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TECHNICAL
MEMORANDUM

EFFECTS OF PHYSICAL LIBRATIONS OF THE MOON
ON THE ORBITAL ELEMENTS
OF A LUNAR SATELLITE

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ABSTRACT

Physical librations of the moon, which cause selenographic axes fixed in the true moon to have a different orientation than similar axes fixed in the mean moon, are small cyclic perturbations with periods of one month and longer, and amplitudes of 100 arc seconds or less.

These librations have two types of effects of present interest. If the orbital elements of a lunar satellite are referred to selenographic axes in the true moon as it rotates and librates, then the librations cause changes in the orientation angles (node, inclination and periapsis argument) large enough that long period planetary perturbation theory cannot be used without compensation for such geometrical effects. As a second effect, the gravitic potential of the moon is actually wobbled in inertial space, a condition not included in the potential expression used in planetary perturbation theory.

The paper gives data on the magnitude of the physical librations, the geometrical effects on the orbital elements and the equivalent changes in the coefficients in the potential. Fortunately, the last effect is shown to be small.

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TECHNICAL MEMORANDUM

INTRODUCTION

The long-term selenodesy method for determining lunar gravity coefficients [1] uses a time history of orbital elements in conjunction with the long-period form of the Lagrange planetary equations. The disturbing function for spherical harmonics used in such work, typically as derived by Kaula [2], is referenced to a selenographic coordinate system, fixed in the moon. The x-y axes of this system are in the lunar equatorial plane and the z axis is along the lunar polar axis. The analytical derivation of the Lagrange perturbation equations requires that the osculating orbital elements be referenced to an inertial coordinate system. In order to make the inertial and selenographic systems simply relatable, it is assumed that they have a common z axis and the moon rotates about this z axis.

But, in addition to its normal rotation about its polar axis, the selenographic system undergoes additional rotations about all three axes due to precession and physical librations. These small rotations can affect long-term selenodesy calculations in two ways. First, if the selenographic axes are used as the reference system, some of the orbital elements will change simply because the axes librate and precess (geometrical effects). Second, since perturbation theory (in its present form) accommodates only polar rotations of the selenographic system, there are also physical effects induced by lunar librations and precession.

This paper illustrates the geometrical and physical effects associated with the changing orientation of the selenographic axes. It is shown that geometrical effects can be accommodated either by using an inertial axes system or by compensating for the lunar librations and precession when the selenographic axes are used. Further, it is shown that physical effects are small and negligible for all but the most exacting endeavors.

MOTION OF THE MOON

Standard angles [3,4,5,6] which yield the orientation of the true selenographic of date (TSD) axes at any particular instant are referenced to the mean earth equator-equinox (MOE) axes of some chosen epoch. These two systems are relatable using a direction cosine matrix M_T so that a position vector $\underline{r}$ measured in the TSD axes is transformed into the MOE axes as follows:

$$\underline{r}_{MOE} = M_T\ \underline{r}_{TSD}$$

If the rotation matrices $R_1(\theta)$, $R_2(\theta)$, and $R_3(\theta)$ are defined as

$$R_1(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C & -S \\ 0 & S & C \end{bmatrix},\ R_2(\theta) = \begin{bmatrix} C & 0 & S \\ 0 & 1 & 0 \\ -S & 0 & C \end{bmatrix},\ R_3(\theta) = \begin{bmatrix} C & -S & 0 \\ S & C & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C = \cos\theta \qquad S = \sin\theta$$

the overall direction cosine matrix, M_T, between the MOE axes and the TSD axes is given by the following cascade of rotation matrices:

$$M_T = R_3(\tfrac{\pi}{2}-\zeta_o)R_1(\theta)R_3(-\tfrac{\pi}{2}-z)R_1(\varepsilon)R_3(\Omega+180°+\sigma)R_1(I+\rho)R_3(☾+\tau-\Omega-\sigma)$$

(All angles are illustrated in Figure 1.)

