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ERROR ANALYSIS OF THE MARS GRAVITATIONAL FIELD ESTIMATION USING RANGE AND RANGE-RATE DATA FROM A VIKING-TYPE ORBITER

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16. Abstract

Standard deviations associated with a weighted least-squares process for estimating the spherical harmonic coefficients in the Mars gravitational potential function are presented for the coefficients through degree and order four. These standard deviations were calculated by using a priori uncertainties on the assumed estimated coefficients and model error uncertainties in the tracking-station location, the Mars ephemeris, the astronomical unit, the Mars gravitational constant, and the gravitational coefficients form degree five and order zero through degree and order seven. The tracking data types were assumed to be earth-based range and range-rate measurements from an artificial satellite about Mars. A comparison of the results for the use of each data type independently and in combination is presented.

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ERROR ANALYSIS OF THE MARS GRAVITATIONAL FIELD ESTIMATION USING RANGE AND RANGE-RATE DATA FROM A VIKING-TYPE ORBITER

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SUMMARY

An error analysis of a weighted least-squares process for estimating the gravitational coefficients through degree and order four in the Mars potential function is presented in this report. Earth-based range (light time travel) and range-rate (Doppler) measurements from a Viking-type orbiter were assumed. The results are presented in terms of standard deviations calculated by assuming a priori statistics on the assumed estimated coefficients and model error uncertainties for tracking-station locations, the Mars ephemeris, the astronomical unit, the Mars gravitational constant, and the gravitational coefficients from degree five and order zero through degree and order seven.

After 30 orbits of tracking, the standard deviations for the second-degree gravitational coefficients showed an improvement of an order of magnitude over their corresponding a priori values, but the third- and fourth-degree gravitational coefficients did not show a significant improvement over their corresponding a priori values. Model error effects for the Viking-type orbit were shown to be small in the presence of a priori information; however, for an orbit similar to the Viking type but with a period not synchronous with the rotational period of Mars, these effects are shown to be very important.

A comparison of range and range-rate shows that the two data types are similar as far as the accuracy of estimating the gravitational coefficients of Mars is concerned. Range-rate appears to be slightly better of the two and the addition of range with range-rate resulted in very little improvement in the accuracy of estimating the coefficients.

INTRODUCTION

Current plans for planetary research missions include the Mars 1975 Viking Project. The purpose of this project is to increase the scientific knowledge of Mars by analyzing scientific data returned from experiments carried on two artificial satellites in orbit around Mars and two unmanned spacecraft on the surface of Mars. Since the orbiter and the lander both are to be used for scientific exploration and since the orbiter is to be used for interrogation of the lander, it is then necessary to calculate a very accurate position of the landed spacecraft and an accurate ephemeris of the orbiter. In order to do this, certain physical constants must be included in the planet model used for orbit determination. One set of such physical constants are the gravitational coefficients $C_{n,m}$ and $S_{n,m}$ of the spherical harmonics in the expansion of the Mars gravitational potential function U. This function is

$$U = \frac{G_M}{r}\left[1 + C_{0,0} + \sum_{n=2}^{\infty} \sum_{m=0}^{n} \left(\frac{R}{r}\right)^n P_{n,m}(\sin \phi)\left(C_{n,m} \cos m\lambda + S_{n,m} \sin m\lambda\right)\right]$$

where

G_M Mars gravitational constant

r distance from center of Mars to satellite

ϕ areographic latitude of satellite

λ areographic longitude of satellite

R mean radius of Mars

$P_{n,m}$ associated Legendre polynomials (n is degree and m is order)

The coefficients $C_{0,0}$ represents a deviation of the mass of Mars from a nominal value assumed in G_M. An accurate knowledge of these gravitational coefficients would be used in all Mars missions planning and would be an important contribution to the scientific community. This knowledge could also be used by those who are concerned with the shape and internal structure of Mars.

Present knowledge of the Mars gravitational field is mainly limited to estimates of the mass of Mars and its second-degree zonal harmonic $C_{2,0}$ (ref. 1). The purpose of this paper is to present the results of an error analysis in which a method for determining the gravity coefficients to a higher degree and order by using range and range-rate tracking from a Viking-type orbiter is analyzed. This method is a weighted least-squares differential correction process from which the standard deviations associated with solution parameters can be obtained. The assumed observables, data types, in this process are range (light time travel) and range-rate (Doppler) measurements between earth-based tracking stations and the Mars orbiter.

A parametric error analysis was made to investigate how the accuracies of estimating the gravitational coefficients are affected by the length of the tracking data arc, the a priori uncertainty on the coefficients estimated, the parameters considered but not estimated (model errors), and the synchronous orbital geometry. The relative advantages of using range and range-rate as independent data types and in combination were also investigated. It should be noted that the coefficients were never actually determined and, in particular, only the covariance matrix for the assumed estimated parameters was calculated.

In keeping with a constraint of the Viking mission, a satellite orbit synchronous with the rotational period of Mars was used as the nominal throughout the study. The Kepler elements of this orbit are:

$$a = 20\ 455 \text{ kilometers}$$

$$e = 0.765\ 681$$

$$i = 45^{\circ}$$

$$\Omega = 143.526^{\circ}$$

$$\omega = 10.05^{\circ}$$

The angles are referenced to an aerographic coordinate system for July 23, 1976.

SYMBOLS

A	matrix containing partial derivatives of a given data type with respect to estimated parameters
a	semimajor axis of Mars satellite orbit
C	matrix containing partial derivatives of a given data type with respect to model error parameters
$C_{n,m}, S_{n,m}$	gravitational coefficients (n is degree of coefficient and m is order)
e	eccentricity of Mars satellite orbit
G_M	gravitational constant of Mars
i	areographic inclination of the Mars satellite orbital plane
R	mean radius of Mars

r distance from center of Mars to Mars satellite

$P_{n,m}$ associated Legendre polynomials (n is degree and m is order)

Λ covariance matrix

λ areographic longitude of Mars satellite

Ω areographic longitude of ascending node of Mars satellite orbital plane

ϕ areographic latitude of Mars satellite

ω argument of periapse, angle measured in orbital plane from ascending node to point of periapse

Superscripts:

-1 matrix inverse

T transpose of matrix

Subscripts:

$\hat{x},\hat{x}_1,\hat{x}_2,\hat{x}_3,\alpha,\epsilon$ parameters associated with a particular covariance matrix (α represents model error parameters and ϵ represents tracking data noise)

ANALYSIS

The results presented in this report are for the simultaneous estimation of six state variables and the gravitational coefficients through degree and order four except $C_{0,0}$. These state variables and gravitational coefficients were assumed to have been estimated in a weighted least-squares process by using simulated range and range-rate measurements from three earth-based tracking stations located at Goldstone, California; Madrid, Spain; and Woomera, Australia. For this analysis, "simulated measurements" means that the measurements were only assumed to have been made and only the statistical properties of the measurements were analyzed. Several simplifying assumptions were made in the least-squares process in order to reduce the complexity of the calculations. All measurement errors of a given data type were assumed to be uncorrelated, unbiased, and of equal weight and the weighting matrix in the estimation process was assumed to be the inverse of the measurement covariance matrix.

The primary data type assumed in this analysis is range-rate; however, the covariance matrices associated with the assumed estimates of the gravitational coefficients were calculated for range and range-rate separately and in combination. They included a priori uncertainties on the estimated parameters and the effects of model errors associated with the uncertainty in tracking station location, the astronomical unit, the Mars ephemeris, the Mars gravitational constant, and the gravitational coefficients from degree five and order zero through degree and order seven. All a priori estimates were assumed to be uncorrelated. In this analysis, "model errors" include only those parameters which were not solved for but which were considered in the calculation of the covariance matrices because of their possible degradation of the solution.

The covariance matrix for a single data type, either range or range-rate, was calculated by using the equation for the covariance of a weighted least-squares estimate as given in reference 2.

$$\Lambda_{\hat{x}} = \left(A^T\Lambda_\epsilon^{-1}A\right)^{-1} + \left(A^T\Lambda_\epsilon^{-1}A\right)^{-1}A^T\Lambda_\epsilon^{-1}C\Lambda_\alpha C^T\Lambda_\epsilon^{-1}A\left(A^T\Lambda_\epsilon^{-1}A\right)^{-1}$$

The matrices A and C contain the partial derivatives of the data type with respect to the estimated parameters and the model error parameters, respectively. The covariance matrix on the parameters estimated, the measurement errors, and the model error parameters is given by $\Lambda_{\hat{x}}$, Λ_ϵ, and Λ_α, respectively. Also given in reference 2 is the equation for the covariance of the linear, minimum variance combination of estimates obtained from independent data types as follows:

$$\Lambda_{\hat{x}_3} = \Lambda_{\hat{x}_1}\left(\Lambda_{\hat{x}_1} + \Lambda_{\hat{x}_2}\right)^{-1}\Lambda_{\hat{x}_2}$$

where $\Lambda_{\hat{x}_1}$ and $\Lambda_{\hat{x}_2}$ are the covariance matrices for the two independent data types and $\Lambda_{\hat{x}_3}$ is the covariance matrix for the combination of data types. Actually this equation can be used for combining the covariance matrices associated with any two independent estimates of the same parameters or for the combining of a priori information on a solution set with the covariance on the solution set.

For the analysis presented in this report, it was necessary to decide which parameters should be estimated. To aid in this decision, some preliminary analyses of psuedo observation data were made. This psuedo data were range-rate observations generated by a computer program using the nominal trajectory given in a previous section of this report and a gravitational potential function which included the gravitational coefficients through degree and order seven. Various combinations of parameters were then used as a solution vector for fitting the psuedo observations. It was found by solving simultaneously for the six state variables and the gravitational coefficients through degree and order four except for $C_{0,0}$ that the psuedo observations could be fitted such that the average residual was less than the assumed noise on the data. This represents a complete removal of all the signal in the data due to the gravitational field of degree and order seven and it was concluded that nothing could be gained by going to a solution vector which contained gravitational coefficients higher than degree and order four. The gravitational coefficient $C_{0,0}$, which represents a deviation of the mass of Mars from some nominal value, was in many cases linearly correlated with other parameters to the extent that the normal matrix could not be inverted. Even in cases where the normal matrix was invertible with $C_{0,0}$ as one of the solution parameters, the standard deviation associated with $C_{0,0}$ was very large. Therefore $C_{0,0}$ was not included in the solution vector and hence all results presented in this report are for a solution vector that includes only the six state variables and the gravitational coefficients through degree and order four except for $C_{0,0}$.

As stated earlier, the nominal orbit used throughout the study was one which was synchronous with the rotational period of Mars. Range and range-rate tracking data with standard deviations of 15 meters and 0.001 meter per second, respectively, were assumed to have been taken at 10-minute intervals over the arc of the orbit which was not occulted by Mars. For this particular orbit there was no occultation and hence there were approximately 150 tracking data points per orbit. Tracking was assumed for an integral number of orbital periods up to 30.

The model error assumed for the Mars ephemeris was 5 kilometers for each component of position and 5×10^{-7} km/sec for each component of velocity (ref. 3). This represents the anticipated uncertainty in the ephemeris by 1975; however, the current uncertainty in the Mars position is estimated to be of the order 200 to 400 kilometers. (See ref. 4.) The tracking-station location errors used were 0.0015 kilometer in the distance off the earth spin axis, 0.025 kilometer along the earth spin axis, and 4.7×10^{-7} radian in longitude. The astronomical unit was assumed to be in error by 2 kilometers, and the uncertainty used for the gravitational constant of Mars was 1.43 km^3/sec^2. These model error uncertainties are presented in table I along with the a priori standard deviations used for the six state variables that were assumed to be estimated. They are the same as those used in reference 3.

The a priori standard deviations for the Mars gravitational coefficients assumed to have been estimated are given in table II along with the standard deviations assumed for those gravity coefficients which were treated as model errors. Two different sets of values for these standard deviations are given in table II, both of which are based on information presented in reference 5 which gives a method for calculating the gravity coefficients for Mars. The two different sets are results of the use of two different normalization factors for the gravity coefficients. The values given as set 1 were chosen as the nominal values for this study.

It should be emphasized that there are basically two types of parameters involved in the analysis in this report. There are those which were assumed to have been estimated and those which were used as model errors. In the discussion of the figures, the phrase "a priori information" applies to only those parameters which were assumed to have been estimated unless otherwise noted.

RESULTS AND DISCUSSION

Effects of Tracking Time and Model Errors

The variation in standard deviation for the gravity coefficients with the number of orbits tracked is given in figure 1. The results presented in the three curves shown on each plot in the figure were obtained from three different calculations of the covariance matrix where range-rate is assumed to be the tracking data type. The covariance matrix was first calculated with the assumption of no a priori information on the solved-for parameters and without consideration of model errors. It was then calculated assuming a priori information on the solution set only and finally it was calculated with the assumption of a priori information on the solution set and with model errors considered. Also shown in each of the plots is the a priori standard deviation for the gravity coefficient estimated.

The curve for no a priori or model errors (circular symbols) is presented to indicate how the information contained in the tracking data or how the sensitivity of the tracking data with respect to the coefficients vary with tracking time. It can be seen by referring to figure 1 that the standard deviations for this case vary by at least 2 orders of magnitude over a period of 30 orbits. In order to obtain a standard deviation equal to or less than the a priori value assumed in this study, five to seven orbits of tracking were required for the second-degree coefficients, 10 to 12 orbits for the third-degree coefficients, and 10 to 20 orbits for the fourth-degree coefficients. Of course the number of orbits of tracking required to obtain standard deviations equal to or less than the a priori values would be different for different a priori values; however, the information rate and content would remain the same. After 30 orbits of tracking, the standard deviations for no a priori or model errors are virtually the same as those with a priori and no model errors (triangular symbols). However, over the first few orbits of tracking they are different by as much as 2 orders of magnitude. This difference indicates that, although the a priori causes a significant reduction in the standard deviations for the first orbits of tracking, the a priori becomes less significant as far as accuracy is concerned when long tracking periods are available and no model errors are considered.

The curve for a priori only represents the minimum uncertainty that can be expected for the method and approach presented in this report whereas the curve for both a priori and model errors (square symbols) is the most representative of what might be expected in the method and approach. By comparing these two curves in figure 1, it can be seen that model errors of the order used in this report degrade the standard deviations after 30 orbits of tracking by a factor of 2 to 6 depending upon the coefficients of interest. Most of this degradation occurs after 10 orbits of tracking and appears to increase with tracking time. Although model errors do not appear to significantly affect the standard deviations in the first 30 orbits of tracking, there is some masking of the model error effects by the a priori information on the parameters estimated. It was found, although not shown in this report, in analyzing the results presented in this report that without a priori information the model errors could cause as much as 2 orders of magnitude increase in the standard deviations, depending upon the coefficients of interest and the number of orbits tracked. This indicates that a priori information of the type used in this study (statistically uncorrelated) helps reduce the linear correlation between the solution parameters and to reduce the standard deviations in the presence of model errors.

With the assumption of a priori information and model errors, the variation of the standard deviation with the orbits tracked was small. After 30 orbits of tracking, the standard deviations for the second-degree coefficients were about 1 order of magnitude less than the a priori standard deviations. After 30 orbits, the standard deviations for the third-degree coefficients differed from a priori values by only a factor of 3 to 4, whereas those for the fourth-degree coefficients were almost no different than the a priori values.

The results presented in figure 2 were obtained exactly as those presented in figure 1 except that range was assumed to be the tracking data type. As for range-rate, the curve for no a priori or model errors is presented to indicate how the information content of the tracking data varies with tracking time. The variation of the standard deviation presented in this curve over 30 orbits is about the same as that for range-rate, 2 orders of magnitude. Second-degree coefficients require 10 to 18 orbits of tracking to reduce the standard deviations to the a priori level, whereas the third-degree coefficients require from approximately 20 to 30 orbits. Except for $C_{4,4}$ and $S_{4,4}$ the standard deviations for the fourth-degree coefficients were never less than the a priori values for up to 30 orbits of tracking. By comparing the curve for no a priori or model errors with that for a priori and no model errors, it is seen that in general the a priori causes a significant reduction in the standard deviations for up to about 15 orbits of tracking.

A comparison of the curve in figure 2 for a priori and no model errors with the curve for a priori and model errors indicates that the effects of model errors are about the same as those for range-rate. However, without a priori information of the type used in this report, these model error effects would be very large, mainly due to the uncertainty in the Mars ephemeris and the astronomical unit. Again it appears that a priori information, uncorrelated statistically, serves to reduce the linear correlation between the solution parameters and to reduce the standard deviations in the presence of model errors.

