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FOURIER TRANSFORMABLE PROPERTIES OF PARABOLOIDAL MIRROR SEGMENTS

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GSFC

GODDARD SPACE FLIGHT CENTER
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ABSTRACT

Theoretical and experimental results are presented to demonstrate advantages of paraboloidal mirror segments over lenses as Fourier transforming and image reconstructing elements in an optical data processing system. Experimental results of spatial filtering are also presented.

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ILLUSTRATIONS

ILLUSTRATIONS (Continued)

FOURIER TRANSFORMABLE PROPERTIES OF
PARABOLOIDAL MIRROR SEGMENTS

1. INTRODUCTION

Ernst Abbe[1] in 1873 realized the phenomenon that the diffraction pattern in the focal plane of a converging lens has all the properties of two dimensional Fourier transform of the light distribution in the input information. Since then lenses are exclusively used in optical data processing systems as Fourier transforming and image reconstructing elements.[2,3]

The quality and resolution of Fourier transform relationship is very much affected by lens aberrations. The undesirable front surface reflections and axial symmetry of the optical system are a few examples of problems normally encountered in optical data processing systems with lenses. The purpose of this paper is to demonstrate theoretically as well as experimentally the advantages of paraboloidal mirror segments over lenses for producing Fourier transform relationship in the focal plane of the optical system and for image reconstructing in an optical data processing system.

2. PARABOLOIDAL MIRROR AS A FOURIER TRANSFORMING ELEMENT

Let us consider the parabolic as a linear element in an optical signal processing system. Referring to Figure 1 light rays entering from the right parallel to the Z-axis are reflected from the mirror and intersect the Z-axis at the focal point, f. The relationship between the incident and reflected scalar light fields at any point in the (x, y, zo) plane can be described in terms of a transfer function, t (x, y, zo). For mirrors, (as well as for lenses) the transfer function is strictly a phase function, and we have

$$\begin{aligned} t(x, y, zo) &= e^{[j\phi(x, y, z_0)]} \\ &= e^{[jk\Delta(x, y, z_0)]} \end{aligned} \qquad (1)$$

The term $\phi(x, y, zo)$ is the phase difference in radians between the incident and reflected fields, k is the wave number $2\pi/\lambda$ and $\Delta(x, y, z_0)$ is the difference in path lengths between the incident and reflected waves.

Again referring to Figure 1, we see that the total path difference is

$$\Delta(x, y, z_0) = 2(z_0 - \Delta z) \qquad (2)$$

(this assumes no ray bending to the left of the (x, y, zo) plane, analogous to the thin lens approximation in lens analysis).

Now from the equation for a paraboloid of revolution,

$$\Delta z = \frac{x^2 + y^2}{4f} \tag{3}$$

therefore

$$\begin{aligned} t(x, y, z_0) &= e^{[jk\Delta(x, y, z_0)]} \\ &= e^{[j2kz_0]}\, e^{-jk\left[\frac{x^2 + y^2}{2f}\right]} \end{aligned} \tag{4}$$

In the case of a spherical thin lens, if the paraxial approximation is made, i.e. we consider only light rays close to the lens axis, the transfer function is found to be:[4]

$$t(x, y, z_0) = e^{[jknz_0]}\, e^{-jk\left[\frac{x^2 + y^2}{2f}\right]} \tag{5}$$

Here n is the index of refraction of the lens material, and zo is the lens thickness along the optical axis. It is important to note that the focal length f of a lens is a function of the index of refraction of the lens material.

Referring to the deviation of the transfer function, several points for the parabolic mirror as a system element can be made. First, since the focal length f of a mirror is not a function of index of refraction, the parabolic mirror system will not have the chromatic aberration that is inherent in the lens system.

Secondly all the light rays parallel to the z-axis will intersect the point f for a parabolic mirror, while only the paraxial rays will intersect the focal point for a spherical lens. This means that the parabolic mirror is inherently free from spherical aberrations (while the lens, of course, is not).

The absence of paraxial approximation for mirrors has several pratical advantages, such as, that the aperture of the optical signal being processed can be larger for a parabolic mirror than for a lens of the same diameter. More importantly, it means that one can cut out off-axis segments from a parabolic mirror, and these segments will have the same transforming properties of the original mirror and the same relative axis. Thus one can construct folded optical processing system.

Two other aberrations also deserve some attention. Let us consider astigmatism. This aberration arises when the incoming light rays make a large angle with the z-axis of a mirror. This is not a serious drawback, however, since for most optical signal processing, the incoming light is a coherent plane wave whose rays are parallel to z-axis. Furthermore all interelement light paths can be kept at very small angles with respect to the element axes. Astigmatism therefore, can be made negligible in a parabolic processing system.

The second aberration is coma, the aberration that occurs for light rays at small angles to the Z-axis. In experiments using parabolic mirrors as Fourier transforming elements, the effects of coma were not observable at the normal working angles.

3. EXPERIMENTAL PERFORMANCE OF PARABOLOIDAL MIRROR SEGMENTS

Figure 2 illustrates the experimental arrangement used to record the Fourier transform relationships in the focal plane of the parabola. Light from a He-Ne laser radiating at 6328 Å wavelength was brought to a point focus by a microscope objective. A thin metal plate with a 10 micron round aperture was placed in this plane for elimination of laser beam noise and also to exclude stray light. Another lens was placed a focal length away from this plane to collimate the laser light. The photographic transparencies were placed in the "input plane," placed a focal length away from the paraboloidal mirror segment. The off-axis paraboloidal mirror focuses the incident light in the "Fourier transform plane." The Fourier transforms of the various input information were recorded in this plane on Polaroid films.