ζ_o, θ, z are earth precession angles and ε is the mean obliquity. Together these angles establish the mean Ecliptic of Date (MED) axes to which the lunar angles are referenced. These angles are needed only to establish the correct orientation of the TSD axes: their changes over the time periods of interest in selenodesy are negligible.

The angles Ω, I, and $☾$ are the mean node, inclination, and anomaly of the moon. The rotational rates of the mean selenographic of date (MSG) axes are $\dot{☾}-\dot{\Omega}(1-\cos I)$ about the polar axis (27.3 day period), and $\dot{\Omega}\sin I\cos(☾-\Omega)$ and $\dot{\Omega}\sin I\sin(☾-\Omega)$ about the x and y mean lunar axes respectively. I is constant and $\dot{\Omega}$ measures the precession of the lunar node (18.6 year period).

The angles σ, ρ, and τ are the physical librations of the moon, which distinguish the true from the mean lunar axes. They are cyclic (see Appendix for formulas) with principal components having periods of 27.5 days, 1 year, and 18.6 years in duration.

Figures 2 and 3 show the location of the true selenographic z and x axes relative to mean selenographic axes, both at the same instant. The differences can be given as three small (cyclic) angles

$$\alpha_3 = \tau - \sigma(1-\cos I)$$

$$\alpha_2 = \sigma\sin I\cos(☾-\Omega+\tau-\tfrac{\sigma}{2}) - \rho\sin(☾-\Omega+\tau-\tfrac{\sigma}{2})$$

$$\alpha_1 = \sigma\sin I\sin(☾-\Omega+\tau-\tfrac{\sigma}{2}) + \rho\cos(☾-\Omega+\tau-\tfrac{\sigma}{2})$$

with direction cosines for mean-to-true axes of

$$R_3(\alpha_3)\ R_2(\alpha_2)\ R_1(\alpha_1)$$

The figures are for the 28 day period following August 30, 1967, a date typical of the selenodesy phase of the Lunar Orbiter III satellite.

The bias between the mean and true axes that appears in the figures is due to the components of physical libration which have a period longer than the 28 day time span in the figures.

The three small angles between true selenographic axes of a fixed epoch (t_o) and true selenographic axes at a different instant (t), are:

$$\beta_3 = \Delta☾ + \Delta\tau - (\Delta\Omega+\Delta\sigma)(1-\cos(I+\rho))$$

$$\beta_2 = (\Delta\Omega+\Delta\sigma)\ \sin(I+\rho)\ \cos(☾'+\tau'-(\Omega'+\sigma'))$$

$$-\Delta\rho\ \sin(☾'+\tau'-(\Omega'+\sigma'))$$

$$\beta_1 = (\Delta\Omega+\Delta\sigma)\ \sin(I+\rho)\ \sin(☾'+\tau'-(\Omega'+\sigma'))$$

$$+\Delta\rho\ \cos(☾'+\tau'-(\Omega'+\sigma'))$$

Here ☾' (for example) applies at t and Δ☾ is ☾(t) - ☾(t_o). The direction cosine matrix (t_o to t) is

$$R_3(\beta_3)\ R_2(\beta_2)\ R_1(\beta_1)$$

Although the α's are all cyclic, the β's are not, because ☾ and Ω both monotonically change. β_1 and β_2 are treated as small angles, but if enough time passes β_3 may be sizeable enough that small angle approximations are insufficient for it. The Δ☾ - $\Delta\Omega$(1-cos I) part of β_3 does not show up in Figures 2 and 3 since in that figure the mean and true axes are always at the same instant.

PERTURBATION THEORY

Lagrange's perturbation theory [2] applies for the basic vector differential equation

$$\ddot{\underline{r}} + \frac{\mu}{r^3}\underline{r} = \nabla R$$

where R is now the disturbing potential function. To account for, say, irregularities in density and shape of the moon, R can have the usual spherical harmonic function form:

$$R = \frac{\mu}{r} \sum_{\ell=2}^{\infty} \sum_{m=0}^{\ell} \left(\frac{R_e}{r}\right)^{\ell} P_{\ell m}(\sin\phi)\,[C_{\ell m} \cos m\lambda + S_{\ell m} \sin m\lambda]$$

where R_e is the reference body radius, P the associated Legendre function, ϕ is latitude, λ is longitude and $C_{\ell m}$ and $S_{\ell m}$ are the specific constants which describe the body's potential. Using $\ell=2$ as the least index assumes the center of mass is at the origin of the coordinate axes.

