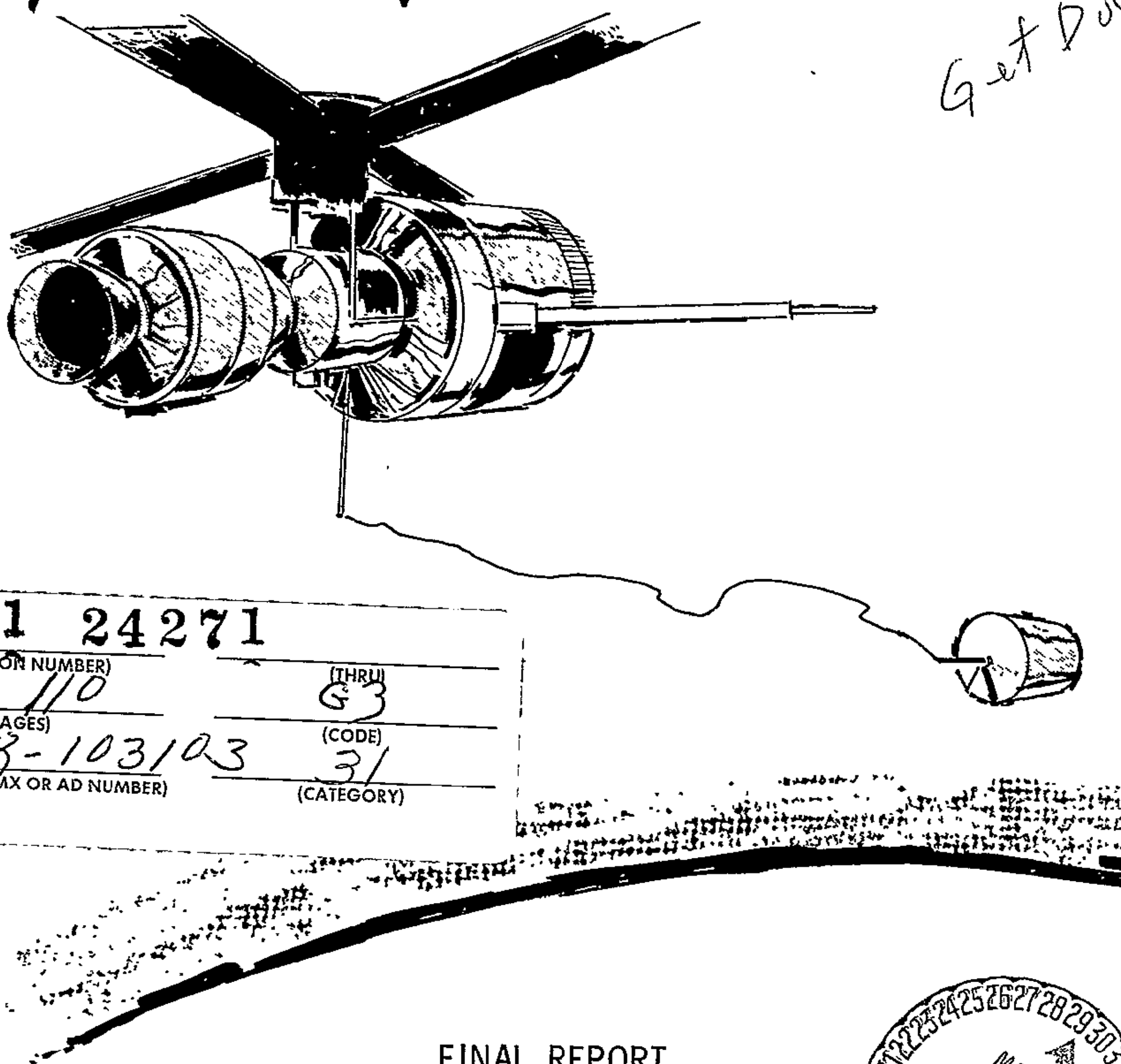
MCR-71-77 Copy No.

Tether Docking of Orbiting Spacecraft

Get DRA



FINAL REPORT

March 1971

MARTIN MARIETTA
DENVER DIVISION

MCR-71-77

TETHER DOCKING OF ORBITING SPACECRAFT

Final Report

March 1971

Fredrick J. Greeb

Prepared Under Contract No. NAS8-25353

For

National Aeronautics and Space Administration
Marshall Space Flight Center
Alabama

Approval

W. L. DeRocher, Jr.
Program Manager

Martin Marietta Corporation
Denver Division
Denver, Colorado 80201

FOREWORD

The research described in this report (Tether Docking of Orbiting Spacecraft, National Aeronautics and Space Administration Contract NAS8-25353) was performed by Martin Marietta Corporation, Denver Division, for the NASA Marshall Space Flight Center, Huntsville, Alabama. Dr. Eugene Worley was the NASA Contracting Officer's Representative.

Mr. W. L. DeRocher, Jr. was Martin Marietta's Program Manager and Mr. F. J. Greeb was the Technical Director for the program. Mr. B. J. Harbick developed the digital computer program logic and programmed the equations of motion.

ABSTRACT

This report describes a study of Tether Docking of Orbiting Spacecraft. The study was divided into two basic parts. In the first, a digital computer program was developed to simulate the problem of two orbital spacecraft attached by a tether. A detailed description of this program, along with the program operating instructions, is contained under separate cover in the Computer Program User's Manual.

In the second part of the study, the feasibility of using a single tether to provide all of the control forces and moments required to dock two vehicles was investigated. A tether control law to accomplish this task was developed using the linearized equations of orbital motion, and was verified using the developed computer program which solves the exact equations of motion.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

SECTION I

INTRODUCTION

This document is one product of a NASA Contract (NAS8-25353) with the Denver Division of the Martin Marietta Corporation. Two objectives were to be met under this contract. The first was to develop a digital computer program simulating the dynamics of two interrelated orbital rigid space vehicles attached to each other by a tether line, and the second was to perform a feasibility study on the problem of tether docking, using the developed computer program for verification of the docking techniques.

The purpose of this report is to provide a summary of the digital computer program which was developed to meet the first contract objective, and to present a detailed description of the system dynamics and the results of the feasibility study. A detailed description of the computer program, along with the program operating instructions, is contained in the Computer Program User's Manual (Ref. 2).

The primary effort for the feasibility study was the development of a control law which would allow retrieval and docking of a Chase Vehicle such as a supply package or experiment module to a Target Vehicle which would typically be a Skylab or Space Station. This problem, as depicted in Figure I-1, involved three separate control functions. The first was control of a boom on the Target Vehicle which allowed variation in the direction of the applied tether forces; the second was control of a boom on the chase vehicle, which allowed control of the roll motion; and the third was control of the tether force, which supplied all of the forces

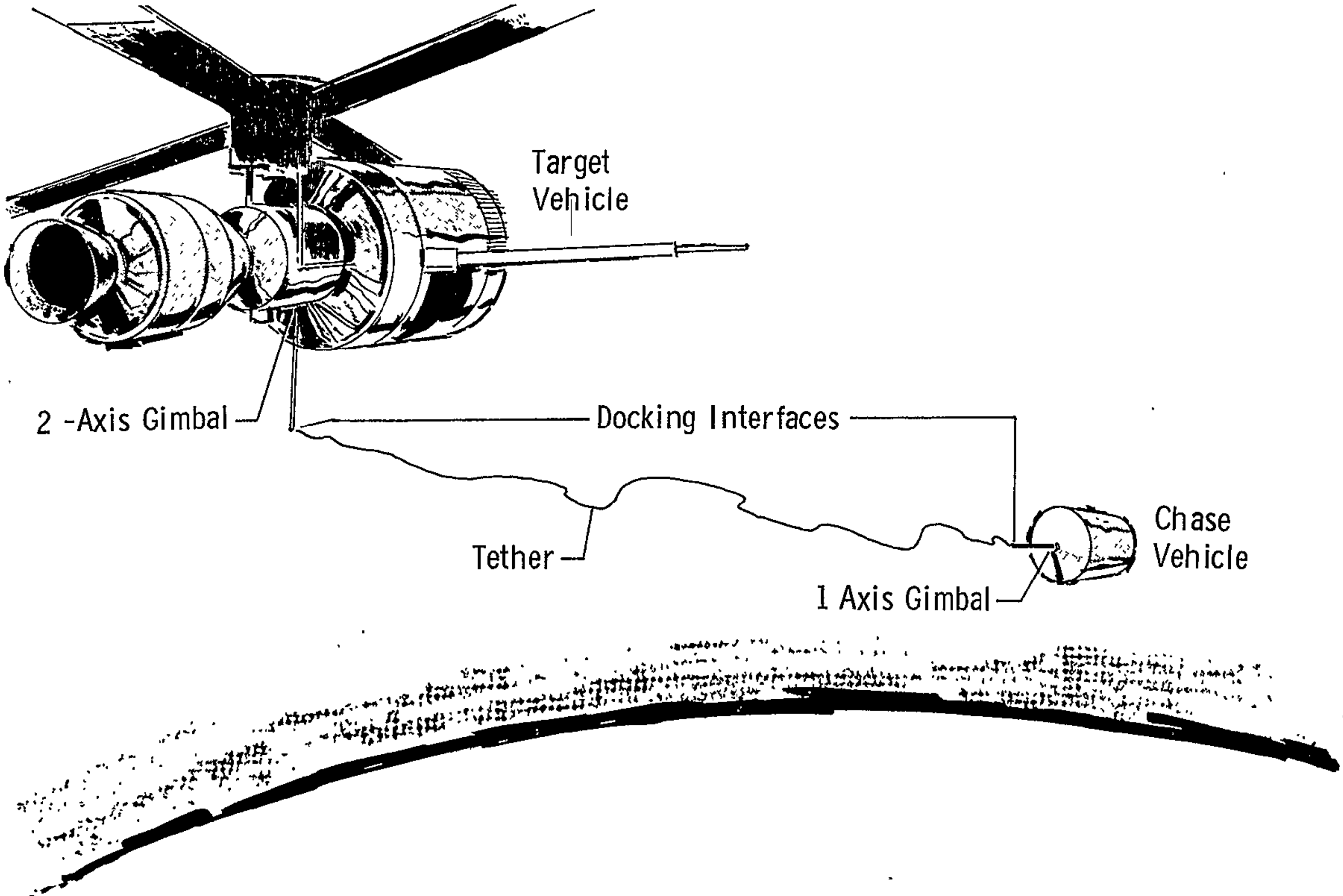


Figure I-1   Tether Docking Problem

and moments required for docking the two vehicles. With these three controlled variables, six degree of freedom control is accomplished using a single tether.

This type of control technique has a wide variety of applications. The primary application considered in the development of the control law was the retrieval and docking of cargo and experiment modules. By using the tether to provide the control forces and moments, it is not necessary to incorporate propulsion or attitude control systems into these modules for the purpose of docking. Thus, the modules can be designed to accomplish their primary functions, with docking achieved using the tether.

Other possible applications lie in the area of orbital assembly and satellite capture and retrieval. Also, by combining the retrieval and docking technique with stationkeeping techniques, a wide variety of tasks in the area of experiment or facility operation can be accomplished using the tether as the prime mode of control. These applications include deployment and orbit establishment, and stationkeeping and maneuvering. Temporary cargo storage and emergency isolation of modules are other potential applications for tether control.

The body of this report is divided into four major sections. Section II discusses the equations of motion and the digital computer program which solves these equations; Section III describes the control law which was developed to accomplish retrieval and docking; Section IV presents the results which were obtained using this control law; and Section V presents the conclusions and the requirements for future studies.

SECTION II

SYSTEM DYNAMICS

This section of the report deals with the equations of motion which define the problem of two rigid orbital spacecraft attached by a tether and with the digital computer program which was developed to solve these equations. The equations may be divided into four basic areas: 1) orbital motion, 2) rotational motion, 3) perturbations, and 4) tether forces and moments.

The equations of orbital motion define the position and velocity of each vehicle relative to an unperturbed reference orbit. In this manner, the relative position and velocity between the two vehicles is determined without forming the differences of large numbers, which would be the case if the orbit of each vehicle was determined independently.

The target vehicle is assumed to be stabilized and controlled, and the chase vehicle is unstabilized and controlled. The rotational motion of the chase vehicle is determined using the quaternion technique. This allows rotations of greater than 90 degrees to be accomplished without encountering the singularity problem present in the Euler angle rate transformation.

The two perturbation sources which are considered are gravity gradient and aerodynamics. The gravity gradient effects place moments on both vehicles, and the aerodynamics cause moments on the target vehicle only. In addition, the aerodynamic effects cause the orbital motion of both vehicles to deviate from the reference orbit.

The tether forces and moments affect the orbital motion of both vehicles, and also affect the rotational motion of the chase vehicle.

The digital computer program which was developed to solve these equations of motion is designed to allow the incorporation of variour tether control laws. Thus, the program can be tailored to any particular tether control problem desired.

Each of the four areas concerning the equations of motion, and the digital computer program, are discussed in the following section of this chapter.

1. Orbital Motion

The motion of the target vehicle and chase vehicle is defined relative to an unperturbed reference point. The reference point is normally initially located at the center-of-mass of the target vehicle and describes the nominal trajectory (reference orbit) of the target vehicle unaffected by the disturbance forces.

Referring to Figure II-1, let $\bar{R}_I$, $\bar{R}_T$, and $\bar{R}_C$ denote the position vectors from the center of the inverse-square force field to the reference point, the target vehicle, and the chase vehicle, respectively. Letting $\bar{R}_V$ denote the radius vector to the vehicle in question (e.g., $\bar{R}_T$ or $\bar{R}_C$), the motion of the reference point, and of each vehicle, is then governed by the vector equations:

$$\frac{d^2\bar{R}_I}{dt^2} = \frac{-\mu}{R_I^3}\bar{R}_I \qquad (2.1)$$

$$\frac{d^2\bar{R}_V}{dt^2} = \frac{-\mu}{R_V^3}\bar{R}_V + \frac{\bar{D}_V}{M_V} \qquad (2.2)$$

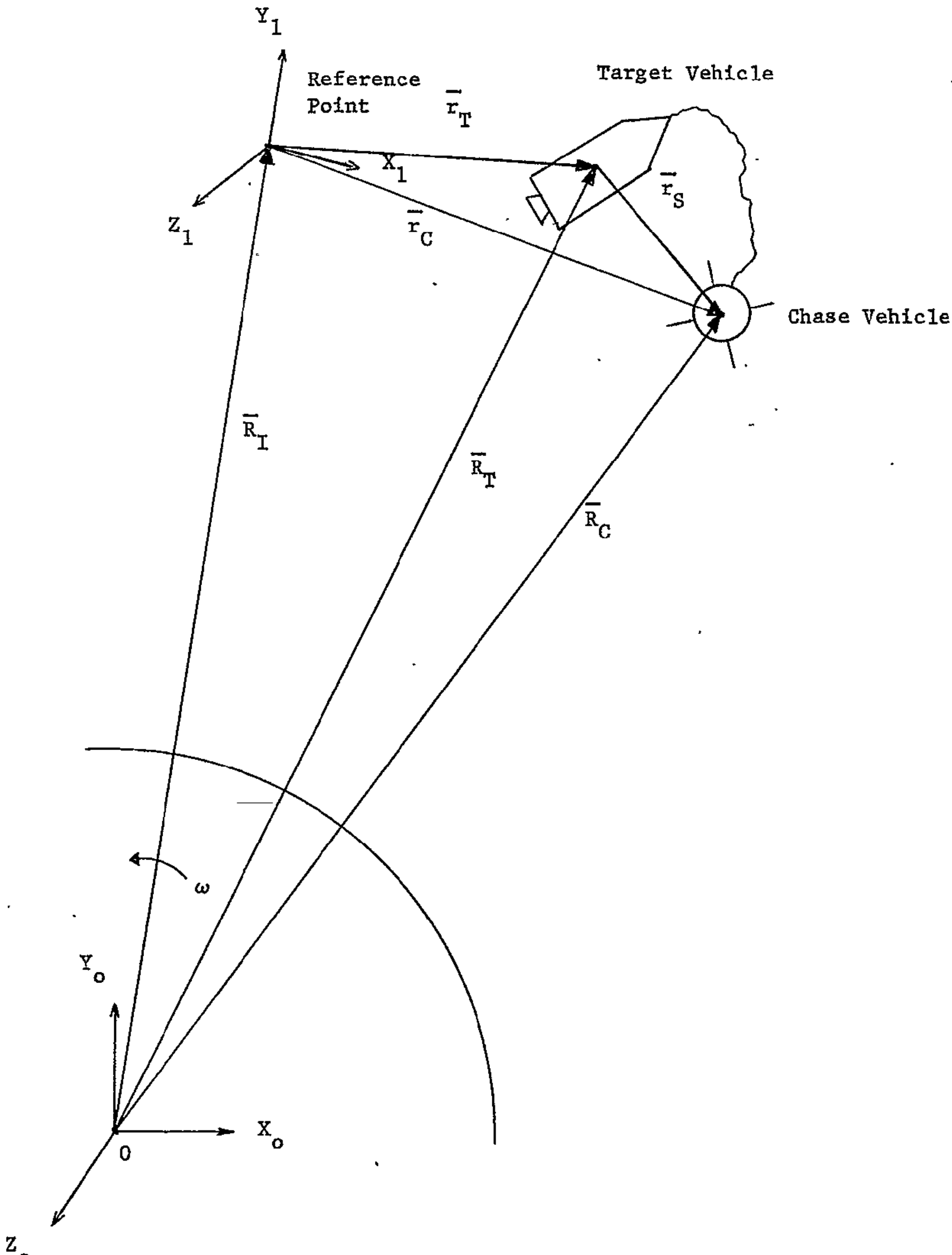


Figure II-1   Relative Coordinate System where μ denotes the gravitational constant of the earth, $\overline{D}_V$ denotes the sum of the external forces acting on the vehicle, and M_V denotes the mass of the vehicle.

Defining the position vector from either vehicle to the reference point as $\overline{r}_V = \overline{R}_V - \overline{R}_I$, the differential equation for $\overline{r}_V$ is obtained as:

$$\frac{d^2\overline{r}_V}{dt^2} = \mu\left(\frac{\overline{R}_V}{R_V^3} - \frac{\overline{R}_I}{R_I^3}\right) + \frac{\overline{D}_V}{M_V} \tag{2.3}$$

Four coordinate systems, as defined in Table II-1, are used in the development of the equations of motion. The scalar form of equation (2.3) is formed by expanding the equation into component form in the rotating local vertical coordinate system.

Table II-1 Coordinate Systems Definition

Coordinate System	Definition
Inertial Coordinates (X_0, Y_0, Z_0)	Earth centered, non-rotating system. X_0, Y_0 in orbit plane.
Local Vertical Coordinates (X_1, Y_1, Z_1)	Rotating system with origin at unperturbed reference point. X_1 opposite to orbital velocity of reference point. Y_1 outward along radius vector from the center of the earth. Z_1 normal to the orbit plane to form a right hand orthogonal system.
Target Vehicle Body Coordinates (X_2, Y_2, Z_2)	Body fixed rotating system with origin at the c.m. of the target vehicle.
Chase Vehicle Body Coordinates (X_3, Y_3, Z_3)	Body fixed rotating system with origin at the c.m. of the chase vehicle

Let $\bar{i}$, $\bar{j}$, $\bar{k}$ denote unit vectors along the X_1, Y_1, Z_1 axes, respectively. Then:

$$\bar{R}_I = \bar{R}_I j \qquad (2.4)$$

$$\bar{R}_V = X_V \bar{i} + Y_V \bar{j} + Z_V \bar{k} \qquad (2.5)$$

so that

$$\bar{r}_V = X_V \bar{i} + (R_I + Y_V)\,\bar{j} + Z_V \bar{k} \qquad (2.6)$$

In addition, the following relationship is used for the time derivative of the vector which occurs in Equation (2.3)

$$\frac{d^2 \bar{r}_V}{dt^2} = \ddot{\bar{r}}_V + 2\,\bar{\omega} \times \dot{\bar{r}}_V + \bar{\omega} \times (\bar{\omega} \times \bar{r}_V) + \dot{\bar{\omega}} \times \bar{r}_V \qquad (2.7)$$

where the dots indicate differentiation with respect to time, as viewed in the rotating X_1, Y_1, Z_1 system. Also, the angular velocity of the rotating system can be expressed as:

$$\bar{\omega} = \omega \bar{k} \qquad (2.8)$$

It follows from Equations (2.5) - (2.8) that the acceleration can be written as:

$$\frac{d^2 \bar{r}_V}{dt^2} = (\ddot{X}_V - 2\omega \dot{Y}_V - \omega^2 X_V - \dot{\omega}\, Y_V)\,\bar{i} + (\ddot{Y}_V + 2\omega \dot{X}_V - \omega^2 Y_V + \dot{\omega}\, X_V)\,\bar{j} + \ddot{Z}\,\bar{k} \qquad (2.9)$$

In addition, the external forces can be represented as:

$$\overline{D}_V = D_{VX}\overline{i} + D_{VY}\overline{j} + D_{VZ}\overline{k} \tag{2.10}$$

By use of Equations (2.9) and (2.10) the scalar form of Equation (2.3) is obtained as:

$$\ddot{X}_V - 2\omega\dot{Y}_V - \omega^2 X_V - \dot{\omega} Y_V = \frac{-\mu X_V}{R_V^3} + \frac{D_{VX}}{M_V} \tag{2.11}$$

$$\ddot{Y}_V - 2\omega\dot{X}_V - \omega^2 Y_V + \dot{\omega} X_V = \frac{-\mu (Y_V + R_I)}{R_V^3} + \frac{\mu}{R_I^2} + \frac{D_{VY}}{M_V} \tag{2.12}$$

$$\ddot{Z}_V = \frac{-\mu Z_V}{R_V^3} + \frac{D_{VZ}}{M_V} \tag{2.13}$$

Equations (2.11) - (2.13) are the exact equations of motion in relative coordinates for an inverse square force field and the sum of all perturbing forces. They are applicable for the chase and target vehicles, and provide for elliptical as well as circular reference orbits. These equations are coupled with the equations which define the reference trajectory. This motion is given by Equation (2.1) which yields the familiar equations of planar motion in a central force field, expressed in polar form as:

$$\ddot{R}_I - \frac{h^2}{R_I^3} = \frac{-\mu}{R_I^2} \tag{2.14}$$

$$R_I^2 \omega = h \tag{2.15}$$

The reference orbit is defined by the perigee altitude, A_p, and the apogee altitude, A_a. From these two parameters, the length of the major axis, denoted as 2a, and the orbit eccentricity, denoted as e, are found from the relations

$$2a = 2R_E + A_a + A_p \tag{2.16}$$

$$e = 1 - (R_E + A_p)/a \tag{2.17}$$

where R_E denotes the radius of the earth.

The average orbital rate, n, and the orbit period, T, are then given by:

$$n = \mu^{1/2} a^{-3/2} \tag{2.18}$$

$$T = \frac{2\pi}{n} \tag{2.19}$$

The orbital parameters at a specific instant of time are determined by first solving for the mean anomaly, denoted M, using the relation:

$$M = n\,t \tag{2.20}$$

Next, the eccentric anomaly, denoted E, is determined using the relation:

$$E - e \text{ Sin } E = M \tag{2.21}$$

Equation (2.21) is solved using a series of successive approximations. For small values of eccentricity, ($e \leq 0.1$), the convergence of the solution is quite rapid, requiring only four or five iterations.

Once the eccentric anomoly is found, the orbital radius is determined as:

$$R_I = a\,(1 - e \,\mathrm{Cos}\, E) \tag{2.22}$$

The orbital angle (true anomaly), θ_I, and the orbital velocity, V, are given as:

$$\theta_I = 2\,\mathrm{Tan}^{-1}\left[\left(\frac{1+e}{1-e}\right)^{1/2} \mathrm{Tan}\,\frac{E}{2}\right] \tag{2.23}$$

$$V^2 = \mu\left(\frac{2}{R_I} - \frac{1}{a}\right) \tag{2.24}$$

The angle of the orbital velocity relative to a perpendicular to the radius vector, denoted as β, is given as:

$$\beta = \mathrm{Cos}^{-1}\left[\frac{a^2(1 - e^2)}{R_I(2a - R_I)}\right]^{1/2} \tag{2.25}$$

The velocity along the radius vector, $\dot{R}_I$, and the velocity perpendicular to the radius vector, $R_I\,\omega$, are given as:

$$\dot{R}_I = V\,\mathrm{Sin}\,\beta \tag{2.26}$$

$$R_I\,\omega = V\,\mathrm{Cos}\,\beta \tag{2.27}$$

from which the orbital angular rate, ω, is determined as:

$$\omega = V\,\mathrm{Cos}\,\beta / R_I \tag{2.28}$$

The orbital angular acceleration is found by differentiating Equation (2.15). Since h is a constant, this differentiation yields:

$$\dot{\omega} = \frac{-2\dot{R}_I\omega}{R_I} \tag{2.29}$$

Equations (2.16) - (2.29) specify all of the parameters required for the solution of the equations of relative motion. Calculation of the orbital parameters in this manner avoids the problem of propagation of errors which could occur with a numerical integration of Equation (2.14). This solution also allows easy incorporation of logic to simplify the solution for circular orbits, in which case many of the terms are constant or zero.

