Science and the Weighing of Evidence

by

P. A. Sturrock

March 1971

SUIPR Report No. 413

Air Force Office of Scientific Research
Contract F44620-69-C-0008
NASA Grant NGL-05-020-272
Office of Naval Research
Contract N00014-67-A-0112-0006

INSTITUTE FOR PLASMA RESEARCH
STANFORD UNIVERSITY, STANFORD, CALIFORNIA

SCIENCE AND THE WEIGHING OF EVIDENCE

by

P. A. Sturrock

Air Force Office of Scientific Research
Contract F44620-69-C-0008
NASA Grant NGL-05-020-272
Office of Naval Research
Contract N00014-67-A-0112-0006

SUIPR Report No. 413

March 1971

Institute for Plasma Research
Stanford University
Stanford, California

ABSTRACT

Progress in science depends on the appraisal of hypotheses by reference to observational data. This procedure is formalized in such a way that judgements made by theorists and judgements made by observers may be combined to yield probabilities for the considered hypotheses.

1. INTRODUCTION

Sciences are often divided into two parts: observational (or experimental) and theoretical. This article is concerned with the interplay between the two. More specifically, the aim is to set up a bookkeeping procedure for organizing the judgements involved in comparing a scientific theory with scientific data for the purpose of appraising one or more theories. When there is reasonable doubt as to which theory is correct, due either to uncertainties in the theories or to uncertainties in the data, the nature of the problem is much the same, whether one is dealing with an "exact" science, an "inexact" science, or a detective story.

The most convenient outcome of the bookkeeping would be the assigning of a number to each theory. Since we shall be dealing with uncertain knowledge, a convenient and appropriate calculus is that of probability theory. An example of the kind of result we seek is given in a monograph on quasars by Kahn and Palmer[1]. Table 3, on page 111 of that monograph, gives the "estimated probability of correctness" of six hypotheses concerning quasars. The authors do not indicate how these estimates were arrived at. Furthermore, they do not consider the case (which is logically possible) that none of their specified hypotheses is correct.

In order to arrive at a bookkeeping procedure for judging scientific hypotheses, it will be necessary to propose a "model" for the scientific procedure of evaluating a theoretical hypothesis in terms of observational data. Such a model will be proposed in Section 3. However, before taking up this topic, a short statement will be interjected (in Section 2) on the interpretation of probability which appears to be appropriate to the problem in hand.

2. PROBABILITY, INDUCTION AND BAYES' THEOREM

Our purpose will be to make a judgement about a theory which is admittedly uncertain and incomplete by comparing it with data which are uncertain and incomplete. In developing a science, such an evaluation must be made not once, but progressively, as the data come in and as theories develop. The following remark of Jeffreys[2] is relevant: "Either we can learn from experience or we cannot. The ability to learn from experience demands the concept of probability in relation to varying data, and the recognition of the meanings of more probable than and less probable than."

In science, as in anything else, we can make judgements only on the basis of available information. We therefore introduce the notation $(A|B)$ to denote the probability that proposition A is true on the basis of the knowledge that proposition B is true. As is usual, the measure of probability extends over the range 0 to 1: $(A|B) = 0$ if A is impossible given B, and $(A|B) = 1$ if A is certain given B.

Notation AB stands for the "product" of the two proposition A and B. Then AB is true if and only if both A and B are true. The "product rule" of probability theory[3] is

$$(AB|C) = (A|BC)(B|C) \ . \tag{2.1}$$

Since AB = BA, we see also that

$$(AB|C) = (B|AC)(A|C) \ . \tag{2.2}$$

From these two equations, we derive Bayes' theorem:

$$(A|BC) = \frac{(B|AC)}{(B|C)} (A|C) \quad . \tag{2.3}$$

According to Jeffreys,[4] "This theorem is to the theory of probability what Pythagoras's theorem is to geometry." It is the basic algorithm for up-dating judgements on the basis of new information.

Suppose, for instance, that C is an existing body of scientific information and A is a certain hypothesis. A new statement B is made as a result of a new observation. Then the probability of the hypothesis A should be revised in accordance with Bayes' theorem. Note that we shall need to consider the probability that the result B could have been "predicted" on the basis of information C. For instance, if B could be predicted with certainty on the basis of information C, and therefore on the basis of information AC, so that $(B|C) = 1$ and $(B|AC) = 1$, then the new knowledge has no effect on our evaluation of hypothesis A. We may note also that, if $(B|C) = 0$, there is something wrong with our information C, since B is incompatible with C, yet B is observed to be true.

