilde{b} \hat{\phi}^2$

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I, C, Equations (cont.)

The nozzle admittance coefficients can be defined from the auxiliary functions at the desired value of Z.

$$A = \frac{1}{\gamma} \left(\frac{\gamma + 1}{2} \right)^{\frac{1}{2}(\gamma+1)/(\gamma-1)} \frac{\bar{q}}{c^{2/(\gamma-1)}} \frac{\overline{c^2 \hat{\xi}}(1) - \hat{\xi}}{q^2 (\overline{c^2 \hat{\xi}}(1) - \hat{\xi}) - \frac{1}{2} i \hat{\omega}} \quad (46)$$

$$B = iK \frac{\hat{\omega} \bar{q}^{-\frac{1}{2}}}{c^{1/(\gamma-1)}} \frac{\overline{c^2 \hat{\xi}}(1)}{q^2 (\overline{c^2 \hat{\xi}}(1) - \hat{\xi}) - \frac{1}{2} i \hat{\omega}} \quad (47)$$

$$C = \left(\frac{2}{\gamma + 1} \right)^{\frac{1}{2}} \frac{\bar{q}}{c^2} \left[\frac{\frac{1}{2}(1 - \bar{q}^2) \hat{\xi}(1) - \frac{1}{2} i \hat{\omega} \hat{\xi}(2) + f_3 \hat{\xi}}{q^2 (\overline{c^2 \hat{\xi}}(1) - \hat{\xi}) - \frac{1}{2} i \hat{\omega}} \right] \quad (48)$$

$$E = \gamma A + \frac{iB}{\omega_c} \quad (49)$$

I, C, Equations (cont.)

4. Program D - Expansion of Results of Programs A & B

Take h from Program B

$$\tilde{h} = \frac{h}{A_{v\eta} + i \left(\frac{B_{v\eta} \frac{1_R}{n}}{\gamma\omega} + \frac{C_{v\eta} \frac{1_\theta}{n}}{\gamma\omega} \right)}$$

5. Program E - Injector Nonuniformity Coefficients

For standing mode

$$A_{v\eta} = \frac{\int_0^{2\pi} \int_0^1 \mu(r, \theta) F_P(r, \theta) J_v^2(s_{v\eta} r) \cos^2 v\theta r dr d\theta}{\int_0^{2\pi} \int_0^1 J_v^2(s_{v\eta} r) \cos^2 v\theta r dr d\theta} \quad (50)$$

$$B_{v\eta} = \frac{\int_0^{2\pi} \int_0^1 \mu(r, \theta) F_R(\omega, r, \theta) J_v'(s_{v\eta} r) J(s_{v\eta} r) \cos^2 v\theta r dr d\theta}{\int_0^{2\pi} \int_0^1 J_v^2(s_{v\eta} r) \cos^2 v\theta r dr d\theta} \quad (51)$$

$$C_{v\eta} = - \frac{\int_0^{2\pi} \int_0^1 \mu(r, \theta) \frac{F_T(r, \theta, \omega)}{r} J_v^2(s_{v\eta} r) v \sin v\theta \cos v\theta r dr d\theta}{\int_0^{2\pi} \int_0^1 J_v^2(s_{v\eta} r) \cos^2 v\theta r dr d\theta} \quad (52)$$

For a spinning mode make the following replacements:

$$\begin{aligned} A_{v\eta} &= \frac{A_{v\eta}}{2} \\ B_{v\eta} &= \frac{B_{v\eta}}{2} \\ C_{v\eta} &= \frac{i C_{v\eta}}{2} \end{aligned}$$

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I, C, Equations (cont.)

$$\mu(r, \theta) = \frac{\sigma(r, \theta)}{\sigma_{ave}} \quad (53)$$

$\sigma(r, \theta)$ is the injection density (weight flow per unit area)

σ_{ave} is the total weight flow divided by the total injector area

6. Program F - Stability Zone Mapping

$$n (1 - e^{-i\omega\tau}) = \tilde{h}$$

7. Program I - Nonlinear Combustion Response

For non-linear combustion response, input tables of burning rate versus pressure, radial velocity, and tangential velocity, ϕ_P , ϕ_R , ϕ_L . Additional data needed are: P_{oo} (amplitude of pressure oscillation being considered divided by the mean chamber pressure) TFLP, TFLR, and TFLT (the linear transfer functions for equivalent linear operation associated with the pressure, radial velocity and tangential velocity dependent nonlinear element.) A describing function is calculated at each point on the injector.

$$f_p(r, \theta) = \frac{1}{\pi P_o (TF_{LP})} \int_{-\pi}^{\pi} \phi_P(\omega\tau, r, \theta) \cos(\omega\tau) d(\omega\tau) \quad (54)$$

where

$$\phi_P(\omega\tau, r, \theta) = \phi_P(P')$$

$$P' = P_o \cos \omega\tau$$

$$P_o = |P_{oo} J_v(s_{v\eta}) \cos v \theta|$$

$$F_R(r, \theta, \omega) = \frac{1}{\pi V_o (TF_{LR})} \int_{-\pi}^{\pi} \phi_R(\omega\tau, r, \theta) \cos(\omega\tau) d(\omega\tau)$$

I, C, Equations (cont.)

where

$$\phi_R(\omega\tau, r, \theta) = \phi_R(v')$$

$$v' = V_o \cos \omega\tau$$

$$V_o = |V_{oo} \left[\frac{v J_v(s_{v\eta} r) - s_{v\eta} J_{v+1}(s_{v\eta} r)}{r} \right] \cos v\theta|$$

$$V_{oo} = \frac{P_{oo}}{\gamma\omega}$$

$$f_T(r, \theta, \omega) = \frac{1}{\pi W_o(TF_{LT})} \int_{-\pi}^{\pi} \phi_T(\omega\tau, r, \theta) \cos(\omega\tau) d(\omega\tau)$$

where

$$\phi_T(\omega\tau, r, \theta) = \phi_T(w')$$

$$w' = W_o \cos \omega\tau$$

$$W_o = \left| \frac{V_{oo} J_v(s_{v\eta} r) v \sin v\theta}{r} \right|$$

8. Program J, Injected Mass Distribution Effects

This program calculates mass flow distribution $\mu(r, \theta)$ used in the program for calculating the $A_{v\eta}$, $B_{v\eta}$ and $C_{v\eta}$. It also calculates useful design parameters such as pressure drops, injection velocities, mixture ratios, and flow areas. Input consists of type and location of elements, description of each element type in terms of orifice diameters and number of orifices, and overall information including propellant densities, orifice discharge coefficients, amount of film cooling, injector radius, total flow rates and total mixture ratio.

First the total orifice area, A_T , for each propellant is calculated:

$$A_T = \sum_i 0.7853891 d_i^2$$

I, C, Equations (cont.)

Then the area of the film coolant orifices, A_{FCT} , is calculated

$$A_{FCT} = \sum_i 0.7853891 d_{FC_i}^2$$

Total flows of oxidizer and fuel are then calculated. The equations used depend on how the information on the film coolant is input.

If percent film coolant is given; P_{ffc} , percent fuel film coolant; P_{xfc} percent oxidizer film coolant, the total fuel flow rate in injector matrix, W_{fT} , is given by:

$$W_{fT} = \frac{W_T}{MR+1} (1 - P_{ffc})$$

The total oxidizer flow rate in the injector matrix, W_{XT} , is given by:

$$W_{XT} = W_T - \frac{W_T}{MR+1} (1 - P_{xfc})$$

If the number and size of the film cooling orifices are given in terms of the fuel hole area in the matrix A_F , oxidizer hole area in the matrix A_X , fuel film coolant hole area A_{FFC} , and oxidizer film coolant area A_{XFC} .

$$W_{fT} = \frac{W_T}{MR+1} \frac{A_{FT}}{A_{FT} + A_{FFC}}$$

$$W_{XT} = W_T - \frac{W_T}{MR+1} \frac{A_{XT}}{A_{XT} + A_{XFC}}$$

The total weight flow per element is given by:

$$W_{E_i} = \frac{A_{Fi}}{A_{FT}} W_{fT} + \frac{A_{Xi}}{A_{XT}} W_{XT}$$

I, C, Equations (cont.)

W_{Ei} is used in Program E as follows:

$$\frac{W_{Ei}}{W_T} \quad XM \quad XN = \mu_i \text{ for each element.}$$

See Equations 51 to 53.

This program also calculates injector pressure drop ΔP :

$$\Delta P = \frac{W^2}{C_d^2 A^2} \frac{(144)}{\rho (64.4)}$$

where C_d is the orifice discharge coefficient, A is the orifice area in in.², W is the orifice flow rate in lbm/sec and ρ is the propellant density in lbm/ft³. Injection velocity V is calculated from ΔP .

$$V = \sqrt{\frac{(144)(64.4) \Delta P}{\rho}}$$

I, Problem (cont.)

D. DEFINITION OF TERMS

$a^*(z^*)$	Area of annular nozzle, in. ²
A_{Fi}	Area of the ith fuel element, in. ²
A_{xi}	Area of the ith oxidizer element, in. ²
A_{XFC}	Total area of oxidizer film coolant orifices, in. ²
A_{FFC}	Total area of fuel film coolant orifices, in. ²
A_{XT}	Total area of oxidizer orifices excluding film coolant, in. ²
A_{FT}	Total area of fuel orifices excluding film coolant, in. ²
A_{FCT}	Total film coolant area, in. ²
A_T	Total orifice area, in. ²
$A_{\nu\eta}$	Factor measuring the effect of nonuniform injection on stability (see Equation 37)
b	Variable used in nozzle analysis
B	Term used in longitudinal analysis
$B_{\nu\eta}$	Factor measuring the effect of nonuniform injection on stability (see Equation 37)
c	Speed of sound nondimensionalized by the stagnation speed of sound in the chamber analysis or the throat speed of sound in the nozzle analysis
C	Term used in longitudinal analysis
C_d	Injector orifice discharge coefficient
C_{STAR}	Characteristic Exhaust Velocity, ft/sec
C_o^*	Dimensional stagnation gas speed of sound, feet per second
C_{th}^*	Dimensional speed of sound at the nozzle throat, feet per second
C_1, C_2	Constants in radial dependent factor of pressure in an annular chamber
$C_{\nu\eta}$	Factor measuring the effect of nonuniform injection on stability (see Equation 37) (complex number, $C_{\nu\eta R} + i C_{\nu\eta I}$)
d_i	Diameter of the ith orifice, in.
d_{FCi}	Diameter of the ith film in coolant orifice, in.
D	Term used in longitudinal analysis
$D_1(z)$	Combustion distribution used in transverse analysis

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I, D, Definition of Terms (cont.)

e	Base the natural logarithms
E	Term used in longitudinal analysis
f*	Dimensional frequency in Hertz
f_3	Auxiliary function in nozzle analysis (complex)
f_p	$\frac{\partial f}{\partial p}$
f_T	$\frac{\partial f}{\partial T}$
f_v	$\frac{\partial f}{\partial v}$
f()	Rate of preparatory processes before burning
F	Term used in longitudinal analysis
F(ω)	Describing function defined by Equation 34
F_p	Describing function for pressure oscillations
F_R	Describing function for velocity oscillations in the radial direction
F_T	Describing function for velocity oscillations in the tangential direction
g	Variable used in nozzle analysis
$\hat{h}$	Variable used in nozzle analysis
h	Part of characteristic equation which includes all acoustic effects (Equation 40) can also be viewed as a chamber admittance (complex number, $h = h_r + i h_i$)
h_i	Imaginary part of h
h_r	Real part of h
$\tilde{h}$	Includes acoustic and nonuniform injection effects in the characteristic equation (see Equation 40)
$\tilde{h}_i$	Imaginary part of $\tilde{h}$
$\tilde{h}_r$	Real part of $\tilde{h}$
h_s	Total enthalpy of gas nondimensionalized by $R^* T_o^*$
i	Unit imaginary number $\sqrt{-1}$
I	Input to a nonlinear element
I_1	Term used in longitudinal analysis

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I, D, Definition of Terms (cont.)

j	Variable used in nozzle analysis
J	Term used in the longitudinal analysis
$J_v()$	Bessel function of the first kind of order v
$J'_v()$	Derivative with respect to the argument of the Bessel function of the first kind of order v
k	Variable used in nozzle analysis
$\hat{K}$	Velocity gradient at the throat, nondimensional (see Equation 45)
K	Factor involving droplet drag on frequency defined in Equation 10f
K_1	Term used in longitudinal analysis
l_r	Velocity interaction index measuring the amount of interaction between gas velocity in the radial direction and the burning rate
l_θ	Velocity interaction index measuring the amount of interaction between gas velocity in the tangential direction and the burning rate
L	Term used in longitudinal analysis
L_c^*	Chamber length, in.
$\dot{m}_b$	Mass burning rate, lbf-sec/ft
$\dot{m}_i$	Mass injection rate, lbf-sec/ft
m_r	Displacement interaction index measuring the amount of interaction between gas displacement in the radial direction and the burning rate
m_θ	Displacement interaction index measuring the amount of interaction between gas displacement in the tangential direction and the burning rate
M	Mach number
M_1	Term used in longitudinal analysis
M_e	Mach number at nozzle entrance
MR	Mixture ratio
n	Pressure interaction index measuring the amount of interaction between pressure oscillations and burning rate oscillations
n_{min}	Minimum value of the pressure interaction index for a particular mode
N	Term used in the longitudinal analysis

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I, D, Definition of Terms (cont.)

O	Output of a nonlinear element
O_1	Component of output of a nonlinear element at the input frequency ω_f
O_j	Component of output of a nonlinear element of a frequency $j\omega_f$: $j = 2, 3, 4$, etc.
p	Pressure nondimensionalized by the stagnation gas pressure
P_o	Zeroth order solution for pressure. Defined by Equations 11 and 12a
P_1	First order solution for pressure defined by Equations 11 and 12b
P	Z dependent factor in pressure perturbation
P_o	Amplitude of pressure oscillation lbf/in^2
P_{oo}	Constant coefficient measuring amplitude of pressure defined in Equation 13a
P_{ffc}	Percent fuel film coolant
P_{xfc}	Percent oxidizer film coolant
$\vec{q}$	Velocity nondimensionalized by the stagnation gas speed of sound, C_o^* in chamber. In the nozzle it is nondimensionalized by the speed of sound at the throat
Q	Burning rate per unit volume nondimensionalized by $\frac{\rho_o^* C_o^*}{r_c^*}$ or $\frac{\rho_o^* C_o^*}{L_c^*}$ for transverse or longitudinal modes.
r	Coordinate in the radial direction. Nondimensionalized by the chamber radius r_c^*
r_c^*	Dimensional chamber radius in inches
r_d^*	Droplet radius, feet
r_i^*	Inner radius of an annular chamber, feet
r_o^*	Outer radius of an annular chamber, feet
R^*	Gas constant, $\text{ft-lbf/lbm-}^\circ\text{R}$
R	Ratio of inner radius and outer radius of an annular nozzle
R_{ACi}^*	Radius of inner contour of an annular chamber at the chamber entrance, inches (see Figures 14,15)
R_{ACo}^*	Radius of outer contour of an annular chamber at the chamber entrance, inches (see Figures 14,15)

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I, D, Definition of Terms (cont.)

R_{ATi}^*	Radius of the throat of the inner contour of an annular nozzle, inches (see Figures 14,15)
R_{ATo}^*	Radius of the throat of the outer contour of an annular nozzle, inches (see Figures 14,15)
R_{CCi}^*	Radius of curvature at the chamber entrance of the inner contour of an annular nozzle, inches (see Figures 14,15)
R_{CCo}^*	Radius of curvature at the chamber entrance of the outer contour of an annular nozzle, inches (see Figures 14,15)
R_{CTi}^*	Radius of curvature at the throat of an inner contour of an annular nozzle, inches (see Figures 14,15)
R_{CTo}^*	Radius of curvature at the throat of the outer contour of an annular nozzle, inches (see Figures 14,15)
R_{ST1i}^*	Radius of inner contour of annular nozzle at transition between throat curvature and tangent, inches $[r_i^* (Z_{1i}^*)]$
R_{ST1o}^*	Radius of outer contour of annular nozzle at transition between throat curvature and tangent, inches $[r_o^* (Z_{1o}^*)]$
R_{ST2i}^*	Radius of inner contour of annular nozzle at transition between chamber curvature and tangent, inches $[r_i^* (Z_{2i}^*)]$
R_{ST2o}^*	Radius of outer contour of annular nozzle at transition between chamber curvature and tangent, $[r_o^* (Z_{2o}^*)]$
S	Z dependent factor in entropy perturbation
s_{vn}	Mode number for a transverse mode defined by Equations 19 thru 21 tabulated for a cylindrical chamber in Table I
$\hat{s}_{vn}$	Mode number used in nozzle analysis (see Equation 45)
t	time nondimensional as τ_i
T	Temperature nondimensionalized by T_o^*
T_1	Term in longitudinal analysis
T_o^*	Stagnation gas temperature, °R
TF	Equivalent linear transfer function (see Equation 33)
$(TF)_{LIM}$	Limiting linear transfer function
u	Gas velocity in the axial direction nondimensionalized by C_o^*
$\bar{u}_e$	Mean value of the gas velocity at the nozzle entrance nondimensionalized by the stagnation gas speed of sound
u_L	Axial velocity of liquid drops, ft/sec
$\bar{u}_{Lo}^*$	Liquid injection velocity, ft/sec (see Equation 75)
U	Z dependent factor in axial velocity perturbation

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I, D, Definition of Terms (cont.)

v	Gas velocity in the radial direction nondimensionalized by C_o^*
V_o	Amplitude of radial velocity oscillation
V_{oo}	Constant coefficient measuring the amplitude of radial velocity
V	Z-dependent factor in radial velocity perturbation
w	Gas velocity in the tangential direction nondimensionalized by C_o^*
W_o	Amplitude of tangential velocity oscillation
W_T	Total propellant weight flow rate, lb/sec
W_{fT}	Total fuel flow rate excluding film coolant, lb/sec
W_{XT}	Total oxidizer flow rate excluding film coolant, lb/sec
W_{Ei}	Flow rate of the ith element, lb/sec
$Y_v()$	Bessel function of the second kind of order v
$Y'_v()$	Derivative with respect to the argument of the Bessel function of the second kind of order v
Y_x	Mass fraction of vaporized oxidizer
Y_1, Y_2, Y_3	Variables used in transverse analysis
Z	Length along the chamber axis nondimensionalized by the chamber length L^* for a longitudinal mode or by the chamber radius r_c^* for a transverse mode
Z_e	Axial plane at nozzle entrance
Z_{li}^*	Axial distance from throat to first transition from curve to tangent for inner contour of an annular nozzle, feet
Z_{lo}^*	Axial distance from throat to first transition from curve to tangent for outer contour of an annular nozzle, feet
Z_{2i}^*	Axial distance from throat to second transition from tangent to curve for inner contour of an annular nozzle, feet
Z_{2o}^*	Axial distance from throat to second transition from tangent to curve for outer contour of an annular nozzle, feet
Z_{3i}^*	Axial distance from throat to transition from chamber to nozzle of the inner contour of an annular nozzle, feet
Z_{3o}^*	Axial distance from throat to transition from chamber to nozzle, feet

I, D, Definition of Terms (cont.)

α_o	Convergence angle of outer contour of an annular nozzle (see Figure 14)
α_i	Convergence angle of inner contour of an annular nozzle (see Figure 14)
α_1	Real part of derivative of ζ at throat
α_o	Real part of ζ at throat
β	Angle of orientation relative to the θ coordinate (see Figure 10)
β_1	Imaginary part of derivative of ζ at throat
β_N	Longitudinal admittance coefficient for entropy, complex, $\beta_N = \beta_{Nr} + i \beta_{Ni}$
β_o	Imaginary part of ζ at throat
γ	Ratio of specific heats
Γ	Z-dependent factor in the first order solution for pressure defined by Equation 13
δ_r	Displacement of gas in the radial direction due to pressure oscillations
δ_θ	Displacement of gas in the tangential direction due to pressure oscillations
δ	Term used in longitudinal analysis
δ_m	Term used in longitudinal analysis
ΔP	Injector pressure drop, lb/in. ²
ζ	Variable in Z direction, used in various integrals
$\hat{\zeta}$	Auxiliary function in nozzle analysis complex
η	Index numbering the zeros in the derivative of the Bessel function
θ	Coordinate in the tangential direction
θ_1	Term used in longitudinal analysis
θ_F	Fuel angle of injection measured from Z axis
θ_x	Oxidizer angle of injection measured from Z axis
Θ	θ dependent factor in the solution for pressure defined in Equation 13
Θ^*	Complex conjugate of Θ
κ	Momentum interchange coefficient between droplets and gas * = $\frac{9}{2} \frac{\mu^*}{r_d^{*2} \rho_L^*}$ nondimensionalized by $\frac{C_o^*}{r_c^*}$ or $\frac{C_o^*}{L_c^*}$ for transverse or longitudinal modes

I, D, Definition of Terms (cont.)

λ	Real part of complex frequency σ
μ	Burning rate distribution function = $\frac{\sigma}{\sigma_{\text{mean}}}$
μ^*	Viscosity in lb-sec/ft ²
ν	Order of the Bessel function which indicates the number of cycles of pressure oscillation when traversing in the tangential direction at a fixed radius, axial length and time
ν_F	Fuel injection velocity, ft/sec
ν_x	Oxidizer injection velocity, ft/sec
ξ	Factor involving droplet drag, frequency and liquid inertance defined in Equation 10f
ξ_1	Distance from injector to concentrated combustion front non-dimensionalized by chamber length
ξ_L	Term used in longitudinal analysis
$\xi^{(1)}$	Auxiliary function in nozzle analysis (complex)
$\xi^{(2)}$	Auxiliary function in nozzle analysis (complex)
ρ	Density of chamber gases nondimensionalized by the stagnation gas density ρ_o^*
ρ_L^*	Density of liquid droplets, lb-sec ² /ft ⁴
σ	Complex frequency nondimensionalized as the reciprocal of τ_i
σ	Injection density
σ_{mean}	Mean injection density
τ_i	Portion of time lag which is insensitive to pressure oscillations nondimensionalized by $\frac{L_c^*}{C_o^*}$ for a longitudinal mode and $\frac{r_c^*}{C_o^*}$ for a transverse mode
τ	Mean sensitive time lag, nondimensional as τ_i
τ_T	Total time delay between injection of propellant and burning of that same propellant, nondimensional as τ_i
τ^*	Sensitive time lag, milliseconds
ϕ	Velocity potential in the exhaust nozzle
ϕ_p	Function relating input pressure to the combustion to the output (burning rate)

I, D, Definition of Terms (cont.)

ϕ_R	Function relating input radial velocity to the combustion to the output (burning rate)
ϕ_T	Function relating input tangential velocity to the combustion to the output (burning rate)
ϕ_1	Term used in longitudinal analysis
χ	ωt
ψ	Stream function in the exhaust nozzle
Ψ	r dependent factor in the solution for pressure defined by Equation 13 c
ω	Imaginary part of complex Laplace variable σ
	For a transverse mode in chamber $\omega = \frac{2 f^* r_c^*}{C_o^* 12}$
	For a longitudinal mode in chamber $\omega = \frac{2 f^* L_c^*}{C_o^* 12}$
	For the nozzle $\omega = \frac{2 f^* R_{AT}^*}{C_{th}^* 12}$
ω_f	Frequency of input signal to a nonlinear element
$\hat{\omega}$	Nondimensional frequency used in nozzle analysis (see Equation 44)
Ω	$(s_{vn}^2 - \omega^2)^{1/2}$
A	Nozzle admittance coefficient (see Equation 38) (complex number $A = A_r + i A_i$)
B	Nozzle admittance coefficient (see Equation 38) (complex number $B = B_r + i B_i$)
C	Nozzle admittance coefficient (see Equation 38) (complex number $C = C_r + i C_i$)
E	Combined nozzle admittance coefficient defined by Equation 49 (complex number $E = E_r + i E_i$)
P	Pressure sensitive combustion response defined as $n(1-e^{-\sigma\tau})$
R	Radial velocity sensitive combustion response defined by $l_r(1-e^{-\sigma\tau})$
R_δ	Radial displacement sensitive combustion response defined by $m_r(1-e^{-\sigma\tau})$
T_δ	Tangential displacement sensitive combustion response defined by $m_\theta(1-e^{-\sigma\tau})$

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I, D, Definition of Terms (cont.)

- τ Tangential velocity sensitive combustion response defined by $\bar{1}_\theta(1-e^{-\sigma\tau})$
- A bar over a variable indicates it is a mean quantity and does not depend on time
- ' A prime on a variable indicates it is a perturbation quantity
- A (+) when used as a subscript, means before the combustion front
- A (-) when used as a subscript, means after the combustion front

I, Problem (cont.)

E. SPECIAL OPTIONS

The stability solution of any given liquid rocket engine must consider the three main portions of that engine: (1) the injector, (2) the combustion chamber, and (3) the exhaust nozzle. It is clear that these considerations serve as the basis for building the computer program.

The complete program consists of nine separate subprograms which may be used independently or in various combinations depending upon the data available and the information required by the designer. Figure 16 shows the interrelationships of the programs and the optional routes for proceeding with the calculation. The individual programs are:

- A. Distributed Combustion Analysis for Transverse Modes
(not combined with longitudinal)
- B. Concentrated Combustion Analysis Valid for any Mode
- C. Nozzle Admittance. Here the resistive effects of the nozzle are calculated for both longitudinal and transverse modes.
- D. Expansion of Results from Programs A and B by interpolation and add nonuniformity and velocity effects.
- E. Injector Nonuniformity Coefficients. Calculates modulating coefficients for the effects resulting from uneven injection distribution and nonlinear combustion response.
- F. Stability Zone Mapping (n , τ)
- G. Not in use
- H. Not in use
- I. Nonlinear Combustion Response. Input to Program E.
- J. Injected Mass Distribution Effects

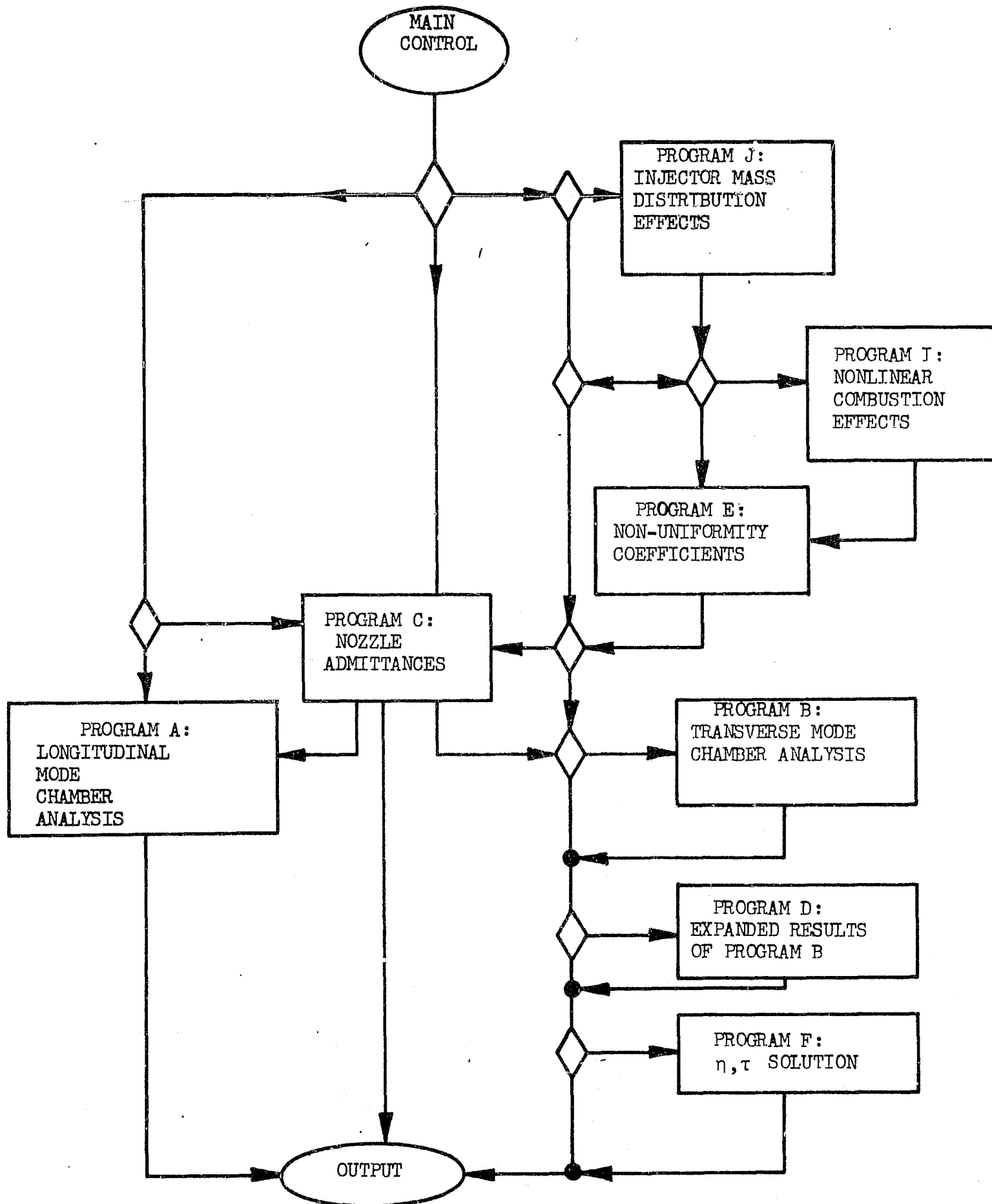


Figure 16 -- General Flow Diagram of the Computer Program

I, E, Special Options (cont.)

To assure that the proper subprograms are requested it is necessary to introduce a regulatory device that monitors the course of the entire program as it proceeds from one portion of the engine to another. This regulatory device is called MAIN CONTROL and serves to direct the flow of the solution in a consistent manner. Figure 16 illustrates the basic framework of the computer program. The diamond-shaped symbols can be considered as switches that are turned on if the particular item is to be analyzed by the program or turned off if the item is not needed. It is the function of MAIN CONTROL to turn these switches on or off according to the dictates of the problem.

The versatility of the program is illustrated in Figure 16 by the various flow-paths that can be constructed. For example, if the required injector and nozzle data were known beforehand, one could determine the transverse stability of the engine by merely running the combustion chamber program. If, on the other hand, all the data were unknown, proper use of MAIN CONTROL would initiate the injector, nozzle, and chamber programs. Therefore, the program is as suitable for parametric studies as it is for analysis of specific designs.

The injector requires programs J, I, and E to evaluate the injector parameters that affect stability. The chamber analysis requires A or B. Program C is run if the nozzle admittance is needed.

I, E, Special Options (cont.)

Following is an outline which will help determine which programs are to be run when evaluating an engine design.

I. Stability evaluation of an engine.

A. Is the mode a pure transverse mode?

Yes - either program A or B can be used.

No - program B only must be used.

B. Is the Mach number greater than about .3?

Yes - program B must be used.

No - programs A or B can be used.

C. Is injection distribution uniform?

Yes - A or B is all that is necessary.

No - run (A or B) and D.

1. Are distribution coefficients known?

a. Yes - run (A or B) and D.

b. No - run (A or B) and D and E.

(1) Is injection distribution known?

(a) Yes - run (A or B) and D and E.

(b) No - run (A or B) and D and E and J.

(2) Are non-linear effects desired?

(a) Yes - run (A or B) and D and E and I.

(b) No - run (A or B) and D and E.

D. Are nozzle admittance coefficients known?

A. Yes - run A or B

B. No - run (A or B) and C

II. Evaluation of Nozzle Admittance Only.

Run program C alone.

There are many more possible combinations of programs that can be run but this should serve as a useful guide.

I, Problem (cont.)

F. NUMERICAL METHODS OF SOLUTION*

As seen in Section C, the nozzle admittance coefficients are defined in terms of a system of non-linear differential equations which must be integrated from the throat to the nozzle entrance. These equations are integrated using Adams-Bashforth integration. Adams-Bashforth integration, as developed at Aerojet, uses several subroutines: ADSET, ADINT, ADCOR, ADRES and ADPAR. A description of Adams-Bashforth integration, which is reproduced from an Aerojet manual, follows:

PURPOSE

To integrate a system of N simultaneous first order differential equations.

RESTRICTIONS

N 200

METHOD

Adam's method of integration is used. Accurate computation of functional values between integration steps is possible using partial step integration. The set of differential equations must be written as a system of first order equations in the form:

$$\begin{aligned} \frac{dy_1}{dx} &= f_1(x, y_1, y_2, \dots, y_n) \\ \frac{dy_n}{dx} &= f_n(x, y_1, y_2, \dots, y_n) \end{aligned}$$

*Nomenclature in this section does not correspond to the rest of the report.

I, F, Numerical Methods of Solution (cont.)

USAGE

This subroutine has five separate entries:

1. The initial setup of the integration which specified number of equations and storage locations is done at the beginning of the program with the statement:

CALL ADSET (N, F, D, P, T, X, H, E)

with

N = number of first order differential equations to be integrated.

F = location of integrated y_1, y_2, y_n .

D = location of derivatives $\frac{dy_1}{dx}, \frac{dy_2}{dx}, \frac{dy_n}{dx}$

P = location of interpolated or partial step

$y_1^P, y_2^P, \dots, y_n^P$.

T = temporary storage having 8N cells

X = independent variable

H = integration step (initially set at approximate value)

E = location of desired accuracy, E (1) = accuracy of y_1 ...

E (n) = accuracy of y_n . If E (I) = 0 then program sets

$E(I) = 10^{-8}$

I, F, Numerical Methods of Solution (cont.)

2. To integrate forward a single integration step use:

```

CALL ADINT
100  D(1) = ...
      D(2) = ...
      .
      .
      D(N) = ...

CALL ADCOR (& 100)
    
```

The ADINT entry will predict values of $y_1, \dots, y_n$ at $x+h$ and the ADCOR corrects these values. After the CALL ADCOR statement one has available the values of y_1, y_2, y_n at the new value of x .

3. To obtain functional values $y_1, \dots, y_n$ at a discontinuity or print point x_p use the statement:

```
CALL ADPAR (XP)
```

This will compute $y_1^p, \dots, y_n^p$ and put these values in the P(N) array. This will not affect the values in F, D, T.

4. To restart the integration such as at a discontinuity:

```
CALL ADRES
```

This will zero out the difference table, compute new values for desired accuracy (e.g., if numbers in E have been changed) and set a flag to restart the integration.

The following two equations are the predictor and corrector equations written in terms of the backward difference operator $\nabla_n = y_n(x_k) - y_n(x_{k-1})$:

$$y_n^p(x_{k+1}) = y_n(x_k) + H \left(1 + \frac{1}{2} \nabla + \frac{5}{12} \nabla^2 + \frac{3}{8} \nabla^3 + \frac{251}{720} \nabla^4 \right) \frac{dy_n(x_k)}{dx}$$

I, F, Numerical Methods of Solution (cont.)

$$y_n^c(x_{k+1}) = y_n(x_k) + H \left[\frac{251}{720} \frac{dy_n(x_{k+1})}{dx} + \left(\frac{469}{720} + \frac{109}{720} \nabla + \frac{49}{720} \nabla^2 + \frac{32}{720} \nabla^3 \right) \frac{dy_n(x_k)}{dx} \right]$$

The predictor and corrector are in addition modified to account for the previous error. This effectively increases the order of the predictor and corrector. In these equations the integration error is controlled by changing H, the integration step size. The table of backward differences is modified whenever the integration step is changed. This table of backward differences is also used to interpolate for intermediate values of the function by using a 4th order interpolating polynomial.

Also in the nozzle admittance calculation a table of velocity potential is calculated by integrating the steady state velocity profile (Equation 43). Simpson's one third rule is used to evaluate this integral. This method approximates the curve to be integrated by a parabola.

In calculating the sensitive time lag τ an inverse tangent function of a fraction is required (Eq. 42). The inverse tangent in the computer library picks an angle between plus ninety degrees and minus ninety degrees. The correct quadrant of τ is determined by the signs of the numerator and denominator of the fraction. Subroutine QUAD calculates the correct quadrant.

I, F, Numerical Methods of Solution (cont.)

Boole's Integration

The Boole's integration subroutine for evaluating a definite integral in program A uses two entries: INTGR, INTGS. This subroutine uses Boole's formula and Simpson's rule to find the integral of a function between known limits. Boole's formula is used to extrapolate two Simpson's rule values with $H = \Delta X$ and $2 \Delta X$ to obtain an answer for the integral which is more accurate than either of the two alone.

$$\int_a^b y \, dx = I_H = \frac{H}{3} \left[y(x_0) + 4 y(x_1) + 2 y(x_2) + 4 y(x_3) + \dots + y(x_k) \right]$$

$$I_{\text{Boole}} = I_{H = \Delta X} + \frac{I_{H = \Delta X} - I_{H = 2\Delta X}}{15}$$

The subroutines are used as follows:

```
CALL INTGR (A,B,X,N)
```

```
100 CONTINUE
```

```
F =
```

```
CALL INTGS ($ 100,F,Y,E,M)
```

A = lower limit of independent variable

B = upper limit of independent variable

X = independent variable

N = initial number of increments

F = integrand

Y = integral

E = allowable relative error

M = final number of increments used

I, F, Numerical Methods of Solution (cont.)

Bessel Functions

The BESJ subroutine computes the J Bessel function for a given argument and integer order by using the recurrence relationship:

$$F_{n+1}(x) + F_{n-1}(x) = \left(\frac{2n}{x}\right) F_n(x)$$

The desired Bessel function is:

$$J_n(x) = \frac{F_n(x)}{\alpha}$$

where

$$\alpha = F_0(x) + 2 \sum_{m=1}^{M-2} F_{2m}(x)$$

M is initialized at M_0 .

M_0 is the greater of M_A and M_B where:

$$M_A = \begin{cases} [X+6] & \text{for } X < 5 \\ [1.4X + 60/X] & \text{for } X \geq 5 \end{cases}$$

$$M_B = [n+x/4+2]$$

$F_{M-2}, F_{M-3}, \dots, F_2, F_1, F_0$ is evaluated using the recurrence relationship above with $F_M = 0$ and $F_{M-1} = 10^{-30}$.

I, F, Numerical Methods of Solution (cont.)

α and $J_n(x)$ are then computed.

The computation is repeated for $M+3$.

The values of $J_n(x)$ for M and $M+3$ are compared.

$$\text{If } |J_n(x)_M - J_n(x)_{M+3}| \leq \delta |J_n(x)_{M+3}|$$

this value is accepted as $J_n(x)$; if not, the computation is repeated by adding 3 to M and using this as a new value for M . If M reaches M_{MAX} before the desired accuracy is obtained, execution is terminated. M_{MAX} is defined as:

$$M_{MAX} = \begin{cases} [20 + 10x - \frac{x^2}{3}] & \text{for } x \leq 15 \\ [90 + x/2] & \text{for } x > 15 \end{cases}$$

The BESY subroutine computes the Y Bessel function for a given argument x and order n . The recurrence relation:

$$Y_{n+1}(x) = (\frac{2n}{x}) \cdot Y_n(x) - Y_{n-1}(x)$$

is used for this evaluation.

I, F, Numerical Methods of Solution (cont.)

For $x > 4$

$$Y_0(x) = \left(\frac{2}{\pi x} \right)^{1/2} \left(P_0(x) \sin \left(x - \frac{\pi}{4} \right) + Q_0(x) \cos \left(x - \frac{\pi}{4} \right) \right)$$

$$Y_1(x) = \left(\frac{2}{\pi x} \right)^{1/2} \left(-P_1(x) \cos \left(x - \frac{\pi}{4} \right) + Q_1(x) \sin \left(x - \frac{\pi}{4} \right) \right)$$

$P_0(x)$, $Q_0(x)$, $P_1(x)$, and $Q_1(x)$ are:

$$\frac{1}{\sqrt{2\pi}} P_0 \left(\frac{4}{t} \right) = 0.3989422793 - 0.0017530620t^2 + 0.0001734300t^4 - 0.0000487613t^6 + 0.0000173565t^8 - 0.0000037043t^{10}$$

$$\frac{1}{t\sqrt{2\pi}} Q_0 \left(\frac{4}{t} \right) = -0.124669441 + 0.0004564324t^2 - 0.0000869791t^4 + 0.0000342468t^6 - 0.0000142078t^8 + 0.0000032312t^{10}$$

$$\frac{1}{\sqrt{2\pi}} P_1 \left(\frac{4}{t} \right) = 0.3989422819 + 0.0029218256t^2 - 0.0002232030t^4 + 0.0000580759t^6 - 0.0000200920t^8 + 0.0000042414t^{10}$$

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I, F, Numerical Methods of Solution (cont.)

$$\frac{1}{t\sqrt{2\pi}} Q_1\left(\frac{4}{t}\right) = 0.0374008364 - 0.0006390400t^2 \\ + 0.0001064741t^4 - 0.0000398708t^6 \\ + 0.0000162200t^8 - 0.0000036594t^{10}$$

where $t = \frac{4}{x}$

For $x \leq 4$

$$Y_0(x) = \frac{2}{\pi} \sum_{m=0}^{15} (-1)^m \left(\frac{x}{2}\right)^{2m} \frac{1}{(m!)^2} \\ \left[\log \frac{x}{2} + \gamma - H_m \right]$$

where

$$H_m = \begin{cases} \sum_{r=1}^m \frac{1}{r} & \text{for } m \geq 1 \\ 0 & \text{for } m = 0 \end{cases}$$

and γ = Euler's constant = 0.5772156649

$$Y_1(x) = -\frac{2}{\pi x} + \frac{2}{\pi} \sum_{m=1}^{16} (-1)^{m+1} \left(\frac{x}{2}\right)^{2m-1}$$

$$\frac{1}{m! (m-1)!} \cdot \left[\log \frac{x}{2} + \gamma - H_m + \frac{1}{2m} \right]$$

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I, Problem (cont.)

G. TECHNICAL REFERENCES

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I, Problem (cont.)

H. RELATED PROJECTS

This program and its predecessors have been used to analyze many engines and proposed engines. The parts of the program concerned with velocity effects and non-linear response have not been used since not enough is known about either of these effects to define the parameters involved. These parts of the program are intended to be research tools.

One of the major programs on which this program has been used is the Gemini Stability Improvement Program (AF 04(695)-517), during which the effect of nonuniform injection on combustion stability was demonstrated. A series of contracts (NAS8-4008, NAS8-11741, NAS8-20672) which were concerned with the stability characteristics of hydrogen - oxygen at high chamber pressures (500 to 2500 psi) used this program to help correlate stability data. The program was also used on Apollo (NAAI-M6J7XAA-400000A), M-1 (NAS3-2555) and Transtage (F04695-68-C-0297).

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II. PROGRAMMING

A. PROGRAM SUBROUTINES

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SUB05	INTD4(X,Y,XI,YO,DY)		120
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SUB16	CORE(X,N,CODE)		123
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II, A, Program Subroutines (cont.)

<u>Header</u>	<u>Subroutine</u>	<u>Description</u>	<u>Page</u>
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SUB29	ORTHLS	Curve Fit	127
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CGJR	CGJR	Solves Simultaneous Equations	141
BESJ	BESJ(X,N,BJ,D,IER)	Calculates Function	147
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BESY	BESY(X,N,BY,IER)	Calculates Function	149

II, A, Program Subroutines (cont.)

1. MAIN
 - a. Purpose:

Overall controlling subroutine
 - b. User Procedure:

EXECUTE MAIN
2. SUB01
 - a. Purpose:

Storage subroutine
 - b. User Procedure:

CALL BLOCK DATA
3. SUB02 - no longer used
4. SUB03 - no longer used
5. SUB04
 - a. Purpose

Table interpolation in a table given by a set of x's and a set of y's.
 - b. User procedure
 - (1) Entry

CALL INT4(X,Y,XI,YO)

where:

	<u>DESCRIPTION</u>	<u>TYPE</u>
X	is the first location of the table of X values	real; input
Y	is the first location of the table of Y values	real; input
XI	is the input X	real; input
YO	is the interpolated Y output	real; input

II, A, Program Subroutines (cont.)

The end of the table is sensed when both $x = 0$ and $y = 0$. Thus in setting up the tables of x and y the last entry should be followed by a zero.

(2) Restrictions

There must be at least 3 points in the table. The last point in the table should be followed by a 0,0 point.

(3) Mathematical Method

The interpolation is done using an interpolating function giving a continuous function and a continuous derivative. Consider an example of six points. For a general interior point, such as between 3,4, a cubic interpolating polynomial is used passing through 3 and 4 and having the average of the slopes 2-3 and 3-4 at the point 3 and the average of the slopes 3-4 and 4-5 at the point 4. For the end regions, a quadratic interpolating polynomial is used passing through the two outermost points and having the average slope at the interior of the two.

The interior regions are controlled by the following equations.

$$D = x_3 - x_2$$

$$A1 = x - x_2$$

$$A2 = x - x_3$$

$$M_{12} = (Y_2 - Y_1) / (X_2 - X_1)$$

$$M_{23} = (Y_3 - Y_2) / (X_3 - X_2)$$

$$M_{34} = (Y_4 - Y_3) / (X_4 - X_3)$$

$$Y = \frac{(A1)(A2)^2}{2D^2} (M_{12} - M_{23}) + \frac{(A2)(A1)^2}{2D^2} (M_{34} - M_{23}) - \frac{A_2 Y_2}{D} + \frac{A_1 Y_3}{D}$$

II, A, Program Subroutines (cont.)

Likewise the end regions are controlled by

$$D = X_2 - X_1$$

$$A_1 = X - X_1$$

$$A_2 = X - X_2$$

$$Y = \frac{(A_1)(A_2)}{2D} \left(\frac{Y_3 - Y_2}{X_3 - X_2} - \frac{Y_2 - Y_1}{X_2 - X_1} \right) - \frac{A_2}{D} Y_1 + \frac{A_1}{D} Y_2$$

6. SUB05

a. Purpose

This subroutine interpolates in a table given by a set of x's and a set of y's. Both the function Y0 and the derivative DY are obtained at the point XI.

b. User procedure

(1) Entry

CALL INT4D(X,Y,XI,YO,DY)

where:

	<u>DESCRIPTION</u>	<u>TYPE</u>
X	is first location of x table	real; input
Y	is first location of y table	real; input
XI	is input x	real; input
YO	is interpolated y	real; input
DY	is derivative at interpolated point	real; input

(2) Restrictions

A minimum of 3 points on the curve is required. Last point in table should be followed by a 0,0 point.

(3) Mathematical Method

See writeup for SUB04.

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II, A, Program Subroutines (cont.)

7. SUB06

a. Purpose

Calls new page

b. User procedure

CALL PAGE (LINES)

where:

	<u>DESCRIPTION</u>	<u>TYPE</u>
LINES	Page indexing variable	input

8. SUB07

a. Purpose

Subroutine used to control logical flow of program

b. User procedure

CALL CHAMBER

9. SUB08

a. Purpose

To generate frequencies used in program

b. User procedure

CALL GENMEG(W)

where:

	<u>DESCRIPTION</u>	<u>TYPE</u>
W	Numerical value associated with specific acoustic mode under consideration	Floating point; input

10. SUB09

a. Purpose

Calculates characteristic function for chamber with concentrated combustion.

II, A, Program Subroutines (cont.)

b. User procedure

CALL TRANS(BIN,XOUT,CB,KER)

where:

	<u>DESCRIPTION</u>	<u>TYPE</u>
BIN	Input variables (see program listing)	floating point; input
XOUT	Chamber admittance	floating point; output
CB	Used to control amount of output	floating point; input
KER	Number of frequencies	floating point; input

11. SUB10

a. Purpose:

Includes injection distribution and velocity effects in chamber admittance calculations.

b. User procedure

CALL DDD(DIN,DOUT,CD,NER,ERR)

where:

	<u>DESCRIPTION</u>	<u>TYPE</u>
DIN	See BIN SUB09	
DOUT	See XOUT SUB09	
CD	See CB SUB09	
NER	See KER SUB09	
ERR	error message	output

12. SUB14

a. Purpose

Calculates n and τ values

II, A, Program Subroutines (cont.)

b. User procedure

CALL FFF(FIN,FOUT,CF,NER,W)

where:

	<u>DESCRIPTION</u>	<u>TYPE</u>
FIN	See BIN SUB09	
FOUT	See XOUT SUB09	
CF	See CB SUB09	
NER	See KER SUB09	
W	No longer used	

13. SUB15

a. Purpose

Determines quadrant of τ

b. User procedure

CALL QUAD(A,B,ANGLE)

where:

	<u>DESCRIPTION</u>	<u>TYPE</u>
A	Chamber admittance-real part	floating point; input
B	Chamber admittance-imaginary part	floating point; input
ANGLE	Value of τ	floating point; output

14. SUB16

a. Purpose

Input data dump

b. User procedure

CALL CORE(X,N,CODE)

where:

	<u>DESCRIPTION</u>	<u>TYPE</u>
X	Value to be dumped	floating point; output
N	Number of dumps	integer; input
CODE	Code to determine if dump is performed	floating point; input

II, A, Program Subroutines (cont.)

15. SUB17
 - a. Purpose
Subroutine used to control logic of injector calculations
 - b. User procedure
CALL INJCTR
16. SUB18
 - a. Purpose
Calculates injection parameters
 - b. User procedure
CALL JJJ
17. SUB19
 - a. Purpose
Injection distribution calculations
 - b. User procedure
CALL INJDIS
18. SUB20
 - a. Purpose
Calculates table of velocity potential vs square of velocity
used in nozzle admittance calculations.
 - b. User procedure
CALL TBLCAL
19. SUB22
 - a. Purpose
To Process input

II, A, Program Subroutines (cont.)

b. User procedure

CALL AS138(C,ITITLE,IERR)

where:

	<u>DESCRIPTION</u>	<u>TYPE</u>
C	Input array	Real; output
ITITLE	Job title	output
IERR	Error indicator	integer; output

20. SUB24

a. Purpose

Performs Booles integration

b. User procedure

CALL INTGR(A,B,X,N)

See Section I,F for description of terms and methods of use

21. SUB25

a. Purpose

Calculates chamber admittance for transverse modes with distributed combustion

b. User procedure

CALL DISTR

22. SUB26

a. Purpose

Performs Adams Integration

b. User procedure

CALL ADSET(N,F,D,FP,T,X,HA,E)

See Section I,F for description of terms and methods of use

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II, A, Program Subroutines (cont.)

23. SUB27

a. Purpose

Calculates Nozzle Admittance

b. User procedure

CALL CCC

24. SUB28

a. Purpose

Calculates term for nozzle admittance

b. User procedure

CALL BTLDA

II, A, Program Subroutines (cont.)

25. SUB29 (Ref 16)

a. Purpose

This subroutine finds the least squares polynomial which best approximates a weighted set of data points using orthogonal polynomials.

b. User Procedure

(1) Entry

CALL ORTHLS(X,Y,W,N,L,J,C,ALPHA,BETA,K,T1,T2,T3,IND1)

where:

	DESCRIPTION	TYPE
X	is the array of N data points for the independent variable.	floating-point array; input and output
Y	is the array of N data points for the dependent variable.	floating-point array; input and output
W	is the array of N weights for the data points.	floating-point array; input
N	is the number of elements in the X,Y, and W arrays. Also, N is the number of elements in the T1, T2 and T3 arrays.	FORTTRAN integer; input
L	is the weight switch: = 0 if weights are all set equal to one; = 1 if weights are input in the W array.	FORTTRAN integer; input
J	is the number of low-order coefficients to be set equal to zero	FORTTRAN integer; input
C	is the array of K + 1 computed polynomial coefficients.	floating-point array; output

II, A, Program Subroutines (cont.)

	DESCRIPTION	TYPE
ALPHA	is the computed α array.	floating-point array; output
BETA	is the computed β array.	floating-point array; output
K	is the maximum degree of the polynomial to be fitted. Also, $K + 1$ is the number of elements in the C array and K is the number of elements in the ALPHA and BETA arrays.	FORTTRAN integer; input
T1	is an array of N elements used for temporary storage by ORTHLS. Its contents upon return from ORTHLS are of no significance to the user.	floating-point array
T2	is an array of N elements used for temporary storage by ORTHLS. The contents upon return from ORTHLS are of no significance to the user.	floating-point array
T3	is an array of N elements for temporary storage by ORTHLS. The contents upon return from ORTHLS are of no significance to the user.	floating-point array
IND1	is the error indicator: = -1, when $J > K$. = +1, when $J \leq K$.	FORTTRAN integer: output

(2) Restrictions

(1) $J \leq K$

(2) $N > K$, since a minimum of $N + 1$ data points are required to fit a polynomial of degree K.

II, A, Program Subroutines (cont.)

(3) Special Considerations

- (a) If $L = 1$ in the calling sequence (i.e., non-unit weights), the input data points, X and Y , are altered when ORTHLS is executed.
- (b) Since there are $K + 1$ coefficients for a K th degree polynomial, the C array must be dimensioned for a minimum of $K + 1$ elements in the calling program. The computed coefficients are atored as:

$$\begin{array}{l} c_0 \dots C(1) \\ c_1 \dots C(2) \\ \cdot \quad \cdot \\ \cdot \quad \cdot \\ c_k \dots C(K + 1) \end{array}$$

(4) Error Returns

If, upon entering ORTHLS, $J > K$:

- (a) the error indicator IND1 is set equal to -1;

II, A, Program Subroutines (cont.)

- (b) all the elements in the C, ALPHA and BETA arrays are set equal to zero.
- (c) control returns to the calling program through the normal exit.

c. Supporting Information

(1) Mathematical Method

Let

$$y = \sum_{j=0}^k C_j P_j(X) \quad (55)$$

where $P_j(x)$ is a polynomial of degree j . To obtain the best fit in the weighted least-squares sense to the set of data points (X_i, Y_i) , $i = 1, 2, \dots, N$ with positive non-zero weights (W_i) , $i = 1, 2, \dots, N$, then

$$\sum_{i=1}^N W_i^2 \left[Y_i - \sum_{j=0}^K C_j P_j(X_i) \right]^2 = \text{minimum} \quad (56)$$

By letting

$$r_{jm} = \sum_{i=1}^N W_i^2 P_j(X_i) P_m(X_i) \quad (57)$$

$$s_j = \sum_{i=1}^N W_i^2 Y_i P_j(X_i) \quad j = 0, 1, 2, \dots, K \quad (58)$$

II, A, Program Subroutines (cont.)

A set of normal equations of the following form is obtained.

$$S_j = \sum_{m=0}^K C_m r_{jm} \quad j = 0, 1, 2, \dots, K \quad (59)$$

Equation (59) consists of a set of $K + 1$ simultaneous equations with $K + 1$ unknowns. However, suppose that the polynomials $P_j(X)$ can be chosen to be orthogonal in the summation; i.e.,

$$r_{jm} = \begin{cases} 0 & \text{for } j \neq m \\ \sum_{i=1}^N W_i^2 P_j^2(X_i) & \text{for } j = m \end{cases} \quad (60)$$

Under the condition of orthogonality, the unknown coefficients C_j can now be evaluated by

$$C_j = \frac{S_j}{r_{jj}} \quad j = 0, 1, 2, \dots, K \quad (61)$$

Consequently, the use of orthogonal polynomials makes the least-squares analysis appear as if each parameter were the only one used. Also, the coefficients C_j are independent of each other; as K is increased, the previously calculated C_j do not change. However, the problem of generating the orthogonal polynomials must be solved.

II, A, Program Subroutines (cont.)

The polynomials $P_j(X)$ are defined recursively as

$$\left. \begin{aligned} P_{-1}(X) &= 0 \\ P_0(X) &= 1 \\ P_1(X) &= (X - \alpha_1)P_0(X) - \beta_0 P_{-1}(X) \\ &\vdots \\ P_{J+1}(X) &= (X - \alpha_{j+1})P_j(X) - \beta_j P_{j-1}(X) \end{aligned} \right\} \quad (62)$$

with $\beta_0 = 0$. The α and β must be chosen in such a way that the polynomials $P_0(X), P_1(X), \dots, P_j(X)$ are all orthogonal to each other in the sense of Equation (60).

The $(J + 1)$ th polynomial is defined as

$$P_{J+1}(X_i) = (X_i - \alpha_{j+1})P_j(X_i) - \beta_j P_{j-1}(X_i) \quad (63)$$

Multiplying Equation (63) through by $W_i^2 P_j(X_i)$ and summing over all i yields

$$\begin{aligned} \sum_{i=1}^N W_i^2 P_{j+1}(X_i) P_j(X_i) &= \sum_{i=1}^N W_i^2 (X_i - \alpha_{j+1}) P_j^2(X_i) + \\ &- \sum_{i=1}^N W_i^2 \beta_j P_{j-1}(X_i) P_j(X_i) \end{aligned} \quad (64)$$

The term on the left-hand side of Equation (64), is zero since $j + 1 \neq j$ and Equation (60) requires the summation to vanish for $j \neq m$. Likewise, the second term on the right-hand side vanishes leaving

II, A, Program Subroutines (cont.)

$$\sum_{i=1}^N W_i^2 (X_i - \alpha_{j+1}) P_j^2(X_i) = 0 \quad (65)$$

Since α_{j+1} does not depend on i , Equation (65) can be rearranged to yield

$$\alpha_{j+1} = \frac{\sum_{i=1}^N W_i^2 X_i P_j^2(X_i)}{\sum_{i=1}^N W_i^2 P_j^2(X_i)} \quad j = 0, 1, \dots, K \quad (66)$$

By multiplying Equation (63) through by $W_i^2 P_{j-1}(X_i)$ and again using Equation (60) it can be shown that

$$\beta_j = \frac{\sum_{i=1}^N W_i^2 X_i P_{j-1}(X_i) P_j(X_i)}{\sum_{i=1}^N W_i^2 P_{j-1}^2(X_i)} \quad j = 0, 1, 2, \dots, K-1 \quad (67)$$

Multiplying Equation (63) through by $W_i^2 P_{j+1}(X_i)$ and summing over all i , yields the identity

$$\sum_{i=1}^N W_i^2 P_{j+1}^2(X_i) = \sum_{i=1}^N W_i^2 X_i P_j(X_i) P_{j+1}(X_i) \quad (68)$$

Thus, substituting Equation (68) into Equation (67) β_j may be computed by an alternative formula,

$$\beta_j = \frac{\sum_{i=1}^N W_i^2 P_j^2(X_i)}{\sum_{i=1}^N W_i^2 P_{j-1}^2(X_i)} \quad j = 0, 1, 2, \dots, K-1 \quad (69)$$

II, A, Program Subroutines (cont.)

(2) Programming Method

The subroutine ORTHLS has the option of placing certain constraints on the form of the fitted polynomial, that is, if J is a non-zero positive integer, then the fitted polynomial has the form:

$$Y = \sum_{j=0}^{K-J} C_j \bar{P}_j(X) \quad (70)$$

where the $\bar{P}_j(X)$ are defined recursively as

$$\left. \begin{aligned} \bar{P}_{-1}(X) &= 0 \\ \bar{P}_0(X) &= X^J \\ \bar{P}_1(X) &= (X - \alpha_1) \bar{P}_0(X) - \beta_0 \bar{P}_{-1}(X) \\ &\vdots \\ \bar{P}_{j+1}(X) &= (X - \alpha_{j+1}) \bar{P}_j(X) - \beta_j \bar{P}_{j-1}(X) \end{aligned} \right\} \quad (71)$$

It can be shown that the set $\bar{P}_0, \bar{P}_1, \dots, \bar{P}_{j+1}$, are orthogonal by requiring that α and β satisfy the same relations (Equations (66) and (67)) as before. These constraints allow the first J coefficients in Equation (70) to be set equal to zero thus forcing the fitted polynomial and its first $J - 1$ derivatives to be zero at $X = 0$.

The subprogram ORTHLS uses Equations (61), (66), and (69) to compute the orthogonal coefficients C_j defined by Equation (55) and the parameters α_j and β_j , respectively.

Further information can be found in Reference 15.

II, A, Program Subroutines (cont.)

(3) Storage

ORTHLS	285
NEXT5\$	<u>33</u>
Total Storage: 318 positions.	

(4) Nomenclature

B	is the recursive β used in computing the polynomials.
I	is the index used for summing over all the data points.
II	is the index used to determine the successful completion of the calculation.
KJ1	is the value of $K - J + 1$.
R	is the parameter defined by Equation (57).
RO	is the previous value of R.
S	is the parameter defined by Equation (58).
SUM	is the initial value of RO.
SUMXPS	is the numeration of Equation (66).
TEMP	is the variable used for computing the $(j + 1)$ th polynomial from the j th and $(j - 1)$ th polynomials.

All other variables are defined in paragraph II.A.25.b.(1).

26. SUB30 (Ref 16)

a. Purpose

This subroutine computes the coefficients of the least-squares polynomial which best approximates the weighted set of data points.

Given the coefficients, c_j , and the orthogonality parameters of the polynomials $P_j(x)$ for the equation

$$y = \sum_{j=0}^{K-J} C_j P_j(x)$$

II, A, Program Subroutines (cont.)

COEFS computes the coefficients a_j for the equation

$$y = \sum_{j=J}^K a_j x^j$$

b. User Procedure

(1) Entry

```
CALL COEFS(J,C,ALPHA,BETA,KC,A,T1,T2,T3,IND2)
```

where:

	DESCRIPTION	TYPE
J	is the number of low-order coefficients to be set equal to zero.	FORTTRAN integer; input
C	is the array of KC + 1 polynomial coefficients.	floating-point array; input
ALPHA	is the α array of KC elements.	floating-point array; input
BETA	is the β array of KC elements.	floating-point array; input
KC	is the degree of the polynomial for which the coefficients are to be found. Also, KC + 1 is the number of elements in the arrays C, A, T1, T2, and T3. KC is also the number of elements in the ALPHA and BETA arrays.	FORTTRAN integer; input
A	is the computed coefficient array with KC + 1 elements.	floating-point array; output
T1	is an array of KC + 1 elements used for temporary storage by COEFS. The contents upon return from COEFS are of no significance to the user.	floating-point array

II, A, Program Subroutines (cont.)

	DESCRIPTION	TYPE
T2	is an array of $KC + 1$ elements used for temporary storage by COEFS. The contents array upon return from COEFS are of no significance to the user.	floating-point array
T3	is an array of $KC + 1$ elements used for temporary storage by COEFS. The contents array upon return from COEFS are of no significance to the user.	floating-point array
IND2	is an error indicator. = -2, when $J > KC$ = +2, when $J \leq KC$	FORTRAN integer; output

(2) Restrictions

(a) $J \leq KC$.(b) $KC \leq K$, where K is the maximum degree of the polynomial used by subprogram ORTHLS.

(3) Special Considerations

(a) The value of J used by COEFS must be the same as that used by subprogram ORTHLS in computing the coefficients c_j , and the parameters α_j and β_j .(b) The C , A , $T1$, $T2$, $T3$ arrays must be dimensioned for a minimum of $KC + 1$ elements in the calling program. The input coefficients should be stored as: $c_0 \dots C(1)$ $c_1 \dots C(2)$ $c_{KC} \dots C(KC + 1)$

II, A, Program Subroutines (cont.)

The computed coefficients are stored as:

$a_0 \dots A(1)$

$a_1 \dots A(2)$

$a_{KC} \dots A(KC + 1)$

(4) Other Subprograms Required

The subprogram COEFS requires that the subprogram ORTHLS be executed first. ORTHLS computes the coefficients, c_j , and the parameters α_j and β_j used by COEFS in computing the coefficients a_j .

(5) Error Returns

If, upon entering COEFS, $J > KC$:

The error indicator IND2 is set equal to -2; all elements in the A array are set equal to zero; control returns to the calling program through the normal exit.

c. Supporting Information

(1) Mathematical Method

Let

$$y = \sum_{j=0}^{KC-J} C_j P_j(x) \quad (72)$$

II, A, Program Subroutines (cont.)

with the polynomials defined recursively as

$$\begin{aligned}
 P_{-1}(x) &= 0 \\
 P_0(x) &= x^J \\
 P_1(x) &= (x - \alpha_1)P_0(x) - \beta_0 P_{-1}(x) \\
 &\vdots \\
 P_{j+1}(x) &= (x - \alpha_{j+1})P_j(x) - \beta_j P_{j-1}(x)
 \end{aligned}
 \tag{73}$$

with $\beta_0 = 0$.

Given the coefficients C_j , and the parameters J , α_j , β_j and KC , COEFS computes the coefficients, a_j , of the equation

$$y = \sum_{j=J}^{KC} a_j x^j
 \tag{74}$$

(2) Programing Method

See Mathematical Method.

(3) Storage

COEFS: 198 positions.

II, A, Program Subroutines (cont.)

(4) Nomenclature

- B is the recursive β value used in computing the coefficients.
- II is the index used to determine the successful completion of the calculations.
- KCJ1 is the value of $KC - J + 1$.
- N1 is the index used in arranging the coefficients properly if $J > 0$.
- N2 is the index used in arranging the coefficients properly if $J > 0$.
- NN is the index used in accumulating the coefficients.
- All other variables are defined in paragraph II.A.26.b.(1).

II, A, Program Subroutines (cont.)

27. CGJR - (Ref 16)

a. Purpose

This subroutine solves simultaneous equations, computes a determinant, or inverts a matrix or any combination of the three above by using a Gauss-Jordan elimination technique with column pivoting.

b. User Procedure

(1) Entry

```
CALL CGJR(A,NC,NR,N,MC,$k,JC,V)
```

where:

	DESCRIPTION	TYPE
A	is the matrix whose inverse or determinant is to be determined. If simultaneous equations are solved, the last MC - N columns of the matrix are the constant vectors of the equations to be solved. On output, if the inverse is computed, it is stored in the first N columns of A. If simultaneous equations are solved, the last MC - N columns contain the solution vectors.	complex array; input and output
NC	is the maximum number of columns of the array A.	FORTRAN integer; input
NR	is the maximum number of rows of the array A.	FORTRAN integer; input
N	is the number of rows of the array A.	FORTRAN integer; input
MC	is the number of columns of the array A. This entry is a dummy argument, if simultaneous equations are not to be solved.	FORTRAN integer; input

II, A, Program Subroutines (cont.)

	DESCRIPTION	TYPE
k	is a statement number in the calling program to which control is returned if an overflow is detected. It must be preceded by \$ in the calling sequence.	input
JC	is a one-dimensional permutation array of N elements used for permuting the rows and columns of A if an inverse is computed. If an inverse is not computed this argument must have at least one cell for the error return identification. On output, the first element of the array is N if control is returned normally. If an overflow is detected, the first element is the negative of the last correctly completed row of the reduction. If matrix singularity is detected, the entry contains the value of the last row before the singularity was detected.	FORTTRAN integer array; input and output
V	on input REAL (V) is the option indicator, its values are set as follows:	complex variable; input and output

Operation	REAL(V)						
	1.	2.	3.	4.	5.	6.	7.
Compute Determinant	no	yes	yes	no	no	yes	yes
Invert Matrix	yes	no	yes	no	yes	no	yes
Solve Equations	no	no	no	yes	yes	yes	yes

On normal return from the program V contains the value of the natural logarithm of the determinant. If an error return is made and the determinant to be computed is set to (0,0), or if an overflow return was made, V contains the last correct partially-computed value of the determinant.

(2) Restrictions

None

II, A, Program Subroutines (cont.)

c. Special Considerations

- (1) If the matrix is singular or ill-conditioned, roundoff error may cause large discrepancies in the results.
- (2) In the case of a singular matrix, return may not be made through the singularity exit because of roundoff error.
- (3) See 7.1.2.3. Ref 16 for notes on usage of the row-dimension arguments N and NR.

d. Error Returns

- (1) If a singularity is detected, the first element of the array JC is set to the number of the last correctly completed row; and V is set to (0.,0.), if the determinant was to be computed. Control is then returned to the calling program at the statement number specified.
- (2) If an overflow is detected, JC(1) is set to the negative of the last correctly completed row of the reduction. Control is then returned to the calling program at the statement number specified.

e. Mathematical Method

For any matrix A, if a matrix B exists such that $BA = AB = I$, where I is the unit matrix, then $B = A^{-1}$.

If $AX = C$ where A is n by n, X is n by p, and C is n by p, then the solution to these sets of simultaneous equations is $X = A^{-1}C$.

II, A, Program Subroutines (cont.)

The determinant of A is defined by the following equation

$$A = \sum (-1)^{f(j_1, \dots, j_N)} \prod_{i=1}^N a_{ij_i}$$

where

a_{ij} is the (i,j)th element of the matrix A; $f(j_1, \dots, j_N)$ is the number of transpositions required to transform (1, ..., N) to $(j_1, \dots, j_N)$; and the summation is over all permutations $(j_1, \dots, j_N)$ of the integers (1, ..., N).

The solution to all of these problems is found by using a Gauss-Jordan elimination scheme with maximal pivoting by columns. More information can be found in References 13 and 14.

f. Programming Method

- (1) For each diagonal, the program searches for a pivotal element for each column below the diagonal by finding the element of maximum norm in the remaining rows of the column. The norm is defined as $\text{Norm}(Z) = |X| + |Y|$, where a complex number is represented as $Z = X + iY$.
- (2) This row is interchanged with the row of the diagonal.
- (3) Each of the elements of the pivotal row is divided by the pivot except the pivot which is replaced by its reciprocal.

II, A, Program Subroutines (cont.)

- (4) All other rows of the array are changed by the formula

$$a_{ij} = a_{ij} - a_{ik}a_{kj}$$

where

a_{kk} is the pivotal element. If $i = k$, a_{ij} is replaced by 0.

- (5) When this process has been completed for each diagonal of the array, the columns of the matrix are repermuted to give the inverse in the first N columns of the array A.
- (6) If the determinant is to be found, each permutation of rows and columns changes the value of its sign. The complex natural logarithm of the absolute value of the diagonal element is summed after step 2.
- (7) Only the computations necessary for the options specified are carried out.

g. Storage

GJR 557 positions, not including the Library Subroutines OVERFL, CLOG, NERR\$2, and NCDP\$.

h. Nomenclature

ANQRM is used for storing the complex norm of elements of the matrix A.

IBIT is the number of sign changes in the calculation of the determinant.

IFL is the overflow test indicator.

II, A, Program Subroutines (cont.)

IW	is the FORTRAN integer value of the option indicator.
KD	is the option key for determinant evaluation.
KI	is the option key for matrix inversion.
L	is the column control for solution of simultaneous equations.
M	is the column control for matrix inversion.
MU	is a dummy variable.
S	is the sign control for determinant evaluation.
X	is a dummy variable.
XC	is a dummy complex variable used in the permutation of rows of the matrix A.
Z	is a complex variable used to store the value of $\text{CLOG}(A(I,I))$.

All other variables are either simple loop indices or as defined earlier in this section.

II, A, Program Subroutines (cont.)

28. BESJ

a. Purpose

To calculate the Bessel function of the first kind

b. User Procedure

CALL BESJ (X,N,BJ,D,IER)

where:

	DESCRIPTION	TYPE
X	argument	floating point; input
N	order	integer; input
BJ	value of Bessel function	floating point; output
D	error criteria	floating point; input
ER	error code	integer; output

See Section I,F for further information.

II, A, Program Subroutines (cont.)

29. BESSEL

a. Purpose

Calculates Bessel functions

b. User Procedure

CALL BESSEL (J,Y,V,X,K)

where :

	DESCRIPTION	TYPE
J	Bessel function of 1st kind	floating point; output
Y	Bessel function of 2nd kind	floating point; output
V	Order	integer; input
X	argument	floating point; input
K	error code	integer; output

See Section I.F. for further information

II, A, Program Subroutines (cont.)