In Lagrange's work appropriate "orbital elements" are found - quantities which describe the satellite orbit completely, which are constants if R = 0 and if the axes in which $\underline{r}$ is measured are inertial. The classical set of orbital elements is suggested in Figure 4. Ω is the node on the planet's equator, i the inclination, ω the argument of periapse, f the true anomaly, a the semi-major axis and e the eccentricity. All are constant except f, but M_o and n are constant in the equation for the mean anomaly M, and it is related to f through the eccentric anomaly E. The equations are

$$n^2 a^3 = \mu$$

$$M = E - e \sin E = M_o + n(t-t_o)$$

$$\tan \frac{f}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2}$$

A typical equation in perturbation theory is

$$\frac{d\Omega}{dt} = \frac{3n\, R_e^{\,2} \cos i}{2(1-e^2)^2\, a^2}\, C_{20} + (\text{other } C_{\ell m} \text{ and } S_{\ell m} \text{ effects})$$

so if only C_{20} were non-zero, then accurate knowledge of $d\Omega/dt$ would yield accurate knowledge of C_{20}.

GEOMETRICAL EFFECTS

The geometrical effects can be avoided completely if the axes used are inertial or have only a polar rotation. It is common, however, and suggested in many developments of perturbation theory [2] that the reference axes be the true planetary axes of a local epoch plus the planetary rotation after that epoch. If the axes used were the true selenographic axes at every instant, which is this same assumption except for the effects of lunar precession and physical libration, the changes in the orbit orientation angles would be, for a purely conic orbit, calculated by

$$\gamma_1 = \beta_1 \cos(\Omega_0 - \beta_3) + \beta_2 \sin(\Omega_0 - \beta_3)$$

$$\gamma_2 = \beta_1 \sin(\Omega_0 - \beta_3) - \beta_2 \cos(\Omega_0 - \beta_3)$$

$$\Delta\Omega = -\beta_3 + \text{arc tan } (\gamma_2 \cos i_0/(\sin i_0 - \gamma_1 \cos i_0))$$

$$\Delta i = -\gamma_1$$

$$\Delta\omega = \text{arc tan}(-\gamma_2/(\sin i_0 - \gamma_1 \cos i_0))$$

Of these the mean lunar motion part of β_3 in $\Delta\Omega$ would be expected, but all the other parts would be due to physical librations.

As an idea of the magnitude of these changes, take orbital elements typical of the Lunar Orbiter III satellite during its selenodesy phase. Table 1 shows the elements and tabulates the element rates due to the currently used L-1 potential coefficients (Table 2) and due to geometrical effects. In the node, the geometrical effects are about 0.6% of the L-1 effects, in inclination about 7%, and in argument of perilune about 3%. For other orbital elements, of course, the percentages would be different, but they quite apparently are not trivial: potential coefficients deduced from the rates without compensating for the geometrical effects would be in error.

LIBRATION OF THE MOON'S POTENTIAL

Suppose the reference axes used are inertial and are the true selenographic axes at a fixed epoch in the range of interest. Then the geometric effects of librations are zero but the polar axis at the fixed epoch is not always coincident with the polar axis of the true moon (because of the β_1 and β_2 angles).

One way of coping with this problem is to refer the instantaneous lunar potential to the reference axes. That is, the coefficients which apply for the instantaneous moon are "rotated" through the angles β_1, β_2, β_3, becoming time varying as β_1, β_2, β_3 vary with time. Several authors [7,8,9] have shown how to calculate the rotated coefficients. In particular Levie [9] shows that a coefficient with ℓ=k can cause coefficients with ℓ=k only, but with all values of m.

