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FORTRAN PROGRAM FOR QUASI-THREE-DIMENSIONAL CALCULATION OF SURFACE VELOCITIES AND CHOKING FLOW FOR TURBOMACHINE BLADE ROWS

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16. Abstract

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FORTRAN PROGRAM FOR QUASI-THREE-DIMENSIONAL CALCULATION OF SURFACE VELOCITIES AND CHOKING FLOW FOR TURBOMACHINE BLADE ROWS

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SUMMARY

A quasi-three-dimensional compressible flow analysis is reported in NASA TM X-1394 for axial flow turbines. This analysis has been generalized herein to allow for mixed or radial flow and for nonuniform inlet conditions. The velocity gradient method is used, with one velocity gradient equation used for radial equilibrium and another velocity gradient equation for blade-to-blade variation. In addition to a description of the quasi-three-dimensional flow analysis, a FORTRAN IV computer program is included for the solution of these equations.

INTRODUCTION

The aerodynamic design of turbomachinery blades requires the determination of the blade surface velocity distribution. Also, it is often necessary to know the choking mass flow. In many blade designs there are significant velocity gradients both from blade to blade and from hub to tip; this condition necessitates consideration of three-dimensional effects.

One of the useful techniques for calculating surface velocities where three-dimensional effects are of importance is the velocity gradient (stream filament) method. The general velocity gradient equation (ref. 5) determines the velocity variation in any direction. In particular, the velocity gradient equation can be reduced to special cases to determine both the blade to blade and hub to tip variation in velocity. A combination of the velocity variation in two directions with a specified mass flow will determine the velocities at a passage cross section. This method works well in a well-guided passage. Reference 1 applies this method to axial flow turbines. The computer program (CTTD) in reference 1 has been used at Lewis Research Center for 17 years and has long been available to industry for their own use in turbine design.

A new program, CHANEL, has been written. CHANEL is more general than CTTD and can obtain quasi-three-dimensional solutions in any well-guided channel. Some of the things that can be handled by the CHANEL program that could not be handled before are nonuniform inlet temperature, pressure, and prewhirl, and nonaxial flow where meridional flow angle, meridional streamline curvature, and radius can vary as desired from the hub to tip. Also, output has been clarified, and additional output information is given.

This report gives a description of the analysis procedure and of the use of the CHANEL program. The CHANEL program listing is also given.

SYMBOLS

a	coefficient, eqs. (B1) and (B2)
a_b	coefficient, eqs. (A3) and (A4)
a_n	coefficient, eqs. (A1) and (A2)
b	coefficient, eqs. (B1) and (B2)
b_b	coefficient, eqs. (A3) and (A4)
b_n	coefficient, eqs. (A1) and (A2)
c	coefficient, eqs. (B1) and (B2)
c_n	coefficient, eqs. (A1) and (A2)
c_p	specific heat at constant pressure, J/(kg-K) (ft-lbf/(slug-$^{\circ}$R))
h	enthalpy, J/kg ((ft-lbf)/slug)
m	meridional streamline distance, meters (ft)
n	distance from hub along normal to meridional streamline, meters (ft)
n_{b-b}	distance from suction surface along normal to blade-to-blade streamline, meters (ft)
p	pressure, kg/meter^2 (lbf/ft^2)
q	distance along an arbitrary curve
R	gas constant, J/(kg-K) (ft-lbf/(slug-$^{\circ}$R)
r	radius, meters (ft)

r_c	radius of curvature of meridional streamline, meter (ft)
$(r_c)_{b-b}$	radius of curvature of blade-to-blade streamline, meter (ft)
$r_h(r_t)$	radius at intersection of normal to meridional streamlines with hub (tip), meters (ft)
T	temperature, K ($^{\circ}$R)
V_θ	tangential component of absolute velocity, meters/sec (ft/sec)
W	velocity relative to blade, meters/sec (ft/sec)
W_{cr}	critical velocity relative to blade, meters/sec (ft/sec)
W_m	meridional component of velocity relative to blade, meters/sec (ft/sec)
$W_{mid,mean}$	velocity relative to blade at midchannel between suction and pressure surfaces and at the mean section between hub and tip, meters/sec (ft/sec)
W_θ	tangential component of velocity relative to blade, meters/sec (ft/sec)
w	mass flow through the channel, kg/sec (slugs/sec)
z	axial coordinate, meter (ft)
α	angle between meridional streamline and axis of rotation, rad, see fig. 3
β	angle between relative velocity vector and meridional plane, rad, see fig. 3
γ	specific heat ratio
θ	relative angular coordinate, rad, see fig. 3
λ	prerotation $(rV_\theta)_i$, meter^2/sec (ft^2/sec)
ρ	density, kg/meter^3 (slugs/ft^3)
ω	rotational speed, rad/sec

Subscripts:

i	inlet or upstream
isen	isentropic
loss	difference between isentropic and actual

Superscripts:

'	absolute stagnation condition
''	relative stagnation condition

METHOD AND ASSUMPTIONS

The objective of the analysis method is to calculate the quasi-three-dimensional velocity distribution satisfying continuity at a given channel orthogonal surface (see fig. 1). The mass flow may be specified for the calculation, or the maximum (choking) mass flow for that channel orthogonal surface may be calculated. The velocity variation on the orthogonal surface is given by differential equations for the rate of change of velocity. These equations are called velocity gradient equations. One velocity gradient equation gives the blade-to-blade variation, and the other gives the hub-to-tip variation. The velocity gradient equations are solved simultaneously with the condition of either a specified or choking weight flow. This determines the blade surface velocities on a particular channel orthogonal surface. When this is done for several orthogonal surfaces, a velocity distribution over the blade surface is determined.

The velocity distribution can be obtained in this way for the guided channel formed by the portion of the passage where the blade-to-blade orthogonals extend from suction to pressure surface. The guided channel will not cover the entire suction surface. To obtain the velocity distribution on the uncovered portion of the blade, the location of the stagnation streamline would have to be known. Therefore, CHANEL cannot be used to obtain velocities on the uncovered portion of the blade. (However, other methods are available for this problem, e.g., refs. 3 and 4.)

The basic simplifying assumptions used in deriving the equations are the following:

(1) The flow relative to the blade is steady.

(2) The fluid is a perfect gas with constant c_p.

(3) The fluid is a nonviscous gas.

(4) The midchannel line is a streamline, hereinafter referred to as the midchannel streamline.

(5) There is a linear variation of streamline <u>curvature</u> along an orthogonal or there is a linear variation of <u>radius of curvature</u> along an orthogonal. An option is provided for this assumption in the program.

(6) There is no change in radius along a blade-to-blade orthogonal. (The change in radius usually has a negligible effect on the solution.)

(7) There is no change in flow angle along a blade-to-blade orthogonal. (The change in flow angle usually has a negligible effect on the solution.)

The method used here is very similar to that in reference 1. The main difference is that the assumptions of axial flow and uniform inlet conditions have been dropped. The analytical equations and the details of the solution procedure are given in appendix A.

DESCRIPTION OF INPUT AND OUTPUT

The computer program requires, as input, operating conditions, gas constants, and flow passage geometry. The usual procedure for obtaining this input includes the following steps:

(1) Define inlet absolute stagnation temperature and density, prerotation from hub to tip, mass flow per passage, rotational speed, and gas constants.

(2) Lay out graphically (or determine analytically) the meridional profile of the passage and determine hub to tip meridional normals as desired. Each meridional normal is a separate case requiring a separate set of input. Any number of cases may be included in a single computer run. From the meridional profile, the distance along the meridional normal from the hub, the radius from axis of rotation, the meridional flow angle, and the meridional streamline curvature must be obtained several points from hub to tip.

(3) Lay out the blade-to-blade passage at hub, mean, and tip to determine blade-to-blade flow angle and orthogonal distance, and the suction and pressure surface curvatures.

(4) Prepare input sheets, one input sheet for each case.

Input

The input data sheet is shown in figure 2. The quantities shown are for a sample problem. The output for this sample problem is presented later. Certain items (marked *) are discussed in greater detail in the section Instructions for Preparing Input.

The input variables are as follows:

JX* controls blade-to-blade curvature variation assumption
- JX = 1, linear variation of curvature
- JX = 2, linear variation of radius of curvature

JZ* controls type of solution obtained
- JZ = 1, subsonic solution
- JZ = 2, supersonic solution
- JZ = 3, choked flow solution

KR1* controls use of card starting with GAM
- KR1 = 0, omit card with GAM
- KR1 = 1, supply card with GAM

KR2* controls use of cards starting with RHOIP
- KR2 = 0, omit cards with RHOIP
- KR2 = 1, supply one card with RHOIP
- KR2 = 3, supply three cards with RHOIP

NSP* number of meridional input points between hub and tip for the NAMELIST arrays (see ref. 8) (minimum of 2, maximum of 25)

GAM specific heat ratio, γ

AR gas constant, J/kg-K (ft-lb/slug-^{o}R)

OMEGA rotational speed, ω, rad/sec (Note that ω is negative if rotation is in the opposite direction of that shown in fig. 3.)

WTFL mass flow per blade, kg/sec (slug/sec) (not required for choked solution)

RHOIP inlet stagnation density, ρ_i', kg/meter^3 (slug/ft^3)

PLOSS* fractional loss of relative stagnation pressure $(1 - (p''/p''_{isen}))$

NBB* length of streamline normal from blade to blade, meter (ft) (fig. 6)

CS* blade surface curvature at intersection of orthogonal with suction surface, 1/meter (1/ft) (fig. 6) (Sign is negative if surface is concave downward.)

CPR* blade surface curvature at intersection of orthogonal with pressure surface, 1/meter (1/ft) (fig. 6) (Sign is negative if surface is concave downward.)

