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SONIC RADIATION FROM A CIRCULAR PISTON WITH IMPULSE EXCITATION

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16. Abstract

The sonic pressure as a function of space and time has been calculated in closed analytic form for a circular piston whose displacement is a step function in time. The method of calculation involves approximating the piston by a uniform array of very small spherical sources.

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SONIC RADIATION FROM A CIRCULAR PISTON WITH IMPULSE EXCITATION

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SUMMARY

Closed analytic expressions have been derived for the sonic pressure as a function of space and time when a circular piston is excited so that its velocity varies with time as a Dirac delta function. These expressions are valid not only for the far field but also for the near field. Moreover a number of typical plots of the sonic pressure versus time for various special points have been made using the results of the calculations. The basis of the method of calculation is the approximation of the circular piston by an array of very small spheres uniformly distributed over the area of the piston. Each of these spheres are assumed to have their radius increase as a step function in time at the instant of excitation. The sonic field for such an excited sphere is well known and is relatively simple in form.

* * * * * * *

The sonic field for a circular disk with impulse excitation has been solved for distances large compared to the disk radius (ref. 1). For small distances and distances comparable to the radius, the evaluation becomes too complex when conventional methods are employed.

In the work to be reported here, a different approach to the solution has been taken which allows its evaluation for both the near and far field without too much difficulty. The first step in attaining the solution is to illustrate the configuration of the geometric system and this is done in Figure 1. As can be seen, the flat excitor disk lies in the XY plane with its center at the origin. The field measurement point, M, lies in the YZ plane, and has the coordinates (y,z). It is assumed that all points in the front face of the disk move in the +z direction during excitation, and that these points move with a velocity, u, given by:

$$u = \Delta\delta(t) \qquad (1a)$$

As can be seen from studying Eq. (1a), the whole disk face suddenly moves a distance, Δ, in the z direction at t=0, and then remains stationary thereafter. ($\delta(t)$, incidentally, is the Dirac delta function).

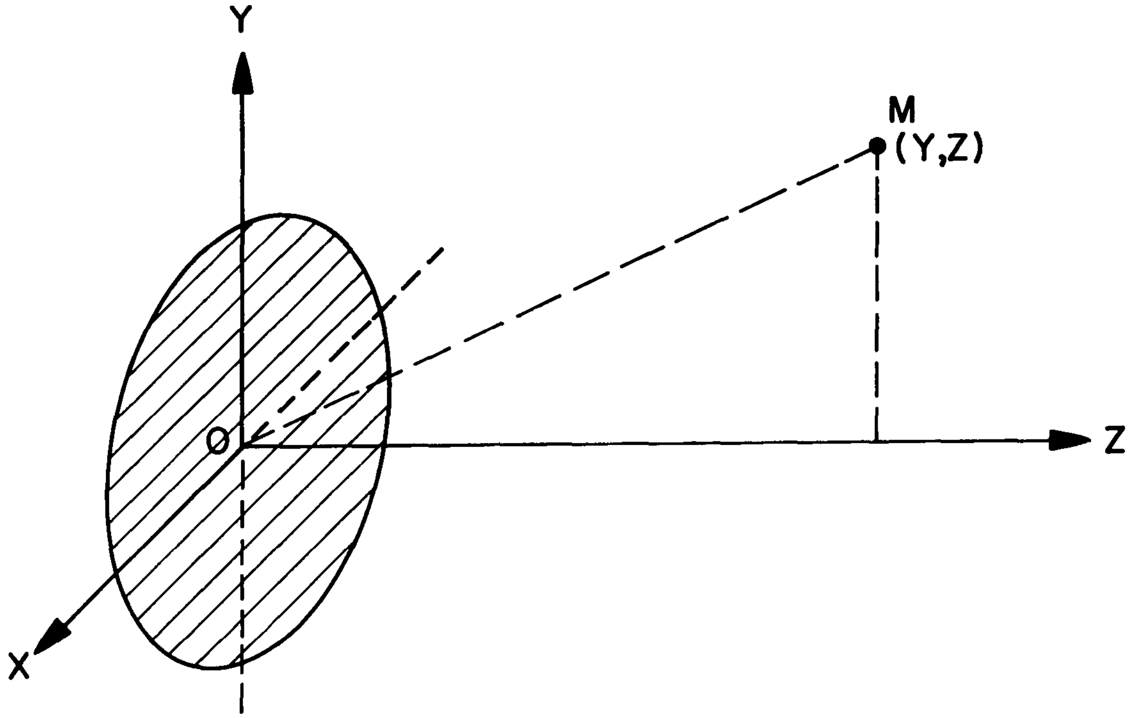

Figure 1.- System geometric configuration

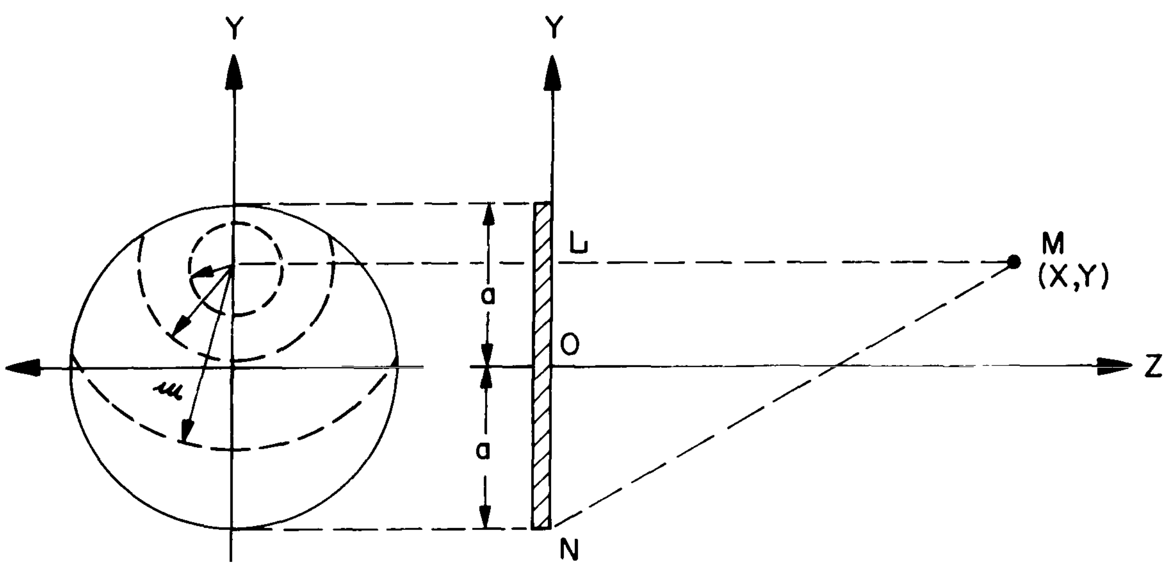

Figure 2.- Case I- $0 \leq y \leq 1$

For the method adopted here, the starting point of the whole calculation is the determination of the radiation field from a very small sphere. The radius of the sphere is to suddenly increase by an amount, Δ_S. The assumption is now made that a very large number of these small spheres with their centers distributed uniformly over the area of the disk shown in Figure 1 would approximate the action of a disk whose thickness suddenly increases by the amount, 2Δ, at $t=0$. One disk face moves in the $+z$ direction by an amount, Δ, and the other in the $-z$ direction by the same amount. The justification of this assumption forms the basis for the validity of the whole treatment and will now be gone into in detail.