TABLE 1

GEOMETRICAL EFFECTS ON ORBITAL ELEMENTS

Quantity	Value	Rate due to L-1 field (deg/sec)	Rate due to Geometrical Effects (deg/sec)
Ω	63°72	-11.69E-6	0.073E-6
i	20°82	0.52E-6	0.038E-6
ω	354°59	2.19E-6	0.078E-6

$a = 1965$ km, $e = 0.0436$, $M_o = 194°6$, $t = $ J.D. 2439733.37

As an example, using the L1 field (Table 2), one can show the effect of β_1, β_2 and β_3 over the 28 day time period used in Figure 2. Table 2 shows characteristics of the rotated field, using true selenographic axes at the initial time as reference axes, and with the mean motion part of the β_3 rotation removed.

As another illustration, using the orbital elements used before, we have calculated the instantaneous rates due to the L-1 field - first as if the physical lunar axes were identical to the inertial axes and then as if the physical axes were displaced from the inertial axes by the amounts given by β_1, β_2 and β_3 (less mean monthly motion) at a time 14 days after the initial epoch. For this particular time these angles are $\beta_1 = 0\overset{\circ}{.}062$, $\beta_2 = 0\overset{\circ}{.}0039$ and β_3 - mean motion = $-0\overset{\circ}{.}0116$. Rotating the L-1 coefficients through these angles gives the "smudged" L-1 field. The rates are shown in Table 3: the differences are small but do occur in the third place of some of the rates.

Since 14 days is somewhat of a worst case for β_1, β_2 and β_3, it is also of interest to see the integrated effect of time varying smudging of the field. And so the Lagrangian long-period equations were integrated numerically for a 28 day period. In one case it was assumed the inertial and true selenographic axes were identical except for mean lunar motion. In the second the axes were identical at the start but the physical librations and precession were also introduced, giving a time varying smudging to the L-1 potential coefficients. The orbital elements at the start were those given above and Table 4 shows them 14 and 28 days later. The changes are trivial and are far less than those due to the uncertainty in the coefficients in the L-1 field. In short, the long period effects of the smudging seem ignorably small for practical work.

CONCLUSIONS

The geometrical effects of the lunar precession and physical libration are appreciable in selenodetic work if the elements of a satellite's orbit are referenced to true selenographic axes moving with the moon. But if true selenographic axes of a fixed epoch in the period of interest are used, then the geometrical effects are eliminated and the physical effects which are introduced are negligible.

TABLE 2

VARIATION IN COEFFICIENTS DUE TO VARIATION IN PHYSICAL LIBRATION AND LUNAR PRECESSION

Coefficient	L-1 Field Value	Variation	
C_{20}	-0.207108×10^{-3}	$+0.32\times10^{-9}$,	$-0.$
C_{21}	0	$+0.139\times10^{-6}$,	-0.220×10^{-6}
S_{21}	0	$+0.111\times10^{-6}$,	-0.179×10^{-6}
C_{22}	0.20716×10^{-4}	$+0.46\times10^{-10}$,	-0.49×10^{-10}
S_{22}	0	$+0.902\times10^{-8}$,	$-0.$
C_{30}	0.210×10^{-4}	$+0.114\times10^{-6}$,	-0.180×10^{-6}
C_{31}	0.340×10^{-4}	$+0.185\times10^{-7}$,	-0.118×10^{-7}
S_{31}	0	$+0.301\times10^{-7}$,	-0.110×10^{-7}
C_{32}	0	$+0.818\times10^{-8}$,	-0.517×10^{-8}
S_{32}	0	$+0.268\times10^{-7}$,	-0.167×10^{-7}
C_{33}	0.2583×10^{-5}	$+0.140\times10^{-11}$,	-0.790×10^{-11}
S_{33}	0	$+0.169\times10^{-8}$,	$-0.$