Another important point is that if $(A|C) = 0$, $(A|BC) = 0$. That is, an impossible hypothesis remains impossible no matter what the evidence. Similarly, one may show that a certain hypothesis remains certain, despite new information. This means that one must be very careful about assigning probability zero or unity to any proposition, since this entails that one may never change these estimates, no matter what subsequent information may turn up. Good[5] offers some shrewd advice on this point: "Probability judgements can be sharpened by laying bets at suitable odds. If people always felt obliged to back their opinions when challenged, we would be spared a few of the 'certain' predictions that are so fre‑ made." One must therefore be very cautious about making "certain" theoretical predictions or stating "certain" observational facts. Theorists sometimes find a calculation to be wrong, and observers sometimes find that their results are not supported by subsequent observations by other groups.

3. MODEL OF THE INDUCTIVE PROCESS IN SCIENCE

Equation (2.3) can be used to update scientific judgements in the simple situation in which a theory could predict, for instance, the reading which would be obtained from a particular measuring device. This is not the usual situation in science. It may take a great deal of thought and some compromise to find a quantity which can be both measured and calculated. Furthermore, although one thinks of measurement as being the key process in exact sciences, many of the comparisons between theory and observation are not normally expressed in terms of measurable quantities. For instance, the nature of pulsars for some time hinged upon the question, "Is a pulsar a white dwarf or a neutron star?"

To assist in setting up a model, let us consider a specific situation in which an observer, with a briefcase full of observational data, and a theorist, with a briefcase full of calculations, agree to meet to determine whether the observations support a particular theory. Let us assume that the observer will entertain no criticism of his observations from the theorist, and vice versa. Two problems arise: (a) How will they communicate? and (b) Who will make the final decision?

Let us suppose that the scientists are rational men and aware of their own limitations. Then they may recognize that, although the observer is excellent at his job, and the theorist is good at his, neither feels properly qualified to bridge the gap. For this reason, they call in a third person who we will call the "referee".

The referee will see his goal as that of organizing the discussion in such a way that information provided to him by the observer and the theorist will lead to an unambiguous estimate of the probability that the theory is correct, given the observational data. He will recognize that, to make any progress, it is necessary to set up precise and meaningful communication between his colleagues. We will assume that, after due deliberation, the referee proposes the following scheme, and that it is adopted.

There is to be an "interface" between information provided by the observer and information provided by the theorist. The interface will comprise a list of statements so chosen that each statement may be either (a) a possible result of data reduction of observations, or (b) a possible consequence of theoretical analysis of the hypothesis or hypotheses under consideration. It is agreed that the observer will assign to each statement S a probability $(S|ROX)$ based upon his reduction R of his observations O, and that the theorist will assign to each statement a probability $(S|AHX)$ based upon his analysis A of a hypothesis H. Here and elsewhere, X denotes the body of scientific knowledge which both parties agree to accept without question. (Maxwell's equations, the speed of light, etc., will usually come under this heading.) It is agreed that no exchange of information is to be taken into account except that which takes place at the interface. More specifically, this means that the referee will note the probabilities which the observer assigns to the specified statements, and the probabilities which the theorist assigns to the statements. It is necessary for him to arrange the statements in such a way that he can arrive at the probability $(H|AROX)$ that the hypothesis H is true on the basis of knowledge involving the theorist's analysis, the observer's reduction of observations, and the "general knowledge" X.

In order to be able to complete his task, the referee will need to set out further rules. We suppose that the following are proposed and agreed to.

Statements are arranged in groups, and each group is termed an "item". There is a finite set of items I_α, $\alpha = 1, 2, \ldots, A$.

With each item I_α there is associated a group of statements which, for present convenience, we assume to be finite in number. This set of statements is represented by $S_{\alpha n}$, $n = 1, 2, \ldots, N_\alpha$. For any item, the group of statements are to form a mutually exclusive and complete set. That is, for any item I_α, it is <u>logically</u> demonstrable that one and only one of the statements $S_{\alpha n}$ is true. It follows, from the sum rule of probability theory[3] that

$$\sum_{n=1}^{N_\alpha} (S_{\alpha n}|X) = 1 \; , \; \sum_{n=1}^{N_\alpha} (S_{\alpha n}|ROX) = 1, \text{ etc.} \qquad (3.1)$$

It is, furthermore, to be agreed that neither the observer nor the theorist will use information concerning one item in assigning probabilities to the statements of another item.