The remaining input is given in NAMELIST format as shown on the last two lines of figure 2. These variables are all arrays, with NSP items in each array. NSP must be at least 2 but not over 25. All the NAMELIST arrays are initialized to zero by the program so that zero input items may be omitted. Any input given in NAMELIST format remains unchanged from the previous case unless given again as input.

NMERID* array of distances from hub along normal to the meridional streamlines, meter (ft) (These values must be in increasing order. The first value must be zero, and the last value must be the total distance along the normal from hub to tip. A meridional streamline is the projection on a plane through the axis of rotation of a streamline midway between the two blade surfaces.) (See fig. 5.)

TEMPIP* array of inlet stagnation temperatures T_i' for streamline corresponding to the NMERID array, K (^{o}R)

LAMBDA* array of values of prerotation λ for streamlines corresponding to the NMERID values, meter^2/sec (ft^2/sec)

RADIUS* array of radii r corresponding to the NMERID array, meter (ft) (fig. 5)

CURV* array of meridional streamline curvatures $1/r_c$ corresponding to the NMERID array, 1/meter (1/ft) (fig. 5)

ALPHA* array of meridional streamline angle α corresponding to the NMERID array, deg (fig. 5)

BETA* array of relative flow angle β measured from the meridional plane, positive in direction of rotation, midchannel only, and corresponding to the NMERID array, deg (fig. 6)

Instructions for Preparing Input (see fig. 2)

Units of measurement. - The International System of Units (ref. 2) is used throughout this report. However, the program does not use any constants which depend on the system of units being used. Therefore, any consistent set of units may be used in preparing input for the program. In particular, U.S. Customary Units may be used, and these are specifically noted in the list of input variables.

Title cards. - The first card is a general title card for the entire run. Several cases may be submitted on a single run. And each of these cases may be labeled by means of the label card. All 80 columns may be used.

Option code card. - The next card provides for several options in running the program. The first option (JX) is to choose an assumption of how the blade-to-blade streamline curvature varies. The usual assumption is linear variation of curvature (JX = 1). This assumption must be used if the blade curvature is zero on either blade or if the curvature changes sign between the blades, since the curvature must be zero ($r_c = \infty$) at some point. On the other hand, there are cases when the assumption of linear variation of curvature is not satisfactory. For example, if the curvature is very large on one surface, and very small on the other, it may be much more reasonable to assume linear variation of radius of curvature (JX = 2). Ordinarily, there is not a large change in curvature, and in this case either assumption will give good results.

The next option (JZ) specifies the type of solution desired. If there is a solution for a given mass flow through the channel, there are usually two distinct solutions. These two solutions have two different velocity levels. The solution at the lower level has mostly subsonic velocities, although some velocities may be supersonic. On the other hand, the solution at the higher level has mostly supersonic velocities although there may be some subsonic velocities. The solution with lower velocities will be referred to as the subsonic solution and the greater solution as the supersonic solution. The option JZ = 1 (=2) will result in obtaining the subsonic (supersonic) solution. If the maximum possible or choking mass flow solution is desired, use JZ = 3. The program will compute the choking mass flow and the corresponding velocities. If the mass flow specified as input is larger than the choking mass flow, the result will be the same as when JZ = 3.

For the first case of a given run KR1 must be 1. Then each item on the card starting with GAM must be specified. For successive cases, if nothing on the GAM card changes from the previous case the card is omitted and KR1 is 0. If it is desired to change some items, then let KR1 = 1 and completely fill in the GAM card.

The next option (KR2) specifies whether information is given on the RHOIP cards. On the first case of a given run, KR2 must be either 1 or 3 corresponding to 1 or 3 RHOIP cards. For successive cases, if nothing on the RHOIP cards changes from the previous case, the cards are omitted and KR2 is 0. Use one RHOIP card if all quantities are constant from hub to tip. When 3 RHOIP cards are needed, the quantities on these cards are given at hub, mean, and tip, in that order, where the mean is taken to be midway along the meridional normal between hub and tip.

The next quantity (NSP) specifies the number of meridional input points for the NAMELIST arrays. A great deal of flexibility is provided here. NSP must always be at least 2, since information must be provided at hub and tip. Any additional points, up to a total of 25, are entirely at the discretion of the user, both as to the number and the location.

Loss assumption. - A loss correction may be made by specifying the fraction, PLOSS, of the relative stagnation pressure which has been lost. This is equivalent in a simplified calculation like this to specifying the fraction of the passage which is occupied by the displacement thickness of the boundary layer. PLOSS is zero if there is no loss correction.

Meridional profile layout. - The meridional profile should include hub, shroud, and mean line, as well as blade leading and trailing edge, as shown in figure 5. Any number of meridional normals can then be drawn. The meridional normals should be normal to the hub, mean, and tip lines. If a meridional solution has been previously obtained, the meridional normal should be normal to the meridional streamlines. Each orthogonal will be a separate case using one input sheet. The quantities which are obtained from the meridional layout are NMERID, RADIUS, CURV, and ALPHA. Normally these four quantities are obtained at hub, mean, and tip. However, if a meridional solution has been previously obtained more values can be given.

NMERID is the distance along the meridional normal, so the first value must be zero and the last value must be the total length of the meridional normal. Any desired intermediate points may be added, up to a maximum total of 25. The total number of points is NSP. After the desired points have been chosen, all the lengths are measured from the hub. Then the corresponding radii are measured for the RADIUS array. The angles from horizontal, in degrees, go in the ALPHA array. If the hub and tip are straight, the wall curvature (CURV) is zero, so that no entry need be made for CURV. The example input in figure 2 is for straight axial flow so that both CURV and ALPHA are zero, and hence omitted from the input. If the hub or tip are not straight, the wall curvature should be measured at the ends of the meridional normal. It is suggested that the meridional streamline curvature be assumed to vary linearly between hub and tip, unless a meridional streamline solution is available. The curvature is considered to be positive if the streamline is concave upwards.

After the NMERID distances have been determined, the inlet conditions of inlet stagnation temperature (TEMPIP) and prerotation (LAMBDA) can be specified at corresponding points. TEMPIP and LAMBDA must each have NSP values given, although these values may be constant.

Blade-to-blade channel layouts. - The blade-to-blade channel layouts are usually the most difficult step in obtaining the input data. If the hub and tip are straight (in the meridional profile layout), the blade-to-blade layouts can be laid out as a flat surface. If either the hub or tip is not straight, then the corresponding blade-to-blade surface is not developable and cannot be laid out as a flat surface with correct angles and linear measurements. There are several ways that a nondevelopable blade-to-blade surface of revolution can be laid out. However, these layouts are distorted, so that corrections must be made to determine orthogonal directions; also, distance measurements from the layout may require a graphical integration procedure. With the proper corrections, the desired blade-to-blade normals can be determined, and the lengths calculated with sufficient accuracy. Since this portion of the graphical procedure can be done in any of several ways, no further explanation is given here for determining blade-to-blade normals when hub or tip are not straight.

Blade-to-blade layouts at hub, mean, and tip should be drawn as shown in figure 6. The layouts in figure 6 correspond to the blade shown in figure 5. The intersection point of the meridional normal with the mean line on the meridional layout (marked 1 in fig. 5) should be marked on the midchannel line on the mean blade-to-blade layout (marked 1 in fig. 6). This determines the reference θ coordinate for the meridional normal. This reference θ coordinate can be marked on the hub and tip layouts as shown in figure 6. Now the intersection point of the meridional normal with the tip on the meridional layout is marked on the tip blade-to-blade layout (marked 2 in figs. 5 and 6). The corresponding hub point is marked 3 in figures 5 and 6. Now the true blade-to-blade normals are drawn to be normal to suction and pressure surfaces and to pass through the points 1, 2, and 3 on the hub, mean, and tip layouts as shown in figure 6. Then the blade-to-blade normal length (NBB) is measured or calculated from each layout. Also, the blade suction surface curvature (CS) and blade pressure surface curvature (CPR) are measured at the ends of the blade-to-blade normal. Blade surface curvatures are important, but they are difficult to measure accurately; therefore, extra care should be taken to measure blade curvatures accurately. Finally, the angle (BETA) of the midchannel line from axial is measured (with correction if necessary) at the hub, mean, and tip. The angle BETA is positive if the tangential component of velocity is in the direction of rotation. Since BETA is given as NAMELIST input, values are usually given at hub, mean, and tip, but more values must be given if NSP is greater than 3.

Output

An example of the output obtained is given in figure 7. This output corresponds to the output given on the input form shown in figure 2. U.S. Customary Units of measurement are used. Each case will normally be printed on one page. The first output is a listing of all the input, pretty much in the same format as on the input sheet (fig. 2). All input items are given, including those which are repeated from the previous case.

After the input listing, the velocities across the passage from blade to blade and from hub to tip are printed. Then the critical velocity ratio W/W_{cr} across the passage is printed. If the passage is choked, the choking mass flow is also printed.

Finally, information on the meridional streamline spacing is printed. Information is given at 10 equally spaced points from hub to tip. The information is given in three ways. First is the integrated mass flow fraction between the hub and the given point. The next column gives the ratio of the actual streamline spacing to uniform spacing. Finally, the streamline spacing (or normal stream sheet thickness) for 1 percent of the mass flow is given. This last item is useful if it is desired to obtain a blade-to-blade solution using the TSONIC or TURBLE program of reference 3 or 4.

Error Messages

A number of error messages have been incorporated into the program. These error messages are listed here. Where necessary, suggestions for finding and correcting the error are given.

NSP MUST BE GREATER THAN OR EQUAL TO 2.

NSP is the number of points in the NAMELIST arrays. Since the first point must be at the hub and the last point at the tip, NSP must be at least 2.

NSP CANNOT BE GREATER THAN 25.

JX MUST BE EITHER 1 OR 2.

WHEN JX = 2, CS AND SPR MUST HAVE THE SAME SIGN.

When JX = 2 it is assumed that the radius of curvature of blade-to-blade streamlines varies linearly. This is not possible if CS and CPR are of opposite sign, or if either CS or CPR is zero.

