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DEUTERIUM - HELIUM-3 FUSION POWER BALANCED CALCULATIONS

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16. Abstract

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SUMMARY

Steady-state operating parameters were calculated for the D-He^3 cycle. Energy equations were written for the electrons and the two ion species in the plasma. The effects of varying amounts of reflection and reabsorption of cyclotron radiation were studied by the inclusion of a variable cyclotron radiation parameter. Other parameters were varied to assess the effects of (1) unequal confinement times of plasma electrons and ions, (2) plasma ion heating by fusion products at an enhanced rate, and (3) injection of fuel ions with varying amounts of initial energy. Results are presented in the form of equilibrium values of a containment parameter, $n\tau$, the electron and ion temperatures, and a breakdown of the output power into bremsstrahlung, cyclotron radiation, and power carried out by escaping ions and electrons. Results are plotted as a function of plasma ion temperature.

INTRODUCTION

Power from controlled fusion reactors may be important in the future not only for ground but also for space applications (ref. 1). Present concepts of fusion propulsion systems, based on a steady-state D-He^3 cycle, are presented in references 1 to 3. This cycle permits most of the fusion energy, carried by the charged reaction products to be retained within the magnetically confined plasma and also reduces the flux of energetic neutrons which may impose intolerable heat loads on superconductive magnet elements. The energy carried by escaping charged particles, primarily the heated reactant ions, can be used directly to produce thrust (ref. 4).

A better understanding of the probable range of operating conditions for such a space-application reactor can be obtained by examining the energy balance of a D-He^3 plasma. Energy balance studies of the DT cycle have been reported (see, e.g., ref. 5).

These equate the energy produced by fusion within the plasma to the net energy loss from the plasma and determine the conditions of density, confinement time, and ion and electron temperatures which correspond to such an equilibrium. Studies envisioning ground applications emphasize the DT reaction, which has the lowest ignition temperature. The D-He^3 reaction needed for space applications has not received equivalent attention.

The purpose of the study reported here is to determine the required steady-state operating parameters for the D-He^3 reactor. The approach parallels that of reference 5. Energy equations were written for the electrons and the two ion species in the plasma. Energy exchange rates were based on assumed Maxwellian distributions for these three species, as were the rates at which energy was added to each from the fusion product ions. Bremsstrahlung and cyclotron radiation losses from the electrons were also based on an assumed Maxwellian distribution, even through as Rose has pointed out there is reason to expect departures from such a distribution both at the very low and very high velocities. The cyclotron radiation parameter, C_2/D, of reference 5, was varied through a range of values to represent the effects of varying amounts of reflection and reabsorption of the radiation within the plasma. Other parameters were varied to permit an assessment of the effects to be expected when (1) confinement times of electrons and ions are unequal, (2) ion heating by the fusion products is at an enhanced rate, and (3) the reactant ions are injected with varying amounts of initial energy.

The effect of He^4 and proton ash buildup was not considered in this initial study. Such ashes may be considered as having either of two effects: First, the positively charged He^4 and proton ashes will be accompanied by an equal number of electron charges. Hence, for a given fuel-ion density and temperature there will be proportionately larger bremsstrahlung and cyclotron radiation losses. Second, if the reactor density is limited by the plasma pressure, the attainable density of fuel ions must be decreased and the fusion power reduced. If the ash pressure is not too large, the results of this study should still be substantially correct.

For this initial study, only the D-He^3 reaction was included in the calculations. For ion temperatures greater than 30 keV, which applies to all cases in this report with zero ion injection energy, the D-D reaction energy was less than 5 percent of the D-He^3 reaction energy. Below 20 keV, the D-D reaction might become significant. For the case of energetic ion injection, steady-state operation at temperatures below 20 keV is possible. However, in most of these low temperature regions fusion power is insignificant compared to injection power. Therefore the trends predicted for energetic injection will still be valid.

The results of the analyses are presented in the form of values of the containment parameter $n\tau$ and electron and ion temperatures for which the energy balance is satisfied over a range of values of the parameters.

PLASMA POWER BALANCE

The work in this section is essentially the same as the power balance calculations given in reference 5. The equations for fusion reaction rates were taken from reference 6. The fast-ion energy-loss equations, and the plasma-component energy-exchange equations are in the forms given in the review article of reference 7. The bremsstrahlung and cyclotron radiation loss equations are from reference 5. Units are SI throughout, except as noted.

The block diagram for the plasma power balance is given in figure 1. The plasma components are electrons, deuterium ions and helium-3 ions, which are injected with energy E_{Ie}, E_{ID}, and E_{IH}, respectively. In all of the calculations of this report, E_{Ie} is set equal to zero. The energy distribution of all plasma components as assumed to be Maxwellian. The three plasma components exchange energy with each other, and they are heated by the fusion protons and α-particles - all by means of Coulomb collisions. The electrons lose energy by bremsstrahlung $\dot{E}_B$ and cyclotron radiation $\dot{E}_C$. As plasma particles escape from the reaction chamber they carry their kinetic energy with them.

We assume a 50 percent D - 50 percent He^3 plasma in the reaction chamber. The number density of deuterium ions, helium-3 ions and electrons is, respectively,

$$n_D = \frac{n}{2} \tag{1}$$

$$n_H = \frac{n}{2} \tag{2}$$

$$n_e = \frac{3n}{2} \tag{3}$$

Since helium-3 has a double positive charge, there must be more electrons than ions in the plasma to maintain charge neutrality. The number density of protons and α-particles is assumed to be negligible.

The mean confinement times of the deuterium ions, helium-3 ions, and electrons are, respectively, τ_D, τ_H, and τ_e. The proton and α-particle confinement times are assumed to be equal to their slowing down times as will be explained later. The escape rate of the s-component is n_s/τ_s. The mean kinetic energy carried by each particle is assumed to be $(3/2)\ kT_s$.

Thermonuclear Reaction Rates

Reaction rates $\langle \sigma v \rangle$ for the deuterium - helium-3 reaction were taken from reference 6. The empirical formula for $\langle \sigma v \rangle$, which Kozlov estimates to be accurate to within 15 percent, is

$$\langle \sigma v \rangle = 0.90\times 10^{-19} \frac{(1 + 3\,\theta^{3/4})}{\sqrt{1 + 4\,\theta^{3.25}}} \frac{\exp\left(\frac{-6.98}{\theta^{1/2}}\right)}{\theta^{2/3}} \left(\frac{m^3}{sec}\right) \tag{4}$$

where

$$\theta = \frac{T}{93.82}$$

and where T is in units of keV.

In our calculations T was taken as $(T_D + T_H)/2$. Since the deuterium and helium-3 ions are strongly coupled, their temperatures are approximately equal for all the results in this report.

