

General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.
- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.
- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.
- This document is paginated as submitted by the original source.
- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Memorandum 33-476

*Computation of Structural Modes
of a Rollout Array Spacecraft
for Attitude Control Study*

Evan O. Weiner

FACILITY FORM 602

N71-25155

(ACCESSION NUMBER)

(THRU)

G 3

(PAGES)

OR-118327

(CODE)

(NASA CR OR TMX OR AD NUMBER)

31
(CATEGORY)

**JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA**

May 1, 1971



NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Memorandum 33-476

*Computation of Structural Modes
of a Rollout Array Spacecraft
for Attitude Control Study*

Evan O. Weiner

JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

May 1, 1971

PREFACE

The work described in this report was performed by the Engineering Mechanics Division for the Guidance and Control Division of the Jet Propulsion Laboratory.

ACKNOWLEDGMENT

The author gratefully acknowledges the assistance of R. M. Bramford, J. Garba, and J. Chisholm during the course of this investigation.

CONTENTS

I. Introduction	1
II. Analysis	2
III. Inertias and Stiffnesses	4
IV. Out-of-Plane Motion	5
A. Programming the OP YS Modes	5
B. Interpretation of the OP YS Mode Output	7
C. Programming the OP YA Modes	8
D. Interpretation of the OP YA Mode Output	8
V. In-Plane Motion	9
VI. Summary of Results	10
Appendix A. Normalization	11
Appendix B. Derivation of Eq. (18)	14
References	17
Tables	
1. Types of modes	18
2. Program for out-of-plane y-symmetric modes	19
3. Program for out-of-plane y-antisymmetric modes	24
4. Program for in-plane modes	28
5. Frequencies and full spacecraft rigid-elastic coupling terms	33
A-1. Linear operators A, B, C	35
Figures	
1. Spacecraft configuration	36
2. Stiffness element	37
3. Membrane element	37
4. String element	37
5. Model for out-of-plane modes	38
6. Model for in-plane modes	39

ABSTRACT

A study is presented of the structural modes required to determine the interaction between an attitude control system and a flexible structure. The flexible structure considered has a low stiffness that leads to natural frequencies in the range of the frequency response of the attitude control system, producing a coupling between the response of the structure and the attitude control system.

I. INTRODUCTION

This report is a study of the structural modes required (Ref. 1) to determine the interaction between an attitude control system and a flexible structure. The flexible structure considered has a low stiffness that leads to natural frequencies in the range of the frequency response of the attitude control system, producing a coupling between the response of the structure and the attitude control system.

The configuration to be analyzed is the deployed solar electric multimission spacecraft configuration (Ref. 2) suitable for an asteroid belt mission. A simplified sketch of the configuration is given in Fig. 1. The four blankets are Kapton sheets covered with solar cells on one side and partially covered with micrometeoroid impact sensors on the other. They are stretched between

the beams and the reference body with a tension along the y-axis furnished by springs in the rollout devices (not shown) of the reference body.

Each beam is attached to a boom through a bearing having an axis of rotation fixed in the end of the boom. Only the blanket tensions prevent the beam from "windmilling" about the boom axes. Each boom is a stainless steel bitem attached to the reference body through a rollout device. One beam, boom, and pair of blankets will be called a rollout array, or simply an array; the two arrays shown are symmetric about the xz-plane. Each array has a 5 deg dihedral from the xy-plane to accommodate a star tracker field of view. This dihedral is not considered in the analysis. The reference body (bus) represents a spacecraft assembly and will be modeled as a rigid body.

II. ANALYSIS

The arrays are to be modeled by a collection of subbodies $A_1, \dots, A_n$, each considered rigid for the purposes of determining inertia properties. In general, let u_1^i, u_2^i , and u_3^i be the linear deflections of the i th subbody relative to the reference body along the x -, y -, and z -axes. Similarly, let β_1^i, β_2^i , and β_3^i be the small angle rotations of the i th subbody about the x -, y -, and z -axes. Then any displacement field of the arrays may be defined by a vector

$$\{q\} = [u_1^i, u_2^i, u_3^i, \beta_1^i, \beta_2^i, \beta_3^i, u_1^n, \dots, \beta_3^n]^T \quad (1)$$

Let m_i be the mass of A_i and put

$$[m^i] = \begin{bmatrix} m_i & 0 & 0 \\ 0 & m_i & 0 \\ 0 & 0 & m_i \end{bmatrix} \quad (2)$$

If $\bar{m}$ is the reference body mass, the total spacecraft mass is

$$\mathcal{M} = \bar{m} + \sum_{i=1}^n m_i \quad (3)$$

Letting I^i be the 3×3 matrix giving the inertia tensor components of A^i (Ref. 3, Section 47), the matrix

$$[M] = \begin{bmatrix} m^1 & 0 & \dots & 0 \\ 0 & I^1 & & \vdots \\ \vdots & & \ddots & m^n \\ 0 & \dots & \dots & I^n \end{bmatrix} \quad (4)$$

is called the mass matrix for the arrays. Let $[K]$ be the $6n \times 6n$ symmetric stiffness matrix which relates structural interaction forces and torques to displacement fields of the array, and let

$$[M'] = [M] - \frac{1}{\mathcal{M}} [m^i m^j] \quad (5)$$

where

$$[m^i m^j] = \begin{bmatrix} m^1 m^1 & 0 & m^1 m^2 & 0 & \dots & m^1 m^n & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ m^2 m^1 & 0 & m^2 m^2 & 0 & \dots & m^2 m^n & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ m^n m^1 & 0 & m^n m^2 & 0 & \dots & m^n m^n & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix} \quad (6)$$

The matrix $[M']$ is displayed in detail in Fig. 5 of Ref. 1.

The reference body is constrained in rotation but free in translation. With no external forces on the craft, and without any spin associated with the subbodies, the equations of motion reduce to

$$[M'] \{\ddot{q}\} + [K] \{q\} = 0 \quad (7)$$

provided the origin of the coordinate axes coincides with the mass center of the undeformed total spacecraft. A detailed derivation for more general cases is given in Chapter III of Ref. 1. It is assumed that the row and column associated with any zero diagonal term in $[M_A]$ is zero. It follows that the rank R of $[M_A]$ is simply the number of nonzero diagonal terms of $[M_A]$, and there are R linearly independent solutions

$$\{q^j\} = \{\phi^j\} e^{i\omega_j t}, \quad j = 1, \dots, R \quad (8)$$

to Eq. (7). The amplitudes ϕ_j are called the hybrid elastic modes of the spacecraft, and terms $\omega_j/2\pi$ are the associated frequencies in Hz. They may be determined by solving the eigenvalue/eigenvector problem given by

$$[-\omega_j^2 M' + K] \{\phi^j\} = 0 \quad (9)$$

Solutions to this equation may be normalized so that

$$\{\phi^i\}^T [M'] \{\phi^j\} = [\delta^{ij}] \quad (10)$$

where δ^{ij} is one if $i = j$ and zero otherwise. On defining the $6n \times R$ matrix $[\phi_H]$ by

$$[\phi_H] = [\phi^1 | \phi^2 | \dots | \phi^R]$$

Eq. (10) is equivalent to

$$[\phi_H]^T [M'] [\phi_H] = [E] \quad (11)$$

where $[E]$ is the $R \times R$ identity matrix.

Let the vectors $r^1, r^2, \dots, r^6$ be those vectors obtained from Eq. (1) when the u_j^i and β_j^i terms are determined by imposing on the spacecraft six rigid body displacements corresponding to unit translations along the x, y, and z axes, and unit rotations about the x, y, and z axes, respectively. These vectors are called the rigid body modes of the arrays. Putting

$$[\phi_R] = [r^1 | r^2 | \dots | r^6]$$

the $R \times 6$ matrix

$$[C_{RE}] \equiv [\phi_H]^T [M] [\phi_R] \quad (12)$$

is called the matrix of rigid-elastic coupling terms.

For the purpose of constructing a stiffness matrix to be described in the next section, it is convenient to partition the reference body into subbodies $B^1, \dots, B^P$, even though the reference body itself is considered rigid. As in Eqs. (2) and (4), let $\bar{m}^i$ and $\bar{I}^i$ denote the mass and inertia tensor matrices of B^i , and let

$$[M_B] = \begin{bmatrix} \bar{m}^1 & \dots & 0 \\ \vdots & \bar{I}^1 & \vdots \\ 0 & \dots & \bar{I}^P \end{bmatrix} \quad (13)$$

Let

$$[\bar{M}] = \begin{bmatrix} M_A & \vdots \\ \vdots & \vdots \\ \vdots & M_B \end{bmatrix}$$

and let $\bar{r}^1, \bar{r}^2, \dots, \bar{r}^6$ describe the displacements of the mass centers of the subbodies when six rigid body displacements corresponding to unit xyz-translations and unit xyz-rotations are imposed on the reference body. Define

$$[\bar{\phi}_R] = \begin{bmatrix} r^1 & r^2 & \dots & r^6 \\ \bar{r}^1 & \bar{r}^2 & \dots & \bar{r}^6 \end{bmatrix}$$

Then

$$[G_{RR}] = [\bar{\phi}_R]^T [\bar{M}] [\bar{\phi}_R]$$

is called the matrix of rigid-rigid coupling terms. It can be shown that

$$[\bar{\phi}_R]^T [\bar{M}] [\bar{\phi}_R] = \begin{bmatrix} \mathcal{M}E & 0 \\ 0 & I_s \end{bmatrix}$$

where each block shown here is 3×3 , where $\mathcal{M}$ is the total spacecraft mass, $[E]$ is the identity, and $[I_s]$ is the matrix of inertia tensor components for the total spacecraft.

The objectives of the analysis may now be stated as:

- (1) Determine inertia and stiffness properties of the spacecraft, partition the arrays, and obtain the mass and stiffness matrices for the resulting model.
- (2) Obtain the hybrid elastic modes, normalized per Eq. (10), for the lower frequencies.
- (3) Obtain the rigid-elastic coupling terms using the hybrid-elastic modes determined in (2).
- (4) Obtain the rigid-rigid coupling terms.

Units to be used in the program input are pounds for weights and forces, inches for lengths, and radians for angles. Conversion rules for obtaining output in the slug-ft-sec system, which preserve the normalization of Eq. (10), are given in Ref. 4. Data and results are indicated in both the International and the British system of units.

III. INERTIAS AND STIFFNESSES

With information from Ref. 5 along with a desire to avoid roundoff errors in the program input, a blanket tension (total force in one blanket) of 7.2 N (1.632 lb) was chosen. With the use of information from Ref. 2, the following estimates for inertia terms and stiffnesses were made:

$$\begin{aligned}
 &\text{Weight of one blanket, } M_{BL} = 332 \text{ N (75 lb)} \\
 &\text{Weight of one boom, } M_{BO} = 39.8 \text{ N (9 lb)} \\
 &\text{Weight of one beam, } M_{BE} = 17.7 \text{ N (4 lb)} \\
 &\text{Weight of reference body, } M_0 = 4420 \text{ N (997 lb)} \\
 &\text{Inertia tensor components of reference body:} \\
 &\quad I_{\text{oxx}} = 1.875 \times 10^3 \text{ N-m}^2 (0.650 \times 10^6 \text{ lb-in.}^2) \\
 &\quad I_{\text{oyy}} = 3.550 \times 10^3 \text{ N-m}^2 (1.225 \times 10^6 \text{ lb-in.}^2) \\
 &\quad I_{\text{ozz}} = 4.950 \times 10^3 \text{ N-m}^2 (1.718 \times 10^6 \text{ lb-in.}^2) \\
 &\text{Boom stiffness, } (EI)_{BO} = 6.55 \times 10^2 \text{ N-m}^2 (0.227 \times 10^6 \text{ lb-in.}^2) \\
 &\text{Beam stiffness, } (EI)_{BE} = 1.66 \times 10^3 \text{ N-m}^2 (0.576 \times 10^6 \text{ lb-in.}^2)
 \end{aligned} \tag{14}$$