A SOLUTION CANNOT BE OBTAINED AT THIS STATION.

This message is printed when the solution procedure breaks down and no solution can be found. The input as listed should be examined carefully for errors.

SPLINT USED FOR EXTRAPOLATION. EXTRAPOLATED VALUE = X.XXX

This message is followed by a printout of the SPLINT input and output arguments. SPLINT is used for interpolation of the input values in the NAMELIST arrays. If

SPLING is used for extrapolation it must be due to an input error, most likely in the NMERID array.

CHANEL COMPUTER PROGRAM

The program consists of the main program and the three subroutines SPLINT, PABC, and CONTIN. Subroutine SPLINT is used for interpolation by a spline fit curve, and is described more fully in reference 5 and 6. Subroutine PABC calculates the coefficients A, B, and C of the parabola $y=Ax^2+Bx+C$ passing through three given points. Subroutine PABC is called by both the main program and subroutine CONTIN. Subroutine CONTIN finds the value of $W_{mid, mean}$ which will give either the desired value of mass flow w or the choking mass flow, depending on the input value of JZ.

Main Program

```
      COMMON SRW
      DIMENSION NMERID(25),TEMPIP(25),LAMBDA(25),RADIUS(25),CURV(25),
     1   ALPHA(25),BETA(25),AAA(25)
      DIMENSION NMER(9),TIP(9),LBDA(9),RAD(9),CRV(9),ALPH(9),BTA(9),
     1   AA(9),BB(9),CC(9),W(9),WBTB(9)
      DIMENSION RHOIP(3),PLOSS(3),NBB(3),CS(3),CPR(3),SUM(3),CPTIP(3),
     1   WCR(3),X(3),Y(3),BBB(3)
      DIMENSION WWCR(3,3),WFIN(3,3)
      DIMENSION AAB(9,3)
      REAL NMERID,LAMBDA,NMER,LBDA,NBB
      INTEGER BTB,HMT,SRW
      NAMELIST /NAM1/NMERID,TEMPIP,LAMBDA,RADIUS,CURV,ALPHA,BETA
      DO 5 I=1,200
    5 NMERID(I) = 0.
      WRITE(6,1000)
      READ (5,1010)
      WRITE(6,1010)
   10 READ (5,1010)
      WRITE(6,1010)
      READ (5,1020) JX,JZ,KR1,KR2,NSP
      WRITE (6,1100)
      WRITE(6,1020) JX,JZ,KR1,KR2,NSP
      IF(NSP.GT.1) GO TO 20
      WRITE(6,1500)
      STOP
   20 IF(NSP.LE.25) GO TO 30
      WRITE(6,1510)
      STOP
   30 WRITE (6,1110)
      IF(KR1.EQ.1) READ (5,1030) GAM,AR,OMEGA,WTFL
      WRITE(6,1040) GAM,AR,OMEGA,WTFL
      WRITE(6,1120)
      IF(KR2.GT.0) READ (5,1030) RHOIP(1),PLOSS(1),NBB(1),CS(1),CPR(1)
      IF(KR2-1) 60,40,35
   35 CONTINUE
      READ (5,1030) RHOIP(2),PLOSS(2),NBB(2),CS(2),CPR(2)
```

```
      READ (5,1030) RHOIP(3),PLOSS(3),NBB(3),CS(3),CPR(3)
      GO TO 60
   40 DO 50 HMT=2,3
      RHOIP(HMT) = RHOIP(HMT-1)
      PLOSS(HMT) = PLOSS(HMT-1)
      NBB(HMT) = NBB(HMT-1)
      CS(HMT) = CS(HMT-1)
   50 CPR(HMT) = CPR(HMT-1)
   60 WRITE(6,1130) (RHOIP(HMT),PLOSS(HMT),NBB(HMT),CS(HMT),CPR(HMT),
     1    HMT=1,3)
      READ (5,NAM1)
   70 WRITE (6,1140)
      WRITE(6,1040) (NMERID(I),I=1,NSP)
      WRITE(6,1150)
      WRITE(6,1040) (TEMPIP(I),I=1,NSP)
      WRITE(6,1160)
      WRITE(6,1040) (LAMBDA(I),I=1,NSP)
      WRITE(6,1170)
      WRITE(6,1040) (RADIUS(I),I=1,NSP)
      WRITE(6,1180)
      WRITE(6,1040) (CURV  (I),I=1,NSP)
      WRITE(6,1190)
      WRITE(6,1040) (ALPHA (I),I=1,NSP)
      WRITE(6,1200)
      WRITE(6,1040) (BETA  (I),I=1,NSP)
      IF (JX.EQ.1) GO TO 78
      IF (JX.EQ.2) GO TO 72
      WRITE (6,1000)
      GO TO 10
   72 DO 74 I=1,3
   74 IF (CS(I)*CPR(I).LE.0.) GO TO 76
      GO TO 78
   76 WRITE (6,1530)
      WRITE (6,1000)
      GO TO 10
C
C     ALL INPUT HAS BEEN READ
C
C     CALCULATE CONSTANTS AND INTERPOLATE MERIDIONAL VARIABLES AT
C          EIGHT EQUAL INTERVALS FROM HUB TO TIP
C
   78 CP = AR/(GAM-1.)*GAM
      EXPON = 1./(GAM-1.)
      TGROG = 2.*GAM*AR/(GAM+1.)
      DO 80 I=1,9
   80 NMER(I) = FLOAT(I-1)*NMERID(NSP)/8.
      DELTA = NMERID(NSP)/8.
      NINE = 9
      CALL SPLINT(NMERID,TEMPIP,NSP,NMER,NINE,TIP,AAA)
      CALL SPLINT(NMERID,LAMBDA,NSP,NMER,NINE,LBDA,AAA)
      CALL SPLINT(NMERID,RADIUS,NSP,NMER,NINE,RAD,AAA)
      CALL SPLINT(NMERID,CURV  ,NSP,NMER,NINE,CRV,AAA)
      CALL SPLINT(NMERID,ALPHA ,NSP,NMER,NINE,ALPH,AAA)
      CALL SPLINT(NMERID,BETA  ,NSP,NMER,NINE,BTA,AAA)
  110 DO 120 HMT=1,3
      CPTIP(HMT) = 2.*CP*TIP(HMT)
      TWLMR = 2.*OMEGA*LBDA(4*HMT-3)-(OMEGA*RAD(4*HMT-3))**2
      TTIP = 1.-TWLMR/CPTIP(HMT)
```

```
  120 WCR(HMT) = SQRT(TGROG*TTIP*TIP(HMT))
      DELMAX = WCR(2)/20.
C   CALCULATE COEFFICIENTS FOR MERIDIONAL VELOCITY GRADIENT EQUATION
      DO 125 I=1,9
      ALPH(I) = ALPH(I)/57.295779
      CAL = COS(ALPH(I))
      BTA(I) = BTA(I)/57.295779
      SBETA = SIN(BTA(I))
      CBETA = COS(BTA(I))
      AA(I) = CRV(I)*CBETA**2-SBETA**2/RAD(I)*CAL
      BB(I) = -2.*OMEGA*SBETA*CAL
      CC(I) = CP*(TIP(I+1)-TIP(I))-OMEGA*(LBDA(I+1)-LBDA(I))
  125 CONTINUE
C   CALCULATE COEFFICIENTS FOR BLADE TO BLADE VELOCITY GRADIENT EQUATION
      DO 140 HMT=1,3
      SAL = SIN(ALPH(4*HMT-3))
      TEMP = SAL*SIN(BTA(4*HMT-3))/RAD(4*HMT-3)
      BBB(HMT) = 2.*OMEGA*SAL
      IF(JX.EQ.2) GO TO 132
      DO 130 BTB=1,9
  130 AAB(BTB,HMT) = TEMP+CS(HMT)+(CPR(HMT)-CS(HMT))*FLOAT(BTB-1)/8.
      GO TO 140
  132 DO 134 BTB=1,9
      RC = 1./CS(HMT)+(1./CPR(HMT)-1./CS(HMT))*FLOAT(BTB-1)/8.
  134 AAB(BTB,HMT) = TEMP+1./RC
  140 CONTINUE
      WEST = WTFL/RHOIP/NMERID(NSP)/(NBB(1)+4.*NBB(2)+NBB(3))*6.
      IF(WEST.GT.WCR(2)) WEST = WCR(2)
      WTFLSV = WTFL
      IF(JZ.EQ.3) WTFL = WCR(2)*RHOIP*NMERID(NSP)*(NBB(1)+NBB(2)+NBB(3))
      IF(JZ.EQ.3) WEST = WCR(2)
      NCOUNT = 0
  145 IND = 1
  150 W(5) = WEST
      IP = 5
      IM = 5
C   CALCULATE W ON MID STREAMSHEET FROM MEAN TO HUB AND FROM MEAN TO TIP
      DO 160 I=1,4
      IP = IP+1
      WAS = W(IP-1)+(W(IP-1)*AA(IP-1)+BB(IP-1))*DELTA+CC(IP-1)/W(IP-1)
      WASS = W(IP-1)+(WAS*AA(IP)+BB(IP))*DELTA+CC(IP-1)/WAS
      W(IP) = (WAS+WASS)/2.
      IM = IM-1
      WAS = W(IM+1)-(W(IM+1)*AA(IM+1)+BB(IM+1))*DELTA-CC(IM)/W(IM+1)
      WASS = W(IM+1)-(WAS*AA(IM)+BB(IM))*DELTA-CC(IM)/WAS
  160 W(IM) = (WAS+WASS)/2.
C
C   CALCULATE RHO*W THROUGHOUT PASSAGE CROSS SECTION
C
      DO 180 HMT=1,3
      WBTB(5) = W(4*HMT-3)
      WFIN(2,HMT) = WBTB(5)
      WSQ = WBTB(5)**2
      TWLMR = 2.*OMEGA*LBDA(4*HMT-3)-(OMEGA*RAD(4*HMT-3))**2
      TTIP = 1.-(WSQ+TWLMR)/CPTIP(HMT)
      IF(TTIP.LT.0.) GO TO 190
      RHOW = RHOIP(HMT)*(1.-PLOSS(HMT))*TTIP**EXPON*WBTB(5)
      SUM(HMT) = RHOW
      DELTAB = NBB(HMT)/8.
```

```
      IP = 5
      IM = 5
C   CALCULATE W FROM BLADE TO BLADE (HUB, MEAN, OR TIP)
      DO 170 I=1,4
      IP = IP+1
      WAS = WBTB(IP-1)+(WBTB(IP-1)*AAB(IP-1,HMT)+BBB(HMT))*DELTAB
      WASS = WBTB(IP-1)+(WAS*AAB(IP,HMT)+BBB(HMT))*DELTAB
      WBTB(IP) = (WAS+WASS)/2.
      IM = IM-1
      WAS = WBTB(IM+1)-(WBTB(IM+1)*AAB(IM+1,HMT)+BBB(HMT))*DELTAB
      WASS = WBTB(IM+1)-(WAS*AAB(IM,HMT)+BBB(HMT))*DELTAB
      WBTB(IM) = (WAS+WASS)/2.
      IE = IP
  165 WSQ = WBTB(IE)**2
      TTIP = 1.-(WSQ+TWLMR)/CPTIP(HMT)
      IF(TTIP.LT.0.) GO TO 190
      RHOW = RHOIP(HMT)*(1.-PLOSS(HMT))*TTIP**EXPON*WBTB(IE)
      SUM(HMT) = SUM(HMT)+RHOW
      IF(I.EQ.4) SUM(HMT) = SUM(HMT)-RHOW/2.
      IF(IE.EQ.IM) GO TO 170
      IE = IM
      GO TO 165
  170 CONTINUE
      WFIN(1,HMT) = WBTB(1)
      WFIN(3,HMT) = WBTB(9)
  180 CONTINUE
      WTFLES = (SUM(1)*NBB(1)+4.*SUM(2)*NBB(2)+SUM(3)*NBB(3))/48.*
     1   NMER(9)
      IF(IND.GE.6.AND.ABS(WTFL-WTFLES).LE.WTFL/10.E5) GO TO 200
      CALL CONTIN(WEST,WTFLES,IND,JZ,WTFL,DELMAX)
      IF(IND.LT.10) GO TO 150
      IF(IND.NE.10) GO TO 195
      WRITE(6,2000) WTFLES
      GO TO 200
  190 WEST = WEST-DELMAX*3.
      NCOUNT = NCOUNT+1
      IF(NCOUNT.LT.50) GO TO 145
  195 WRITE(6,2010)
      GO TO 240
C   SOLUTION HAS BEEN OBTAINED - PRINT SOLUTION
  200 DO 210 HMT=1,3
      DO 210 BTB=1,3
  210 WWCR(BTB,HMT) = WFIN(BTB,HMT)/WCR(HMT)
      WRITE(6,2020)
      WRITE(6,2040)((WFIN(BTB,HMT),BTB=1,3),HMT=1,3)
      WRITE(6,2030)
      WRITE(6,2040)((WWCR(BTB,HMT),BTB=1,3),HMT=1,3)
C   CALCULATE MASS FLOW DISTRIBUTION AND STREAMLINE SPACING
      DO 220 I=1,3
      Y(I) = SUM(I)*NBB(I)/8.0
  220 X(I) = FLOAT(I-1)/2.
      CALL PABC(X,Y,A,B,C)
      SL100 = NMER(9)/100.
      YAVE = WTFLES/NMER(9)
      WRITE(6,2050)
      DO 230 I=1,11
      FRSLD = FLOAT(I-1)/10.
      WFFR = A*FRSLD**3/3.+B*FRSLD**2/2.+C*FRSLD
      WFFR = WFFR/YAVE
```

```
      RSLSP = YAVE/(A*FRSLD**2+B*FRSLD+C)
      B100 = RSLSP*SL100
  230 WRITE(6,2060) FRSLD,WFFR,RSLSP,B100
  240 WRITE(6,1000)
      WTFL = WTFLSV
      GO TO 10
 1000 FORMAT(1H1)
 1010 FORMAT(1X,80H
     1
 1020 FORMAT(16I5)
 1030 FORMAT(8F10.5)
 1040 FORMAT(1X,8G16.5)
 1100 FORMAT(25H   JX   JZ  KR1  KR2  NSP)
 1110 FORMAT(8X,3HGAM,14X,2HAR,11X,5HOMEGA,12X,4HWTFL)
 1120 FORMAT(1HL,11X,5HRHOIP,11X,5HPLOSS,12X,3HNBB,14X,2HCS,13X,3HCPR)
 1130 FORMAT(6H HUB  ,5G16.5/6H MEAN ,5G16.5/6H TIP  ,5G16.5)
 1140 FORMAT(1HL,9X,12HNMERID ARRAY)
 1150 FORMAT(10X,12HTEMPIP ARRAY)
 1160 FORMAT(10X,12HLAMBDA ARRAY)
 1170 FORMAT(10X,12HRADIUS ARRAY)
 1180 FORMAT(10X,10HCURV ARRAY)
 1190 FORMAT(10X,11HALPHA ARRAY)
 1200 FORMAT(10X,10HBETA ARRAY)
 1500 FORMAT(39HLNSP MUST BE GREATER THAN OR EQUAL TO 2)
 1510 FORMAT(30HLNSP CANNOT BE GREATER THAN 25)
 1520 FORMAT(26HL JX MUST BE EITHER 1 OR 2)
 1530 FORMAT(49HL WHEN JX = 2, CS AND CPR MUST HAVE THE SAME SIGN)
 2000 FORMAT(42HLTHE PASSAGE IS CHOKED WITH A MASS FLOW OF,G16.5)
 2010 FORMAT(46HLA SOLUTION CANNOT BE OBTAINED AT THIS STATION)
 2020 FORMAT(1HL,10X,25HVELOCITIES ACROSS PASSAGE)
 2030 FORMAT(1HL,10X,39HCRITICAL VELOCITY RATIOS ACROSS PASSAGE)
 2040 FORMAT(9X,7HSUCTION,11X,3HMID,13X,8HPRESSURE/6H HUB  ,3G16.5/
     1   6H MEAN ,3G16.5/6H TIP  ,3G16.5)
 2060 FORMAT(3X,5G25.5)
 2050 FORMAT(105HL          FRACTIONAL STREAMLINE          MASS FLOW
     1   RATIO OF ACTUAL TO AVERAGE   NORMAL THICKNESS B /15X,
     2   8HDISTANCE,17X,8HFRACTION,13X,18HSTREAMLINE SPACING,6X,
     3   21H(FOR 100 STREAMLINES) )
      END
```