We next assume that the referee has read the following statement by Jeffreys[6]: "We get no evidence for a hypothesis by merely working out its consequences and showing that they agree with some observations, because it may happen that a wide range of other hypotheses would agree with those observations equally well. To get evidence for it we must also examine its various contradictories and show that they do not fit the observations. This elementary principle is often overlooked in alleged scientific work, which proceeds by stating a hypothesis, quoting masses of results of observation that might be expected on the hypothesis and possibly on several contradictory ones, ignoring all that would not be expected on it, but might be expected on some alternative, and claiming that the observations support the hypothesis. . . . So long as alternatives are not examined and compared with the whole of the relevant data, a hypothesis can never be more than a considered one."

The referee therefore persuades the theorist to revise his normal working habits. Instead of taking one hypothesis and analyzing it theoretically, he is asked to draw up a complete set of mutually exclusive hypotheses H_i, $i = 1, 2, \ldots I$, and to subject each of these hypotheses in turn to an analysis.

It is likely that, after a few months, the theorist would call for a new meeting, at which he would object to this procedure as being unreasonable and impractical. He could argue that no mortal can specify all the possible explanations of a natural phenomenon and that, even if he could, he might have neither the time nor competence to examine the hypotheses thoroughly. He also could quote Jeffreys[7] to support his case: "The chief advances in modern physics . . . were achieved by the method of Euclid and Newton: to state a set of hypotheses, work out their consequences, and assert them if they accounted for most of the outstanding variation."

The referee is now faced with a dilemma. He foresees that, in order to be able to use probability theory to evaluate any one hypothesis H, he must be able to treat this hypothesis as a member of a complete set of hypotheses. On the other hand, it is no use expecting the theorist to draw up such a complete set, much less analyze them all.

The observer might point out that one can always form a complete set by adding to the hypothesis H the statement $\overline{H}$ (H not true), but the theorist can retort that this is of no help unless $\overline{H}$ can be expressed as one or more specific calculable hypotheses.

In the end, the referee and the theorist might arrive at the following solution. If the theorist can think of a set of exclusive hypotheses $H_1, H_2, \ldots H_I$, each of which he is able and willing to calculate, it is formally possible to add a hypothesis H_0 which is to represent whatever hypotheses may be necessary to make $H_0, H_1, \ldots H_I$ a complete set. This causes difficulty only if the theorist is to be asked to make a sensible calculation of the consequences of H_0. In order to avoid this impossible task, it is agreed that this hypothesis is to be subject to a "null" analysis A_0 which gives no information whatever about the consequences of the hypothesis to which it is applied. If the theorist is allowed to express complete ignorance concerning the outcome of the hypothesis H_0, he need not concern himself with what H_0 really means. We will simply assume that, for each item I_α, the probabilities of the various statements $(S_{\alpha n}|A_0H_0X)$ are to be chosen so as to be "maximally noncommittal", subject only to restrictions imposed by the information X.

The manner in which one may ascribe "maximally noncommittal" values to a set of statements, taking account of possible information about the statements, has been discussed by Jaynes.

At this point, to simplify the mathematics, we suppress the symbols A and R (but we should remember that they are implicitly always present) so that in referring to a hypothesis H we have in mind a particular theory of that hypothesis, and in referring to an observation O we have in mind certain reduction of that data. In accordance with this change in notation, we may refer to H_0 as either the "null hypothesis" or the "null theory".

In passing, it may be noted that it is not an acceptable procedure to identify $(S_{\alpha n}|A_0H_0X)$ with $(S_{\alpha n}|X)$ for the following reason. Since the hypotheses H_0, H_1, ... H_I form a complete set, we know that $H_0 + H_1 + \ldots + H_I$ is true, where the summation sign here indicates a "logical sum", i.e. "and/or". Hence

$$\left(S_{\alpha n} \sum_{i=0}^{I} H_i \middle| X\right) = \left(S_{\alpha n} \middle| X\right) . \quad (3.2)$$

