X-552-71-111
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ANALYTICAL STUDIES OF LUNI-SOLAR EFFECTS ON THE MOTION OF ARTIFICIAL SATELLITES

THEODORE L. FELSENTREGER
JAMES P. MURPHY

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GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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ANALYTICAL STUDIES OF LUNI-SOLAR EFFECTS ON THE MOTION OF ARTIFICIAL SATELLITES

By

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Abstract

Theoretical and experimental studies indicate that long periodic lunar and solar forces produce perturbations in the orbital elements of some close earth satellites which are comparable to long period geopotential zonal harmonic effects. Analytical formulations for third body, solar radiation pressure, tidal, and precession and nutation effects have been successfully used to account for observable long period variations in the orbital elements of such satellites as Relay 1, Relay 2, and Telstar 2. For example, in the case of Relay 1, a variation in the eccentricity with an amplitude of about .0007 and a period of about 400 days was successfully attributed to luni-solar forces using these formulations.

Remaining variations in some orbital elements (after removal of the luni-solar, etc. effects) provide strong evidence for adjustment of geopotential parameters (used in the orbit determinations) to more recent values. For example, use of the value -2.504×10^{-6} for J_3 would account for the remaining variation in the eccentricity of Relay 2 (the value -2.285×10^{-6} was used in the orbit determination).

Preliminary attempts to deduce seasonal variations in the geopotential have been promising. This study involves an analysis of the long period variations remaining in the orbital elements of satellites after removal of all zonal harmonic, luni-solar gravitation, solar radiation pressure, tidal, and precession and nutation effects.

Discussion

Introduction

Most orbit theories for close earth satellites neglect lunar and solar forces with respect to the effects caused by the Earth's gravity field. However, an examination of the parameters in the long period parts of the disturbing functions indicates that some luni-solar forces give rise to perturbations which are comparable to long period zonal harmonic effects. Moreover, these perturbations are observable and predictable in the orbital element histories of some earth satellites over time spans covering hundreds of days.

In order to demonstrate this, analytic expressions for lunar and solar gravitational (secular and long period), solar radiation pressure (long period), tidal (long period), and precession and nutation (long period) effects have been formulated (References 1, 2, and 3). These formulations were obtained from a straightforward integration of the variation equations for the Keplerian orbital elements (semi-major axis, eccentricity, inclination, mean anomaly, argument of perigee, and longitude of ascending node). The elements were referred to the equator of date and the equinox at an initial epoch; the integration was performed by assuming only the arguments of trigonometric terms to be functions of time. Resonances were treated as special cases.

The satellite elements used in the study were obtained from a reduction of tracking data using an orbit determination computer program based upon the 1959 artificial satellite orbit theory of Brouwer (Reference 4). Hence, these elements are the so called "mean," or "double-primed," elements of Brouwer. The theory involves only the geopotential zonal harmonics through J_5; the following values were used:

$$J_2 = 1.08219 \times 10^{-3} \qquad J_4 = -2.123 \times 10^{-6}$$

$$J_3 = -2.285 \times 10^{-6} \qquad J_5 = -2.32 \times 10^{-7}.$$

The study was carried out on a number of satellites. However, only some of the more interesting results for the satellites Relay 1, Relay 2, Telstar 2, and Nimbus 2 will be reported here. There was a particularly interesting resonance phenomenon in the case of Relay 2 for which a special analytic formulation was derived (Reference 5). Also, an observable long period variation in the semi-major axis of Telstar 2 was found to be attributable to an indirect solar radiation pressure effect caused by passage of the satellite in and out of the Earth's shadow (Reference 6).

Perturbations in Orbital Elements

The secular and long period perturbations in the Keplerian orbital elements were obtained from integration of the following variation equations, which include coupling effects with zonal harmonics:

$$\left.\begin{aligned}\frac{de}{dt} &= -\frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial \omega}, \\ \frac{di}{dt} &= \frac{\cos i}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial \omega} - \frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial \Omega},\end{aligned}\right\} (1)$$

$$\left.\begin{aligned}\frac{d\ell}{dt} &= -\frac{2}{na}\frac{\partial R}{\partial a} - \frac{1-e^2}{na^2 e}\frac{\partial R}{\partial e} + \frac{d\dot{\ell}}{de}\delta e + \frac{d\dot{\ell}}{di}\delta i, \\ \frac{d\omega}{dt} &= -\frac{\cos i}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial i} + \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial e} + \frac{d\dot{\omega}}{de}\delta e + \frac{d\dot{\omega}}{di}\delta i, \\ \frac{d\Omega}{dt} &= \frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial i} + \frac{d\dot{\Omega}}{de}\delta e + \frac{d\dot{\Omega}}{di}\delta i.\end{aligned}\right\} (2)$$

The integrations were performed by considering the arguments of trigonometric terms to be linear functions of time – hence, resonances had to be treated as special cases. In Equations (2), δe and δi are the perturbations obtained by integration of Equations (1), and $\dot{\ell}$, $\dot{\omega}$, and $\dot{\Omega}$ are given by (see References 1 and 4)

$$\dot{\ell} = n\left\{1 - \frac{3J_2 a_e^2}{4a^2(1-e^2)^{3/2}}(1-3\cos^2 i) + \frac{3J_2^2 a_e^4}{128a^4(1-e^2)^{3/2}}\left[10-25e^2+16\sqrt{1-e^2}\right.\right.$$

$$\left. -6(10-15e^2+16\sqrt{1-e^2})\cos^2 i + (130-25e^2+144\sqrt{1-e^2})\cos^4 i\right]$$

$$\left. - \frac{45J_4 a_e^4 e^2}{128a^4(1-e^2)^{3/2}}(3-30\cos^2 i + 35\cos^4 i)\right\},$$

$$\dot{\omega} = n \left\{ - \frac{3J_2 a_e^2}{4a^2(1-e^2)^2} (1 - 5\cos^2 i) + \frac{3J_2{}^2 a_e^2}{128a^4(1-e^2)^4} [-10 - 25e^2 + 24\sqrt{1-e^2} \right.$$

$$-6(6 - 21e^2 + 32\sqrt{1-e^2}) \cos^2 i + 5(86 - 9e^2 + 72\sqrt{1-e^2}) \cos^4 i]$$

$$\left. - \frac{15J_4 a_e^4}{128a^4(1-e^2)^4} [3(4+3e^2) - 18(8+7e^2)\cos^2 i + 7(28+27e^2)\cos^4 i] \right\},$$

$$\dot{\Omega} = n \left\{ - \frac{3J_2 a_e^2 \cos i}{2a^2(1-e^2)^2} + \frac{3J_2{}^2 a_e{}^4 \cos i}{32a^4(1-e^2)^4} [4 - 9e^2 + 12\sqrt{1-e^2} - (40 - 5e^2 \right. \quad (3)$$

$$\left. + 36\sqrt{1-e^2}) \cos^2 i] - \frac{15J_4 a_e^4 (2+3e^2)\cos i}{32a^4(1-e^2)^4} (3 - 7\cos^2 i) \right\}.$$

Also, in Equations (1) and (2), the function "R" represents, in general, any of the secular and/or long period "disturbing" functions due to lunar and solar forces. An equation for the semi-major axis does not appear because only secular and long period perturbations are being considered here. Perturbations in e, i, ℓ, ω, and Ω arise from lunar and solar gravitation and solar radiation pressure; tidal forces and precession and nutation cause effects in i, ℓ, ω, and Ω.