As an example of single simple shapes, Figures 3, 4 and 5 show the Fourier transforms of circular, rectangular and square apertures. In Figure 3, the well known Airy disk can be seen distinctly in the center of the transform, and also in Figure 4 the transform is centered at the optical axis and is in a direction orthogonal to the length of the rectangular aperture.

Figures 6 to 10 illustrate the Fourier transforms of multiple simple shapes. The well known Fourier transform theorem, that the Fourier transform of a product of two functions is equal to the convolution of the Fourier transform of the two functions, can be proved optically. Let us consider Figure 6. The object transparency is an equally spaced parallel lines in a circular aperture. The Fourier transform of the combination will be the convolution of two individual transforms. The Fourier transform of parallel lines is a series of impulses with separation inversely related to the spacing of the lines and orthogonal to the direction of the lines. The Fourier transform of a circular aperture is an

Airy disk. Their resulting Fourier transform should be an Airy disk at the origin and also at the locations of each impulse. This is exactly what we have in Fourier transform plane and is shown in Figure 6 (b).

Figure 7 (a, b) shows the object transparency of a triangle and its Fourier transform. Figure 8 (a, b, c) shows a number of squares in a circular aperture, the Fourier transform and an enlarged view of the central portion of the Fourier transform. Figures 9 and 10 are two other examples of simple multiple shapes, their Fourier transforms and enlarged view of the central portions of Fourier transforms is also shown in these pictures.

Spatial filtering in the Fourier transform plane was also performed. The experimental arrangement is shown in Figure 11.

A wire grid as shown in Figure 12 (a) was chosen as an object. The Fourier transform of this object is shown in Figure 12 (b). For spatial filtering purposes, a slit was placed in the Fourier transform plane so that only the vertical structure of the Fourier transform was allowed to pass through the Fourier transform plane. The reconstructed filtered image is shown in Figure 12(c), and is without any vertical information. The vertical information of the object which was in the horizontal plane of the Fourier transform was suppressed in the Fourier transform plane. Figure 12 (d) shows the reconstructed image without any filtering.

4. CONCLUSIONS

A study of paraboloidal mirror segments as Fourier transforming and image reconstructing elements in optical data processing system have shown that they have several advantages over lenses. Some of them can be enumerated as follows:

1. They are inherently free from spherical and chromatic aberrations.

2. The unwanted front surface reflections are non-existent.

3. The optical data processing system is capable of folding resulting in a considerable saving of space.

4. Astigmatism and coma can be minimized to a far greater extent than in lenses.

5. Since the light rays do not pass through the material of a front surface reflecting paraboloidal mirror, it is not necessary that optical material

for fabrication purposes should be homogeneous and isotropic. The use of paraboloidal mirrors overcomes all the critical optical limitations.

6. The use of paraboloidal mirror segments provides a method for performing optical data processing using electromagnetic waves of any frequencies. (Lenses for ultraviolet, infrared and especially in radio frequencies are, if not impossible very difficult to fabricate.)

5. ACKNOWLEDGEMENTS

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One of the authors, Akram S. Husain-Abidi is a "Postdoctoral Resident Research Associate," of National Academy of Sciences. The other author, Thomas F. Krile is a Professor of Electrical Engineering at Rose Polytechnic, Indiana. He was at GSFC on a NASA summer fellowship program at the time the work reported here was undertaken.