After the position and velocity of each vehicle relative to the reference point has been calculated, the separation distances and velocities between the two vehicles can be found using the relation $\bar{r}_S = \bar{r}_C - \bar{r}_T$, which yields the following scalar equations:

$$\left.\begin{aligned} X_S &= X_C - X_T \\ Y_S &= Y_C - Y_T \\ Z_S &= Z_C - Z_T \end{aligned}\right\} \qquad (2.30)$$

and

$$\left.\begin{aligned} \dot{X}_S &= \dot{X}_C - \dot{X}_T \\ \dot{Y}_S &= \dot{Y}_C - \dot{Y}_T \\ \dot{Z}_S &= \dot{Z}_C - \dot{Z}_T \end{aligned}\right\} \qquad (2.31)$$

Thus, by numerical integration of Equations (2.11) - (2.13) for each vehicle, coupled with the solution of the reference orbit, the motion of both vehicles relative to the reference orbit and the motion of the chase vehicle relative to the target vehicle is determined.

2. Rotational Motion

The attitude of each vehicle is defined by Euler angles relating each vehicle's body axes to the local vertical reference axes. A standard Yaw-Pitch-Roll sequence is used. The orbital angle, θ_I, defines the rotation of the local vertical axes with respect to the earth centered inertial axes. The attitude of the chase vehicle relative to the target vehicle is also determined.

The target vehicle is assumed to be perfectly stabilized and controlled, and thus does not deviate from its initial attitude. The computer program allows the option of placing the target vehicle in either an earth oriented or inertially oriented attitude. The magnitude and integral of the disturbance torques acting on the target vehicle are calculated, allowing sizing of the control system requirements.

The chase vehicle is assumed to be unstabilized and uncontrolled and thus its attitude is a function of the disturbance torques. The equations of rotational motion for the chase vehicle are obtained by equating the sum of all external torques applied to the vehicle to the time rate of change of the angular momentum:

$$\frac{d\overline{H}_C}{dt} = \sum_i T_{iC} \tag{2.32}$$

where $\overline{H}_C$ is the angular momentum of the chase vehicle, and $\overline{T}_{iC}$ is the $i^{\underline{th}}$ external torque on the chase vehicle.

Since the torques are measured in the rotating chase vehicle body axis system, Equation (2.32) may be rewritten as:

$$\sum_i \bar{T}_{iC} = \bar{\bar{I}}_C \dot{\bar{\omega}}_C + \bar{\omega}_C \times \bar{H}_C \tag{2.33}$$

where $\bar{\omega}_C$ is the chase vehicle body angular rates, $\bar{\bar{I}}_C$ is the chase vehicle inertia dyadic, and the multiplication symbol, X, denotes the cross-product of the two vectors. Equation (2.33) is rewritten as:

$$\dot{\bar{\omega}} = \bar{\bar{I}}_C^{-1} \left[\sum_i \bar{T}_{iC} - \bar{\omega}_C \times \bar{H}_C \right] \tag{2.34}$$

and numerically integrated to obtain the chase vehicle body angular rates.

Rather than using the conventional Euler angle rate transformation, the quaternion technique is used to determine the attitude of the chase vehicle (Ref. 4). Quaternions were selected because of their greater inherent accuracy and to circumvent the singularity problem that is present with the Euler angle rate transformation.

The quaternions are found by numerical integration of the following equations:

$$\left.\begin{aligned}
\dot{e}_0 &= -1/2\ (e_1\omega_X + e_2\omega_Y + e_3\omega_Z) \\
\dot{e}_1 &= 1/2\ (e_0\omega_X + e_2\omega_Z - e_3\omega_Y) \\
\dot{e}_2 &= 1/2\ (e_0\omega_Y - e_1\omega_Z + e_3\omega_X) \\
\dot{e}_3 &= 1/2\ (e_0\omega_Z + e_1\omega_Y - e_2\omega_X)
\end{aligned}\right\} \tag{2.35}$$

where e_i are the terms of the quaternions and $\omega_X, \omega_Y, \omega_Z$ are the components of the chase vehicle angular velocity. The initial values of the quaternions are calculated from the initial attitude of the chase vehicle, which is input as conventional Euler angles, by first setting up the chase vehicle body axes to inertial axes transformation matrix, and then using the relations to determine the initial magnitude of the quaternions:

$$\left.\begin{aligned} 4\,e_0^{\,2} &= 1 + d_{11} + d_{22} + d_{33} \\ 4\,e_1^{\,2} &= 1 + d_{11} - d_{22} - d_{33} \\ 4\,e_2^{\,2} &= 1 - d_{11} + d_{22} - d_{33} \\ 4\,e_3^{\,2} &= 1 - d_{11} - d_{22} + d_{33} \end{aligned}\right\} \tag{2.36}$$

where the d_{ij} terms are the components of the transformation matrix.

The initial sign of the quaternions is found by assuming that e_0 is always positive and using the relations:

$$\left.\begin{aligned} 4\,e_0\,e_2 &= d_{13} - d_{31} \\ 4\,e_0\,e_3 &= d_{21} - d_{12} \\ 4\,e_0\,e_1 &= d_{32} - d_{23} \end{aligned}\right\} \tag{2.37}$$

The special case of $e_0 = 0$, which corresponds to a rotation of 180° about the quaternion axis of rotation results in an ambiguity in the determination of the initial signs, and thus should be avoided as an initial condition.

The terms of the quaternion are used to generate the transformation matrix relating the chase vehicle body axes to the inertial axes. The quaternions are structured such that normalization after each step of the numerical integration assures orthogonality of the transformation matrix.

The terms of the transformation matrix are given by:

$$
\begin{aligned}
d_{11} &= e_0^2 + e_1^2 - e_2^2 - e_3^2 \\
d_{12} &= 2(e_1e_2 - e_0e_3) \\
d_{13} &= 2(e_1e_3 + e_0e_2) \\
d_{21} &= 2(e_1e_2 + e_0e_3) \\
d_{22} &= e_0^2 - e_1^2 + e_2^2 - e_3^2 \\
d_{23} &= 2(e_2e_3 - e_0e_1) \\
d_{31} &= 2(e_1e_3 - e_0e_2) \\
d_{32} &= 2(e_2e_3 + e_0e_1) \\
d_{33} &= e_0^2 - e_1^2 - e_2^2 + e_3^2
\end{aligned}
\tag{2.38}
$$

The transformation matrix relating the chase vehicle body axes to the local vertical axes, denoted as D_{31}, is determined using the relation:

$$D_{31} = D_{01}\,D_{30} \tag{2.39}$$

where D_{30} denotes the chase vehicle to inertial transformation defined by Equation (2.38) and D_{01} denotes the transformation from the inertial axes to the local vertical axes.

The transformation D_{01} is calculated as:

$$D_{01} = \begin{bmatrix} \text{Cos } \theta_I & \text{Sin } \theta_I & 0 \\ -\text{Sin } \theta_I & \text{Cos } \theta_I & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (2.40)$$

Other transformation matrices which are used in the solution of the equations of motion include the target vehicle to local vertical transformation, denoted D_{21}, which is calculated from the target vehicle Euler angles, and is constant for an earth oriented target vehicle. The transformation from the chase vehicle body axes to the target vehicle body axes, denoted D_{32}, is calculated as:

$$D_{32} = D_{12} D_{31} \qquad (2.41)$$

where D_{12} is the inverse of the D_{21} transformation matrix. Since the transformation matrices are orthogonal, the inverse is the transpose of the matrix.

Once the terms of the transformation matrix are determined, the Euler angles can be found by equating the terms of the matrix to trigonometric functions which form the matrix. For a Yaw-Pitch-Roll Euler angle sequence, this yields:

$$
\begin{aligned}
d_{11} &= \text{Cos } \psi \text{ Cos } \theta \\
d_{12} &= \text{Cos } \psi \text{ Sin } \theta \text{ Sin } \phi - \text{ Sin } \psi \text{ Cos } \phi \\
d_{13} &= \text{Cos } \psi \text{ Sin } \phi \text{ Cos } \phi + \text{ Sin } \psi \text{ Sin } \phi \\
d_{21} &= \text{Sin } \psi \text{ Cos } \theta \\
d_{22} &= \text{Sin } \psi \text{ Sin } \theta \text{ Sin } \phi + \text{ Cos } \psi \text{ Cos } \phi \\
d_{23} &= \text{Sin } \psi \text{ Sin } \theta \text{ Cos } \phi - \text{ Cos } \psi \text{ Sin } \phi \\
d_{31} &= -\text{Sin } \theta \\
d_{32} &= \text{Cos } \theta \text{ Sin } \phi \\
d_{33} &= \text{Cos } \theta \text{ Cos } \phi
\end{aligned}
\tag{2.42}
$$

where ϕ, θ, ψ denote the Roll, Pitch, and Yaw angles, respectively.

The pitch angle is found as:

$$\theta = \text{Sin}^{-1}(-d_{31}) \tag{2.43}$$

Assuming that the pitch angle is in quadrant 1 or 4, the Yaw and Roll angles can be determined as:

$$\psi = \text{Tan}^{-1}\left(\frac{d_{21}}{d_{11}}\right) \tag{2.44}$$

$$\phi = \text{Tan}^{-1}\left(\frac{d_{32}}{d_{33}}\right) \tag{2.45}$$

Since these two functions are sine over cosine functions, the quadrant of the angles can be determined. If the assumption that the pitch angle is in quadrant 1 or 4 is incorrect, the Yaw and Roll angles will show a 180 degree discontinuity from their previous values. By utilizing this check, the values of all three angles, including the quadrant, are determined.

3. Perturbations

The two perturbation sources which are considered are aerodynamics and gravity gradient. Aerodynamics are assumed to affect the orbital motion of both vehicles, and to generate moments acting on the target vehicle. Gravity gradient moments are assumed to act on both vehicles.

a. Aerodynamics - The atmospheric density is calculated using a simplified version of Jacchia's Model Atmosphere (Ref. 1). The simplifications consist of neglecting the geomagnetic activity correction, and of performing all calculations at zero latitude. The result is a density function which decreases approximately exponentially as a function of altitude, and includes the diurnal bulge as a function of longitude. The longitude of each spacecraft is assumed to be the same as the longitude of the reference orbit. This is a reasonable assumption, since a separation distance of 100 kilometers results in an error of less than one degree at a 100 km altitude. The orbit is assumed to be located at zero longitude at time zero, which corresponds to midnight, Greenwich Mean Time.

The aerodynamic disturbances on the target vehicle are generated using the six dimensionless aerodynamic coefficients CA, CY, CN, CL, CM, and CN1. The coefficients are converted to body forces and moments, in the aerodynamic axis system, using the following equations:

$$
\begin{aligned}
F_X &= -1/2 \ CA \rho V^2 \ \frac{\pi (D_r)^2}{4} \\
F_Y &= 1/2 \ CY \rho V^2 \ \frac{\pi (D_r)^2}{4} \\
F_Z &= 1/2 \ CN \rho V^2 \ \frac{\pi (D_r)^2}{4} \\
M_X &= 1/2 \ CL \rho V^2 \ \frac{\pi (D_r)^3}{4} + D_Y F_Z - D_Z F_Y \\
M_Y &= 1/2 \ CM \rho V^2 \ \frac{\pi (D_r)^3}{4} + D_Z F_X - D_X F_Z \\
M_Z &= 1/2 \ CN1 \rho V^2 \ \frac{\pi (D_r)^3}{4} + D_X F_Y - D_Y F_X
\end{aligned}
\qquad (2.46)
$$

where ρ is the atmospheric density, V is the orbital velocity, D_r is the reference diameter of the vehicle, and D_X, D_Y, D_Z are the separation distances from the aero axes origin to the c.g. of the vehicle. The resulting forces are transformed into the local vertical axis system for use in the equations of orbital motion, and the moments are transformed into the target vehicle body axis systems.

The aerodynamic coefficients are programmed as double fourier series functions of the angle of attack and roll angle. Data concerning the fourier series functions were supplied by NASA. The angle of attack and roll angle are calculated from the components of the velocity vector of the target vehicle in the target vehicle aerodynamic axis system.

For the chase vehicle, the aerodynamic drag force is calculated using the equation:

$$F_X = 1/2 \ CD \ A \rho V^2 \tag{2.47}$$

where CD is the drag coefficient and A is the frontal area of the vehicle. The chase vehicle is represented as a cylinder, and thus the frontal area can be calculated directly as a function of the vehicle attitude. Aerodynamic lift, side force, and moments are not calculated for the chase vehicle.

b. Gravity Gradient - Gravity gradient moments result from the center-of-gravity of an orbiting vehicle being located at a different point in the vehicle than the center of mass. The orbit of the c.m. is also affected by gradient moments, but this effect is very small and therefore only the rotational gravity gradient effects are considered.

The gravity gradient moments are calculated using the standard first order approximation, which yields:

$$\bar{M}_G = \frac{3\mu}{R^5} (\bar{R} \ X \ \bar{\bar{I}} \cdot \bar{R}) \tag{2.48}$$

where $\bar{M}_G$ denotes the moments, $\bar{R}$ is the vector from the center of the earth to the vehicle c.m., and $\bar{\bar{I}}$ is the inertia dyadic of the vehicle.

This equation is programmed using standard vector-matrix manipulation routines, and is used to calculate the gravity gradient moments for both the target and chase vehicles. The vector $\overline{R}$, which is known in the local vertical axis system, is transformed into the appropriate vehicle body axes for use in the gravity gradient calculations. The resulting moments are thus given in the vehicle body axes, which is the desired form.

4. Tether Forces and Moments

The tether is assumed to be a massless, perfectly flexible, spring-dashpot combination. With these assumptions, the tether force always acts along a straight line between the attach points on the two vehicles. The attach point on either of the vehicles may be input to the digital computer program as a constant or calculated by the tether control law module as a function of the problem dynamics. Applied forces are assumed to be transmitted instantaneously through the tether. In this case, the spring effect is used only to determine the stretched length of the tether, and the dashpot effect is not used. The main effect of the spring-dashpot combination is in those cases where the tether length, rather than the tether force, is the controlled parameter. The tether force is then a function of the tether length and retrieval rate, and of the separation distance and velocity between the two vehicles.

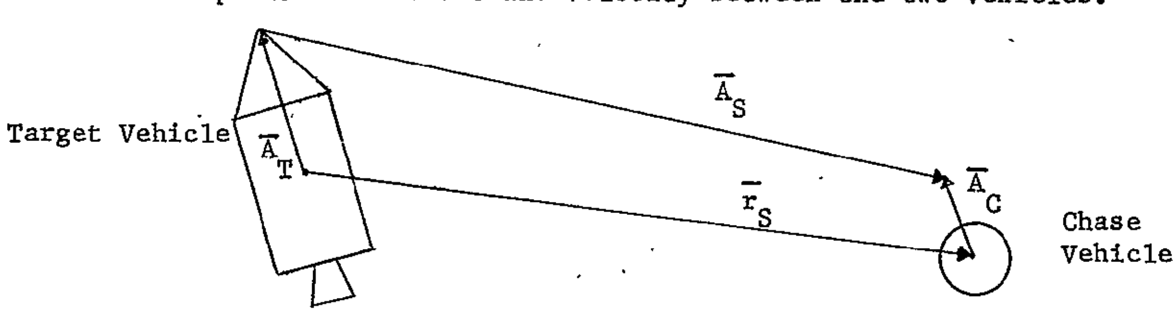


Figure. II-2 Tether Attachment Geometry

Referring to Figure II-2, the vector from the tether attach point on the target vehicle to the attach point on the chase vehicle, $\bar{A}_S$, is found as:

$$ {}_1\bar{A}_S = -{}_1\bar{A}_T + {}_1\bar{r}_S + {}_1\bar{A}_C \tag{2.49} $$

Since $\bar{A}_T$ and $\bar{A}_C$ are specified in the target and chase vehicle body coordinate systems respectively, it is necessary to transform these vectors to the local vertical axis system before the addition is performed.

$$ {}_1\bar{A}_T = D_{21} \ {}_2\bar{A}_T \tag{2.50} $$

$$ {}_1\bar{A}_C = D_{31} \ {}_3\bar{A}_T \tag{2.51} $$

The tether force acting on the target vehicle is then given as:

$$ \bar{F}_T = \underline{F_T} \ \frac{\bar{A}_S}{A_S} \tag{2.52} $$

where F_T is the magnitude of the applied force and A_S is the magnitude of $\bar{A}_S$. This force vector is expressed in the reference point local vertical axis system, which is the required form for the equations of orbital motion.

The tether force acting on the chase vehicle is the same magnitude but opposite direction of the force acting on the target vehicle. This force is therefore given by:

$$ \bar{F}_C = -\bar{F}_T \tag{2.53} $$

In order to determine the moments acting on each vehicle, in body coordinates, it is necessary to transform the force vector into the appropriate body coordinate system and form the cross product of the moment arm from the vehicle c.g. to the tether attach point and the force.

For the target vehicle the result is:

$$_2\overline{M}_T = \overline{A}_T \times (D_{12}\overline{F}_T) \tag{2.54}$$

and for the chase vehicle

$$_3\overline{M}_C = \overline{A}_C \times (D_{13}\overline{F}_C) \tag{2.55}$$

The tether control is calculated in a separate module of the digital computer program, allowing incorporation of any type of control law without modification to any other portion of the program. The control law module calculates the tether attach point on each of the vehicles, such as would be obtained using pivoted booms, and the magnitude of the tether force. The components of the force and the moments acting on the two vehicles are then calculated in a separate module of the program, using Equations (2.49) - (2.55).

Typical control laws which can be applied are:

a. Applying constant tension

b. Reel-in at a constant rate

c. Applying force (or controlling tether length) as a function of the state of the system

In general, the first two techniques will not result in feasible retrieval techniques for a wide range of initial conditions. The first method results in a constant acceleration of the chase vehicle relative to the target vehicle, while the second method can result in extremely high tether forces in order to maintain the constant reel-in rate. Also, neither of these two methods provides compensation for the build-up of relative angular momentum between the two vehicles.

The most practical techniques appear to be control of the tether force, or length, as a function of the state of the system, such as time, orientation of the spacecraft relative to the orbital frame, tether length, relative velocities, etc. A detailed description of the tether control law generated during the feasibility study is given in Section III of this report.

5. Digital Computer Program

The equations of motion defined in the previous sections of this chapter are solved using the digital computer program which was developed to meet the first contract objective. A brief description of this program is contained in Appendix A, and a detailed description, along with the program operating instructions, is contained in the Computer Program User's Manual (Ref. 2).

The important features of the program include the modular format, which allows various tether control laws to be incorporated into the program without affecting the logic or computational sequence of the remainder of the program. The program also contains the option of omitting the aerodynamic and gravity gradient perturbations. Inclusion of the aerodynamic perturbations increases the running time of the program by a factor of two, and thus this perturbation source should be included only for long term effects or for the final verification of a particular tether control law.

The compatibility of the program with the MSFC Univac 1108 digital computer was demonstrated by running five test cases at the MSFC computer facility. For all five cases, the results were in agreement with those obtained on the MMC CDC 64/6500 digital computer, with only minor differences attributable to the differences in the two machines.

SECTION III

FEASIBILITY STUDY

The objective of the feasibility study was to develop a control law applicable for the retrieval and docking of a supply package or experiment module to a Skylab or a Space Station. This control law was developed using the linear equations of orbital motion and was verified using the general computer program which solves the exact equations of motion. A listing of the control law module for the general computer program is contained in Appendix B.

1. Linear Equations of Orbital Motion

The linear equations which were used for the development of the control law are the familiar rendezvous equations with a constant aerodynamic drag acceleration included. These equations were developed in previous soft tether stationkeeping studies (Refs. 3, 5).

These equations are limited to circular reference orbits, but this does not seriously restrict the missions under consideration, since most long term space missions which would require retrieval and docking capabilities would be designed for circular orbits. The linear equations also assume a constant aerodynamic drag on each vehicle. Since the retrieval time is short and the aerodynamic forces are several orders of magnitude smaller than the tether forces, the aerodynamic effects are not significant and were omitted from most of the data runs. Therefore, the assumption of constant drag does not restrict the validity of the results. Verification of the control law with the general computer program confirms this conclusion.

The linearized equations of relative motion, with the constant aerodynamic drag and the tether forces included, are:

$$\ddot{X} - 2\omega\dot{Y} = d + \frac{F_X}{M} \qquad (3.1)$$

$$\ddot{Y} + 2\omega\dot{X} - 3\omega^2 Y = \frac{F_Y}{M} \qquad (3.2)$$

$$\ddot{Z} + \omega^2 Z = \frac{F_Z}{M} \qquad (3.3)$$

where d is the differential aerodynamic drag acceleration, F_X, F_Y, F_Z are the components of the tether force, and M is defined as

$$M = M_T M_C/(M_C + M_T) \qquad (3.4)$$

If the target vehicle mass is much larger than the chase vehicle mass, equation (3.4) can be replaced by using the chase vehicle mass.

Equations (3.1) - (3.4), and the equations of rotational motion defined previously, were used to develop the control logic.

2. Assumptions and Guidelines

One of the primary assumptions for the feasibility study was that the control law should be applicable for retrieval and docking of a supply package or experiment module. Therefore, it was necessary to specify initial conditions which would be applicable to this type of problem. Also, since actual docking mechanisms and post contact reactions were not investigated, it was necessary to speicfy final condition limits which were acceptable for docking. Table III-1 lists the initial and final condition limits which were specified for the feasibility study.

Table III-1 Retrieval and Docking Limits

Parameter	Initial Condition Limit	End Condition Limit
Range	30 to 50 Meters	Docked
Radial Velocity	Zero	0.305 m/sec
Tangential Velocity	Zero	0.152 m/sec
Relative Attitude	Non-Zero	10 degrees
Rotational Rates	1 deg/sec	10 deg/sec

Although no specific limits were placed on the initial relative attitude errors, they were held under 90 degrees, since anything greater leads to a condition where the tether can wrap around the chase vehicle. With this condition, the logic in the computer program assumes the tether passes through the chase vehicle and generates incorrect force and moment components.

The docked condition was achieved when the tether attach points on the two vehicles coincided. The control law drives the separation distance to zero.

3. Problem Definition

In the development of the tether control law, it was necessary to introduce an additional coordinate system, and to define several parameters concerning the chase vehicle position and attitude. The logic used in the development of these additional parameters is discussed in part four of this section of the report.

As illustrated in Figure III-1, the problem was divided into two separate control regions. In Control Region 1 the main objective is to build up the radial velocity, with only gross attitude control maintained. In Control Region 2, which begins at a range of 5 meters from the docking interface, the emphasis is on attitude control, with the objective being to drive the attitude errors to within the acceptable limits for docking.

The additional coordinate system, denoted as the A-E axis system, is also depicted in Figure III-1. This system is an Azimuth Elevation system, with the origin located at the base of the target vehicle boom. The X axis is aligned along the range vector to the c.g. of chase vehicle, the Z axis is the elevation axis, and the Y axis is placed to form a right hand system. Thus, the range between the two vehicles is given as the X component of the $\overline{R}$ vector in the A-E axis system, and the range rate is given as $\dot{X}$.

The chase vehicle attitude errors are defined relative to the A-E axis system. Thus, the objective of the control law is to align the chase vehicle body axes parallel with the A-E axes when docking is achieved. Two angles are used to define the attitude errors, rather than using the three conventional Euler angles. The first angle, referred to as the attitude angle and denoted θ_1, is the half-cone angle between the chase vehicle X axis and the A-E X axis. This angle, which is a combination of pitch and yaw, is used since the target vehicle boom cannot be positioned to allow independent control of the pitch and yaw angles.