Comparison with Geopotential Zonal Harmonics Effects

One can obtain an estimate of the magnitude of the secular and long period luni-solar perturbations relative to those due to the Earth's zonal harmonics by a study of the parameters appearing as factors in the disturbing functions.

It can be seen from the 1959 paper of Brouwer (Reference 4) that the long-period part of the Hamiltonian F_{2p}^* appears with the small factors J_2^2/a^5, J_3/a^4, J_4/a^5, and J_5/a^6, and the secular parts F_{1s} and F_{2s}^* have J_2/a^3, J_2^2/a^5, and J_4/a^5 as small factors (F_{2p}^*, although second order, gives rise to first-order long-period perturbations). The secular and long-period gravitational disturbing functions of the sun and moon have the factors $m_\odot n_\odot^2 a^2$ and $m_☾ n_☾^2 a^2$, respectively. The long-period solar radiation pressure disturbing function appears with $-Fa$ as a multiplier (for illustration, the value of A/m for Telstar 2, which is about 0.074860427 cm²/gm, was chosen for use in this analysis).

Table 1 gives a listing of these parameters for increasing values of a; here, the values of the zonal harmonic coefficients used were those adopted by the Goddard Computing Center (see Introduction). In addition, the radius of the Earth has been chosen as the unit of distance, time is expressed in the Vanguard unit (806.832 sec.), and angular measure is in radians.

Assuming that divisors of approximately the same order arise in the integration of both the zonal harmonic and luni-solar disturbing functions,

Table 1
Parameters in the Secular and Long-Period Disturbing Functions
(Telstar 2)

a (earth radii)	J_2/a^3	J_2^2/a^5	J_3/a^4	J_4/a^5	J_5/a^6	$m_☾ n_☾^2 a^2$	$m_☉ n_☉^2 a^2$	$-Fa$
1.00	10.082×10^{-4}	11.70×10^{-7}	-22.85×10^{-7}	-21.23×10^{-7}	-232.0×10^{-9}	0.5507×10^{-7}	2.579×10^{-8}	3.537×10^{-9}
1.05	9.347	9.173	-18.81	-16.64	-173.1	.6071	2.843	3.714
1.10	8.129	7.269	-15.62	-13.18	-131.0	.6663	3.121	3.891
1.15	7.114	5.820	-13.06	-10.56	-100.3	.7283	3.411	4.068
1.20	6.262	4.705	-11.02	-8.535	-77.72	.7930	3.714	4.244
1.25	5.540	3.836	-9.360	-6.957	-60.82	.8605	4.030	4.421
1.30	4.925	3.153	-8.000	-5.718	-48.06	.9307	4.359	4.598
1.35	4.398	2.611	-6.879	-4.735	-38.33	1.004	4.700	4.775
1.40	3.943	2.176	-5.948	-3.947	-30.81	1.079	5.055	4.952
1.45	3.549	1.826	-5.169	-3.312	-24.96	1.158	5.422	5.129
1.5	3.206	1.542	-4.513	-2.796	-20.37	1.239	5.803	5.306
1.6	2.642	1.117	-3.487	-2.024	-13.83	1.410	6.602	5.659
1.7	2.202	.8244	-2.736	-1.495	-9.611	1.592	7.453	6.013
1.8	1.855	.6196	-2.177	-1.123	-6.822	1.784	8.356	6.367
1.9	1.577	.4727	-1.753	-.8574	-4.932	1.988	9.310	6.720
2.0	1.352	.3657	-1.428	-.6635	-3.625	2.203	10.32	7.074
2.2	1.016	.2272	-.9755	-.4120	-2.046	2.665	12.48	7.781
2.4	.7827	.1470	-.6888	-.2666	-1.214	3.172	14.86	8.489
2.6	.6156	.09856	-.5000	-.1787	-.7510	3.723	17.43	9.196
2.8	.4929	.06801	-.3718	-.1234	-.4814	4.317	20.22	9.904
3.0	.4007	.04816	-.2821	-.08737	-.3182	4.956	23.21	10.61
3.5	.2524	.02229	-.1523	-.04042	-.1262	6.746	31.59	12.38
4.0	.1691	.01143	-.08925	-.02073	-.05665	8.811	41.26	14.15
4.5	.1187	.006342	-.05572	-.00151	-.02794	11.15	52.22	15.92
5.0	.08656	.003746	-.03656	-.006794	-.01485	13.77	64.48	17.69

we can see that at about $a \approx 1.2$ the long-period effects due to luni-solar gravitation are about the same as those arising from J_5. Further, at $a \approx 1.9$ the luni-solar long-period effects are just as important as those due to any one of J_2^2, J_3, J_4, or J_5. Also, at $a \approx 3$, solar radiation pressure is as significant as the zonal harmonics of the Earth. For increasing values of a, of course, the luni-solar forces became progressively larger while the effects due to the Earth's zonal harmonics diminish.

As far as the secular terms are concerned, the luni-solar effects became comparable to the zonal harmonic effects at about $a \approx 5$. Therefore, the integration becomes invalid at this point, and the formulas should not be used for satellites whose orbits have semimajor axes exceeding this value.

Analysis of Relay 1 and Telstar 2

An examination of the mean orbital elements of Relay 1 and Telstar 2 reveals quite substantial long-period variations remaining in the eccentricity, inclination, argument of perigee, and longitude of ascending node. These elements are the "double-primed" variables of Brouwer (see Reference 4), so the effects due to the earth's zonal harmonics though J_5 have been accounted for. Since the two satellites are high enough so that air drag may be neglected, it was felt that these perturbations might be due to luni-solar forces. For these two satellites tidal and precession and nutation effects were found to be negligible compared to lunar and solar gravitational effects and solar radiation pressure.

Data for Relay 1 and Telstar 2 spanning 630 and 525 days, respectively, were analyzed (Reference 1). Figures 1 to 4 give plots of the uncorrected and corrected values of eccentricity and inclination versus time. It is clear that the major portions of the perturbations were indeed due to luni-solar forces.