The mass centers of the arrays and the reference body are assumed to be coincident on a line containing both booms and midway between the beams. The beam and boom are modeled by a standard beam column element (Fig. 2), given in Ref. 5, having the following elemental stiffness matrix:

$$[K] = \frac{2EI}{L^3} \begin{bmatrix} 6 & 3L & -6 & 3L \\ 3L & 2L^2 & -3L & L^2 \\ -6 & -3L & 6 & -3L \\ 3L & L^2 & -3L & 2L^2 \end{bmatrix} + \frac{P}{5} \begin{bmatrix} \frac{6}{L} & \frac{1}{2} & -\frac{6}{L} & \frac{1}{2} \\ \frac{1}{2} & \frac{2L}{3} & -\frac{1}{2} & -\frac{L}{6} \\ -\frac{6}{L} & -\frac{1}{2} & \frac{6}{L} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{L}{6} & -\frac{1}{2} & \frac{2L}{3} \end{bmatrix} \tag{15}$$

Note that $P = 0$ for the beam elements. For out-of-plane motion, the blankets are modeled with rectangular membrane elements with uniaxial tension as shown in Fig. 3. The element stiffness matrix, given by Ref. 5, is

$$[K] = \frac{T}{L} \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} & -\frac{1}{6} & \frac{1}{6} \\ -\frac{1}{3} & \frac{1}{3} & \frac{1}{6} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{1}{6} & \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{6} & -\frac{1}{6} & -\frac{1}{3} & \frac{1}{3} \end{bmatrix} \tag{16}$$

Cases involving in-plane motion of the blanket will have each blanket modeled by a string under a tension T as shown in Fig. 4. The element stiffness matrix is simply

$$[K] = \frac{T}{L} \begin{bmatrix} w_1 & w_2 \\ 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{17}$$

IV. OUT-OF-PLANE MOTION

The mode shapes are divided into two categories: out-of-plane motion (OP), in which the only linear displacements are those along the z axis, and in-plane motion (IP), in which all linear deflections are in the xy -plane. An OP mode is said to be symmetric with respect to the yz -plane if the z -displacement at any point (x, y) on the array is equal to the z displacement at the point $(-x, y)$. The mode is then called an OP YS mode. If the z -displacements at (x, y) and $(-x, y)$ are always equal but opposite, the mode is called an OP YA mode. Each of the classes of OP YS and OP YA modes may be further subdivided by considering symmetry or antisymmetry with respect to the xz -plane: if, in an OP mode, the z -displacements at (x, y) and $(x, -y)$ are always equal, the mode is said to be x -symmetric (XS), and if they are always opposite, the mode is said to be x -antisymmetric (XA). The four categories of OP modes are summarized in Table 1.

All OP modes are obtained by analyzing the portion of the spacecraft in the first quadrant of the xy -plane, subject to the constraint that the resulting quarter of the reference body is free to translate but not free to rotate. The model used is shown in Fig. 5. Single arrows indicate z -translational degrees of freedom; double arrows indicate rotational degrees of freedom, the axis of rotation being parallel to the double arrow. In the numbering of the degrees of freedom, the last digit indicates the type of degree of freedom (1 indicates z -translations, 2 indicates x -rotations, and 3 indicates y -rotations). The remaining digits indicate the node associated with the degree of freedom. The inner edge of the blanket should be understood to be adjacent to the boom. The only points of the blanket that are attached to the boom are those at the beam and reference body (nodes 27 and 3, respectively).

Half the boom mass and bending stiffness values given by Eq. (14) should be assigned to the model given by Fig. 5. Similarly, half the mass and the full stiffness of Eq. (14) should be assigned to the beam. The full blanket mass of Eq. (14) should be assigned to the blanket, and one-quarter of the reference body inertia properties should be assigned to node 36. From Eqs. (15) and (16),

$T = -P/2 = 3.61 \text{ N (0.816 lb)}$, and $L = 1.72 \text{ m (68 in.)}$ for the boom and blanket elements. For the beam elements, $P = 0$ and $L = 0.635 \text{ m (25 in.)}$.

A. Programming the OP YS Modes

The SAMIS deck (Ref. 6) used to obtain these modes (both the OP YS XS and OP YS XA categories) is shown in Table 2. In Table 2, the cards are divided into 13 groups to aid in a detailed description of the deck. The first three groups are the pseudo-instructions. Section 4 of Ref. 6 describes the format of these instructions; Section 6 describes the functions of the pseudo-instructions.

The pseudo-instructions, represented by groups ① through ③, serve to determine the hybrid-elastic modes (normalized per Eq. (10)), the rigid elastic coupling terms, and the rigid-rigid coupling terms for the structure represented by the input given in groups ④ through ⑬. The structure here is that given by Fig. 5 with degrees of freedom 11, 21, 31, 351, 352, 361, 362, and 363 identifying the rigid reference body and with the degree of freedom 273 constrained to zero. Group ① is essentially concerned with reading the input and constructing the stiffness matrix for the structure. Group ② determines the first 15 cantilever modes corresponding to fixing the reference body. The rigid body modes corresponding to unit displacements of the reference body are also determined here. Group ③, using a coordinate transformation associated with the cantilever modes, determines the hybrid elastic modes, rigid-elastic coupling terms, and rigid-rigid coupling terms.

Group ④ starts the input section and provides two matrices to be read in by pseudo-instruction 1.0 (see Section 7.4 of Ref. 6). The matrix KER 1 has one card of data and is row-listed and coded. As in all coded matrices, only nonzero elements need be punched. KER 1 has unit diagonal terms corresponding to the 361, 362, and 363 degrees of freedom. This only serves to give the final stiffness matrix the same order as that of the mass matrix. The matrix KER 2 has six cards of data and is row-listed and coded. The 4-digit row and column codes are dummy codes not associated

with any of the degrees of freedom. Each of these codes must not coincide with any of the degrees of freedom associated with the model. The first digit of each dummy code corresponds to the row or column number of the blanket element stiffness matrix given by Eq. (16), while the elements of KER 2 are the elements of Eq. (16) subject to the assignment of values given previously.

The cards of group (5) are used to replace the dummy codes of KER 2 with the appropriate degrees of freedom for each of the 16 blanket elements. The number of blanket elements must be entered in the E field (Section 4.1, Ref. 6) of instruction 1.1. The first card amounts to a matrix identification card (Section 7.4, Ref. 6) associated with the second card. The second card has the format 1216 and, together with instruction 1.3, replaces every 1001 dummy code of KER 2 by 11, replaces every 2001 dummy code of KER 2 by 41, etc., and the resulting matrix is named KED 1 and stored on tape-group 12001 (Section 4.3, Ref. 6) in accordance with the C and 3 fields of instruction 1.3. Thus KED 1 is the blanket element stiffness matrix for the element in the lower right corner in Fig. 5. Instructions 1.1 through 1.4 correspond to a Fortran DO loop (Section 4.5, Ref. 6), and the remaining 15 pairs of cards in group (5) result in the generation of the blanket element stiffness matrices KED 2 through KED 16 by replacing the dummy codes in KER 2. The code replacement option of the DECO instruction is not given in Ref. 6.

Using the same scheme of dummy codes, group (6) provides the beam and boom element stiffness matrices KER 3 and KER 4, respectively. The elements of KER 3 are those of (3.2) divided by 100, and the elements of KER 4 are those of (3.2) subject to the assignment of values given previously. The division by 100 of elements in KER 3 was done to alleviate an ill-condition problem attributed to the rather stiff beam existing between the blanket and boom elements.

The first two pairs of cards in group (7) are used with instructions 1.6 through 1.9 to replace the dummy codes of KER 3 with the appropriate degrees of freedom resulting in the two-beam element stiffness matrices KED 17 and KED 18. The number of beam elements must be entered in the E field of instruction 1.6, while the group

number (Section 4.3, Ref. 6) in the C field (Section 4.3, Ref. 6) of instruction 1.8 must be the number of blanket elements plus 1. The remaining eight pairs of cards in group (7) are used with instructions 2.0 through 2.3 to obtain the boom element matrices KED 19 through KED 26 by replacing codes of KER 4. Again, the number of boom elements must be entered in the E field of instruction 2.0, while the group number in the C field of instruction 2.2 must be the number of beam and blanket elements plus 1.

By means of instructions 2.4 and 2.5, the element matrices KED 1 through KED 26 are coded, sorted, and row-listed to matrices KER 2 through KER 27, respectively. The E fields of instructions 2.4 and 2.5 must contain the numbers 100N and $-(100N + 1)$, respectively, where N is the total number of blanket, beam, and boom elements. The resulting KER 1 through KER 27 matrices are added (see ADD operation link of Section 6, Ref. 6) by instruction 3.0 to form the overall stiffness matrix KTR 1. The E field of instruction 3.0 must contain the number 100N.

Group (8) is the mass matrix WTR 1 for the model given by Fig. 1. With the exception of node 36, each node is modeled by a mass point determined by the sum of one-fourth of the mass of any adjacent blanket element and one-half of any adjacent beam or boom element. This mass appears as the diagonal element corresponding to the z translation degree of freedom associated with the node. The reference body mass assignment appears as the diagonal term corresponding to degree of freedom 361, while the appropriate inertia tensor components are associated with degrees of freedom 362 and 363.

Group (9) gives the column-listed matrix FOC 1, which is necessary to evaluate the rigid body modes. For a meaningful discussion, group (10) must be described first. This group consists of two identical sets of cards having the format 1216. Each set contains all degrees of freedom whose displacements (relative to the reference body) are known to be zero in any hybrid elastic mode. These degrees of freedom are called prescribed codes; 273 appears here since the beam obviously does not rotate about the boom axis in any out-of-plane y-symmetric mode. The prescribed codes must be listed in ascending

order. With N being the number of cards required to make up one set of prescribed codes, the numbers $51000 + N$ and N must appear in the E fields of instructions 4.0 and 6.0, respectively. With the rows and columns of the stiffness matrix corresponding to the prescribed codes deleted, the resulting matrix must be positive definite, i.e., having displacements associated with prescribed codes set to zero; any nonzero displacement field on the remaining structure must result in a positive work. In group (9), each column of FOC 1 provides displacements associated with prescribed codes and forces associated with unprescribed codes, resulting in a rigid-body mode. The first column (with the 10001 code) gives the rigid-body mode associated with a unit translation of the reference body, while the 10002 and 10003 codes correspond to unit rotations about the x- and y-axes, respectively.

The elements of FOC 1 having the codes (251, 10002), (261, 10002) and (271, 10002) represent forces applied at nodes 25, 26, and 27 along the z-axis. Unusual as it may seem to use forces to identify the rigid-body mode corresponding to a unit rotation about the x-axis, a run made without these forces gave results which indicated that the model of Fig. 5 has the bearing axis remaining parallel to the y-axis in all deformations of the structure. Considering such a restraint along with the blanket tension and boom compression acting on the beam, it is not difficult to determine the forces on the beam required to produce a rigid-body mode.

The card of group (11) has no meaning in the present analysis. It is associated with a more general analysis from which the instructions of Table 2 were obtained.

The number of cantilever modes desired in the output is designated N , which should be somewhat larger than the number of hybrid-elastic modes desired for the quarter structure of Fig. 5. The number $-N$ must appear in the E field of instruction 11.0, and $-M$, where $M \leq N$ is the number of hybrid-elastic modes desired, must appear in the E field of instruction 206.0. The cards of group 12 have a 12I6 format and contain the numbers 1, ..., N together with the column codes of FOC 1 corresponding to the rotational rigid body modes. The numbers here must be listed

in ascending order and, letting P be the number of cards required to make up this group, $52000 + P$ must appear in the E field of instruction 41.0. Finally, group (13) consists of two identical sets of cards with format 12I6. Each set contains the column codes of FOC 1 listed in ascending order; with Q being the number of cards in each set, the numbers Q and $50000 + Q$ must appear in the E fields of instruction 201.0 and 203.0.

B. Interpretation of the OP YS Mode Output

The OP YS XA hybrid-elastic modes are obtained from the cantilever modes given by the matrix DTC 1 printed by instruction 22.1. Only columns 1 through 15 are applicable; the remaining columns give rigid-body modes which may be useful as a check. Each column of DTC 1 is a cantilever mode of the quarter spacecraft model given by Fig. 5, and an OP YS XA hybrid-elastic mode for the complete spacecraft may be obtained from such a cantilever mode simply by symmetry considerations. The mode shape obtained in this way is not normalized in accordance with (2.10); the appropriate normalization is accomplished by dividing the elements of the i^{th} column in DTC 1 by twice the i^{th} entry in the second column (titled ROOT + R*R) of the output ROOTS OF MATRIX DDC 1 associated with instruction 11.0. The frequency, in Hz, associated with the mode is given by the i^{th} entry in the last column of the output ROOTS OF MATRIX DDC 1.

