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PROBLEMS IN STRESS ANALYSIS AND OPTIMUM DESIGN

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The George Washington University

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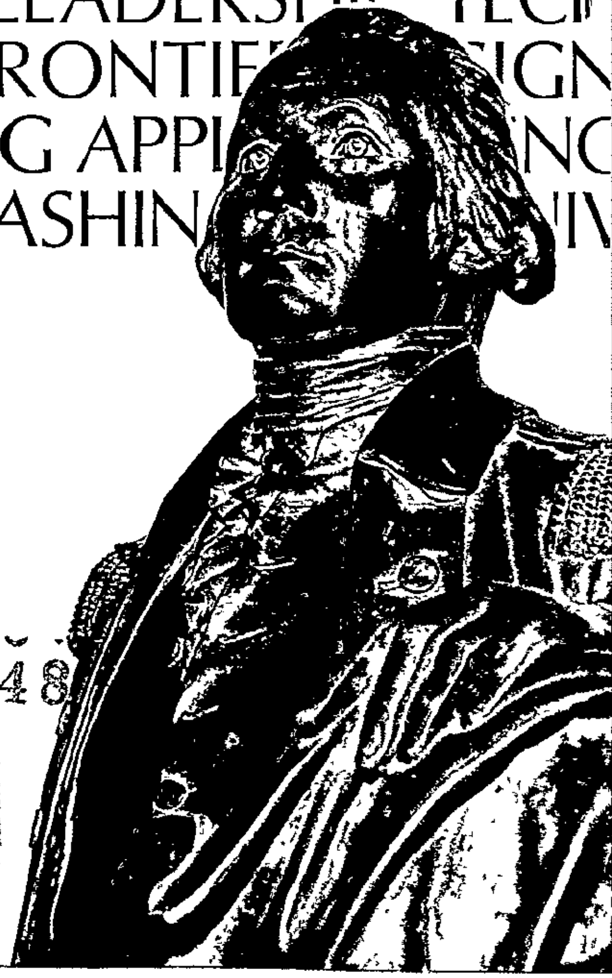

SCHOOL OF ENGINEERING
AND APPLIED SCIENCE

PROBLEMS
IN
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AND
OPTIMUM DESIGN

By

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Final Report to National Aeronautics and Space Administration
on research performed under NASA Grant NGR-09-010-053.

This report presents the results of research conducted by and under the supervision of B. I. Hyman, Associate Professor, Engineering Mechanics Department, and under the sponsorship of NASA Grant NGR-09-010-053. The research was conducted during the period Feb. 1, 1969 - Feb. 1, 1970.

I. STRESS ANALYSIS OF ELASTO-PLASTIC SHELLS

The manuscript "Stress Analysis of Thin Elasto-Plastic Shells" by O. Lomacky and B. I. Hyman has been prepared. This manuscript is based on a D.Sc. dissertation submitted to The George Washington University by O. Lomacky in September 1968. This manuscript (enclosed as Appendix A of this report) will be submitted for publication in the International Journal of Solids and Structures.

In this paper, the theory of thin shells undergoing large deflections and loaded into the strain hardening range is presented. Plastic strain incompressibility is assumed. The two governing differential equations in terms of the stress function and the normal displacement are derived in a form where the corresponding equations of the elastic problem are modified only by the addition of the integrals of the plastic strains. The equations can be utilized in conjunction with any yield criterion, flow rule and hardening law.

The theory is applied to the problem of stress concentration around a circular opening in a pressurized spherical shell. A numerical solution is obtained by an iterative procedure using the finite difference technique for the special case of small displacements, bilinear stress-strain curve, and deformation theory of plasticity. Results for a typical shell are obtained for stress concentration factor, strain concentration factor, and extent of plastic deformation as functions of pressure.

II. FIBER REINFORCED PLATES WITH HOLES

The technical note "Exploratory Tests on Fiber-Reinforced Plates with Circular Holes under Tension" by B. I. Hyman, A. DeTurk, R. Diaz, and G. DiGiovanni was published in Vol. 7, No. 9 of the AIAA Journal. This paper (enclosed as Appendix B) describes work which was completed prior to February 1, 1969. Financial support for the manuscript preparation was provided by the subject grant.

The work reported in Appendix B indicated that an approximately 40% increase in tensile strength of bilateral fiber reinforced plates can be achieved by routing continuous fibers around the hole instead of drilling holes in prefabricated plates. As a result of these findings, further studies were conducted in an attempt to determine the details of the stress distribution in the vicinity of the hole.

A crucial phase of any further work in this area is the derivation of appropriate stress-strain relations for a composite plate with fibers routed around the hole. Such a study was conducted and is included as Appendix C of this report. This analysis assumes that both the matrix and the fibers are elastic and isotropic, and that the composite is locally orthotropic. Expressions for fiber density and orientation at any point in the plate are derived from the assumption that the fiber distribution in the plate is analogous to the stream line pattern for two dimensional potential flow past a circular cylinder.