Energy Loss from Fusion Protons and α-Particles

The deuterium - helium-3 reaction is

$$D + He^3 \rightarrow \text{Proton } (14.7\ \text{MeV}) + He^4\ (3.6\ \text{MeV})$$

The proton and α-particle (He^4) are born with energy far in excess of the plasma thermal energy, and for this reason they are referred to as fast ions. Before leaving the reaction chamber, the fusion particles will give up most of their energy to the three plasma components. The mean rate of change of kinetic energy of a fast ion k moving through a plasma with N-components is (see, e.g., ref. 7)

$$\frac{dE_k}{dt} = -\frac{e^4 Z_k^2}{4\pi\epsilon_0^2 v_k} \sum_{s=1}^{N} h_s \frac{n_s Z_s^2 L_s}{m_s}\left[\Phi(X_s) - \frac{2}{\sqrt{\pi}}(1 + \gamma_s) X_s \exp\left(-X_s^2\right)\right] \left(\frac{w}{m^3}\right) \tag{5}$$

where s refers to the plasma components.

All quantities are defined in the symbol list. The quantity h_s, which does not appear in the usual slowing-down equation, is the enhancement coefficient. It is included to take into account the possibility that energy transfer from the fast ions to the plasma components may occur more rapidly than predicted by binary Coulomb collision theory. In all calculations, the Coulomb logarithm L_s was set equal to 20.

The energy transfer rate from the fast ion k to the plasma component s is defined as

$$\frac{dE_{ks}}{dt} \equiv r_{ks} = \frac{e^4 Z_k^2}{4\pi \epsilon_0^2 v_k} \frac{h_s n_s Z_s^2 L_s}{m_s} \left[\Phi(X_s) - \frac{2}{\sqrt{\pi}} (1 + \gamma_s) X_s \exp\left(-X_s^2\right)\right] \tag{6}$$

Then let R_k be defined as

$$R_k = - \sum_{s=1}^{N} r_{ks} \tag{7}$$

The ratio $-r_{ks}/R_k$ is the fraction of energy loss from the k ion that is absorbed by the s plasma component as the k ion energy is changed from E_{ko} to E_k. The energy transfer from the fast ion k to the plasma component s is

$$E_{ks} = \int_{E_{ko}}^{E_k} \frac{r_{ks}}{R_k} dE_k \tag{8}$$

To determine the total energy transferred from the fast ion k to the plasma component s, we must select the upper limit of integration E_k. As the energy of the fast ion decreases, it will reach a critical energy $\left(E_{kcr}\right)_s$ where there is zero mean energy exchange between the k ion and the s plasma component, that is,

$$\frac{dE_{ks}}{dt} = r_{ks} = 0$$

The critical energy $\left(E_{kcr}\right)_s$ depends on the mass ratio m_s/m_k and on the temperature of the s plasma component T_s. Since there are three plasma components, there are three values of $\left(E_{kcr}\right)_s$. The largest of the three values of $\left(E_{kcr}\right)_s$ is used as the upper limit of integration for equation (8). When the fast ion energy reaches $\left[\left(E_{kcr}\right)_s\right]_{largest}$ it is assumed to disappear from the plasma. Values of $\left(E_{kcr}\right)_s/T_s$ are taken from reference 7 and appear in table I. Equation (8) is integrated numerically.

TABLE I. - VALUES OF $(E_{kcr})_s/T_s$

Plasma component	Proton	α-particle
Electron	1.5	1.5
Deuteron	.757	1.17
Helium-3	.626	1.065

The fast ion slowing down time τ_k is defined as the time it takes to reach the energy $\left[\left(E_{kcr}\right)_s\right]_{largest}$. To obtain τ_k, we begin by combining equations (5) and (7) to give

$$dt = \frac{dE_k}{R_k} \tag{9}$$

After multiplying equation (9) by n, it can be integrated for $n\tau_k$ to give

$$n\tau_k = \int_{E_{ko}}^{\left[\left(E_{kcr}\right)_s\right]_{largest}} n\,\frac{dE_k}{R_k} \tag{10}$$

After the power balance equations are solved for the equilibrium values of T_D, T_H, and T_e, the quantity $n\tau_k$ is calculated from equation (10). The power balance equations embody the assumption that the fast ion is contained within the plasma for a time equal to or greater than this slowing-down time τ_k. If the fast-ion confinement time is about the same as that of the plasma ions τ_s the assumption can be tested by examining the ratio $n\tau_s/n\tau_k$. The larger this ratio, the greater is the probability that the assumption is justified. The actual confinement time for the fast particles will depend on factors not included in the analysis. In some cases the confinement time may increase for fast ions because of their slower diffusion; in other cases the larger gyroradio may lead to more rapid loss rates. The ratio τ_s/τ_k was calculated for the cases reported; when it drops below unity that fact is noted.

Energy Exchange Between Plasma Components

For the plasma components characterized by Maxwellian distributions, the energy transfer rate per unit volume from component a to component b is given as (see, e.g., ref. 5)

$$Q_{a\to b} = \frac{3}{2}\,\frac{n_a(T_a - T_b)}{\tau_{\epsilon(a\to b)}} \qquad \left[\frac{\text{keV}}{(\text{sec})(\text{m}^3)}\right] \tag{11}$$

where

$$\tau_{\epsilon(a\to b)} = \frac{3(1.6021)^{3/2}\times 10^{-10}}{8\sqrt{2\pi}\,c^4}\,\frac{m_a m_b}{n_b L Z_a^2 Z_b^2 e^4}\left(\frac{T_a}{m_a} + \frac{T_b}{m_b}\right)^{3/2} \tag{12}$$

where T_a and T_b are in keV. The Coulomb logarithm L was set equal to 20 in all calculations.

As pointed out in reference 5, the assumption of Maxwellian distribution for the electron population is not strictly sound. In particular, the energy transfer from the fast ions is primarily to those electrons with velocities less than the ion velocity. Consequently, the slow electrons are heated, and the distribution at low velocities depleted. This results in fewer electrons capable of receiving the energy from the fast ions, and a reduced energy transfer rate. Although this effect should be taken into account to obtain the best approximation to the energy balance, it is quite difficult to do so in a parametric analysis such as the present one. Moreover, the effect appears to be small at conditions of interest. The low velocity electrons diffuse rapidly in velocity space and tend to fill in any deficiency from the Maxwellian distribution. The assumption of Maxwellian distribution for the electrons is also risky at the other extreme - the high velocity electron population will be depleted both by bremsstrahlung and cyclotron radiation losses as will be discussed later.

Bremsstrahlung Loss

The bremsstrahlung loss term, for a Maxwellian electron distribution, is

$$\dot{E}_B = 3\times10^{-21}\, n_e \left(n_D Z_D^2 + n_H Z_H^2\right) \sqrt{T_e} \qquad \left[\frac{\text{keV}}{(\text{sec})(\text{m}^3)}\right] \tag{13}$$

where T_e is in keV.

In this initial study, normal bremsstrahlung was assumed, which corresponds to setting $C_1 = 1$ in reference 5.