The argument of perigee and the longitude of ascending node were handled in a somewhat different fashion because of the presence of secular effects. First, the existence of long-period variations in the double-primed elements was detected by examining the residuals from least-squares fits to a constant plus a secular term. Then the long-period luni-solar perturbations were subtracted from the double-primed elements, additional least squares analyses of the same type were made, and the residuals were again examined. The least squares results are as follows:

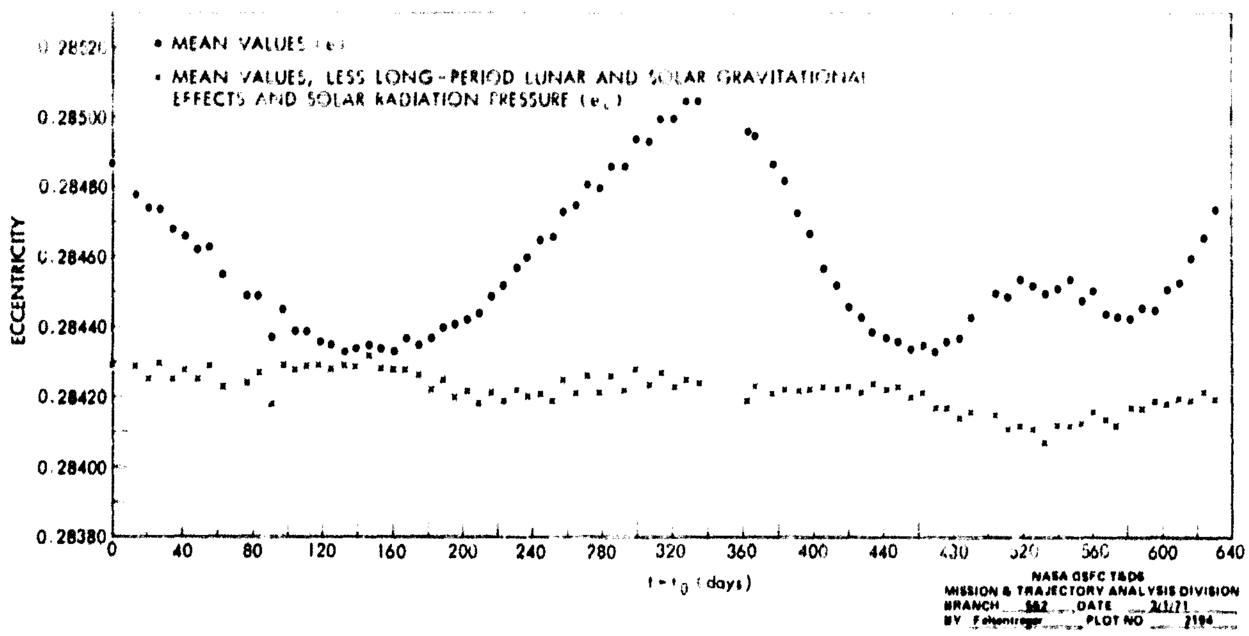

Figure 1. Eccentricity of Relay 1.

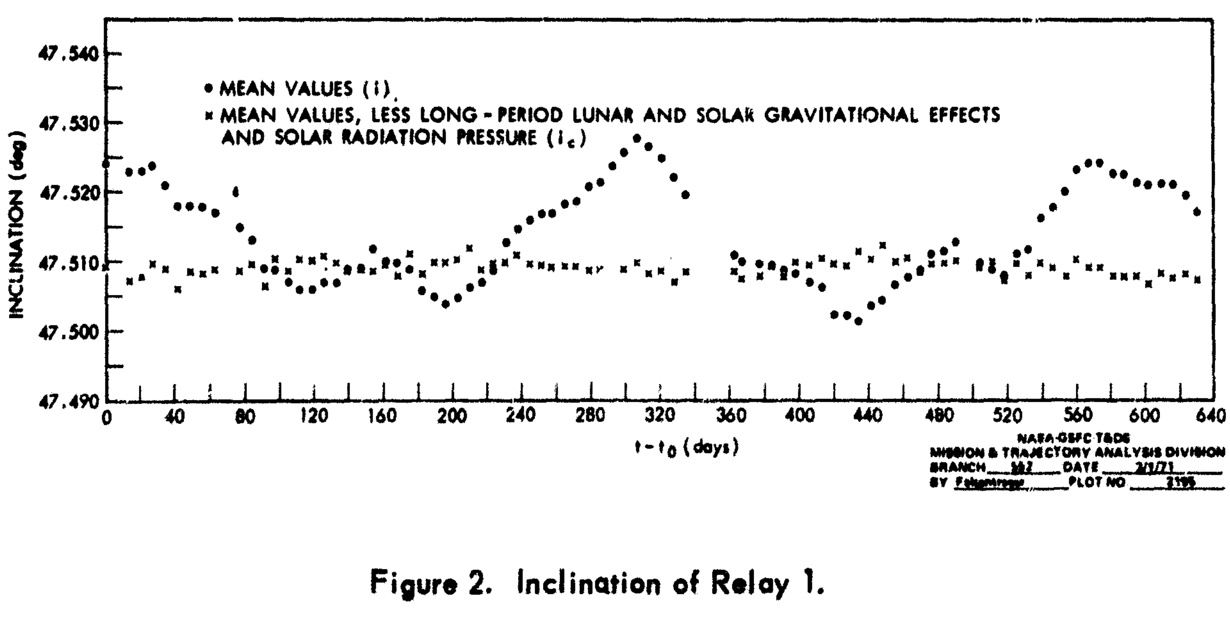

Figure 2. Inclination of Relay 1.

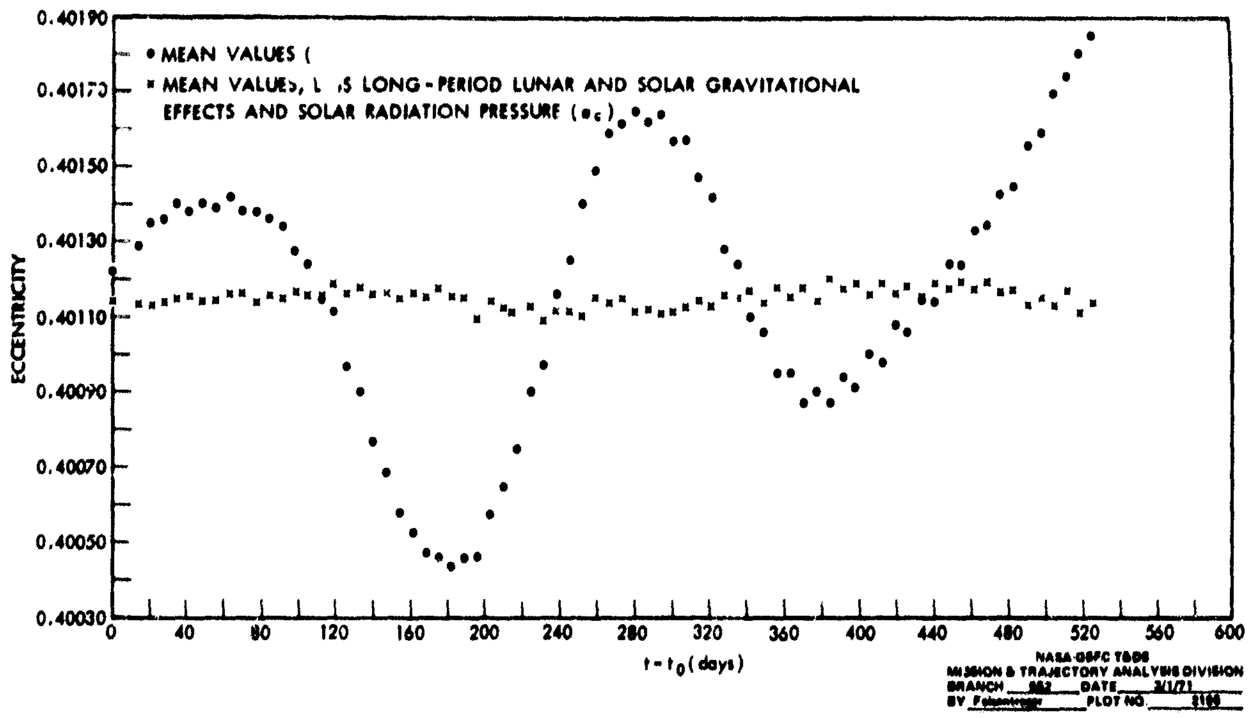

Figure 3. Eccentricity of Telstar 2.