In general, it was found with the assumption of a priori information and model errors and range tracking data that the a priori standard deviations of the estimated coefficients were not significantly improved for up to 30 orbits of tracking.

Comparison of Data Types

Since it might be feasible to supplement the Doppler tracking data with range tracking data, it is of interest to compare the results of the two data types. Such a comparison is given in figure 3 where in each plot are presented three curves representing the results of using range only data, range-rate only, and the two in combination. These three curves are the standard deviations obtained assuming a priori information on the solution set and with model errors considered. There is very little difference in the accuracy obtained from the two data types and range-rate is slightly better than range. Also, there is virtually no improvement when the two data types are combined. Hence, it appears that supplemental range measurements would add very little to the accuracy obtained from range-rate measurements.

From the discussion of figures 1, 2, and 3, range and range-rate are similar data types as far as the accuracy of estimating the coefficients is concerned and the addition of range with range-rate does not significantly improve the accuracy of estimating the coefficients. Therefore, the remaining results presented in this report are for range-rate only.

Effects of Gravity Coefficient A Priori Information and Model Errors

From the discussion of figure 1, it appears that except for the second-degree coefficients the assumed a priori knowledge of the coefficients was not significantly improved after 30 orbits of tracking and that the results obtained from the process used in this analysis are strongly influenced by the a priori knowledge. For example, the standard deviations for up to about 10 orbits of tracking were affected significantly by a priori knowledge. Also a priori knowledge helped to reduce the degradation caused by model errors. It, therefore, may be of interest to know how a change in the a priori knowledge of the coefficients to be estimated affects the accuracy of estimating the coefficients. These effects are illustrated in figure 4. The uppermost curve (circular symbols) in each plot is the same as that presented in figure 1 for nominal a priori, set 1 in table II, on the estimated parameters and consideration of nominal model errors, set 1 in table II. The middle curve (triangular symbols) in each plot is the same as the uppermost curve except that the a priori values on the estimated coefficients are taken to be those given in set 2 of table II. The bottommost curve (square symbols) in each plot represents the standard deviations associated with set 2 a priori on the estimated coefficients and consideration of set 2 gravity coefficient model errors. Also shown in each plot is the a priori standard deviation for the coefficient estimated, both set 1 and set 2. The set 2 a priori standard deviations and model errors are from about a factor of 4 to 7 smaller than the nominal values of set 1.

The difference between corresponding values of the standard deviations presented in the uppermost and bottommost curves of each plot in figure 4 is due entirely to the use of two different sets of a priori values on the estimated gravity coefficients and two different sets of gravity coefficient model errors in the calculations of the covariance matrices. The difference between corresponding values of the standard deviations in the uppermost and middle curve is due entirely to the two different sets of a priori values used for the estimated gravity coefficients. The middle and bottom curves are not very different; therefore, it can be said that the a priori values on the estimated coefficients is the main contributor to the difference shown between the top and bottom curves. Also, a given percentage change in the a priori on the estimated coefficient results in nearly the same percentage change in the solution standard deviations. This change indicates that the solutions associated with the process used in this report are very sensitive to the a priori on the estimated coefficients. This is essentially what has been found in previous sections of this paper.

Effects of Synchronous Orbit

As stated earlier, the nominal orbit used in this report was synchronous with the rotational period of Mars. In this situation, the perturbations on a Mars orbiter due to the Mars gravitational field change very little from orbit to orbit. In other words the orbiter samples almost identically the same gravitational field each orbit. For gravitational field determination, it is desirable to pass over as many different planet latitudes and longitudes as possible. A small change in the period of the synchronous orbit would allow the orbiter to sample different longitudes throughout the tracking phase. It is therefore of interest to know how the accuracy of estimating the gravitational coefficients is affected when the period of the satellite orbit is changed. In order to study this effect, the semimajor axis and eccentricity of the nominal orbit was changed slightly so as to have an orbit which was almost identical to the nominal except for a 1-hour decrease in the period. For this particular orbit, the longitude of a given subsatellite point changes by about 15° per orbit and hence a satellite in this orbit could pass over about 360° of longitude in about 24 orbits. Solutions for the gravitational coefficients were assumed to have been obtained by using range-rate data from the perturbed orbit. A comparison of the standard deviations for the synchronous and nonsynchronous orbits is given in figure 5. In each of the plots there are two curves for the synchronous and two curves for the nonsynchronous orbit. For each set of two curves, one curve represents the results for no a priori or model errors while the other represents the results for a priori and model errors. If the two curves for no a priori or model errors are compared, it is seen that the information content and rate is much larger for the nonsynchronous orbit. After only 10 orbits of tracking, the standard deviations associated with the nonsynchronous orbit are from one to two orders of magnitude smaller than those associated with the synchronous orbit; thus, it appears that it would be more advantageous to have tracking data from a nonsynchronous orbit for a gravity field determination. However, on comparing the remaining two curves in the plots, it is obvious that the effects of model errors are such that the accuracies of the solutions for the two different orbits are not very different and that the variation of the standard deviations over 30 orbits of tracking in the nonsynchronous orbit is very small. A nonsynchronous orbit is more favorable for gravitational field estimation but in the presence of model errors such as those assumed in this report, the advantages of this particular orbit may be nulled by model errors.

CONCLUDING REMARKS

The results of an error analysis of a weighted least-squares process for estimating the gravitational field of Mars has been presented in this report. The assumption of a Viking-type orbiter is very restrictive and hence the results might be improved with a less restrictive orbital geometry. The a priori information on the coefficients is only approximate and it has been shown that the a priori strongly influences the results.

In general, the a priori assumptions for the estimated coefficients were not significantly improved for tracking up to 30 orbital periods. The largest improvement, about an order of magnitude, was noted for the second-degree coefficients. The a priori information can be used to significantly reduce the standard deviations for the first orbits of tracking as might be expected, and a priori information has a tendency to mask or eliminate model error effects. For the Viking-type orbit, the model error effects do not appear to be significant; however, for a nonsynchronous type orbit, these effects are such that they might null the advantages of having a nonsynchronous orbit.

A comparison of the accuracies obtained from the use of the two data types, range and range-rate, shows the two to be approximately equivalent with range-rate being slightly better as far as the accuracy is concerned. An analysis of the two data types in combination indicated very little advantage for supplementing range-rate tracking with range. With range only and without a priori information, model error effects would be very large mainly due to the uncertainty in the astronomical unit and the Mars ephemeris.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., March 10, 1971.

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TABLE I.- PARAMETER STANDARD DEVIATIONS

	Standard deviations
Solution parameters:	
Orbiter position (each component)	1000 km
Orbiter velocity (each component)	1 km/sec
Model error parameters:	
Tracking station distance off the spin axis	0.0015 km
Tracking station Z-component	0.025 km
Tracking station longitude	4.7×10^{-7} rad
Ephemeris position (each component)	5 km
Ephemeris velocity (each component)	5×10^{-7} km/sec
Astronomical unit	2 km
Mars gravitational constant, G_M	1.43 km^3/sec^2

TABLE II.- STANDARD DEVIATIONS OF MARS GRAVITATIONAL FIELD COEFFICIENTS

Solution coefficients, C and S		Standard deviations	
n	m	Set 1	Set 2
2	0	3.88×10^{-5}	1.00×10^{-5}
2	1	2.24×10^{-5}	5.79×10^{-6}
2	2	1.12×10^{-5}	2.89×10^{-6}
3	0	2.04×10^{-5}	3.86×10^{-6}
3	1	8.33×10^{-6}	1.58×10^{-6}
3	2	2.64×10^{-6}	4.98×10^{-7}
3	3	1.08×10^{-6}	2.03×10^{-7}
4	0	1.30×10^{-5}	1.94×10^{-6}
4	1	4.12×10^{-6}	6.14×10^{-7}
4	2	9.71×10^{-7}	1.45×10^{-7}
4	3	2.59×10^{-7}	3.87×10^{-8}
4	4	9.17×10^{-8}	1.37×10^{-8}
Model error coefficients, C and S		Standard deviations	
n	m	Set 1	Set 2
5	0	9.21×10^{-6}	1.13×10^{-6}
5	1	2.38×10^{-6}	2.93×10^{-7}
5	2	4.50×10^{-7}	5.53×10^{-8}
5	3	9.18×10^{-8}	1.13×10^{-8}
5	4	2.16×10^{-8}	2.66×10^{-9}
5	5	6.84×10^{-9}	8.42×10^{-10}
6	0	6.96×10^{-6}	7.29×10^{-7}
6	1	1.52×10^{-6}	1.59×10^{-7}
6	2	2.40×10^{-7}	2.52×10^{-8}
6	3	4.00×10^{-8}	4.19×10^{-9}
6	4	7.30×10^{-9}	7.65×10^{-10}
6	5	1.56×10^{-9}	1.63×10^{-10}
6	6	4.49×10^{-10}	4.71×10^{-11}
7	0	5.49×10^{-6}	5.01×10^{-7}
7	1	1.04×10^{-6}	9.47×10^{-8}
7	2	1.41×10^{-7}	1.29×10^{-8}
7	3	2.00×10^{-8}	1.82×10^{-9}
7	4	3.01×10^{-9}	2.75×10^{-10}
7	5	5.02×10^{-10}	4.58×10^{-11}
7	6	9.84×10^{-11}	8.98×10^{-12}
7	7	2.63×10^{-11}	2.40×10^{-12}
0	0	3.33×10^{-5}	3.33×10^{-5}

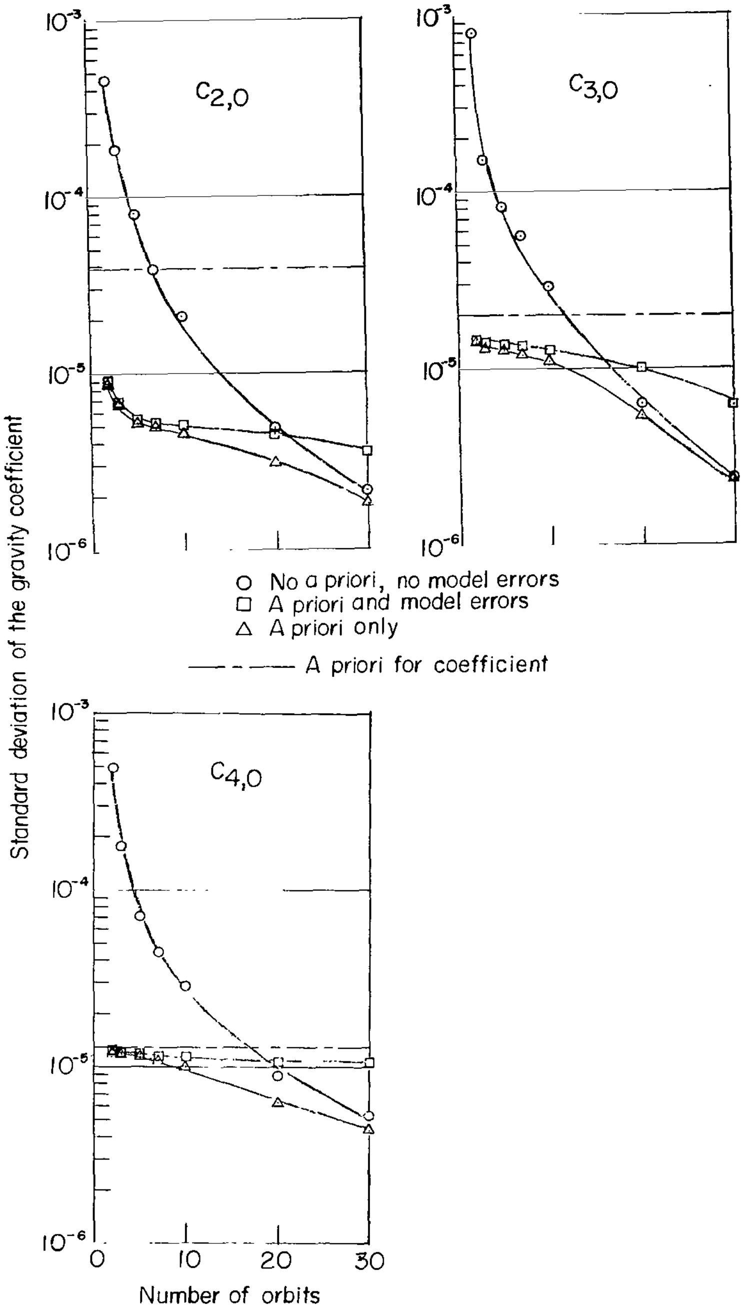


Figure 1.- Variation of standard deviation of Mars gravity coefficients with number of orbits assumed to have been tracked for range-rate data only.

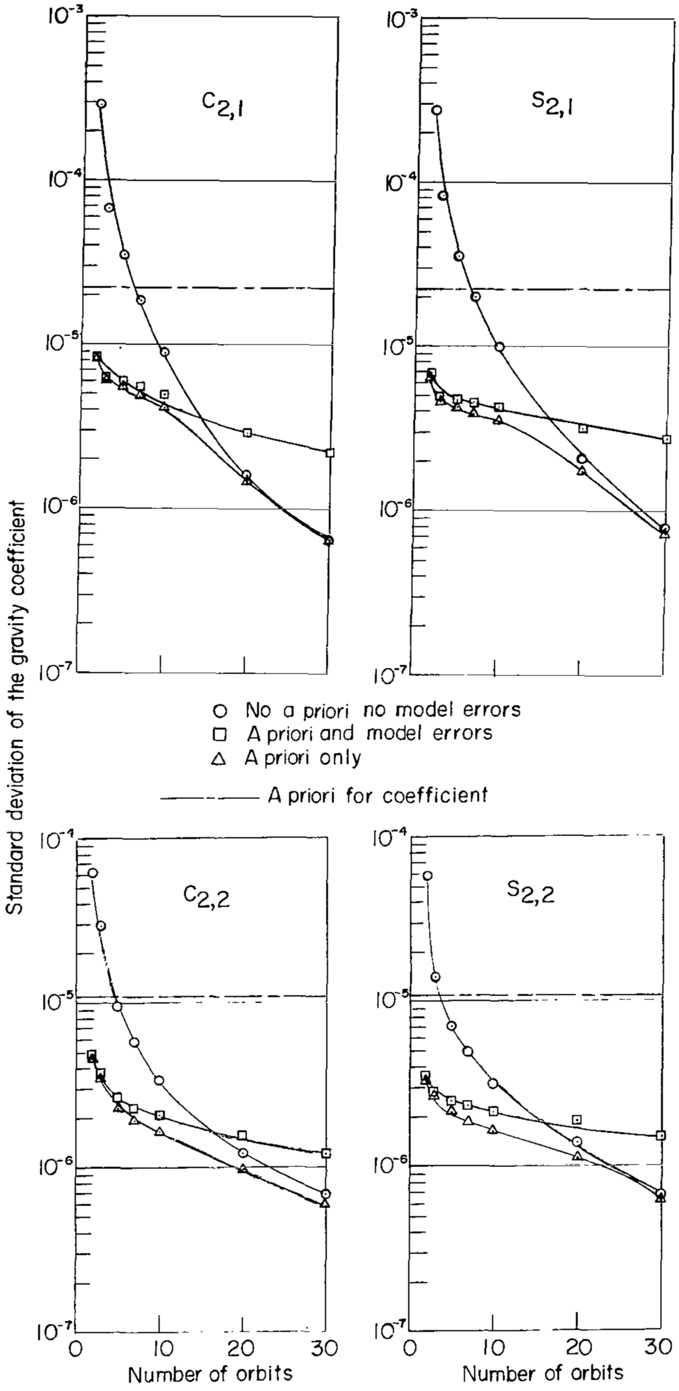


Figure 1.- Continued.