On noting that the hypotheses H_0, H_1, ... H_I are mutually exclusive, we see from the sum rule of probability theory[3] that

$$\left(S_{\alpha n} \sum_{i=0}^{I} H_i \middle| X\right) = \sum_{i=0}^{I} \left(S_{\alpha n} H_i \middle| X\right) . \quad (3.3)$$

We may now use the product rule[3] to write

$$(S_{\alpha n}H_i|X) = (S_{\alpha n}|H_iX)(H_i|X) , \quad (3.4)$$

so that

$$(S_{\alpha n}|X) = \sum_{i=0}^{I} (S_{\alpha n}|H_i X)(H_i|X) \quad . \tag{3.5}$$

If we make the choice

$$(S_{\alpha n}|H_0 X) = (S_{\alpha n}|X) \ , \tag{3.6}$$

we see that

$$(S_{\alpha n}|H_0 X) = [1 - (H_0|X)]^{-1} \sum_{i=1}^{I} (S_{\alpha n}|H_i X)(H_i|X). \tag{3.7}$$

It follows from this equation that if a particular statement $S_{\alpha' n'}$ is impossible on the basis of hypotheses H_1, ... H_I, then it must be considered to be impossible on the basis of hypothesis H_0 also. This is an unacceptable restriction on the interpretation of the null theory represented by H_0, so that we should not identify $(S_{\alpha n}|H_0 X)$ with $(S_{\alpha n}|X)$.

The model which we have now developed, for the interplay of observation and theory in scientific research, may be represented schematically as in Figure 1.

4. EVALUATION BASED ON ONE FACT

We now consider the simple case that the theorist and observer agree that their work may be compared by considering only one item comprising a complete and mutually exclusive set of statements S_1, S_2, ..., S_n. The observer has represented his knowledge of these statements, based on reduction of observational data O, by the probabilities $(S_n | OX)$. The theorist has drawn up a set of hypotheses, which we assume to include a null hypothesis, $H_0, H_1, \ldots, H_I$. On the basis of his analysis of hypotheses $H_1, \ldots, H_I$, and of his professed ignorance of the implications of H_0, he has prepared the set of probabilities $(S_n | H_i X)$, $i = 0, 1, \ldots, I$.

The referee is required to calculate $(H_i | OX)$, the "post probabilities" of the various hypotheses as determined by the prior information and the observations O, following the rules set down in the previous section. In order to proceed, "prior probabilities" $(H_i | X)$ must be assigned to the various hypotheses on the basis of the general knowledge X alone. We will assume that everyone will attempt to be as open-minded as possible with the aim of making the prior probabilities as noncommittal as possible.

Since the statements S_n are mutually exclusive and form a complete set, we may write

$$(H_i | OX) = \left(H_i \sum_n S_n \middle| OX\right) . \tag{4.1}$$

Again using the sum rule of probability theory, this equation becomes

$$(H_i | OX) = \sum_n (H_i S_n | OX) . \tag{4.2}$$

The product rule enables us to rewrite this equation as

$$(H_i | OX) = \sum_n (H_i | S_n OX)(S_n | OX) . \tag{4.3}$$

According to the rules set up in Section 3, the connection between the hypotheses and the observations occur only via the statements S_n. If it is asserted that S_n is true, all other knowledge about the observations is irrelevant, as far as the hypotheses are concerned. This property of our model therefore implies that

$$(H_i|S_n OX) = (H_i|S_n X) . \tag{4.4}$$

In consequence of which equation (4.3) becomes

$$(H_i|OX) = \sum_n (H_i|S_n X)(S_n|OX) . \tag{4.5}$$

At this stage we use Bayes' theorem (equation 2.3) to obtain the following equation:

$$(H_i|S_n X) = \frac{(S_n|H_i X)}{(S_n|X)} (H_i|X) . \tag{4.6}$$

This step introduces the probabilities $(S_n|X)$, which do not appear among our given data. It is at this point that we profit from the assumption that the hypotheses $H_0, H_1, \ldots, H_I$ are mutually exclusive and form a complete set. We saw in Section 3 that these assumptions lead to equation (3.5), which is now written as

$$(S_n|X) = \sum_j (S_n|H_j X)(H_j|X) . \tag{4.7}$$

On combining equations (4.5), (4.6), and (4.7), we finally arrive at

$$(H_i|OX) = \sum_n \left[\frac{(S_n|H_iX)(S_n|OX)}{\sum_j (S_n|H_jX)(H_j|X)} \right] (H_i|X) . \qquad (4.8)$$

