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MASCOT - A NEW CONCEPT IN GUIDANCE

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This report documents one such guidance concept which permits practical realization of many heretofore unrealizable optimal guidance tasks, namely, on-board optimization of ascent trajectories through the atmosphere, self-targeting, optimal rendezvous, optimal coast-burn-coast-burn orbital transfer, and reentry for high lift-to-drag-ratio type of vehicles.

This collection of techniques combined into one guidance program has been called MASCOT (Manned Shuttle Comprehensive Optimization and Targeting).

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SUMMARY

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This report documents one such guidance concept which permits practical realization of many heretofore unrealizable optimal guidance tasks, namely, on-board optimization of ascent trajectories through the atmosphere, self-targeting, optimal rendezvous, optimal coast-burn-coast-burn orbital transfer, and reentry for high lift-to-drag-ratio type of vehicles.

This collection of techniques combined into one guidance program has been called MASCOT (Manned Shuttle Comprehensive Optimization and Targeting).

I. INTRODUCTION

The advent of a new generation of space vehicles (specifically NASA's post-Apollo space shuttle) with advanced features such as completely reusable stages, sophisticated on-board digital computers, and airplane-type landing characteristics for both booster and orbiter has required a re-evaluation of the space vehicle guidance concepts used in the past. When re-examined, these traditional guidance concepts have been found to be inadequate for the new generation of space vehicles. In response to this need, new concepts, procedures, and techniques have recently been developed which provide a significant improvement in optimal guidance applications. In particular, this new approach yields a physically realizable optimal guidance scheme which yields nearly optimal performance for the boost phase, the rendezvous phase, and the reentry phase of the post-Apollo shuttle vehicle flight path. This new system and concept has been given the name MASCOT.

The MASCOT concept is made possible by the development of new mathematical techniques and the significant improvement of others. It also takes advantage of the improved speed, memory capacity, size, and reliability of the new flight-worthy digital computers.

In this paper, the evolution and mathematical theory of the MASCOT guidance technique are explained. In addition, some computer-simulated trajectories obtained by means of the MASCOT program are presented to demonstrate the accuracy, flexibility, and optimality of the MASCOT system. Some comparisons will be made, whenever practical, with results obtained from the present Saturn Iterative Guidance Mode (IGM).

II. SOME BACKGROUND ON GUIDANCE, NAVIGATION, AND CONTROL

The operation of a guidance system involves three interrelated divisions of effort, navigation, guidance signal generation, and control. Navigation is concerned with the determination of the current position and velocity of the vehicle. Navigation is normally an open-loop process for short flights but must be made closed-loop by navigation update in some instances, i.e., long flight times.

With an accurate knowledge of all physical parameters of vehicle and environment and current position and velocity and an accurate knowledge of the desired terminal conditions, it is possible to determine the proper directions of the engines thrust to obtain the desired end conditions. This closed-loop process of thrust-direction determination is referred to as guidance signal generation.

The control system implements the guidance signals in addition to employing such maneuvers that may be required to preserve the structural integrity of the vehicle. Thus, the control system may temporarily ignore or minimize the influence of the guidance signal in order to protect the vehicle from winds, wind gusts, wind shears, vibrational disturbances, or fuel sloshing problems. Control, too, is a closed-loop process.

III. THE GENERAL CONCEPT OF GUIDANCE

Implementation of the guidance signal will determine the vehicle flight path to reach a desired destination or set of terminal conditions. Perfect operation of the navigation system, exact generation of the guidance signal, and exact implementation of the control system, together with an exact knowledge of the vehicle physical parameters and an exact knowledge of the physical environment during the entire flight to the desired end conditions, would make it possible to solve for the guidance signal only once. This signal would, of course, be a function of time or some other convenient variable so that the guidance signal would change continuously during the flight.

In practice, perfect operation of the three systems cannot be achieved. Thus, a practical guidance system may make the computations at the initiation of guidance, and predict the required guidance signal or function to guide to the required end condition. Since the initial information will have some inaccuracies, the process must be repeated periodically during the flight, treating each instant at which the new computations occur as a new set of initial conditions.