Cyclotron Radiation Loss

The cyclotron radiative loss is the same as Rose's. It is

$$\dot{E}_C = C_2 \times 0.387\, B^2 n_e T_e \left(1 + \frac{T_e}{204}\right) K_{\mathscr{L}} \qquad \left[\frac{\text{keV}}{(\text{sec})(\text{m}^3)}\right] \tag{14}$$

where $K_{\mathscr{L}}$ is the plasma transparency coefficient.

$$K_{\mathscr{L}} = \frac{2.1\times10^{-3}\, T_e^{\;7/4}}{\mathscr{L}^{1/2}} \tag{15}$$

The quantity $\mathscr{L}$ introduced in reference 5 is

$$\mathscr{L} = l n_e e c \frac{B\beta}{2 \sum\limits_{s=1}^{N} n_s k T_s}$$

and for the D-He^3 reactor case $\mathscr{L}$ becomes

$$\mathscr{L} = \frac{3}{2} \times 10^5 \frac{l B\beta}{\left(\frac{T_D}{3} + \frac{T_H}{3} + T_e\right)} \tag{16}$$

Substituting equations (15) and (16) into equation (14), we get the cyclotron radiative loss term

$$\dot{E}_C = \frac{C_2}{\sqrt{lB\beta^3}} \, 3.38\times10^{-27} \, \frac{n_e^2}{4} \, T_e^{11/4} \left(\frac{T_D}{3} + \frac{T_H}{3} + T_e\right)^{3/2} \left(1 + \frac{T_e}{204}\right) \left[\frac{\text{keV}}{(\text{sec})(\text{m}^3)}\right] \tag{17}$$

where T is in keV.

In the power balance equations that follow, a coefficient C_2' is used which is defined as

$$C_2' = \frac{C_2}{\sqrt{lB\beta^3}} \tag{18}$$

where l is the plasma size for an equivalent slab, "perhaps equal to the radius for cylindrical geometry" (ref. 5), B is the local magnetic field strength in the plasma, and β is the ratio of plasma pressure to the local magnetic field pressure.

If the external magnetic field pressure balances the sum of the internal magnetic field pressure and plasma pressure, then C_2' can be written as

$$C_2' = \frac{C_2}{\sqrt{\dfrac{lB_0\beta_0^3}{(1 - \beta_0)^{5/2}}}}$$

where B_0 is the external magnetic field strength, and β_0 is the ratio of plasma pressure to external magnetic field pressure.

In reference 5, the quantity C_2 accounts for the fact that the cyclotron radiation is partly reflected at the electrically conducting vacuum wall. In reference 10, it was pointed out that the effect of reflectors is to increase the photon path length by a factor $(1 - \Gamma)^{-1}$ where Γ is the reflectivity. Since photon path length is proportional to the system dimension l it is reasonable to assume (as is done in ref. 10) that the radiation loss is reduced by about $(1 - \Gamma)^{1/2}$. Thus the quantity C_2 of reference 5 may be approximated by the quantity $(1 - \Gamma)^{1/2}$. The preceding equation then becomes

$$C_2' = \frac{(1 - \Gamma)^{1/2}}{\sqrt{lB_0\left[\frac{\beta_0^3}{\left(1 - \beta_0\right)^{5/2}}\right]}}$$

Solving this last equation for Γ, gives

$$\Gamma = 1 - \left(C_2'\right)^2 lB_0 \frac{\beta_0^3}{\left(1 - \beta_0\right)^{5/2}} \tag{19}$$

Figure 2, which is a plot of the cyclotron reflectivity Γ against β_0 with C_2' as a parameter, is obtained from equation (19). The acceptable range of values of C_2' are obtained from energy balance considerations and from space propulsion requirements. Figure 2 will be referred to again in the RESULTS section.

We have used approximation for $K_{\mathscr{L}}$ as given in reference 5 without checking its validity in the range of electron temperatures and $\mathscr{L}$ values used in this study. Hopefully, any errors should be adequately covered by the variable coefficient C_2, which combines the effects of partial reflection of the cyclotron radiation at the vacuum chamber wall and the effects of $K_{\mathscr{L}}$. Departures from the assumed Maxwellian distribution may have particularly important effects on cyclotron radiation. In the high-velocity tail of the distribution the radiative losses are the greatest while the thermalization proceeds most slowly. The exact equilibrium will be determined by the balancing of the depopulating effects of cyclotron radiation and bremsstrahlung against the repopulating effects of diffusion in velocity space. (And even that balance should be modified to account for the probable velocity-dependence of the particle loss mechanisms.) The problem is made even more complex by the self-absorption of the cyclotron radiation. Energy is transferred within the electron population as the electrons emit and absorb the (doppler-broadened) cyclotron fundamental and its many harmonics. Exact solutions for such a system seem to be beyond our present capabilities. One technique that may be used is to assume a net energy loss, for a given electron, equal to a fixed fraction of its single particle radiation rate.

Thermal Power Balance Equations

With the equations presented in the previous sections it is a simple matter to write power balance equations for the three plasma components D, He^3, and electrons. First we write the particle balance equations for the deuterons and helium-3 ions, and obtain the expression for fuel fractional burnup. A word-form equation for the steady-state particle balance is

$$\begin{bmatrix}\text{Rate of change}\\ \text{of particles in}\\ \text{a given volume}\end{bmatrix} = \begin{bmatrix}\text{Injection}\\ \text{rate}\\ \ \end{bmatrix} - \begin{bmatrix}\text{Nuclear}\\ \text{reaction}\\ \text{loss rate}\end{bmatrix} - \begin{bmatrix}\text{Geometrical}\\ \text{escape rate}\\ \ \end{bmatrix} = 0 \tag{20}$$

The deuteron particle balance equation is

$$\frac{dn_D}{dt} = S_D - n_D n_H \langle \sigma v \rangle - \frac{n_D}{\tau_D} = 0 \tag{21}$$

The helium-3 particle balance equation is

$$\frac{dn_H}{dt} = S_H - n_D n_H \langle \sigma v \rangle - \frac{n_H}{\tau_H} = 0 \tag{22}$$