The desired expression for the sonic pressure, P_d, for the present disk problem must satisfy only two conditions. The first is that it be a solution of the differential equation usually referred to as the wave equation (i.e., $\nabla^2 P_d - (1/c^2)(\partial^2/\partial t^2) P_d = 0$). The second condition is that P_d satisfy the following relations in the time interval, $0<t<\varepsilon(2/C)$.

$$P_d \cong \mathcal{C}\ \delta(t \pm z/c) \text{ for } y \leq a \tag{1b}$$

$$P_d = 0 \text{ for } y > a \tag{1c}$$

$\mathcal{C}$ is a constant, a is the disk radius, C is the velocity of sound, and ε is to be chosen so that it is a very small number (typically having a value around 1/100).

The radiation field from a small sphere has been treated in several places (ref. 2). (This result is only valid if the radius, r_o, of the small sphere is such that $\Delta_S \leq r_o << c\tau_p$ where τ_p is the duration of the excitation pulse.) This condition can be satisfied in most practical cases. The result for the sonic pressure, P_S, as a function of space and time is:

$$P_S(\underline{r},t) = \frac{\mu \Delta_S S}{4\pi r} \frac{d}{dt}\left[\delta(t - r/c)\right] \tag{1d}$$

where μ is the density of the fluid, and S is the surface area of the sphere.

Since the sonic pressure, P_S, for a single small sphere is a solution to the wave equation everywhere except on the disk, the total sonic pressure from a large array of these spheres will be such a solution also, because it is merely the linear super-position of a large number of the functions, P_S, (and the wave equation is a linear differential equation). Thus, the total array pressure satisfies the first condition mentioned above. It satisfies the second condition also, but this cannot become apparent until the total array pressure is calculated. The calculation has actually been performed in the treatment that follows. By temporarily skipping ahead, it can be verified that the total array pressure (to be labeled P_D) satisfied the boundary conditions at times near t=0. Thus, it can be seen that the P_D, as determined from Eqs. (9b) and (19) show that it agrees with Eq. (1c). Thus, since P_D satisfies the two conditions of acceptance given above, it follows that $P_D = P_d$, the desired solution.

From the description of the method of calculation given above, it would appear that the sonic pressure, P_D, generated by the action of the disk can be given by the following expression:

$$P_D(\underline{r},t) = (q/S)\int_0^a \int_0^{2\pi} P_S(\underline{r},t;\rho,\phi)\ \rho d\phi d\rho \tag{2}$$

where $\rho \equiv [x^2 + y^2]^{1/2}$; $\tan\phi = (x/y)$; and q is a source strength constant to be determined.

In evaluating the integral of Eq. (2), the ϕ integration is to be performed first. The result of the ϕ integration is dp_D which is the pressure wave that would result if only a ring of radius, ρ, and width, $d\rho$, were excited. Using Eqs. (2) and (1d), the explicit form of dp_D is:

$$dp_D = \left(\frac{q\mu\Delta_S}{4\pi}\right)(\rho d\rho)\ \frac{d}{dt}\left[\int_0^{2\pi}(d\phi/r)\,\delta(t - r/c)\right]_L \tag{3}$$

where $r = \left[z^2 + y^2 + \rho^2 - 2y\rho\cos\phi\right]^{1/2}$

The evaluation of Eq. (3) involves using the properties of a Dirac delta function. It can thus be seen that $dp_D \neq 0$ only for those values of t where t = r/c. Moreover, since only r appears explicitly in the argument of the delta function and the variable of integration is ϕ, a factor of $c|dr/d\phi|^{-1}$ must appear. The result of the ϕ integration of Eq. (3) is then:

$$dP_D = 0 \text{ for } t<\check{\tau} \text{ and } t>\hat{\tau} \tag{4a}$$

$$dP_D = -\left(\frac{\mu\Delta_S\rho d\rho}{4\pi}\right)\frac{d}{dt}\left(\frac{c}{r|dr/d\phi|}\right)_{\phi=\phi_t} \tag{4b}$$

where $\hat{\tau} \equiv \sqrt{z^2 + y^2 + \rho^2 + 2y\rho}$; $\check{\tau} \equiv \sqrt{z^2 + y^2 + \rho^2 - 2y\rho}$

and $ct = \sqrt{z^2 + y^2 + \rho^2 - 2y\rho \cos \phi_t}$

Note that there are two values of ϕ that can satisfy the ϕ_t equation. From Eqs. (3) and (4b) it follows that:

$$\left| r\frac{dr}{d\phi} \right|_{\phi = \phi_t} = \rho y |\sin \phi_t| = \frac{1}{2} \sqrt{4\rho^2 y^2 - (D^2 - \rho^2)^2} \qquad (5)$$

where $D^2 \equiv c^2t^2 - z^2 - y^2$

Let η be defined so that:

$$c^2t^2 = z^2 + y^2 + \rho^2 - 2\rho y\eta \qquad (6)$$

It then follows from Eqs. (4b) and (5) that:

$$dP_D = -\left(\frac{2q\mu c\Delta_s \rho d\rho}{2\pi}\right) \frac{d}{dt} \left[\frac{1}{2\rho y\sqrt{1 - \eta^2}}\right] \qquad (7)$$

It is to be noted that there is an extra factor of 2 in Eq. (7) to account for the fact that there are two values of ϕ satisfying the ϕ_t equation. η is a function of t only with $\eta = -1$ when $t = \check{\tau}$; $\eta = 1$ when $t - \hat{\tau}$ and $-1<\eta<1$ when $\check{\tau}<t<\hat{\tau}$. Equation (7), therefore, provides a more suggestive form for dP_D. Finally, it is of interest to note that:

$$\int_{\check{\tau}}^{\hat{\tau}} \frac{dt}{2\rho y \sqrt{1 - \eta^2}} = -\left[\frac{1}{2c \sqrt{z^2 + y^2 + \rho^2}}\right] \int_{-1}^{1} \frac{d\eta}{\sqrt{1 - \kappa\eta} \sqrt{1 - \eta^2}} \qquad (8)$$

where $\kappa \equiv 2\rho y / \left(z^2 + y^2 + \rho^2\right)$

With the use of Eqs. (7) and (8) it is possible to see that for large distances from the ring (i.e., where $z^2 + y^2 >> \rho^2$) the time and space dependence of dP_D begins to resemble that of Eq. (1d), the pressure wave from a small sphere. This is what is to be expected and therefore provides an accuracy check on the calculations.

