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ANALOG ANALYSIS OF
THE HEAVE RESPONSE AND CONTROL OF
A PLENUM-TYPE AIR-CUSHION VEHICLE

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16. Abstract

An analytical investigation was conducted to determine the basic heave response characteristics of a plenum-type tracked air-cushion vehicle as a function of vehicle operating parameters. In addition to determining basic characteristics, the effect of an active lip control system on the vehicle dynamic response was investigated. Nonlinear equations describing the dynamic and thermodynamic state of the air-cushion system were derived, and the response of the system to sinusoidal perturbations of the guideway was obtained with the use of an analog computer. Results are presented in terms of vehicle-to-guideway motions and vehicle accelerations experienced during fundamental and subharmonic oscillations.

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ANALOG ANALYSIS OF THE HEAVE RESPONSE AND CONTROL

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SUMMARY

An analog computer study was conducted to determine the basic heave response characteristics of a plenum-type air-cushion vehicle as a function of vehicle operating parameters. The effect of an active lip control system on the vehicle dynamic response was also investigated. Nonlinear equations describing the dynamic and thermodynamic state of the air-cushion system were derived and solved.

Results indicate that vehicle responses to sinusoidal perturbations of the simulated guideway depend significantly upon the choice of vehicle operating parameters. Vehicle-guideway contact may readily occur and vehicle acceleration responses may be quite large. The most stable vehicle responses occurred for the cushion operating at low footprint pressures with small dead volume. The active lip control system effectively reduced vehicle acceleration responses and virtually eliminated vehicle-guideway contact for the range of parameters studied.

INTRODUCTION

The tracked air-cushion vehicle (TACV) is being studied in detail as a candidate mode of high-speed ground transportation where speeds up to 134 m/sec (300 mph) are desired. Preliminary design studies (refs. 1 to 5) performed in this country and experimental model studies (ref. 6) conducted abroad have demonstrated the advantages and feasibility of the TACV. Its advantages include the virtual elimination of moving friction, dynamic unbalances, and contact deformations associated with wheeled vehicles. Furthermore, the low footprint pressure should not require elaborate guideway construction or extensive maintenance, which will be some of the most critical cost factors in any high-speed ground transportation system. To examine the TACV concept in more detail, the U.S. Department of Transportation is currently pursuing programs having several objectives: the design, construction, and testing of a 134-m/sec (300-mph) research vehicle and the development of a lower speed (67 m/sec (150 mph)) demonstration vehicle (ref. 5, for example).

The ultimate success of the TACV research and demonstration programs will be determined to a large extent by human factors such as noise levels, safety, and ride quality. In terms of ride quality the TACV presents many unknowns, since there is little experience with the unique suspension system and/or the 134-m/sec (300-mph) dynamic environment. A detailed understanding of the air-cushion dynamics is essential for the design and control of the cushions to achieve acceptable levels of vehicle response and ride quality.

Several theoretical studies (refs. 7 to 11) have been performed to study the heave motion and stability of vehicles supported by peripheral-jet and plenum-chamber air cushions. These studies, however, have been based primarily upon approaches which employ linear analyses to predict air-cushion heave responses. An investigation of the basic nonlinear nature of air-cushion heave responses was conducted both analytically and experimentally in reference 10 for a small, low-pressure (0.69 kN/m^2 (0.10 psig)) plenum-type air cushion. Results of reference 10 demonstrated that large subharmonic oscillations of the plenum fluid suspension may readily occur and that the heave response could be reduced by using acceleration feedback to modulate the air-cushion supply flow.

Presented herein are the results of a parametric analysis describing the heave response characteristics of a representative TACV supported by plenum-type air cushions. This analysis is similar to that presented in reference 10 in that an analog computer model has been used to obtain responses of a plenum-type air-cushion vehicle to sinusoidal guideway excitations. However, this study has been based on realistic TACV parameters and includes the effect of a nonconstant supply flow rate. (Ref. 10 was restricted to a system with a constant supply flow rate.) In addition, an alternate active control concept employing movable cushion lips is introduced, and its effectiveness in controlling vehicle heave responses is described. The system parameters that are varied in the analysis include equilibrium gap height, equilibrium cushion support pressure, supply flow characteristics, cushion dead volume, and the magnitude of the input disturbances. Results are presented to illustrate the effect of these various system parameters on the problem of vehicle-guideway contact and on the nature of the vehicle acceleration responses. Additional data to indicate the range of oscillatory excursions which may be experienced by a vehicle during resonance have been found to be important for the design of TACV's requiring wayside power and are included in this report.

SYMBOLS

The units used for the physical quantities in this report are given in the International System of Units (SI) and parenthetically in U.S. Customary Units. Measurements were

made in U.S. Customary Units; however, any consistent system of units may be used in the analysis.

A	cushion support (base) area
A_s	area of supply duct
C_d	air-cushion discharge coefficient
$C_{d,s}$	supply-duct (orifice) discharge coefficient
g	acceleration due to gravity ($1g = 9.8$ m/sec^2 (32 ft/sec^2))
h	instantaneous vehicle hover height
h_o	depth of dead volume
K	feedback gain
l	perimeter of air cushion
m	total supported (vehicle) mass
N	ratio of supply pressure to cushion pressure
p	cushion pressure
p_a	ambient pressure
p_s	air-cushion supply pressure
R	universal gas constant
s	Laplace transform
t	time
T	temperature

V	total cushion volume
V_L	air-cushion live volume
V_0	air-cushion dead volume
W_0	weight flow rate out of air-cushion periphery
W_s	weight flow rate supplied to air cushion
x	air-cushion absolute displacement
y	actuator piston deflection
y_0	mean operating position of actuator piston
z	guideway input amplitude
z_{max}	maximum value of guideway input amplitude
α	guideway contact parameter
γ	polytropic gas constant, $\gamma = 1.4$
δ	guideway separation parameter
ρ	air density
τ	compensator time constant
τ_a	actuator time constant
ω	driving (input) frequency

A bar over a symbol denotes equilibrium conditions, and a dot over a symbol denotes differentiation with respect to time.

4

ANALYSIS

Mathematical Model

A diagram of the air-cushion vehicle studied in this analysis is illustrated in figure 1. The vehicle is represented as a one-degree-of-freedom system consisting of a mass supported by a single open-plenum air cushion operating at some gap height h above the guideway. Although real systems, such as the TACV, will have angular as well as translational degrees of freedom, this simplified representation is considered adequate for study of basic heave response characteristics. The guideway is assumed to be the major source of input disturbances, which result from local roughness, local waviness, and static and live-load deflections associated with guideway flexibility. The deflection inputs attributed to local guideway roughness and irregularities are generally high frequency and low amplitude (ref. 9) and may therefore be attenuated by the low-frequency characteristic of the air cushions. The inputs due to static and live-load deflections, however, may be of relatively large amplitude with frequencies well within the range of expected air-cushion natural frequencies and consequently cannot be ignored. They are represented in this analysis as vertical sinusoidal motions of a large surface.

The plenum device being analyzed contains an adjustable base plate to allow the air-cushion dead volume to be parametrically varied. The dead volume V_O is defined as the volume of air contained between the tip of the cushion lips and the base plate. The live volume V_L is the volume of air located directly beneath the air-cushion structure and is equal to the product of the cross-sectional area of the cushion base and the gap height h. The total volume V is

$$V = V_O + V_L = A(h_O + h) \tag{1}$$

where h_O is the depth of the dead volume.

The lower (plenum) chamber of the air cushion is supplied with air via suitable ducts (or orifices) connecting the upper and lower chambers. The upper chamber is assumed to be a constant pressure source, and the weight flow rate (supply flow) W_s into the lower chamber is assumed to be described by a simple orifice flow equation,

$$W_s = C_{d,s} A_s g \sqrt{2\rho(p_s - p)} \tag{2}$$

where $C_{d,s}$ is the orifice discharge coefficient, A_s is the orifice (or duct) area, ρ is the air density (assumed constant in the duct), p_s is the absolute supply pressure, and

p is the absolute cushion pressure. Equation (2) gives the supply flow characteristics of the system and can be written in terms of a useful system parameter called the supply pressure ratio defined by

$$N = \frac{p_s}{\bar{p}} \tag{3}$$

where $\bar{p}$ is the equilibrium cushion pressure. In terms of equation (3), the supply flow rate may be rewritten as

$$W_s = C_{d,s} A_{sg} \sqrt{2\rho\bar{p}\left(N - \frac{p}{\bar{p}}\right)} \tag{4}$$

Notice that for small values of N, the supply flow rate can be sensitive to changes in cushion support pressure p, whereas for large values of N the supply flow rate is relatively unaffected by changes in cushion pressure.

The weight flow rate W_o of air escaping from around the periphery of the air cushion is also described by a simple orifice equation in which the orifice area is now a function of gap height. This equation is

$$W_o = C_d \ell hg \sqrt{2\rho\left(p - p_a\right)} \tag{5}$$

where C_d is the discharge coefficient, ℓ is the perimeter of the plenum base, h is the gap height, ρ is the mass density of the air contained in the cushion volume, g is the acceleration due to gravity, and p_a is the ambient atmospheric pressure.

The coordinate system and nomenclature used in the development of the nonlinear equation of motion are shown in figure 2. Summing the forces acting on the vehicle mass gives

$$m\ddot{x} = A\left(p - p_a\right) - mg \tag{6}$$

where m is the vehicle mass, x is the absolute displacement of the vehicle mass, and A is the cushion support (base) area. The equation for cushion pressure is developed by assuming that the time derivative of momentum per unit volume within the air-cushion volume is equal to the change in mass flow into the volume. That is,

$$\frac{d}{dt}(\rho V) = \left(W_s - W_o\right)\frac{1}{g} \tag{7}$$

Performing the differentiation gives

$$\rho\dot{V} + V\dot{\rho} = \left(W_s - W_o\right)\frac{1}{g} \tag{8}$$

The isentropic gas law is now used to calculate the derivative of ρ in equation (8). The isentropic gas law states that

$$\frac{p}{\rho^\gamma} = \text{Constant} \tag{9}$$

where γ is the polytropic exponent ($\gamma = 1.4$). Taking the time derivative of equation (9) gives

$$\dot{\rho} = \frac{1}{\gamma}\frac{\rho}{p}\dot{p} \tag{10}$$

Substituting equation (10) into equation (8) and applying the perfect gas law ($p/\rho g = RT$) yield the following pressure equation:

$$\dot{p} = \frac{\gamma(W_s - W_0)}{V} RT - \frac{\gamma p \dot{V}}{V} \tag{11}$$

The final equation necessary to describe the motion of the system completely is the constraint relationship between coordinates. As seen in figure 2, this relationship can be written as

$$h = x - z \tag{12}$$

where $z = z_{max} \sin \omega t$ for harmonic guideway disturbances.