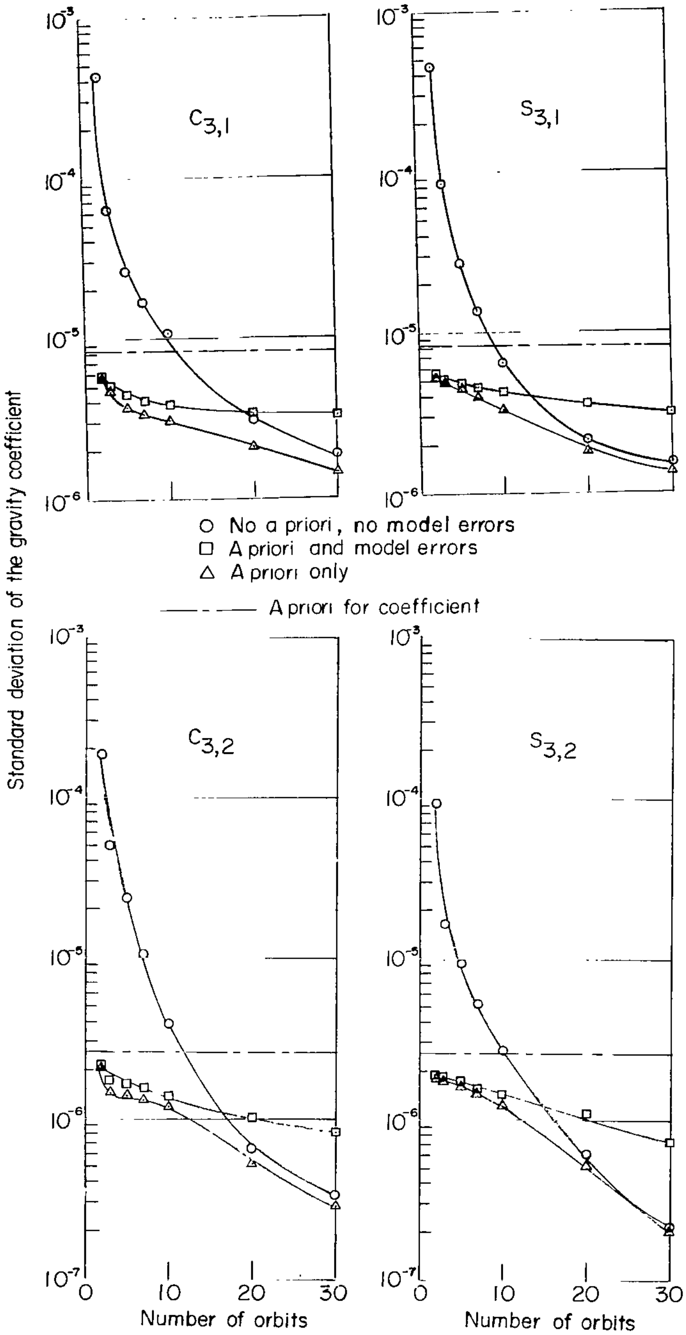
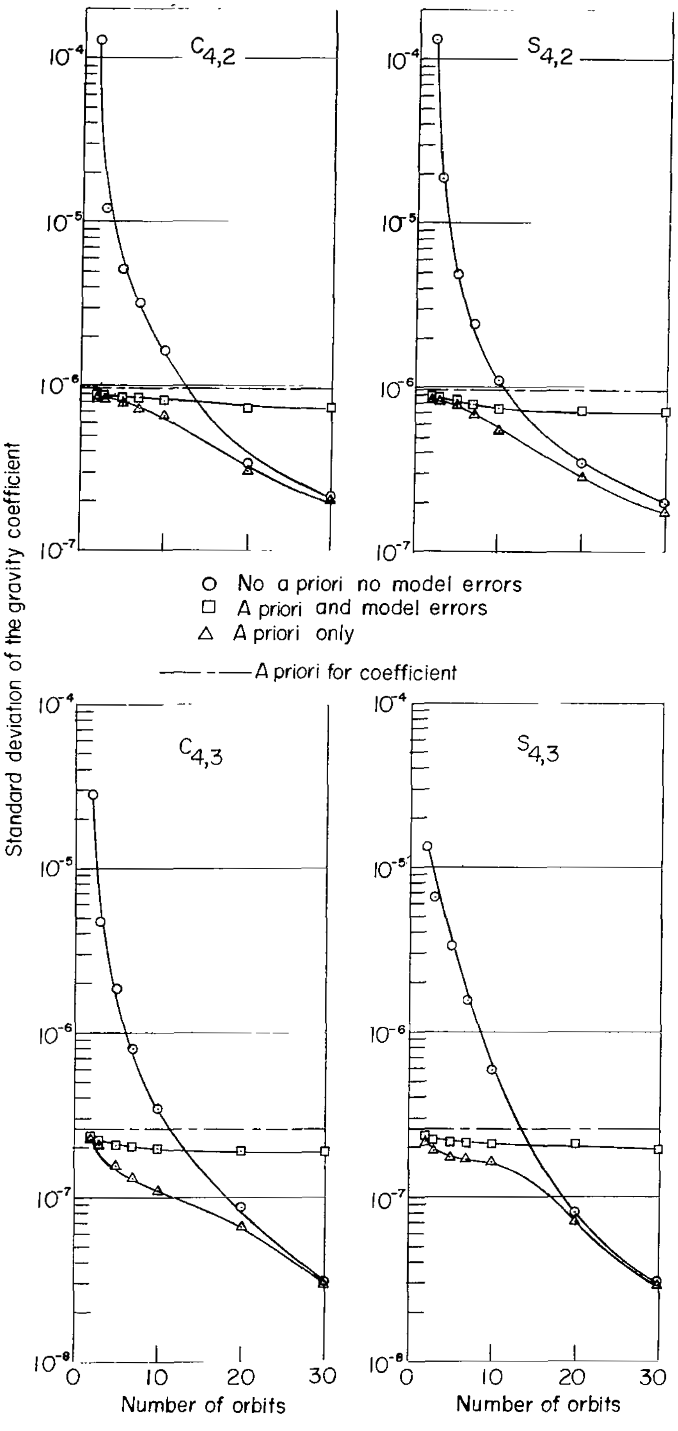


Figure 1.- Continued.



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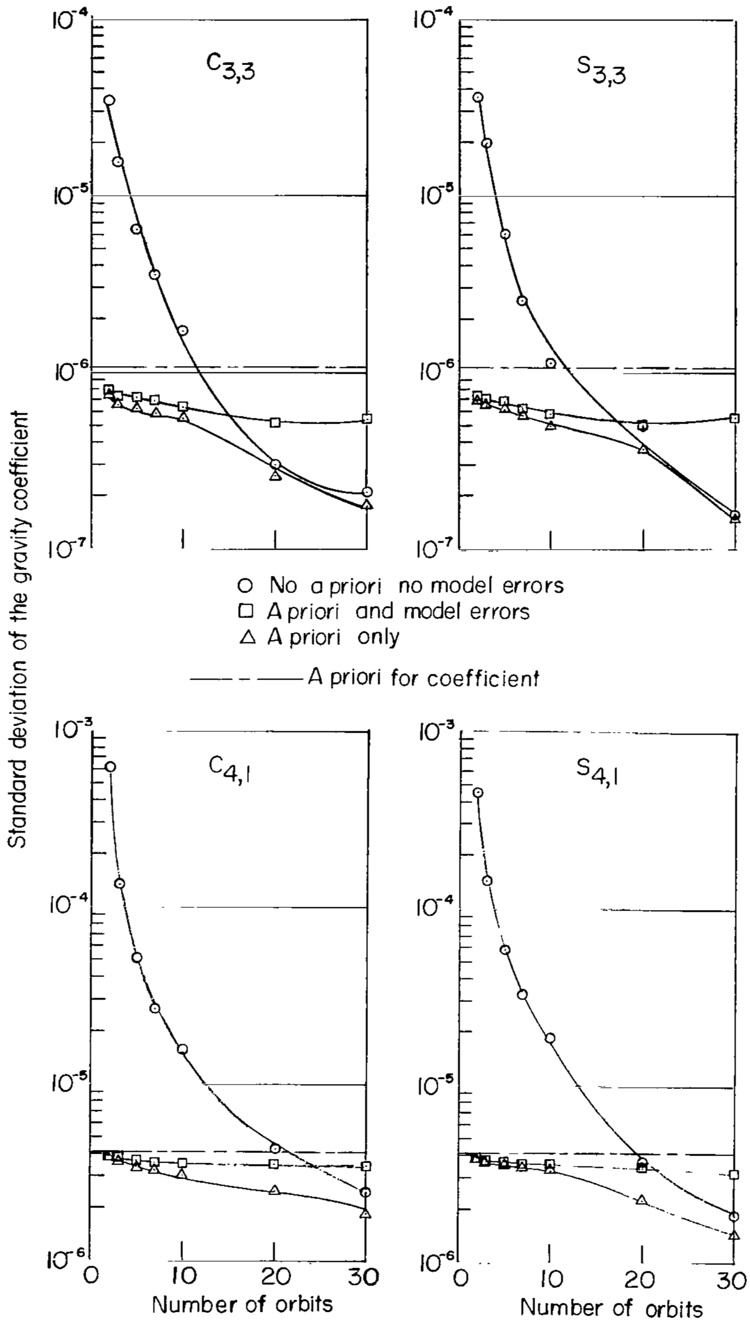

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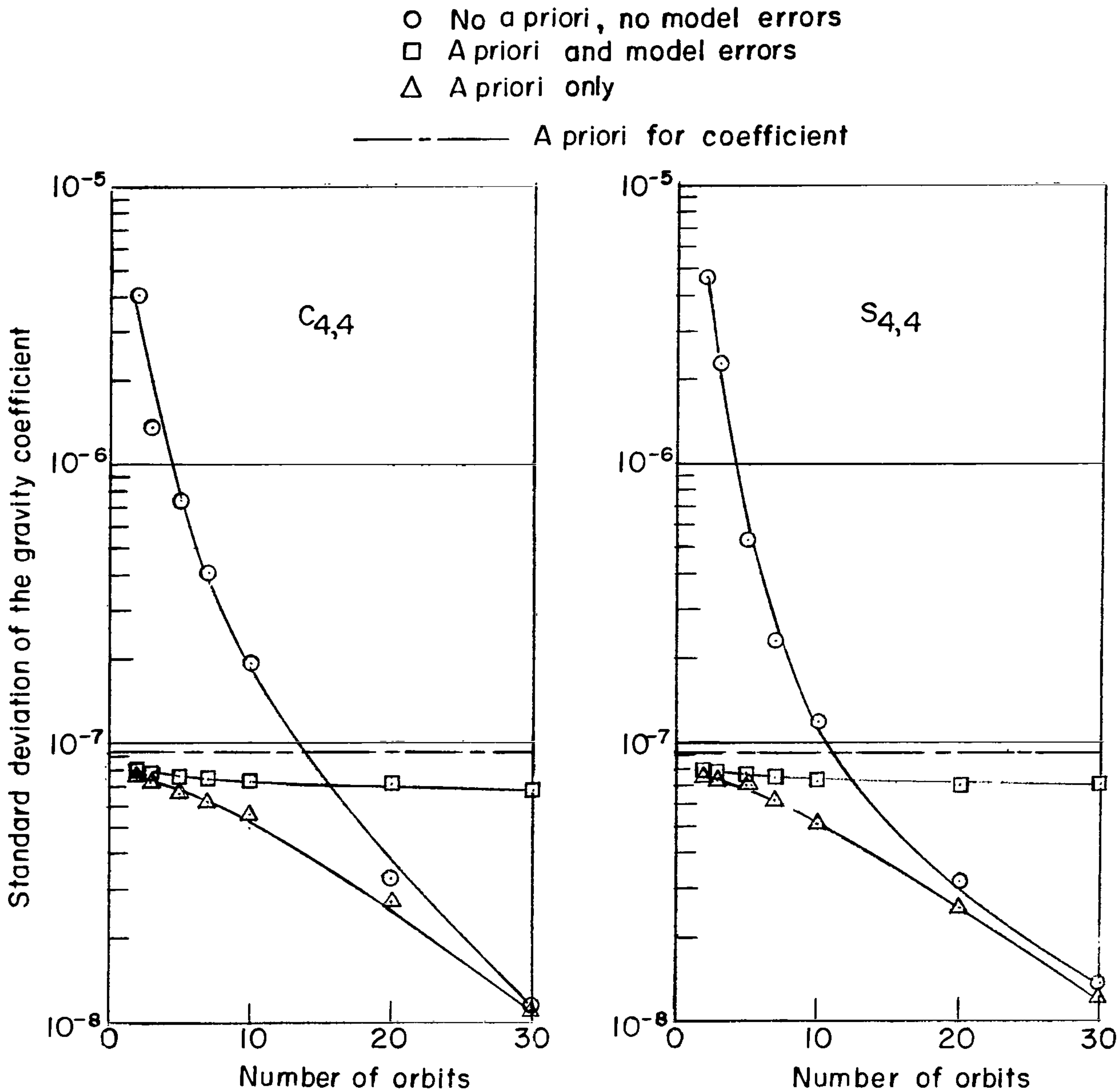

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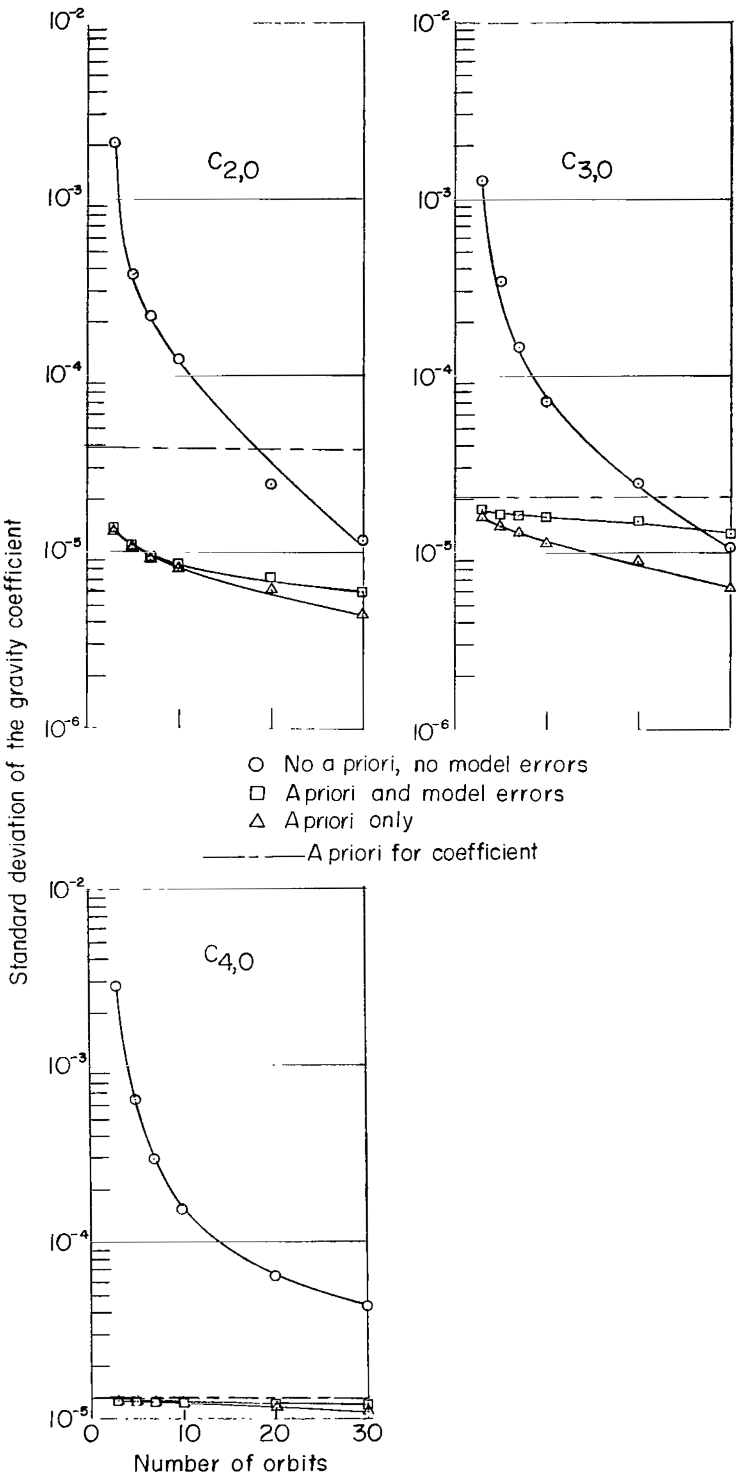
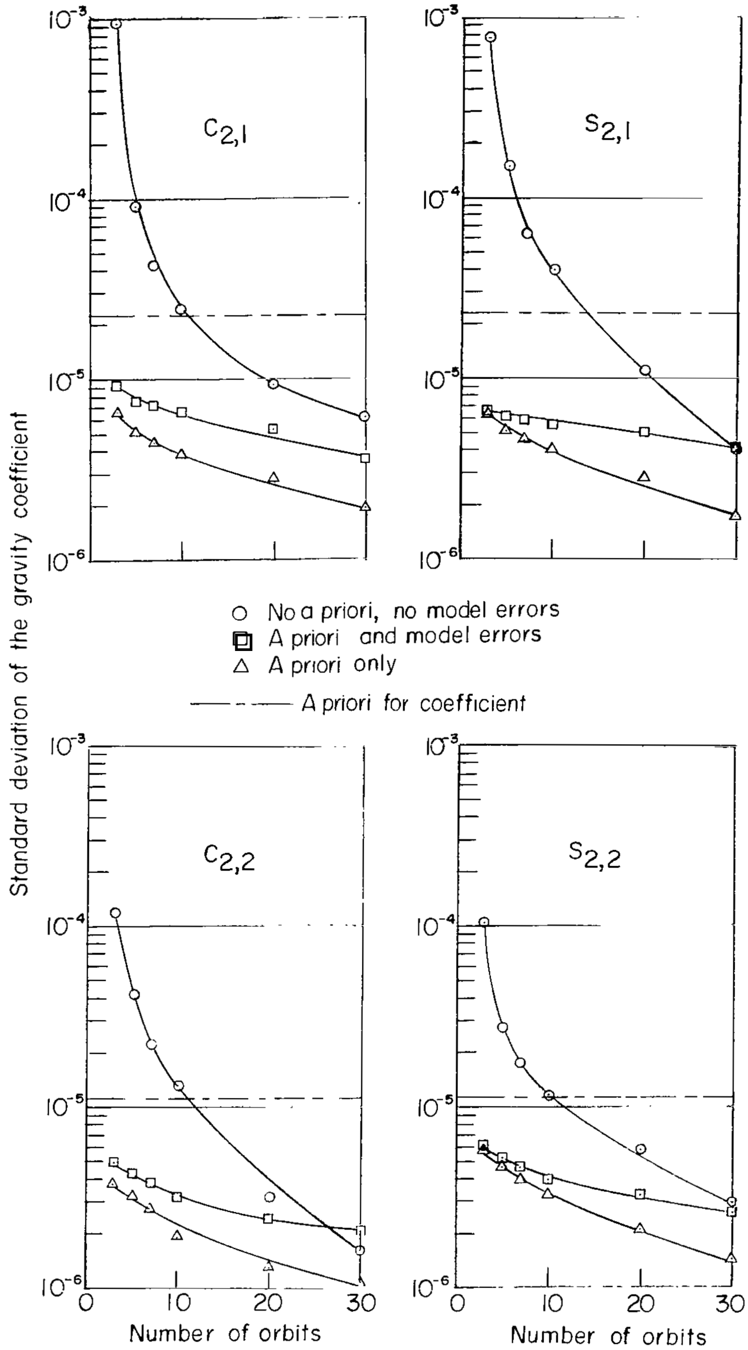