TABLE 3

PHYSICAL EFFECTS OF LIBRATION AND PRECESSION
ON RATE-OF-CHANGE OF ORBITAL ELEMENTS

Orbital Element	Value	Rate due to L-1 field	Change in Rate due to "smudged" L-1 field
Ω	63°72	-11.69E-6 °/s	0.00082E-6
i	20°82	0.52E-6 °/s	0.00023E-6
ω	354°59	2.19E-6 °/s	-0.023E-6
e	0.0436	- 0.64E-8	0.0013E-8
M	194°60	0.046 °/s	0.2E-7

a = 1965 km

TABLE 4

LONG TERM EFFECT OF PRECESSION AND PHYSICAL LIBRATION

Orbital Element	Value after 14 Days L-1 field	Change due to "Smudged" L-1 field	Value after 28 days L-1 field	Change due to Smudged L-1 field
Ω	226°17	-0°000038	37°65	-0°00012
i	20°91	-0°000090	20°86	-0°000013
ω	2°09	-0°0085	13°64	-0°0016
e	0.0579	-0.0000040	0.0585	+0.0000020
M	48°56	+0.0083	48°08	+0°0016

$a = 1965$ km

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M. H. Gittes and S. B. Watson wrote the computer programs used to generate the data for this analysis.

A. J. Ferrari

W. G. Heffron

$2014\text{-}\frac{\text{AJF}}{\text{WGH}}\text{-ksc}$

Attachment
Appendix
Figures 1-4

REFERENCES

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APPENDIX

Formulas for the physical librations are adapted from those of Eckhardt [5] although the angular arguments used here are different from those given in the reference. Further, Eckhardt gives 11 terms in the $I\sigma$ and ρ expansions and 18 for τ [4]; the expansions given below (and used in this study) carry only those terms with amplitudes greater than 10", following a JPL suggestion [6].

The formulas used are

$$I\sigma = -100''.63 \sin g' + 23''.75 \sin(g'+2\omega') - 10''.58 \sin(2g'+2\omega')$$

$$\rho = -98''.36 \cos g' + 23''.84 \cos(g'+2\omega') - 10''.77 \cos(2g'+2\omega')$$

$$\tau = -16''.87 \sin g' + 91''.57 \sin g - 15''.32 \sin 2\omega'$$

$$+10''.0 \sin(2g+2\omega-2\omega')$$

$$+ 14''.27 \sin(360°(0.53733431 - 1.0104982 \times 10^{-5}(36525T))$$

where

g' = mean anomaly of the moon = $☾ - \Gamma'$

g = mean anomaly of the sun = $L - \Gamma$

ω = argument of periapsis of the sun measured from the ascending node of the orbit of the moon = $\Gamma - \Omega$