The attitude angle is determined by first transforming a unit vector along the chase vehicle X axis into the A-E axis system. Denoting the components of this transformed unit vector as X_1, Y_1, Z_1, the mangitude of

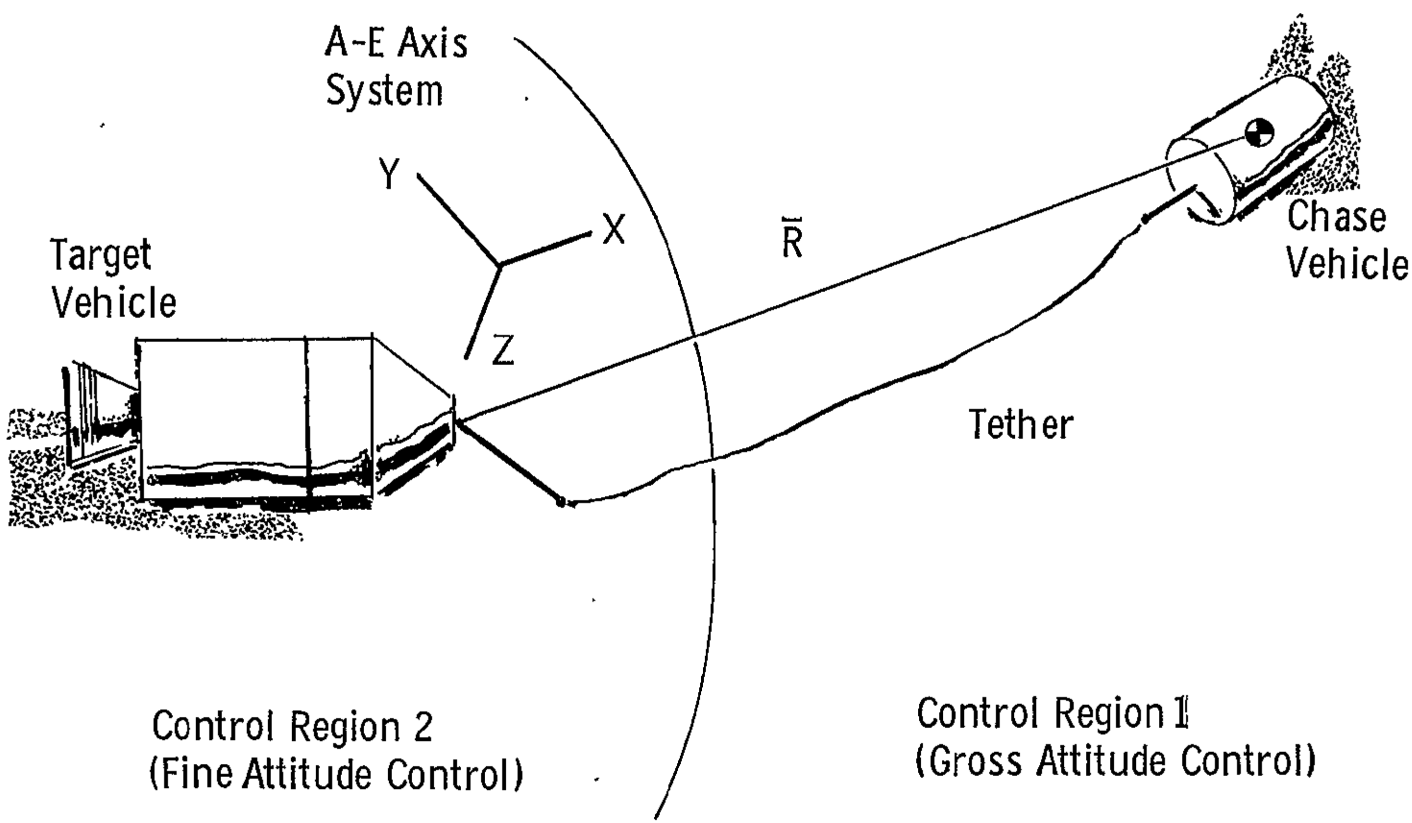


Figure III-1 Control Law Geometry

θ_1 is given as:

$$\theta_1 = \operatorname{Sin}^{-1}\left(\sqrt{Y_1^2 + Z_1^2}\right) \qquad (3.5)$$

The two components Y_1 and Z_1 define the location of the chase vehicle X-axis on the cone, and are used in the control logic for positioning the target vehicle boom.

The second angle which is used in the control logic, denoted as ϕ_1, defines the rotation of the chase vehicle about the A-E X-axis with a zero attitude angle. This angle, referred to as the roll angle is determined by transforming a unit vector along the chase vehicle Y axis into the A-E axis system.. Since the objective of the control law is to align the two coordinate systems, the roll error is defined as:

$$\phi_1 = \operatorname{Sin}^{-1} Z_2 \qquad (3.6)$$

where Z_2 is the Z component of the transformed unit vector. Thus, with a zero attitude angle, and a zero roll angle, the two coordinate systems will be parallel.

4. Control Law Development

The initial effort in the development of the control logic was directed towards planar motion of the chase vehicle, coupled with one degree of rotational motion. This simplified case was chosen since it would allow investigation of control of the radial and tangential velocity in the orbit plane, and would also provide a preliminary investigation of the attitude control problem.

Since positive control can only be applied to the separation distance between the tether attach points on the two vehicles (i.e. only radial pulls can be applied with a flexible tether), the docking interfaces are located at the tether attach points on the two vehicles. This assures that separation distance between the two vehicles can be driven to zero.

Using only the tether force, it was not possible to maintain the tangential velocity within acceptable limits. With the chase vehicle initially behind the target vehicle, the tether force adds energy to the chase vehicle orbit, causing it to rise. Conversely, with the chase vehicle initially ahead of the target vehicle, the tether force removes energy from the chase vehicle orbit, causing it to drop. This rising (or dropping) effect results in a high tangential velocity when the separation distance is reduced to zero.

A target vehicle boom allows compensation to be included for control of the tangential velocity. With the chase vehicle behind the target vehicle, the boom can be positioned down so that the tangential component of the applied tether force pulls the chase vehicle down, which tends to offset the rising effect of the increased energy orbit. Conversely, with the chase vehicle ahead of the target vehicle, the boom can be positioned up to pull the chase vehicle up, thus offsetting the dropping effect of the decreased energy orbit. In this manner, the radial velocity can be increased while maintaining the tangential velocity at a level acceptable for docking. Thus, a target vehicle boom with an elevation axis provides a simple means of controlling the chase vehicle tangential velocity.

Control of the chase vehicle attitude poses a more difficult problem. The first step which is necessary is to attach the tether to a fixed boom on the target vehicle, so that the applied tether force generates moments of sufficient magnitude to allow attitude control to be accomplished. With the docking interfaces located at the tether attach points, this means that docking will be achieved when the tip of the chase vehicle boom reaches the tip of the target vehicle boom. Therefore, the attitude is controlled relative to the vector from the base of the target vehicle boom to the c.g. of the chase vehicle. With zero attitude error, the chase vehicle X axis will then be aligned along this vector, and docking can be accomplished by positioning the target vehicle boom to lie along the same vector when the separation distance is reduced to zero.

If a constant force is applied to reduce the separation distance to zero, the attitude error will oscillate, and a small attitude error at docking can be assured only if the initial attitude error is small. For this reason, a pull and coast mode of control was adopted. This technique requires the force to be applied as a discrete function of time, and allows large initial attitude errors to be reduced to small values when docking is achieved.

Forces applied to control the attitude error also increase the radial velocity. If the force required for attitude control is minimized by positioning the target vehicle boom such that the applied tether force generates the maximum moments (e.g. if the chase vehicle is rotated down, the boom is positioned up), the control of the tangential velocity is affected.

In order to overcome these problems, the docking maneuver was divided into the two control regions previously described. In Control Region 1, which is that region beyond 5 meters from the docking interface, the target vehicle boom is positioned to control the tangential velocity of the chase vehicle. Force is initially applied to increase the closing velocity to 0.1 meters per second. This velocity level assures that time required to complete the docking maneuver will not be excessive, and allows additional force to be applied for attitude control without increasing the radial velocity to unacceptable values.

In Control Region 1, force is applied for attitude control only if the attitude error exceeds 45 degrees. This limit was chosen for two reasons. With this limit, the force for attitude control is applied for a short time duration, since relatively large moments will be placed on the chase vehicle (i.e. the angle between the chase vehicle boom and the tether will be large), and thus the radial velocity increase will be small. This limit also results in an attitude error which can be reduced to near zero in Control Region 2. In other words, if a lower limit is used, excessive force is applied for attitude control which increases the radial velocity to an unacceptable value, and if a higher limit is used, the final attitude error cannot be corrected in Control Region 2.

If the attitude error exceeds the 45 degree limit, force is applied until the error is decreasing at 3 deg/sec. The chase vehicle then coasts at this rate until the attitude error reaches 45 degrees in the opposite direction. Thus, if no additional force is applied, 30 seconds elapse between the applications of force for attitude control in Control Region 1.

This spacing of the tether force maintains the radial and tangential velocities at levels such that the force applied for the final attitude correction results in final values suitable for docking.

In Control Region 2, the emphasis is on attitude control of the chase vehicle, with the objective of driving the attitude error to zero when docking is achieved. Therefore, the boom is positioned up or down as required, to result in the generation of the maximum moments from the applied tether force. Since the range between the two vehicles is less than 5 meters in Control Region 2, the target vehicle boom can be positioned to place the tether at a large angle relative to the chase vehicle boom even with small attitude errors, which results in the generation of moments sufficiently large to accomplish fine attitude control. In this region, an attitude error of six degrees will cause force to be applied for attitude control.

In both control regions, the force required to stop the rotational motion is calculated if the attitude errors are within acceptable limits. A minimum limit of zero and a maximum limit of the maximum allowable tether force is applied to this calculation. The objective of this calculation is to reduce the force required for attitude control. If the rotational rate can be reduced to a small value, the attitude error limits will not be exceeded as soon, which reduces the force required for attitude control. This aids in maintaining the radial velocity at a level acceptable for docking.

With only one attitude error to control, this calculation results in a force less than the maximum level in a majority of cases, and reduces the rotational rate to a small value. The maximum force is selected to provide an acceptable combination of translational and rotational accelerations, and thus is a function of the chase vehicle mass, inertia, and tether attach point.

This combination of target vehicle boom position control and tether force control resulted in a successful three-degree-of-freedom technique as illustrated by a number of test cases. The tether force control logic results in a pull and coast mode of control, wherein tether force is applied only if the radial velocity or attitude error exceeds the specified bounds.

Expansion of the control law to the five-degree-of-freedom case involved inclusion of the out-of-plane (Z) translational motion and expanded the attitude control problem to include both pitch and yaw. The attitude angle controlled is the half cone angle defined previously.

Control of the out-of-plane translational motion posed no significant problems, since this motion is essentially uncoupled from the orbital effects for the time duration required to complete the docking maneuver. The tether force pulls the chase vehicle directly towards the target vehicle boom in the Z direction.

The target vehicle boom position control was expanded to include azimuth angle control to allow control of the chase vehicle attitude. In Control Region 1, the boom elevation is maintained at $\pm$ 45 degrees for control of the tangential velocity, and the azimuth angle is positioned at $\pm$ 45 degrees as required for attitude control. In Control Region 2, both the boom azimuth and elevation angles are independently positioned at $\pm$ 45 degrees, such that the maximum moments are generated on the chase vehicle from the applied tether force. The $\pm$ 45 degree positions were chosen in order to prevent large elevation angles which reduce the azimuth angle effects. For example, with a 90 degree elevation angle, the azimuth angle has no effect on the boom tip position.

With the inclusion of the chase vehicle roll motion, several changes are necessary in the control law. A three position pivot is necessary on the chase vehicle boom to allow generation of roll moments on the chase vehicle. With three positions, the boom can be oriented so that the tether force generates either positive, negative, or zero roll moments on the chase vehicle. Without the ability to move the tether attach point off the chase vehicle X axis it is impossible to generate chase vehicle roll moments with the flexible tether.

With all three rotational modes, and the resulting coupling between axes, it was found necessary to reduce the attitude rate limit in Control Region 1 to one deg/sec for control of the attitude error and 1/2 deg/sec for roll control. These rate limits prevented the force applied for attitude control for increasing the radial velocity to an unacceptable level. It was also necessary to observe rate limits of 3 deg/sec during the application of force for the initial buildup of the radial velocity and for control of the roll motion. Without these limits, the rates would increase to high levels and the force required to correct the rotational motion would result in an excessive radial velocity.

These rate limits reduce the force applied for attitude control, resulting in a slower closing velocity and a corresponding increase in the time required to complete the docking maneuver. This increased time results in a higher tangential velocity caused by the orbital effects which are time dependent. For this reason, the radial velocity is increased from 0.1 m/sec to 0.15 m/sec at a range of 10 meters from the docking interface. This range is in Control Region 1, and thus the target vehicle boom elevation angle is positioned to reduce the tangential velocity.

The target vehicle boom positioning logic is essentially the same as that used in the five-degree-of-freedom case. This logic, coupled with the chase vehicle boom positioning logic, and the tether force logic, allows six-degree-of-freedom control of the chase vehicle.

The control logic applies the tether force as a discrete function of time, which results in a pull and coast mode of control. Since there are only four independent control variables, (the two components of the target vehicle boom position, the chase vehicle boom position, and the tether force), exact control cannot be applied to all of the 12 dependent variables. Thus, the three translational positions are controlled by driving the separation distance to zero, and bounds are placed on the linear velocities, the attitude errors, and the rotational rates.

5. Six-Degree-of-Freedom Control Logic

Four separate sets of control logic are used to accomplish the six-degree-of-freedom docking maneuver. The logic controls the target vehicle boom azimuth angle, the target vehicle boom elevation angle, the chase vehicle boom position, and the magnitude of the applied tether force.

a. Target Vehicle Boom Control Logic - Figure III-2 illustrates the logic used in both control regions to control the target vehicle boom azimuth angle. This angle is calculated relative to the A-E axis system. Thus, at zero azimuth angle, the boom will be contained in the A-E X-Y plane.

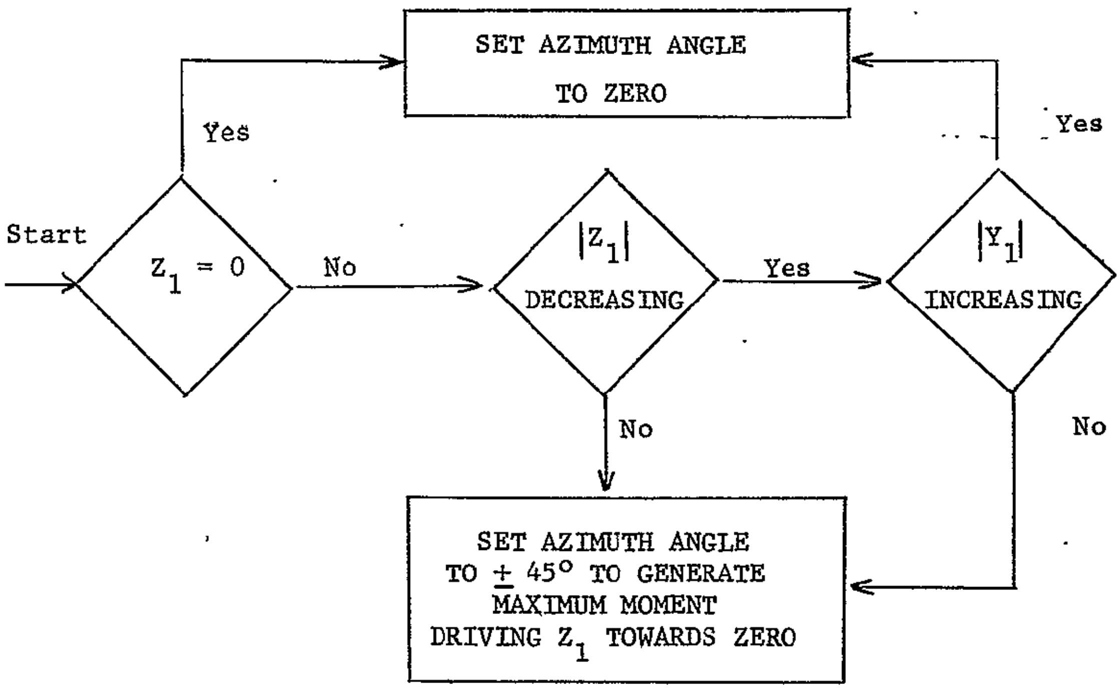


Figure III-2 Target Vehicle Boom Azimuth Angle Control Logic

The two parameters, Y_1 and Z_1, which were used previously to define the attitude angle, specify the rotation of the chase vehicle X axis relative to the A-E X axis. Thus, the Y_1 component is related to rotation of the chase vehicle about the A-E Z axis (elevation), and the Z_1 component is related to rotation about the A-E Y axis (azimuth).

The first check tests Z_1 for a zero condition. If Z_1 is zero, the chase vehicle X axis is contained in the A-E X-Y plane. Therefore, the boom azimuth is set to zero to position the boom in the X-Y plane, which keeps Z_1 at zero when tether forces are applied.

If Z_1 is non-zero and is increasing in magnitude, the boom azimuth is positioned right or left 45 degrees to generate the maximum moment from the tether force driving Z_1 towards zero. For example, if the chase vehicle is rotated to the right, the target vehicle boom is positioned to the left. This generates the maximum separation distance between the tips of the booms on the two vehicles in the Z direction, and thus applied tether forces will generate the maximum effect on the Z_1 component of the attitude error.

If Z_1 is non-zero and decreasing in magnitude, Y_1 is checked. If Y_1 is increasing in magnitude, the azimuth angle is set to zero to generate the maximum effect on the Y_1 component of the attitude angle. If both Z_1 and Y_1 are decreasing, the azimuth angle is set to $\pm$ 45 degrees to generate the maximum effect on Z_1. This, coupled with the elevation angle control, will cause the applied tether forces to continue to drive the attitude angle towards zero.

For the target vehicle boom elevation angle, different control logic is applied in Control Region 1 than in Control Region 2. In Control Region 1 the prime objective is to build up the radial velocity while maintaining the tangential velocity at an acceptable level. The target vehicle boom elevatio angle is therefore positioned up or down 45 degrees dependent on whether the chase vehicle is ahead of the target vehicle or behind it. This offsets the buildup of tangential velocity caused by the orbital effects and the tangential velocity at docking is maintained within the specified limit.

In Control Region 2, the target vehicle boom elevation angle control logic is similar to that used for the azimuth angle. As illustrated in

Figure III-3, the two parameters Y_1 and Z_1, which define the chase vehicle attitude angle, are used in this logic.

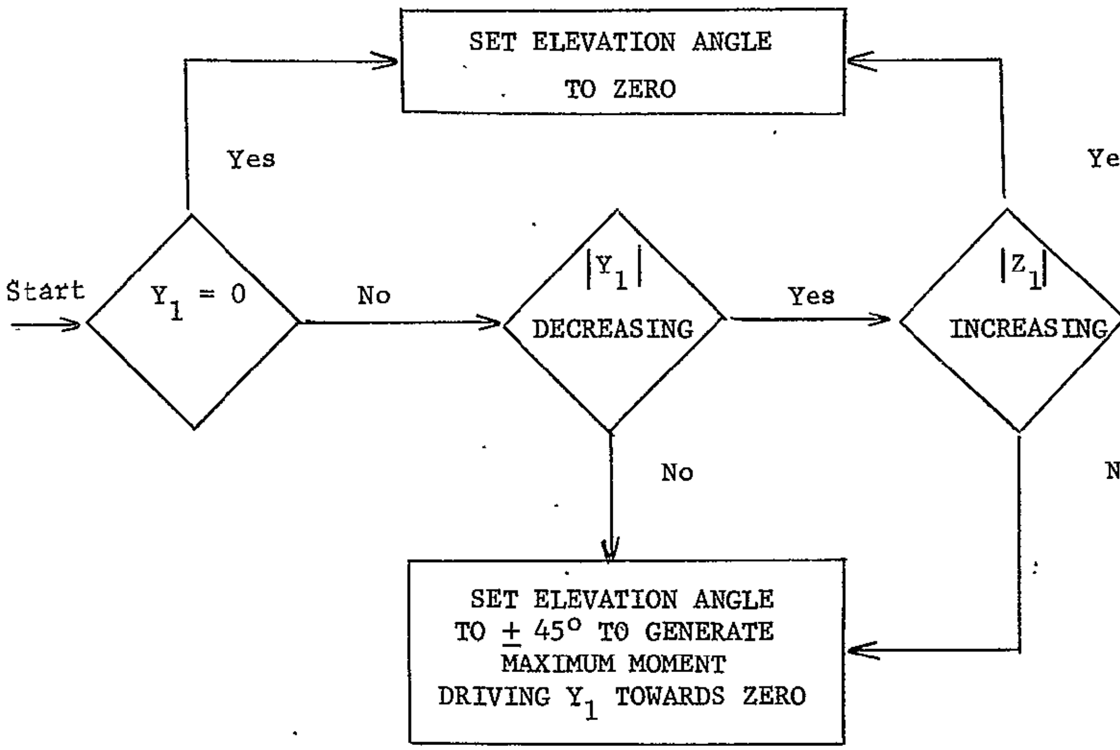


Figure III-3 Target Vehicle Boom Elevation Angle Control Logic - Control Region 2

The first check tests Y_1 for a zero condition. If Y_1 is zero, the chase vehicle X axis is contained in the A-E X-Z plane. Therefore, the target vehicle boom elevation angle is set to zero, which places the boom in the A-E X-Z plane and the applied tether forces will not affect the Y_1 component of the cone angle.

If Y_1 is non-zero and is increasing, the target vehicle boom elevation angle is positioned at $\pm$ 45 degrees so that the tether force generates the maximum moment driving Y_1 towards zero. For example, if the chase vehicle is rotated up, which will generate a positive Y_1 component, the target vehicle boom elevation is positioned down 45 degrees. This results in the maximum separation distance between the tips of the two booms, and thus the tether force generates the maximum moment driving Y_1 towards zero.

If Y_1 is non-zero and decreasing, and Z_1 is increasing, the boom elevation angle is set to zero for generation of the maximum effect on Z_1 from the applied force. If both Y_1 and Z_1 are decreasing, the elevation angle is positioned at $\pm$ 45 degrees to keep both variables decreasing when tether force is applied.

The combination of the target vehicle boom azimuth and elevation control laws allows positioning of the applied tether force to control the radial velocity, the tangential velocity, and the chase vehicle attitude angle relative to the A-E X axis. The azimuth and elevation angles are both set to zero in Control Region 2 only if the Y_1 and Z_1 components of the attitude angle are both zero, which corresponds to zero attitude error.

b. Chase Vehicle Boom Control Logic - The chase vehicle boom is mounted on a three position pivot, and can be set at $\pm$ 30 degrees relative to the center position to allow generation of moments to control the roll motion of the chase vehicle.

The boom is normally set at either the $\pm$ 30 degree position dependent on the sign of the roll error. The selection of the + 30 or - 30 degree position is determined by checking the sign of the moment which is generated. For example, if the roll error is positive, the chase vehicle boom is positioned to generate a negative moment from the applied tether force, thus driving the roll error towards zero.

Table III-2 lists the limits used to set the chase vehicle boom at the zero position. If any one of the conditions listed in the table is met, the boom is set at the zero position, which will nominally align the boom along the chase vehicle X axis. Thus, the application of tether force will not generate and the roll motion will not be affected. As shown in the table, tigher limits are used in Control Region 2 than in Control Region 1, since the emphasis is on attitude control in the second region.

Table III-2 Limits for Chase Vehicle Boom Zero Position

Condition	Control Region 1	Control Region 2
Roll Error Greater Than and Decreasing at Greater Than	20° 1 deg/sec	6° 2 deg/sec
Roll Error Less Than	20°	2°
Roll Error Less Than and Roll Rate Less Than	-- --	6° 1 deg/sec

These limits are not the limits which are used to determine if force should be applied to control the roll motion. These limits are used to position the chase vehicle boom so that if force is applied for any reason the roll motion will be controlled. Thus, the chase vehicle roll motion can be affected if force is applied to control the radial velocity or the attitude error, as well as if the force is applied directly for roll control.

c. Tether Force Control Logic - The tether provides all of the forces and moments used to complete the docking maneuver. Separate logic is used for control of the tether force in each of the two control regions.

Figure III-4 illustrates the control logic used to determine if tether force should be applied in Control Region 1. Three options are available in the logic. The path through the logic can lead to the return, with the tether force set to zero; the tether force can be set to the maximum allowable value (TF = FMX); or the tether force can be set to a calculated value less than the maximum value. In general, if all of the parameters are within acceptable limits, the logic path will lead to the return, with the tether force at zero. If any one of the variables is outside of the prescribed limits, the tether force will be set to a non-zero value.

Since the prime emphasis in Control Region 1 is control of the radial velocity, only gross attitude control is maintained. Thus, tether force is applied to control the attitude error (θ_1) or the roll error (ϕ_1) only if these errors are greater than 45 degrees. Two rate limits are used in the logic. The first limits the attitude rate ($\dot{\theta}_1$) to a maximum value of three degrees per second. The second limits the chase vehicle body rates (ω_j) to a maximum of three degrees per second. Two rate limit checks are required since it is possible to have a chase vehicle body rate greater than the limit even though the attitude rate (cone angle rate) is within the limit.