Figure 2.- Variation of standard deviation of Mars gravity coefficients with number of orbits assumed to have been tracked for range data only.



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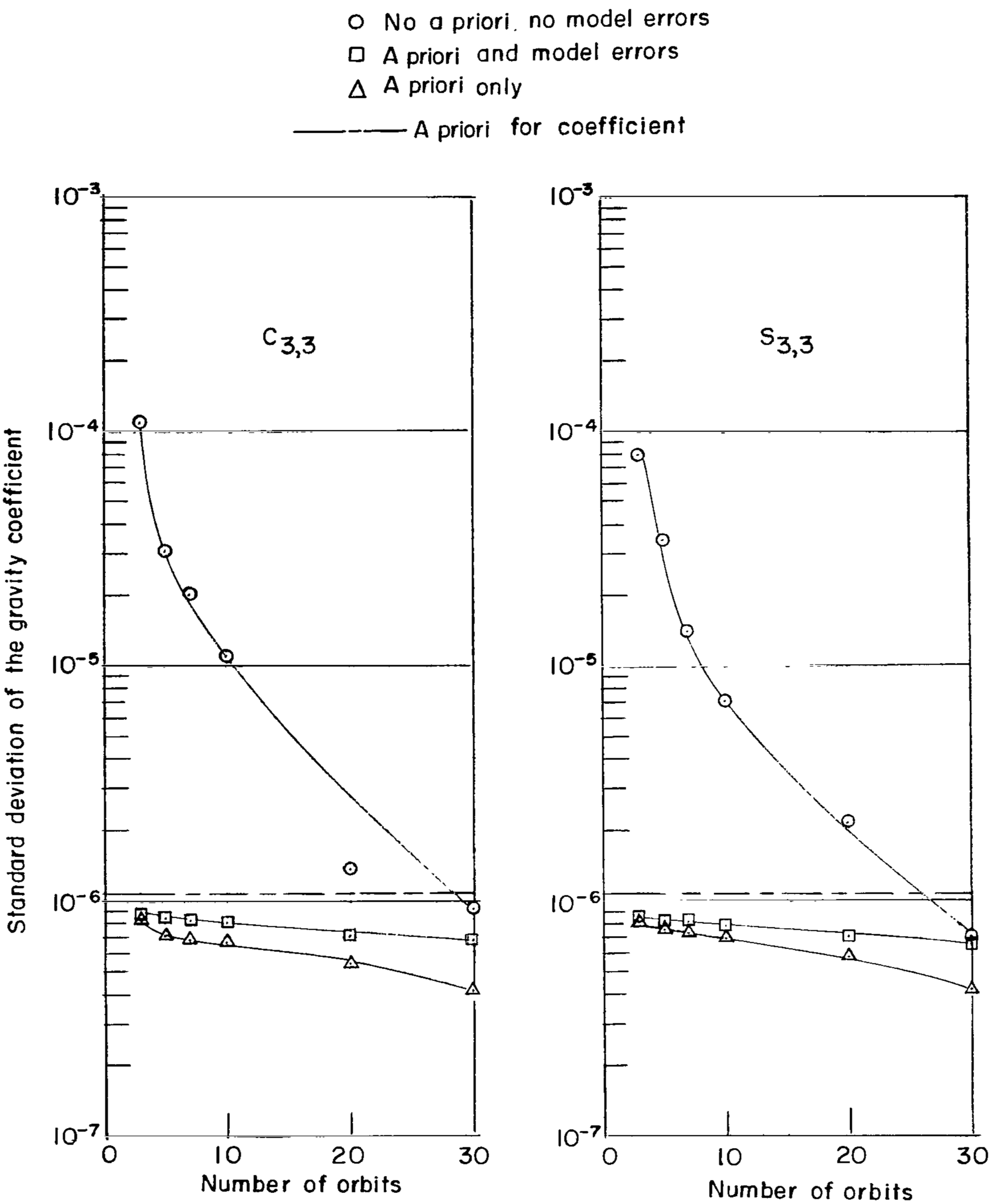

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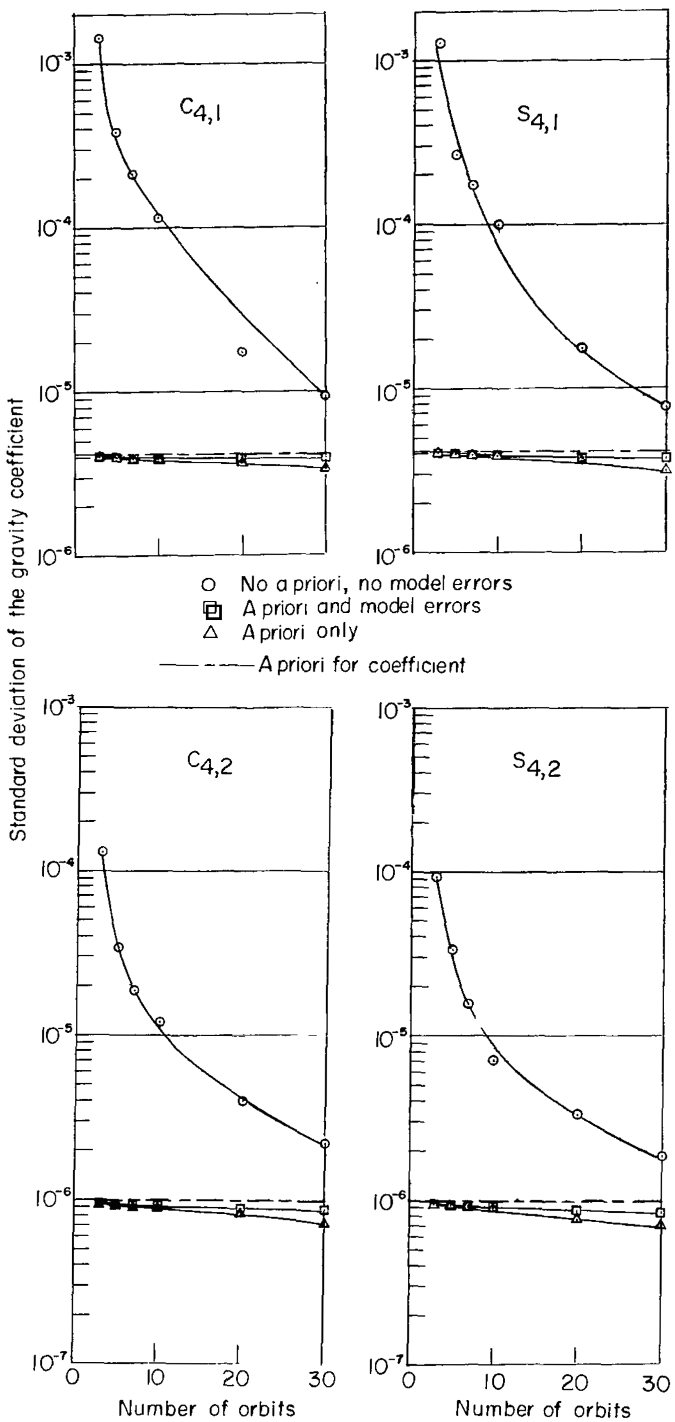

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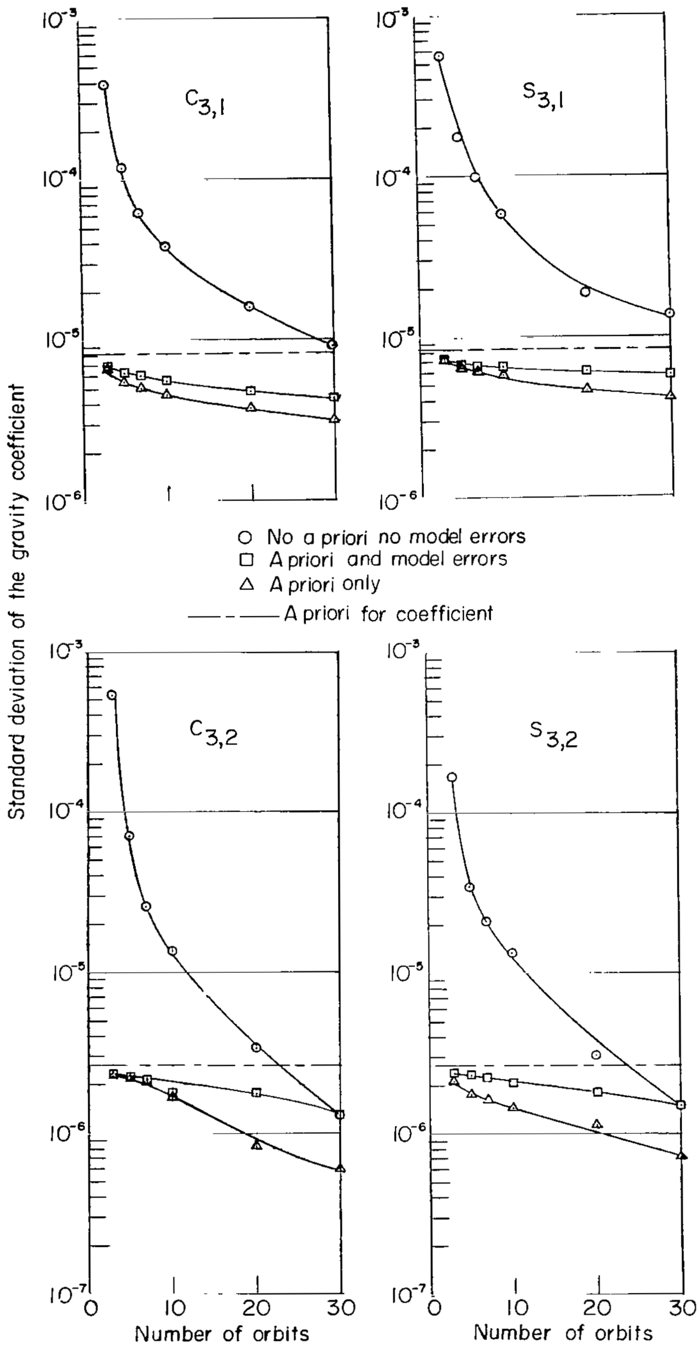

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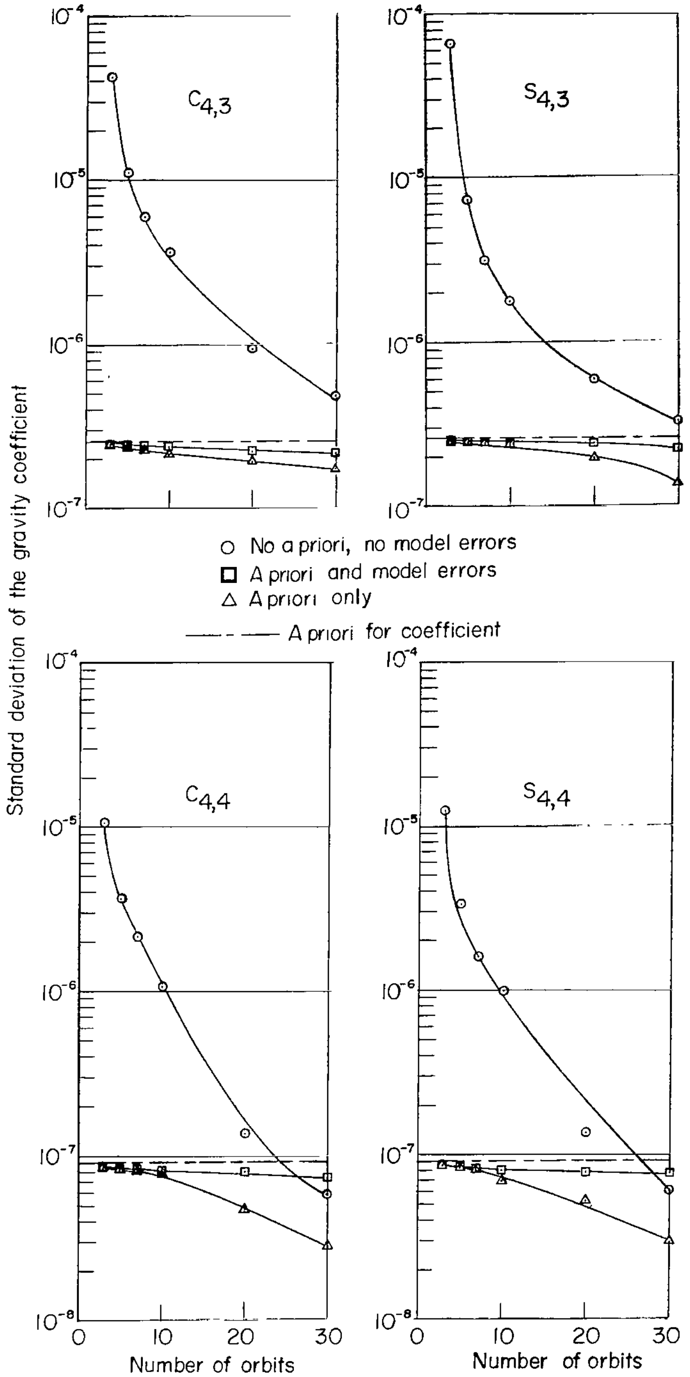