Dictionary of Variables in Main Program

A	coefficient of X^2 calculated by subroutine PABC
AA	array of coefficients a_n from eq. (A2)
AAA	array of dummy values not used in calculation
AAB	array of coefficients a_b from eq. (A4)
ALPH	array of values of α at eight equal intervals from hub to tip
ALPHA	input array
AR	input

B	coefficient of X calculated by subroutine PABC
B100	normal streamline spacing for 100 streamlines
BB	array of coefficients b_n from eq. (A2)
BBB	array of coefficients b_b from eq. (A4)
BETA	input array
BTA	array of values of β at eight equal intervals from hub to tip
BTB	integer subscript used to indicate blade-to-blade position
C	constant coefficient calculated by subroutine PABC
CAL	$\cos \alpha$
CBETA	$\cos \beta$
CC	array of coefficients c_n from eq. (A2)
CP	specific heat at constant pressure, c_p
CPR	input array
CPTIP	array of values of $2c_pT_i'$
CRV	array of values of $1/r_c$ at eight equal intervals from hub to tip
CS	input array
CURV	input array
DELMAX	maximum change in WEST permitted in subroutine CONTIN
DELTA	one-eighth the distance from hub to tip
DELTAB	one-eighth the blade-to-blade streamline normal distance
EXPON	$1/(\gamma-1)$
FRSLD	fractional distance from hub to tip
GAM	input
HMT	integer subscript used to indicate hub, mean, or tip position
I	DO loop index
IE	integer subscript
IM	integer subscript
IND	integer switch - used for logical control in subroutine CONTIN
IP	integer subscript
JX	input

JZ	input
KR1	input
KR2	input
LAMBDA	input array
LBDA	array of values of λ at eight equal intervals from hub to tip
NBB	input array
NCOUNT	number of times that a solution could not be obtained because of excessive velocities
NINE	9
NMER	array of distances from hub
NMERID	input array
NSP	input
OMEGA	input
PLOSS	input array
RAD	array of values of r at eight equal intervals from hub to tip
RADIUS	input array
RC	$(r_c)_{b-b}$
RHOIP	input array
RHOW	ρW
RSLSP	ratio of actual to average streamline spacing
SAL	$\sin \alpha$
SBETA	$\sin \beta$
SL100	one-hundredth the distance from hub to tip
SRW	integer code variable that will cause SPLINT to write out data useful for debugging (If SRW = 16, SPLINT will write input and output data. SRW is not an input item.)
SUM	array of integrated values of ρW at hub, mean, and tip
TEMP	temporary storage
TEMPIP	input array
TGROG	$2\gamma R/(\gamma+1)$

TIP	array of values of T_i' at eight equal intervals from hub to tip
TTIP	T/T_i'
TWLMR	$2\omega\lambda - (\omega r)^2$
W	array of values of W_{mid} at eight equal intervals from hub to tip
WAS	initial estimate for W at next point
WASS	secondary estimate for W at next point
WBTB	array of values of W at eight equal intervals from blade to blade
WCR	array of values of W_{cr} at hub, mean, and tip
WEST	estimated value for $W_{mid, mean}$
WFFR	weight flow fraction between some point and hub
WFIN	array of final calculated velocities to be printed as output
WSQ	W^2
WTFL	input
WTFLES	calculated mass flow if $W_{mid, mean}$ = WEST
WTFLSV	used to save input value of WTFL when choking mass flow is to be calculated
WWCR	array of values of W/W_{cr} for output
X	array of fractional distances from hub
Y	array of values of mass flow per unit radial distance
YAVE	average value of mass flow per unit radial distance

Subroutine CONTIN

Subroutine CONTIN finds the value of $W_{mid, mean}$ which will give either the desired value of mass flow WTFL or the choking mass flow, depending on the input value of JZ. The actual calculation of mass flow corresponding to a given value of $W_{mid, mean}$ is done by the main program; CONTIN determines the value of $W_{mid, mean}$ which should be used. If WTFL is larger than the choking mass flow, the choking mass flow will be automatically calculated, and a message to this effect will be printed.