If the attitude error is increasing at greater than three degrees per second, the tether force required to stop the attitude rate is calculated, with a minimum limit of zero, since the tether cannot push, and a maximum

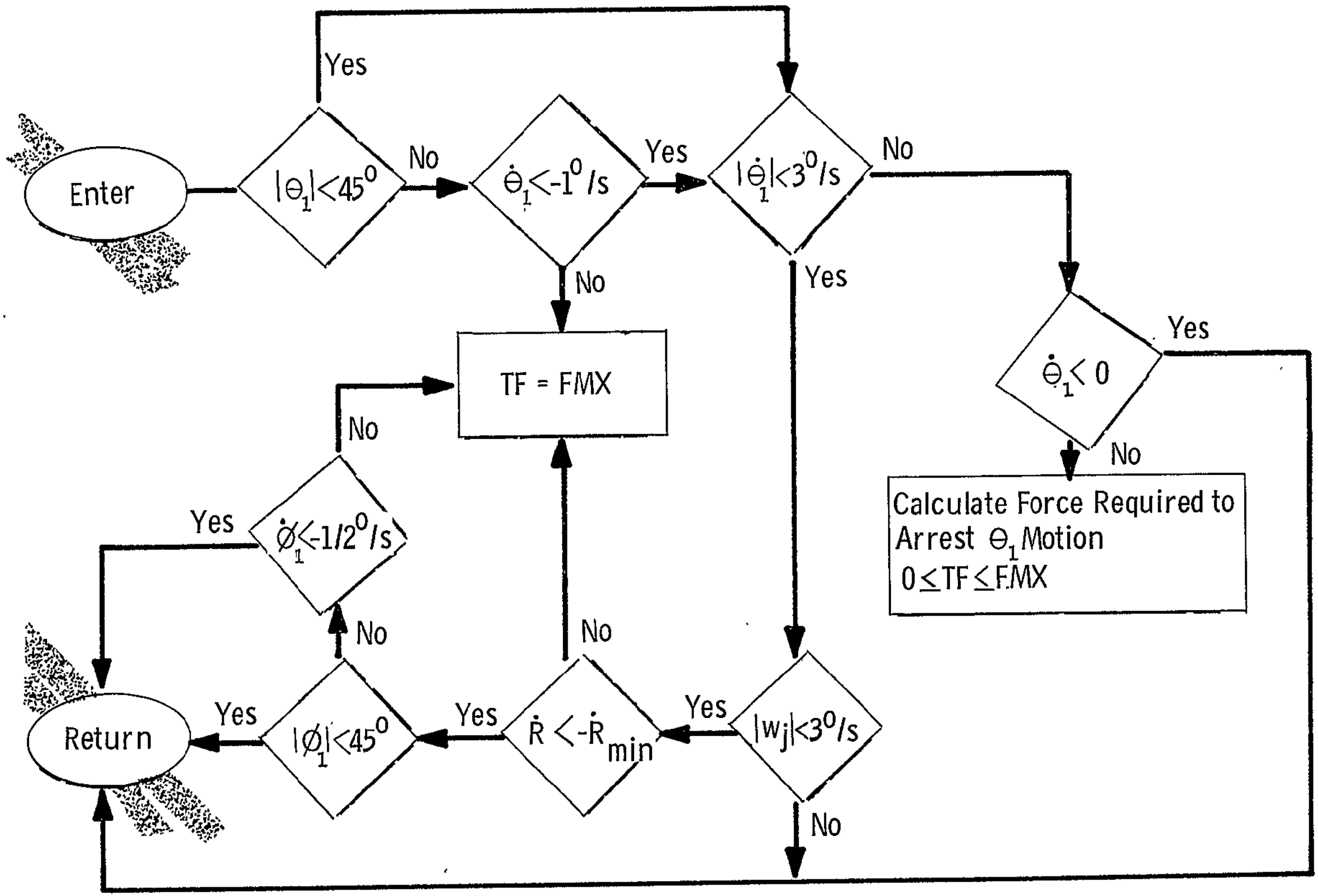


Figure III-4 Tether Force Control Law-Control Region 1 limit of the maximum allowable tether force. The result of this calculation is normally the maximum limit, due to the three deg/sec rate limit which must be exceeded before this calculation is performed.

The primary function of the logic in Control Region 1 is control of the radial velocity ($\dot{R}$). The radial velocity is initially increased to 0.1 meters per second, bringing the two vehicles together. At a range of 10 meters from the docking interface, the radial velocity is increased to 0.15 meters per second. Thus, unless the rotational rate limits are continually exceeded, the chase vehicle will be approaching the target vehicle at a rate of at least 0.15 meters per second when entering Control Region 2.

Figure III-5 illustrates the logic used to determine if tether force should be applied in Control Region 2. The same three options exist as in Control Region 1; return with the tether force at zero; set the tether force to the maximum allowable value; or set the force to a calculated value less than the maximum. The general flow of the logic results in zero force if all of the variables are within acceptable limits, and sets the force to a finite value if any one variable is outside the prescribed limits.

Since the emphasis is on attitude control in Control Region 2, the attitude rate ($\dot{\theta}_1$) is the first parameter which is checked. The preferred condition is that the attitude error is decreasing at a rate greater than the prescribed minimum value ($\dot{\theta}_m$). The rate limit is either 1.25 deg/sec or 3.25 deg/sec dependent on the value of the attitude error. If the attitude error is greater than 22.5 degrees, the upper limit is used.

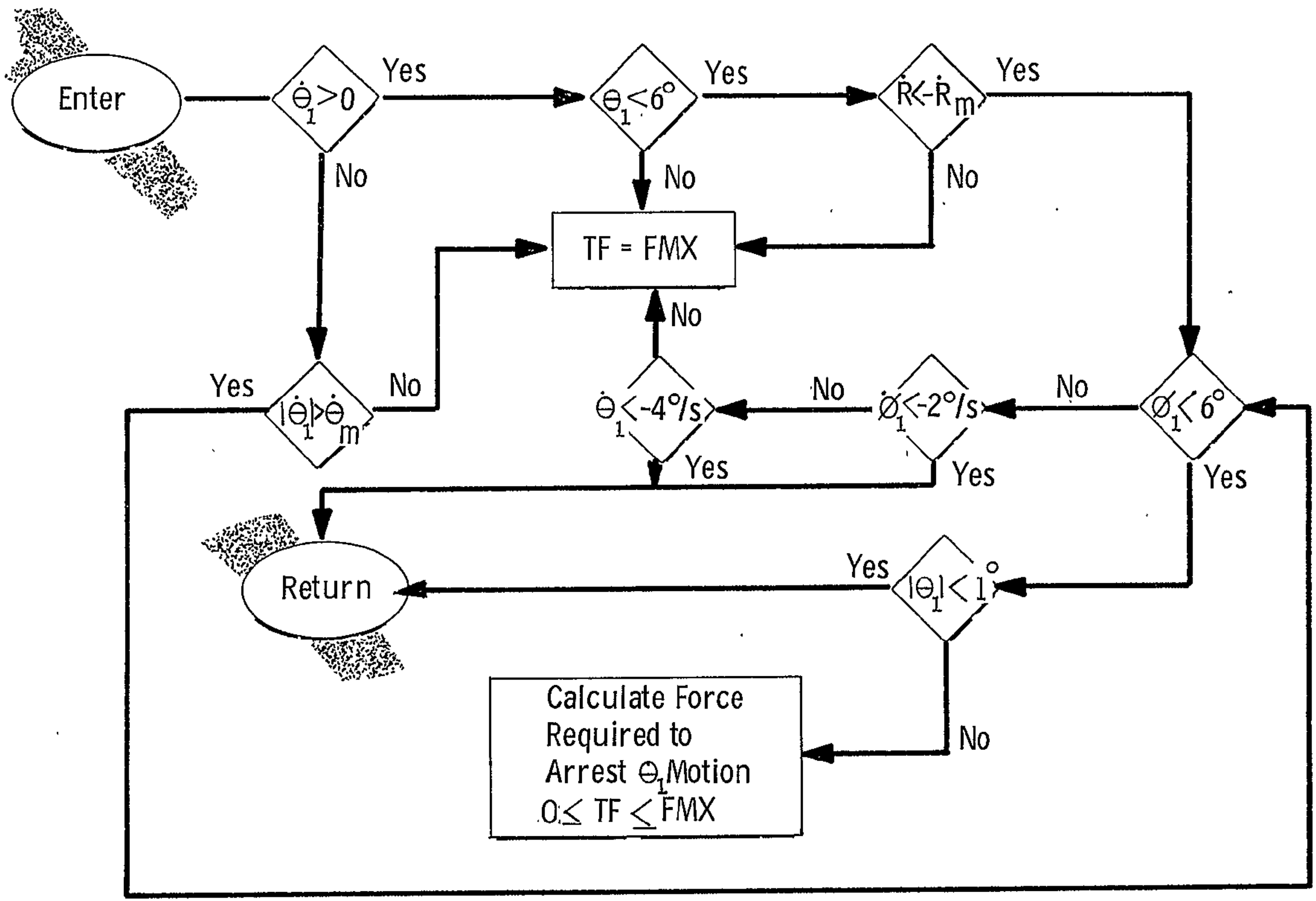


Figure III-5 Tether Force Control Law-Control Region 2

If the attitude rate is acceptable, the roll error is checked. If force is to be applied for control of the roll motion, the roll rate and the attitude rate are checked to assure that unacceptable limits are not reached.

If the roll error is less than six degrees and the attitude error is greater than one degree, the force required to arrest the attitude rate is calculated, with the limits of $0 \leq TF \leq FMX$ maintained. In Control Region 2, this calculation can result in tether forces less than the maximum, which will reduce the attitude rate to a small value.

The remainder of the logic sets the tether force to the maximum value if the attitude rate is greater than zero and the attitude error is greater than six degrees, or if the range rate drops below the minimum value.

Thus, with the logic controlling the target vehicle boom, the chase vehicle boom, and the tether force, six-degree-of-freedom control of the chase vehicle is maintained during the docking maneuver. The magnitudes of the control parameters used and the results which were obtained with this control law are discussed in the next section of this report.

SECTION IV

FEASIBILITY STUDY RESULTS

In order to test the validity of the control logic, 39 data cases were run using the linear equations of motion and 8 data cases were run using the general equations of motion. All of the data cases were run in a low earth (352 kilometer) circular orbit, since the orbital effects on the tangential velocity are more significant at low altitude. The initial separation distance between the two vehicles was normally set near 30 meters, since this was the maximum range specified, and thus represents a worst case condition for control of the tangential velocity. The initial chase vehicle attitude and rotational rates were chosen at random, with maximum attitude errors of 70 degrees and maximum body rates of 1.25 deg/sec. The final conditions for all of these cases were within the limits specified for a successful docking.

1. Control Parameter Magnitudes

For the majority of the test cases, the chase vehicle was assumed to be cylindrical, with equal moments of inertia about the pitch and yaw axes. For use in the general equations of motion the Skylab mass and inertia data was used for the target vehicle. The mass and inertia characteristics of these two vehicles is given in Table IV-1.

A boom length of one meter is used on the chase vehicle. The boom pivot is located one meter from the center of gravity, along the chase vehicle X axis. Thus, a maximum moment arm of 2.0 meters is used for control of the attitude error and a maximum moment arm of 0.5 meters is used for control of the roll error.

Table IV-1 Vehicle Mass and Inertia Data

Parameter	Chase Vehicle	Target Vehicle
Mass (kilograms)	351	41592
Roll Inertia (kg-m^2)	58	341900
Pitch Inertia (kg-m^2)	161	4597600
Yaw Inertia (kg-m^2)	161	4746700

The target vehicle boom length is three meters. This length assures that the tangential velocity control can be maintained in Control Region 1. For example, with the chase vehicle behind the target vehicle and the boom elevation positioned down, the tangential component of the applied force will always pull the chase vehicle down. A longer boom length provides more control of the tangential velocity, with three meters being the minimum length for control of the chase vehicle previously defined.

The maximum tether force is set at 3.5 newtons, which results in a linear acceleration of 0.01 m/sec^2. The resulting maximum rotational accelerations, which occur when the tether is at right angles to the chase vehicle boom, are 2.5 deg/sec^2 for pitch and yaw, and 1.75 deg/sec^2 for roll. A higher force level results in excessive radial velocity from the attitude control functions or excessive rotational rates from the radial velocity control, and a lower force reduces the rotational accelerations such that the attitude errors cannot be corrected.

2. Ideal Case

The first case investigated, referred to as the ideal case, involved use of the linear equations of orbital motion, and the cylindrical chase vehicle, with the boom pivot located on the X axis. The boom base on the target vehicle is located at the vehicle center of gravity. While this is physically impossible, it does not place any restrictions on the results, since the boom can be moved out to the target vehicle surface, and can still be positioned to apply the force in the desired direction relative to the base of the boom.

a. Chase Vehicle Behind Target Vehicle - Figure IV-1 illustrates a typical retrieval trajectory for the ideal case with the chase vehicle behind the target vehicle. The parameter plotted is the chase vehicle c.g. position relative to the c.g. of the target vehicle, and thus docking occurs at a range of 5 meters.

The plot of the in-plane motion illustrates the increase in the tangential velocity caused by the orbital effects. The initial tether force applied to start the chase vehicle moving towards the target vehicle pulls the chase vehicle down in the Y direction. The chase vehicle then coasts to a range of 15 meters, with two short duration tether pulls used for attitude corrections. During this coasting phase, the chase vehicle begins to rise. At a range of 15 meters, force is applied to increase the radial velocity to 0.15 m/sec which decreases the rising effect. Upon entering Control Region 2 at a separation distance of 10 meters, the force applied for the final attitude corrections pulls the chase vehicle down towards the target vehicle, and results in a final tangential velocity of 0.11 m/sec.

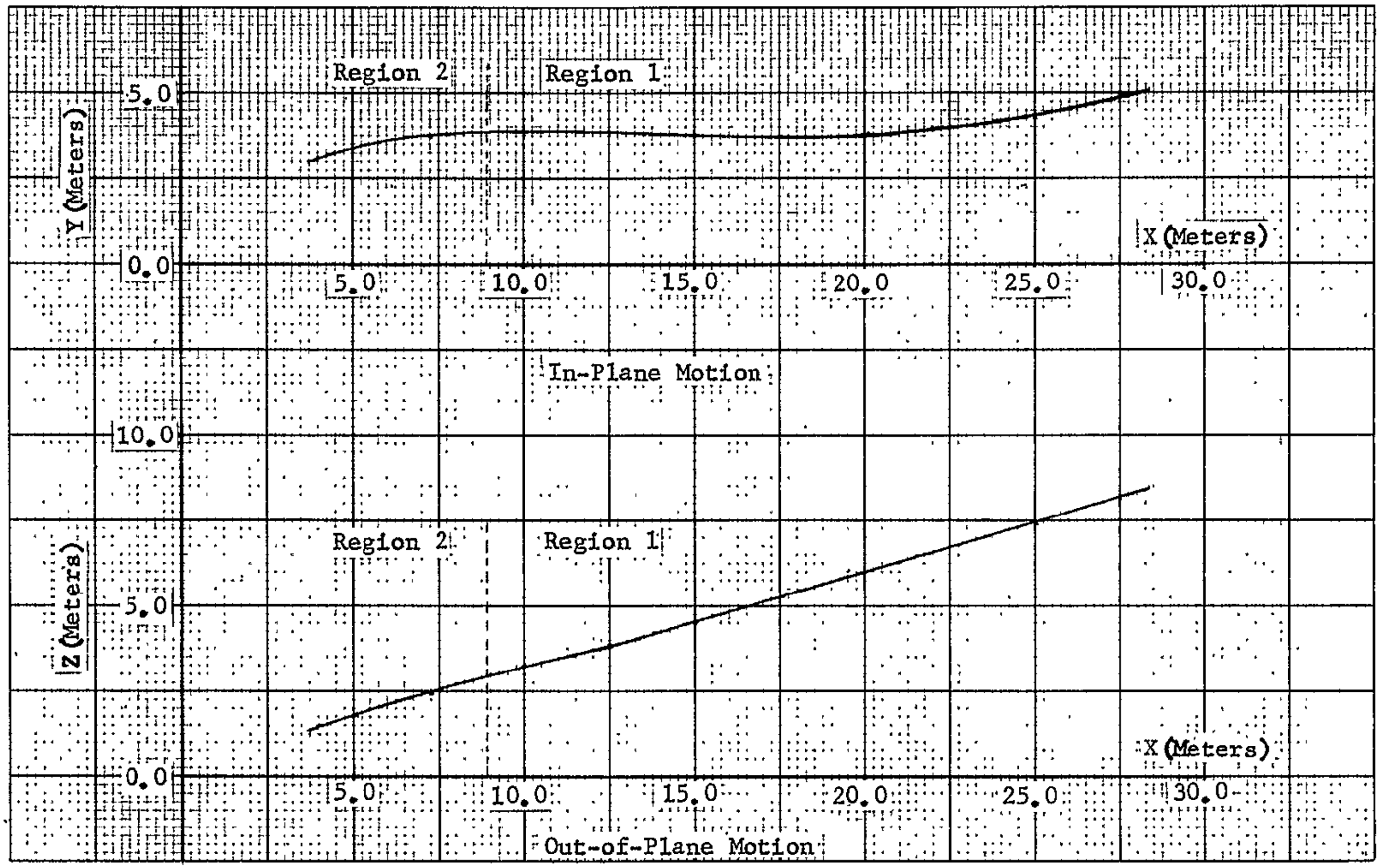


Figure IV-1 Retrieval Trajectory-Run Number 15

The plot of the out-of-plane (Z) motion illustrates the fact that this motion is not coupled with the orbital effects. This component of the range vector tends to come straight in towards the origin, with only minor deviations in Control Region 2, caused by the target vehicle boom positioning required for attitude control.

Figure IV-2 is a plot of the rotational motion as a function of time for this retrieval trajectory. The initial attitude error of 26 degrees is decreased to near zero at 10 seconds as force is applied to increase the radial velocity. The initial tether pulls cause the rotational rate limits to be exceeded, so that two pulls are required to achieve the desired radial velocity of 0.1 m/sec. The second pull reduces the attitude rate to approximately one deg/sec. The attitude error increases at this rate until the 45 degree limit is exceeded. This occurs after 49 seconds have elapsed from the start of the retrieval maneuver. At this time force is applied which causes the attitude error to decrease at one deg/sec. The chase vehicle then coasts until the attitude error again reaches 45 degrees at 140 seconds, when force is again applied. At 150 seconds, force is applied to increase the radial velocity, which increases the rotational rates above the three deg/sec limit. At 174 seconds the desired radial velocity of 0.15 m/sec is achieved, which causes the attitude error to decrease.

Control Region 2 is entered at 186 seconds, at which time the attitude error is rapidly driven towards zero, with a final value of 6.8 degrees when docking is achieved. The total time required to complete the docking maneuver is 212 seconds.

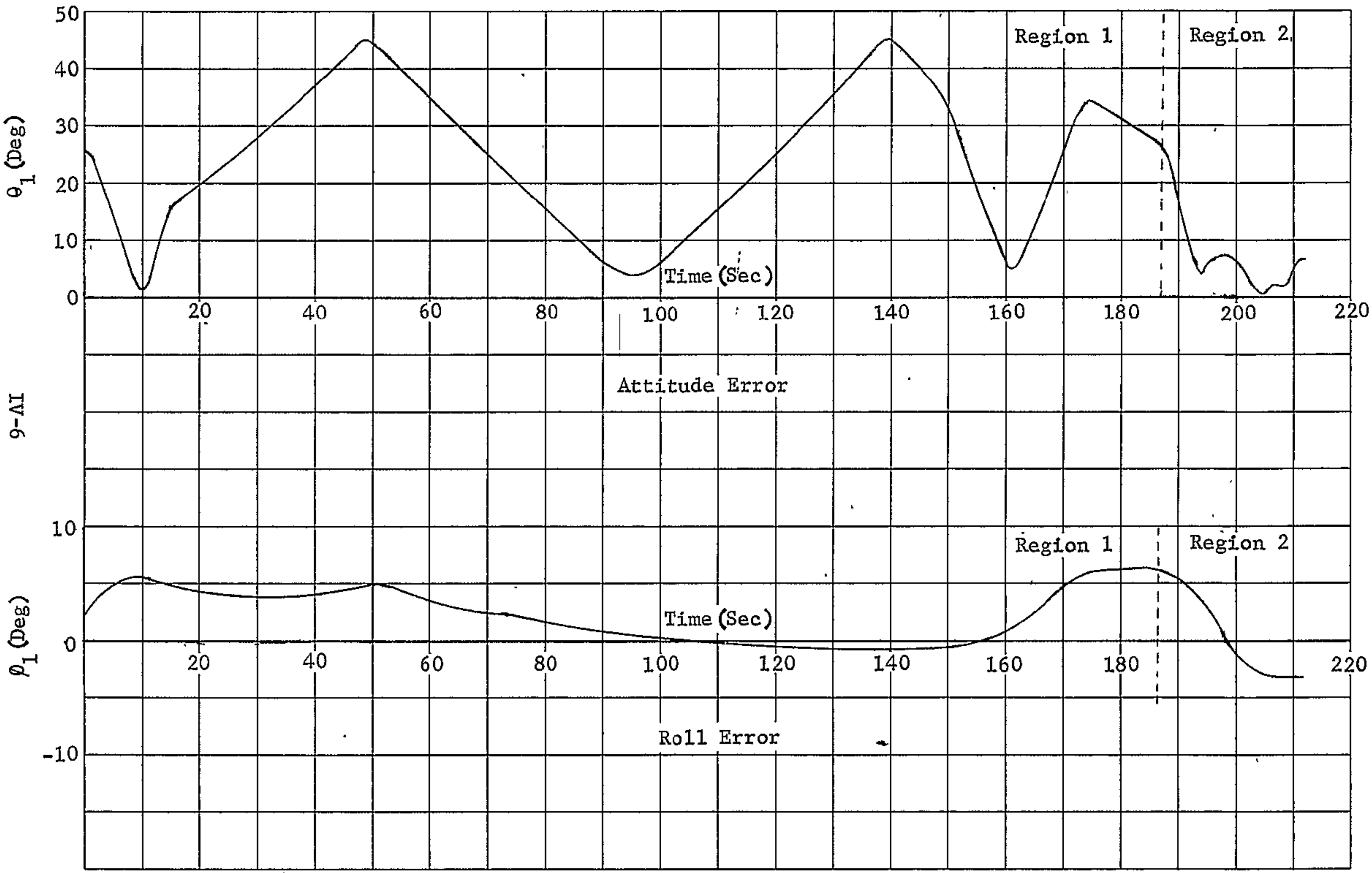


Figure IV-2 Rotational Motion-Run Number 15

No control is applied to the roll motion until Control Region 2 is entered, since the roll error remains small. In Control Region 2, the roll error is reduced to a final value of -3.1 degrees when docking is achieved.

Figure IV-3 illustrates the tether force and the target vehicle boom positioning required to complete the docking maneuver. As shown in the figure, the force is applied as a discrete function of time. The two pulses in the first 20 seconds increase the radial velocity to 0.1 m/sec. The pulses at 50 seconds and 140 seconds are applied for attitude control. The two pulses at 150 seconds and 172 seconds, as well as the 5 impulsive forces applied from 168 to 186 seconds increase the radial velocity to 0.15 m/sec. The time duration of the impulse forces is 0.1 seconds each, which is the integration interval used in the computer program. In Control Region 2, a series of force pulses are applied to reduce the final attitude errors.

The target vehicle boom azimuth angle location is dictated by the attitude error. This angle is moved to one of its three positions (45°, 0°, -45°) by use of the control logic described previously. The boom elevation angle is maintained at -45 degrees in Control Region 1 for control of the tangential velocity. In Control Region 2, the elevation angle is controlled by the logic, and is positioned as required to control the attitude errors.

The chase vehicle boom position is not plotted since it only deviates from the zero position twice during the entire maneuver. The non-zero positions both occur in Control Region 2. The first causes the roll error

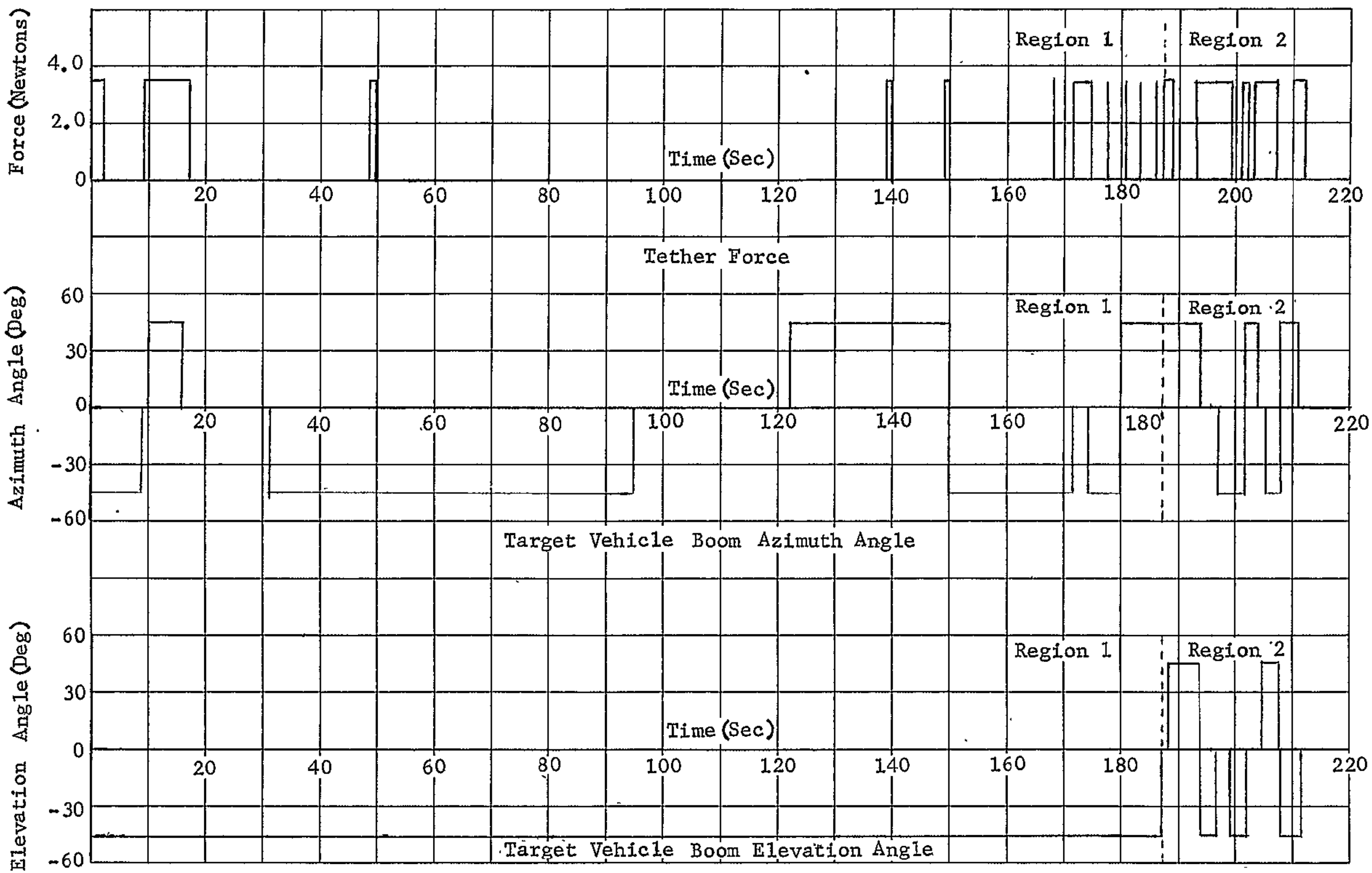


Figure IV-3 Tether Force and Target Vehicle Boom Angles-Run Number 15 to decrease towards zero with the application of force and the second causes the tether force to reduce the roll rate to less than one deg/sec.