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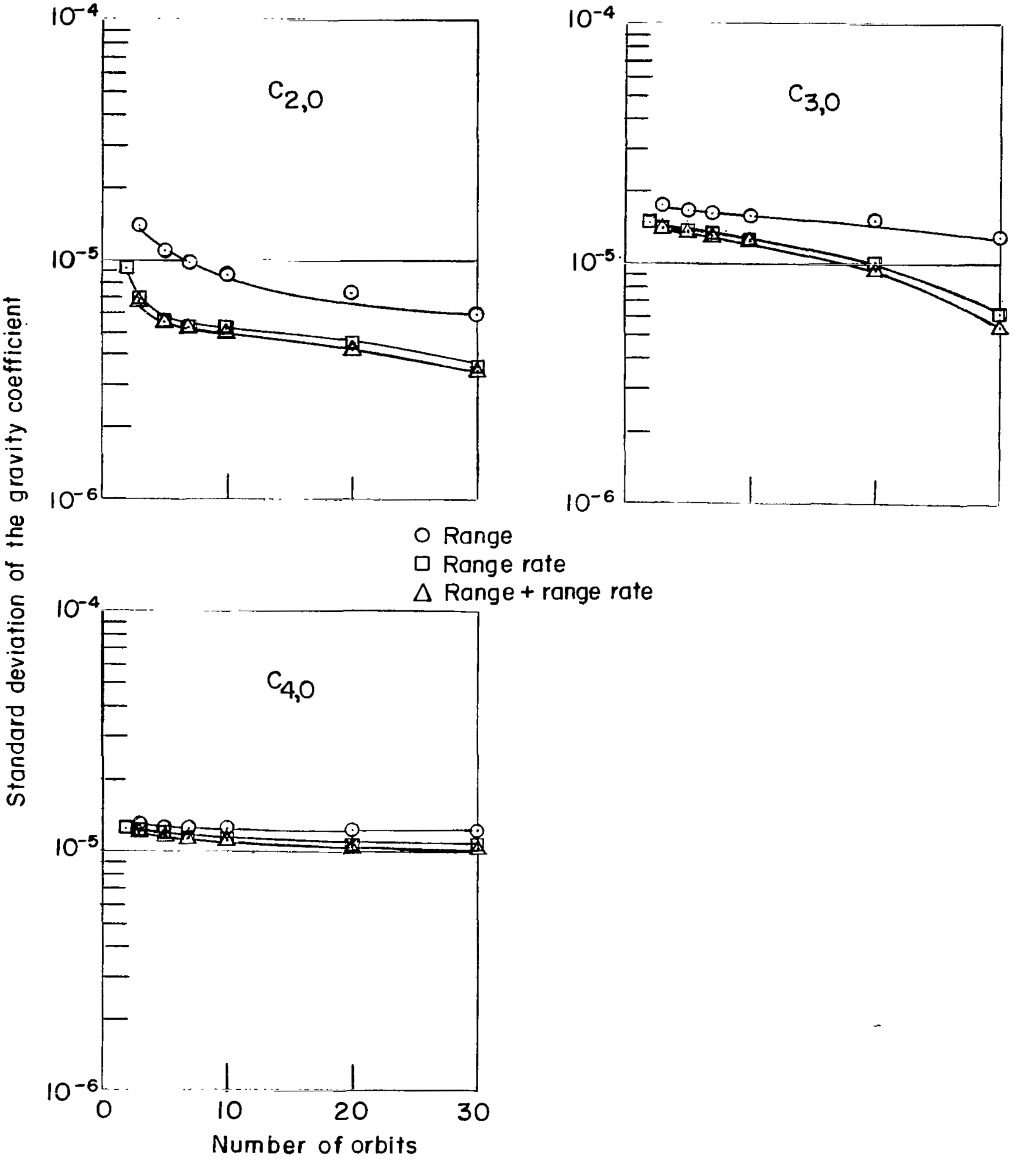


Figure 3.- Comparison of range and range-rate as data types for estimating the Mars gravity coefficients, a priori information and model errors being assumed.

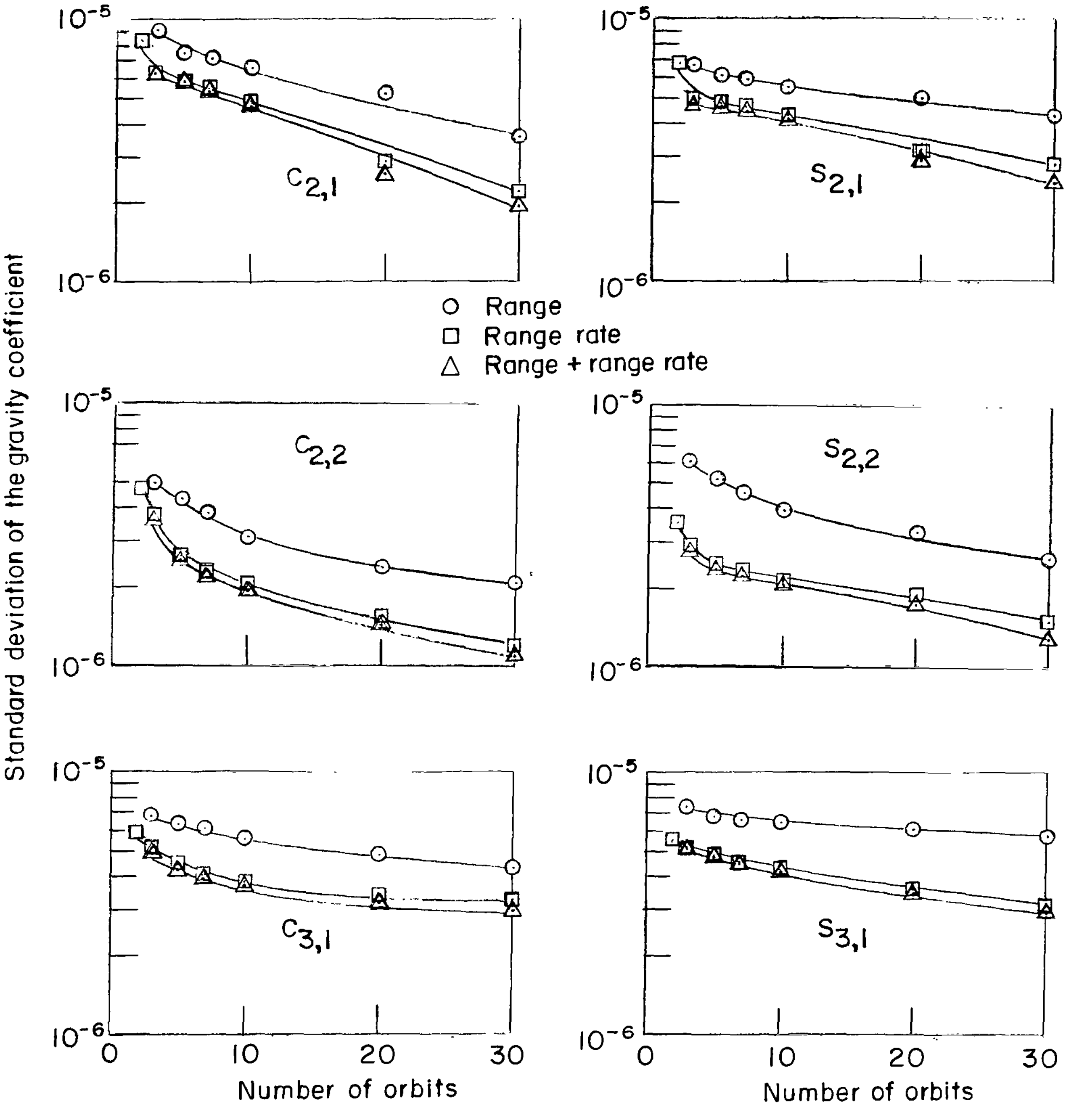


Figure 3.- Continued.

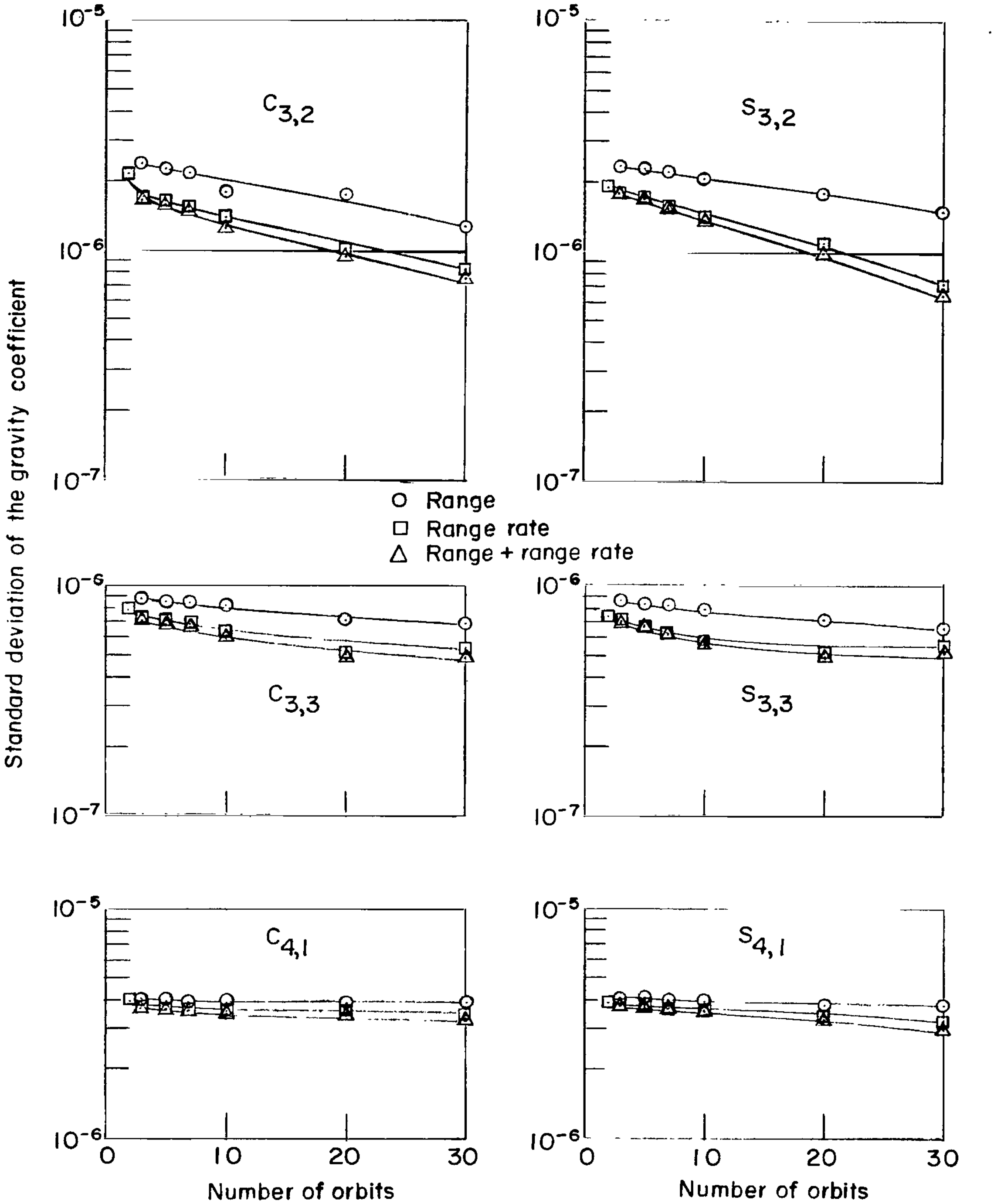
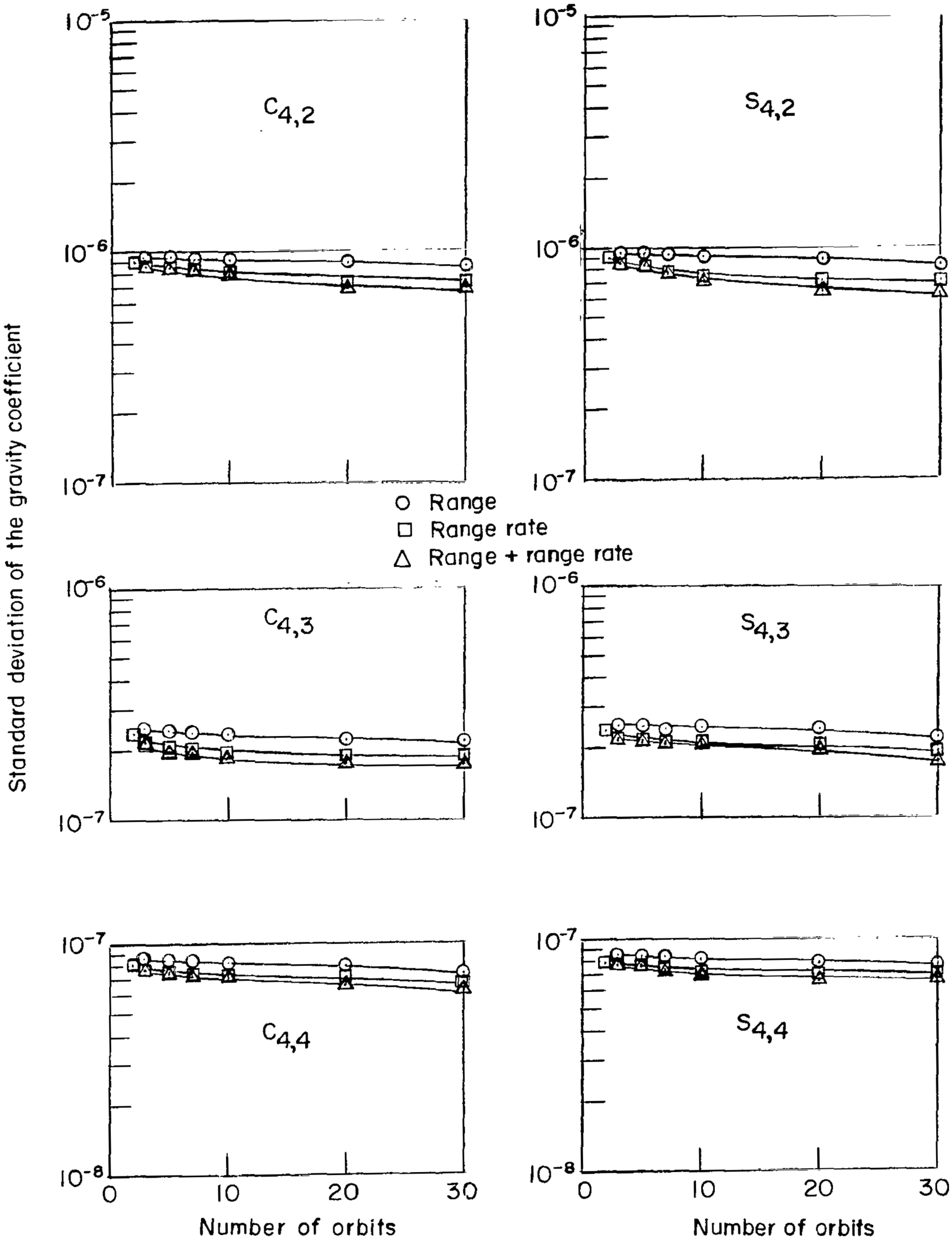


Figure 3.- Continued.