Active Lip Control

Active control of air-cushion heave response is based on reducing vehicle response by introducing effective damping and/or spring forces into the system. These forces are introduced by manipulating a quantity describing the dynamic state of the system according to a prescribed control law. In reference 10, for example, the system state was goverened when the input supply flow W_s was modulated by placing a controllable valve in the feed lines. The valve was activated continuously by electronically compensated measurements of vehicle acceleration response and was effective in minimizing undesirable heave responses. Two important factors which may limit the practical application of the input flow modulation concept, however, are (1) the inherent time lag between valve action and the generation of cushion pressure forces and (2) the size, complexity, power requirements, and reliability of state-of-the-art valves. An alternate concept, requiring less power (or size), would involve modulation of the cushion outflow by moving the cushion lip according to a specified control law. This concept, as illustrated in figure 3, offers the additional important advantage that lip motions will cause more rapid cushion pressure changes. Thus, the time lag between control action and the resulting cushion force should be reduced.

7

In figure 3 an accelerometer is assumed to be attached to the vehicle mass and a circumferential ring, free to slide vertically, is attached to the base of the air cushion by means of an electrohydraulic actuator. The lip is assumed to be positioned at a distance y_o below the cushion base and to respond only to acceleration signals. The operation of the active lip control system is as follows: The vehicle acceleration response is monitored continuously by the accelerometer, the output of which is applied to a compensator; the compensator output, in turn, activates the actuator to deflect the circumferential ring; the deflections of the ring modify the flow rate out of the cushion periphery and thereby generate variations in cushion pressure or support forces. By careful selection of feedback compensation, these variations in support forces will have in-phase and out-of-phase components which will alter the damping of the system response as well as reduce the effective air-cushion spring rate.

A simplified block diagram of the active lip control system is shown in figure 4. The accelerometer transfer function is assumed to be unity for the frequency range of interest. The actuator is represented as a first-order time lag, and the compensator is in the form of a lead network (to reject dc signals).

Solution of Equations

Because the dynamic and thermodynamic equations of the cushion are nonlinear, an analog computer was used to obtain solutions. Equations (1), (2), (4), (5), (6), (11), and (12) were solved for the response of the system to harmonic excitation by use of the basic circuit diagram of figure 5. The parameters varied in the analysis are input amplitude z_{max}, air-cushion dead volume V_o, supply pressure ratio N, and equilibrium support pressure $\Delta \bar{p}$ (defined as $\bar{p} - p_a$). The values of these parameters together with other pertinent numerical data are listed in table I, which includes some of the proposed operating parameters outlined in references 1 and 2 for the TACV research vehicle. The analog computer produced time histories of vehicle acceleration, absolute displacement, differential cushion pressure, gap height, and active lip displacements for each set of parameters listed in table I.

RESULTS AND DISCUSSION

This section presents the results of the analog computer solutions of the sinusoidally disturbed nonlinear equations describing the heave response of an air-cushion vehicle. Primary emphasis is devoted to results which show the effects of systematic variations of operating and design parameters on the relative motion between the vehicle and the guideway and the acceleration levels which can develop. The effect of the active lip control technique in reducing the probability of vehicle-guideway contact and vehicle accelerations is also demonstrated.

8

Vehicle Motions

Typical response time histories of the plenum air-cushion vehicle to a sinusoidal input frequency sweep (sweep rate of 0.10 Hz/sec) are presented in figures 6(a) and 6(b). The upper trace of each figure is the simulated guideway input displacement, the middle trace is the response in terms of the relative displacement between the vehicle and guideway (gap height), and the lower trace is the vehicle mass acceleration response. Figure 6(a) shows the response from excitation frequencies below, to slightly above the system natural frequency. Below the system natural frequency the response is basically sinusoidal. As the natural frequency is approached, however, the displacement and acceleration responses are large and become quite nonlinear. Above the natural frequency the response amplitude decreases but becomes large again, as shown in figure 6(b). The second resonant frequency oscillates at one-half the input frequency and has a peak amplitude at twice the system natural frequency. Such response is termed subharmonic response. Higher order subharmonic responses were not observed for the range of operating conditions covered in this study. The results of reference 10 showed that subharmonic responses were found to be highly sensitive to the nature of the input. Because of the response sensitivity, the results presented herein related to the subharmonic response are considered typical and are presented to show trends only.

Vehicle-guideway contact.- A very important consideration affecting vehicle performance and safety is the likelihood of vehicle-guideway contact during resonant cushion oscillations. A nondimensional parameter which is a measure of the minimum dynamic gap clearance between the air-cushion lip and guideway at resonance is defined as follows:

$$\frac{h_{min}}{\bar{h}} = \alpha \tag{13}$$

where α is called the guideway contact parameter, h_{min} denotes the minimum gap clearance measured at resonance, and $\bar{h}$ is the equilibrium hover height. A second nondimensional parameter defining the maximum dynamic gap clearance between the air-cushion lip and guideway is

$$\frac{h_{max}}{\bar{h}} = \delta \tag{14}$$

where δ is called the guideway separation parameter and h_{max} is the maximum gap clearance measured at resonance. The parameter δ, together with the guideway contact parameter α, defines the envelope of vertical travel the TACV may experience under worst-case conditions.

The effect of variations in all the system parameters (i.e., input amplitude, supply pressure ratio, dead volume, and equilibrium support pressure) on the guideway contact

parameter α is presented in figure 7. The range of variation of each parameter is listed in table I. The envelope of response defined by the shaded parts of each plot of figure 7 represents the effect of varying supply ratio over the range considered (N = 1.5 to 4.0) and is presented for small dead volumes (approximately zero) and dead volumes equal to twice the live volume $\left(V_O \to 0 \quad \text{and} \quad V_O = 2V_L\right)$.

Effect of input amplitude and dead volume: Inspection of figures 7(a) to 7(f) indicates that the guideway contact parameter α decreases with increasing input amplitude and at some input amplitude becomes zero and thereby indicates vehicle-guideway contact. The magnitude of input amplitude at which contact will occur is considerably smaller for the larger dead volume. This result may be partially attributed to the fact that increasing the dead volume tends to significantly reduce the effective damping of the air-cushion system (refs. 9 and 10). In fact, if dead volume is increased sufficiently, dynamic instability can result. (Dynamic instability in this report, is defined as an oscillation that builds up until vehicle-guideway contact occurs.) For example, the air cushion operating at the conditions indicated in figure 7(f) was unstable at all input amplitudes for $V_O = 2V_L$. These results show that for a plenum-supported vehicle, the air-cushion dead volume should be kept as small as possible in order to minimize the likelihood of vehicle-guideway contact. Furthermore, a cushion operating at a particular equilibrium with stable dead-volume conditions can become unstable if, during operation, the hover height is reduced. Such conditions may occur for an air-cushion vehicle operating over a wide range of equilibrium hover heights.

Effect of supply pressure ratio: The shaded parts of each plot of figure 7 indicate the effect of varying supply pressure ratio N for the two dead volumes considered. For small or near-zero dead volume, the magnitude of input amplitude at which contact occurs increases with increasing supply pressure ratio. This result is to be expected, since for the large values of N, the supply flow rate is relatively unaffected by cushion pressure changes and increases both the magnitude and rapidity of pressure buildup as the cushion approaches the guideway. Much larger vertical restoring forces are generated which act to decelerate the downward cushion motion more effectively and thereby reduce the possibility of contact. When the dead volume is large, the response trends associated with changes of N are not consistent with those for small dead volume but depend upon the particular value of equilibrium support pressure, probably because of the nonlinear nature of the cushion responses. It is therefore difficult to draw a general conclusion from these results regarding a preferable value of supply pressure ratio.

Equilibrium support pressure: From the standpoint of vehicle-guideway contact, the most desirable response for both hover heights occurs at the lowest value of equilibrium support pressure $\Delta\bar{p}$. At the largest value of equilibrium support pressure $(\Delta\bar{p} = 28.68 \text{ kN/m}^2 \ (4.16 \text{ psig}))$ and at large dead volume, the system may become dynamically unstable, and oscillations will rapidly build up until contact occurs. For example,

when $V_O = 2V_L$ in figure 7(f), the system is unstable at all input amplitudes and all pressure ratios. One reason for this behavior is that increasing cushion support pressures tends to decrease the effective cushion damping. (See ref. 10.)

The nondimensional input amplitude at which each curve of figures 7(a) to 7(f) intersects the abscissa is plotted as a function of equilibrium support pressure in figure 8 for two values of supply pressure ratio (N = 1.5 and 4.0) and two values of dead volume $\left(V_O \to 0 \text{ and } V_O = 2V_L\right)$. These curves define the maximum input levels that an uncontrolled air cushion can tolerate under these conditions. The maximum tolerable input amplitudes are much higher at the lower values of equilibrium support pressure for both values of pressure ratio and dead volume. The trend associated with pressure ratio, however, is reversed when the dead volume is large, the lower value of pressure ratio being the more desirable condition.