30. BESY

a. Purpose

Calculates Bessel function of the second kind

b. User Procedure

CALL BESY (X,N,BY,IER)

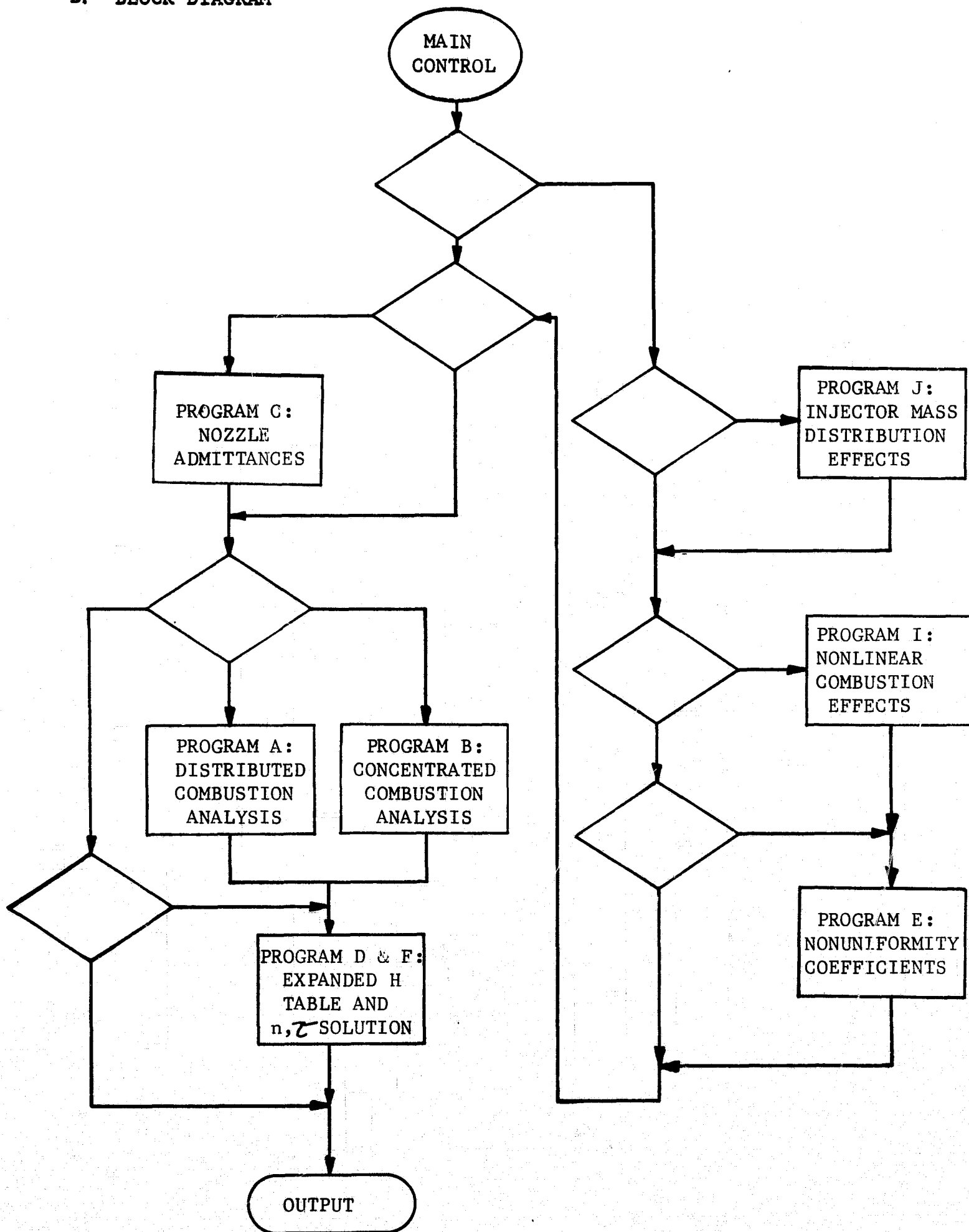
where :

	DESCRIPTION	TYPE
X	argument	floating point; input
N	order	integer; input
BY	value of Bessel function	floating point; output
IER	error code	integer; output

See Section I.F. for further information

II, Programing (cont.)

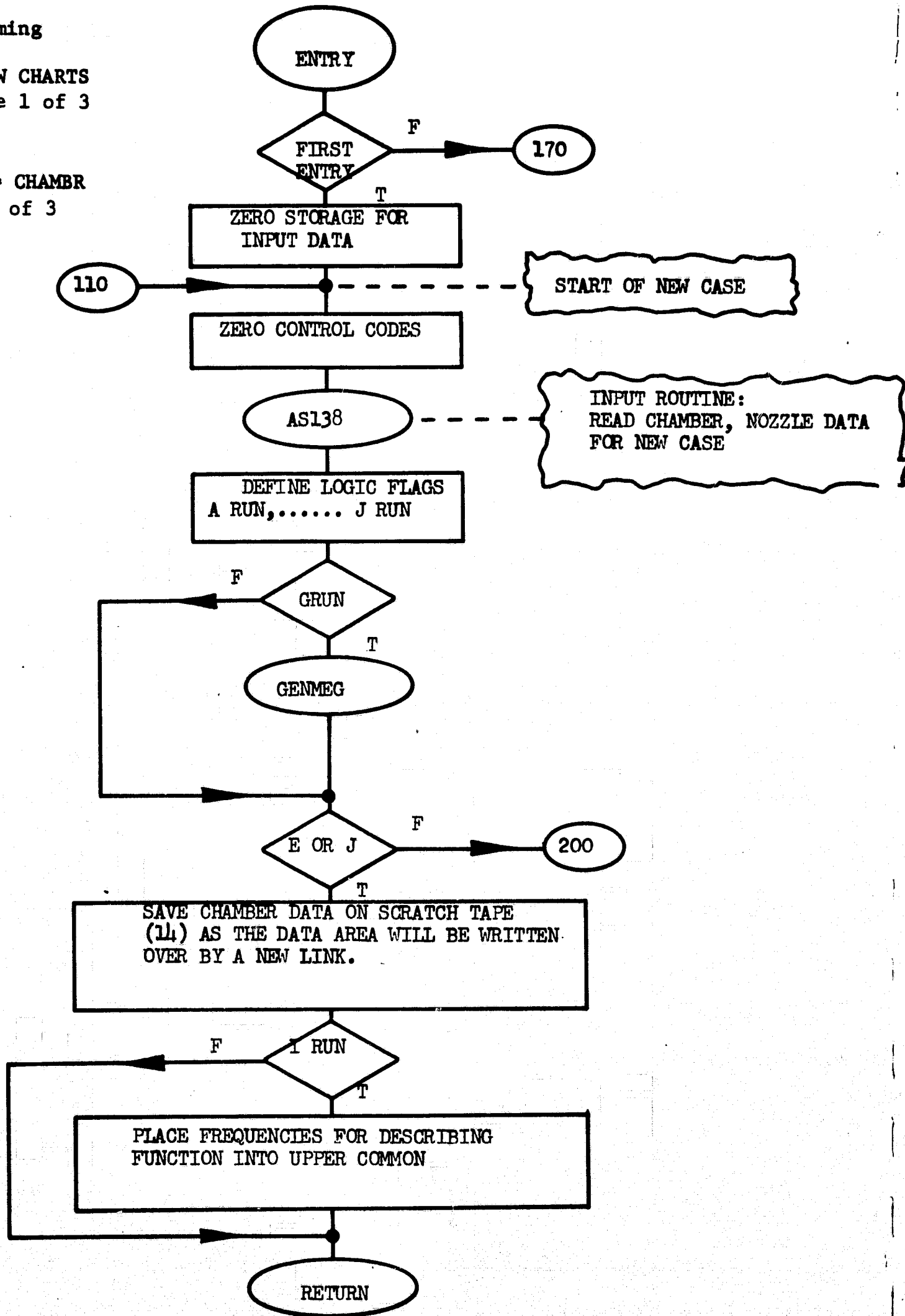
B. BLOCK DIAGRAM

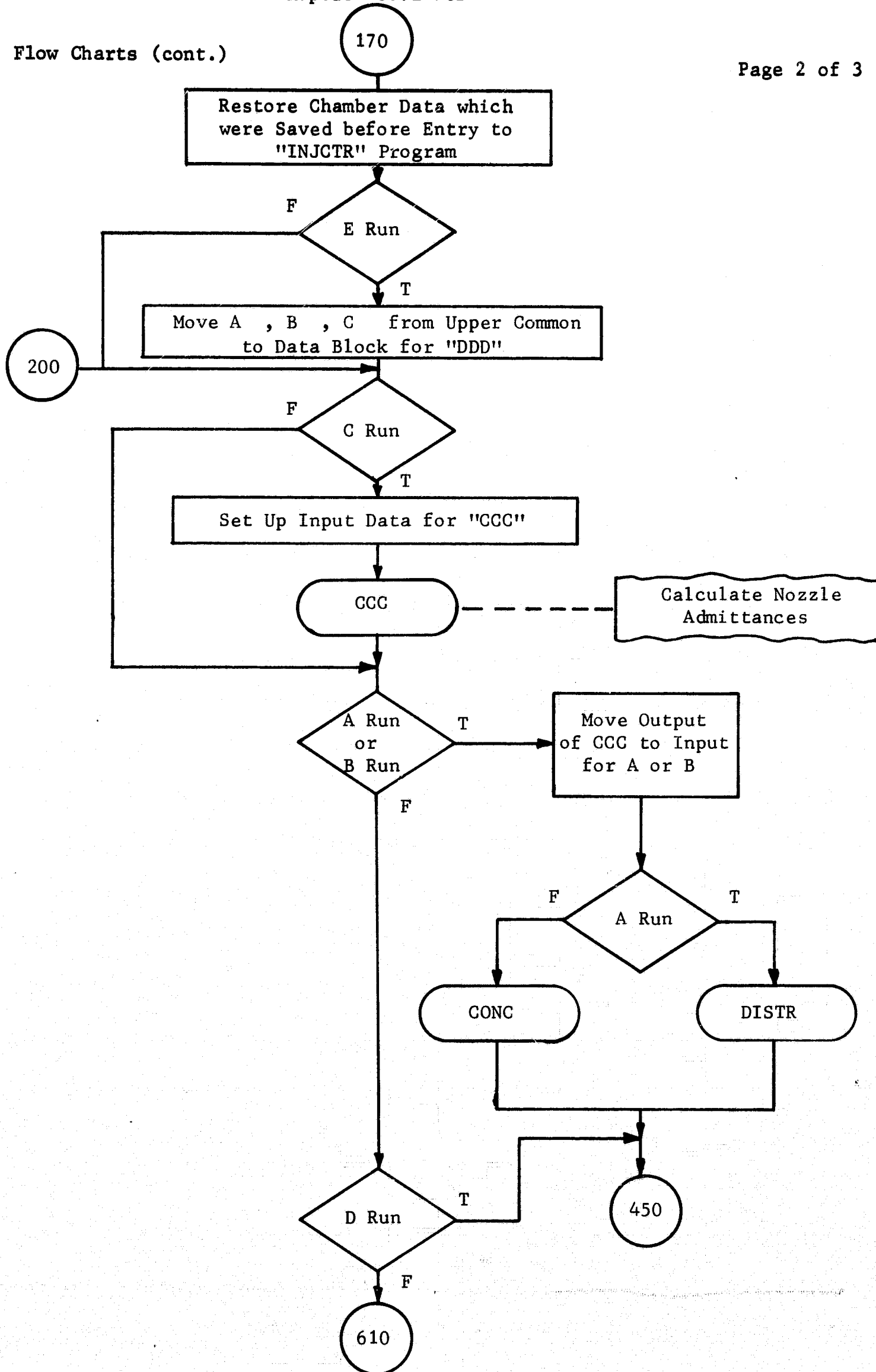


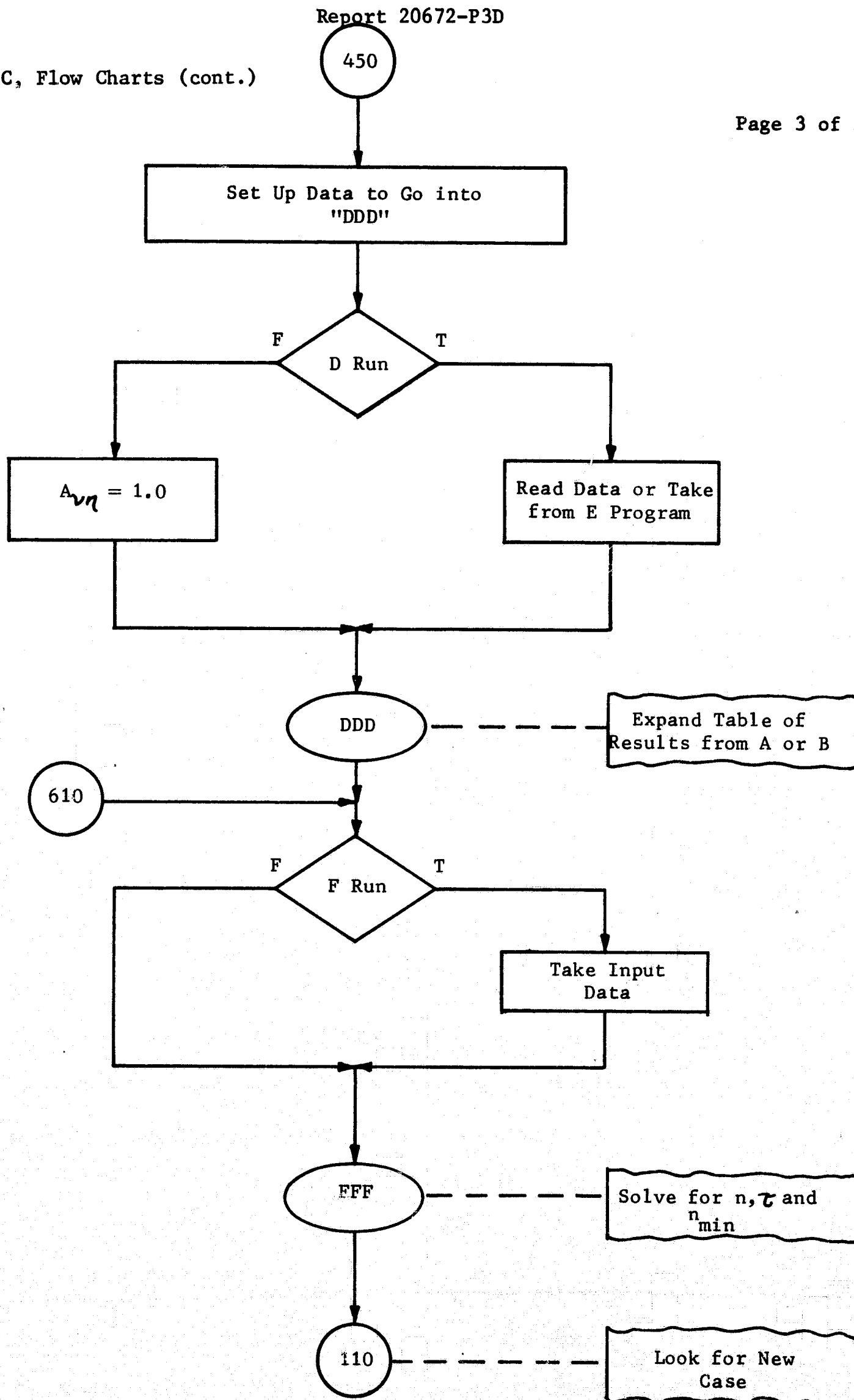
II, Programming

C. FLOW CHARTS Page 1 of 3

ENTRY = CHAMBR
Page 1 of 3

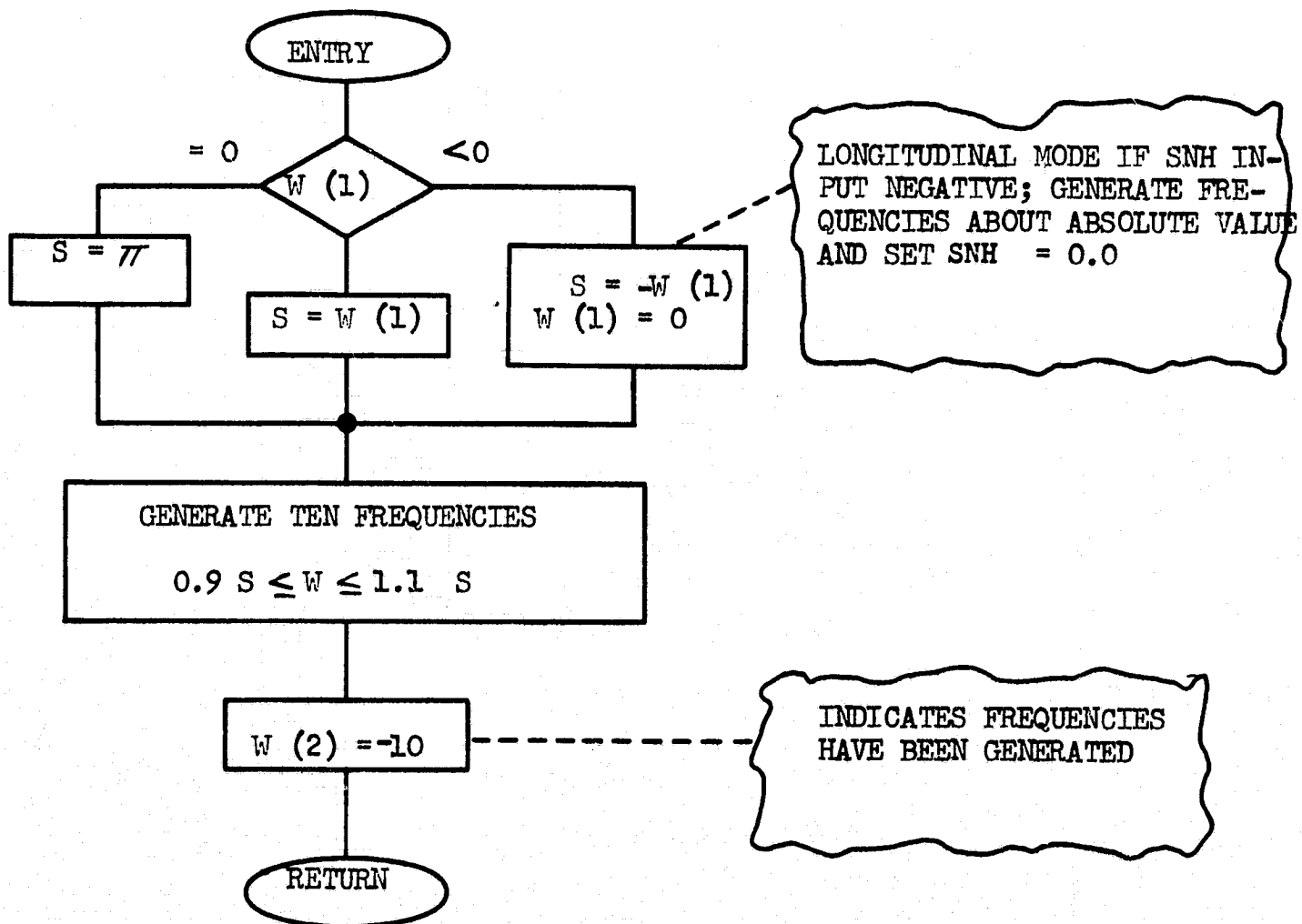






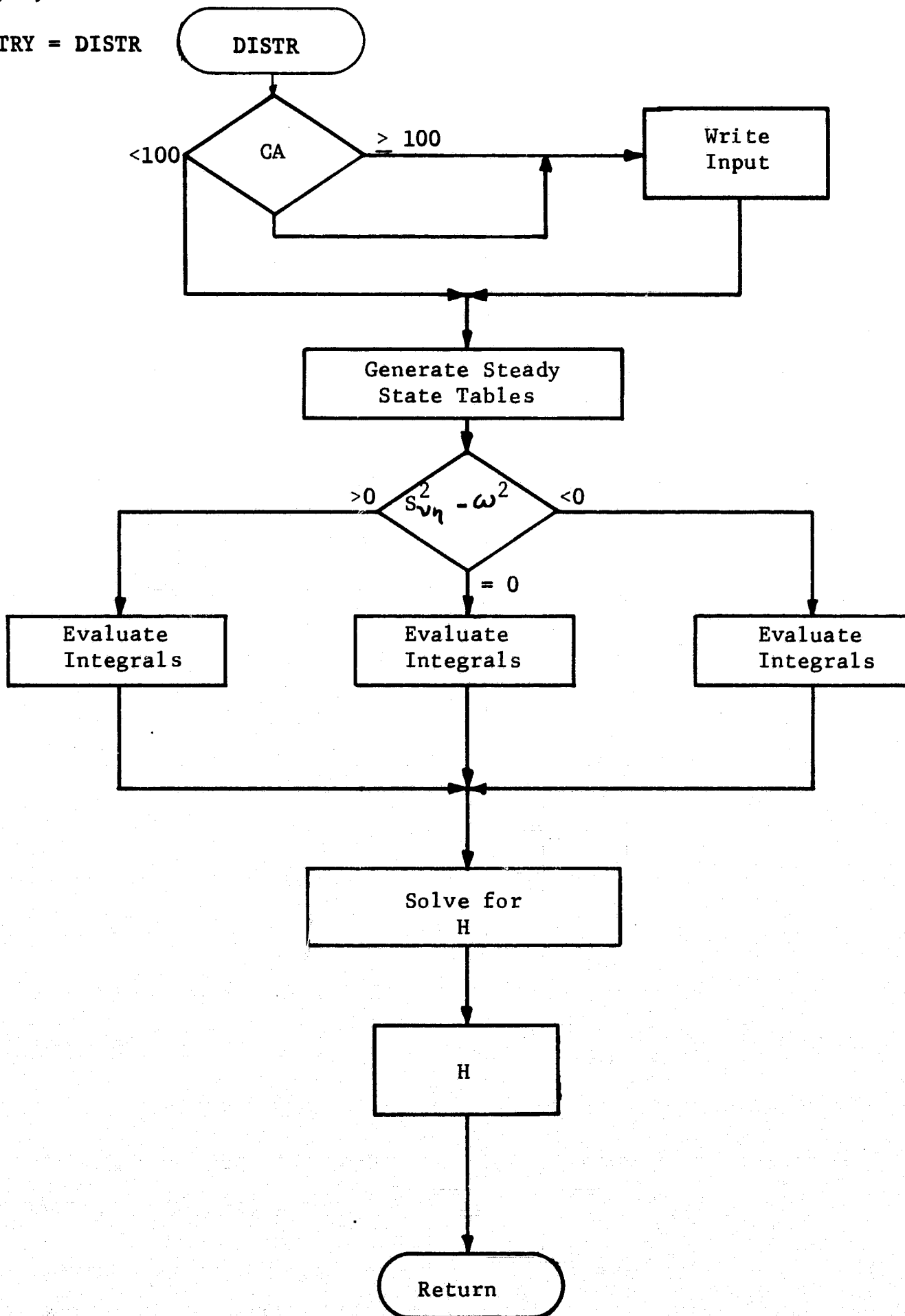
II, C, Flow Charts (cont.)

ENTRY = GEMMEG



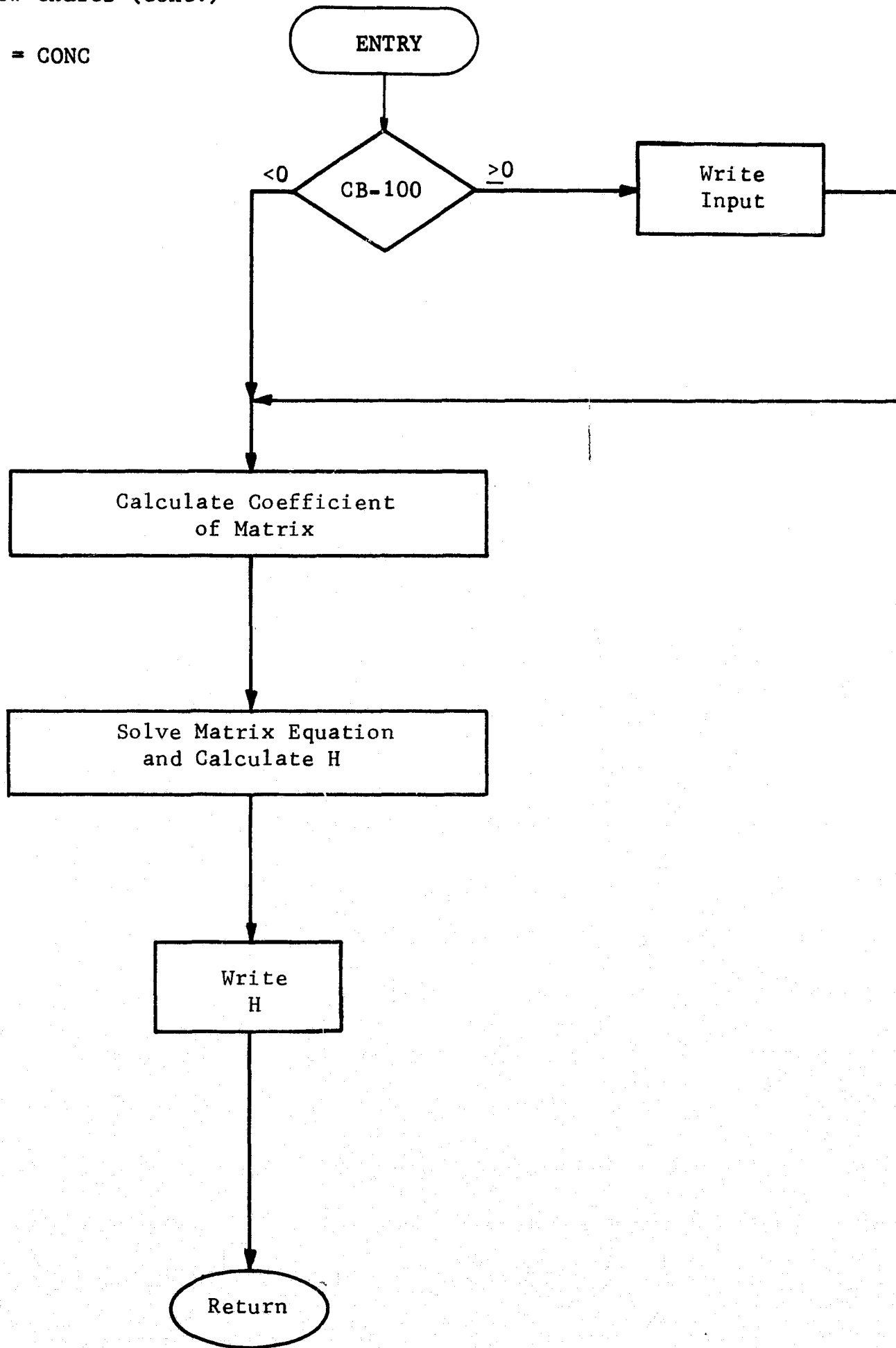
II, C, Flow Charts (cont.)

ENTRY = DISTR



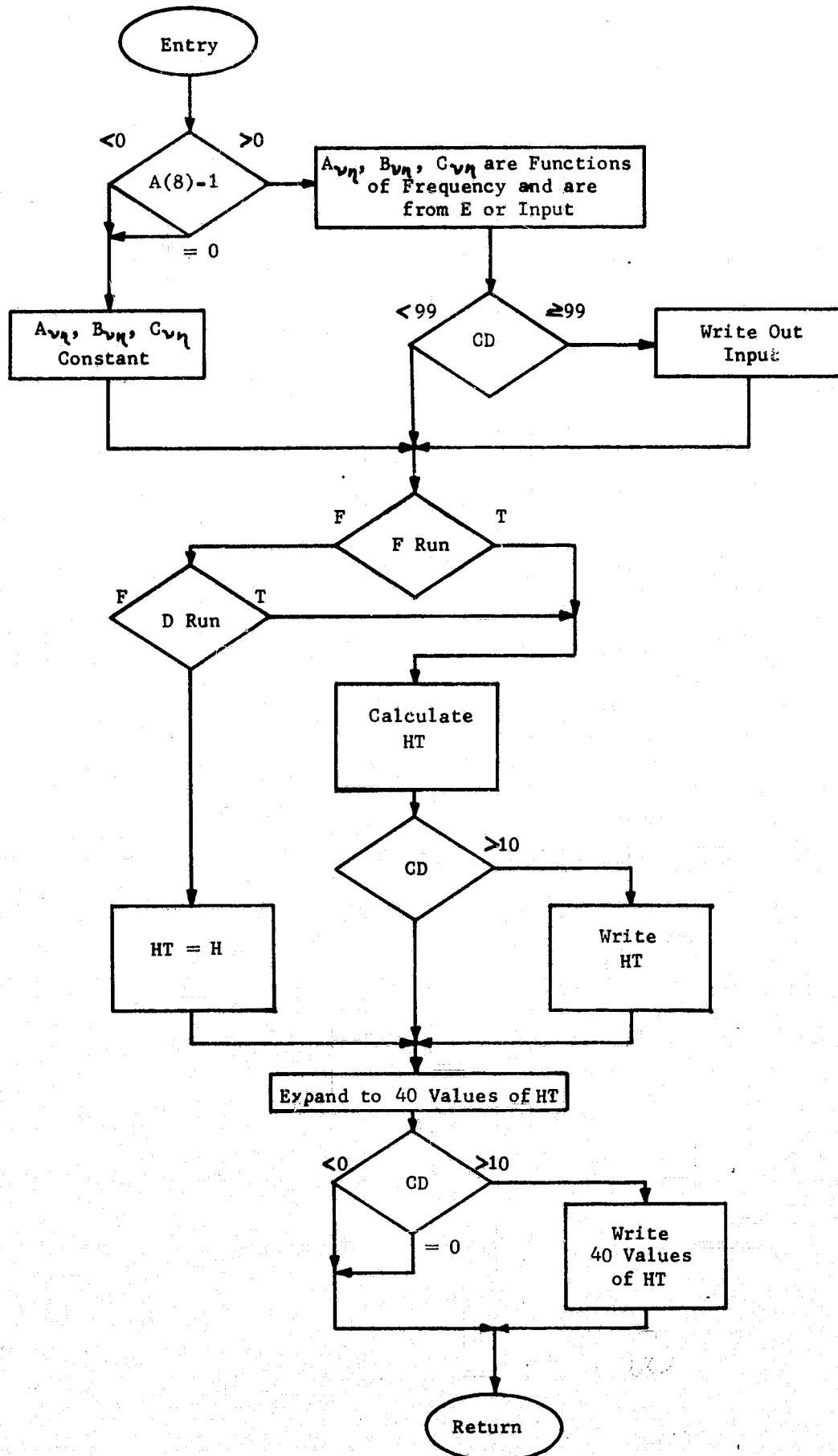
II, C, Flow Charts (cont.)

ENTRY = CONC



Report 20672-P3D

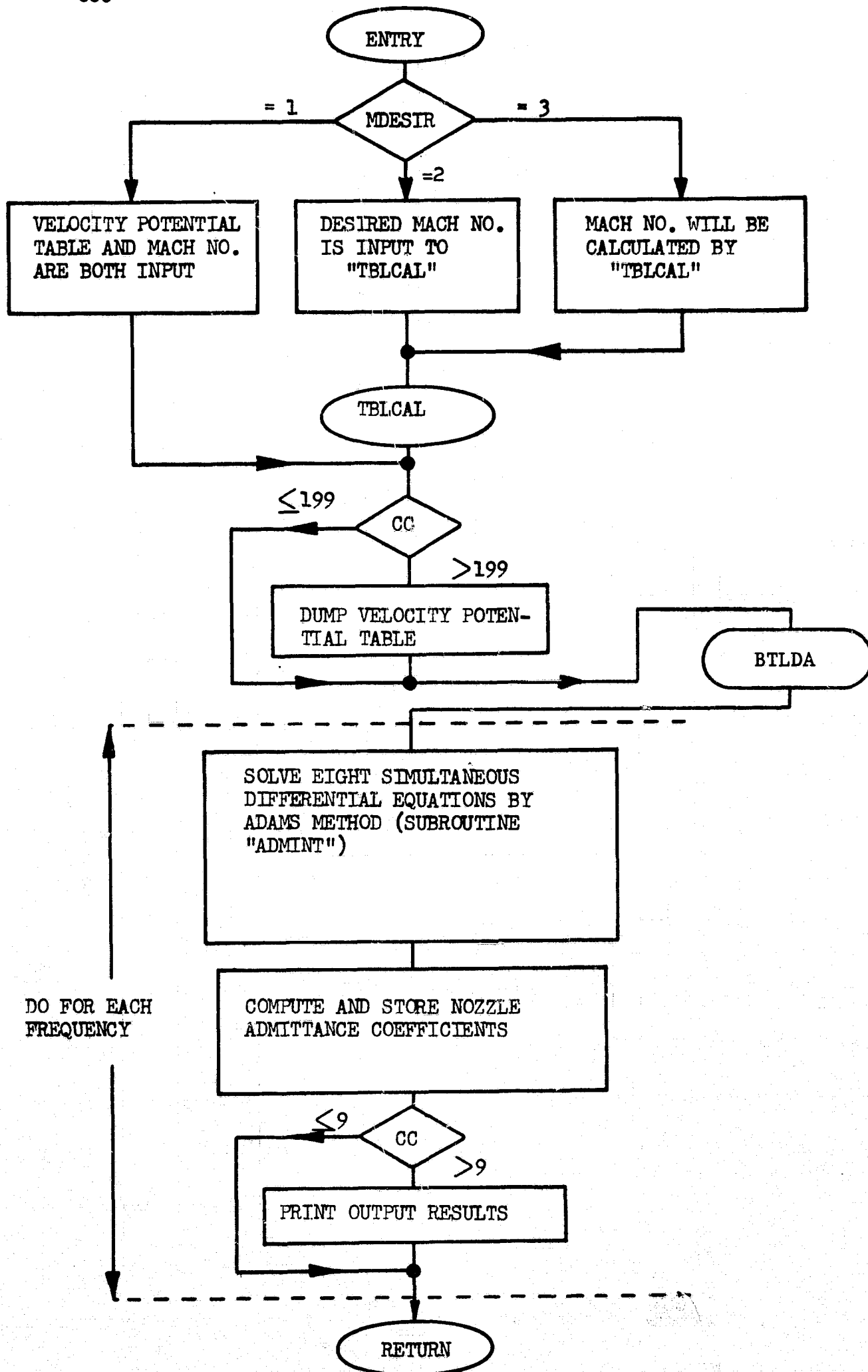
ENTRY = DDD



II, C, Flow Charts (cont.)

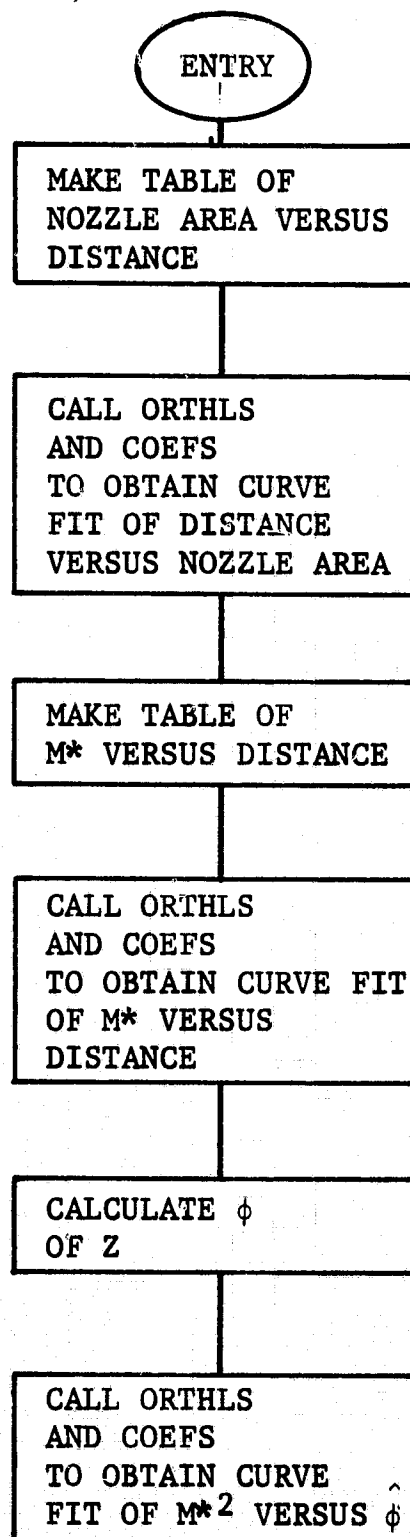
Report 20672-P3D

ENTRY = CCC

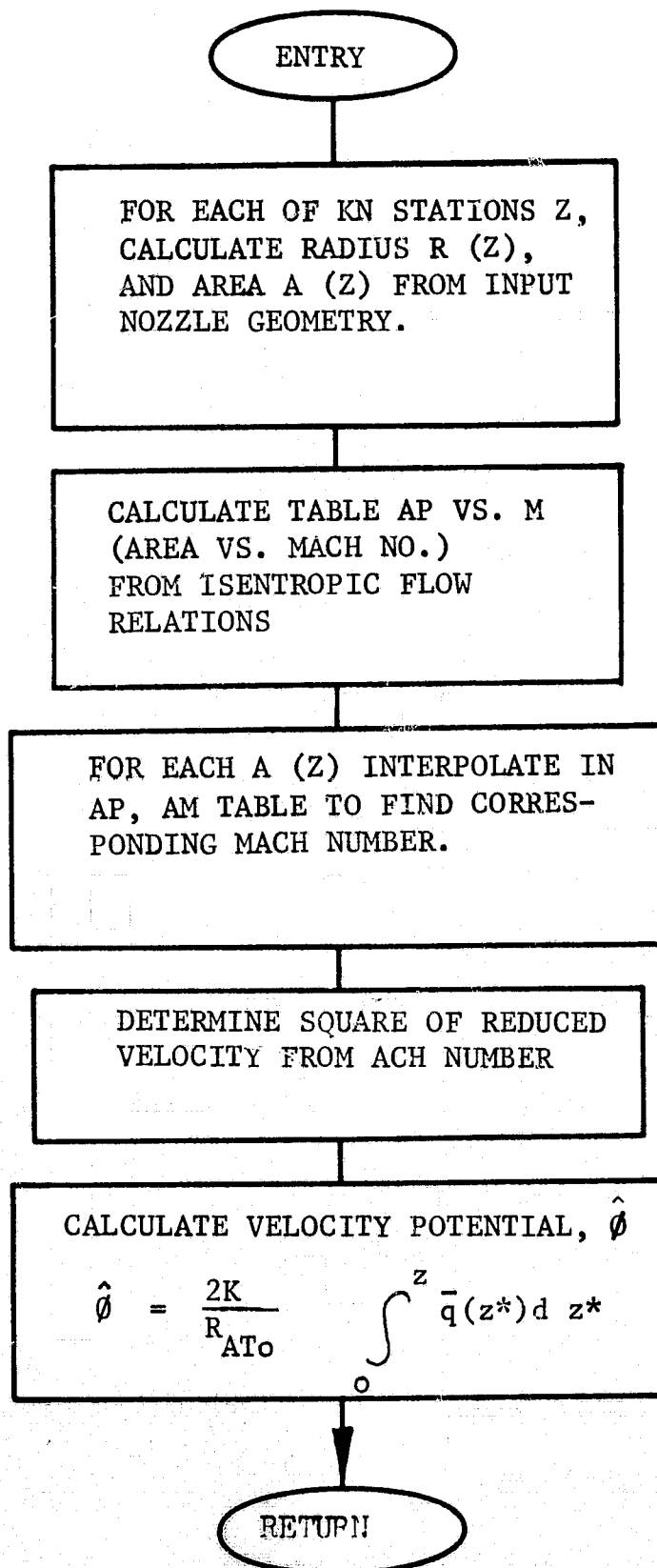


II, C, Flow Charts (cont.)

BTLDA



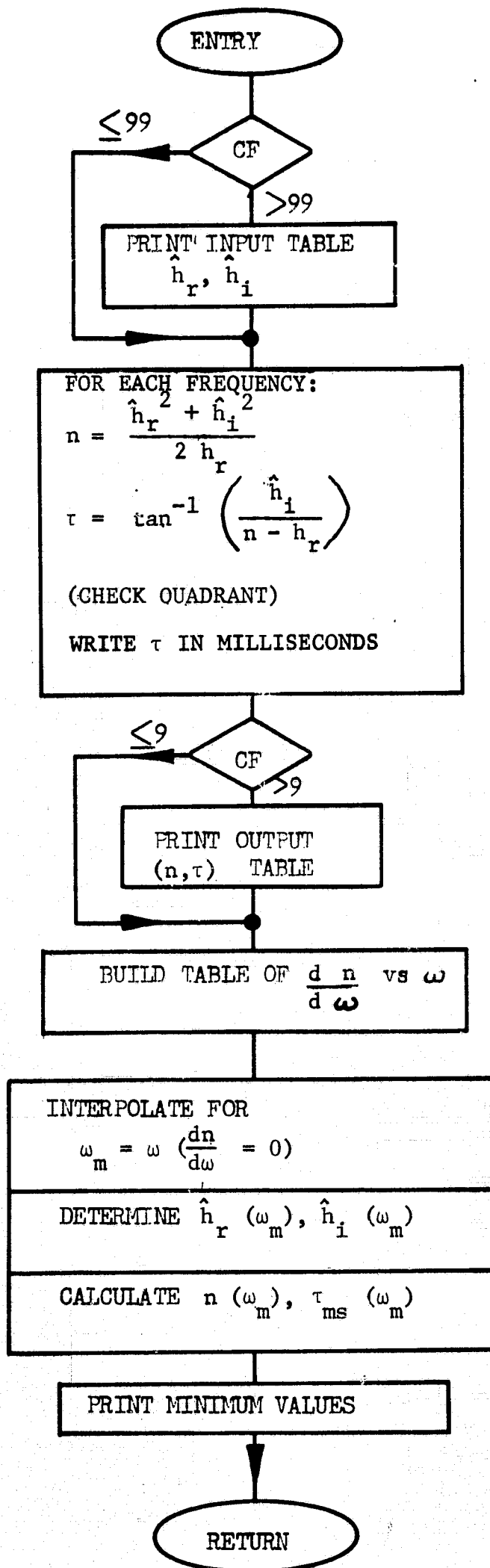
ENTRY = TBLCAL



II, C, Flow Charts (cont.)

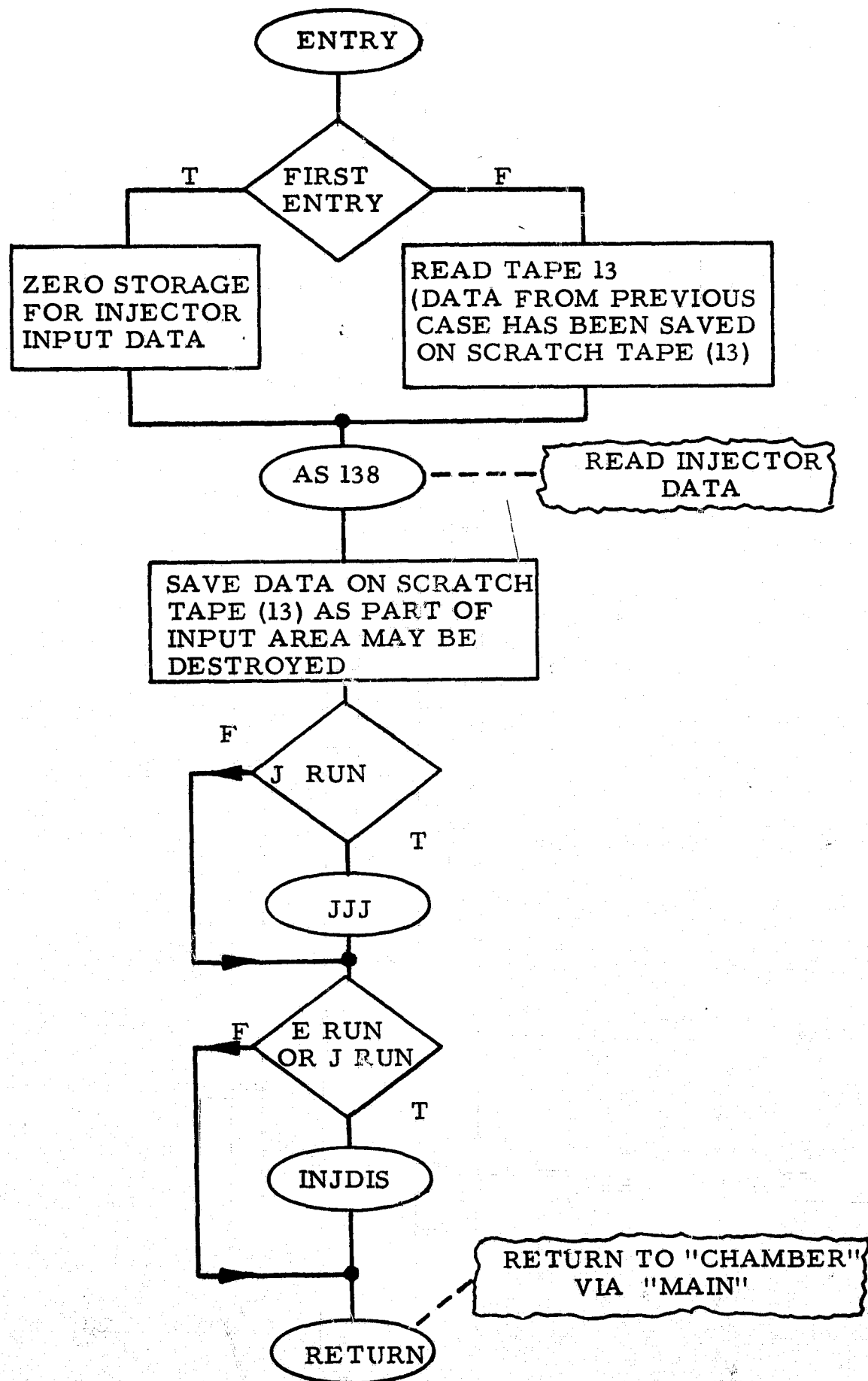
Report 20672-P3D

ENTRY = FFF



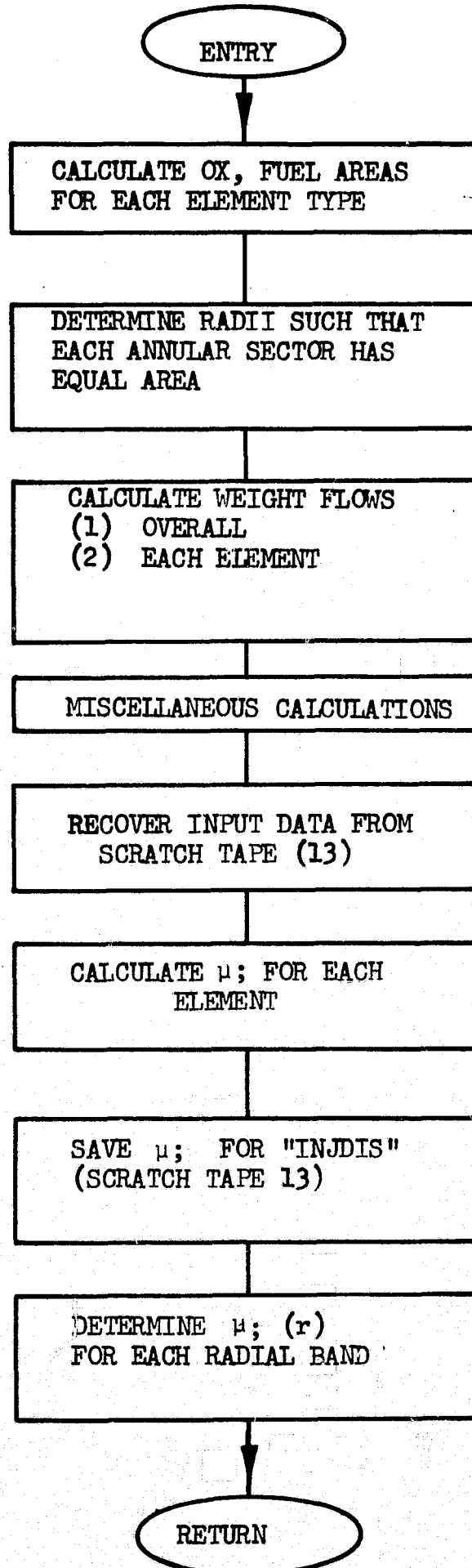
II, C, Flow Charts (cont.)

ENTRY = INJCTR



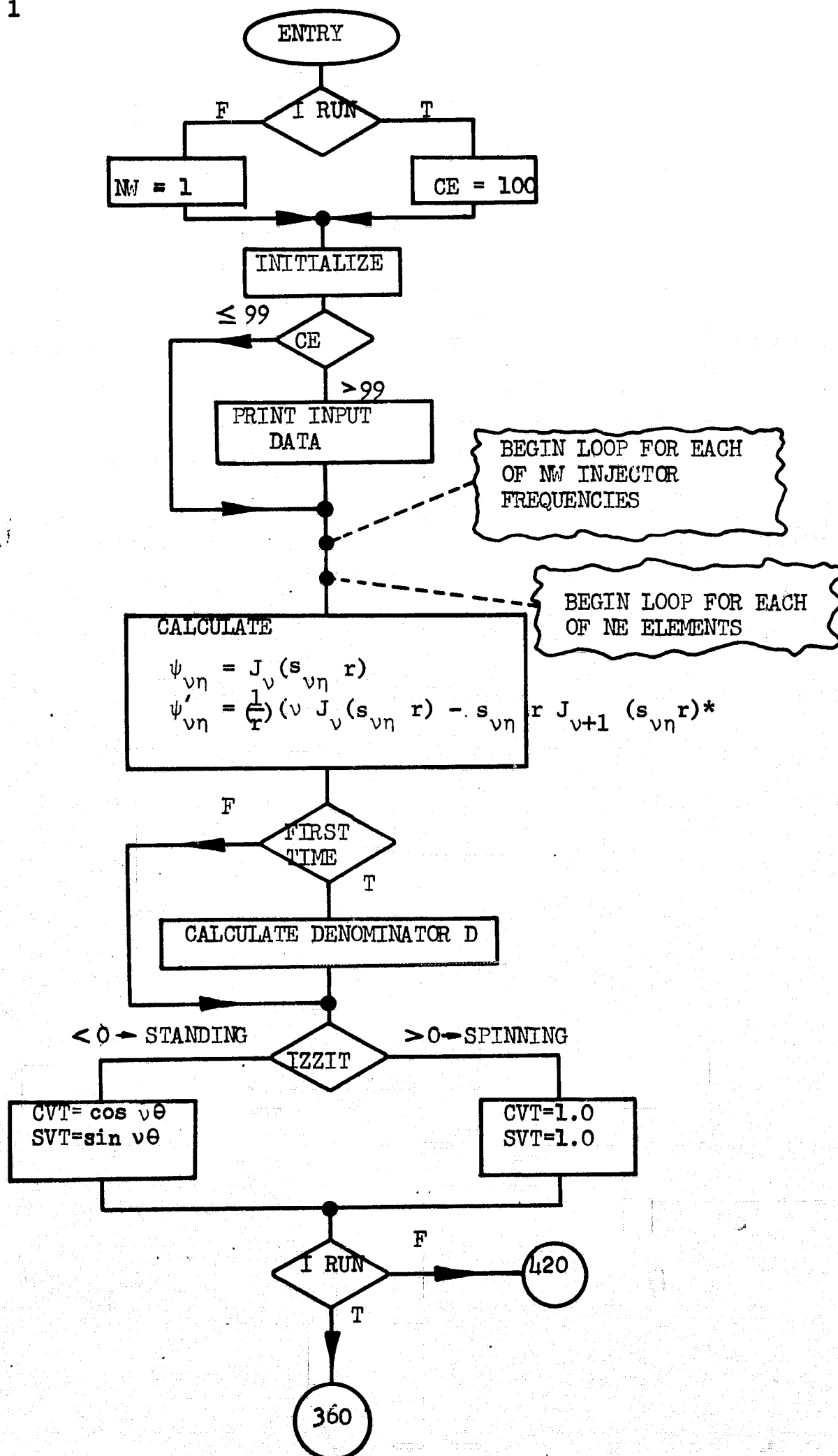
II, C, Flow Charts (cont.)

ENTRY = JJJ



II, C, Flow Charts (cont.)

ENTRY = INJDIS
Page 1 of 1

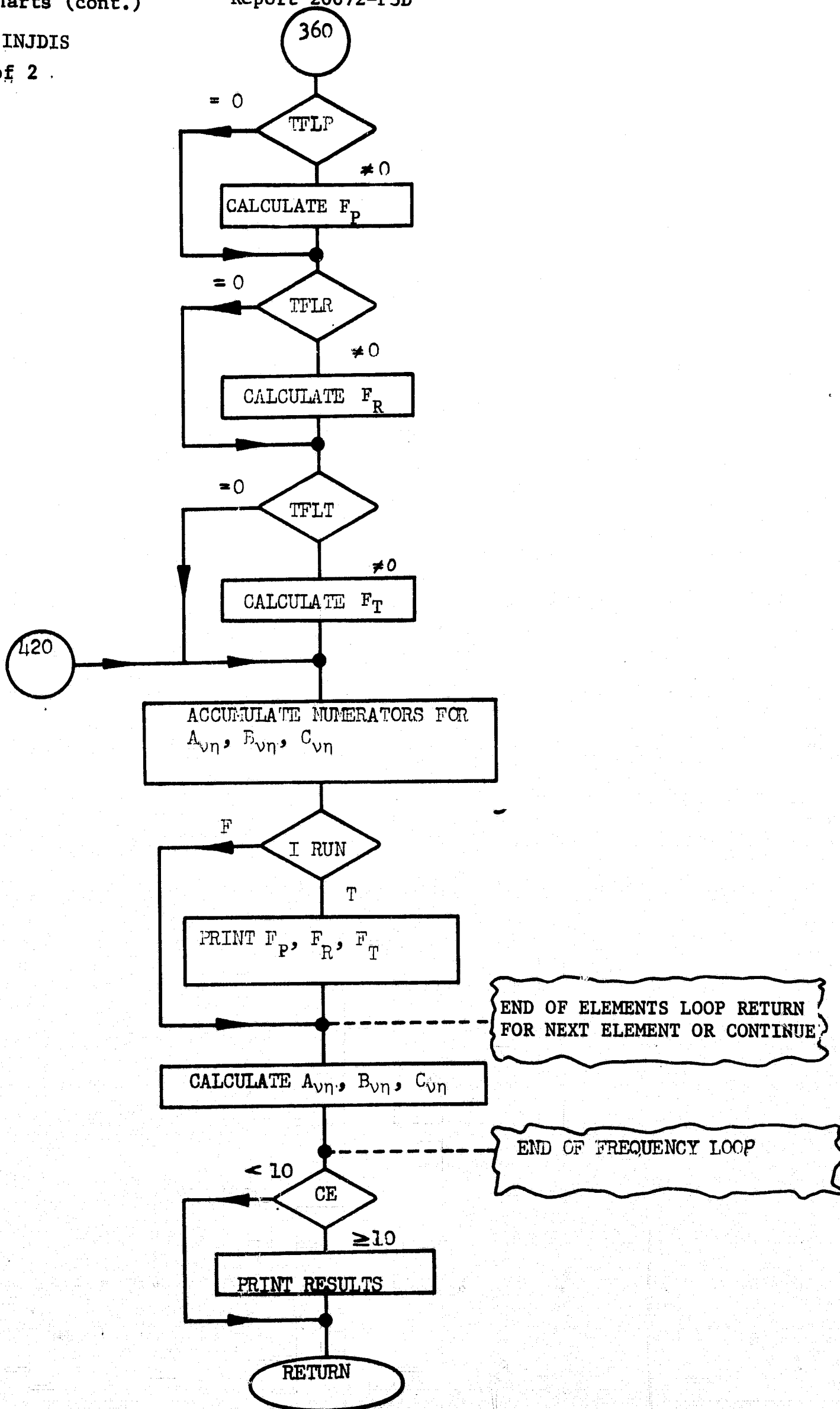


II, C, Flow Charts (cont.)

Report 20672-P3D

ENTRY = INJDIS

Page 2 of 2



II, Programming (cont.)

D. PROGRAM LISTING

10. ELT MAIN, 1,6907, 2, 33249

000001		COMMON /PROLOG/ LOGIK(50), SL1, SL2, EORJ		10
000002		LOGICAL LOGIK, SL1, SL2, EORJ		20
000003	C			30
000004		10 CALL CHAMDR		40
000005	C			50
000006	C	RETURNS FROM MAJOR PROGRAM IF AND ONLY IF AN INJECTOR PROGRAM		60
000007	C	IS TO BE CALLED... INJCTR DETERMINES WHICH, WRITES SCRATCH		70
000008	C	TAPE, AND CALLS PROPER ROUTINE.		80
000009	C			90
000010		CALL INJCTR		100
000011	C			110
000012		GO TO 10		120
000013		END		130

II, D, Program Listing (cont.)

ELT SUB01,1,690702, 33250

000001		BLOCK DATA		BLOK	30
000002	C			BLOK	40
000003		COMMON /PROLOG/ LOGIK(50), SL1, SL2, EORJ		BLOK	50
000004		LOGICAL LOGIK, SL1, SL2, EORJ		BLOK	60
000005	C			BLOK	70
000006		DATA SL1, SL2 / .FALSE., .FALSE. /		BLOK	80
000007		END		BLOK	90

II, D, Program Listing (cont.)

ELT SUB02,1,690702, 33250

```
000001      SUBROUTINE OUTAS8(A,K)
000002      DIMENSION A(12)
000003      WRITE(6,10)(A(I),I=1,K)
000004 10  FORMAT(1X,12A6)
000005      RETURN
000006      END
```

```
OUT 10
OUT 20
OUT 30
OUT 40
OUT 50
OUT 60
```

Report 20672-P3D

II, D, Program Listing (cont.)

W ELT SUB03,1,690702, 33251

000001	SUBROUTINE INAS58(A)	IN	10
000002	DIMENSION A(12)	IN	20
000003	READ(5,10)A	IN	30
000004	10 FORMAT(12A6)	IN	40
000005	RETURN	IN	50
000006	END	IN	60

II, D, Program Listing (cont.)

w ELI SUB04,1,690717, 38451

000001	SUBROUTINE INT4(X,Y,XI,YO)	INT4	10
000002	DIMENSION X(9),Y(9),XC(4),YC(4)	INT4	20
000003	EQUIVALENCE (XC(1),X1),(XC(2),X2),(XC(3),X3),(XC(4),X4),(YC(1),Y1)	INT4	30
000004	1,(YC(2),Y2),(YC(3),Y3),(YC(4),Y4)	INT4	40
000005	10 NA=1		
000006	J=2	INT4	60
000007	B=XI	INT4	70
000008	20 IF(X(J).NE.0..OR.Y(J).NE.0.) GO TO (90,160),NA		
000009	IF(J.GT.2) GO TO 70		
000010	YE=0.		
000011	GO TO 180		
000012	70 NB=1		
000013	J=J-1	INT4	150
000014	80 X1=X(J)	INT4	160
000015	X2=X(J-1)	INT4	170
000016	X3=X(J-2)	INT4	180
000017	Y1=Y(J)	INT4	190
000018	Y2=Y(J-1)	INT4	200
000019	Y3=Y(J-2)	INT4	210
000020	GO TO (150,170),NB		
000021	90 IF(X(J)-B)120,100,100	INT4	230
000022	100 IF(J.LE.2) GO TO 130		
000023	110 NA=2		
000024	120 J=J+1	INT4	260
000025	GO TO 20	INT4	270
000026	130 DO 140 J=1,3	INT4	280
000027	XC(J)=X(J)	INT4	290
000028	140 YC(J)=Y(J)	INT4	300
000029	150 D=X2-X1	INT4	310
000030	A1=B-X1	INT4	320
000031	A2=B-X2	INT4	330
000032	YE=A1*A2/2.0/D*((Y3-Y2)/(X3-X2)-(Y2-Y1)/D)-A2/D*Y1+A1/D*Y2	INT4	340
000033	GO TO 180	INT4	350
000034	160 NB=2		
000035	GO TO 80	INT4	370
000036	170 X4=X(J-3)	INT4	380
000037	Y4=Y(J-3)	INT4	390
000038	D=X3-X2	INT4	400
000039	A1=B-X2	INT4	410
000040	A2=B-X3	INT4	420
000041	XM12=(Y2-Y1)/(X2-X1)	INT4	430
000042	XM23=(Y3-Y2)/D	INT4	440
000043	XM34=(Y4-Y3)/(X4-X3)	INT4	450
000044	YE=A1*A2**2/2.0/D**2*(XM12-XM23)+A2*A1**2/2.0/D**2*(XM34-XM23)-A2*	INT4	460
000045	1Y2/D+A1*Y3/D	INT4	470
000046	180 YO=YE	INT4	480
000047	RETURN	INT4	490
000048	END	INT4	500

II, D, Program Listing (cont.)

ELT SUB05,1,690717, 38453

```

000001      SUBROUTINE INT4D(X,Y,XI,YO,DY)
000002      DIMENSION X(9),Y(9),XC(4),YC(4)
000003      EQUIVALENCE (XC(1),X1),(XC(2),X2),(XC(3),X3),(XC(4),X4),(YC(1),Y1)
000004      1,(YC(2),Y2),(YC(3),Y3),(YC(4),Y4)
000005
10      NA=1
000006      J=2
000007      B=XI
000008      20 IF(X(J).NE.0..OR.Y(J).NE.0.) GO TO (90,160),NA
000009      IF(J.GT.2) GO TO 70
000010      YE=0.
000011      GO TO 180
000012      70 NB=1
000013      J=J-1
000014      80 X1=X(J)
000015      X2=X(J-1)
000016      X3=X(J-2)
000017      Y1=Y(J)
000018      Y2=Y(J-1)
000019      Y3=Y(J-2)
000020      GO TO (150,170),NB
000021      90 IF(X(J)-B)120,100,100
000022      10 IF(J.LE.2) GO TO 130
000023      110 NA=2
000024      120 J=J+1
000025      GO TO 20
000026      130 DO 140 J=1,3
000027      XC(J)=X(J)
000028      140 YC(J)=Y(J)
000029      150 D=X2-X1
000030      A1=B-X1
000031      A2=B-X2
000032      XM23=(Y3-Y2)/(X3-X2)
000033      XM12=(Y2-Y1)/(X2-X1)
000034      XM2B=(XM23-XM12)/2.0/D
000035      YO=A1*A2*XM2B-A2*Y1/D+A1*Y2/D
000036      DY=XM2B*(A1+A2)+XM12
000037      GO TO 180
000038      160 NB=2
000039      GO TO 80
000040      170 X4=X(J-3)
000041      Y4=Y(J-3)
000042      D=X3-X2
000043      A1=B-X2
000044      A2=B-X3
000045      XM12=(Y2-Y1)/(X2-X1)
000046      XM23=(Y3-Y2)/D
000047      XM34=(Y4-Y3)/(X4-X3)
000048      AM2=A2*(XM12-XM23)
000049      AM1=A1*(XM34-XM23)
000050      YO=(A1*A2/2.0/D*(AM2+AM1)-A2*Y2+A1*Y3)/D
000051      DY=(AM2*(2.0*A1+A2)+AM1*(2.0*A2+A1))/2.0/D**2+XM23
000052      180 RETURN
000053      END

```

INT4D 10
INT4D 20
INT4D 30
INT4D 40

INT4D 60
INT4D 70

*NEW
**-1

INT4D150
INT4D160
INT4D170
INT4D180
INT4D190
INT4D200
INT4D210

INT4D230

INT4D260
INT4D270
INT4D280
INT4D290
INT4D300
INT4D310
INT4D320
INT4D330
INT4D340
INT4D350
INT4D360
INT4D370
INT4D380
INT4D390

INT4D410
INT4D420
INT4D430
INT4D440
INT4D450
INT4D460
INT4D470
INT4D480
INT4D490
INT4D500
INT4D510
INT4D520
INT4D530
INT4D540
INT4D550

II, D, Program Listing (cont.)

ELF SUB06,1,690807, 35835

000001		SUBROUTINE PAGE(LINES)	PAGE 10
000002	C		PAGE 20
000003	C	HEAD MOVED TO /PROLOG/ AND PAGE MODIFIED TO PRINT HEAD 25JUL 67	PAGE 30
000004	C		PAGE 40
000005		COMMON /PROLOG/ LOGIK(38), HEAD(12), SL1, SL2, EORJ	PAGE 50
000006		DIMENSION TODAY (2)	PAGE 60
000007		DATA KPG / 0 /	PAGE 70
000008	C		PAGE 80
000009		IF (LINES-60) 20,10,10	PAGE 90
000010	10	L=2	PAGE 100
000011		GO TO 60	PAGE 110
000012	20	K=L+LINES	PAGE 120
000013		IF (K-60) 30,30,50	PAGE 130
000014	30	L=K	PAGE 140
000015	40	RETURN	PAGE 150
000016	C		PAGE 160
000017	50	L=LINES+2	PAGE 170
000018	60	IF (KPG.EQ.0) GO TO 90	*NEW
000019	70	KPG = KPG + 1	PAGE 190**1
000020		WRITE (6,80) TODAY, HEAD, KPG	PAGE 200
000021	80	FORMAT (1H1 3X 6HDATE 2A6, 12X 12A6, 11X 5HPAGE 15)	PAGE 210
000022		GO TO 40	PAGE 220
000023	90	TODAY(1)=6H	*NEW
000024		TODAY(2)=6H	*NEW
000025		CALL DATE (12,TODAY)	*NEW
000026		GO TO 70	*NEW
000027		END	PAGE 230

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II, D, Program Listing (cont.)

W ELI SUB07,1,700722, 38212

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000001      SUBROUTINE CHAMBR                                CHAM 10
000002      C                                                CHAM 20
000003      C          20 SEP 67  MODIFIED FOR TABULAR INJECTOR COEFFICIENTS    CHAM 30
000004      C                                                CHAM 40
000005      LOGICAL LOGIK, ARUN, BRUN, CRUN, DRUN, ERUN, FRUN, GRUN, HRUN, IRUN, JRUN CHAM 50
000006      LOGICAL SL1, SL2, EORJ                                CHAM 60
000007      REAL MACH                                            CHAM 70
000008      COMMON / /                                           CHAM 80
000009      1 GAM, NWI, WIT(30), AVN(30), BVN(30), CVNR(30), CVNI(30), EE, DSCCHAM 90
000010      COMMON /PROLOG/ LOGIK(38), HEAD(12), SL1, SL2, EORJ    CHAM 100
000011      COMMON/ABCD/ DIMP, STOW                                CHAM 110
000012      DIMENSION EXTRA(100),                                WC(75)      CHAM 120
000013      DIMENSION DIMP(4300), A(1), B(1), C(1), D(1)          CHAM 130
000014      DIMENSION X(133), Y(133), Q(134), STOW(222), STODAT(4607) CHAM 140
000015      DIMENSION ZZ(205)                                       CHAM 150
000016      DIMENSION G(1), DISTL(20), DISTM(20)                 CHAM 160
000017      DIMENSION AMIT(90)                                       CHAM 170
000018      EQUIVALENCE ( EXTRA, DIMP, STODAT )                  CHAM 180
000019      EQUIVALENCE                                             CHAM 190
000020      1 (LOGIK(1), ARUN), (LOGIK(2), BRUN), (LOGIK(3), CRUN), CHAM 200
000021      2 (LOGIK(4), DRUN), (LOGIK(5), ERUN), (LOGIK(6), FRUN), CHAM 210
000022      3 (LOGIK(7), GRUN), (LOGIK(8), HRUN), (LOGIK(9), IRUN), CHAM 220
000023      4 (LOGIK(10), JRUN)                                     CHAM 230
000024      EQUIVALENCE (EXTRA(1), CA), (EXTRA(2), CB), (EXTRA(3), CC), (EXTRA(4), CHAM 240
000025      1CD), (EXTRA(5), CE), (EXTRA(6), CF), (EXTRA(7), CG), (EXTRA(8), CH) CHAM 250
000026      EQUIVALENCE (EXTRA(9), CI)                             CHAM 260
000027      EQUIVALENCE (DIMP(1), A), (DIMP(3001), B), (DIMP(3401), D), CHAM 270
000028      1(DIMP(3501), G), (DIMP(3801), C), (EXTRA(21), WC), (B(4), UE) CHAM 280
000029      EQUIVALENCE (DIMP(601), AMIT), (DIMP(2813), Q), (DIMP(3501), X), CHAM 290
000030      1 (DIMP(3651), Y)                                       CHAM 300
000031      EQUIVALENCE                                             CHAM 310
000032      1 (STOW(1), ZZ), (STOW(213), YH)                       CHAM 320
000033      2 , (STOW(214), J), (STOW(215), NE), (STOW(216), YL), (STOW(217), K1) CHAM 330
000034      3 , (STOW(218), N), (STOW(219), KER), (STOW(220), XH) CHAM 340
000035      4 , (STOW(221), KQUAD), (STOW(222), XL)                CHAM 350
000036      5 , (EXTRA(51), DISTL), (EXTRA(71), DISTM)            CHAM 360
000037      6 , (EXTRA(10), CJ), (EXTRA(21), SNH ), (EXTRA(12), MACH ) CHAM 370
000038      7 , (DIMP(107), U1BAR)                                   CHAM 380
000039      C                                                        CHAM 390
000040      C*****                                                CHAM 400
000041      C                                                        CHAM 410
000042      10 FORMAT(1H0,60H      A      B      C      D      E      F      G      H      CHAM 420
000043      1 I      J      // 3X,(10F6.0))                      CHAM 430
000044      20 FORMAT(///,9X,108H***** THE FOLLOWING MAIN CONTCHAM 440
000045      1ROL DATA WILL BE USED IN THIS CASE ***** ,//,CHAM 450
000046      245X,33H RATIO OF SPECIFIC HEAT (GAMMA) = ,F7.4, //,45X,22H DESIRED MACHAM 460
000047      3CH NUMBER = ,E12.5,26H (=0 IF BEING CALCULATED) //45X 17HCHCHAM 470
000048      5AMBER RADIUS = ,F7.3, 9H (INCHES), //,45X,17HCHAMBER LENGTH = ,F7.3CHAM 480
000049      6, 9H (INCHES), //,45X, 17HSPEED OF SOUND = ,F10.3, 9H (FT/SEC), //,CHAM 490
000050      745X, 27HCHAMBER MODE DESCRIPTION = ,F8.5, 28H (=0 FOR LONGITUDINALCHAM 500
000051      8 MODES), //,)                                           CHAM 510
000052      30 FORMAT(///,33X,48H***** CHAMBER FREQUENCIES (WC) ***** CHAM 520
000053      1 ,//,)                                                  CHAM 530
000054      40 FORMAT(5F20.5, / )                                     CHAM 540
000055      50 FORMAT(30X, 64H***** MACH DISTRIBUTION IN CHAMBER AS A FUNCTION OFCHAM 550

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II, D, Program Listing (cont.)

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000056      1 LENGTH ****,/,11X,7HCHAMBER,15X,4HMACH,14X,7HCHAMBER,15X,4HMACHCHAM. 560
000057      2,14X,7HCHAMBER,15X,4HMACH,/,12X,6HLENGTH,11X,12HDISTRIBUTION,11X, CHAM. 570
000058      3,6HLENGTH,11X,12HDISTRIBUTION,11X,6HLENGTH,11X,12HDISTRIBUTION,/,) CHAM 580
000059      60 FORMAT(6(10X,F10.5)) CHAM 590
000060      65 FORMAT ( 11X,42HDISTANCE FROM INJECTOR TO COMBUSTION FRONT ,3X,
000061      1F10.5,6HINCHES)
000062 C***** CHAM 600
000063 C***** CHAM 610
000064 C CHAM 620
000065      IF ( SL2 ) GO TO 170 CHAM 630
000066 C CHAM 640
000067      SL2 = .TRUE. CHAM 650
000068      DO 70 I = 1, 4300 CHAM 660
000069      70 DIMP (I) = 0.0 CHAM 670
000070      GO TO 110 CHAM 680
000071 C CHAM 690
000072 C***** CHAM 700
000073 C CHAM 710
000074      80 WRITE (6,100) NE CHAM 720
000075      90 CALL EXIT CHAM 730
000076      100 FORMAT (1H010X17HINPUT ERROR, NE = I3, 19H, HENCE TERMINATION ) CHAM 740
000077 C CHAM 750
000078 C***** CHAM 760
000079 C***** CHAM 770
000080 C CHAM 780
000081      110 KER = 0 CHAM 790
000082      CALL DVCHK (KCHK) CHAM 800
000083 C CHAM 810
000084      DO 120 I = 1, 10 CHAM 820
000085      120 DIMP(I) = 0.0 CHAM 830
000086      CALL AS130 ( DIMP(1), HEAD(1), NE ) CHAM 840
000087      IF ( NE .NE. 1 ) GO TO 80 CHAM 850
000088      ARUN = CA .NE. 0.0 CHAM 860
000089      BRUN = CB .NE. 0.0 CHAM 870
000090      CRUN = CC .NE. 0.0 CHAM 880
000091      DRUN = CD .NE. 0.0 CHAM 890
000092      ERUN = CE .NE. 0.0 CHAM 900
000093      FRUN = CF .NE. 0.0 CHAM 910
000094      IRUN = CI .NE. 0.0 CHAM 920
000095      JRUN = CJ .NE. 0.0 CHAM 930
000096      OGRUN = DIMP(22) .LE. 0.0 CHAM 940
000097      1 .AND. ( ARUN .OR. BRUN .OR. CRUN .OR. IRUN ) CHAM 950
000098 C CHAM 960
000099 C PRINT NEW MAIN CONTROL DATA CHAM 970
000100 C***** CHAM 980
000101      CALL PAGE ( 60 ) CHAM 990
000102      WRITE (6,10) (DIMP(I), I=1,10) CHAM1000
000103      WRITE (6,20) DIMP(11), MACH, DIMP(14), DIMP(15), DIMP(16), SNH CHAM1010
000104 C***** CHAM1020
000105 C ARE FREQUENCIES TO BE CALCULATED... IF SO, CALCULATE AND PRINT. CHAM1030
000106 C***** CHAM1040
000107      IF ( .NOT. GRUN ) GO TO 130 CHAM1050
000108      CALL GENMEG ( WC(1) ) CHAM1060
000109      IF ( CRUN ) CC = CC + 11.0 CHAM1080
000110      130 JOMEGA=ABS(EXTRA(22))+22.0001 CHAM1090
000111      WRITE (6,30) CHAM1100
000112      WRITE (6,40)(EXTRA(I),I=23,JOMEGA) CHAM1110
000113      IF((.NOT. ARUN) .AND. (.NOT. BRUN)) GO TO 132 *NEW
000114      IF ( BRUN ) GO TO 131

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II, D, Program Listing (cont.)