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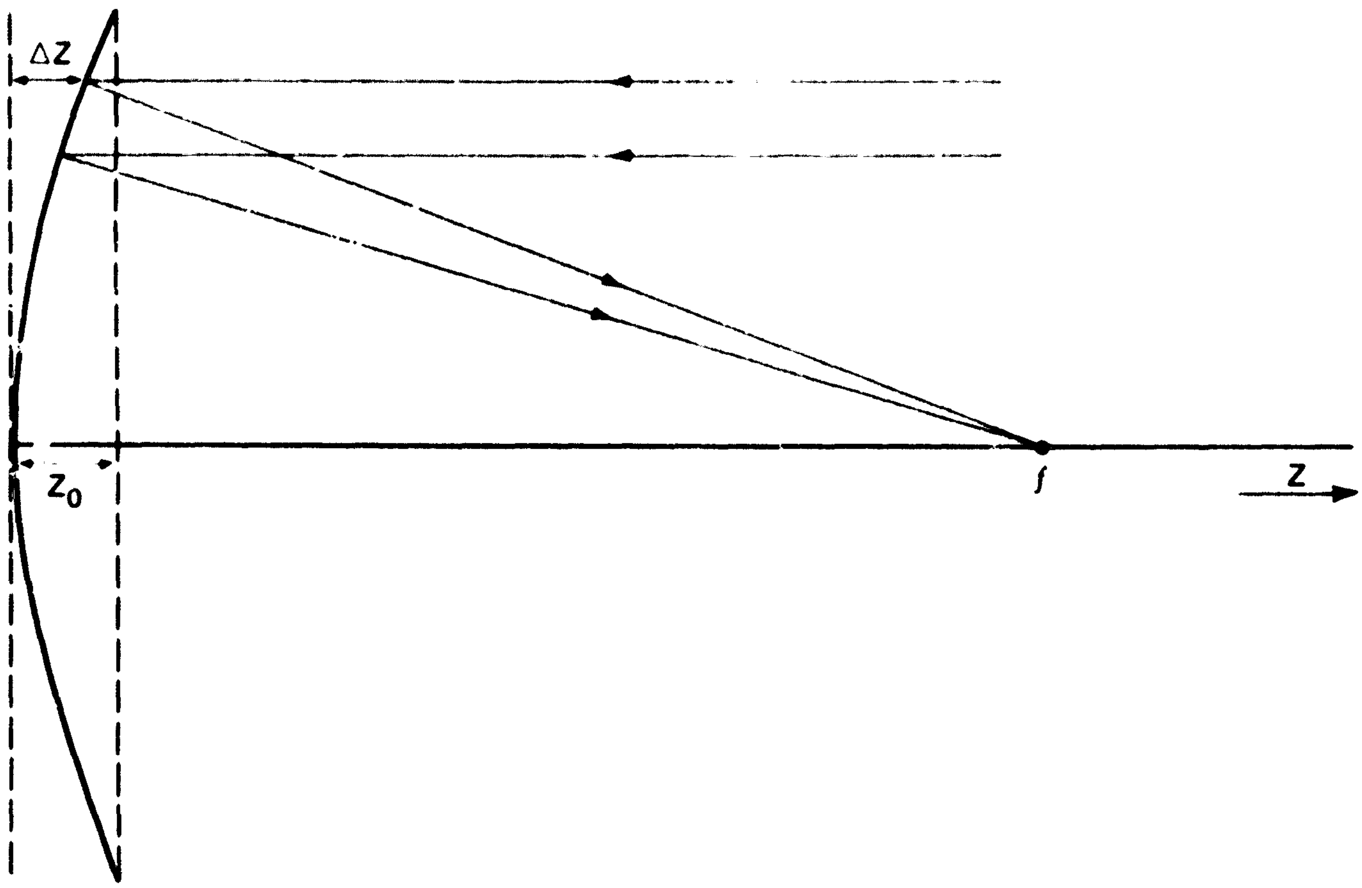
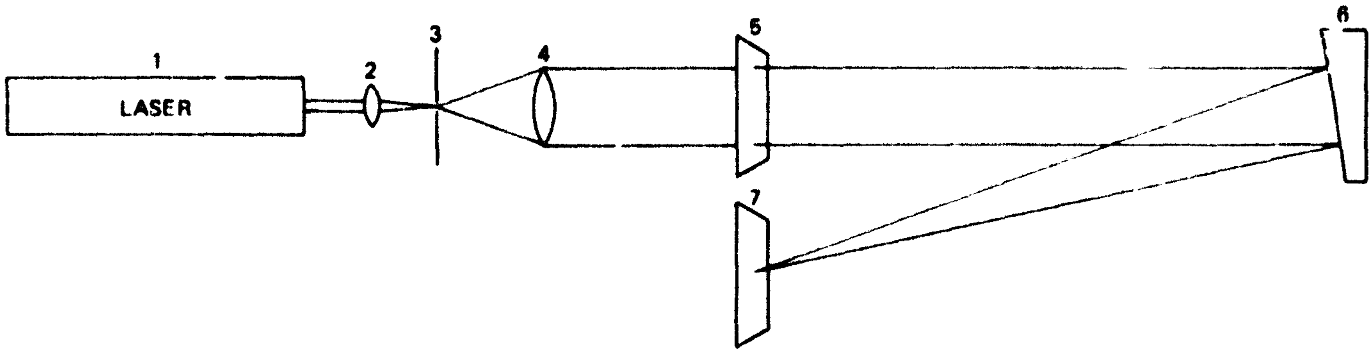


Figure 1. Transfer Function for Parabolic Mirror



1. He-Ne LASER
2. MICROSCOPE LENS
3. 10 MICRON PINHOLE
4. COLLIMATING LENS
5. INPUT PLANE
6. PARABOLOIDAL MIRROR SEGMENT
7. FOURIER TRANSFORM PLANE

Figure 2. Experimental Arrangement for Recording Fourier Transform Relationship in the Focal Plane of the Parabolic Mirror Segment