This formula for the post-probabilities $(H_i|OX)$ involves only the prior probabilities $(H_i|X)$ and the probabilities of statements S_n as determined on the one hand by reduction of the observations O and on the other hand by analysis of the hypotheses H_i. It is easily verified that

$$\sum_i (H_i|OX) = 1 . \qquad (4.9)$$

In scientific work, the comparison of theory and observation often involves consideration of continuous variables, which means that we must consider continuous sequences of statements. For instance, the statement S_ν may be the statement that the measurable quantity ϕ has the value $F(\nu)$. If we now denote by "S_ν to $S_{\nu+d\nu}$" the logical sum of all statements enumerated by ν as it runs from ν to $\nu+d\nu$, we can introduce the notation

$$(S_\nu \text{ to } S_{\nu+d\nu}|H_iX) = (S_\nu|H_iX)_\nu \, d\nu \text{ , etc.} \qquad (4.10)$$

Using this notation, and replacing the summation sign in equation (4.8) by an integration sign, we obtain the formula

$$(H_i|OX) = \left[\int d\nu \frac{(S_\nu|H_iX)_\nu (S_\nu|OX)_\nu}{\sum_j (S_\nu|H_jX)_\nu (H_j|X)} \right] (H_i|X) . \qquad (4.11)$$

Similarly, a theory may have one or more adjustable parameters. This possibility may be treated by supposing that we are dealing with a continuous sequence of hypotheses H_λ, where λ is a continuous variable. With the notation

$$(H_\lambda \text{ to } H_{\lambda+d\lambda}|X) = (H_\lambda|X)_\lambda \, d\lambda, \text{ etc.,} \tag{4.12}$$

equation (4.8) becomes

$$(H_\lambda|OX)_\lambda = \left[\sum_n \frac{(S_n|H_\lambda X)(S_n|OX)}{\int d\mu (S_n|H_\mu X)(H_\mu|X)_\mu}\right](H_\lambda|X)_\lambda \,. \tag{4.13}$$

5. EVALUATION BASED ON MANY FACTS

We now consider how one might combine information obtained from several items. We assume that these items are "independent", in the sense specified in Section 3.

We introduce the symbol F_α to denote the "fact" associated with observational evidence concerning item I_α. Thus the fact F_α comprises the set of probabilities $(S_{\alpha n}|OX)$, $n = 1, \ldots, N_\alpha$.

We now suppose that a group of hypotheses H_i have been evaluated in terms of two facts F_1 and F_2, considered separately and independently. In this way we have arrived at probabilities which may be written as $(H_i|F_1X)$, $(H_i|F_2X)$. The problem which we now consider is that of determining the probabilities $(H_i|F_1F_2X)$. The sense in which F_1 and F_2 are considered to be independent is the following: knowledge of F_1 will influence our interpretation of F_2 only through the effect which F_1 has on our evaluation of the hypotheses H_i and the influence of knowledge of hypotheses H_i on our interpretation of F_2, and conversely.

We first note that $(H_i|F_1F_2X)$ may be expressed as follows:

$$(H_i|F_1F_2X) = \frac{(H_iF_1|F_2X)}{(F_1|F_2X)} \quad . \tag{5.1}$$

By an argument parallel to that leading to equation (4.4), we see that

$$(H_iF_1|F_2X) = \sum_j (H_iF_1|H_jF_2X)(H_j|F_2X) \tag{5.2}$$

and

$$(F_1|F_2X) = \sum_j (F_1|H_jF_2X)(H_j|F_2X) \quad . \tag{5.3}$$

We now note that the first term on the right-hand side of (5.2) may be expressed as

$$(H_iF_1|H_jF_2X) = (H_i|F_1H_jF_2X)(F_1|H_jF_2X) \quad . \tag{5.4}$$

However, since the set of hypotheses is assumed to be mutually exclusive and complete,

$$(H_i|F_1H_jF_2X) = \delta_{ij}. \tag{5.5}$$

Furthermore, our specification of the sense in which F_1 and F_2 are taken to be independent implies that

$$(F_1|H_jF_2X) = (F_1|H_jX) \ . \tag{5.6}$$

If we now note from Bayes' theorem that

$$(F_2|H_jX) = \frac{(H_j|F_2X)}{(H_j|X)}(F_2|X) \ , \tag{5.7}$$

we find from equations (5.2) through (5.7) that equation (5.1) may be expressed as

$$(H_i|F_1F_2X) = \frac{(H_i|F_1X)(H_i|F_2X)[(H_i|X)]^{-1}}{\sum_j (H_j|F_1X)(H_j|F_2X)[(H_j|X)]^{-1}} \ . \tag{5.8}$$