IV. OPTIMAL GUIDANCE

Optimal guidance, a subclass of the broad class of guidance concepts, is characterized usually by the deliberate maximization or minimization of some aspect of vehicle performance, for example, maximum payload to a given orbit or equivalently minimum fuel required for a given payload.

To accomplish optimal guidance of a vehicle, a mathematical model of the vehicle motions must first be chosen. Appearing in these equations of motion are the so-called control variables. These variables are the equation parameters which can be given an arbitrary (within prescribed limits that depend on the problem) value at any instant along the vehicle's flight path. This large amount of freedom of selection for the control variables allows a time history for the control variables to be chosen which achieves a desired destination and also optimizes some specified aspect of the vehicle performance. For a typical rocket-powered vehicle, the control variable is usually the thrust direction, and a time history of this thrust direction is determined (as described previously) to maximize the payload delivered to a particular destination.

A. Mathematical Techniques of Optimal Guidance

The different mathematical techniques used to select the optimum control variable are usually grouped into a body of knowledge known as optimization theory or calculus-of-variations theory.

When some type of optimization theory is applied to the differential equations simulating the motion of a rocket-powered vehicle, a two-point boundary value problem is produced. This means that not all of the initial conditions for the differential equations involved are known at the initial time. The unknown initial conditions must be determined so that they cause the results of the integration of the differential equations to satisfy some pre-specified conditions at the final time or destination. To accomplish this, in practice, one usually attempts to discover, empirically, some mathematical relations which connect the initial conditions with the final conditions. Then, the desired conditions at the destination are placed in these relations, and the resulting equations are solved, hopefully, to yield values for the missing initial conditions. These initial conditions and the relations connecting the initial conditions with the final conditions yield the time history of the control variables that optimize the chosen criterion of the vehicle's performance. In practice, there may be additional relations or constraints to be satisfied by a particular vehicle's trajectory which produce a multi-point boundary value problem instead of the simple two-point boundary value problem described above. These aspects of optimization theory are examined in more detail in the section on mathematical development.

B. The Philosophy of Optimal Guidance: Old and New

In the past, computer technology and mathematical techniques were inadequate to allow the consideration of a realistic computational algorithm for trajectory optimization as a practical on-board guidance signal generator. Thus, older techniques for on-board guidance signal generation relaxed the reality of the mathematical simulation of the vehicle's motion or degraded the optimality of the solution path in order to solve the resultant boundary value problem rapidly enough to control a particular vehicle's flight. The new MASCOT concept (which is a realistic computational algorithm for trajectory optimization) can be considered as a guidance signal generator for all phases of a vehicle's flight (boost phase, rendezvous phase, and reentry phase). This advance is possible at the present time because of improvements in computer technology and mathematical techniques for the solution of optimal trajectory problems.

Before going directly into the explanation and results associated with MASCOT, we briefly outline the historical developments which led to the present form of the MASCOT guidance technique. For readers not familiar with some of the ideas and terminology contained in this historical development, an extensive list of references is included at the end of the report.

V. HISTORICAL BACKGROUND

A. Delta Minimum Guidance

Beginning with the V-2 rockets in the mid-1940's and continuing through the Redstone, Jupiter, and Pershing missiles, the "delta-minimum" guidance concept was used successfully to guide rocket flights. The delta-minimum concept required that both nominal and perturbed trajectories follow essentially the same geometrical trajectory regardless of other considerations. On-board flight computations were simple, and analog computation was used for the execution of the delta-minimum concept.

B. Iterative Guidance Mode

In 1960, research work was begun to develop new guidance concepts for the Saturn space vehicles [1,2,3]. This work was motivated by the development of new mathematical techniques for maximization of payload through optimization methods and by the development of digital computers to replace analog computers as on-board hardware. It was also obvious that space trajectories would require greater flexibility to cope with sudden changes, such as engine out, and that more options must be permitted in the selection of flight profiles. The guidance law which resulted from the research work begun in 1960 and which has been flown on Saturn vehicles was given the name IGM, Iterative Guidance Mode.