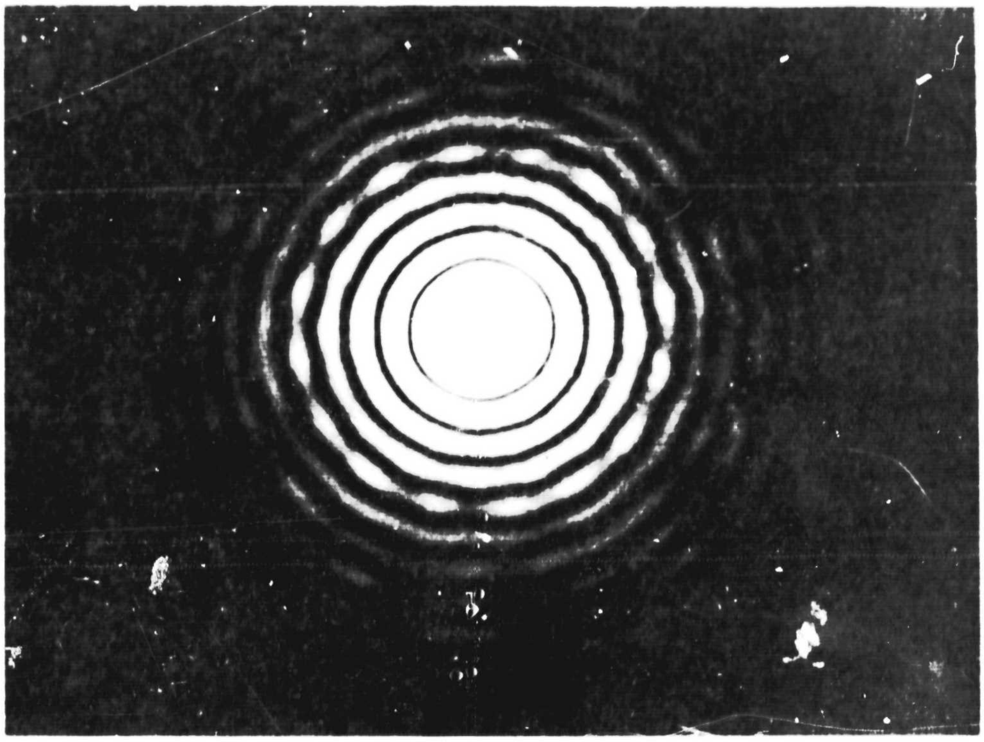


Figure 3. Fourier Transform of a Circular Aperture

Figure 4. Fourier Transform of a Rectangular Aperture

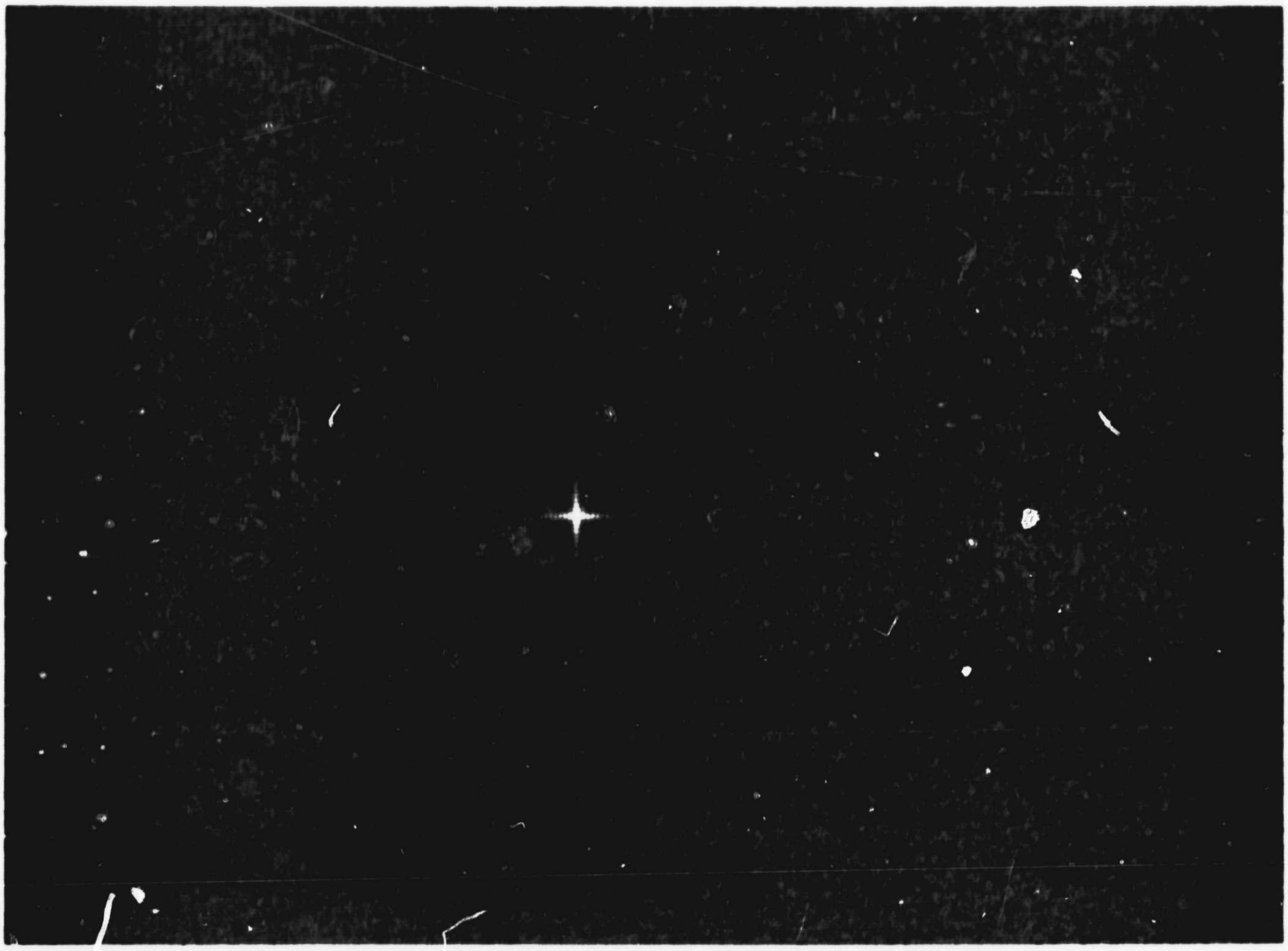
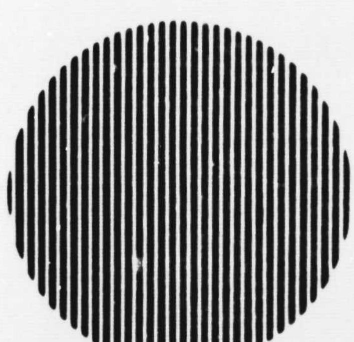
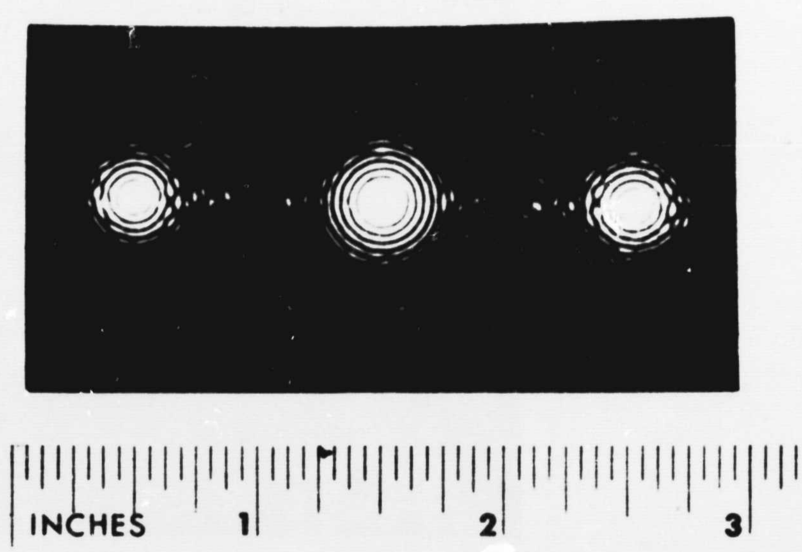