ω' = argument of periapsis of the moon = $\Gamma - \Omega$

I = $5521\overset{''}{.}5$

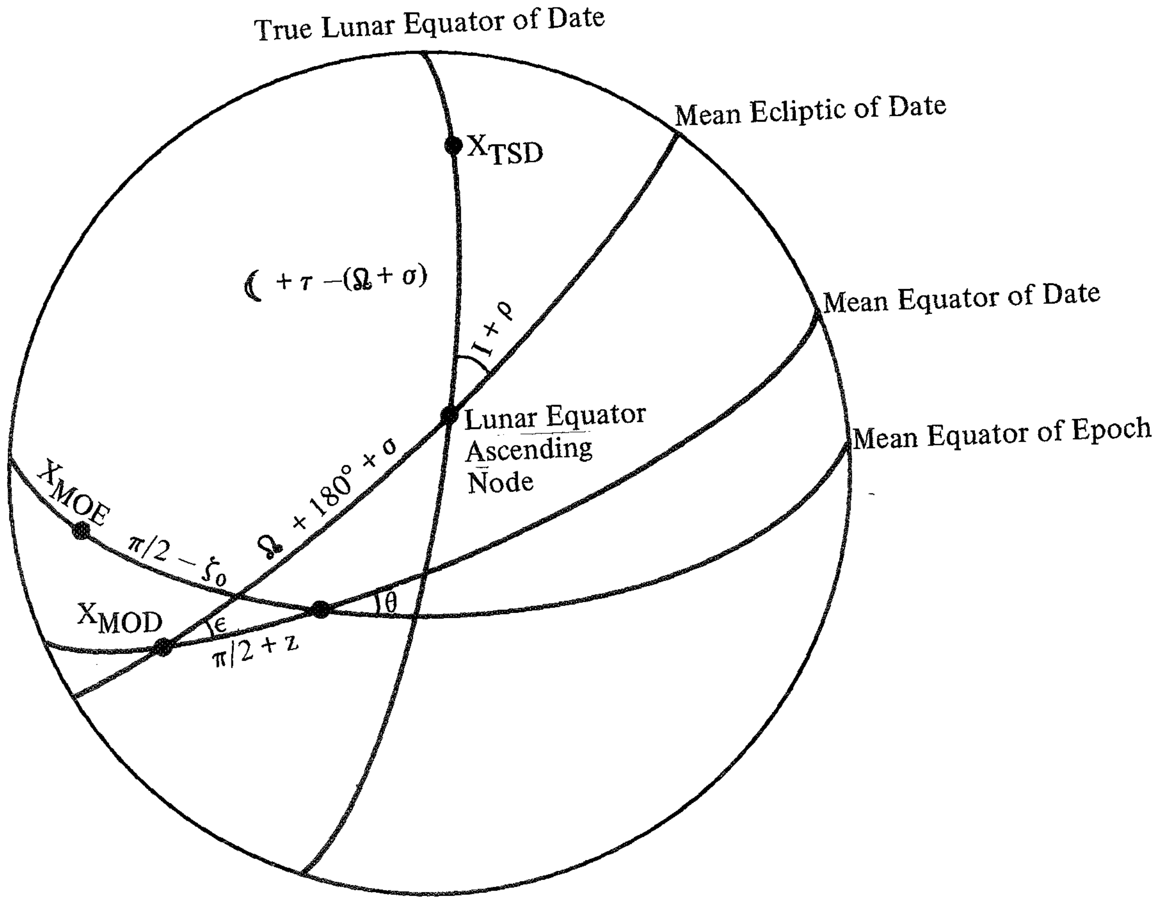

FIGURE 1 - ROTATIONS BETWEEN MOE AND TSD AXES