IGM is essentially an approximate formulation of the calculus-of-variations problem that allows analytic construction of major parts of the solution, so that only a simple iterative numerical process is required for solution. This approach avoids the time-consuming numerical integration procedures that have been required to compute a general solution to the fundamental calculus-of-variations problems. The speed needed for real-time application has been the primary motivation for the derivation of semi-explicit methods of this type.

However, as a result of the approximations, the accuracy and flexibility of such flight schemes are limited, primarily in that they are nearly optimal only for short arcs of powered flights and for specialized mission (boundary value) conditions in a restricted coordinate system. This limitation can be relaxed somewhat in practice by use of special purpose adjustments, but only at the expense of additional preflight analysis and simulation.

The basic simplifications made in the Iterative Guidance Mode to obtain analytic construction of major parts of the solution were (1) to assume a uniform gravitational field rather than an inverse square law and (2) to apply the same transversality condition at engine cutoff regardless of the mission flown. There were, incidentally, some small-angle approximations and ingenious special purpose adjustments made to improve the optimality of the scheme and to reduce the computational effort required for on-board mechanization. However, the basic concept of the simplifications is best seen in view of the assumption concerning the gravitational field and the assumption of the adequacy of a single transversality condition.

C. QUOTA

Next came the QUasi Optimal Trajectory Analysis, QUOTA [4], which belongs in the same category with IGM since it too is an approximation to the calculus-of-variations (COV) solution.

The fundamental approximation in QUOTA may be viewed in at least two different ways. The first way may be said to be an approximation to the COV in the sense that the Euler-Lagrange equations are replaced by a slightly different set of equations, the purpose of which is to allow analytic integration of both the "state" and "co-state" equations. Of course, the "co-state" variables must be interpreted in a generalized sense because they are not obtained from the Euler-Lagrange equations.

A second manner of viewing the approximations to achieve the solutions will be mentioned. From many observations of the behavior of the Lagrange multipliers (λ's) in the calculus of variations, it has been noted that for rocket flights in a wide variety of missions, these λ's are very nearly linear functions of time. The QUOTA equations may be derived by assuming linear λ's and by expanding the gravity term in series.

The advantage of this approach is a very rapid computational scheme which is much more flexible than IGM and also leads to a smaller loss of optimality. Even this small loss of optimality may be regained by making the assumption that the λ's are linear only for a portion of the flight; thus, the λ versus time curve is represented by a series of connecting straight lines where the slopes of the linear λ segments are determined from the Euler-Lagrange equations. For an Apollo-type mission, the λ's may be assumed to be linear for the entire flight with only a negligible loss in payload.

With either of the viewpoints mentioned above, the true gravitational field may be accurately represented and transversality conditions may be fitted to the particular end conditions. Needless to say, the optimality of the solution must be checked because of the substitution of approximate equations for the Euler-Lagrange equations.

Computationally, QUOTA offered much more flexibility than IGM, required less preflight analysis and only slightly increased the amount of on-board computation required. Because IGM was entirely adequate for all Saturn flights, there has been no need to change from IGM to QUOTA and, therefore, QUOTA has never been tested on any actual flights. It is still available for any mission which requires flexibility beyond IGM capability and for which the same computational equipment must be used.

D. OPGUID AND SWITCH

Before 1965, general (flexible) numerical procedures for computing precise optimal trajectories were too unreliable in convergence and costly in computational requirements to be considered for real-time guidance. However, an indirect method for computing optimal trajectories [5,6] was developed in 1965 incorporating improved techniques to obtain a substantial gain in speed, convergence, and flexibility. A simple scaling rule for the amount of the Newton correction that was permitted per iteration resulted in an extremely large region of convergence that was surprisingly insensitive to accurate initialization of the boundary value problem.

The indirect approach is particularly well suited for real-time use, where the continual adjustment of the guidance scheme to accommodate perturbations in initial conditions is readily accomplished by a single Newton iteration on the boundary value problem. In 1966, the feasibility of this approach as a real-time guidance scheme for optimizing single-burn-arc orbital injection missions was demonstrated [7] and named OPGUID (OPtimal GUIDance).