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000115      WRITE (6,50)
000116      WRITE (6,60) ( DISTL(I), DISTM(I), DISTL(I+7), DISTM(I+7), CHAM1120
000117      1, DISTL(I+14), DISTM(I+14), I=1,6 ), DISTL(7), DISTM(7), CHAM1130
000118      2DISTL(14), DISTM(14) CHAM1140
000119      GO TO 132 CHAM1150
000120
000121      131 WRITE(6,65) DINP(52)
000122 C***** CHAM1160
000123 C CHAM1170
000124      132 CONTINUE
000125      EORJ = ERUN .OR. JKUN .OR. IRUN CHAM1180
000126      IF ( CRUN .OR. EORJ ) CALL PAGE ( 60 ) CHAM1190
000127      140 IF ( .NOT. EORJ ) GO TO 200 CHAM1200
000128      WRITE (14) STODAT CHAM1210
000129      BACKSPACE 14 CHAM1220
000130      EE = CE CHAM1230
000131      DSC = CI CHAM1240
000132      GAM = DINP(11) CHAM1250
000133      IF ( .NOT. IRUN ) GO TO 160 CHAM1260
000134      NWI = ( JOMEGA-22 )/2 +1 CHAM1270
000135      DO 150 I = 1, NWI CHAM1280
000136      150 WIT(I) = DINP( 2*I+21 ) CHAM1290
000137      WIT(NWI) = DINP( JOMEGA ) CHAM1300
000138 C CHAM1310
000139      160 RETURN CHAM1320
000140 C ***** CHAM1330
000141 C PROGRAM RETURNS IF AND ONLY IF INJECTOR PROGRAM IS TO BE RUN CHAM1340
000142 C CHAM1350
000143 C***** CHAM1360
000144 C***** CHAM1370
000145 C CHAM1380
000146 C CHAM1390
000147      170 READ (14) STODAT CHAM1400
000148      BACKSPACE 14 CHAM1410
000149      IF ( .NOT. ERUN ) GO TO 200 CHAM1420
000150 C CHAM1430
000151      DINP(3408) = NWI CHAM1440
000152      DO 180 I = 1, NWI CHAM1450
000153      DINP(I+4522) = WIT(I) CHAM1460
000154      DINP(I+4539) = AVN (I) CHAM1470
000155      DINP(I+4556) = BVN (I) CHAM1480
000156      DINP(I+4573) = CVNR(I) CHAM1490
000157      180 DINP(I+4590) = CVNI(I) CHAM1500
000158      DO 190 I = NWI, 84, 17 CHAM1510
000159      190 DINP(I+4523) = 0.0 CHAM1520
000160 C CHAM1530
000161 C***** CHAM1540
000162 C CHAM1550
000163      200 WC(2) = ABS( WC(2) ) CHAM1560
000164 C***** CHAM1570
000165 C FIX NUMBER OF FREQUENCIES AND TEST DESIRE FOR INTERNAL C FLAG CHAM1580
000166 C***** CHAM1590
000167      210 JOMEG=WC(2)+.0001 CHAM1600
000168      IF (EXTRA(12))230,220,230 CHAM1610
000169      220 DINP(3802)=3.0 CHAM1620
000170      GO TO 240 CHAM1630
000171      230 U1BAR = MACH CHAM1640
000172      UE = MACH CHAM1650
000173      240 IF ( .NOT. CRUN ) GO TO 390 CHAM1660
C***** CHAM1670

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II, D, Program Listing (cont.)

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000174 C      SET UP DATA FOR PROGRAM C.
000175 C*****
000176 250 DINP(3801)=EXTRA(11)
000177 NMHAF = NOMEQ/2 + 1
000178 ZZ(1)=EXTRA(14)
000179 ZZ(2)=EXTRA(15)
000180 ZZ(3)=EXTRA(16)
000181 DINP(3805) = EXTRA(14)
000182 DINP(3803)=NMHAF
000183 IF ( DINP(3809) .EQ. 0.0 )          DINP(3809) = 101.0
000184 K = 0
000185 RATO=DINP(3804)
000186 RACO=DINP(3805)
000187 RCCO=DINP(3806)
000188 RCTO=DINP(3807)
000189 RATI=DINP(91)
000190 RCTI=DINP(92)
000191 RACI=DINP(93)
000192 RCCI=DINP(94)
000193 IF(RCTI.EQ.0.) RCTI=1.
000194 GRAD=RATO*SQRT(2./(1.+EXTRA(11)))*(RATO/RCTO-RATI/RCTI)/(RATO*
000195 1RATO-RATI+RATI)
000196 ZAVE=SQRT((1.+EXTRA(11))/2.)*RATO/GRAD
000197 ZZ(4)=ZAVE
000198 IF(EXTRA(21))270,260,270
000199 260 SNOZ = 0.0
000200 C*****
000201 C      USING HALF OF CHAMBER FREQUENCIES, CALCULATE NOZZLE FREQUENCIES
000202 C      FOR USE IN PROG C.
000203 C*****
000204 RORL = EXTRA(15)
000205 GO TO 280
000206 270 RORL = EXTRA(14)
000207 SNOZ = SNI/GRAD
000208 280 DO 290 I=1,NMHAF
000209 KN=(2*I)+1
000210 MN=4210+K
000211 DINP(MN) = ZAVE*WC(KN)/RORL
000212 DINP(MN+1) = SNOZ
000213 DINP(MN+2) = MACH
000214 K=K+3
000215 290 CONTINUE
000216 DINP(MN)=ZAVE*(WC(NOMEQ+2)/EXTRA(14))
000217 300 CALL CCC(C(1),ZZ(1),WC(1),CC,KER)
000218 C      SET UP NOZZ ADM FOR A,B
000219 IF(KER) 310,330,310
000220 310 WRITE (6,320)
000221 320 FORMAT (1H0 30X39H ERROR PROGRAM C, ALL CASES TERMINATED )
000222 CALL CORE(C(1),420,CC)
000223 GO TO 90
000224 330 GO TO 360
000225 360 CONTINUE
000226 C*****
000227 C      MOVE OUTPUT FROM PROG C INTO INPUT BLOCK FOR PROG B.
000228 C*****
000229 370 DO 380 I=18,200
000230 380 B(I)=ZZ(I)
000231 B(4)=ZZ(205)
000232 C

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CHAM1680
CHAM1690
CHAM1700
CHAM1710
CHAM1720
CHAM1730
CHAM1740
CHAM1750
CHAM1760
CHAM1770
CHAM1780

CHAM1800
CHAM1810
CHAM1820
CHAM1830
CHAM1840
CHAM1850
CHAM1860
CHAM1870
CHAM1880
CHAM1900
CHAM1910
CHAM1920
CHAM1930
CHAM1940
CHAM1950
CHAM1960
CHAM1970
CHAM1980
CHAM1990
CHAM2000
CHAM2010
CHAM2020
CHAM2030
CHAM2040
CHAM2050
CHAM2060

CHAM2270
CHAM2280
CHAM2290
CHAM2300
CHAM2310
CHAM2320
CHAM2330

II, D, Program Listing (cont.)

000233	390 CONTINUE	CHAM2340
000234	CONTINUE	
000235	C NOZZLE ADMITTANCE IS INPUT TO PROGS A, B.	CHAM2360
000236	C*****	CHAM2370
000237	C	CHAM2380
000238	C*****	CHAM2490
000239	C SET UP MAIN CONTROL DATA FOR USE IN B OR A	
000240	C*****	CHAM2510
000241	450 IF ((.NOT. BRUN) .AND. (.NOT. ARUN)) GO TO 530	
000242	B(1)=WC(1)	CHAM2530
000243	B(2)=EXTRA(15)/EXTRA(14)	CHAM2540
000244	B(3)=EXTRA(11)	CHAM2550
000245	B(5)=EXTRA(16)	CHAM2560
000246	B(6) = DIMP(17)	CHAM2570
000247	B(7) = DIMP(18)	CHAM2580
000248	B(8) = DIMP(19)	CHAM2590
000249	B(19)=WC(2)	CHAM2600
000250	DO 460 I=1,NOMEG	CHAM2610
000251	B(I+169)=WC(I+2)	CHAM2620
000252	460 CONTINUE	CHAM2630
000253	B(200) = DISTL(1) / DIMP(14)	CHAM2640
000254	B(220) = DISTM(1)	CHAM2650
000255	B(17) = 1.0	CHAM2660
000256	DO 480 I = 2, 20	CHAM2670
000257	IF(DISTM(I))490,490,470	CHAM2680
000258	470 B(1+199)=DISTL(I)/EXTRA(14)	CHAM2690
000259	B(1+219)=DISTM(I)	CHAM2700
000260	B(17)=B(17)+1.0	CHAM2710
000261	480 CONTINUE	CHAM2720
000262	490 CONTINUE	CHAM2730
000263	IF(.NOT. ARUN) GO TO 492	
000264	CALL DISTR(B,X,CA,KER)	
000265	GO TO 499	
000266	492 CALL CONC (B,X,CB,KER)	
000267	499 DO 500 I=9,100	
000268	500 D(I) = X(I)	CHAM2760
000269	IF(KER)530,510,530	CHAM2770
000270	510 WRITE (6,520)	CHAM2780
000271	520 FORMAT (1H0,40X,21H ERROR PROGRAM A OR B)	
000272	CALL CORE(D(1),400,CB)	CHAM2800
000273	GO TO 110	CHAM2810
000274	C*****	CHAM2820
000275	530 CONTINUE	
000276	IF((.NOT. ARUN) .AND. (.NOT. BRUN) .AND. (.NOT. DRUN)) GO TO 610	
000277	IF(.NOT. DRUN) GO TO 575	
000278	540 DO 550 I=1,8	CHAM2840
000279	X(I)=D(I)	CHAM2850
000280	550 CONTINUE	CHAM2860
000281	560 X(5)=EXTRA(11)	CHAM2870
000282	DO 570 I = 9, 100	CHAM2880
000283	570 X(I) = D(I)	CHAM2890
000284	575 CALL DDD (X(1),Y(1), CD,KER, ERR)	
000285	IF (ERR) 110,580,580	CHAM2910
000286	580 IF (KER) 610,590,610	CHAM2920
000287	590 WRITE (6,600)	CHAM2930
000288	600 FORMAT (1H0,40X,18H ERROR PROGRAM D)	CHAM2940
000289	CALL CORE(X(1),100,CD)	CHAM2950
000290	GO TO 110	CHAM2960
000291	610 CONTINUE	CHAM2970

II, D, Program Listing (cont.)

000292	620 IF ((.NOT. ARUN) .AND.(.NOT. BRUN) .AND.(.NOT. FRUN)) GO TO 670	
000293	C*****	CHAM2990
000294	C SET UP DATA FOR PROG F (N, TAU)	CHAM3000
000295	C*****	CHAM3010
000296	630 Y(1)=EXTRA(14)	CHAM3020
000297	IF(SNH) 632,631,632	
000298	631 Y(1)=EXTRA(15)	
000299	632 CONTINUE	
000300	Q(101) = EXTRA(16)/Y(1) * 12.0/6.2831853	CHAM3030
000301	640 Y(2)=EXTRA(16)	CHAM3040
000302	CALL FFF (Y(1), Q(1), CF, KER, WC(1))	CHAM3050
000303	IF (KER) 670,650,670	CHAM3060
000304	650 WRITE (6,660)	CHAM3070
000305	660 FORMAT (1H0,40X,18H ERROR PROGRAM F)	CHAM3080
000306	CALL CORE(Y(1),100,CF)	CHAM3090
000307	670 GO TO 110	CHAM3100
000308	C	CHAM3110
000309	C	CHAM3120
000310	C*****	CHAM3130
000311	C*****	CHAM3140
000312	C*****	CHAM3150
000313	END	CHAM3160

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II, D, Program Listing (cont.)

W LIT SUB08,1,690702, 33259

000001	SUBROUTINE GENMEG(W)	WGEN 10
000002	C	WGEN 20
000003	C*****	WGEN 30
000004	DIMENSION W(1)	WGEN 40
000005	C	WGEN 50
000006	C LONGITUDINAL WILL HAVE NEG SNH WHICH WILL BE SET TO ZERO BEFOR RETURN	WGEN 60
000007	C IF SNH 0 LONGITUDINAL AND GENERATE + OR - 10 PERCENT OF PIE	WGEN 70
000008	C*****	WGEN 80
000009	IF(W(1))40,50,10	WGEN 90
000010	C*****	WGEN 100
000011	C FOR TRANSVERSE GENERATE 10 VALUES AROUND SNH 9 PERCENT BELOW AND	WGEN 110
000012	C 11 PERCENT ABOVE	WGEN 120
000013	C*****	WGEN 130
000014	10 SNH=W(1)	WGEN 140
000015	20 DELMEG=.1*SNH	WGEN 150
000016	W(3)=SNH-DELMEG	WGEN 160
000017	DELMEG=DELMEG/5.0	WGEN 170
000018	W(3)=W(3)+DELMEG	WGEN 180
000019	DO 30 I=4,12	WGEN 190
000020	W(1)=W(I-1)+DELMEG	WGEN 200
000021	30 CONTINUE	WGEN 210
000022	C*****	WGEN 220
000023	C NEGATIVE 10 INDICATES TO PROGRAM THAT FREQUENCES ARE GENERATED INTERN	WGEN 230
000024	C*****	WGEN 240
000025	W(2)=-10.0	WGEN 250
000026	RETURN	WGEN 260
000027	C*****	WGEN 270
000028	C IF NEG GENERATE + OR - 10 PERCENT OF POSITIVE INITIAL GUESS	WGEN 280
000029	C*****	WGEN 290
000030	40 SNH=-W(1)	WGEN 300
000031	W(1)=0.0	WGEN 310
000032	GO TO 20	WGEN 320
000033	50 SNH=3.141592	WGEN 330
000034	GO TO 20	WGEN 340
000035	END	WGEN 350

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II, D, Program Listing (cont.)

W LIT SUB09,1,700717, 49719

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000001      SUBROUTINE CONC (BIN, XOUT, CB, KER )
000002      C
000003      C
000004      C
000005      C
000006      C
000007      C
000008      C*****
000009      C
000010      LOGICAL LOGIK, HRUN, CKOUT, SIMPL, KNOT, LIMIT, TABLR
000011      C
000012      COMMON /PROLOG/ LOGIK(38), HEAD(12), SL1, SL2, EORJ
000013      COMMON /ABCD/ / DIN (4300)
000014      1 ,SPACE(222)
000015      DIMENSION B(240),THR (30),THI (30)
000016      DIMENSION
000017      1 BIN ( 240 ) , XOUT ( 100 ) , WC ( 28 ) ,
000018      2 WET ( 30 ) , ERT ( 30 ) , EIT ( 30 ) ,
000019      3 CRT ( 30 ) , CIT ( 30 )
000020      EQUIVALENCE
000021      1 ( HRUN , LOGIK( 8) ), ( CKOUT, LOGIK(13) ), ( SIMPL, LOGIK(14) ),
000022      2 ( KNOT, LOGIK(15) ), ( LIMIT, LOGIK(16) ), ( TABLR, LOGIK(17) )
000023      EQUIVALENCE
000024      1 ( B(1) , SNH ) , ( DIN(14) , RCH ) , ( B(18) , XNE ) ,
000025      3 ( B(19) , XNW ) ,
000026      4 ( B(4) , UE ) , ( B(170) , WC ) , ( B(20) , WET ) ,
000027      5 ( B(5) , SOUND ) , ( B(50) , ERT ) ,
000028      6 ( B(6) , ULM ) , ( B(80) , EIT ) ,
000029      7 ( B(7) , XK ) , ( B(110) , CRT ) ,
000030      8 ( B(8) , XCML ) , ( B(140) , CIT )
000031      EQUIVALENCE
000032      1 ( DIN(11) , GAMMA ) , ( DIN(14) , RCSTAR ) , ( DIN(16) , CBAROS ) ,
000033      2 ( DIN(17) , VBARLS ) , ( DIN(21) , SNUETA ) , ( DIN(51) , PBAROS ) ,
000034      3 ( DIN(52) , XSTAR ) , ( DIN(53) , VBARM ) , ( DIN(54) , WBARS )
000035      COMPLEX A, EOMEGA, COMEGA, Y, H, K1, K2, S, DUMV, V
000036      DIMENSION A(6,7) , JC(6)
000037      DATA A /42*(0.,0.)/
000038      DATA PBARP, RHOBPR, HBARP, HBARL /4*1.0/
000039      C*****
000040      C
000041      10 FORMAT ( / 65H TRANSVERSE STABILITY PROGRAM... CALCULATES HR, HI
000042      1
000043      20 FORMAT ( // 11H INPUT DATA// 9X3H5NH12X2HZE10X5HGAMMA9X2HUE9X
000044      1 14HSOUND (FT/SEC) 2X13H ULM (FT/SEC)// F15.8, F14.8, F12.5, F14.
000045      2, F16.2, 2X F15.6 // )
000046      30 FORMAT ( // 25H NOZZLE ADMITTANCES INPUT// 15X9HOMEGA(CH) 12X
000047      1 3HERT 17X3HEIT 17X3HCRT 17X3HCIT // )
000048      40 FORMAT ( /// 22H CALCULATED RESULTS // 17X5HOMEGA 12X6HH REAL
000049      114X6HH IMAG )
000050      50 FORMAT ( 9XF15.7, 2E20.8 )
000051      60 FORMAT ( 9XF15.7, 4E20.8 )
000052      110 FORMAT ( 6H ERROR 6X6E18.8 )
000053      160 FORMAT ( 7E18.8 )
000054      C
000055      C*****

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II, D, Program Listing (cont.)

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000056 C***** HYMN 860
000057 DO 170 I = 1, 240 HYMN 910
000058 170 B(I) = BIN(I) HYMN 920
000059 180 CONTINUE
000060 190 CALL PAGE ( 70 ) HYMN 940
000061 KOUT = 2 HYMN 950
000062 NET = XNE HYMN 960
000063 WRITE (6,10) HYMN 970
000064 IF ( CB-100.0 ) 220,200,200 HYMN 980
000065 200 WRITE(6,20)(B(I),I=1,6) *NEW
000066 WRITE (6,30) HYMN1000**-1
000067 WRITE (6,60) ( WET(I), ERT(I),EIT(I), CRT(I),CIT(I), I=1,NET ) HYMN1010
000068 CKOUT = ( CB .GT. 200.0 ) HYMN1020
000069 C HYMN2340
000070 C***** HYMN2350
000071 C HYMN2360
000072 220 GAM=GAMMA-1.
000073 VBARM=UE/SQRT(1.+GAM/2.*UE*UE)
000074 VBARM2=VBARM*VBARM
000075 VBARL=VBARLS/CHAROS
000076 VBARL2=VBARL*VBARL
000077 RHBROS=PBAROS*GAMMA/(CBAROS*CBAROS)
000078 RHOBRM=WBARIS/(RHBROS*101.065*RCSTAR*RCSTAR*CBAROS*VBARM)
000079 PBARM=1.-GAMMA*(RHOBRM*VBARM2-RHOBRM*VBARM*VBARL)
000080 HBARM=1.+VBARL2/2.-VBARM2/2.
000081 IF(SNUETA) 222,222,221
000082 221 X=XSTAR/RCSTAR
000083 GO TO 223
000084 222 X=XSTAR/DIN(15)
000085 223 TBARM=HBARM
000086 NW=XNW+.1
000087 DO 820 IW=1,NW
000088 DO 85 LLL=1,42
000089 85 A(LLL)=(0.,0.)
000090 W=WC(IW)
000091 A(1,7)=(1.,0)
000092 A(3,7)=HBARL+GAM*VBARL2/2.
000093 A(2,7)=VBARL
000094 A(1,2)=-VBARM
000095 A(1,3)=-RHOBRM
000096 A(2,1)=-1./GAMMA
000097 A(2,2)=-VBARM2
000098 A(2,3)=-2.*RHOBRM*VBARM
000099 A(2,4)=1./GAMMA
000100 A(3,2)=-GAM*VBARM2*VBARM/2.-HBARM*VBARM
000101 A(3,3)=-GAM*3.*RHOBRM*VBARM2/2.-HBARM*RHOBRM
000102 A(3,5)=-RHOBRM*VBARM
000103 A(4,1)=(1.,0.)
000104 A(4,2)=(-1.,0.)
000105 A(4,5)=(-1.,0.)
000106 A(5,5)=(-1.,0.)
000107 A(5,1)=GAM/(GAMMA*RHOBRM)
000108 A(5,6)=TBARM
000109 A(6,3)=GAMMA
000110 C HYMN2420
000111 CALL INT4 ( WET, ERT, W, ER ) HYMN2430
000112 CALL INT4 ( WET, EIT, W, EI ) HYMN2440
000113 C HYMN4240
000114 CALL INT4 ( WET, CRT, W, CR ) HYMN4250

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II, D, Program Listing (cont.)

000115		CALL INT4 (WET, CIT, W, CI)	
000116	C		HYMN4260
000117		LOMEGA=CMPLX(ER ,EI)	HYMN4270
000118		COMEGA=CMPLX(CR ,CI)	
000119		S=CMPLX(0 ,W)	
000120		DUMV=CSQRT(S*S+SNUETA*SNUETA)	
000121		Y=-CTANH(DUMV*X)*DUMV/(S*GAMMA)	
000122		A(1,4)=Y	
000123		A(3,4)=Y*RHOBRP*HBARP	
000124		A(6,1)=LOMEGA	
000125		IF(SNUETA)120,120,121	
000126	120	A(6,1)=LOMEGA*GAM	*NEW
000127	121	CONTINUE	**1
000128		A(6,6)=GAMMA*COMEGA	*NEW
000129		V=(5,0.)	**1
000130		CALL CGJR (A,7,6,6,7,\$123,JC,V)	
000131		GO TO 124	
000132	123	WRITE (6,125) A	
000133	125	FORMAT (/1X'OVERFLOW',/(6E20,6/))	
000134	124	K1=-A(4,7)	
000135		K2=-A(1,7)	
000136		H=2.*PBARM/(RHOBRM*VBARM*(K1+K2))	
000137		HR=REAL(H)	
000138		HI=AIMAG(H)	
000139		I=IW	
000140	C		HYMN7080
000141		THR (IW)=HR	
000142		THI (IW)=HI	
000143	820	CONTINUE	
000144	C	END OF W-LOOP	HYMN7090
000145	C	*****	HYMN7100
000146	C		HYMN7110
000147		KER = NW	HYMN7120
000148		XOUT(9) = XNW	HYMN7130
000149	C		HYMN7140
000150	C		HYMN7150
000151		DO 830 IW = 1, NW	HYMN7170
000152		XOUT(IW+ 9) = WC (IW)	HYMN7180
000153		XOUT(IW+39) = THR(IW)	HYMN7190
000154		XOUT(IW+69) = THI(IW)	HYMN7200
000155	830	CONTINUE	HYMN7210
000156	840	IF (CKOUT) CALL PAGE (70)	HYMN7220
000157		WRITE (6,40)	HYMN7240
000158		WRITE (6,50) (WC(I), THR(I), THI(I), I = 1, NW)	HYMN7250
000159	C		HYMN7260
000160	C		HYMN7280
000161	C		HYMN7490
000162	C		HYMN7500
000163	C		HYMN7510
000164	C	THIS SUBROUTINE HAS CALCULATED HR AND HI. NO INJECTION DISTRIBUTIO	
000165	C	N OR VELOCITY EFFECTS HAVE BEEN INCLUDED. THESE EFFECTS CAN BE	
000166	C	ADDED IN SUBROUTINE DDD WHICH MUST BE RUN TO GET AN N, TAU PLOT	
000167	C		HYMN7570
000168	C		HYMN7580
000169	890	RETURN	HYMN7590
000170		END	HYMN7600
			HYMN7610

II, D, Program Listing (cont.)

W ELT SUB10,1,700708, 29819

000001	SUBROUTINE DDD(DIN,DOUT,CD,NER,ERR)	HTNT	10
000002		HTNT	20
000003	C PROGRAM D COMPUTE HTR,HTI + INTERPOLATE 40 POINTS	HTNT	30
000004	C 20 SEP 67 MODIFIED FOR TABULAR INJECTOR COEFFICIENTS	HTNT	40
000005	C	HTNT	50
000006	LOGICAL LOGIK,TABLR,DRUN,ERUN		
000007	COMMON /PROLOG/ LOGIK(50)	HTNT	70***-1
000008	COMMON /ABCOF/ EXTRA(100), ABLOK(600), A , B , SPACE(3556),	HTNT	80
000009	1 WLT(17), AVN(17), BVN(17), CVNR(17), CVNI(17)	HTNT	90
000010	DIMENSION DIN(1),DOUT(1),A(133),B(133),OMEGA(1),HR(1),HI(1)	HTNT	100
000011	1,HTR(30),HTI(30),HTRINT(1),HTIINT(1), OMGA(1)	HTNT	110
000012	2,DUM(30)	HTNT	120
000013	EQUIVALENCE (LOGIK(4), DRUN),(LOGIK(5), ERUN),(LOGIK(17),TABLR)		*NEW
000014	EQUIVALENCE (A(1),ANH),(A(2),BNH),(A(3),CNHRE),(A(4),CNHIM),	HTNT	140***-1
000015	1(A(5),GAMMA),(A(6),XLRN),(A(7),XLON),(A(9),XNW),(A(10),OMEGA),	HTNT	150
000016	2(A(40),HR),(A(70),HI),(B(9),ONW),(B(10),OMGA),(B(51),HTRINT),	HTNT	160
000017	3(B(92),HTIINT), (OMEGA, DOM)	HTNT	170
000018		HTNT	180
000019	10 FORMAT (21H0 INPUT TO PROGRAM D // 10X8H GAMMA = F8.4,5X6HLR/N =	HTNT	190
000020	1 F12.8, 5X6HLT/N = F12.8)	HTNT	200
000021	20 FORMAT (16X3HANH12X3HBNH11X6HCNH RE 9X6HCNH IM // 9X 4F15.7 //)	HTNT	210
000022	30 FORMAT (// 38H INJECTOR DISTRIBUTION COEFFICIENTS... //	HTNT	220
000023	117X5HOMEGA 11X3HANH 12X3HBNH 11X6HCNH RE 9X6HCNH IM // (9X5F15.7))	HTNT	230
000024	40 FORMAT (1H0, 19X,8HOMEGA(C),9X3H HR,13X3H HI)	HTNT	240
000025	50 FORMAT (1H0,19X, 8HOMEGA(C),9X3HHTR 13X3HHTI)	HTNT	250
000026	60 FORMAT(1H ,10X,3F16.6)	HTNT	260
000027	70 FORMAT (1H0 ,20H PROGRAM D OUTPUT // 19X,8HOMEGA(C) 9X	HTNT	270
000028	16H HTR ,10X,6H HTI //)	HTNT	280
000029	80 FORMAT (91H0ALL VALUES OF HTR ARE NEGATIVE- (I.E. OUT OF RANGE OF	HTNT	290
000030	1INTEREST- WILL PROCEED TO NEXT CASE))	HTNT	300
000031	90 FORMAT(//,30X,69H FOLLOWING WILL BE INTERPOLATION WITHIN HTR HTI	HTNT	310
000032	1 TABLE GIVEN ABOVE)	HTNT	320
000033	100 FORMAT (19X,8H OMEGA ,9X6HHTRINT,10X,6HHTIINT)	HTNT	330
000034	110 FORMAT (11X,F10.5,10X,F10.5,10X,F10.5)	HTNT	340
000035		HTNT	350
000036	C LRR=0.0	HTNT	360
000037	C=0.0	HTNT	370
000038	CALL DVCHK (K000FX)	HTNT	380
000039	120 DO 130 I=1,133	HTNT	390
000040	A(I)= DIN(I)	HTNT	400
000041	130 B(I)=0.0	HTNT	410
000042	NER=IFIX(XNW)	HTNT	420
000043	IF(NER)140,140,160	HTNT	430
000044	140 WRITE (6,150)NER	HTNT	440
000045	150 FORMAT (1H0,10X,31H NUMBER OF OMEGAS IN ERROR = ,3X,I4)	HTNT	450
000046	GO TO 510	HTNT	460
000047	160 IF(NER-29)170,170,140	HTNT	470
000048	170 ONW = 40.0	HTNT	480
000049	CONTINUE		
000050	NWI = A(8) + 0.01	HTNT	500***-1
000051	IF (NWI .GT. 2) GO TO 180	HTNT	510
000052	ASSIGN 280 TO NT	HTNT	520
000053	TABLR = .FALSE.	HTNT	530
000054	ANH = AVN (1)	HTNT	540
000055	BNH = BVN (1)	HTNT	550

II,D, Program Listing (cont.)

000056	CNHRE = CVNR(1)	HTNT 560
000057	CNHIM = CVNI(1)	HTNT 570
000058	GO TO 190	HTNT 580
000059	180 ASSIGN 270 TO NT	HTNT 590
000060	TABLR = .TRUE.	HTNT 600
000061	190 IF (CD-99.0) 260,260,200.	HTNT 610
000062	200 CALL PAGE (60)	HTNT 620
000063	WRITE (6,10) GAMMA, XLRN, XLON	HTNT 630
000064	CONTINUE	
000065	IF (TABLR) GO TO 210	HTNT 650**NEW
000066	WRITE (6,20) ANH, BNH, CNHRE, CNHIM	HTNT 660
000067	GO TO 220	HTNT 670
000068	210 WRITE (6,30) (WIT(I), AVN(I), BVN(I), CVNR(I), CVNI(I),	HTNT 680
000069	1 I = 1, NWI)	HTNT 690
000070	220 WRITE (6,40)	HTNT 700
000071	GO TO 240	HTNT 710
000072	240 DO 250 I = 1,NER	HTNT 730**NEW
000073	250 WRITE (6,60) OMEGA(I), HR(I), HI(I)	HTNT 740
000074		HTNT 750
000075	C 260 IF ((.NOT. DRUN) .AND. (.NOT. ERUN)) GO TO 350	HTNT 770**NEW
000076	DO 340 I=1,NER	HTNT 780
000077	W = OMEGA(I)	HTNT 790
000078	GO TO NT,(270,280)	HTNT 800
000079	270 CALL INT4 (WIT, AVN, W, ANH)	HTNT 810
000080	CALL INT4 (WIT, BVN, W, BNH)	HTNT 820
000081	CALL INT4 (WIT, CVNR, W, CNHRE)	HTNT 830
000082	CALL INT4 (WIT, CVNI, W, CNHIM)	HTNT 840
000083	280 DEN = GAMMA*W	HTNT 850
000084	X = ANH - CNHIM/DEN*XLON	HTNT 860
000085	Y = BNH / DEN * XLRN + CNHRE * XLON / DEN	HTNT 870
000086	SQD = X * X + Y * Y	HTNT 880
000087	HTR(I) = (X * HR(I) + Y * HI(I)) / SQD	HTNT 890
000088	HTI(I) = (X * HI(I) - Y * HR(I)) / SQD	HTNT 900
000089	CALL DVCHK (K000FX)	HTNT 910
000090	GO TO (310,290),K000FX	HTNT 920
000091	290 IF (CD-10.0) 340,300,300	HTNT 930
000092	300 IF (I-1) 330,320,330	HTNT 940
000093	310 C=10.0	HTNT 950
000094	320 LIN=NER+6	HTNT 960
000095	CALL PAGE (LIN)	HTNT 970
000096	WRITE (6,70)	HTNT 980
000097	330 WRITE (6,60) DOM(I), HTR(I), HTI(I)	HTNT 990
000098	340 CONTINUE	HTNT 1000
000099	GO TO 370	HTNT 1010
000100		HTNT 1020
000101	C 350 DO 360 I = 1,NER	HTNT 1030
000102	HTI(I) = HI(I)	HTNT 1040
000103	360 HTR(I) = HR(I)	HTNT 1050
000104	370 JOMEG1 = 0	HTNT 1060
000105	JOMEG2=0	HTNT 1065
000106	IF (CD .LE. 99.0) CALL PAGE (60)	HTNT 1070
000107	DO 390 I =1,NER	HTNT 1080
000108	IF (HTR(I)) 390,390,380	HTNT 1090
000109	380 JOMEG1 = I	HTNT 1100
000110	GO TO 410	HTNT 1110
000111	390 CONTINUE	HTNT 1120
000112	IF (JOMEG1) 400,400,410	HTNT 1130
000113	400 WRITE (6,80)	HTNT 1140
000114	ERR= -1.0	

II, D, Program Listing (cont.)

000115	GO TO 520	HTNT1150
000116	410 DO 430 J = I, NER	HTNT1160
000117	IF (HTR(J)) 420, 430, 430	HTNT1170
000118	420 JOMEG2 = J	HTNT1180
000119	GO TO 440	HTNT1190
000120	430 CONTINUE	HTNT1200
000121	JOMEG2 = NER	HTNT1210
000122	440 DLTMEG = (DOM(JOMEG2) - DOM(JOMEG1)) / 39.0	HTNT1220
000123	L = NER + 1	HTNT1230
000124	HTI(L) = 0.0	HTNT1240
000125	HTR(L) = 0.0	HTNT1250
000126	DOM(L) = 0.0	HTNT1260
000127	OMGA(1) = DOM(JOMEG1)	HTNT1270
000128	DO 490 I = 1, 40	HTNT1280
000129	CALL INT4 (DOM(1), HTR(1), OMGA(I), HTRINT(I))	HTNT1290
000130	CALL INT4 (DOM(1), HTI(1), OMGA(I), HTIINT(I))	HTNT1300
000131	IF (CD-10, 0) 480, 450, 450	HTNT1310
000132	450 IF (I-1) 470, 460, 470	HTNT1320
000133	460 CALL PAGE (60)	
000134	WRITE (6, 90)	*NEW
000135	WRITE (6, 100)	HTNT1340***-1
000136	470 WRITE (6, 60) OMGA(I), HTRINT(I), HTIINT(I)	HTNT1350
000137	480 OMGA(I+1) = OMGA(I) + DLTMEG	HTNT1360
000138	490 CONTINUE	HTNT1370
000139	DO 500 I = 1, 133	HTNT1380
000140	500 DOUT (I) = B(I)	HTNT1390
000141	IF (C) 510, 520, 510	HTNT1400
000142	510 NER = 0	HTNT1410
000143	520 RETURN	HTNT1420
000144	END	HTNT1430

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II, D, Program Listing (cont.)

LL1 SUB14,1,700708, 29821

000001	SUBROUTINE FFF (FIN, FOUT, CF, NER, W)	NTAU 10
000002	C*****	NTAU 20
000003	DIMENSION FIN(1),FOUT(1),A(133),B(133),OMGA(1),XN(1),TAU(1),OMX(1)	NTAU 30
000004	1,HTRINT(1),HTIINT(1)	NTAU 40
000005	DIMENSION XNNEW(41),FCCPS(41),W (1)	NTAU 50
000006	COMMON /ABCD / EXTRA(100), ABLOK(600), A , B, XNNEW, FCCPS	NTAU 60
000007	EQUIVALENCE (A(9),ONW),(A(10),OMGA),(A(51),HTRINT),(A(92),HTIINT),	NTAU 70
000008	1(B(9),XNW),(B(10),OMX),(B(51),TAU),(B(92),XN)	NTAU 80
000009	C	NTAU 90
000010	C	NTAU 100
000011	CONST=FOUT(101)	NTAU 110
000012	XNMIN=1000.0	NTAU 120
000013	CALL PAGE(70)	NTAU 130
000014	CALL DVCHK (K000FX)	NTAU 140
000015	10 DO 20 I=1,133	NTAU 150
000016	A(I)=FIN(I)	NTAU 160
000017	20 B(I)=0.0	NTAU 170
000018	NER=IFIX(ONW)	NTAU 180
000019	XNW=ONW	NTAU 190
000020	IF(CF-99.0)70,70,30	NTAU 200
000021	30 CALL PAGE (44)	NTAU 210
000022	WRITE (6,40)	NTAU 220
000023	40 FORMAT (1H0,50H PROGRAM F INPUT SOLVE FOR N(W) AND TAU(W)	NTAU 230
000024	1//19X,8H(OMEGA)D,9X,6HHTRINT,10X,6HHTIINT)	NTAU 240
000025	DO 50 I =1,40	NTAU 250
000026	50 WRITE (6,60)OMGA(I),HTRINT(I), HTIINT(I)	NTAU 260
000027	60 FORMAT(1H ,10X,3F16.6)	NTAU 270
000028	70 CONTINUE	NTAU 280
000029	DO 120 I =1,40	NTAU 290
000030	XN(I)=(HTRINT(I)*HTRINT(I) + HTIINT(I)*HTIINT(I))/(2.0*HTRINT(I))	NTAU 300
000031	DNOM = XN(I) - HTRINT(I)	NTAU 310
000032	CALL QUAD (DNOM,HTIINT(I),TAU(I))	NTAU 320
000033	TAU(I)=(TAU(I)*A(1)*83.333333)/(A(2)*OMGA(I))	NTAU 330
000034	C A(1)=L,A(2)=C0	NTAU 340
000035	OMX(I)=OMGA(I)	NTAU 350
000036	CALL DVCHK (K000FX)	NTAU 360
000037	GO TO (80,90),K000FX	NTAU 370
000038	80 NER=0	NTAU 380
000039	GO TO 220	NTAU 390
000040	90 IF(XN(I))120,120,100	NTAU 400
000041	100 IF(XN(I)-XNMIN) 110,120,120	NTAU 410
000042	110 XNMIN=XN(I)	NTAU 420
000043	IMIN=I	NTAU 430
000044	120 CONTINUE	NTAU 440
000045	DO 130 I=1,100	NTAU 450
000046	130 FOUT(I) = B(I)	NTAU 460
000047	CONTINUE	
000048	140 CALL PAGE (45)	NTAU 480
000049	WRITE (6,150)	NTAU 490
000050	150 FORMAT(1H0,20H PROGRAM F OUTPUT //21X,7HFC(CPS),13X,8H(OMGA)D,	NTAU 500
000051	113X,7HTAU(MS),16X,1HN)	NTAU 510
000052	DO 160 I=1,40	NTAU 520
000053	FCCPS(I)=CONST*OMX(I)	NTAU 530
000054	160 WRITE (6,170)FCCPS(I),OMX(I),TAU(I),XN(I)	NTAU 540
000055	170 FORMAT(21X,F7.1 ,11X,F10.5,10X,F10.5,10X,F10.5)	NTAU 550

II, D, Program Listing (cont.)

000056	180 HTIINT(41) = 0.0	NTAU 560
000057	HTRINT(41)=0.0	NTAU 570
000058	XN(41)=0.0	NTAU 580
000059	OMX(41)=0.0	NTAU 590
000060	FCCPS(41)=0.0	NTAU 600
000061	190 FORMAT(36X,5HNMNIN=,F10.5,/36X,8HTAU(MS)=,F10.5,/36X,9H(OMEGA)D=,F10.5,/36X,8HFC(CPS)=,F10.1,)	NTAU 610
000062	DO 200 I =1,40	NTAU 620
000063	200 CALL INT4D(OMX(1),XN(1),OMX(1),SAVNOT,XNNEW(1))	NTAU 630
000064	XNNEW(41)=0.0	NTAU 640
000065	CALL INT4(XNNEW(1),OMX(1),0.0,DOMMIN)	NTAU 650
000066	FCMIN=CONST*DOMMIN	NTAU 660
000067	CALL INT4(FCCPS(1),HTRINT(1),FCMIN,HTR1)	NTAU 670
000068	CALL INT4(FCCPS(1),HTIINT(1),FCMIN,HTI1)	NTAU 680
000069	HTRMIN=(HTR1*HTR1+HTI1*HTI1)/(2.0*HTR1)	NTAU 690
000070	DNOM=HTRMIN-HTR1	NTAU 700
000071	CALL QUAD(DNOM,HTI1,TAUMIN)	NTAU 710
000072	TAUMIN=(TAUMIN+A(1)*83.333333)/(A(2)*DOMMIN)	NTAU 720
000073	CALL PAGE (8)	NTAU 730
000074	WRITE (6,210)	NTAU 740
000075	210 FORMAT(//21X,58H THE FOLLOWING ARE VALUES INTERPOLATED AT SLOPE OF	NTAU 750
000076	1 N=0.0 ,/)	NTAU 760
000077	WRITE (6,190)HTRMIN,TAUMIN,DOMMIN,FCMIN	NTAU 770
000078	220 RETURN	NTAU 780
000079	END	NTAU 790
000080		NTAU 800

II, D, Program Listing (cont.)

ELF 50,15,1,690702, 39149

000001	SUBROUTINE QUAD (A,B,ANGLE)	QUAD 10
000002	IF(B) 10,50,80	QUAD 20
000003	10 IF(A) 20,30,40	QUAD 30
000004	20 ROTATE = 3.1415927	QUAD 40
000005	GO TO 110	QUAD 50
000006	30 ANGLE = 4.7123890	QUAD 60
000007	GO TO 120	QUAD 70
000008	40 ROTATE = 6.2831853	QUAD 80
000009	GO TO 110	QUAD 90
000010	50 IF (A) 60,70,70	QUAD 100
000011	60 ANGLE = 3.1415927	QUAD 110
000012	GO TO 120	QUAD 120
000013	70 ANGLE = 0.0	QUAD 130
000014	GO TO 120	QUAD 140
000015	80 IF(A) 20,90,100	QUAD 150
000016	90 ANGLE = 1.5707963	QUAD 160
000017	GO TO 120	QUAD 170
000018	100 ROTATE = 0.0	QUAD 180
000019	110 ANGLE = ATAN(B/A) + ROTATE	QUAD 190
000020	120 RETURN	QUAD 200
000021	END	QUAD 210

II, D, Program Listing (cont.)

W ELT SUB16,1,690702, 39150

000001	SUBROUTINE CORE(X,N,CODE)	KORE	10,
000002	DIMENSION X(1)	KORE	20,
000003	IF(CODE-500.0)40,10,10	KORE	30
000004	10 CODE=100.0	KORE	40
000005	CALL PAGE(70)	KORE	50
000006	WRITE (6,20)	KORE	60
000007	20 FORMAT(10X,37HINPUT DATA DUMP FOR PROGRAM FAILER //	KORE	70
000008	WRITE (6,30)(X(I),I=1,N)	KORE	80
000009	30 FORMAT(5X,10(F10.4,2X))	KORE	90
000010	40 RETURN	KORE	100
000011	END	KORE	110

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II, D, Program Listing (cont.)

ELT SUB17,1,690702, 39151

000001		SUBROUTINE INJCTR	INJ	10
000002	C		INJ	20
000003		COMMON /PROLOG/ LOGIK(38), HEAD(12), SL1, SL2, EORJ	INJ	30
000004		COMMON /JECTOR/ EJDATA(9600)	INJ	40
000005	C		INJ	50
000006		EQUIVALENCE (LOGIK(5),ERUN) , (LOGIK(10),JRUN) , (LOGIK(9),IRUN)	INJ	60
000007		LOGICAL LOGIK, ERUN, JRUN, SL1, SL2, EORJ, IRUN	INJ	70
000008		DIMENSION HEAD1(12)	INJ	80
000009	C		INJ	90
000010		IF (SL1) GO TO 20	INJ	100
000011		DO 10 I = 1, 9600	INJ	110
000012	10	EJDATA(I) = 0.0	INJ	120
000013		SL1 = .TRUE.	INJ	130
000014		GO TO 30	INJ	140
000015	C		INJ	150
000016	20	READ (13) EJDATA	INJ	160
000017		BACKSPACE 13	INJ	170
000018	30	CALL AS138 (EJDATA, HEAD1, NE)	INJ	180
000019		IF (NE .NE. 1) CALL EXIT	INJ	190
000020		WRITE (13) EJDATA	INJ	195
000021		BACKSPACE 13	INJ	196
000022		IF (.NOT. JRUN) GO TO 40	INJ	200
000023		CALL JJJ	INJ	210
000024	40	IF (ERUN .OR. IRUN) CALL INJDIS	INJ	220
000025	C		INJ	230
000026	50	RETURN	INJ	240
000027		END	INJ	250

II, D, Program Listing (cont.)

W ELT SUB18,1,690708, 48741

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000001      SUBROUTINE JJJ                                JECT 10
000002      C                                           JECT 20
000003      C ***** DECK MODIFIED 20 AUG 67 ***** JECT 30
000004      C                                           JECT 40
000005      DIMENSION THET1(1450), R1(1450), TMU(1000)    JECT 50
000006      DIMENSION NELE(1000), X(1000), Y(1000), NTYPE(1000), R(22) JECT 60
000007      1, THETA(182), AS(20), WTE(1000), AAX(1000), AAF(1000), NBAND(1000), XMRE( JECT 70
000008      21000), XMUTOT(20), AXTY(200), AFTY(200), XX(1000), YY(1000) JECT 80
000009      COMMON /JECTOR/ DATA ( 9600)                 JECT 90
000010      C                                           JECT 130
000011      EQUIVALENCE (DATA(3), XM), (DATA(4), XN)       JECT 140
000012      EQUIVALENCE (DATA(1021), THET1), (DATA(2471), R1), (DATA(3921), TMU) JECT 150
000013      EQUIVALENCE (NTYPE, DATA(2001)), (AXTY, DATA(1001)), (AFTY, DATA(1201) JECT 160
000014      1), (DATA(1921), XX), (DATA(2921), YY), (DATA(4921), XMUTOT) JECT 170
000015      C                                           JECT 180
000016      NERROR=0                                       JECT 200
000017      SECT=DATA(5)                                    JECT 210
000018      WT=DATA(9573)                                   JECT 220
000019      RINJ=DATA(9574)                                  JECT 230
000020      XMR=DATA(9575)                                   JECT 240
000021      FFC=DATA(9576)                                   JECT 250
000022      DFFC=DATA(9577)                                 JECT 260
000023      NT=DATA(9570)+.0001                             JECT 270
000024      POLAR=DATA(9572)                                 JECT 280
000025      NE=DATA(9571)+.0001                             JECT 290
000026      ROX=DATA(9579)                                   JECT 300
000027      ROF=DATA(9580)                                   JECT 310
000028      EMUMAX=5.0                                       JECT 320
000029      PXADJ=1.0                                         JECT 330
000030      PFADJ=1.0                                         JECT 340
000031      CDX=DATA(9585)                                   JECT 350
000032      CDF=DATA(9586)                                   JECT 360
000033      XFC=DATA(9591)                                   JECT 370
000034      DXFC=DATA(9592)                                  JECT 380
000035      PFFC = DATA(9589)/100.0                         JECT 390
000036      PXFC = DATA(9590)/100.0                         JECT 400
000037      IF (XM .EQ. 0.0 ) XM = 20.0                      JECT 410
000038      IF ( XN .EQ. 0.0 ) XN = 180.0                   JECT 420
000039      10 K=319                                           JECT 430
000040      THFNL=6.2831853/SECT                               JECT 440
000041      IF (DATA(322)) 20, 30, 30                        JECT 450
000042      20 ROTATE=3.1415926/SECT                          JECT 460
000043      GO TO 40                                           JECT 470
000044      30 ROTATE=0.0                                       JECT 480
000045      40 IF (POLAR) 50, 180, 50                         JECT 490
000046      50 IF (SECT-1.0) 80, 80, 60                      JECT 500
000047      60 DO 70 I=1, NE                                   JECT 510
000048      KK=K+1                                             JECT 520
000049      NELE(I)=DATA(KK)+.0001                             JECT 530
000050      X(I)=DATA(KK+1)                                     JECT 540
000051      Y(I)=DATA(KK+2)                                     JECT 550
000052      NTYPE(I)=DATA(KK+3)+.0001                         JECT 560
000053      SAVE=SQRT(X(I)*X(I)+Y(I)*Y(I))                   JECT 570
000054      Y(I)=(ATAN(Y(I)/X(I)))+ROTATE                      JECT 580
000055      X(I)=SAVE                                           JECT 590

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II, D, Program Listing (cont.)

000056	K=K+4	JECT 600
000057	70 CONTINUE	JECT 610
000058	GO TO 220	JECT 620
000059	80 DO 170 I=1,NE	JECT 630
000060	KK=K+1	JECT 640
000061	NELE(I)=DATA(KK)+.0001	JECT 650
000062	X(I)=DATA(KK+1)	JECT 660
000063	Y(I)=DATA(KK+2)	JECT 670
000064	NTYPEE(I)=DATA(KK+3)+.0001	JECT 680
000065	C THE FOLLOWING SEGMENT OF CODING REPLACES SUBROUTINE MOVXY...	JECT 690
000066	IF (Y(I)) 90,130,140	JECT 700
000067	90 IF (X(I)) 100,110,120	JECT 710
000068	100 ROTATE = 3.1415927	JECT 720
000069	GO TO 160	JECT 730
000070	110 X(I) = 1.0E-15	JECT 740
000071	120 ROTATE = 6.2831853	JECT 750
000072	GO TO 160	JECT 760
000073	130 ROTATE = 0.0	JECT 770
000074	GO TO 160	JECT 780
000075	140 IF (X(I)) 100,150,130	JECT 790
000076	150 X(I) = 1.0E-15	JECT 800
000077	ROTATE = 0.0	JECT 810
000078	160 SAVE = SQRT (X(I)*X(I)+Y(I)*Y(I))	JECT 820
000079	SAVE=SQRT(X(I)*X(I)+Y(I)*Y(I))	JECT 830
000080	Y(I)=(ATAN(Y(I)/X(I)))+ROTATE	JECT 840
000081	X(I)=SAVE	JECT 850
000082	K=K+4	JECT 860
000083	170 CONTINUE	JECT 870
000084	GO TO 220	JECT 880
000085	180 IF (SECT-1.0)190,190,200	JECT 890
000086	190 ROTATE=0.0	JECT 900
000087	200 DO 210 I=1,NE	JECT 910
000088	KK=K+1	JECT 920
000089	NELE(I)=DATA(KK)+.0001	JECT 930
000090	X(I)=DATA(KK+1)	JECT 940
000091	Y(I)=(DATA(KK+2)*.0174532)+ROTATE	JECT 950
000092	NTYPEE(I)=DATA(KK+3)+.0001	JECT 960
000093	K=K+4	JECT 970
000094	210 CONTINUE	JECT 980
000095	CC	JECT 990
000096	C CALCULATING AREAS OF ELEMENTS	JECT1000
000097	CC	JECT1010
000098	220 NN=4944	JECT1020
000099	DO 280 I=1,NT	JECT1030
000100	KK=NN+1	JECT1040
000101	NST=DATA(KK)+.0001	JECT1050
000102	KK1=KK+1	JECT1060
000103	JJ=KK1	JECT1070
000104	NX=DATA(KK1)+.0001	JECT1080
000105	AXTY(NST)=0.0	JECT1090
000106	IF (NX)230,250,230	JECT1100
000107	230 DO 240 JX=1,NX	JECT1110
000108	JJ=KK1+JX	JECT1120
000109	AXTY(NST)=AXTY(NST)+(.7853891*(DATA(JJ)*DATA(JJ)))	JECT1130
000110	240 CONTINUE	JECT1140
000111	250 NN1=JJ+1	JECT1150
000112	NN=NN1	JECT1160
000113	AF=DATA(NN1)+.0001	JECT1170
000114	AFTY(NST)=0.0	JECT1180

II, D, Program Listing (cont.)

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000115      IF(NF)260,280,260                                JECT1190
000116      260 DO 270 JF=1,NF                                JECT1200
000117          NN=NN1+JF                                        JECT1210
000118          AFTY(NST)=AFTY(NST)+(.7853981*(DATA(NN)*DATA(NN))) JECT1220
000119      270 CONTINUE                                          JECT1230
000120      280 CONTINUE                                          JECT1240
000121          AXTOT=0.0                                        JECT1250
000122          AFTOT=0.0                                        JECT1260
000123          DO 290 I=1,NE                                    JECT1270
000124              NN=NTYPEE(I)                                  JECT1280
000125              AAX(I)=AXTY(NN)                               JECT1290
000126              AAF(I)=AFTY(NN)                               JECT1300
000127              AXTOT=AXTOT+AAX(I)                            JECT1310
000128              AFTOT=AFTOT+AAF(I)                            JECT1320
000129      290 CONTINUE                                          JECT1330
000130          AXTOT=AXTOT*SECT                                    JECT1340
000131          AFTOT=AFTOT*SECT                                    JECT1350
000132      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC JECT1360
000133      C      CALCULATING RADII AND ANGLE BOUNDRIES          JECT1370
000134      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC JECT1380
000135      300 NPTS=180.0/SECT                                    JECT1390
000136          TPS=NPTS                                           JECT1400
000137          DELTH=THFN/TPS                                       JECT1410
000138          AREA = 3.141592 * RINJ**2                          JECT1420
000139          ASECT = AREA/(XM*XN)                                JECT1430
000140          R(1) = 0.0                                           JECT1440
000141          ANP = ASECT * 180.0 / 3.141592                    JECT1450
000142          DO 310 I = 2,21                                     JECT1460
000143              R(I) = SQRT ( ANP + R(I-1)**2)                 JECT1470
000144      310 CONTINUE                                          JECT1480
000145          R(22) = 0.0                                         JECT1490
000146          XNUM=0.0                                           JECT1500
000147          NPTS1=NPTS+1                                         JECT1510
000148          DO 320 J=2,NPTS1                                    JECT1520
000149              XNUM=XNUM+1.0                                    JECT1530
000150              THETA(J)=DELTH*XNUM                             JECT1540
000151      320 CONTINUE                                          JECT1550
000152      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC JECT1560
000153      C      CALCULATING WEIGHT FLOW                          JECT1570
000154      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC JECT1580
000155          WFT1=WT/(XMR+1.0)                                    JECT1590
000156          WXT1=WT-WFT1                                        JECT1600
000157          WFT2=WFT1                                           JECT1610
000158          WXT2=WXT1                                           JECT1620
000159          IF(PFFC)340,340,330                                  JECT1630
000160      330 WFT2=WFT1*(1.0-PFFC)                                JECT1640
000161      340 IF(PXFC)360,360,350                                  JECT1650
000162      350 WXT2=WXT1*(1.0-PXFC)                                JECT1660
000163      360 SIGMEN=WT/(3.141592*RINJ*RINJ)                    JECT1670
000164      370 AXIMAX=AXTOT                                         JECT1680
000165          AXIMIN=AXTOT                                         JECT1690
000166          AFIMAX=AFTOT                                         JECT1700
000167          AFIMIN=AFTOT                                         JECT1710
000168      380 AFFC=0.0                                           JECT1720
000169          AXFC=0.0                                           JECT1730
000170          IF(DFFC)400,400,390                                  JECT1740
000171      390 AFFC=FFC*.785398*DFFC*DFFC                        JECT1750
000172          AFIMIN=AFIMIN-AFFC                                  JECT1760
000173          AFIMAX=AFIMAX-AFFC                                  JECT1770

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II. D, Program Listing (cont.)

000174	400 IF (DXFC) 420, 420, 410	JECT1780
000175	410 AXFC=XFC*.785398*DXFC*DXFC	JECT1790
000176	AXTMIN=AXTMIN-AXFC	JECT1800
000177	AXTMAX=AXTMAX-AXFC	JECT1810
000178	420 AXNOM=(AXTMAX+AXTMIN)/2.0	JECT1820
000179	AFNOM=(AFTMAX+AFTMIN)/2.0	JECT1830
000180	IF (DFFC+DXFC) 440, 440, 430	JECT1840
000181	430 WXT2=(WXT2*AXNOM)/(AXNOM+AXFC)	JECT1850
000182	WFT2=(WFT2*AFNOM)/(AFNOM+AFFC)	JECT1860
000183	440 XMR1=WXT2/WFT2	JECT1870
000184	450 ETOF=0.0	JECT1880
000185	ETFF=0.0	JECT1890
000186	QX=WXT2/AXTOT	JECT1900
000187	OF=WFT2/AFTOT	JECT1910
000188	CALL DVCHK (K000FX)	JECT1920
000189	460 DO 480 I=1, NE	JECT1940
000190	NBAND(I)=0	JECT1950
000191	WFE1=AAF(I)*OF	JECT1960
000192	WXE1=AAX(I)*QX	JECT1970
000193	WTE(I)=WFE1+WXE1	JECT1980
000194	XMRE(I)=WXE1/WFE1	JECT1990
000195	ETOF=ETOF+WXE1	JECT2000
000196	ETFF=ETFF+WFE1	JECT2010
000197	CALL DVCHK (K000FX)	JECT2020
000198	GO TO (470, 480), KU00FX	JECT2030
000199	470 NBAND(I)=-1	JECT2040
000200	480 CONTINUE	JECT2050
000201	ETOF=SECT*ETOF	JECT2060
000202	ETFF=SECT*ETFF	JECT2070
000203	WMRELM=ETOF/ETFF	JECT2080
000204	AXXTT=AXTOT+AXFC	JECT2090
000205	AFFTT=AFTOT+AFFC	JECT2100
000206	WFT2 = ETFF	JECT2110
000207	WXT2 = ETOF	JECT2120
000208	WFT1=WFT2/(1.0-PFFC)	JECT2130
000209	WXT1=WXT2/(1.0-PXFC)	JECT2140
000210	WFT1 = WFT1*AFFTT/AFTOT	JECT2150
000211	WXT1 = WXT1*AXXTT/AXTOT	JECT2160
000212	XMRINJ=WXT1/WFT1	JECT2170
000213	D= ((WXT1/(CDX*AXXTT))*2)*2.2360248	JECT2180
000214	WTOT=WXT1+WFT1	JECT2190
000215	DPX1=D/ROX	JECT2200
000216	D= ((WFT1/(CDF*AFFTT))*2)*2.2360248	JECT2210
000217	DPF1=D/ROF	JECT2220
000218	TOFCF = WXT1*(1.0 - PXFC)	JECT2230
000219	TOFCF = (TOFCF*(AXXTT - AXFC))/AXXTT	JECT2240
000220	TOFCF = WXT1 - TOFCF	JECT2250
000221	TFFC = WFT1*(1.0 - PFFC)	JECT2260
000222	TFFC = (TFFC*(AFFTT - AFFC))/AFFTT	JECT2270
000223	TFFC = WFT1 - TFFC	JECT2280
000224	VINJX=SQRT((9273.6*DPX1 /ROX))	JECT2290
000225	VINJF=SQRT((9273.6*DPF1 /ROF))	JECT2300
000226	XXY=TPS*SIGMEN*SECT	JECT2310
000227	490 WRITE (6, 820)	JECT2320
000228	GO TO 500	JECT2330
000229	500 WRITE (6, 760) AXTOT, AFTOT, AXFC, AFFC, AXXTT, AFFTT, DPX1, DPF1	JECT2340
000230	WRITE (6, 770) WTOT, WMRELM, XMRINJ, ETOF, ETFF, TOFCF, TFFC, WXT1, WFT1	JECT2350
000231	1 VINJX, VINJF	JECT2360
000232	PFFC=PFFC*100.0	JECT2370

II, D, Program Listing (cont.)

000233	PXFC=PXFC+100.0	JECT2380
000234	510 WRITE (6,830)WT,NT,NE,CDX	JECT2390
000235	WRITE (6,780)CDF,PFPC,PXFC,DFFC,DXFC,FFC,XFC,ROX,ROF	JECT2400
000236	CALL PAGE(70)	JECT2410
000237	WRITE (6,810)	JECT2420
000238	NPGE=49	JECT2430
000239	DO 540 I=1,NE	JECT2440
000240	IF (NPGE-I)520,520,530	JECT2450
000241	520 CALL PAGE(70)	JECT2460
000242	NPGE=NPGE+48	JECT2470
000243	WRITE (6,810)	JECT2480
000244	530 DEGREE=Y(I)/.0174532	JECT2490
000245	B=X(I)*SIN(Y(I))	JECT2500
000246	A=X(I)*COS(Y(I))	JECT2510
000247	WRITE (6,800)NELE(I),NTYPEE(I),X(I),DEGREE,A,B	JECT2520
000248	540 CONTINUE	JECT2530
000249	CALL PAGE(70)	JECT2540
000250	NPGE=46	JECT2550
000251	JCONT=0	JECT2560
000252	WRITE (6,840)	JECT2570
000253	NN=4944	JECT2580
000254	DO 670 I=1,NT	JECT2590
000255	KK=NN+1	JECT2600
000256	NST=DATA(KK)+.0001	JECT2610
000257	KK1=KK+1	JECT2620
000258	JJ=KK1	JECT2630
000259	NX=DATA(KK1)+.0001	JECT2640
000260	JCONT=JCONT+NX+1	JECT2650
000261	IF (NPGE-JCONT)550,550,560	JECT2660
000262	550 CALL PAGE(70)	JECT2670
000263	WRITE (6,840)	JECT2680
000264	JCONT=0	JECT2690
000265	560 AXMAX=0.0	JECT2700
000266	AFMAX=0.0	JECT2710
000267	WFE1=0.0	JECT2720
000268	WXE1=0.0	JECT2730
000269	WRITE (6,850)NST,NX	JECT2740
000270	IF (NX)570,590,570	JECT2750
000271	570 DO 580 JX=1,NX	JECT2760
000272	JJ=KK1+JX	JECT2770
000273	A=.7853891*DATA(JJ)*DATA(JJ)	JECT2780
000274	B=A*QX	JECT2790
000275	WRITE (6,860)DATA(JJ),A,B	JECT2800
000276	AXMAX=AXMAX+A	JECT2810
000277	WXE1=WXE1+B	JECT2820
000278	580 CONTINUE	JECT2830
000279	590 NN1=JJ+1	JECT2840
000280	NN=NN1	JECT2850
000281	NF=DATA(NN1)+.0001	JECT2860
000282	JCONT=JCONT+NF+2	JECT2870
000283	IF (NPGE-JCONT)600,600,610	JECT2880
000284	600 CALL PAGE(70)	JECT2890
000285	WRITE (6,840)	JECT2900
000286	JCONT=0	JECT2910
000287	610 WRITE (6,870)NF	JECT2920
000288	IF (NF)620,640,620	JECT2930
000289	620 DO 630 JF=1,NF	JECT2940
000290	NN=NN1+JF	JECT2950
000291	A=.7853891*DATA(NN)*DATA(NN)	JECT2960

II, D, Program Listing (cont.)

000292	B=A*QF	JECT2970
000293	WRITE (6,880) DATA(NN),A,B	JECT2980
000294	AFMAX=AFMAX+A	JECT2990
000295	WFE1=WFE1+B	JECT3000
000296	630 CONTINUE	JECT3010
000297	640 WTOT1=WFE1+WFE1	JECT3020
000298	IF(WFE1)660,650,660	JECT3030
000299	650 WRITE (6,900) AXMAX,AFMAX,WTOT1	JECT3040
000300	GO TO 670	JECT3050
000301	660 XMR1=WFE1/WFE1	JECT3060
000302	WRITE (6,890) AXMAX,AFMAX,WTOT1,XMR1	JECT3070
000303	670 CONTINUE	JECT3080
000304	680 READ (13) DATA	JECT3090
000305	BACKSPACE 13	JECT3100
000306	LINKNT = 0	JECT3110
000307	AJS = XM*XN/WT	JECT3120
000308	DO 700 J = 1,NE	JECT3130
000309	TMU(J) = WFE(J)*AJS	JECT3140
000310	IF (LINKNT .GT. 0) GO TO 690	JECT3150
000311	CALL PAGE(70)	JECT3160
000312	WRITE (6,910)	JECT3170
000313	WRITE (6,930)	JECT3180
000314	LINKNT = 50	JECT3190
000315	690 WRITE(6,920) J, X(J), Y(J), TMU(J)	JECT3200
000316	LINKNT = LINKNT - 1	JECT3210
000317	700 CONTINUE	JECT3220
000318	DO 705 I = 1, NE	JECT3222
000319	XX(I) = X(I)	JECT3224
000320	705 YY(I) = Y(I)	JECT3225
000321	WRITE (13) DATA	JECT3230
000322	BACKSPACE 13	JECT3240
000323	XMUMAX=0.0	JECT3250
000324	AJS = AJS*SECT/XN	JECT3260
000325	DO 730 J=1,20	JECT3270
000326	TOTW=0.0	JECT3280
000327	XXX = XXY * ASECT	JECT3290
000328	DO 710 I=1,NE	JECT3310***-1
000329	IF (X(I) .LE. R(J+1) .AND. X(I) .GT. R(J)) TOTW = TOTW + WTE(I)	JECT3320
000330	710 CONTINUE	JECT3330
000331	XMUTOT(J) = TOTW*AJS	JECT3340
000332	IF(XMUTOT(J)-XMUMAX)730,730,720	JECT3350
000333	720 XMUMAX=XMUTOT(J)	JECT3360
000334	730 CONTINUE	JECT3370
000335	CALL PAGE(70)	JECT3380
000336	WRITE (6,790)	JECT3390
000337	DO 740 J=1,20	JECT3400
000338	XXMAX=XMUTOT(J)/XMUMAX	JECT3410
000339	WRITE (6,750) R(J+1),XMUTOT(J),XXMAX	JECT3420
000340	740 CONTINUE	JECT3430
000341	RETURN	JECT3440
000342	750 FORMAT(32X,F6.3,14X,F7.3,13X,F6.4)	JECT3450
000343	760 FORMAT(//,5X,28HA....PROPELLANT ORFICE AREAS,//,9X,34HELEMENT TOTAL	JECT3460
000344	1L OXIDIZER AREA =,F11.8,8H SQ. IN.,15X,30HELEMENT TOTAL FUEL	JECT3470
000345	2AREA =,F11.8,8H SQ. IN.,//,9X,34HTOTAL OXIDIZER FILM COOLING AREA	JECT3480
000346	3REA =,F11.8,8H SQ. IN.,15X,30HTOTAL FUEL FILM COOLING AREA =,F11.8	JECT3490
000347	4,8H SQ. IN.,//,9X,34HINJECTOR TOTAL OXIDIZER AREA =,F11.8,8H SQ	JECT3500
000348	5. IN.,15X,30HINJECTOR TOTAL FUEL AREA =,F11.8,8H SQ. IN.,//,5X	JECT3510
000349	6,54H8....INJECTOR PRESSURE DROPS FOR ABOVE INJECTOR DESIGN,//,9X,	JECT3520
000350	724HOXIDIZER PRESSURE DROP =,F6.1,3HPSI,35X,20HFUEL PRESSURE DROP =	JECT3530

II, D, Program Listing (cont.)

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000351      8,F6.1,4H PSI,///,5X,71HC....PROPELLANT FLOWS AND INJECTOR VELOCITIES JECT3540
000352      9ES FOR ABOVE INJECTOR DESIGN,/) JECT3550
000353 770 FORMAT(/,48X,19HTOTAL WEIGHT FLOW =,F6.1,7H LB/SEC,/,42X,39HAVERAG JECT3560
000354 1E MIXTURE RATIO OF THE ELEMENTS =,F7.3,/,41X,40HOVERALL MIXTURE RATIO JECT3570
000355 2TIO FOR THE INJECTOR =,F7.3,/,9X,34HELEMENT TOTAL OXIDIZER FLOW JECT3580
000356 3 =,F6.1,7H LB/SEC,21X,26HELEMENT TOTAL FUEL FLOW =,F6.1,7H LB/JECT3590
000357 4SEC,/,9X,34HTOTAL OXIDIZER FILM COOLING FLOW =,F6.1,7H LB/SEC,21X,JECT3600
000358 526HTOTAL FUEL FILM COOLING =,F6.1,7H LB/SEC,/,9X,34HINJECTOR TOTAL FLOW JECT3610
000359 6L OXIDIZER FLOW =,F6.1,7H LB/SEC,21X,25HINJECTOR TOTAL FUEL FLOW JECT3620
000360 7OW=,F7.1,7H LB/SEC,/,9X,37HOXIDIZER OVERALL INJECTION VELOCITY =,FJECT3630
000361 86.1,7H FT/SEC,18X,33HFUEL OVERALL INJECTION VELOCITY =,F6.1,7H FT/JECT3640
000362 9SEC,/) JECT3650
000363 780 FORMAT(9X,23HFUEL LOSS COEFFICIENT =,F5.3,/,9X,27HPERCENT FUEL FIL JECT3660
000364 14 COOLING =,F5.1,/,9X,31HPERCENT OXIDIZER FILM COOLING =,F5.1,/,9X JECT3670
000365 2,38HDIAMETER OF FUEL FILM COOLING ORFICE =,F7.5,75H IN. (NOTE..THIJECT3680
000366 3S MIGHT BE AN EQUIVALENT DIAMETER FOR MULTIPLE-ROW COOLING),/,9X,JECT3690
000367 442HDIAMETER OF OXIDIZER FILM COOLING ORFICE =,F7.5,21H IN. (SEE ABJECT3700
000368 5OVE NOTE),/,9X,51HNUMBER OF FUEL FILM COOLING ORIFICES PER INJECTOJECT3710
000369 6R =,F5.0,/,9X,54HNUMBER OF OXIDIZER FILM COOLING ORIFICES PER INJECTJECT3720
000370 7TOK =,F5.0,/,9X,18HOXIDIZER DENSITY =,F7.2,4H PCF,/,9X,14HFUEL DENJECT3730
000371 8SITY =,F6.1,4H PCF,/) JECT3740
000372 790 FORMAT(///,3X,114HSECTION 4..RESULTANT FLOW DISTRIBUTION (MU) AS JECT3750
000373 1A FUNCTION OF ONLY THE RADIUS, I.E.. AVERAGE OF MU IN A RADIAL BANJECT3760
000374 2D, ///,32X,6HRADIUS,15X,5H:J(R),11X,13HMU(R)/MU(MAX),/,34X,3HINJECT3770
000375 3,/) JECT3780
000376 800 FORMAT(13X,13,18X,12,4X,4(10X,F10.5)) JECT3790
000377 810 FORMAT(/,2X, 49HSECTION 2.... ELEMENT LOCATION AND INJECTION TYPEJECT3800
000378 1,///,11X, 7HELEMENT,15X, 4HTYPE,17X,1HR,17X, 5HTHETA,17X,1HX,19X,1JECT3810
000379 2HY,/,13X, 3HNO.,18X, 3HNO.,14X, 8H(INCHES),12X, 8H(DEGREE),12X, 8HJECT3820
000380 3(INCHES),12X,8H(INCHES),/) JECT3830
000381 820 FORMAT(///,3X, 74HSECTION 1....MISCELLANEOUS INFORMATION FOR INJECT3840
000382 1TOR DESIGNED BY PROJECTS ,/) JECT3850
000383 830 FORMAT(/,5X, 46HD.....INPUT INFORMATION USED IN COMPUTATIONS ,/JECT3860
000384 1/,9X, 23HTOTAL PROPELLANT FLOW =,F6.1,7H LB/SEC,/,9X, 59HTOTAL NUMJECT3870
000385 2BER OF ELEMENT TYPES (SYMMETRICAL SECTION ONLY) = ,13,/,9X, 54HTOTJECT3880
000386 3AL NUMBER OF ELEMENTS (SYMMETRICAL SECTION ONLY) = ,14,/,9X, 27HOXJECT3890
000387 4IDIZER LOSS COEFFICIENT =,F5.3,) JECT3900
000388 840 FORMAT(/,2X, 78HSECTION 3....TYPE DESCRIPTION,ORIFICE AREA, PROPEJECT3910
000389 1LLANT FLOW, AND MIXTURE RATIO,/,3X, 84HTYPE *----- OXIDIZER JECT3920
000390 2ORIFICE DATA -----*----- FUEL ORIFICE DATA -----*,/,93X,JECT3930
000391 3 5HTOTAL,5X, 5HTOTAL,5X, 5HTOTAL,4X, 7HMIXTURE,/,12X, 6HNUMBER,34XJECT3940
000392 4, 6HNUMBER,33X, 8HOXIDIZER,4X, 4HFUEL,3X,10HPROPELLANT,3X, 5HRATIOJECT3950
000393 5,/,14X,2HOF,5X, 8HDIAMETER,4X, 4HAREA,6X, 4HFLOW,7X,2HOF,5X, 8HDIAJECT3960
000394 6METER,4X, 4HAREA,6X, 4HFLOW,6X, 4HAREA,6X, 4HAREA,4X, 9HFLOW RATE,JECT3970
000395 7/,11X, 7HORIFICE,6X,3HIN.,5X, 7HSQ. IN.,4X, 6HLB/SEC,3X, 8HORIFICEJECT3980
000396 85,5X, 3HIN.,4X, 7HSQ. IN.,4X, 6HLB/SEC,3X, 7HSQ. IN.,3X, 7HSQ. IN.JECT3990
000397 9,4X, 6HLB/SEC,/) JECT4000
000398 850 FORMAT(3X,13,7X,12) JECT4010
000399 860 FORMAT(22X,F6.4,3X,F8.6,2X,F8.6) JECT4020
000400 870 FORMAT(54X,12) JECT4030
000401 880 FORMAT(62X,F6.4,3X,F8.6,2X,F8.6) JECT4040
000402 890 FORMAT(91X,F8.5,2X,F8.5,2X,F8.4,3X,F7.4) JECT4050
000403 900 FORMAT(91X,F8.5,2X,F8.5,2X,F8.3,2X,8HINFINITY) JECT4060
000404 910 FORMAT (///10X15HELEMENT RESULTS ) JECT4070
000405 920 FORMAT (32X15,11XF10.3,10XF10.4,11XF10.4 ) JECT4080
000406 930 FORMAT (/31X7HELEMENT14X6HRADIUS15X5HANGLE11X12HDISTRIBUTION/90X JECT4090
000407 111HCOEFFICIENT/34X3HNO.35X7HRADIANS15X2HMU// ) JECT4100
000408 END JECT4110

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II, D, Program Listing (cont.)