The rigid-elastic coupling terms corresponding to the OP YS XA modes are obtained from the output GCC 3 associated with instruction 24.1 subject to the following considerations. Rows with rigid-body mode codes and columns with hybrid-elastic mode codes are to be considered. All rows are zeroed out except the 10002 row, and the elements of that row are multiplied by $2/(\text{ROOT})_i$, where i is the column number and $(\text{ROOT})_i$ is the i^{th} entry in the second column (titled ROOT = R*R) of the output ROOTS OF MATRIX DDC 1. The submatrix obtained in this way is the transpose of the contribution of the OP YS XA hybrid elastic modes and the z-translational and xy-rotational rigid-body modes to the matrix C_{RE} given by Eq. (12). All other contributions from the OP YS XA modes are zero.

The OP YS XS hybrid-elastic mode shapes are obtained from columns 1 through 15 of output matrix DDC 1, associated with instruction 218.0. The mode shapes given here are those for the quarter spacecraft model of Fig. 5, with the reference body permitted to translate freely. The OP YS XS modes for the full craft are obtained by symmetry conditions, and the appropriate normalization of Eq. (10) is achieved by dividing the elements of DDC 1 by 2. The natural frequencies (in Hz) of these modes are given by the last column of the output associated with instruction 206.0.

With care being taken not to confuse rows with columns in the output matrix AAA 1 associated with instruction 231.1, the rigid-elastic coupling terms corresponding to the OP YS XS modes are obtained as follows. Rows with rigid-body codes and columns with hybrid-elastic codes are considered. This is important since AAA 1 is not symmetric. The elements of the 10001 row are multiplied by 2 and the remaining two rows are zeroed out. The resulting submatrix is the transpose of the contribution of the OP YS XS modes and the z-translational and xy-rotational rigid-body modes to the matrix $[C_{RE}]$ given by Eq. (12). All other contributions from the OP YS XA modes are zero.

Finally, the total mass and the two inertia tensor components I_{xx} and I_{yy} are 4 times the elements with (rows, column) codes (10001, 10001), (10002, 10002), and (10003, 10003), respectively.

C. Programming the OP YA Modes

Clearly, all the OP YA modes do not involve any motion of the reference body. Both the OP YA XS and OP YA XA modes may be obtained from the cantilever modes determined by fixing the reference body and constraining all z-translational and x-rotational degrees of

freedom of the boom shown in Fig. 5. The SAMIS deck used to obtain these modes are shown in Table 3. Groups (3), (11), (12), and (13) are missing since, in Table 2, they only play a role in determining modes in which the reference body must be allowed to translate. In Table 3, groups (1), (4), (5), (6), (7), and (8) are precisely those of Table 2. Group (2) differs from Table 2 only in the E fields of instructions 4.0, 6.0, and 11.0. Here, two cards were required to make up each of the two sets of prescribed codes given in group (10), and only nine modes are requested. Note that 273 does not appear as a prescribed code since the beam must rotate freely about the bearing axis. In group (9), each column of FOC 1 provides displacements associated with prescribed codes and forces associated with unprescribed codes, resulting in a rigid-body mode.

D. Interpretation of the OP YA Mode Output

In the output matrix DTC 1 associated with instruction 22.1, the i^{th} column is divided by $2(\text{ROOT})_i$, where $(\text{ROOT})_i$ is the i^{th} entry in the second column (titled ROOT = R R) of the output associated with instruction 11.0. From these results, both the OP YA XS and the OP YA XA hybrid-elastic modes are determined from symmetry considerations. There are no nonzero rigid-elastic coupling terms associated with the OP YA XA modes. In the matrix GCC 3 of instruction 24.1, the rows and columns are considered as was done with the OP YS XA modes. All rows are zeroed out except the 10003, which should have its elements multiplied by $2/(\text{ROOT})_i$. The resulting submatrix is the transpose of the contribution of the OP YA XS modes and the z-translational and xy-rotational rigid-body modes to the C_{RE} matrix of (2.10). All other contributions are zero.

V. IN-PLANE MOTION

The in-plane (IP) modes are subdivided into those which are symmetric in x-displacements with respect to the xz-plane (IP XS modes) and those which are antisymmetric (IP XA modes). This subdivision does not cover modes with zero x-displacements everywhere. Such modes have frequencies of about 4 Hz or higher and are not of interest. Figure 6 shows the model of the half of the spacecraft above the xz-plane used for the IP modes.

The assignment of inertia and stiffness properties is in accordance with Section III. The beam is not weakened since no ill-condition problem was apparent from the IP results. Considering Eqs. (15) and (17), we have $T = -P/2 = 7.22 \text{ N}$ (1.632 lb) and $L = 1.72 \text{ m}$ (68 in.) for the boom and blanket elements. For the beam elements we have $P = 0$ and $L = 0.635 \text{ m}$ (25 in.).

The SAMIS deck used for obtaining the IP modes and rigid-elastic coupling terms is shown in Table 4. The basic scheme is that described for the OP YS modes, and the only noteworthy difference is that the codes 10001, 10002, and

10003 represent the rigid-body modes corresponding to the x- and y-translations of the z-rotation. The KER 4 and KER 5 matrices are identical to the KER 4 matrix of Table 2. They are simply added by instruction 1.55 to obtain the boom stiffness matrix prior to the code replacement procedure.

The IP XA modes are determined by the output matrix DTC 1 associated with instruction 22.1 after the i^{th} column has been divided by $\sqrt{2}/(\text{ROOT})_i$. Only the 1003 row of the GCC 3 output matrix associated with instruction 24.1 contributes nonzero rigid-elastic coupling terms. Here, the i^{th} element in this row must be multiplied by $\sqrt{2}/(\text{ROOT})_i$. The IP XS modes are given by the matrix DDC 1 associated with instruction 218.0 after each element is divided by $\sqrt{2}$. For these modes, only the 10001 and 10002 rows of the AAA 1 output associated with instruction 231.1 will contribute nonzero rigid-elastic coupling terms. These terms must be multiplied by $\sqrt{2}$ (Appendix A).

VI. SUMMARY OF RESULTS

Frequencies and rigid-elastic coupling terms of the output from the decks shown in Tables 2, 3, and 4 are listed in Table 5 by ascending frequencies. The matrices $[\Phi_{TRL}]$ and $[\Phi_{ROT}]$ are the translational and rotational rigid-body modes, respectively. Note that $[\Phi_R] = [\Phi_{TRL} | \Phi_{ROT}]$. The hybrid-elastic mode shapes are not shown in Table 5.

It can be shown (Appendix B) that

$$[C_{RE}]^T [C_{RE}] = \begin{bmatrix} AE & 0 \\ 0 & I_A \end{bmatrix} \quad (18)$$

where $A \ll M$, provided that $[C_{RE}]$ has the rigid-elastic coupling terms corresponding to all possible modes associated with the models of Figs. 5

and 6. Here, A is the mass and $[I_A]$ is the matrix of inertia tensor components of the arrays less those of elements assigned to the reference body. Also, $[E]$ is the 3×3 identity matrix. If not all the modes are represented in $[C_{RE}]$, then the diagonal terms of $[C_{RE}]^T [C_{RE}]$ can only be less than or equal to those of a full representation. Thus, Eq. (18) would serve as a useful check on the results of Table 5. The diagonal terms of $[C_{RE}]^T [C_{RE}]$ are listed as sums of squares in each column. The weight terms do not compare very well with $A = 1305 \text{ N}$ (306 lb). There are two reasons for this: (1) the lowest mode with any significant inertia along the y-axis has a natural frequency of about 4 Hz; (2) Eq. (18) is accurate only if $A \ll M$ and only if the mass center of the arrays coincides with that of the spacecraft (see Appendix B).

APPENDIX A NORMALIZATION

The SAMIS decks of Tables 2, 3, and 4 represent the calculations performed to obtain the hybrid-elastic mode shapes of one-fourth or one-half spacecraft configurations. Assuming that the results of these calculations are normalized in the sense of Eq. (10) with $[M']$ representing the one-fourth or one-half spacecraft, it is obvious from symmetry considerations how one should obtain the mode shapes for the full spacecraft; but the normalization factor required to normalize the full spacecraft mode shapes per Eq. (10) with $[M']$ representing the full spacecraft is not obvious.

It will first be shown that Eq. (10) is equivalent to another normalization, and the notation and terminology of Section II will be adopted. For simplicity, the arrays are assumed to be modeled by mass points, and inertia tensor components appearing in $[M]$ and $[M']$ are ignored. Relative to an inertial frame, let $\{v_i^p\}$ be the displacement vector of the i th mass point in the p th mode. The displacement vector for the reference body in the p th mode is denoted $\{\bar{v}^p\}$. The solutions $\{\phi^p\}$ of Eq. (9) are written

$$\{\phi^p\} = \begin{Bmatrix} \phi_1^p \\ \phi_2^p \\ \vdots \\ \phi_n^p \end{Bmatrix}$$

where $\{\phi_i^p\}$ denotes the displacement vector of the i th mass point relative to the reference body.

Using this notation, it follows that

$$\{\phi_i^p\} = \{v_i^p\} - \{\bar{v}^p\} \quad (A-1)$$

The condition that the mass center of the deformed structure remains stationary relative to the inertial frame gives

$$\sum_{i=1}^n m^i \{v_i^p\} + \bar{m} \{\bar{v}^p\} = 0 \quad (A-2)$$

By substituting Eq. (A-1) into Eq. (A-2), we obtain

$$\{\bar{v}^p\} = - \frac{1}{\bar{m}} \sum_{i=1}^n m^i \{\phi_i^p\} \quad (A-3)$$

Let

$$\{v^p\} = \begin{Bmatrix} v_1^p \\ v_2^p \\ \vdots \\ v_n^p \\ \bar{v}^p \end{Bmatrix}$$

Then

$$\begin{aligned} \{v^p\}^T [M] \{v^q\} &= \begin{Bmatrix} v_1^p \\ \vdots \\ v_n^p \\ \bar{v}^p \end{Bmatrix}^T \begin{bmatrix} m^1 E_3 & \dots & 0 & 0 \\ \vdots & & \vdots & \vdots \\ 0 & & m^n E_3 & 0 \\ 0 & \dots & 0 & \bar{m} E_3 \end{bmatrix} \begin{Bmatrix} v_1^q \\ \vdots \\ v_n^q \\ \bar{v}^q \end{Bmatrix} \\ &= \sum_i \{v_i^p\}^T m^i \{v_i^q\} + \{\bar{v}^p\}^T \bar{m} \{\bar{v}^q\} \\ &= \sum_i \{\phi_i^p + \bar{v}^p\}^T m^i \{\phi_i^q + \bar{v}^q\} + \{\bar{v}^p\}^T \bar{m} \{\bar{v}^q\} \\ &= \sum_i \{\phi_i^p\}^T m^i \{\phi_i^q\} + \{\bar{v}^p\}^T \left\{ \sum_i m^i \{\phi_i^q\} \right\} \\ &\quad + \left\{ \sum_i \{\phi_i^p\} m^i \right\}^T \{\bar{v}^q\} \\ &\quad + \{\bar{v}^p\}^T \left\{ \sum_i m^i \right\} \{\bar{v}^q\} + \{\bar{v}^p\}^T \bar{m} \{\bar{v}^q\} \end{aligned}$$

By noting that the m^i matrices are multiples of the identity matrices, we reduce the last two terms in the expression to $\mathcal{M} \{v^p\}^T \{v^q\}$. By using Eq. (A-3) to eliminate the $\{v^p\}$ and $\{v^q\}$ terms, we reduce the resulting expression for $\{v^p\}^T [\bar{M}] \{v^q\}$ to

$$\begin{aligned} \{v^p\}^T [\bar{M}] \{v^q\} &= \sum_i \{ \phi_i^p \}^T m^i \{ \phi_i^q \} - \frac{1}{\mathcal{M}} \sum_i \sum_j \{ \phi_i^p \}^T m^i m^j \{ \phi_j^q \} \\ &= \sum_i \sum_j \{ \phi_i^p \}^T \left[m^i \delta^{ij} - \frac{m^i m^j}{\mathcal{M}} \right] \{ \phi_j^q \} \\ &= \{ \phi^p \}^T [M'] \{ \phi^q \} \end{aligned}$$

That is,

$$\{v^p\}^T [\bar{M}] \{v^q\} = \{ \phi^p \}^T [M'] \{ \phi^q \} \quad (A-4)$$

If the $\{ \phi^p \}$ vectors are called the hybrid-elastic modes and the $\{v^p\}$ vectors are called the natural modes, then Eq. (A-4) shows that the natural modes are normalized with respect to $[\bar{M}]$ if, and only if, the hybrid-elastic modes are normalized with respect to $[M']$. It must be noted that this result depends on Eqs. (A-1) and (A-2) between the two types of modes.