However, many orbit transfer problems require the use of several burn arcs separated by relatively long optimal coast arcs. A multi-burn-arc version of OPGUID, developed in 1967 [8], demonstrated that the attractive fundamental approach of OPGUID could successfully converge a general formulation of this problem, with variable boundary conditions. A sophisticated version of the multi-burn program (SWITCH) has been developed [9] that has successfully converged a variety of orbital transfer problems with an efficiency and reliability comparable to that of the original OPGUID.

As a result, the indirect method of SWITCH is not only feasible but considerably superior to existing implementations of quasilinearization in convergence as well as efficiency. A principal feature of SWITCH is the use of classical two-body theory to render the computations for coast arcs explicit. Since high-thrust multi-burn orbit transfers usually involve coast arcs many times longer in duration than burn arcs, the explicit method results in a substantial savings in computation per iteration. A universal variable formulation of the two-body problem with closed-form expressions for the state transition matrix is used. This formulation was adapted from the work in references 8, 10, 11 and 12 in a novel way to avoid the cumbersome computation of the three-dimensional tensor of second partial derivatives of final state with respect to initial state that is required when computing the partial derivative of final co-state with respect to initial state.

Since all the partial derivatives are available at each iteration, as was the case for the original OPGUID, the SWITCH algorithm is appropriate for computing real-time corrections to in-flight perturbations.

Unlike the OPGUID algorithm, the SWITCH algorithm does require reasonable initialization. That is, it is not possible with SWITCH as it was with OPGUID to misalign the thrust direction by 90 or 180 degrees and retain convergence. However, rough estimates of impulsive solutions have proved more than adequate as initialization in every trial case.

E. MASCOT

With the advent of the space shuttle with its high aerodynamic lifting characteristics both on ascent and descent, it has become imperative that optimization programs be devised to take advantage of these atmospheric lifting capabilities.

This has been accomplished essentially by introducing into the SWITCH program the aerodynamic effects of both lift and drag [13]. Additionally, the effect on thrust of pressure variation with altitude has been included in the propulsion computations.

These changes have recently been carried out, and some of the observed results may be summarized as follows:

1. Maximum payload and maximum sensitivity to convergence occur when atmospheric effects are introduced into both the state and co-state equations.

2. Approximately two-thirds of the payload increases obtained under (1) may be kept by introducing the atmospheric effects into the state equations only. In this case, convergence is far easier.

3. Extremely difficult convergence cases may be approached gradually by multiplying the atmospheric model by $0 \leq K \leq 1$ where K starts at zero and gradually increases to 1. For each value of K, the case is converged before increasing the value of K.

When the atmospheric effects, together with changes to be described later, were added to the SWITCH concept, the name MASCOT was chosen for the overall scheme to handle multiple burn-coast-burn optimal trajectories both in and out of the atmosphere.

VI. MATHEMATICAL DEVELOPMENT FOR MASCOT

The first step in the development of an optimal guidance technique for a rocket-powered vehicle is the mathematical modeling of the vehicle's trajectory. To do this, a second-order three-dimensional vector of ordinary differential equations is used which consists of the sums of the vector accelerations produced by all the forces to be considered acting on the vehicle. For the MASCOT scheme, this sum of vector accelerations must consist of terms produced by forces acting during the boost phase, the exo-atmospheric rendezvous phase, and the reentry phase. During the computation of a portion of a trajectory for a particular phase, all of the terms in the sum of all of the phases do not have to be considered, and the unneeded terms can be easily ignored with the logic of the computer program.