The radiation pressure, P_D, for the total disk can now be obtained by integrating the result for the ring (i.e. dP_D given by Eqs. (4a), (4b) and (5)) with respect to the variable, ρ. When this is done, the result can be presented in the form:

$$P_D = 0 \text{ when } t>\hat{t} \text{ and } t<\check{t} \tag{9a}$$

$$P_D = \left(\frac{qc\mu\Delta_S}{\pi}\right)\frac{d}{dt}\int_{\check{\rho}}^{\hat{\rho}}\left[\frac{\rho d\rho}{\sqrt{4\rho^2 y^2 - (D^2 + \rho^2)^2}}\right]$$

$$\text{for } \check{t} \leq t \leq \hat{t} \tag{9b}$$

The limits of integration, $\check{\rho}$ and $\hat{\rho}$ will in general depend on t as well as y. The exact determination of these limits is rather complicated and, therefore, must be deferred until later. The times, $\check{t}$ and $\hat{t}$, are also to be determined.

It is possible to perform the ρ integration in Eq. (9b) by changing variables from ρ to σ so that $\sigma = D^2 - \rho^2$. If the quantity in the square bracket in Eq. (9b) is designated as I, then it can be shown that after substituting σ for ρ, I has the form:

$$I = \frac{1}{2}\int_{D^2 - \hat{\rho}^2}^{D^2 - \check{\rho}^2} \frac{d\sigma}{\sqrt{4y^2D^2 - 4y^2\sigma - \sigma^2}} \tag{10}$$

The integral of Eq. (10) can be evaluated exactly and the result is:

$$I = \frac{1}{2}\left\{\text{arc sin}\left[\frac{D^2 + 2y^2 - \check{\rho}^2}{2y\sqrt{D^2 + y^2}}\right] - \text{arc sin}\left[\frac{D^2 + 2Y^2 - \hat{\rho}^2}{2y\sqrt{D^2 + y^2}}\right]\right\} \tag{11}$$

There now remains the task of evaluating the limits, $\check{\rho}$ and $\hat{\rho}$. It can be seen that in doing this, two distinct cases naturally arise. Case I is where y lies in the range, $0\leq y\leq a$, and Case II is where $y>a$. Case I is to be considered first and this can best be done by referring to Figure 2. This is seen as a representation on the XY plane of the measurement point, M, and the disk trace (shown by the cross-hatched bar). There is also a representation of the disk on the YZ plane. Upon studying this figure, it is apparent that the point on the disk closest to the point, M, is the point, L, which is the intersection of the line, LM, with the surface of the disk. The line, LM, lies in the XY plane and is parallel to the x axis. It is obvious that the minimum time, $\check{t}$, in the pulse is determined by the product of (1/c) and the line length, LM. Similarly, it can be seen that the point on the disk having the greatest distance to M is N, the point at the bottom of the disk. Thus, the maximum pulse time, $\hat{t}$, is (1/c) times the length, $\overline{MN}$. From the above considerations it follows that:

$$\check{t} = z/c \tag{12a}$$

$$\hat{t} = (1/c)\sqrt{z^2 + (y + a)^2} \tag{12b}$$

It can also be seen from a study of Figure 2 that for any given t (in the range $\check{t}$ to $\hat{t}$), the locus of all points on the surface of the disk having a distance, ct, to the point, M, is a circle whose center is at the point, (0,y,0) and which has a radius ξ, given by

$$\sqrt{c^2t^2 - z^2} \; .$$

It follows from this that the minimum radius, $\check{\rho}$, of any ring contributing to the integral of Eq. (9b) is determined by the shortest distance from the origin to any point on the locus circle. Similarly, the maximum radius, $\hat{\rho}$, is determined by the greatest distance of any point on the locus circle to the origin.

From these considerations and a study of Figure 2, it can be shown that $\check{\rho}$ and $\hat{\rho}$ are given by:

$$\hat{\rho} = y + \xi \qquad \text{for } \xi < a - y \tag{13a}$$

$$\hat{\rho} = a \qquad \text{for } \xi > a - y \tag{13b}$$

$$\check{\rho} = y - \xi \qquad \text{for } \xi < y \tag{14a}$$

$$\check{\rho} = \xi - y \qquad \text{for } \xi > y \tag{14b}$$

$$\check{\rho}^2 = D^2 + 2y^2 - 2y\sqrt{D^2 + y^2} \quad \text{for all } \xi \tag{14c}$$

where $\xi \equiv \sqrt{c^2t^2 - z^2} = \sqrt{D^2 + y^2}$

Using the results of Eq. sets (13) and (14) in Eq. (11), the final result for I when y<a is:

$$I = \pi/2 \qquad \text{for } \check{t} < t < \bar{t} \tag{15a}$$

$$I = \left\{ \frac{\pi}{4} - \frac{1}{2} \text{ arc sin} \left[\frac{D^2 + 2y^2 - a^2}{2y\sqrt{D^2 + y^2}} \right] \right\} \tag{15b}$$

where $\bar{t} \equiv (1/c)\sqrt{z^2 + (a - y)^2}$ for $\bar{t} < t < \hat{t}$

For Case II where y>a, it is possible to determine $\check{\rho}$ and $\hat{\rho}$ in the same manner that they were determined in Case I. The main difference is that instead of using Figure 2 to determine the various geometric relations, Figure 3 must now be used. Just as in Case I, the length, $\overline{MN}$, represents the longest distance between any disk point and the point, M. However, for Case II, $\overline{QN}$ represents the shortest distance instead of $\overline{LM}$. From an analysis of Case II similar to the one already performed for Case I, it is possible to show that the following relations are valid:

$$\check{t} = (1/c)\sqrt{z^2 + (y - a)^2} \tag{16a}$$

$$\hat{t} = (1/c)\sqrt{z^2 + (y + a)^2} \tag{16b}$$

$$\check{\rho} = y - \xi \qquad \text{for } \xi < y \tag{17a}$$

$$\check{\rho} = \xi - y \qquad \text{for } \xi > y \tag{17b}$$

$$\check{\rho}^2 = D^2 + 2y^2 - 2y\sqrt{y^2 + D^2} \qquad \text{for all } \xi \tag{17c}$$