A total of 18 cases were run for the ideal case with the chase vehicle initially behind the target vehicle. The initial conditions for these cases are listed in Table IV-2, and the final conditions are listed in Table IV-3. The initial conditions for the rotational motion are given as the body rates and Euler angles and as the controlled attitude errors and rates. The force sum listed in the final conditions table is the integral of the applied tether force, and thus is a measure of the energy required for completion of the docking maneuver.

The final conditions for all of these cases meet the limits specified for successful docking. Run number 18 is for a smaller chase vehicle than was used in the rest of the runs. The chase vehicle mass for this run is 200 kilograms, with pitch and yaw inertias of 117 $kg\text{-}m^2$ and roll inertia of 100 $kg\text{-}m^2$. The maximum force level for this case was set at 2.0 newtons, to yield the linear acceleration of 0.01 m/sec^2. The resulting rotational accelerations were 2.0 deg/sec^2 in the pitch and yaw axes, and 0.6 deg/sec^2 in the roll axis.

b. Chase Vehicle Ahead of Target Vehicle - Figure IV-4 illustrates the separation distance between the c.g.'s of the two vehicles for a typical retrieval trajectory for the ideal case with the chase vehicle ahead of the target vehicle. These plots also illustrate the coupling of the orbital effects for the in-plane motion, and the absence of this coupling for the out-of-plane motion.

Table IV-2 Initial Conditions - Chase Vehicle Behind Target Vehicle

Run Number	Position (meters)			Euler Angles (degrees)			Body Rates (deg/sec)			Attitude		Roll	
	X	Y	Z	Yaw	Pitch	Roll	ω_x	ω_y	ω_z	Error (deg)	Rate (deg/sec)	Error (deg)	Rate (deg/sec)
1	29.9	1.9	0.8	54.4	54.4	3.2	0.03	0.80	-0.47	69.9	-0.15	-6.1	-0.87
2	28.1	-7.5	7.1	-25.2	-39.5	20.6	0.71	0.48	0.82	27.3	-0.49	12.4	-0.06
3	29.9	-2.6	2.4	-3.4	-6.3	33.8	0.77	0.05	0.08	2.3	0.07	33.5	0.76
4	29.9	-2.4	-0.2	-54.4	-12.6	-41.3	-0.60	0.77	0.22	51.0	0.20	-39.7	-0.52
5	26.8	0.3	1.2	13.8	39.5	0.6	0.23	-0.08	-1.24	43.6	-0.47	1.0	-0.79
6	28.4	11.2	7.1	28.7	12.6	8.0	0.43	-0.53	-0.71	27.1	-0.59	14.3	0.26
7	29.9	9.4	-0.6	4.6	-33.2	56.7	0.20	0.33	0.59	36.8	0.05	43.5	-0.20
8	29.9	1.4	-0.5	-40.1	30.4	-11.5	-0.79	0.06	-0.53	50.1	0.45	-9.3	-1.09
9	29.6	6.6	-0.5	1.7	-23.5	41.3	1.14	-0.35	-0.61	26.7	0.16	36.8	1.44
10	29.6	4.3	-1.1	17.8	-6.9	-20.1	0.60	0.52	0.85	13.0	-0.09	-20.5	0.52
11	29.8	-3.0	-0.5	-55.0	0.6	-36.7	-0.13	0.36	0.47	49.2	-0.17	-35.8	-0.13
12	29.3	4.1	-4.8	-41.8	-28.7	-23.5	0.03	0.54	0.39	61.2	-0.45	-12.9	-0.04
13	29.3	6.0	-0.8	-48.7	42.4	-17.8	-0.22	-0.36	0.50	67.4	-0.61	-12.0	0.31
14	28.7	-4.9	-7.0	-9.7	9.7	12.6	0.05	0.36	0.04	3.9	-0.34	14.9	0.07
15	28.3	5.1	8.4	-10.3	0.0	5.7	0.00	0.00	0.00	26.0	0.00	2.6	0.00
16	20.7	21.2	4.6	30.4	-6.9	2.3	0.00	0.00	0.00	15.3	0.00	8.5	0.00
17	21.2	-18.3	10.9	-27.2	-32.7	16.6	0.00	0.00	0.00	16.6	0.00	4.4	0.00
18	30.0	1.4	-0.8	1.7	-19.5	7.5	-0.34	-0.80	0.02	22.1	0.74	6.9	-0.43

Table IV-3 Final Conditions - Chase Vehicle Behind Target Vehicle

Run Number	Radial Velocity (m/sec)	Tangential Velocity (m/sec)	Attitude Error (deg)	Attitude Rate (deg/sec)	Roll Error (deg)	Roll Rate (deg/sec)	Force Sum (Newton-sec)	Time (seconds)
1	-0.25	0.12	0.7	0.07	-6.8	0.00	114.1	233.6
2	-0.25	0.09	0.9	-0.07	-5.2	0.48	109.9	195.0
3	-0.25	0.10	10.0	-2.36	3.0	0.64	103.6	181.9
4	-0.28	0.09	2.8	-2.23	-6.4	-0.05	119.0	176.3
5	-0.23	0.11	3.4	0.36	4.0	0.22	102.8	223.8
6	-0.27	0.12	5.2	-2.55	-4.2	-0.90	117.6	181.3
7	-0.27	0.13	1.6	0.43	4.2	-2.50	120.1	196.5
8	-0.28	0.12	1.2	0.26	6.6	0.10	123.9	195.0
9	-0.27	0.12	2.5	1.36	6.9	-0.02	114.8	177.8
10	-0.23	0.11	2.8	-3.08	5.8	0.05	106.1	192.8
11	-0.25	0.12	2.2	-1.51	5.1	0.99	112.0	203.1
12	-0.26	0.14	0.9	0.37	-7.4	0.63	123.6	223.6
13	-0.27	0.14	3.5	-1.37	6.4	0.00	127.8	228.5
14	-0.24	0.09	3.9	0.26	-4.2	0.42	105.2	208.1
15	-0.24	0.11	6.8	-2.40	-3.1	-0.82	109.2	212.1
16	-0.25	0.14	5.9	-1.1	8.1	-0.09	122.0	213.4
17	-0.26	0.09	6.1	1.22	3.7	-0.31	108.2	194.3
18	-0.25	0.14	6.3	-0.28	8.8	0.64	67.8*	201.5

*The maximum force level was 2.0 newtons for this case

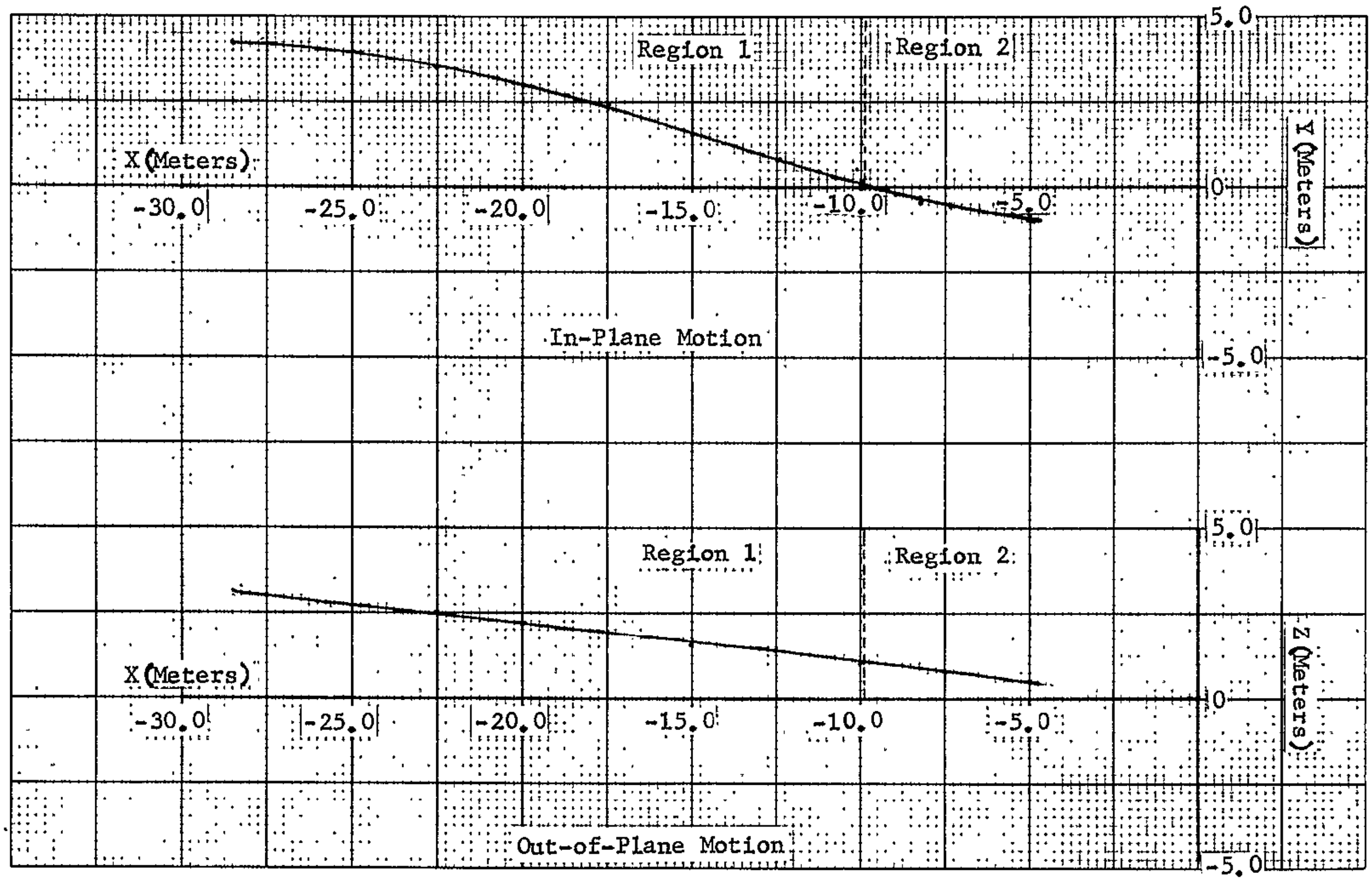


Figure IV-4 Retrieval Trajectory-Run Number 25

The rotational motion for this case is plotted in Figure IV-5. The attitude error oscillates as force is initially applied to increase the radial velocity, reaches the 45 degree limit three times in Control Region 1, and is driven towards zero in Control Region 2 with a final value of 5.7 degrees. Control is applied to the roll error during the initial application of force; at 130 seconds; and in Control Region 2, where the final roll error is reduced to 6.7 degrees.

The tether force required for this maneuver is plotted in Figure IV-6. As in the case previously plotted, the force is applied at discrete intervals of time, with the chase vehicle coasting during the no-force periods. During the final 18 seconds, a constant force is applied, causing the attitude error to oscillate.

Figure IV-7 illustrates the angles for the booms on the two vehicles. The chase vehicle boom is initially positioned to decrease the roll error, and is returned to the zero position when the roll rate reaches -0.5 deg/sec At 101 seconds, the boom is moved to the -30 degree position, and remains there until force is applied at 133 seconds, driving the roll error towards zero. In Control Region 2, the chase vehicle boom is positioned to drive the roll error to zero.

The target vehicle boom azimuth and elevation angles are controlled by the appropriate logic, and are similar to the case previously plotted. The elevation angle is maintained at + 45 degrees in Control Region 1 for control of the tangential velocity.

Figure IV-8 illustrates the attitude errors for another case with the chase vehicle ahead of the target vehicle. For this case, roll control is

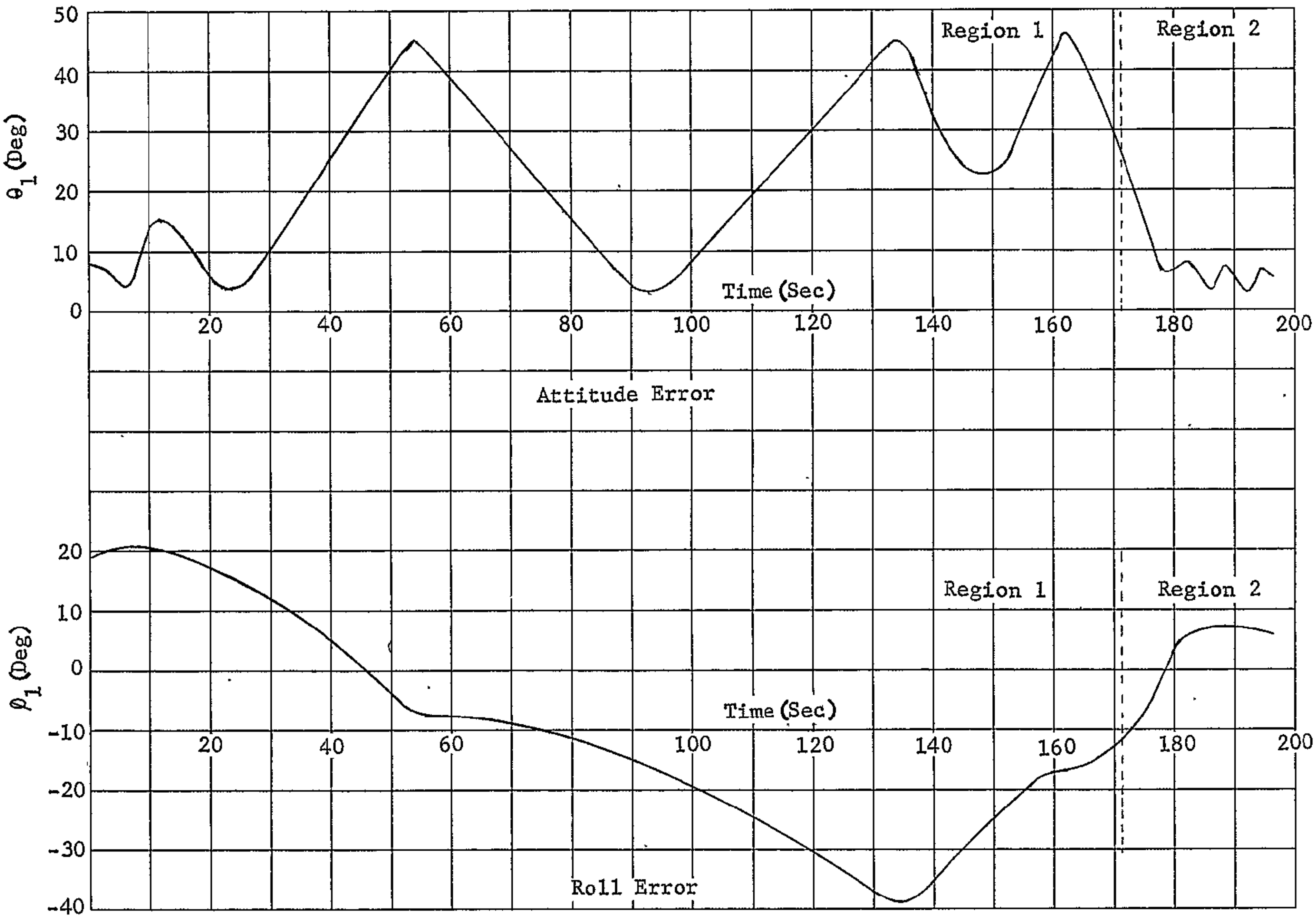


Figure IV-5 Rotational Motion-Run Number 25

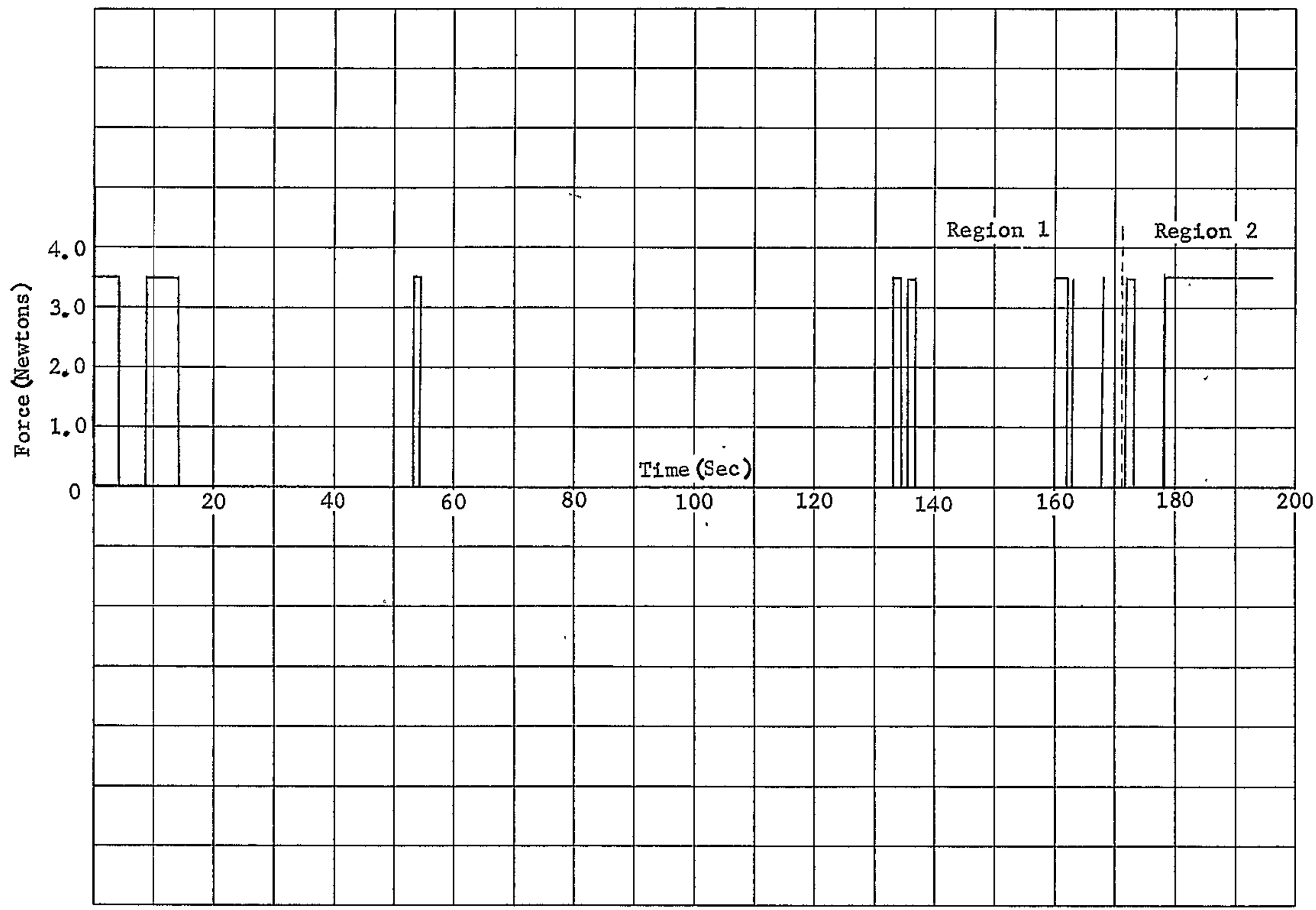


Figure IV-6 Tether Force-Run Number 25

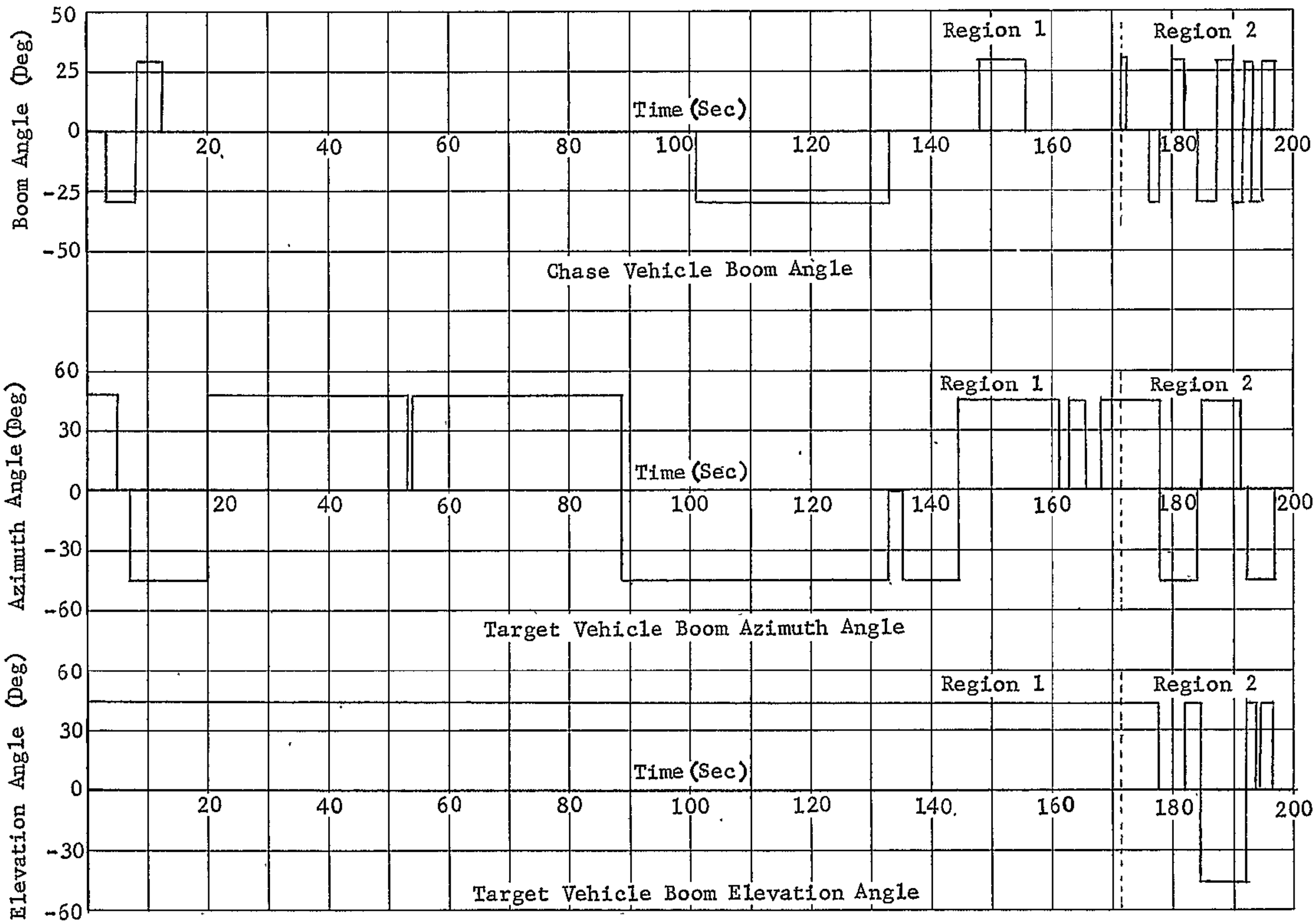


Figure IV-7 Chase and Target Vehicle Boom Angles-Run Number 25 applied in Control Region 1 during the initial application of force; at 117 seconds; and at 142 seconds.

This case presents a more difficult control problem in Control Region 2. Upon entering Control Region 2 at 177 seconds, the attitude error is at 43 degrees and is increasing at approximately one deg/sec. The roll error at this time is 11 degrees, which is also not an acceptable limit in Control Region 2. Thus, force is applied driving both of the angles towards zero. The roll error shows considerable overshoot, reaching a value of -18 degrees before the rate is reversed, reducing the final roll error to 1.0 degree. The force applied to reverse the roll rate causes the attitude error to oscillate, with a final value of 9.6 degrees and a rate of -1.6 deg/sec.