In an arbitrarily oriented earth-centered Cartesian coordinate system, the differential equations to be used for simulating the motion of a point mass subject to gravitational, aerodynamic, and thrusting forces can be written as follows:

$$\ddot{\bar{x}} = (F/m)\bar{u} + \frac{\bar{L}-\bar{D}}{m} - \frac{(GM)\bar{x}}{R^3} + \bar{g}(\bar{x}). \tag{1}$$

In the above vector differential equation, the first term on the right is the thrusting term where F is the instantaneous magnitude of the thrust, m is the instantaneous mass, and $\bar{u}$ is a unit vector which is considered to be a control variable for the direction of the thrust. In the second term, $\bar{L}$ is the vector of aerodynamic forces due to lift, and $\bar{D}$ is a vector of aerodynamic forces due to drag. The determination and control of the thrusting force and the aerodynamic forces will be explained below. The third term in equation (1) is the gravitational force term for a spherical central body, where GM is the gravitational constant and R is the magnitude of the position vector of the point mass under consideration. The fourth term which is considered to be a function of the position vector is a symbolic representation of corrections to the gravitational force expression which are needed when a non-spherical shape for the central body is considered. For simplicity in the rest of this development, the fourth term on the right-hand side of equation (1) will be ignored, and no further explanation of the derivation of the third term will be given since the representation is the standard expression for the gravitational force due to a spherical central body.

To begin an explanation of the first term (the term due to thrust) in equation (1), the thrust force F will be assumed to be constant for exo-atmospheric flight and have the following form for atmospheric flight:

$$F = F_s + A_e(P_o - P), \tag{2}$$

where

F_s is the vehicle's thrust measured to sea level ($P = P_o$)

A_e is the exit area of the engine

P_o, P are the atmospheric pressures at sea level and at any altitude, respectively.

In either case, the mass flow rate $\dot{m}$ is assumed to be a constant and determined by the relation

$$\dot{m} = \frac{F}{g_o\, I_{sp}}, \tag{3}$$

where the variable I_{sp} indicates the efficiency of a particular set of engines and

$$g_o = \frac{GM}{R_o^2}$$

is a constant giving the acceleration due to gravity at the assumed R_o (radius of the spherical central body). For exo-atmospheric flight, the I_{sp} is assumed to be constant and thus $\dot{m}$ as given by equation (3) is constant. In this case, the mass at any time t referenced to time t_o is given by

$$m(t) = m(t_o) - \dot{m}(t - t_o). \tag{4}$$

In atmospheric flight $\dot{m}$ is still assumed to be constant, and thus as the force increases as given in equation (2), the I_{sp} used in equation (3) to give $\dot{m}$ must also increase. Then m at any time is still given by equation (4).

Optimization theory could be applied directly to the equations of motion (equation (1)) in its present form to determine the correct time history of $\bar{u}$, but the resulting computer program would be much too complicated to consider for guidance purposes. For this reason, some simplifying assumptions will now be made that result in a more compact and efficient computer program for the application of optimization theory.

The original equations of motion (equation (1)) are completely accurate representations of the entire trajectory of the shuttle vehicle's flight. For the rendezvous portion of the flight, an accurate representation is obtained by leaving out the term

$$\frac{\bar{L} - \bar{D}}{m},$$

and for the reentry phase, an accurate representation is obtained by leaving out the term $(F/m)\bar{u}$.

Before applying optimization theory to equation (1), the expression for $\bar{L}$ and $\bar{D}$ which will be used in both ascent and reentry phases will be simplified. Additionally, earth oblateness effects will be neglected throughout the entire flight, at least in the optimization procedures.

Furthermore, the rotational dynamics of the vehicle will be neglected, and a perfect control system with instantaneous reaction will be assumed. This means that no swiveling of the engines will be required for guidance and that the thrust may be aligned along the zero aerodynamic lift axis of the vehicle. With these assumptions and some other assumptions about the aerodynamics of the vehicle explained in reference [13], the expressions for $\bar{L}$ and $\bar{D}$ can be written as:

$$\begin{aligned} \bar{L} &= \frac{1}{2} A_r\, C_{L\alpha}\, \rho[(\bar{V}_r \cdot \bar{V}_r)\bar{u} - (\bar{V}_r \cdot \bar{u})\bar{V}_r] \\ \bar{D} &= \frac{1}{2} A_r\, \rho\, [(\bar{V}_r \cdot \bar{V}_r)^{\frac{1}{2}} (C_A + 2\eta\, C_{L\alpha}) - (2\eta\, C_{L\alpha})(\bar{V}_r \cdot \bar{u})]\bar{V}_r \end{aligned} \qquad (5)$$

where

$$R = (\bar{x} \cdot \bar{x})^{1/2}$$

$\bar{V}_r = \dot{\bar{x}} - \bar{\omega} \times \bar{x}$ where $\bar{\omega}$ is the earth's rotation vector.