where $\xi = \sqrt{D^2 + y^2}$

$$\hat{\rho} = a \qquad \text{for } \check{t} < t < \hat{t} \tag{18}$$

Using the results of Eq. sets (17) and (18) to obtain $\hat{\rho}$ and $\check{\rho}$, the value of the quantity, I, of Eq. (11) can be seen to be (for y>a):

$$I = \left\{ \frac{\pi}{4} - \frac{1}{2} \text{ arc sin} \left[\frac{D^2 + 2y^2 - a^2}{2y\sqrt{D^2 + y^2}} \right] \right\} \tag{19}$$

$$\text{for } \check{t} < t < \hat{t}$$

With the derivation of Eq. sets (15) and (19), the quantity, I, has been completely determined and, therefore, with the use of Eq. set (9), the form of the pressure, P_D, can be derived. To completely determine P_D, however, it is necessary to find the

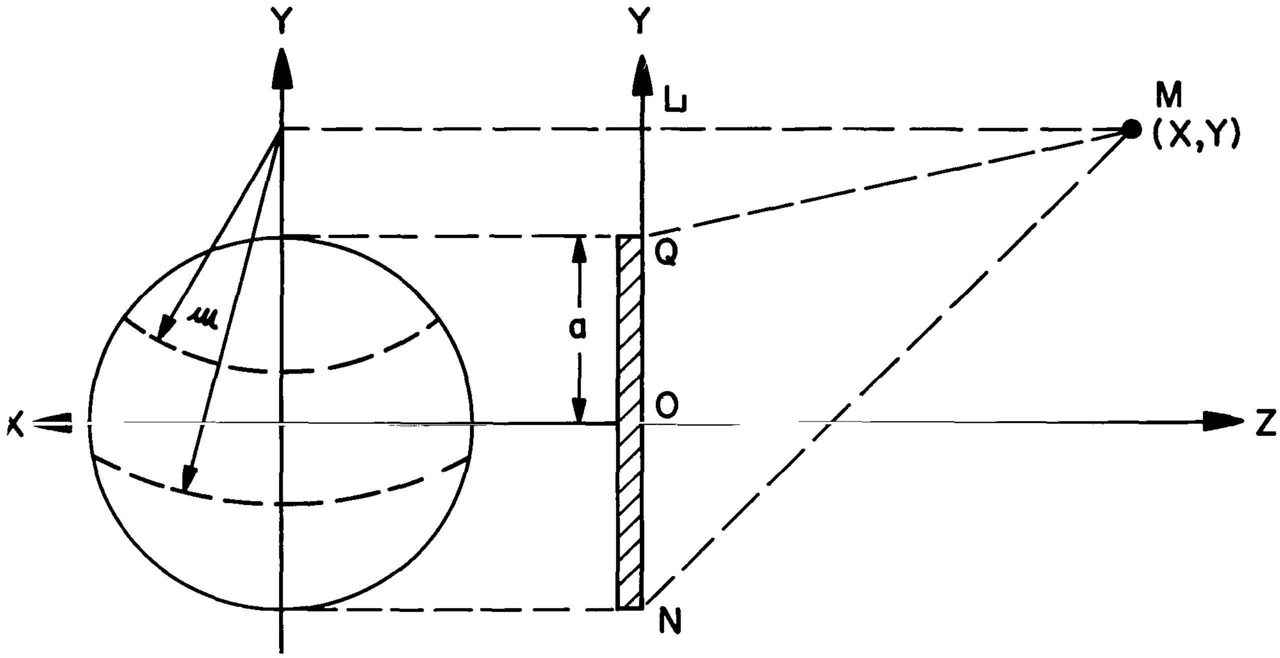

Figure 3.- Case - y > a

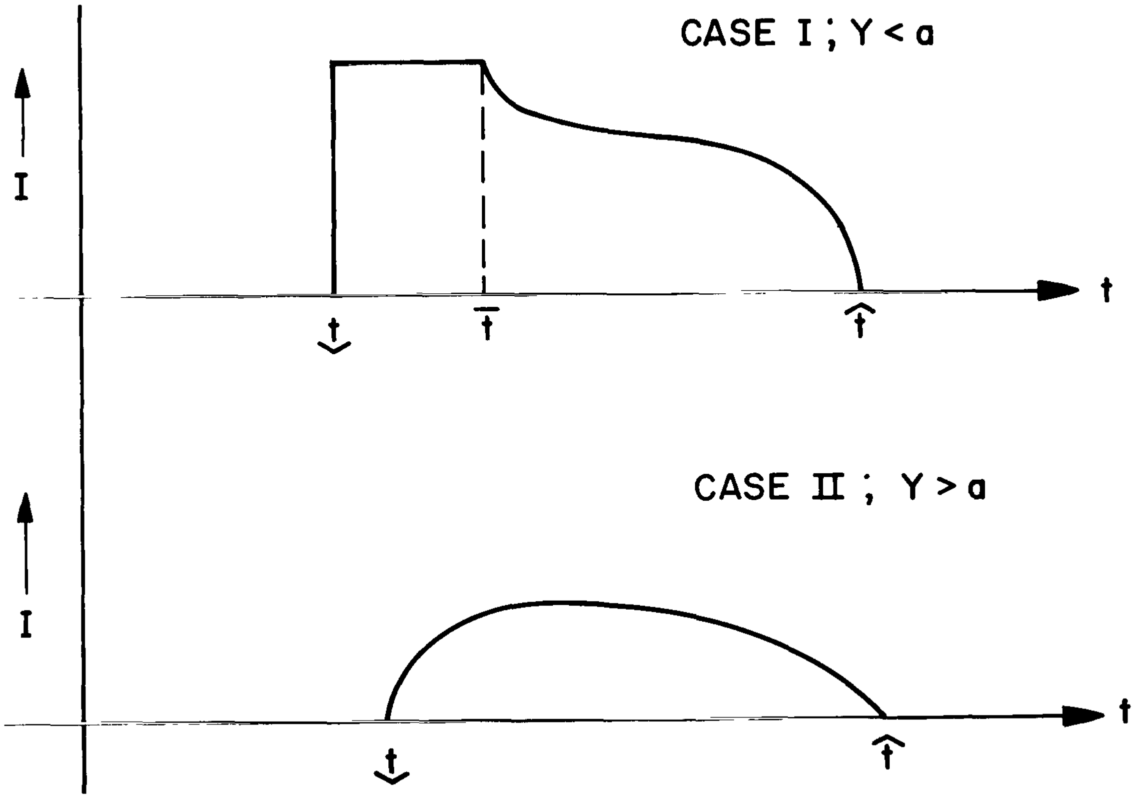

Figure 4.- Plot of I vs t for Cases I and II

value of the source strength constant, q. To do this and also to provide a check for some of the calculations presented above, a simple special case of the sonic radiation field is now to be treated. This case is where y = 0 (i.e. M lies on the X axis). Here it is obvious that

$$\check{t} = z/c \text{ and } \hat{t} = \frac{1}{c}\sqrt{z^2 + a^2}.$$

Moreover it follows that:

$$P_D = 2\pi\left[\frac{\mu\Delta_S q}{4\pi}\right]\frac{d}{dt}\left[\int_0^a\left(\frac{\rho d\rho}{r}\right)\delta\,(t - r/c)\right] \tag{20}$$