The fuel fractional burnup f_B is defined as

$$f_B = \frac{\text{Number of fuel particles reacting per unit time}}{\text{Number of fuel particles injected per unit time}}$$

$$f_B = \frac{2\, n_D n_H \langle \sigma v \rangle}{S_D + S_H} \tag{23}$$

Using equations (21) and (22) for S_D and S_H, respectively (noting that $\tau_D = \tau_H$ was assumed in all of the calculations) results in the following expression for f_B:

$$f_B = \frac{1}{1 + \dfrac{2}{n\tau_D \langle \sigma v \rangle}} \tag{24}$$

Deuteron energy equation. - The steady-state deuteron energy equation is represented in word-form as

$$\begin{bmatrix}\text{Net D}\\\text{heat-}\\\text{ing}\\\text{rate}\end{bmatrix} = \begin{bmatrix}\text{D injec-}\\\text{tion}\\\text{power}\end{bmatrix} + \begin{bmatrix}\alpha\text{-fusion}\\\text{power}\\\text{to D}\end{bmatrix} + \begin{bmatrix}\text{Proton}\\\text{fusion}\\\text{power}\\\text{to D}\end{bmatrix} - \begin{bmatrix}\text{Power}\\\text{lost}\\\text{by escap-}\\\text{ing D}\end{bmatrix} - \begin{bmatrix}\text{Power}\\\text{from}\\\text{D to}\\\text{elec-}\\\text{trons}\end{bmatrix} - \begin{bmatrix}\text{Power}\\\text{from}\\\text{D to}\\\text{He}^3\end{bmatrix} = 0 \tag{25}$$

Noting that the energy content of deuterons per unit volume is $(3/2)n_D T_D$ the deuteron energy equation becomes

$$\frac{d}{dt}\left(\frac{3}{2} n_D T_D\right) = \left(n_D n_H \langle \sigma v \rangle + \frac{n_D}{\tau_D}\right) E_{ID} + n_D n_H \langle \sigma v \rangle (E_{\alpha D} + E_{pD}) - \frac{n_D}{\tau_D} \frac{3}{2} T_D$$

$$- \frac{3}{2} \frac{n_D(T_D - T_e)}{\tau_{\epsilon(D \to e)}} - \frac{3}{2} \frac{n_D(T_D - T_H)}{\tau_{\epsilon(D \to H)}} = 0 \qquad \left[\frac{\text{keV}}{(\text{sec})(\text{m}^3)}\right] \tag{26}$$

where T and E are in keV.

Helium-3 energy equation. - The steady-state energy equation for helium-3 is the same as equation (25) with the subscripts D and H interchanged. Therefore the helium-3 energy equation is

$$\frac{d}{dt}\left(\frac{3}{2} n_H T_H\right) = \left(n_D n_H \langle \sigma v \rangle + \frac{n_H}{\tau_H}\right) E_{IH} + n_D n_H \langle \sigma v \rangle (E_{\alpha H} + E_{pH}) - \frac{n_H}{\tau_H} \frac{3}{2} T_H$$

$$- \frac{3}{2} \frac{n_H(T_H - T_e)}{\tau_{\epsilon(N \to e)}} - \frac{3}{2} \frac{n_H(T_H - T_D)}{\tau_{\epsilon(N \to D)}} = 0 \qquad \left[\frac{\text{keV}}{(\text{sec})(\text{m}^3)}\right] \tag{27}$$

where T and E are in keV.

Electron energy equation. - In all calculations the electrons were assumed to be injected with negligible energy ($E_{Ie} = 0$). In word-form the energy balance equation for the plasma electrons is

$$\begin{bmatrix}\text{Net}\\ \text{elec-}\\ \text{tron}\\ \text{heat-}\\ \text{ing}\\ \text{rate}\end{bmatrix} = \begin{bmatrix}\alpha\text{-fusion}\\ \text{power to}\\ \text{electrons}\\ \\ \\ \\ \end{bmatrix} + \begin{bmatrix}\text{Proton}\\ \text{fusion}\\ \text{power}\\ \text{to elec-}\\ \text{trons}\\ \\ \end{bmatrix} - \begin{bmatrix}\text{Power}\\ \text{lost by}\\ \text{escap-}\\ \text{ing}\\ \text{elec-}\\ \text{trons}\end{bmatrix} - \begin{bmatrix}\text{Power}\\ \text{from}\\ \text{elec-}\\ \text{trons}\\ \text{to D}\\ \\ \end{bmatrix} - \begin{bmatrix}\text{Power}\\ \text{from}\\ \text{elec-}\\ \text{trons}\\ \text{to He}^3\\ \\ \end{bmatrix} - \begin{bmatrix}\text{Brems-}\\ \text{strahlung}\\ \text{power}\\ \text{loss}\\ \\ \\ \end{bmatrix} - \begin{bmatrix}\text{Cyclo-}\\ \text{tron}\\ \text{power}\\ \text{loss}\\ \\ \\ \end{bmatrix} = 0 \tag{28}$$

The electron energy equation is

$$\frac{d}{dt}\left(\frac{3}{2} n_e T_e\right) = n_D n_H \langle \sigma v \rangle \left(E_{\alpha e} + E_{pe}\right) - \left(3\, n_D n_H \langle \sigma v \rangle + \frac{n_e}{\tau_e}\right) \frac{3}{2} T_e - \frac{3}{2} \frac{n_e (T_e - T_D)}{\tau_{\epsilon(e \to D)}}$$

$$- \frac{3}{2} \frac{n_e (T_e - T_H)}{\tau_{\epsilon(e \to H)}} - \dot{E}_B - \dot{E}_C = 0 \qquad \left[\frac{\text{keV}}{(\text{sec})(\text{m}^3)}\right] \tag{29}$$

where T and E are in keV.

Solution of Three Simultaneous Energy Equations

We make use of equations (1), (2), and (3) to get a common number density n in the three energy equations. Equations (26), (27), and (29) can be written in the following functional forms:

$$\frac{1}{n\tau_D} = f(T_D,\ T_H,\ T_e,\ E_{ID}) \tag{26a}$$

$$\frac{1}{n\tau_H} = g(T_D,\ T_H,\ T_e,\ E_{IH}) \tag{27a}$$

$$\frac{1}{n\tau_e} = h(T_D,\ T_H,\ T_e,\ C_2') \tag{29a}$$

There are also the assumed relations

$$n\tau_D = n\tau_H \tag{30}$$

and

$$n\tau_e = Jn\tau_D \tag{31}$$

Equations (26), (27), (29), (30), and (31) are five simultaneous equations in terms of the 10 variables $n\tau_D$, $n\tau_H$, $n\tau_e$, T_D, T_H, T_e, E_{ID}, E_{IH}, C_2', and J. Therefore, the quantities E_{ID}, E_{IH}, C_2', J, and T_e are chosen as input parameters, and the variables to be determined are $n\tau_D$, $n\tau_H$, $n\tau_e$, T_D, and T_H. These five simultaneous equations can be reduced to two simultaneous equations with T_D and T_H as unknown variables.