$$\rho = \rho_o \, e^{-a(R-R_o)}$$

$$P = P_o \, e^{-b(R-R_o)}$$

$$C_{L_\alpha} = \sum_{i=0}^{6} a_i e^{-iM}$$

$$C_A = \sum_{i=0}^{6} b_i e^{-iM}$$

$$\eta = C$$

$$F = F_s + A_e(P_o - P)$$

$$M = \frac{(\bar{V}_r \cdot \bar{V}_r)^{1/2}}{V_s}$$

$$g_o = \frac{GM}{R_o^2}$$

$$V_s = d \, T_M^{1/2}$$

$$T_M = \sum_{i=0}^{6} c_i (R-R_o)^i .$$

Note that the constants ρ_o , a, P_o, b, a_i, b_i, C, d, and c_i must be determined to curve fit the atmospheric model and the aerodynamic coefficients.

Now that the equations of motion and associated constants have been defined, a transition to state vector notation will be made by the following definitions:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \\ m \end{bmatrix}$$

Then

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \dot{m} \end{bmatrix}$$

The next step in applying optimization theory to the equations of motion is the definition of the function to be minimized. For the MASCOT development, the function to be minimized will be written as

$$J = K_1 m(t_f) + K_2 \int_{t_o}^{t_f} \dot{Q}\, dt + K_3 \int_{t_o}^{t_f} \frac{|\overline{L}|^2 + |\overline{D}|^2}{m^2}\, dt \tag{6}$$

where $\dot{Q} = e\rho^{1/2}|\overline{V}r|^3$ is the convective heating rate per unit area of the vehicle and e is a given constant. The weighting factors K_1, K_2, and K_3, for the different components of J can be chosen arbitrarily (positive or negative) to achieve a particular relation between the minimum of a particular trajectory. In the above expression for J, the term $(|\bar{L}|^2 + |\bar{D}|^2)$ is not very convenient for consideration when the operations needed to minimize J are applied to J. With a bit of algebraic and trigonometric manipulation, the expression for $|\bar{L}|^2 + |\bar{D}|^2$ is closely approximated by

$$|\bar{L}|^2 + |\bar{D}|^2 = \left(\frac{\rho A_r}{2m}\right)^2 \left[(C_A^2 + 2\eta\, C_A\, C_{L_\alpha} + 2\, C_{L_\alpha}^2)\, |\bar{V}_r|^4 \right.$$

$$\left. - 2|\bar{V}_r|^3 (C_{L_\alpha}^2 - 2\eta\, C_A\, C_{L_\alpha})(\bar{V}_r \cdot \bar{u})\right] . \tag{7}$$

Now a Hamiltonian (denoted by H) can be written which is linear in $\bar{u}$. That is,

$$H = K_2[e\rho^{1/2}|\bar{V}_r|^3] + K_3 \left\{\left(\frac{\rho A_r}{2m}\right)^2 \left[(C_A^2 + 2\eta\, C_A\, C_{L_\alpha} + 2\, C_{L_\alpha}^2)|\bar{V}_r|^4\right.\right.$$

$$\left.\left. - 2|\bar{V}_r|^3 (C_{L_\alpha}^2 - 2\eta\, C_A\, C_{L_\alpha})(\bar{V}_r \cdot \bar{u})\right]\right\}$$

$$+ \begin{bmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\end{bmatrix} \cdot \begin{bmatrix}x_4\\ x_5\\ x_6\end{bmatrix} + \begin{bmatrix}\lambda_4\\ \lambda_5\\ \lambda_6\end{bmatrix} \cdot \left\{(F/m)\bar{u} - (GM/R^3)\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}\right.$$

$$+ \frac{\rho A_r}{2m} |\bar{V}_r| (C_A + 2\eta\, C_{L_\alpha})\bar{V}_r + \frac{\rho A_r}{2m} \left[C_{L_\alpha} |\bar{V}_r|^2 (I) + \right.$$

$$\left.\left.(2\eta - 1)\, C_{L_\alpha} (\bar{V}_r\, \bar{V}_r^T)\right]\bar{u}\right\} + \lambda_7\, \dot{m} \tag{8}$$