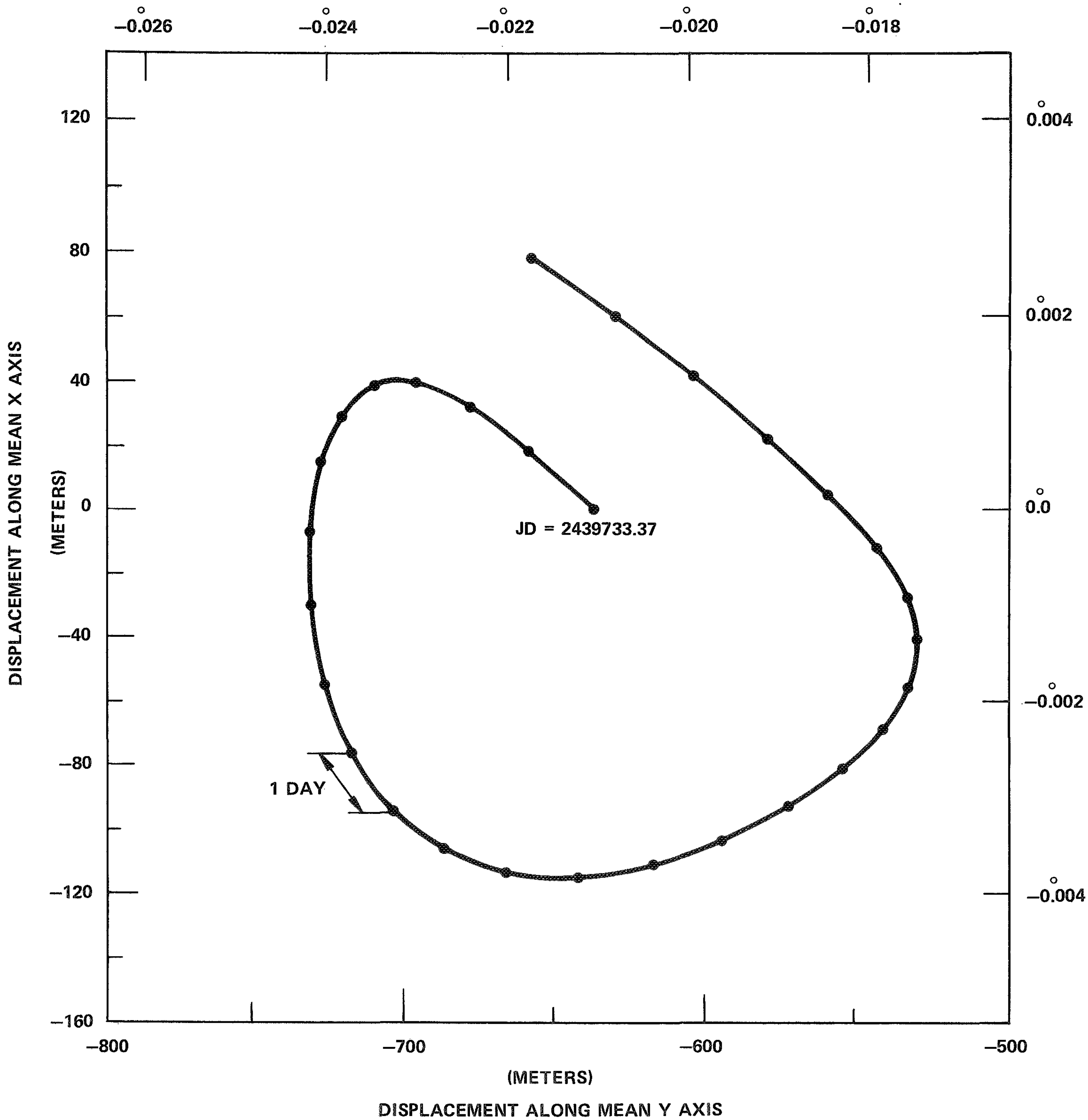

FIGURE 2 - PATH OF THE TRUE Z AXIS IN THE MEAN X-Y PLANE. DISPLACEMENT GIVEN IN METERS (AT THE POLE OF THE MOON) AND IN DEGREES.