Since, from equation (30), $\tau_D = \tau_H$, we can eliminate $n\tau_D$ from equations (26) and (28). After some rearrangement the following equation results:

$$\langle\sigma v\rangle (E_{IH} + E_{\alpha H} + E_{pH}) - \frac{3(T_H - T_e)}{n\tau_{\epsilon(H\to e)}} - \frac{3(T_H - T_D)}{n\tau_{\epsilon(H\to D)}} - \left(\frac{3\ T_H - 2\ E_{IH}}{3\ T_D - 2\ E_{ID}}\right)$$

$$\times \left[\langle\sigma v\rangle (E_{ID} + E_{\alpha D} + E_{pD}) - \frac{3(T_D - T_e)}{n\tau_{\epsilon(D\to e)}} - \frac{3(T_D - T_H)}{n\tau_{\epsilon(D\to H)}}\right] = 0 \tag{32}$$

Use equation (31) to replace $n\tau_e$ by $Jn\tau_D$ in the electron energy equation (29). Then eliminate $n\tau_D$ between equations (26) and the electron energy equation (29) to obtain the following result.

$$\langle\sigma v\rangle (E_{\alpha e} + E_{pe}) - \langle\sigma v\rangle \frac{9}{2} T_e - \frac{9(T_e - T_D)}{n\tau_{\epsilon(e\to D)}} - \frac{9(T_e - T_H)}{n\tau_{\epsilon(e\to H)}} - \frac{4\dot{E}_B}{n^2} - \frac{4\dot{E}_C}{n^2} - \frac{9\ T_e}{J(3\ T_D - 2\ E_{ID})}$$

$$\times \left[\langle\sigma v\rangle (E_{\alpha D} + E_{pD} + E_{ID}) - \frac{3(T_D - T_e)}{n\tau_{\epsilon(D\to e)}} - \frac{3(T_D - T_H)}{n\tau_{\epsilon(D\to H)}}\right] = 0 \tag{33}$$

Equations (32) and (33) are two simultaneous equations with T_D and T_H as unknowns, and E_{ID}, E_{IH}, C_2', J, and T_e are inputs. The solutions, T_D and T_H, are substituted into equation (26) to obtain the corresponding value of $n\tau_D$. Similarly, the values of fast-ion slowing down times $n\tau_k$, fractional fuel burnup f_b, and the various forms of output power are all calculated after T_D and T_H are obtained. When E_{ID}

and E_{IH} are zero, there is only one solution for T_D and T_H. When E_{ID} and E_{IH} are not zero there are sometimes two solutions. Solutions are obtained on a high-speed digital computer.

RESULTS

Zero Ion Injection Energy

The cases with zero injection energy have the results shown on figures 3 to 5. Figures 3(a), 4(a), and 5(a) are for the normal or most probable case, with equal confinement times for electrons and ions ($\tau_e = \tau_D$) and with heating of the ions and electrons by the fusion product ions occurring at the basic rates calculated from binary collisions ($h_e = h_D = h_H = 1$). Figures 3(b), 4(b), and 5(b) explore the effect of assumed enhanced fast-ion heating of the deuterons and helium-3 ions. The deuterons and helium-3 ions are assumed to gain energy from the fast ions at 1000 times the normal rate ($h_D = h_H = 1000$, $h_e = 1$). Finally, figures 3(c), 4(c), and 5(c) return to normal ion heating but assume the electron confinement time to be 10 times that of the ions. The opposite assumption, that the confinement time of the electrons is one-tenth that of the ions, gives the results shown on figures 3(d), 4(d), and 5(d). Throughout figures 3 to 5, the results are plotted as a function of deuteron temperature. The cyclotron radiation coefficient C_2' varies as a parameter from zero to 2.5.

Two points to keep in mind throughout this discussion are that (1) the confinement times of the deuterons and helium-3 ions were assumed to be equal (i.e., $\tau_D = \tau_H$) and (2) because of the strong coupling between the deuterons and helium-3 ions, their temperatures were always approximately the same to within 10 percent or less.

Confinement Parameter, $n\tau_D$

Figure 3 shows the confinement parameter $n\tau_D$ as a function of deuterium ion temperature. This relation alone does not determine the operating conditions for any particular reactor. The reactor will have a confinement geometry which will impose some relation between n, τ_D, and T. If this relation can be expressed as $n\tau_D = f(T)$, to take a very simple example, the actual operating points would be represented by the intersections of such a curve with the energy balance $n\tau$ curve. In some respects, such a system can be likened to the case of the electronic vacuum tube, where the tube characteristics must be combined with the load line representing the external circuitry.

There are several interesting pieces of information on figure 3(a). First, the minimum value of $n\tau_D$ is 7×10^{20} seconds per cubic meter, for zero cyclotron radiation loss. This is about a factor of five above the minimum value for a closed DT system. Second, the minimum ion temperature for steady-state operation is about 30 keV. Near this point, the particle loss rate approaches zero. For lower ion temperatures, the bremsstrahlung loss alone exceeds the fusion power released, precluding steady-state operation (without energetic ion injection). Third, the effect of increased cyclotron radiation loss is to increase the minimum $n\tau_D$ and to impose an upper limit on ion temperature. This upper limit represents the case wherein the total energy produced by fusion is lost by the bremsstrahlung plus cyclotron radiation, leaving none to escape with particles. At higher temperatures, the cyclotron radiation increases more rapidly than the fusion power so that the radiative losses exceed the fusion power. Hence steady-state operation at higher temperatures is not possible.

All four cases shown in figure 3 exhibit the same trends as the cyclotron radiation parameter, C_2', is varied. Figure 3(b) shows the effect of enhanced heating of the plasma ions by the fusion products. Since less energy goes directly to the electrons, the electron temperature should be suppressed and the radiative losses reduced particularly at the higher ion temperatures. This effect is indeed noticed. The upper limit on ion temperature is increased over the normal case, pronouncedly so at the higher values of C_2'. The minimum values of $n\tau$ are lower than the normal case: only slighter lower for $C_2' = 0$ but lower by a factor of 10 for $C_2' = 2.5$. Any enhanced ion heating effect would be very welcome.

Although it seems likely that in most reactor schemes the confinement times of the electrons and ions will be about equal, occasionally the idea of unequal throughputs of electrons and ions is considered. Figure 3(c) shows results obtained when $\tau_e/\tau_D = 10$. The values of $n\tau_D$ are slightly less than the normal case, because the energy lost with the escaping electrons is less, which must be compensated by increased ion loss rates. However, the changes are small. Even if one could devise a means for preferentially confining the electrons, the incentive seems inadequate.

Figure 3(d) represents the opposite case $\tau_e/\tau_D = 0.1$. It seems more plausible, from an engineering viewpoint, to consider increasing the flow of cold electrons into the plasma by a factor of 10. The first thing we note is that $n\tau_D$ for the ions is about a factor of five greater than for our normal case, while the allowable range of ion temperature is about the same. The effect may be viewed as an increase in required ion-confinement time; hence, a reduction in energy transported out with the ions which compensates for the increased energy lost with the increased electron flow.

Electron Temperature

The values of electron temperature calculated for the four cases are shown on figure 4. The normal case (fig. 4(a)) shows that, for zero cyclotron radiation, the electron temperature slightly exceeds that of the ions up to about 175 keV. At still higher temperatures, the combination of increased bremsstrahlung losses and proportionately less fast-ion heating of the electrons causes the electron temperature to fall below that of the ions. Increased cyclotron radiation losses act to cool the electrons, especially at the higher temperatures.