It is a straightforward matter to prove (by induction!) that the general formula is

$$(H_i|F_1\ldots F_AX) = \frac{(H_i|F_1X)\ldots(H_i|F_AX)[(H_i|X)]^{-(A-1)}}{\sum_j (H_j|F_1X)\ldots(H_j|F_AX)[(H_j|X)]^{-(A-1)}} \ . \tag{5.9}$$

We note from this equation that

$$\sum_j (H_i|F_1\ldots F_AX) = 1 \ . \tag{5.10}$$

By adopting the notation introduced at the end of Section 4, we may modify equation (5.9) for the case that the hypotheses must be enumerated by a continuous variable.

6. DISCUSSION

An example of the application of the foregoing procedure to the evaluation of a scientific problem is presented in a subsequent article. However, there are a few lessons to be drawn from this exercise which may be stated briefly.

1. Comparison of theory and observation requires an interface. The definition of an interface is a job neither for the theorist nor for the observer, but for both.
2. We know we must be cautious about accepting a theorists' analysis of his hypothesis. We must be cautious also if an observer draws inferences which are not plainly deducible from his data.
3. We may assign to a set of statements an "entropy" E given by

$$E = -\sum_n p_n \ln p_n \qquad (6.1)$$

   where p_n are the probabilities defined either by observations or by analysis of a hypothesis. More evidence is provided by a low-entropy fact, which may be termed a "hard fact", than by a high-entropy fact (a soft fact). Similarly one can make a more definite appraisal of a theory if it leads to hard (low-entropy) conclusions rather than soft (high-entropy) conclusions.
4. To get a good test of a theory, it is necessary to compare one or more hard facts with one or more hard conclusions. In the case that we are matching a hard fact with a weak conclusion, or vice versa, we are no better off than if we were comparing a soft fact with a soft conclusion. In this case we could say

that the strength of the inference is "theory limited" or "observation limited", respectively.

5. It is generally recognized that theorists' conclusions are likely to be biased by knowledge of the observations. The reason that great weight is attached to predictions is that these are manifestly free from bias. It is equally important that facts stated by an observer should be free from bias due to knowledge of theory, but theorists are not so concerned about this possibility that they demand observers to make observations before a theory is proposed. There is in fact a double standard applied to theorists and observers. However, it would be difficult to demand at the same time both prediction by the theorists and "pre-observation" by the observers!

6. Since the null theory assigns a non-zero probability to each statement, no combination of observational evidence will ever reduce to zero the post probability of the null theory. This means that (as long as the null theory is one of the admitted possibilities) none of the specified theories can ever be conclusively established. Hence a specific theory can be proved incorrect by appropriate evidence, but it cannot be proved correct.

7. It therefore appears that the normal situation concerning a scientific phenomenon is that there is never a "correct" theory - there is at best a currently accepted theory. If, however, an explanation comes to be so well accepted that the phenomenon is <u>redefined</u> <u>theoretically</u>, then what was a phenomenon becomes

merely a theoretical construct to use in the evaluation of real phenomena.

8. Although the present scheme for evaluating scientific theories and weighing scientific evidence has been presented as a dialogue between a theorist and an observer, with a referee to call the scores, the scheme can prove helpful when only one person is involved. The rules which have been set out will then enable him to identify and classify the various judgements which he must make. Although it will not remove all bias, it may make his bias more evident, and so make it somewhat easier to keep himself honest.

An article which applies this model of the scientific process to the evaluation of pulsar theories will be published shortly.