A total of 8 runs were made for the ideal case with the chase vehicle ahead of the target vehicle. Table IV-4 lists the initial conditions for these runs, and Table IV-5 lists the final conditions at docking. Initial Euler angles greater than 90 degrees are used for these cases, since the chase vehicle is located in the negative X direction. Thus, the chase vehicle is rotated to point at the target vehicle.

All of the final conditions are within the specified limits. For run number 20, the radial velocity is exactly 0.305 m/sec, which is the maximum allowable value. The time history of this run leads to no definite conclusions as to why the closing velocity is this high. For the remainder of the cases, the final radial velocity is 0.28 m/sec or less.

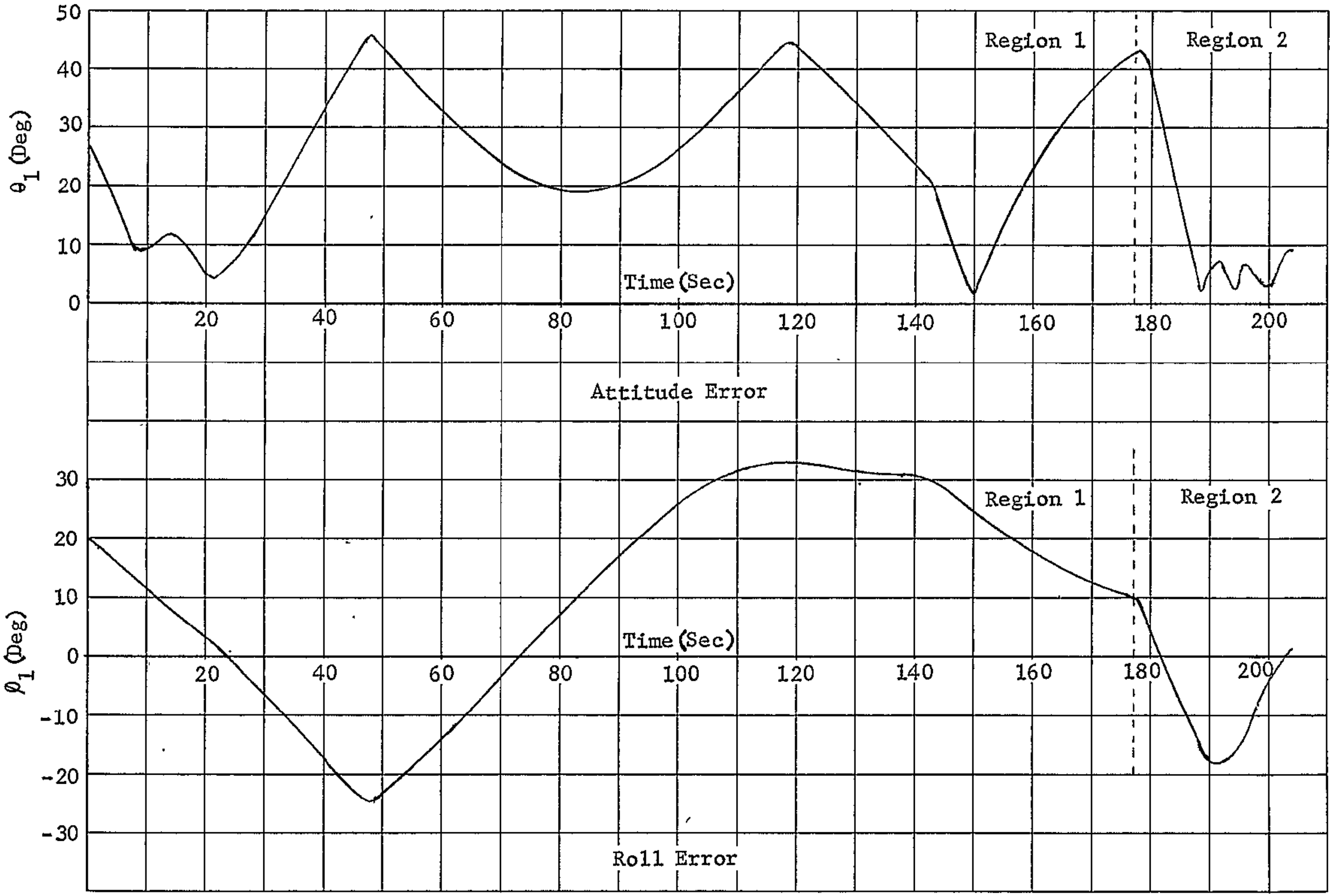


Figure IV-8 Rotational Motion-Run Number 26

Table IV-4 Initial Conditions - Chase Vehicle Ahead of Target Vehicle

Run Number	Position (meters)			Euler Angles (degrees)			Body Rates (deg/sec)			Attitude		Roll	
	X	Y	Z	Yaw	Pitch	Roll	ω_x	ω_y	ω_z	Error (deg)	Rate (deg/sec)	Error (deg)	Rate (deg/sec)
19	-30.0	0.3	1.2	13.8	219.5	0.6	0.23	-0.08	-1.24	33.0	1.03	1.1	0.17
20	-19.3	18.7	-13.2	128.3	13.2	-170.7	0.00	0.00	0.00	14.8	0.00	35.4	0.00
21	-29.1	-1.3	6.8	1.2	179.3	24.6	0.00	0.00	0.00	13.9	0.00	24.1	0.00
22	-23.7	-18.2	0.3	22.4	-151.3	-14.9	0.00	0.00	0.00	31.7	0.00	-12.8	0.00
23	-29.9	1.9	0.8	54.4	-151.8	-13.2	0.03	0.80	-0.47	61.4	0.79	-10.5	-0.31
24	-29.9	-2.4	-1.2	-167.9	8.0	-183.4	-0.17	0.16	-0.41	9.4	0.26	-3.8	-0.11
25	-28.3	4.3	3.1	-12.0	179.3	20.6	0.45	-0.52	0.59	7.7	0.79	19.2	0.44
26	-30.0	-3.2	6.5	22.6	170.7	16.8	-0.21	0.69	0.22	27.2	-0.73	20.5	-0.28

Table IV-5 Final Conditions - Chase Vehicle Ahead of Target Vehicle

Run Number	Radial Velocity (m/sec)	Tangential Velocity (m/sec)	Attitude Error (deg)	Attitude Rate (deg/sec)	Roll Error (deg)	Roll Rate (deg/sec)	Force Sum (Newton-sec)	Time (seconds
19	-0.25	0.08	5.2	-4.10	6.9	0.05	105.4	217.8
20	-0.305	0.08	2.7	2.23	-2.6	0.92	124.6	171.4
21	-0.24	0.13	1.4	-1.34	6.8	-0.21	124.6	190.6
22	-0.25	0.15	1.5	-1.30	-6.6	0.04	127.4	202.8
23	-0.24	0.11	3.9	-2.40	-3.8	-0.02	108.2	221.8
24	-0.27	0.14	6.8	-0.46	6.2	0.99	123.1	204.9
25	-0.28	0.09	5.7	-3.53	6.7	-0.11	121.8	195.8
26	-0.24	0.13	9.6	-1.64	1.0	1.23	114.8	203.2

3. Non-Cylindrical Chase Vehicle

Four runs were made using a non-symmetrical chase vehicle configuration to determine the effects of unequal pitch and yaw accelerations. Table IV-6 lists the rotational accelerations which were used for these runs.

Table IV-6 Rotational Accelerations for Non-Symmetrical Chase Vehicle

Run Number	Yaw Acceleration (deg/sec^2)	Pitch Acceleration (deg/sec^2)	Roll Acceleration (deg/sec^2)
27	2.4	2.6	1.75
28	2.8	2.2	1.75
29	2.4	2.6	1.75
30	11.1	10.9	1.75

For run number 29, inertia cross products were included, to increase the coupling between the rotational axes. The interia matrix for this run is:

$$I = \begin{bmatrix} 58.0 & -1.36 & 3.74 \\ -1.36 & 157.0 & 2.68 \\ 3.74 & 2.68 & 165.0 \end{bmatrix}$$

For run number 30, a completely different chase vehicle configuration was used. This vehicle has a mass of 99 kg, yaw inertia of 7.74 $kg\text{-}m^2$, pitch inertia of 7.88 $kg\text{-}m^2$, and roll inertia of 8.23 $kg\text{-}m^2$. The maximum tether force was reduced to 1.0 newtons, to result in a linear acceleration of 0.01 m/sec^2. In addition, the chase vehicle boom length was reduced to 0.5 meter, with the pivot point remaining 1.0 meter from the c.g. of the vehicle. This reduced the roll acceleration to the same level used in the rest of the runs. The pitch and yaw accelerations remained high, but a successful docking was achieved with these high acceleration levels.

Table IV-7 lists the initial conditions used, in terms of the controlled parameters, and Table IV-8 lists the final conditions at docking.

Table IV-7 Initial Conditions - Non-Cylindrical Chase Vehicle

Run Number	Position (meters)			Attitude		Roll	
	X	Y	Z	Error (deg)	Rate (deg/sec)	Error (deg)	Rate (deg/sec)
27	30.0	0.0	0.0	31.7	0.00	5.3	0.00
28	30.0	0.0	0.0	31.7	0.00	5.3	0.00
29	30.0	0.0	0.0	31.7	0.00	5.3	0.00
30	29.7	-2.7	4.8	27.1	0.35	6.3	0.13

Table IV-8 Final Conditions - Non-Cylindrical Chase Vehicle

Run Number	Radial Velocity (m/sec)	Tangential Velocity (m/sec)	Attitude Error (deg)	Attitude Rate (deg/sec)	Roll Error (deg)	Roll Rate (deg/sec)	Force Sum (Newton-sec)	Time (seconds)
27	-0.25	0.08	4.5	-2.18	-4.3	-0.94	102.6	197.2
28	-0.24	0.13	3.8	1.45	-3.7	-0.54	112.2	213.6
29	-0.25	0.12	1.4	-0.17	7.1	-0.90	114.3	214.3
30	-0.16	0.14	2.3	-2.77	-3.9	.0.17	22.9	232.0

All of the final conditions are within the limits specified for successful docking. The low closing velocity for run number 30 is caused by the high pitch and yaw accelerations. With these high accelerations, only a small amount of force is required for attitude corrections, and thus the radial velocity increase is small during these corrections.

4. Offset Chase Vehicle Boom

For all of the previous data cases, the chase vehicle boom pivot is located on the X axis, which produces zero roll moments with the boom at the zero position. Since it would often be physically impossible to place the boom on the X axis due to vehicle constraints, five runs were performed using the cylindrical chase vehicle with the boom pivot offset from this axis. Table IV-9 lists the pivot point offset for these runs.

Table IV-9 Chase Vehicle Boom Pivot Location

Run Number	Pivot Location (meters)		
	X	Y	Z
31	1.0	0.025	-0.025
32	1.0	-0.050	-0.050
33	1.0	-0.076	0.076
34	1.0	0.100	0.100
35	1.0	0.130	0.0

The initial conditions for the first four of these runs were identical, with an initial separation distance of 30 meters along the X axis, and zero attitude errors and rates. For run number 35, an initial roll error of 5.4 degrees was included, to assure that out-of-plane motion would be included in the retrieval trajectory.

The final conditions for these runs are listed in Table IV-10. All of the final limit requirements are met, indicating that it is not necessary to position the chase vehicle boom pivot exactly on the X axis.

Table IV-10 Final Conditions - Offset Chase Vehicle Boom

Run Number	Radial Velocity (m/sec)	Tangential Velocity (m/sec)	Attitude Error (deg)	Attitude Rate (deg/sec)	Roll Error (deg)	Roll Rate (deg/sec)	Force Sum (Newton-sec)	Time (seconds)
31	-0.20	0.12	3.0	0.10	4.1	0.56	98.7	211.4
32	-0.23	0.12	3.0	-2.63	5.8	0.25	108.0	212.0
33	-0.22	0.14	0.1	1.62	-7.0	0.36	107.6	216.9
34	-0.26	0.11	1.4	1.89	-3.5	0.04	113.4	193.8
35	-0.29	0.07	2.2	3.75	6.3	0.01	118.3	185.9

5. Offset Target Vehicle Boom

Since it is physically impossible to place the target vehicle boom gimbal at the c.g. of the vehicle, four runs were made with the gimbal offset a known distance. Table IV-11 lists the offset distances which were used. The two cases with the boom offset 12 meters from the c.g. were selected to be representative of attachment to the end of a large vehicle such as the Skylab.

Table IV-11 Target Vehicle Boom Offsets

Run Number	Gimbal Location (meters) X	Y	Z
36	2.0	1.0	12.0
37	3.0	-2.0	-2.0
38	-3.0	0.0	-12.0
39	2.5	-0.3	3.5

Figure IV-9 illustrates a typical retrieval trajectory with the target vehicle boom offset from the c.g. of the vehicle. The in-plane motion rises above the boom base, and the out-of-plane movement is directly towards the base. With the boom offset, the position and radial velocity is controlled relative to the base of the boom.

Figure IV-10 illustrates the rotational motion for this run. The attitude error is typical of all the runs made, with initial oscillations as the radial velocity is increased and, in this case, three attitude corrections applied in Control Region 1. In Control Region 2, the attitud error is reduced to a final value of 0.8 degrees.

The chase vehicle boom is positioned for roll control when force is applied at 10 seconds and at 140 seconds in Control Region 1. In Control Region 2 the roll error is driven towards zero, with a final value of 6.2 degrees when docking is achieved.

Table IV-12 lists the initial conditions for the four runs with the target vehicle boom offset. The position data is relative to the base of the boom, and the rotational errors are relative to the vector from the boom base to the c.g. of the chase vehicle.

The final conditions for these runs are listed in Table IV-13. All of the final conditions are within the limits specified for successful docking.

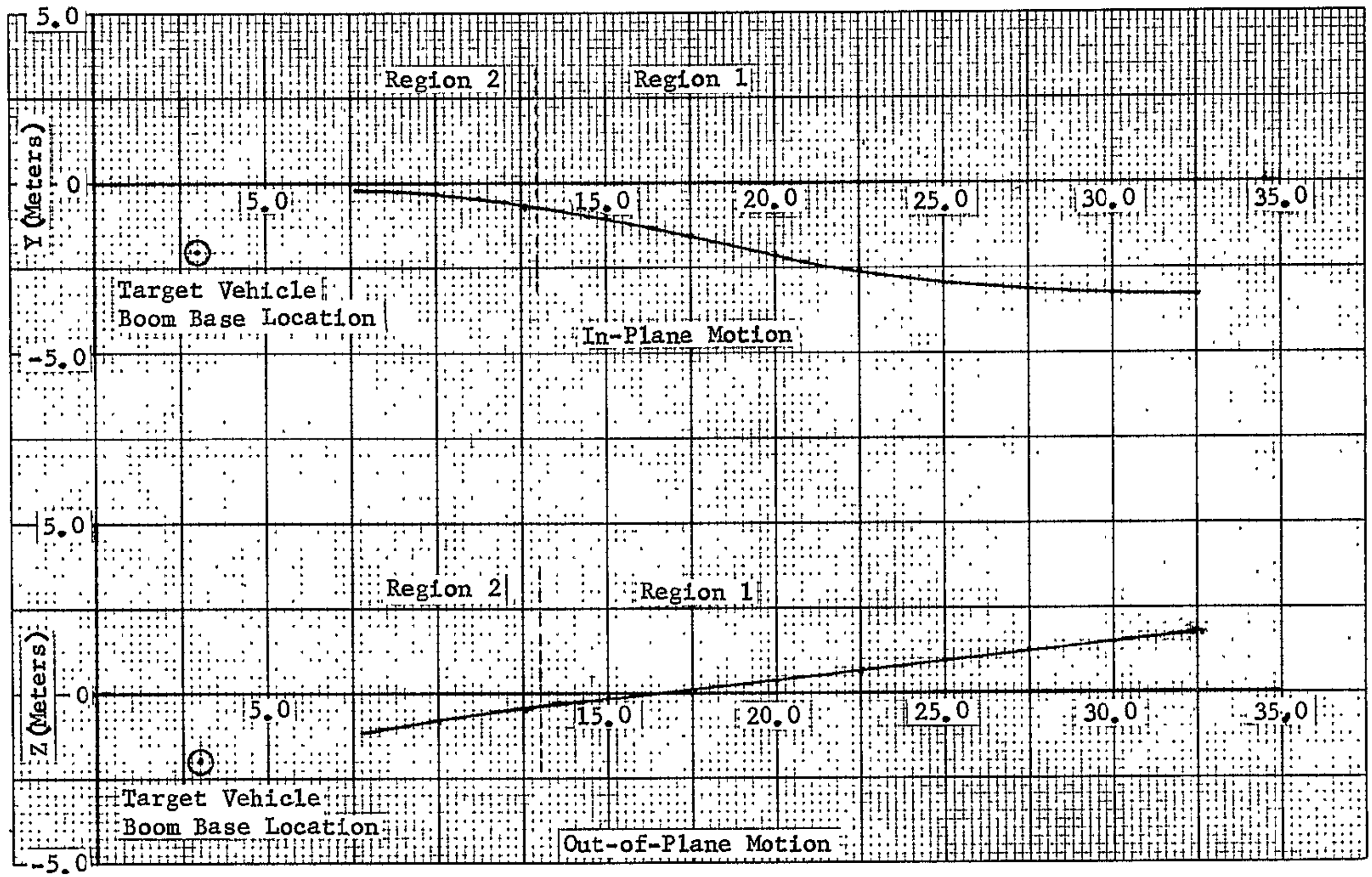


Figure IV-9 Retrieval Trajectory-Run Number 37

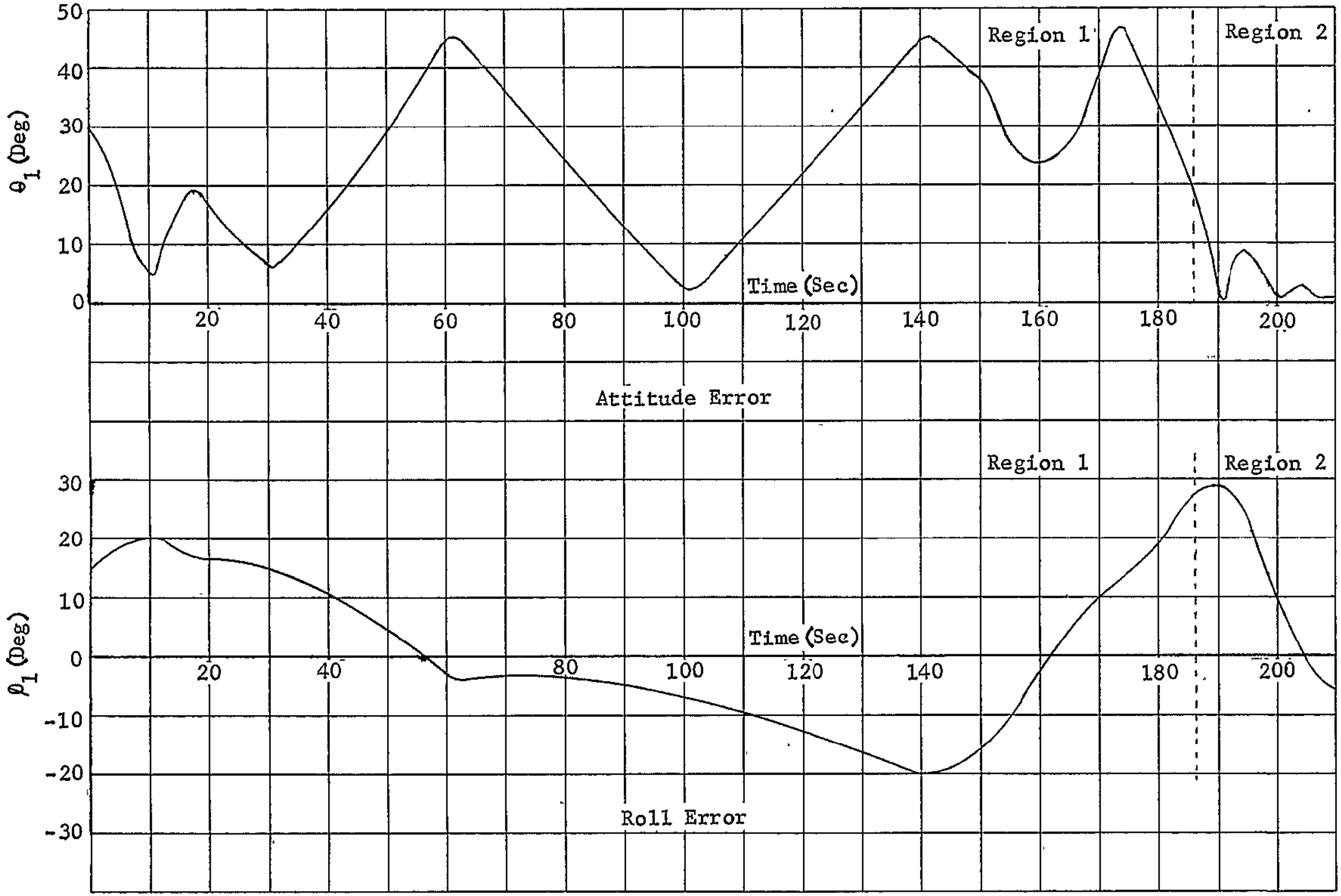


Figure IV-10 Rotational Motion-Run Number 37

Table IV-12 Initial Conditions - Target Vehicle Boom Offset

Run Number	Position (meters)			Attitude		Roll	
	X	Y	Z	Error (deg)	Rate (deg/sec)	Error (deg)	Rate (deg/sec)
36	29.3	3.3	2.3	20.7	-0.15	7.6	0.11
37	29.6	-1.3	3.8	29.9	-0.54	15.4	0.01
38	-30.1	1.5	-0.3	9.7	-0.03	-0.1	0.69
39	30.0	0.3	-3.5	6.3	0.00	0.0	0.00

Table IV-13 Final Conditions - Target Vehicle Boom Offset

Run Number	Radial Velocity (m/sec)	Tangential Velocity (m/sec)	Attitude		Roll		Force Sum (Newton-sec)	Time (seconds)
			Error (deg)	Rate (deg/sec)	Error (deg)	Rate (deg/sec)		
36	-0.27	0.13	3.2	0.18	5.8	0.01	124.3	184.0
37	-0.25	0.10	0.8	-1.53	-6.2	-0.08	111.2	210.3
38	-0.29	0.10	4.5	-2.06	7.1	0.04	122.4	168.7
39	-0.22	0.09	0.2	-2.20	1.8	0.03	91.7	203.0

6. General Equations of Motion

As a final check on the validity of the control logic, 8 runs were made using the developed digital computer program which solves the exact equations of motion. Several of these runs included target vehicle boom base offsets and perturbations due to gravity gradient or aerodynamic effects. Table IV-14 lists the features which were included for each run.

Table IV-14 Test Case Features - General Equations

Run Number	Target Vehicle Boom Offset	Gravity Gradient	Aerodynamics
40	No	No	No
41	No	Yes	Yes
42	No	Yes	No
43	No	No	No
44	Yes	No	No
45	Yes	No	No
46	Yes	Yes	Yes
47	No	No	No

No significant differences are present for the cases which included the perturbations. Since the retrieval time is short, and the tether forces and moments are several orders of magnitude greater than the aerodynamic or gravity gradient effects, the perturbations do not affect the solution.

Figure IV-11 illustrates a typical retrieval trajectory obtained using the general computer program with the aerodynamic and gravity gradient effects included. The plot of the in-plane motion illustrates the coupling of the orbital effects in the same manner as was obtained with the linear equations, and the out-of-plane motion illustrates the absence of this coupling.