In figure 4(b), the effect of enhanced ion heating is to transfer proportionately more fusion energy directly to the plasma ions so that the electron temperature is seen to fall below that of the ions. Most of the electron heating now comes from the plasma ions, rather than from the fast fusion products. This was confirmed by calculations setting $h_e = 0$, that is, no electron heating directly from the fast ions. The resulting electron temperatures, for the $C_2' = 0$ case, were approximately the same as in figure 4(b). Energy exchange with the plasma ions is sufficient to keep the electron temperature from dropping below about one-half of the plasma ion temperature.

Figure 4(c) demonstrates the effect of increased confinement of the electrons. For zero cyclotron radiation, the electron temperature rises sharply above the ion temperature. This permits the increased loss by bremsstrahlung and the increased energy lost with each escaping electron to partly compensate for the reduced number of electrons escaping. Because of the low values of $n\tau_D$, for the case of $C_2' = 0$, the ratio of the plasma ion confinement time to the fast-proton confinement time is less than unity for T_D above 62 keV. However, the ratio is always greater than one-third, so that the electrons at least are confined longer than the proton slowing down time. The results for $C_2' \neq 0$ are much closer to the corresponding normal cases. The cyclotron radiation losses are quite sensitive to electron temperature, so that a small increase in electron temperature serves to restore the balance at the higher C_2'.

Results for the case of rapid electron throughflow are plotted on figure 4(d). With $\tau_e/\tau_D = 0.1$, the electron temperature is strongly suppressed. The large flux of electrons serves to carry off most of the energy the electrons receive, even at a lower value of the mean energy per electron.

Distribution of Output Power

In the introduction of this report it was noted that for a fusion propulsion system, the majority of fusion energy must appear in the escaping plasma particles. When these particles are emitted preferentially in one direction, they produce thrust. This thrust may be augmented by using the escaping particles to ionize and heat additional propellant, followed by an expansion in a magnetic nozzle. A theoretical analysis of such a process (ref. 4) revealed that the plasma ions of our energy range could transfer energy effectively to the propellant, but that electrons of equal energy were relatively ineffective. We are interested, then, in how the total energy output from a fusion reactor is distributed among the various forms, that is, in bremsstrahlung and cyclotron radiation, in escaping electrons and in escaping ions. These quantities have been calculated and are presented in figure 5. The plots in figure 5 are the fraction of fusion energy released to the plasma that appear in various forms of output energy.

The results for the normal case, with zero cyclotron radiation, are presented in figure 5(a-1). Although the ideal case of zero cyclotron radiation is unattainable, it establishes an upper limit on the fraction of the output in the form of charged particle kinetic energy. At best about 84 percent of the output will be in the form of charged particles. The electron and ion temperatures for this case are approximately equal, so that the plasma components carry out energy in proportion to their number densities. The electrons carry about 1.5 times the energy of the ions. Also, as T_D is reduced to about 30 keV, all of the energy appears as bremsstrahlung. At the higher temperatures, the bremsstrahlung fraction is nearly constant. The bremsstrahlung increase with $T_e^{1/2}$ is nearly matched by a corresponding slow increase in the fusion reaction rate $\langle \sigma v \rangle$. The effect of cyclotron radiative loss on the distribution of output power is shown in figures 5(a-2) to 5(a-4) for C_2' between 0.1 and 2.5. The fraction of power output in charged particle kinetic energy varies with ion temperature, with the maximum fraction shifting to lower ion temperatures as C_2' is increased. The maximum assumed cyclotron radiation loss ($C_2' = 2.5$) would result in very poor operation, for only about 11 percent of the energy output is in charged particles. Worse, only about 4.4 percent is in the ions.

We conclude two things from the results thus far. First, large values of cyclotron radiation imply higher confinement times; second, for the normal case of equal electron and ion confinement times the electrons carry away a disproportionate share of the fusion energy. Perhaps, as is common in open-ended systems, some of this electron kinetic energy can be transferred to the ions by means of an ambipolar potential drop at the exit of the reactor. Third, if at least 40 percent of the output energy is to be in the form of charged particles, C_2' must be less than 1.0, and the D-He^3 reactor must operate somewhere between 40 to 70 keV.

In our attempts to increase the fraction of energy carried out by the plasma ions, we considered two extreme variations from the normal conditions. First, it may be possible to get enhanced heating of the plasma ions by the fast ions. Figure 5(b) shows the result of assuming 1000 times the normal energy transfer rates between these species. (Results are substantially the same for any enhancement factor of 1000 or greater.)

Figure 5(b-1) is again our unachievable ideal ($C_2' = 0$). The electron temperatures are lower, thus the bremsstrahlung fraction is now only 11 percent at the best condition. For the same reason, the energy carried by the electrons has dropped. Now the ions carry about 1.25 times as much as the electrons. The effect of increasing C_2' is much less pronounced than in the normal case because of the lower electron temperatures. These results are very attractive, even for the worst case of $C_2' = 2.5$, the charged particles carry about 47 percent of the energy. The minimum $n\tau_D$ is about 9×10^{20} seconds per cubic meter. At present, we know of no physical process which would provide enhancement of this magnitude.

Another scheme for getting a higher fraction of the fusion output into the escaping ions is by reducing the electron loss-rate relative to the ion loss rate. This scheme also is studied in figure 5(c) for the case of $\tau_e/\tau_D = 10$. Again, for the ideal $C_2' = 0$ (fig. 5(c-1)) we see that indeed the distribution of energies is good. Although the fraction of bremsstrahlung has increased to about 22 percent minimum, the plasma ions escape with about 1.4 times the energy of the escaping electrons. Unfortunately, increasing τ_e also causes an increased electron temperature. Hence, the radiation losses are large, causing the fraction of power output in escaping particles to be low at high C_2' values. If values of C_2' on the order of 0.1 are achievable, then there would be some advantage to this scheme, namely, more energy output in the ions.

For completeness, we include the results of the case where the electron throughput is 10 times the ion throughput in figure 5(d). One might hope for some encouraging results because the electron temperatures are suppressed. However, looking at the $C_2' = 0$ case, figure 5(d-1) we see that although bremsstrahlung is slightly lower than the normal case, the fraction of energy carried out by the escaping electrons is about 10 times that carried by the ions. This trend persists as C_2' is increased. Since cycloton radiation losses are still very high as C_2' is increased, there seems to be nothing to recommend increasing the ratio of ion confinement time to electron confinement time.