A computer program was written which can be used to determine at any point in the composite plate the six elastic coefficients needed for the two dimensional generalized Hookes' Law. Results for a typical composite are shown in Appendix C.

The stress-strain relations described above can be combined with the equilibrium equations and compatability condition to determine the stress distribution in the composite plate. For a homogeneous isotropic plate, these equations lead to the biharmonic equation in terms of the Airy stress function. For a homogeneous, orthotropic plate, the solution can still be formulated in terms of a single fourth order partial differential equation involving the Airy stress function. However, for the nonhomogeneous, locally orthotropic plate, the introduction of an Airy stress function does not lead to any simplifications.

One approach is to write the compatability condition in terms of the stresses and the elastic coefficients where the elastic coefficients are determined at any location from the computer program described in Appendix C. Then the compatability condition plus the two equilibrium equations can be solved for the three stress components $(\sigma_x, \sigma_y, \tau_{xy})$. A finite difference solution to these equations in terms of cartesian coordinates is presented in Appendix D of this report. The computer program included in Appendix D can be used to obtain a solution for the general nonhomogeneous case. However, the numerical calculations were carried out only for the homogeneous isotropic and orthotropic cases. These numerical results compared very poorly with the existing analytical solutions to these problems. The discrepancies are attributed to the difficulty in handling the boundary conditions at the hole. Various schemes described in Appendix D to modify the finite difference scheme in the vicinity of the hole proved to be unsuccessful.

An alternative approach to that described in Appendix D was also investigated. This consisted of writing the finite difference equations in terms of polar coordinates. This simplifies the handling of the boundary conditions at the hole. A computer program for the case of a homogeneous isotropic plate has been written and is currently being debugged. However, the usefulness of this approach for the general case of the nonhomogeneous anisotropic plate is limited because of the extremely complex form that the compatability equation takes in this case. This approach has been temporarily abandoned in favor of a finite element analysis. Since the nonhomogenity and anisotropy do not cause any particular difficulty using the finite element method, it is expected that this approach will be successful and it is planned to have the analysis completed by June 1970.

The analytical model for the plate with the fibers routed around the hole involves several major assumptions. The two most important assumptions are:

1) The fiber distribution can be obtained from the stream line pattern for potential flow around a cylinder.

2) The elastic properties for a woven fabric can be obtained by replacing the woven fabric by an appropriate lamination of unidirectional layers.

In order to assess the accuracy of these assumptions and the results obtained from the analysis based on these assumptions, an experimental program was conducted. This work, included as Appendix E of this report, consisted of an experimental determination of the strain distribution in the vicinity of the hole. The investigation consisted of taking strain gage data at seven locations around the hole of each test specimen, while the plate was subjected to a tensile load in a universal testing machine. Data on the elastic properties of the material and on the matrix content by volume were also obtained. As described in Appendix E, two specimens of drilled plates (Type B) and two specimens of plates with routed fibers (Type C) were tested. The results presented in Appendix E were based on the assumption that the elastic properties of all the specimens were equal to the values obtained on reference specimens with no holes. It was only after the data was evaluated that it was realized that differences in the curing times during fabrication of the specimens could cause enough difference in the material properties to make this assumption invalid. Therefore, before the results presented in Appendix E can be used, additional tests will have to be run. It is hoped that these tests will be conducted before June 1970 so that valid experimental data will be available at the time the finite element analysis is completed. The additional tests will be in the form of tensile tests on coupons cut from the same Type B and Type C specimens from which the strain distribution data was obtained. In this manner, the elastic properties of each specimen will be determined.

The problem of stress concentration around a hole in an infinite plate under tension is only one of many problems to which the fiber reorientation concept can be applied. Preliminary work has been started on an experimental program to determine the effectiveness of this concept for a plate with a hole in which the load is applied through a bolt inserted in the hole, hence producing a bearing load on the edge of the hole. This work is expected to be completed by August 1970. A finite element solution to this problem is also contemplated.

III. BUCKLING OF SHELLS WITH VARIABLE THICKNESS

This study consisted of the development of a direct approach for optimum design of shells to resist buckling. The general approach is illustrated by solving the particular problem of optimum design of a variable thickness cylindrical shell subject to external lateral pressure. The analysis is described in Appendix F to this report. The constraint of constant weight is introduced into the expression for the change in potential energy of the system and the Rayleigh-Ritz technique is used to reduce the energy expression to a set of nonlinear algebraic equations. The solution of these equations yields the collapse pressure for the strongest shell of a given weight, the corresponding thickness variation, and the associated buckling mode. The values obtained for the buckling pressures are from 6% to 112% greater than the buckling pressures for constant thickness shells of the same weight.

A manuscript based on Appendix F has been submitted for publication in the Journal of Applied Mechanics and for presentation at the 6th U.S. National Congress of Applied Mechanics. A copy of the manuscript is included as Appendix G to this report.