Assuming that the reference body is constrained in rotation, then the result of Eq. (A-4) is obtained again if nonzero inertia tensors of the subbodies comprising the arrays are considered. In this case, the 3×3 inertia tensor matrices are centered on the diagonals of both $[\bar{M}]$ and $[M']$, and corresponding degrees of freedom of a hybrid-elastic mode and its related natural mode associated with small angle rotations of the subbodies are equal. Finally, Eq. (A-4) is valid for the one-fourth, one-half, or full spacecraft model, provided the proper mass matrices and mode vectors are used.

Let $\{ \phi^p \}$, $p = 1, \dots, 15$ be the hybrid-elastic mode shapes for the one-fourth spacecraft model obtained from the OP YS XS mode output mentioned in Section IV-B. Letting m^i , $i = 1, \dots, n$, be

the mass matrix for the i^{th} subbody comprising the array portion of the one-fourth model, the $\{ \phi^p \}$ mode output is normalized so that

$$\{ \phi^p \}^T [M'] \{ \phi^q \} = [\delta^{pq}]$$

where $[M']$ is given by Eq. (B-3). According to Eq. (A-4), this is equivalent to

$$\{v^p\}^T [\bar{M}] \{v^q\} = [\delta^{pq}]$$

where

$$[M] = \begin{bmatrix} m^1 & \dots & 0 & 0 \\ \vdots & & \vdots & \vdots \\ 0 & & m^n & 0 \\ 0 & \dots & 0 & \bar{m} \end{bmatrix}$$

and where $\bar{m}$ is one-fourth the reference body mass and $\{v^p\}$ is related to $\{ \phi^p \}$ by Eq. (A-1). Put

$$[\bar{M}_f] = \begin{bmatrix} \bar{M} & 0 & 0 & 0 \\ 0 & \bar{M} & 0 & 0 \\ 0 & 0 & \bar{M} & 0 \\ 0 & 0 & 0 & \bar{M} \end{bmatrix}$$

and define

$$\{v^p\} = \begin{Bmatrix} v^p \\ A v^p \\ B v^p \\ C v^p \end{Bmatrix}$$

where the linear operators A, B, and C multiply components of $\{v^p\}$ associated with degrees of freedom (DOF) by factors shown in Table A-1. The $\{v^p\}$ vector represents a natural mode for the full spacecraft generated from $\{ \phi^p \}$ by symmetry

conditions, while $[\bar{M}_f]$ is the mass matrix for the full spacecraft. Its normalization is determined as follows:

$$\begin{aligned} \{v^p\}^T [\bar{M}_f] \{v^q\} &= \{v^p\}^T [\bar{M}] \{v^q\} + \{A v^p\}^T [\bar{M}] \{A v^q\} \\ &+ \{B v^p\}^T [\bar{M}] \{B v^q\} + \{C v^p\}^T [\bar{M}] \{C v^q\} = [4\delta^{pq}] \end{aligned} \quad (A-5)$$

If inertia tensor components of the subbodies are considered, the last inequality may not be valid unless the principal axes of the subbodies are parallel with the reference coordinate axes. From Eq. (A-5) it follows that

$$\left\{ \frac{1}{2} v^p \right\}^T [\bar{M}_f] \left\{ \frac{1}{2} v^q \right\} = [\delta^{pq}]$$

18.1	9001	EVC 1			FLIP	12001	EVR 1	
18.2	12001	EVR 1			CHOL	11005	EIC 1	50000
18.3	10004	DWC 5			ROWS	12001	DWR 5	
18.4	12001	DWR 5	11005	EIC 1	MULT	10004	DWC 5	

In view of Eq. (A-4), the full spacecraft hybrid-elastic modes generated from the OP YS XS mode output by symmetry considerations must be divided by 2 in order to achieve the desired normalization (see Section II).

Similar arguments may be furnished to obtain the appropriate factors for the remaining mode classes. Once the OP modes are normalized with respect to the mass matrix for the one-fourth model, they should be divided by 2 before generating the full spacecraft modes by symmetry considerations. The corresponding number for the IP modes is $\sqrt{2}$.

The cantilever modes printed out by instruction 22.1 of Tables 2, 3, and 4 are not normalized with respect to the mass matrix of the appropriate one-fourth or one-half model. If such a normalization is desired, it is suggested that instruction number 18.2 be changed to 18.5 and that the following instructions be inserted:

APPENDIX B DERIVATION OF EQ. (18)

Using Eq. (12) to define the rigid-elastic coupling terms, we will develop an expression for $[C_{RE}]^T [C_{RE}]$ which will give an understanding of the limitations of the test mentioned in Section II. For convenience, it will be assumed that the subbodies $A_1, \dots, A_n$ modeling the arrays are mass points. There is no point in considering rotations of the subbodies, and the mass matrix of Eq. (4) is rewritten

$$[M] = \begin{bmatrix} m^1 & 0 & \dots & 0 \\ 0 & m^2 & & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & m^n \end{bmatrix} \quad (B-1)$$

Now, $[M]$ is a $3n \times 3n$ diagonal matrix. Also, Eq. (6) becomes

$$[m^i m^j] = \begin{bmatrix} m^1 m^1 & m^1 m^2 & \dots & m^1 m^n \\ m^2 m^1 & m^2 m^2 & \dots & m^2 m^n \\ \vdots & \vdots & \dots & \vdots \\ m^n m^1 & m^n m^2 & \dots & m^n m^n \end{bmatrix} \quad (B-2)$$

We define $[M']$ as

$$[M'] = [M] - \frac{1}{\mathcal{M}} [m^i m^j] \quad (B-3)$$

where

$$[M] = [\bar{m}] + [A]$$

$$A = \sum_{i=1}^n m_i$$

and $\bar{m}$ is the mass of the reference body. It is assumed that $A < M$ and that there are $3n$ linearly independent solutions of Eq. (9) which may be

arranged in columns to form the matrix $[\Phi_H]$ satisfying Eq. (11).

The inverse of Eq. (B-3) will be required. Under the assumption

$$\|M\| < \frac{1}{\mathcal{M}} \| [m^i m^j] \|$$

where $\|\cdot\|$ denotes the supremum norm, it is easy to show that

$$[M']^{-1} = \left[E + \frac{N}{\mathcal{M} - A} \right] [M]^{-1} \quad (B-4)$$

where

$$[N] = \begin{bmatrix} m^1 & m^2 & \dots & m^n \\ m^1 & m^2 & \dots & m^n \\ \vdots & \vdots & & \vdots \\ m^1 & m^2 & \dots & m^n \end{bmatrix}$$

It will now be shown that Eq. (B-4) holds if $\mathcal{M} > A$ and if $[M]$ is invertible by simply proving

$$[M'] \left[E + \frac{N}{\mathcal{M} - A} \right] [M]^{-1} = [E] \quad (B-5)$$

We define

$$[J] = \begin{bmatrix} E_3 & E_3 & \dots & E_3 \\ E_3 & E_3 & \dots & E_3 \\ \vdots & \vdots & & \vdots \\ E_3 & E_3 & \dots & E_3 \end{bmatrix}$$

where $[E_3]$ is the 3×3 identity matrix. Then

$$[N][M]^{-1} = [J]$$

and

$$[M'] \left[E + \frac{N}{\mathcal{M} - A} \right] [M]^{-1} = \left[M - \frac{[m^i m^j]}{\mathcal{M} - A} \right] [M]^{-1} + \frac{J}{\mathcal{M} - A} = [E] - \frac{[m^i m^j][M]^{-1}}{\mathcal{M}} + \frac{[M][J]}{\mathcal{M} - A} - \frac{[m^i m^j][J]}{\mathcal{M}(\mathcal{M} - A)} \quad (B-6)$$

It is easy to verify that

$$[m^i m^j][M]^{-1} = [N]^T$$

$$[M][J] = [N]^T$$

$$[m^i m^j][J] = A[N]^T$$

Hence it follows from Eq. (B-6) that

$$[M'] \left[E + \frac{N}{\mathcal{M} - A} \right] [M]^{-1} = [E] - \frac{[N]^T}{\mathcal{M}} + \frac{[N]^T}{\mathcal{M} - A} - \frac{A}{\mathcal{M}(\mathcal{M} - A)} [N]^T = [E]$$

which proves Eq. (B-4).

The matrix of rigid-elastic coupling terms is given by Eq. (12), which is repeated here for convenience:

$$[C_{RE}] = [\phi_H]^T [M] [\phi_R] \quad (12)$$

where $[\phi_H]$ is the matrix whose columns constitute the assumed $3n$ linearly independent hybrid-elastic mode shapes satisfying Eq. (11), also repeated here for convenience:

$$[\phi_H]^T [M'] [\phi_H] = [E] \quad (11)$$

and where $[\phi_R]$ is the $3n \times 6$ matrix of rigid-body modes corresponding to unit translations and rotations of the spacecraft mass center. Using Eq. (11), it is possible to write

$$\begin{aligned} [G_{RE}]^T [G_{RE}] &= [\phi_R]^T [M] [\phi_H] [\phi_H]^{-1} [M']^{-1} \left([\phi_H]^T \right)^{-1} [\phi_H]^T [M] [\phi_R] \\ &= [\phi_R]^T [M] [M']^{-1} [M] [\phi_R] \end{aligned}$$

By using Eq. (B-4), this becomes

$$[C_{RE}]^T [C_{RE}] = [\phi_R]^T [M] [\phi_R] + [\phi_R]^T \frac{[M][N]}{\mathcal{M} - A} [\phi_R]$$

which, since $[m^i m^j] = [M][N]$, may be put in the form

$$[C_{RE}]^T [C_{RE}] = [\phi_R]^T [M] [\phi_R] + [\phi_R]^T \frac{[m^i m^j]}{\mathcal{M} - A} [\phi_R] \quad (B-7)$$

Clearly,

$$[\phi_R]^T [M] [\phi_R] = \begin{bmatrix} AE_3 & | & 0 \\ \hline 0 & | & I_A \end{bmatrix} \quad (B-8)$$

is the expression in Section II, and in view of Eq. (B-7) it need not be equal to $[C_{RE}]^T [C_{RE}]$.

The expression

$$[\Phi_R]^T \frac{[m^i m^j]}{M-A} [\Phi_R] \quad (B-9)$$

will be evaluated by considering the rigid-body modes individually. As in Section II, put

$$[\Phi_R] = \begin{bmatrix} r^1 & | & r^2 & | & \dots & | & r^6 \end{bmatrix}$$

Then, the pq^{th} element of Eq. (B-9) is simply

$$\{r^p\}^T \frac{[m^i m^j]}{M-A} \{r^q\}$$

Clearly,

$$\{r^1\} = \begin{Bmatrix} t_1 \\ t_1 \\ \vdots \\ t_1 \end{Bmatrix}, \quad t_1 = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

It is easy to show that

$$\{r^p\}^T \frac{[m^i m^j]}{M-A} \{r^1\} = \frac{A^2}{M-A}$$

Similarly,

$$\{r^p\}^T \frac{[m^i m^j]}{M-A} \{r^q\} =$$

$$\begin{cases} \frac{A^2}{M-A} & \text{if } r, p = 1, 2, 3 \text{ and } r = p \\ 0 & \text{if } r, p = 1, 2, 3 \text{ and } r \neq p \end{cases}$$

Let (x_i, y_i, z_i) be the coordinates of the i^{th} node having the point mass m_i . Then

$$\{r^4\}^T = [0, -z_1, y_1, \dots, 0, -z_n, y_n]$$

and it is easy to verify that

$$\{r^4\}^T [m^i m^j] \{r^4\} = \left(\sum_{i=1}^n m_i z_i \right)^2 + \left(\sum_{i=1}^n m_i y_i \right)^2$$

Let $(\bar{x}, \bar{y}, \bar{z})$ denote the coordinates of the mass center of the arrays. Then

$$A_{\bar{x}} = \sum_{i=1}^n m_i x_i, \quad A_{\bar{y}} = \sum_{i=1}^n m_i y_i,$$

$$A_{\bar{z}} = \sum_{i=1}^n m_i z_i$$

and it is a straightforward exercise to derive the elements of Eq. (B-9). The result is that the matrix $[C_{RE}]^T [C_{RE}] = [\Phi_R]^T [M] [\Phi_R]$ has the form

$$\frac{A^2}{M-A} =$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \bar{z} & -\bar{y} \\ 0 & 1 & 0 & -\bar{z} & 0 & \bar{x} \\ 0 & 0 & 1 & \bar{y} & -\bar{x} & 0 \\ 0 & -\bar{z} & \bar{y} & \bar{y}^2 + \bar{z}^2 & -\bar{x}\bar{y} & -\bar{x}\bar{z} \\ \bar{z} & 0 & -\bar{x} & -\bar{x}\bar{y} & \bar{x}^2 + \bar{z}^2 & -\bar{y}\bar{z} \\ -\bar{y} & \bar{x} & 0 & -\bar{x}\bar{z} & -\bar{y}\bar{z} & \bar{x}^2 + \bar{y}^2 \end{bmatrix}$$

(B-10)

The approximation $[C_{RE}]^T [C_{RE}] = [\Phi_R]^T [M] [\Phi_R]$ is accurate only to the extent that $A \ll M$ and that the mass center of the arrays coincides with that of the spacecraft.

