NASA TECHNICAL NOTE

NASA TN D-7035

C.1

NASA TN D-7035

TEMPERATURE DISTRIBUTION OF A THIN-WALLED, TRANSPARENT, SPHERICAL, EARTH SATELLITE

by Joseph T. Skladany

Goddard Space Flight Center
Greenbelt, Md. 20771

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • MARCH 1971

1. Report No. NASA TN D-7035	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle: Temperature Distribution of a Thin-Walled, Transparent, Spherical, Earth Satellite		5. Report Date: March 1971
		6. Performing Organization Code
7. Author(s): Joseph T. Skladany		8. Performing Organization Report No. G-1008
9. Performing Organization Name and Address: Goddard Space Flight Center, Greenbelt, Maryland 20771		10. Work Unit No.
		11. Contract or Grant No.
12. Sponsoring Agency Name and Address: National Aeronautics and Space Administration, Washington, D.C. 20546		13. Type of Report and Period Covered: Technical Note
		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

This paper is concerned with deriving an expression for the temperature distribution of a transparent, spherical, earth satellite. Lateral and radial conduction are assumed negligible; the satellite is assumed to be in a steady-state condition at a fixed point in orbit.

The thermal-radiation heat-balance problem is solved in a closed form by Poljak's radiosity method–somewhat modified to include the transmittance of the satellite skin and to include solar and earth radiation as external inputs.

A general solution is derived for skin temperatures; this consists of three simultaneous integral equations at each node and holds for any shape enclosure. The spherical shape is then introduced; expressions for mean skin temperature and the temperature at any point on the sphere are obtained.

Three numerical examples are included, two opaque and one transparent, to illustrate the use of the derived expressions and point out the differences in temperature distribution between opaque and transparent spheres. Temperature distribution curves for each sphere are plotted at sub-solar position in orbit and in the earth's shadow. Although the temperature gradient is not reduced in the transparent sphere, a more symmetrical distribution is obtained for transparent than for opaque spheres.

17. Key Words Suggested by Author	18. Distribution Statement
Satellite Thermal Design Satellite Temperature Distribution Transparent Surface Temperature Distribution	Unclassified–Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	31	$3.00

*For sale by the National Technical Information Service, Springfield, Virginia 22151

CONTENTS

	Page
INTRODUCTION	1
THE OPAQUE SPHERE–MEAN TEMPERATURE	2
General Discussion	2
Earth Inputs	3
Albedo	6
THE OPAQUE SPHERE–TEMPERATURE DISTRIBUTION	6
General Discussion	6
Earth Inputs to Differential Element on Sphere	9
Approximation for Earth Inputs	10
THE TRANSPARENT SPHERE	11
General Discussion	11
Net Internal Energy Leaving an Element of an Enclosure	11
Mean Temperature	13
Temperature Distribution	14
EXAMPLES	16
CONCLUDING REMARKS	22
References	23
Appendix A–Angle Between Earth-Satellite Line and Earth-Sun Line	25
Appendix B–Satellite Coordinate System and Relations Between Surface-Normal and Solar Vector and Between Unit Normal and Earth-Satellite Line	27
Appendix C–Nomenclature	29

TEMPERATURE DISTRIBUTION OF A THIN-WALLED, TRANSPARENT, SPHERICAL, EARTH SATELLITE

by
Joseph T. Skladany
Goddard Space Flight Center

INTRODUCTION

Investigations of the temperature distribution of spherical satellites began almost twenty years ago. As early as 1955, Sandorff and Prigge (Reference 1) wrote a paper that included conduction in the skin but excluded radiation within the hollow sphere. The sphere was considered as heated by solar input only, earth input being neglected. The skin was divided into a number of "stations" or "nodes," and the temperature distribution was obtained by solution of a number of simultaneous equations. A paper on the general thermal design of satellites by Schach and Kidwell (Reference 2) made use of a similar subdivision method; this nodal technique is still in use today. Hrycak (Reference 3) considered both skin conduction and internal radiation effects and concluded that, for thin-walled satellites, internal radiation plays the dominant role.

Wood and Carter (Reference 4) gave equations for the mean temperature and hot- and cold-spot temperatures on a thin-walled spherical shell at the sub-solar position and at a position in the earth's shade. Conduction was neglected, and an approximate method was used to account for albedo inputs. In the computation of the local body temperatures, distribution of the earth inputs over the skin of the satellite was ignored. It is generally acknowledged that, for a sphere at 1000-km altitude, earth-emitted and albedo energy incident on part of the sphere can be as much as three times the average over the entire sphere; this may drastically affect local temperatures.

A proposal (Reference 5) to NASA suggested the use of a partially transparent sphere of polypropylene with an etched pattern of aluminum on both sides to increase reflectivity. The temperature at a few special points on the sphere and the mean temperature were obtained from tracings of the infinite number of reflections at the inner surface. It was this proposal that prompted the present investigation.

This paper is concerned with temperature distribution over the surface of a thin-walled, transparent sphere in an earth orbit. The only mode of heat transfer considered is radiation, since lateral and radial conduction in the skin can be shown to be negligible (Reference 3). It is assumed that the thermal mass is small and that transient effects are therefore negligible. The sphere is assumed to be a gray body; only specular transmittance is considered. This paper is not concerned with thermal-property measurements; it is assumed that the materials have the required properties.

Before making a thermal analysis of a transparent sphere, we examine the case of an opaque sphere in order to introduce the theory and display the necessary thermal inputs. Once the expression for the temperature distribution of an opaque sphere is derived, attention is focused on a transparent enclosure. The heat transfer equation for radiation, including an infinite number of internal reflections, is solved by Poljak's radiosity method, described by Eckert and Drake (Reference 6). This yields three simultaneous integral equations which, by means of the thermal model technique (Reference 2), can be solved for any shape enclosure. Spherical geometry is then introduced, and a solution for the mean radiation temperature of a sphere is obtained. Finally, an expression for the temperature at any point on the sphere is derived. Both expressions are found to be general in that they yield mean temperature and temperature distribution for any sphere, whether opaque or transparent, or for a sphere having opaque portions. The solution of this last case can be derived from the effective thermal properties (Reference 5).

THE OPAQUE SPHERE—MEAN TEMPERATURE

General Discussion

The mean temperature of a satellite in equilibrium with its environment is found when the thermal energy absorbed by the satellite is equated to the thermal energy radiated by the surface of the satellite.

The external thermal inputs of an earth satellite consist of radiant energy from the sun, solar radiation reflected by the earth (albedo), and radiation emitted by the earth (Figure 1). Since the solar radiation and albedo extend over the entire solar spectrum, the fraction of this energy absorbed by the surface is given by its absorptivity. Also, since the earth-emitted energy lies almost wholly in the infrared portion of the spectrum (as from a 250 K blackbody, Reference 7), the fraction of this energy absorbed by the surface is given by its emissivity.

The rate of solar energy impinging on an element normal to the sun's rays is the well-known solar constant (0.0333 cal/sec-cm^2) and at any other orientation is simply a function of the cosine between the solar vector and the normal to the surface (Reference 2). If this input is integrated over the surface of a sphere, the total solar energy impinging on the sphere equals the solar constant multiplied by the projected area of the sphere. The albedo- and earth-radiation inputs to a sphere can likewise be found as functions of the projected area of the sphere. The energy radiated by the satellite, however, is a function of its entire surface area. The energy balance on the sphere can be written in integrated form as in Reference 2,

$$a_0 S_0 A_p + a_0 P_0 A_p + \epsilon_0 G_0 A_p = A\sigma\epsilon_0 T_m^4 , \tag{1}$$

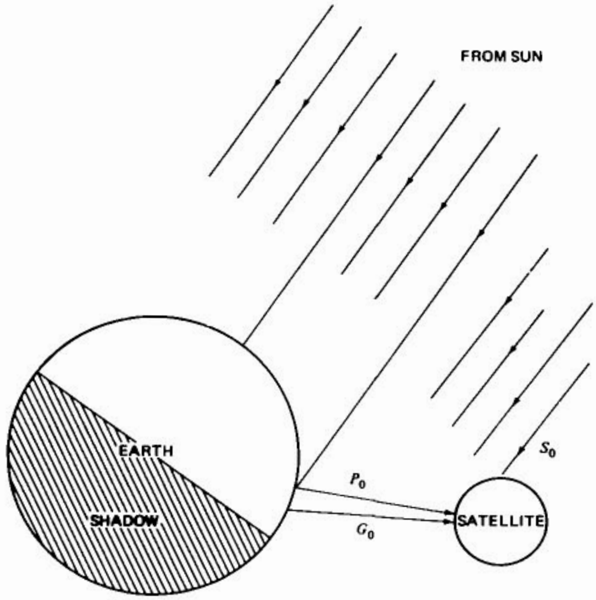

Figure 1—Energy incident on an earth satellite.

where

$$\epsilon_0 T_m^4 = \frac{\int_A \epsilon_{0i} T_i^4 \, di}{A} .$$

(For explanation of symbols, see Appendix C—Nomenclature.) This equation defines the satellite mean temperature for radiation exchanges, or, solving Equation 1 for T_m, we obtain

$$T_m = \left\{ \frac{1}{4\sigma} \left[\left(\frac{a_0}{\epsilon_0} \right) \left(S_0 + P_0 \right) \right] + \frac{G_0}{4\sigma} \right\}^{1/4} . \tag{2}$$

If the satellite is far removed from the earth, this reduces to

$$T_m = \left(\frac{a_0 S_0}{4\sigma\epsilon_0} \right)^{1/4} . \tag{3}$$

Earth Inputs

In the previous section, earth radiation and albedo were introduced, but no mention was made as to how they are determined. In this section, earth radiation will be discussed. Albedo will be discussed in the following section.