ELI SUB19,1,691029, 58811

000001		SUBROUTINE INJDIS	INJD 10
000002	C		INJD 20
000003	C	***** DECK MODIFIED 20 AUG 67 *****	INJD 30
000004	C		INJD 40
000005		REAL IPP,IPR,IPT,INTEG,IPX	INJD 50
000006		INTEGER DSCRIB, TIME	INJD 60
000007		LOGICAL LOGIK, SL1, SL2, EORJ, IRUN	INJD 70
000008		COMMON /PROLOG/ LOGIK(50), SL1, SL2, EORJ	INJD 80
000009		COMMON /JECTOR/ DATA (9600)	INJD 90
000010		COMMON GAMMA, NW, WC, AVN, BVN, CVNR, CVNI, CE, CI	INJD 110
000011		DIMENSION	INJD 120
000012	1	U (1000), AVN (30), BVN (30), CVNR (30),	INJD 130
000013	2	CVNI (30), RR (1000), THATA (1000), FIRST (30),	INJD 140
000014	3	SECOND(30), Z (2), WC (30),	INJD 150
000015	4	IPP(50), OPP(50), IPR(50), OPR(50), IPT(50), OPT(50),	INJD 160
000016	5	X (1000), Y (1000)	INJD 170
000017		EQUIVALENCE (IRUN, LOGIK(9)),	INJD 180
000018	1	(E1 ,DATA(1)), (XM ,DATA(3)),	INJD 190
000019	2	(XN ,DATA(4)),(V ,DATA(6)),(SVN ,DATA(8)),	INJD 200
000020	3	(POO ,DATA(13)), (Z1 ,DATA(15)),	INJD 210
000021	4	(RR, X, DATA(1921)),(THATA, Y, DATA(2921)),	INJD 220
000022	5	(U ,DATA(3921)),	INJD 230
000023	6	(IPP ,DATA(20)),	INJD 240
000024	7	(OPP ,DATA(70)),(IPR ,DATA(120)), (OPR ,DATA(170)),	INJD 250
000025	8	(IPT ,DATA(220)),(OPT ,DATA(270)),(TFLP,DATA(9596)),	INJD 260
000026	9	(TFLR,DATA(9597)),(TFLT,DATA(9598))	INJD 270
000027	C		INJD 280
000028	10	FORMAT (//12X30HRESULTS OF DESCRIBING FUNCTION // 23X10HELEMENT	INJD 290
000029	1	6HRADIUS4X5HANGLE3X10HFRACTIONAL /13X5HOMEGA7X3HNO.7X3H	INJD 300
000030	2	4X9HFLOW-RATE10X2HFP12X2HFR12X2HFT //)	INJD 310
000031	20	FORMAT (10XF9.4,I9,F11.3,F10.4,F11.5,2X3F14.6)	INJD 320
000032	30	FORMAT (// 12X41HRESULTS OF INJECTION DISTRIBUTION EFFECTS //	INJD 330
000033	14	3X5HOMEGA6X3HAVN8X3HBVN7X3HCVN7X3HCVN / 48X3(6X4HREAL),6X4HIMAG/	INJD 340
000034	2/)		INJD 350
000035	40	FORMAT (39X5F10.4)	INJD 360
000036	50	FORMAT (44X5HALL 4F10.4)	INJD 370
000037	60	FORMAT (// 5X39HINPUT TO INJECTION DISTRIBUTION PROGRAM//10X9HCON	INJD 380
000038	1	STANTS//14X18HNUMBER OF OMEGAS = I3//14X20HNUMBER OF ELEMENTS =I5,INJD	INJD 390
000039	1	13H FOR EACH OF I4, 20H SYMMETRIC SECTIONS.	INJD 391
000040	2	//14X22HRADIAL DIVISIONS(XM) = F5.0//14X24HANGULAR DIVISIONS (XN) INJD	INJD 400
000041	3	=F5.0//14X27HACOUSTIC MODE NUMBER(SVN) =F7.4//14X30HORDER OF BESSEINJD	INJD 410
000042	4	L FUNCTIONS(V) = F3.0//14X17HINJECTOR RADIUS =F8.3,5H, IN.//14X32HINJD	INJD 420
000043	5	RATIO OF SPECIFIC HEATS(GAMMA) =F7.4//14X39HMAXIMUM PRESSURE AMPLIINJD	INJD 430
000044	6	ITUDE RATIO(POO) =F7.3//14X39HTRANSFER FUNCTIONS FOR LINEAR OPERATINJD	INJD 440
000045	7	ION//20X16HPRESSURE(TFLP) =F7.3//20X23HRADIAL VELOCITY(TFLR) =F7.3INJD	INJD 450
000046	8	//20X27HTANGENTIAL VELOCITY(TFLT) =F7.3)	INJD 460
000047	70	FORMAT (// 10X17HINPUT FREQUENCIES)	INJD 470
000048	80	FORMAT (// 7X 5F20.4)	INJD 480
000049	90	FORMAT (///10X19HELEMENT INFORMATION)	INJD 490
000050	100	FORMAT (//31X7HELEMENT14X6HRADIUS15X5HANGLE11X12HDISTRIBUTION/90X	INJD 500
000051	11	1HCoefficient/34X3HNO.17X3HIN.15X7HRADIANS15X2HMO//)	INJD 510
000052	110	FORMAT (32X15,11XF10.3,10XF10.4,11XF10.4)	INJD 520
000053	120	FORMAT (14,1//26H TABULAR NONLINEAR EFFECTS//11X8HPRESSURE11X10HCOMINJD	INJD 530
000054	1	JUSTION12X6HRADIAL12X10HCOMBUSTION10X10HTANGENTIAL10X10HCOMBUSTIONINJD	INJD 540
000055	2	/51X8HVELOCITY32X8HVELOCITY/ 9X 3(12HPERTURBATION13X4HGAIN12X)) INJD	INJD 550

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II, D, Program Listing (cont.)

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000056 130 FORMAT ( 6(F18.3,2X) ) INJD 560
000057 140 FORMAT (/33X65H***** THESE VALUES PERTAIN TO A STANDING MOINJD 570
000058 1DE ***** ) INJD 580
000059 150 FORMAT (/33X65H***** THESE VALUES PERTAIN TO A SPINNING MOINJD 590
000060 1DE ***** ) INJD 600
000061 160 FORMAT (23X42H THE VALUE OF IZZIT IS ZERO AND NOT ALLOWED) INJD 610
000062 170 FORMAT(23X45H VALUE OF BESSEL ARGUMENT TOO HIGH OR LOW Z = F10.4) INJD 620
000063 C INJD 630
000064 180 DSCRB = 1 INJD 640
000065 IF ( .NOT. IRUN ) GO TO 190 INJD 650
000066 CE = 100.0 INJD 660
000067 DSCRB = 2 INJD 670
000068 GO TO 200 INJD 680
000069 190 NW = 1 INJD 690
000070 WC(1) = 1.0 INJD 700
000071 200 TIME = 1 INJD 710
000072 RINJ = DATA(9574) INJD 720
000073 NE = DATA(9571) + 0.0001 INJD 730
000074 NS = DATA(5) + 0.0001 INJD 731
000075 NUMBR = DATA(9599) + 0.0001 INJD 740
000076 IZZIT = DATA(2) INJD 750
000077 K = V + 0.0001 INJD 760
000078 KK = K + 1 INJD 770
000079 LEK
000080 M=L+1
000081 N=L+2
000082 C ONLY THE REAL PART IS USED.
000083 IF ( E1 .LE. 0.0 ) E1 = 0.0001 INJD 810
000084 IF ( NUMBR .LE. 0 ) NUMBR = 10 INJD 820
000085 IF ( CE .LE. 99.0 ) GO TO 260 INJD 830
000086 CALL PAGE ( 70 ) INJD 840
000087 WRITE (6,60) NW,NE,NS,XM,XN,SVN,V,RINJ,GAMMA,POO,TFLP,TFLR,TFLT INJD 850
000088 GO TO (220,210), DSCRB INJD 860
000089 210 WRITE (6,70) INJD 870
000090 WRITE (6,80) (WC(IW), IW = 1,NW) INJD 880
000091 220 LINKNT = 0 INJD 890
000092 DO 240 J = 1,NE INJD 900
000093 IF (LINKNT .GT. 0) GO TO 230 INJD 910
000094 CALL PAGE ( 70 ) INJD 920
000095 WRITE (6,90) INJD 930
000096 WRITE (6,100) INJD 940
000097 LINKNT = 50 INJD 950
000098 230 WRITE (6,110) J, RR(J), THATA(J), U(J) INJD 960
000099 LINKNT = LINKNT + 1 INJD 970
000100 240 CONTINUE INJD 980
000101 GO TO (260,250), DSCRB INJD 990
000102 250 WRITE (6,120) INJD 1000
000103 WRITE (6,130) (IPP(I),OPP(I),IPR(I),OPR(I),IPT(I),OPT(I),I=1,50) INJD 1010
000104 CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX INJD 1020
000105 C RUNNING DESCRIBING FUNCTION IMPLIES THAT THE EXPANSION INJD 1030
000106 C COEFFICIENTS AVN, BVN, AND CVN ARE FUNCTIONS OF THE FREQUENCY, OMEGA. INJD 1040
000107 C THEREFORE, AN OMEGA LOOP MUST BE ESTABLISHED. OTHERWISE, THIS LOOP INJD 1050
000108 C IS GONE THROUGH ONLY ONCE. INJD 1060
000109 CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX INJD 1070
000110 260 DO 470 IW = 1, NW INJD 1080
000111 WCR = WC(IW) INJD 1090
000112 VOO = POO/(GAMMA*WCR) INJD 1100
000113 SUMP = 0.0 INJD 1110
000114 SUMR = 0.0 INJD 1120

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II, D, Program Listing (cont.)

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000115      SUMT = 0.0                                INJD1130
000116      LINKNT = 0                                INJD1140
000117      DO 450 J = 1, NE                          INJD1150
000118      R = RR(J)/RINJ                             INJD1160
000119      ETA = U(J)/(XM*XN)                          INJD1170
000120      C*****
000121      C THE ANSWERS WILL BE
000122      C STORED IN THE FOLLOWING FASHION, J(V-1) = FIRST(L), J(V) = INJD1230
000123      C FIRST(M), AND J(V+1) = FIRST(N).              INJD1240
000124      C*****
000125      KERR = 0                                       INJD1250
000126      Z(1) = SVI/R                                  INJD1260
000127      Z(2) = 0.0                                    INJD1270
000128      CALL BESSL ( FIRST(1), SECOND(1), KK, Z(1), KERR ) INJD1280
000129      IF ( KERR ) 270,270,550                      INJD1290
000130      270 SIVN = FIRST(M)                            INJD1300
000131      DSIVN = ( V*FIRST(M) - Z(1)*FIRST(N) )/R      INJD1310
000132      Z(1) = SVI                                     INJD1320
000133      Z(2) = 0.0                                    INJD1330
000134      CALL BESSL ( FIRST(1), SECOND(1), KK, Z(1), KERR ) INJD1340
000135      IF ( KERR ) 280,280,550                      INJD1350
000136      280 GO TO (290,320), TIME                     INJD1360
000137      290 IF ( K ) 300,300,310                      INJD1370
000138      300 D = ( FIRST(M)*FIRST(M) + FIRST(L)*FIRST(L) ) / 2.0 INJD1380
000139      GO TO 320                                       INJD1390
000140      310 D = ( FIRST(M)*FIRST(M) - FIRST(L)*FIRST(N) ) * 3.14159 / (2.0*DATA(5)) INJD1400
000141      320 IF ( IZZIT ) 330,560,340                  INJD1410
000142      CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX INJD1420
000143      C IZZIT IS NEGATIVE FOR STANDING MODES AND POSITIVE FOR SPINNING MODES INJD1430
000144      CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX INJD1440
000145      330 VT = V*THAIA(J)                            INJD1450
000146      CVT = COS(VT)                                  INJD1460
000147      SVT = SIN(VT)                                  INJD1470
000148      GO TO 350                                       INJD1480
000149      340 CVT = 1.0                                    INJD1490
000150      SVT = 1.0                                       INJD1500
000151      350 FP = 1.0                                    INJD1510
000152      FR = 1.0                                       INJD1520
000153      FT = 1.0                                       INJD1530
000154      GO TO (420,360), DSCRB                         INJD1540
000155      360 PO = ABS(P00*SIVN*CVT)                     INJD1550
000156      VO = ABS(V00*DSIVN*CVT)                       INJD1560
000157      W0 = ABS(V00*SIVN*V*SVT/R)                   INJD1570
000158      A = -3.14159                                   INJD1580
000159      B = -A                                          INJD1590
000160      IF ( TFLP ) 370,380,370                       INJD1600
000161      370 SAVER = 1.0/(3.14159*P0*TFLP)             INJD1610
000162      CALL INTGR(A,B,PSI,NUMBR)                     INJD1620
000163      371 CONTINUE                                    INJD1630
000164      CPSI = COS(PSI)                                INJD1640
000165      IPX = P0*CPSI                                  INJD1650
000166      CALL INT4(IPP(1), OPP(1), IPX, OPX)           INJD1660
000167      F = OPX*CPSI                                   INJD1670
000168      CALL INTGS(5371,F,INTEG,E1,MM)
000169      FP = SAVER*INTEG                                INJD1690
000170      380 IF ( TFLR ) 390,400,390                   INJD1700
000171      390 SAVER = 1.0/(3.14159*V0*TFLR)            INJD1710
000172      CALL INTGR(A,B,PSI,NUMBR)                     INJD1720
000173      391 CONTINUE

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II, D, Program Listing (cont.)

000174	CPSI = COS(PSI)	INJD1730
000175	IPX = VO*CPSI	INJD1740
000176	CALL INT4 (IPR(1), OPR(1), IPX, OPX)	INJD1750
000177	F = OPX*CPSI	INJD1760
000178	CALL INTGS(4391,F,INTEG,E1,MM)	
000179	FR = SAVER*INTEG	INJD1780
000180	400 IF (TFLT) 410,420,410	INJD1790
000181	410 SAVET = 1.0/(3.14159*W0*TFLT)	*NEW
000182	CALL INTGR(A,B,PSI,NUMBR)	INJD1810**1
000183	411 CONTINUE	
000184	CPSI = COS(PSI)	INJD1820
000185	IPX = W0*CPSI	INJD1830
000186	CALL INT4 (IPT(1), OPT(1), IPX, OPX)	INJD1840
000187	F = OPX*CPSI	INJD1850
000188	CALL INTGS(4411,F,INTEG,E1,MM)	
000189	FT = SAVET*INTEG	INJD1870
000190	420 TERMP = ETA*FP*SIVN**2*CVT**2	INJD1880
000191	SUMP = SUMP + TERMP	INJD1890
000192	TERMR = ETA*FR*SIVN*DSIVN*CVT**2	INJD1900
000193	SUMR = SUMR + TERMR	INJD1910
000194	TERMT = -ETA*FT*SIVN**2*CVT*V*SVT/R	INJD1920
000195	SUMT = SUMT + TERMT	INJD1930
000196	TIME = 2	INJD1940
000197	GO TO (450,430), DSCRIB	INJD1950
000198	430 IF (LINKNT .GT. 0) GO TO 440	INJD1960
000199	CALL PAGE (70)	INJD1970
000200	WRITE (6,10)	INJD1980
000201	LINKNT = 50	INJD1990
000202	440 WRITE (6,20) WCR, J, R, THATA(J), ETA, FP, FR, FT	INJD2000
000203	LINKNT = LINKNT - 1	INJD2010
000204	450 CONTINUE	INJD2020
000205	DD = D	INJD2030
000206	IF (IZZIT .GT. 0) DD = D/2.0	INJD2040
000207	AVN(IW) = SUMP/DD	INJD2050
000208	BVN(IW) = SUMR/DD	INJD2060
000209	IF (IZZIT .GT. 0) GO TO 460	INJD2070
000210	CVNR(IW) = SUMT/DD	INJD2080
000211	CVNI(IW) = 0.0	INJD2090
000212	GO TO 470	INJD2100
000213	460 CVNR(IW) = 0.0	INJD2110
000214	CVNI(IW) = SUMT/DD	INJD2120
000215	470 CONTINUE	INJD2130
000216	IF (CE .LT. 10.0) GO TO 530	INJD2140
000217	CALL PAGE (70)	INJD2150
000218	WRITE (6,30)	INJD2160
000219	GO TO (480,490), DSCRIB	INJD2170
000220	480 WRITE (6,50) AVN(1), BVN(1), CVNR(1), CVNI(1)	INJD2180
000221	GO TO 500	INJD2190
000222	490 WRITE (6,40) (WC(I), AVN(I), BVN(I), CVNR(I), CVNI(I), I=1,NW)	INJD2200
000223	500 IF (IZZIT) 510,560,520	INJD2210
000224	510 WRITE(6,140)	INJD2220
000225	GO TO 530	INJD2230
000226	520 WRITE (6,150)	INJD2240
000227	530 CONTINUE	INJD2250
000228	540 RETURN	INJD2260
000229	550 WRITE (6,170) Z(1)	INJD2270
000230	GO TO 540	INJD2280
000231	560 WRITE (6,160)	INJD2290
000232	GO TO 540	INJD2300

II, D, Program Listing (cont.)

000233

END

INJD2310

II, D, Program Listing (cont.)

000233

END

INJD2310

II, D, Program Listing (cont.)

J ELT SUB20,1,700707, 29972

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000001      SUBROUTINE TBLCAL
000002      C TBLCAL SUBROUTINE CALCULATES XX VS U2TBL FROM NOZZLE GEOMETRY      *ANNU0030
000003      C SIMPSONS RULE IS USED WITH KN INPUT ODD AND CHANGED TO KN/2+1 IN MAIN*ANNU0040
000004      C DECK ANNULR DERIVED FROM DECK VELPOT... TREATS ANNULAR NOZZLES      ANNU0050
000005      C      ANNU0060
000006      LOGICAL LOGIK, SL1, SL2, EORJ      ANNU0070
000007      COMMON /PROLOG/ LOGIK(50), SL1, SL2, EORJ      ANNU0080
000008      C      ANNU0090
000009      COMMON /ABCD/ EXTRA(100), ABLOK(600)      ANNU0100
000010      1 , XX , U2TBL , DESIRE , RAT , RAC , RCC      ANNU0110
000011      2 , RCT , HANG , G , KN , Y , YP      ANNU0120
000012      3 , YOUT , TEMP , E , XTABLE , YTABLE , A      ANNU0130
000013      4 , R , AM , AP , AMP , ZZ , AMM      ANNU0140
000014      5 , AMP2 , CALFA , CTALFA , DELAM , DELTZ , FKN      ANNU0150
000015      6 , G1 , G2 , G3 , G4 , JFLAG1 , KNM1      ANNU0160
000016      7 , K , NN , PI , PROD , RSTA1 , RSTA2      ANNU0170
000017      8 , SALFA , T1 , T2 , T3 , XINT , XK      ANNU0180
000018      9 , ZZ1 , ZZ2 , ZZ3 , A1 , ABC , ABD      ANNU0190
000019      COMMON /ABCD/      ANNU0200
000020      1 , A11 , ALPHAI , ALPHAR , AR1 , B101 , B102      ANNU0210
000021      2 , B10 , B1 , B2 , B3 , B4 , B5      ANNU0220
000022      3 , B6 , B7 , B8 , B91 , B92 , B9      ANNU0230
000023      4 , B11 , BR1 , C2 , C3 , CH11 , CHIR      ANNU0240
000024      5 , C11 , CR1 , C , D10 , D11 , D1      ANNU0250
000025      6 , D2 , D3 , D4 , D5 , D6 , D7      ANNU0260
000026      7 , D8 , D9 , DC2 , D , DU2 , EI      ANNU0270
000027      8 , ER , F3I , F3R , FI , FR , H1      ANNU0280
000028      9 , H , I , IWO , IW , IWW , J      ANNU0290
000029      COMMON /ABCD/      ANNU0300
000030      1 , MDISIR , NK , NP , S2 , S , TT      ANNU0310
000031      2 , U2 , U , W2 , W , XI01 , XI0R      ANNU0320
000032      3 , XI2I , XI2R , XI , XJI , XJR , XMNEW      ANNU0330
000033      4 , XMOLD , XNEW , XOLD , XPT , X , ZI      ANNU0340
000034      5 , ZR      ANNU0350
000035      C      ANNU0360
000036      DIMENSION XX(200),U2TBL(200),XTABLE(200),YTABLE(200),ZZ(200)      ANNU0370
000037      DIMENSION Y(8),YP(8),YOUT(8),TEMP(16,8),E(8)      *NEW
000038      DIMENSION A(200),R(200),AM(200),AP(200),AMP(200)      ANNU0390**-1
000039      C      ANNU0400
000040      EQUIVALENCE (XK,GRAD),(EXTRA(91),RAT),      ANNU0410
000041      1 ( EXTRA(93),RAC),(EXTRA(94),RCCI),(EXTRA(95),HANG1)      ANNU0420
000042      C      ANNU0430
000043      FKN = KN      ANNU0440
000044      KNM1 = KN - 1      ANNU0450
000045      DELAM = 1.0/(FKN+1.0)      ANNU0460
000046      PI = 3.1415927      ANNU0470
000047      C      ANNU0480
000048      DO 10 J = 1,200      ANNU0490
000049      ZZ(J) = 0.0      ANNU0500
000050      A(J) = 0.0      ANNU0510
000051      R(J) = 0.0      ANNU0520
000052      AMP(J) = 0.0      ANNU0530
000053      AM(J) = 0.0      ANNU0540
000054      XX(J) = 0.0      ANNU0550
000055      U2TBL(J) = 0.0      ANNU0560

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II, D, Program Listing (cont.)

000056	10 AP(J) = 0.0	ANNU0570
000057	C	ANNU0580
000058	R(1) = RAT	ANNU0590
000059	RI = RATI	ANNU0600
000060	RCTI = EXTRA(92)	ANNU0610
000061	A(1) = PI*(R(1)*R(1)-RI*RI)	ANNU0620
000062	U2BL(1)=1.0	ANNU0630
000063	R(KN) = RAC	ANNU0640
000064	ALFA = HANG*.01745329	ANNU0650
000065	CALFA=COS(ALFA)	ANNU0660
000066	SALFA=SIN(ALFA)	ANNU0670
000067	CTALFA=CALFA/SALFA	ANNU0680
000068	RSTA1=RAT+RCT*(1.0-CALFA)	ANNU0690
000069	RSTA2=RAC-RCC*(1.0-CALFA)	ANNU0700
000070	ZZ1=RCT*SALFA	ANNU0710
000071	ZZ2=ZZ1+CTALFA*(RSTA2-RSTA1)	ANNU0720
000072	ZZ3=ZZ2+RCC*SALFA	ANNU0730
000073	C	ANNU0740
000074	JFLAG1=1	ANNU0750
000075	JFLAG2 = 1	ANNU0760
000076	IF (RACI .EQ. 0.0 .AND. RATI .EQ. 0.0) JFLAG2 = 4	ANNU0770
000077	ALFI = HANGI*.01745329	ANNU0780
000078	CALFI = COS(ALFI)	ANNU0790
000079	SALFI = SIN(ALFI)	ANNU0800
000080	CTALFI = CALFI/SALFI	ANNU0810
000081	RI1 = RATI + RCTI*(1.0-CALFI)	ANNU0820
000082	RI2 = RACI - RCCI*(1.0-CALFI)	ANNU0830
000083	ZI1 = RCTI*SALFI	ANNU0840
000084	ZI2 = ZI1 + CTALFI*(RI2-RI1)	ANNU0850
000085	ZI3 = ZI2 + RCCI*SALFI	ANNU0860
000086	Z4 = ZZ3	ANNU0870
000087	IF (ZI3 .GT. ZZ3) Z4 = ZI3	ANNU0880
000088	DELTZ = Z4 / (FKN-1.0)	ANNU0890
000089	C	ANNU0900
000090	DO 80 I = 2,KNM1	ANNU0910
000091	ZZ(I) = ZZ(I-1) + DELTZ	ANNU0920
000092	Z = ZZ(I)	ANNU0930
000093	GO TO (20, 40, 60, 65), JFLAG1	ANNU0940
000094	20 R(I)=RAT+RCT-SQRT(RCT**2-ZZ(I)**2)	ANNU0950
000095	IF(R(I)-RSTA1)70,70,30	ANNU0960
000096	30 JFLAG1=2	ANNU0970
000097	40 R(I)=RSTA1+(RSTA2-RSTA1)*(ZZ(I)-ZZ1)/(ZZ2-ZZ1)	ANNU0980
000098	IF (Z-ZZ2) 70, 70, 50	ANNU0990
000099	50 JFLAG1 = 3	ANNU1000
000100	60 IF (Z-ZZ3) 62, 65, 65	ANNU1010
000101	62 R(I) = RAC - RCC + SQRT (RCC*RCC - (ZZ3-Z)**2)	ANNU1020
000102	GO TO 70	
000103	65 R(I) = RAC	ANNU1030
000104	JFLAG1 = 4	ANNU1040
000105	70 GO TO (72, 74, 76, 77), JFLAG2	ANNU1050
000106	72 TX = SQRT (RCTI*RCTI - Z*Z)	ANNU1060
000107	IF (RCTI .LT. 0.0) TX = -TX	ANNU1070
000108	RI = RATI + RCTI - TX	ANNU1080
000109	IF (Z-ZI1) 78, 78, 73	ANNU1090
000110	73 JFLAG2 = 2	ANNU1100
000111	74 RI = RI1 + (RI2-RI1)*(ZZ(I)-ZI1)/(ZI2-ZI1)	ANNU1110
000112	IF (Z-ZI2) 78, 78, 75	ANNU1120
000113	75 JFLAG2 = 3	ANNU1130
000114	76 TX = SQRT (RCCI*RCCI - (ZI3-Z)**2)	ANNU1140

II, D, Program Listing (cont.)

000115	IF (RCCI .LT. 0.0)	TX = -TX	ANNU1150
000116	RI = RACI - RCCI + TX		ANNU1160
000117	IF (Z-ZI3)	78, 77, 77	ANNU1170
000118	77 RI = RACI		ANNU1180
000119	JFLAG2 = 4		ANNU1190
000120	78 A(I) = PI*(R(I)*R(I) - RI*RI)		ANNU1200
000121	80 CONTINUE		ANNU1210
000122	C		ANNU1220
000123	ZZ(KN) = ZZ(KNM1) + DELTZ		ANNU1230
000124	A(KN) = PI*(RAC*RAC - RACI*RACI)		ANNU1240
000125	AMM = 1.0+ DELAM		ANNU1250
000126	G1=2.0/(G +1.0)		ANNU1260
000127	G2 =(G - 1.0)/2.0		ANNU1270
000128	G3 = (G + 1.0)/(2.0*G - 2.0)		ANNU1280
000129	G4=1.0/G1		ANNU1290
000130	C		ANNU1300
000131	DO 90 J = 1,KN		ANNU1310
000132	AMM = AMM - DELAM		ANNU1320
000133	AM(J) = AMM		ANNU1330
000134	AP(J)=(A(1)/AMM)*(G1*(1.0+G2*AMM**2))**G3		ANNU1340
000135	90 CONTINUE		ANNU1350
000136	C		ANNU1360
000137	DO 100 K = 2,KN		ANNU1370
000138	CALL INT4(AP(1),AM(1),A(K),AMP(K))		ANNU1380
000139	AMP2=AMP(K)**2		ANNU1390
000140	U2TBL(K)=(G4*AMP2)/(1.0+G2*AMP2)		ANNU1400
000141	100 CONTINUE		ANNU1410
000142	C		ANNU1420
000143	DESIRE = AMP(KN)		ANNU1430
000144	XINT = 0.0		ANNU1440
000145	ZZ(1)=0.0		
000146	NN = KN - 2		ANNU1450
000147	K = 1		ANNU1460
000148	IF (RCTI .EQ. 0.0)	RCTI = 1.0	ANNU1470
000149	GRAD = SQRT (G1*(RAT/RCT - RATI/RCTI)/(RAT*RAT - RATI*RATI))		ANNU1480
000150	1*RAT		
000151	PROD = 2.0*XK*DELTZ/3.0		ANNU1490
000152	DO 110 J = 1,NN,2		ANNU1500
000153	T1 = SQRT(U2TBL(J))		ANNU1510
000154	T2 = SQRT(U2TBL(J+1))		ANNU1520
000155	T3 = SQRT(U2TBL(J+2))		ANNU1530
000156	XINT = XINT + PROD*(T1+4.0*T2+T3)		ANNU1540
000157	K = K + 1		ANNU1550
000158	XX(K) = -XINT/RAT		ANNU1560
000159	ZZ(K)=ZZ(K-1)+2.*DELTZ		
000160	U2TBL(K-1) = U2TBL(J)		ANNU1570
000161	110 CONTINUE		ANNU1580
000162	U2TBL(K) = U2TBL(KN)		ANNU1590
000163	C*****		ANNU1600
000164	RETURN		ANNU1610
000165	END		ANNU1620

II, D, Program Listing (cont.)

W LIT SUB22,1,690715, 33520

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000001 SUBROUTINE AS138 (C,ITITLE,IERR)
000002 C THIS ROUTINE PROVIDES A HIGHLY SIMPLIFIED INPUT PROCEDURE WHICH IS CTP 20
000003 C NON-CARD-COLUMN-ORIENTED. DEVELOPED AT THE AIR FORCE ROCKET CTP 30
000004 C PROPULSION LABORATORY BY RPMMD/CAPT V.L.OLIVIER/553-2819 CTP 40
000005 C LAST REVISED APRIL 1968 CTP 50
000006 C THE CONTROL CHARACTERS ARE DEFINED AS FOLLOWS** CTP 60
000007 C L ...SPECIFIES A LOCATION NUMBER WILL FOLLOW CTP 70
000008 C + OR - AND/OR , ARE USED TO SEPARATE DATA ENTRIES CTP 80
000009 C E ...SPECIFIES A POWER OF TEN WILL FOLLOW CTP 90
000010 C T ...TERMINATES THE DATA READ IN AND IS FOLLOWED BY BCD CTP 100
000011 C INFORMATION THAT CAN BE PRINTED CTP 110
000012 C BLANKS AND ZEROES PRECEDING A NUMBER ARE IGNORED. CTP 120
000013 C CTP 130
000014 C EXAMPLES** 1 2 3 4 CTP 140
000015 C COLUMN 123456789012345678901234567890123456789.....CTP 150
000016 C L 0002300,24+-1,++E12,E-2 ,10.001 L12 + +-1CTP 160
000017 C LU+ 1 ,1, 100.1E20+L50 ,, , CTP 170
000018 C T (T IS THE ONLY CONTROL THAT MUST BE IN COLUMN 1) CTP 180
000019 C CTP 190
000020 C THE RESULTS ARE STORED IN THE C-ARRAY AND ITITLE AS FOLLOWS** CTP 200
000021 C C(2301)=24. CTP 210
000022 C C(2302)=0. CTP 220
000023 C C(2303)=-1. CTP 230
000024 C C(2304)=0. CTP 240
000025 C C(2305)=1.E+12 CTP 250
000026 C C(2306)=1.E-02 CTP 260
000027 C C(2307)=10.001 CTP 270
000028 C C(13)=0. CTP 280
000029 C C(14)=0. CTP 290
000030 C C(15)=0. CTP 300
000031 C C(16)=-1. CTP 310
000032 C C(1)=1. CTP 320
000033 C C(2)=1. CTP 330
000034 C C(3)=100.1E+20 CTP 340
000035 C C(51)=0. CTP 350
000036 C C(52)=0. CTP 360
000037 C C(53)=0. CTP 370
000038 C ITITLE=T (T IS THE ONLY CONTROL THAT MUST BE IN COLUMN 1) CTP 380
000039 C CTP 390
000040 DIMENSION C(1),DCARD(72),SYMB(17),XNUM(16)
000041 DIMENSION IDCARD(72), ITITLE(12), MULT1(6), MASK1(6) CTP1400
000042 EQUIVALENCE (IDCARD(1),DCARD(1)) CTP1410
000043 DATA MULT1/01,0100,010000,01000000,0100000000,010000000000/ CTP1420
000044 DATA MASK1/0,040000000000,0400000000,04000000,04000,040/ CTP1430
000045 DATA MASK/07700000000000,BLNK/6H 00000/ CTP1440
000046 LOGICAL NUMBER,DECIMAL,EXPON,LOCATE CTP 410
000047 DATA BLANK,ICARD,SYMB/1H ,1HT,1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9, CTP 420
000048 11H0,1H+,1,-,1H,,1H,,1HE,1HL,1HS/ *NEW
000049 IERR=1 CTP 440
000050 NUMBER=.TRUE. CTP 450
000051 DECIMAL=.FALSE. CTP 460
000052 EXPON=.FALSE. CTP 470
000053 LOCATE=.FALSE. CTP 480
000054 ASIGN=0.0 CTP 490
000055 LUCN=0 CTP 500

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II, D, Program Listing (cont.)

000056	C		CTP 510
000057		10 READ (5,240) DCARD	CTP 520
000058		IF (DCARD(1).EQ.TCARD) GO TO 200	CTP 530
000059		DO 190 I=1,73	CTP 540
000060		IF (I.EQ.73) GO TO 80	CTP 550
000061		IF (DCARD(1).EQ.BLANK) GO TO 190	CTP 560
000062		DO 20 J=1,17	
000063		IF (DCARD(I).EQ.SYMB(J)) GO TO (40,40,40,40,40,40,40,40,40,30,100,CTP 580 **-1	
000064		10,80,50,70,90,100),J	
000065		20 CONTINUE	CTP 600
000066		GO TO 230	CTP 610
000067	C		CTP 620
000068		30 J=0	CTP 630
000069		IF (IDIG.EQ.0) GO TO 190	CTP 640
000070	C		CTP 650
000071		40 IDIG=IDIG+1	CTP 660
000072		IF (.NOT.DECMAL) NDEC=IDIG-1	CTP 670
000073		XNUM(IDIG)=J	CTP 680
000074		GO TO 190	CTP 690
000075	C		CTP 700
000076		50 IF (DECMAL) GO TO 230	CTP 710
000077		DECMAL=.TRUE.	CTP 720
000078		IF (IDIG.GT.0) GO TO 190	CTP 730
000079		IDIG=1	CTP 740
000080		XNUM(1)=0.0	CTP 750
000081		GO TO 190	CTP 760
000082	C		CTP 770
000083		60 KSIGN=-1	CTP 780
000084		GO TO 100	CTP 790
000085	C		CTP 800
000086		70 IF (EXPON) GO TO 230	CTP 810
000087		EXPON=.TRUE.	CTP 820
000088		NUMBER=.FALSE.	CTP 830
000089	C		CTP 840
000090		80 KSIGN=0	CTP 850
000091		GO TO 110	CTP 860
000092	C		CTP 870
000093		90 IF (LOCATE) GO TO 230	CTP 880
000094		LOCATE=.TRUE.	CTP 890
000095		KSIGN=1	CTP 900
000096		IF (XSIGN.EQ.0.0.AND.IDIG.EQ.0) GO TO 180	CTP 910
000097		NUMBER=.FALSE.	CTP 920
000098		GO TO 110	CTP 930
000099	C		CTP 940
000100		100 IF (XSIGN.EQ.0.0.AND.IDIG.EQ.0) GO TO 180	CTP 950
000101	C		CTP 960
000102		110 X=0.0	CTP 970
000103		IF (IDIG.EQ.0) GO TO 130	CTP 980
000104		DO 120 K=1,IDIG	CTP 990
000105		X=X+10.**NDEC*XNUM(K)	CTP1000
000106		120 NDEC=NDEC-1	CTP1010
000107	C		CTP1020
000108		IDIG=0	CTP1030
000109		NDEC=0	CTP1040
000110		DECMAL=.FALSE.	CTP1050
000111	C		CTP1060
000112		IF (XSIGN.LT.0.0) X=-X	CTP1070
000113		130 IF (.NOT.NUMBER) GO TO 140	CTP1080
000114		IF (LOCATE) GO TO 160	CTP1090

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II, D, Program Listing (cont.)

000115	IF (EXPON) GO TO 170	CTP1100
000116	GO TO 150	CTP1110
000117	140 NUMBER=.TRUE.	CTP1120
000118	ESIGN=XSIGN	CTP1130
000119	IF (ESIGN.EQ.0.0) ESIGN=1.0	CTP1140
000120	C	CTP1150
000121	C	CTP1160
000122	150 LOCN=LOCN+1	CTP1170
000123	C(LCN)=X	CTP1180
000124	GO TO 180	CTP1190
000125	C	CTP1200
000126	160 LOCN=ABS(X)	CTP1210
000127	LOCATE=.FALSE.	CTP1220
000128	GO TO 180	CTP1230
000129	C	CTP1240
000130	170 EXPON=.FALSE.	CTP1250
000131	IF (C(LCN).EQ.0.0) C(LCN)=ESIGN	CTP1260
000132	C(LCN)=C(LCN)*10.**IFIX(X)	CTP1270
000133	C	CTP1280
000134	180 XSIGN=KXSIGN	CTP1290
000135	KXSIGN=1	CTP1300
000136	C	CTP1310
000137	190 CONTINUE	CTP1320
000138	C	CTP1330
000139	GO TO 10	CTP1340
000140	C	CTP1350
000141	C	CTP1360
000142	C THE FOLLOWING LOGIC IS TO PACK THE 'DCARD' DATA INTO 'ITITLE'	CTP1370
000143	C FOR A 'FORMAT(1X,12A6)' PRINTOUT.	CTP1380
000144	C	CTP1390
000145	200 DCARD(1)=BLNK	CTP1450
000146	L=0	CTP1460
000147	DO 220 I=1,72,6	CTP1470
000148	L=L+1	CTP1480
000149	KK=1-1	CTP1490
000150	DO 210 J=1,6	CTP1500
000151	K=KK+J	CTP1510
000152	DCARD(K)=AND(IDCARD(K),MASK)	CTP1520
000153	IF (J.EQ.1) GO TO 210	CTP1530
000154	IDCARD(K)=IDCARD(K)/MULT1(J)	CTP1540
000155	IF (ISIGN(1,IDCARD(K)).LT.0) IDCARD(K)=IABS(IDCARD(K))+MASK1(J)	CTP1550
000156	210 CONTINUE	CTP1560
000157	220 ITITLE(L)=IDCARD(I)+ISIGN((IDCARD(I+1)+IDCARD(I+2)+IDCARD(I+3)+IDC	CTP1570
000158	ARD(I+4)+IDCARD(I+5)),IDCARD(I))	CTP1580
000159	RETURN	CTP1590
000160	C	CTP1600
000161	230 IERR=2	CTP1610
000162	WRITE (6,250) DCARD	CTP1620
000163	GO TO 10	CTP1630
000164	C	CTP1640
000165	C	CTP1650
000166	240 FORMAT (72A1)	CTP1660
000167	250 FORMAT (1X,72A1)	CTP1670
000168	END	CTP1680-

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II, D, Program Listing (cont.)

ELT SUB24,1,690708, 37499

```

000001      SUBROUTINE INTGR (A,B,X,N)
000002      X=A
000003      I=1
000004      NL=128*N
000005      RETURN
000006      ENTRY INTGS (S,F,Y,E,M)
000007      GO TO (1,2,3),I
000008      C FIRST TIME
000009      1 X=B
000010      F0 = F
000011      I=2
000012      RETURN1
000013      C
000014      2 FEND = F+F0
000015      N= ((N+3)/4)*4
000016      20 DX = (B-A) / N
000017      F1=0.
000018      F2=0.
000019      F3=0.
000020      F4=0.
000021      K=1
000022      I=3
000023      X=A+ DX
000024      RETURN1
000025      C
000026      3 KF = MOD(K - 1,4)+1
000027      GO TO (10,11,12,13),KF
000028      10 F1=F1 +F
000029      GO TO 14
000030      11 F2=F2 +F
000031      GO TO 14
000032      12 F3=F3 +F
000033      GO TO 14
000034      13 F4=F4 +F
000035      14 X=X+DX
000036      K=K+1
000037      IF (K.NE.N ) RETURN1
000038      SUM1 = DX/3.*(FEND+ 2.*(F2+F4) + 4.*(F1+F3))
000039      SUM2 = 2*DX/3.*(FEND+ 2.*F4+4.*F2)
000040      ER= (SUM1-SUM2) /15.
000041      IF (ABS(ER/SUM1).LT.E) GO TO 15
000042      IF (N.GT.NL) GO TO 15
000043      N=2*N
000044      GO TO 20
000045      15 Y=SUM1 + ER
000046      M=N
000047      RETURN
000048      END

```


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II, D, Program Listing (cont.)

W ELT SUB25,1,700717, 49712

```

000001      SUBROUTINE DISTR (BIN, XOUT, CA, KER )
000002      C
000003      C      CALCULATES COMBUSTION PARAMETERS HR, HI
000004      C      PROGRAM BY LW VERNON FROM 11MAY67 ANALYSIS OF AJ SMITH, JR
000005      C      INCORPORATES CORRECTIONS TO ANALYSIS THRU 1 JUN 67
000006      C      BOOLE INTEGRATION AS OF 12 JUNE 67
000007      C      RHO, RHOL CONVENTIONS CORRECTED 22JUN 67
000008      C      20 SEP 67 MODIFIED FOR TABULAR INJECTOR COEFFICIENTS
000009      C
000010      C*****
000011      C
000012      LOGICAL LOGIK, HRUN, CKOUT, SIMPL, KNOT, LIMIT, TABLR
000013      COMPLEX E, SIGMA, Q1SI, Q1CI, Q2SI, Q2CI, H, OMEGC, SWC, CSQRT
000014      C
000015      COMMON /PROLOG/ LOGIK(38), HEAD(12), SL1, SL2, LORJ
000016      COMMON /ABCD/ D1N (4300)
000017      DIMENSION B ( 240 ), ZZ ( 102 ),
000018      1 U ( 102 ), DU (102 ), UL ( 102 ),
000019      2 RHO ( 102 ), RHOL (102 ), ZING ( 6 ),
000020      3 QBAR ( 102 ), SIMP ( 6 ), ZIP3 ( 102 ),
000021      4 ERRZ ( 6 ), ZIP5 ( 102 ), ZF ( 6 ),
000022      5 THR ( 30 ), THI ( 30 ), BIN ( 240 ),
000023      6 XOUT ( 100 ), WC ( 28 ), WET ( 30 ),
000024      7 ERT ( 30 ), EIT ( 30 ), ZDIST ( 20 ),
000025      8 DISTM ( 20 )
000026      EQUIVALENCE
000027      1 ( HRUN , LOGIK( 8) ), ( CKOUT, LOGIK(13) ), ( SIMPL, LOGIK(14) ),
000028      2 ( KNOT , LOGIK(15) ), ( LIMIT, LOGIK(16) ), ( TABLR, LOGIK(17) )
000029      EQUIVALENCE
000030      1 ( B(1) , SNH ), ( B( 10 ), ZINC ), ( D1N( 14 ), RCH ),
000031      2 ( B(2) , ZE ), ( B( 18 ), XNE ), ( UL(1) , ULO ),
000032      3 ( B(3) , GAM ), ( B( 19 ), XNW ), ( RHOL(1) , RHOL ),
000033      4 ( B(4) , UE ), ( B(170) , WC ), ( B(20) , WET ),
000034      5 ( B(5) , SOUND ), ( B(200) , ZDIST ), ( B(50) , ERT ),
000035      6 ( B(6) , ULM ), ( B(220) , DISTM ), ( B(80) , EIT ),
000036      7 ( B(7) , XK ), ( B(8) , XCML )
000037      C*****
000038      C
000039      NAMELIST /TAB5/SWC, SNH, W, OMEGC, E, SIGMA, Q1SI, Q1CI, Q2SI, Q2CI, H
000040      10 FORMAT ( / 65H TRANSVERSE STABILITY PROGRAM... CALCULATES HR, HI
000041      1
000042      20 FORMAT ( // 11H INPUT DATA// 9X3HSNH12X2HZE10X5HGAMMA9X2HUE9X
000043      1 14HSOUND (FT/SEC) 2X13H ULM (FT/SEC) 5X 9HXK (DRAG)4X11HXCOMPL
000044      2(IN) // F15.8, F14.8, F12.5, F14.8, F16.2, 2X 3F15.6 //
000045      3 5X10HINCREMENTS/ 5XF8.1//)
000046      30 FORMAT ( // 25H NOZZLE ADMITTANCES INPUT// 15X9HOMEGA(CH) 12X
000047      1 3HERT 17X3HEIT//)
000048      40 FORMAT ( ///22H CALCULATED RESULTS... 10X 29HFIRST-ORDER SOLN ( UHYMN 650
000049      1NIF INJ )// 17X5HOMEGA 12X6HH REAL 14X6HH IMAG )
000050      50 FORMAT ( 9XF15.7, 2E20.8 )
000051      60 FORMAT ( 9XF15.7, 2E20.8 )
000052      100 FORMAT ( 12H0INTEGRALS 6E18.8 )
000053      110 FORMAT ( 6H ERROR 6X6E18.8 )
000054      C
000055      C*****

```

HYMN	30
HYMN	40
HYMN	50
HYMN	60
HYMN	70
HYMN	80
HYMN	90
HYMN	100
HYMN	110
HYMN	120
HYMN	140
	*NEW
HYMN	150
HYMN	160
HYMN	170
HYMN	360
HYMN	370
HYMN	380
HYMN	390
HYMN	400
HYMN	410
HYMN	420
HYMN	430
HYMN	440
HYMN	450
HYMN	550
HYMN	560
	*NEW
HYMN	570
HYMN	590
HYMN	600
HYMN	610
HYMN	630
HYMN	650
HYMN	680
HYMN	730
HYMN	740
HYMN	840
HYMN	850

II, D, Program Listing (cont.)

000056	C*****		HYMN 860
000057	CALL DVCHK (KCHK)		HYMN 880
000058	NWI = DIN(3408) + 0.01		HYMN 890
000059	TABLR = NWI ,GE. 3		HYMN 900
000060	DO 170 I = 1, 240		HYMN 910
000061	170 B(I) = BIN(I)		HYMN 920
000062	180 CONTINUE		
000063	190 CALL PAGE (70)		HYMN 940
000064	KOUT = 2		HYMN 950
000065	NET = XNE		HYMN 960
000066	WRITE (6,10)		HYMN 970
000067	IF (CA-100.0) 220,200,200		
000068	200 WRITE (6,20) (B(I), I=1,8),ZINC		
000069	WRITE (6,30)		HYMN1000
000070	WRITE (6,60) (WET(I), ERT(I),EIT(I),I=1,NET)		
000071	CKOUT = (CA .GT. 200.0)		
000072	C		HYMN1100
000073	C	GENERATE NORMALIZED DISTRIBUTION AND OTHER TABULAR FUNCTIONS	HYMN1110
000074	C		HYMN1120
000075	220 ZC = XCMPL / RCH		HYMN1130
000076	ZC = ZE		HYMN1140
000077	IDZ = ZINC + 0.001		HYMN1150
000078	CALL INT4 (ZDIST, DISTM, ZC, UENRM)		HYMN1160
000079	SCALE = UE / UENRM		HYMN1170
000080	SIMPL = IDZ .LE. 0		HYMN1180
000081	IF (SIMPL)	GO TO 230	HYMN1190
000082	INT = 2		HYMN1200
000083	IDZ = IDZ/4 * 4		HYMN1210
000084	C	MUST BE POSITIVE MULTIPLE OF FOUR, LESS THAN 101	HYMN1220
000085	GO TO 240		HYMN1230
000086	C		HYMN1240
000087	230 INT = 1		HYMN1250
000088	IDZ = -IDZ/2 * 2		HYMN1260
000089	C		HYMN1270
000090	240 IF ((IDZ .EQ. 0) .OR. (IDZ .GT. 100)) IDZ = 80		HYMN1280
000091	C		HYMN1290
000092	C	IDZ IS NUMBER OF Z-INCREMENTS.	HYMN1300
000093	C		HYMN1310
000094	IDZP = IDZ + 1		HYMN1320
000095	DZ = ZC / FLOAT(IDZ)		HYMN1330
000096	C		HYMN1340
000097	ZZ(1) = 0.0		HYMN1350
000098	U(1) = 1.0E-10		HYMN1360
000099	DU(1) = 0.0		HYMN1370
000100	RHO(1) = 1.0		HYMN1380
000101	ULO = ULM/SOUND		HYMN1390
000102	GF1 = -1.0 / (GAM-1.0)		HYMN1400
000103	GF2 = (GAM-1.0) / 2.0		HYMN1410
000104	RHOZE = (1.0 + GF2*UE*UE) **GF1		HYMN1420
000105	RHOLO = RHOZE * UE / ULO		HYMN1430
000106	QHAR(1) = 0.0		HYMN1440
000107	ZIP3(1) = 0.0		HYMN1450
000108	ZIP5(1) = RHOLO		HYMN1460
000109	C	ABOVE ARE FIRST TABULAR ENTRIES.	HYMN1470
000110	C		HYMN1480
000111	Z = 0.0		HYMN1490
000112	C		HYMN1500
000113	DO 250 IZ = 2, IDZP		HYMN1510
000114	Z = Z + DZ		HYMN1520

II, D, Program Listing (cont.)

```

000115      CALL INT4D ( ZDIST, DISTM, Z, U(IZ), DU(IZ) )
000116      ZZ(IZ) = Z
000117      U(IZ) = U(IZ)*SCALE
000118      DU(IZ) = DU(IZ)*SCALE
000119      UL(IZ) = UL(IZ-1) + XK*DZ*( U(IZ-1)-UL(IZ-1) )/UL(IZ-1)
000120      TEMP = 1.0 + GAM*( U(IZ)-UL(IZ) )*U(IZ)
000121      RHO(IZ) = ( 1.0 + GF2*U(IZ)*U(IZ) ) **GF1
000122      RHOL(IZ) = ( RHOZE*UE - RHO(IZ)*U(IZ) ) / UL(IZ)
000123      0 QBAR(IZ) = ( ( 1.0 - GAM*U(IZ)*U(IZ) ) * RHO(IZ) * DU(IZ)
000124      1 - GAM*U(IZ)*RHOL(IZ)*XK*( U(IZ)-UL(IZ) ) ) / TEMP
000125      ZIP3(IZ) = DU(IZ) + DU(IZ)
000126      ZIP5(IZ) = RHOL(IZ) / RHO(IZ)
000127      250 CONTINUE
000128      ZZ(IDZP) = ZC
000129      U(IDZP) = UE
000130      DU(IDZP) = 0.0
000131      RHO(IDZP) = RHOZE
000132      RHOL(IDZP) = 0.0
000133      QBAR(IDZP) = 0.0
000134      ZIP3(IDZP) = 0.0
000135      ZIP5(IDZP) = 0.0
000136      C
000137      C QBAR, ZIP3, ZIP5 AS TABULATED ABOVE ARE FREQ-INDEPENDENT PARTS
000138      C OF INTEGRANDS.
000139      C
000140      NW = XNW
000141      C
000142      C*****
000143      C
000144      C CALCULATIONS FOR EACH FREQUENCY, W
000145      C
000146      C*****
000147      C
000148      DO 820 IW = 1, NW
000149      W = WC(IW)
000150      OMEG2 = SINH*SNH - W*W
000151      OMEG = SQRT(ABS(OMEG2))
000152      LIMIT = .FALSE.
000153      C
000154      CALL INT4 ( WET, ERT, W, ER )
000155      CALL INT4 ( WET, EIT, W, EI )
000156      IF ( OMEG2 ) 280,290,300
000157      280 IM = 1
000158      GO TO 310
000159      C
000160      290 IM = 2
000161      OMEG = 0.5E-10
000162      OMEG2 = 0.25E-20
000163      GO TO 310
000164      C
000165      300 IM = 3
000166      C
000167      C EVALUATION OF SIX INTEGRALS BY BOOLE FORMULA ( OR SIMPSON )
000168      C BOOLE INTEGRATION IF ZINC INPUT POSITIVE, ELSE SIMPSON RULE
000169      C ( BOOLE IS SIMPSON RULE WITH ERROR FORMULA )
000170      C
000171      C IM IS ( 1, 2, 3 ) AS OMEG2 IS ( -, 0, + ),
000172      C THEN USE ( CIRCULAR, LIMIT, HYPERBOLIC ) FUNCTIONS
000173      C

```

HYMN1530
 HYMN1540
 HYMN1550
 HYMN1560
 HYMN1570
 HYMN1580
 HYMN1590
 HYMN1600
 HYMN1610
 HYMN1620
 HYMN1630
 HYMN1640
 HYMN1650
 HYMN1660
 HYMN1670
 HYMN1680
 HYMN1690
 HYMN1700
 HYMN1710
 HYMN1720
 HYMN1730
 HYMN1740
 HYMN1750
 HYMN1760
 HYMN1770
 HYMN2290
 HYMN2300
 HYMN2310
 HYMN2320
 HYMN2330
 HYMN2340
 HYMN2350
 HYMN2360
 HYMN2370
 HYMN2380
 HYMN2390
 HYMN2400
 HYMN2410
 HYMN2420
 HYMN2430
 HYMN2440
 HYMN2450
 HYMN2460
 HYMN2470
 HYMN2480
 HYMN2490
 HYMN2500
 HYMN2510
 *NEW
 HYMN2520
 HYMN2530
 HYMN2540
 HYMN2550
 HYMN2560
 HYMN2570
 HYMN2580
 HYMN2590
 HYMN2600
 HYMN2610

II, D, Program Listing (cont.)

000174	310	DO 320 I = 1, 6	HYMN2620
000175	320	ZING(I) = 0.0	HYMN2630
000176		NOD = 3	HYMN2640
000177		DO 430 IZ = 1, IDZP, INT	HYMN2650
000178		Z = ZZ(IZ)	HYMN2660
000179		GO TO (350,340,330), NOD	HYMN2670
000180	C		HYMN2680
000181	330	WT = 1.0	HYMN2690
000182		NOD = 1	HYMN2700
000183		GO TO 360	HYMN2710
000184	C		HYMN2720
000185	340	WT = 2.0	HYMN2730
000186		IF (IZ .EQ. IDZP) GO TO 330	HYMN2740
000187		NOD = 1	HYMN2750
000188		GO TO 360	HYMN2760
000189	C		HYMN2770
000190	350	WT = 4.0	HYMN2780
000191		NOD = 2	HYMN2790
000192	C		HYMN2800
000193	360	PSI = OMEG * (ZE-Z)	HYMN2810
000194		GO TO (390,370,380), IM	HYMN2820
000195	C		HYMN2830
000196	370	ZEZ = ZE - Z	HYMN2840
000197		ZF(1) = QBAR(IZ) * ZEZ	HYMN2850
000198		ZF(2) = QBAR(IZ)	HYMN2860
000199		ZF(3) = ZIP3(IZ) * ZEZ	HYMN2870
000200		ZF(4) = ZIP3(IZ)	HYMN2880
000201		ZF(5) = ZIP5(IZ) * ZEZ	HYMN2890
000202		ZF(6) = ZIP5(IZ)	HYMN2900
000203		GO TO 410	HYMN2910
000204	C		HYMN2920
000205	380	CF = COSH(PSI)	HYMN2930
000206		SF = SINH(PSI)	HYMN2940
000207		GO TO 400	HYMN2950
000208	C		HYMN2960
000209	390	CF = COS (PSI)	HYMN2970
000210		SF = SIN (PSI)	HYMN2980
000211	C		HYMN2990
000212	400	ZF(1) = QBAR(IZ) * SF	HYMN3000
000213		ZF(2) = QBAR(IZ) * CF	HYMN3010
000214		ZF(3) = ZIP3(IZ) * SF	HYMN3020
000215		ZF(4) = ZIP3(IZ) * CF	HYMN3030
000216		ZF(5) = ZIP5(IZ) * SF	HYMN3040
000217		ZF(6) = ZIP5(IZ) * CF	HYMN3050
000218	C	ZF ARE COMPLETE INTEGRANDS	HYMN3060
000219	C		HYMN3070
000220	410	DO 420 I = 1, 6	HYMN3080
000221		ZING(I) = ZING(I) + WT*ZF(I)	HYMN3090
000222	420	CONTINUE	HYMN3100
000223	430	CONTINUE	HYMN3110
000224	C		HYMN3120
000225		GO TO (460,440), INT	HYMN3130
000226	C		HYMN3140
000227	440	INT = 1	HYMN3150
000228		DO 450 I = 1, 6	HYMN3160
000229	450	SIMP(I) = ZING(I)*DZ/1.5	HYMN3170
000230	C	SIMP IS SIMPSON INTEGRAL FOR IDZ/2 INCREMENTS	HYMN3180
000231		GO TO 310	HYMN3190
000232	C		HYMN3200

II, D, Program Listing (cont.)

[illegible]

II, D, Program Listing (cont.)

000292	C		HYMN7490
000293	C		HYMN7500
000294	C		HYMN7510
000295	C	N.B. HTR, HTI ARE FINAL RESULTS OF THIS SUBROUTINE WHEN PLACED	HYMN7520
000296	C	IN XOUT. HTR, HTI INCLUDE INJECTOR EFFECTS, WHILE HR, HI	HYMN7530
000297	C	DO NOT. HENCE IF SUBROUTINE DDD IS ENTERED WITH HTR, HTI,	HYMN7540
000298	C	DDD WILL EXPAND TABLE (BY INTERPOLATION), PRINT,	HYMN7550
000299	C	AND RETURN.	HYMN7560
000300	C		HYMN7570
000301	C		HYMN7580
000302	C		HYMN7590
000303	C	890 RETURN	HYMN7600
000304		END	HYMN7610

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II, D, Program Listing (cont.)

W ELT SUB26,1,700610, 37330

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000001      SUBROUTINE ADSET(N,F,D,FP,T,X,HA,E)
000002      IMPLICIT REAL*8(A-H,O-Z)
000003      REAL F,D,FP,X,HA,E,TP
000004      DIMENSION F(1),D(1),FP(1),T(8,8),HA(1),E(1),AC(6,3),AD(10),AH(10) 00000030
000005      1,ACC(6),ER(60)
000006      EQUIVALENCE (ACC(1),AC(13))
000007      DATA AC/ 0.D0,1.D0,0.5D0,0.4166666667D0,0.375D0,2*0.3486111111D0, 00000060
000008      1 0.6513888889D0,0.1513888889D0,0.06805555556D0,0.02638888889D0, 00000070
000009      2 0.D0,0.D0,0.D0,0.D0,0.D0,0.D0,0.D0,0.D0/ 00000080
000010      DATA AD /2.0D0,-1.0D0,0.D0,0.D0,4.0D0,-4.0D0,1.0D0,8.0D0,-12.0D0, 00000090
000011      1 16.0D0/ 00000100
000012      DATA AH /0.5D0,0.125D0,0.0625D0,0.0390625D0,0.25D0,0.125D0, 00000110
000013      1 0.078125D0,0.125D0,0.09375D0,0.0625D0/ 00000120
000014      DATA CON1,CON2/ 0.9462151D0,0.0537848D0/ 00000130
000015      C 00000140
000016      C**** THIS SUBROUTINE WILL INTEGRATE N DIFFERENTIAL EQUATIONS. 00000150
000017      C WHERE N MAY BE NO LARGER THAN THE SUBSCRIPT ON ER 00000160
000018      C**** THE CALLING SEQUENCE IS AS FOLLOWS. 00000170
000019      C**** N IS THE NUMBER OF EQUATIONS 00000180
000020      C**** F IS THE ARRAY OF FUNCTIONS 00000190
000021      C**** D IS THE ARRAY OF DERIVATIVES OF THE FUNCTIONS F 00000200
000022      C**** FP IS THE ARRAY OF THE PARTIAL STEP VALUES OF F AT X = TP 00000210
000023      C**** T IS AN ARRAY OF 8*N WORDS 00000220
000024      C**** HA IS EITHER AN ARRAY OR A SINGLE WORD 00000230
000025      C**** IN EITHER CASE HA(1) IS THE INITIAL STEP SIZE GUESS 00000240
000026      C**** IF THE ARRAY FEATURE IS USED THE FOLLOWING CONDITIONS HOLD. 00000250
000027      C**** IF HA(2) IS EQUAL TO 1111 THEN STEP SIZE IS LIMITED 00000260
000028      C**** HA(3) IS THE LOWEST VALE OF THE STEP SIZE ALLOWED 00000270
000029      C**** HA(4) IS THE LARGEST VALUE OF THE STEP SIZE ALLOWED 00000280
000030      C**** IF HA(2) IS EQUAL TO 2222 BOTH THE LIMIT AND THE CORRECTED 00000290
000031      C**** DERIVATIVES ARE USED 00000300
000032      C**** IF HA(2) IS EQUAL TO 3333 ONLY THE CORRECTED DERIVATIVES ARE 00000310
000033      C**** IF HA(2) IS NOT DEFINED THEN NEITHER ARE USED 00000320
000034      KKF=0 00000330
000035      C**** SET FOR NO RECAL OF DERIVATIVES 00000340
000036      CALL ELT1 00000350
000037      H=HA(1) 00000360
000038      HMIN=0.D0 00000370
000039      HMAX=1.D30 00000380
000040      IF(HA(1).LT.0.D0) HMIN=-HMAX 00000390
000041      IF(HA(1).LT.0.D0) HMAX=0.D0 00000400
000042      C**** SET MAX-MIN STEP SIZE 00000410
000043      IF(HA(2).EQ.1111.D0) GO TO 400 00000420
000044      IF(HA(2).EQ.2222.D0) GO TO 401 00000430
000045      IF(HA(2).EQ.3333.D0) KKF=1 00000440
000046      403 GO TO 4 00000450
000047      400 HMIN=HA(3) 00000460
000048      HMAX=HA(4) 00000470
000049      GO TO 403 00000480
000050      401 KKF=1 00000490
000051      GO TO 400 00000500
000052      C 00000510
000053      C 00000520
000054      ENTRY ADINT 00000530
000055      IENT=1 00000540

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II, D, Program Listing (cont.)

000056	IDF=0	0000550
000057	IF(INT.EQ.0) GO TO 20	0000560
000058	21 KF=1	0000570
000059	X=X+H	
000060	GO TO 100	0000590
000061	20 H=HA(1)	0000600
000062	506 DO 504 I=1,N	0000610
000063	IF(E(I).LT.1.D-9) E(I)=1.D-9	0000620
000064	504 CONTINUE	0000630
000065	520 IF(IENT.NE.0) RETURN	0000640
000066	RETURN1	0000650
000067	C	0000660
000068	ENTRY ADCOR(*)	0000670
000069	IENT=0	0000680
000070	2 IF(INT.EQ.0) GO TO 23	0000690
000071	22 IF(KKF.EQ.2) GO TO 34	0000700
000072	DO 41 I=1,N	0000710
000073	T(8,I)=F(I)	0000720
000074	41 T(1,I)=D(I)	0000730
000075	KF=2	0000740
000076	GO TO 100	0000750
000077	C	0000760
000078	23 INT=1	0000770
000079	DO 40 I=1,N	0000780
000080	T(7,I)=F(I)	0000790
000081	40 T(2,I)=D(I)	0000800
000082	GO TO 21	0000810
000083	C	0000820
000084	C	0000830
000085	42 TEST ERROR TERM HERE.	0000840
000086	CONTINUE	0000850
000087	IDF=2	0000860
000088	DO 16 I=1,N	0000870
000089	BOT=F(I)	
000090	IF(BOT.EQ.0.D0) BOT=1.D0	
000091	ERT=T(8,I)-F(I)	
000092	IF(NS.EQ.6) ERT=ERT+CON1*ER(I)	
000093	ER(I)=ERT	
000094	ERR=DABS(ER(I)/BOT)	
000095	IF(NS.EQ.6) F(I)=F(I)+CON2*ER(I)	0000930
000096	IF(ERR.LT.E(I)) GO TO 18	0000940
000097	19 IDF=1	0000950
000098	GO TO 500	0000960
000099	18 IF(128.D0*ERR.LT.E(I)) GO TO 16	
000100	33 IDF=0	0000980
000101	16 CONTINUE	0000990
000102	340 IF(KKF.NE.0) GO TO 404	0001000
000103	C NEW DIFFERENCES	0001010
000104	34 IF(NS.GT.5) NS=5	0001020
000105	12 DO 17 I=1,N	0001030
000106	T(7,I)=F(I)	0001040
000107	TE=D(I)	0001050
000108	DO 15 K=2,NS	0001060
000109	TF=TE-T(K,I)	0001070
000110	T(K,I)=TE	0001080
000111	15 TE=TF	0001090
000112	17 T(NS+1,I)=TE	0001100
000113	IF(KKF.NE.0) KKF=1	0001110
000114	NS=NS+1	0001120
	IF(IDF.EQ.2) GO TO 25	0001130

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II, D, Program Listing (cont.)

000115	501	RETURN	00001140
000116	C		00001150
000117	404	KKF=2	00001160
000118		RETURN1	00001170
000119	C		00001180
000120	C	HALF	00001190
000121	500	IF(H.LE.HMIN) GO TO 340	00001200
000122	502	INT =INT +1	00001210
000123		IF(INT.GE.10) GO TO 340	00001220
000124		X=X-H	
000125	24	H=H/2.D0	00001240
000126		HA(1)=H	00001250
000127		GO TO 28	00001260
000128	C	DOUBLE	00001270
000129	25	INT=1	00001280
000130		IF(2.D0*H.GE.HMAX) RETURN	00001290
000131	510	H=H+H	00001300
000132		HA(1)=H	00001310
000133	28	NN=NS-2	00001320
000134		IF(NN.EQ.0) GO TO 260	
000135		DO 26 I=1,N	00001330
000136		ER(I)=ER(I)*64.D0**(2>IDF-3)	
000137		KKK=1	00001340
000138		DO 26 L=1,NN	00001350
000139		TE=0.D0	00001360
000140		DO 27 K=L,NN	00001370
000141		GO TO (44,45),IDF	00001380
000142	44	TF=AH(KKK)	00001390
000143		GO TO 29	00001400
000144	45	TF =AD(KKK)	00001410
000145	29	TE = TE +TF*T(K+2,I)	00001420
000146	27	KKK=KKK+1	00001430
000147	26	T(L+2,I) =TE	00001440
000148	260	IF(IDF-2)21,501,21	00001450
000149	C		00001460
000150		ENTRY ADPAR(TP)	
000151	3	KF = 3	
000152		P= (TP-X)/H	
000153		P2 =P*P	00001500
000154		P3 =P*P2	00001510
000155		P4 = P*P3	00001520
000156		P5 = P*P4	00001530
000157		ACC(2)=P	00001540
000158		ACC(3)=P2/2.D0	00001550
000159		ACC(4) =(2.D0*P3+3.D0*P2)/12.D0	00001560
000160		ACC(5) =(P4+4.D0*(P3+P2))/24.D0	00001570
000161		ACC(6) =(5.D0*P5 +45.D0*P4+110.D0*P3 +90.D0*P2)/720.D0	00001580
000162		GO TO 100	00001600
000163	C		00001610
000164	43	DO 46 I=1,N	
000165	46	FP(I) = F(I)	
000166		RETURN	00001650
000167	C		00001660
000168		ENTRY ADRES	00001670
000169	4	INT=0	00001680
000170		H=H/32.D0	
000171		HA(1)=H	00001720
000172		NS=2	00001730
000173		RETURN	00001740

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II, D, Program Listing (cont.)

000174	C		00001750
000175	C		00001760
000176	100	DO 101 I=1,N	00001770
000177		YT=0.00	00001780
000178		DO 102 K=1,NS	00001790
000179	102	YT = YT + AC(K,KF)* T(K,I)	00001800
000180		F (I) = YT *H + T(7,I)	00001810
000181		IF(KF.EQ.1.AND.NS.EQ.6) F(I)=F(I)-CON1*ER(I)	00001820
000182	101	CONTINUE	00001830
000183		GO TO (520,42,43),KF	00001840
000184	C		00001850
000185	C		00001860
000186		END	00001870

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II, D, Program Listing (cont.)

Line	Code	Text	NOZM	NEW
00101	1*			
00101	2*	SUBROUTINE CCC(DIN,DOUT,WC,CODE,NER)	NOZM 10	
00101	3*	C		
00101	4*	C*****NOZM	15	
00101	5*	C***** THIS PROGRAM WAS WRITTEN FROM A REPORT ON NOZZLE ADMITTANCE*****NOZM	20	
00101	6*	C***** THEORY FROM PRINCETON UNIVERSITY. THE ANALYSIS WAS DONE BY*****NOZM	25	
00101	7*	C***** CARL LUNDELIUS AND THE PROGRAMMING BY JERRY HOWARD.JOB 8052*****NOZM	30	
00101	8*	C*****		
00101	9*	C NOZMIT MODIFIED 25 JUL 67 TO SUPPLY CR1,CI1 IN PLACE OF BR1,BI1.		
00101	10*	C NOZMIT MODIFIED BY WAUGH AND DUDLEY MARCH 70 TO EXTEND NOZZLE		
00101	11*	C AS A CYLINDER STN INCHES		
00101	12*	C*****NOZM	35	
00101	13*	C	NOZM 40	
00103	14*	LOGICAL LOGIK, SL1, SL2, EORJ	NOZM 185	
00104	15*	COMPLEX GH,SC,XJK,F3,X1,X2,ZETA		
00105	16*	COMPLEX T1X,UQ,F3P,X1P,X2P,ZETAP,DZETP,DF3P,DX1P,DX2P		
00106	17*	COMPLEX ASCRP,BSCR,P,CSGRP,ESGRP		
00107	18*	COMMON /ABCD/ EXTRA(100), ABLOK(600)	NOZM 45	
00107	19*	1 , XX , U2TBL , DESIRE , RAT , RAC , RCC	NOZM 50	
00107	20*	2 , RCT , ALFA , G , KN , Y , YP	NOZM 55	
00107	21*	3 , YOUT , TEMP , E , XTABLE , YTABLE , A	NOZM 60	
00107	22*	4 , R , AM , AP , AMP , ZZ , AMM	NOZM 65	
00107	23*	5 , AMP2 , CALFA , CTALFA , DELAM , DELTZ , FKN	NOZM 70	
00107	24*	6 , G1 , G2 , G3 , G4 , JFLAG1 , KNM1	NOZM 75	
00107	25*	7 , K , NN , PI , PROD , RSTA1 , RSTA2	NOZM 80	
00107	26*	8 , SALFA , T1 , T2 , T3 , XINT , XK	NOZM 85	
00107	27*	9 , ZZ1 , ZZ2 , ZZ3 , A1 , ABC , ABD	NOZM 90	
00110	28*	COMMON /ABCD/	NOZM 95	
00110	29*	1 , A11 , ALPHAI , ALPHAR , AR1 , B101 , B102	NOZM 100	
00110	30*	2 , B10 , B1 , B2 , B3 , B4 , B5	NOZM 105	
00110	31*	3 , B6 , B7 , B8 , B91 , B92 , B9	NOZM 110	
00110	32*	4 , BI1 , BR1 , C2 , C3 , CHII , CHIR	NOZM 115	
00110	33*	5 , CI1 , CR1 , C , D10 , D11 , D1	NOZM 120	
00110	34*	6 , D2 , D3 , D4 , D5 , D6 , D7	NOZM 125	
00110	35*	7 , D8 , D9 , DC2 , D , DU2 , EI	NOZM 130	
00110	36*	8 , ER , F3I , F3R , FI , FR , H1	NOZM 135	
00110	37*	9 , H , I , IWO , IW , IWW , J	NOZM 140	
00111	38*	COMMON /ABCD/	NOZM 145	
00111	39*	1 , MDESIR , NK , NP , S2 , S , TT	NOZM 150	
00111	40*	2 , U2 , U , W2 , W , XI0I , XI0R	NOZM 155	
00111	41*	3 , XI2I , XI2R , XI , XJI , XJR , XMNEW	NOZM 160	
00111	42*	4 , XMOLD , XNEW , XOLD , XPT , X , ZI	NOZM 165	
00111	43*	5 , ZR	NOZM 170	
00112	44*	COMMON /PROLOG/ LOGIK(50), SL1, SL2, EORJ	NOZM 180	
00112	45*	C	NOZM 235	
00113	46*	DIMENSION XX(200),U2TBL(200),XTABLE(200),YTABLE(200),ZZ(200)	NOZM 195	
00114	47*	DIMENSION Y(8),YP(8),YOUT(8),TEMP(16,8),E(8)		
00115	48*	DIMENSION A(200),R(200),AM(200),AP(200),AMP(200)	NOZM 205	
00116	49*	DIMENSION DIN(1), DOUT(1),WC(1)	NOZM 210	
00117	50*	EQUIVALENCE (VEPOT, EXTRA(101)), (STN, EXTRA(102))		
00120	51*	EQUIVALENCE (GRAD , XK)		
00120	52*	C	NOZM 320	
00120	53*	C*****		
00120	54*	C	NOZM 330	
00120	55*	C	NOZM 335	

II, D, Program Listing (cont.)