Figure 5. Fourier Transform of a Square Aperture



6 (a)



6 (b)

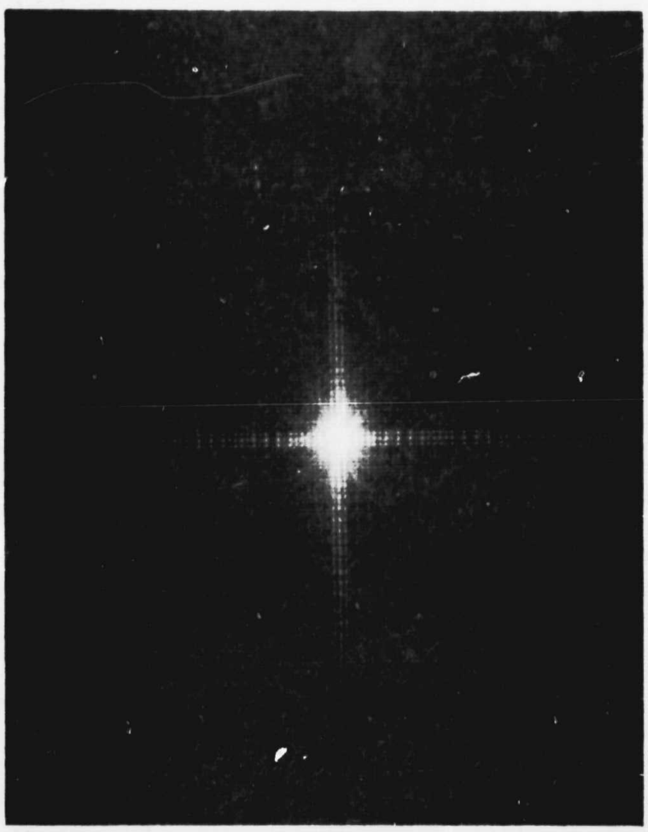
Figure 6. (a) The Object Transparency, Parallel Lines in a Circular Aperture
(b) The Fourier Transform of (a)

7 (a)

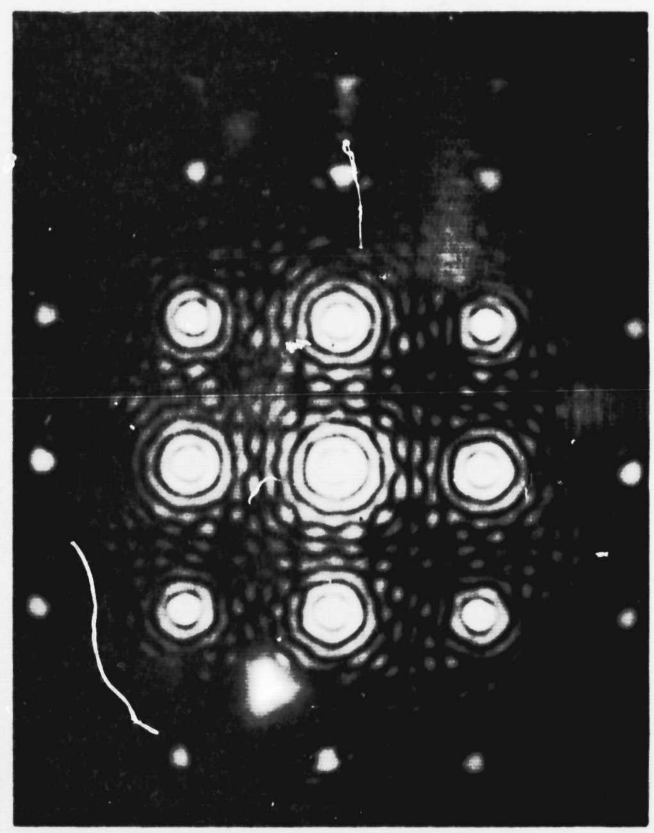
7 (b)

Figure 7. (a) The Object Transparency of a Triangle
(b) The Fourier Transform of the Triangle

8 (a)