To at least a first approximation, the earth can be regarded as a blackbody radiating diffusely at 250 K (Reference 7). The earth-radiated energy directly incident on the satellite's surface can therefore be derived from the equation

$$G_0 = \sigma T_e^4 F_{1-2} , \tag{4}$$

where 1 and 2 refer to the satellite and the earth, respectively, as shown in Figure 2 and F_{1-2} is the shape factor from area 1 to area 2. To compute F_{1-2} accurately it would be necessary to set up an expression for the shape factor from a point on the satellite's surface, integrate over that portion of the earth viewed from the point, and finally integrate over the satellite's surface. However, if two surfaces obey Lambert's law, the shape factor from a point on one surface to the whole of the other is merely the solid angle subtended by the second surface at the point on the first surface. Since the earth is much larger than the satellite and the distance between them is great in comparison to the satellite's diameter, the solid angle subtended by the earth at any point on the satellite disk is very nearly a constant. Hence, it is sufficient to consider the satellite a disk and compute one shape factor for a point on the satellite disk (as shown in Figure 2) and multiply this factor by the projected area of the sphere to obtain the fraction of the earth-radiated energy that impinges on the sphere.

By definition under the above assumption,

$$F_{di-A_D} = F_{1-2} = \frac{1}{\pi A_p} \int_{A_D} \int_{A_p} \frac{\cos\phi \cos\phi \, di \, dk}{l^2} , \tag{5}$$

where

l = distance between the satellite center and an element on the earth's disk,

ϕ = angle between l and the normal to an element on the earth's disk (or satellite disk),

A_D = area of earth's disk visible from the satellite,

and

A_p = projected area of satellite.

However, $di \cos\phi / l^2$ is the solid angle $d\omega$ subtended by the satellite disk at a point on the earth's disk. When the satellite's diameter is small compared to the distance between the earth and satellite, $d\omega$ is very nearly a constant for any point on the earth's disk and is equal to A_p / l^2. With this substitution, Equation 5 becomes

$$F_{1-2} = \frac{1}{\pi} \int_{A_D} \frac{\cos\phi \, dk}{l^2} . \tag{6}$$

By reference to Figure 2, we obtain

$$l = \frac{r \cos^2\theta}{\cos\phi}$$

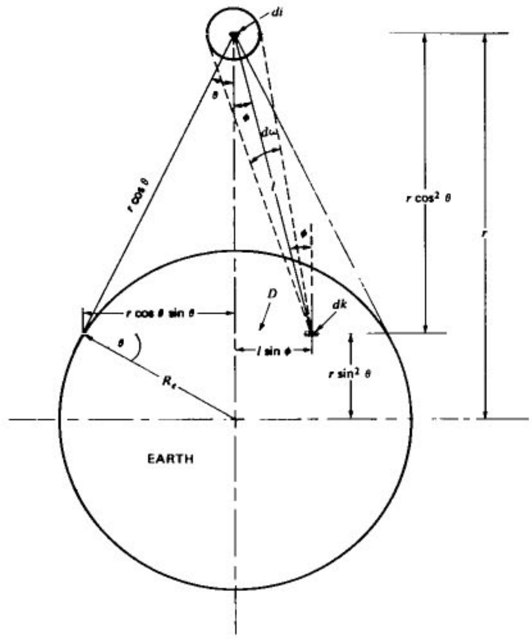

Figure 2—Earth radiation to a sphere.

and

$$dk = 2\pi l \sin \phi [d(l \sin \phi)] = 2\pi r^2 \cos^4 \phi \tan \phi \sec^2 \phi d\phi \,,$$

where θ is the angle between the line from the satellite's center to the earth's center and the tangent from the satellite's center to the earth's surface. Substituting these expressions into Equation 6 and simplifying, we obtain

$$F_{A_p\text{-}A_D} = F_{1\text{-}2} = \int_{\phi=0}^{\phi=\theta} 2 \sin \phi d\phi = 2(1 - \cos \theta) \,, \tag{7}$$

but

$$\cos \theta = \left(\frac{r^2 - R_e^2}{r^2} \right)^{1/2} \quad . \tag{8}$$

When we define $h = r/R_e$, i.e., when h is in the dimensionless form of earth radii, Equation 8 becomes

$$F_{1\text{-}2} = 2 \left[1 - \frac{(h^2 - 1)^{1/2}}{h} \right] . \tag{9}$$

Finally,

$$G_0 = 2\sigma T_e^4 \left[1 - \frac{(h^2 - 1)^{1/2}}{h} \right] . \tag{10}$$

(A plot of this expression is shown in Figure 8 in a later portion of this report.)

Albedo

The earth-reflected solar radiation incident on a spherical satellite can be obtained in much the same manner as the earth radiation. The reflectance of the earth varies over a wide range, with the mean ranging from 0.34 to 0.5 (Reference 8), depending on the portion of sky covered by clouds.

The problem is set up as in Figure 2, but now the flux from the earth is the reflected solar energy instead of the earth's radiated energy, and the integration must be limited to the sunlit portion of the earth; this complicates the limits of integration. This problem has been investigated in some detail by Cunningham (Reference 9); we shall make use of his results. To do this we must know the altitude of the satellite and the angle between the earth-sun line and earth-satellite line. The altitude is known from orbital predictions; the angle can be computed from latitude and longitude observations of the sun and satellite, as is explained in Appendix A.

THE OPAQUE SPHERE—TEMPERATURE DISTRIBUTION

General Discussion

The temperature distribution of an opaque spherical satellite can be found by means of an energy balance set up at a differential element on the sphere's surface. Again, the inputs to the external surface of the sphere consist of direct solar energy, albedo, and earth radiation. The net energy leaving the internal surface of the element must also be considered, since this is not necessarily the same at all internal surface elements. A schematic of the thermal energy balance is shown in Figure 3. The energy balance equation at any element *di* is

$$a_{0i} S_{0i} \, di + a_{0i} P_{0i} \, di + \epsilon_{0i} G_{0i} \, di = \sigma \epsilon_{0i} T_i^4 + dQ_i \, , \tag{11}$$

where dQ_i is the net, inward-directed energy leaving the internally facing surface of *di*. Using Poljak's radiosity method (Reference 6), we obtain

$$dQ_i = (B_i - H_i) di \, , \tag{12}$$

$$\begin{aligned} B_i &= e_i + \rho_i^* H_i \, , \\ B_k &= e_k + \rho_k^* H_k \, , \end{aligned} \tag{13}$$

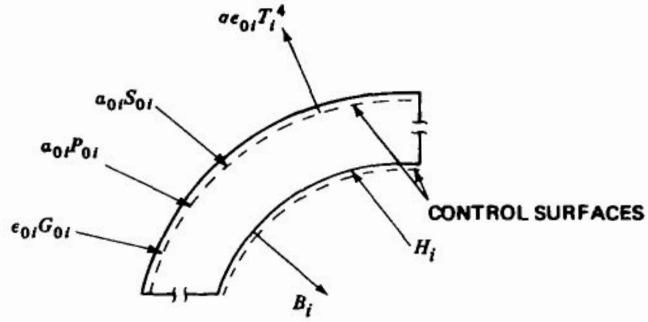

Figure 3—Thermal balance on an opaque element of an earth satellite.

and

$$diH_i = \int_k F_{dk-di} B_k \, dk \ , \tag{14}$$

and, substituting 13 in 14,

$$diH_i = \int_k F_{dk-di}(e_k + \rho_k^* H_k) \, dk \ . \tag{15}$$

(Differential element *di* is not unique; an equation written for *di* is true for all other elements.)

Since at any instant the sphere is at equilibrium with its environment, the total energy incident on the inside of the sphere is equal to the total energy leaving the internal surface of the sphere; that is, in equation form,

$$\int_i dQ_i = 0 = \int_k B_k \, dk - \int_k H_k \, dk \ . \tag{16}$$

After B_k is substituted from Equation 13, Equation 16 becomes

$$\int_k H_k \, dk = \int_k (e_k \rho_k^* H_k) \, dk \ , \tag{17}$$

or, rearranged,

$$\int_k e_k \, dk = \int_k (1 - \rho_k^*) H_k \, dk \ . \tag{18}$$

Next we define $F_{dk\text{-}di}$, the shape factor from element di to element dk.