00121	56*	NER = 0	NOZM 340
00121	57*	C *****	NOZM 345
00121	58*	C READ INPUT*	NOZM 350
00121	59*	C *****	NOZM 355
00122	60*	G = DIN(1)	NOZM 360
00123	61*	NOMEG=WC(2)+2.0001	NOZM 365
00124	62*	MDESIR = DIN(2) + .0001	NOZM 370
00125	63*	GEO=.5*(G+1.0)*SQRT(DIN(4)*DIN(7))	NOZM 375
00126	64*	IF (CODE) 10,30,50	NOZM 380
00131	65*	10 NER = 1	NOZM 385
00132	66*	30 RETURN	NOZM 390
00133	67*	50 IF(WC(1))70,90,70	NOZM 395
00136	68*	70 WCONST=DOUT(3)/(6.2831853*DOUT(1))	NOZM 400
00137	69*	GEO=GEO/DOUT(1)	NOZM 405
00140	70*	GO TO 110	NOZM 410
00141	71*	90 WCONST=DOUT(3)/(6.2831853*DOUT(2))	NOZM 415
00142	72*	GEO=GEO/DOUT(2)	NOZM 420
00143	73*	110 IF(CODE-99.0)190,190,130	NOZM 425
00143	74*	C *****	NOZM 430
00143	75*	C PRINT OUT INPUT*	NOZM 435
00143	76*	C *****	NOZM 440
00146	77*	130 CALL PAGE(5)	NOZM 445
00147	78*	WRITE (6,150)	NOZM 450
00151	79*	150 FORMAT (1H0,75H PROGRAM C INPUT - CALCULATES NOZZLE ADMITTANCE	NOZM 455
00151	80*	1COEFFICIENTS USING 8052)	NOZM 460
00152	81*	WRITE (6,170)G, MDESIR	NOZM 465
00156	82*	170 FORMAT (1H0,5X,3HG =,F9.3,11H , MDESIR =,I2)	NOZM 470
00157	83*	190 CONTINUE	NOZM 475
00157	84*	C *****	NOZM 480
00157	85*	C MDESIR=1,INPUT TABLE,INPUT DESIRE MDESIR=2,CALCULATE TABLE,INPUT*	NOZM 485
00157	86*	C DESIRE MDESIR=3,CALCULATE TABLE,CALCULATE DESIRE (AT LAST POINT)*	NOZM 490
00157	87*	C *****	NOZM 495
00160	88*	GO TO (290,210,210),MDESIR	NOZM 500
00161	89*	210 RAT = DIN(4)	NOZM 505
00162	90*	RAC = DIN(5)	NOZM 510
00163	91*	RCC = DIN(6)	NOZM 515
00164	92*	RCT = DIN(7)	NOZM 520
00165	93*	ALFA = DIN(8)	NOZM 525
00166	94*	KN = DIN(9) + .0001	NOZM 530
00167	95*	IF (CODE - 99.0) 270,270,230	NOZM 535
00172	96*	230 CALL PAGE (1)	NOZM 540
00173	97*	WRITE (6,250) RAT,RAC,RCC,RCT,ALFA,KN,STN	
00204	98*	250 FORMAT (1H ,5X5HRAT =,F7.3,6H,RAC =,F7.3,6H,RCC =,F7.3,6H,RCT =,	NOZM 550
00204	99*	1 F7.3, 7H,ALFA = F7.3,5H,KN = I4,6H,STN = F7.3)	
00205	100*	270 CONTINUE	NOZM 560
00206	101*	CALL TBLCAL	NOZM 565
00207	102*	KN = KN/2 + 1	NOZM 570
00210	103*	*DIAGNOSTIC* THE TEST FOR EQUALITY BETWEEN NON-INTEGERS MAY NOT BE MEANINGFUL.	
00210	104*	IF (STN .EQ. 0.0) GO TO 370	
00212	105*	H= SQRT (U2TBL(KN))/RAT*GRAD*2.0	
00213	106*	KN1 = KN+1	
00214	107*	KN3 = KN+3	
00215	108*	DO 280 NK = KN1, KN3	
00220	109*	XX(NK) = XX(NK-1) - H*0.01	
00221	110*	ZZ(NK) = ZZ(NK-1)+.01	
00222	111*	280 U2TBL(NK) = U2TBL(KN)	
00224	112*	XX(KN3) = XX(KN) - H*STN	
00225	113*	ZZ(KN3) = ZZ(KN)+STN	
00226	114*	KN = KN3	
00227	114*	GO TO 370	NOZM 575

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II, D, Program Listing (cont.)

00227	115*	C *****	NOZM 580
00227	116*	C READ VELOCITY POTENTIAL TABLE. FIRST POINT IS (0,1)*	NOZM 585
00227	117*	C *****	NOZM 590
00230	118*	290 J = 10	NOZM 595
00231	119*	DO 330 I = 1,200	NOZM 600
00234	120*	XX(I) = DIN(J)	NOZM 610
00235	121*	U2TBL(I) = DIN(J+1)	NOZM 615
00236	122*	J = J+2	NOZM 620
00237	123*	IF (I) 330,330,310	NOZM 625
00242	124*	310 IF (XX(I)) 330,350,350	NOZM 630
00245	125*	330 CONTINUE	NOZM 635
00245	126*	C *****	NOZM 640
00245	127*	C KN = COUNT OF TOTAL NO. OF POINTS IN THE TABLE.*	NOZM 645
00245	128*	C *****	NOZM 650
00247	129*	350 KN = I - 1	NOZM 655
00250	130*	370 CONTINUE	
00251	131*	FORLT=U2TBL(2)	
00252	132*	IF (CODE-199.0) 470,470,390	
00255	133*	390 CALL PAGE(70)	NOZM 665
00256	134*	WRITE (6,430)	NOZM 670
00260	135*	KKKN = KN/2	
00261	136*	DO 410 I=1,KKKN	NOZM 680
00264	137*	KKK=KKKN+I	NOZM 685
00265	138*	VOL1=SQRT(U2TBL(I))	NOZM 690
00266	139*	VOL2=SQRT(U2TBL(KKK))	NOZM 695
00267	140*	WRITE (6,450) I,ZZ(I),XX(I),VOL1,KKK,ZZ(KKK),XX(KKK),VOL2	
00301	141*	410 CONTINUE	NOZM 705
00303	142*	430 FORMAT(//,40X, 25HVELOCITY POTENTIAL TABLE ,//,35X, 46HZ = AXIAL PNOZM 710	
00303	143*	10SITION IN NOZZLE (INCHES) ,//,35X,39HX = VELOCITY POTENNOZM 715	
00303	144*	2TIAL (NONDIMENSIONAL) ,//,35X, 69HV = AXIAL VELOCITY (UNDIMENSIONALNOZM 720	
00303	145*	3IZED BY SPEED OF SOUND AT THROAT)	NOZM 725
00303	146*	4 ,//,5X,5HPOINT,11X,1HZ,16X,1HX,15X,1HV,16X,5HPOINT,11X,1HZ,16X,1HNOZM 730	
00303	147*	5X,16X,1HV,//,)	NOZM 735
00304	148*	450 FORMAT(6X,13,6X,3(E12.5,5X),6X,13,6X,2(E12.5,5X),E12.5,)	NOZM 740
00305	149*	470 CONTINUE	NOZM 745
00305	150*	C *****	NOZM 750
00305	151*	C REVERSE ORDER OF TABLE VALUES AND SET UP FOR INT40*	NOZM 755
00306	152*	490 NK = KN+1	NOZM 760
00307	153*	XTABLE(NK) = 0.0	NOZM 765
00310	154*	YTABLE(NK) = 0.0	NOZM 770
00311	155*	DO 510 I = 1,KN	NOZM 775
00314	156*	NK = NK - 1	NOZM 780
00315	157*	XTABLE(I) = XX(NK)	NOZM 785
00316	158*	YTABLE(I) = U2TBL(NK)	NOZM 790
00317	159*	510 CONTINUE	NOZM 795
00317	160*	C	NOZM 800
00321	161*	VELPOT = XX(KN)	
00321	162*	C *****	NOZM 805
00321	163*	C READ ONE CASE AT A TIME*	NOZM 810
00321	164*	C *****	NOZM 815
00322	165*	IWW = 410	NOZM 820
00323	166*	IWO = 2	NOZM 825
00324	167*	NP = DIN(3) + .0001	NOZM 830
00325	168*	IF (CODE - 99.0) 570,570,530	NOZM 835
00330	169*	530 NPR = NP + 3	NOZM 840
00331	170*	NPT = 3 * NP + 409	NOZM 845
00332	171*	CALL PAGE (NPR)	NOZM 850
00333	172*	WRITE (6,550) (DIN(I), I = 410, NPT)	NOZM 855
00341	173*	550 FORMAT(1H0,39X, 6X,4H WN,12X,6H(SNH)N,10X,4H DES //	NOZM 860
00341	174*	1 (40X,3E16.6)	NOZM 865
00342	175*	570 CONTINUE	NOZM 870
00343	176*	590 IF (CODE - 9.0) 670,670,610	NOZM 875

II, D, Program Listing (cont.)

00343	177*	C *****	NOZM 880
00343	178*	C PRINT HEADER AND OUTPUT SYMBOLS*	NOZM 885
00343	179*	C *****	NOZM 890
00346	180*	610 CALL PAGE(70)	NOZM 895
00347	181*	KPAGE=48	NOZM 900
00350	182*	KCOUNT=0	NOZM 905
00351	183*	WRITE (6,630)	NOZM 910
00353	184*	630 FORMAT (1H0, 20H PROGRAM C OUTPUT)	NOZM 915
00354	185*	WRITE (6,650)	NOZM 920
00356	186*	650 FORMAT(/4X,6H(SNH)C,5X2HWC,6X,7HMACH NO,9X,2HAR,14X,2HAI,14X,2HBRNOZM 925	
00356	187*	1,14X,2HBI,14X,2HCR,14X,2HCI/4X,6H(SNH)N,5X2HWN,9X,1HG,6X,13H-AR/(MNOZM 930	
00356	188*	2ACH NO),3X,13H-AI/(MACH NO),9X,2HT1,14X,2HT2,8X,13H-CR/(MACH NO),3NOZM 935	
00356	189*	3X,13H-CI/(MACH NO)/)	NOZM 940
00357	190*	670 CONTINUE	NOZM 945
00360	191*	DO 1310 I = 1,NP	NOZM 950
00363	192*	W = DIN(IWW)	NOZM 955
00364	193*	S = DIN(IWW+1)	NOZM 960
00365	194*	GO TO (690,690,710), MDESIR	NOZM 965
00366	195*	690 DESIRE = DIN(IWW+2)	NOZM 970
00367	196*	710 IWW = IWW + 3	NOZM 975
00367	197*	C	NOZM 980
00370	198*	MX=1	
00371	199*	XPR=0.0	
00372	200*	XMOLD=0.0	NOZM 985
00373	201*	XMNEW=0.0	NOZM 990
00374	202*	XOLD=0.0	NOZM 995
00375	203*	XPT = 0.0	NOZM1000
00376	204*	DXI = -.001	
00377	205*	X = 0.0	
00400	206*	A1 = .5*(G+1.0)	
00401	207*	XNEW=0.0	NOZM1010
00402	208*	XI = -.001	NOZM1015
00403	209*	H=XI	NOZM1020
00404	210*	W2=W*W	NOZM1025
00405	211*	S2=S*S	NOZM1030
00406	212*	G2=G*G	NOZM1035
00406	213*	C *****	NOZM1040
00406	214*	C INITIALIZE 8 EQUATIONS AT X=0.*	NOZM1045
00406	215*	C *****	NOZM1050
00407	216*	DO 730 I = 1,8	NOZM1055
00412	217*	730 E(I) = 1.E-05	NOZM1060
00414	218*	I = 0	NOZM1065
00415	219*	SC = CMPLX(0.0,W)	
00416	220*	CALL BTLOA (EXTRA(91),EXTRA(92),RAT,RCT,G,BT)	
00417	221*	ZETA = ((W2-S2)/4.0-(G-1.)/4.0*SC)/(A1+SC)	
00420	222*	ZETAP = ((SC+A1*(A1+2.*BT))*ZETA+A1*ZETA**2+S2*(1.-G)/8.	
00420	223*	1+SC*(G-1.)/4.*(A1+2.*BT))/(-2.*A1-SC)	
00421	224*	X1 = 0.0	
00422	225*	X1P = S2/4.0/A1/(1.0+SC/A1)	
00423	226*	X2 = 0.0	
00424	227*	X2P = 1.0/(2.0*A1+2.0*SC)	
00425	228*	Y(1) = 0.0	
00426	229*	Y(2) = 0.0	
00427	230*	Y(3) = REAL (X1)	
00430	231*	Y(4) = AIMAG (X1)	
00431	232*	Y(5) = REAL (X2)	
00432	233*	Y(6) = AIMAG (X2)	
00433	234*	Y(7) = REAL(ZETA)	
00434	235*	Y(8) = AIMAG(ZETA)	

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II, D, Program Listing (cont.)

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00435 236*      YP(1) = .5
00436 237*      YP(2) = 0.0
00437 238*      YP(3) = REAL (X1P)
00440 239*      YP(4) = AIMAG (X1P)
00441 240*      YP(5) = REAL (X2P)
00442 241*      YP(6) = AIMAG (X2P)
00443 242*      YP(7) = REAL (ZETAP)
00444 243*      YP(8) = AIMAG (ZETAP)
00445 244*      X = DXI
00446 245*      CALL ADSET(B,Y,YP,YOUT,TEMP,X,H,E)
00447 246*      Y(1)=Y(1)+DXI*YP(1)
00450 247*      Y(2)=Y(2)+DXI*YP(2)
00451 248*      Y(3)=Y(3)+DXI*YP(3)
00452 249*      Y(4)=Y(4)+DXI*YP(4)
00453 250*      Y(5)=Y(5)+DXI*YP(5)
00454 251*      Y(6)=Y(6)+DXI*YP(6)
00455 252*      Y(7)=Y(7)+DXI*YP(7)
00456 253*      Y(8)=Y(8)+DXI*YP(8)
00457 254* 750 CALL ADINT
00457 255* C *****
00457 256* C INTERPOLATING ON VELOCITY POTENTIAL TABLE*
00457 257* C *****
00460 258* 770 CALL INT4D(XTABLE(1),YTABLE(1),X,U2,DU2)
00461 259*      IF (U2=FORLT) 772,772,771
00464 260* 771 U2 = 1.+X+BT*X**2
00465 261*      DU2 = 1.+2.*BT*X
00466 262*      772 C2 = .5*(G+1.0-U2*(G-1.0))
00467 263*      C=SQRT(C2)
00470 264*      U=SQRT(U2)
00471 265*      DC2=-.5*(G-1.0)*DU2
00471 266* C *****
00471 267* C SINGULARITY AT THROAT-FAKE EQUATIONS 7 AND 8 UNTIL PAST*
00471 268* C *****
00472 269*      C2P = C2**((1.0/(G-1.0))
00473 270*      SG = A1*U2/C2*DU2
00474 271*      SH = W*U2
00475 272*      XJ = .25*(W2-U*C2P**G*S2)
00476 273*      XK1=- (G-1.)/4.*U2/C2*DU2*W
00477 274*      GH = CMPLX(SG,SH)
00500 275*      QQ = SC/2./U2
00501 276*      B = U2*(C2-U2)
00502 277*      XJK=CMPLX(XJ,XK1)
00503 278*      F3P= CMPLX(Y(1),Y(2))
00504 279*      X1P= CMPLX(Y(3),Y(4))
00505 280*      X2P= CMPLX(Y(5),Y(6))
00506 281*      ZETAP= CMPLX(Y(7),Y(8))
00507 282*      DZETP = (GH*ZETAP-XJK)/B-ZETAP**2
00510 283*      T1X =ZETAP-SC*(.5/U2+1./A1/(1.0-U2))
00511 284*      DX1P = -T1X*X1P+S2/A1/4.0/U*C2P
00512 285*      DF3P = QQ*F3P+.5*DU2
00513 286*      DX2P = -T1X*X2P+1.0/A1*((DF3P-F3P/C2*DC2)/C2+S2*C2P*U/2.0
00513 287*      1/SC*((1.0-U2)/2.0/U2+F3P/U2))
00514 288*      YP(1) = REAL (DF3P)
00515 289*      YP(2) = AIMAG (DF3P)
00516 290*      YP(3) = REAL (DX1P)
00517 291*      YP(4) = AIMAG (DX1P)
00520 292*      YP(5) = REAL (DX2P)
00521 293*      YP(6) = AIMAG (DX2P)
00522 294*      YP(7) = REAL (DZETP)
00523 295*      YP(8) = AIMAG (DZETP)
00524 296* 930 FORMAT (3X,4E16.5/3X,4E16.5/3X,4E16.5/3X,4E16.5/3X,2E16.5/)

```

NOZM1245
NOZM1250
NOZM1255
NOZM1260

NOZM1270
NOZM1275
NOZM1280
NOZM1285
NOZM1290
NOZM1295

II, D, Program Listing (cont.)

00524	297*	C		NOZM1445
00525	298*		CALL ADCOR(\$770)	
00525	299*	C		NOZM1455
00525	300*	C*****	X AND VELPOT BOTH NEGATIVE	
00525	301*	C*****	TERMINATE AT FINAL VELOCITY POTENTIAL	
00526	302*		IF (X - VELPOT) 990, 990, 750	
00531	303*	990	CALL ADPAR(VELPOT)	
00531	304*	C		NOZM1630
00531	305*	C		NOZM1635
00531	306*	C	*****	NOZM1640
00531	307*	C	COMPUTE ADMITTANCE COEFFICIENTS AND PRINT FINAL RESULTS*	NOZM1645
00531	308*	C	*****	NOZM1650
00532	309*		1010 CALL INT4D (XTABLE, YTABLE, VELPOT, U2, DU2)	
00533	310*		1030 C2=.5*(G+1.0-U2*(G-1.0))	NOZM1660
00534	311*		C=SQRT(C2)	NOZM1665
00535	312*		U=SQRT(U2)	NOZM1670
00536	313*		C2P=C2**(.1/(G-1.))	
00536	314*	C		NOZM1675
00537	315*		F3R = YOUT(1)/C2	
00540	316*		F3I = YOUT(2)/C2	
00541	317*		X1R = YOUT(3)/(1.-U2)	
00542	318*		X1I = YOUT(4)/(1.-U2)	
00543	319*		X2R = YOUT(5)/(1.-U2)	
00544	320*		X2I = YOUT(6)/(1.-U2)	
00545	321*		ZETAR = YOUT(7)	
00546	322*		ZETAI = YOUT(8)	
00547	323*		F3 = CMPLX(F3R,F3I)	
00550	324*		X1 = CMPLX(X1R,X1I)	
00551	325*		X2 = CMPLX(X2R,X2I)	
00552	326*		ZETA = CMPLX(ZETAR,ZETAI)	
00553	327*		ASCRP=1./G*A1**(.1/(G-1.))*U/C2P*(C2*X1-ZETA)/(U2*(C2*X1-ZETA)	
00553	328*		1-SC/2.)	
00554	329*		BSCRIP = SC*GRAD*SQRT(U/C2P)*C2*X1/(U2*(C2*X1-ZETA)-SC/2.)	
00555	330*		CSCRIP=1./SQRT(A1)*U*C2*(.5*(1.-U2)*X1-.5*SC*X2+F3*ZETA)/(U2*(C2*	
00555	331*		X1-ZETA)-.5*SC)	
00556	332*		WCP=GRAD*RAC/RAT/A1**(.5*W	
00557	333*		ESCRP=G*ASCRP+(0.,1.)*BSCRIP/WCP	
00560	334*		AR1 = REAL(ASCRP)	
00561	335*		AI1 = AIMAG(ASCRP)	
00562	336*		BR1 = REAL(BSCRIP)	
00563	337*		BI1 = AIMAG(BSCRIP)	
00564	338*		CR1 = REAL(CSCRIP)	
00565	339*		CI1 = AIMAG(CSCRIP)	
00566	340*		T1 = REAL(ESCRP)	
00567	341*		T2=AIMAG(ESCRP)	
00567	342*	C		NOZM1825
00570	343*		1110 ALPHAR = -AR1/DESIRE	NOZM1830
00571	344*		ALPHAI = -AI1/DESIRE	NOZM1835
00572	345*		CHIR = -CR1/DESIRE	NOZM1840
00573	346*		CHII = -CI1/DESIRE	NOZM1845
00573	347*	C		NOZM1880
00573	348*	C	*****	NOZM1885
00573	349*	C	CASE IS COMPLETED	NOZM1890
00573	350*	C	*****	NOZM1895

A vertical strip of 20 small, rectangular, black-and-white images, likely film frames or microfilm segments, showing various patterns and textures. The images are arranged in a single column, separated by thin white lines. Each frame contains a different pattern, ranging from solid black to complex, abstract shapes and textures. The overall appearance is that of a film strip or a series of microfilm frames.

II, D, Program Listing (cont.)

NO	LINE	CODE	TEXT	NOZM
00573	351*	C		NOZM1900
00573	352*	C	FOR S=0 USE COEFFICIENTS A AND C IN CONC	
00573	353*	C		
00573	354*	C	FOR S=N USE COEFFICIENTS T AND C IN CONC OR DISTR	
00573	355*	C		
00574	356*		IF(S) 1150,1130,1150	
00577	357*	1130	DOUT(IW+19)=W/GE0	
00600	358*		DOUT(IW+49)=AR1	
00601	359*		DOUT(IW+79)=AI1	
00602	360*		DOUT(IW+109)=CR1	
00603	361*		DOUT(IW+139)=CI1	
00604	362*		GO TO 1170	NOZM1950
00605	363*	1150	CONTINUE	NOZM1955
00606	364*		DOUT(IW+19)=W/GE0	NOZM1960
00607	365*		DOUT(IW+49)=T1	NOZM1965
00610	366*		DOUT(IW+79)=T2	NOZM1970
00611	367*		DOUT(IW+109) = CR1	
00612	368*		DOUT(IW+139) = CI1	
00613	369*	1170	IWO =IWO+2	NOZM1985
00614	370*		IF (CODE - 9.0) 1310,1310,1190	NOZM1990
00617	371*	1190	CONTINUE	NOZM1995
00620	372*		KKKN=(2*IW)+1	NOZM2000
00621	373*		IF (HOMEG-KKKN)1210,1230,1230	NOZM2005
00624	374*	1210	KKKN=KKN-1	NOZM2010
00625	375*	1230	KCOUNT=KCOUNT+4	NOZM2015
00626	376*		IF (KCOUNT-KPAGE)1270,1270,1250	NOZM2020
00631	377*	1250	CALL PAGE(70)	NOZM2025
00632	378*		WRITE (6,650)	NOZM2030
00634	379*		KCOUNT=0	NOZM2035
00635	380*		KPAGE=48	NOZM2040
00635	381*	C	*****	NOZM2045
00635	382*	C	PRINT FINAL RESULTS*	NOZM2050
00635	383*	C	*****	NOZM2055
00636	384*	1270	WRITE (6,1290)WC(1),WC(KKKN),DESIRE,AR1,AI1,BR1,BI1,CR1,CI1,S,W,G,	NOZM2060
00636	385*		1ALPHAR,ALPHA1,T1,T2,CHIR,CHII,VELPOT	
00663	386*	1290	FORMAT(2(3X,F7.4,2X,F7.4,3X,F7.4,6E16.5/))	NOZM2070
00664	387*		FCCPS=,WCONST*WC(KKKN))*12.0	NOZM2075
00665	388*		WRITE (6,1330)FCCPS	NOZM2080
00670	389*	1310	CONTINUE	NOZM2085
00672	390*	1330	FORMAT(4X,8HFC(CPS)=F10.4//)	NOZM2090
00673	391*		DOUT(205)=DESIRE	NOZM2095
00674	392*		IF(S) 1370,1350,1370	NOZM2100
00677	393*	1350	DOUT(1)=NP	NOZM2105
00700	394*		DOUT(101)=NP	NOZM2110
00701	395*		RETURN	NOZM2115
00702	396*	1370	DOUT(18)=NP	NOZM2120
00703	397*		DOUT(NP+21)=0.0	NOZM2125
00704	398*		DOUT(NP+51)=0.0	NOZM2130
00705	399*		DOUT(NP+81)=0.0	NOZM2135
00706	400*		DOUT(NP+111)=0.0	NOZM2140
00707	401*		DOUT(NP+141)=0.0	NOZM2145
00710	402*		RETURN	NOZM2150
00711	403*		END	NOZM2155

II, D, Program Listing (cont.)

LL1 SUB28,1,700708, 29803

```

000001      SUBROUTINE BTLDA(RSATI,RSCTI,RSATO,RSCTO,G,BT)
000002      DIMENSION XM(10),ZS(10),XMS(10),XMS2(10),PHI(10),
000003      1 T1(10),T2(10),T3(10),W(10),C(10),ALPHA(10),BETA(10),A(10)
000004      2,B(10)
000005      DIMENSION E(10),ARAT(10),ZSA(10),ARA(10)
000006      L = 0
000007      J = 0
000008      N = 5
000009      KC = 2
000010      K = 2
000011      ARAT(1) = 1.
000012      XM(1) = 1.
000013      XMS(1) = 1.
000014      XM(2) = .99
000015      XMS2(1) = 1.
000016      ZSG=-ABS(RSCTO/2.)
000017      DO 20 I = 2,5
000018      XM(I+1) = XM(I)-.01
000019      ARAT(I) = 1./XM(I)*(2./(G+1.)*(1.+(G-1.)/2.*XM(I)*XM(I))**((G+1.)
000020      1/2./(G-1.))
000021      20 XMS(I) = SQRT((G+1.)/2.*XM(I)**2/(1.+(G-1.)/2.*XM(I)**2))
000022      48 RSO = RSATO+RSCTO-SQRT(ABS(RSCTO*RSCTO-ZSG*ZSG))
000023      RSI = RSATI + RSCTI-SQRT(ABS(RSCTI*RSCTI-ZSG*ZSG))
000024      IF (RSCTI) 47,49,46
000025      49 IF (RSATI) 51,51,46
000026      51 RSI = 0.
000027      GO TO 46
000028      47 RSI = RSATI + RSCTI+SQRT(ABS(RSCTI*RSCTI-ZSG*ZSG))
000029      46 ARATI = (RSO*RSO-RSI*RSI)/(RSATO*RSATO-RSATI*RSATI)
000030      IF(ARATI-ARAT(5)) 50,60,52
000031      52 IF(M.EQ. 2) GO TO 60
000032      ZSG=ZSG/2.
000033      M = 1
000034      GO TO 48
000035      50 ZSG=ZSG*2.
000036      IF(M.EQ. 1) GO TO 60
000037      M = 2
000038      GO TO 48
000039      60 ZSA(2) = ZSG/5.
000040      DZ = ZSA(2)
000041      ZSA(1) = 0.
000042      ARA(1) = 1.
000043      DO 80 I = 2,5
000044      RSO = RSATO +RSCTO-SQRT(ABS(RSCTO*RSCTO-ZSA(I)*ZSA(I)))
000045      RSI = RSATI+RSCTI-SQRT(ABS(RSCTI*RSCTI-ZSA(I)*ZSA(I)))
000046      IF(RSCTI) 82,83 ,81
000047      83 IF (RSATI) 84,84,81
000048      84 RSI = 0.
000049      GO TO 81
000050      82 RSI = RSATI+RSCTI-SQRT(ABS(RSCTI*RSCTI-ZSA(I)*ZSA(I)))
000051      81 ARA(I)=(RSO*RSO-RSI*RSI)/(RSATO*RSATO-RSATI*RSATI)
000052      80 ZSA(I+1)=ZSA(I)+DZ
000053      CALL ORTHLS(ZSA,ARA,W,N,L,J,C,ALPHA,BETA,K,T1,T2,T3,IND1)
000054      CALL COEFS(J,C,ALPHA,BETA,KC,A,T1,T2,T3,IND2)
000055      DO 90 I=2,5

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*NEW
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II, D, Program Listing (cont.)

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000056      90  ZS(1) =-SQRT(-(A(1)-ARAT(I))/A(3))
000057      CALL ORTHLS(ZS,XMS,W,N,L,J,C,ALPHA,BETA,K,T1,T2,T3,IND1)
000058      CALL COEFS(J,C,ALPHA,BETA,KC,B,T1,T2,T3,IND2)
000059      PHI(1) = 0.
000060      DO 30 I = 2,5
000061      PHI(I) = 2.*B(2)      *(B(1)*ZS(I)+B(2)/2.*ZS(I)**2+B(3)*ZS(I)**3
000062      1/3.)
000063      30  XMS2(I) = XMS(I)**2
000064      CALL ORTHLS(PHI,XMS2,W,N,L,J,C,ALPHA,BETA,K,T1,T2,T3,IND1)
000065      CALL COEFS(J,C,ALPHA,BETA,KC,F,T1,T2,T3,IND2)
000066      BT = E(3)
000067      RETURN
000068      END

```

II, D, Program Listing (cont.)

* ELI SUB29,1,700707, 30007

```

000001      SUBROUTINE ORTHLS (X,Y,W,N,L,J,C,ALPHA,BETA,K,T1,T2,T3,IND1)      ORTHLS
000002      C-----ORTHLS
000003      C   THIS SUBROUTINE COMPUTES THE COEFFICIENTS OF THE POLYNOMIAL      ORTHLS
000004      C   EQUATION OF DEGREE K AND THE ALPHA AND BETA PARAMETERS.          ORTHLS
000005      C-----ORTHLS
000006      DIMENSION X(N),Y(N),W(N),C(K),ALPHA(K),BETA(K),T1(N),T2(N),T3(N)      ORTHLS
000007      C-----ORTHLS
000008      C   PROGRAM INITIALIZATION.                                          ORTHLS
000009      C-----ORTHLS
000010      KJ1=K-J+1                                                            ORTHLS
000011      IF (KJ1.LE.0) GO TO 16                                              ORTHLS
000012      SUM=0.0                                                            ORTHLS
000013      IF (L.EQ.1) GO TO 3                                                 ORTHLS
000014      DO 2 I=1,N                                                         ORTHLS
000015      T3(I)=X(I)                                                         ORTHLS
000016      IF (J.GT.0) GO TO 1                                               ORTHLS
000017      SUM=SUM+1.0                                                        ORTHLS
000018      GO TO 2                                                            ORTHLS
000019      1 SUM=SUM+X(I)**(2*J)                                                ORTHLS
000020      2 W(I)=1.0                                                         ORTHLS
000021      GO TO 7                                                            ORTHLS
000022      3 DO 6 I=1,N                                                         ORTHLS
000023      T3(I)=X(I)                                                         ORTHLS
000024      IF (J.GT.0) GO TO 4                                                 ORTHLS
000025      SUM=SUM+W(I)                                                       ORTHLS
000026      GO TO 5                                                            ORTHLS
000027      4 SUM=SUM+W(I)*X(I)**(2*J)                                          ORTHLS
000028      5 X(I)=W(I)*X(I)                                                  ORTHLS
000029      6 Y(I)=W(I)*Y(I)                                                  ORTHLS
000030      7 B=0.0                                                            ORTHLS
000031      R0=SUM                                                             ORTHLS
000032      DO 9 I=1,N                                                         ORTHLS
000033      IF (J.GT.0) GO TO 8                                                 ORTHLS
000034      T2(I)=1.0                                                         ORTHLS
000035      GO TO 9                                                            ORTHLS
000036      8 T2(I)=T3(I)**J                                                  ORTHLS
000037      9 T1(I)=0.0                                                       ORTHLS
000038      C-----ORTHLS
000039      C   BEGIN COMPUTATION.                                              ORTHLS
000040      C-----ORTHLS
000041      II=1                                                                ORTHLS
000042      10 S=0.0                                                            ORTHLS
000043      DO 11 I=1,N                                                         ORTHLS
000044      11 S=S+Y(I)*T2(I)                                                  ORTHLS
000045      C-----ORTHLS
000046      C   COMPUTATION OF A COEFFICIENT IN THE POLYNOMIAL EQUATION.      ORTHLS
000047      C-----ORTHLS
000048      C(II)=S/R0                                                         ORTHLS
000049      IF (II.GE.KJ1) GO TO 15                                             ORTHLS
000050      C-----ORTHLS
000051      C   COMPUTATION OF AN ALPHA FOR THE POLYNOMIAL EQUATION.          ORTHLS
000052      C-----ORTHLS
000053      SUMXPS=0.0                                                         ORTHLS
000054      DO 12 I=1,N                                                         ORTHLS
000055      12 SUMXPS=SUMXPS+X(I)*T2(I)*T2(I)                                  ORTHLS

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II, D, Program Listing (cont.)

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000056      ALPHA(II)=SUMXPS/RO                                ORTHLS
000057 C-----                                                ORTHLS
000058 C      COMPUTATION OF A NEW POLYNOMIAL.                    ORTHLS
000059 C-----                                                ORTHLS
000060      DO 13 I=1,N                                           ORTHLS
000061      TEMP=T2(I)                                           ORTHLS
000062      T2(I)=(T3(I)-ALPHA(II))*T2(I)-B*T1(I)              ORTHLS
000063      13 T1(I)=TEMP                                         ORTHLS
000064 C-----                                                ORTHLS
000065 C      COMPUTATION OF A BETA FOR THE POLYNOMIAL EQUATION.  ORTHLS
000066 C-----                                                ORTHLS
000067      R=0.0                                                  ORTHLS
000068      DO 14 I=1,N                                           ORTHLS
000069      14 R=R+W(I)*T2(I)*T2(I)                               ORTHLS
000070      BETA(II)=R/RO                                         ORTHLS
000071      RO=R                                                  ORTHLS
000072      B=BETA(II)                                           ORTHLS
000073      II=II+1                                              ORTHLS
000074      GO TO 10                                             ORTHLS
000075 C-----                                                ORTHLS
000076 C      SUCCESSFUL RETURN.                                    ORTHLS
000077 C-----                                                ORTHLS
000078      15 IND1=+1                                             ORTHLS
000079      RETURN                                                 ORTHLS
000080 C-----                                                ORTHLS
000081 C      ERROR RETURN. SET ALL C COEFFICIENTS, ALPHA AND BETA TO ZERO. ORTHLS
000082 C-----                                                ORTHLS
000083      16 DO 17 II=1,K                                       ORTHLS
000084      C(II)=0.0                                              ORTHLS
000085      ALPHA(II)=0.0                                         ORTHLS
000086      17 BETA(II)=0.0                                       ORTHLS
000087      C(K+1)=0.0                                           ORTHLS
000088      IND1=-1                                               ORTHLS
000089      RETURN                                               ORTHLS
000090      END                                                    ORTHLS

```

II, D, Program Lising (cont.)

* ELT SUB30,1,700/07, 30008

```

000001      SUBROUTINE COEFS (J,C,ALPHA,BETA,KC,A,T1,T2,T3,IND2)      COEFS
000002      C-----COEFS
000003      C   THIS SUBROUTINE COMPUTES THE A COEFFICIENTS FOR A POLYNOMIAL      COEFS
000004      C   OF DEGREE KC WHERE KC IS LESS THAN OR EQUAL TO K.      COEFS
000005      C-----COEFS
000006      DIMENSION C(KC),ALPHA(KC),BETA(KC),A(KC),T1(KC),T2(KC),T3(KC)      COEFS
000007      C-----COEFS
000008      C   PROGRAM INITIALIZATION.      COEFS
000009      C-----COEFS
000010      KCJ1=KC-J+1      COEFS
000011      IF(KCJ1.LE.0) GO TO 9      COEFS
000012      B=0.0      COEFS
000013      DO 1 NN=1,KCJ1      COEFS
000014      A(NN)=C(NN)      COEFS
000015      T1(NN)=0.0      COEFS
000016      T2(NN)=0.0      COEFS
000017      1 T3(NN)=0.0      COEFS
000018      IF (KC.LE.J) GO TO 5      COEFS
000019      II=2      COEFS
000020      C-----COEFS
000021      C   BEGIN COMPUTATION.      COEFS
000022      C-----COEFS
000023      2 T2(II)=1.0      COEFS
000024      DO 3 NN=2,II      COEFS
000025      T3(NN)=T2(NN-1)-T2(NN)*ALPHA(II-1)-B*T1(NN)      COEFS
000026      C-----COEFS
000027      C   COMPUTATION OF AN A COEFFICIENT.      COEFS
000028      C-----COEFS
000029      3 A(NN-1)=A(NN-1)+C(II)*T3(NN)      COEFS
000030      IF (II.GE.KCJ1) GO TO 5      COEFS
000031      C-----COEFS
000032      C   RESETTING THE VECTORS FOR THE NEXT COEFFICIENT.      COEFS
000033      C-----COEFS
000034      DO 4 NN=1,II      COEFS
000035      T1(NN)=T2(NN)      COEFS
000036      4 T2(NN)=T3(NN)      COEFS
000037      B=BETA(II-1)      COEFS
000038      II=II+1      COEFS
000039      GO TO 2      COEFS
000040      5 IF (J.LE.0) GO TO 8      COEFS
000041      C-----COEFS
000042      C   ARRANGE COEFFICIENTS PROPERLY IF J IS NON ZERO.      COEFS
000043      C-----COEFS
000044      DO 6 NN=1,KCJ1      COEFS
000045      N1=KCJ1-NN+1      COEFS
000046      N2=N1+J      COEFS
000047      6 A(N2)=A(N1)      COEFS
000048      DO 7 NN=1,J      COEFS
000049      7 A(NN)=0.0      COEFS
000050      C-----OEFS
000051      C   SUCCESSFUL RETURN.      OEFS
000052      C-----OEFS
000053      8 IND2=+2      OEFS
000054      RETURN      OEFS
000055      C-----OEFS

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II, D, Program Listing (cont.)

000056	C	ERROR RETURN. SET ALL THE A COEFFICIENTS EQUAL TO ZERO.	COEFS
000057	C	-----	COEFS
000058	9	DO 10 NN=1,KC	COEFS
000059	10	A(NN)=0.0	COEFS
000060		A(KC+1)=0.0	COEFS
000061		IND2=-2	COEFS
000062		RETURN	COEFS
000063		END	COEFS

II, D, Program Listing (cont.)

* ELT BESJ,1,690714, 35959

000001	SUBROUTINE BESJ(X,N,BJ,D,IER)	BESJ	1
000002	BJ=.0	BESJ	2
000003	IF(N)10,20,20	BESJ	3
000004	10 IER=1	BESJ	4
000005	RETURN	BESJ	5
000006	20 IF(X)30,30,31	BESJ	6
000007	30 IER=2	BESJ	7
000008	RETURN	BESJ	8
000009	31 IF(X-15.)32,32,34	BESJ	9
000010	32 NTEST=20.+10.*X-X** 2/3	BESJ	10
000011	GO TO 36	BESJ	11
000012	34 NTEST=90.+X/2.	BESJ	12
000013	36 IF(N-NTEST)40,38,38	BESJ	13
000014	38 IER=4	BESJ	14
000015	RETURN	BESJ	15
000016	40 IER=0	BESJ	16
000017	N1=N+1	BESJ	17
000018	BPREV=.0	BESJ	18
000019	C COMPUTE STARTING VALUE OF M	BESJ	19
000020	IF(X-5.)50,60,60	BESJ	20
000021	50 MA=X+6.	BESJ	21
000022	GO TO 70	BESJ	22
000023	60 MA=1.4*X+60./X	BESJ	23
000024	70 MB=N+IFIX(X)/4+2	BESJ	24
000025	MZERO=MA	BESJ	25
000026	IF(MA-MB)80,90,90	BESJ	26
000027	80 MZERO=MB	BESJ	27
000028	C SET UPPER LIMIT OF M	BESJ	28
000029	90 MMAX=NTEST	BESJ	29
000030	100 DO 190 M=MZERO,MMAX,3	BESJ	30
000031	C SET F(M),F(M-1)	BESJ	31
000032	FM1=1.0E-28	BESJ	32
000033	FM=.0	BESJ	33
000034	ALPHA=.0	BESJ	34
000035	IF(M-(M/2)*2)120,110,120	BESJ	35
000036	110 JT=-1	BESJ	36
000037	GO TO 130	BESJ	37
000038	120 JT=1	BESJ	38
000039	130 M2=M-2	BESJ	39
000040	DO 160 K=1,M2	BESJ	40
000041	MK=M-K	BESJ	41
000042	BMK=2.*FLOAT(MK)*FM1/X-FM	BESJ	42
000043	FM=FM1	BESJ	43
000044	FM1=BMK	BESJ	44
000045	IF(MK-N-1)150,140,150	BESJ	45
000046	140 BJ=BMK	BESJ	46
000047	150 JT=-JT	BESJ	47
000048	S=1+JT	BESJ	48
000049	160 ALPHA=ALPHA+BMK*S	BESJ	49
000050	BMK=2.*FM1/X-FM	BESJ	50
000051	IF(N)180,170,180	BESJ	51
000052	170 BJ=BMK	BESJ	52
000053	180 ALPHA=ALPHA+BMK	BESJ	53
000054	BJ=BJ/ALPHA	BESJ	54
000055	IF(ABS(BJ-BPREV)-ABS(D*BJ))200,200,190	BESJ	55

Report 20672-P3D

II, D, Program Listing (cont.)

```
000056      190 BPREV=BJ
000057          IER=3
000058      200 RETURN
000059          END
```

```
BESJ 56
BESJ 57
BESJ 58
BESJ 59
```

II, D, Program Listing (cont.)

• ELT BESY,1,690714, 35961

000001	SUBROUTINE BESY(X,N,BY,IER)	BESY	1
000002	C CHECK FOR ERRORS IN N AND X	BESY	2
000003	IF(N)180,10,10	BESY	3
000004	10 IER=0	BESY	4
000005	IF(X)190,190,20	BESY	5
000006	20 P1=3.141592653	BESY	6
000007	C BRANCH IF X LESS THAN OR EQUAL 4	BESY	7
000008	IF(X-4.)40,40,30	BESY	8
000009	C COMPUTE Y0 AND Y1 FOR X GREATER THAN 4	BESY	9
000010	30 I=4./X	BESY	10
000011	P0=.3989422793	BESY	11
000012	Q0=-.0124669441	BESY	12
000013	P1=.3989422819	BESY	13
000014	Q1=.0374008364	BESY	14
000015	A=T*T	BESY	15
000016	B=A	BESY	16
000017	P0=P0-.0017530620*A	BESY	17
000018	Q0=Q0+.0004564324*A	BESY	18
000019	P1=P1+.0029218256*A	BESY	19
000020	Q1=Q1-.00003904*A	BESY	20
000021	A=A*A	BESY	21
000022	P0=P0+.00017343*A	BESY	22
000023	Q0=Q0-.0000869791*A	BESY	23
000024	P1=P1-.000223203*A	BESY	24
000025	Q1=Q1+.0001064741*A	BESY	25
000026	A=A*B	BESY	26
000027	P0=P0-.0000487613*A	BESY	27
000028	Q0=Q0+.0000342468*A	BESY	28
000029	P1=P1+.0000580759*A	BESY	29
000030	Q1=Q1-.0000398708*A	BESY	30
000031	A=A*B	BESY	31
000032	P0=P0+.0000173565*A	BESY	32
000033	Q0=Q0-.0000142078*A	BESY	33
000034	P1=P1-.000020092*A	BESY	34
000035	Q1=Q1+.00001622*A	BESY	35
000036	A=A*B	BESY	36
000037	P0=P0-.0000037043*A	BESY	37
000038	Q0=Q0+.0000032312*A	BESY	38
000039	P1=P1+.0000042414*A	BESY	39
000040	Q1=Q1-.0000036594*A	BESY	40
000041	A=SQRT(2.*PI)	BESY	41
000042	B=4.*A	BESY	42
000043	P0=A*P0	BESY	43
000044	Q0=B*Q0/X	BESY	44
000045	P1=A*P1	BESY	45
000046	Q1=B*Q1/X	BESY	46
000047	A=X-PI/4.	BESY	47
000048	B=SQRT(2./(PI*X))	BESY	48
000049	Y0=B*(P0*SIN(A)+Q0*COS(A))	BESY	49
000050	Y1=B*(-P1*COS(A)+Q1*SIN(A))	BESY	50
000051	GO TO 90	BESY	51
000052	C COMPUTE Y0 AND Y1 FOR X LESS THAN OR EQUAL TO 4	BESY	52
000053	40 XX=X/2.	BESY	53
000054	X2=XX*XX	BESY	54
000055	T=ALOG(XX)+.5772156649	BESY	55

Report 20672-P3D

II, D, Program Listing (cont.)

000056	SUM=0.	BESY	56
000057	TERM=T	BESY	57
000058	Y0=T	BESY	58
000059	DO 70 L=1,15	BESY	59
000060	IF(L-1)50,60,50	BESY	60
000061	50 SUM=SUM+1./FLOAT(L-1)	BESY	61
000062	60 FL=L	BESY	62
000063	TS=T-SUM	BESY	63
000064	TERM=(TERM*(-X2)/FL**2)*(1.-1./(FL*TS))	BESY	64
000065	70 Y0=Y0+TERM	BESY	65
000066	TERM = XX*(T-.5)	BESY	66
000067	SUM=0.	BESY	67
000068	Y1=TERM	BESY	68
000069	DO 80 L=2,16	BESY	69
000070	SUM=SUM+1./FLOAT(L-1)	BESY	70
000071	FL=L	BESY	71
000072	FL1=FL-1.	BESY	72
000073	TS=T-SUM	BESY	73
000074	TERM=(TERM*(-X2)/(FL1*FL))*((TS-.5/FL)/(TS+.5/FL1))	BESY	74
000075	80 Y1=Y1+TERM	BESY	75
000076	PI2=2./PI	BESY	76
000077	Y0=PI2*Y0	BESY	77
000078	Y1=-PI2/X+PI2*Y1	BESY	78
000079	C CHECK IF ONLY Y0 OR Y1 IS DESIRED	BESY	79
000080	90 IF(N-1)100,100,130	BESY	80
000081	C RETURN EITHER Y0 OR Y1 AS REQUIRED	BESY	81
000082	100 IF(N)110,120,110	BESY	82
000083	110 BY=Y1	BESY	83
000084	GO TO 170	BESY	84
000085	120 BY=Y0	BESY	85
000086	GO TO 170	BESY	86
000087	C PERFORM RECURRENCE OPERATIONS TO FIND YN(X)	BESY	87
000088	130 YA=Y0	BESY	88
000089	YB=Y1	BESY	89
000090	K=1	BESY	90
000091	140 T=FLOAT(2*K)/X	BESY	91
000092	YC=T*YB-YA	BESY	92
000093	K=K+1	BESY	93
000094	IF(K-N)150,160,150	BESY	94
000095	150 YA=YB	BESY	95
000096	YB=YC	BESY	96
000097	GO TO 140	BESY	97
000098	160 BY=YC	BESY	98
000099	170 RETURN	BESY	99
000100	180 IER=1	BESY	100
000101	RETURN	BESY	101
000102	190 IER=2	BESY	102
000103	RETURN	BESY	103
000104	END	BESY	104

II, D, Program Listing (cont.)

W ELT BESSEL,1,690716, 76335

```

000001      SUBROUTINE BESSEL(J,Y,V,X,K)
000002      INTEGER V
000003      REAL J,Y
000004      DIMENSION J(1),Y(1)
000005      D=1.E-6
000006      NV=V+1
000007      DO 1 I=1,NV
000008      CALL BESJ(X,I-1,J(I),D,IER )
000009      IF(IER.NE.0) GO TO 10
000010      CALL BESY(X,I-1,Y(I),IER)
000011      IF(IER.NE.0) GO TO 11
000012      1  CONTINUE
000013      RETURN
000014      10  WRITE(6,100)X,V
000015      GO TO 12
000016      11  WRITE(6,101)X,V
000017      12  K=1
000018      RETURN
000019      100 FORMAT(10X,28HERROR IN BESJ, X AND V ARE. ,2E15.7)
000020      101 FORMAT(10X,28HERROR IN BESY, X AND V ARE. ,2E15.7)
000021      END

```

*NEW

**-1

*NEW

*NEW

**-1

II, Programming (cont.)

E. METHODS OF VERIFICATION

For most sections of this program, the only method of verification is the "reasonableness" of the numbers output. This is a nebulous statement and the only way an engineer can know what is reasonable is from experience with the sensitive time lag theory and its application. There are some guidelines that can be given, however, that may help those new to the program.

1. If A_{vn} , B_{vn} , and C_{vn} are 1.0, the n minimum will be on the order of 0.5 to 1.0. It will always be positive.
2. The frequency of the n minimum is near the acoustic mode frequency of a cylinder. This frequency is given in a formula on Figure 2.
3. The calculation of A_{vn} , B_{vn} , and C_{vn} can be checked by running a case with combustion concentrated at a particular location. The answer can then be easily checked using the formulas in Section I,B,4,b (5) and Figure 8.
4. When τ at n minimum is given in seconds, it can be converted to an equivalent frequency by the relation $\tau^* = \frac{1}{2f^*}$.
5. The test cases given in this manual are a convenient reference to establish whether the program and computer are working correctly.

III. DECK SETUP

A. COMPUTER CONFIGURATION

1. Univac 1108 computer, 65K of core minimum
2. 46211₈ for code, 72245₈ for data
3. FORTRAN V
4. Executive II monitor system
5. No plot output required
6. No punch output required
7. Units 12, 13, 14 are used for temporary storage.

B. ESTIMATED RUNNING TIME

Because of the extremely large number of ways of running this program, it would be very difficult to give a formula for running time. As a guide, the sample case given in Section III,L ran in 2.30 minutes. The nozzle admittance calculation is the single most time consuming portion of the program taking about 80% of the above time.

III, Deck Setup (cont.)

C. DECK SEQUENCE

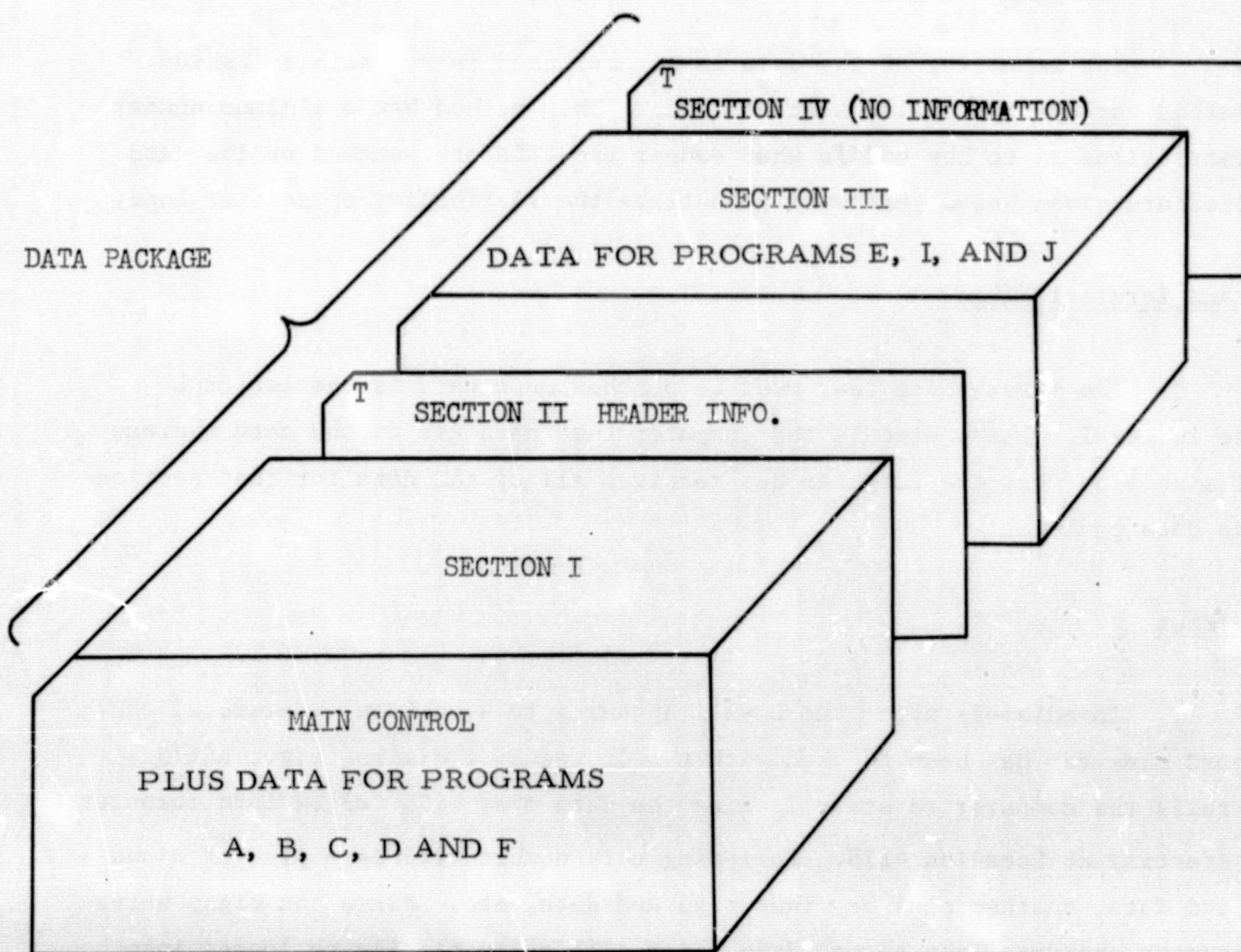
It is necessary to discuss the general organization of the data as well as how the data actually get into the computer before proceeding with any further discussion of the computer program. Hereafter, the input data deck will be referred to as a set of data packages.

The manner in which the data deck is assembled is shown in Figure 17. It can be seen that the data package (one package required per case), is divided into four sections. The first section contains the data for MAIN CONTROL and for Programs A, B, C, D and F. The second section is one card that has the letter T punched in card column one. This card serves the purpose of an end-data signal and of a header card such that all information that follows the T will be printed on the top of each output page. The third section contains the data for Programs E, I, and J. The fourth section is a T-card that is used only to signal the end of that data.

It should be noted that if programs E, I, or J are not run, Section III and IV are not needed. Sections I and II, however, must always be in the data package.

Running Multiple Cases

The flexibility of the program is such that stacking cases (i.e., running multiple cases) is an easy task. All that is required is to add the required number of data packages to the data deck. There is no restriction as to the flow paths of individual data packages; therefore, data packages of parametric studies involving the use of one program can be intermingled with data packages that use any of the other program combinations for an analysis.



IF SECTION III IS NOT USED, SECTION IV MUST NOT BE USED.

Figure 17 -- Input Load Sequence Required by the Computer Program

III, C, Deck Sequence (cont.)

Getting the Data Onto the Card

The inputting of the data is accomplished in a flexible fashion. The method employed is called scatter load.* This method has a minimum number of restrictions as to how and in what manner the data are punched on the card. Examples are given below that will illustrate the flexibility of scatter load.

Load and Terminal Flags

The first thing that must be punched on a card is the letter L or the letter T. The L signals the computer that data are on the card whereas the T signifies that the computer has received all of the data for that section of the data package.

Data Input

Immediately after the L will appear 1 to 4 numbers; therefore, the card thus far has been punched with L followed by 4 digits, e.g., L4175. This tells the computer to start loading the data that will follow into computer core starting at Location 4175. Following this number will be a plus or minus sign and data, another plus or minus sign and data, etc. Since the signs serve to separate the data, the second data point will automatically be loaded into core location 4176. This process continues across the first 72 columns of the data card.

Consider the following example of input data

*Except for the input to main control.

III, C, Deck Sequence (cont.)

L0 + 10.0

L1 + 19.4

L2 + 0.0

L3 - 0.92

L8 + 6.4

This may be punched on a single card as

(Example 1) L0 + 10.0 + 19.4 + 0.0 - 0.92 L8 + 6.4

(Example 2) L0 + 10.0 + 19.4 + -0.92 + + + + 6.4

(Example 3) L8 + 6.4 L0 + 10.0 + 19.4 + -0.92

(Example 4) L39 + 0.21 + 3.14 + 5.0 + 1.5 -35.5 L39 + 1545.

Example 1 illustrates a normal load sequence for the given input. No data are transferred into core locations L4 thru L7; therefore, any previous data stored there remains there. Example 2 illustrates how algebraic signs can be used to index the core location to the proper station. Note also the L2, which is identically zero, has no number punched on the card. This illustrates that the computer, when it sees no number following the sign, loads the value of zero into that core location. If data have been previously stored in Locations L4 thru L7 and are to be used again, the use of indexing presented in Example 2 will load zeros into those core locations; consequently, the data will be lost.

Example 3 illustrates that the L-numbers do not have to be punched in sequential order. Example 4 illustrates the method of correcting data. Specifically, L39 was initially loaded with 0.21 and later loaded with 1545.0; therefore, the former number is erased on core and the latter included in its place. The most convenient method for correcting data cards is to punch all the corrections on one or more cards and place these cards immediately before the T-card of that data section.

III, C, Deck Sequence (cont.)

Presented below are some basic rules to employ when punching data cards:

1. DO start each input card with L. The L does not have to be punched in card column 1 but it must appear before the data.
2. DO pay close attention to the sequence of the remaining data included on the card so that data are not loaded into the wrong core location.
3. DO NOT use more than 8 significant digits (including decimal point) for the input data. Exponential notation is permitted. That is, the number 0.0625 can be loaded as 6.25 E-02.
4. DO NOT start the data on one card and complete it on another.
5. DO NOT use more than 72 card columns for data input. The computer looks at 73 to 80 but does not transfer that information to core. Therefore, columns 73 and 80 can be used for the card sequence number in the data deck or identification of some significant aspect of the data so that it can be identified at some later date.
6. DO include a T-card at the end of a data section in the data package. The T must be punched in card column 1. If the T appears anywhere else on the card, the computer will dump the entire run.
7. DO include a description of the case on the T-card of Section 2. This information serves as a header for each output page. This is a convenience more than a necessity.

III, C, Deck Sequence (cont.)

8. DO NOT use two or more T cards for a header. This can confuse the computer's input logic.

Thus far the storage locations for the individual data bits have not been noted. They will be provided in Section III,E. It is sufficient to state here that the input data have reserved locations in core and that these data remain in those locations unless over-written by the next set of input data. Therefore, it is not necessary to input all the data on the next run if only one parameter (e.g., the ratio of the specific heats) is to be changed. Only the data that changes from one case to the next have to be input on successive data packages (except for main control).

Operation of MAIN CONTROL

Since the computer obeys every command explicitly, it is necessary to indicate correctly to the computer the programs that are desired for a particular case. Furthermore, this must be done for every case because the computer will turn off the switch that activated the program after it is through with the program. This is done purposely to avoid using a program that is not needed for the second case.

The first 10 core locations are reserved for MAIN CONTROL in the following order:

- L0: Program A, Distributed Combustion Analysis (Transverse Modes Only)
- L1: Program B, Concentrated Combustion Analysis
- L2: Program C, Exhaust Nozzle Admittance Coefficients for Longitudinal and Transverse Modes

III, C, Deck Sequence (cont.)

- L3: Program D, Expansion of Results from Program B.
- L4: Program E, Injector Nonuniformity Coefficients
- L5: Program F, Final Solution of Instability Zones
- L6: Program G, Not used in this package
- L7: Program H, Not used in this package.
- L8: Program I, Nonlinear Combustion Response Analysis
- L9: Program J, Injected Mass Distribution Effects

A number greater than zero instructs the computer to execute this program at the time it is required. The program knows when to execute the program and a number in the proper place tells the computer whether or not to execute the program. Running Program A or B automatically runs D and F as well. This is done to simplify the operation of this program.

Program Options

Most programs have various options concerning the desired output to be printed. These options are exercised by the same control number that executes the program and are keyed by the magnitude of the number. These print options are presented in the discussion of the individual programs. However, a few examples are presented here to illustrate this point:

EXAMPLE 1: EXECUTE PROGRAM C - PRINT NO OUTPUT

LO + + + 9.0 + + + + + (0<L2≤9.0)

EXAMPLE 2: EXECUTE PROGRAM C - PRINT OUTPUT

LO + + + 99.0 + + + + + (10<L2 ≤ 99.0)

III, C, Deck Sequence (cont.)

EXAMPLE 3: EXECUTE PROGRAM C - PRINT INPUT AND OUTPUT

L0 + + + 199.0 + + + + + (100 < L2 ≤ 199.0)

III, Deck Setup (cont.)

D. INPUT DATA

1. Program A: Distributed Combustion Stability Analysis

a. Input Requirements: The following data go into Section I of data package.

<u>Core Location</u>	<u>Mnemonic</u>	<u>Description</u>
L0	--	To execute Program A L0 must be a positive number. The magnitude of the number determines the print options as shown in paragraph III,D,1,b.
L10	γ	Ratio of the specific heats.
L11	M_e	Steady-state Mach number at the entrance to the exhaust nozzle. This can be determined by the contraction ratio or by Program C. In the latter case, leave L11 blank.
L13	r_c^*	Chamber radius, in. also loaded in L3804, Program C.
L14	L_c^*	Length of cylindrical portion of chamber, in.
L15	c_o^*	Speed of sound in the chamber, ft/sec.
L16	u_{LO}^*	Weighted liquid injection velocity, ft/sec. See Equation (75) at the end of this section.
L17	κ	Gas/liquid momentum interchange coefficient. ($\kappa = 0$ for no droplet momentum effects)
L20	$s_{v\eta}$	Mode number. These numbers are given in Section III,D,1,e, figure 9. Must not be zero for this program.
L21	N_{wc}	Number of chamber frequencies to be used. Leaving this column blank will turn on Program GENMEC which will select 10 frequencies according to the relationship given by Eq. (76).
L22	ω	Table of nondimensional, chamber frequencies arranged in ascending order. A zero must be included at the end of the table. The maximum number of frequencies is 28 (including the zero point). This is left blank if L21 is blank.
.		
.		
.		
.		
.		
L49		

III, D, Input Data (cont.)

<u>Core Location</u>	<u>Mnemonic</u>	<u>Description</u>
L50 . . . L69	Z*	Table of ascending axial positions at which the combustion distribution (i.e., the variation of the local Mach number with respect to Z) is known. Dimensioned in inches. A zero value must appear at the last point. Minimum of four real data points is required.
L70 . . L89	M(Z*)	The local steady-state Mach number that corresponds to the given L_z . A zero <u>must</u> appear as the last value.
L3017 L3019 . . . L3048	N_ω	The number of input frequencies at which the nozzle admittance coefficients are known. This number must be greater than 2 and less than 30. If the admittance for a nozzle is unknown and Program C is run, C will provide this number.
L3049 . . . L3078	ω	The ascending table of chamber frequencies for which the real and imaginary parts of the nozzle admittance coefficient E_r, E_i . If Program C is run to determine the admittances, these frequencies will be automatically transferred. The last point in the table must be zero.
L3049 . . . L3078	E_r	The real part of the admittances corresponding to the above frequencies. The last point in the table must be zero. These can be determined by running Program C.
L3079 . . . L3108	E_i	The imaginary part of the admittance corresponding to the above frequencies. The last point in the table must be zero. These can be determined by running Program C.

b. Print Options

$0 \leq L0 \leq 99$:	Execute A, print output
$100 \leq L0 \leq 100$:	Execute A, print input and output (recommended)
$200 \leq L0 \leq 299$:	Execute A, print input and output, print steady state tables and values of integrals used in calculations.

c. Placement of the Data into the Data Package

These data are inserted anywhere in Section I of the data package.

III, D, Input Data (cont.)

d. Output

<u>Label in Output</u>	<u>Symbol in Analysis</u>
ULM	$\bar{u}_{L0}^*$
XK	κ
OMEGA (CH)	ω
ERT	E_r
EIT	E_i
H REAL	h_r
H IMAG	h_i

e. Tabulation of Transverse Acoustic Mode Number ($s_{\nu\eta}$)

(1) Tangential Modes

First tangential:	$s_{11} = 1.8413$
Second tangential:	$s_{21} = 3.0543$
Third tangential:	$s_{31} = 4.2012$
Fourth tangential:	$s_{41} = 5.3175$
Fifth tangential:	$s_{51} = 6.4154$

(2) Radial Modes

First radial:	$s_{02} = 3.8317$
Second radial:	$s_{03} = 7.0156$
Third radial:	$s_{04} = 10.1734$

III, D, Input Data (cont.)

2T-1R:	s_{22}	=	6.7060
2T-2R:	s_{23}	=	9.9695
2T-3R:	s_{24}	=	13.1705
3T-1R:	s_{32}	=	8.0151
3T-2R:	s_{33}	=	11.3459
3T-3R:	s_{34}	=	14.5858

f. Auxiliary Equations

(1) Axial Liquid Velocity (Weighted)

$$\bar{u}_{L0}^* = \frac{(MR)v_x \cos \theta_x + v_F \cos \theta_F}{MR+1}, \text{ ft/sec} \quad (75)$$

MR = mixture ratio = $\dot{w}_x / \dot{w}_F$

v_x = oxidizer injection velocity, ft/sec

v_F = fuel injection velocity, ft/sec

$\theta_{x,F}$ = oxidizer and fuel impingement angle

(2) Selection of Frequencies by GENMEG

$$(\text{Transverse}) = (1 \pm 0.10) s_{v\eta} \quad (76)$$

where $s_{v\eta}$ = transverse acoustic mode number
given in section III,D,1,e.

g. Comments:

When L0 is made positive and all the other required input for A is supplied an n, τ plot for uniform injection is produced. This can be modified to include non-uniform injection or non-linear effects by making L3 or L4 positive with the corresponding data for programs D and E.

III, D, Input Data (cont.)

2. Program B: Concentrated Combustion Stability Analysis

a. Input Requirements: the following data go into Section I of the data package.

<u>Core Location</u>	<u>Mnemonic</u>	<u>Description</u>
L1	--	Execute Program B. Magnitude of number depends on print options given in Paragraph III,D,2,b.
L10	γ	Ratio of the specific heats.
L11	M_e	Steady-state Mach number at the entrance to the exhaust nozzle. This can be determined by the contraction ratio or by Program C. In the latter case, leave L11 blank.
L13	r_c^*	Chamber radius, in. also loaded in L3804, Program C.
L14	L_c^*	This length is used to determine the frequency of a longitudinal mode. $\frac{\omega^* L_c^*}{c_o^*} = \pi$ determining the resonant frequency. It may not correspond to any physical dimension on the chamber. Table I can be used to estimate the longitudinal mode frequency. L^* is used as a non-dimensionalizing factor for longitudinal modes ($s_{v\eta}=0$).
L15	c_o^*	Speed of sound in the chamber, ft/sec.
L16	u_{L0}^*	Weighted liquid injection velocity, ft/sec (see Equation (75)).
L20	$s_{v\eta}$	Mode number. These numbers are given in section III,D,2,e, and Figure 9.
L21	$N_{\omega c}$	Number of chamber frequencies to be used. Leaving this column blank will turn on Program GENMEC which will select 10 frequencies according to the relationship given by Equation (77). GENMEC gives first mode only.
L22	ω	Table of nondimensional, chamber frequencies arranged in ascending order. A zero must be included at the end of the table. The maximum number of frequencies is 28 (including the zero point). This is left blank if L21 is blank.
L49		
L50	$\bar{P}_c^*$	Mean chamber pressure in pounds per square inch.
L51	L_{jc}^*	Length from injector to combustion front in inches.
L53	$\bar{W}^*$	Mean chamber weight flow in pounds per second.

III, D, Input Data (cont.)

<u>Core Location</u>	<u>Mnemonic</u>	<u>Description</u>
L3017	N_{ω}	The number of input frequencies at which the nozzle admittance coefficients are known. This number must be greater than 2 and less than 30. If the admittance for a nozzle is unknown and Program C is run, C will provide this number.
L3019 : : L3048	ω	The values of the chamber frequencies, arranged in ascending order, for which the admittances are known. Program C will provide them if it is run. The last value must be zero.
L3049 : L3078	A_r or E_r	The real part of the nozzle admittance coefficient. If $s_{\omega} = 0$ use A_r , otherwise use E_r . These values must correspond to the frequencies given above. The last point in the table must be zero. Program C will provide these values if it is run.
L3079 : : L3108	A_i or E_i	Imaginary part of the nozzle admittance coefficient. If $s_{\omega} = 0$ use A_i , otherwise use E_i . These values must correspond to the frequencies given above. The last point in the table must be zero.
L3109 : : L3138	C_r	The real part of the entropy nozzle admittance coefficient. These values correspond to the frequencies given above. The last point in the table must be zero. Program C will provide these values if it is run.
L3139 : : L3168	C_i	The imaginary part of the entropy nozzle admittance coefficient. These values correspond to the frequencies given above. The last point in the table must be zero. Program C will provide these values if it is run.

b. Print Options

- $0 \leq L1 \leq 99$: Execute B, print output
 $100 \leq L1 \leq 199$: Execute B, print input and output (recommended)
 $200 \leq L1 \leq 299$: Execute B, print input and output, and matrix before inversion.

c. Placement of the Data into the Data Package

These data are inserted anywhere in Section I of the data package.

III, D, Input Data (cont.)

d. Output

<u>Label in Output</u>	<u>Symbol in Analysis</u>
GAMMA	γ
DESIRED MACH NUMBER	M_e
CHAMBER RADIUS	R_{AC}^*
CHAMBER LENGTH	L_c^*
SPEED OF SOUND	c_o^*
CHAMBER MODE DESCRIPTION	$s_{v\eta}$
CHAMBER FREQUENCIES	ω
WC	ω
LJC	L_{jc}
SNH	$s_{v\eta}$
ZE	L_c
UE	M_e
SOUND	c_o^*

III, D, Input Data (cont.)

e. Tabulation of Transverse Acoustic Mode Number ($s_{v\eta}$)

(1) Tangential Modes

First tangential:	s_{11}	=	1.8514
Second tangential:	s_{21}	=	3.0543
Third tangential:	s_{31}	=	4.2012
Fourth tangential:	s_{41}	=	5.3175
Fifth tangential:	s_{51}	=	6.4154

(2) Radial Modes

First radial:	s_{02}	=	3.8317
Second radial:	s_{03}	=	7.0156
Third radial:	s_{04}	=	10.1734

(3) Combined tangential-radial modes

1T-1R:	s_{12}	=	5.3313
1T-2R:	s_{13}	=	8.5263
1T-3R:	s_{14}	=	11.7059
2T-1R:	s_{22}	=	6.7060
2T-2R:	s_{23}	=	9.9695
2T-2R:	s_{24}	=	13.1705
3T-1R:	s_{32}	=	8.0151
3T-2R:	s_{33}	=	11.3459
3T-3R:	s_{34}	=	14.5858

(4) Longitudinal Mode

$$s_{v\eta} = 0$$

III, D, Input Data (cont.)

e. Auxiliary Equations

(1) Axial Liquid Velocity (Weighted)

$$\bar{u}_{LO}^* = \frac{(MR)v_x \cos \theta_x + v_F \cos \theta_F}{MR+1}, \text{ ft/sec} \quad (75)$$

$$\begin{aligned} MR &= \text{mixture ratio} = \dot{w}_x / \dot{w}_F \\ v_x &= \text{oxidizer injection velocity, ft/sec} \\ v_F &= \text{fuel injection velocity, ft/sec} \\ \theta_{x,F} &= \text{oxidizer and fuel impingement angle} \end{aligned}$$

(2) Selection of Frequencies by GENMEG

$$\begin{aligned} (\text{Longitudinal}) &= (1 \pm 0.10) \pi \\ (\text{Transverse}) &= (1 \pm 0.10) s_{v\eta} \end{aligned} \quad (77)$$

where

$$\begin{aligned} \pi &= 3.14159 + = \text{resonant frequency} \\ &\quad \text{(non-dimensional) for longitudinal} \\ &\quad \text{modes} \\ s_{v\eta} &= \text{transverse acoustic mode number} \\ &\quad \text{given in the Program B writeup.} \end{aligned}$$

(3) Other Information

For a longitudinal mode a resonant frequency of π is true for a very short nozzle. It may be necessary to override GENMEG to find an n minimum. Also GENMEG gives only the first mode. Other modes, such as 1T-1L, can be obtained by specifying the frequencies to be run.

When L1 is made positive and all other required data for program B is supplied an n, τ plot for uniform injection is produced. This can be modified to include non-uniform injection effects or non-linear effects by making L3 or L4 positive with the corresponding data for programs D and E.

L101 in the nozzle input is generally required with

III, D, Input Data (cont.)

(4) Estimate of Chamber Weight Flow

$$\dot{w}^* = \frac{\bar{P}_c A_{\text{throat}} g}{C_{\text{star}}}$$

$$C_{\text{star}} = \frac{\bar{C}_o^*}{\gamma \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

III, D, Input Data (cont.)

3. Program C: Exhaust Nozzle Admittance Coefficients for Longitudinal and Transverse Modes

a. Input Requirements: The following data go into Section I of the data package.

<u>Core Location</u>	<u>Mnemonic</u>	<u>Description</u>
L2		Run program C if positive.
L3801	M	= 1.: the table of velocity potential values within the nozzle is input and the Mach number to the entrance of the nozzle is input. = 2.: the Mach number at the entrance to the nozzle is input but the velocity potential table must be calculated. = 3.: the Mach number and the velocity potential table must be calculated.
L3803	R* _{ATO}	Radius of the throat, in.
L3804	R* _{ACO}	Radius of the chamber, in., also loaded in L13, Program A.
L3805	R* _{CCO}	Radius of chamber curvature at the nozzle entrance, in.
L3806	R* _{CTO}	Radius of curvature at the throat, in.
L3807	α_o	Nozzle convergent half-angle, deg.
L90*	R* _{AT1}	Radius of centerbody throat, in.
L91*	R* _{CT1}	Radius of curvature of the centerbody throat, in.
L101		Length from end of nozzle to combustion zone when using Program B (zero for Program A).

*Note 1: These values may be plus or minus, see Figures 14 and 15.

Note 2: For cylindrical chambers, leave L90 through L94 blank.

III, D, Input Data (cont.)