This derivation does not include cases in which the subbodies have nonzero inertia tensor components. Although not derived, it is suspected that Eq. (B-9), as well as Eq. (B-8), holds when the inertia tensor components of the subbodies are incorporated in M as shown in Eq. (4).

REFERENCES

1. Likins, P. W., Dynamics and Control of Flexible Space Vehicles, Technical Report 32-1329. Jet Propulsion Laboratory, Pasadena, Calif., Feb. 15, 1969.
2. Study of a Solar Electric Multimission Spacecraft (four volumes), prepared for JPL under Contract No. 952394. TRW Systems, Jan. 15, 1970.
3. Housner, G. W., and Hudson, D. E., Applied Mechanics - Dynamics, Van Nostrand Reinhold Co., New York, 1950.
4. Marsh, E. L., Conversion Factors for Spacecraft Structural Data, Engineering Memo 344-257-ELM. Jet Propulsion Laboratory, Pasadena, Calif., Feb. 4, 1970.
5. Coyner, J. V., Jr., and Ross, R. G., Jr., Parametric Study of the Performance Characteristics and Weight Variations of Large-Area Roll-Up Solar Arrays, Technical Report 32-1502. Jet Propulsion Laboratory, Pasadena, Calif., Dec. 15, 1970.
6. Melosh, R. J., Diether, P. A., and Brennan, M., Structural Analysis and Matrix Interpretive System (SAMIS) Program Report, Technical Memorandum 33-307, Revision 1. Pasadena, Calif., Dec. 15, 1966.

Table 1. Types of modes

Category	CP YS XS	OP YS XA	OP YA XS	OP YA XA
The z-displacements at (x,y) and (-x,y) are	Equal	Equal	Opposite	Opposite
The z-displacements at (x,y) and (x,-y) are	Equal	Opposite	Equal	Opposite

Table 2. Program for out-of-plane y-symmetric modes

1.0	9001	KER	1	10001	KER	2	READ	-1	16		
1.1							PREP				
1.2							VARY	1	1		
1.3	10001	KER	2		COD	1	DECO	12001	KFD	1	3
1.4							BACK				
1.5	10002	KER	3	10003	KER	4	READ	-1	2		
1.6							PREP				
1.7							VARY	1	1	1	
1.8	10002	KER	3		COD	1	DECO	12017	KED	17	3
1.9							BACK				
2.0							PREP				8
2.1							VARY	1	1	1	
2.2	10003	KER	4		COD	1	DECO	12019	KED	19	3
2.3							BACK				
2.4	12001	KED	1				CODE	10001	KEV	1	2600
2.5	10001	KEV	1				SORT	9002	KER	2	-2601
2.6	10001	WTR	1	12001	FOC	1	READ	-1	27		
3.0	9001	KER	1				ADDS	10002	KTR	1	1
3.5	10002	KTR	1				INKS				
4.0	10002	KTR	1	12001	FOC	1	CHOL	10003	DWC	4	51001
4.1				11001	KUR	1	CONT				
5.0							ERRS				
5.1	11001	KUR	1				CHOL	9001	RSC999	92000	
5.2	9001	RSC999					INKS				
5.3							STOP				
6.0	10001	WTR	1				CHOL				1
6.1				11002	WUR	1	CONT				
7.0	11002	WUR	1				FLIP	12001	WLC	1	
8.0	11001	KUR	1	12001	WLC	1	CHOL	9001	TEC	1	90000
9.0	11002	WUR	1	9001	TEC	1	MULT	11003	DCC	1	
10.0	11003	DCC	1				DECO	12002	DDC	1	
11.0	12002	DCC	1	11004	VDC	1	ROOT	9001	EVC	1	-15
12.0	11004	VDC	1				CODE	9002	VCC	1	
13.0	11002	WUR	1	9002	VCC	1	CHOL	12001	DWC	1	89000
14.0	10001	WTR	1	12001	DWC	1	MULT	9002	FOC	2	
15.0	11001	WUR	1	9002	FOC	2	CHOL	10004	DWC	5	90000

Table 2 (contd)

22.2	10003	DWC	4	9002	DIC	1	ADDS	9002	DIC	1	200
22.3	10001	WIR	1	9002	DIC	1	MULT	12001	FOC	3	
22.1	9002	DIC	1				INKS				
23.0	12001	FOC	3				FLIP	10005	GCR	2	
24.0	10005	GCR	2	9002	DIC	1	MULT	11005	GCC	3	
24.1	11005	GCC	3				INKS				
32.0	9002	DIC	1				ROWS	10006	GCR	1	
33.0	11005	GCC	3				FLIP	10007	GCR	2	
34.0							READ	10008	GCR	3	
35.0	9001	EVC	1				FLIP	10009	GCR	4	
41.0	10007	GCR	2				CHOL	12001	TRC	3	52002
41.1				11006	TUR	1	CONT				
44.0	12001	TRC	3				FILL				
45.0	10006	GCR	1		TRC	3	MULT	9001	GCR	6	200
46.0	9001	GCR	6				ROWS	10010	GCR	2	
49.0	12001	TRC	3				FLIP	11007	TIR	1	
50.0	11007	TIR	1	9002	GCR	7	MULT	10011	GCR	3	
54.0	11007	TIR	1				COLS	10012	TEC	10	
199.1	10011	GCR	3				ROWS	13001	CCR	2	
201.0	13001	CCR	2				CHOL				1
201.1				11007	XUR	1	CONT				
202.0	11007	XUR	1				FLIP	12001	WLC	1	
203.0	10009	GCR	4	12001	WLC	1	CHOL	9001	TEC	1	50001
203.5	12001	WLC	1				COLS	13001	WLC	1	
204.0	11007	XUR	1	9001	TEC	1	MULT	12001	DCC	1	
205.0	12001	DCC	1				DECO	11008	DCC	1	
206.0	11008	DCC	1	11009	VDC	1	ROOT	11010	GCC	13	-15
207.0	11009	VDC	1				CODE	9001	VCC	1	
208.0	11007	XUR	1	9001	VCC	1	CHOL	12001	DWC	1	89000
209.0	10008	GCR	3	12001	DWC	1	MULT	11011	TEC	4	
210.0	10006	GCR	1	12001	DWC	1	MULT	11012	TEC	5	
216.0	10003	DWC	4	11012	TEC	5	ADDS	9001	DWC	8	
217.0	9001	DWC	6				DECO	12001	DCC	1	
218.0	12001	DCC	1				INKS				
220.0	10005	GCR	2				ROWS	13001	GCR	2	
220.0	13001	GCR	2	9001	DWC	8	MULT	12001	TEC	9	

Table 2 (contd)

231.0 231.1 242.0	12001 9001	TEC 9 AAA 1	DECO INRS HALT	9001 363 363	AAA 1 363 363	1 1. 1.	③ (cont)
KFR 1	1	1	-1	362	362	1.	-1
KFR 2	6	6	-1	362	362	1.	-1
1001	1001	1001	1001	2001	2001	-0.004	1001 3001 -0.002
1001	4001	4001	2001	1001	2001	-0.004	2001 2001 0.004
2001	3001	3001	2001	4001	3001	-0.002	3001 1001 -0.002
3001	2001	2001	3001	3001	3001	0.004	3001 4001 -0.004
4001	1001	1001	4001	2001	4001	-0.002	4001 3001 -0.004
4001	4001	4001					
1001	11	2001	41	3001	51	4001	21
1001	21	2001	51	3001	61	4001	31
1001	41	2001	71	3001	81	4001	51
1001	51	2001	91	3001	91	4001	61
1001	71	2001	101	3001	111	4001	81
1001	81	2001	111	3001	121	4001	91
1001	101	2001	121	3001	141	4001	111
1001	111	2001	141	3001	151	4001	121
1001	131	2001	161	3001	171	4001	141
1001	141	2001	171	3001	181	4001	151
1001	151	2001	191	3001	201	4001	171
1001	171	2001	201	3001	211	4001	131

Table 2 (contd)

⑤ (cont'd)									
COB 1	1								
1001	191	2001	221	3001	231	4001	201		
COB 1	1								
1001	201	2001	231	3001	241	4001	211		
COB 1	1								
1001	221	2001	251	3001	261	4001	231		
COB 1	1								
1001	231	2001	261	3001	271	4001	241		
REF 3	6								
1001	1001	4.44		1001	2001	55.5	1001	3001	-4.44
1001	4001	55.5		2001	1001	55.5	2001	2001	925.
2001	3001	-55.5		2001	4001	462.5	3001	1001	-4.44
3001	2001	-55.5		3001	3001	4.44	3001	4001	-55.5
4001	1001	55.5		4001	2001	462.5	4001	3001	-55.5
4001	4001	925.							
REF 4	6								
1001	1001	4.3032		1001	2001	147.1248	1001	3001	-4.3032
1001	4001	147.1248		2001	1001	147.1248	2001	2001	6662.2592
2001	3001	-147.1248		2001	4001	3342.2272	3001	1001	-4.3032
3001	2001	-147.1248		3001	3001	4.3032	3001	4001	-147.1248
4001	1001	147.1248		4001	2001	3342.2272	4001	3001	-147.1248
4001	4001	6662.2592							
COB 1	1								
1001	251	2001	253	3001	261	4001	253		
COB 1	1								
1001	261	2001	263	3001	271	4001	273		
COB 1	1								
1001	291	2001	282	3001	271	4001	272		
COB 1	1								
1001	291	2001	292	3001	281	4001	282		
COB 1	1								
1001	301	2001	302	3001	291	4001	292		
COB 1	1								
1001	311	2001	312	3001	321	4001	302		
COB 1	1								
1001	321	2001	322	3001	311	4001	312		

⑥									
COB 1	1								
1001	191	2001	221	3001	231	4001	201		
COB 1	1								
1001	201	2001	231	3001	241	4001	211		
COB 1	1								
1001	221	2001	251	3001	261	4001	231		
COB 1	1								
1001	231	2001	261	3001	271	4001	241		
REF 3	6								
1001	1001	4.44		1001	2001	55.5	1001	3001	-4.44
1001	4001	55.5		2001	1001	55.5	2001	2001	925.
2001	3001	-55.5		2001	4001	462.5	3001	1001	-4.44
3001	2001	-55.5		3001	3001	4.44	3001	4001	-55.5
4001	1001	55.5		4001	2001	462.5	4001	3001	-55.5
4001	4001	925.							
REF 4	6								
1001	1001	4.3032		1001	2001	147.1248	1001	3001	-4.3032
1001	4001	147.1248		2001	1001	147.1248	2001	2001	6662.2592
2001	3001	-147.1248		2001	4001	3342.2272	3001	1001	-4.3032
3001	2001	-147.1248		3001	3001	4.3032	3001	4001	-147.1248
4001	1001	147.1248		4001	2001	3342.2272	4001	3001	-147.1248
4001	4001	6662.2592							
COB 1	1								
1001	251	2001	253	3001	261	4001	253		
COB 1	1								
1001	261	2001	263	3001	271	4001	273		
COB 1	1								
1001	291	2001	282	3001	271	4001	272		
COB 1	1								
1001	291	2001	292	3001	281	4001	282		
COB 1	1								
1001	301	2001	302	3001	291	4001	292		
COB 1	1								
1001	311	2001	312	3001	321	4001	302		
COB 1	1								
1001	321	2001	322	3001	311	4001	312		