The geometry for this problem is shown in Figure 4. By reciprocity, $F_{dk\text{-}di}dk = F_{di\text{-}dk}di$. By definition,

$$F_{di\text{-}dk} = \frac{\cos\phi_i \cos\phi_k \, dk}{\pi l^2}, \tag{19}$$

where

ϕ_i = the angle between the normal to area i and the line joining area i to area k,

ϕ_k = the angle between the normal to area k and the line joining area k to area i,

and, from geometry,

$$\cos\phi_i = \cos\phi_k = \frac{1}{2}\frac{l}{r'},$$

where r' is the radius of the satellite; therefore,

$$F_{di\text{-}dk} = \frac{l^2 \, dk}{4\pi^2 r'^2 l^2} = \frac{dk}{A}. \tag{20}$$

Substituting this expression in Equation 15, we obtain

$$H_i = \frac{1}{A}\int_k (e_k + \rho_k^* H_k) dk. \tag{21}$$

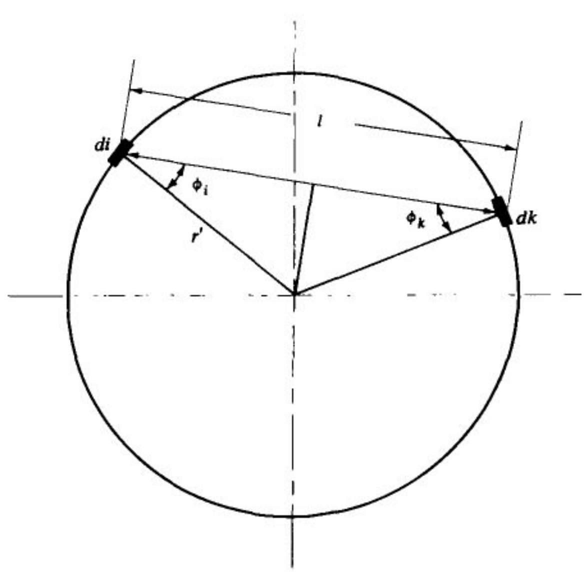

Figure 4—Geometry for shape factors in a sphere.

If $\rho_k^* =$ constant ρ^*, Equation 18 can be combined with Equation 21 to give

$$H_i = \frac{1}{A} \int_k \frac{e_k \, dk}{1 - \rho^*} . \tag{22}$$

Substituting for H_i in Equations 12 and 13 yields

$$dQ_i = e_i di - di \int_k \frac{e_k \, dk}{A} , \tag{23}$$

but

$$\int_k \frac{e_k \, dk}{A} = \epsilon \sigma T_m^4 ,$$

and finally

$$dQ_i = \sigma \epsilon \left(T_i^4 - T_m^4 \right) di \; ; \tag{24}$$

Equation 11 becomes

$$a_0 S_{0i} + a_0 P_{0i} + \epsilon_0 G_{0i} = \sigma(\epsilon_{0i} + \epsilon_i) T_i^4 - \sigma \epsilon T_m^4 . \tag{25}$$

Therefore, to find the temperature at any point on the surface of the sphere we need only define the external inputs at that point and solve Equation 2 for T_m. The next section deals with defining earth inputs.

Earth Inputs to Differential Element on Sphere

To define the earth inputs to a differential element on the satellite, it is necessary to find a shape factor from the element to the earth's surface (sunlit portion of the earth for the albedo) (refer to Figure 5). Again, by definition,

$$F_{di\text{-}dk} = \frac{1}{\pi} \int_k \frac{\cos a_1 \, \cos a_2 \, dk}{D^2} , \tag{26}$$

where

a_1 = the angle between the normal $\mathbf{N}_1$ to the element on the satellite and the line joining the element to an element on the earth's surface,

a_2 = the angle between the normal $\mathbf{N}_2$ to an element on the earth and the line joining this element to the element on the satellite's surface,

and

D = the distance between the element on the satellite and the element on the earth.

The integration takes place over that part of the surface of the earth which is visible from *di* (and also illuminated by the sun in the case of albedo).

From vector analysis we obtain

$$\cos a_1 = \frac{\mathbf{N}_1 \cdot \mathbf{D}}{|\mathbf{N}_1||\mathbf{D}|} \quad \text{and} \quad \cos a_2 = \frac{\mathbf{N}_2 \cdot \mathbf{D}}{|\mathbf{N}_2||\mathbf{D}|}. \tag{27}$$

These expressions can be written in terms of θ, ϕ, and R_0. These, when substituted in Equation 26, yield an unwieldy integral that requires numerical integration. Rather than attempt this, we will make an approximation that allows use of existing data.

Approximation for Earth Inputs

The approximation for earth inputs consists of imagining that each element of the earth is located at the sphere's center, as shown in Figure 6. Because of the relative sizes of the earth and satellite, locating an element in this manner has very little effect on D, a_1, or a_2 as long as the element's orientation remains unchanged. It is now permissible to use existing literature on earth-radiation and albedo inputs to a small flat plate (References 7 and 10).

To do this we must find λ, the angle between the surface normal and the earth-satellite line, and (for the albedo) θ_s, the angle between the earth-satellite line and the earth-sun line. (Expressions for these angles can be found in Appendices B and A, respectively.)

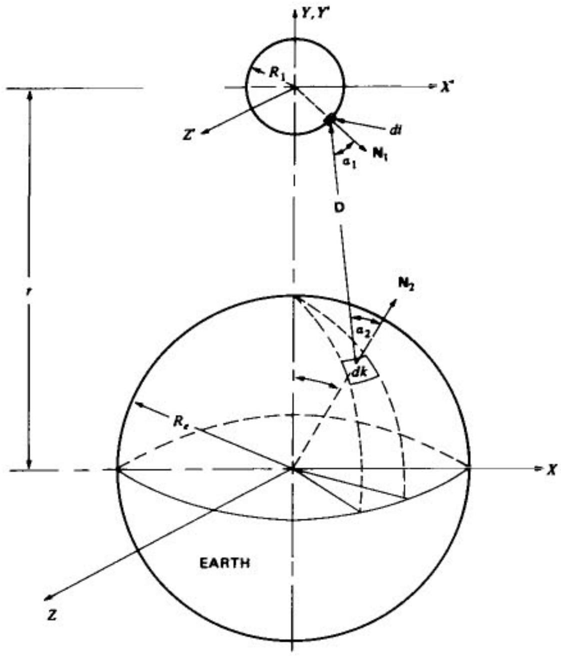

Figure 5–Earth inputs to an element on a sphere.

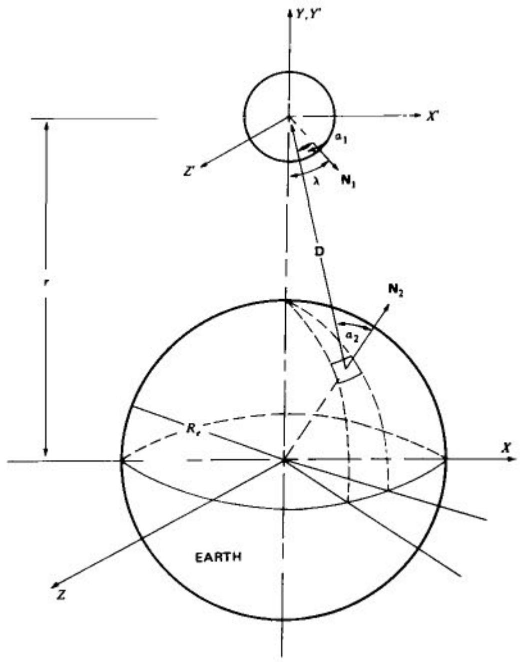

Figure 6–Approximation for earth inputs to element on a sphere.

THE TRANSPARENT SPHERE

General Discussion

In finding the temperature distribution of a transparent sphere, we deal with external inputs in the same manner as with an opaque sphere. However, with a transparent sphere, part of the energy passes through the surface and impinges on the inner surface. The problem of accounting for this additional energy will be discussed in this part of the report.

Besides the assumptions made previously, we will also assume that energy is neither scattered nor diffracted as it passes through the surface of the sphere. This means that the path of a ray of energy is not changed as it passes through the skin of the satellite. We find the external energy that impinges on the interior skin of the satellite by multiplying the external energy that impinges on the exterior skin by the appropriate transmissivity.

Poljak's radiosity method (Reference 6) is again extensively used in this section and enlarged to include solar energy and albedo as well as infrared energy.

The thermal balance is treated generally at first, so that the method may be used for any shape enclosure. Later, the spherical shape is introduced. This allows some simplifications to be made.

Net Internal Energy Leaving an Element of an Enclosure

Figure 7 shows the thermal balance on a transparent element of a satellite. Note that this figure includes the following energy terms: I, the solar energy incident on the interior skin; L, the solar energy leaving the interior skin; τI, the solar energy transmitted through the element; and $\tau^* H$, the infrared energy transmitted through the element. By redefining dQ_i so that it includes not only net energy leaving the interior skin but also transmitted energy, we can see that Equation 11 still holds for a transparent surface. We can now write

$$\left.\begin{array}{r}\text{Total energy incident}\\ \text{on an interior element}\end{array}\right\} = di(H_i + I_i) \,. \tag{28}$$

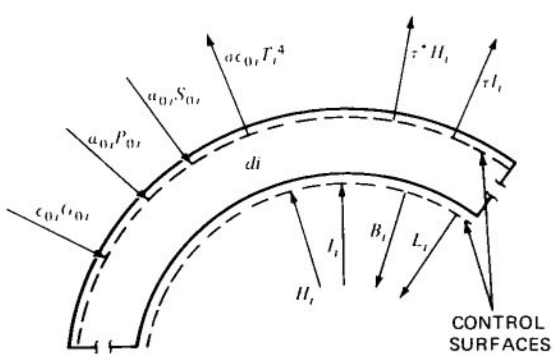

Figure 7–Thermal balance on a transparent element of an earth satellite.