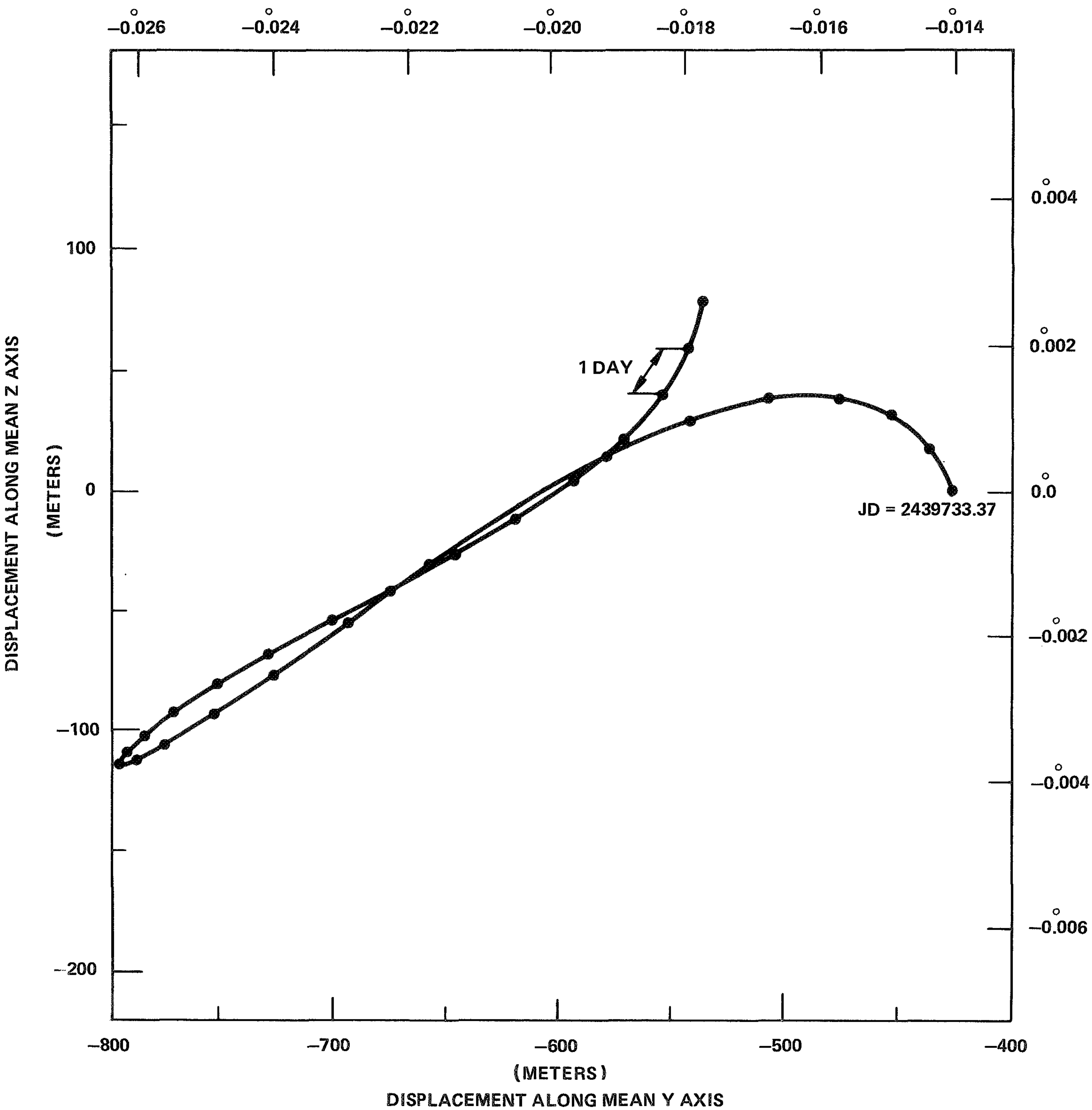

FIGURE 3 - PATH OF THE TRUE X AXIS IN THE MEAN Y-Z PLANE. DISPLACEMENT GIVEN IN IN METERS (AT THE FACE OF THE MOON) AND IN DEGREES.

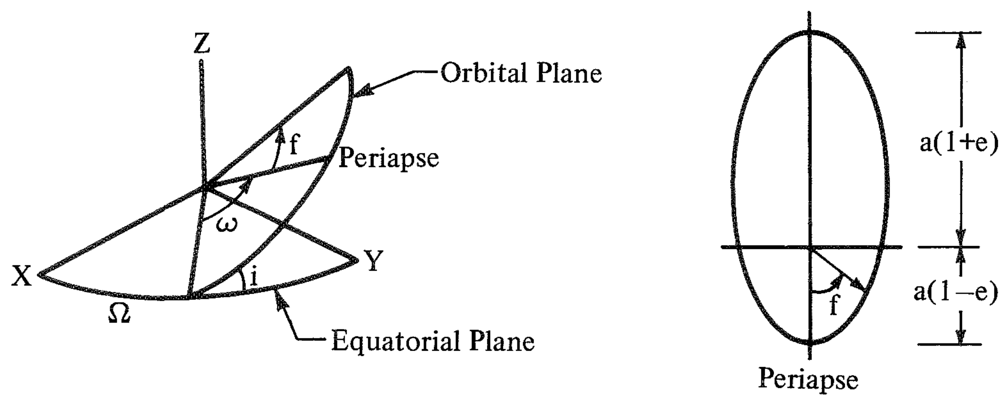

FIGURE 4 - CLASSICAL ORBITAL ELEMENTS