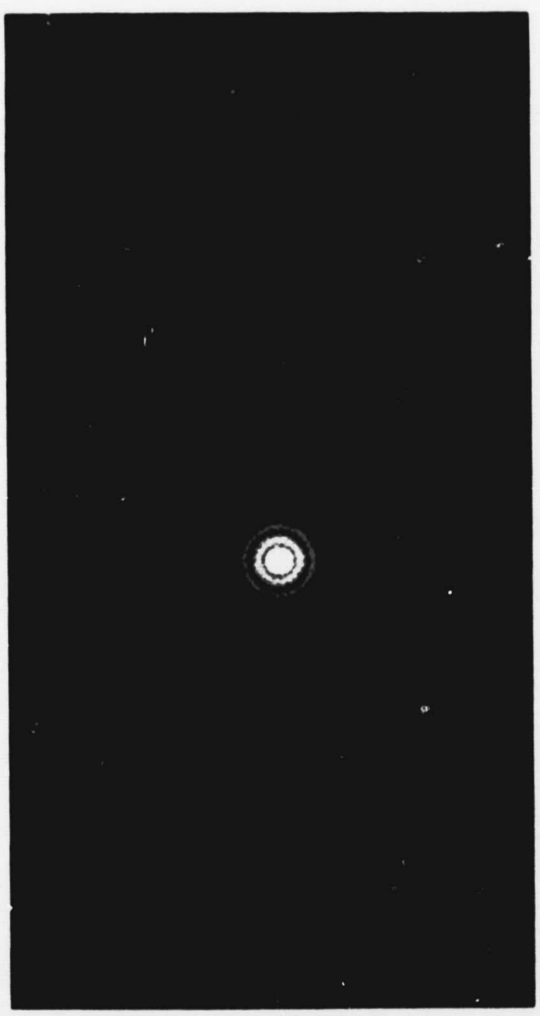
8 (b)



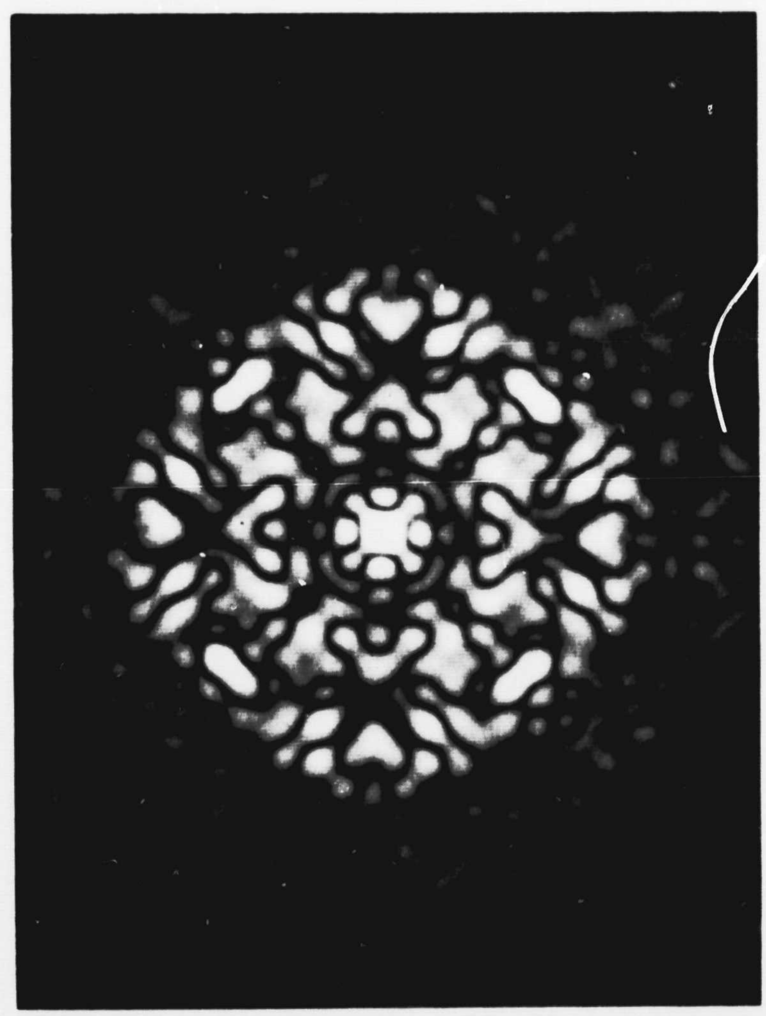
8 (c)

Figure 8. (a) The Object Transparency
(b) The Fourier Transform of (a)
(c) The Enlarged View of the Central Area of (a)

9 (a)

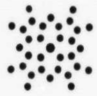


9 (b)

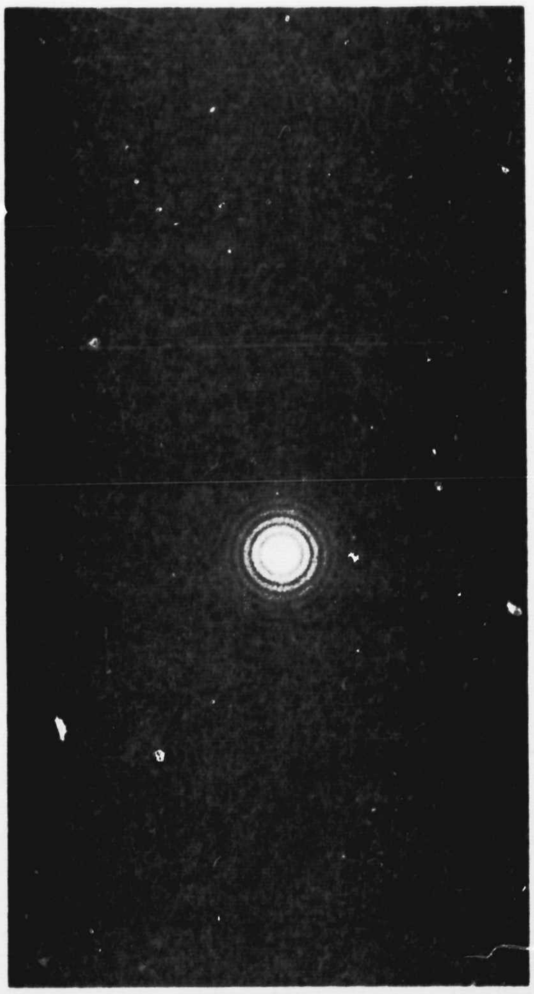


9 (c)