Since some of this energy passes through the surface, we can write

$$\left.\begin{array}{l}\text{Total energy leaving}\\ \text{the element (includ-}\\ \text{ing transmitted)}\end{array}\right\} = di(B_i + L_i + \tau_i I_i + \tau_i^* H_i)\ , \tag{29}$$

where the last two terms take into account that incident energy which passes through the surface.

Subtracting Equation 29 from Equation 28 gives

$$dQ_i = di[B_i + L_i - (1 - \tau_i)I_i - (1 - \tau_i^*)H_i]\ . \tag{30}$$

From the definitions of the terms in Equation 30, we obtain

$$B_i = e_i + \rho_i^* H_i\ , \tag{31}$$

$$L_i = \rho_i I_i\ , \tag{32}$$

$$di I_i = di(S_i + P_i) + \int_k dk F_{dk\text{-}di} L_k\ , \tag{33}$$

and

$$di H_i = di G_i + \int_k dk F_{dk\text{-}di} B_k\ . \tag{34}$$

Substituting Equations 31 and 32 in Equation 30 gives

$$dQ_i = di\Big\{[e_i + (\rho_i^* + \tau_i^* - 1)H_i] + (\rho_i + \tau_i - 1)I_i\Big\} \tag{35}$$

and, using reciprocity, $dkF_{dk\text{-}di} = diF_{di\text{-}dk}$. Substituting Equations 31 and 32 in Equations 34 and 33, respectively, gives

$$I_i = (S_i + P_i) + \int_k F_{di\text{-}dk} \rho_k I_k \tag{36}$$

and

$$H_i = G_i + \int_k (e_k + \rho_k^* H_k) F_{di\text{-}dk}\ . \tag{37}$$

Equations 35, 36, and 37 give three simultaneous equations at a differential element. These equations are completely general and therefore hold for an enclosure of any shape. Since the integrals are difficult to solve analytically, we can most easily solve these expressions by subdividing the surface into a number of finite nodes, changing the integral signs to summations, and solving the resulting set of equations simultaneously. This network method is similar to the method discussed by Schach and

Kidwell (Reference 2). It is usually true that accuracy increases with the number of nodes chosen; obviously solution very soon becomes unwieldly and requires a computer.

When the surface in question is a sphere, some simplifications can be made, resulting in a closed-form solution without the need to subdivide the surface into a number of finite nodes. This will be investigated in the remainder of the report.

Mean Temperature

To find the mean temperature of a transparent sphere, integrate Equation 11 over the sphere's surface. Then the left-hand side becomes the same as in Equation 1; the right-hand side must now include the net energy leaving the interior of the sphere. When dQ_i is integrated over the surface we obtain

$$Q = A[e + (\rho^* + \tau^* - 1)H + (\rho + \tau - 1)I] \ , \tag{38}$$

where the properties are constant over the surface.

From the definitions of H and I, from the assumption that the transmitted energy passes through the surface with no diffraction, and from the fact that the shape factor from the interior of the sphere to itself equals unity (since the sphere is a closed boundary), Equations 36 and 37 become

$$AI = \tau A_p(S_0 + P_0) + \rho AI = \frac{\tau A_p(S_0 + P_0)}{1 - \rho} \tag{39}$$

and

$$AH = \tau^* A_p G_0 + AB \ . \tag{40}$$

If we integrate Equation 30 over the surface of the sphere with ρ_i^* constant, B_i becomes B; we can substitute for B to obtain

$$AH = \frac{\tau^* A_p G_0 + Ae}{1 - \rho^*} \ . \tag{41}$$

Hence the energy balance becomes

$$a_0 A_p(S_0 + P_0) + \epsilon_0 A_p G_0 = A\sigma\epsilon_0 T_m^4 + Ae + \frac{(\rho^* + \tau^* - 1)(\tau^* A_p G_0 + Ae)}{1 - \rho^*}$$

$$+ \frac{(\rho + \tau - 1)\tau A_p(S_0 + P_0)}{1 - \rho} \ . \tag{42}$$

Noting that

$$\rho^* + \tau^* - 1 = -a^* = -\epsilon$$

and

$$\rho + \tau - 1 = -a$$

and rearranging terms, we rewrite Equation 42 as

$$A_p(S_0 + P_0)\left(a_0 + \frac{a\tau}{1 - \rho}\right) + A_p G_0\left(\epsilon_0 + \frac{\epsilon\tau^*}{1 - \rho^*}\right) = A\sigma T_m^4\left(\epsilon_0 + \frac{\epsilon\tau^*}{1 - \rho^*}\right), \tag{43}$$

which reduces to Equation 2 for an opaque sphere. If the inner surface has the same properties as the outer surface, i.e., $a = a_0$ and $\epsilon = \epsilon_0$, then

$$aA_p(S_0 + P_0)\left(1 + \frac{\tau}{1 - \rho}\right) + \epsilon A_p G_0\left(1 + \frac{\tau^*}{1 - \rho^*}\right) = \epsilon A\sigma T_m^4\left(1 + \frac{\tau^*}{1 - \rho^*}\right). \tag{43a}$$

Temperature Distribution

As on the opaque sphere, the temperature at any point on the transparent sphere can be obtained from Equation 11. Note that dQ_i must be derived from Equations 35, 36, and 37.

With constant surface properties, and making use of the expression for the shape factor between two elements on a sphere (Equation 20), we can rewrite Equations 36 and 37 as

$$I_i = (S_i + P_i) + \rho I \tag{44}$$

and

$$H_i = G_i + e + \rho^* H\,. \tag{45}$$

If substitutions from Equations 39 and 41 are made for I and H, Equations 44 and 45 become

$$I_i = S_i + P_i + \rho\left[\frac{A_p\tau(S_0 + P_0)}{A(1 - \rho)}\right] \tag{46}$$

and

$$H_i = G_i + e + \rho^*\left[\frac{\tau^*(A_p/A)G_0 + e}{1 - \rho^*}\right]. \tag{47}$$

Therefore Equation 35 can be rewritten as

$$\frac{dQ_i}{di} = e - \epsilon\left[e + G_i + \rho^*\left(\frac{\tau^*(A_p/A)G_0 + e}{1 - \rho^*}\right)\right] - a\ (S_i + P_i) + \frac{\rho\tau A_p(S_0 + P_0)}{A(1 - \rho)} \tag{48}$$

and, simplified,

$$\frac{dQ_i}{di} = e_i - a(S_i + P_i) - \epsilon G_i - a\tau\left(\frac{A_p}{A}\right)\left(\frac{\rho}{1 - \rho}\right)(S_0 + P_0)$$

$$- \epsilon\tau^*\left(\frac{A_p}{A}\right)\left(\frac{\rho^*}{1 - \rho^*}\right)G_0 - \epsilon\tau\left(1 + \frac{\rho^*}{1 - \rho^*}\right)e\,. \tag{49}$$

By the use of Equation 43, one of the input terms in Equation 49 can be eliminated. Since G_0 is usually the least important term, we will choose to retain this term. This allows the user to simplify the result, when G_0 becomes negligible (at altitudes above approximately 1000 km), by neglecting this term (see Figure 8).

Solving Equation 43 for $A_p(S_0 + P_0)$ and substituting in Equation 49 yields

$$\frac{dQ_i}{di} = \epsilon_i - a(S_i + P_i) - \epsilon(G_i) - \frac{A_p}{A}(G_0)\left[\frac{\epsilon\rho^*\tau^*}{1-\rho^*} - \left(\frac{a\rho\tau}{1-\rho}\right)\frac{\left(\epsilon_0 + \frac{\epsilon\tau^*}{1-\rho^*}\right)}{\left(a_0 + \frac{a\tau}{1-\rho}\right)}\right]$$

$$- \sigma T_m^4 \left(\frac{a\rho\tau}{1-\rho}\right)\left(\frac{\epsilon_0 + \frac{\epsilon\tau^*}{1-\rho^*}}{a_0 + \frac{a\tau}{1-\rho}}\right) + \frac{\epsilon^2}{1-\rho^*} \quad . \tag{50}$$