Subharmonic response.- A typical plot illustrating the likelihood of vehicle-guideway contact due to subharmonic oscillations is shown in figure 9 for the system with small dead volume. The guideway contact parameter α is presented as a function of nondimensional input amplitude for two values of equilibrium support pressure, $\Delta\bar{p} = 7.17$ and 14.34 kN/m^2 (1.04 and 2.08 psig), and the full range of supply pressure ratio N. Examination of these curves shows that the vehicle can readily contact the guideway while undergoing subharmonic oscillations, and in fact, contact will generally occur much easier (less input amplitude is required) than it does during fundamental resonant oscillations. These curves also show that the lower value of equilibrium support pressure still permits larger inputs before contact.

An interesting point observed during the analog computer runs was that very small transient disturbances could trigger very large subharmonic responses, especially when the system was being forced sinusoidally at frequencies above fundamental resonance. In many cases (for the smaller sinusoidal input amplitudes), the subharmonic responses would not occur without the transient being applied. This observation is important, since it is very possible that a TACV operating along a practical guideway will experience combined excitations and will therefore be susceptible to oscillations of this nature. Of special note is that the results of figures 7 to 9 suggest that a low-pressure cushion with small dead volume V_O and a large supply pressure ratio N (corresponding to a relatively constant supply flow rate) will minimize the likelihood of vehicle-guideway contact.

Vehicle-Guideway Separation

The effect of input amplitude on the vehicle-guideway separation parameter δ during fundamental resonance is presented in figure 10 for the two equilibrium hover heights and for small dead volume. Each set of curves corresponds to a specific value of equilibrium support pressure and contains the four values of supply pressure ratio.

Each curve is plotted for a range of input amplitudes that extend to the amplitude at which vehicle-guideway contact occurs, as denoted by the symbols. These data indicate (particularly for the higher pressure ratios) that during fundamental resonant oscillation, the maximum vehicle-guideway separation distance may be several times larger than the equilibrium hover height. Note also that the actual magnitudes of the separation parameter at any specific input amplitude is relatively unaffected by supply pressure ratio N but is very dependent upon equilibrium support pressure, the lower equilibrium support pressures resulting in the smaller vehicle-guideway excursions.

A typical example of the effect of input amplitude on the guideway separation parameter during subharmonic oscillations is shown in figure 11 for an equilibrium hover height of 2.54 cm (1.0 in.), an equilibrium support pressure of 14.34 kN/m^2 (2.08 psig), and small dead volume. These curves indicate that vehicle-guideway separation during subharmonic oscillations may also be quite large and, in addition, may depend to a greater extent on supply pressure ratio.

Vehicle Acceleration Response

The maximum vehicle accelerations, or decelerations since they occur as the vehicle approaches the guideway, are caused by pressure buildup in the air cushions. Figure 12 presents the maximum vehicle accelerations as a function of the input amplitude and supply pressure ratio for an equilibrium support pressure of 14.34 kN/m^2 (2.08 psig) and small dead volume. Similar curves were obtained for the other values of equilibrium support pressure. Each of the curves shown in this figure extends to the input amplitude at which contact occurs. The data for both hover heights indicate that the magnitude of the vehicle accelerations may be quite large and that they do increase significantly with increasing supply pressure ratio. By examining figure 12, it can be seen that although the accelerations for both hover heights are roughly of the same order of magnitude, the input amplitudes differ by an order of magnitude. Thus, operation at small hover heights will not offer any appreciable advantage from the standpoint of vehicle acceleration response. In fact, it will be detrimental because of the severely reduced input amplitude tolerances and the fact that the dynamic amplifications range (that is, the input frequency bandwidth at which the input is amplified) is extended to much higher frequencies, as shown in reference 10. A typical example of the acceleration levels attributed to the air-cushion subharmonic response is presented in figure 13 for a hover height of 2.54 cm (1.0 in.) and an equilibrium support pressure of 14.34 kN/m^2 (2.08 psig). It is seen that in the absence of transients, subharmonic acceleration responses did not begin to build up until the input amplitudes exceeded approximately 0.254 cm (0.10 in.). Beyond this point, however, the vehicle accelerations did rise quite rapidly with input amplitude. Comparison of these data with that of figure 12 indicates that for the identical range of input amplitudes, the subharmonic accelerations are generally somewhat larger than the

accelerations associated with fundamental resonance. Thus, from the standpoint of vehicle dynamics, stability, and ride comfort, the possibility of subharmonic responses being generated and sustained cannot be ignored.

Several additional comments of a qualitative nature regarding the subharmonic response phenomena are of interest. It was observed that the inclusion of dead volume on the air cushion had a very pronounced destabilizing effect at subharmonic resonance, much more so than at fundamental resonance. Tolerable input amplitudes were greatly reduced, and both stable and unstable behavior readily occurred. The cushion operating at large static support pressures $\Delta \bar{p}$ (and with some nominal dead volume) was likely to become unstable at virtually all input amplitudes (with the exception of very small inputs) and for all values of supply pressure ratio N.

Active Lip Control

The likelihood of vehicle-guideway contact, the presence of large fundamental and subharmonic acceleration responses, and the large excursions of the vehicle from the guideway make it desirable to apply some method of minimizing vehicle response. The approach considered in this analysis was to employ the controllable cushion lip using acceleration feedback as described in the section "Analysis." The use of acceleration feedback was chosen because it is probably the most practical feedback mode in that it is a relatively simple matter to measure vehicle acceleration; whereas the use of position (gap height) may require more sophisticated sensors. Since the primary intent of the analysis was to show the feasibility and possible effectiveness of an active lip control system, no attempt was made to design in detail or to optimize the system.

Figure 14 shows a typical effect of the control technique on the guideway contact parameter at the fundamental resonant condition. The data presented in this figure (and fig. 15) fall within the bands shown on the plot, which represent the envelope of vehicle response for the full range of supply pressure ratios and dead volume. The results of figure 14 show that the active lip control system significantly increases the guideway contact parameter α and virtually eliminates the possibility of contact. No subharmonic oscillations were found to occur. In essence, the active control system is isolating the vehicle from the guideway disturbance inputs and has reduced the sensitivity of vehicle response to variations in operating parameters. Figure 15 presents the acceleration response envelope as a function of acceleration feedback gain. As indicated, the acceleration response drops off rather rapidly with increasing feedback gain and above a gain of approximately 0.5 remains essentially at about 0.25g. The displacement of the cushion lip required to achieve the reduction in vehicle response indicated in figures 14 and 15 is illustrated in figure 16 as a function of acceleration feedback gain. As expected, the lip displacements increase with increasing feedback gain. The region bounded by the dashed

lines of figure 16 represents the probable range of operating parameters for the active lip control systems under study. Figures 14 to 16 show that the active lip control technique is very effective in reducing undesirable vehicle responses.

CONCLUDING REMARKS

A parametric analysis of the heave response of a plenum-type air-cushion vehicle with and without an active lip control has been conducted. Results have been presented to demonstrate the effect of systematic variations of operating parameters on the likelihood of vehicle-guideway contact and vehicle acceleration response.

The air-cushion dynamic response is nonlinear and depends significantly on operating parameters such as equilibrium hover height, dead volume, supply pressure ratio (ratio of supply pressure to equilibrium cushion pressure) and equilibrium support pressure (pressure above ambient). Depending on operating parameters, the air cushion may contact the guideway during resonant oscillations at input amplitudes much smaller than the equilibrium hover height. The likelihood of vehicle-guideway contact may be reduced by operating the air cushion at low pressures with small dead volume and a large supply pressure ratio. These same operating conditions will also tend to minimize vehicle excursions during resonance. The addition of dead volume to the air cushion increased the likelihood of vehicle-guideway contact.

Vehicle acceleration responses of the air cushions may be rather large and for constant values of equilibrium support pressure were found to increase considerably with increasing supply pressure ratio. Thus, the use of large supply pressure ratios, although reducing the possibility of contact, will increase acceleration response levels.

Subharmonic responses of the uncontrolled vehicle may readily occur and could possibly be more detrimental from the standpoint of vehicle-guideway contact and acceleration response than the fundamental resonant responses. Subharmonic response was found to be easily initiated when transient disturbances were superimposed upon a steady-state sinusoidal driving function and was also found to be significantly dependent on operating parameters.

Active lip control was found to be very effective in reducing vehicle acceleration responses and may eliminate the possibility of vehicle-guideway contact and subharmonic response.

Langley Research Center,
    National Aeronautics and Space Administration,
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TABLE I.- NUMERICAL VALUES

(a) SI Units

Parameter	Range	
	$\bar{h}$ = 0.254 cm	$\bar{h}$ = 2.54 cm
Cushion base area, A	1.16 to 4.64 m^2	1.16 to 4.64 m^2
Equilibrium support pressure, $\Delta\bar{p}$ ...	7.17 to 28.7 kN/m^2	7.17 to 28.7 kN/m^2
Dead volume, V_O	0.003 to 0.012 m^3	0.03 to 0.24 m^3
Supply pressure ratio, p_s/p	1.5 to 4.0	1.5 to 4.0
Supported weight, mg	33.4 kN	33.4 kN
Live volume, V_L	0.006 m^3	0.06 m^3
Absolute cushion pressure, p	108.5 to 130 kN/m^2	108.5 to 130 kN/m^2
Ambient pressure, p_a	101.4 kN/m^2	101.4 kN/m^2
Feedback gain, K		0 to 2.0
Actuator time constant, τ_a		1.59 sec
Compensator time constant, τ		1.59 sec
Air density, ρ	1.23 kg/m^3	1.23 kg/m^3
Temperature, T	289 K	289 K
Universal gas constant, R	8.31 J/mol-K	8.31 J/mol-K