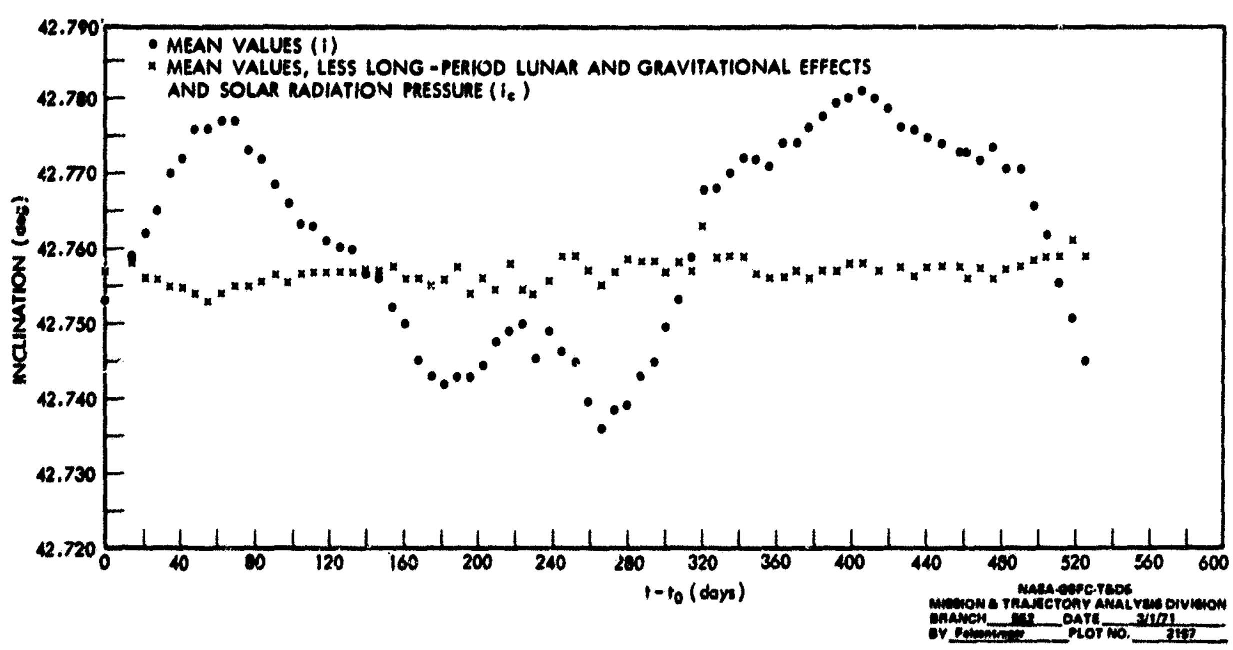

Figure 4. Inclination of Telstar 2.

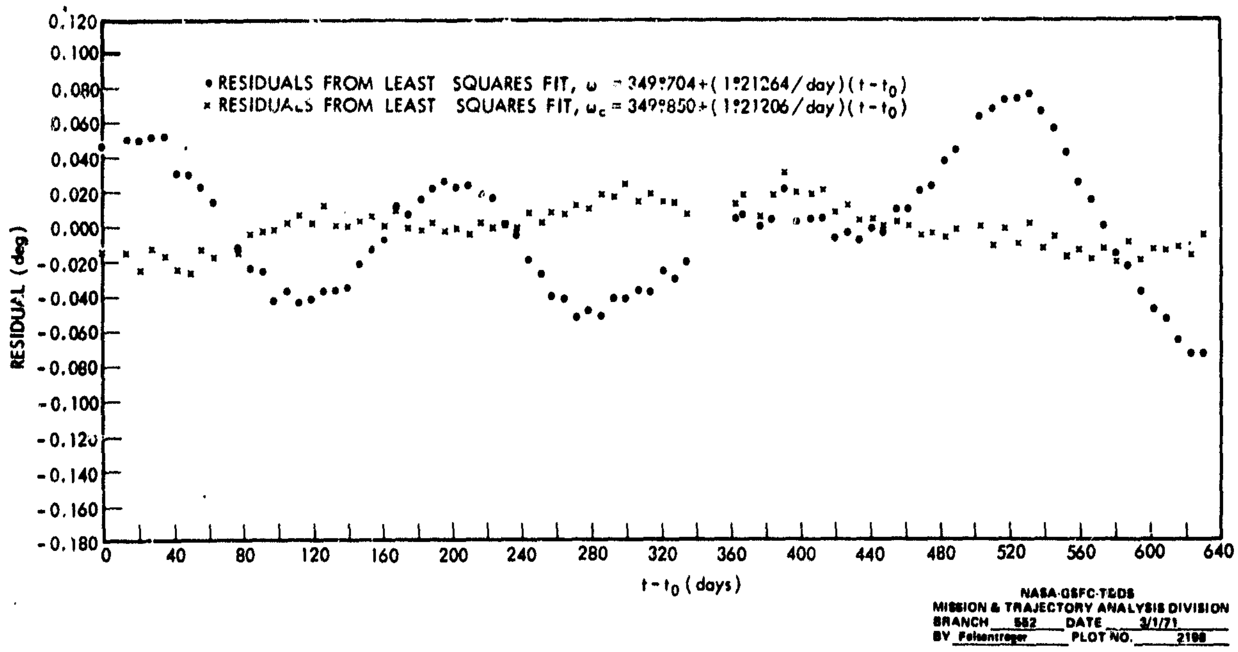

Figure 5. Residuals from least squares fits to argument of perigee for Relay 1.