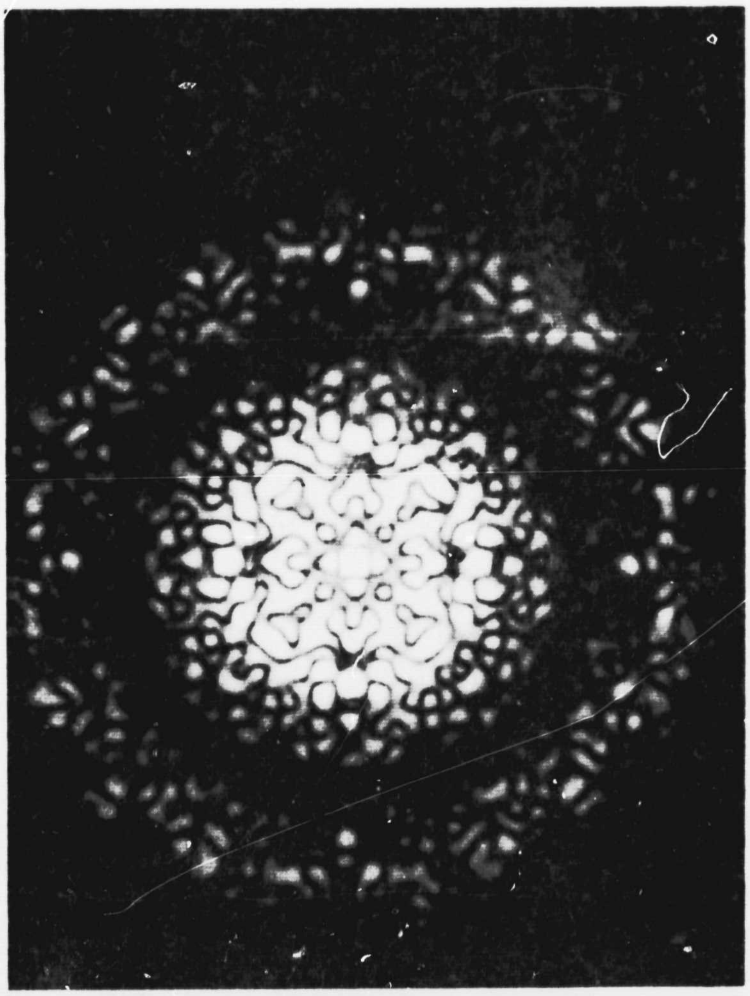
Figure 9. (a) The Object Transparency
(b) The Fourier Transform of (a)
(c) The Enlarged View of Central Portion of (b)



10 (a)

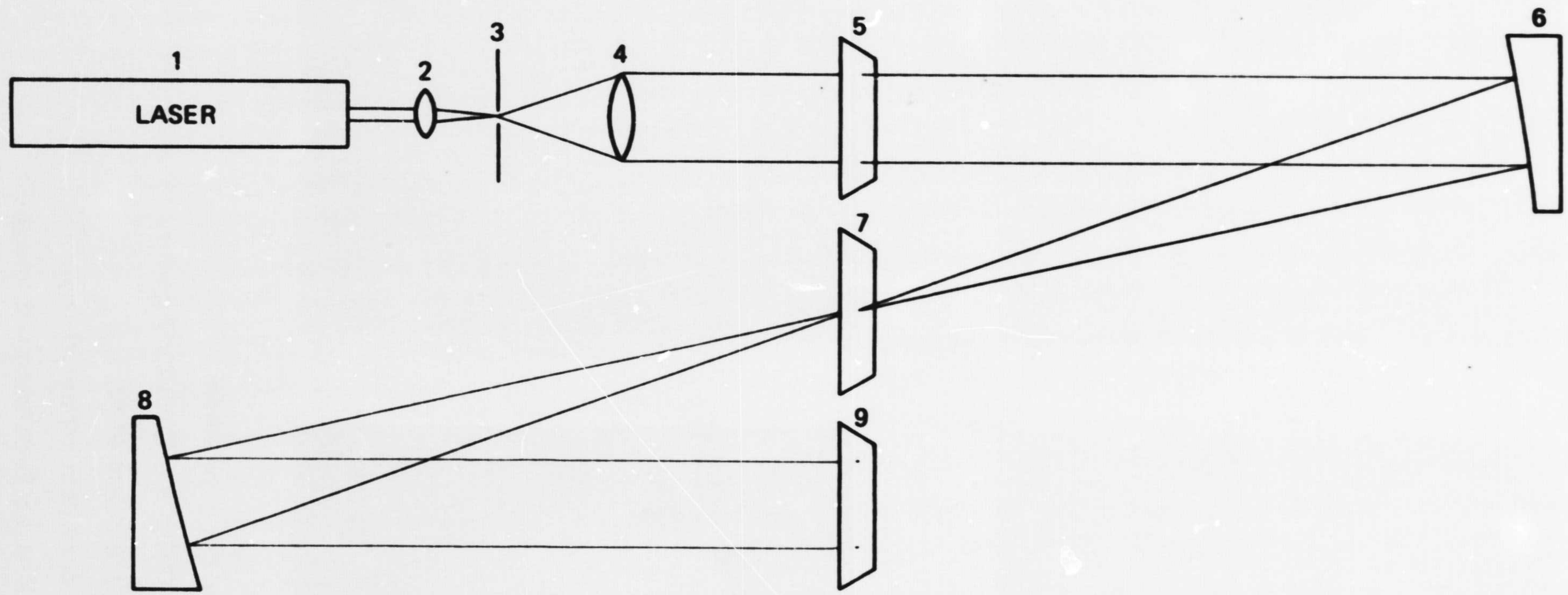


10 (b)