(b) U.S. Customary Units

Parameter	Range	
	$\bar{h}$ = 0.10 in.	$\bar{h}$ = 1.0 in.
Cushion base area, A	1800 to 7200 in^2	1800 to 7200 in^2
Equilibrium support pressure, $\Delta\bar{p}$ ...	1.04 to 4.16 psig	1.04 to 4.16 psig
Dead volume, V_O	180 to 720 in^3	1800 to 14 400 in^3
Supply pressure ratio, p_s/p	1.5 to 4.0	1.5 to 4.0
Supported weight, mg	7500 lb	7500 lb
Live volume, V_L	360 in^3	3600 in^3
Absolute cushion pressure, p	15.74 to 18.86 psig	15.74 to 18.86 psig
Ambient pressure, p_a	14.7 psig	14.7 psig
Feedback gain, K		0 to 2.0
Actuator time constant, τ_a		1.59 sec
Compensator time constant, τ		1.59 sec
Air density, ρ	$11.5 \times 10^{-8} \dfrac{\text{lb-sec}^2}{\text{in}^4}$	$11.5 \times 10^{-8} \dfrac{\text{lb-sec}^2}{\text{in}^4}$
Temperature, T	520° R	520° R
Universal gas constant, R	640 $\dfrac{\text{in-lbf}}{\text{lbm-}°\text{R}}$	640 $\dfrac{\text{in-lbf}}{\text{lbm-}°\text{R}}$
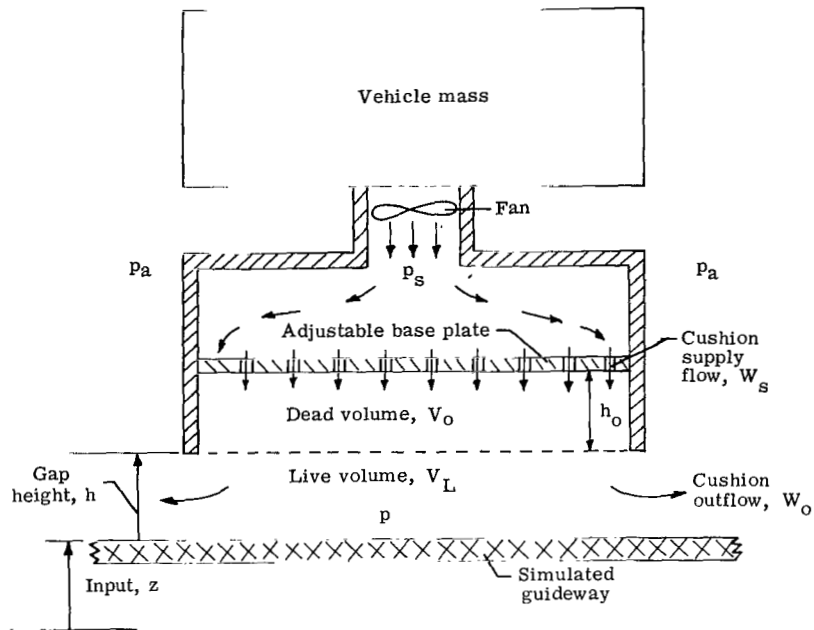
Figure 1.- Diagram of air-cushion vehicle.
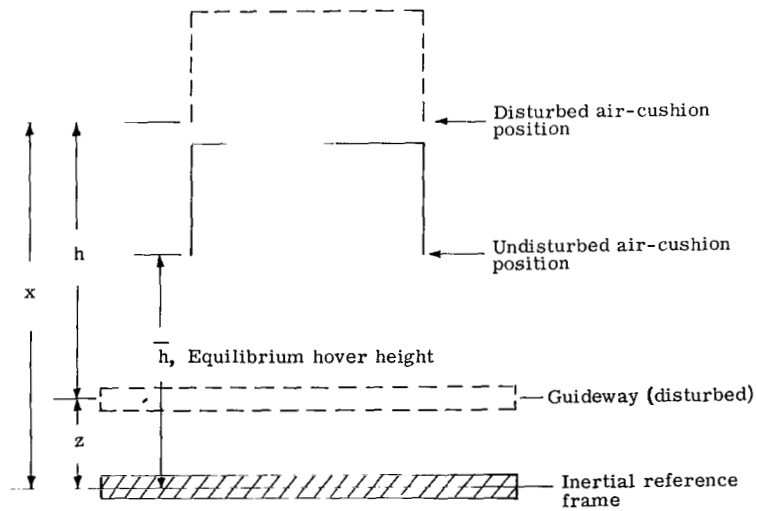
Figure 2.- Air-cushion coordinate system.
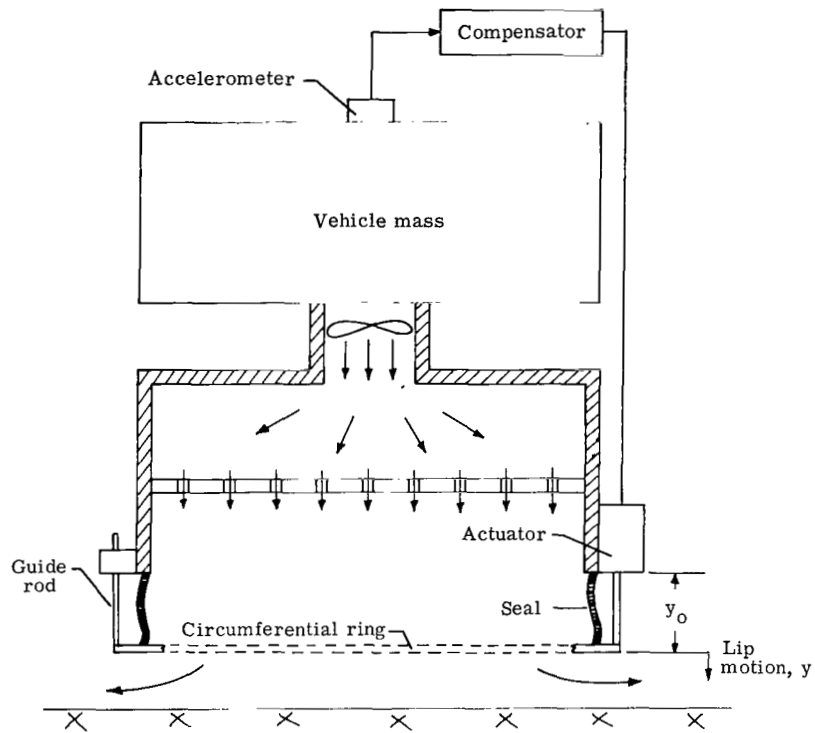
Figure 3.- Schematic of active lip control system.
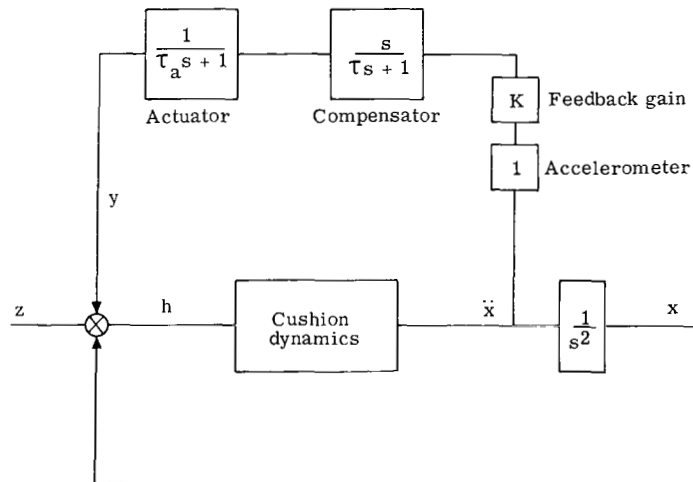
Figure 4.- Simplified block diagram of active lip
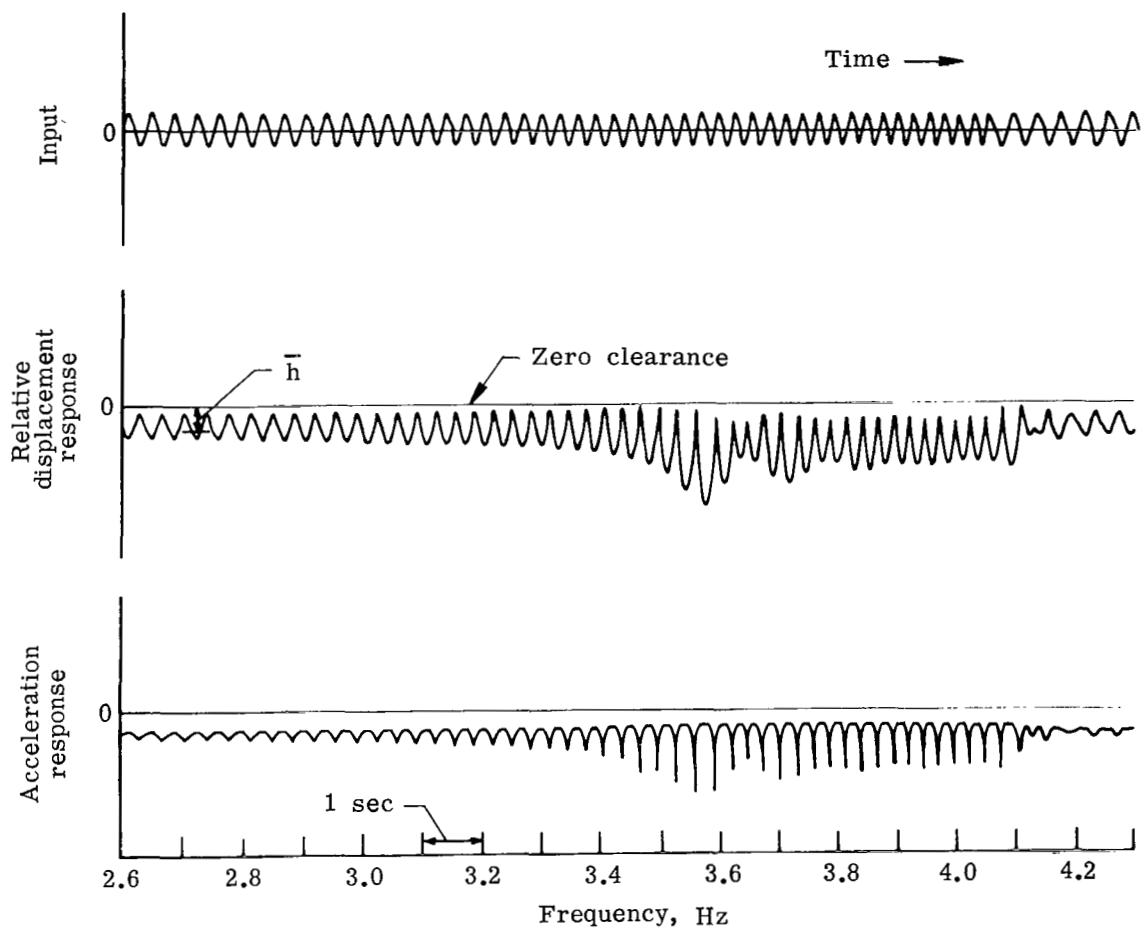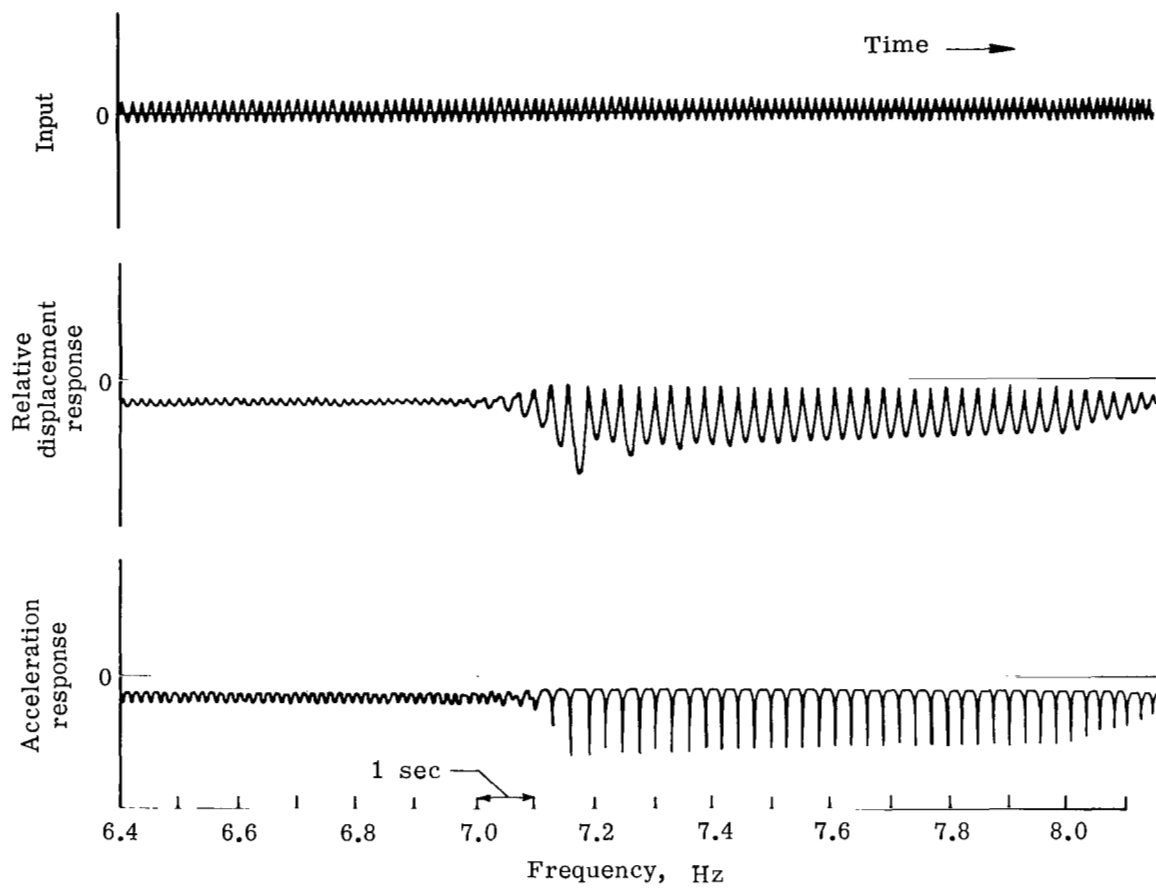control system with acceleration feedback.
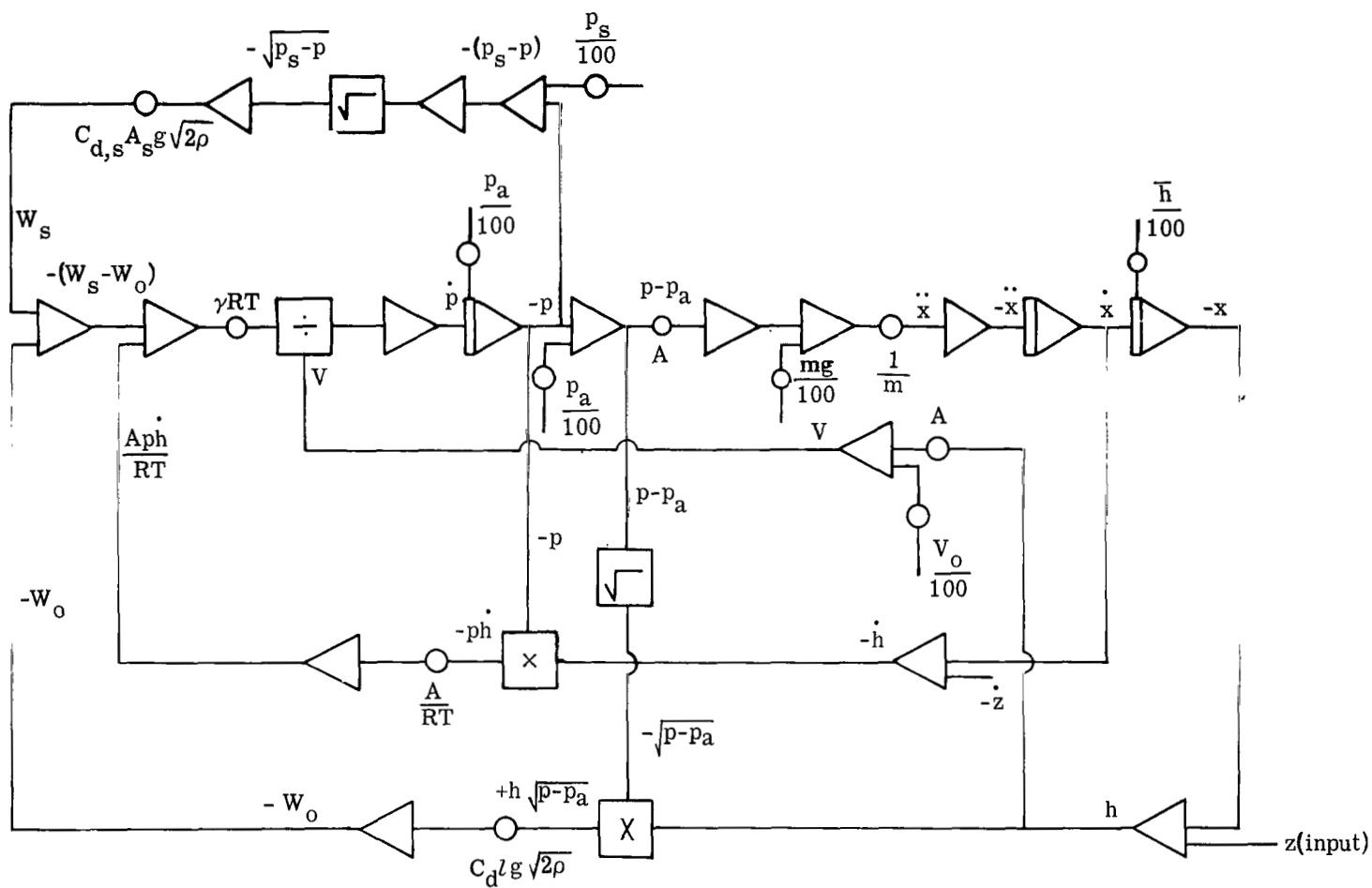
Figure 5.- Basic circuit diagram of analog computer.