<u>Core Location</u>	<u>Mnemonic</u>	<u>Description</u>
L92*	R_{ACi}^*	Radius of chamber of centerbody, in.
L93*	R_{CCi}^*	Radius of curvature of centerbody at the nozzle entrance.
L94*	α_i	Centerbody nozzle convergent half-angle, deg.
L20	$s_{v\eta}$	Transverse acoustic mode number is given on the previous page for cylindrical chambers. For annular chambers see Table 1, Section I,B.
L3808	KN	If $L3801 \geq 2$, then KN is either (1) An odd integer less than 200 telling the desired size of the program generated velocity potential table, or (2) Blank and program will assume $K_N = 101$ (recommended)
L3809	$\phi(Z)$	Dimensionless velocity potential table (199 values maximum) in the odd numbered locations only. If $L3801 \geq 2$, this is not necessary.
L3811		
L3813		
L4205		
L3810	$q^{-2}(Z)$	Squares of the reduced velocity table (199 values maximum) in the even numbered locations only. If $L3801 \geq 2$, this not necessary.
L3812		
L3814		
L4206		

*Note 1: These values may be plus or minus, see Figures 14 and 15.

Note 2: For cylindrical chambers, leave L90 through L94 blank.

When this program is run by itself the frequencies to be run must either be input or generated by GENMEG. As discussed later this admittance calculation is done for every other input or generated frequency. To run Program C by itself the above data must be supplemented as follows.

<u>Core Location</u>	<u>Mnemonic</u>	<u>Description</u>
L10	γ	Ratio of specific heats
L11	M_e	Steady state Mach number at the entrance to the nozzle. Not needed if L3801 is 3.

III, D, Input Data (cont.)

<u>Core Location</u>	<u>Mnemonic</u>	<u>Description</u>
L13	r_c^*	Chamber radius, in.
L14	L_c^*	Chamber length, in.
L15	c_o^*	Speed of sound in the chamber, ft/sec.
L21	$N_{\omega c}$	Number of chamber frequencies to be used leaving this column blank will turn on Program GENMEG. Admittances will actually be calculated for every other frequency as discussed later.
L22 . L49	ω	Table of non-dimensional chamber frequencies arranged in ascending order. Last point must be zero.

b. Print Options

$0 < L2 \leq 9:$	Execute Program C, print no output
$10 \leq L2 \leq 99:$	Execute Program C, print output
$100 \leq L2 \leq 199:$	Execute Program C, print input and output (recommended)
$200 \leq L2 \leq 299:$	Execute Program C, print input and output, and print the nondimensional velocity potential table.

c. Placement of the Data in the Data Package

This data is inserted anywhere in Section I of the data package.

d. Output

<u>Label In Output</u>	<u>Symbol In Analysis</u>
(SNH)C	$s_{v\eta}$
(SNH)N	$\hat{s}_{v\eta}$
WC	ω
WN	$\hat{\omega}$

III, D, Input Data (cont.)

<u>Label In Output</u>	<u>Symbol In Analysis</u>
MACH NO.	M_e
G	γ
AR	A_r
AI	A_i
-AR/(MACH NO.)	α_{Nr}
-AI/(MACH NO.)	α_{Ni}
BR	B_r
BI	B_i
T1	E_r
T2	E_i
CR	C_r
CI	C_i
-CR/(MACH NO.)	β_{Nr}
-CI/(MACH NO.)	β_{Ni}
FC (CPS)	f^*

e. Auxiliary Equations

- (1) The Selection of the Chamber Frequencies to be Used

Because of the long execution time of this program for a given value of ω , the program starts with the first chamber frequency and uses every other frequency thereafter; that is, if n is the total number of chamber frequencies, Program C will use m number of frequencies according to the relationship

III, D, Input Data (cont.)

$$m = \frac{n}{2} + 1$$

In computer language, m and n are whole numbers.

Therefore, if n = 11

$$m = \frac{11}{2} + 1$$

$$m = 5 + 1 \text{ (where 0.5 has been truncated out)}$$

$$m = 6$$

(2) Nondimensional chamber frequency

$$\omega = \frac{(2\pi f^*) (v_{AC}^*)}{c_o^*} \quad (78)$$

where

* = denotes dimensional variables
subscript c denotes chamber conditions

f* = chamber frequency, cps

c_o* = chamber speed of sound ft/sec

v_{AC}* = chamber length L_c* or radius r_c*
(depending if longitudinal or transverse
modes respectively are desired).

(3) Nondimensional nozzle frequency

$$\hat{\omega} = \frac{\omega}{\hat{k}} \quad (79)$$

where

$$\hat{k} = R_{ATO}^* \left[\frac{2}{\gamma+1} \left(\frac{R_{ATo}^*}{R_{CTo}^*} - \frac{R_{ATi}^*}{R_{CTi}^*} \right) \right] / \left(R_{ATo}^{*2} - R_{ATi}^{*2} \right)^{1/2}$$

(4) Nozzle transverse acoustic nozzle number

$$\hat{s}_{v\eta} = \frac{s_{v\eta}}{\hat{k}} \quad (80)$$

III, D, Input Data (cont.)

4. Program D: Expansion of Results from Program B

a. Input Requirements

The following data go into Section I of the data package:

<u>Core Location</u>	<u>Mnemonic</u>	<u>Description</u>
L3		Must be greater than zero to run this program. Use only if $A_{v\eta}$, $B_{v\eta}$, $C_{v\eta}$ are input or calculated in Program E.
L3405	l_r/η	The ratio of the radial velocity coefficient to the pressure interaction index. Generally set equal to zero.
L3406	l_θ/η	The ratio of the tangential velocity coefficient to the pressure interaction index. Generally set equal to zero.
L3407		= 1 for linear combustion response. >1 for nonlinear combustion response. This parameter indicates the number of frequencies for which the injector nonuniformity coefficients, $A_{v\eta}$, $B_{v\eta}$, and $C_{v\eta}$, are known. The running of Program I will determine this number. (Not required if Program B is run.)
L3408		The number of chamber frequencies (Not required if Program B is run.)
L3409	ω	The table of frequencies arranged in ascending order.
...		The last value must be zero. (Not required if
L3437		Program B is run.)
...		
L3439	$h_r(\omega)$	Real part of the damping effects computed in Program A or B.
...		The last value must be zero. (Not required if Program A or B
L3467		is run.)
...		
L3469	$h_i(\omega)$	Imaginary part of the damping effects computed in Program B.
...		The last value must be zero. (Not required if Program A
L3497		or B is run.)
...		
L4522		The values of chamber frequencies for which the
...	ω	nonuniformity coefficients are known. Use only if
L4538		data are available.
...		

III, D, Input Data (cont.)

<u>Core Location</u>	<u>Mnemonic</u>	<u>Description</u>
L4539 : L4555 : L4556 : L4572 : L4573 : L4589 : L4590 : L4606	$A_{v\eta}$ $B_{v\eta}$ $C_{v\eta R}$ $C_{v\eta I}$	The pressure nonuniformity coefficient corresponding to the above frequencies (ω). $A_{v\eta} = 1.0$ for uniform injection distribution. The radial velocity nonuniformity coefficient corresponding to the above frequencies ω. Generally set equal to zero. The real part of the tangential velocity nonuniformity coefficient corresponding to the above frequencies ω. Generally set equal to zero. The imaginary part of the tangential velocity nonuniformity coefficient corresponding to the above frequencies ω. Generally set equal to zero.

- NOTES: a. All these data, with the exception of l_r/η and l_θ/η , can be calculated using Programs B, E, and I.
- b. This program was constructed specifically to include the effects of the nonuniformity coefficients for the low Mach number cases.

b. Print Options

- $0 < L3 \leq 9$: Execute D, print no output or input
- $10 \leq L3 \leq 99$: Execute D, print output only
- $100 \leq L3 \leq 199$: Execute D, print input and output

c. Placement of the Data into the Data Package

These data can be inserted anywhere in Section I of the data package.

III, D, Input Data (cont.)

d. Output

<u>Label on Output</u>	<u>Symbol in Analysis</u>
LR/N	$\frac{l_r}{n}$
LT/N	$\frac{l_\theta}{n}$
ERT	E_r
EIT	E_i
CRT	C_r
CIT	C_i
OMEGA (C)	ω
OMEGA (CH)	ω
HTR	$\tilde{h}_r$
HTI	$\tilde{h}_i$
HTRINT	$\tilde{h}_r$ (interpolated)
HTIINT	$\tilde{h}_i$ (interpolated)
OMEGA	ω

5. Program E: Injector Nonuniformity Coefficients: $A_{v\eta}$, $B_{v\eta}$, $C_{v\eta}$

a. Input Requirements (not programmed for annular chambers)

MAIN CONTROL requires L4 in Section I of data package.
The remaining data go into Section III of the data package.

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III, D, Input Data (cont.)

<u>Core Location</u>	<u>Mnemonic</u>	<u>Description</u>
L1	IZZIT	Indicates whether the mode is standing or spinning: Standing: IZZIT <0 Spinning: IZZIT >0
L2	X_M	The number of radial divisions desired on the injector face ($10 \leq X_M \leq 21$).
L3	X_N	The number of angular divisions desired around the injector face ($20 \leq X_N \leq 181$).
L4	SECTOR	The number of symmetrical sectors that the injector can be divided into, ($1 \leq \text{SECTOR} \leq 180$).
L5	v	Order of transverse mode in tangential direction 1, 2, 3, etc. For radial modes $v = 0$.
L7	$s_{v\eta}$	Transverse acoustic mode number given by tabulation presented in Program B write-up.
L9573	R_{inj}	Injector radius, in.
L9570	NE	Number of elements per symmetrical sector. The maximum number of elements is 1000. Ignore if Program J is run.
L1920 : L2919 : L2920 : : L3919 : : L3920 : : L4919	r θ μ	The radial position of each element within the symmetrical sector, in. Program J supplies these data when they are run. The angular displacement (from any convenient reference line on the injector face) of each element within the symmetrical sector, radians. Program J supplies these data when they are run. The element injection distribution given by Equation (81) at the end of this section. Usually Program J provides this information to Program E.

III, D, Input Data (cont.)

b. Print Options

$0 \leq L4 \leq 9$: Execute E, do not print input or output
 $10 \leq L4 \leq 99$: Execute E, print output only
 $100 \leq L4 \leq 199$: Execute E, print input and output

c. Placement of the Data into the Data Package

Main control data: Section I of data package.

All other data: Section III of data package. Section IV must be included in the data package.

d. Output

<u>Label on Output</u>	<u>Symbol in Analysis</u>
AVN	$A_{v\eta}$
BVN	$B_{v\eta}$
CVN	$C_{v\eta}$
MU	μ

e. Auxiliary Equations

Distribution coefficient, μ_E

$$\mu_E = \frac{(\dot{w}_T)_E / (A_s)_E}{\dot{w}_{T/A_{inj}}} = \left(\frac{1}{x_M \cdot x_N} \right)^{-1} \frac{(w_T)_E}{\dot{w}_T} \quad (81)$$

III, D, Input Data (cont.)

where

- $(w_T)_E$ = total weight flow rate of element, lb/sec
 w_T = total weight flow rate of injector, lb/sec
 $(A_s)_E$ = surface area serviced by the element, in.²
 A_{inj} = total surface area of injector, in.²

6. Program F: Final Solution for Instability Zones

a. Input Requirements

The input for this program comes directly from Program D and consists of the following parameters:

- The frequency in the chamber, ω
 The radius of the chamber R_{AC}^* , in.
 Speed of sound in the chamber, c_o^* , ft/sec
 Real and imaginary parts of the damping parameters h corresponding to the above frequencies.

This program can be run by itself if the data are inserted into those locations specified by Program D.

b. Print Options

- | | |
|-------------------------|--|
| $0 < L5 \leq 9:$ | Execute Program F, print no input and output |
| $10 \leq L5 \leq 99:$ | Execute Program F, print output only |
| $100 \leq L5 \leq 199:$ | Execute Program F, print input and output |

III, D, Input Data (cont.)

c. Placement of the Data into the Data Package

These data are included into Section I of the data package.

d. Output

The following output results from this program:

- f^* = frequency of oscillation, Hz
- ω = nondimensional frequency
- τ^* = sensitive time lag corresponding to the above frequency, millisec
- n = pressure interaction index

At the bottom of the page, an effort is made to locate the minimum interaction index. In most cases, this information is valid. However, effects from an adjacent mode will intervene occasionally thereby invalidating this information.

e. Other Information

The program obtains its result by the solution of the following equations:

Frequency, cps

$$f^* = \frac{\omega}{2\pi} \frac{c^* 12}{R_{AC}^*} \text{ for transverse modes.}$$

for a longitudinal mode replace R_{AC}^* by L_c^* .

III, D, Input Data (cont.)

Pressure interaction index, n

$$n = \frac{\tilde{h}_r^2 + \tilde{h}_i^2}{2\tilde{h}_r}$$

Sensitive time lag, τ , millisec

$$\tau^* = (83.33) \frac{R_{ACo}^*}{c_o^*} \frac{1}{f^*} \tan^{-1} \frac{h_i}{n-h_r}$$

for a longitudinal mode replace R_{ACo}^* by L_c^* .

7. Program G: Obsolete and deleted from the listing.

8. Program H: Obsolete and deleted from the listing.

Program H is not operational. Recommended procedure is to place a negative number in load location L7 of MAIN CONTROL.

9. Program I: Nonlinear Combustion Response (Option for Program E)

a. Input Requirements

MAIN CONTROL requires L8 for Section I of data package. The remaining data go into Section III of data package:

III, D, Input Data (cont.)

<u>Core Location</u>	<u>Mnemonic</u>	<u>Description</u>
ALL THE DATA REQUIRED TO RUN PROGRAM E AS WELL AS:		
L0	E1	The permissible error. If blank, the program will assume $E1 = 0.001$.
L12	P_{oo}	The ratio of the maximum pressure amplitude at the injector face to the steady-state pressure value.
L19 L68	IPP	The values of the pressure perturbation associated with the pressure-dependent nonlinear element.
L69 L118	OPP	The values of the combustion perturbation corresponding to the above IPP values.
L119 L168	IPR	The values of the radial velocity perturbation associated with the velocity-dependent nonlinear element.
L169 L218	OPR	The values of the combustion perturbation corresponding to the above IPR values.
L219 L268	IPT	The values of the tangential velocity perturbation associated with the velocity dependent nonlinear element.
L269 L318	OPT	The values of the combustion perturbation corresponding to the above IPT values.
L9595	TFLP	The linear transfer function for equivalent linear operation associated with the pressure-dependent nonlinear element.
L9596	TFLR	The same as TFLP except for radial velocity-dependent nonlinear element.
L9597	TFLT	The same as TFLP except for tangential velocity-dependent nonlinear elements.
L9598	NUMBR	The number of steps to be used in the integration scheme associated with this problem. If left blank, the computer will assume 20, which is sufficient for most cases.

III, D, Input Data (cont.)

b. Print Options

$0 \leq L8 \leq 9$: Execute I, print no input or output data
 $10 \leq L8 \leq 99$: Execute I, print output only
 $100 \leq L8 \leq 199$: Execute I, print input and output

c. Placement of the Data into the Data Package

These data are included in Section III of the data package.
Along with these data, Section IV must be included.

d. Output

The output of this program is as follows:

The frequency, ω

The element number, location, fractional flow rate,
 F_p , F_R , and F_T necessary for Program E.

This will automatically change the output from Program E
as follows:

The frequency, ω

The frequency dependent expansion coefficients $A_{v\eta}$,
 $B_{v\eta}$, and $C_{v\eta}$.

III, D, Input Data (cont.)

10. Program J: Injected Mass Distribution Effects (Not Programed for Annular Chambers)

a. Input Requirements

MAIN CONTROL requires L9 in Section I of data package. The remaining data go in Section III of data package.

<u>Core Location</u>	<u>Mnemonic</u>	<u>Description</u>
L319		
L323	n	Element number (maximum = 1000) within symmetrical sector.
L327		
L319 + 4n		
L320		
L324		
L328	X_n or r_n	Element location, in.
L320 + 4n		
L321		
L325		
L329	Y_n or θ_n	Element location, in. or degrees
L321 + 4n		
L322		
L326		
L330	T_n	Type of injection element. The details concerning this type number are given next.
L322 + 4n		
L4	DX	Number of symmetrical sectors, DX = 1 for no symmetry.
L4944 to L9568	-	The data input in this section serves to define the element types and consists of a maximum of 100 variable length data sets. Each set contains:

III, D, Input Data (cont.)

- a. The element type number (≤ 100)
- b. Number of oxidizer orifices for that type
- c. Diameters of all the oxidizer orifices
- d. Number of fuel orifices for that type
- e. Diameters of all the fuel orifices

EXAMPLE

let T_i = i^{th} element type
 NX_i = number of oxidizer orifices for the i^{th} type element
 DX_{ij} = diameter of the j^{th} oxidizer orifice in the i^{th} type element
 NF_i = number of fuel orifices for the i^{th} type element
 DF_{ij} = diameter of the j^{th} fuel orifice in the i^{th} type element

Then a typical input would read

L4944 + T_1 + NX_1 + $DX_{1,1}$ + $DX_{1,2}$ + $DX_{1,3}$ + NF_1 + $DF_{1,1}$ + $DF_{1,2}$ + T_2 + NX_2
 + $DX_{2,1}$ +

If NF_i or NX_i is zero, then do not set $DF_{i,1} = 0$ or $DX_{i,1} = 0$.

Assume $NX_1 \equiv 0.0$, then

L4944 + T_1 + NX_1 + NF_1 + $DF_{1,1}$ + $DF_{1,2}$ + $DF_{1,3}$ + T_2 + NX_2 + $DX_{2,1}$ + ...

III, D, Input Data (cont.)

<u>Core Location</u>	<u>Mnemonic</u>	<u>Description</u>
L9569	NT	Total number of injection types
L9570	NE	Number of elements per symmetrical sector
L9571	COORD	COORD = 0: elements are located using polar coordinates COORD > 0: elements are located using cartesian coordinates
L9572	WT	Total injector weight flow, lb/sec
L9573	R _{inj}	Injector radius, in.
L9574	MR	Injector mixture ratio
L9575	NFFC	Total number of fuel film cooling holes
L9576	DFFC	Diameter of fuel film cooling holes

In the event of fuel or oxidizer film cooling, either the percent film cooling or the actual orifice dimensions can be used to describe the cooling. Place a zero(s) in the one(s) not used.

L9578	ROX	Oxidizer density, lb/ft ³
L9579	ROF	Fuel density, lb/ft ³
L9584	CDX	Oxidizer orifice loss coefficient
L9585	CDF	Fuel orifice loss coefficient
L9588	PFFC	Percent fuel film cooling
L9589	PXFC	Percent oxidizer film cooling
L9590	XFC	Total number of oxidizer film cooling orifices
L9591	DFFX	Diameter of film cooling orifices

L2, L3, and L4 from Program E.

III, D, Input Data (cont.)

b. Print Options

$0 < L9 \leq 9$:	Execute J, do not print output or input data
$10 \leq L9 \leq 99$:	Execute J, print output only
$100 \leq L9 \leq 199$:	Execute J, print input and output
$L9 \leq 500$:	Execute J, print input and output, and, if error occurs in J, dump error message with input data.

c. Placement of the Data into the Data Package

Main control data: Section I of the data package.

All other data: Section III of the data package.

Section IV must be included in the data package.

d. Output

The following output results as a consequence of this program:

Miscellaneous information concerning the total orifice area, circuit pressure drops, overall mixture ratio, etc.

Element number, injection type, and location.

Description of various parameters associated with the different types of injection elements.

Element number, location, and distribution coefficient.

The distribution coefficient as a function of the radius across the face of the injector.

III, Deck Setup (cont.)

E. RESTRICTIONS AND LIMITATIONS

1. One limitation of the distributed combustion analysis (Program A) which cannot easily be implied from the information given in this manual is the frequency limit in transverse mode analysis. This limit is implied from the relationship

$$\Omega \sinh \Omega Z \leq 0 (M_e)$$

This limitation means that combined transverse-longitudinal modes cannot be analyzed with this program. Long chambers (long compared to their diameter) may also produce erroneous results for the pure transverse modes. This limitation does not apply to the concentrated combustion analysis (Program B).

2. The restrictions of the annular nozzle analysis have already been discussed in Section I,B,4,b,(6) of this manual.

3. The distributed combustion (Program A) analysis is restricted to low Mach numbers, those less than approximately 0.3. It is also limited to low combustion intensity by the assumption that $\frac{d\bar{u}}{dz}$ is the same order of magnitude as $\bar{u}$.

4. The sensitive time lag theory is developed for liquid propellant rocket engines. Before applying it to gas injection, many assumptions must be examined critically.

5. Calculations of the distribution coefficients A_{vn} , B_{vn} , and C_{vn} are not applicable to annular chambers.

III, Deck Setup (cont.)

F. OUTPUT DEFINITION

1. Program A

<u>Label in Output</u>	<u>Symbol in Analysis</u>
GAMMA	γ
DESIRED MACH NUMBER	M_e
CHAMBER RADIUS	R_{AC}^*
CHAMBER LENGTH	L_c^*
SPEED OF SOUND	c_o^*
CHAMBER MODE DESCRIPTION	$s_{v\eta}$
MACH DISTRIBUTION	$M(Z)^*$
UIBAR	M_e
K	κ
x	ξ_1
FC(CPS)	f^*
OMEGA	ω
TAU (MS)	τ^*
N	n
ULM	$\bar{u}_{Lo}^*$
WC	ω

2. Program B

<u>Label in Output</u>	<u>Symbol in Analysis</u>
GAMMA	γ
DESIRED MACH NUMBER	M_e
CHAMBER RADIUS	R_{AC}^*
CHAMBER LENGTH	L_c^*
SPEED OF SOUND	c_o^*

III, F, Output Definition (cont.)

<u>Label in Output</u>	<u>Symbol in Analysis</u>
CHAMBER MODE DESCRIPTION	$s_{v\eta}$
CHAMBER FREQUENCIES	ω
WC	ω
MACH DISTRIBUTION	$M(Z^*)$
SNH	$s_{v\eta}$
ZE	L_c
UE	M_e
SOUND	c_o^*
ULM	$\bar{u}_{LO}^*$
XK	κ
OMEGA (CH)	ω
ERT	E_r
EIT	E_i
H REAL	h_r
H IMAG	h_i

3. Program C

<u>Label in Output</u>	<u>Symbol in Analysis</u>
(SNH)C	$s_{v\eta}$
(SNH)N	$s_{v\eta}$
WC	ω
WN	$\hat{\omega}$

III, F, Output Definition (cont.)

<u>Label in Output</u>	<u>Symbol in Analysis</u>
MACH NO.	M_e
G	γ
AR	A_r
AI	A_i
-AR/(MACH NO.)	α_r
-AI/(MACH NO.)	α_i
BR	B_r
BI	B_i
T1	E_r
T2	E_i
CR	C_r
CI	C_i
-CR/(Mach No.)	β_{nr}
-CI/(Mach No.)	β_{ni}
FC (CPS)	f^*

4. Program D

<u>Label on Output</u>	<u>Symbol in Analysis</u>
LR/N	$\frac{l_r}{n}$
LT/N	$\frac{l_\theta}{n}$
ERT	E_r
EIT	E_i
CRT	C_r
CIT	C_i
OMEGA (C)	ω
OMEGA (CH)	ω

III, F, Output Definition (cont.)

<u>Label On Output</u>	<u>Symbol in Analysis</u>
HTR	$\tilde{h}_1$
HTI	$\tilde{h}_r$
HTRINT	$\tilde{h}_1$ (interpolated)
HTIINT	$\tilde{h}_r$ (interpolated)
OMEGA	ω

5. Program E

<u>Label on Output</u>	<u>Symbol in Analysis</u>
AVN	$A_{v\eta}$
BVN	$B_{v\eta}$
CVN	$C_{v\eta}$
MU	μ

6. Program F

<u>Label on Output</u>	<u>Symbol in Analysis</u>
FC(CPS)	f^*
(OMGA)D	ω
TAU(MS)	τ^*
N	n
NMIN	n_{min}

III, F, Output Definition (cont.)

7. Program I

<u>Label on Output</u>	<u>Symbol in Analysis</u>
PRESSURE PERTURBATION	p'
COMBUSTION GAIN	ϕ_p
RADIAL VELOCITY PERTURBATION	v'
COMBUSTION GAIN	ϕ_R
TANGENTIAL VELOCITY PERTURBATION	w'
COMBUSTION GAIN	ϕ_T
OMEGA	ω
FP	F_p
FR	F_R
FT	F_T
AVN REAL	A_{vn}
BVN REAL	B_{vn}
CVN REAL	C_{vnR}
CVN IMAG	C_{vnI}

8. Program J

<u>Label on Output</u>	<u>Symbol in Analysis</u>
PRESSURE (TFLP)	TF_{LP}
RADIAL VELOCITY (TFLR)	TF_{LR}
TANGENTIAL VELOCITY (TFLT)	TF_{LT}
DISTRIBUTION COEFFICIENT MU	μ

III, Deck Setup (cont.)

G. INPUT FORM

A tabulation of the input for the illustrative problem given below is shown immediately following the discussion of the problems in Figure 19.

1. Illustrative Design Problem

a. Longitudinal Mode Analysis

The following data concerning a hypothetical engine will be used for this analysis:

Engine Geometry: See Figure 19

$$\gamma = 1.218$$

$$C^*_o = 3800 \text{ ft/sec}$$

Mach Number: unknown

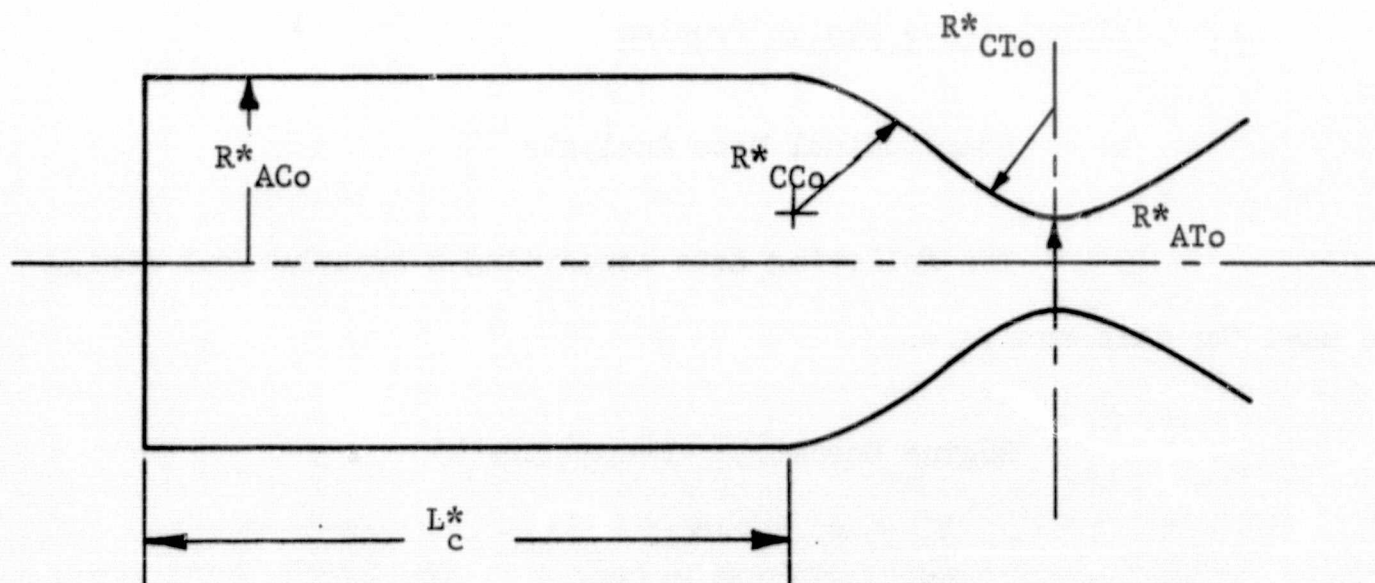
Momentum interchange coefficient = 0.0

Combustion front location: 2 inches from injector face

Combustion distribution: a linear combustion distribution with complete combustion occurring at 6.4 in.

$$P_c = 600 \text{ psia}$$

The problem is to find the n , zone for the first longitudinal mode.



$$\begin{aligned} L^*_c &= 11.8, \text{ in.} \\ R^*_{ACo} &= 6.1, \text{ in.} \\ R^*_{CCo} &= 3.0, \text{ in.} \\ R^*_{CTo} &= 7.16, \text{ in.} \\ R^*_{ATo} &= 3.58, \text{ in.} \\ \alpha &= 15.0^\circ \end{aligned}$$

Figure 18 -- Definition of the Geometrical Factors used in Example Problem

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III, G, Input Form (cont.)

b. Transverse Mode

For the transverse case, the problem will be to find the n, r zones for the first tangential and the second tangential modes. The data given in the longitudinal case will be supplemented by the following information:

$$s_{11} = 1.8413 \text{ and } s_{12} = 3.0543$$

Injector

Number of radial baffle compartments = 180

Total weight flow = 150 lb/sec

Mixture ratio = 2.0

Injection occurs at three radii:

Unlike doublets at $r = 2.0$ in.

Triplets (X-F-X) at $r = 4.0$ in.

Pentads (4F-X) at $r = 6.0$ in.

Injector radius = chamber radius

No film cooling

Storable propellants

Fuel and oxidizer loss coefficients = 0.75

Spinning modes

Tangential mode numbers = 1 and 2

No radial or tangential velocity effects

III, G, Input Form (cont.)

c. Nonlinear Combustion Response

For this case, the first tangential mode will be examined using a deadband nonlinear element. In addition to the above injector data the following additional information will be used:

$$P_{oo} = 1.0$$

$$TFLP = 1.0$$

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III, Deck Setup (cont.)

```

L1+201.+201.
L10+1.218+0
L13+6.4+14.8+3800+20.42+0.0
L20+0.0
L21+15+2.0+2.5+3.0+3.25+3.5+3.75+4.0+4.25+4.5+4.75+5.0
+5.5+6.0+6.5+7.0
L3801+3.0+0.+3.58+0.+3.0+7.16+15.0+0.
L50+600.+2.++151.
L101+12.8
TEAMPLE PROBLEM... LONGITUDINAL MODE FOR HYPOTHETICAL ENGINE

L0+100++10+10+100L9+100
L101+0.0
L3407+1
L20+1.8411838+0.0
L50+0.0+2.5+5.0+6.3+6.35+6.38+6.39+6.40+6.41+6.42+6.43+6.5
+7.5+12.5+25.+50.+0.00000
L70+0.0+2.5+5.0+6.3+6.35+6.38+6.39+6.395+6.4+6.4+6.4+6.4
+6.4+6.4+6.4+6.4+0.00000
T EXAMPLE PROBLEM... FIRST TANGENTIAL MODE, HYPOTHETICAL ENGINE
L0+0+1+20+180+180+1++1.8411838
L4944+1+1+.0785+1+.0785
+2+2+.0785+.0785+1+.0785
+3+1+.0785+4+.0785+.0785+.0785+.0785
L319+1+2.0+1.0+1
L323+2+4.0+1+2
L327+3+0.0+1.0+3
L9509+3+3+0+150+6.4+2.0++
L9578+89.52+50.1
L9584+0.75+0.75
T

L1+100+ +10+100+10
L20+3.0542369+0.0
L50+600.+2.++151.
L101+12.8
T EXAMPLE PROBLEM... SECOND TANGENTIAL MODE, HYPOTHETICAL ENGINE
L0+0+1+20+180+180+2++3.0542369
T

L4+200
L8+50
L20+1.8411838+0.0
L3405+1.0
T IF MODE, SHOWING EFFECTS OF DEADBAND NONLINEARITY
L0+0+1+20+180+180+1++1.8411838
L12+1.0L9598+1.0
L119-5.1-2.1-1.1-0.60-0.20-0.15-0.11-0.10-0.09-0.05
L169-3.0-2.0-1.0-0.50-0.10-0.05-0.01+0.00+0.00+0.00
L129-0.01+0.01+0.05+0.09+0.10+0.11+0.15+0.20+0.60
L179+0.00+0.00+0.00+0.00+0.00+0.01+0.05+0.10+0.50
L138+1.10+2.1+5.1+0.000000
L188+1.00+2.0+5.0+0.000000
T

```

Figure 19. Tabulation of Input for Illustrative Design Problem

III, Deck Setup (cont.)

H. SAMPLE OUTPUT

The following output is run from the sample input given in the previous section:

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EXAMPLE PROBLEM K-1: LONGITUDINAL MODE FOR HYPOTHETICAL ENGINE

PAGE 1

A	B	C	D	E	F	G	H	I	J
0.	201.	201.	0.	0.	0.	0.	0.	0.	0.

***** THE FOLLOWING MAIN CONTROL DATA WILL BE USED IN THIS CASE *****

RATIO OF SPECIFIC HEAT (GAMMA) = 1.2180

DESIRED MACH NUMBER = .00000 (=0 IF BEING CALCULATED)

CHAMBER RADIUS = 6.400 (INCHES)

CHAMBER LENGTH = 14.800 (INCHES)

SPEED OF SOUND = 3800.000 (FT/SEC)

CHAMBER MODE DESCRIPTION = .00000 (=0 FOR LONGITUDINAL MODES)

***** CHAMBER FREQUENCIES (WC) *****

2.00000	2.50000	3.00000	3.25000	3.50000
3.75000	4.00000	4.25000	4.50000	4.75000
5.00000	5.50000	6.00000	6.50000	7.00000

DISTANCE FROM INJECTOR TO COMBUSTION FRONT 2.00000 INCHES

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DATE 25 JUL 70

EXAMPLE PROBLEM<-//,:24ITUDINAL MODE FOR HYPOTHETICAL ENGINE

PAGE 2

PROGRAM C INPUT - CALCULATES NOZZLE ADMITTANCE COEFFICIENTS USING 8052

G = 1.218 , MDESIR = 3

RAT = 3.580,RAC = 6.400,RCC = 3.000,RCT = 7.160,ALFA = 15.000,KN = 101,STN = 12.800

DATE 25 JUL 70

EXAMPLE PROBLEM<-/'/:74ITUDINAL MODE FOR HYPOTHETICAL ENGINE

PAGE 3

VELOCITY POTENTIAL TABLE

Z = AXIAL POSITION IN NOZZLE (INCHES)
X = VELOCITY POTENTIAL (NONDIMENSIONAL)
V = AXIAL VELOCITY (UNDIMENSIONALIZED BY SPEED OF SOUND AT THROAT)

POINT	Z	X	V	POINT	Z	X	V
1	.00000	.00000	.10000+01	28	.64055+01	-.13654+01	.32922+00
2	.23724+00	-.87012-01	.95571+00	29	.66427+01	-.13943+01	.32022+00
3	.47448+00	-.17011+00	.91188+00	30	.68799+01	-.14224+01	.31162+00
4	.71172+00	-.24933+00	.86862+00	31	.71172+01	-.14498+01	.30340+00
5	.94896+00	-.32473+00	.82602+00	32	.73544+01	-.14764+01	.29553+00
6	.11862+01	-.39637+00	.78416+00	33	.75917+01	-.15024+01	.28790+00
7	.14234+01	-.46432+00	.74314+00	34	.78289+01	-.15277+01	.28075+00
8	.16607+01	-.52867+00	.70303+00	35	.80661+01	-.15523+01	.27381+00
9	.18979+01	-.58948+00	.66399+00	36	.83034+01	-.15764+01	.26714+00
10	.21352+01	-.64698+00	.62899+00	37	.85406+01	-.15999+01	.26072+00
11	.23724+01	-.70156+00	.59834+00	38	.87779+01	-.16228+01	.25455+00
12	.26096+01	-.75357+00	.57103+00	39	.90151+01	-.16452+01	.24861+00
13	.28469+01	-.80327+00	.54637+00	40	.92523+01	-.16671+01	.24289+00
14	.30841+01	-.85088+00	.52390+00	41	.94896+01	-.16885+01	.23737+00
15	.33214+01	-.89657+00	.50326+00	42	.97268+01	-.17093+01	.23205+00
16	.35586+01	-.94050+00	.48420+00	43	.99641+01	-.17298+01	.22691+00
17	.37958+01	-.98279+00	.46650+00	44	.10201+02	-.17497+01	.22195+00
18	.40331+01	-.10236+01	.45000+00	45	.10439+02	-.17693+01	.21714+00
19	.42703+01	-.10629+01	.43456+00	46	.10676+02	-.17884+01	.21253+00
20	.45075+01	-.11009+01	.42006+00	47	.10913+02	-.18071+01	.20805+00
21	.47448+01	-.11377+01	.40642+00	48	.11150+02	-.18254+01	.20377+00
22	.49820+01	-.11733+01	.39356+00	49	.11387+02	-.18434+01	.20060+00
23	.52193+01	-.12078+01	.38139+00	50	.11625+02	-.18612+01	.19876+00
24	.54565+01	-.12412+01	.36987+00	51	.11862+02	-.18788+01	.19815+00
25	.56937+01	-.12736+01	.35893+00	52	.11872+02	-.18796+01	.19815+00
26	.59310+01	-.13051+01	.34854+00	53	.11882+02	-.18803+01	.19815+00
27	.61682+01	-.13357+01	.33865+00	54	.24662+02	-.28302+01	.19815+00

WN	(SNH)N	DES
.758748+00	.000000	.000000
.113812+01	.000000	.000000
.132781+01	.000000	.000000
.151750+01	.000000	.000000
.170718+01	.000000	.000000
.189687+01	.000000	.000000
.227625+01	.000000	.000000
.614112+01	.000000	.000000

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EXAMPLE PROBLEM<-//:24ITUDINAL MODE FOR HYPOTHETICAL ENGINE

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PROGRAM C OUTPUT

(SNH)C (SNH)N	WC WN	MACH NO G	AR -AR/(MACH NO)	AI -AI/(MACH NO)	BR T1	BI T2	CR -CR/(MACH NO)	CI -CI/(MACH NO)
.0000 .0000	2.0000 .7587	.1885 1.2180	-.26984+00 .14313+01	.27232+00 -.14444+01	.00000 -.32866+00	.00000 .33168+00	-.61422-01 .32580+00	-.26865-01 .14250+00
-2.8302 FC(CPS)= 980.7386								
.0000 .0000	3.0000 1.1381	.1885 1.2180	-.97749+00 .51849+01	-.28783+00 .15267+01	.00000 -.11906+01	.00000 -.35057+00	.34497-01 -.18298+00	.57043-01 -.30257+00
-2.8302 FC(CPS)= 1471.1079								
.0000 .0000	3.5000 1.3278	.1885 1.2180	-.87266+00 .46288+01	.37683-01 -.19988+00	.00000 -.10629+01	.00000 .45898-01	-.35382-01 .18767+00	.30422-01 -.16136+00
-2.8302 FC(CPS)= 1716.2925								
.0000 .0000	4.0000 1.5175	.1885 1.2180	-.82148+00 .43574+01	-.12272+00 .65093+00	.00000 -.10006+01	.00000 -.14947+00	-.18879-01 .10014+00	-.32612-01 .17298+00
-2.8302 FC(CPS)= 1961.4772								
.0000 .0000	4.5000 1.7072	.1885 1.2180	-.10790+01 .57234+01	-.61946-01 .32858+00	.00000 -.13142+01	.00000 -.75451-01	.31844-01 -.16891+00	-.11966-01 .63468-01
-2.8302 FC(CPS)= 2206.6618								
.0000 .0000	5.0000 1.8969	.1885 1.2180	-.92186+00 .48898+01	.25215+00 -.13375+01	.00000 -.11228+01	.00000 .30712+00	.84186-02 -.44655-01	.20991-01 -.11134+00
-2.8302 FC(CPS)= 2451.8464								
.0000 .0000	6.0000 2.2762	.1885 1.2180	-.67979+00 .36058+01	-.75759-01 .40185+00	.00000 -.82799+00	.00000 -.92274-01	.10422-02 -.55280-02	-.13956-01 .74026-01
-2.8302 FC(CPS)= 2942.2157								
.0000	7.0000	.1885	-.80277+00	.30663-02	.00000	.00000	.93632-03	.12451-02

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.0000 6.1411 1.2180 .42581+01 -.16265-01 -.97777+00 .37348-02 -.49665-02 -.66043-02
-2.8302
FC(CPS)= 3432.5850

DATE 25 JUL 70 EXAMPLE PROBLEM<-'/:?4ITUDINAL MODE FOR HYPOTHETICAL ENGINE
TRANSVERSE STABILITY PROGRAM... CALCULATES HR, HI

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INPUT DATA

SIN	ZE	GAMMA	UE	SOUND (FT/SEC)	ULM (FT/SEC)
.00000000	2.31250000	1.21800	.18852762	3800.00	20.420000

NOZZLE ADMITTANCES INPUT

OMEGA(CH)	ERT	EIT	CRT	CIT
2.0000000	-.26983859+00	.27231600+00	-.61422292-01	-.26865192-01

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EXAMPLE PROBLEM<-/'':?4ITUDINAL MODE FOR HYPOTHETICAL ENGINE

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CALCULATED RESULTS

OMEGA	H REAL	H IMAG
2.0000000	.11152828+01	.93327805+00
2.5000000	.15325213+01	.16800883+01
3.0000000	.17270751+01	.21513920+01
3.2500000	.16893139+01	.21996202+01
3.5000000	.16139799+01	.22259782+01
3.7500000	.15737817+01	.24372959+01
4.0000000	.15769181+01	.26913456+01
4.2499999	.17041927+01	.29135248+01
4.4999999	.18208100+01	.31073267+01
4.7499999	.17901384+01	.32061581+01
5.0000000	.17180099+01	.33261167+01
5.4999999	.15904034+01	.39081204+01
6.0000000	.14842304+01	.46183871+01
6.4999999	.14287092+01	.53301924+01
7.0000000	.13770885+01	.61530102+01

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EXAMPLE PROBLEM<--/:?4ITUDINAL MODE FOR HYPOTHETICAL ENGINE

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EXAMPLE PROBLEM<-//:24ITUDINAL MODE FOR HYPOTHETICAL ENGINE

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PROGRAM F OUTPUT

FC(CPS)	(OMGA)D,	TAU(MS)	N
980.7	2.00000	.28367	.94813
1043.6	2.12821	.25133	1.15447
1106.5	2.25541	.22662	1.35069
1169.3	2.38462	.20686	1.53419
1232.2	2.51282	.19050	1.70348
1295.1	2.64103	.17653	1.86052
1357.9	2.76923	.16442	2.00293
1420.8	2.89744	.15386	2.12482
1483.7	3.02564	.14461	2.21896
1546.5	3.15385	.13655	2.25803
1609.4	3.28205	.12894	2.28362
1672.3	3.41026	.12177	2.30069
1735.2	3.53846	.11377	2.37642
1798.0	3.66667	.10469	2.54619
1860.9	3.79487	.09652	2.74482
1923.8	3.92308	.08928	2.96544
1986.6	4.05128	.08449	3.14990
2049.5	4.17949	.08205	3.27774
2112.4	4.30769	.07984	3.39624
2175.2	4.43590	.07776	3.50808
2238.1	4.56410	.07488	3.61243
2301.0	4.69231	.07128	3.71256
2363.8	4.82051	.06752	3.83567
2426.7	4.94872	.06359	3.98963
2489.6	5.07692	.05927	4.24519
2552.4	5.20513	.05486	4.58604
2615.3	5.33333	.05062	4.99461
2678.2	5.46154	.04671	5.45309
2741.0	5.58974	.04315	5.95343
2803.9	5.71795	.03982	6.52140
2866.8	5.84615	.03680	7.14338
2929.6	5.97436	.03413	7.79593
2992.5	6.10256	.03184	8.45198
3055.4	6.23077	.02984	9.11516
3118.2	6.35897	.02803	9.81155
3181.1	6.48718	.02632	10.57552
3244.0	6.61538	.02469	11.42604
3306.8	6.74359	.02317	12.34961
3369.7	6.87179	.02174	13.35083
3432.6	7.00000	.02042	14.43477

THE FOLLOWING ARE VALUES INTERPOLATED AT SLOPE OF N=0.0

NMIN= .40249
 TAU(MS)= .47258
 (OMEGA)D= 1.65950
 FC(CPS)= 813.8

DATE 25 JUL 70 EXAMPLE PROBLEM+/'41%ST TANGENTIAL MODE!'2-%?THETICAL ENGINE

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A	B	C	D	E	F	G	H	I	J
100.	0.	10.	10.	100.	0.	0.	0.	0.	100.

***** THE FOLLOWING MAIN CONTROL DATA WILL BE USED IN THIS CASE *****

RATIO OF SPECIFIC HEAT (GAMMA) = 1.2180

DESIRED MACH NUMBER = .00000 (=0 IF BEING CALCULATED)

CHAMBER RADIUS = 6.400 (INCHES)

CHAMBER LENGTH = 14.800 (INCHES)

SPEED OF SOUND = 3800.000 (FT/SEC)

CHAMBER MODE DESCRIPTION = 1.84118 (=0 FOR LONGITUDINAL MODES)

***** CHAMBER FREQUENCIES (WC) *****

1.69389	1.73071	1.76754	1.80436	1.84118
1.87801	1.91483	1.95165	1.98848	2.02530

***** MACH DISTRIBUTION IN CHAMBER AS A FUNCTION OF LENGTH *****

CHAMBER LENGTH	MACH DISTRIBUTION	CHAMBER LENGTH	MACH DISTRIBUTION	CHAMBER LENGTH	MACH DISTRIBUTION
.00000	.00000	6.40000	6.39500	25.00000	6.40000
2.50000	2.50000	6.41000	6.40000	50.00000	6.40000
5.00000	5.00000	6.42000	6.40000	.00000	.00000
6.30000	6.30000	6.43000	6.40000	.00000	.00000
6.35000	6.35000	6.50000	6.40000	.00000	.00000
6.38000	6.38000	7.50000	6.40000	.00000	.00000
6.39000	6.39000	12.50000	6.40000		

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EXAMPLE PROBLEM+/'41%ST TANGENTIAL MODE!'2-%?THETICAL ENGINE

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SECTION 1....MISCELLANEOUS INFORMATION FOR INJECTOR DESIGNED BY PROJECTS

A....PROPELLENT ORFICE AREAS

ELEMENT TOTAL OXIDIZER AREA = 3.48462972 SQ. IN.
TOTAL OXIDIZER FILM COOLING AREA = .00000000 SQ. IN.
INJECTOR TOTAL OXIDIZER AREA = 3.48462972 SQ. IN.

ELEMENT TOTAL FUEL AREA = 5.22700441 SQ. IN.
TOTAL FUEL FILM COOLING AREA = .00000000 SQ. IN.
INJECTOR TOTAL FUEL AREA = 5.22700441 SQ. IN.

B....INJECTOR PRESSURE DROPS FOR ABOVE INJECTOR DESIGN

OXIDIZER PRESSURE DROP = 36.6PSI

FUEL PRESSURE DROP = 6.5 PSI

C....PROPELLANT FLOWS AND INJECTOR VELOCITIES FOR ABOVE INJECTOR DESIGN

TOTAL WEIGHT FLOW = 150.0 LB/SEC
AVERAGE MIXTURE RATIO OF THE ELEMENTS = 2.000
OVERALL MIXTURE RATIO FOR THE INJECTOR = 2.000

ELEMENT TOTAL OXIDIZER FLOW = 100.0 LB/SEC
TOTAL OXIDIZER FILM COOLING FLOW = .0 LB/SEC
INJECTOR TOTAL OXIDIZER FLOW = 100.0 LB/SEC
OXIDIZER OVERALL INJECTION VELOCITY = 61.5 FT/SEC

ELEMENT TOTAL FUEL FLOW = 50.0 LB/SEC
TOTAL FUEL FILM COOLING = .0 LB/SEC
INJECTOR TOTAL FUEL FLOW = 50.0 LB/SEC
FUEL OVERALL INJECTION VELOCITY = 32.7 FT/SEC

D.....INPUT INFORMATION USED IN COMPUTATIONS

TOTAL PROPELLENT FLOW = 150.0 LB/SEC
TOTAL NUMBER OF ELEMENT TYPES (SYMMETRICAL SECTION ONLY) = 3
TOTAL NUMBER OF ELEMENTS (SYMMETRICAL SECTION ONLY) = 3
OXIDIZER LOSS COEFFICIENT = .750
FUEL LOSS COEFFICIENT = .750
PERCENT FUEL FILM COOLING = .0
PERCENT OXIDIZER FILM COOLING = .0
DIAMETER OF FUEL FILM COOLING ORFICE = .00000 IN. (NOTE..THIS MIGHT BE AN EQUIVALENT DIAMETER FOR MULTIPLE-ROW COOLING)
DIAMETER OF OXIDIZER FILM COOLING ORFICE = .00000 IN. (SEE ABOVE NOTE)
NUMBER OF FUEL FILM COOLING ORIFICES PER INJECTOR = 0.
NUMBER OF OXIDIZER FILM COOLING ORIFICES PER INJECTOR = 0.
OXIDIZER DENSITY = 89.52 PCF
FUEL DENSITY = 56.1 PCF

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EXAMPLE PROBLEM+//41%ST TANGENTIAL MODE! '2-%?THETICAL ENGINE

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SECTION 2.... ELEMENT LOCATION AND INJECTION TYPE

ELEMENT NO.	TYPE NO.	R (INCHES)	THETA (DEGREE)	X (INCHES)	Y (INCHES)
1	1	2.00000	1.00000	1.99970	.03490
2	2	4.00000	1.00000	3.99939	.06981
3	3	6.00000	1.00000	5.99909	.10471

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EXAMPLE PROBLEM+//W'41%ST TANGENTIAL MODE!'2-%?THETICAL ENGINE

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SECTION 3....TYPE DESCRIPTION,ORIFICE AREA, PROPELLANT FLOW, AND MIXTURE RATIO

TYPE *----- OXIDIZER ORIFICE DATA -----*----- FUEL ORIFICE DATA -----*

	NUMBER OF ORIFICE	DIAMETER IN.	AREA SQ. IN.	FLOW LB/SEC	NUMBER OF ORIFICES	DIAMETER IN.	AREA SQ. IN.	FLOW LB/SEC	TOTAL OXIDIZER AREA SQ. IN.	TOTAL FUEL AREA SQ. IN.	TOTAL PROPELLANT FLOW RATE LB/SEC	MIXTURE RATIO
1	1	.0785	.004840	.138889	1	.0785	.004840	.046296	.00484	.00484	.1852	3.0000
2	2	.0785	.004840	.138889	1	.0785	.004840	.046296	.00968	.00484	.3241	6.0001
		.0785	.004840	.138889								
3	1	.0785	.004840	.138889	4	.0785	.004840	.046296	.00484	.01936	.3241	.7500
						.0785	.004840	.046296				
						.0785	.004840	.046296				
						.0785	.004840	.046296				
						.0785	.004840	.046296				

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EXAMPLE PROBLEM+/'41%ST TANGENTIAL MODE!'2-%?THETICAL ENGINE

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ELEMENT RESULTS

ELEMENT NO.	RADIUS	ANGLE RADIANS	DISTRIBUTION COEFFICIENT MU
1	2.000	.0175	4.4444
2	4.000	.0175	7.7778
3	6.000	.0175	7.7778

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EXAMPLE PROBLEM+/'41%ST TANGENTIAL MODE!'2-%?THETICAL ENGINE

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SECTION 4..RESULTANT FLOW DISTRIBUTION (MU) AS A FUNCTION OF ONLY THE RADIUS, I.E.. AVERAGE OF MU IN A RADIAL BAND

RADIUS IN.	MU(R)	MU(R)/MU(MAX)
1.431	.000	.0000
2.024	4.444	.5714
2.479	.000	.0000
2.862	.000	.0000
3.200	.000	.0000
3.505	.000	.0000
3.786	.000	.0000
4.048	7.778	1.0000
4.293	.000	.0000
4.525	.000	.0000
4.746	.000	.0000
4.957	.000	.0000
5.160	.000	.0000
5.355	.000	.0000
5.543	.000	.0000
5.724	.000	.0000
5.901	.000	.0000
6.072	7.778	1.0000
6.238	.000	.0000
6.400	.000	.0000

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EXAMPLE PROBLEM+/W'41%ST TANGENTIAL MODE!'2-%?THETICAL ENGINE

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INPUT TO INJECTION DISTRIBUTION PROGRAM

CONSTANTS

NUMBER OF OMEGAS = 1

NUMBER OF ELEMENTS = 3 FOR EACH OF 180 SYMMETRIC SECTIONS.

RADIAL DIVISIONS(XM) = 20.

ANGULAR DIVISIONS (XN) = 180.

ACOUSTIC MODE NUMBER(SVN) = 1.8412

ORDER OF BESSEL FUNCTIONS(V) = 1.

INJECTOR RADIUS = 6.400, IN.

RATIO OF SPECIFIC HEATS(GAMMA) = 1.2180

MAXIMUM PRESSURE AMPLITUDE RATIO(P00) = .000

TRANSFER FUNCTIONS FOR LINEAR OPERATION

PRESSURE(TFLP) = .000

RADIAL VELOCITY(TFLR) = .000

TANGENTIAL VELOCITY(TFLT) = .000

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EXAMPLE PROBLEM+/'41%ST TANGENTIAL MODE!'2-%?THETICAL ENGINE

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ELEMENT INFORMATION

ELEMENT NO.	RADIUS IN.	ANGLE RADIANS	DISTRIBUTION COEFFICIENT MU
1	2.000	.0175	4.4444
2	4.000	.0175	7.7778
3	6.000	.0175	7.7778

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EXAMPLE PROBLEM+/'41%ST TANGENTIAL MODE!'2-%?THETICAL ENGINE

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RESULTS OF INJECTION DISTRIBUTION EFFECTS

OMEGA	AVN REAL	BVN REAL	CVN REAL	CVN IMAG
ALL	1.2745	.8766	.0000	-1.8125

***** THESE VALUES PERTAIN TO A SPINNING MODE *****

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PROGRAM C OUTPUT

EXAMPLE PROBLEM+/W*41%ST TANGENTIAL MODE!*2-%?THETICAL ENGINE

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(SINH) (SINH)	WC WN	MACH NO G	AR -AR/(MACH NO)	AI -AI/(MACH NO)	BR T1	BI T2	CR -CR/(MACH NO)	CI -CI/(MACH NO)
1.8412 2.7421	1.6939 1.4861	.1885 1.2180	-.49834-03 .26433-02	.39852+00 -.21139+01	-.42140-01 .23613+00	-.40101+00 .46052+00	.43929-02 -.23301-01	-.38922-03 .20645-02
-1.8788 FC(CPS)= 1920.8346								
1.8412 2.7421	1.7675 1.5507	.1885 1.2180	.11057-01 -.58648-01	.33418+00 -.17726+01	-.53721-01 .22195+00	-.36851+00 .37664+00	.38370-02 -.20353-01	-.21846-02 .11588-01
-1.8788 FC(CPS)= 2004.3491								
1.8412 2.7421	1.8412 1.6153	.1885 1.2180	.23791-01 -.12619+00	.26500+00 -.14056+01	-.56067-01 .20942+00	-.33223+00 .29232+00	.23592-02 -.12514-01	-.36299-02 .19254-01
-1.8788 FC(CPS)= 2087.8636								
1.8412 2.7421	1.9148 1.6799	.1885 1.2180	.35725-01 -.18950+00	.18962+00 -.10058+01	-.47438-01 .19851+00	-.29679+00 .20618+00	.17772-03 -.94266-03	-.42791-02 .22698-01
-1.8788 FC(CPS)= 2171.3781								
1.8412 2.7421	1.9885 1.7445	.1885 1.2180	.44991-01 -.23865+00	.10706+00 -.56789+00	-.28209-01 .18923+00	-.26732+00 .11622+00	-.22948-02 .12172-01	-.37970-02 .20140-01
-1.8788 FC(CPS)= 2254.8926								
1.8412 2.7421	2.0253 1.7768	.1885 1.2180	.48110-01 -.25519+00	.62812-01 -.33317+00	-.15334-01 .18518+00	-.25637+00 .68934-01	-.34685-02 .18398-01	-.30747-02 .16309-01
-1.8788 FC(CPS)= 2296.6498								

TRANSVERSE STABILITY PROGRAM... CALCULATES HR, HI

INPUT DATA

SNH	ZE	GAMMA	UE	SOUND (FT/SEC)	ULM (FT/SEC)	XK (DRAG)	XCOMPL (IN)
1.84118374	2.31250000	1.21800	.18852762	3800.00	20.420000	.000000	.000000
INCREMENTS							
.0							

NOZZLE ADMITTANCES INPUT

	OMEGA(CH)	ERT	EIT
	1.6938890	.23612978+00	.46052134+00
	1.7675364	.22195471+00	.37663660+00
	1.8411837	.20942160+00	.29232068+00
	1.9148310	.19851034+00	.20618085+00
	1.9884783	.18923368+00	.11621683+00
	2.0253020	.18518268+00	.68934425-01
STABS			
S*C	= (.52069750E+00,	.00000000E+00),	
SNH	= .18411837E+01,		
W	= .16938890E+01,		
OMEGC	= (.72159372E+00,	.00000000E+00),	
E	= (.23612978E+00,	.46052134E+00),	
SIGMA	= (.00000000E+00,	.16938890E+01),	
Q1SI	= (.00000000E+00,	-.12950017E+01),	
Q1CI	= (.00000000E+00,	-.14993784E+01),	
Q2SI	= (.00000000E+00,	.71299743E+00),	
Q2CI	= (.00000000E+00,	.82565935E+00),	
H	= (.17504949E+01,	-.25148608E+01),	
SEND			
STABS			
S*C	= (.39459112E+00,	.00000000E+00),	
SNH	= .18411837E+01,		
W	= .17507127E+01,		
OMEGC	= (.62816488E+00,	.00000000E+00),	
E	= (.22883699E+00,	.41863286E+00),	
SIGMA	= (.00000000E+00,	.17307127E+01),	
Q1SI	= (.00000000E+00,	-.10085253E+01),	
Q1CI	= (.00000000E+00,	-.12387506E+01),	
Q2SI	= (.00000000E+00,	.55528116E+00),	
Q2CI	= (.00000000E+00,	.68218827E+00),	
H	= (.17280367E+01,	-.22175635E+01),	
SEND			
STABS			
S*C	= (.26577279E+00,	.00000000E+00),	
SNH	= .18411837E+01,		
W	= .17675364E+01,		
OMEGC	= (.51553155E+00,	.00000000E+00),	

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E      = ( .22195471E+00, .37663660E+00),
SIGMA  = ( .00000000E+00, .17675364E+01),
Q1SI   = ( .00000000E+00, -.78727534E+00),
Q1CI   = ( .00000000E+00, -.10742069E+01),
Q2SI   = ( .00000000E+00, .43349116E+00),
Q2CI   = ( .00000000E+00, .59163223E+00),
H      = ( .17021257E+01, -.17422520E+01),
$END
$TABS
SWC    = ( .13424250E+00, .00000000E+00),
SNH    = .18411837E+01,
W      = .18043000E+01,
OMEGC  = ( .36039119E+00, .00000000E+00),
E      = ( .21548416E+00, .33461957E+00),
SIGMA  = ( .00000000E+00, .18043000E+01),
Q1SI   = ( .00000000E+00, -.53023429E+00),
Q1CI   = ( .00000000E+00, -.91210546E+00),
Q2SI   = ( .00000000E+00, .29197411E+00),
Q2CI   = ( .00000000E+00, .50241572E+00),
H      = ( .16037479E+01, -.12504508E+01),
$END
$TABS
SWC    = ( .23841858E-06, .00000000E+00),
SNH    = .18411837E+01,
W      = .18411837E+01,
OMEGC  = ( .48828125E-03, .00000000E+00),
E      = ( .20942160E+00, .29232068E+00),
SIGMA  = ( .00000000E+00, .18411837E+01),
Q1SI   = ( .00000000E+00, -.66701029E-03),
Q1CI   = ( .00000000E+00, -.75323950E+00),
Q2SI   = ( .00000000E+00, .36731105E-03),
Q2CI   = ( .00000000E+00, .41497515E+00),
H      = ( .16040049E+01, -.73145898E+00),
$END
$TABS
SWC    = (-.13695401E+00, .00000000E+00),
SNH    = .18411837E+01,
W      = .18780073E+01,
OMEGC  = ( .00000000E+00, .37007298E+00),
E      = ( .20376244E+00, .24960378E+00),
SIGMA  = ( .00000000E+00, .18780073E+01),
Q1SI   = ( .00000000E+00, -.47517942E+00),
Q1CI   = ( .00000000E+00, -.59836961E+00),
Q2SI   = ( .00000000E+00, .26168950E+00),
Q2CI   = ( .00000000E+00, .32972904E+00),
H      = ( .15051324E+01, -.16932303E+00),
$END
$TABS
SWC    = (-.27662018E+00, .00000000E+00),
SNH    = .18411837E+01,
W      = .19148310E+01,
OMEGC  = ( .00000000E+00, .52594693E+00),
E      = ( .19851034E+00, .20618086E+00),
SIGMA  = ( .00000000E+00, .19148310E+01),
Q1SI   = ( .00000000E+00, -.63188915E+00),
Q1CI   = ( .00000000E+00, -.44821967E+00),
Q2SI   = ( .00000000E+00, .34801620E+00),
Q2CI   = ( .00000000E+00, .24707574E+00),
H      = ( .13280004E+01, .45938596E+00),

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$END
$TABS
SWC      = (-.41899833E+00, .00000000E+00),
SWH      = (.18411837E+01,
W         = .19516547E+01,
OMEGC    = (.00000000E+00, .64730080E+00),
E         = (.19369643E+00, .16172541E+00),
SIGMA     = (.00000000E+00, .19516547E+01),
Q1SI     = (.00000000E+00, -.72407361E+00),
Q1CI     = (.00000000E+00, -.30347320E+00),
Q2SI     = (.00000000E+00, .39881757E+00),
Q2CI     = (.00000000E+00, .16739143E+00),
H         = (.97758906E+00, .11801624E+01),
$END
$TABS
SWC      = (-.56408846E+00, .00000000E+00),
SWH      = (.18411837E+01,
W         = .19884783E+01,
OMEGC    = (.00000000E+00, .75105823E+00),
E         = (.18923368E+00, .11621683E+00),
SIGMA     = (.00000000E+00, .19884783E+01),
Q1SI     = (.00000000E+00, -.77798069E+00),
Q1CI     = (.00000000E+00, -.16477004E+00),
Q2SI     = (.00000000E+00, .42854564E+00),
Q2CI     = (.00000000E+00, .91028284E-01),
H         = (.21287662E+00, .19509883E+01),
$END
$TABS
SWC      = (-.71189046E+00, .00000000E+00),
SWH      = (.18411837E+01,
W         = .20253020E+01,
OMEGC    = (.00000000E+00, .84373601E+00),
E         = (.18518267E+00, .68934424E-01),
SIGMA     = (.00000000E+00, .20253020E+01),
Q1SI     = (.00000000E+00, -.80446833E+00),
Q1CI     = (.00000000E+00, -.32703382E-01),
Q2SI     = (.00000000E+00, .44317819E+00),
Q2CI     = (.00000000E+00, .18312837E-01),
H         = (-.14011466E+01, .22003332E+01),
$END

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EXAMPLE PROBLEM+/W'41%ST TANGENTIAL MODE?'2-X?THEYICAL ENGINE

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CALCULATED RESULTS...

FIRST-ORDER SOLN (UNIF INJ)

OMEGA	H REAL	H IMAG
1.6938890	.17504949+01	-.25148608+01
1.7307127	.17280367+01	-.22175635+01
1.7675364	.17021257+01	-.17422520+01
1.8043600	.16637479+01	-.12504508+01
1.8411837	.16040049+01	-.73145898+00
1.8780073	.15051324+01	-.16932303+00
1.9148310	.13280004+01	.45938596+00
1.9516547	.97758906+00	.11801624+01
1.9884783	.21287662+00	.19509883+01
2.0253020	-.14011486+01	.22003332+01

PROGRAM D OUTPUT

OMEGA(C)	HTR	HTI
1.693889	1.373502	-1.973252
1.730713	1.355881	-1.739982
1.767536	1.335550	-1.367035
1.804360	1.305438	-.981150
1.841184	1.258561	-.573930
1.878007	1.180982	-.132857
1.914831	1.041998	.360451
1.951655	.767052	.925999
1.988478	.167031	1.530817
2.025302	-1.099393	1.726462

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EXAMPLE PROBLEM+/'41%ST TANGENTIAL MODE!'2-%?THETICAL ENGINE

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EXAMPLE PROBLEM+741%ST TANGENTIAL MODE!*2-%?THETICAL ENGINE

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FOLLOWING WILL BE INTERPOLATION WITHIN HTR HTI TABLE GIVEN ABOVE

OMEGA	HTRINT	HTIINT
1.693889	1.373502	-1.973252
1.702387	1.369676	-1.931818
1.710885	1.365706	-1.882945
1.719382	1.361591	-1.826634
1.727880	1.357333	-1.762885
1.736378	1.353000	-1.690428
1.744876	1.348704	-1.607302
1.753373	1.344205	-1.517776
1.761871	1.339244	-1.426523
1.770369	1.333600	-1.337834
1.778867	1.327555	-1.249955
1.787364	1.321018	-1.161421
1.795862	1.313732	-1.071923
1.804360	1.305438	-.981151
1.812858	1.296393	-.889327
1.821355	1.286685	-.796572
1.829853	1.275798	-.702425
1.838351	1.263219	-.606423
1.846849	1.248932	-.508461
1.855346	1.233754	-.409131
1.863844	1.216690	-.307847
1.872342	1.196607	-.203932
1.880840	1.172675	-.096822
1.889338	1.147202	.012709
1.897835	1.118764	.124939
1.906333	1.084612	.240606
1.914831	1.041999	.360448
1.923329	.994491	.485222
1.931826	.942842	.614384
1.940324	.880079	.746717
1.948822	.799229	.881007
1.957320	.699321	1.020978
1.965817	.590285	1.174381
1.974315	.461141	1.326201
1.982813	.299305	1.459906
1.991311	.093280	1.560390
1.999808	-.151651	1.634594
2.008306	-.432072	1.687007
2.016804	-.747981	1.717630
2.025302	-1.099378	1.726462

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PROGRAM F OUTPUT

EXAMPLE PROBLEM+/W*41%ST TANGENTIAL MODE!*2-%?THETICAL ENGINE

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FC(CPS)	(OMGA)D:	TAU(MS)	N
1920.8	1.69389	.41984	2.10419
1930.5	1.70239	.41631	2.04717
1940.1	1.71088	.41248	1.98089
1949.7	1.71938	.40831	1.90605
1959.4	1.72788	.40377	1.82347
1969.0	1.73638	.39875	1.73250
1978.7	1.74488	.39308	1.63209
1988.3	1.75337	.38691	1.52898
1997.9	1.76187	.38042	1.42937
2007.6	1.77037	.37384	1.33784
2017.2	1.77887	.36705	1.25222
2026.8	1.78736	.35995	1.17106
2036.5	1.79586	.35250	1.09418
2046.1	1.80436	.34463	1.02143
2055.7	1.81286	.33632	.95324
2065.4	1.82136	.32752	.88992
2075.0	1.82985	.31817	.83127
2084.7	1.83835	.30819	.77717
2094.3	1.84685	.29751	.72797
2103.9	1.85535	.28610	.68471
2113.6	1.86384	.27389	.64729
2123.2	1.87234	.26080	.61568
2132.8	1.88084	.24672	.59033
2142.5	1.88934	.23173	.57367
2152.1	1.89784	.21588	.56636
2161.7	1.90633	.19915	.56899
2171.4	1.91483	.18145	.58334
2181.0	1.92333	.16300	.61562
2190.7	1.93183	.14433	.67160
2200.3	1.94032	.12545	.75682
2209.9	1.94882	.10612	.88519
2219.6	1.95732	.08613	1.09495
2229.2	1.96582	.06651	1.46337
2238.8	1.97432	.04758	2.13759
2248.5	1.98281	.02863	3.71011
2258.1	1.99131	.00842	13.09772
2267.7	1.99981	.20750	-8.88517
2277.4	2.00831	.18451	-3.50946
2287.0	2.01680	.16146	-2.34613
2296.6	2.02530	.13912	-1.90531

THE FOLLOWING ARE VALUES INTERPOLATED AT SLOPE OF N=0.0

NMIN= .56602
TAU(MS)= .21213
(OMGA)D= 1.89978
FC(CPS)= 2154.3

DATE 25 JUL 70

EXAMPLE PROBLEM+//W'S590ND TANGENTIAL MODE2 HY00THETICAL ENGINE

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A	B	C	D	E	F	G	H	I	J
0.	100.	0.	10.	100.	10.	0.	0.	0.	0.

***** THE FOLLOWING MAIN CONTROL DATA WILL BE USED IN THIS CASE *****

RATIO OF SPECIFIC HEAT (GAMMA) = 1.2180

DESIRED MACH NUMBER = .00000 (=0 IF BEING CALCULATED)

CHAMBER RADIUS = 6.400 (INCHES)

CHAMBER LENGTH = 14.800 (INCHES)

SPEED OF SOUND = 3800.000 (FT/SEC)

CHAMBER MODE DESCRIPTION = 3.05424 (=0 FOR LONGITUDINAL MODES)

***** CHAMBER FREQUENCIES (WC) *****

2.80990	2.87098	2.93207	2.99315	3.05424
3.11532	3.17641	3.23749	3.29858	3.35966

DISTANCE FROM INJECTOR TO COMBUSTION FRONT 2.00000INCHES

DATE 25 JUL 70

EXAMPLE PROBLEM+/W'S590ND TANGENTIAL MODE2 HY00THETICAL ENGINE

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DATE 25 JUL 70

EXAMPLE PROBLEM+/W/5590ND TANGENTIAL MODE2 HY00THETICAL ENGINE

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INPUT TO INJECTION DISTRIBUTION PROGRAM

CONSTANTS

NUMBER OF OMEGAS = 1

NUMBER OF ELEMENTS = 3 FOR EACH OF 180 SYMMETRIC SECTIONS.

RADIAL DIVISIONS(XM) = 20.

ANGULAR DIVISIONS (XN) = 180.

ACOUSTIC MODE NUMBER(SVN) = 3.0542

ORDER OF BESSEL FUNCTIONS(V) = 2.

INJECTOR RADIUS = 6.400, IN.