If the variable of integration is changed from ρ to r, and the relation ρdρ = rdr is used, Eq. (20) becomes transformed to:

$$P_D = 2\pi\left[\frac{\mu\Delta_S q}{4\pi}\right]\frac{d}{dt}\left[\int_x^{\sqrt{x^2+a^2}} dr\;\delta(t - r/c)\right] \tag{21}$$

$$\text{for } \check{t} < t < \hat{t}$$

The integration indicated in Eq. (21) can be readily performed and it produces a pair of step funtions. These become delta functions again when differentiated with respect to t so that the final result for P_D is:

$$P_D = \left[(1/2)qc\mu\Delta_S\right]\left[\delta(t - z/c) - \delta\left(t - (1/c)\sqrt{z^2 + a^2}\right)\right] \tag{22}$$

$$\text{for } \check{t} < t < \hat{t}$$

The result of Eq. (22) is now to be subjected to two qualitative checks. It is to be expected that when $a\to\infty$, the form of P_D approximates that where the piston is an infinite plane with all the surface points moving with the velocity defined by Eq. (1a). The solution for this can readily be obtained and is found to have the form of the delta function, $\delta(t - z/c)$. It can be seen that as $a\to\infty$, the form of P_D does indeed approach the required form of $\delta(t - z/c)$, since the second term in Eq. (22) appears only after a very long time. Thus, Eq. (22) is able to pass the first check.

As a second check it is to be expected that when $z\to\infty$, P_D approaches the form obtained from a pulsating spherical monopole as given by Eq. (1d). It is to be noted, first, that $[\delta(u+\Delta)-\delta(u)\cong\Delta d/du[\delta(u)]]$ when $\Delta<<u$. Thus it follows from

Eq. (22) when z>>a:

$$P_D \cong \left[\frac{1}{2} q \mu \Delta_S\right]\left[\frac{a^2}{2z}\right] \frac{d}{dt}\left[\delta\ t - \frac{z}{c}\right] \tag{23}$$

Since, in this case z=r, it can be seen that Eq. (23) agrees in form with Eq. (1d) and, thus Eq. (22) is able to pass the second check also.

In order to determine q, the source strength constant, the same boundry conditions used for the small sphere (ref. 2) can be invoked here. These are that on the surface of the piston the relation, $\mu\, du_z/dt = -dP_D/dz$, is valid. Using Eq. (1a) to give u_z and Eq. (22) to give P_D, it follows that the boundry conditions require that the various parameters be related as follows:

$$\Delta = \frac{1}{2}\, q\ \Delta_S \tag{24}$$

Since the general expression for P_D given by Eq. (9b) involves the entire quantity, $(q\Delta_S)$. the source strength constant, q, is, in effect, determined by Eq. (24).

It is of interest to note that the form of P_D given by Eq. (22) agrees with the P_D given in the main text (from Eqs. (9b), (15a), and (15b)) when y = 0. It can be shown that for these forms to agree, I in Eq. set (15) must be a function such that I = 0 for $t < \check{t}$ and $t > \hat{t}$; $I = \pi/2$ for $\hat{t} > t > \check{t}$. This condition is satisfied by Eq. set (15) since it readily follows from Eq. (12b) and Eq. set (15) that when y = 0, $\bar{t} = \hat{t}$. Thus, Eq. (22) is able to pass the final check.

The final task of the treatment is to present the quantity, dI/dt, in explicit form. Doing this requires a straight-forward but rather tedious differentiation of the quantity, I. Without presenting any of the calculational details, the final result is given by the following equations:

$$\frac{dI}{dt} = 0 \text{ for } t > \hat{t} \text{ and } t < \check{t} \tag{25a}$$

$$\frac{dI}{dt} = \frac{1}{2}\left[\frac{-\frac{d}{dt}(G - J)}{\sqrt{1 - (G - J)^2}}\right] \tag{25b}$$

$$\text{for } \check{t} < t < \hat{t}$$

where:

$$G \equiv \frac{c^2t^2 - z^2 + y^2}{2y\sqrt{c^2t^2 - z^2}} \; ; \quad \frac{dG}{dt} \equiv \frac{c^2t\left[c^2t^2 - z^2 - y^2\right]}{2y\left(c^2t^2 - z^2\right)^{3/2}}$$

Case I; $y < a$.

$$J \equiv \frac{a^2}{2y\sqrt{c^2t^2 - z^2}} \qquad \text{for} \quad \bar{t} < t < \hat{t} \tag{26a}$$

$$\frac{dJ}{dt} = \frac{-a^2c^2t}{2y\left(c^2t^2 - z^2\right)^{3/2}} \qquad \text{for} \quad \bar{t} < t < \hat{t} \tag{26b}$$

Case II; $y > a$.

$$J = \frac{a^2}{2y\sqrt{c^2t^2 - z^2}} \tag{27a}$$

$$\frac{dj}{dt} = \frac{-a^2c^2t}{2y\left(c^2t^2 - z^2\right)^{3/2}} \tag{27b}$$

With the derivation of the quantity, dI/dt, the sonic radiation field has now been completely determined. The character of the field can probably be best obtained by substituting in the pertinent equations above to get a plot of I versus t for different values of x and z. This operation has been carried out, and the results are given below. However, it is felt to be more instructive to first obtain a rough qualitative description of the sonic field from the study of the pertinent equations.

Before proceeding with the description, it is necessary that, as an aid to the discussion, a rough general plot of I versus t for both Cases I and II be made. This is done in Figure 4. It can be seen from this figure that it is not necessary to calculate dI/dt for the time interval, $\check{t} < t < \bar{t}$, in Case I, since it always has the same form in this interval.