(a) Fundamental response.

Figure 6.- Typical response time histories.
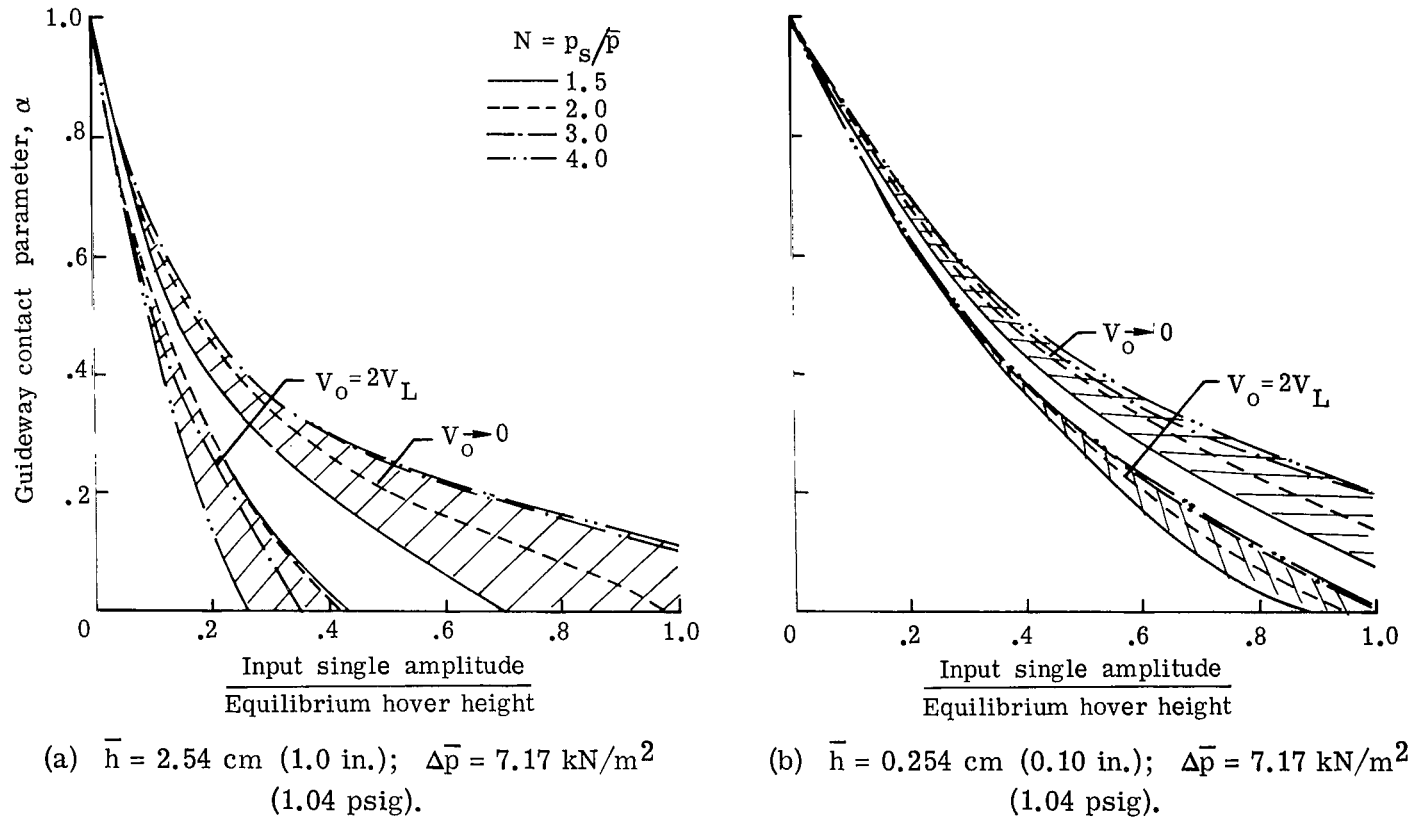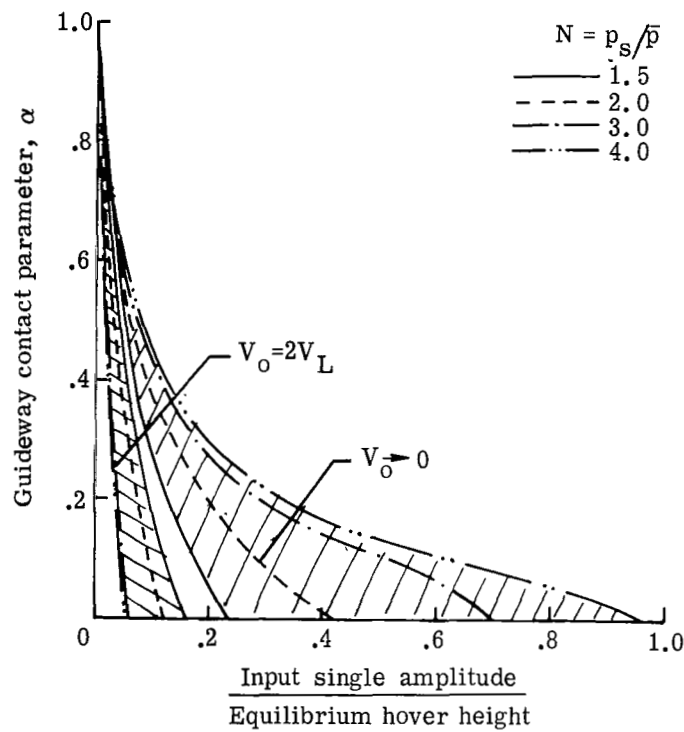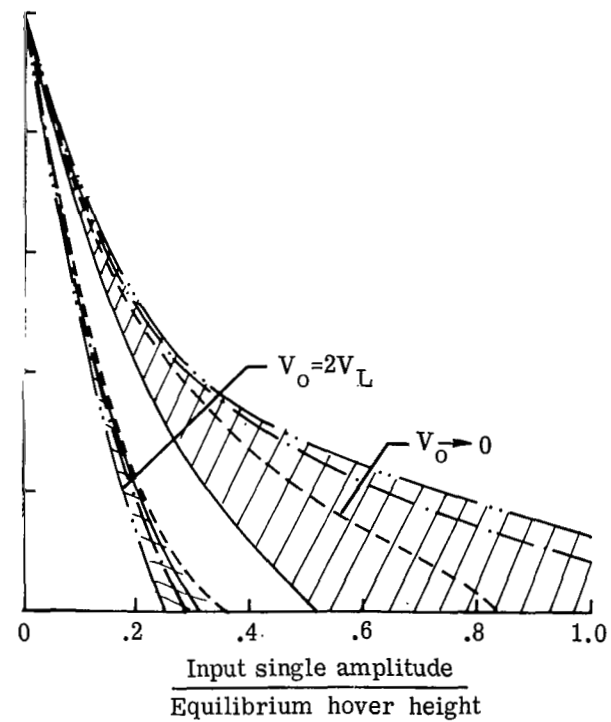
(b) Subharmonic response.

Figure 6.- Concluded.

(a) $\overline{h} = 2.54$ cm (1.0 in.); $\Delta\overline{p} = 7.17$ kN/m^2 (1.04 psig).

(b) $\overline{h} = 0.254$ cm (0.10 in.); $\Delta\overline{p} = 7.17$ kN/m^2 (1.04 psig).

Figure 7.- Effect of variations in system parameters on guideway contact parameter. Guideway contact corresponds to $\alpha = 0$.

(c) $\bar{h} = 2.54$ cm (1.0 in.); $\Delta\bar{p} = 14.34$ kN/m^2 (2.08 psig).

(d) $\bar{h} = 0.254$ cm (0.10 in.); $\Delta\bar{p} = 14.34$ kN/m^2 (2.08 psig).

Figure 7.- Continued.
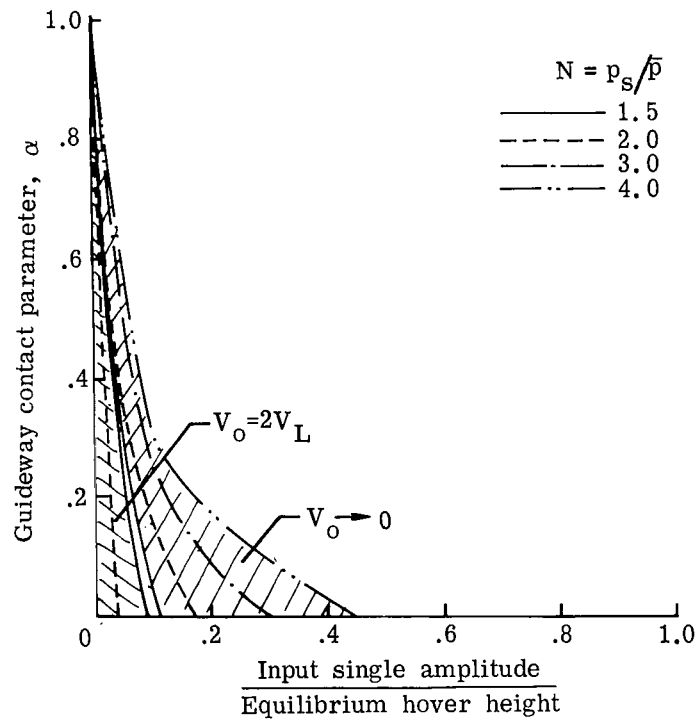
(e) $\bar{h}$ = 2.54 cm (1.0 in.); $\Delta\bar{p}$ = 28.68 kN/m^2 (4.16 psig). Note: This case is unstable at p_s/p = 3.0 and 4.0 for V_O = 2V_L.
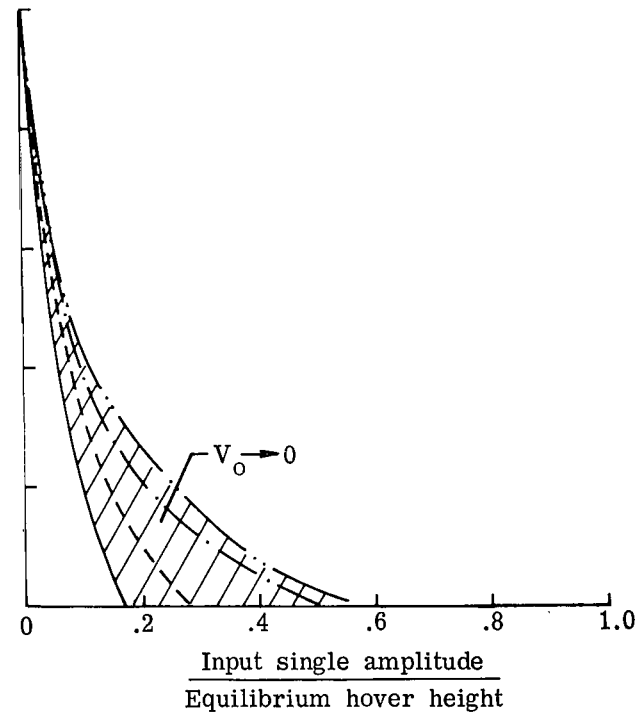
(f) $\bar{h}$ = 0.254 cm (0.10 in.); $\Delta\bar{p}$ = 28.68 kN/m^2 (4.16 psig). Note: This case is unstable for all input at V_O = 2V_L.