⑦									
COB 1	1								
1001	191	2001	221	3001	231	4001	201		
COB 1	1								
1001	201	2001	231	3001	241	4001	211		
COB 1	1								
1001	221	2001	251	3001	261	4001	231		
COB 1	1								
1001	231	2001	261	3001	271	4001	241		
REF 3	6								
1001	1001	4.44		1001	2001	55.5	1001	3001	-4.44
1001	4001	55.5		2001	1001	55.5	2001	2001	925.
2001	3001	-55.5		2001	4001	462.5	3001	1001	-4.44
3001	2001	-55.5		3001	3001	4.44	3001	4001	-55.5
4001	1001	55.5		4001	2001	462.5	4001	3001	-55.5
4001	4001	925.							
REF 4	6								
1001	1001	4.3032		1001	2001	147.1248	1001	3001	-4.3032
1001	4001	147.1248		2001	1001	147.1248	2001	2001	6662.2592
2001	3001	-147.1248		2001	4001	3342.2272	3001	1001	-4.3032
3001	2001	-147.1248		3001	3001	4.3032	3001	4001	-147.1248
4001	1001	147.1248		4001	2001	3342.2272	4001	3001	-147.1248
4001	4001	6662.2592							
COB 1	1								
1001	251	2001	253	3001	261	4001	253		
COB 1	1								
1001	261	2001	263	3001	271	4001	273		
COB 1	1								
1001	291	2001	282	3001	271	4001	272		
COB 1	1								
1001	291	2001	292	3001	281	4001	282		
COB 1	1								
1001	301	2001	302	3001	291	4001	292		
COB 1	1								
1001	311	2001	312	3001	321	4001	302		
COB 1	1								
1001	321	2001	322	3001	311	4001	312		

Table 2 (contd)

⑦ (cont.)												
CON	1	1	2001	222	2001	321	4001	322				
CON	1	1	2001	242	2001	331	4001	332				
CON	1	1	2001	251	2001	341	4001	342				
WTR	1	13	-1									
	11	11	1.172	21	21	21	2.344	31	31	1.172	⑧	
	41	41	2.344	51	51	51	4.638	61	61	2.344		
	71	71	2.344	81	81	81	4.638	91	91	2.344		
	101	101	2.344	111	111	111	4.638	121	121	2.344		
	131	131	2.344	141	141	141	4.638	151	151	2.344		
	161	161	2.344	171	171	171	4.638	181	181	2.344		
	191	191	2.344	201	201	201	4.638	211	211	2.344		
	221	221	2.344	231	231	231	4.686	241	241	2.344		
	251	251	1.672	261	261	261	3.344	271	271	1.95325		
	281	281	.5625	291	291	291	.5625	301	301	.5625		
	311	311	.5625	321	321	321	.5625	331	331	.5625		
	341	341	.5625	351	351	351	.26125	361	361	249.25		
FOC	1	6	162500.	363	363	363	306250.					
⑨												
	11	10001	1.	21	10001	1.	31	10001	1.			
	351	10001	1.	361	10001	1.	11	10002	40.			
	21	10002	40.	31	10002	40.	251	10002	.408			
	251	10002	.816	271	10002	-1.224	351	10002	40.			
	352	10002	1.	362	10002	1.	11	10003	-50.			
	21	10003	-25.	272	10003	1.	363	10003	1.			
	11	21	31	272	351	352	361	362	363			
	11	21	31	272	351	352	361	362	363			
GCR	3	0	-1	5	6	7	8.	9	10	11	12	
	1	2	3	4	5	6	7	8.	9	10	12	
	13	14	15	10002	10003							⑫
	10001	10002	10003									
	10001	10002	10003									⑬
	10001	10002	10003									

Tabl 2 Program for out-of-plane y-antisymmetric modes

1.0	0001	KER 1	0001	KER 2	READ	-1	16
1.1					PREP		
1.2					VARY	1	1
1.3	10001	KER 2		COD 1	DECC 12001	KED 1	3
1.4					SACK		
1.5	10002	KER 3	10003	KER 4	READ	-1	2
1.6					PREP		
1.7					VARY	1	1
1.8	10002	KER 3		COD 1	DECC 12017	KED 17	3
1.9					SACK		
2.0					PREP		8
2.1					VARY	1	1
2.2	10003	KER 4		COD 1	DECC 12019	KED 19	3
2.3					SACK		
2.4	12001	KED 1			CODE 10001	KEV 1	2600
2.5	10001	KEV 1			SORT 9002	KER 2	-2601
2.6	10001	WTR 1	12001	FOC 1	READ	-1	
2.7	0001	KER 1			ADDS 10002	KTR 1	27
3.5	10002	KTR 1			INXS		
4.0	10002	KTR 1	12001	FOC 1	CHOL 10003	DWC 4	51002
4.1			11001	KUR 1	CONT		
5.0					ERRS		
5.1	11001	KUP 1			CHCL 9001	RSC999	92000
5.2	0001	00099			INXS		
5.3					STOP		
6.0	10001	WTR 1			CHOL		2
6.1			11002	WUR 1	CONT		
7.0	11002	WUR 1			FLIP 12001	WLC 1	
8.0	11001	KUR 1	12001	WLC 1	CHOL 9001	TEC 1	90000
9.0	11002	WUR 1	9001	TEC 1	MULT 11003	DCC 1	
10.0	11003	000 1			DECO		
11.0	12002	000 1	11004	VDC 1	ROCI 9001	EVC 1	-09
12.0	11004	VDC 1			CODE 9002	VCC 1	
13.0	11002	WUR 1	9002	VCC 1	CHOL 12001	DWC 1	85000
14.0	10001	WTR 1	12001	DWC 1	MULT 9002	FOC 2	
15.0	11001	WUR 1	9002	FOC 2	CHOL 10004	DWC 5	90000
16.2	10003	DWC 4			ADDS 9002	DTC 1	200
22.0	10001	WTR 1	9002	DTC 1	MULT 12001	FOC 3	

Table 3 (contd)

	9002	DTC	1	9002	DTC	1	9005	GCR	2	1005	GCC	3	
22.1	12001	FLIP	INKS	10005	GCR	2	11005	GCC	3				
23.0	12001	FLIP	INKS	10005	GCR	2	11005	GCC	3				
24.0	12001	FLIP	INKS	10005	GCR	2	11005	GCC	3				(cont)
24.1	12001	FLIP	INKS	10005	GCR	2	11005	GCC	3				
242.0	12001	FLIP	INKS	10005	GCR	2	11005	GCC	3				
KER	1	1	-1	362	362	1.	363	363	1.				
KER	2	6	-1	362	362	1.	363	363	1.				
1001	1001	.004	1001	2001	- .004	1001	3001	- .002					
1001	4001	.002	2001	1001	- .004	2001	2001	.004					
2001	2001	.002	2001	4001	- .002	3001	1001	- .002					
3001	2001	.002	3001	3001	.004	3001	4001	- .004					
4001	1001	.002	4001	2001	- .002	4001	3001	- .004					
4001	4001	.004											
COB	1	1	41	3001	51	4001	21						
1001	11	2001	51	3001	61	4001	31						
COB	1	1	71	3001	81	4001	51						
1001	41	2001	81	3001	91	4001	61						
COB	1	1	101	3001	111	4001	81						
1001	71	2001	111	3001	121	4001	91						
COB	1	1	131	3001	141	4001	111						
1001	101	2001	141	3001	151	4001	121						
COB	1	1	161	3001	171	4001	141						
1001	131	2001	171	3001	181	4001	151						
COB	1	1	191	3001	201	4001	171						
1001	141	2001	201	3001	211	4001	181						
COB	1	1	221	3001	231	4001	191						
1001	151	2001	231	3001	241	4001	201						
COB	1	1	251	3001	261	4001	211						
1001	171	2001	261	3001	271	4001	221						

Table 3 (contd)

⑤ (cont)										
COB 1	1	2001	231	3001	231	4001	201			
COB 1	1									
COB 1	201	2001	231	3001	241	4001	211			
COB 1	1									
COB 1	221	2001	251	3001	261	4001	231			
COB 1	1									
COB 1	231	2001	261	3001	271	4001	241			
KSP 3	6									
COB 1	1001	4.44	1001	2001	55.5	1001	3001	-4.44		
COB 1	4001	55.5	2001	1001	55.5	2001	2001	925.		
COB 1	3001	-55.5	2001	4001	462.5	2001	1001	-4.44		
COB 1	2001	-55.5	3001	3001	4.44	3001	4001	-55.5		
COB 1	1001	55.5	4001	2001	462.5	4001	3001	-55.5		
COB 1	4001	925.								
⑥										
KSP 4	6									
COB 1	1001	4.2032	1001	2001	147.1248	1001	3001	-4.3032		
COB 1	4001	147.1248	2001	1001	147.1248	2001	2001	6662.2592		
COB 1	3001	-147.1248	2001	4001	3342.2272	2001	1001	-4.3032		
COB 1	2001	-147.1248	3001	3001	4.2032	3001	4001	-147.1248		
COB 1	1001	147.1248	4001	2001	3342.2272	4001	3001	-147.1248		
COB 1	4001	6662.2592								
⑦										
COB 1	1	2001	253	3001	251	4001	263			
COB 1	1									
COB 1	261	2001	263	3001	271	4001	273			
COB 1	1									
COB 1	281	2001	282	3001	271	4001	272			
COB 1	1									
COB 1	201	2001	282	3001	231	4001	282			
COB 1	1									
COB 1	301	2001	302	3001	251	4001	292			
COB 1	1									
COB 1	311	2001	312	3001	311	4001	302			
COB 1	1									
COB 1	321	2001	322	3001	311	4001	312			
COB 1	1									
COB 1	331	2001	332	3001	321	4001	322			

Table 3 (contd)

(7)									
CON	1	241	2001	242	3001	331	4001	352	
2001	1								
3001	1								
4001	1								
11	11	1.172			21	21	2.344	31	31 1.172
41	41	2.344			51	51	4.688	61	61 2.344
71	71	2.344			81	81	4.688	91	91 2.344
101	101	2.344			111	111	4.688	121	121 2.344
131	131	2.344			141	141	4.688	151	151 2.344
161	161	2.344			171	171	4.688	181	181 2.344
191	191	2.344			201	201	4.688	211	211 2.344
221	221	2.344			231	231	4.688	241	241 2.344
251	251	1.572			261	261	3.344	271	271 1.95325
281	281	.5625			291	291	.5625	301	301 .5625
311	311	.5625			321	321	.5625	331	331 .5625
341	341	.5625			351	351	.28125	361	361 249.25
362	362	162500.			363	363	306250.		
(8)									
CON	1	241	2001	242	3001	331	4001	352	
2001	1								
3001	1								
4001	1								
11	11	1.172			21	21	2.344	31	31 1.172
41	41	2.344			51	51	4.688	61	61 2.344
71	71	2.344			81	81	4.688	91	91 2.344
101	101	2.344			111	111	4.688	121	121 2.344
131	131	2.344			141	141	4.688	151	151 2.344
161	161	2.344			171	171	4.688	181	181 2.344
191	191	2.344			201	201	4.688	211	211 2.344
221	221	2.344			231	231	4.688	241	241 2.344
251	251	1.572			261	261	3.344	271	271 1.95325
281	281	.5625			291	291	.5625	301	301 .5625
311	311	.5625			321	321	.5625	331	331 .5625
341	341	.5625			351	351	.28125	361	361 249.25
362	362	162500.			363	363	306250.		
(9)									
CON	1	241	2001	242	3001	331	4001	352	
2001	1								
3001	1								
4001	1								
11	11	1.172			21	21	2.344	31	31 1.172
41	41	2.344			51	51	4.688	61	61 2.344
71	71	2.344			81	81	4.688	91	91 2.344
101	101	2.344			111	111	4.688	121	121 2.344
131	131	2.344			141	141	4.688	151	151 2.344
161	161	2.344			171	171	4.688	181	181 2.344
191	191	2.344			201	201	4.688	211	211 2.344
221	221	2.344			231	231	4.688	241	241 2.344
251	251	1.572			261	261	3.344	271	271 1.95325
281	281	.5625			291	291	.5625	301	301 .5625
311	311	.5625			321	321	.5625	331	331 .5625
341	341	.5625			351	351	.28125	361	361 249.25
362	362	162500.			363	363	306250.		
(10)									
CON	1	241	2001	242	3001	331	4001	352	
2001	1								
3001	1								
4001	1								
11	11	1.172			21	21	2.344	31	31 1.172
41	41	2.344			51	51	4.688	61	61 2.344
71	71	2.344			81	81	4.688	91	91 2.344
101	101	2.344			111	111	4.688	121	121 2.344
131	131	2.344			141	141	4.688	151	151 2.344
161	161	2.344			171	171	4.688	181	181 2.344
191	191	2.344			201	201	4.688	211	211 2.344
221	221	2.344			231	231	4.688	241	241 2.344
251	251	1.572			261	261	3.344	271	271 1.95325
281	281	.5625			291	291	.5625	301	301 .5625
311	311	.5625			321	321	.5625	331	331 .5625
341	341	.5625			351	351	.28125	361	361 249.25
362	362	162500.			363	363	306250.		