The input arguments for CONTIN are as follows:

XEST	estimated value of $W_{mid, mean}$
YCALC	calculated mass flow for $W_{mid, mean}$ = XEST

IND switch controlling action to be taken by CONTIN

JZ program input, see fig. 2

YGIV desired mass flow, WTFL, when JZ = 1 or 2

XDEL maximum change in XEST in one iteration

The output arguments for CONTIN are as follows:

XEST estimated value of $W_{mid,mean}$ for next iteration

IND switch indicating whether a solution can or cannot be obtained by CONTIN

The internal variables in CONTIN are as follows:

APA coefficient of X^2 calculated by subroutine PABC

BPB coefficient of X calculated by subroutine PABC

CPC constant coefficient calculated by subroutine PABC

DISCR discriminant, $\sqrt{b^2 - 4ac}$, of parabola obtained by subroutine PABC

NCALL number of times CONTIN has been called for a given case

X array of three values of XEST from previous calls

Y array of three values of YCALC from previous calls

```
      SUBROUTINE CONTIN(XEST,YCALC,IND,JZ,YGIV,XDEL)
      DIMENSION X(3),Y(3)
      NCALL = NCALL+1
      IF(IND.NE.1.AND.NCALL.GT.50) GO TO 160
      GO TO (10,30,40,50,60,80,130),IND
C   FIRST CALL
   10 NCALL = 1
      IF(YCALC.GT.YGIV.AND.JZ.EQ.1) GO TO 20
      IND = 2
      Y(1) = YCALC
      X(1) = XEST
      XEST = XEST+XDEL
      RETURN
   20 IND = 3
      Y(3) = YCALC
      X(3) = XEST
      XEST = XEST-XDEL
      RETURN
C   SECOND CALL
   30 IND = 4
      Y(2) = YCALC
      X(2) = XEST
      XEST = XEST+XDEL
      RETURN
   40 IND = 5
      Y(2) = YCALC
```

```
      X(2) = XEST
      XEST = XEST-XDEL
      RETURN
C   THIRD OR LATER CALL - FIND SUBSONIC OR SUPERSONIC SOLUTION
   50 Y(3) = YCALC
      X(3) = XEST
      GO TO 70
   60 Y(1) = YCALC
      X(1) = XEST
   70 IF(YGIV.LT.AMIN1(Y(1),Y(2),Y(3))) GO TO (90,95),JZ
   75 IND = 6
      CALL PABC(X,Y,APA,BPB,CPC)
      DISCR = BPB**2-4.*APA*(CPC-YGIV)
      IF(DISCR.LT.0.) GO TO 110
      XEST = -BPB-SIGN(SQRT(DISCR),APA)
      IF(JZ.EQ.2.AND.APA.LT.0.) XEST = -BPB-SQRT(DISCR)
      XEST = XEST/2./APA
      IF(XEST.GT.X(3)+XDEL) GO TO 95
      IF(XEST.LT.X(1)-XDEL) GO TO 90
      RETURN
C   FOURTH OR LATER CALL - (NOT CHOKED)
   80 IF(XEST.GT.X(3)) GO TO 50
      IF(XEST.LT.X(1)) GO TO 60
      Y(2) = YCALC
      X(2) = XEST
      GO TO 70
C   THIRD OR LATER CALL - SOLUTION EXISTS,
C         BUT RIGHT OR LEFT SHIFT REQUIRED
   90 IND = 5
C   LEFT SHIFT
      Y(3) = Y(2)
      X(3) = X(2)
      Y(2) = Y(1)
      X(2) = X(1)
      XEST = X(1)-XDEL
      RETURN
   95 IND = 4
C   RIGHT SHIFT
      Y(1) = Y(2)
      X(1) = X(2)
      Y(2) = Y(3)
      X(2) = X(3)
      XEST = X(3)+XDEL
      RETURN
C   THIRD OR LATER CALL - APPEARS TO BE CHOKED
  110 XEST = -BPB/2./APA
      IND = 7
      IF(X(1).LE.XEST.AND.XEST.LE.X(3)) RETURN
      IF(XEST.LT.X(1)) GO TO 90
      GO TO 95
C   FOURTH OR LATER CALL - PROBABLY CHOKED
  130 IF(YCALC.GE.YGIV) GO TO 80
      IND = 10
      RETURN
C   NO SOLUTION FOUND IN 50 ITERATIONS
  160 IND = 11
      RETURN
      END
```

Subroutine SPLINT

This subroutine calculates interpolated values using a cubic spline curve fitted through the given data. The input variables for SPLINT are as follows:

X	array of spline point ordinates
Y	array of function values at spline points
N	number of X and Y values given
Z	array of ordinates at which interpolated function values are desired
MAX	number of Z values given

The output variable for SPLINT is as follows:

YINT	array of interpolated function values

If SRW = 16 in COMMON, or if some element of the z-array falls outside of the interval for the x-array, input and output data for SPLINT are printed. This is useful in debugging.

```
      SUBROUTINE SPLINT (X,Y,N,Z,MAX,YINT,DYDX)
C
C  SPLINT CALCULATES INTERPOLATED POINTS AND DERIVATIVES
C  FOR A SPLINE CURVE
C   END CONDITION - SECOND DERIVATIVE AT EITHER END POINT IS ONE-HALF
C   THAT AT THE ADJACENT POINT
C
      COMMON SRW
      DIMENSION X(N),Y(N),Z(MAX),YINT(MAX),DYDX(MAX)
      DIMENSION G(100),SB(100),EM(100)
      INTEGER SRW
      IF(MAX.LE.0) RETURN
      III = SRW
      SB(1) = -.5
      G(1) = 0
      NO=N-1
      IF(NO.LT.2) GO TO 20
      DO 10 I=2,NO
      A = (X(I)-X(I-1))/6.
      C = (X(I+1)-X(I))/6.
      W = 2.*(A+C)-A*SB(I-1)
      SB(I) = C/W
      F = (Y(I+1)-Y(I))/(X(I+1)-X(I))-(Y(I)-Y(I-1))/(X(I)-X(I-1))
   10 G(I) = (F-A*G(I-1))/W
   20 EM(N) = G(N-1)/(2.+SB(N-1))
      DO 30 I=2,N
      K = N+1-I
   30 EM(K) = G(K)-SB(K)*EM(K+1)
      DO 140 I=1,MAX
```

```
      K=2
      IF(Z(I)-X(1)) 70,60,90
   60 YINT(I)=Y(1)
      SK = X(K)-X(K-1)
      GO TO 130
   70 IF(Z(I).GE.(1.1*X(1)-.1*X(2))) GO TO 120
      WRITE (6,1000) Z(I)
      SRW = 16
      GO TO 120
   80 K=N
      IF(Z(I).LE.(1.1*X(N)-.1*X(N-1))) GO TO 120
      WRITE (6,1000) Z(I)
      SRW = 16
      GO TO 120
   90 IF(Z(I)-X(K)) 120,100,110
  100 YINT(I)=Y(K)
      SK = X(K)-X(K-1)
      GO TO 130
  110 K=K+1
      IF(K-N) 90,90,80
  120 CONTINUE
      SK = X(K)-X(K-1)
      YINT(I) = EM(K-1)*(X(K)-Z(I))**3/6./SK   +EM(K *(Z(I)-X(K-1))**3/6.
     1 /SK+(Y(K)/SK   -EM(K)*SK   /6.)*(Z(I)-X(K-1))+(Y(K-1)/SK   -EM(K-1)
     2 *SK/6.)*(X(K)-Z(I))
  130 DYDX(I)=-EM(K-1)*(X(K)-Z(I))**2/2.0/SK   +EM(K)*(X(K-1)-Z(I))**2/2.
     1   /SK+(Y(K)-Y(K-1))/SK   -(EM(K)-EM(K-1))*SK/6.
  140 CONTINUE
      MXA = MAX0(N,MAX)
      IF(SRW.EQ.16) WRITE(6,1010) N,MAX,(X(I),Y(I),Z(I),YINT(I),DYDX(I),
     1   I=1,MXA)
      SRW = III
      RETURN
 1000 FORMAT (54H SPLINT USED FOR EXTRAPOLATION.  EXTRAPOLATED VALUE = ,
     1G14.6)
 1010 FORMAT (2X,21HNO. OF POINTS GIVEN =,I3,30H, NO. OF INTERPOLATED PO
     1INTS =,I3/10X,1HX,19X,1HY,16X,11HX-INTERPOL.,9X,11HY-INTERPOL.,
     28X,14HDYDX-INTERPOL./(5G20.8))
      END
```

Subroutine PABC

Subroutine PABC calculates coefficients A, B, and C of the parabola $y = Ax^2 + Bx + C$ passing through three given X, Y points.

```
      SUBROUTINE PABC(X,Y,A,B,C)
C     SUBROUTINE PABC CALCULATES COEFFICIENTS A, B, C OF THE PARABOLA
C     Y = A*X**2 + B*X + C, PASSING THROUGH THREE GIVEN X,Y POINTS
      DIMENSION X(3),Y(3)
      C1 = X(3)-X(1)
      C2 = (Y(2)-Y(1))/(X(2)-X(1))
      A = (C1*C2-Y(3)+Y(1))/C1/(X(2)-X(3))
      B = C2-(X(1)+X(2))*A
      C = Y(1)-X(1)*B-X(1)**2*A
      RETURN
      END
```

Lewis Research Center,

National Aeronautics and Space Administration,

Cleveland, Ohio, October 27, 1970,

720-03.