Figure 7.- Concluded.
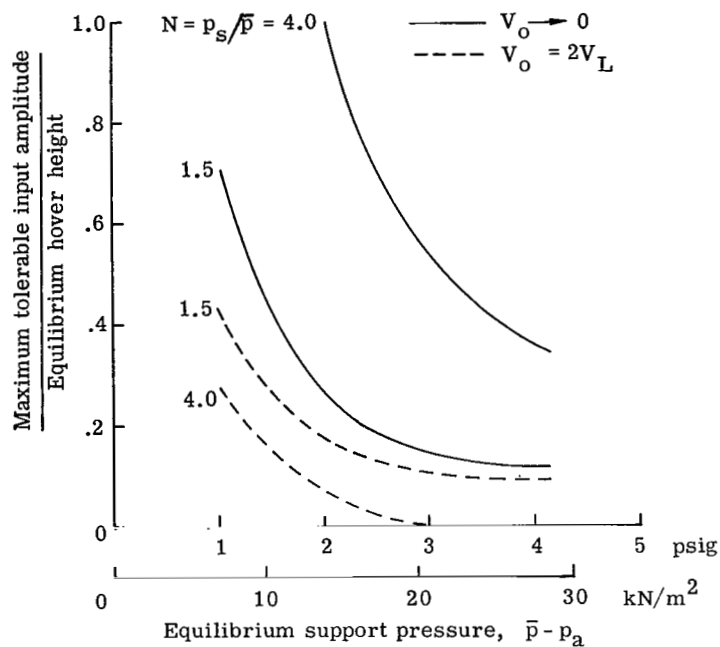
(a) $\bar{h} = 2.54$ cm (1.0 in.).
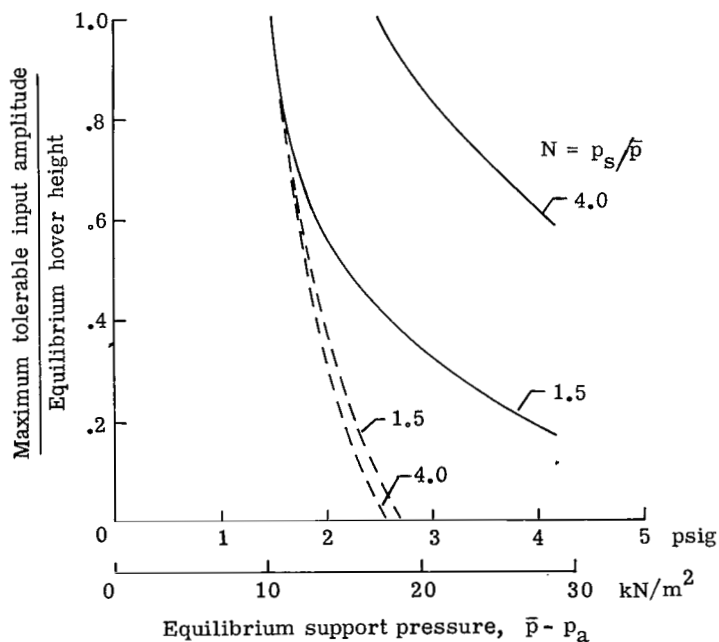
(b) $\bar{h} = 0.254$ cm (0.10 in.).

Figure 8.- Effect of equilibrium support pressure and supply pressure ratio on maximum tolerable input amplitude for hover heights of 2.54 cm (1.0 in.) and 0.254 cm (0.10 in.) and for dead volumes approaching zero and $2V_L$.
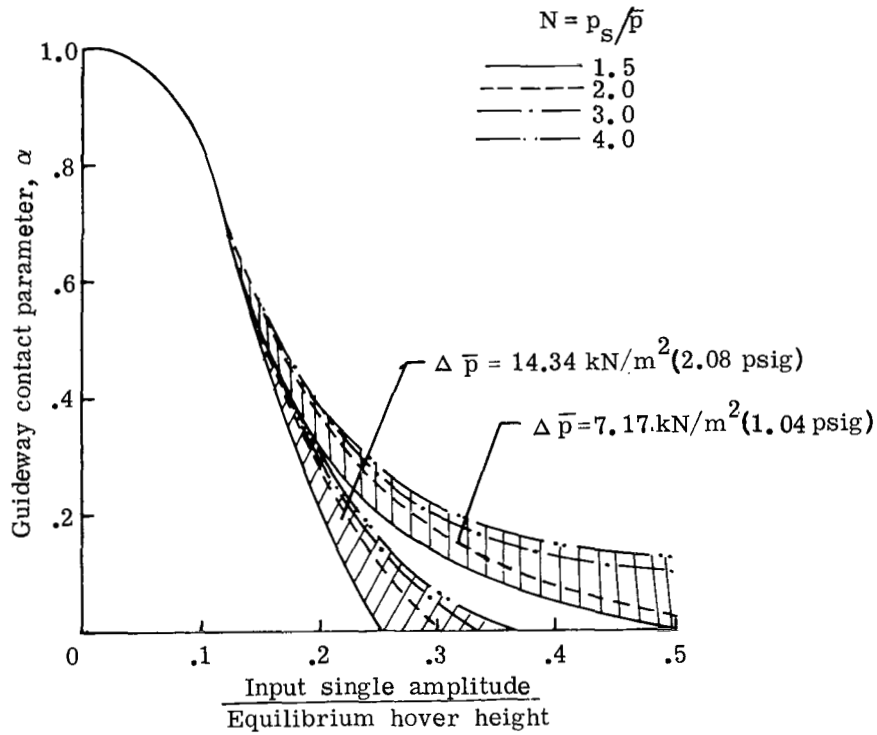
Figure 9.- Effect of input amplitude and supply pressure ratio on guideway contact parameter at subharmonic resonance. $\bar{h}$ = 2.54 cm (1.0 in.); $V_0 \to 0$; $\alpha = 0$ corresponds to contact. Note: For $\Delta\bar{p}$ = 28.68 kN/m^2 (4.16 psig), the cushion is stable only for very small input amplitude.

(a) $\overline{h}$ = 2.54 cm (1.0 in.).

(b) $\overline{h}$ = 0.254 cm (0.10 in.).
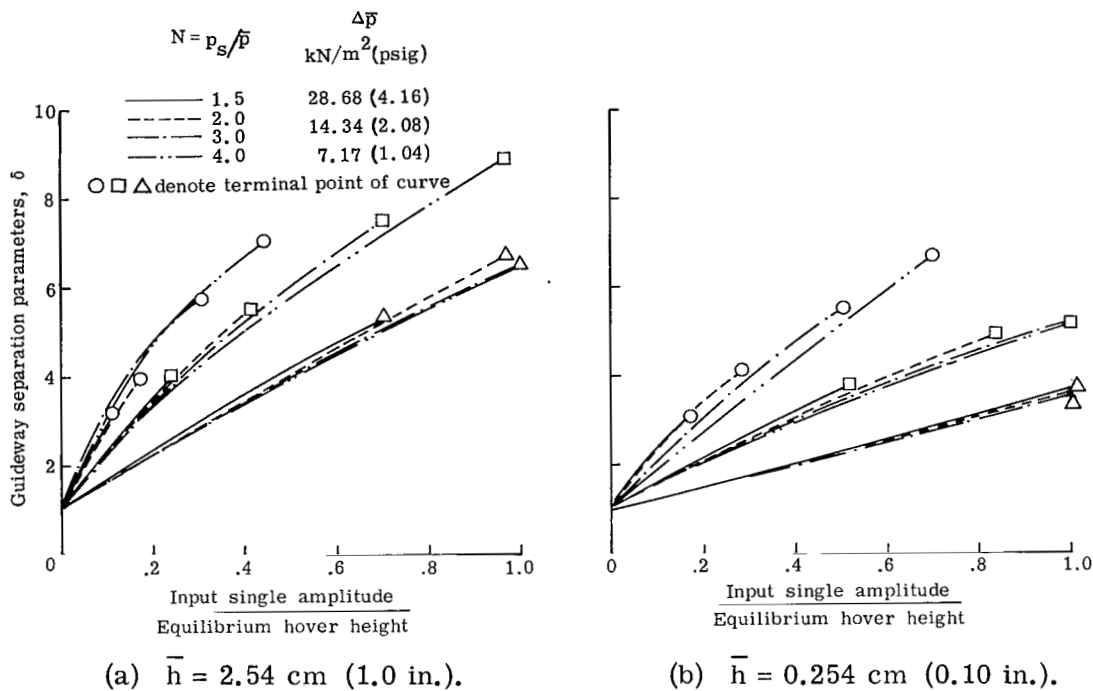
Figure 10.- Effect of input amplitude, equilibrium support pressure, and supply
pressure ratio on guideway separation parameter. $V_O \rightarrow 0$.
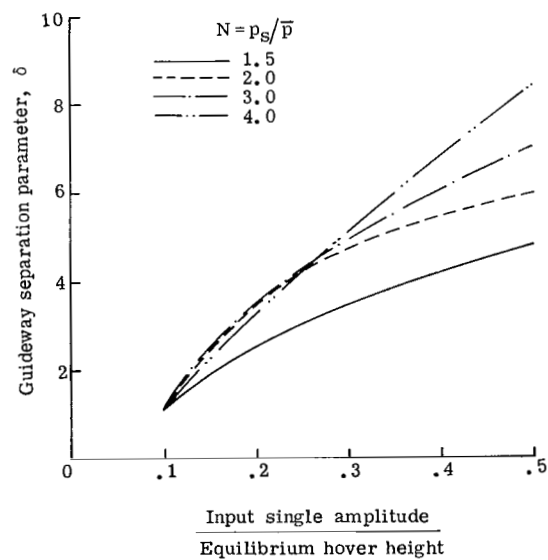
Figure 11.- Guideway separation parameter during
subharmonic oscillations. $\overline{h}$ = 2.54 cm (1.0 in.);
$V_O \rightarrow 0$; $\Delta\overline{p}$ = 14.34 kN/m^2 (2.08 psig).