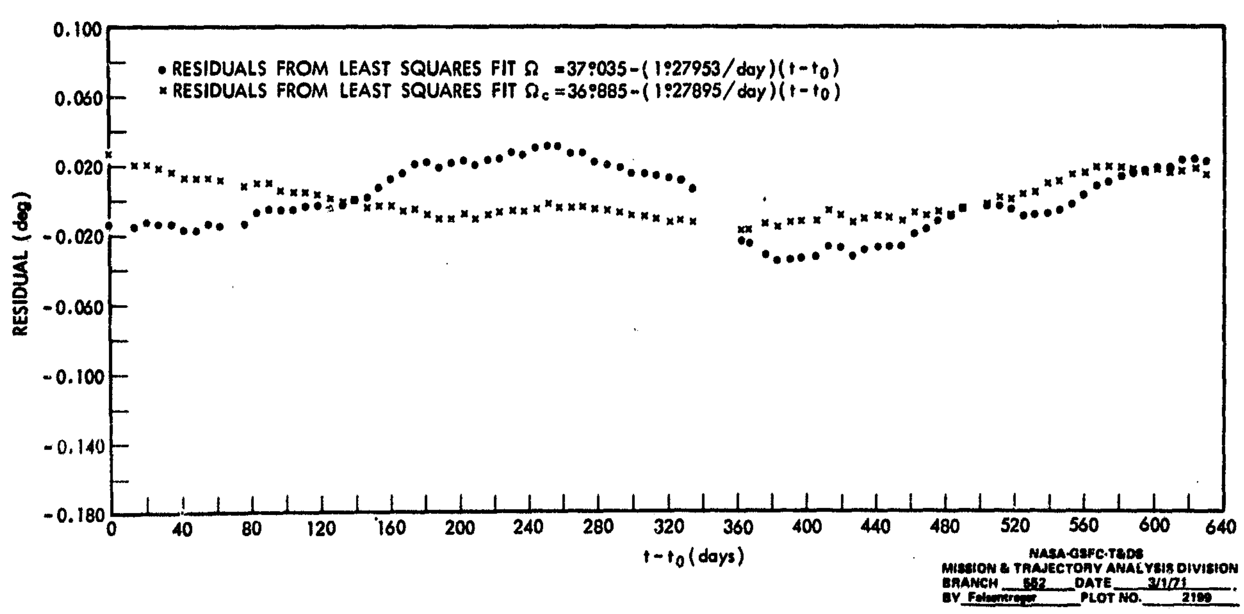

Figure 6. Residuals from least squares fits to longitude of ascending node for Relay 1.

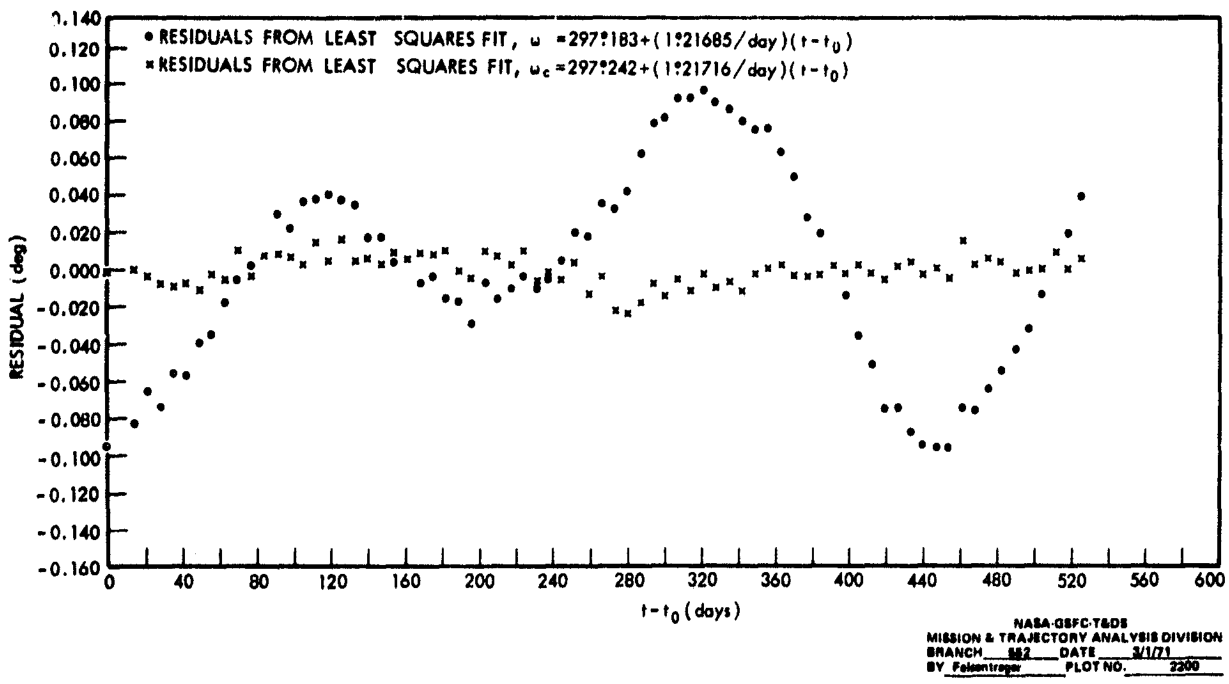

Figure 7. Residuals from least squares fits to argument of perigee for Telstar 2.

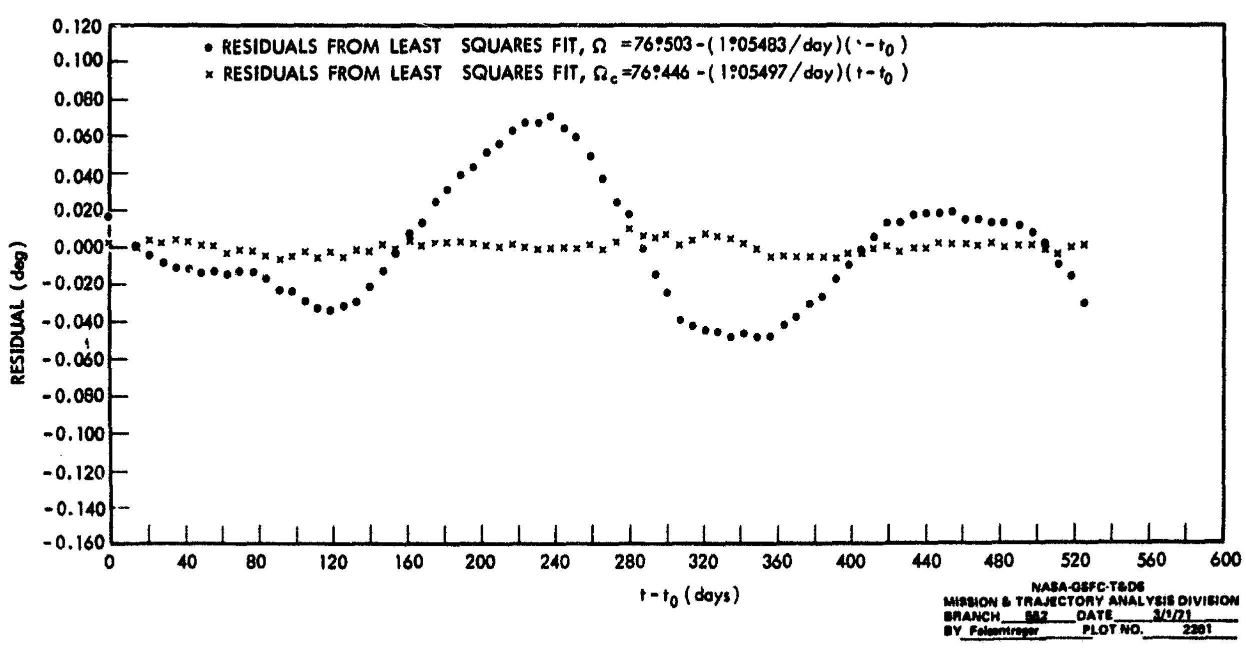

Figure 8. Residuals from least squares fits to longitude of ascending node for Telstar 2.