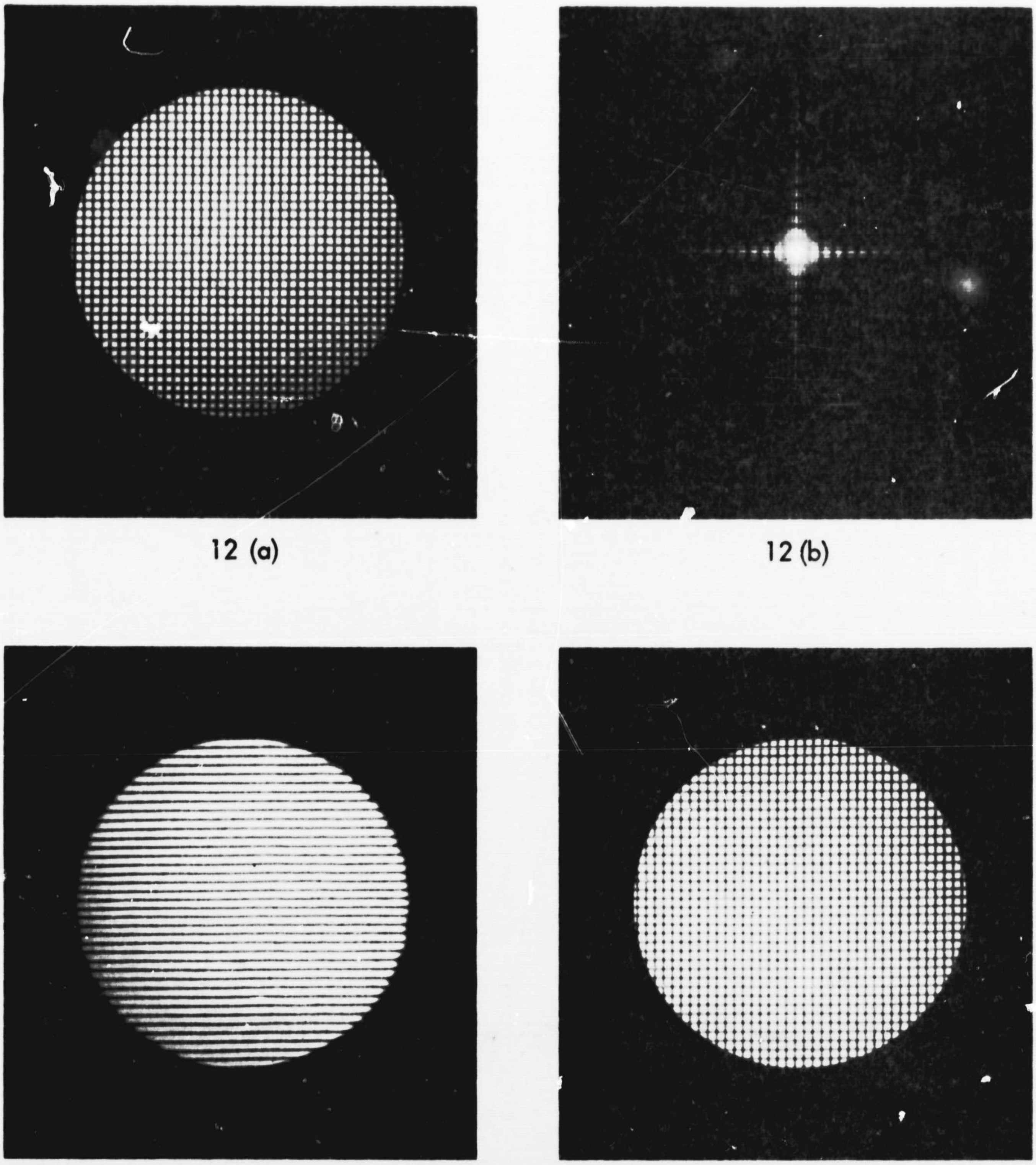
10 (c)

Figure 10. (a) The Object Transparency
(b) The Fourier Transform of (a)
(c) The Enlarged View of the Central Portion of (b)



1. He-Ne LASER
2. MICROSCOPE LENS
3. 10 MICRON PINHOLE
4. COLLIMATING LENS
5. INPUT PLANE
6. PARABOLOIDAL MIRROR SEGMENT, (FOURIER TRANSFORMING ELEMENT)
7. FOURIER TRANSFORMING AND SPATIAL FILTERING PLANE
8. PARABOLOIDAL MIRROR SEGMENT (IMAGE RECONSTRUCTING ELEMENT)
9. IMAGE RECONSTRUCTING PLANE

Figure 11. Experimental Arrangement for Spatial Filtering



12 (a) 12 (b)

12 (c) 12 (d)

Figure 12. Spatial Filtering Experiment

(a) A Wire Grid in a Circular Aperture (The Object Transparency)

(b) Fourier Transform of (a)

(c) The Filtered Reconstructed Image

(d) The Unfiltered Reconstructed Image