27
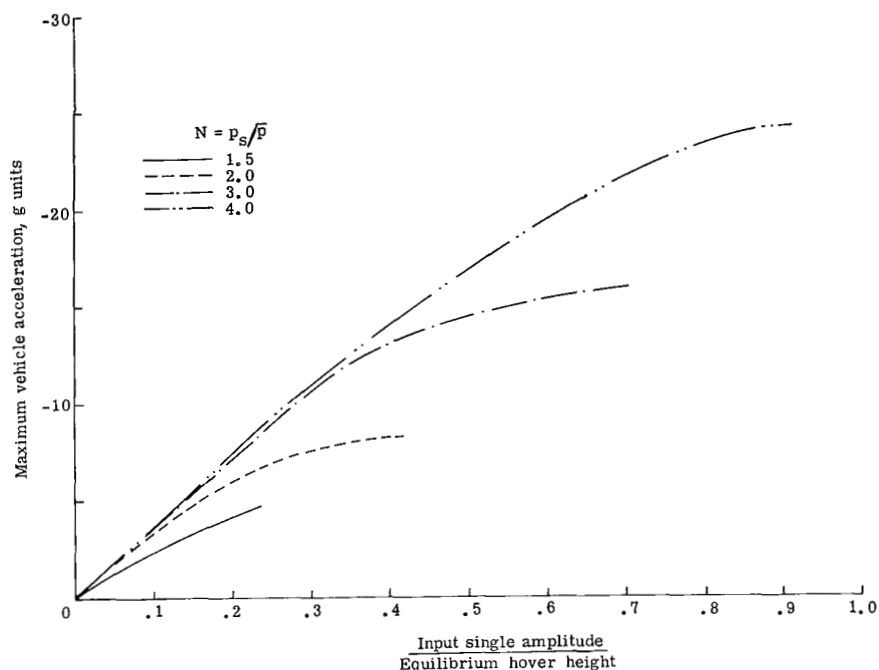
(a) $\overline{h}$ = 2.54 cm (1.0 in.).
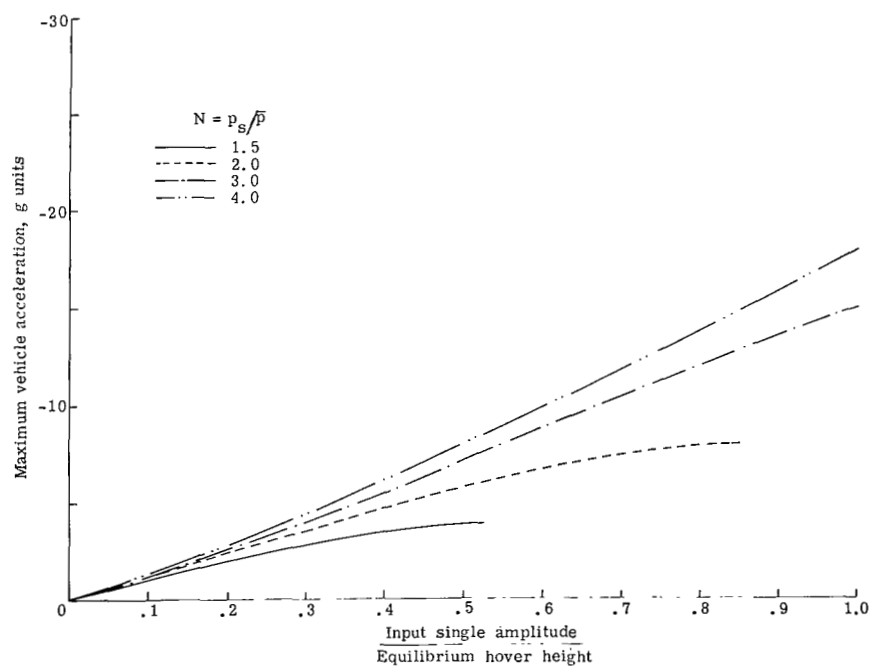
(b) $\overline{h}$ = 0.254 cm (0.10 in.).

Figure 12.- Vehicle acceleration response at fundamental resonance as a function of input amplitude and supply pressure ratio. $\Delta\overline{p}$ = 14.34 kN/m^2 (2.08 psig); $V_o \rightarrow 0$.
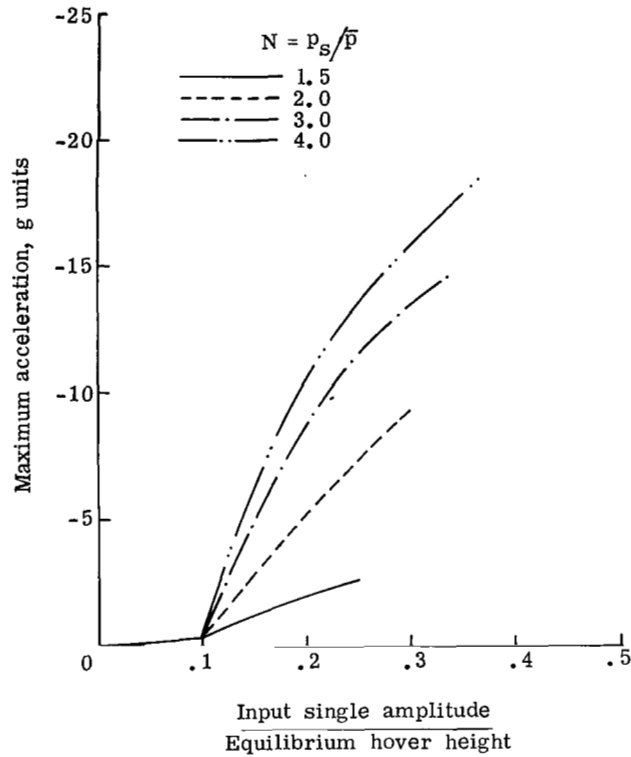
Figure 13.- Accelerations experienced during subharmonic oscillations.
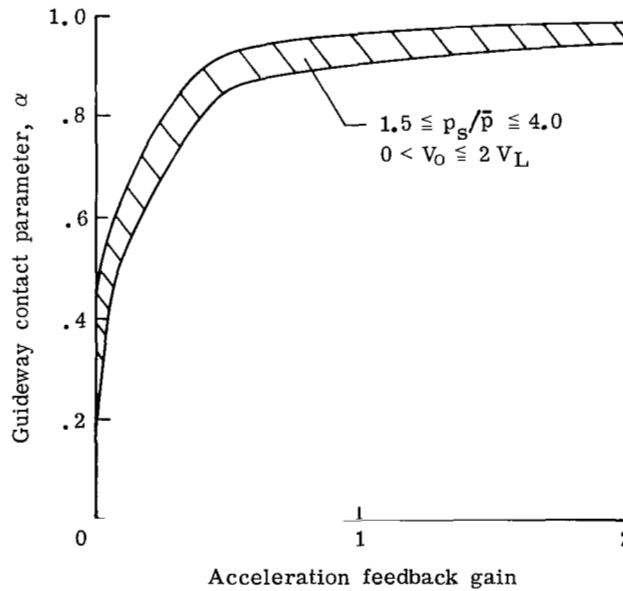$\overline{h}$ = 2.54 cm (1.0 in.); $\Delta\overline{p}$ = 14.34 kN/m^2 (2.08 psig).

Figure 14.- Effect of active lip control on guideway contact parameter.
$\overline{h}$ = 2.54 cm (1.0 in.); $\Delta\overline{p}$ = 14.34 kN/m^2 (2.08 psig). Note: Sub-
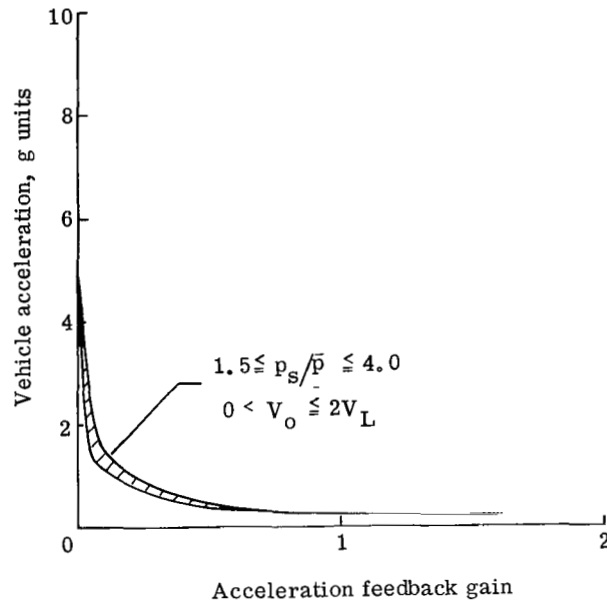harmonic responses did not occur with the active lip control operating.

29

Figure 15.- Effect of active lip control on vehicle accelera-
tion response. $\bar{h}$ = 2.54 cm (1.0 in.); $\Delta\bar{p}$ = 14.34 kN/m^2
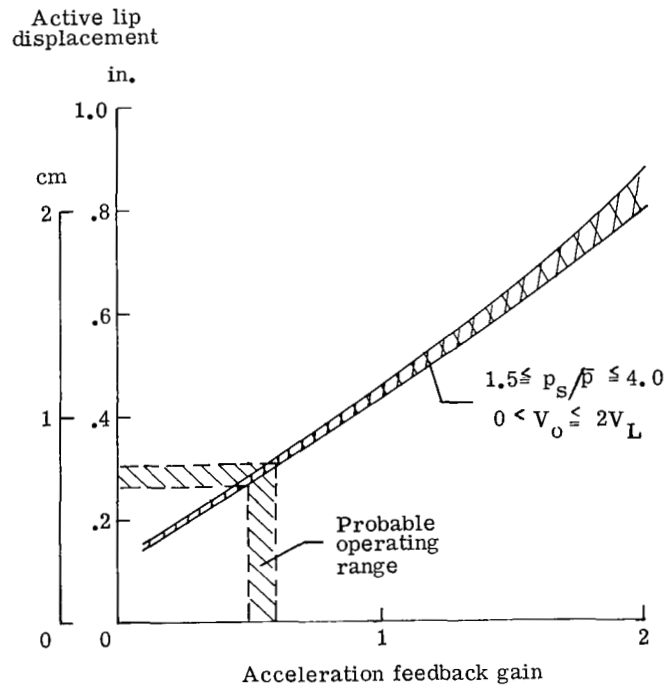(2.08 psig).

Figure 16.- Effect of active lip control on required
active lip displacement. $\bar{h}$ = 2.54 cm (1.0 in.);
$\Delta\bar{p}$ = 14.34 kN/m^2 (2.08 psig).