1.0	9001	KER	1	10001	KER	2	READ	-1	16		
1.1							PREP				
1.2							VARY	1			
1.3	10001	KER	2		CON	1	PREC	12001	KER	1	3
1.4							BACK				
1.5	10002	KER	3	10003	KER	4	READ	10004	KER	5	-1
1.55	10003	KER	4	10004	KER	5	CODE	11001	KER	6	
1.6							PREP				2
1.7							VARY	1		1	
1.8	10002	KER	3		CON	1	PREC	12017	KER	17	3
1.9							BACK				
2.0							PREP				8
2.1							VARY	1		1	
2.2	11001	KER	6		CON	1	PREC	12019	KER	19	3
2.3							BACK				
2.4	12001	KER	1				CODE	10001	KER	1	2600
2.5	10001	KER	1				CODE	9002	KER	2	-2601
2.6	10001	KER	1	12001	KER	1	READ				-1
2.7	0001	KER	1				CODE	10002	KER	1	27
2.8	10002	KER	1				INXS				
3.0	10002	KER	1	12001	KER	1	CHCL	10003	KER	4	52001
4.0	10002	KER	1	12001	KER	1	CONT				
4.1				11001	KER	1	ERRS				
5.0							CHCL	9001	KER	999	92000
5.1	11001	KER	1				INXS				
5.2	9001	KER	999				STOP				
5.3							CHCL				
6.0	10001	KER	1				CHCL				1
6.1				11002	KER	1	CONT				
7.0	11002	KER	1				FLIP	12001	KER	1	
8.0	11001	KER	1	12001	KER	1	CHCL	9001	KER	1	50000
9.0	11002	KER	1	0001	KER	1	MULT	11003	KER	1	
10.0	11003	KER	1				PREC	12002	KER	1	
11.0	12002	KER	1	11004	KER	1	CODE	9001	KER	1	-13
12.0	11004	KER	1				CODE	9002	KER	1	
13.0	11002	KER	1	9002	KER	1	CHCL	12001	KER	1	55000
14.0	10001	KER	1	12001	KER	1	MULT	9002	KER	2	

Table 4 (contd)

18.0	11001	WUR	1	9002	FDC	2	CHOL	10004	DWC	5	90000	
18.2	10003	DWC	4				ADDS	9002	DTC	1	200	
22.0	10001	WTR	1	9002	OTC	1	WULT	12001	FDC	3		
22.1	9002	OTC	1				INXS					
23.0	12001	FDC	3				FLIP	10005	GCR	2		
24.0	10005	GCR	2	9002	OTC	1	WULT	11005	GCC	3		
24.1	11005	GCC	3				INXS					
32.0	9002	DTC	1				ROWS	10006	GCR	1		
33.0	11005	GCC	3				FLIP	10007	GCR	2		
34.0							READ	10008	GCR	3		
35.0	9001	FVC	1				FLIP	10009	GCR	4		
41.0	10007	GCR	2				CHOL	12001	TRC	3	52002	
41.1				11006	TUR	1	CONT					
42.2	12001	TRC	3	11006	TUR	1	INXS					
44.0	12001	TRC	3				FILL					
45.0	10006	GCR	1			TRC	3	WULT	9001	GCR	6	200
46.0	9001	GCR	6				ROWS	10010	GCR	2		
49.0	12001	TRC	3				FLIP	11007	TRC	1		
50.0	11007	TRC	1	9002	GCR	7	WULT	10011	GCR	3		
50.1	10011	GCR	3				INXS					
52.0	11007	TRC	1				COLS	10012	TRC	10		
199.1	10011	GCR	3				ROWS	13001	CCR	2		
201.0	13001	CCR	2				CHOL				1	
201.1				11007	WUR	1	CONT					
202.0	11007	WUR	1				FLIP	12001	WLC	1		
203.0	10009	GCR	4	12001	WLC	1	CHOL	9001	TRC	1	50001	
203.6	12001	WLC	1				COLS	13001	WLC	1		
204.0	11007	WUR	1	9001	TRC	1	WULT	12001	DCC	1		
204.5	12001	DCC	1				INXS					
205.0	12001	DCC	1				DECO	11008	DCC	1		
206.0	11008	DCC	1	11009	VDC	1	ROCT.	11010	GCC	13	-18	
207.0	11009	VDC	1				CODE	9001	VCC	1		
208.0	11007	WUR	1	9001	VCC	1	CHOL	12001	DWC	1	89000	
209.0	10008	GCR	3	12001	DWC	1	WULT	11011	TRC	4		
210.0	10008	GCR	1	12001	DWC	1	WULT	11012	TRC	5		
216.0	10003	DWC	4	11012	TRC	5	ADDS	9001	DWC	9		

Table 4 (contd)

217.0	9001	04C	8	95CC	12001	DDC	1
218.0	12001	00C	1	INXS			
219.0	10005	GCQ	2	ROWS	13001	GCQ	2
220.0	12001	GCQ	2	WULT	13001	TFC	9
221.0	12001	TEC	9	DECO	9001	AAA	1
221.1	0001	AAA	1	INXS			
242.0				HALT			
KER 1	1			-1			
361	361	1.		362	362	1.	
362	362	1.		363	363	1.	
1001	1001	.024		1001	2001	-.024	
2001	2001	.024					
1001	11	2001	21				
1001	21	2001	31				
1001	31	2001	41				
1001	41	2001	51				
1001	51	2001	61				
1001	61	2001	71				
1001	71	2001	81				
1001	81	2001	271				
1001	111	2001	121				
1001	121	2001	131				
1001	131	2001	141				
1001	141	2001	151				

Table 4 (contd)

[illegible]

[illegible]

Table 5. Frequencies and full spacecraft rigid-elastic coupling terms

Mode	Description	Frequency, Hz	$[\Phi]^T [M] [\Phi_{TRL}]$			$[\Phi]^T [M] [\Phi_{ROT}] \equiv \delta$		
			x	y	z	x	y	z
1	IP XA	0.021293	-36.5 (-17.31)		36.8 (17.46)	343 (6436)	25.5 (-482.4)	343 (6442)
2	OP XA YS	0.021410						
3	IP XS	0.023589						
4	OP XS YS	0.023756						
5	OP XS YA	0.028375						
6	OP XA YA	0.028375						
7	OP XA YS	0.035654						
8	OP XS YS	0.035654						
9	OP XS YA	0.035654						
10	OP XA YA	0.035654						
11	CP XA YS	0.050340						
12	OP XS YS	0.050340						
13	OP XS YA	0.053247						
14	OP XA YA	0.053247						
15	IP XA	0.061764						
16	IP XS	0.061764	15.85 (7.52)		15.4 (7.34)	0.05 (1)	-8.25 (-156.4)	4.3 (-81)
17	OP XA YS	0.069938						
18	OP XS YS	0.069938						
19	OP XS YA	0.069938						
20	OP XA YA	0.069938						
21	OP XS YA	0.084006						
22	OP XA YA	0.084006						
23	IP XA	0.085803						
24	OP XA YS	0.086226						
25	IP XS	0.087274						
26	OP XS YS	0.087630						
27	OP XA YS	0.098764						
28	OP XS YS	0.098764						
29	OP XA YS	0.10154						
30	OP XS YS	0.10154						
31	OP XS YA	0.10154						
32	OP XA YA	0.10154						
33	OP XS YA	0.10445						
34	OP XA YA	0.10445						
35	IP XA	0.12115						
36	IP XS	0.12115						
37	OP XA YS	0.12923						
38	OP XS YS	0.12923						
39	OP XS YA	0.12923						
40	OP XA YA	0.12923						

Table 5 (contd)

Mode	Description	Frequency, Hz	$[\Phi]^T [M] [\Phi_{TRL}]$			$[\Phi]^T [M] [\Phi_{ROT}] \equiv \delta$		
			x	y	z	x	y	z
41	OP XS YA ^a	0.13605	7.65 (3.64)		8.02 (3.82)	14 (264)	-4.65 (-87.5)	11.8 (224)
42	OP XA YA ^a	0.13605						
43	IP XA	0.14179						
44	IP XS	0.14234						
45	OP XA YS	0.14241						
46	OP XS YS	0.14302						
Sum of squares in each column, modes 1-46			1630 (369.4)		1650 (373.3)	11.87 E04 (41.51 E06)	0.076 E04 (0.2649 E06)	11.89 E04 (41.56 E06)
Corresponding inertias of appendages less elements assigned to central body			1355 (306.2)	1355 (306.0)	1355 (306.2)	11.94 E04 (41.74 E06)	0.2710 E06	11.99 E04 (41.93 E06)
Corresponding inertias of appendages			1450 (326.0)	1450 (326.0)	1450 (326.0)	11.95 E04 (41.77 E06)	0.083 E04 (0.2886 E06)	12.0 E04 (41.96 E06)
Note: OP = out-of-plane; IP = in-plane; XS = symmetric with respect to xz-plane; XA = antisymmetric with respect to xz-plane; YS = symmetric with respect to yz-plane; YA = antisymmetric with respect to yz-plane. Results in International System of Units; numbers in parentheses are lb-in. -s system. ^a No higher modes of this description obtained in output.								

Table A-1. Linear operators A, B, C

Operator	A			B			C		
DOF	xx1	xx2	xx3	xx1	xx2	xx3	xx1	xx2	xx3
Factor	+1	+1	-1	+1	-1	+1	+1	-1	-1