Reactor Beta Requirements

It was pointed out in a previous section that C_2' must be less than 1.0, for the normal case, while for enhanced fast-ion heating, values of C_2' up to 2.5 might be acceptable. Some insight into the consequences of these requirements on C_2' can be obtained from figure 2. Figure 2 is a plot of the reflectivity for cyclotron radiation Γ against β_0 with C_2' as a parameter. The product lB_0 is taken to be 10.0. For the normal case where C_2' must be less than 1.0, values of Γ must lie above the curve for $C_2' = 1.0$. Hence, at very low values of β_0 near-perfect reflectivity is required. For β_0 greater than 0.1, less stringent values of Γ are required. Values of lB_0 greater than 10.0 would also help. Actual values of Γ for cyclotron radiation are not presently available, but a possible method for determining Γ has been presented in reference 8. Clearly low β_0 operation demands the difficult requirement of obtaining a surface, operated at extremely high material temperatures, with a high reflectivity to cyclotron radiation. On the other hand, if Γ is low, there is the equally difficult requirement of operating a closed reactor at high β_0 (ref. 9).

Energetic Ion Injection

Next, we explore the effects of energetic ion injection on steady-state operation of a D-He^3 reactor. For space propulsion application, it seems doubtful that energetic ion injection would be acceptable. The energy conversion system and other appurtenances required would so increase the system mass as to make it noncompetitive with other advanced propulsion systems. However, the results with injection may be of interest to those considering the D-He^3 cycle in conjunction with direct energy conversion concepts.

The results, with various assumed ion injection energies, are displayed on figure 6, where $n\tau_D$ is plotted against deuterium ion temperature. The electron temperature contours are also shown. These results were obtained assuming zero cyclotron radiation, normal bremsstrahlung, equal electron and ion confinement times and cold electron injection. One of the interesting effects of energetic ion injection is that it significantly expands the range of possible steady-state operation. In fact, for some combinations of electron temperature and ion injection energy there are two different operating points possible: One is at a high value of $n\tau_D$ and low ion temperature; the other occurs at a lower $n\tau_D$ and a higher ion temperature.

Some of the features of this map can be readily understood in physical terms. It is helpful to think of the density n as being held constant so that variations in $n\tau_D$ imply variations in τ_D only. Figure 6 can be divided for discussion into three regions. First consider the upper-middle and upper-right region. At the higher operating temperatures, the curves for ion injection energies from 0 to 90 keV are essentially alike. Fusion remains the dominant energy source, but the addition of energy via injection permits operation with lower confinement times. Near 30 keV the $n\tau_D$ again increases without bound; here the injection power and the charged particle output power become negligible, and a balance is found between the bremsstrahlung and the fusion terms. At the higher injection energies, as T_D decreases the curves turn down instead of up and provide operating conditions at low values of $n\tau_D$ which will be discussed later.

In this upper-right region the ratio of ion confinement time to fast-ion slowing down time is often less than one because of the lower values of $n\tau_D$. Hence the numerical values of $n\tau_D$ in these regions are less reliable, but still are useful for indicating trends. If less fusion energy is absorbed by the plasma than the calculations indicate, the $n\tau_D$ curves would be squeezed together and pushed upward toward the zero injection case. In those cases where the higher injection energy curves turn downward, the fusion power becomes negligible compared to the injection power so that the curves should not be significantly changed by the fact that the slowing down time of the reaction products is very much greater than the ion confinement time.

In the upper-left part of figure 6 is a set of curves heretofore absent, but of little practical significance. For very low ion temperatures and very long confinement times, the injection energy term is low. The fusion term is much lower. Consequently, a small injection energy is balanced by a small bremsstrahlung with electron and ion temperatures equalized at a low value.

In this same region, near 30 keV ion temperature the confinement times are long and injection power is small compared with fusion power. Again, the fusion energy is balanced against bremsstrahlung losses.

The last region to consider is the lower portion of figure 6. Here the throughput of ions and electrons becomes so large that the injection energy and the particle loss terms dominate. It might correspond to an experimental machine with very poor confinement. As the injected ions pass through the machine, they become thermalized (according to our model) and exit with a temperature approaching 2/3 of the injection energy. These possible operating conditions have little practical importance, because the ratios of recirculating power to fusion power become very large. For example, with a 90-keV injection energy, this ratio has already reached 10 at an $n\tau_D$ of about 1×10^{20} seconds per cubic meter. For a 180-keV injection energy, the ratio reached 10 at about $n\tau_D = 2.7\times10^{19}$. These are high values of $n\tau_D$ to expect in an open-ended machine.

Other assumptions in our model start to break down as $n\tau_D$ becomes low. For values of $n\tau_D$ less than 10^{19} or 10^{20} some cases will not permit confinement times long enough to thermalize the plasma ions, or to slow down the fast ions and transfer their energy to the plasma. Although the model could perhaps be modified to account for the resulting non-Maxwellian distributions and partial energy transfer, it seems hardly worth the effort.

CONCLUDING REMARKS

Solutions to the energy balance equations for the D-He^3 fusion plasma have been obtained. These indicate possible operating conditions required for such a fusion reaction. The validity of these solutions for an actual plasma depends on the validity of the many assumptions embodied in the original equations. Among these assumptions are (1) that the electrons, deuterons, and He^3 ions within the plasma are characterized by a

Maxwellian distribution, (2) that the energy interchanges among plasma species as well as the fast-ion heating could be derived from the simplest, binary interaction (3) that the net cyclotron radiation loss may be represented by the approximate expression of reference 5, and (4) that the confinement time of the fast ions produced by fusion will about equal that of the plasma ions, or at least will exceed the time required for their energy to be lost to the plasma.

The results show a minimum value of the containment parameter $n\tau$ of about 7×10^{20} seconds per cubic meter for the normal case with neither injection energy nor cyclotron radiation loss. This is about five times greater than the value for the DT reaction, and occurs at a high value of about 100 keV ion temperature. When cyclotron radiation loss is included, the minimum value of $n\tau$ is greater still and solutions exist only within a band of ion temperatures. The lower limit of this band, about 30 keV, is relatively insensitive to the magnitude of the cyclotron radiation loss. However, if the coefficient C_2' exceeds a value of about 2.5 steady-state operation is no longer possible at any ion temperature.

The minimum required $n\tau$ may be reduced if it is possible to produce enhanced fast-ion heating. The reduction is particularly dramatic when the cyclotron radiation coefficient C_2' is large. Lower values of $n\tau$ may also be obtained by resorting to energetic ion injection. Such solutions are of limited interest-when $n\tau$ is reduced by more than about a factor of 10 the injection power exceeds the fusion power by a like factor.

For space propulsion or for direct conversion applications, the fraction of the total power carried by the escaping charged particles is important. When cyclotron radiation is a factor, this fraction exhibits a maximum at a particular value of ion temperature. The magnitude of this maximum varies from 0.1 to 0.7 for the normal case over the range of cyclotron losses considered. To maximize the particle energy losses, we must select an operating temperature just slightly below that for which $n\tau_D$ is minimum. The optimum operating temperatures range from 65 to 45 keV as C_2 varies from 0.1 to 2.5.