RATIO OF SPECIFIC HEATS(GAMMA) = 1.2180

MAXIMUM PRESSURE AMPLITUDE RATIO(P00) = .000

TRANSFER FUNCTIONS FOR LINEAR OPERATION

PRESSURE(TFLP) = .000

RADIAL VELOCITY(TFLR) = .000

TANGENTIAL VELOCITY(TFLT) = .000

DATE 25 JUL 70

EXAMPLE PROBLEM+/W*590ND TANGENTIAL MODE2 HY00THETICAL ENGINE

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ELEMENT INFORMATION

ELEMENT NO.	RADIUS IN.	ANGLE RADIANS	DISTRIBUTION COEFFICIENT MU
1	2.000	.0175	4.4444
2	4.000	.0175	7.7778
3	6.000	.0175	7.7778

DATE 25 JUL 70

EXAMPLE PROBLEM+/W\$590ND TANGENTIAL MODE2 HYDOTHETICAL ENGINE

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RESULTS OF INJECTION DISTRIBUTION EFFECTS

OMEGA	AVN REAL	BVN REAL	CVN REAL	CVN IMAG
ALL	1.2761	1.2828	.0000	-3.2524

***** THESE VALUES PERTAIN TO A SPINNING MODE *****

TRANSVERSE STABILITY PROGRAM... CALCULATES HR, HI

INPUT DATA

SNH	ZE	GAMMA	UE	SOUND (FT/SEC)	ULM (FT/SEC)
3.05423680	2.31250000	1.21800	.18852762	3800.00	20.420000

NOZZLE ADMITTANCES INPUT

OMEGA(CH)	ERT	EIT	CRT	CIT
1.6938890	.23612978+00	.46052134+00	.43929329-02	-.38921888-03
1.7675364	.22195471+00	.37663660+00	.38370105-02	-.21846309-02
1.8411837	.20942160+00	.29232068+00	.23591529-02	-.36299161-02
1.9148310	.19851034+00	.20618085+00	.17771693-03	-.42791335-02
1.9884783	.18923368+00	.11621683+00	-.22948344-02	-.37970401-02
2.0253020	.18518268+00	.68934425-01	-.34685028-02	-.30747288-02

CALCULATED RESULTS

OMEGA	H REAL	H IMAG
2.8098978	.84986281+00	.75610643+00
2.8709826	.89863663+00	.10063908+01
2.9320673	.94631281+00	.12515626+01
2.9931520	.99286398+00	.14920600+01
3.0542367	.10382651+01	.17283221+01
3.1153214	.10824934+01	.19607932+01
3.1764061	.11255292+01	.21899250+01
3.2374909	.11673558+01	.24161806+01
3.2985756	.12079595+01	.26400370+01
3.3596603	.12473300+01	.28619903+01

PROGRAM D OUTPUT

OMEGA(C)	HTR	HTI
2.809898	.665967	.592498
2.870983	.704187	.788625
2.932067	.741547	.980746
2.993152	.778025	1.169204
3.054237	.813602	1.354343
3.115321	.848260	1.536511
3.176406	.881984	1.716063
3.237491	.914760	1.893361
3.298576	.946578	2.068778
3.359660	.977429	2.242705

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EXAMPLE PROBLEM+/W'S590ND TANGENTIAL MODE2 HY00THETICAL ENGINE

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EXAMPLE PROBLEM+/W'S590ND TANGENTIAL MODE2 HY00THETICAL ENGINE

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FOLLOWING WILL BE INTERPOLATION WITHIN HTR HTI TABLE GIVEN ABOVE

OMEGA	HTRINT	HTIINT
2.809898	.665967	.592498
2.823994	.674863	.638114
2.838091	.683714	.683516
2.852187	.692519	.728705
2.866284	.701278	.773681
2.880380	.709991	.818439
2.894477	.718659	.862976
2.908573	.727281	.907302
2.922670	.735856	.951431
2.936766	.744384	.995372
2.950863	.752866	1.039112
2.964959	.761300	1.082655
2.979055	.769687	1.126015
2.993152	.778025	1.169204
3.007248	.786316	1.212215
3.021345	.794558	1.255045
3.035441	.802753	1.297705
3.049538	.810898	1.340208
3.063634	.818994	1.382556
3.077731	.827042	1.424743
3.091827	.835040	1.466772
3.105924	.842969	1.508659
3.120020	.850888	1.550414
3.134117	.858737	1.592024
3.148213	.866536	1.633493
3.162310	.874285	1.674835
3.176406	.881984	1.716062
3.190502	.889632	1.757170
3.204599	.897230	1.798151
3.218695	.904777	1.839019
3.232792	.912273	1.879790
3.246888	.919717	1.920466
3.260985	.927111	1.961033
3.275081	.934454	2.001504
3.289178	.941745	2.041891
3.303274	.948985	2.082210
3.317371	.956173	2.122452
3.331467	.963310	2.162616
3.345564	.970395	2.202700
3.359660	.977429	2.242704

DATE 25 JUL 70

EXAMPLE PROBLEM+/W/\$590ND TANGENTIAL MODE2 HY00THETICAL ENGINE

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PROGRAM F OUTPUT

FC(CPS)	(OMGA)D,	TAU(MS)	N
3186.4	2.80990	.08428	.59655
3202.4	2.82399	.08085	.63911
3218.3	2.83809	.07769	.68352
3234.3	2.85219	.07479	.72965
3250.3	2.86628	.07211	.77742
3266.3	2.88038	.06964	.82672
3282.3	2.89448	.06734	.87747
3298.3	2.90857	.06521	.92958
3314.2	2.92267	.06323	.98301
3330.2	2.93677	.06138	1.03768
3346.2	2.95086	.05964	1.09353
3362.2	2.96496	.05802	1.15048
3378.2	2.97906	.05650	1.20850
3394.2	2.99315	.05506	1.26754
3410.2	3.00725	.05371	1.32756
3426.1	3.02134	.05244	1.38848
3442.1	3.03544	.05123	1.45029
3458.1	3.04954	.05009	1.51296
3474.1	3.06363	.04900	1.57646
3490.1	3.07773	.04797	1.64072
3506.1	3.09183	.04699	1.70573
3522.1	3.10592	.04605	1.77148
3538.0	3.12002	.04516	1.83796
3554.0	3.13412	.04430	1.90511
3570.0	3.14821	.04349	1.97290
3586.0	3.16231	.04270	2.04135
3602.0	3.17641	.04195	2.11045
3618.0	3.19050	.04123	2.18017
3633.9	3.20460	.04054	2.25046
3649.9	3.21870	.03987	2.32135
3665.9	3.23279	.03923	2.39284
3681.9	3.24689	.03861	2.46493
3697.9	3.26098	.03801	2.53755
3713.9	3.27508	.03744	2.61073
3729.9	3.28918	.03688	2.68449
3745.8	3.30327	.03634	2.75883
3761.8	3.31737	.03582	2.83373
3777.8	3.33147	.03531	2.90917
3793.8	3.34556	.03482	2.98515
3809.8	3.35966	.03434	3.06165

THE FOLLOWING ARE VALUES INTERPOLATED AT SLOPE OF N=0.0

NMIN= .27513
TAU(MS)= .19057
(OMEGA)D= 2.60202
FC(CPS)= 2950.6

DATE 25 JUL 70

AT LODE2 RHOWING EFFECTS OF DEADBAND NONLINEARITY

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A	B	C	D	E	F	G	H	I	J
0.	0.	0.	0.	200.	0.	0.	0.	50.	0.

***** THE FOLLOWING MAIN CONTROL DATA WILL BE USED IN THIS CASE *****

RATIO OF SPECIFIC HEAT (GAMMA) = 1.2180

DESIRED MACH NUMBER = .00000 (=0 IF BEING CALCULATED)

CHAMBER RADIUS = 6.400 (INCHES)

CHAMBER LENGTH = 14.800 (INCHES)

SPEED OF SOUND = 3800.000 (FT/SEC)

CHAMBER MODE DESCRIPTION = 1.84118 (=0 FOR LONGITUDINAL MODES)

***** CHAMBER FREQUENCIES (WC) *****

1.69389	1.73071	1.76754	1.80436	1.84118
1.87801	1.91483	1.95165	1.98848	2.02530

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AT LODE2 RHOWING EFFECTS OF DEADBAND NONLINEARITY

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AT LOE2 SHOWING EFFECTS OF DEADBAND NONLINEARITY

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INPUT TO INJECTION DISTRIBUTION PROGRAM

CONSTANTS

NUMBER OF OMEGAS = 6

NUMBER OF ELEMENTS = 3 FOR EACH OF 180 SYMMETRIC SECTIONS.

RADIAL DIVISIONS(XM) = 20.

ANGULAR DIVISIONS (XN) = 180.

ACOUSTIC MODE NUMBER(SVN) = 1.8412

ORDER OF BESSEL FUNCTIONS(V) = 1.

INJECTOR RADIUS = 6.400 IN.

RATIO OF SPECIFIC HEATS(GAMMA) = 1.2180

MAXIMUM PRESSURE AMPLITUDE RATIO(P00) = 1.000

TRANSFER FUNCTIONS FOR LINEAR OPERATION

PRESSURE(TFLP) = .000

RADIAL VELOCITY(TFLR) = 1.000

TANGENTIAL VELOCITY(TFLT) = .000

INPUT FREQUENCIES

1.6939

1.7675

1.8412

1.9148

1.9885

2.0253

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AT LODE2 RHOWING EFFECTS OF DEADBAND NONLINEARITY

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ELEMENT INFORMATION

ELEMENT NO.	RADIUS IN.	ANGLE RADIAN	DISTRIBUTION COEFFICIENT MU
1	2.000	.0175	4.4444
2	4.000	.0175	7.7778
3	6.000	.0175	7.7778

TABULAR NONLINEAR EFFECTS

[illegible]

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AT LODE2 SHOWING EFFECTS OF DEADBAND NONLINEARITY

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RESULTS OF DESCRIBING FUNCTION

OMEGA	ELEMENT NO.	RADIUS	ANGLE RAD	FRACTIONAL FLOW-RATE	FP	FR	FT
1.6939	1	.313	.0175	.00123	1.000000	.678814	1.000000
1.6939	2	.625	.0175	.00216	1.000000	.493693	1.000000
1.6939	3	.938	.0175	.00216	1.000000	.000000	1.000000

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VT LODE2 RHOWING EFFECTS OF DEADBAND NONLINEARITY

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RESULTS OF DESCRIBING FUNCTION

OMEGA	ELEMENT NO.	RADIUS	ANGLE RAD	FRACTIONAL FLOW-RATE	FP	FR	FT
1.7675	1	.313	.0175	.00123	1.000000	.665192	1.000000
1.7675	2	.625	.0175	.00216	1.000000	.472795	1.000000
1.7675	3	.938	.0175	.00216	1.000000	.000000	1.000000

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VT LODE2 RHOWING EFFECTS OF DEADBAND NONLINEARITY

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RESULTS OF DESCRIBING FUNCTION

OMEGA	ELEMENT NO.	RADIUS	ANGLE RAD	FRACTIONAL FLOW-RATE	FP	FR	FT
1.8412	1	.313	.0175	.00123	1.000000	.651640	1.000000
1.8412	2	.625	.0175	.00216	1.000000	.452589	1.000000
1.8412	3	.938	.0175	.00216	1.000000	.000000	1.000000

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VT LODE2 RHOWING EFFECTS OF DEADBAND NONLINEARITY

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RESULTS OF DESCRIBING FUNCTION

OMEGA	ELEMENT NO.	RADIUS	ANGLE RAD	FRACTIONAL FLOW-RATE	FP	FR	FT
1.9148	1	.313	.0175	.00123	1.000000	.637977	1.000000
1.9148	2	.625	.0175	.00216	1.000000	.432330	1.000000
1.9148	3	.938	.0175	.00216	1.000000	.000000	1.000000

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VT LODE2 RHOWING EFFECTS OF DEADBAND NONLINEARITY

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RESULTS OF DESCRIBING FUNCTION

OMEGA	ELEMENT NO.	RADIUS	ANGLE RAD	FRACTIONAL FLOW-RATE	FP	FR	FT
1.9885	1	.313	.0175	.00123	1.000000	.624504	1.000000
1.9885	2	.625	.0175	.00216	1.000000	.412330	1.000000
1.9885	3	.938	.0175	.00216	1.000000	.000000	1.000000

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NY LODE2 RHOWING EFFECTS OF DEADBAND NONLINEARITY

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RESULTS OF DESCRIBING FUNCTION

OMEGA	ELEMENT NO.	RADIUS	ANGLE RAD	FRACTIONAL FLOW-RATE	FP	FR	FT
2.0253	1	.313	.0175	.00123	1.000000	.617796	1.000000
2.0253	2	.625	.0175	.00216	1.000000	.402424	1.000000
2.0253	3	.938	.0175	.00216	1.000000	.000000	1.000000

DATE 25 JUL 70

\T LODE2 RHOWING EFFECTS OF DEADBAND NONLINEARITY

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RESULTS OF INJECTION DISTRIBUTION EFFECTS

OMEGA	AVN REAL	BVN REAL	CVN REAL	CVN IMAG
1.6939	1.2745	.4300	.0000	-1.8125
1.7675	1.2745	.4158	.0000	-1.8125
1.8412	1.2745	.4020	.0000	-1.8125
1.9148	1.2745	.3881	.0000	-1.8125
1.9885	1.2745	.3744	.0000	-1.8125
2.0253	1.2745	.3676	.0000	-1.8125

***** THESE VALUES PERTAIN TO A SPINNING MODE *****

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N71-20417

Appendix A

ANALYSIS OF THE LOW FREQUENCY STABILITY OF A GAS-TAPOFF CYCLE USING
GASEOUS HYDROGEN AND GASEOUS OXYGEN

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SECTION I - PROBLEM

A. ABSTRACT

This analysis deals with a feed system coupled one-dimensional stability problem for a gas tapoff cycle using gaseous hydrogen and oxygen. The tapoff cycle considers that a portion of the combustion product gases are "tapped off" for use elsewhere. This program can be run with or without the tapoff cycle included. It includes the wave motion and heat motion (entropy wave) in the chamber. The burning process is treated as a process of temperature, molecular weight and specific heat change. A very general expression relating injection rate to burning rate is used.

The many equations obtained are solved using matrices. The matrix operations however are used only on the equations involving real variables. The complex arithmetic used on the complex equations has been built into the program so the program can be used on a variety of computers.

The final result of the analysis is a system characteristic equation. Stability of the system can be determined using the Nyquist criteria, Ref. 1.

B. TECHNICAL DESCRIPTION

The feed system coupled stability problem at low frequencies can be analyzed conveniently by dividing the problem into two parts, the feed system and the chamber. Each part can be analyzed separately and then combined to determine the system stability. Method of analyzing the feed system is well developed, Ref. 2, but the traditional method of analyzing the chamber is quite primitive. The traditional method, Ref. 3 and 4, considers the volume of propellant injected into the chamber to be negligible and the burning to be a process of gas generation. In other words the volume of unburned propellant is assumed negligible compared to the volume of burned gases. This is a good assumption for liquid propellants burning at low chamber pressures. But when the propellants are gaseous, especially gaseous hydrogen, this assumption is not good. Actually unburned gases flow into the chamber and when burning occurs gas properties (such as molecular weight, temperature and specific heat) change. The same weight flow of gas exists before and after burning.

B, Technical Description (cont.)

This analysis looks only at the chamber analysis but has provision for adding the feed system effects. For simplicity the combustion is assumed to occur at one discrete plane. This is justified since the combustion does occur over a length that is small compared to the wave length of the oscillation. After combustion the gas goes through the sonic exhaust nozzle. The dynamics of a sonic exhaust nozzle have been well analyzed in reference 5. The results of that analysis will be used here. In addition to the propellant feed lines an additional feature must be accounted for in the gas tapoff cycle, the dynamics of the tapoff duct. To analyze the combustion front the following relationships must be used:

Conservation of mass

$$\dot{w}_t + \dot{w}_x + \dot{w}_f = \dot{w}_p$$

Conservation of Momentum

$$P_f = P_x$$

$$P_x = P_p$$

Conservation of energy

$$\dot{w}_x h_x + \dot{w}_f h_f + \dot{w}_t h_t = \dot{w}_p h_p$$

Definition of enthalpy for a chemical system

$$h_x = h_{O_x} + \int_{T_R}^{T_x} CP_x dT$$

$$h_f = h_{O_f} + \int_{T_R}^{T_f} CP_f dT$$

B, Technical Description (cont.)

$$h_p = h_{O_p} + \int_{T_R}^{T_P} C_{P_p} dT$$

$$h_{O_p} = Y_{H_2} h_{O_{H_2}} + Y_{H_2O} h_{O_{H_2O}}$$

Definition of the product specific heat in terms of its composition

$$C_{P_p} = Y_{H_2} C_{P_{H_2}} + Y_{H_2O} C_{P_{H_2O}}$$

Definition of the mass fraction of species in the products

$$Y_{H_2O} = \frac{9}{8} \frac{\dot{w}_x}{\dot{w}_p}$$

$$Y_{H_2} = 1 - Y_{H_2O}$$

Equation of state

$$P = R_p T$$

$$R = \frac{R_g}{MW}$$

Definition of the molecular weight of the products

$$MW_p = X_{H_2} MW_{H_2} + X_{H_2O} MW_{H_2O}$$

B, Technical Description (cont.)

Definition of the mole fraction of species in products

$$X_{H_2} = \frac{\frac{Y_{H_2}}{MW_{H_2}}}{\frac{Y_{H_2}}{MW_{H_2}} + \frac{Y_{H_2O}}{MW_{H_2O}}}$$

$$X_{H_2O} = 1 - X_{H_2}$$

The analysis of the sonic exhaust nozzle results in a relationship between perturbations in pressure, velocity and entropy:

$$\gamma_p v_p' + \alpha p_p' + \beta s_p' = 0$$

The entropy can be related to other properties through the thermodynamics equation

$$T ds = dh - \frac{1}{g\rho} dp$$

The injection rates are related to the feed line dynamics by the following equations:

B, Technical Description (cont.)

$$\frac{w'_x}{p'_x} = \frac{e^{-i\omega\tau_x}}{Z_x}$$

$$\frac{w'_f}{p'_f} = \frac{e^{-i\omega\tau_f}}{Z_f}$$

$$\frac{w'_t}{p'_x} = \frac{1}{Z_t}$$

The final equation which completely characterizes the system stability can be written as

$$\frac{F e^{-i\omega\tau_x}}{Z_x} + \frac{G e^{-i\omega\tau_f}}{Z_f} + \frac{H}{Z_t} + 1 = 0$$

The problem then amounts to the determination of F, G and H from the previous equations. To do this, all variables must be written as a mean value plus a perturbation value which varies with time. Each equation can then be divided into a mean equation and a perturbation equation. There are 27 perturbation equations which must be solved simultaneously. This is done with the help of matrices. Since 22 of the equations contain only real coefficients they will be solved using matrices as follows:

$$[AA] [BB] + [CC] \dot{w}'_x + [DD] \dot{w}'_f + [EE] \dot{w}'_t + [FF] P'_p + [GG] S'_p = [0]$$

The matrix $[AA]$ is a 22 x 22 matrix and the others are column matrices. All the coefficients of these matrices are real. The remaining 5 equations involve complex variables and are solved simultaneously to obtain the final equation.

There are some important assumptions that are implied by the basic equations presented. First in the momentum and energy equations, the gas momentum and kinetic energy terms have been dropped. This assumes that the Mach number is small compared to unity. Also, in developing the perturbation equations, products of perturbations are neglected which means that only small amplitude oscillations can be treated (small compared to the mean variables). In defining the properties of the products only hydrogen and water are assumed to be in the products which infers that the engine must be running fuel rich. The analysis is limited to an engine using gaseous oxygen and gaseous hydrogen.

C. EQUATIONS

The following equations are used to determine some of the mean variables which are used as coefficients in the perturbation equations:

$$\bar{T}_p = \bar{T}_o \left(\frac{\gamma_p - 1}{2} M_p^2 + 1 \right)^{-1}$$

$$\bar{C}_p = \left(\gamma_p \frac{R_g}{MW_p} \bar{T}_o \right)^{1/2}$$

$$\bar{w}_f = \bar{w}_p (1 + MR)^{-1}$$

$$\bar{w}_x = \bar{w}_f (MR)$$

$$\bar{C}_x^2 = \gamma_{xg} \frac{R}{MW_x} \bar{T}_x$$

$$\bar{C}_p = \bar{C}_o \left(\frac{\gamma_p - 1}{2} M_p^2 + 1 \right)^{-1/2}$$

$$\bar{v}_p = M_p \bar{C}_p$$

C, Equations (cont.)

$$\bar{p}_p = \frac{\bar{P}_p \bar{MW}_p}{R_g \bar{T}_p}$$

$$\bar{h}_x = CP_{O_2} (\bar{T}_x - T_R)$$

$$\bar{h}_f = CP_{H_2} (\bar{T}_f - T_R)$$

$$\bar{p}_f = \frac{\bar{P}_f \bar{MW}_f}{R_g \bar{T}_f}$$

$$\bar{p}_x = \frac{\bar{P}_x \bar{MW}_x}{R_g \bar{T}_x}$$

$$\bar{h}_p = (\bar{h}_x \bar{w}_x + \bar{h}_f \bar{w}_f) / \bar{w}_p$$

$$\bar{Y}_{H_2} = \left(\bar{w}_f - \frac{\bar{w}_x}{8} \right) / \bar{w}_p$$

$$\bar{Y}_{H_2O} = 1 - \bar{Y}_{H_2}$$

$$R_x = \frac{R_g}{MW_x}$$

$$R_f = \frac{R_g}{MW_f}$$

$$\bar{R}_p = \frac{R_g}{\bar{MW}_p}$$

$$\alpha = \frac{\bar{C}_o}{\bar{P}_o} A \tau_p$$

$$\beta = \frac{\bar{C}_o}{CP_p} C \tau_p$$

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The perturbation equations are:

$$w_t' + w_x' + w_f' - w_p' = 0$$

$$P_x' - P_p' = 0$$

$$\bar{h}_x \dot{w}_x' + \dot{\bar{w}}_x h_x' + \dot{w}_f' \bar{h}_f + \dot{\bar{w}}_f h_f' + \dot{w}_t h_t + \dot{w}_t' \bar{h}_t - \dot{\bar{w}}_p h_p' - \dot{w}_p' \bar{h}_p = 0$$

$$h_x' - CP_x T_x' = 0$$

$$h_f' - CP_f T_f' = 0$$

$$h_p' - h_{p0}' - \overline{CP}_p T_p' - CP_p' (\bar{T}_p - \bar{T}_R) = 0$$

$$h_{p0}' - Y_{H_2}' \overline{h_{0H_2}} - Y_{H_20}' \overline{h_{0H_20}} = 0$$

$$CP_p' - Y_{H_2}' \overline{CP_{H_2}} - Y_{H_20}' \overline{CP_{H_20}} = 0$$

$$Y_{H_2}' + Y_{H_20}' = 0$$

$$Y_{H_20}' - \frac{9}{88} \frac{\dot{w}_x}{\bar{w}_p} + \frac{9}{8} \frac{\dot{\bar{w}}_x}{\bar{w}_p^2} \dot{w}_p' = 0$$

$$P_x' - R_x \bar{\rho}_x T_x' - R_x \bar{T}_x \rho_x' = 0$$

$$P_f' - R_f \bar{\rho}_f T_f' - R_f \bar{T}_f \rho_f' = 0$$

$$P_x' - \frac{\gamma_x \bar{P}_x}{\bar{\rho}_x} \rho_x' = 0$$

$$v_x' - \frac{\dot{w}_x'}{\bar{\rho}_x g A_x} + \frac{\bar{v}_x}{\bar{\rho}_x} \rho_x' = 0$$

$$v_p' - \frac{\dot{w}_p'}{\bar{\rho}_p A_c g} + \frac{\bar{v}_p}{\bar{\rho}_p} \rho_p' = 0$$

$$P_p' - \bar{R}_p \bar{\rho}_p T_p' - R_p T_p \rho_p' - \bar{\rho}_p T_p R_p' = 0$$

$$\bar{T}_p S_p' - h_p' + \frac{P_p'}{g \bar{\rho}_p} = 0$$

$$R_p' + \frac{g}{MW_p^2} MW_p' = 0$$

$$MW_p' - X'_{H_2} MW_{H_2} - X'_{H_2O} MW_{H_2O} = 0$$

$$X'_{H_2} = Y'_{H_2} \left[\frac{\frac{1}{MW_{H_2}}}{\frac{\bar{Y}_{H_2}}{MW_{H_2}} + \frac{\bar{Y}_{H_2O}}{MW_{H_2O}}} - \frac{\frac{\bar{Y}_{H_2}}{MW_{H_2}^2}}{\left(\frac{\bar{Y}_{H_2}}{MW_{H_2}} + \frac{\bar{Y}_{H_2O}}{MW_{H_2O}} \right)^2} \right] + Y'_{H_2O} \frac{\frac{\bar{Y}_{H_2}}{MW_{H_2O} MW_{H_2}}}{\left(\frac{\bar{Y}_{H_2}}{MW_{H_2}} + \frac{\bar{Y}_{H_2O}}{MW_{H_2O}} \right)^2} = 0$$

$$X'_{H_2O} + X'_{H_2} = 0$$

The 22 equations above will be solved using matrices. The remaining perturbation equations are:

$$P_x' - P_f' = 0$$

$$\dot{w}_x' - \frac{e^{-i\omega\tau_x}}{Z_x} P_x' = 0$$

$$\dot{w}_f' - \frac{e^{-i\omega\tau_f}}{Z_f} P_f' = 0$$

$$\dot{w}_t' - \frac{1}{Z_t} P_t' = 0$$

$$v_p' + \alpha P_p' + \beta S_p' = 0$$

There are also several constants which are built into the program:

$$\gamma_x = 1.4$$

$$\gamma_f = 1.4$$

$$MW_x = 32 \frac{\text{lb}}{\text{lb - mole}}$$

$$MW_f = 2 \frac{\text{lb}}{\text{lb - mole}}$$

$$R = 1545 \frac{\text{lb}}{\text{lb - mole } ^\circ\text{R}}$$

$$g = 32.172 \text{ ft/sec}^2$$

$$C_{p_x} = \frac{\gamma_x R}{(\gamma_x - 1) MW_x} = 169 \frac{\text{ft-lb}}{\text{lb } ^\circ\text{R}}$$

$$C_{p_f} = \frac{\gamma_f R}{(\gamma_f - 1) MW_f} = 2710 \frac{\text{ft-lb}}{\text{lb } ^\circ\text{R}}$$

$$h^0_{H_2O} = -4.498 \times 10^6 \frac{\text{ft-lb}}{\text{lb}}$$

$$C_{p_{H_2O}} = 555 \frac{\text{ft-lb}}{\text{lb } ^\circ\text{R}}$$

$$T_r = 537 ^\circ\text{R}$$

$$MW_{H_2} = 2$$

$$MW_{H_2O} = 18$$

$$h^0_{H_2} = 0.0$$

For convenience a diagram of the matrices is shown on page 13. The coefficients $A_{21, 12}$, $A_{21, 13}$, $A_{22, 12}$, $A_{22, 13}$ are defined as follows:

$$A_{21, 12} = - \frac{\frac{1}{MW_{H_2}}}{\frac{\bar{Y}_{H_2}}{MW_{H_2}} + \frac{\bar{Y}_{H_2O}}{MW_{H_2O}}} + \left(\frac{\frac{\bar{Y}_{H_2}}{MW_{H_2}} + \frac{\bar{Y}_{H_2O}}{MW_{H_2O}}}{\frac{\bar{Y}_{H_2}}{MW_{H_2}} + \frac{\bar{Y}_{H_2O}}{MW_{H_2O}}} \right)^2$$

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$$A_{21, 13} = \frac{\frac{\bar{Y}_{H_2}}{MW_{H_2}} \quad \frac{1}{MW_{H_2O}}}{\left(\frac{\bar{Y}_{H_2}}{MW_{H_2}} + \frac{\bar{Y}_{H_2O}}{MW_{H_2O}} \right)^2}$$

$$A_{22, 12} = \frac{\frac{\bar{Y}_{H_2O}}{MW_{H_2O}} \quad \frac{1}{MW_{H_2}}}{\left(\frac{\bar{Y}_{H_2}}{MW_{H_2}} + \frac{\bar{Y}_{H_2O}}{MW_{H_2O}} \right)^2}$$

$$A_{22, 13} = - \frac{\frac{1}{MW_{H_2O}}}{\frac{\bar{Y}_{H_2O}}{MW_{H_2O}} + \frac{\bar{Y}_{H_2}}{MW_{H_2}}} + \frac{\frac{\bar{Y}_{H_2O}}{(MW_{H_2O})^2}}{\left(\frac{\bar{Y}_{H_2}}{MW_{H_2}} + \frac{\bar{Y}_{H_2O}}{MW_{H_2O}} \right)^2}$$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	
1	1.0																						$\bar{W}_p'$
2		$-\bar{W}_x$	$-gA_c$																				V_x'
3		$-\bar{h}_p$		$\bar{W}_x$	$\bar{W}_p$	$\bar{W}_x \bar{W}_p$																	P_x'
4				1.0			$-\bar{C}_p$																$\bar{h}_x'$
5					1.0		$-\bar{C}_p$																$\bar{h}_x'$
6						1.0		-1.0	$-\bar{C}_p$	$\bar{T}_a - \bar{T}_p$													$\bar{h}_x'$
7								1.0				$\bar{h}_{x0}$											$\bar{T}_x'$
8										1.0	$-\bar{C}_p$	$-\bar{C}_{p0}$											$\bar{T}_x'$
9											1.0	1.0											hQ'
10													1.0										$\bar{T}_x'$
11			1.0				$-R_x \bar{P}_x$						$-R_x \bar{P}_x$										CP_x'
12							$-R_x \bar{P}_x$							1.0	$-R_x \bar{T}_x$								V_x'
13			1.0										$\bar{T}_x \bar{P}_x$										V_{x0}'
14														1.0	$\bar{T}_x \bar{P}_x$								P_x'
15			1.0											$\bar{V}_x$									P_x'
16																1.0	$\bar{V}_x$						P_x'
17									$-R_x \bar{P}_x$							$-\bar{R}_x \bar{T}_x$	$-\bar{P}_x \bar{T}_x$						V_x'
18									$\bar{C}_p$							$-\bar{P}_x$	$\bar{P}_x^2$						P_x'
19																1.0	$\frac{R}{MH_x}$						P_x'
20																1.0	$-MH_x$	$-MH_{x0}$					MW_x'
21												$A_{21,12}$	$A_{21,13}$						1.0				X_{x0}'
22												$A_{22,12}$	$A_{22,13}$								1.0		X_{x0}'

=

$W_x' +$

$W_x' +$

$W_x' +$

$P_x' +$

S_x'

D. DEFINITION OF TERMS

A	Area
AA	22 x 22 real matrix
A	nozzle admittance coefficient
BB	column matrix multiplying AA
C	speed of sound
c*	characteristic exhaust velocity
C	nozzle admittance coefficient
CC	column matrix multiplying $\dot{w}_x'$
CP	specific heat at constant pressure
DD	column matrix multiplying $\dot{w}_f'$
e	base of the natural logarithm
EE	column matrix multiplying $\dot{w}_t'$
F	coefficient in characteristic equation
FF	column matrix multiplying P_p'
g	acceleration of gravity
G	coefficient in characteristic equation
GG	column matrix multiplying S_p'
h	enthalpy
h0	enthalpy of element at reference temperature of 77°F
H	coefficient in characteristic equation
i	$\sqrt{-1}$
M	Mach number
MR	mixture ratio
MW	molecular weight
P	pressure
$\mathcal{R}$	universal gas constant
R	gas constant

D. Definition of Terms (cont.)

S	entropy
T	temperature
v	velocity
$\dot{w}$	weight flow rate
X	mole fraction
Y	mass fraction
Z	axial coordinate
α	nozzle admittance coefficient
β	nozzle admittance coefficient
γ	ratio of specific heats
ρ	density
τ	total time lag between injection and combustion
ω	frequency

Subscripts

x	oxidizer
f	fuel
t	tapoff
H ₂	hydrogen
H ₂ O	water
p	products
o	stagnation conditions
R	reference

E. REFERENCES

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4. Crocco, L., Cheng, S. I., "Theory of Combustion Instability In Liquid Propellant Rocket Motors", AGARD Monograph No. (8), Butterworths Scientific Publications, Ltd., London, 1956.
5. Crocco, L., Sirignano, W. A., "Behavior of Supercritical Nozzles Under Three-Dimensional Oscillatory Conditions", AGARDograph 117, North Atlantic Treaty Organization, 1967.
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7. "Stability Characterization of Advanced Injectors" Contract NAS 8-20672, Report 20672-II-M6, Aerojet-General Corp., Liquid Rocket Operations, Sacramento, Calif. 7, March 1969.
8. "Stability Characterization of Advanced Injectors Design Guide", Vol. 2, Operation of the Computer Program 20672-P2D January 1970.
9. General Reference Manual, UNIVAC 1108, Sperry-Rand Corp. 1966.

SECTION II - PROGRAMMING

A. SUBROUTINES

This program consists mostly of subroutines. Subroutine CMPLX is really part of the main line of the program and is therefore flow charted in a subsequent section of this report. The subroutines in this program are used for matrix inversion, matrix multiplication, complex multiplication, addition, subtraction, division, and to output results of the programs. These subroutines go by the following names:

MINV
MVMLT
CMULT
CCMLT
CADD
CSUB
CDVID
OUTPT
GSTAB (Main Program)

The subroutine MINV is used to invert the square matrix AA. This subroutine was developed at Aerojet-General Corp. The method was taken from Ref. 6. The following is excerpted from an Aerojet reference Manual.

IDENTIFICATION

MINV - Matrix Inversion Subroutine

Written by F. Yee
Aerojet-General Corporation, Sacramento, California
17 April 1967

PURPOSE

To find the inverse of an $n \times n$ real matrix .

RESTRICTIONS

The subroutine could compute an inverse to a singular matrix because of round-off errors introduced in the numerical computations.

METHOD

This subroutine uses the Gauss-Jordan method with complete pivoting.

USAGE

The subroutine is called with the statement:

CALL MINV (A, N, DET, NDET, NA, ITEM)

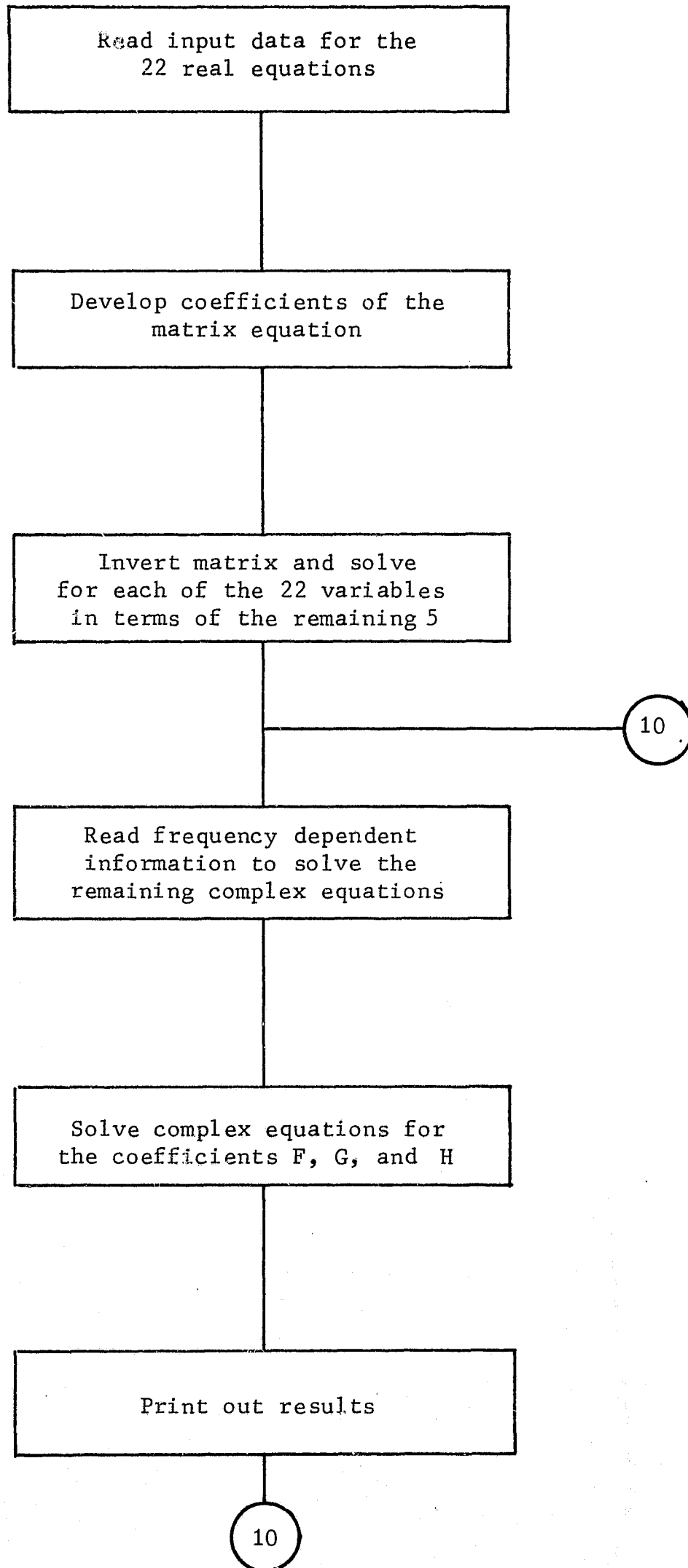
where:

A = initially the input matrix, finally, the matrix inverse.
N = the input matrix order.
DET = the scaled determinant.
NDET = exponent in powers of ten for the scaled determinant
NA = 1st dimension of A; i.e., A(NA,...)
ITEM = temporary storages with dimension size at least equal to N. After the return from MINV, if ITEM (1) = -1, the inverse has been computed; if ITEM (1) = 0, 1, 2, ..., the matrix is singular and the positive integer is the rank of the matrix.

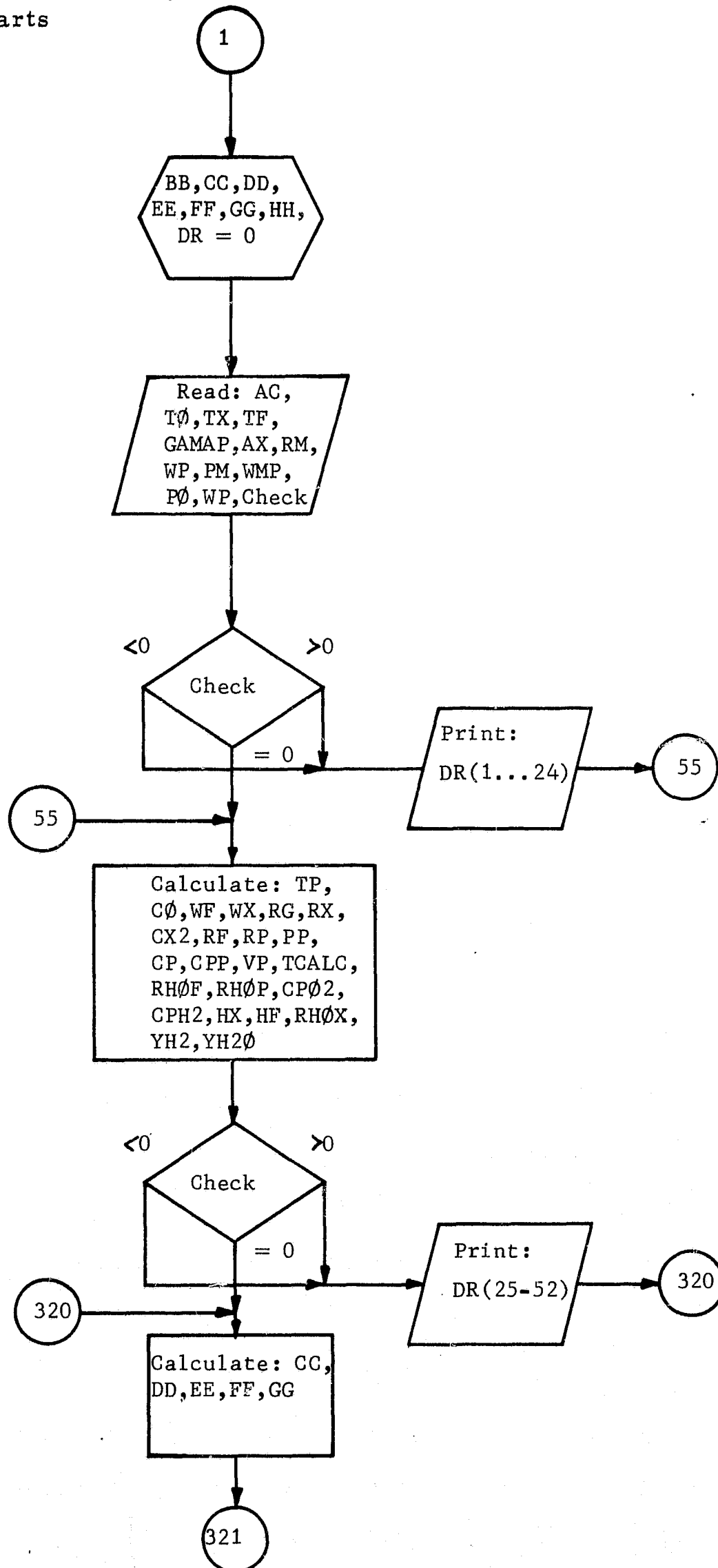
The elements a_{ij} of the matrix A are stored according to the matrix convention

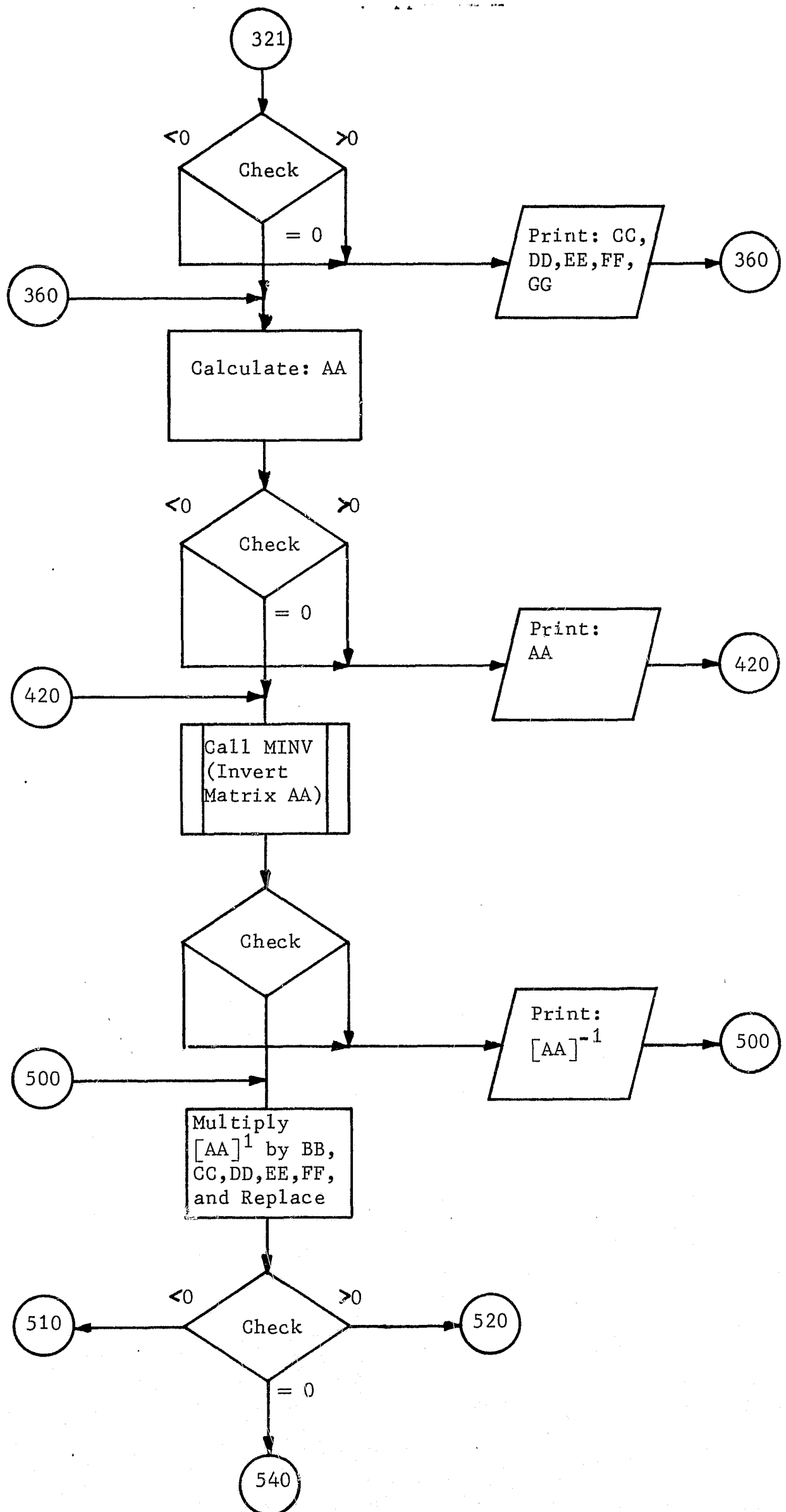
$$A(I, J) = a_{ij}, \quad I \leq N, \quad J \leq N.$$

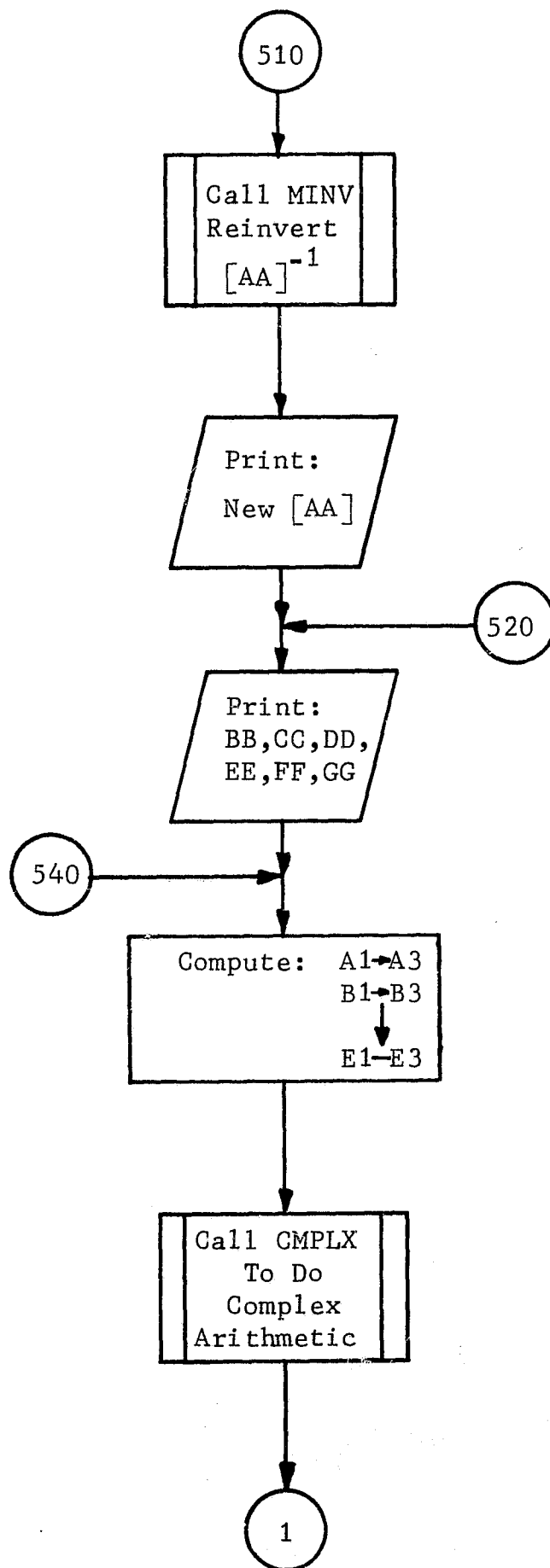
B. Block Diagrams

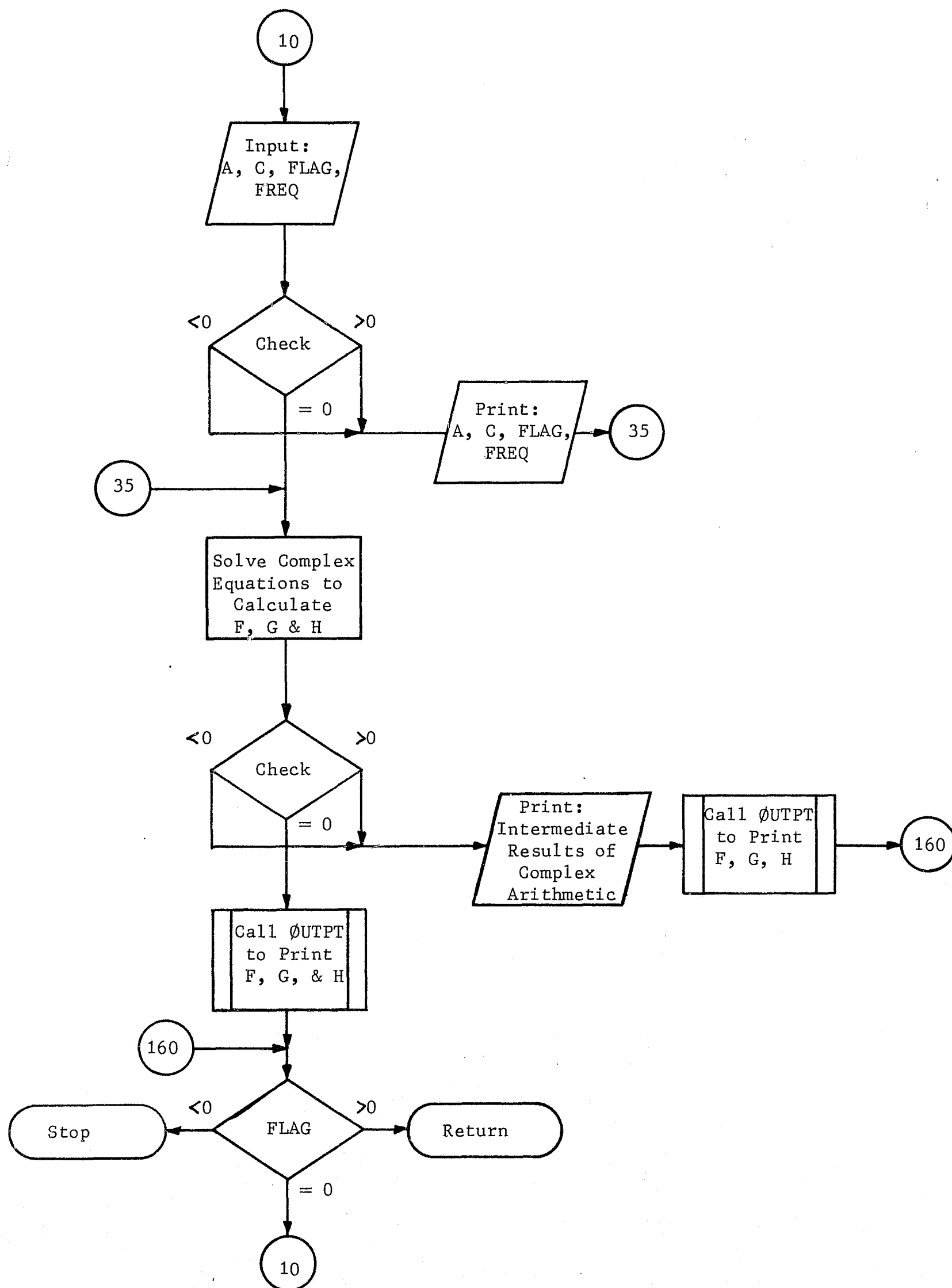


C. Flow Charts









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W ELT GSTAB,1,700416, 60795

```

000001      C      PROGRAM 021504      GAS-GAS FEED SYSTEM COUPLED STABILITY PROGRAM
000002      C
000003      C      CODED BY      R.H. MARLER
000004      C      CUSTOMER      R.C. WAUGH
000005
000006      DIMENSION LL(22),MM(22),HH(22)
000007      DIMENSION AA(22,22),BB(22),CC(22),DD(22),EE(22),FF(22),GG(22)
000008      COMMON      DR(72)
000009      X      ,A(2),C(2),ALPHA(2),BETA(2)
000010      X      ,F(2),G(2),H(2)
000011      EQUIVALENCE
000012      X      (GAMAX,DR( 1)), (GAMAF,DR( 2)), (WMX ,DR( 3)), (WMF ,DR( 4))
000013      X      , (R ,DR( 5)), (GRVTY,DR( 6)), (CPX ,DR( 7)), (CPF ,DR( 8))
000014      X      , (HOH20,DR( 9)), (CPH20,DR(10)), (TR ,DR(11)), (WMH2 ,DR(12))
000015      X      , (WMH20,DR(13)), (AC ,DR(14)), (TO ,DR(15)), (TX ,DR(16))
000016      X      , (TF ,DR(17)), (GAMAP,DR(18)), (RM ,DR(19)), (WP ,DR(20))
000017      EQUIVALENCE
000018      X      (PM ,DR(21)), (WMP ,DR(22)), (PO ,DR(23)), (AX ,DR(24))
000019      X      , (TP ,DR(25)), (CO ,DR(26)), (WX ,DR(27)), (WF ,DR(28))
000020      X      , (RX ,DR(29)), (RF ,DR(30)), (PP ,DR(31)), (CP ,DR(32))
000021      X      , (CPP ,DR(33)), (VP ,DR(34)), (PF ,DR(35)), (CAY1 ,DR(36))
000022      X      , (CVP ,DR(37)), (PX ,DR(38)), (RHOP ,DR(39)), (HX ,DR(40))
000023      EQUIVALENCE
000024      X      (HF ,DR(41)), (RHOF ,DR(42)), (RHOX ,DR(43)), (VX ,DR(44))
000025      X      , (HP ,DR(45)), (YH2 ,DR(46)), (YH20 ,DR(47)), (DENOM,DR(48))
000026      X      , (CP02 ,DR(49)), (CPH2 ,DR(50)), (CX2 ,DR(51)), (RP ,DR(52))
000027      X      , (A1 ,DR(53)), (B1 ,DR(54)), (C1 ,DR(55)), (D1 ,DR(56))
000028      X      , (E1 ,DR(57)), (A2 ,DR(58)), (B2 ,DR(59)), (C2 ,DR(60))
000029      EQUIVALENCE
000030      X      (D2 ,DR(61)), (E2 ,DR(62)), (A3 ,DR(63)), (B3 ,DR(64))
000031      X      , (C3 ,DR(65)), (D3 ,DR(66)), (E3 ,DR(67)), (FREQ ,DR(68))
000032      X      , (WT ,DR(69))
000033      C
000034      I IN = 5
000035      I OUT= 6
000036      GAMAX= 1.4
000037      GAMAF= 1.4
000038      WMX = 32.0
000039      WMF = 2.0
000040      R = 1545.0
000041      GRVTY= 32.172
000042      CPX = 169.
000043      CPF = 2710.
000044      HOH20= -4.498 E+06
000045      HOH2 = 0.
000046      CPH20= 555.
000047      TR = 537.
000048      WMH2 = 2.
000049      WMH20= 10.
000050      C
000051      1 DO 10 J=1,22
000052      BB(J)=0.
000053      CC(J)=0.
000054      DD(J)=0.
000055      EE(J)=0.
000056      FF(J)=0.

```

*NEW

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```

000056      GG(J)=0.
000057      DO 10 I=1,22
000058 10      AA(I,J)=0.
000059      DO 20 I=14,62
000060 20      DR(I) = 0.
000061
000062      C      READ (IIN,40)AC,TO,TX,TF,GAMAP,AX,RM,WP,PM,WMP,PO,WT,CHECK
000063 40      FORMAT(1X,4E15.6)
000064      IF(CHECK)45,55,45
000065 45      WRITE(I OUT,50)
000066 50      FORMAT( 16H1 INITIAL VALUES /)
000067      WRITE(I OUT,40)(DR(I),I=1,22)
000068
000069      C
000070 55      AC = AC / 144.
000071      AX = AX / 144.
000072      PO = PO * 144.
000073      CON1 =(GAMAP-1.0 )*PM/2.0 *PM + 1.0
000074      IEQ= 1
000075      IF(CON1 )140,70,100
000076 70      WRITE(I OUT,80)IEQ
000077 80      FORMAT(21H1 DIVIDE CHECK IN EQ ,I2)
000078 90      WRITE(I OUT,40)( DR(I),I=25,52)
000079 91      READ(IIN,92) FLAG
000080 92      FORMAT(61X,F2.0)
000081      IF(FLAG)93,91,1
000082 93      STOP
000083 100     TP = TO/CON1
000084
000085      IF(WMP)120,70,120
000086 120     TCALC=GAMAP*R/WMP*GRVTY*TO
000087      IF(TCALC)140,170,160
000088 140     WRITE(I OUT,150)IEQ
000089 150     FORMAT(29 H1 SQRT OF -X ATTEMPTED IN EQ ,I2)
000090      GO TO 90
000091 160     CO = SQRT(TCALC)
000092
000093      C
000094 170     WF = WP/(RM+1.0 )
000095      WX = WF* RM
000096      RG = R*GRVTY
000097      RX = RG/WMX
000098      CX2=GAMAP*RX*TX
000099      RF = RG/WMF
000100      RP = RG/ WMP
000101      PP = PO/CON1*(GAMAP/(GAMAP-1.0 ))
000102      CP = CO / SQRT(CON1)
000103      CPP= GAMAP/(GAMAP-1.0 ) * R / WMP
000104      VP = CP* PM
000105      TCALC = GRVTY * AC * AX
000106      IF(TCALC)190,180,190
000107 180     GO TO 70
000108 190     CAY1=WX**2 /TCALC*R / WMX *TX
000109
000110      C      TCALC= PP**2-4.0 *CAY1
000111
000112      IF(TCALC)140,210,200
000113 200     TCALC= SQRT(TCALC)
000114 210     PX=PO

```

*NEW
***-1

IEQ = 2

IEQ = 12

IEQ = 11

*NEW

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000115			IEQ = 13	**-1
000116	IF(GAMAP)220,70,220			
000117	220 CVP=CPP/GAMAP			
000118	PF = PX			
000119			IEQ = 15	
000120	IF(TP)230,70,230			
000121	230 RHOP = PP/RG * WMP/ TP			
000122	CP02= CPX			
000123	CPH2= CPF			
000124	HX=(TX-TR) * CP02			
000125	HF=(TF-TR) * CPH2			
000126			IEQ = 18	
000127	IF(TF)240,70,240			
000128	240 RHOF = PF / RG * 2.0 /TF			
000129			IEQ = 19	
000130	IF(TX)250, 70,250			
000131	250 RHOX = PX /RG *32.0 / TX			
000132	TCALC=RHGX*GRVTY*AX			
000133			IEQ = 20	
000134	IF(TCALC)260,70 ,260			
000135	260 VX = WX / TCALC			
000136			IEQ = 21	
000137	IF(WP)270, 70,270			
000138	270 HP=(HX*WX+WF*HF)/WP			
000139			IEQ = 22	*NEW
000140	TCALC = WF + WX			**-1
000141	IF(TCALC)280, 70,280			
000142	280 YH2 = (WF - WX/8.)/TCALC			
000143	YH20= 1. -YH2			
000144	C			
000145	DENOM = YH20/WMH20 + YH2 / WMH2			
000146			IEQ = 24	
000147	IF(DENOM)290, 70,290			
000148	C			
000149	290 IF(CHECK)300,320,300			
000150	300 WRITE(I OUT,310)			
000151	310 FORMAT(21H0 COMPUTED PARAMETERS)			
000152	WRITE(I OUT,40)(DR(I),I=25,52)			
000153	320 CC(1)=-1.0			*NEW
000154	CC(3)=+HX			*NEW
000155	CC(10)=-9./B./WP			*NEW
000156	CC(15)=-1.0/GRVTY/AX/RHOX			*NEW
000157	DD(1)=-1.0			*NEW
000158	DD(3)=HF			*NEW
000159	EE(1)=-1.0			*NEW
000160	EE(3)=HP			*NEW
000161	FF(2)=GRVTY*AC			*NEW
000162	FF(17)=1.0			*NEW
000163	FF(18)=1./RHOP/GRVTY			*NEW
000164	GG(18)=TP			*NEW
000165	C			*-13
000166	IF(CHECK)330,360,330			
000167	330 WRITE(I OUT,340)			
000168	340 FORMAT(53H0 COLUMN VECTORS CC, DD, EE, FF AND GG, RESPECTFULLY)			
000169	350 FORMAT(1H., 5E15.6)			
000170	WRITE(I OUT,350)(CC(I),DD(I),EE(I),FF(I),GG(I),I=1,22)			
000171	C			
000172	360 TCALC = RHOF*RHOP			
000173			IEQ = 25	

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```

000174      IF(TCALC)370, 70,370
000175 370 AA( 1, 1)= 1.0
000176      AA( 2, 3)= 0. -GRVTY * AC
000177      AA( 3, 1)= 0. -HP
000178      AA( 3, 4)= WX
000179      AA( 3, 5)= WF
000180      AA( 3, 6)= WT-WP
000181      AA( 4, 4)= 1.0
000182      AA( 4, 7)= 0. -CPX
000183      AA( 5, 5)= 1.0
000184      AA( 5, 8)= 0. -CPF
000185      AA( 6, 6)= 1.0
000186      AA( 6, 9)= 0. -1.0
000187      AA( 6,10)= 0. -CPP
000188      AA( 6,11)= TR-TP
000189      AA( 7, 9)= 1.0
000190      AA( 7,12)= 0. -H0H2
000191      AA( 7,13)= 0. -H0H20
000192      AA( 8,11)= 1.0
000193      AA( 8,12)= 0. -CPH2
000194      AA( 8,13)= 0. -CPH20
000195      AA( 9,12)= 1.0
000196      AA( 9,13)= 1.0
000197      AA(10, 1)= WX / WP * 1.125 / WP
000198      AA(10,13)= 1.0
000199      AA(11, 3)= 1.0
000200      AA(11, 7)= 0. -RX * RHOX
000201      AA(11,14)= 0. -RX * TX
000202      AA(12, 8)= 0. -RF * RHOF
000203      AA(12,15)= 1.0
000204      AA(12,16)= 0. -RF * TF
000205      AA(13, 3)= 1.0
000206      AA(13,14)= 0. -GAMAX / RHOX * PX
000207      AA(14,15)= 1.0
000208      AA(14,16)= 0. -GAMAF / RHOF * PF
000209      AA(15, 2)= 1.0
000210      AA(15,14)= VX / RHOX
000211      AA(16, 1)= -1.0/ GRVTY /AC / RHOP
000212      AA(16,17)= 1.0
000213      AA(16,18)= VP / RHOP
000214      AA(17,10)= 0. -RP * RHOP
000215      AA(17,18)= 0. -RP * TP
000216      AA(17,19)= 0. -RHOP * TP
000217      AA(18, 6)=-1.0
000218      AA(19,19)= 1.0
000219      AA(19,20)= RG / WHP**2
000220      AA(20,20)= 1.0
000221      AA(20,21)= 0. -WMH2
000222      AA(20,22)= 0. -WMH20
000223      TEM1 = 1./WMH2 /DENOM
000224      AA(21,12)= (TEM1*YH2 -1. )*TEM1
000225      AA(21,13)= TEM1/DENOM / WMH20 * YH2
000226      AA(21,21)= 1.0
000227      TEM1 = 1./WMH20/DENOM
000228      AA(22,13)= (TEM1 * YH20 -1. )* TEM1
000229      AA(22,12)= TEM1/DENOM / WMH2 * YH20
000230      AA(22,22)= 1.0
000231
000232      IF(CHECK)380,420,380

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***-1

***-1

*NEW

***-1

*NEW

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*NEW

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[illegible]

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W ELT CMLX,1,700724, 29535

```

000001      SUBROUTINE CMLX(IIN,IOUT,CHECK)
000002      DIMENSION CREG1(2),CREG2(2),CREG3(2),CREG4(2)
000003      DIMENSION A4(2),B4(2),C4(2),D4(2),E4(2)
000004      COMMON DR(72)
000005      X      ,A(2),C(2),ALPHA(2),BETA(2)
000006      X      ,F(2),G(2),H(2)
000007      EQUIVALENCE
000008      X      (GAMAX,DR( 1)), (GAMAF,DR( 2)), (WMX ,DR( 3)), (WMF ,DR( 4))
000009      X      , (R ,DR( 5)), (GRVY,DR( 6)), (CPX ,DR( 7)), (CPF ,DR( 8))
000010      X      , (HOH20,DR( 9)), (CPH20,DR(10)), (TR ,DR(11)), (WMH2 ,DR(12))
000011      X      , (WMH20,DR(13)), (AC ,DR(14)), (TO ,DR(15)), (TX ,DR(16))
000012      X      , (TF ,DR(17)), (GAMAP,DR(18)), (RM ,DR(19)), (WP ,DR(20))
000013      EQUIVALENCE
000014      X      (PM ,DR(21)), (WMP ,DR(22)), (PO ,DR(23)), (AX ,DR(24))
000015      X      , (TP ,DR(25)), (CO ,DR(26)), (WX ,DR(27)), (WF ,DR(28))
000016      X      , (RX ,DR(29)), (RF ,DR(30)), (PP ,DR(31)), (CP ,DR(32))
000017      X      , (CPP ,DR(33)), (VP ,DR(34)), (PF ,DR(35)), (CAY1 ,DR(36))
000018      X      , (CVP ,DR(37)), (PX ,DR(38)), (RHOP ,DR(39)), (HX ,DR(40))
000019      EQUIVALENCE
000020      X      (HF ,DR(41)), (RHOF ,DR(42)), (RHOX ,DR(43)), (VX ,DR(44))
000021      X      , (HP ,DR(45)), (YH2 ,DR(46)), (YH20 ,DR(47)), (DENOM,DR(48))
000022      X      , (CPO2 ,DR(49)), (CPH2 ,DR(50)), (CX2 ,DR(51)), (RP ,DR(52))
000023      X      , (A1 ,DR(53)), (B1 ,DR(54)), (C1 ,DR(55)), (D1 ,DR(56))
000024      X      , (E1 ,DR(57)), (A2 ,DR(58)), (B2 ,DR(59)), (C2 ,DR(60))
000025      EQUIVALENCE
000026      X      (D2 ,DR(61)), (E2 ,DR(62)), (A3 ,DR(63)), (B3 ,DR(64))
000027      X      , (C3 ,DR(65)), (D3 ,DR(66)), (E3 ,DR(67)), (FREQ ,DR(68))
000028      X      , (WT ,DR(69))
000029      10 READ(IIN,20)A,C,FLAG,FREQ
000030      20 FORMAT(1X,4E15.6,F2.0,F9.2)
000031      IF(CHECK)25,35,25
000032      25 WRITE(IOUT,30)
000033      30 FORMAT(34H1INITIAL DATA, COMPLEX EQUATIONS )
000034      WRITE(IOUT,20)A,C,FLAG,FREQ
000035      35                                     IEQ = 26
000036      IF(PO) 80, 40,80
000037      40 WRITE(IOUT,50)IEQ
000038      50 FORMAT( 20H0DIVIDE CHECK IN EQ ,I3)
000039      WRITE(I OUT,70)(DR(I),I=53,68),ALPHA,BETA
000040      70 FORMAT(1H ,4E15.6)
000041      GO TO 160
000042      80 TCALC = CO/PO*GAMAP
000043      CALL CCMLT(TCALC,A,ALPHA)
000044                                     IEQ = 27
000045      IF(CPP)90,40,90
000046      90 TCALC = CO/PP*GAMAP
000047      CALL CCMLT(TCALC,C,BETA)
000048                                     IEQ = 28
000049      IF(E3)92,40,92
000050      92 E4(1) =(BETA(1)/GAMAP + E2) / E3
000051      E4(2) = BETA(2)/GAMAP / E3
000052                                     IEQ = 29
000053      CREG2(1)=ALPHA(1)/GAMAP + D2
000054      CREG2(2)=ALPHA(2)/GAMAP
000055      CALL CCMLT(D3, E4 ,CREG3)

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```

000056      CALL CSUB (CREG2,CREG3,D4)
000057
000058      CALL CCMLT(C3, E4 ,CREG3)
000059      C4(1) = C2 - CREG3(1)
000060      C4(2) =      - CREG3(2)
000061
000062      CALL CCMLT(B3, E4 ,CREG3)
000063      B4(1) = B2 - CREG3(1)
000064      B4(2) =      - CREG3(2)
000065
000066      CALL CCMLT(A3, E4 ,CREG3)
000067      A4(1) = A2 - CREG3(1)
000068      A4(2) =      - CREG3(2)
000069
000070      A5= A1-E1/E3*A3
000071      B5= B1-E1/E3*B3
000072      C5= C1-E1/E3*C3
000073      D5= D1-E1/E3*D3
000074      E5= E1/E3-1.0
000075      IF(D5)105,40,105
000076 105  TCALC = 1.0/ D5
000077      CALL CCMLT(TCALC,D4,CREG4)
000078      CALL CCMLT(A5,CREG4,CREG1)
000079      CALL CCMLT(B5,CREG4,CREG2)
000080      CALL CCMLT(C5,CREG4,CREG3)
000081      CALL CCMLT(E5,CREG4,CREG4)
000082      CALL CSUB(A4,CREG1,CREG1)
000083      CALL CSUB(B4,CREG2,CREG2)
000084      CALL CSUB(C4,CREG3,CREG3)
000085      CALL CSUB(E4,CREG4,CREG4)
000086      CALL CCMLT(144.00,CREG4,CREG4)
000087      CALL COVID(CREG1,CREG4,F,IERR)
000088      IF(IERR)40,110,40
000089 110  CALL COVID(CREG2,CREG4,G,IERR)
000090      CALL COVID(CREG3,CREG4,H,IERR)
000091      IF(CHECK)120,150,120
000092 120  WRITE(IOUT,130)
000093 130  FORMAT('COMPUTED VALUES OF A4,B4,C4,D4,CREG1,CREG2,CREG3,CREG4, (
000094      CUR(I),I=5,68),A5,B5,C5,D5,E5 ' )
000095      WRITE(IOUT,140)A4,B4,C4,D4,E4,CREG1,CREG2,CREG3,CREG4,(DR(I),I=5,
000096      X      67),A5,B5,C5,D5,E5
000097 140  FORMAT(1H E16.6,E14.6,E16.6,E14.6)
000098 150  CALL OUTPT(IOUT)
000099 160  IF(FLAG)180, 10,170
000100 170  RETURN
000101 180  STOP
000102      END

```

IEQ = 30

IEQ = 31

IEQ = 32

IEQ = 33

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W ELT OUTPT,1,700416, 60799

```

000001      SUBROUTINE OUTPT(IOUT)
000002      COMMON DR(72)
000003      X ,A(2),C(2),ALPHA(2),BETA(2)
000004      X ,F(2),G(2),H(2)
000005      EQUIVALENCE
000006      X (GAMAX,DR( 1)), (GAMAF,DR( 2)), (WMX ,DR( 3)), (WMF ,DR( 4))
000007      X , (R ,DR( 5)), (GRVTY,DR( 6)), (CPX ,DR( 7)), (CPF ,DR( 8))
000008      X , (H0H20,DR( 9)), (CPH20,DR(10)), (TR ,DR(11)), (WMH2 ,DR(12))
000009      X , (WMH20,DR(13)), (AC ,DR(14)), (TO ,DR(15)), (TX ,DR(16))
000010      X , (TF ,DR(17)), (GAMAP,DR(18)), (RM ,DR(19)), (WP ,DR(20))
000011      EQUIVALENCE
000012      X (PM ,DR(21)), (WMP ,DR(22)), (PO ,DR(23)), (AX ,DR(24))
000013      X , (TP ,DR(25)), (CO ,DR(26)), (WX ,DR(27)), (WF ,DR(28))
000014      X , (RX ,DR(29)), (RF ,DR(30)), (PP ,DR(31)), (CP ,DR(32))
000015      X , (CPP ,DR(33)), (VP ,DR(34)), (PF ,DR(35)), (CAY1 ,DR(36))
000016      X , (CVP ,DR(37)), (PX ,DR(38)), (RHOP ,DR(39)), (HX ,DR(40))
000017      EQUIVALENCE
000018      X (HF ,DR(41)), (RHOF ,DR(42)), (RHOF ,DR(43)), (VX ,DR(44))
000019      X , (HP ,DR(45)), (YH2 ,DR(46)), (YH20 ,DR(47)), (DENOM,DR(48))
000020      X , (CPO2 ,DR(49)), (CPH2 ,DR(50)), (CX2 ,DR(51)), (RP ,DR(52))
000021      X , (A1 ,DR(53)), (B1 ,DR(54)), (C1 ,DR(55)), (D1 ,DR(56))
000022      X , (E1 ,DR(57)), (A2 ,DR(58)), (B2 ,DR(59)), (C2 ,DR(60))
000023      EQUIVALENCE
000024      X (D2 ,DR(61)), (E2 ,DR(62)), (A3 ,DR(63)), (B3 ,DR(64))
000025      X , (C3 ,DR(65)), (D3 ,DR(66)), (E3 ,DR(67)), (FREQ ,DR(68))
000026      X , (WT ,DR(69))
000027      10 FORMAT(1H1,//////,47X 27HAEROJET-GENERAL CORPORATION
000028      X /1H ,46X 27HCOMPUTING SCIENCES DIVISION
000029      Y /1H ,48X 23HSACRAMENTO, CALIFORNIA
000030      W /1H0,52X,15HPROGRAM E21504
000031      Z /1H ,37X 45HGAS-GAS FEED SYSTEM COUPLED STABILITY PROGRAM )
000032      20 FORMAT(//1H0,49X21HREAL INPUT CONDITIONS
000033      X /1H0 18X2HAC14X2HT014X2HTF11X7HGAMMA P12X2HAX14X2HMR)
000034      30 FORMAT(13XE13,6,5E16,6
000035      X /1H0,34X2HWP14X2HMP12X4HWP P14X2HPO14X2HWT
000036      Y /28XE13,6,4E16,6 )
000037      40 FORMAT(//1H0,41X38HCOMPLEX NOZZLE ADMITTANCE COEFFICIENTS
000038      X /1H0,34X4HKEAL11X4HIMAG14X4HREAL11X4HIMAG
000039      Y / 40X8HSCRIPT A25X8HSCRIPT C
000040      Z / 30XE13,6,E15,6,E18,6,E15,6
000041      W /1H0 40X5HALPHA29X4HBETA
000042      Z / 30XE13,6,E15,6,E18,6,E15,6 )
000043      50 FORMAT(//1H0 41X36HCOMPUTED COMPLEX COEFFICIENTS IN THE
000044      X /1H 48X23HCHARACTERISTIC EQUATION
000045      Y /1H039X41H(F)* WX' + (G)* WF' + (H)* WT' + PX' = 0. /
000046      Z 1H0 45X 19HFOR A FREQUENCY OF ,F10,3/)
000047      60 FORMAT(//20X 8H( F ) = E14.7,3H + E14.7,6H * I ,3HOR E14.7,18H TIM
000048      VES E TO THE I E14.7/
000049      W 20X 8H( G ) = E14.7,3H + E14.7,6H * I ,3HOR E14.7,18H TIM
000050      XES E TO THE I E14.7/
000051      Y 20X 8H( H ) = E14.7,3H + E14.7,6H * I ,3HOR E14.7,18H TIM
000052      ZES E TO THE I E14.7 )
000053
000054      C
000055      C
      WRITE(IOUT,10)

```

*NEW

*NEW

*NEW

**2

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**3

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000056	WRITE(IOUT,20)	
000057	TPO = PO / 144.	
000058	TAC = AC * 144.	
000059	TAX = AX * 144.	
000060	FA = SQRT(F(1)**2+F(2)**2)	*NEW
000061	FP = ATAN(F(2)/F(1))	*NEW
000062	IF (F(1)) 101,102,102	*NEW
000063	101 FP = FP+3.1415927	*NEW
000064	102 CONTINUE	*NEW
000065	GA = SQRT(G(1)**2+G(2)**2)	*NEW
000066	GP = ATAN(G(2)/G(1))	*NEW
000067	IF (G(1)) 201,202,202	*NEW
000068	201 GP = GP+3.1415927	*NEW
000069	202 CONTINUE	*NEW
000070	HA = SQRT(H(1)**2+H(2)**2)	*NEW
000071	HP = ATAN(H(2)/H(1))	*NEW
000072	IF (H(1)) 301,302,302	*NEW
000073	301 HP = HP+3.1415927	*NEW
000074	302 CONTINUE	*NEW
000075	WRITE(IOUT,30) TAC, TO, TF, GAMAP, TAX, RM, WP, PM, WMP, TPO, WT	*NEW
000076	WRITE(IOUT,40) A, C, ALPHA, BETA	*NEW
000077	WRITE(IOUT,50) FREQ	** -1
000078	WRITE(IOUT,60) F, FA, FP, G, GA, GP, H, HA, HP	*NEW
000079	RETURN	** -1
000080	END	

Report 20672-P3D, Appendix A

W ELT MINV,1,700724, 29525

000001	SUBROUTINE MINV(A,N,D,L,M)	MINV	1
000002	DIMENSION A(1),L(1),M(1)	MINV	2
000003	C SEARCH FOR LARGEST ELEMENT	MINV	3
000004	D=1.0	MINV	4
000005	NK=-N	MINV	5
000006	DO 80 K=1,N	MINV	6
000007	NK=NK+N	MINV	7
000008	L(K)=K	MINV	8
000009	M(K)=K	MINV	9
000010	KK=NK+K	MINV	10
000011	BIGA=A(KK)	MINV	11
000012	DO 20 J=K,N	MINV	12
000013	IZ=N*(J-1)	MINV	13
000014	DO 20 I=K,N	MINV	14
000015	IJ=IZ+I	MINV	15
000016	10 IF(ABS(BIGA)-ABS(A(IJ))) 15,20,20	MINV	16
000017	15 BIGA=A(IJ)	MINV	17
000018	L(K)=I	MINV	18
000019	M(K)=J	MINV	19
000020	20 CONTINUE	MINV	20
000021	C INTERCHANGE ROWS	MINV	21
000022	J=L(K)	MINV	22
000023	IF(J-K) 35,35,25	MINV	23
000024	25 KI=K-N	MINV	24
000025	DO 30 I=1,N	MINV	25
000026	KI=KI+N	MINV	26
000027	HOLD=-A(KI)	MINV	27
000028	JI=KI-K+J	MINV	28
000029	A(KI)=A(JI)	MINV	29
000030	30 A(JI)=HOLD	MINV	30
000031	C INTERCHANGE COLUMNS	MINV	31
000032	35 I=M(K)	MINV	32
000033	IF(I-K) 45,45,38	MINV	33
000034	38 JP=N*(I-1)	MINV	34
000035	DO 40 J=1,N	MINV	35
000036	JK=NK+J	MINV	36
000037	JJ=JP+J	MINV	37
000038	HOLD=-A(JK)	MINV	38
000039	A(JK)=A(JJ)	MINV	39
000040	40 A(JJ)=HOLD	MINV	40
000041	C DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT ELEMENT IS	MINV	41
000042	C CONTAINED IN BIGA)	MINV	42
000043	45 IF(BIGA) 46,46,48	MINV	43
000044	46 D=0.0	MINV	44
000045	RETURN	MINV	45
000046	48 DO 55 I=1,N	MINV	46
000047	IF(I-K) 50,55,50	MINV	47
000048	50 IK=NK+I	MINV	48
000049	IF(A(IK)) 52,55,52		
000050	52 A(IK)=A(IK)/(-BIGA)	MINV	49
000051	55 CONTINUE	MINV	50
000052	C REDUCE MATRIX	MINV	51
000053	DO 65 I=1,N	MINV	52
000054	IK=NK+I	MINV	53
000055	IJ=I-N	MINV	54

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000056	DO 65 J=1,N	MINV 55
000057	IJ=IJ+N	MINV 56
000058	IF(I=K) 60,65,60	MINV 57
000059	60 IF(J=K) 62,65,62	MINV 58
000060	62 KJ=IJ-I+K	MINV 59
000061	IF(A(IK))63,65,63	
000062	63 IF(A(KJ))64,65,64	
000063	64 A(IJ)=A(IK)*A(KJ)+A(IJ)	MINV 60
000064	65 CONTINUE	MINV 61
000065	C DIVIDE ROW BY PIVOT	MINV 62
000066	KJ=K-N	MINV 63
000067	DO 75 J=1,N	MINV 64
000068	KJ=KJ+N	MINV 65
000069	IF(J=K) 70,75,70	MINV 66
000070	70 IF(A(KJ))72,75,72	
000071	72 A(KJ)=A(KJ)/BIGA	MINV 67
000072	75 CONTINUE	MINV 68
000073	C PRODUCT OF PIVOTS	MINV 69
000074	D=D*BIGA	MINV 70
000075	C REPLACE PIVOT BY RECIPROCAL	MINV 71
000076	A(KK)=1.0/dIGA	MINV 72
000077	80 CONTINUE	MINV 73
000078	C FINAL ROW AND COLUMN INTERCHANGE	MINV 74
000079	K=N	MINV 75
000080	100 K=(K-1)	MINV 76
000081	IF(K) 150,150,105	MINV 77
000082	105 I=L(K)	MINV 78
000083	IF(I=K) 120,120,108	MINV 79
000084	108 JO=N*(K-1)	MINV 80
000085	JR=N*(I-1)	MINV 81
000086	DO 110 J=1,N	MINV 82
000087	JK=JO+J	MINV 83
000088	HOLD=A(JK)	MINV 84
000089	JI=JR+J	MINV 85
000090	A(JK)=-A(JI)	MINV 86
000091	110 A(JI)=HOLD	MINV 87
000092	120 J=M(K)	MINV 88
000093	IF(J=K) 100,100,125	MINV 89
000094	125 KI=K-N	MINV 90
000095	DO 130 I=1,N	MINV 91
000096	KI=KI+N	MINV 92
000097	HOLD=A(KI)	MINV 93
000098	JI=KI-K+J	MINV 94
000099	A(KI)=-A(JI)	MINV 95
000100	130 A(JI)=HOLD	MINV 96
000101	GO TO 100	MINV 97
000102	150 RETURN	MINV 98
000103	END	MINV 99

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W ELT MVMLT,1,700724, 29526