REFERENCES

1. F. D. Kahn and H. P. Palmer, Quasars (Harvard University Press, Cambridge, Mass., 1967), p. 111.

2. H. Jeffreys, Scientific Inference (Cambridge University Press, 1931), p. 21.

3. I. J. Good, Probability and the Weighing of Evidence (Griffin, London, 1950), p. 16.

4. H. Jeffreys, Scientific Inference (Cambridge University Press, 1931), p. 19.

5. I. J. Good, Probability and the Weighing of Evidence (Griffin, London, 1950), p. 49.

6. H. Jeffreys, Theory of Probability (2nd ed., Oxford University Press, 1948), p. 48.

7. H. Jeffreys, Theory of Probability (2nd ed., Oxford University Press, 1948), p. 388.

8. E. T. Jaynes, IEEE Trans. SSC 4, 227 (1968).

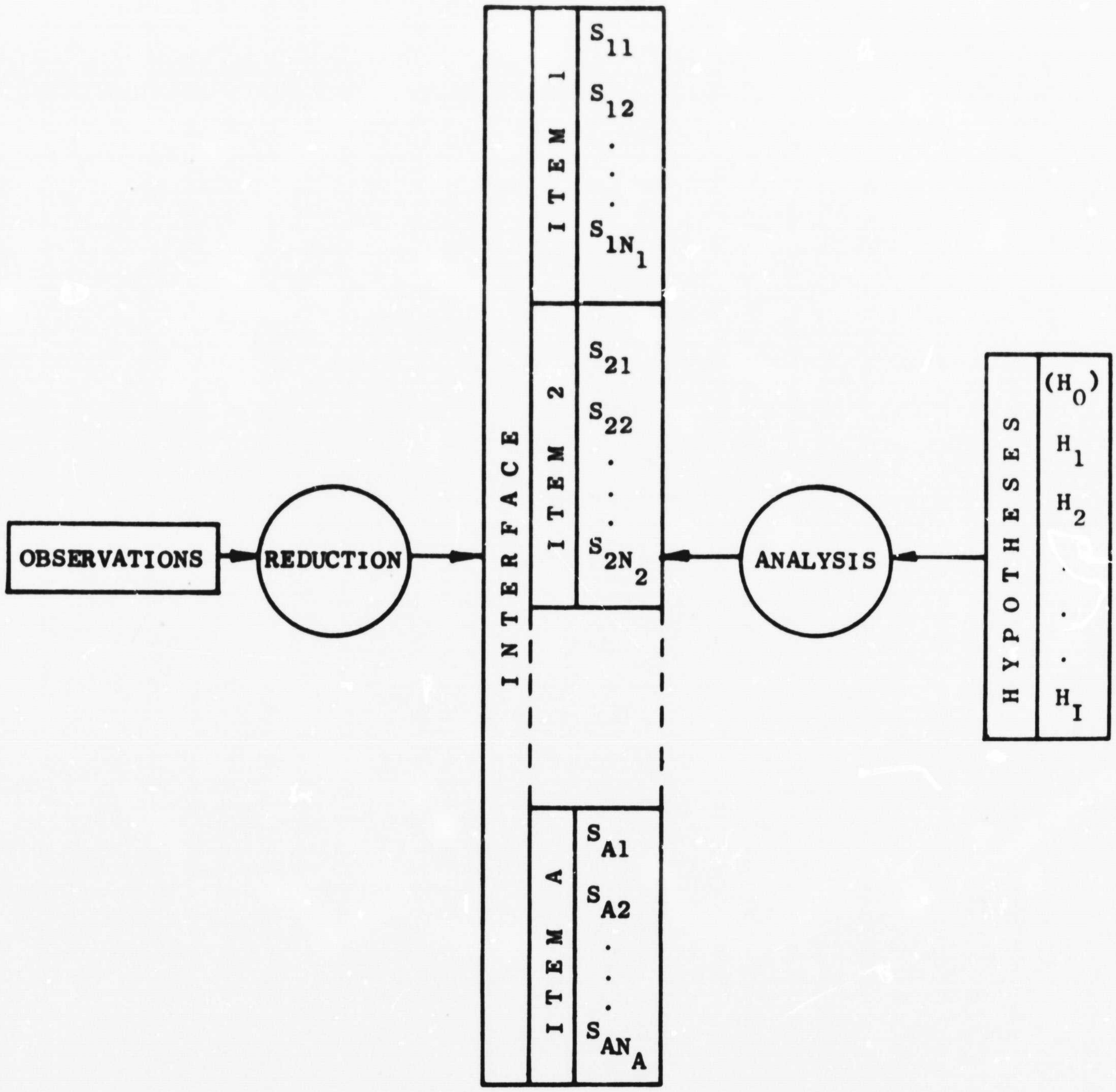

Figure 1.

Schematic Representation of Model used for

Evaluation of

Relationship between Theory and Observation