From Eq. (22) it can be seen that when $y = 0$, P_D is in the form of two delta functions desplaced in time with respect to one another. This is to say that when the piston is excited by a very sharp time pulse, the pressure waves it generates on the x axis will also be sharp pulses. Off the x axis but for $y < a$, the P_D versus t plot remains two pulses displaced in time and the leading pulse stays sharp (i.e. it is a delta function). The lagging pulse, however, gets spread out (to a pulse duration of around a/c). This result is perhaps the most important one in the whole treatment since it accounts for the fact that sharp pulse excitations can produce sharp sonic pulses over extended regions.

When the measuring point, M, is far off the x axis (where y > a) it is found that both the leading and lagging pulse is spread out with pulse durations of the order of ac. Moreover, as y becomes larger, the amplitude of the pulse (as well as that of the quantity, I) decreases in value.

As mentioned above, the derivation obtained here has been used to prepare plots of I versus t and (dI/dt) versus t for various spatial points. Figures 5, 6, and 7 give I versus t and Figures 8, 9, 10, 11, and 12 give (dI/dt) versus t. These curves are self explanitory. Figure 12 essentially gives a picture of the "wave front" at the instant of time, t = (4.0)(a/c). The vertical arrows in the above figures indicate Dirac delta functions.

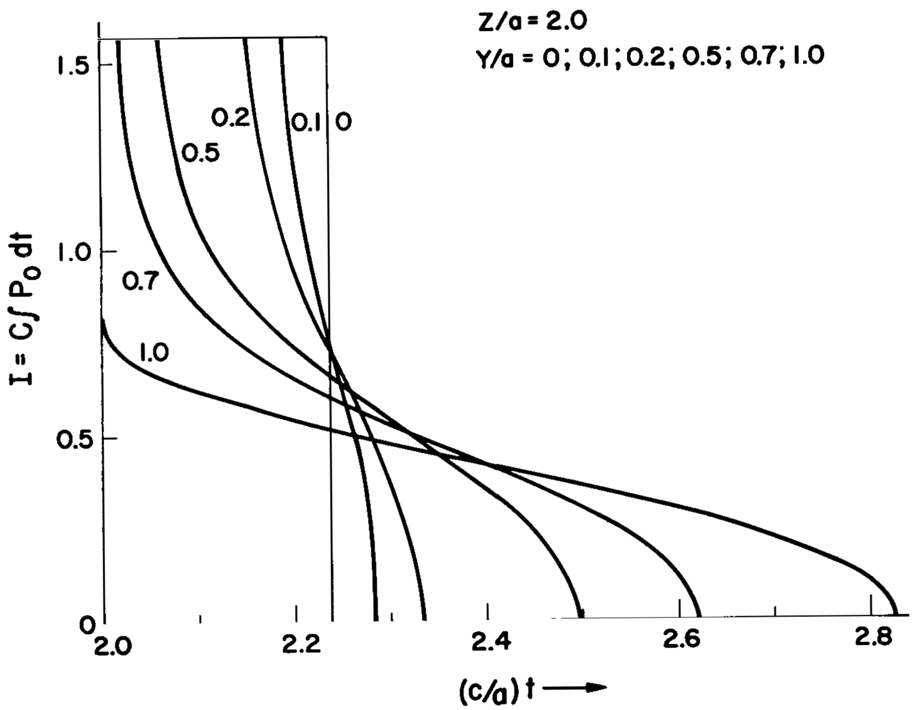

Figure 5.- I vs t

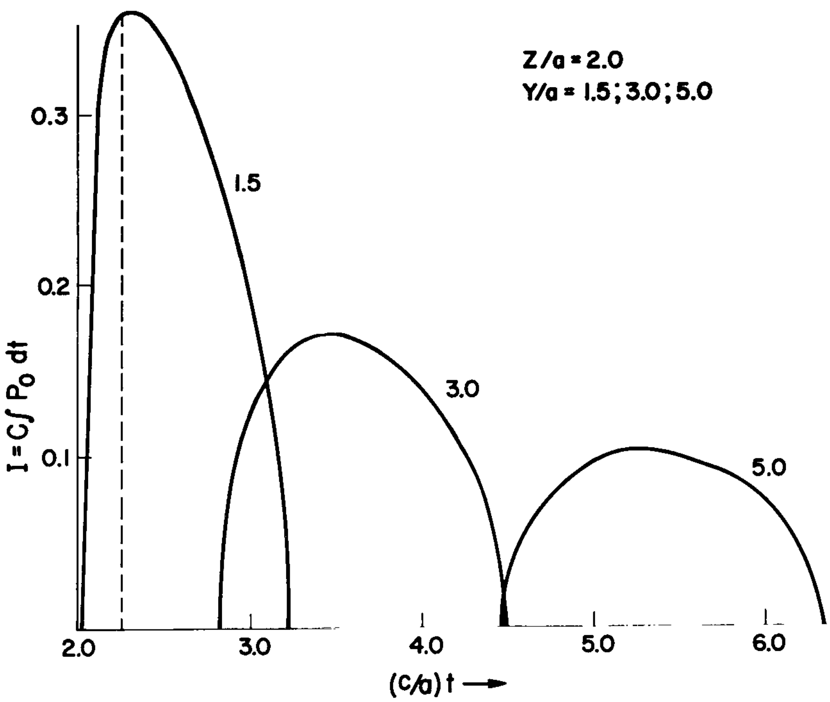

Figure 6.- I vs t

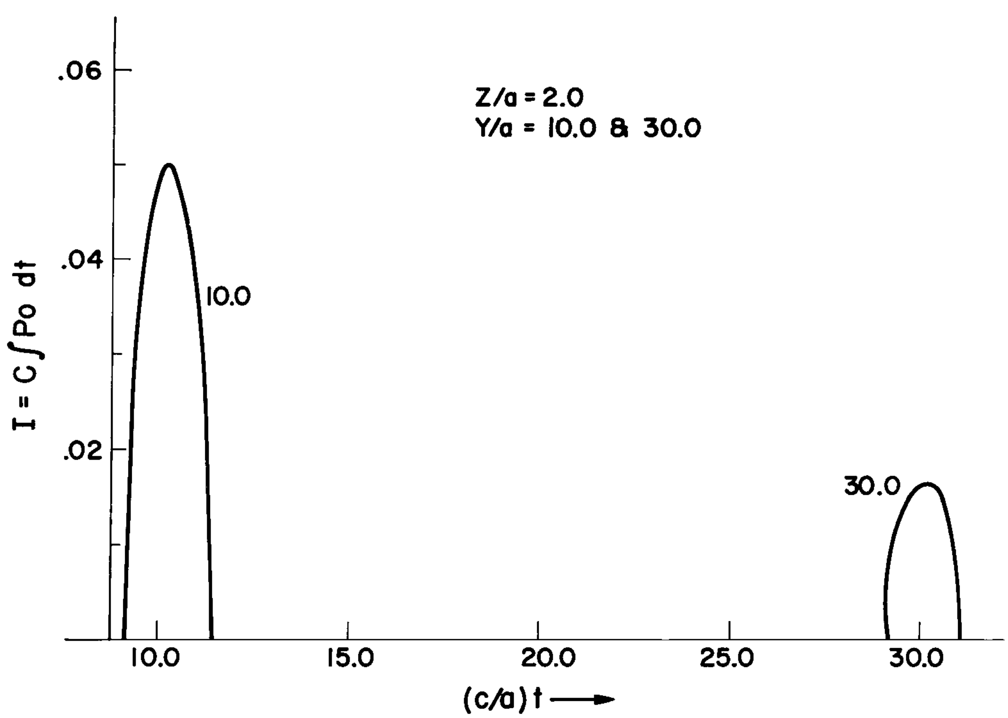

Figure 7.- I vs t

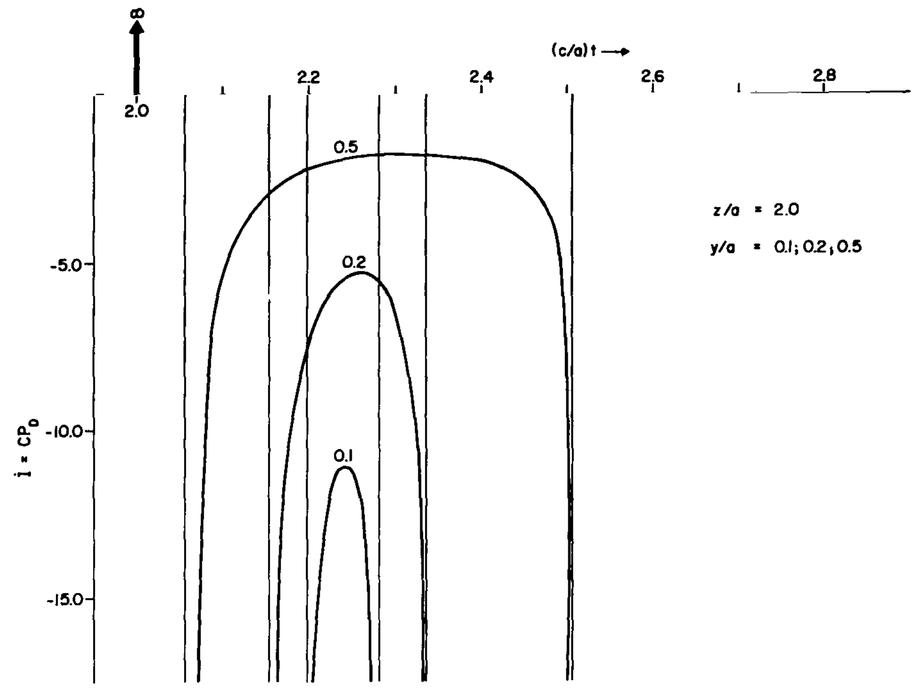

Figure 8.- dI/dt vs t

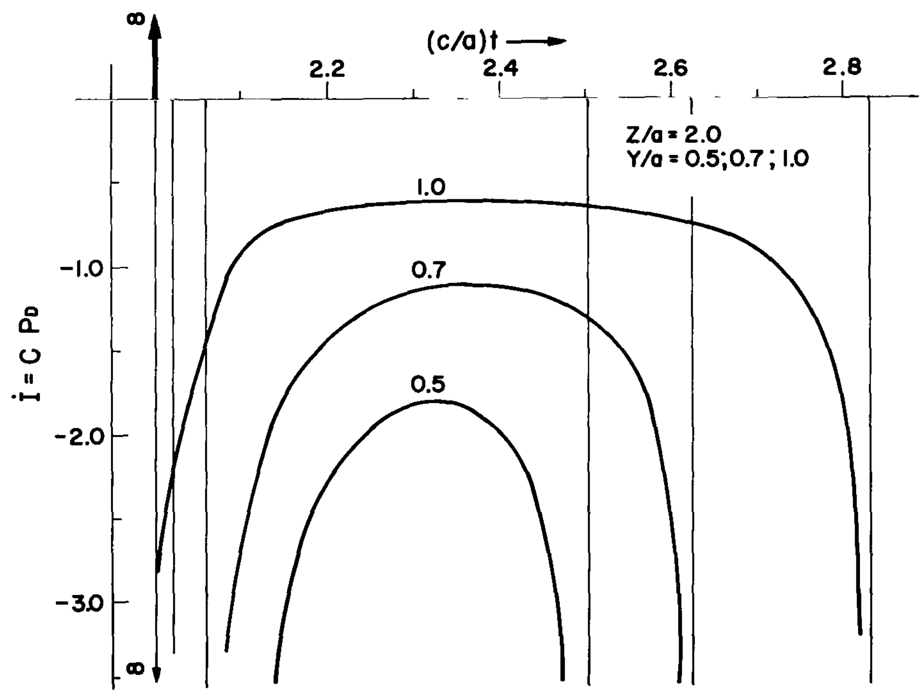

Figure 9.- dI/dt vs t

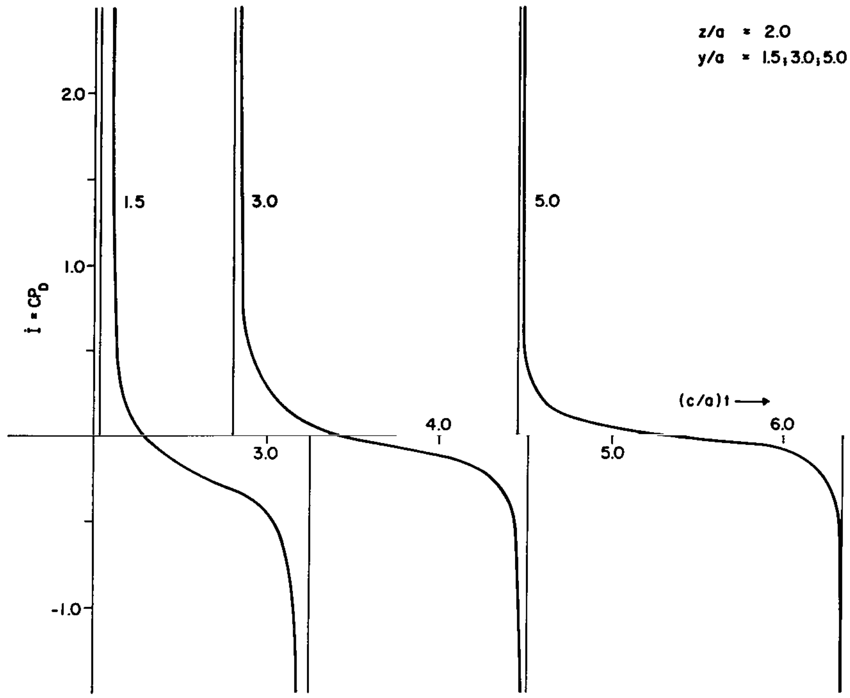

Figure 10.- dI/dt vs t

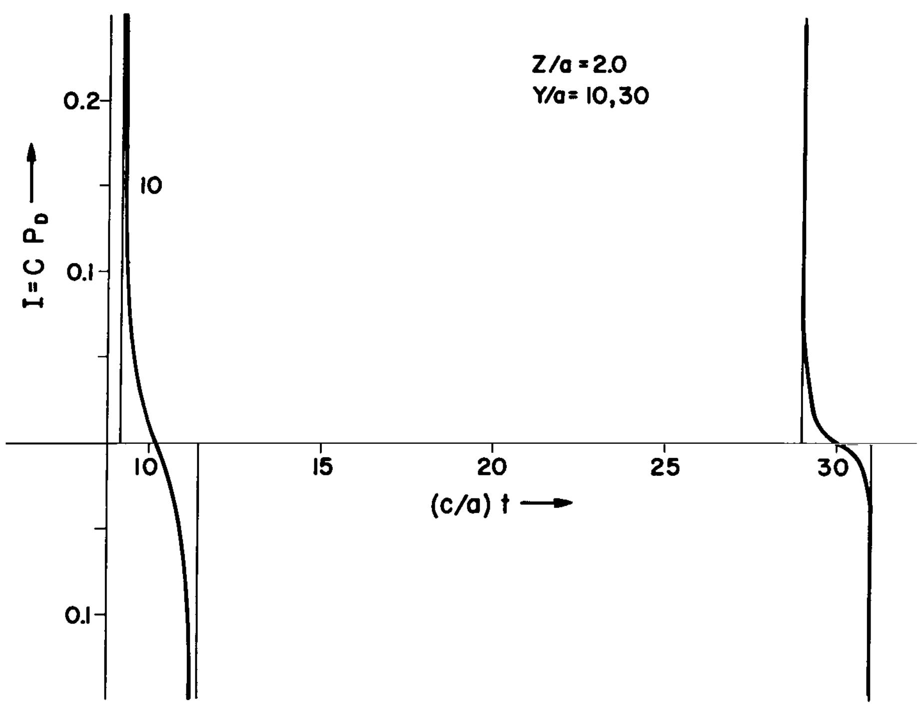

Figure 11.- dI/dt vs t

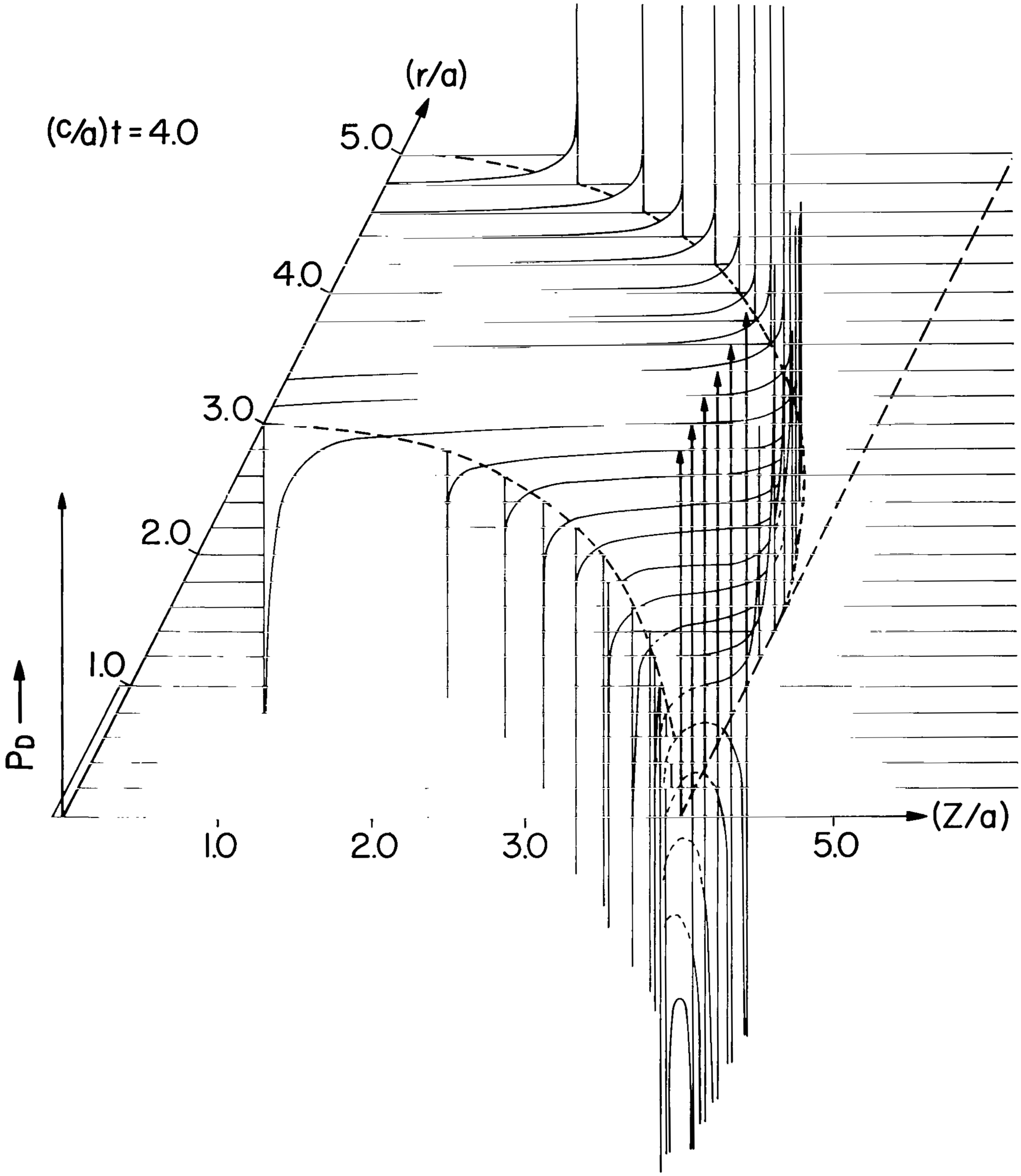

Figure 12.- dI/dt vs t (t = (4.0)(a/c))

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