```

000001      SUBROUTINE MVMLT(A,B,C,N,M)
000002      C SPARSE MATRIX * VECTOR MULTIPLY ROUTINE--MOVES B INTO C, THEN B=A*C
000003      C N = NO. ROWS IN A
000004      C M = NO. COLS IN A = NO. ELEMENTS IN B
000005      C A = N*M MATRIX STORED SEQUENTIALLY BY COLUMN
000006      C B = VECTOR OR COLUMN MATRIX OF LENGTH M
000007      C C = RESULTANT PRODUCT VECTOR OF LENGTH M (MUST BE DIFFERENT FROM B)
000008      C MAP OF MATRIX A STORGE --
000009      C
000010      C      1  N+1  2N+1  .  .  (M-1)N+1
000011      C      2  N+2  2N+2  .  .  (M-1)N+2
000012      C      3  N+3  2N+3  .  .  (M-1)N+2
000013      C      .  .  .  .  .  .
000014      C      .  .  .  .  .  .
000015      C      N  2N  3N  .  .  M*N
000016      C
000017      DIMENSION A(1),B(1),C(1)
000018      DO 5 J=1,M
000019      C(J)=B(J)
000020      5 B(J)=0.
000021      DO 30 I=1,N
000022      DO 30 J=1,M
000023      IF(C(J))10,30,10
000024      10 IJ=(J-1)*N+I
000025      IF(A(IJ))20,30,20
000026      20 B(I)=B(I)+C(J)*A(IJ)
000027      30 CONTINUE
000028      RETURN
000029      END

```

Report 20672-P3D, Appendix A

W ELT CADD,1,700724, 29528

```
000001      SUBROUTINE CADD(A,B,C)
000002      COMPLEX ADDITION C = A + B WHERE EACH VARIABLE IS A 2-CELL
000003      C      I.E., CONSIDER THE COMPLEX NUMBER X = U+V*I,
000004      C      THEN X(1)=U AND X(2)=V.
000005      C
000006      DIMENSION A(2),B(2),C(2)
000007      C(1) = A(1) + B(1)
000008      C(2) = A(2) + B(2)
000009      RETURN
000010      END
```

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W ELT CSUB,1,700724, 29529

```
000001      SUBROUTINE CSUB(A,B,C)
000002      COMPLEX SUBTRACTION C=A-B
000003      C
000004      DIMENSION A(2),B(2),C(2)
000005      C(1) = A(1) - B(1)
000006      C(2) = A(2) - B(2)
000007      RETURN
000008      END
```

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W ELT CMULT,1,700724, 29530

```
000001      SUBROUTINE CMULT(A,B,C)
000002      COMPLEX MULTIPLICATION C = A * B
000003      C
000004      DIMENSION A(2),B(2),C(2)
000005      CR = A(1)*B(1) - A(2)*B(2)
000006      CI = A(1)*B(2) + A(2)*B(1)
000007      C(1)=CR
000008      C(2)=CI
000009      RETURN
000010      END
```

Report 20672-P3D, Appendix A

W ELT CCMLT,1,700304, 44516

```
000001      SUBROUTINE CCMLT(C,X,Y)
000002      COMPLEX SCALAR MULTIPLICATION Y = C*X
000003      C      WHERE X,Y ARE COMPLEX,
000004      C      C IS A REAL SCALAR
000005      DIMENSION X(2), Y(2)
000006      Y(1) = X(1) * C
000007      Y(2) = X(2) * C
000008      RETURN
000009      END
```


Report 20672-P3D, Appendix A

W ELT CQVID,1,700J04, 44517

```
000001      SUBROUTINE CQVID(A,B,C,IERR)
000002      COMPLEX DIVISION C = A / D
000003      C      IERR = 0  IF DIVISION SUCCESSFUL,  IERR=-1 IF DIVIDE CHECK OCCURS
000004      DIMENSION A(2),B(2),C(2)
000005      DEN = B(1)**2 + B(2)**2
000006      IF(DEN )30,20,30
000007      20 IERR=-1
000008      GO TO 40
000009      30 CR  = (A(1)*B(1) + A(2)*B(2)) / DEN
000010      CI  = (A(2)*B(1) - A(1)*B(2)) / DEN
000011      C(1)=CR
000012      C(2)=CI
000013      IERR = 0
000014      40 RETURN
000015      END
```

E. METHOD OF VERIFICATION

The results of this program at zero frequency should be reasonably close to the simple quasi-steady analysis using C^* and the change in C^* with mixture ratio (Ref. 7). This was the guide used in checking out the program. There is no other known reference to measure the validity of these results.

SECTION III - DECK SETUP

A. COMPUTER CONFIGURATION

1. This program should be able to run on any of at least 3 computers: IBM 360-65, IBM 1130, UNIVAC 1108. Only two variables must be changed to change from one computer to another, IIN and IOUT. These variables are defined in the first part of the main program and refer to the UNIT numbers in READ and WRITE statements.

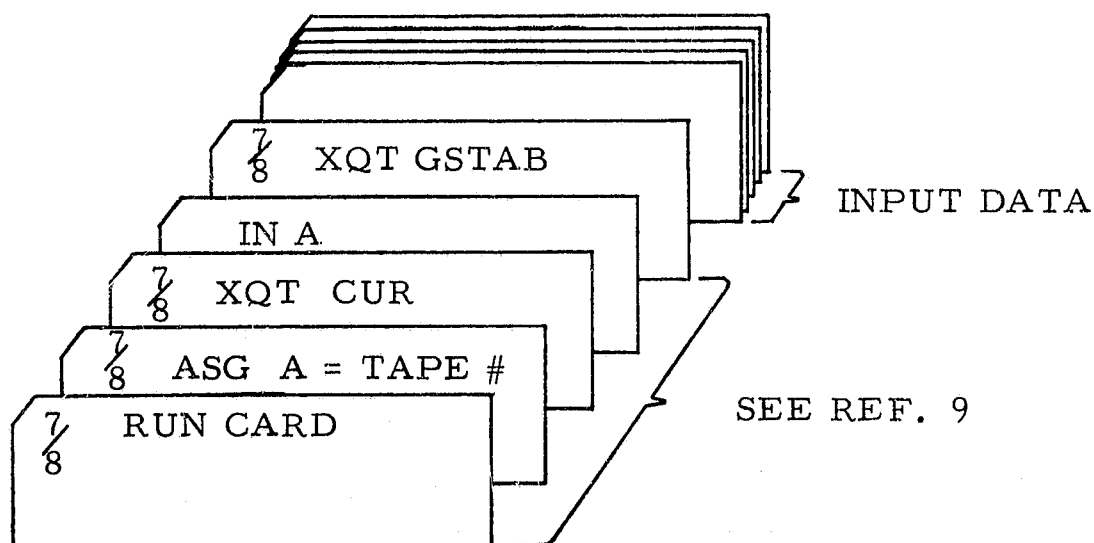
2. FORTRAN language is used in this program.

B. RUN TIME

Run time for one frequency including compilations is 13 sec on the UNIVAC 1108.

C. DECK SEQUENCE

The deck sequence shown below include the control cards as required by the EXEC II Monitor system for the UNIVAC 1108 as described in Ref. 9. For use with other computers the instruction manual for the particular computer system used should be consulted.



To run multiple cases see FLAG under INPUT DATA.

D. INPUT DATA

The following is a list with definitions and units of all input variables including special flags with an explanation of their purpose:

- AC Cross-sectional area of the chamber in square inches.
- TO Stagnation temperature of the burned exhaust gases in degrees Rankine. This comes from thermochemical calculations.
- TX Temperature of gaseous oxygen entering the chamber in degrees Rankine.
- TF Temperature of gaseous hydrogen entering the chamber in degrees Rankine
- GAMAP Ratio of specific heats for the burned exhaust gases. This would be obtained from a thermochemical calculation. It is primarily a function of mixture ratio.
- AX Total oxidizer injection area in square inches.
- RM Mixture Ratio
- WP Total weight flow of exhaust gases, pounds per second.
- PM Mach number of the chamber. This can be calculated from the chamber contraction ratio and GAMAP.
- WMP Molecular weight of the burned exhaust gases. This also comes from thermochemical calculations.
- PO Stagnation pressure in the chamber in pounds per square inch.
- WT Weight flow in tapoff line in pounds per second (normally negative).
- CHECK This is a flag which controls the output of the program.
- If CHECK is less than zero all input variables and variables needed to calculate the coefficients of the matrix are listed. The coefficients of the matrices, and the inverse are listed. The inverted matrix is then reinverted to check the accuracy of the inversion. This result is also listed.
- If CHECK is greater than zero all of the above is done except the reinversion is not performed.

If CHECK is zero only the input variables and final results are listed. CHECK is normally zero.

FLAG

If FLAG is negative only one case is run

If FLAG is zero a new frequency with the same chamber is run

If FLAG is positive a complete new case is run

A, C , FREQ.

These are longitudinal admittance coefficients and their corresponding frequency. A and C are complex. They can be obtained from the computer program described in Reference 8.

E. RESTRICTIONS AND LIMITATIONS

1. Low Mach number in chamber and feed lines.
2. Fuel rich (mixture ratio less than 8).
3. Gaseous hydrogen and gaseous oxygen are used as propellants.
4. Perturbations are assumed to be small.
5. No pressure sensitive combustion is included.

F. DIAGNOSTICS

Any division by zero will cause the program to print DIVIDE CHECK IN EQ, IEQ where IEQ is an equation number in the program. If a square root operation is attempted on a negative number a message is printed SQRT OF -X ATTEMPTED IN* EQ, IEQ. If the matrix is singular a message is printed MATRIX AA IS SINGULAR.

G. QUANTITY OF OUTPUT

If CHECK is zero one page of output per frequency is obtained. If CHECK is not zero 20 to 30 pages of output per frequency are obtained.

H. OUTPUT DEFINITION

<u>LABEL</u>	<u>SYMBOL</u>	<u>UNITS</u>	<u>DESCRIPTION</u>
AC	A_c	in^2	Chamber area
ALPHA	α	--	Nozzle admittance coefficient
AX	A_x	in^2	Oxidizer injection area
BETA	β	--	Nozzle admittance coefficient
F	F	sec/in^2	Coefficient of characteristic equation
G	G	sec/in^2	Coefficient of characteristic equation
GAMMA P	γ_p	--	ratio of specific heats of products of combustion
H	H	sec/in^2	Coefficient of characteristic equation
I	i	--	$\sqrt{-1}$
MP	M_p	--	Mach number of products of combustion
MR	MR	--	Mixture ratio
MWP	MW_p	--	Molecular weight of products of combustion
PO	P_o	lb/in^2	stagnation pressure of products of combustion
SCRIPT A	A	--	nozzle admittance coefficient
SCRIPT C	C	--	nozzle admittance coefficient
TF	T_F	$^{\circ}\text{R}$	temperature of fuel
TO	T_o	$^{\circ}\text{R}$	stagnation temperature of products of combustion
WT	$\dot{w}_t$	lb/sec	weight flow rate in tapoff line
WP	$\dot{w}_p$	lb/sec	weight flow rate of products of combustion

I. INPUT FORM

The format of the input data cards is shown on Page 47 and a listing of the input data cards for the sample case included herein is given on Page 48.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
AC																TO																TX																TF																															
GAMAP																AX																RM																WP																															
PM																WMP																RO																WT																															
CHECK																																																																															

[illegible]

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177.0	6800.	530.	530.	
1.2	29.0	5.0	290.0	
.274	11.74	1000.	-5.0	
-.026049	-.083588	-.12841	.048249	177.89
-.042862	-.24008	-.069302	.11453	553.67

J. SAMPLE OUTPUT

The following pages consist of the output of a typical case for the tapoff cycle. The input data for this case is that given in the previous section.

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AEROJET-GENERAL CORPORATION
COMPUTING SCIENCES DIVISION
SACRAMENTO, CALIFORNIA

PROGRAM E21504
GAS-GAS FEED SYSTEM COUPLED STABILITY PROGRAM

REAL INPUT CONDITIONS

AC .177000+03	TO .680000+04	TF .530000+03	GAMMA P .120000+01	AX .290000+02	MR .500000+01
	WP .290000+03	MP .274000+00	MW P .117400+02	PO .100000+04	WT -.500000+01

COMPLEX NOZZLE ADMITTANCE COEFFICIENTS

REAL	IMAG	REAL	IMAG
SCRIPT A -.260490-01	-.835880-01	SCRIPT C -.128410+00	.482490-01
ALPHA -.127592-02	-.409427-02	BETA -.114705+01	.430995+00

COMPUTED COMPLEX COEFFICIENTS IN THE CHARACTERISTIC EQUATION

$$(F) * WX' + (G) * WF' + (H) * WT' + PX' = 0.$$

FOR A FREQUENCY OF 177.890

(F) =	-.1657541+01 +	.5268061+00 * I ,OR	.1739243+01 TIMES E TO THE I	.2833865+01
(G) =	-.8678599+01 +	.2761513+01 * I ,OR	.9107361+01 TIMES E TO THE I	.2833525+01
(H) =	-.8683373+01 +	.2761445+01 * I ,OR	.9111890+01 TIMES E TO THE I	.2833691+01

Report 20672-P3D, Appendix A

AEROJET-GENERAL CORPORATION
COMPUTING SCIENCES DIVISION
SACRAMENTO, CALIFORNIA

PROGRAM E21504
GAS-GAS FEED SYSTEM COUPLED STABILITY PROGRAM

REAL INPUT CONDITIONS

AC .177000+03	TO .680000+04	TF .530000+03	GAMMA P .120000+01	AX .290000+02	MR .500000+01
	WP .290000+03	MP .274000+00	MW P .117400+02	PO .100000+04	WT -.500000+01

COMPLEX NOZZLE ADMITTANCE COEFFICIENTS

REAL	IMAG	REAL	IMAG
SCRIPT A -.428620-01	-.240080+00	SCRIPT C -.693020-01	.114530+00
ALPHA -.209945-02	-.117595-01	BETA -.619056+00	.102306+01

COMPUTED COMPLEX COEFFICIENTS IN THE CHARACTERISTIC EQUATION

$$(F) * WX' + (G) * WF' + (H) * WT' + PX' = 0.$$

FOR A FREQUENCY OF 553.670

(F) =	-.9958138+00 +	.8208317+00 * I ,OR	.1290508+01 TIMES E TO THE I	.2452220+01
(G) =	-.5208834+01 +	.4300611+01 * I ,OR	.6754791+01 TIMES E TO THE I	.2451413+01
(H) =	-.5214180+01 +	.4301570+01 * I ,OR	.6759525+01 TIMES E TO THE I	.2451807+01

Report 20672-P3D

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Appendix B

ANALYSIS OF THE LONGITUDINAL HIGH FREQUENCY STABILITY OF A
STAGED COMBUSTION CYCLE

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SECTION I -- PROBLEM

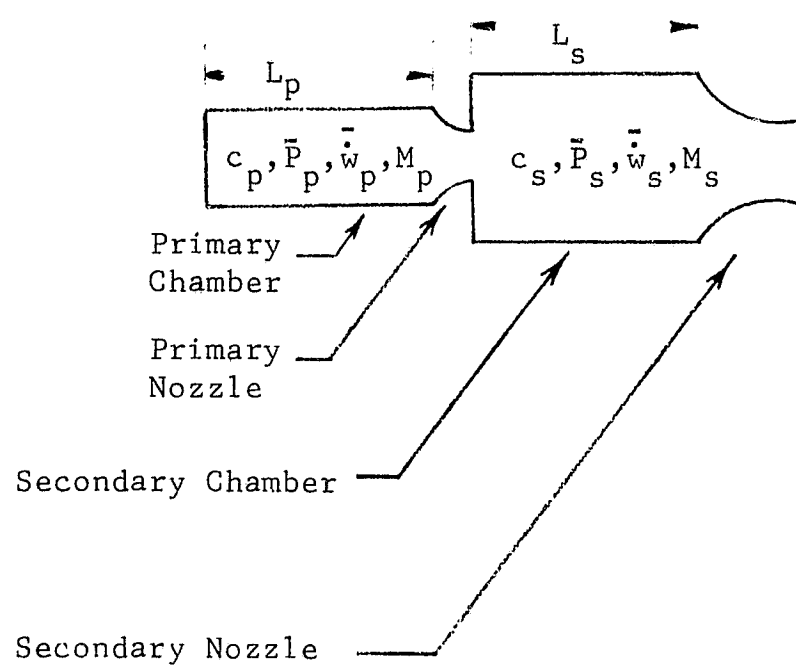
A ABSTRACT

This analysis uses the sensitive time lag theory in both combustors of a staged combustion cycle. It considers only longitudinal modes and no feed system effects are included. The primary and secondary combustors are coupled by including in the secondary analysis the flow oscillation generated in the primary combustor. As in other analyses using the sensitive time lag theory an n and τ curve for neutral stability is obtained (Ref. 1). Since a staged combustion cycle has two combustors there are two sets of n 's and τ 's, one set for the primary combustor and one set for the secondary combustor. In this analysis the n and τ for the primary are assumed known and the analysis solves for the n and τ of the secondary.

The only input parameters needed by the program which are somewhat obscure are the n and τ for the primary combustor and the specific acoustic admittance of the sonic secondary exhaust nozzle, α_s . The first of these, n and τ , must come from experiments on primary combustors fired alone. What limited data that exists is published in Reference 2. The second, the specific acoustic admittance of a sonic nozzle, can be obtained from a computer program described in Reference 1.

B. TECHNICAL DESCRIPTION

There are many possible variations in configuration of staged combustion cycles. The one analyzed in this program is shown on Page 2. Figure 1 also includes dimensions and variables used in the input to the program. The purpose of the primary combustor is to generate gases to drive the turbine which in turn drives the propellant feed pumps. In this analysis the turbine is simulated by a pressure drop which is assumed to be proportional to the square of the flow rate through the turbine. No effect of turbine speed change or vibration is included. The distance between the turbine inlet and the point where the primary gases are injected into the secondary chamber is assumed to be negligible. The flow coming from the turbine that is injected



SCHEMATIC OF STAGED COMBUSTION CYCLE

Figure 1

B, Technical Description (cont.)

into the secondary combustor is assumed to have a time delay τ_{fs} between injection and combustion. This time delay is a mean total time delay as distinguished from the sensitive time lags τ_s and τ_p . The chambers and nozzles are analyzed using one-dimensional acoustics with a finite but small Mach number.

Both chambers are analyzed acoustically the same way so the following development applies to both the primary and secondary combustors. The following equations are used in developing the model:

Conservation of mass.

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial Z} (\rho u) = 0 \quad \text{Eq. 1}$$

Conservation of momentum.

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial Z} + \frac{\partial p}{\partial Z} = 0 \quad \text{Eq. 2}$$

Conservation of energy.

$$\frac{\partial}{\partial t} (\rho e_o) + \frac{\partial}{\partial Z} (\rho u h_o) = 0 \quad \text{Eq. 3}$$

where,

$$e_o = c_v T + \frac{u^2}{2} \quad \text{Eq. 4}$$

$$h_o = c_p T + \frac{u^2}{2} \quad \text{Eq. 5}$$

$$c_v = \frac{RT}{\gamma - 1} \quad \text{Eq. 6}$$

$$c_p = \frac{\gamma RT}{\gamma - 1} \quad \text{Eq. 7}$$

B, Technical Description (cont.)

Ideal gas law.

$$p = \rho RT \quad \text{Eq. 8}$$

The energy equation is simplified if the kinetic energy terms are neglected. These terms become important only when Mach number is near 1. With this assumption the energy equation becomes:

$$\frac{\partial p}{\partial t} + \gamma p \frac{\partial u}{\partial Z} + \gamma u \frac{\partial p}{\partial Z} = 0 \quad \text{Eq. 9}$$

The above equations have built into them the assumption that the area of the chamber does not change with the distance Z. It has also assumed no combustion. The combustion will be assumed to occur at the injector face and will be included in the analysis as a boundary condition.

All the dependent variables in the above equations will be assumed to be composed of a mean part which does not vary with time and a perturbation part which does vary with time.

$$\begin{aligned} p &= \bar{p} + p' e^{St} \\ \rho &= \bar{\rho} + \rho' e^{St} \\ u &= \bar{u} + u' e^{St} \end{aligned} \quad \text{Eq. 10}$$

The variable S is the Laplace variable and is in general complex. When these are substituted into the equations and products of perturbations are neglected (which implies that only small amplitude oscillations will be considered) the following equations result:

$$S p' + \bar{u} \frac{dp'}{dZ} + \bar{p} \frac{du'}{dZ} = 0 \quad \text{Eq. 11}$$

B, Technical Description (cont.)

$$\bar{\rho} S u' + \bar{\rho} \bar{u} \frac{du'}{dZ} + \frac{dp'}{dZ} = 0 \quad \text{Eq. 12}$$

$$S p + \gamma \bar{p} \frac{du'}{dZ} + \gamma \bar{u} \frac{dp'}{dZ} = 0 \quad \text{Eq. 13}$$

In obtaining the above equations $\frac{d\bar{u}}{dZ}$, $\frac{d\bar{p}}{dZ}$, $\frac{d\bar{\rho}}{dZ}$ were all set equal to zero since the area is not changing and no combustion is occurring.

These three equations can be combined to give:

$$\frac{c^2}{S^2} \frac{d^2}{dZ^2} \left(\frac{p'}{p} \right) - (1 - \gamma M^2) \frac{p'}{p} = \frac{\bar{u}}{S} (\gamma + 1) \frac{d}{dZ} \left(\frac{p'}{p} \right) \quad \text{Eq. 14}$$

where M is the Mach number and c is the acoustic velocity, $c = \sqrt{\gamma g R T}$. If the Mach number is small enough γM^2 is negligible compared to 1. For an even smaller Mach number the right-hand side can be neglected. The governing equation then becomes the homogeneous Helmholtz equation:

$$\frac{d^2}{dZ^2} \left(\frac{p'}{p} \right) - \frac{S^2}{c^2} \left(\frac{p'}{p} \right) = 0 \quad \text{Eq. 15}$$

From Equations 11 thru 14, velocity and density can be obtained in terms of pressure.

$$\frac{u'}{\bar{u}} = \frac{1}{\gamma} \frac{p'}{\bar{p}} - \frac{1}{\gamma} \frac{c^2}{S \bar{u}} \frac{d}{dZ} \left(\frac{p'}{\bar{p}} \right) \quad \text{Eq. 16}$$

$$\frac{\rho'}{\bar{\rho}} = \frac{1}{\gamma} \frac{p'}{\bar{p}} \quad \text{Eq. 17}$$

Equation 15 is a second order equation and requires two boundary conditions. At the nozzle entrance the velocity and density are related by the specific acoustic admittance.

$$\frac{u'/\bar{u}}{\rho'/\bar{\rho}} = \alpha \quad \text{Eq. 18}$$

B, Technical Description (cont.)

At the injector the burning rate and pressure are related by the sensitive time lag burning expression.

$$\frac{(\rho u)'}{\bar{\rho} \bar{u}} = \mathcal{P} \frac{p'}{\bar{p}} \quad \text{where} \quad \mathcal{P} = n (1 - e^{-S\tau}) \quad \text{Eq. 19}$$

The solution to equation 15 is:

$$\frac{p'}{\bar{p}} = c_1 e^{\frac{SZ}{c}} + c_2 e^{-\frac{SZ}{c}} \quad \text{Eq. 20}$$

Combining Equation 20 with the 2 boundary conditions, after considerable algebra, the following expression is obtained.

$$M (1 - \gamma \mathcal{P}) = \frac{1 - B \exp\left(\frac{2SL}{c}\right)}{1 + B \exp\left(\frac{2SL}{c}\right)} \quad \text{where} \quad B = \frac{1 + \alpha M}{1 - \alpha M} \quad \text{Eq. 21}$$

This expression applies without qualification to the primary combustor:

$$M_p (1 - \gamma \mathcal{P}_p) = \frac{1 - B_p \exp\left(\frac{2SL_p}{c_p}\right)}{1 + B_p \exp\left(\frac{2SL_p}{c_p}\right)} \quad \text{where} \quad B_p = \frac{1 + \alpha_p M_p}{1 - \alpha_p M_p} \quad \text{Eq. 22}$$

For the secondary combustor Equation 21 must be modified slightly to include the flow injected from the turbine exit as well as the pressure sensitive combustion:

$$M_s \left[1 - \gamma (G e^{-S\tau_{fs}} + \mathcal{P}_s) \right] = \frac{1 - B_s \exp\left(\frac{2SL_s}{c_s}\right)}{1 + B_s \exp\left(\frac{2SL_s}{c_s}\right)} \quad \text{Eq. 23}$$

$$\text{where} \quad B_s = \frac{1 + \alpha_s M_s}{1 - \alpha_s M_s}$$

B, Technical Description (cont.)

The factor $e^{-S\tau_{fs}}$ is the combustion time delay for the fuel entering the secondary. The term G is determined as follows: Define $\hat{G}_p = \frac{\dot{w}_p}{\bar{p}_p}$. This can be related to α_p by:

$$\hat{G}_p = \frac{1}{\gamma} \frac{\bar{w}_p}{\bar{p}_p} (1 + \alpha_p) \quad \text{Eq. 24}$$

The admittance upstream of the turbine is related to the admittance downstream of the turbine by:

$$\frac{1}{\hat{G}_{fs}} = \frac{1}{\hat{G}_p} - \frac{2(\bar{p}_p - \bar{p}_s)}{\bar{w}_p} \quad \text{Eq. 25}$$

The G in Equation 23 is non-dimensional. To convert $\hat{G}_{fs}$ to G divide by $\frac{\bar{w}_s}{\bar{p}_s}$

$$G = \hat{G}_{fs} \frac{\bar{p}_s}{\bar{w}_s} = \frac{\bar{p}_s}{\bar{w}_s} \frac{\frac{\bar{w}_p}{2(\bar{p}_p - \bar{p}_s)}}{\frac{1}{\hat{G}_p} \frac{\bar{w}_p}{2(\bar{p}_p - \bar{p}_s)} - 1}$$

$$G = \frac{-R_s}{1+R_s} \frac{\bar{p}_s}{2(\bar{p}_p - \bar{p}_s)} \left[\frac{1}{1 - \left(\frac{\bar{p}_p}{2(\bar{p}_p - \bar{p}_s)} \right) \left(\frac{\gamma}{1 + \alpha_p} \right)} \right] \quad \text{Eq. 26}$$

where R_s is the mixture ratio of the secondary chamber.

Equations 22, 23 and 26 are the governing equations for the system. These equations can now be solved simultaneously for $\mathcal{P}_s$.

C. EQUATIONS

Equation 22 must first be solved for α_p

$$p = \frac{\Omega (1+\beta) - 1+\beta}{M_p (\Omega (1-\beta) - \beta - 1)}$$

$$\text{where } \beta = \exp\left(\frac{2SL_p}{c_p}\right) \quad \text{and } \Omega = M_p (1 - \alpha_p)$$

Equation 26 can then be used to get G.

$$G = -\left(\frac{R_s}{1+R_s}\right)\left(\frac{\bar{P}_s}{2(\bar{P}_p - \bar{P}_s)}\right)\left[\frac{1}{1 - \left(\frac{\bar{P}_p}{2(\bar{P}_p - \bar{P}_s)}\right)\left(\frac{\gamma}{1 + \alpha_p}\right)}\right]$$

Finally equation 23 can be used to solve for $\mathcal{P}_s$.

$$\mathcal{P}_s = \frac{\left[\frac{1 - B_s \exp\left(\frac{2SL_s}{c_s}\right)}{c_s} - M_s \right]}{1 + B_s \exp\left(\frac{2SL_s}{c_s}\right)} - G e^{-S\tau_{fs}} - \gamma M_s$$

The n and τ come from the definition of $\mathcal{P}_s$.

$$\mathcal{P}_s = n_s (1 - e^{-S\tau_s})$$

Set $S = 0 + i\omega$ and solve for $n(\omega)$ and $\tau(\omega)$

D. DEFINITION OF TERMS

B	Intermediate term defined in equation 21
$C_1 - C_2$	Constants in Equation 20 to be determined by the boundary conditions
c_p	Specific heat at constant pressure
c_v	Specific heat at constant volume
e	Base of Natural Logarithms
e_o	Internal energy of stagnant combustion gases
G	Nondimensional weight flow admittance injected into secondary chamber
G_p	Dimensional weight flow admittance upstream of the primary exhaust nozzle
h_o	Enthalpy of stagnant combustion gases
L	Length of cylindrical portion of chamber
M	Mach number in cylindrical portion of chamber
n	Pressure interaction index measuring the amount of interaction between pressure oscillations and burning rate oscillations
p	Pressure
R	Universal gas constant
R_s	Mixture Ratio in secondary chamber
S	Laplace variable
t	Time
T	Temperature
u	Velocity of gas
$\dot{w}$	Weight flow rate
Z	Length dimension
α	Specific Acoustic admittance (= $-\frac{A\gamma}{M}$ where A is from Ref. 1)
γ	Ratio of specific heats
ρ	Gas density

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D. Definition of Terms (cont.)

τ_{fs} Total time lag of the fuel injected into the secondary chamber in seconds

ω Frequency of oscillation

τ Sensitive time lag in seconds

w dimensional frequency in radians per second

p $n(1 - e^{-S\tau})$

Subscripts

p Primary combustor

s Secondary combustor

Superscripts

$-$ Mean value - does not vary with time

$'$ Perturbation value

E. SPECIAL OPTIONS

This program is restricted to the problem described in Section B.

F. NUMERICAL METHODS OF SOLUTION

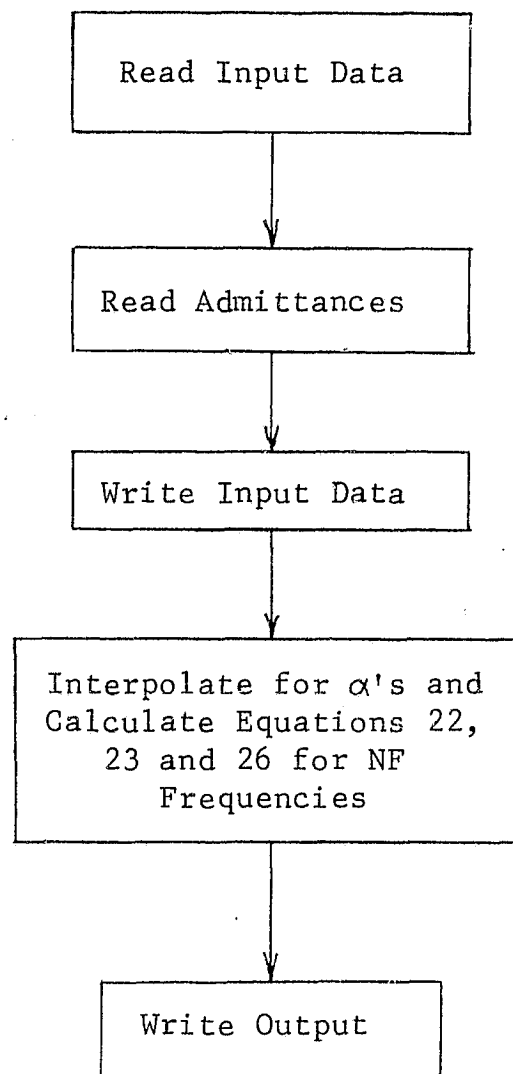
Complex arithmetic is used in this program. The specific acoustic admittance of the second combustor α_s is input as a table. An interpolation function GNINT, taken from the UNIVAC MATH PACK, is used to interpolate for values of α_s between those input in the table. A third order polynomial is used in this interpolation.

G. REFERENCES

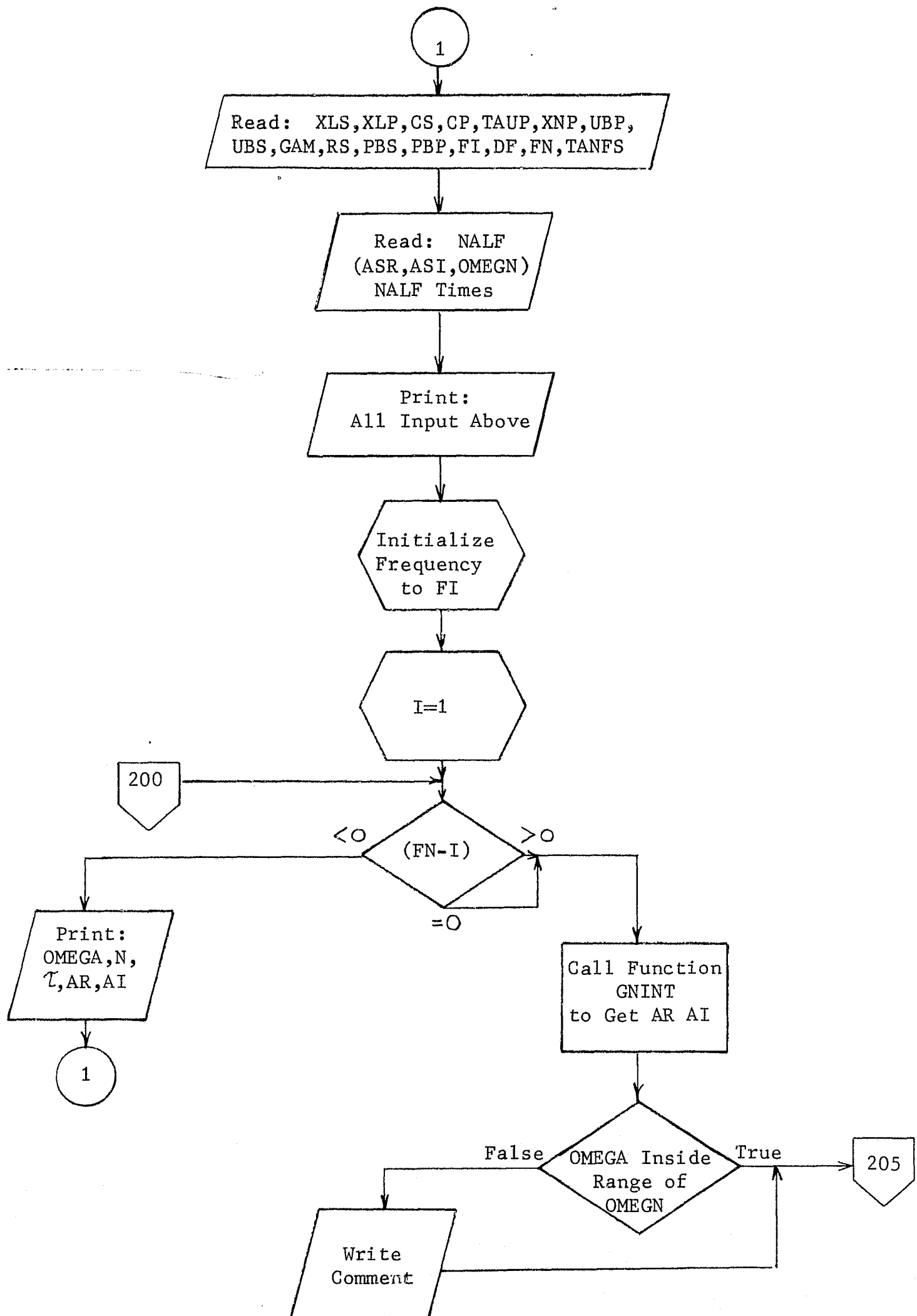
1. Stability Characterization of Advanced Injectors Design Guide Vol. 2 Operation of the Computer Program 20672-P2D Jan. 1970.
2. Stability Characterization of Advanced Injectors Design Guide for Stable H_2/O_2 Combustors. Vol 1: Design Application 20672-P2D May 1970.
3. General Reference Manual, UNIVAC 1108, Sperry-Rand Corp., 1966.

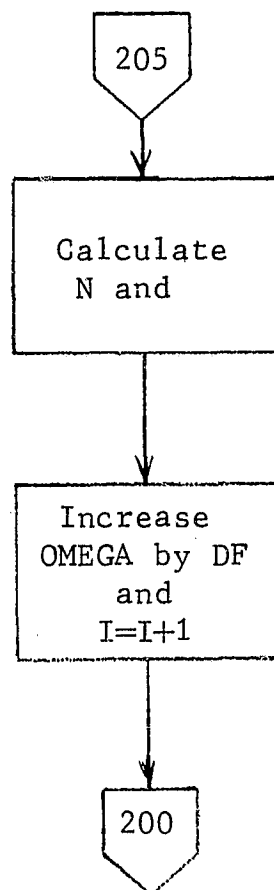
SECTION II - PROGRAMING

A. BLOCK DIAGRAM



B. FLOW CHARTS





C. PROGRAM LISTING

ELT STAGE,1,700724, 29533

```

000001      C  COMMENTS
000002      C  THIS PROGRAM CALCULATES AN EN, TAU ZONE FOR THE SECONDARY INJECTOR OF A
000003      C  STAGED COMBUSTION CYCLE. THE EN AND TAU FOR THE PRIMARY ARE INPUT TO THE
000004      C  PROGRAM ALONG WITH A FUEL TOTAL TIME LAG FOR THE SECONDARY. THE ANALYSIS
000005      C  IS ONE DIMENSIONAL AND APPLIES TO LONGITUDINAL MODES WITHOUT FEED SYSTEM
000006      C  COUPLING. THE ANALYSIS FROM WHICH THIS PROGRAM WAS WRITTEN APPEARS IN THE
000007      C  AEROJET GENERAL REPORT NUMBER 20672-P1F. THE INPUT PARAMETERS ARE AS
000008      C  FOLLOWS:
000009      C  XLS = LENGTH OF SECONDARY CHAMBER IN FEET
000010      C  XPL = LENGTH OF PRIMARY CHAMBER IN FEET
000011      C  CS = SPEED OF SOUND IN SECONDARY IN FEET PER SECOND
000012      C  CP = SPEED OF SOUND IN PRIMARY IN FEET PER SECOND
000013      C  TAUP = SENSITIVE TIME LAG IN PRIMARY CHAMBER IN SECONDS
000014      C  XNP = INTERACTION INDEX IN PRIMARY CHAMBER (DIMENSIONLESS)
000015      C  UBP = MACH NUMBER IN PRIMARY CHAMBER
000016      C  UBS = MACH NUMBER IN SECONDARY CHAMBER
000017      C  GAM = RATIO OF SPECIFIC HEATS
000018      C  RS = MIXTURE RATIO OF SECONDARY CHAMBER
000019      C  PBS = SECONDARY CHAMBER PRESSURE IN POUNDS PER SQUARE INCH
000020      C  PBP = PRIMARY CHAMBER PRESSURE IN POUNDS PER SQUARE INCH
000021      C  FI = INITIAL FREQUENCY AT WHICH THE EN AND TAU ARE TO BE CALCULATED, HZ
000022      C  DF = INCREMENT BETWEEN FREQUENCIES
000023      C  FN = NUMBER OF FREQUENCIES
000024      C  TAUPS = TOTAL TIME LAG OF FUEL IN SECONDARY CHAMBER IN SECONDS
000025      C  ASR, ASI = REAL AND IMAGINARY PARTS OF SPECIFIC ACOUSTIC ADMITTANCE AS
000026      C  DEFINED BY CROCCO (CALLED ALPHA), DIMENSIONLESS
000027      C  OMEGN = FREQUENCY AT WHICH THE ASR, ARE GIVEN - RADIANS PER SECOND
000028      C  DIMENSION ASR(50),ASI(50),OMEGN(50),SN(100),TAUMS(100),HERTZ(100)
000029      C  1,D(10),ALFPR(100),ALFPI(100)
000030      C  COMPLEX SPP,EXPP,G,ALFP,EXPS,RS,ALFS,PHLFS,H,S
000031      C  1 READ(5,101)XLS,XLP,CS,CP,TAUP,XNP,UBP,UBS,GAM,RS,PBS,PBP,FI,DF,FN
000032      C  1,TAUPS
000033      C  101 FORMAT( 6F10.1)
000034      C  READ(5,102) NALF,(ASR(I),ASI(I),OMEGN(I),I=1,NALF)
000035      C  102 FORMAT(I2,/, (3F20.7))
000036      C  WRITE(6,103)XLS,XLP,CS,CP,TAUP, XNP,UBP,UBS,GAM,RS,PBS,PBP,TAUPS,
000037      C  1(OMEGN(I),ASR(I),ASI(I),I=1,NALF)
000038      C  103 FORMAT(1X,15HINPUT DATA //1X,35HLENGTH OF SECONDARY CHAMBER IN
000039      C  1FEET=F10.4/1X,34HLENGTH OF PRIMARY CHAMBER IN FEET=F10.4/
000040      C  21X,55HSPEED OF SOUND IN SECONDARY CHAMBER IN FEET PER SECOND=F10.
000041      C  34/1X,53HSPEED OF SOUND IN PRIMARY CHAMBER IN FEET PER SECOND=F10.
000042      C  44/1X,12HTAU PRIMARY=F10.4/1X,26HINTERACTION INDEX PRIMARY=F10.4/
000043      C  51X,20HMACH NUMBER PRIMARY=F10.4/1X,22HMACH NUMBER SECONDARY=F10.
000044      C  64/1X,6HGAMMA=F10.4/1X,24HMIXTURE RATIO SECONDARY=F10.4/1X,33HMEA
000045      C  74 CHAMBER PRESSURE SECONDARY =F10.4/1X,30HMEAN CHAMBER PRESSURE P
000046      C  81MARY=F10.4/1X,33HTOTAL TIME LAG OF SECONDARY FUEL=F10.4//1X,41
000047      C  9HSECONDARY NOZZLE ADMITTANCE (OMEGA,AR,AI)/(1X,3F20.4))
000048      C  WRITE(6,104)
000049      C  104 FORMAT(1X, 6HOUTPUT,/,1X,98H          FREQUENCY          N
000050      C  1          TAU          APR          API          )
000051      C  NF=FN
000052      C  OMEGI=2.*3.1415927*FI
000053      C  OMEGA=OMEGI
000054      C  DO 200 I=1,NF
000055      C  S=CMPLX(0.0,OMEGA)

```

```

000056      SPP=XNP*(1.0-EXP(-S*TAUP))
000057      EXPP=EXP(2.0*S*XLP/CP)
000058      AR=GNINT(OMEGN,ASR,NALF,3,OMEGA,D,400)
000059      AI=GNINT(OMEGN,ASI,NALF,3,OMEGA,D,400)
000060      GO TO 205
000061  400  WRITE(6,500)
000062  500  FORMAT(1X,47HINTERPOLATION FUNCTION IS TRYING TO EXTRAPOLATE )
000063  205  ALFS=CMPLX(AR,AI)
000064      BS=(1.0+ALFS*UBS)/(1.0-ALFS*UBS)
000065      ALFP=(0.5*(1.0-GAM*SPP)+(1.0+EXPP)-1.0*EXPP/(UBS+(UBS*(1.0-GAM*
000066      1SPP)+(1.0-EXPP)-EXPP-1.0))
000067      G=-KS/(1.0+RS)*PBS/2.0/(PBP-PBS)*(1.0/(1.0-(PBP/2.0/(PBP-PBS))
000068      1*(GAM/(1.0+ALFP))))
000069      PHLFS=EXP(-S*TAUFS)
000070      EXPS=EXP(2.0*S*XLS/CS)
000071      H=-((1.0-BS*EXPS)/(1.0+BS*EXPS)-UBS)/GAM/UBS-G*PHLFS
000072      HI=AIMAG(H)
000073      HR=REAL(H)
000074      ALFPR(I)=REAL(ALFP)
000075      ALFPI(I)=AIMAG(ALFP)
000076      SN(I)=(HR*HR+HI*HI)/HR*G.S
000077      WT=3.1415927-2.0*ATAN(HI/HR)
000078      TAUMS(I)=PI/OMEGA*1000.
000079      HERTZ(I)=OMEGA/6.2831853
000080  200  OMEGA=OMEGA +DF*6.2831853
000081      WRITE(6,300) (HERTZ(I),SN(I),TAUMS(I),ALFPR(I),ALFPI(I),I=1,NF)
000082  300  FORMAT(1X,5F20.5)
000083      GO TO 1
000084  END

```

4. LIST GNINT

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```

000001      FUNCTION GNINT(X,Y,N,LD,XBAR,D,S)      GNINT
000002      C-----GNINT
000003      C  GREGORY-NEWTON INTERPOLATION AND EXTRAPOLATION      GNINT
000004      C  THIS SUBPROGRAM DETERMINES      GNINT
000005      C  1. WHETHER EXTRAPOLATION OR INTERPOLATION IS REQUIRED.      GNINT
000006      C  2. WHETHER FORWARD OR BACKWARD INTERPOLATION IS TO BE PERFORMED.      GNINT
000007      C-----GNINT
000008      C  ARGUMENT DEFINITION      GNINT
000009      C  X  -- ARRAY OF N INDEPENDENT VARIABLE VALUES      GNINT
000010      C  Y  -- ARRAY OF N DEPENDENT VARIABLE VALUES      GNINT
000011      C  N  -- NUMBER OF DATA POINTS      GNINT
000012      C  LD -- DESIRED DEGREE OF INTERPOLATING POLYNOMIAL      GNINT
000013      C  XBAR -- INDEPENDENT VALUE FOR WHICH GNINT IS THE INTERPOLANT      GNINT
000014      C  OR EXTRAPOLANT      GNINT
000015      C  D  -- ARRAY OF LD+1 ELEMENTS USED FOR TEMPORARY STORAGE BY      GNINT
000016      C  GNPOL.      GNINT
000017      C  GNINT STORES SPACING OF X ARRAY ELEMENTS IN D(1).      GNINT
000018      C  S  -- ERROR EXIT IF EXTRAPOLATION NECESSARY, AND XBAR IS MORE      GNINT
000019      C  THAN 5 SPACES FROM LIMIT OF X ARRAY.      GNINT
000020      C-----GNINT
000021      C  OTHER SUBPROGRAMS REFERENCED      GNINT
000022      C  GNPOL -- DETERMINATION OF INTERPOLATING POLYNOMIAL VALUE      GNINT
000023      C  AT XBAR.      GNINT
000024      C  GNEXT -- GREGORY-NEWTON EXTRAPOLATION.      GNINT
000025      C-----GNINT
000026      C  REFERENCES      GNINT
000027      C  F. B. HILDEBRAND, INTRODUCTION TO NUMERICAL ANALYSIS,      GNINT
000028      C  MCGRAW-HILL BOOK CO., 1956.      GNINT
000029      C  J. B. SCARBOROUGH, NUMERICAL MATHEMATICAL ANALYSIS,      GNINT
000030      C  JOHNS HOPKINS PRESS, 1962.      GNINT
000031      C-----GNINT
000032      C  GNINT PERFORMS THE FOLLOWING SET OF OPERATIONS      GNINT
000033      C  1. DETERMINES POSITION OF XBAR WITHIN X ARRAY.      GNINT
000034      C  2. REFERENCES GNEXT IF EXTRAPOLATION IS NECESSARY.      GNINT
000035      C  3. DETERMINES MAXIMUM OBTAINABLE INTERPOLATING POLYNOMIAL DEGREE,      GNINT
000036      C  AS LIMITED BY N AND POSITION OF XBAR.      GNINT
000037      C  4. REDUCES LD IF IT IS GREATER THAN MAXIMUM OBTAINABLE DEGREE.      GNINT
000038      C  5. DETERMINES WHETHER FORWARD OR BACKWARD INTERPOLATION IS TO BE      GNINT
000039      C  PERFORMED.      GNINT
000040      C  6. INITIALIZES ARGUMENTS FOR GNPOL.      GNINT
000041      C  7. REFERENCES GNPOL TO OBTAIN INTERPOLATING POLYNOMIAL VALUE      GNINT
000042      C  AT XBAR.      GNINT
000043      C-----GNINT
000044      C  INITIALIZATION OF D(1) AS X ARRAY SPACING.      GNINT
000045      C  DETERMINATION WHETHER INTERPOLATION OR EXTRAPOLATION IS REQUIRED.      GNINT
000046      C-----GNINT
000047      C  DIMENSION X(N),Y(N),D(1)      GNINT
000048      C  D(1)=X(2)-X(1)      GNINT
000049      C  IF(XBAR.LT.X(1).OR.XBAR.GT.X(N)) GO TO 3      GNINT
000050      C-----GNINT
000051      C  INTERPOLATION REQUIRED.      GNINT
000052      C  DETERMINATION OF X ARRAY ELEMENT, X(J), BELOW XBAR.      GNINT
000053      C-----GNINT
000054      C  J=(XBAR-X(1))/D(1) +1.      GNINT
000055      C-----GNINT

```

```

000056 C      DETERMINATION OF DEGREE OF INTERPOLATING POLYNOMIAL.      GNINT
000057 C      N IS MAXIMUM POSSIBLE DEGREE, LD IS DEGREE USED.      GNINT
000058 C-----GNINT
000059 M=MAX0(J,N-J)      GNINT
000060 LD=MIN0(M,LD)      GNINT
000061 C-----GNINT
000062 C      INITIALIZATION OF ARGUMENTS FOR GNPOL.      GNINT
000063 C-----GNINT
000064 IF(M.EQ.J) J=J+1      GNINT
000065 S=(XBAR-X(J))/D(J)      GNINT
000066 IF(.NOT.ABS(S).GT.U.) GO TO 2      GNINT
000067 IF(M.NE.J-1) GO TO 1      GNINT
000068 J=-J      GNINT
000069 S=-S      GNINT
000070 C-----GNINT
000071 C      REFERENCE FUNCTION SUBPROGRAM GNPOL TO OBTAIN INTERPOLATING      GNINT
000072 C      POLYNOMIAL VALUE AT XBAR.      GNINT
000073 C-----GNINT
000074 1 GNINT=GNPOL(Y,J,S,LD,J)      GNINT
000075 RETURN      GNINT
000076 2 GNINT=Y(J)      GNINT
000077 RETURN      GNINT
000078 C-----GNINT
000079 C      REFERENCE FUNCTION SUBPROGRAM GNEXT TO PERFORM FORWARD OR      GNINT
000080 C      BACKWARD EXTRAPOLATION.      GNINT
000081 3 GNINT=GNEXT(X,Y,N,LD,XBAR,D,S4)      GNINT
000082 RETURN      GNINT
000083 4 RETURN 7      GNINT
000084 END      GNINT

```

5. LIST GNPOL

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W ELT GNPOL,1,700722, 38217

```

000001      FUNCTION GNPOL(Y,J,S,LD,D)                                GNPOL
000002      C-----GPNPOL                                           GNPOL
000003      C  CALCULATION OF GREGORY-NEWTON INTERPOLATING POLYNOMIAL VALUE  GNPOL
000004      C-----GPNPOL                                           GNPOL
000005      C  INITIALIZATION. D ARRAY INITIALIZED AS Y ARRAY VALUES.      GNPOL
000006      C-----GPNPOL                                           GNPOL
000007      DIMENSION Y(1),D(1)                                         GNPOL
000008      COEFF=1.                                                       GNPOL
000009      L=IABS(J)                                                       GNPOL
000010      GNPOL=Y(L)                                                    GNPOL
000011      IF(J.LT.0) L=L-LD                                            GNPOL
000012      DO 1 K=0,LD                                                    GNPOL
000013      1 D(K+1)=Y(L+K)                                               GNPOL
000014      C-----GPNPOL                                           GNPOL
000015      C  LOOP THROUGH ALL DIFFERENCES                               GNPOL
000016      C-----GPNPOL                                           GNPOL
000017      DO 3 K=1,LD                                                    GNPOL
000018      LDM=LD-K+1                                                     GNPOL
000019      C-----GPNPOL                                           GNPOL
000020      C  CALCULATE KTH COLUMN OF DIFFERENCES.                       GNPOL
000021      C-----GPNPOL                                           GNPOL
000022      DO 2 L=1,LDM                                                    GNPOL
000023      2 D(L)=D(L+1)-D(L)                                             GNPOL
000024      C-----GPNPOL                                           GNPOL
000025      C  COMPUTE DIFFERENCE COEFFICIENT(COEFF), POLYNOMIAL TERM(TERM),  GNPOL
000026      C  AND POLYNOMIAL SUM(GNPOL). TERM SUBTRACTED FOR ODD BACKWARD DIFFS GNPOL
000027      C-----GPNPOL                                           GNPOL
000028      COEFF=COEFF*(S-K+1.)/K                                         GNPOL
000029      IF(J.GT.0) LDM=1                                                GNPOL
000030      TERM=COEFF*D(LDM)                                                GNPOL
000031      IF(J.LT.0.AND.MOD(K,2).EQ.1) TERM=-TERM                        GNPOL
000032      3 GNPOL=GNPOL+TERM                                              GNPOL
000033      RETURN                                                         GNPOL
000034      END                                                            GNPOL

```

6. LIST GNEXT

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```

000001      FUNCTION GNEXT(X,Y,N,LD,XBAR,D,S)
000002      C-----GNEXT
000003      C   GREGORY-NEWTON EXTRAPOLATION      GNEXT
000004      C   INITIALIZATION OF VARIABLES AND LOOP PARAMETERS.      GNEXT
000005      C-----GNEXT
000006      DIMENSION TEMP(5),YIN(1),YOUT(1),
000007      SAVL=XBAR      GNEXT
000008      H=0.1      GNEXT
000009      IF(XBAR.LT.X(1)) GO TO 1      GNEXT
000010      J=N      GNEXT
000011      I=-1      GNEXT
000012      N1=1      GNEXT
000013      GO TO 2      GNEXT
000014      1 J=1      GNEXT
000015      I=1      GNEXT
000016      N1=2      GNEXT
000017      2 ITEM=0      GNEXT
000018      FI=I      GNEXT
000019      N2=N1+N-2      GNEXT
000020      NN=N+1-J      GNEXT
000021      S=(XBAR-X(J))/H*FI      GNEXT
000022      C-----GNEXT
000023      C   IF MAGNITUDE OF S LT 1, TABLE DOES NOT HAVE TO BE EXTENDED.      GNEXT
000024      C   RECOMPUTE S.      GNEXT
000025      C-----GNEXT
000026      IF(ABS(S).GT.5.) GO TO 6
000027      IF(.NOT.ABS(S).GT.1.) GO TO 5      GNEXT
000028      XBAR=X(J)-FI*H      GNEXT
000029      3 GNEXT=GNPOL(Y,I*J,FI,LD,D)      GNEXT
000030      ITEM=ITEM+1      GNEXT
000031      TEMP(ITEM)=Y(NN)      GNEXT
000032      DO 4 K=N1,N2      GNEXT
000033      4 Y(K)=Y(K-I)      GNEXT
000034      Y(J)=GNEXT      GNEXT
000035      S=(SAVE-XBAR)/H*FI      GNEXT
000036      IF(.NOT.ABS(S).GT.1.) GO TO 5      GNEXT
000037      XBAR=XBAR-FI*H      GNEXT
000038      GO TO 3      GNEXT
000039      C-----GNEXT
000040      C   ONLY ONE (MORE) EXTRAPOLATION TO BE PERFORMED.      GNEXT
000041      C-----GNEXT
000042      5 XBAR=SAVE      GNEXT
000043      GNEXT=GNPOL(Y,I*J,S,LD,D)      GNEXT
000044      C-----GNEXT
000045      C   RESHIFT Y ARRAY (IF NECESSARY) SO THAT IT IS ORIGINAL INPUT ARRAY.      GNEXT
000046      C-----GNEXT
000047      IF(ITEM.EQ.0) RETURN      GNEXT
000048      N1=N1-I      GNEXT
000049      N2=N2-I      GNEXT
000050      IF(1.EQ.-1) GO TO 9
000051      J=N2      GNEXT
000052      N2=N1      GNEXT
000053      N1=J      GNEXT
000054      9 DO 7 K=ITEM,1,-1      GNEXT
000055      DO 6 J=N2,N1,I      GNEXT
000056      6 Y(J)=Y(J+1)      GNEXT
000057      7 Y(NN)=TEMP(K)      GNEXT
000058      RETURN      GNEXT
000059      8 RETURN 7      GNEXT
000060      END      GNEXT

```


D. METHODS OF VERIFICATION

The only method of verification of the results of this program is based on the experience of the engineer with the sensitive time lag theory and its application. The correctness of the numbers can only be evaluated as reasonable by an engineer who has become familiar with the similar analysis for a single chamber as presented in Program B, described in the main portion of this report. The test case given herein is a convenient method of checking the program and computer for correct operation.

SECTION III - DECK SETUP

A. COMPUTER CONFIGURATION

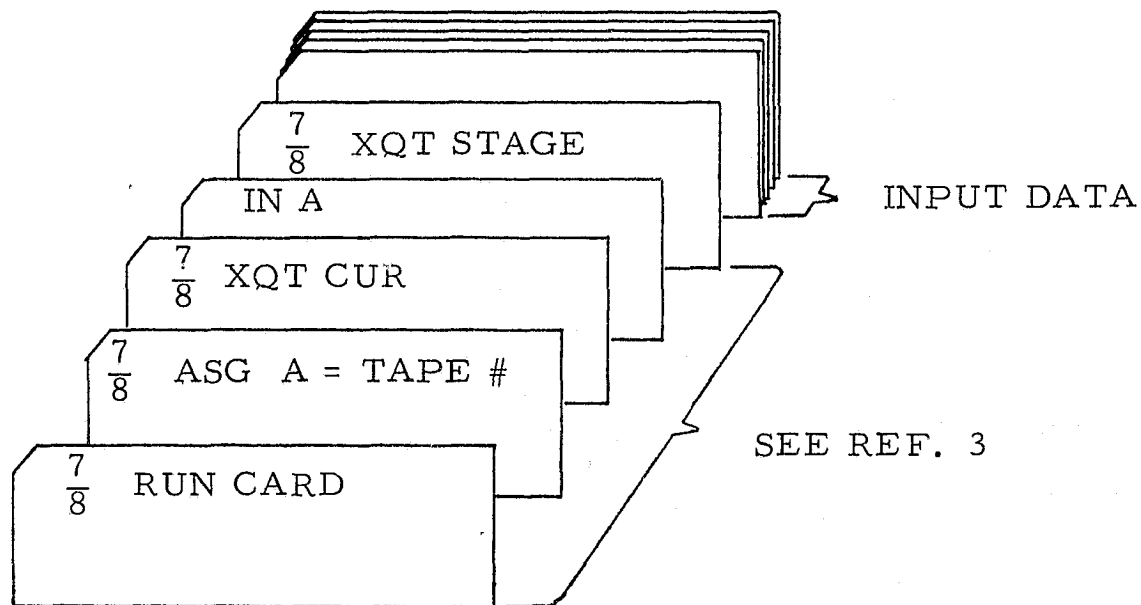
1. Univac 1108 computer, 65 K of core minimum
2. 46211₈ for code, 72245₈ for data
3. FORTRAN V
4. Executive II monitor system
5. No plot output required
6. No punch output required

B. ESTIMATED RUNNING TIME

The sample case presented in this section took 11 seconds to run including compilation. This case can be assumed to be a representative case as far as computer run time is considered.

C. DECK SEQUENCE

The deck sequence shown below includes the control cards as required by the EXEC II Monitor system for the UNIVAC 1108 as described in Ref. 3.



Multiple cases may be run by stacking the data cards for subsequent cases immediately behind the data cards for the original case.

D. INPUT DATA

<u>Symbol</u>	<u>Name in Program</u>	<u>Units</u>	<u>Definition</u>
L_s	XLS	ft.	Length of cylindrical portion of secondary chamber
L_p	XLP	ft.	Length of cylindrical portion of primary chamber
c_s	CS	ft/sec	Speed of sound of burned gases in the secondary chamber
c_p	CP	ft/sec	Speed of sound of burned gases in the primary chamber
τ_p	TAUP	sec	Mean sensitive time lag for the primary
n_p	XNP	—	Combustion interaction index for the primary chamber
M_p	UBP	—	Mach number in primary chamber
M_s	UBS	—	Mach number in secondary chamber
γ	GAM	—	Ratio of specific heats
R_s	RS	—	Mixture Ratio of secondary chamber
$\bar{P}_s$	PBS	psi	Mean pressure in secondary chamber
$\bar{P}_p$	PBP	psi	Mean pressure in primary chamber
F_i	FI	(H_z)	Initial calculation frequency
ΔF	DF	Hz	Frequency increment of calculation
N_f	FN	—	Number of frequencies to be calculated
τ_{fs}	TAUFS	sec	Total time lag of fuel entering the secondary chamber

<u>Symbol</u>	<u>Name in Program</u>	<u>Units</u>	<u>Definition</u>
—	NALF	—	Number of admittance values input
α_s	ASR	—	Real part of specific acoustic admittance
	ASI	—	Imaginary part of specific admittance
ω	OMEGN	$\frac{\text{rad}}{\text{sec}}$	Frequency at which specific acoustic admittance is given

E. RESTRICTION AND LIMITATIONS

Number of input admittances is limited to 50 per case. Number of n, t points output is limited to 100 per case. Applies only to longitudinal modes. Mach number in both chambers must be low.

F. OUTPUT DEFINITION

All input parameters are output as well as the following:

<u>SYMBOL</u>	<u>NAME ON OUTPUT</u>	<u>UNITS</u>	<u>DEFINITION</u>
f	F	Hertz	Frequency
n	N	—	Combustion interaction index for the secondary chamber
τ	TAU	sec	Mean sensitive time lag for the secondary chamber
α_p	APR	—	Real part of specific acoustic admittance of primary nozzle
	API	—	Imaginary part of specific acoustic of primary nozzle

G. INPUT FORM

The format of the input data cards is shown in Figure 2 and a listing of the input data cards for the sample case included herein is given in Figure 3.

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.01	.625	65.0.	5500.	.0005	.6
.04	.274	1.2	6.0	3000.	5660.
200.	100.	59.	.001		
19					
0.11384		0.36194		1120.	
0.42671		1.9776		5000.	
0.97008		3.0084		7840.	
2.3030		4.0984		10100.	
4.9362		3.9735		12320.	
6.1355		1.1506		14560.	
3.8882		-0.44747		17920.	
3.0435		0.010772		20150.	
2.7955		0.64416		22400.	
2.9859		1.2256		24640.	
3.5515		1.5667		26900.	
4.2895		1.3871		29020.	
4.6474		0.65906		31400.	
4.5700		0.30943		32300.	
4.2807		-0.033799		33600.	
3.9100		-0.15899		35150.	
3.5940		-0.10287		36500.	
3.5702		0.068232		37900.	
3.6929		-0.13763		38600.	

Figure 3. Listing of Input for Sample Problem

H. SAMPLE OUTPUT

The following pages consist of the output of a typical case for a staged combustor. The input data for this case is that given the previous section.

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INPUT DATA

LENGTH OF SECONDARY CHAMBER IN FEET= .0100
 LENGTH OF PRIMARY CHAMBER IN FEET= .0250
 SPEED OF SOUND IN SECONDARY CHAMBER IN FEET PER SECOND= 6520.0000
 SPEED OF SOUND IN PRIMARY CHAMBER IN FEET PER SECOND= 5500.0000
 TAU PRIMARY= .0005
 INTERACTION INDEX PRIMARY= .6000
 MACH NUMBER PRIMARY= .0400
 MACH NUMBER SECONDARY= .2740
 GAMMA= 1.2000
 MIXTURE RATIO SECONDARY= 6.0000
 MEAN CHAMBER PRESSURE SECONDARY = 3000.0000
 MEAN CHAMBER PRESSURE PRIMARY= 5060.0000
 TOTAL TIME LAG OF SECONDARY FUEL= .0010

SECONDARY NOZZLE ADMITTANCE (OMEGA, AR, AI)

1120.0000	.1138	.3619
5000.0000	.4267	1.9776
7840.0000	.9707	3.0084
10100.0000	2.3036	4.0984
12320.0000	4.9362	3.9735
14560.0000	6.1353	1.1506
17920.0000	3.8882	-.4475
20150.0000	3.0435	.0108
22400.0000	2.7955	.6442
24640.0000	2.9859	1.2256
26900.0000	3.5515	1.5667
29020.0000	4.2895	1.3871
31400.0000	4.6474	.6597
32300.0000	4.5706	.3094
33600.0000	4.2807	-.0338
35150.0000	3.9166	-.1590
36500.0000	3.5948	-.1029
37900.0000	3.3702	-.0682
38600.0000	3.6929	-.1376

OUTPUT

FREQUENCY	N	TAU	APR	API
200.00000	.61595	2.48136	-.87603	-3.15922
300.00000	.48537	1.52847	-.72905	-4.82700
400.00000	.50185	.77736	-.53712	-6.59965
499.99999	1.06957	.34667	-.31220	-8.51178
599.99999	1.90287	.22842	-.06779	-10.59970
699.99998	2.26283	.21349	.18126	-12.90380
799.99998	2.23552	.22351	.41825	-15.47175
899.99998	2.05917	.23514	.62316	-18.36239
999.99997	1.80242	.24281	.77019	-21.65088
1099.99995	1.61241	.23425	.82390	-25.43552
1199.99995	1.54153	.20549	.73361	-29.84760
1299.99994	1.72290	.15976	.42476	-35.06677
1399.99994	2.32886	.11506	-.21465	-41.34697
1499.99992	3.22977	.08932	-1.35799	-49.06401
1599.99992	3.85329	.08289	-3.28803	-58.80960
1699.99991	4.41323	.09258	-6.50885	-71.59144
1799.99991	4.14735	.10339	-12.04921	-89.29786
1899.99991	3.83689	.11416	-22.44437	-115.90587
1999.99989	3.56174	.12230	-45.82713	-161.12933
2099.99988	3.35459	.12624	-121.87021	-252.18555
2199.99988	3.24237	.12540	-584.02924	-286.57150

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2299.99988	3.25262	.12065	-226.79218	387.01318
2399.99985	3.27948	.13994	-36.76532	209.29594
2499.99985	3.30218	.13662	-8.20351	132.80283
2599.99985	3.36707	.13541	-.88121	94.64927
2699.99982	3.42553	.13704	1.33372	72.09322
2799.99979	3.44515	.14110	1.88899	57.23720
2899.99976	3.41340	.14653	1.81951	46.71402
2999.99976	3.32791	.15214	1.50421	38.85959
3099.99973	2.65861	.16858	1.10022	32.76035
3199.99969	2.41349	.17043	.67847	27.87432
3299.99966	2.13358	.16929	.27351	23.86003
3399.99963	1.86012	.16381	-.09607	20.49134
3499.99960	1.65897	.15394	-.41863	17.61157
3599.99957	1.58462	.14252	-.68556	15.10699
3699.99954	1.62669	.13385	-.88988	12.89088
3799.99954	1.81538	.11898	-1.02619	10.89354
3899.99951	1.89397	.11885	-1.09149	9.05626
3999.99948	1.88617	.12024	-1.08607	7.32792
4099.99945	1.79064	.12099	-1.01427	5.66337
4199.99939	1.62669	.11874	-.88460	4.02308
4299.99939	1.46037	.11025	-.70916	2.37307
4399.99933	1.60701	.09273	-.50249	.68477
4499.99933	1.50722	.10389	-.28028	-1.06560
4599.99933	1.53693	.08613	-.05824	-2.89825
4699.99927	1.79609	.07843	.14847	-4.83124
4799.99927	2.00608	.07721	.32532	-6.88208
4899.99921	2.11058	.07907	.45801	-9.06914
4999.99921	2.11510	.08183	.53173	-11.41282
5099.99915	2.04268	.08393	.53055	-13.93654
5199.99915	2.08463	.08307	.43685	-16.66763
5299.99915	1.98179	.08083	.23146	-19.63777
5399.99908	1.95117	.07610	-.10564	-22.89042
5499.99908	2.03983	.07056	-.59362	-26.47582
5599.99902	2.22894	.06650	-1.24876	-30.47047
5699.99902	2.44108	.06507	-2.08274	-34.98930
5799.99896	2.60493	.06597	-3.10316	-40.21666
5899.99896	2.69594	.07208	-4.31850	-46.45850
5999.99896	2.70202	.07512	-5.75063	-54.23857