(1) **Relay 1:**

$$\omega = 349°704 + (1°21264/\text{day})\ (t - t_0)$$

$$\omega_c = \omega - \delta\omega = 349°850 + (1°21206/\text{day})\ (t - t_0)$$

$$\Omega = 37°035 - (1°27953/\text{day})\ (t - t_0)$$

$$\Omega_c = \Omega - \delta\Omega = 36°885 - (1°27895/\text{day})\ (t - t_0)$$

(2) **Telstar 2:**

$$\omega = 297°183 + (1°21685/\text{day})\ (t - t_0)$$

$$\omega_c = \omega - \delta\omega = 297°242 + (1°21716/\text{day})\ (t - t_0)$$

$$\Omega = 76°503 - (1°05483/\text{day})\ (t - t_0)$$

$$\Omega_c = \Omega - \delta\Omega = 76°446 - (1°05497/\text{day})\ (t - t_0)$$

Figures 5 to 8 indicate that the principal remaining variations in the argument of perigee and longitude of ascending node were caused by the sun and the moon.

To illustrate the fact that long-period luni-solar effects can be as significant as first-order long-period zonal harmonic effects, let us examine the perturbations in the eccentricity of Relay 1 and Testar 2. Tables 2 and 3 present the amplitudes of the principal long-period terms appearing in the formulas for the luni-solar and zonal harmonic effects, along with the corresponding arguments and periods (the formulas for the zonal harmonic terms were obtained from Reference 4). It is clear that the luni-solar effects are just as important as zonal harmonic perturbations.

Analysis of Relay 2

Similar results were obtained in the case of Relay 2. However, the situation was complicated by the existence of a near-resonant condition caused by a longitude of perigee ($\omega + \Omega$) which was practically constant for the time interval studied (about 654 days). The period of the longitude of perigee was found to be on the order of 550 years. This condition caused an almost secular increase in the eccentricity and, to a much lesser extent, in the inclination.

Table 2
Principal Long-Period Terms in Zonal Harmonic and Luni-Solar Perturbations in Eccentricity of Telstar 2.

Source	Amplitude	Argument	Period (days)
(J_3/J_2)	373×10^{-6}	ω	296
(J_4/J_2)	-60	2ω	148
(J_2)	17	2ω	148
(J_5/J_2)	5	ω	296
(J_5/J_2)	-1	3ω	99
Lunar gravity	-220	$2(\Omega_c - \omega - \Omega)$	1121
Lunar gravity	192	$\Omega_c - 2\omega - \Omega$	261
Lunar gravity	-95	2ω	148
Lunar gravity	33	$2(\lambda_c - \omega - \Omega)$	14
Lunar gravity	11	$2\lambda_c - \Omega_c - 2\omega - \Omega$	14
Lunar gravity	-11	$\Omega_c + 2\omega - \Omega$	103
Solar gravity	245	$2(\lambda_{\odot} - \omega - \Omega)$	219
Solar gravity	221	$2\lambda_{\odot} - \Omega_{\odot} - 2\omega - \Omega$	607
Solar gravity	-106	$2(\Omega_{\odot} - \omega - \Omega)$	1111
Solar gravity	90	$\Omega_{\odot} - 2\omega - \Omega$	261
Solar gravity	-42	2ω	148
Solar gravity	-35	$2(\lambda_{\odot} - \Omega_{\odot} - \omega)$	781
Solar gravity	8	$\lambda_{\odot} - \omega_{\odot} - 2\omega - \Omega$	917
Solar gravity	-8	$\Omega_{\odot} + 2\omega - \Omega$	103
Solar radiation pressure	42	$\omega + \Omega - \lambda_{\odot}$	437
Solar radiation pressure	-24	$\omega - \lambda_{\odot}$	1562

Table 3
Principal Long-Period Terms in Zonal Harmonic and Luni-Solar Perturbations in Eccentricity of Relay 1.

Source	Amplitude	Argument	Period (days)
(J_3/J_2)	520×10^{-6}	ω	298
(J_4/J_2)	-71	2ω	149
(J_2)	18	2ω	149
(J_5/J_2)	7	ω	298
(J_5/J_2)	7	3ω	99
Lunar gravity	304	$2(\Omega_c - \omega - \Omega)$	2584
Lunar gravity	147	$\Omega_c - 2\omega - \Omega$	315
Lunar gravity	-68	2ω	149
Lunar gravity	19	$2(\lambda_c - \omega - \Omega)$	14
Lunar gravity	9	$2\lambda_c - \Omega_c - 2\omega - \Omega$	14
Lunar gravity	-7	$\Omega_c + 2\omega - \Omega$	97
Solar gravity	149	$2(\Omega_\odot - \omega - \Omega)$	2691
Solar gravity	109	$2(\lambda_\odot - \omega - \Omega)$	171
Solar gravity	101	$2\lambda_\odot - \Omega_\odot - 2\omega - \Omega$	436
Solar gravity	70	$\Omega_\odot - 2\omega - \Omega$	314
Solar gravity	-30	2ω	149
Solar gravity	-30	$2(\lambda_\odot - \Omega_\odot - \omega)$	795
Solar gravity	12	$\lambda_\odot - \omega_\odot - 2\omega - \Omega$	2256
Solar radiation pressure	31	$\omega + \Omega - \lambda_\odot$	342
Solar radiation pressure	-26	$\omega - \lambda_\odot$	1590

The luni-solar gravitational and solar radiation pressure effects were computed using the formulas in Reference 1. However, the precaution was taken to exclude the near-resonant terms with arguments $2(\Omega_{\odot} - \omega - \Omega)$ and $2(\Omega_{☾} - \omega - \Omega)$ from the initial computations. The portion of the luni-solar gravitational disturbing function which includes the two near-resonant terms is

$$F_R = \frac{15}{64} a^2 e^2 (1 + \cos i)^2 \left[n_{\odot}^2 m_{\odot} \sin^2 i_{\odot} \cos 2(\Omega_{\odot} - \omega - \Omega) \right.$$

$$\left. + n_{☾}^2 m_{☾} \sin^2 i_{☾} \cos 2(\Omega_{☾} - \omega - \Omega) \right] .$$

The method of integration employed in Reference 1 cannot be directly applied to the "moon" term in its present form inasmuch as the motions of $i_{☾}$, $\Omega_{☾}$, and $\omega + \Omega$ are all commensurate; in addition, since $i_{☾}$ and $\Omega_{☾}$ have been referred to the Earth's equatorial plane, their motions are not sufficiently constant over the time interval under study to allow accurate computation of the perturbations.

However, using the ecliptic as a reference plane the "moon" term can be rewritten in terms of angular variables having motions which are essentially constant, and the integration can be carried out (details may be found in Reference 5). Figure 9 illustrates the effect of removing the luni-solar perturbations, including the "near-resonant" perturbation $(\delta e)_R$, from the "mean" values of eccentricity.