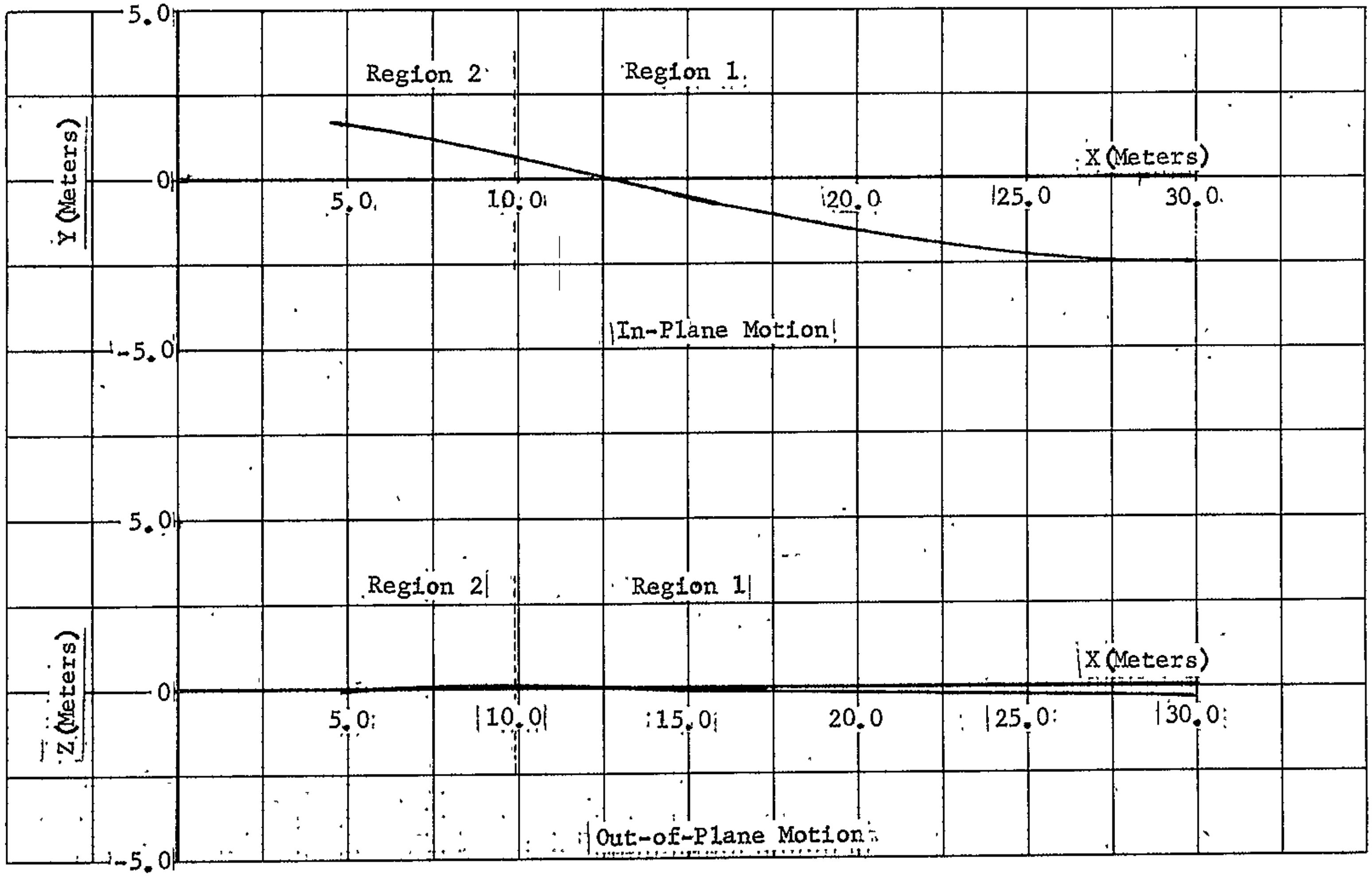


Figure IV-11 Retrieval Trajectory-Run Number 41

Figure IV-12 illustrates the rotational motion for this run. Again, the results are similar to those obtained with the linear program. The attitude error shows oscillation as the initial radial velocity is increased, and, for this case, requires four additional corrections in Control Region 1. In Control Region 2, the attitude error is driven towards zero, with a final value of 0.5 degree at docking. The initial roll error of approximately -40 degrees is driven towards zero during the initial application of force, and is corrected again at 60 and 130 seconds. In Control Region 2, the roll error is driven through zero with approximately 9 degrees overshoot, and is reduced to a final value of 6.5 degrees.

Table IV-15 lists the initial conditions for the runs made with the general computer program. The position data is listed relative to the base of the target vehicle boom, and thus compensates for the boom offsets where applicable. For run number 43, the target vehicle was placed in an inertial orbit, to illustrate the compatibility of the control law with non-earth oriented vehicles.

With the chase vehicle almost directly above the target vehicle, as in run number 47, the target vehicle was rotated up to point at the chase vehicle. The initial conditions listed in the table for this run are in the local vertical coordinate system. The corresponding initial conditions in the target vehicle body axes are X = 29.5, Y = -0.2, and Z = 0.5.

Table IV-16 lists the final conditions for the runs made with the general computer program. All of the final conditions are within the limits specified for successful docking. The time histories for all of the runs are similar to those obtained with the linear equations. This

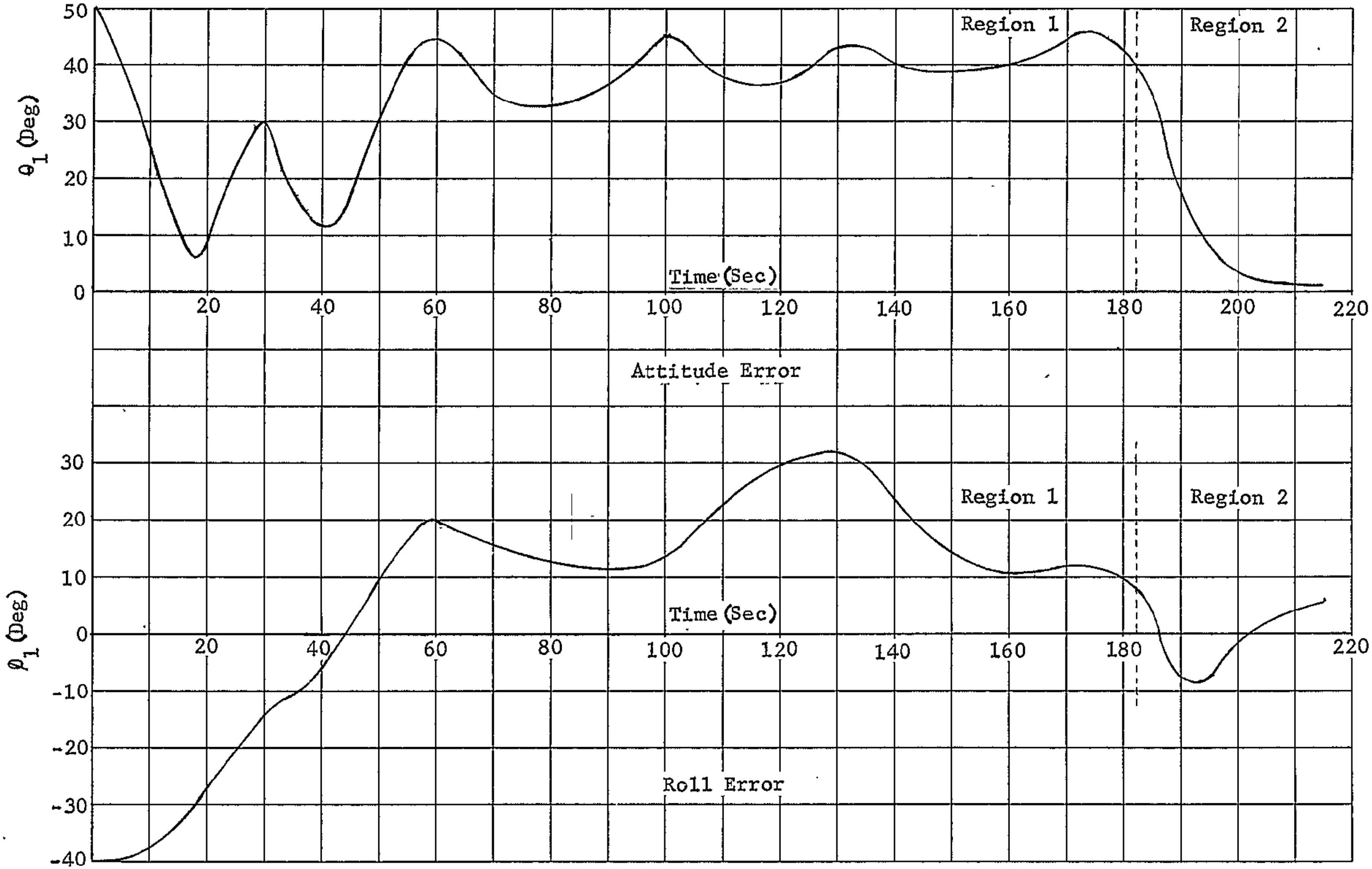


Figure IV-12 Rotational Motion-Run Number 41

was an expected result, since for the separation distances involved, the differences in the two sets of equations are negligible.

Table IV-15 Initial Conditions - General Equations of Motion

Run Number	Position (meters) X	Y	Z	Attitude Error (deg)	Attitude Rate (deg/sec)	Roll Error (deg)	Roll Rate (deg/sec)
40	28.1	-7.6	7.1	27.3	-0.49	12.4	-0.06
41	29.9	-2.5	-0.3	51.0	0.20	-39.7	-0.52
42	28.5	9.5	-0.6	36.8	0.05	43.5	-0.20
43	28.4	-7.4	-6.4	21.3	0.00	-14.7	0.00
44	29.7	-2.3	-1.6	34.0	-0.53	11.4	0.21
45	-30.1	1.5	-0.3	9.7	-0.03	-0.1	0.69
46	29.3	3.4	2.3	20.7	-0.15	7.6	0.11
47	1.4	29.8	0.5	13.1	-0.21	7.2	0.10

Table IV-16 Final Conditions - General Equations of Motion

Run Number	Radial Velocity (m/sec)	Tangential Velocity (m/sec)	Attitude Error (deg)	Attitude Rate (deg/sec)	Roll Error (deg)	Roll Rate (deg/sec)	Force Sum (Newton-sec)	Time (seconds)
40	-0.24	0.09	1.2	-1.12	-6.1	0.10	103.6	192.5
41	-0.24	0.13	0.5	-0.31	6.5	0.70	110.6	207.5
42	-0.23	0.14	0.5	-0.57	-3.9	-0.95	109.2	197.5
43	-0.27	0.13	7.2	1.64	-3.1	0.52	119.9	207.5
44	-0.23	0.11	1.1	2.91	-1.3	0.34	101.2	190.0
45	-0.25	0.11	0.2	-1.32	7.0	0.09	106.4	176.0
46	-0.26	0.12	3.8	0.78	6.9	0.88	116.9	184.0
47	-0.24	0.11	3.2	-1.85	-6.7	-0.10	110.2	211.0

7. Summary

Table IV-17 presents a summary of all the data cases using the 351 kg chase vehicle. Run numbers 18 and 30 are not included in this table since they are for a smaller chase vehicle, and are not representative of the remainder of the cases. The average values listed are for the magnitudes of the parameter under consideration, and thus a positive value is not offset by a negative value.

Table IV-17 Final Conditions Summary

Parameter	Minimum	Average	Maximum
Radial Velocity (m/sec)	0.20	0.25	0.305
Tangential Velocity (m/sec)	0.07	0.11	0.15
Attitude Error (deg)	0.10	2.5	10.0
Attitude Rate (deg/sec)	0.07	1.6	4.1
Roll Error (deg)	1.00	4.4	7.1
Roll Rate (deg/sec)	0.00	0.4	2.5
Force Sum (Newton-sec)	91.7	112.9	127.8
Time (seconds)	171.4	200.0	233.6

For the radial velocity, the tangential velocity, and the attitude error, the maximum values obtained are at the maximum allowable limits specified for completion of a successful docking maneuver. The tangential velocity can be reduced by using a longer boom on the target vehicle to allow more control of the direction of the applied force, but the effect of this change on the remainder of the controlled parameters cannot be predicted.

The maximum attitude rate and roll rate are significantly less than the specified maximum. These maximum rates are directly related to the rate limits which are used in the logic which controls the application of tether force.

Examination of the initial/final conditions does not immediately indicate any means by which the final conditions can be predicted. Thus, the final conditions become known only after the problem is solved using the digital computer program.

SECTION V

CONCLUSIONS/RECOMMENDATIONS

The results of the feasibility study indicate that a single tether can be used to provide all of the control functions necessary for the retrieval and docking of a chase vehicle to a target vehicle. Using this technique, cargo packages and experiment modules can be docked to a parent vehicle without the need for incorporation of either attitude or translational control systems into these modules. This technique also has potential applications in the areas of orbital assembly and satellite capture and retrieval for maintenance.

By expanding the tether control to include stationkeeping techniques, a wide variety of applications can be considered. These applications include the deployment, positioning, maneuvering, and retrieval of experiment modules, and the control of facility operations in the vicinity of a Space Station. The tether can also be used to provide temporary cargo storage and module isolation in the event of an emergency. These applications involve the deployment of a module from the target vehicle, and using the tether to maintain it in a separate orbit near the target vehicle.

Several areas require further investigation before control using a tether can be adopted as an operational technique. These areas of investigation can be categorized into analytical studies and hardware definitions.

In the analytical studies area, the first step which should be taken is to use the digital computer program to perform a sensitivity analysis on the control law which was developed under this contract. The sensitivity analysis would determine the effects of changing the various limits used in the control logic, and would also lead to positive conclusions regarding the range of initial conditions for which docking can be accomplished using the developed control technique. The boom travel and velocity requirements should also be determined and incorporated into the control logic. All of these factors are aimed at developing a better understanding of the problem, whereby inspection of the initial conditions will be all that is required to determine if docking can be accomplished.

After the present control technique has been thoroughly examined, the next step should be the derivation of other control laws. This control law development should investigate different control techniques for retrieval and docking in order to determine the optimum method of control, and should also incorporate the other aspects of tether control, including deployment and orbit establishment and stationkeeping and maneuvering. The area of multiple tethers should also be investigated. This includes the control of two or more separate packages as well as the concept of using multiple tethers for more positive control of a single package.

The other areas which should be investigated include the effects of the tether characteristics (mass, drag, wave effects, etc.) on the control problem and the safety considerations which must be observed to provide for contingency operations. The inclusion of a manual control system, either as the prime or backup mode of control, and the post contact problems, should also be investigated.

In the area of hardware definition, the two areas of prime importance are the tether and tether control mechanisms. Studies in these areas should determine the optimum tether material and its characteristics, and techniques of controlling the tether length, (reel in/out mechanisms), the force application, and control of the slack in the tether without applying force to the chase vehicle.

Another area which requires study is the definition of initial tether attachment devices. These are required for control of packages which were not originally deployed from the target vehicle.

Other hardware aspects include the boom and gimbal systems, the sensor requirements, the displays and controls, and the actual docking mechanisms. Sufficient work has been done in these areas on other programs that the immediate effort on their development should be minimal.

While the above list of development items seems long, it must be remembered that this contract effort is one of the first attempts to study the tether docking problem in detail. This is the first analysis that shows the positive feasibility of space vehicle retrieval and docking using a single tether for six-degree-of-freedom control.

SECTION VI

REFERENCES

1. Models of Earth's Atmosphere (120 to 100 km) NASA SP-8021. May, 1969.

2. Greeb, F. J., Harbick, B. J., Tether Docking of Orbiting Spacecraft; Computer Program User's Manual. Martin Marietta Corporation. MCR-70-401, October, 1970.

3. Perrine, B. S., Jr., A Method of Soft Tether Stationkeeping. NASA TMX-53643, July, 1967.

4. Robinson, A. C., On the Use of Quaternions in Simulation of Rigid-Body Motion. WADC Technical Report 58-17. Wright-Patterson Air Force Base, Ohio, December, 1958.

5. Sasaki, F. T., Greeb, F. J., Soft Tether Stationkeeping. Vol. 1, Analytical Study. Martin Marietta Corporation. Research Report R-68-6, April, 1968.

APPENDIX A

COMPUTER PROGRAM DESCRIPTION

The computer program is written for the Control Data Corporation 6400 or 6500 series computer and is designed to be compatible with the Univac 1108 computer. It is written using the Fortran IV language. Data input to the program utilizes the Namelist feature of Fortran. The program is modularized, that is, broken down into a main calling program and several subroutines. The main program reads input data, controls the order in which the subroutines are called, and assembles the resulting data for output. Subroutines are used throughout the program to calculate each of the various elements which make up the overall tether docking problem.

The modular form of the program permits a clearer understanding of the functions of each portion of the program, and allows modifications to be made to one portion of the program without affecting the computational schemes or logic of the rest of the program. It also allows for the easy incorporation of additional modules, as required, to describe additional features of the dynamical system, such as other external disturbances, additional tethers, or vehicle control system dynamics.

The program utilizes a linear first order numerical integration technique for the solution of the differential equations of motion. Since all of the equations to be integrated are second derivatives, each equation must be integrated twice. Two versions of the integration technique are used. To integrate the second derivative, the new value is weighted more than the old value, using the equation:

$$\int_{t_{n-1}}^{t_n} \ddot{X}\,dt = \frac{\Delta T}{2}\left(3\ddot{X}_n - \ddot{X}_{n-1}\right) \tag{A.1}$$

To integrate the first derivative, a trapezoidal method is used where the new and old derivatives are weighted equally:

$$\int_{t_{n-1}}^{t_n} \ddot{X}\, dt = \frac{\Delta T}{2} \left(\dot{X}_n + \dot{X}_{n-1} \right) \tag{A.2}$$

The value of ΔT is a program input and should be selected on the basis of the type of tether control law being used. During the program checkout, a ΔT of two (2) seconds was normally used for those cases where the orbital effects were the predominant factors, such as in stationkeeping trajectories, and a ΔT of one-tenth (0.1) seconds was used for those cases where the tether forces were predominant.

1. Main Program

The main program has three basic functions. These functions are: 1) read the input data and set up the problem initial conditions; 2) call the necessary subroutines to generate the solution to the problem; and 3) output the results of the calculations. Figure A-1 is a flow chart of the main program illustrating the sequence of calculations performed in the solution of the equations of motion.

Several options are available in the main program. These options, which are controlled by the input data, consist of:

1) Circular or Elliptic Reference Orbit
2) Inertial or Geocentric Target Vehicle Stabilization
3) Frequency of updating reference orbit
4) Perturbations to be used
5) Frequency of Printout

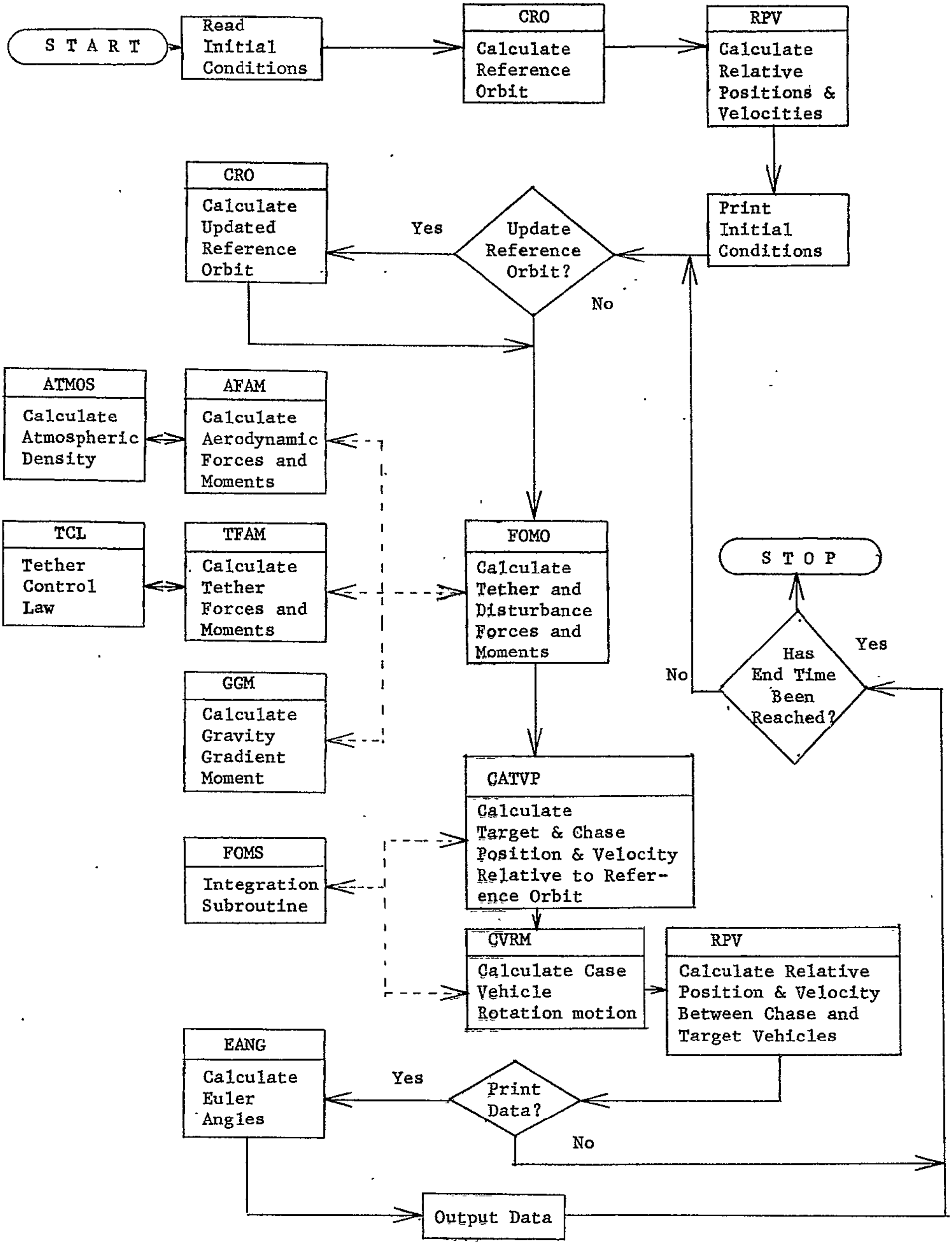


Figure A-1 Main Program Flow Chart

In general, the selection of these options is based on the requirements of the problem under consideration. The most significant option is the selection of the perturbations to be used in the solution of the equations. Inclusion of the aerodynamic perturbations increases the running time of the program by approximately a factor of two. Therefore, for development of retrieval techniques which involve either high reference orbits or the application of a continuous tether force over a short period of time, aerodynamics should not be included. Inclusion of aerodynamics for final verification only will allow the initial development of retrieval techniques to be accomplished at a significant reduction in expended computer time.

2. Subroutines

In addition to the 12 subroutines shown in Figure A-1, 6 additional routines are used to print a visual title page and perform matrix-vector manipulations. Each of the 18 subroutines are described in the following summaries, with flow charts included for those routines which are not direct straight through calculations. These subroutines separate the major functions into separate modules of the computer program.

a. AFAM - Subroutine AFAM is called by subroutine FOMO (if aerodynamics are to be used in the calculations) to calculate the aerodynamic forces and moments on the vehicles. Subroutine AFAM is vehicle dependent, requiring a particular set of initial data for the vehicles that are used in the tether docking problem. The data presently being used is for an Orbital Workshop target vehicle and a cylindrical chase vehicle. This data is used to calculate the aerodynamic coefficients of the vehicle, which in turn are used to generate the forces and moments acting on the vehicles. Subroutine AFAM uses function ATMOS to calculate the atmospheric density.

b. ATMOS - ATMOS is a function subroutine which calculates the atmospheric density as a function of the altitude and longitude of the reference point.

c. CATVP - Subroutine CATVP calculates the position and velocity of the target and chase vehicles relative to the reference orbit. Since the equations of orbital motion are identical for either vehicle, this routine is called twice by the main program, with the carried parameter IVEH indicating which vehicle is being processed.

d. CRO - This subroutine calculates and prints the unperturbed reference orbit as a function of time. The input variable IØTP determines whether the reference orbit is circular or elliptic. Figure A-2 is a flow chart of subroutine CRO.

As illustrated in the flow chart, this subroutine contains an alternate entry point, labeled CROW. The first call to the subroutine enters at the starting point, with the variable ICAL set to zero. Subsequent calls to the subroutine utilize the entry CROW. In this manner, the constant parameters of the orbit are calculated only once.