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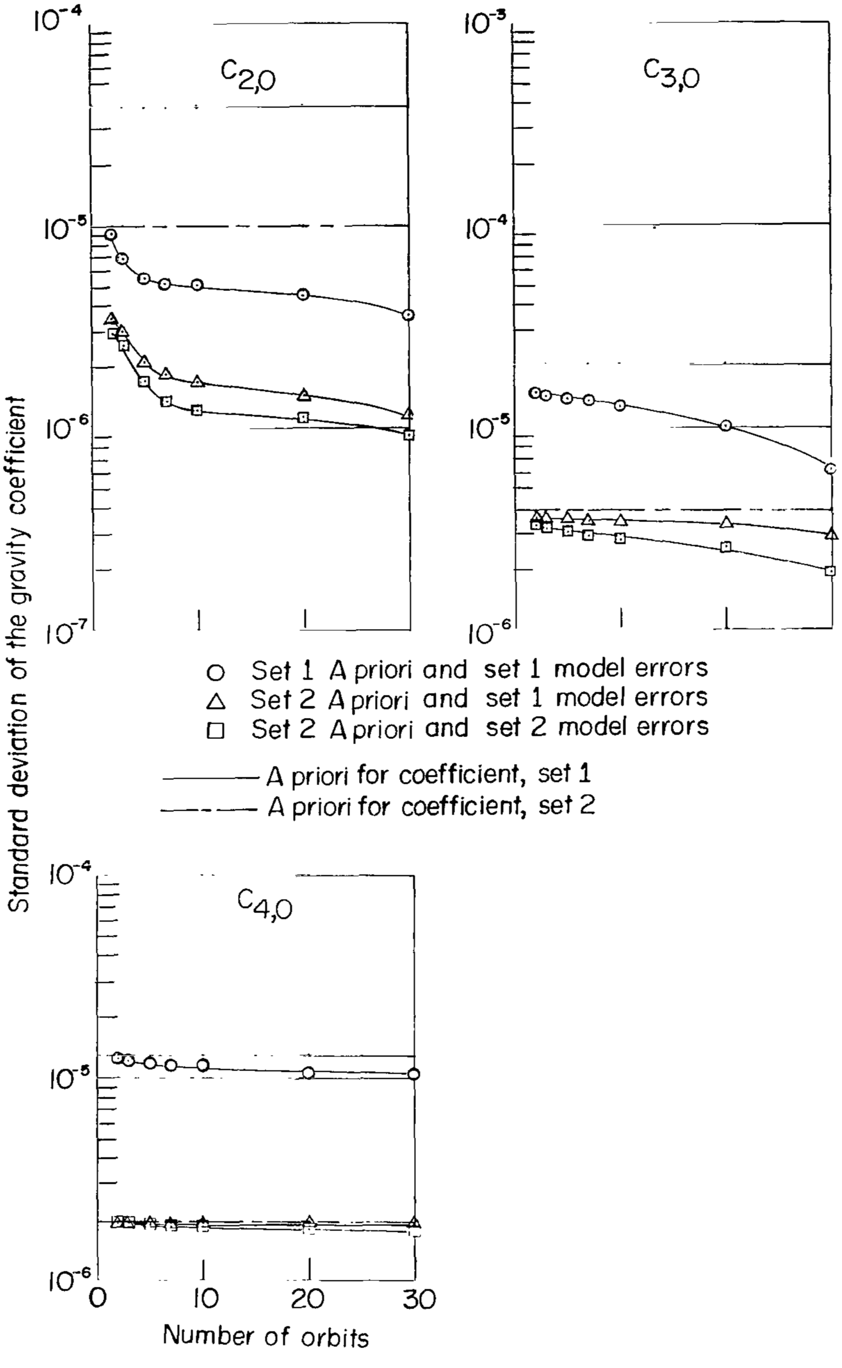
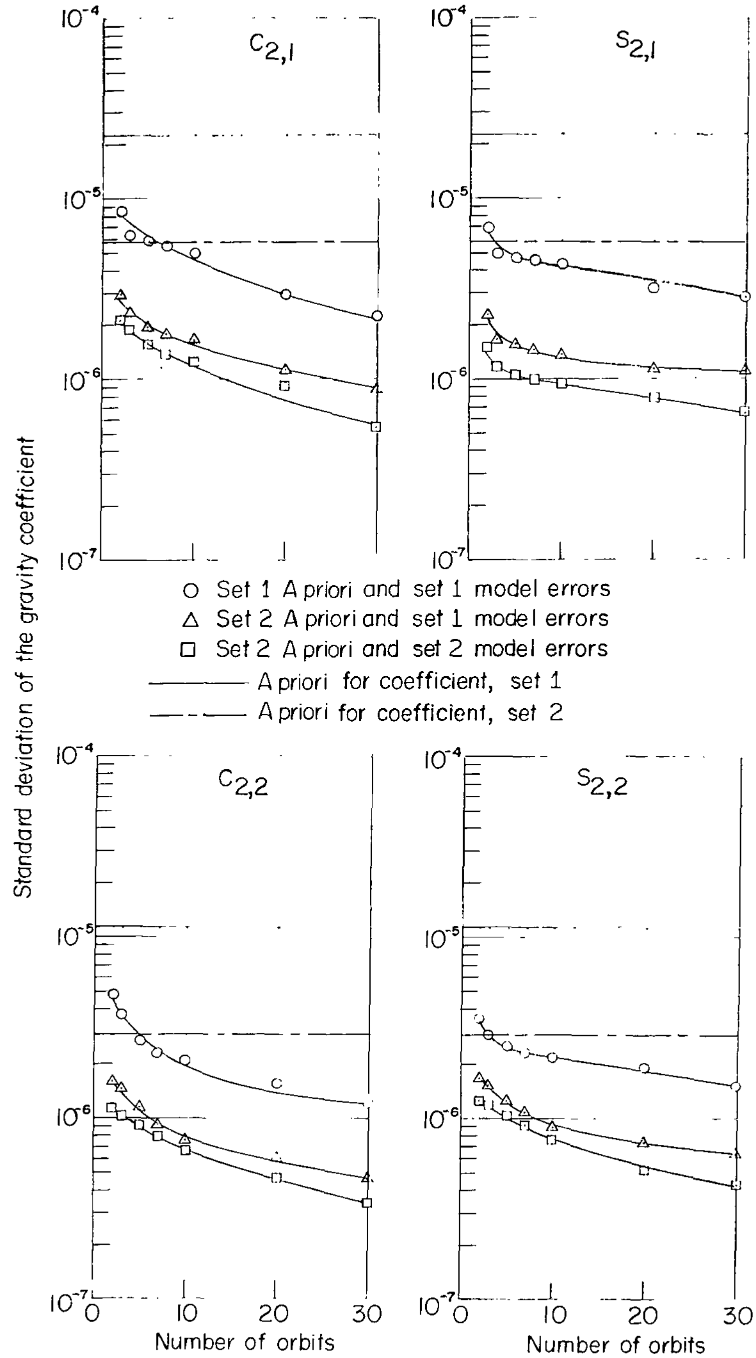


Figure 4.- Comparison of standard deviations of Mars gravity coefficients using two different sets of a priori values.



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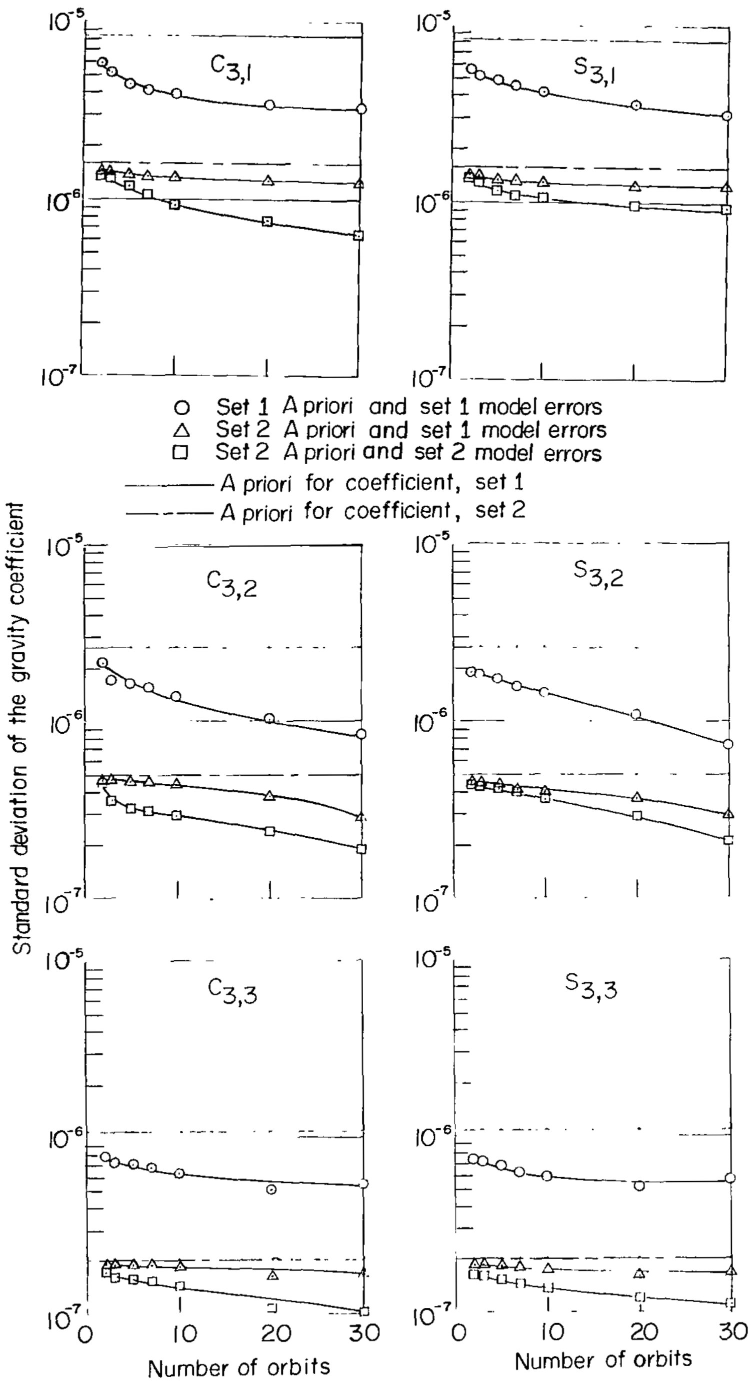

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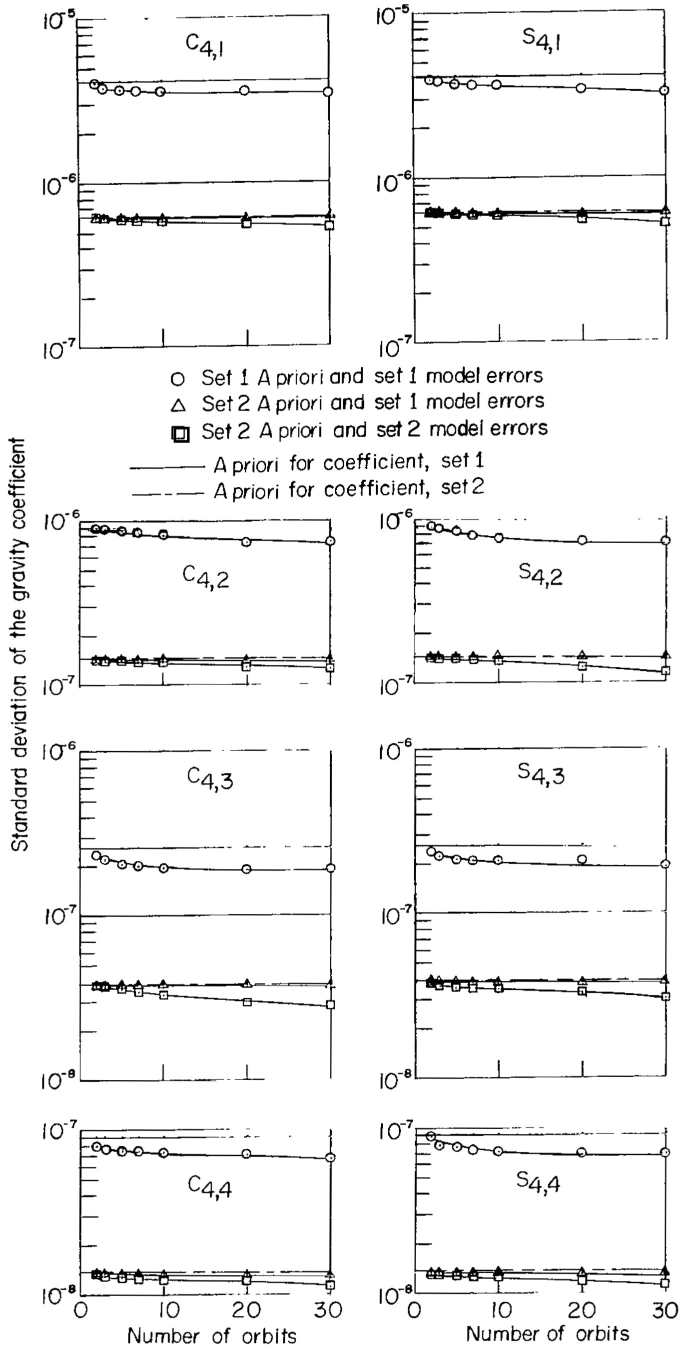

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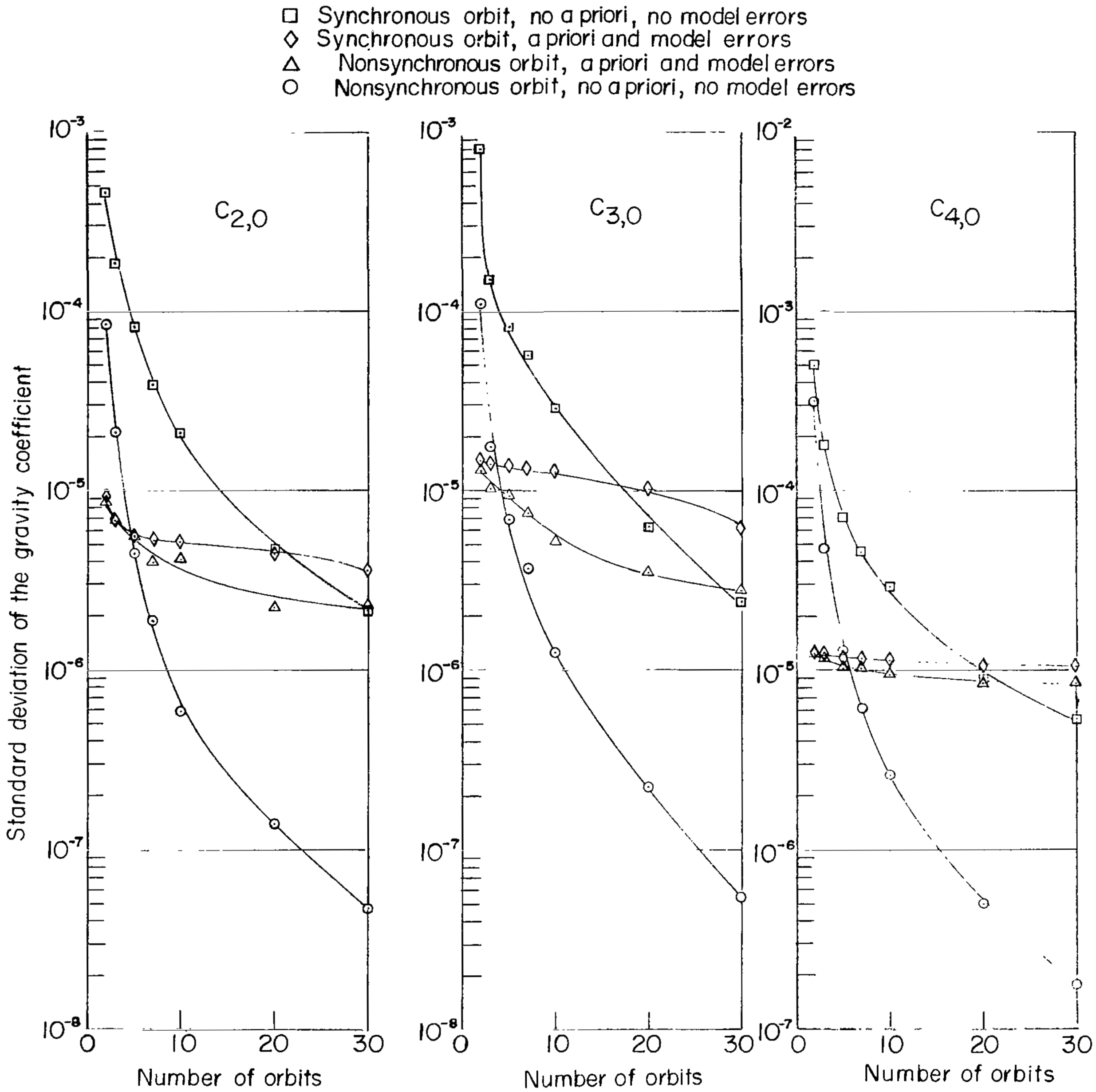


Figure 5.- Comparison of standard deviations of Mars gravity coefficients using two different orbits.