In the above expression, all the vectors that are dotted with $\bar{u}$ can be combined into a vector h as follows:

$$\bar{h}^T = \left\{-2K_3|\bar{V}_r|^3\left(\frac{\rho A_r}{2m}\right)^2 \bar{V}_r^T (C_{L\alpha}^2 - 2\eta\ C_A\ C_{L\alpha}) + \left[\frac{F}{m} + \frac{\rho A_r}{2m} C_{L\alpha}|\bar{V}_r|^2\right][\lambda_4,\lambda_5,\lambda_6]\right\} \times$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \left\{\frac{\rho A_r}{2m} (2\eta - 1)C_{L\alpha} [\lambda_4,\lambda_5,\lambda_6]\right\} [\bar{V}_r\ \bar{V}_r^T] \tag{9}$$

Thus, H can be minimized with respect to $\bar{u}$ by choosing $\bar{u}$ to be a unit vector opposite in direction to $\bar{h}$. That is,

$$\bar{u} = -\frac{\bar{h}}{|\bar{h}|} .$$

H is minimized with respect to $\bar{u}$ with this selection of $\bar{u}$ because

$$H = \bar{z} + \bar{h}^T\bar{u} = \bar{z} + |\bar{h}|\ \cos\ \theta$$

(where θ is the angle between $\bar{u}$ and $\bar{h}$) and thus H is a minimum when $\theta = \pi$ which gives the above expression for $\bar{u}$.

To summarize our problem in state vector notation, we combine the necessary conditions with the original condition to yield the usual form of boundary problems associated with the trajectory optimization problem. That is, determine λ_o, t_f, and γ to satisfy

$$\left.\begin{aligned} &\dot{x} = f(x,u) \\ &\dot{\lambda} = -(\partial f/\partial x)^T \lambda \\ &\text{minimum } \lambda^T f(x,u) \end{aligned}\right\} \text{ System of Differential Equations} \tag{10}$$

$$\left.\begin{aligned} &F[x(t_f),t_f] = 0 \\ &\lambda_f^T = -(\partial J/\partial x_f)^T - (\partial F/\partial x_f)^T\gamma \\ &\partial J/\partial t + \gamma^T \dot{F}_f = 0 \end{aligned}\right\} \text{ Boundary Conditions} \tag{11}$$

where $x(t_o)$ and t_o are assumed fixed.

The MASCOT solution of this complicated boundary-value problem on board a vehicle has been possible for the following reasons.

First of all, an explicit expression has been found for the optimal control $\bar{u}$. This expression saves both computer memory and computer computation time.

Second, the unified set of guidance equations and performance criteria reduced the computer storage requirement.

Third, the techniques for obtaining a solution of the boundary-value problem have been carefully studied and the so-called "shooting method" selected because of its speed and reliability. The shooting method algorithm is easy to program and is compact in size. The alternatives to the shooting method, such as steepest descent and quasilinearization, must use stored time functions so that the size and complexity of the algorithms are increased.

Fourth, the development of the Fehlberg-type, Runge-Kutta numerical integration routines has significantly speeded up the numerical integration process.

Fifth, the boundary conditions obtained by applying the shooting method to obtain a trial trajectory will not usually be satisfied within a specified tolerance. Thus, some method of changing the initial guesses in order to satisfy the boundary conditions is needed. This problem can be considered as the problem of finding the solution to a system of simultaneous nonlinear equations because the boundary conditions can be considered as nonlinear functions of the missing initial conditions. A modified Newton's Method and a modified Secant Method have allowed rapid solution of this sytem of equations.