There still remained a variation in the eccentricity having a period approximately that of the argument of perigee. A harmonic analysis using the corrected values of the argument of perigee $(\omega_{☾})$ was performed, the results of which are shown in Figure 10. A further analysis indicated that this variation could be accounted for by use of the value -2.504×10^{-6} for the J_3 zonal harmonic, instead of the value used in the orbit determination (see Introduction). This "adjusted" value for J_3 is much more in line with recent determinations.

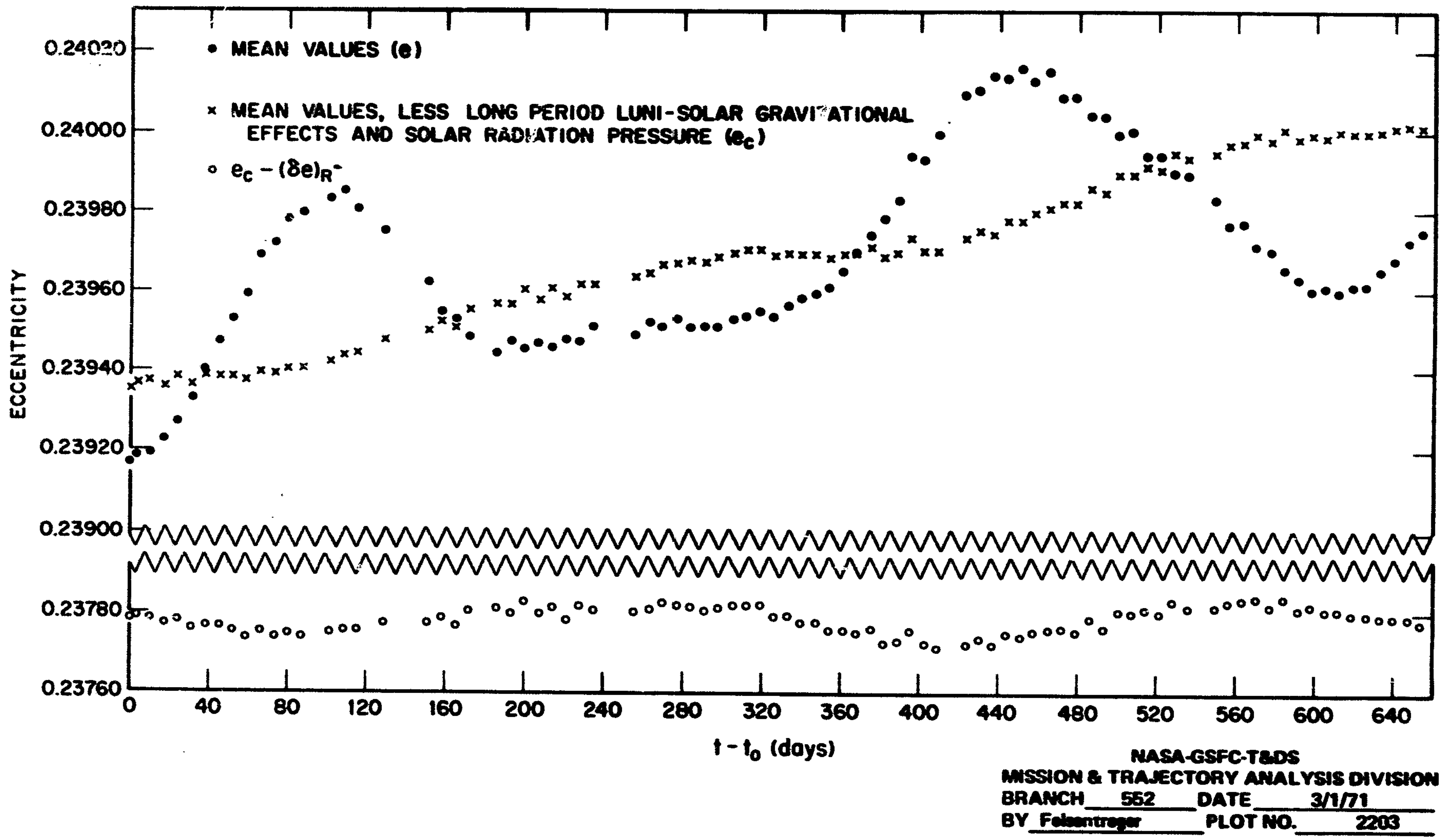

Figure 9. Eccentricity of Relay 2.