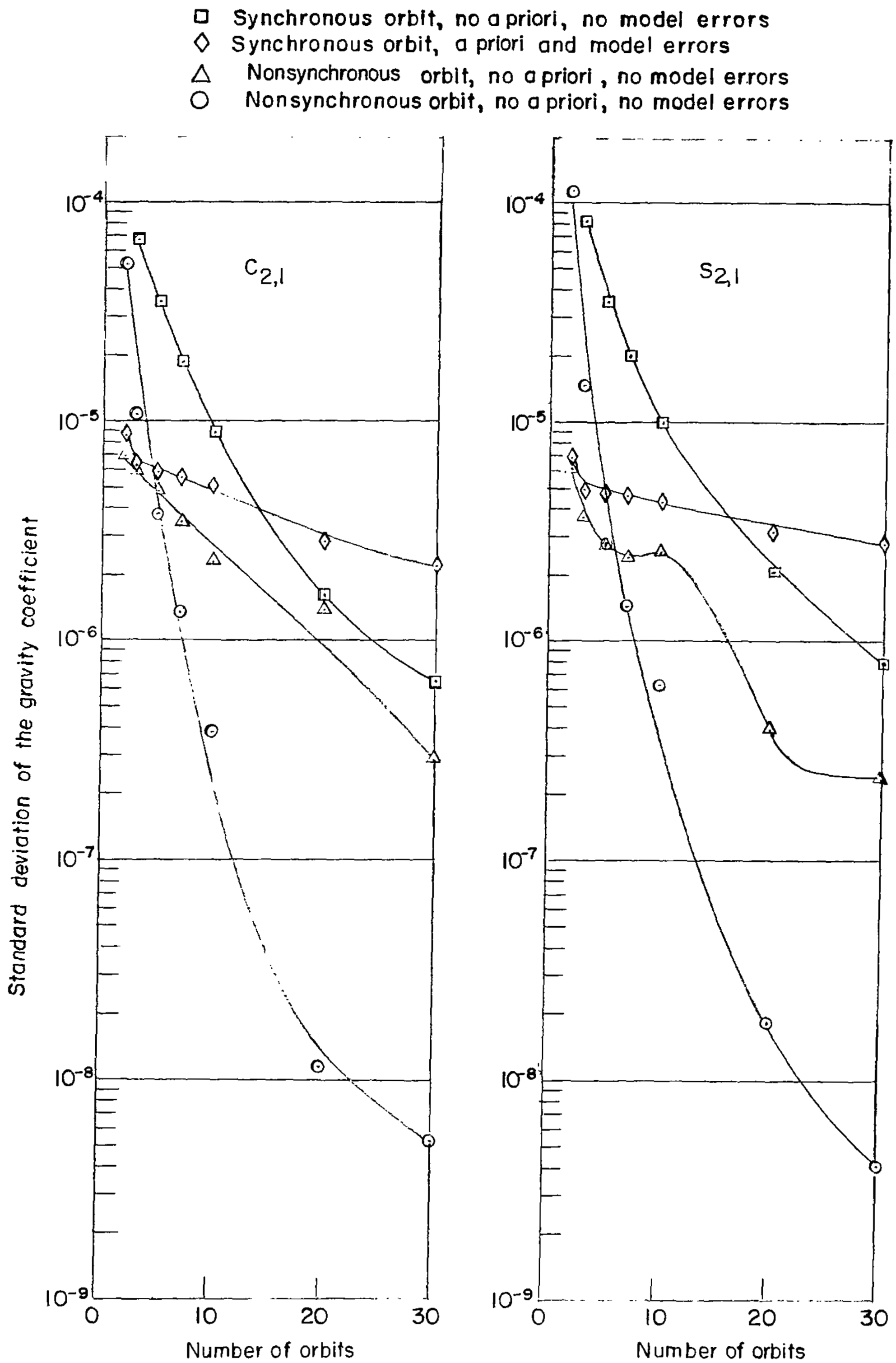
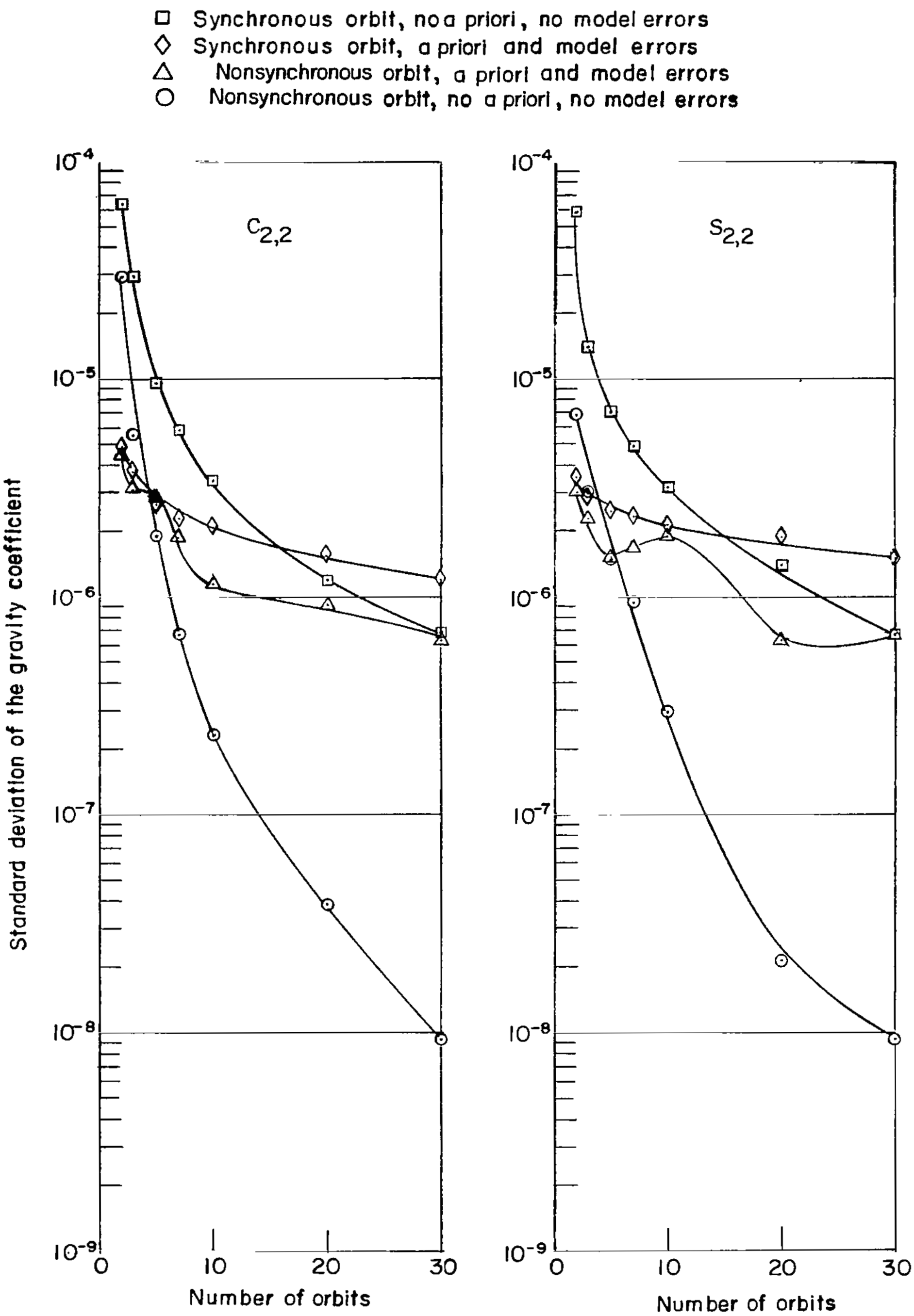


Figure 5.- Continued.



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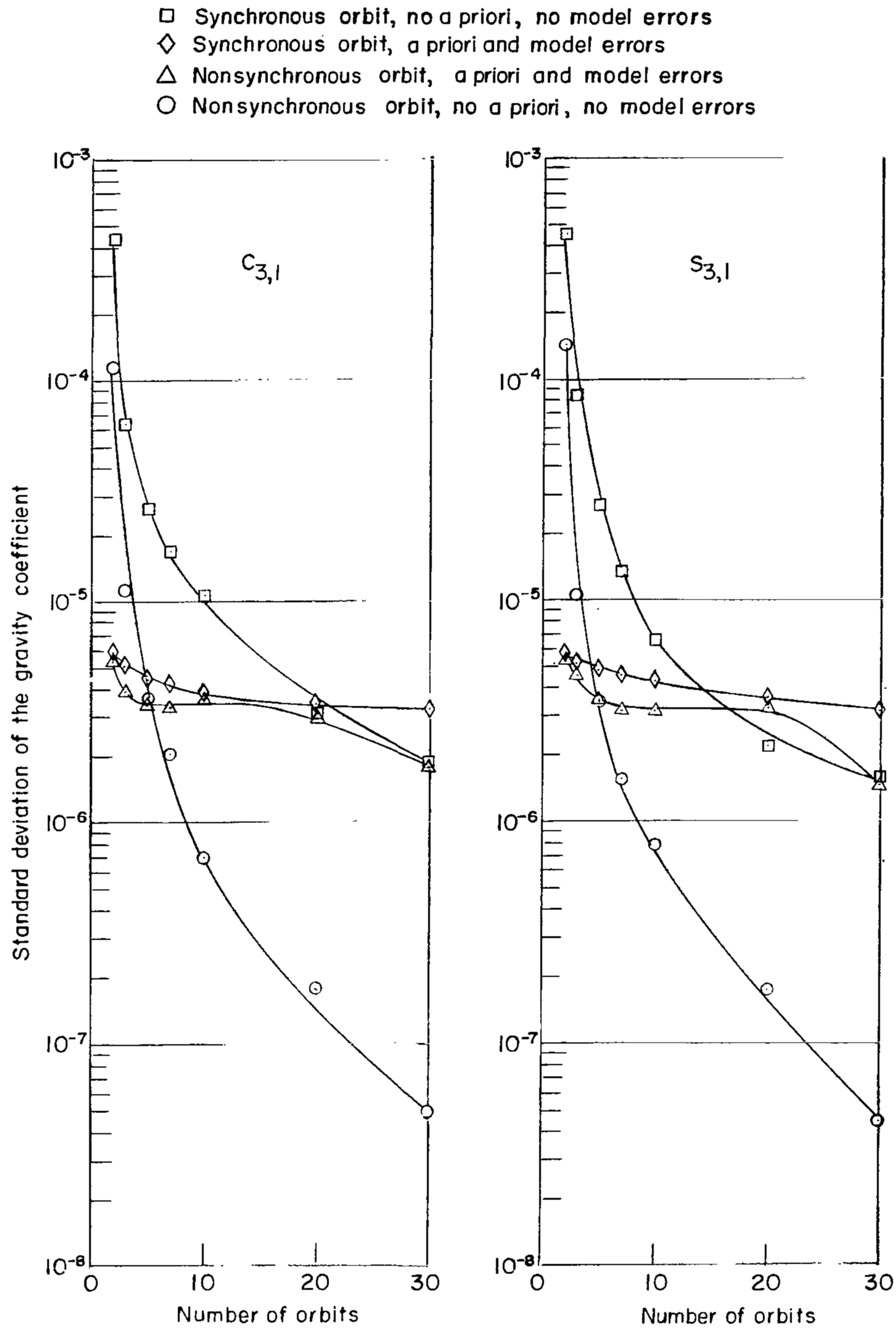

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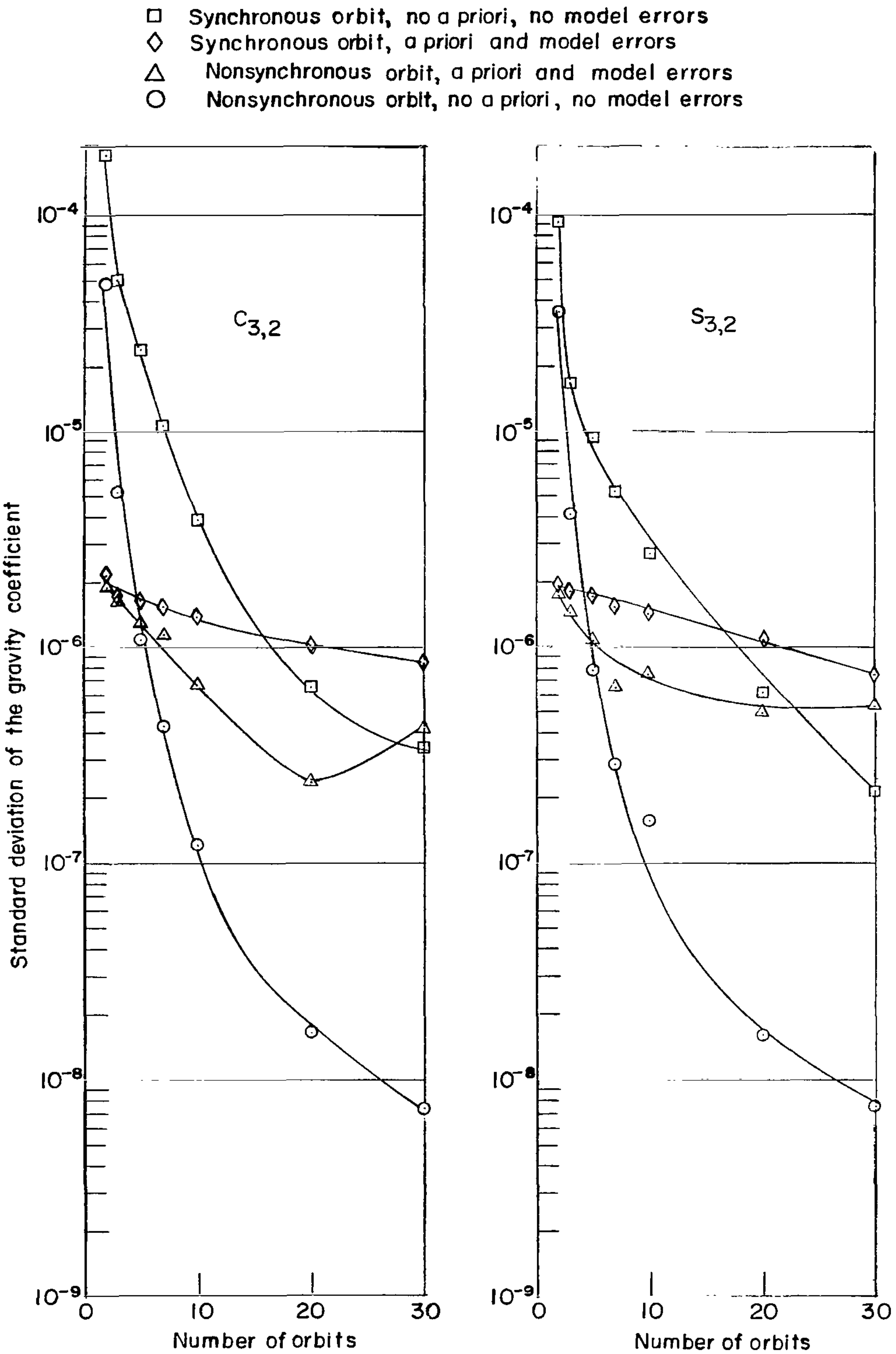

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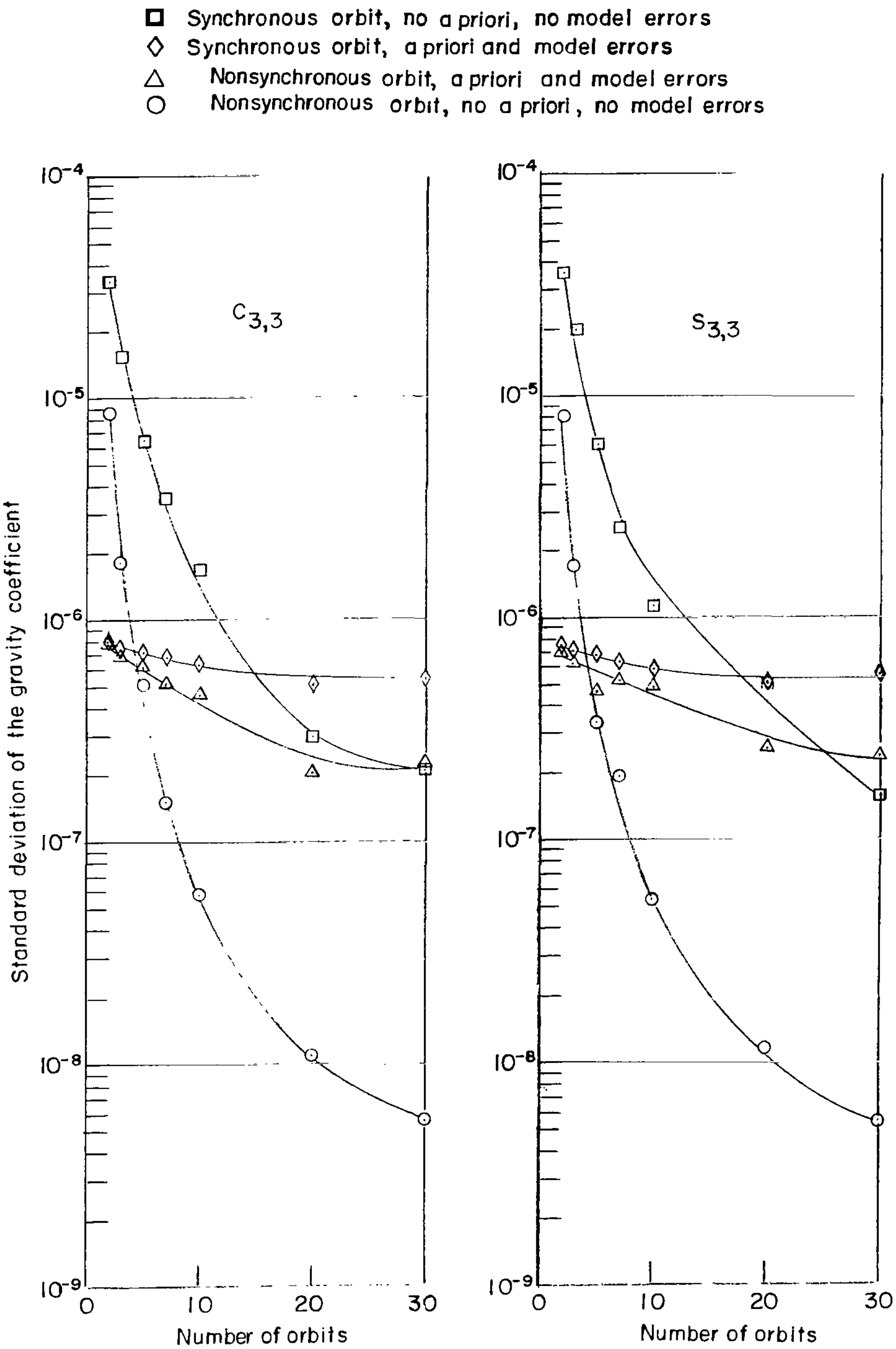