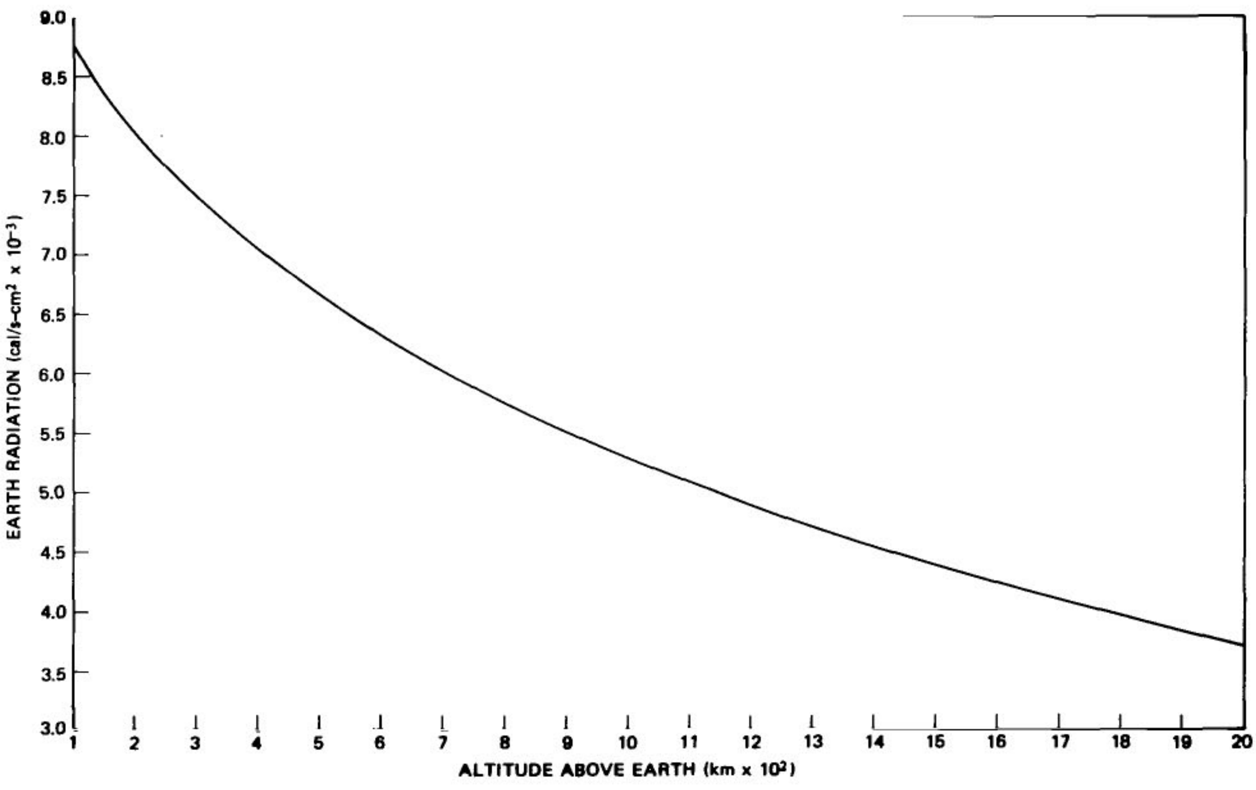

Figure 8—Earth radiation incident per square centimeter projected area of a sphere vs altitude.

Finally, substituting Equation 50 in Equation 11 and solving for T_i^4 yields

$$T_i^4 = \frac{1}{\sigma(\epsilon + \epsilon_0)} \left\{ a_0(S_{0i} + P_{0i}) + a(S_i + P_i) + \epsilon_0 G_{0i} + \epsilon G_i \right.$$

$$+ \left(\frac{A_p}{A}\right) G_0 \left[\frac{\epsilon \rho^* \tau^*}{1 - \rho^*} - \frac{a\rho\tau\left(\epsilon_0 + \frac{\epsilon\tau^*}{1 - \rho^*}\right)}{(1 - \rho)\left(a_0 + \frac{a\tau}{1 - \rho}\right)} \right] \Bigg\}$$

$$+ \frac{T_m^4}{\epsilon + \epsilon_0} \left[\frac{a\rho\tau\left(\epsilon_0 + \frac{\epsilon\tau^*}{1 - \rho^*}\right)}{(1 - \rho)\left(a_0 + \frac{a\tau}{1 - \rho}\right)} + \frac{\epsilon^2}{1 - \rho^*} \right] . \tag{51}$$

EXAMPLES

To illustrate the expressions derived, a fictitious orbit will be used. The temperature distribution in the skin of a satellite will be found for the case of a satellite at the subsolar position in orbit (i.e., where the sun, satellite, and earth are in line) and also 180 degrees from this orbital position, where the only input is earth radiation. These positions were chosen because they are both points of symmetry (i.e., distribution of temperature is only in the polar direction) and because the first case gives the hottest condition and the second gives the coldest, both cases being important in thermal design.

For our fictitious orbit we will assume that the satellite is launched into a circular, polar orbit along the prime meridian at 12 noon GMT on the first day of spring. This positions the sun on the equator and in the plane of the orbit. From Appendix A, for these conditions, the sun's position is given by

$$\mathbf{S} = \mathbf{i}$$

and, when at the subsolar point, the satellite's position is given by

$$\mathbf{L} = \mathbf{i} \; ;$$

therefore

$$\theta_s = 0 \, .$$

From Appendix B,

$$\mathbf{n} = \mathbf{i} \sin\theta \cos\phi + \mathbf{j} \sin\theta \sin\phi + \mathbf{k} \cos\theta \; ;$$

therefore

$$\cos\beta = \sin\theta \cos\phi$$

and

$$\cos\lambda = -\sin\theta \cos\phi \ .$$

However, since we have located the satellite at the subsolar point we can set $\theta = 90$ degrees and vary ϕ from 0 to 180 degrees to cover all points on the sphere, because of symmetry. Then

$$\cos\beta = \cos\phi$$

and

$$\cos\lambda = -\cos\phi \ ,$$

or

$$\beta = \phi$$

and

$$\lambda = 180^\circ - \phi \ .$$

Knowing θ_s and λ, we can now refer to the literature (References 7 and 10) and find the albedo and earth radiation to every point on the exterior of the satellite. Also, since the unit internal normal is $-\mathbf{n}$ and because of our assumption of no diffraction, the interior incident albedo and earth radiation can be found in the same manner if we let $\lambda = \phi$ and multiply the input by the appropriate transmissivity.

Also note that the solar input can be obtained from $Sdi \cos\beta$ for $0^\circ \leqslant \beta \leqslant 90^\circ$ and that the solar input equals zero for $90^\circ < \beta \leqslant 180^\circ$. For the satellite at the subsolar point, this becomes $Sdi \cos\phi$ for $0^\circ \leqslant \phi \leqslant 90^\circ$ and zero for $90^\circ < \phi \leqslant 180^\circ$. Likewise, the solar input to the interior of the sphere is $-\tau Sdi \cos(180^\circ - \phi)$ for $90^\circ < \phi \leqslant 180^\circ$ and zero for $0 \leqslant \phi \leqslant 90^\circ$.

The albedo and earth radiation incident on the exterior of the sphere are plotted in Figure 9 as a function of ϕ. We can use the same figure for internal inputs, substituting ϕ for $(180^\circ - \phi)$ and multiplying albedo values by τ and earth-radiation values by τ^*.

The albedo on the entire sphere at an altitude of 1000 km and $\theta_s = 0$ from Cunningham (Reference 9) is 44.8 W/cm^2 $\times 10^{-3}$ or 10.7×10^{-3} cal/sec-cm^2, and the earth radiation from Figure 8 is 5.28×10^{-3} cal/sec-cm^2.

At 180 degrees from the subsolar point the only input is earth radiation, which is assumed constant over the orbit (5.28×10^{-3} cal/sec-cm^2).

To fully illustrate the equations derived, three spherical satellites will be used, two opaque and one transparent. The surface properties used are summarized in Table 1.

The temperature distributions are shown plotted as T_i/T_m versus ϕ in Figures 10 and 11.

A sample computation for the polypropylene sphere is shown at both positions in orbit. The mean temperature and temperature distribution are computed, and the inputs and results for $0 \leqslant \phi \leqslant 180$ are tabulated in Table 2 for the subsolar point and in Table 3 for shade position.

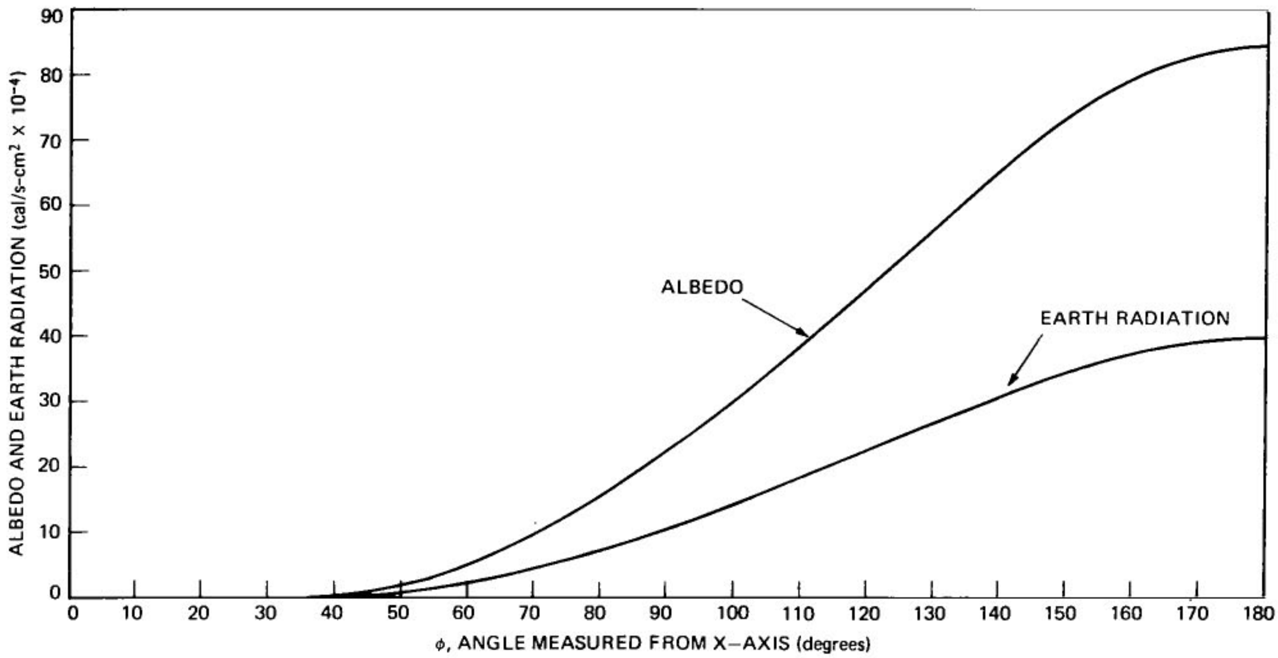

Figure 9—Albedo and earth-radiation distribution incident on a spherical satellite at the subsolar point and 1000-km altitude.