Calculation of the eccentric anomoly for elliptic orbits requires determination of the solution to the equation:

$$E = M + e \operatorname{Sin} E \tag{A.3}$$

where E is the eccentric anomoly, M is the mean anomoly, and e is the orbit eccentricity. The solution to this equation is obtained using a series of successive approximations, using the numerical algorithm:

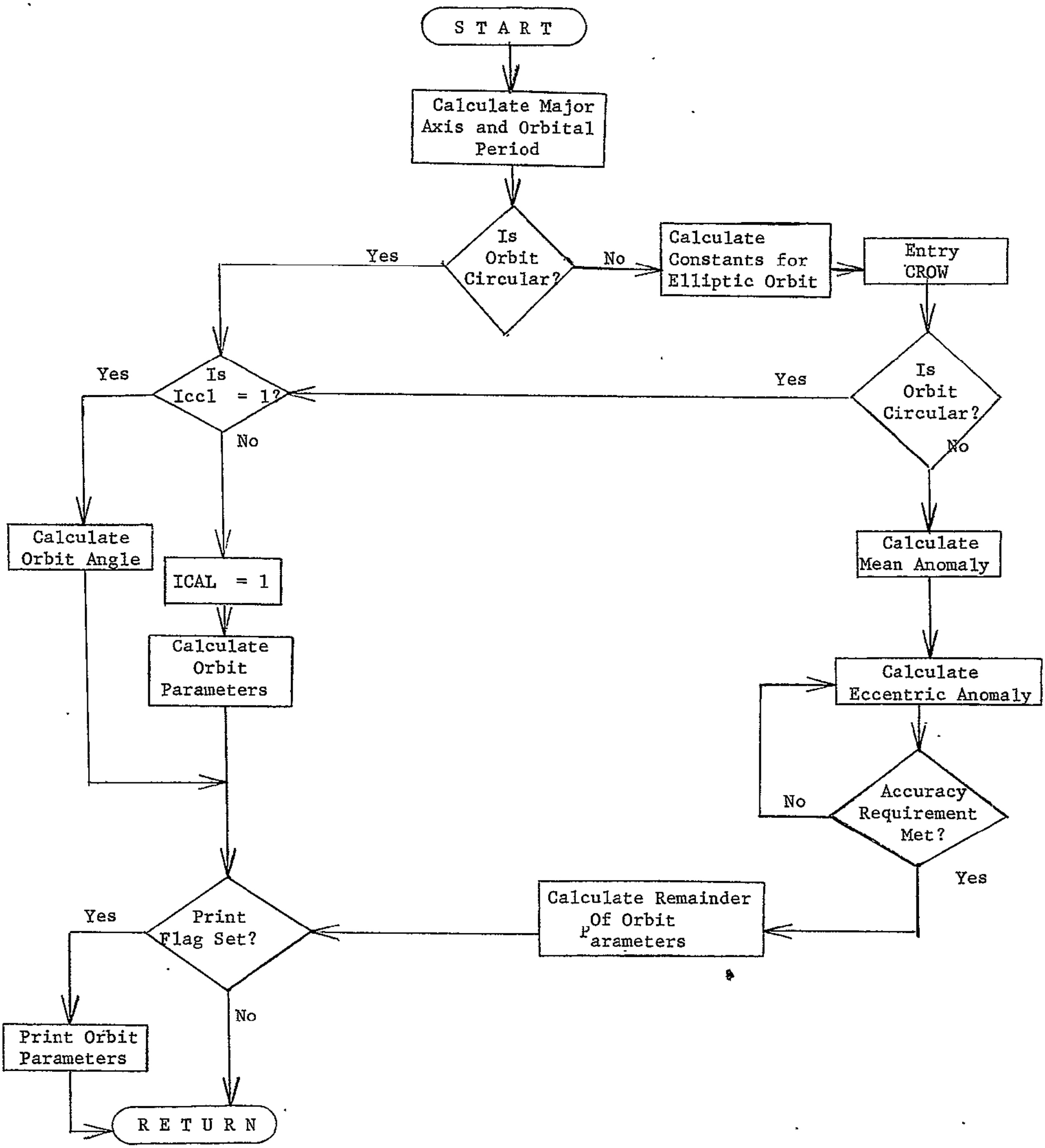


Figure A-2 Subroutine CRO Flow Chart

$$E_n = M + e \sin E_{n-1} \tag{A.4}$$

The starting value of E is set to the mean anomoly for the first entry to the subroutine and to the previous value of E for subsequent matrices. The convergence of the solution is dependent on the orbit eccentricity, requiring only 4 or 5 iterations for low eccentricity orbits ($e \leq 0.1$). As the eccentricity is increased, the speed of convergence decreases.

e. CVRM - This subroutine calculates the rotational motion of the chase vehicle utilizing the quaternion technique, and sets up the chase vehicle body axes to inertial axes transformation matrix. The quaternions are normalized after each step of the numerical integration to assure the orthogonality of the transformation matrix.

f. EANG - Subroutine EANG calculates the vehicle Euler angles from the appropriate transformation matrix. This subroutine is called to calculate the chase vehicle attitude relative to the local vertical, and also is called to calculate the chase vehicle attitude relative to the target vehicle.

g. FOMO - Subroutine FOMO calls the appropriate subroutines to calculate the forces and moments acting on each vehicle due to the disturbances and the tether. As illustrated in Figure A-3, the option of by-passing the gravity gradient or aerodynamic disturbances is available. This option is controlled by the input variable IDD.

h. FOMS - This subroutine is called to perform the required numerical integrations and to store the past values of the derivatives. The two numerical integration techniques previously described are contained in this routine.

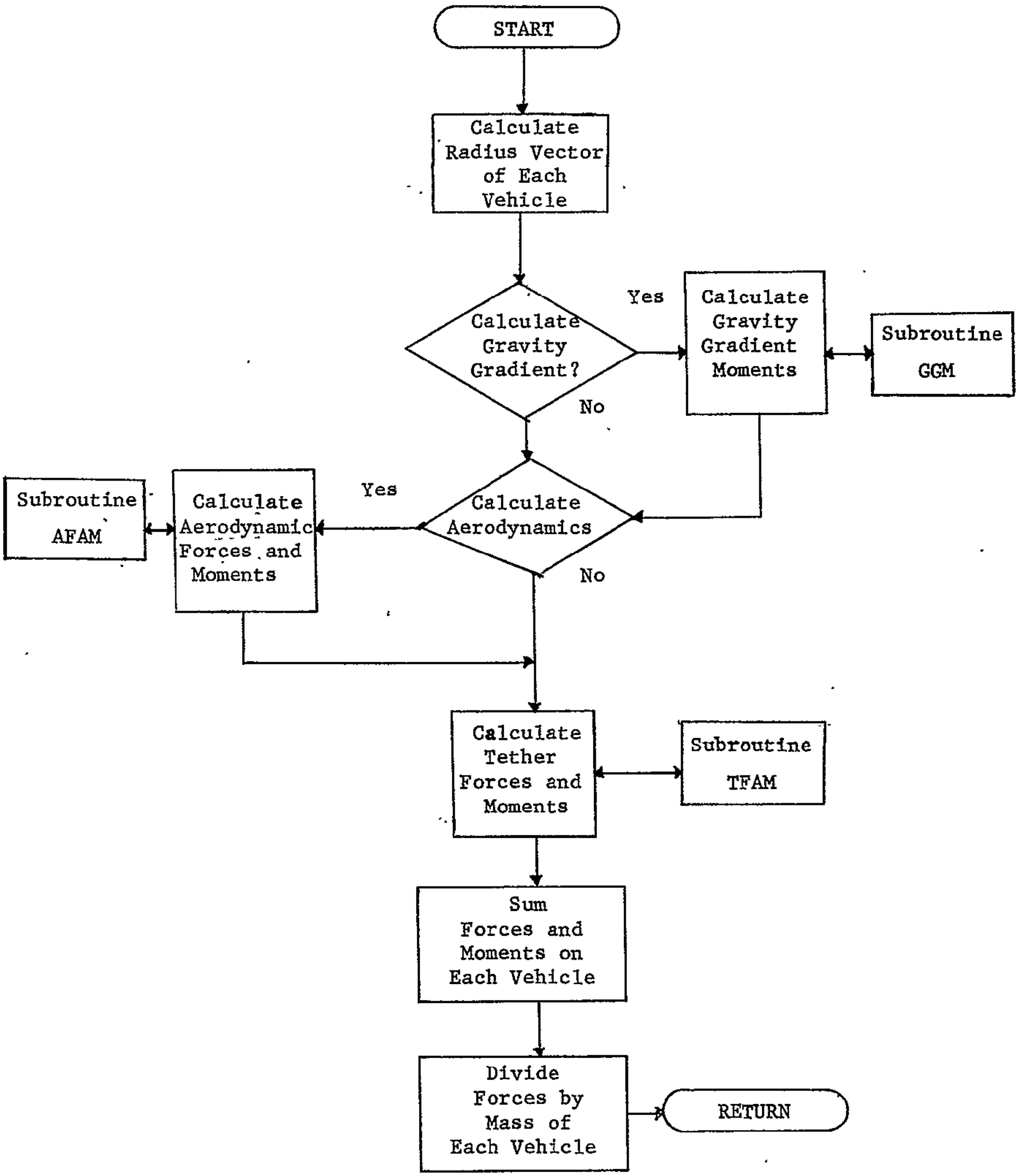


Figure A-3 Subroutine FOMO Flow Chart

i. GGM - Subroutine GGM calculates the gravity gradient moments acting on each vehicle as a function of the vehicles attitude, inertia, and radial distance from the center of the earth.

j. RPV - This subroutine calculates the separation distance and velocity between the chase and target vehicles in the local vertical coordinate system and also transforms these quantities into the target vehicle body axes. For an inertially stabilized target vehicle, this transformation results in the generation of the total velocity vector between the two vehicles, whereas, for an earth oriented target vehicle, the relative velocity does not include the effects of the rotating coordinate system. In both cases, the velocity calculated is the velocity which would be observed (or measured) from the target vehicle.

k. TCL - Subroutine TCL is the tether control law module. This subroutine calculates the tether attach point on the target vehicle (TTETH), the tether attach point on the chase vehicle (CTETH), and the magnitude of the tether force (TFM) as specified by the control law under investigation. This subroutine will normally be user supplied, since it is dependent on the type of control law being used. With the tether control law contained in a separate module, the control logic can be modified for investigation of various control techniques without affecting the computational sequence as the logic of the main program. All pertinent input/output variables to the routine are carried through the common blocks.

l. TFAM - Subroutine TFAM is used to determine tether forces and moments on both vehicles. Subroutine TFAM calls subroutine TCL to generate the magnitude of the tether force and the tether attach point on the two vehicles. Using the vector between tether attach points on both vehicles and the force exerted on the tether, subroutine TFAM calculates the forces and moments acting on both vehicles by the tether and transforms them to the appropriate coordinate systems for use in the equations of motion.

m. Miscellaneous - The remainder of the modules are used to print a visual title page (Subroutine PIC) and to perform matrix-vector manipulations. Subroutine CRPR forms the cross product of two vectors; CTM generates a transformation matrix using the vehicle Euler angles; MATIM forms the product of a matrix and a vector; MATM multiplies two square matrices; and TMAT transposes a square matrix. These routines are used throughout the program in the generation of the solution to the equations of motion.

APPENDIX B
TETHER CONTROL LAW MODULE

```
      SUBROUTINE TCL
C     ******************************************************************
C     TETHER CONTROL LAW ROUTINE
C     TCL CALCULATES THE TETHER FORCE MAGNITUDE AND TETHER ATTACH
C     POINTS ON THE TARGET AND CHASE VEHICLES
C     ******************************************************************
      COMMON/BERN/ AA , AP , DELTAT , GC , ICAL , IFPR , IOTP , PI ,
     1             RE , TIO , TOA
      COMMON/BERN1/ ECC , OAR , ORACC , ORBAN , ORBV , RADI , RADIO ,
     1              RADV , TP , VEL
      COMMON/BERN2/    CAP(3)      , CINTA(3,3) , CINTI(3,3) , CHMAS
     1  , CTETH(3)    , CVEL(3)     , DMOM(3)    , DOVM(2,3)  , D21(3,3)
     2  , D31C(3,3)   , D32(3,3)    , E          , H(3)       , HPR(3)
     3  , INGEO       , NE          , P(2,3)     , PDOT(2,3)  , PDDOT(3)
     4  , RTVA(3)     , QTRP(4)     , QTRN(4)    , RA(3)      , RP(3)
     5  , RV(3)       , TAP(3)      , TARMA      , TCAP(3)    , TRA(3)
     6  , TTETH(3)    , VTVA(3)
      COMMON/BERN3/    ASM          , D12(3,3)   , D13(3,3)   , IDD
     1  , RVCT(2)     , STL         , STRCH      , TELA       , TFM
     2  , TINTA(3,3)  , TMOM(3)     , WAX(3)
      COMMON/BERN4/ ALT  ,  DO1(3,3)  , D30(3,3)
C     ******************************************************************
C
C     6-DOF TETHER CONTROL LAW DEVELOPED UNDER CONTRACT NAS8-25353
C
C     THIS CONTROL LAW ASSUMES A GENERAL BOOM ATTACH POINT ON THE
C     TARGET VEHICLE AND A CHASE VEHICLE BOOM OFFSET OF ONE METER
C     ALONG THE CHASE VEHICLE NEGATIVE X-AXIS.  THE TARGET VEHICLE
C     BOOM LENGTH AND ATTACH POINT ARE SET IN DATA STATEMENTS.
C     THE CHASE VEHICLE BOOM LENGTH IS INPUT AS CTETH(1), AND INCLUDES
C     THE ONE METER OFFSET.  THIS LENGTH IS INPUT AS A NEGATIVE
C     NUMBER IN ORDER TO POINT THE CHASE VEHICLE BOOM AT THE
C     TARGET VEHICLE AT ZERO ATTITUDE ERRORS.  THUS, AN INPUT OF
C     CTETH(1) = -2.0 RESULTS IN A BOOM LENGTH OF ONE METER ATTACHED
C     ONE METER FROM THE CHASE VEHICLE CENTER OF GRAVITY.
C
C     ******************************************************************
      DIMENSION DEA(3,3),CATV(3),CATAE(3),       DAE(3,3),WRF(3),
     1CATDE(3),AOS(3),PTVA(3)
      DIMENSION RLV(3),RAE(3),CT(3),DV(3),DV1(3),DV2(3),D23(3,3)
C     SET TARGET VEHICLE BOOM LENGTH  (ARM)  AND MAXIMUM FORCE  (FMX)
      DATA ARM,FMX,RDMIN,RMIN/3.,3.5,-0.10,10.0/
C     SET TARGET VEHICLE BOOM BASE LOCATION
      DATA AOS/0.0,0.0,0.0/
      IRLER = -5
      RDMIN = -0.1
      IF(E .NE. 0.0) GO TO 2
      DO 1 J=1,3
```

```
    1 CT(J) = CTETH(J)
    2 DO 3 J=1,3
    3 CTETH(J) = CT(J)
C     CALCULATE VECTOR FROM TARGET VEHICLE BOOM BASE
C     TO CHASE VEHICLE C.G.
      DO 4 J=1,3
    4 PTVA(J) = RTVA(J) - AOS(J)
C     CALCULATE CHASE VEHICLE AZIMUTH
      AC = ATAN(PTVA(3)/PTVA(1))
      IF(PTVA(1) .LT. 0.0) AC = AC + SIGN(PI,PTVA(3))
C     CALCULATE CHASE VEHICLE ELEVATION
      EC = ATAN(PTVA(2)/SQRT(PTVA(1)**2 + PTVA(3)**2))
C     CALCULATE TRANSFORMATION FROM CHASE VEHICLE BODY AXES
C     TO AZIMUTH-ELEVATION AXES
      C1=COS(AC)
      C2=COS(EC)
      S1=SIN(AC)
      S2=SIN(EC)
      DAE(1,1)=C1*C2
      DAE(1,2)=-C1*S2
      DAE(1,3)=-S1
      DAE(2,1)=S2
      DAE(2,2)=C2
      DAE(2,3)=0.0
      DAE(3,1)=S1*C2
      DAE(3,2)=-S1*S2
      DAE(3,3)=C1
      DO 10 J=1,3
      DO 10 K=1,3
   10 DEA(J,K)=DAE(K,J)
C     CALCULATE AZIMUTH RATE
      ACD = (PTVA(1)*VTVA(3)-PTVA(3)*VTVA(1))/(PTVA(1)**2+PTVA(3)**2)
      FAC = SQRT(PTVA(1)**2 + PTVA(3)**2)
C     CALCULATE RANGE FROM TARGET VEHICLE BOOM BASE
C     TO CHASE VEHICLE C.G.
      RM = SQRT(PTVA(1)**2 + PTVA(2)**2 + PTVA(3)**2)
C     CALCULATE ELEVATION RATE
      ECD = (FAC*VTVA(2) - PTVA(2)*(PTVA(1)*VTVA(1) + PTVA(3)*VTVA(3))/
     1 FAC)/RM**2
C     CALCULATE RANGE RATE
      RDOT = (PTVA(1)*VTVA(1) + PTVA(2)*VTVA(2) + PTVA(3)*VTVA(3))/RM
C     CALCULATE ATTITUDE ERROR
      DO 20 J=1,3
      CATAE(J)=0.0
      WRF(J)= D32(J,2)*CVEL(3)- D32(J,3)*CVEL(2)
      CATV(J)= D32(J,1)
      DO 20 K=1,3
   20 CATAE(J)=CATAE(J)+DEA(J,K)*CATV(K)
      DO 30 J=1,3
      CATDE(J)=0.0
      DO 30 K=1,3
   30 CATDE(J)=CATDE(J)+DEA(J,K)*WRF(K)
      ERF= CATAE(2)**2 + CATAE(3)**2
```

```
      ER=ASIN(SQRT(ERF))
C     CALCULATE TARGET VEHICLE BOOM POSITION
      SG2=CATAE(2)*CATDE(2)
      SG3=CATAE(3)*CATDE(3)
      ARMA=SIGN(PI/4.0,CATAE(3))
      ARME=SIGN(PI/4.0,CATAE(2))
      IF (SG2*SG3 .GE. 0.0) GO TO 40
      IF (SG2 .GE. 0.0) ARMA=0.0
      IF(SG3 .GE. 0.0) ARME=0.0
   40 IF (ABS(CATAE(2)) .LT. 0.001) ARME=0.0
      IF (ABS(CATAE(3)) .LT. 0.001) ARMA=0.0
C     CALCULATE ATTITUDE RATE
      IF (ERF .LT. 1.0E-16) GO TO 50
      ERD = (CATAE(2)*CATDE(2)+CATAE(3)*CATDE(3))/SQRT((1.0-ERF)*ERF)
      GO TO 60
   50 ERD=0.0
   60 TFM=0.0
C     CALCULATE ROLL ERROR
      DO 61 J=1,3
   61 RLV(J) = D32(J,2)
      CALL MATIM(RAE,DEA,RLV)
      RAER = ASIN(RAE(3))
      IF(RAE(2) .LT. 0.0) RAER = SIGN(PI,RAER) - RAER
      CALL EANG(RA,TRA,D32)
C     CALCULATE ROLL RATE
      TANA = TAN(RA(2))
      WRL = CVEL(1) + TANA*SIN(RA(1))*CVEL(2) + TANA*COS(RA(1))*CVEL(3)
      RTMIN=PI/157.5 + SQRT(ACD**2 + ECD**2)
      IF(ER .GT. PI/8.0) RTMIN=PI/90. + RTMIN
      CALL TMAT(D23,D32,3)
      IF(RM .LE. RMIN) GO TO 200
C     CONTROL REGION 1
      IF(RM .LT.(RMIN + 5.0)) RDMIN = -0.15
C     SET TARGET VEHICLE BOOM ELEVATION
      ARME = -PI/4.0
      IF(ABS(AC) .GT. PI/2.0) ARME = -ARME
C     CALCULATE CHASE VEHICLE BOOM POSITION
      IF(WRL*RAER .LT. 0.0 .AND.(ABS(WRL) .GT. PI/180.0 .AND.
     1 ABS(RAER) .GT. PI/9.00))GO TO 67
      IF(ABS(RAER) .LT. PI/9.00) GO TO 67
      CTETH(1) =-1.0 + 0.86603*(CT(1) + 1.0)
      CTETH(2) = -0.5*(CT(1) + 1.0)
      CALL MATIM(DV,D32,CTETH)
      DV1(1) = COS(ARMA)*COS(ARME)*ARM
      DV1(2) = SIN(ARME)*ARM
      DV1(3) = SIN(ARMA)*COS(ARME)*ARM
      CALL MATIM(TTETH,DAE,DV1)
      DO 315 J=1,3
      TTETH(J) = TTETH(J) + AOS(J)
  315 DV2(J) = DV(J) + RTVA(J) - TTETH(J)
      DV2M = SQRT(DV2(1)**2 + DV2(2)**2 + DV2(3)**2)
      DO 316 J=1,3
  316 DV1(J) =-DV2(J)/DV2M
```

```
      CALL MATIM(DV,D23,DV1)
      TMG = CTETH(2)*DV(3) - CTETH(3)*DV(2)
      IF(TMG*RAER .GT. 0.0) CTETH(2) = -CTETH(2)
      IF(WRL*RAER.LT.0.0.AND. ABS(RAER) .LT. PI/25.0) CTETH(2)=-CTETH(2)
      CTETH(2) = CTETH(2) + CT(2)
C     TEST ATTITUDE ERROR
   67 IF (ER .GT. PI/4.0) GO TO 100
C     TEST ATTITUDE RATE
   70 IF (ABS(ERD) .GT. PI/60.) GO TO 110
C     TEST FOR MAXIMUM ANGULAR VELOCITIES
      DO 75 J=1,3
      IF(ABS(CVEL(J)) .GT. PI/60.0) GO TO 310
   75 CONTINUE
C     TEST RANGE RATE
      IF (RDOT .GT. RDMIN) GO TO 120
C     TEST ROLL ERROR
      IF(ABS(PAER) .GT. PI/4.0 .AND. (WRL*RAER .GT. 0.0 .OR. ABS(WRL)
     1 .LT. PI/360.0))GO TO 120
      GO TO 310
C     SET TETHER FORCE
  100 IF (ERD .LT. 0.0 .AND. ABS(ERD) .GT. PI/180.) GO TO 70
      TFM=FMX
      GO TO 310
  110 IF(ERD .LT. 0.0) GO TO 310
      GO TO 150
  120 TFM=FMX
      GO TO 310
C     CALCULATE FORCE REQUIRED TO STOP ROTATIONAL MOTION
  150 CALL MATIM(DV,D32,CTETH)
      DV1(1) = COS(ARMA)*COS(ARME)*ARM
      DV1(2) = SIN(ARME)*ARM
      DV1(3) = SIN(ARMA)*COS(ARME)*ARM
      CALL MATIM(TTETH,DAE,DV1)
      DO 415 J=1,3
      TTETH(J) = TTETH(J) + AOS(J)
  415 DV2(J) = DV(J) + RTVA(J) - TTETH(J)
      DV2M = SQRT(DV2(1)**2 + DV2(2)**2 + DV2(3)**2)
      DO 416 J=1,3
  416 DV1(J) =-DV2(J)/DV2M
      CALL MATIM(DV,D23,DV1)
      TMGY = CTETH(3)*DV(1) - CTETH(1)*DV(3)
      TMGZ = CTETH(1)*DV(2) - CTETH(2)*DV(1)
      TMG1 = TMGY*CINTA(2,2)*CVEL(2)/DELTAT
      TMG2 = TMGZ*CINTA(3,3)*CVEL(3)/DELTAT
      TFM1 = TMG1/TMGY
      TFM2 = TMG2/TMGZ
      TFM = TFM1
      IF(TFM2 .GT. TFM1) TFM = TFM2
      IF(TFM .GT. FMX) TFM = FMX
      IF(TFM .LT. 0.0) TFM = 0.0
      IF(TFM .GT. 0.0 .AND. ER .GT. PI/30.0) TFM = FMX
      GO TO 310
C     CONTROL REGION 2
```

```
  200 CONTINUE
C       CALCULATE CHASE VEHICLE BOOM POSITION
        IF(WRL*RAER .LT. 0.0 .AND.(ABS(WRL) .GT. PI/90.0 .AND. ABS(RAER)
     1 .GT. PI/30.0))GO TO 220
        IF(ABS(RAER) .LT. PI/90.0) GO TO 220
        IF(ABS(WRL) .LT. PI/180.0 .AND. ABS(RAER) .LT. PI/30.0) GO TO 220
        CTETH(1) = -1.0 + 0.86603*(CT(1) + 1.0)
        CTETH(2) = -0.5*(CT(1) + 1.0)
        IRLER = 5
        CALL MATIM(DV,D32,CTETH)
        DV1(1) = COS(ARMA)*COS(ARME)*ARM
        DV1(2) = SIN(ARME)*ARM
        DV1(3) = SIN(ARMA)*COS(ARME)*ARM
        CALL MATIM(TTETH,DAE,DV1)
        DO 515 J=1,3
        TTETH(J) = TTETH(J) + AOS(J)
  515 DV2(J) = DV(J) + RTVA(J) - TTETH(J)
        DV2M = SQRT(DV2(1)**2 + DV2(2)**2 + DV2(3)**2)
        DO 516 J=1,3
  516 DV1(J) =-DV2(J)/DV2M
        CALL MATIM(DV,D23,DV1)
        TMG = CTETH(2)*DV(3) - CTETH(3)*DV(2)
        IF(TMG*RAER .GT. 0.0) CTETH(2) = -CTETH(2)
        IF(WRL*RAER.LT.0.0.AND. ABS(RAER) .LT. PI/25.0) CTETH(2)=-CTETH(2)
        CTETH(2) = CTETH(2) + CT(2)
C       TEST ATTITUDE RATE
  220 IF(ERD .LT. 0.0) GO TO 250
C       TEST ATTITUDE ERROR
        IF (ER .GT. PI/30.0) GO TO 260
C       TEST RANGE RATE
        IF (RDOT .GT. RDMIN) GO TO 260
C       TEST ROLL ERROR
        IF(IRLER .GT. 0) GO TO 230
C       TEST FOR MINIMUM ATTITUDE ERROR
        IF (ER .LT. PI/180.0) GO TO 310
        GO TO 150
C       SET TETHER FORCE
  230 IF(WRL*RAER .LT. 0.0 .AND. ABS(WRL) .GT. PI/90.0) GO TO 310
        IF(ERD .LT. -PI/45.0) GO TO 310
        IF(IRLER .GT. 0   ) GO TO 260
        GO TO 310
  250 IF(ABS(ERD) .GT. RTMIN) GO TO 230
  260 TFM = FMX
C       CALCULATE TARGET VEHICLE TETHER ATTACH POINT
  310 AX=COS(ARMA)*COS(ARME)*ARM
        AY=SIN(ARME)*ARM
        AZ=SIN(ARMA)*COS(ARME)*ARM
        TTETH(1)=DAE(1,1)*AX + DAE(1,2)*AY + DAE(1,3)*AZ +AOS(1)
        TTETH(2)=DAE(2,1)*AX + DAE(2,2)*AY + DAE(2,3)*AZ +AOS(2)
        TTETH(3)=DAE(3,1)*AX + DAE(3,2)*AY + DAE(3,3)*AZ +AOS(3)
        RETURN
        END
```