APPENDIX A

ANALYTICAL EQUATIONS AND SOLUTION PROCEDURE

Two ordinary differential equations are utilized to determine the velocity gradients in both the blade-to-blade and the hub-to-tip directions. These equations are both derived in appendix B from a general velocity gradient equation given in reference 5. The hub-to-tip velocity gradient equation is

$$\frac{dW}{dn} = a_n W + b_n + \frac{c_n}{W} \tag{A1}$$

where

$$\left.\begin{aligned} a_n &= \frac{\cos^2 \beta}{r_c} - \frac{\cos \alpha \sin^2 \beta}{r} \\ b_n &= -2\omega \cos \alpha \sin \beta \\ c_n &= c_p \frac{dT_i'}{dn} - \omega \frac{d\lambda}{dn} \end{aligned}\right\} \tag{A2}$$

An analytical solution of equation (A1) cannot be obtained, since β is not given as an analytical function. However, a numerical solution can be readily obtained. The numerical method used is the Heun method (ref. 7, p. 236).

With an estimated value for $W_{mid,mean}$ equation (A1) can be integrated from mean to hub and from mean to tip by the Heun method. This gives the entire midchannel velocity W_{mid} distribution from hub to tip at the given station for the estimated value of $W_{mid,mean}$. The first estimate for $W_{mid,mean}$ depends on the input value of JZ. If JZ = 1 or 2, $W_{mid,mean}$ is estimated based on one-dimensional flow using the channel cross section area, input mass flow, and inlet stagnation density ρ_i'. If JZ = 3, $W_{mid,mean}$ is estimated to be W_{cr} at the mean between hub and tip.

The blade-to-blade velocity gradient equation (derived in appendix B) is

$$\frac{dW}{dn_{b-b}} = a_b W + b_b \tag{A3}$$

where

$$\left.\begin{aligned} a_b &= \frac{\sin \alpha \sin \beta}{r} + \frac{1}{(r_c)_{b-b}} \\ b_b &= 2\omega \sin \alpha \end{aligned}\right\} \tag{A4}$$

Equation (A3) could be solved analytically if $(r_c)_{b-b}$ or $1/(r_c)_{b-b}$ is assumed to vary linearly along the blade-to-blade orthogonal. However, this solution is very complex and involves an integral which must be evaluated numerically. Therefore, a numerical solution of equation (A3) requires much less computation and will give at least as good accruacy as the analytical solution. The numerical method used in the program is the Heun method (ref. 7, p. 236). The solution depends on whether JX = 1 ($1/(r_c)_{b-b}$ is assumed to vary linearly from blade to blade) or JX = 2 ($(r_c)_{b-b}$ is assumed to vary linearly from blade to blade). The value of W_{mid} to be used is obtained from the numerical solution to equation (A1). Then equation (A3) is integrated numerically from the midchannel to each blade surface separately at hub, mean, and tip.

With the velocities obtained, the weight flow past the channel cross section can be computed from

$$w = \iint \rho W \, dn_{b-b} \, dn \tag{A5}$$

integrating from blade to blade and from hub to tip.

The density ρ is calculated using the equation for isentropic flow, with a correction for loss in the relative stagnation pressure (given as PLOSS in input). This equation for density is

$$\rho = \rho_i' \left\{1 - \frac{W^2 + 2\omega\lambda - (\omega r)^2}{2c_p T_i'}\right\}^{1/\gamma-1} \left(1 - \frac{P''_{loss}}{P''_{isen}}\right) \tag{A6}$$

(This equation is derived in appendix B.)

The inner integrals are computed at hub, mean, and tip using trapezoidal integration over eight equal intervals from suction to pressure surface.

After the inner integral is computed in equation (A5) at hub, mean, and tip, the outer integral is approximated by Simpson's rule. This results in a value for the mass flow w based on the estimated value for $W_{mid, mean}$. This value of w can be compared with the input value WTFL. The procedure for calculating the mass flow for a given value of $W_{mid,mean}$ is outlined graphically in figure 8.

If JZ = 1 or 2, further calculations are made to determine a value of $W_{mid,mean}$ that will give a mass flow w close to the input value WTFL. If WTFL is less than the choking mass flow, there are two values of $W_{mid,mean}$ which will give w = WTFL. If JZ = 1, the smaller, or subsonic, solution will be found. If JZ = 2, the larger, or supersonic, solution will be found. When $|w - \mathrm{WTFL}| \leq 10^{-6}*\mathrm{WTFL}$ the calculations are stopped, and the solution is printed. If WTFL is larger than the choking mass flow, no solution exists. In this case, the choking mass flow is calculated, and this solution is printed with the message "THE PASSAGE IS CHOKED WITH A MASS FLOW OF XXXX".

If JZ = 3, further calculations are made to determine the choking mass flow, regardless of the input value of WTFL. The second and third estimates for $W_{mid,mean}$ are each increased from the previous estimate by 5 percent of W_{cr} at the mean (the first estimate was $W_{mid,mean} = W_{cr}$). New estimates for $W_{mid,mean}$ are calculated to obtain the maximum weight flow. These new estimates are based on three previous approximations for $W_{mid,mean}$ with the corresponding mass flows. When the new estimate for $W_{mid,mean}$ is estimated to be within the range of the three values used for the last quadratic fit, the corresponding mass flow is calculated and this value is accepted as the choking mass flow.

The final output for a case gives information concerning the mass flow distribution along the meridional normal. A quadratic fit is used from hub to tip (corresponding to Simpson's rule in integrating eq. (A5) from hub to tip). The quadratic fit determines the mass flow fraction at 10 equally spaced intervals from hub to tip, as well as the streamline spacing.

APPENDIX B

DERIVATION OF EQUATIONS

A general velocity gradient equation can be derived from the force equation. If it is assumed that entropy is constant along an arbitrary space curve, with the distance along the curve denoted by q, the following equation can be obtained:

$$\frac{dW}{dq} = a\frac{dr}{dq} + b\frac{dz}{dq} + c\frac{d\theta}{dq} + \frac{1}{W}\left(\frac{dh_i'}{dq} - \omega\frac{d\lambda}{dq}\right) \tag{B1}$$

where

$$\left.\begin{aligned} a &= \frac{W\cos\alpha\cos^2\beta}{r_c} - \frac{W\sin^2\beta}{r} + \sin\alpha\cos\beta\frac{dW_m}{dm} - 2\omega\sin\beta \\ b &= -\frac{W\sin\alpha\cos^2\beta}{r_c} + \cos\alpha\cos\beta\frac{dW_m}{dm} \\ c &= W\sin\alpha\sin\beta\cos\beta + r\cos\beta\left(\frac{dW_\theta}{dm} + 2\omega\sin\alpha\right) \end{aligned}\right\} \tag{B2}$$

Equations (B1) and (B2) are the same as equations (B13) and (B14) of reference 5.

To derive equations (A1) and (A2), it is assumed that q is equal to n, the distance along an orthogonal to the streamlines in a meridional plane. Then $q = n$, $dr/dq = dr/dn = \cos\alpha$, $dz/dq = dz/dn = -\sin\alpha$, and $d\theta/dq = 0$. Since it is assumed that c_p is constant, equations (B1) and (B2) reduce to equations (A1) and (A2).

To derive equations (A3) and (A4), it is assumed that q is equal to n_{b-b}, the distance along a blade-to-blade streamline orthogonal. Along the blade to blade normal, then, r and z are both functions of n_{b-b}. Alternately, r and z can be considered to be functions of m, where m is a function of n_{b-b}. Then we can obtain the following equations:

$$\frac{dr}{dn_{b-b}} = \frac{dr}{dm}\frac{dm}{dn_{b-b}} = -\sin\alpha\sin\beta$$

$$\frac{dz}{dn_{b-b}} = \frac{dz}{dm}\frac{dm}{dn_{b-b}} = -\cos\alpha\ \sin\beta$$

Also,

$$r\frac{d\theta}{dn_{b-b}} = \cos\beta$$

In the blade-to-blade direction uniform upstream conditions are assumed; therefore,

$$\frac{dh_i'}{dn_{b-b}} = \frac{d\lambda}{dn_{b-b}} = 0$$

With these substitutions and simplification, equations (B1) and (B2) become

$$\frac{dW}{dn_{b-b}} = \frac{W\sin\alpha\sin\beta}{r} + 2\omega\sin\alpha - \sin\beta\cos\beta\frac{dW_m}{dm} + \cos^2\beta\frac{dW_\theta}{dm} \tag{B3}$$

Further simplification of equation (B3) is possible by evaluating the derivatives dW_m/dm and dW_θ/dm in terms of angles and blade-to-blade streamline curvatures. We make use of the fact that $W_\theta = W\sin\beta$ and $W_m = W\cos\beta$ (see fig. 3). Also $d\beta/dm = (d\beta/ds)/\cos\beta$ and $d\beta/ds$ is the blade-to-blade streamline curvature $1/(r_c)_{b-b}$. This gives

$$\left.\begin{aligned}\frac{dW_\theta}{dm} &= \sin\beta\frac{dW}{dm} + \frac{W}{(r_c)_{b-b}}\\ \frac{dW_m}{dm} &= \cos\beta\frac{dW}{dm} - \frac{W\tan\beta}{(r_c)_{b-b}}\end{aligned}\right\} \tag{B4}$$

When equations (B4) are substituted in equation (B3) we obtain

$$\frac{dW}{dn_{b-b}} = \frac{W\sin\alpha\sin\beta}{r} + \frac{W}{(r_c)_{b-b}} + 2\omega\sin\alpha \tag{B5}$$

Equations (A3) and (A4) are obtained directly from this equation.