At the subpolar position,

$$T_m^4 = \frac{A_p(S_0 + P_0)\left(a_0 + \dfrac{a\tau}{1-\rho}\right)}{A\sigma\left(\epsilon_0 + \dfrac{\epsilon\tau^*}{1-\rho^*}\right)} + \frac{A_p G_0}{A\sigma}$$

$$= \frac{1(33.3 + 10.7)(10^{-3})\left[0.042 + \dfrac{(0.042)(0.908)}{(1-0.05)}\right]}{4(1.36)(10^{-12})\left[0.103 + \dfrac{(0.103)(0.861)}{(1-0.036)}\right]} + \frac{1(5.28)(10^{-3})}{4(1.36)(10^{-12})}$$

$$= 42.0 \times 10^8 \tag{43}$$

and

$$T_m = 255\ \mathrm{K}\ .$$

Table 1—Thermal properties.

Sphere	a_0	a	ϵ_0	ϵ	ρ	ρ^*	τ	τ^*	Source of measurement
Black	1.0	1.0	1.0	1.0	0	0	0	0	—
Alodine	0.324	—	0.185	0.70	—	—	0	0	GSFC
Polypropylene	0.042	0.042	0.103	0.103	0.05	0.036	0.908	0.861	Reference 5

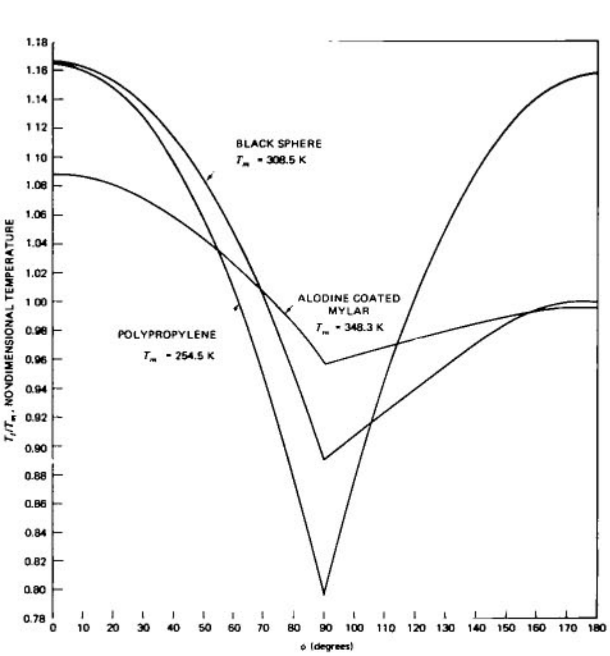

Figure 10—Temperature distribution at subsolar position.

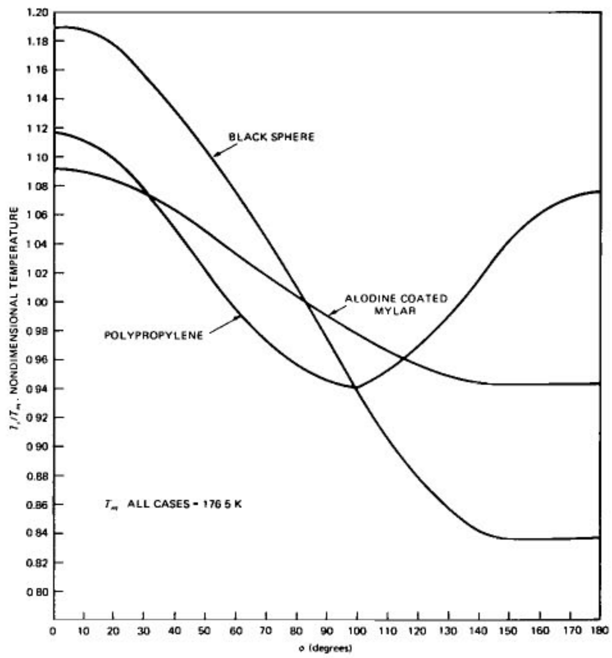

Figure 11—Temperature distribution in earth's shade at 180 degrees from subsolar position.

In earth shade, $(S_0 + P_0) = 0$; therefore

$$T_m^4 = \frac{A_p G_0}{A\sigma} = \frac{1(5.28)(10^{-3})}{4(1.36)(10^{-12})}$$

$$= 9.7 \times 10^8$$

and

$$T_m = 176 \text{ K} .$$

Table 2—Inputs and results for polypropylene sphere at subsolar position.

$\beta = \phi$	Inputs (cal/sec-cm^2 × 10^{-4})						T_i^4 (10^8 K)	T_i/T_m
	S_{0i}	S_i	P_{0i}	P_i	G_{0i}	G_i		
0	333.0	0	0	76.5	0	34.2	77.1	1.165
15	321.6	0	0	74.0	0	33.1	74.6	1.155
30	288.4	0	0	66.5	0	29.6	67.2	1.125
45	235.5	0	1.0	55.2	0.5	24.6	56.2	1.076
60	166.5	0	5.1	47.9	7.4	19.7	43.2	1.007
75	86.2	0	12.2	30.9	5.8	13.9	29.7	0.918
90	0	0	22.2	20.2	10.6	9.1	16.7	0.790
105	0	78.3	34.0	11.1	16.1	5.0	29.4	0.915
120	0	151.2	47.3	4.6	22.3	2.1	42.6	1.005
135	0	213.8	60.8	0.9	28.6	0.4	55.2	1.072
150	0	261.9	73.2	0	34.4	0	66.0	1.119
165	0	292.0	81.5	0	38.4	0	73.2	1.149
180	0	302.4	84.3	0	39.7	0	75.7	1.158

Table 3—Inputs and results for polypropylene sphere in earth-shade position.

$\beta = \phi$	Inputs (cal/sec-cm^2 × 10^{-4})		T_i^4 (10^8 K)	T_i/T_m
	G_{0i}	G_i		
0	39.7	0	15.0	1.116
15	38.4	0	14.6	1.107
30	34.4	0	13.1	1.078
45	28.6	0.4	11.2	1.036
60	22.3	2.1	9.5	0.995
75	16.1	5.0	8.3	0.963
90	10.6	9.1	7.8	0.946
105	5.8	13.9	7.8	0.946
120	2.4	19.2	8.5	0.969
135	0.5	24.6	9.8	1.002
150	0	29.6	11.4	1.042
165	0	33.1	12.6	1.068
180	0	34.2	13.0	1.076

At the subsolar point and $\phi = 0°$, by Equation 51,

$$T_i^4 = \frac{1}{\sigma(\epsilon + \epsilon_0)} \left\{ a_0(S_{0i} + P_{0i}) + a(S_i + P_i) + \epsilon_0 G_{0i} + \epsilon G_i \right.$$

$$\left. + \frac{A_p}{A} \left[G_0 \; \frac{\epsilon \rho^* \tau^*}{1 - \rho^*} - \frac{\left(a\rho\tau \;\; \epsilon_0 + \frac{\epsilon \tau^*}{1 - \rho^*} \right)}{(1 - \rho)\left(a_0 + \frac{a\tau}{1 - \rho} \right)} \right] \right\}$$

$$+ \frac{T_m^4}{\epsilon + \epsilon_0} \left[\frac{a\rho\tau\left(\epsilon_0 + \frac{\epsilon \tau^*}{1 - \rho^*} \right)}{(1 - \rho)\left(a_0 + \frac{a\tau}{1 - \rho} \right)} + \frac{\epsilon^2}{1 - \rho^*} \right] ,$$

or, when numerical values are substituted,

$$T_i^4 = \frac{1 \times 10^{-3}}{(1.36)(10^{-12})(0.103 + 0.103)} \left\{ 0.042(33.3 + 0) + 0.042(0 + 7.65) + 0.103(0 + 3.42) \right.$$

$$\left. + \frac{1}{4}(5.28)(10^{-4}) \left[\frac{(0.103)(0.036)(0.861)}{(1 - 0.036)} - \frac{(0.042)(0.05)(0.908)\left(0.103 + \frac{(0.103)(0.861)}{(1 - 0.036)} \right)}{(1 - 0.05)\left(0.042 + \frac{(0.042)(0.908)}{(1 - 0.05)} \right)} \right] \right\}$$

$$+ \frac{(42.0)(10^8)}{(0.103 + 0.103)} \left[\frac{(0.042)(0.05)(0.908)\left(0.103 + \frac{(0.103)(0.861)}{(1 - 0.036)} \right)}{(1 - 0.05)\left(0.042 + \frac{(0.042)(0.908)}{(1 - 0.05)} \right)} + \frac{(0.103)^2}{1 - 0.036} \right] ,$$

$$T_i^4 = 77.1 \times 10^8 ,$$

$$\left(\frac{T_i}{T_m} \right)^4 = 1.84 ,$$

and

$$\frac{T_i}{T_m} = 1.165 .$$

In the earth-shade position for $\phi = 0°$, all the S and P terms are zero, and hence,