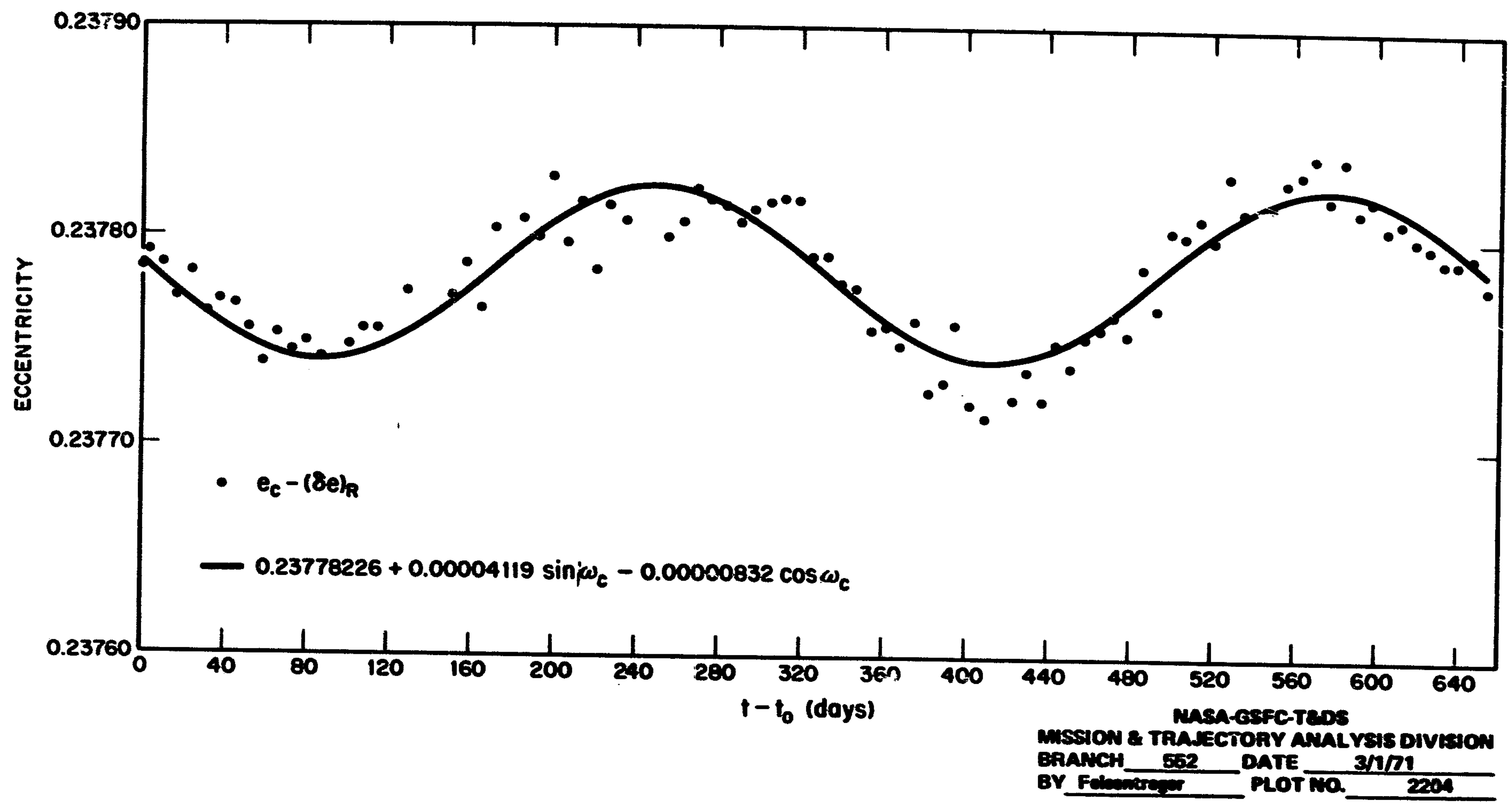

Figure 10. Eccentricity of Relay 2.

Shadow Effects on Telstar 2

An indirect long period solar radiation pressure effect on the orbit of a satellite is caused by the passage of the satellite in and out of the earth's shadow. This effect is best observed in the semi-major axis, which undergoes no long period motion from any other source.

The Telstar 2 satellite provides a good subject for a study of this effect. A long-period variation with approximate amplitude of 0.0000140 earth radii and a period of about a year was observed in the semi-major axis. Analytic formulations derived using a procedure described in Reference 6 successfully accounted for this variation, illustrated in Figure 11.

Seasonal Variations in the Geopotential

A seasonal variation in the second zonal harmonic in the geopotential has been observed (Reference 8). In his paper, Dr. Kozai sought but did not find a variation in the third zonal harmonic. An analysis is underway to use the moderate to highly inclined orbits of the close nearly

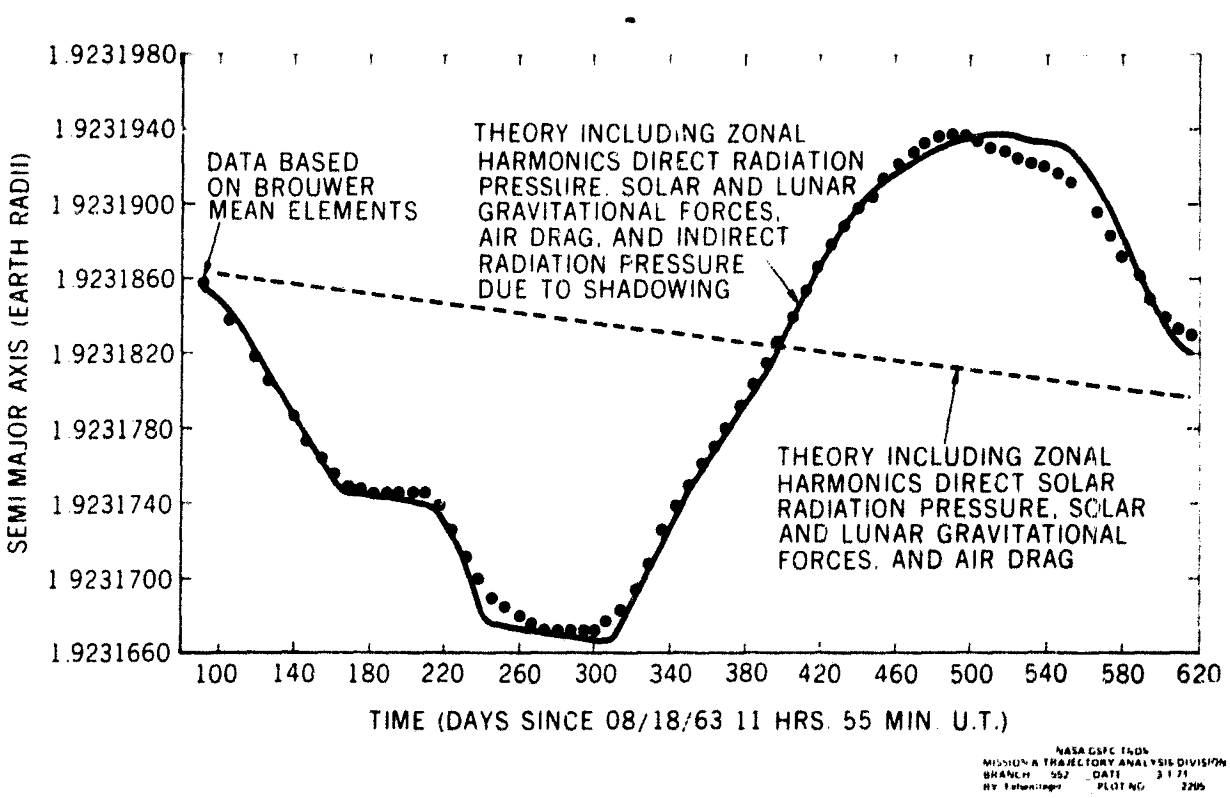

Figure 11. Semi-Major Axis of Telstar 2 vs. Time

circular Tiros 8, Alouette 1, and Nimbus 2 satellites to study these effects. Orbits such as these undergo very substantial periodic perturbations due to the third harmonic in eccentricity and argument of perigee (see Reference 7). Any residual periodicities after all the luni-solar gravitation, precession and nutation, radiation pressure, high degree zonals, and solar and lunar tidal effects are removed may reveal the effects of the time dependent geopotential.

Symbols

A	Effective presentation area of satellite
a	Semi-major axis of satellite's orbit
a_e	Mean equatorial radius of the Earth
e	Eccentricity of satellite's orbit
e_c	Eccentricity corrected for long-period lunar and solar effects
F	Solar radiation pressure force constant
F_R	Resonant disturbing function
i	Inclination of satellite's orbital plane to Earth's equatorial plane
i_c	Inclination corrected for long-period lunar and solar effects
$J_2, \ldots, J_5$	Zonal harmonic coefficients in Earth's gravitational potential
ℓ	Mean anomaly of satellite
$\dot{\ell}$	Mean motion of mean anomaly
m	Mass of satellite
n	Mean motion of satellite

R	Disturbing function
t	Time
ω	Argument of perigee of satellite's orbit
$\dot{\omega}$	Mean motion of argument of perigee
Ω	Longitude of ascending node of satellite's orbit
$\dot{\Omega}$	Mean motion of longitude of ascending node
$\lambda_{\odot}$, $\lambda_{☾}$	Mean longitude of sun or moon

The symbols $e_{\odot}$, $i_{☾}$, etc. refer to the corresponding quantities for the sun ($\odot$) or the moon (☾).

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