The density corresponding to a given velocity may be calculated using the isentropic relation with a correction for loss in total pressure. The equation for this is equation (A6), which is derived here. We start with

$$\rho = \rho''\left(\frac{\rho}{\rho''}\right)$$

and use each of the following relations:

$$\frac{\rho}{\rho''} = \left(\frac{T}{T''}\right)^{1/(\gamma-1)}$$

$$\rho'' = \frac{p''}{RT''}$$

$$\left(\frac{T}{T''}\right)^{1/(\gamma-1)} = \left(\frac{T}{T_i'}\right)^{1/(\gamma-1)} \left(\frac{T_i'}{T''}\right)^{1/(\gamma-1)}$$

$$\left(\frac{T_i'}{T''}\right)^{1/(\gamma-1)} = \frac{\rho_i'}{\rho_{isen}''}$$

$$\rho_{isen}''RT'' = p_{isen}''$$

$$p'' = p_{isen}'' - p_{loss}''$$

Using these relations results in

$$\rho = \rho_i'\left(\frac{T}{T_i'}\right)^{1/(\gamma-1)} \left(1 - \frac{p_{loss}''}{p_{isen}''}\right) \tag{B6}$$

From equation (3) of reference 5

$$\frac{T}{T_i'} = 1 - \frac{W^2 + 2\omega\lambda - (\omega r)^2}{2c_pT_i'}$$

When this is substituted in equation (B6), equation (A6) is obtained.

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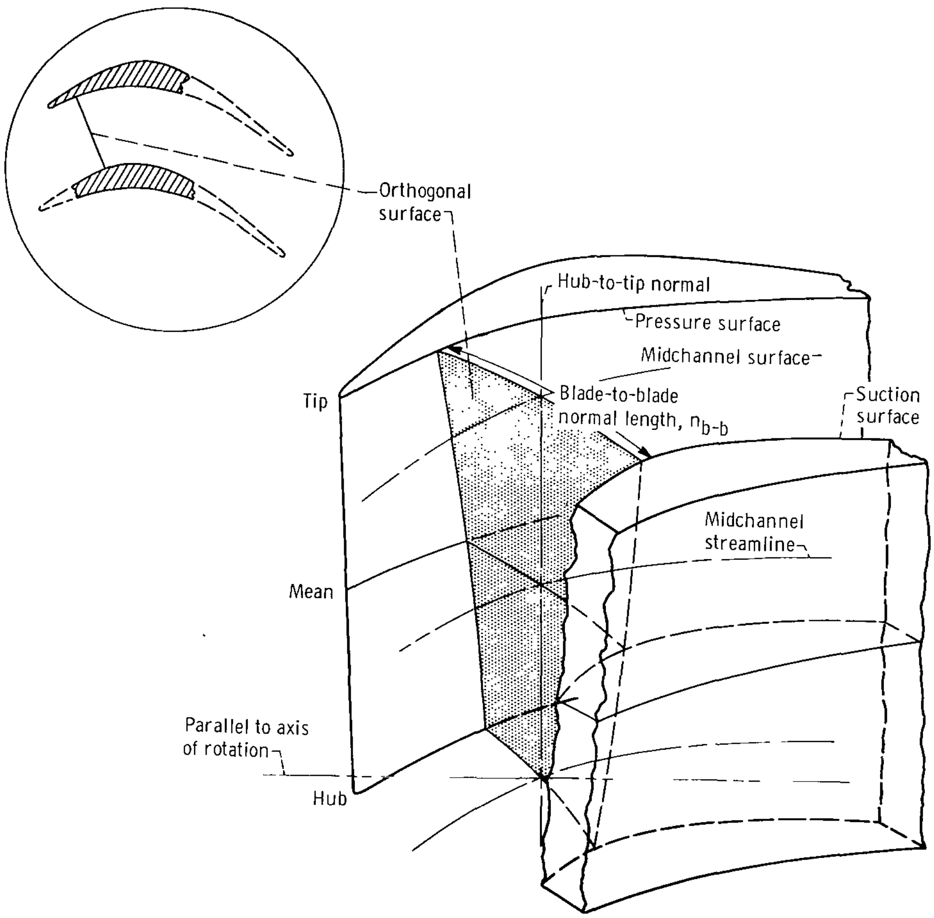


Figure 1. - Pair of typical turbine blades with three-dimensional orthogonal surface across flow passage.

1 5	6 10	11 15	16 20	21 25	26 30	31 40	41 50	51 80

TITLE (first case only)

Sample Problem

LABEL (every case)

Negative Reaction Turbine

JX	JZ	KR1	KR2	NSP
1	1	1	3	3

JX = { 1 - Linear curvature; 2 - Linear radius of curvature }

JZ = { 1 - Subsonic solution; 2 - Supersonic solution; 3 - Choked flow solution }

GAM	AR	OMEGA	WTFL
1.4	1716.57	488.	.01839

KR1 = { 0 - Omit this card; 1 - Supply this card }

Hub

RHOIP	PLOSS	NBB	CS	CPR
.00230557	.0482	.04208	-15.71	-8.82

Mean

RHOIP	PLOSS	NBB	CS	CPR
.00230557	.0482	.0545	-14.16	-8.42

Tip

RHOIP	PLOSS	NBB	CS	CPR
.00230557	.0482	.0715	-12.0	-6.30

KR2 = { 0 - Omit these cards; 1 - Supply one card; 3 - Supply three cards }

Use NAMELIST input for NMERID, TEMPIP, LAMBDA, RADIUS, CURV, ALPHA and BETA

$NAM1 *NMERID = 0., .1875, .375, TEMPIP = 3 * 518.67, LAMBDA = 3 * 1024., RADIUS = .875, 1.0625, 1.25,*

BETA = -5., -16.3, -28.5 $

Figure 2. - CHANEL input form.

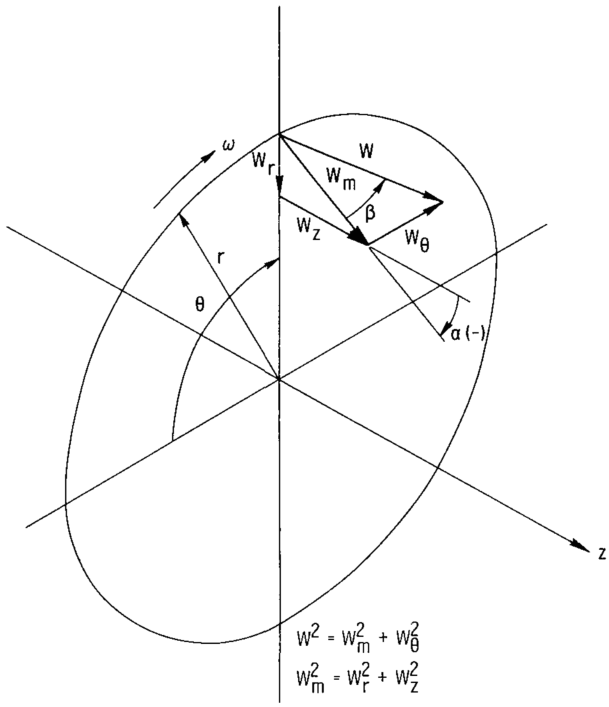


Figure 3. - Cylindrical coordinate system and velocity components.

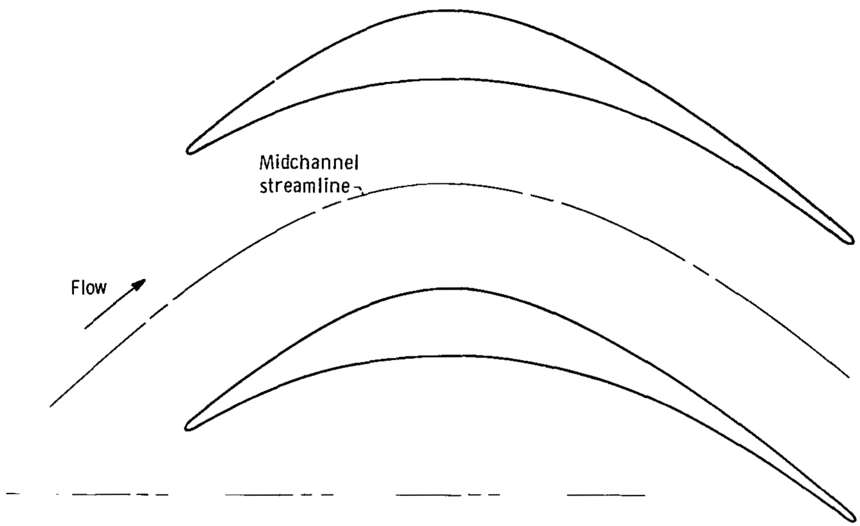


Figure 4. - Blade channel - mean section.