Finally, the improvements in computational hardware which have resulted in flight-worthy high-speed computers have made the whole concept much more attractive.

VII. QUALITATIVE RESULTS FOR NASA SHUTTLE VEHICLE

The feasibility of solving the trajectory optimization problem in real-time for onboard guidance was demonstrated with a computer program for an exoatmospheric ascent to orbit mission in 1966 [8]. Since that time, the multi-burn logic has been added to the program as well as the lift and drag force for atmospheric flight. Guidance studies are being performed for the shuttle vehicle using the new MASCOT program and the detailed results of these studies will be published in a later report. However, some qualitative results will be presented, and they have been broken down into three flight phases.

A. Boost

Our studies have shown a very large sensitivity of the gain in payload to particular vehicle configurations. The most important items identified at this time have been the lift and drag characteristics of the vehicle, and the thrust-to-weight ratio at liftoff of the vehicle. Low take-off accelerations (around 1.2 g's) show higher gain in payload than vehicles with larger take-off accelerations (1.4 g's). Various combinations of lift, drag, and take-off accelerations have resulted in payload gains from near zero to approximately 10,000 pound increases for vehicles with liftoff weights near 3.5 million pounds.

This gain in payload is due to a combination of aerodynamic effects and trajectory shaping and represents a gain with respect to a gravity turn trajectory.

The Apollo guidance system was not designed to optimize trajectories through the atmosphere and could not, in its present form, provide this payload gain.

Optimal lifting trajectories through the atmosphere have shown an increase in the maximum dynamic pressure experienced by the booster during the ascent phase. This increase in dynamic pressure may have an effect on the structural design of the vehicle which has not been evaluated, either from the loads point of view or from the standpoint of heating.

B. On-Orbit, Rendezvous, and Deorbit

The uniform guidance concept proposed in this report allows the use of the same performance criteria during the on-orbit, rendezvous, and deorbit portions of the flight.

Some very essential improvements in the mechanization of the guidance scheme came as a result of the application of transversality conditions to obtain switching functions for the coast-burn-coast-burn type trajectory. The MASCOT formulation permits optimal orbital transfer for non-coplanar transfers. These non-coplanar cases are not possible to achieve with the formulation of present guidance schemes such as the Apollo IGM.

Optimal deorbit maneuvers are also possible with the unified concept regardless of the thrust level involved. Other schemes presently require careful tuning for the particular thrust level available.

C. Orbiter Reentry

During the reentry of the orbiter, it is desirable to maximize the landing footprint, especially the crossrange component, minimize the acceleration, and also to minimize the heating and heating rates during the descent.

The performance criteria make possible a tradeoff of these features, and some early trajectory runs indicate that a satisfactory compromise is possible to achieve all of the desired conditions within satisfactory limits.

Early computations have shown a very high sensitivity of the reentry trajectories to the choice of the initial values of the LaGrange multipliers and consequently a very poor convergence rate.

During recent weeks, an ingenious method has been devised to eliminate this high sensitivity and poor convergence. This method has been the assumption that $\bar{u}(t)$ could adequately be represented as a linear function of time during the atmospheric portions of flight.

Studies are currently underway to determine whether it will be necessary to use the solutions so obtained as first approximations to the true optimum. Presently, no further corrections appear to be necessary.

VIII. CONCLUSIONS

A fast, efficient, compact trajectory algorithm for a total trajectory from liftoff to landing has been made possible by an explicit optimal control, a rapid solution of the boundary value problem, a rapid numerical integration routine, an efficient solution of simultaneous nonlinear equations, and a vastly improved on-board flight computer to achieve a major advance in guidance techniques.

Preflight analysis has been reduced to a minimum thus making it possible to meet the "launch within two hours" constraint. Special tuning constants and functions have been eliminated, which heretofore have been necessary to account for approximations introduced to simplify on-board computation. Optimal trajectories are guaranteed for an extremely wide variety of missions and vehicle characteristics.

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APPROVAL

MASCOT - A NEW CONCEPT IN GUIDANCE

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The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

for

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