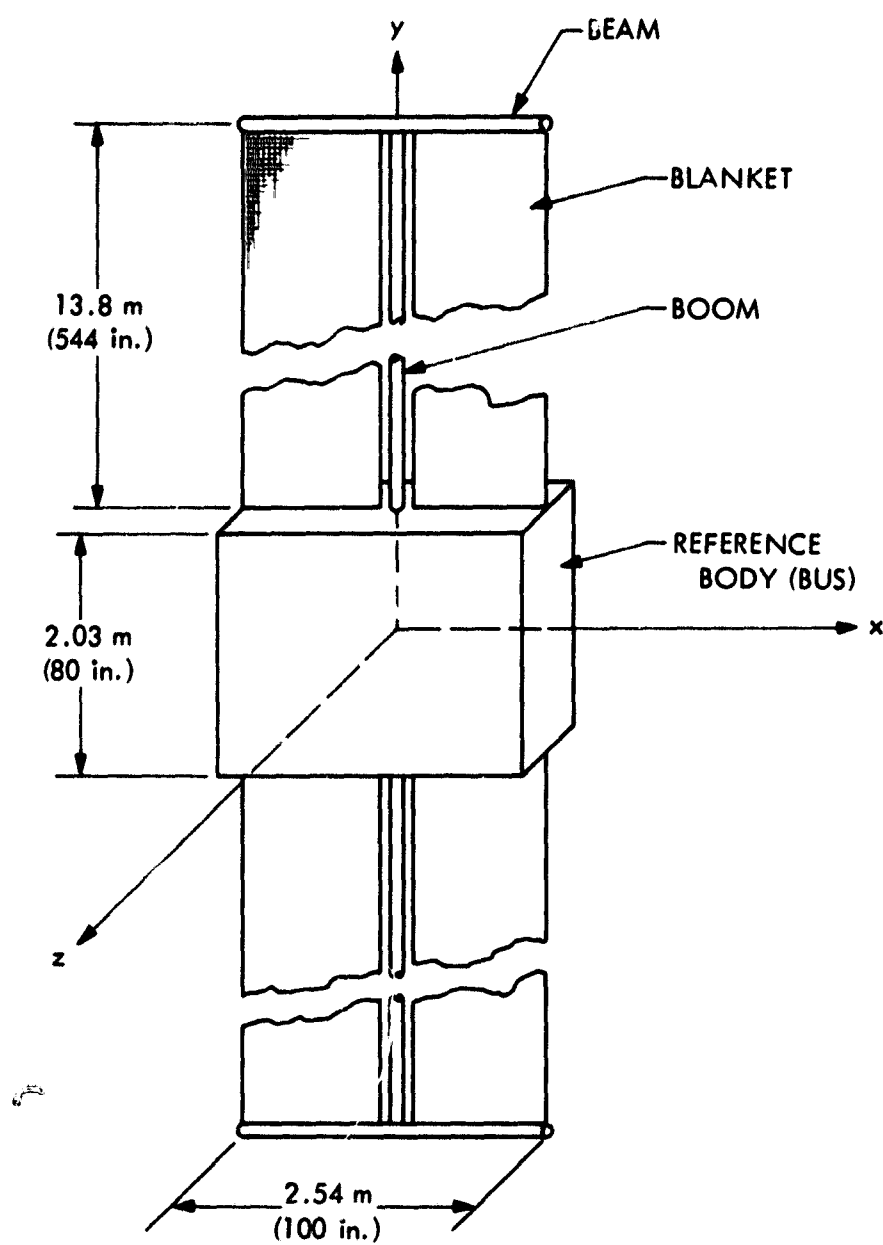


Fig. 1. Spacecraft configuration

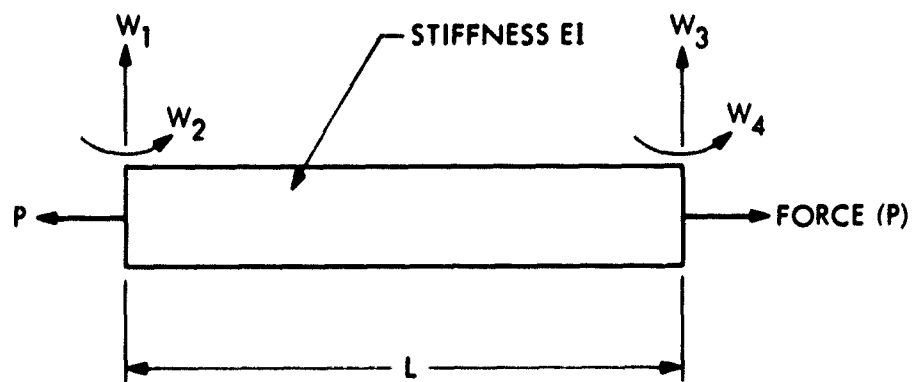


Fig. 2. Stiffness element

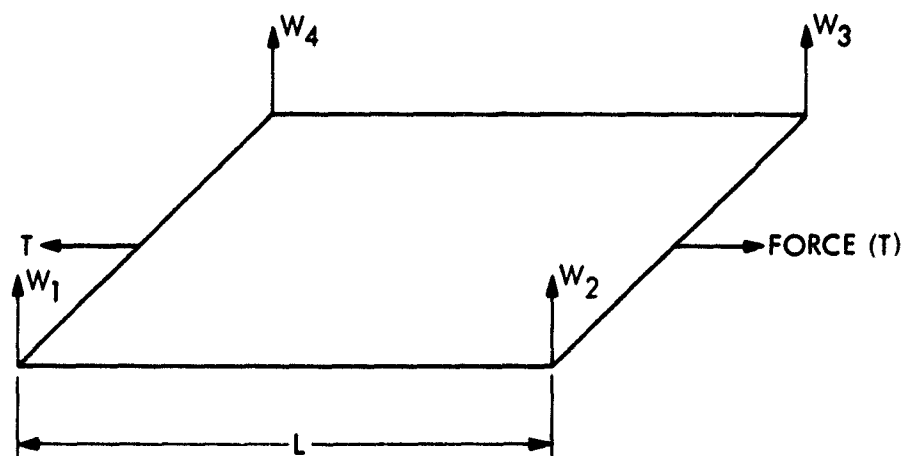


Fig. 3. Membrane element

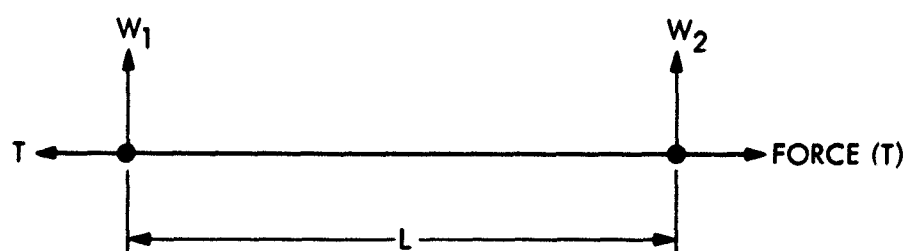


Fig. 4. String element

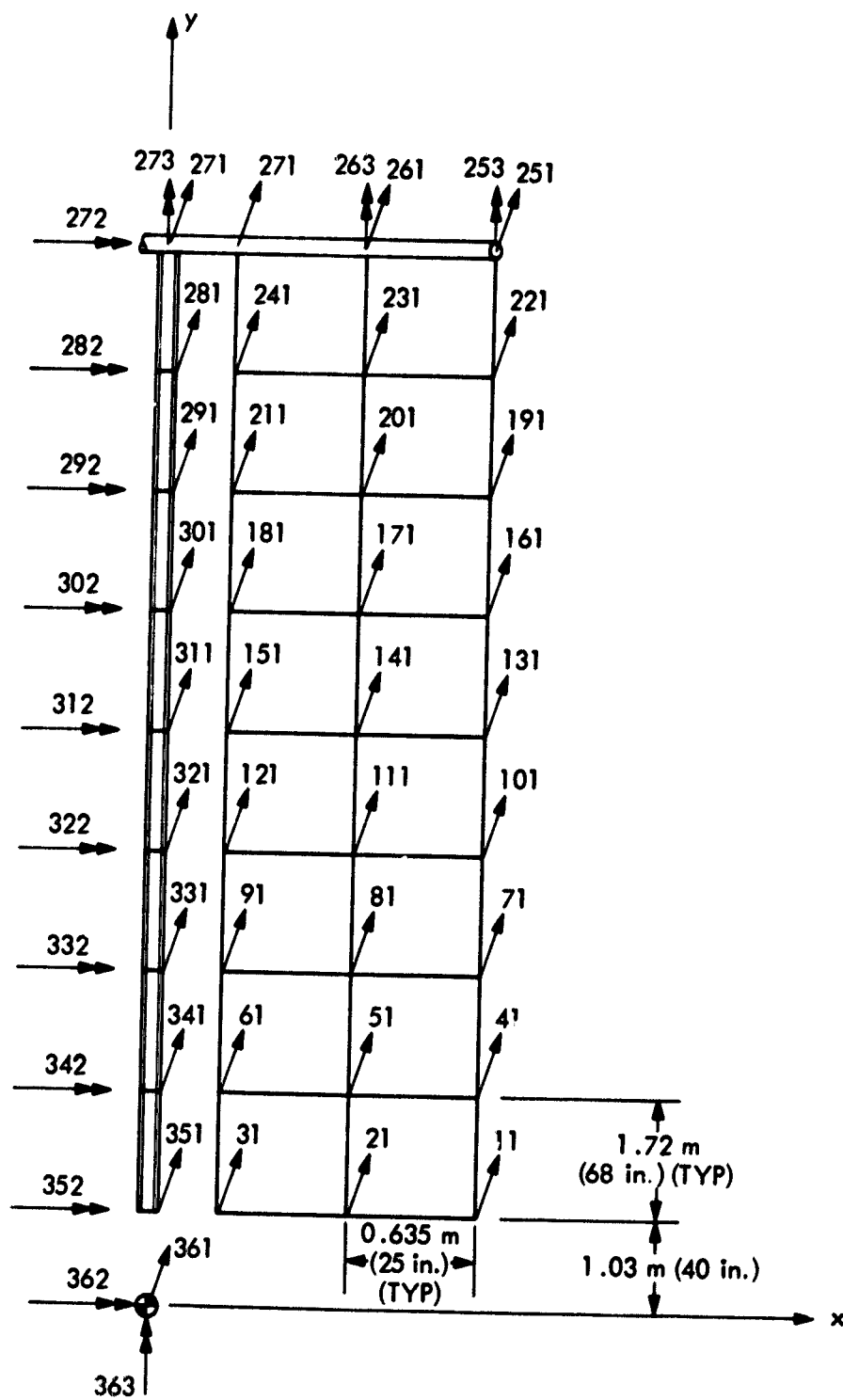


Fig. 5. Model for out-of-plane modes

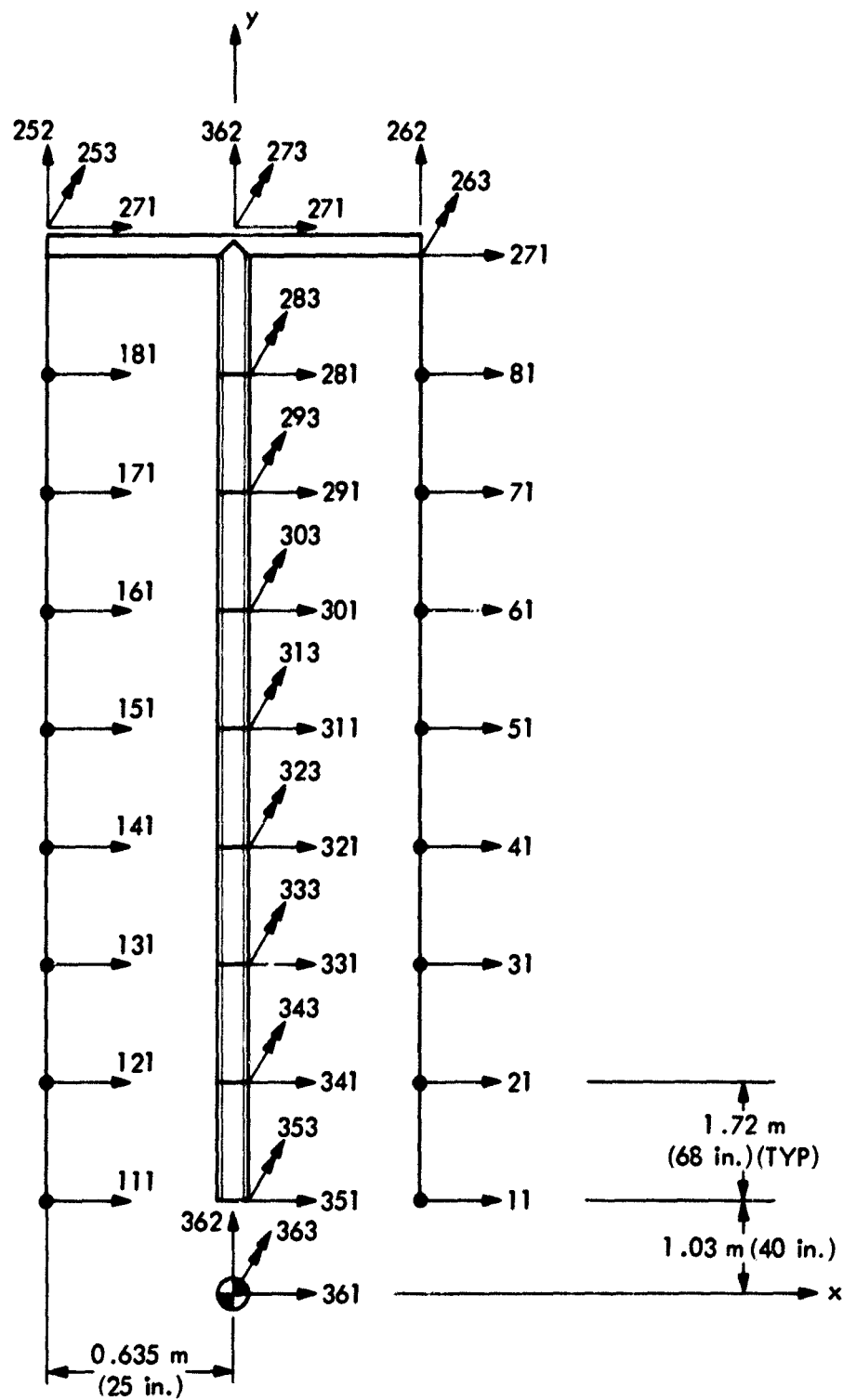


Fig. 6. Model for in-plane modes