$$T_i^4 = \frac{1}{(1.36)(10^{-12})(0.103 + 0.103)} \left\{ (0.103)(39.7)(10^{-4}) \right.$$

$$+ \frac{1}{4}(5.28)(10^{-4}) \left[\frac{(0.103)(0.036)(0.861)}{(1 - 0.036)} - \frac{(0.042)(0.05)(0.908)\left(0.103 + \frac{(0.103)(0.861)}{(1 - 0.036)}\right)}{(1 - 0.05)\left(0.042 + \frac{(0.042)(0.908)}{(1 - 0.05)}\right)} \right] \left. \right\}$$

$$+ \frac{9.7(10^8)}{(0.103 + 0.103)} \left[\frac{(0.042)(0.05)\left(0.103 + \frac{(0.103)(0.861)}{(1 - 0.036)}\right)}{(1 - 0.05)\left(0.042 + \frac{(0.042)(0.908)}{(1 - 0.05)}\right)} + \frac{(0.103)^2}{(1 - 0.036)} \right],$$

or

$$T_i^4 = 15.0 \times 10^8 .$$

Hence,

$$\left(\frac{T_i}{T_m}\right)^4 = 1.55 ,$$

or

$$\frac{T_i}{T_m} = 1.116 .$$

We treat the opaque cases in the same manner, noting that S_i, P_i, G_i, τ, and τ^* are all equal to zero, and thereby reduce Equations 43 and 51 to the simpler forms (Equations 2 and 25) derived for the opaque sphere.

CONCLUDING REMARKS

A method for determining the temperature distribution of a transparent or partially transparent nonconducting satellite has been derived. Equations 11, 35, 36, and 37 can be solved by numerical methods for any satellite configuration with any combination of transparent or opaque surfaces. The only restrictions are that the external thermal inputs and the surface thermal radiation properties of the satellite must be known.

By means of the derived expressions and their application to a sphere with constant surface properties, expressions for mean radiation temperature and temperature distribution were obtained. These are closed-form expressions, easily solved by hand calculation, but their verification in the laboratory is quite difficult; indeed, no such experiments are mentioned in the literature.

The expression for the mean temperature of a transparent sphere is identical with the expression derived in Reference 5. The expressions for hot- and cold-spot temperatures, however, do not agree. Apparently this is because only average earth inputs were used, as suggested by Wood and Carter (Reference 4), rather than inputs distributed over the surface of the sphere as in the present report. The use of average earth inputs would lower hot-spot temperatures, especially in orbits of altitude below 1000 km. Also, it appears that not all the reflections were accounted for in the ray-tracing technique described in Reference 5.

It is of interest to note the differences in the distribution curves between the opaque and transparent cases. From Figures 10 and 11 it can be seen that the position of the sphere 180 degrees from the sun's rays is much cooler than the position directly under the sun's rays for the opaque cases, while the temperatures at these two positions are more nearly equal in the transparent case. This raises the possibility of using transparent surfaces as a means of achieving more uniform temperatures over the spacecraft. However, use of a completely transparent skin does not completely eliminate temperature gradients (sometimes it increases them, as in Figure 10), but it shifts the positions of maximum gradient (see Figure 11). Possibly by some combination of opaque and transparent surfaces, nearly uniform temperatures may be realized.

Goddard Space Flight Center
National Aeronautics and Space Administration
Greenbelt, Maryland, February 26, 1970
861-51-75-01-51

REFERENCES

1. Sandorff, P. E., and Prigge, J. S., Jr., "Thermal Control in a Space Vehicle," *J. Astronaut.* 3:4, Spring 1956.

2. Schach, M., and Kidwell, R. E., Jr., "Thermodynamics of Space Flight (Heat Transfer Phenomena in Space)," Lecture prepared for the Space Environment Technology Series, NASA Technical Memorandum X-51282, National Aeronautics and Space Administration, Washington, D.C., March 1963.

3. Hrycak, P., "Calculation of Satellite Surface Temperature Distribution," *Astronautica Acta*, IX (2): 115–127, 1963.

4. Wood, G. P., and Carter, A. F., "The Design Characteristics of Inflatable Aluminized-Plastic Spherical Earth Satellites with Respect to Ultraviolet, Visible, Infrared, and Radar Radiation," ASME Paper No. 59-AV-38, presented at Aviation Conference, Los Angeles, Calif., March 9–12, 1959, available from the American Society of Mechanical Engineers, New York, N.Y.

5. G. T. Schjedahl Company, "Final Report Development of an Inflatable Rigidizing Satellite," NASA Contract No. NAS5-1190, Goddard Space Flight Center, Greenbelt, Maryland, June 1962.

6. Eckert, E. R. G., and Drake, R. M., Jr., "Heat and Mass Transfer," New York: McGraw-Hill Book Co., Inc., 1959.

7. Cunningham, F. G., "Power Input to a Small Flat Plate from a Diffusely Radiating Sphere, with Application to Earth Satellites," NASA Technical Note D-710, National Aeronautics and Space Administration, Washington, D.C., July 1961.

8. Kreith, F., "Radiation Heat Transfer," Scranton, Pa.: International Textbook Company, 1962.

9. Cunningham, F. G., "Earth Reflected Solar Radiation Input to Spherical Satellites," NASA Technical Note D-1099, National Aeronautics and Space Administration, Washington, D.C., October 1961.

10. Cunningham, F. G., "Earth Reflected Solar Radiation Incident upon an Arbitrarily Oriented Spinning Flat Plate," NASA Technical Note D-1842, National Aeronautics and Space Administration, Washington, D.C., July 1963.

11. Nautical Almanac Office, U.S. Naval Observatory, "The American Ephemeris and Nautical Almanac," Washington, D.C.: U.S. Government Printing Office.

12. Swalley, F. E., "Thermal Radiation Incident on an Earth Satellite," NASA Technical Note D-1524, National Aeronautics and Space Administration, Washington, D.C., December 1962.

Appendix A

Angle Between Earth-Satellite Line and Earth-Sun Line

It is assumed that the satellite's longitude and latitude are known for any time t. In the following equations (see Figure A-1) north latitude and east longitude correspond to positive angles, while south latitude and west longitude correspond to negative angles. The American Ephemeris and Nautical Almanac gives the latitude (or declination) of the sun for any date.

The coordinates are set up as follows: the **i** vector, in the equatorial plane, passes through the Greenwich meridian; the **k** vector, perpendicular to the equatorial plane, passes through the North Pole; the **j** vector, in the equatorial plane, is perpendicular to **i** and **k**. Thus a right-hand system is formed.

The direction of the satellite and sun can now be indicated by unit vectors from the earth's center.

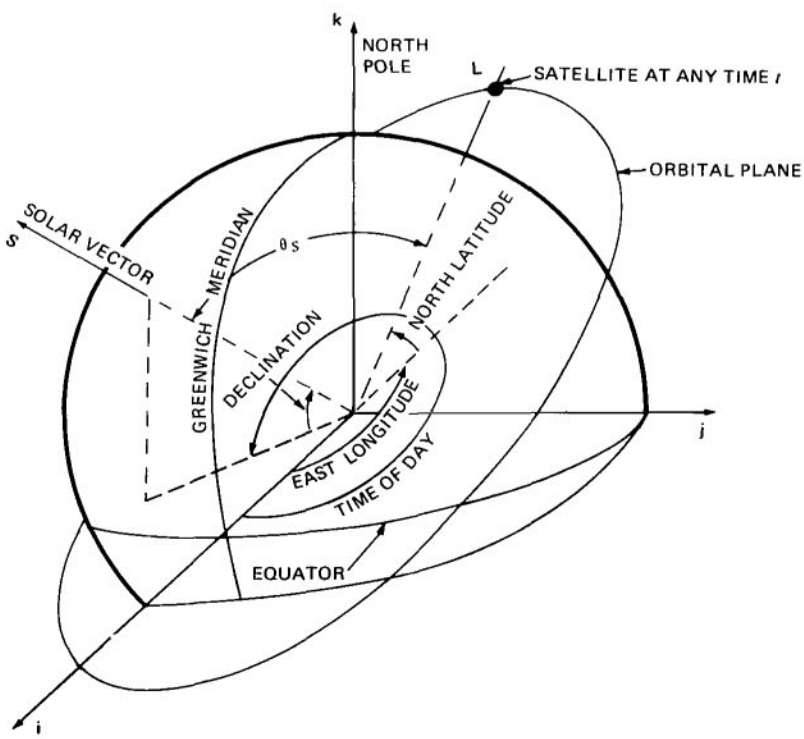

Figure A-1—Geometry of earth-satellite and earth-sun line.

At any time t, given in Greenwich Mean Time on a given day of the year, the following information is available:

For the satellite,

latitude and longitude, from predictions or sighting.

For the sun,

declination, from Reference 11 for date given;
latitude, from time at which satellite is at given latitude and longitude.

Then, referring to Figure A-1, we obtain

$$\mathbf{L} = \cos(\text{lat})\cos(\text{long})\mathbf{L} + \cos(\text{lat})\sin(\text{long})\mathbf{j} + \sin(\text{lat})\mathbf{k} \tag{A-1}$$

and

$$\mathbf{S} = \cos(\text{decl})[\cos(t - 12)15]\mathbf{i} + \cos(\text{decl})[\sin(t - 12)15]\mathbf{j} + \sin(\text{decl})\mathbf{k}\ , \tag{A-2}$$

where

lat = latitude of satellite at time t in degrees,

long = longitude of satellite at time t in degrees,

decl = sun's latitude on the given date in degrees,

and

t = Greenwich Mean Time in hours (0 to 24).