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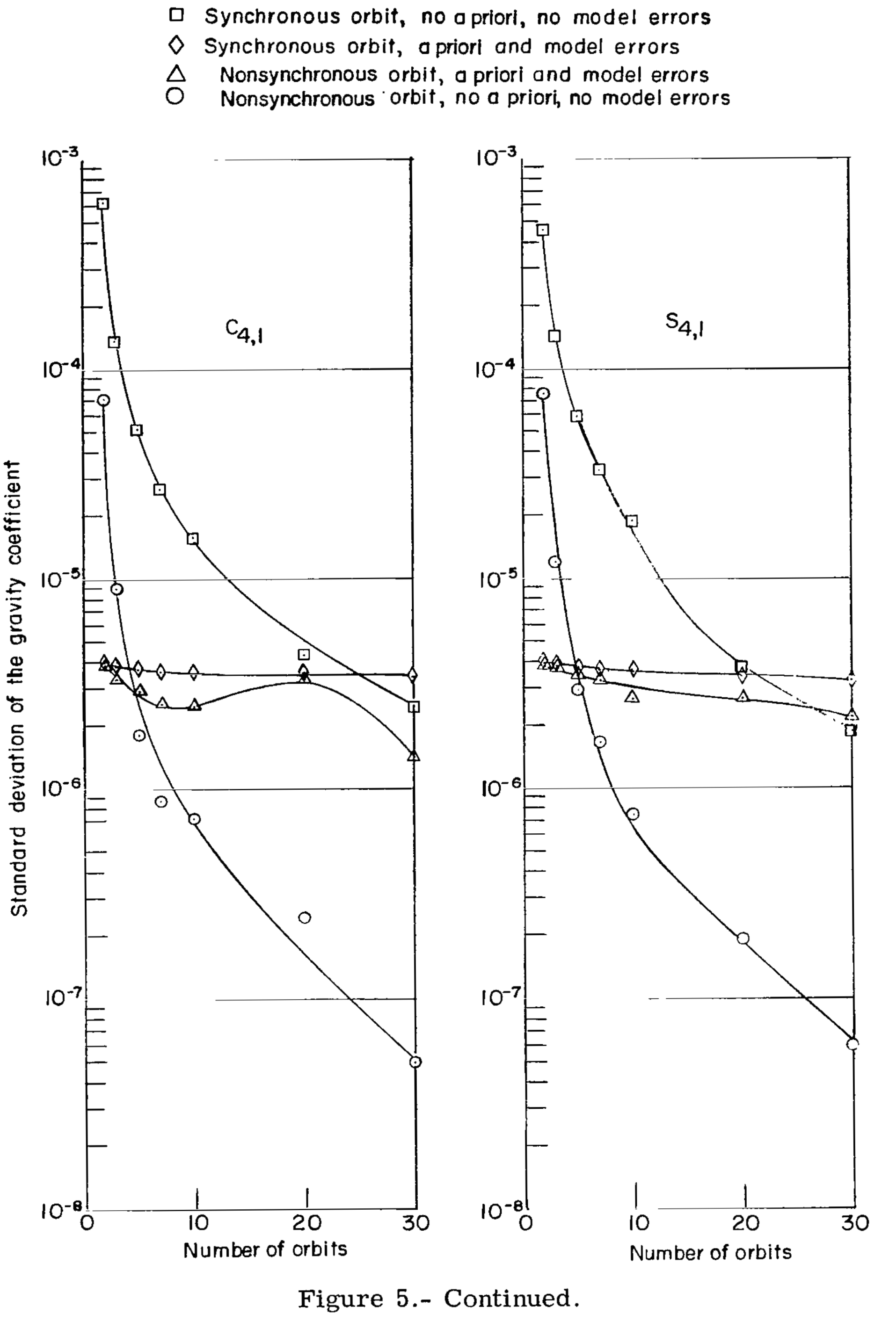

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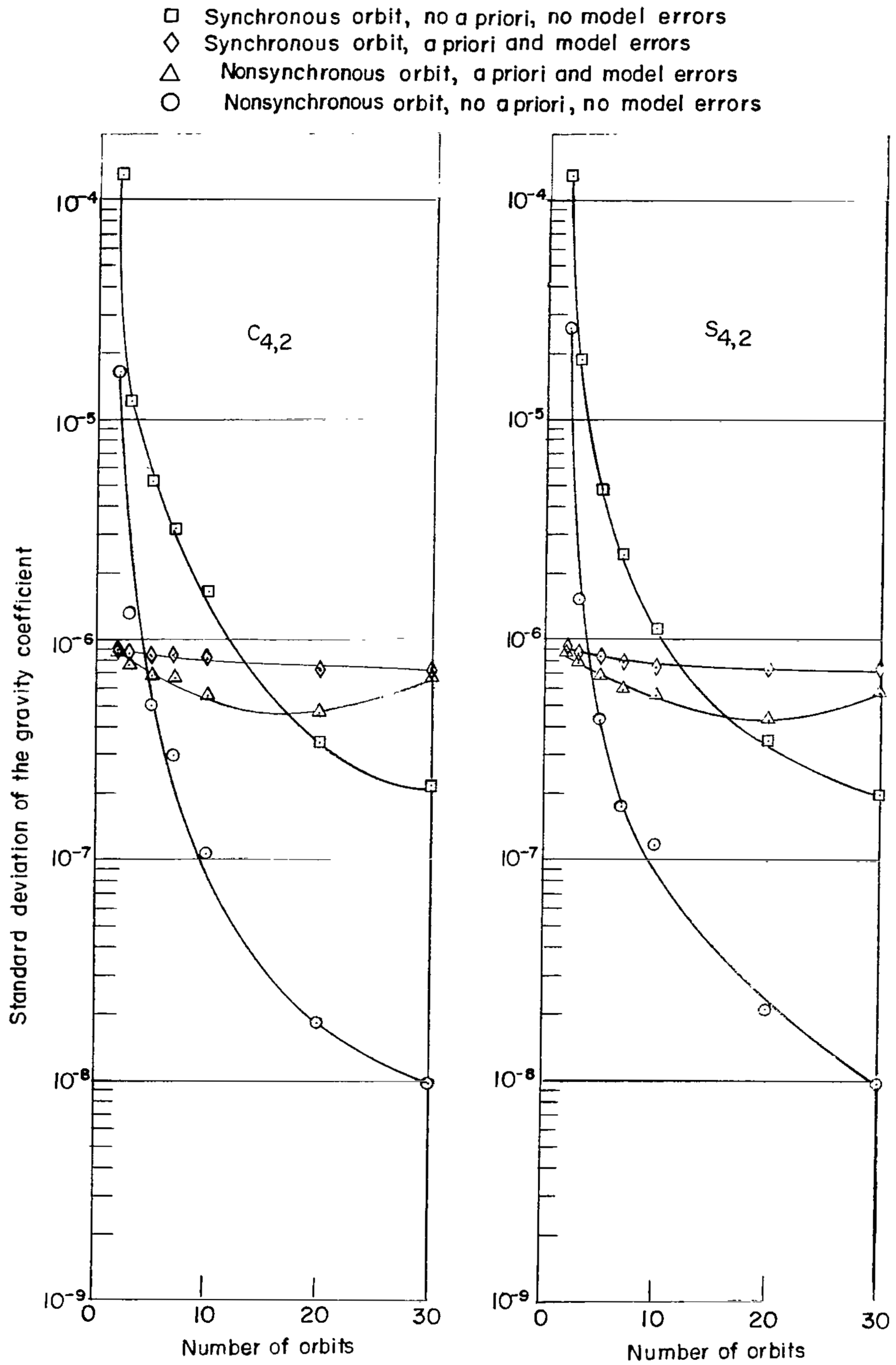

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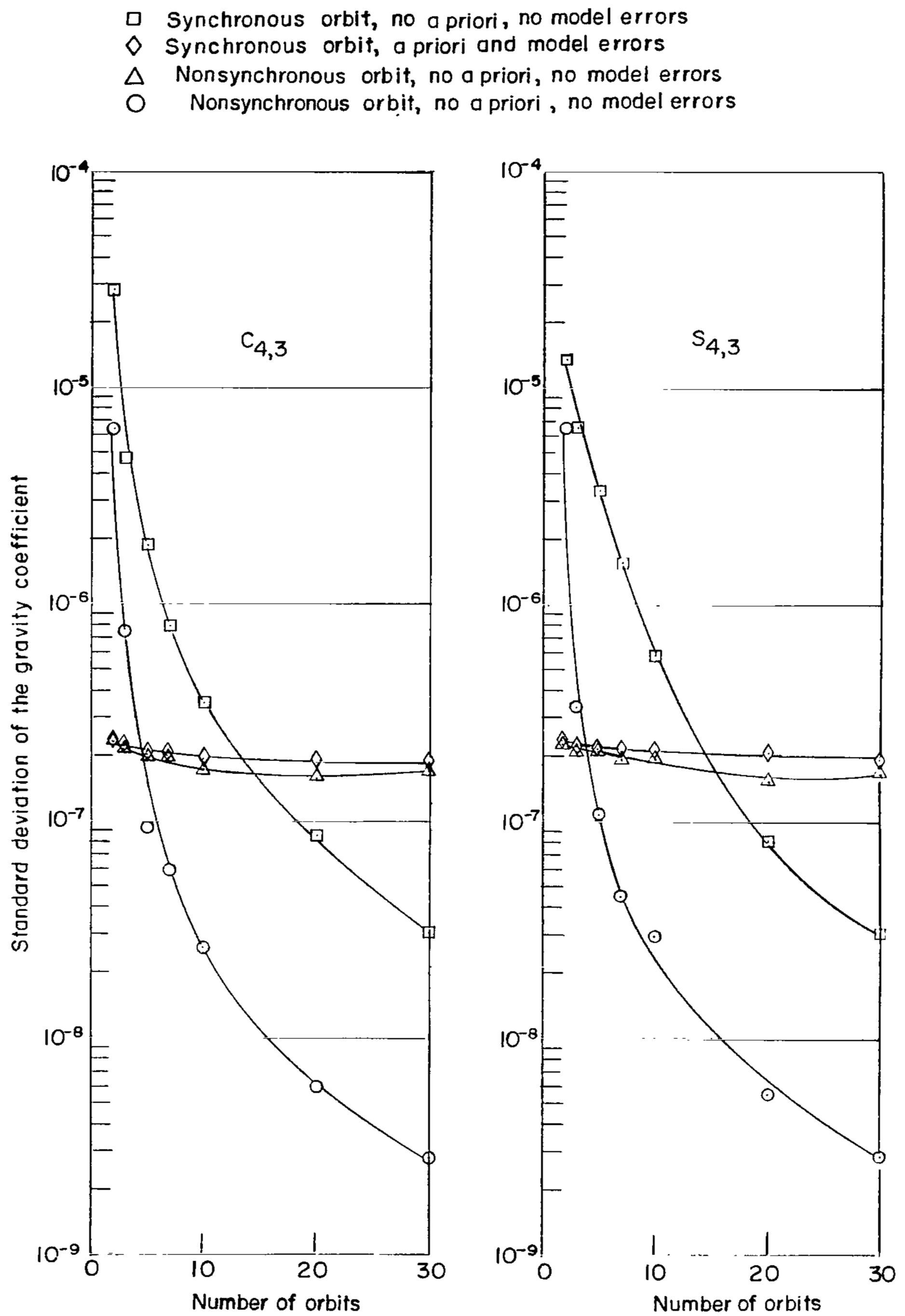

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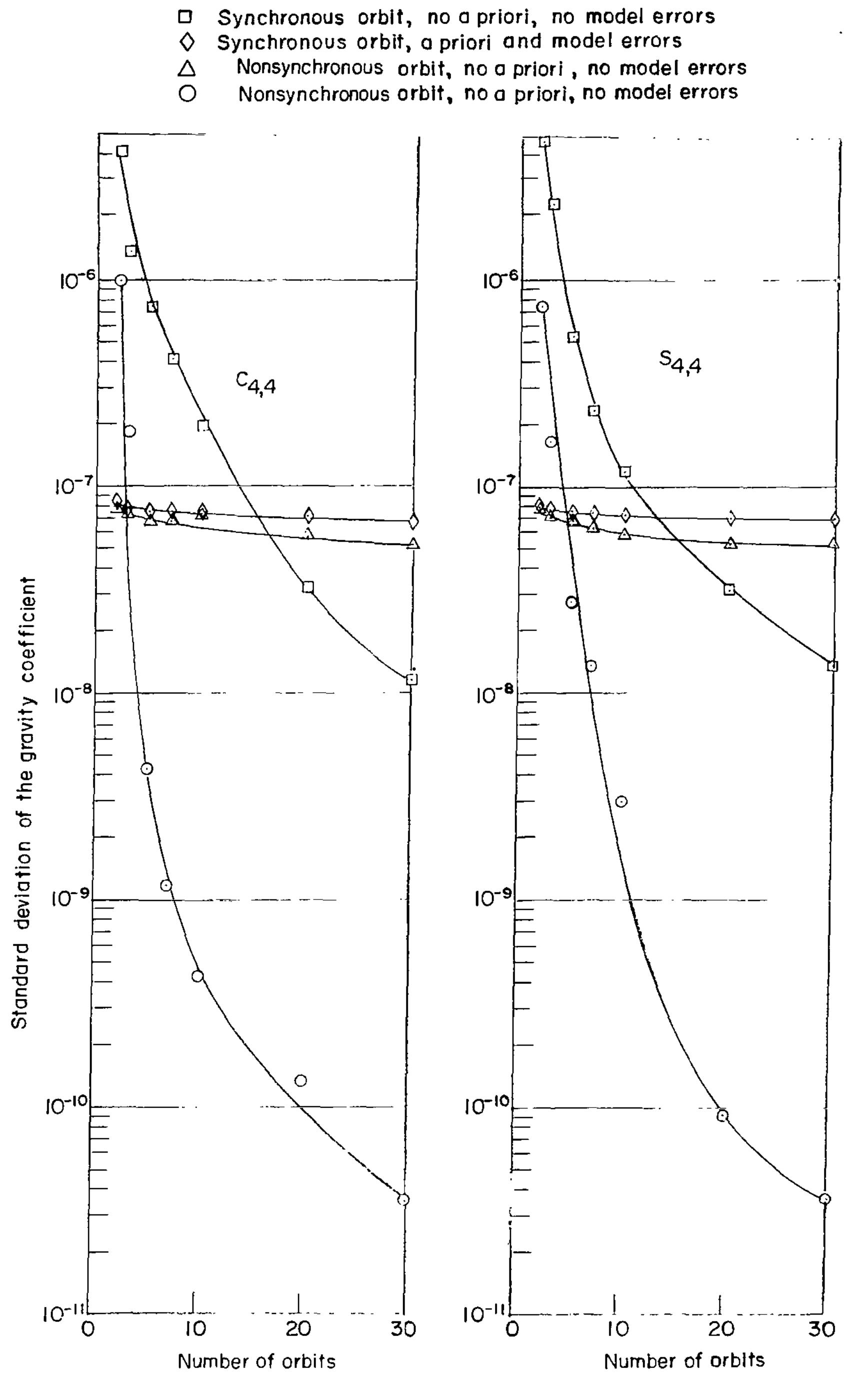

Figure 5.- Concluded.

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— NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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