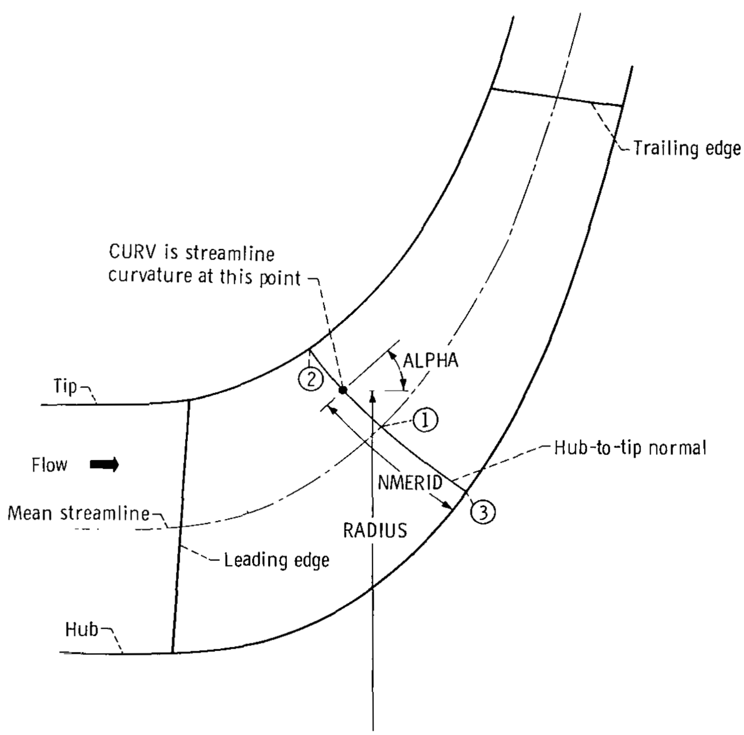


Figure 5. - Meridional section of blade. Points 1, 2, and 3 are used to locate blade-to-blade normals in figure 6.

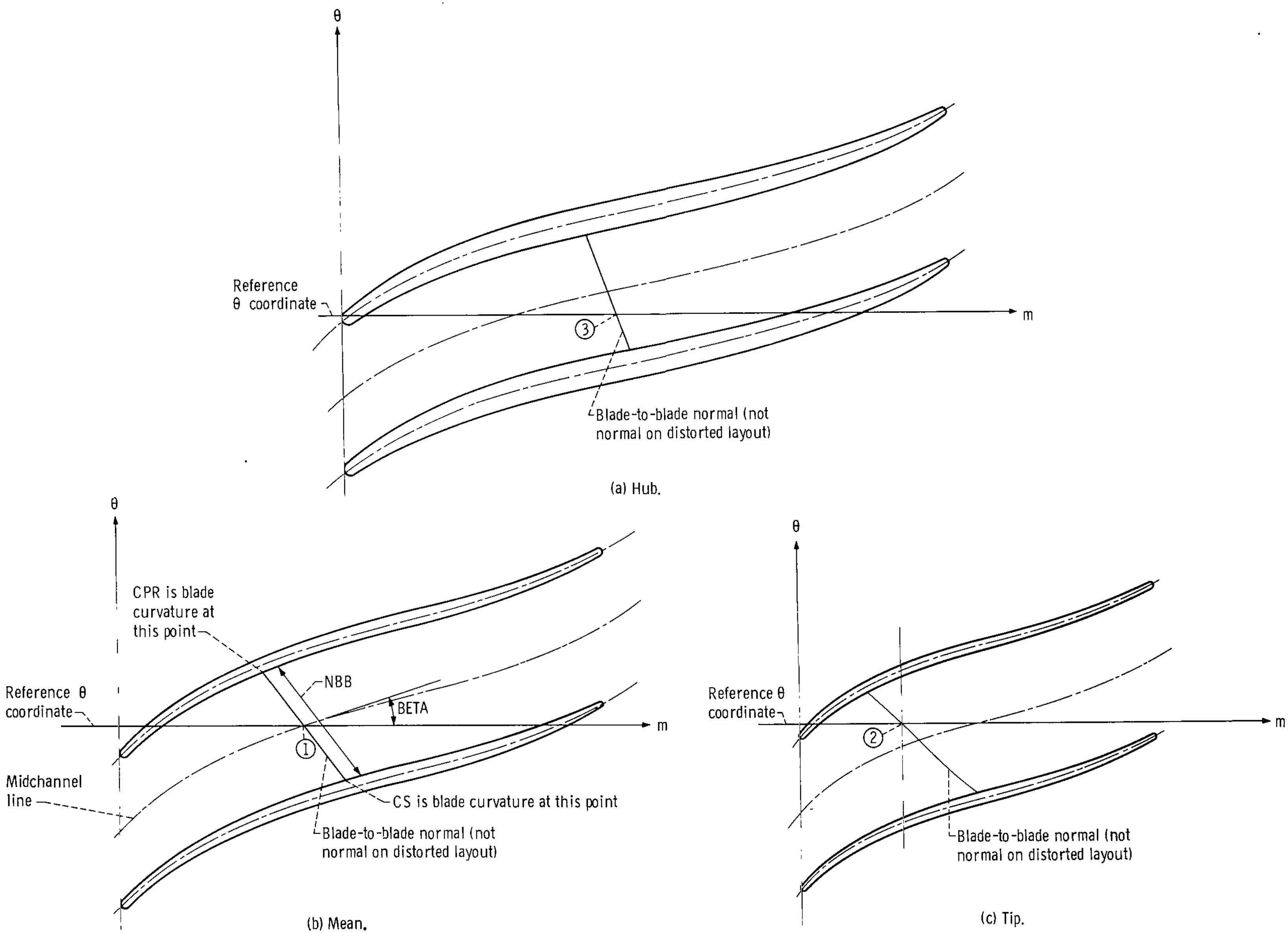
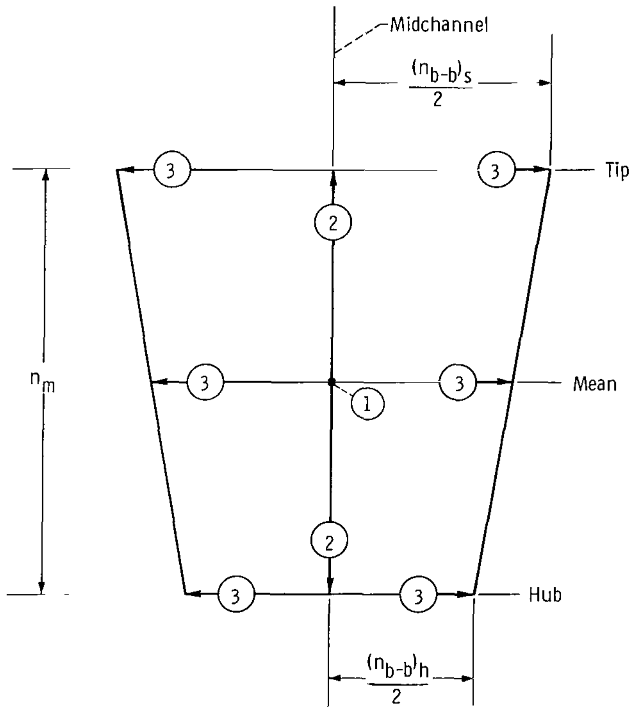


Figure 6. - Blade-to-blade channel layouts. Angular stacking points, ①, ②, and ③ are located from figure 5. Input items BETA, NBB, CS, and CPR must be given as true angle, length, and curvatures, not as quantities measured in m, θ plane. Correct quantities can be calculated from measurements in m, θ plane.

```
       SAMPLE PROBLEM
     NEGATIVE REACTION TURBINE
  JX    JZ   KR1   KR2   NSP
   1     1     1     3     3
     GAM              AR              OMEGA              WTFL
   1.40000         1716.57           488.000          0.18390E-01


           RHOIP           PLOSS           NBB            CS            CPR
HUB      0.23056E-02     0.48200E-01     0.42080E-01    -15.7100       -8.82000
MEAN     0.23056E-02     0.48200E-01     0.54500E-01    -14.1600       -8.42000
TIP      0.23056E-02     0.48200E-01     0.71500E-01    -12.0000       -6.30000


        NMERID ARRAY
   0               0.18750          0.37500
        TEMPIP ARRAY
   518.670         518.670          518.670
        LAMBDA ARRAY
   1024.00         1024.00          1024.00
        RADIUS ARRAY
   0.87500         1.06250          1.25000
        CURV ARRAY
   0               0                0
        ALPHA ARRAY
   0               0                0
        BETA ARRAY
  -5.00000        -16.3000         -28.5000


         VELOCITIES ACROSS PASSAGE
       SUCTION           MID               PRESSURE
HUB      889.531         662.924           531.104
MEAN     978.047         691.750           528.942
TIP      1085.45         744.150           564.698


         CRITICAL VELOCITY RATIOS ACROSS PASSAGE
       SUCTION           MID               PRESSURE
HUB      0.93633         0.69780           0.55904
MEAN     1.02137         0.72239           0.55237
TIP      1.12304         0.76992           0.58426


        FRACTIONAL STREAMLINE          MASS FLOW        RATIO OF ACTUAL TO AVERAGE    NORMAL THICKNESS B
              DISTANCE                 FRACTION            STREAMLINE SPACING        (FOR 100 STREAMLINES)
              0                         0                       1.40942                  0.52853E-02
              0.10000                   0.72987E-01             1.33120                  0.49920E-02
              0.20000                   0.15043                 1.25205                  0.46952E-02
              0.30000                   0.23292                 1.17376                  0.44016E-02
              0.40000                   0.32102                 1.09766                  0.41162E-02
              0.50000                   0.41532                 1.02472                  0.38427E-02
              0.60000                   0.51639                 0.95555                  0.35833E-02
              0.70000                   0.62481                 0.89051                  0.33394E-02
              0.80000                   0.74117                 0.82976                  0.31116E-02
              0.90000                   0.86604                 0.77329                  0.28998E-02
              1.00000                   1.00000                 0.72099                  0.27037E-02
```

Figure 7. - Output listing.



(1) Estimate $W_{mid, mean}$.

(2) Calculate W_{mid} from mean to hub and mean to tip from eq. (A1).

(3) Calculate W from midchannel to suction and pressure blade surfaces at hub, mean, and tip using eq. (A3).

(4) Calculate total mass flow for channel cross section using eq. (A5).

(5) Repeat steps (1) to (4) with new estimates for $W_{mid, mean}$ until either desired mass flow or choking mass flow is obtained.

Figure 8. - Calculation procedure chart.

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