Then,

$$\cos\theta_s = \frac{\mathbf{L}\cdot\mathbf{S}}{|\mathbf{L}||\mathbf{S}|} = \cos(\text{lat})\cos(\text{long})\cos(\text{decl})[\cos(12 - t)15]$$

$$+ \cos(\text{lat})\sin(\text{long})\cos(\text{decl})[\sin(12 - t)15] + \sin(\text{lat})\sin(\text{decl})\ . \tag{A-3}$$

It is important to note the sign convention mentioned earlier.

Appendix B

Satellite Coordinate System and Relations Between Surface-Normal and Solar Vector and Between Unit Normal and Earth-Satellite Line

In general, most satellites are spin-stabilized; hence the spin axis would be the natural choice for one of the coordinate axes. It would then be necessary to transform this coordinate system to a geocentric coordinate system to find the angles necessary for determining earth input and albedo. However, since we are considering a non-spinning satellite, we are free to use any coordinate system we choose and hence will use a geocentric system, thus eliminating the need for any transformation. Figure B-1 illustrates this. For certain applications (such as earth-oriented satellites) this selection of coordinates may prove unsatisfactory. In such cases the reader is referred to Swalley's report (Reference 12) which deals with this problem in much greater detail.

With satellite-centered coordinates, the unit normal at any point is given by

$$\mathbf{n} = \frac{\nabla F}{|\nabla F|} \tag{B-1}$$

where $F(x, y, z) = 0$ is the equation of the surface.

Since the surface under consideration is spherical,

$$F(x, y, z) = x^2 + y^2 + z^2 + R^2 = 0\ , \tag{B-2}$$

where R is the radius of the sphere. Change to spherical coordinates, as shown in Figure B-2; now ∇F becomes the vector $\mathbf{R}$ and $|\nabla F| = R$, which gives the unit normal

$$\mathbf{n} = \mathbf{i} \sin\theta \cos\phi + \mathbf{j} \sin\theta \sin\phi + \mathbf{k} \cos\theta\ , \tag{B-3}$$

where

θ = angle between $\mathbf{k}$ and the normal to the element

and

ϕ = angle measured in the $\mathbf{ij}$th plane from $\mathbf{i}$ to the element (see Figure B-2).

Then β, the angle between the normal to the surface and the solar vector, can be computed from

$$\cos\beta = \mathbf{n} \cdot \mathbf{S}\ ; \tag{B-4}$$

also λ, the angle between the normal to the surface and $-\mathbf{L}$, the satellite-earth line, can be computed from

$$\cos\lambda = \mathbf{n} \cdot (-\mathbf{L}) = -\mathbf{n} \cdot \mathbf{L}\ . \tag{B-5}$$

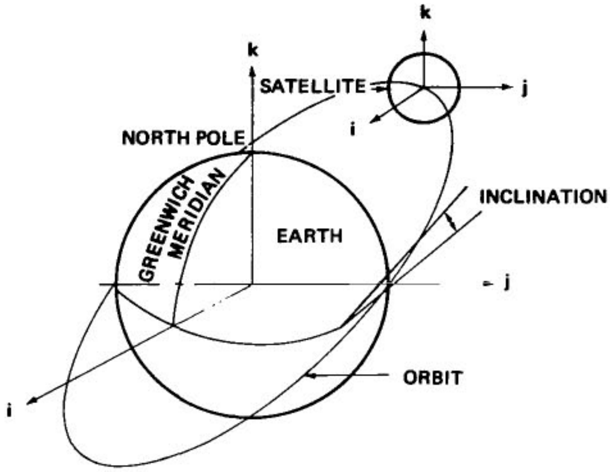

Figure B-1—Geocentric coordinate system.

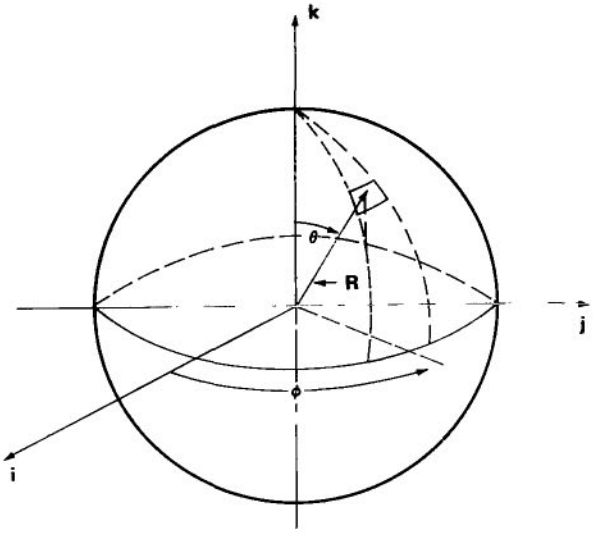

Figure B-2—Satellite coordinates.

At certain positions in the orbit, the satellite may be shaded by the earth. For the satellite to be completely in the earth's shade, two conditions must be satisfied: the satellite's height above the earth's center multiplied by sin θ_s must be less than the earth's radius R_e, and values of cos θ_s must be negative. These conditions can easily be checked at any satellite position.

Appendix C

Nomenclature

A = surface area of satellite

A_D = area of disk on the earth subtended by the satellite

A_p = projected area of satellite

B = infrared energy leaving interior surface of satellite per unit surface area

$B_i(B_k)$ = infrared energy leaving interior surface of area $i(k)$ per unit surface area

$\mathbf{D}$ = vector from an element on the earth's surface to an element on the satellite's surface

$F_{i\text{-}k}$ = shape factor from area i to area k

$F_{di\text{-}dk}$ = shape factor from element di to element dk

G_0 = earth-radiated energy incident on external surface of satellite per unit surface area

G_{0i} = earth-radiated energy incident on external surface of area i per unit surface area

G_i = earth-radiated energy incident on internal surface of area i per unit surface area

H = infrared energy incident on interior surface of satellite (including reflections) per unit surface area

$H_i(H_k)$ = infrared energy incident on interior surface of area $i(k)$ (including reflections) per unit surface area

I = solar energy incident on interior surface of satellite (including reflections) per unit surface area

$I_i(I_k)$ = solar energy incident on interior surface of area $i(k)$ (including reflections) per unit surface area

L = solar energy leaving interior surface of satellite per unit surface area

$L_i(L_k)$ = solar energy leaving interior surface of area $i(k)$ per unit surface area

$\mathbf{L}$ = unit vector from earth to satellite

$\mathbf{N}_1$ = vector normal to element of satellite

$\mathbf{N}_2$ = vector normal to element of earth

P_i = earth-reflected solar energy (albedo) incident on internal surface of area i per unit surface area

P_0 = earth-reflected solar energy (albedo) incident on external surface of satellite per unit surface area

P_{0i} = earth-reflected solar energy (albedo) incident on external surface of area i per unit surface area

$\mathbf{R}$	= satellite radius vector
R_e	= radius of earth
S_i	= direct solar energy incident on the internal surface of area i per unit surface area
S_0	= direct solar energy incident on the external surface of the satellite per unit surface area
S_{0i}	= direct solar energy incident on the external surface of area i per unit surface area
$\mathbf{S}$	= unit vector from earth to sun
T_e	= absolute temperature of earth
T_i	= absolute temperature of area i
T_m	= root mean fourth absolute temperature
di	= differential surface element of area i
dk	= differential surface element of area k
dQ_i	= net, inward-directed energy leaving the internally facing surface of di
e	= emissive power of internal satellite surface = $\sigma\epsilon T^4$
e_i	= emissive power of internal satellite area $i = \sigma\epsilon_i T_i^4$
h	= r/R_e (see below)
l	= distance between two elemental areas
$\mathbf{n}$	= unit vector normal to element of satellite
r	= distance from satellite's center to earth's center
r'	= radius of satellite
a	= mean solar absorptivity of internal surface of satellite
a_i	= solar absorptivity of internal surface of area i
a_0	= mean solar absorptivity of external surface of satellite
a_{0i}	= solar absorptivity of external surface of area i
β	= angle between normal to elemental area on satellite and solar vector
ϵ	= mean infrared emissivity of internal surface of satellite
ϵ_i	= infrared emissivity of internal surface of area i
ϵ_0	= mean infrared emissivity of external surface of satellite
ϵ_{0i}	= infrared emissivity of external surface of area i
θ_s	= angle between earth-sun line and earth-satellite line
λ	= angle between satellite surface normal and earth-satellite line
ρ	= mean solar reflectivity of internal surface of satellite
ρ_i	= solar reflectivity of internal surface of area i
ρ^*	= mean infrared reflectivity of internal surface of satellite
ρ_i^*	= infrared reflectivity of internal surface of area i

σ = Stefan-Boltzmann constant

τ = mean solar transmissivity of satellite

τ_i = solar transmissivity of area i

τ^* = mean infrared transmissivity of satellite

τ_i^* = infrared transmissivity of area i

ω = solid angle

NASA-Langley, 1971 —— 33

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

— NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION OFFICE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546