The big question remains that of determining the appropriate value for the cyclotron radiation losses. At the higher operating temperatures the electrons radiate very strongly and over a large number of harmonics. At the same time, the plasma density and the reabsorption rate is reduced if we hold the plasma pressure constant. On the other hand, at these high temperatures the loss of energy by radiation will tend to distort the electron distribution and reduce the number of energetic electrons which radiate. If, for example, we require that 40 percent of the fusion power be carried by the escaping charged particles, the cyclotron radiation coefficient C_2' has to be 1.0 or less. If we could obtain enhanced ion heating by a factor of 1000, we would meet this goal with C_2' as great as 2.5.

Unless a suitable cyclotron-radiation reflector is developed ($\Gamma > 0.9$), the D-He^3 reactor will have to operate with β_0 in excess of 0.1.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, January 22, 1971,
129-02.

APPENDIX - SYMBOLS

B	magnetic field
C_2	cyclotron radiation loss coefficient
C_2'	C_2/D
c	velocity of light
D	$\sqrt{l B \beta^3}$
$\dot{E}_B$	bremsstrahlung power loss per unit volume
$\dot{E}_C$	cyclotron radiation power loss per unit volume
E_{Is}	injection energy of s plasma component
E_{ks}	energy transferred from fast ion k to plasma component s
E_{k0}	initial energy of k fusion reaction product
E_k	energy of k fusion reaction product at time t
$\left(E_{kcr}\right)_s$	energy of fast ion k at which the mean energy exchange rate with the s plasma component is zero
e	electronic charge
f_B	fractional fuel burnup
h_s	fast-ion heating enhancement coefficient
J	ratio of electron to ion confinement times
$K_{\mathscr{L}}$	plasma transparency coefficient for cyclotron radiation
k	Boltzmann's constant
L	Coulomb logarithm
L_s	Coulomb logarithm for s plasma component
$\mathscr{L}$	cyclotron radiation parameter (see eq. (15))
l	plasma size for an equivalent slab
m_s	mass of s plasma component
m_k	mass of k fusion reaction product
n_s	number density of s plasma component
n	total ion density

$Q_{a \to b}$ energy transfer rate per unit volume from plasma component a to plasma component b

R_k mean energy transfer rate per unit volume from fast ion k to all plasma components

r_{ks} mean energy transfer rate per unit volume from fast ion k to plasma component s

S_s particle injection rate per unit volume of s plasma component

T_s temperature of s plasma component

t time

v_k velocity of k fast ion at energy E_k

X_s dimensionless parameter $\left(\frac{m_s}{m_k} \frac{E_k}{T_s}\right)^{1/2}$

Z_s charge number for s plasma component

Z_k charge number for k fast ion

β ratio of plasma pressure to magnetic pressure

Γ surface reflectivity for cyclotron radiation

γ_s mass ratio m_s/m_k

ϵ_o permittivity of free space numerically equal to 8.854×10^{-12} $coul^2/n\text{-}m^2$

θ normalized plasma ion temperature (see eq. (4))

τ mean particle confinement time

Subscripts:

D deuterium ion

e electron

H helium-3 ion

I injection

k fusion reaction fast ion

p proton

s plasma components

α helium-4 ion

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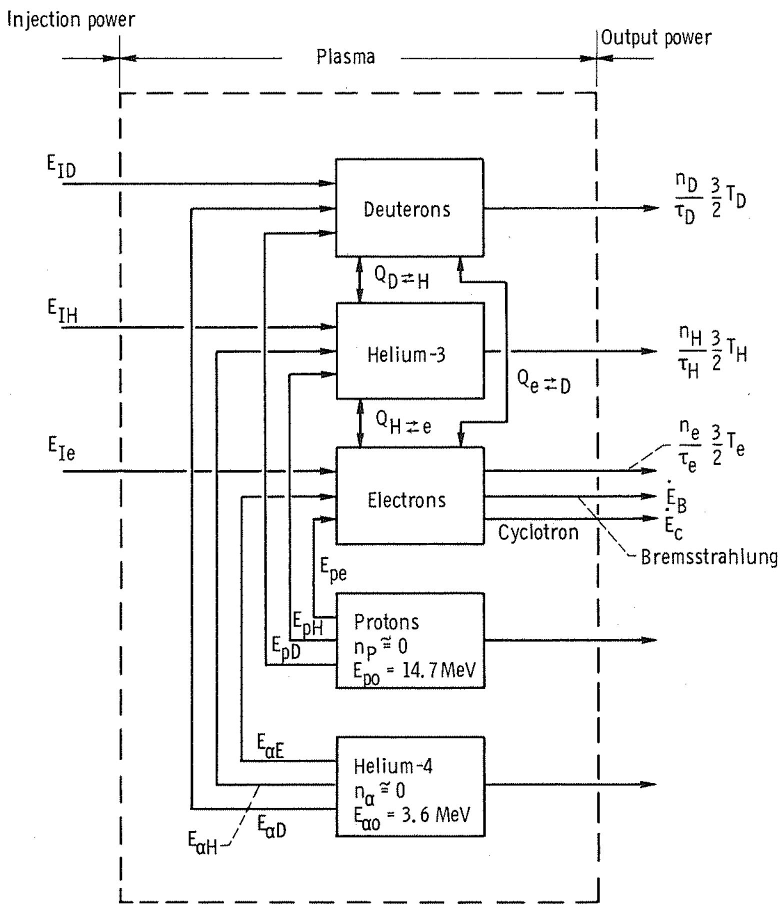


Figure 1. - Power flow diagram.

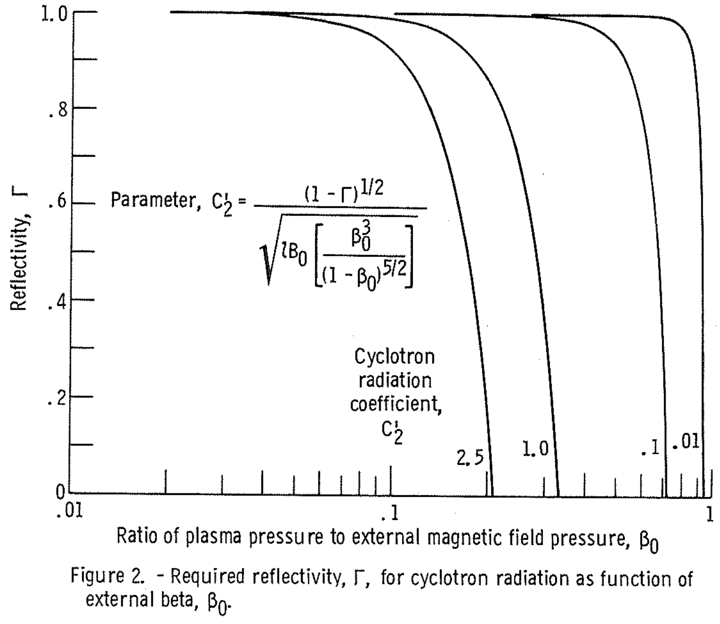
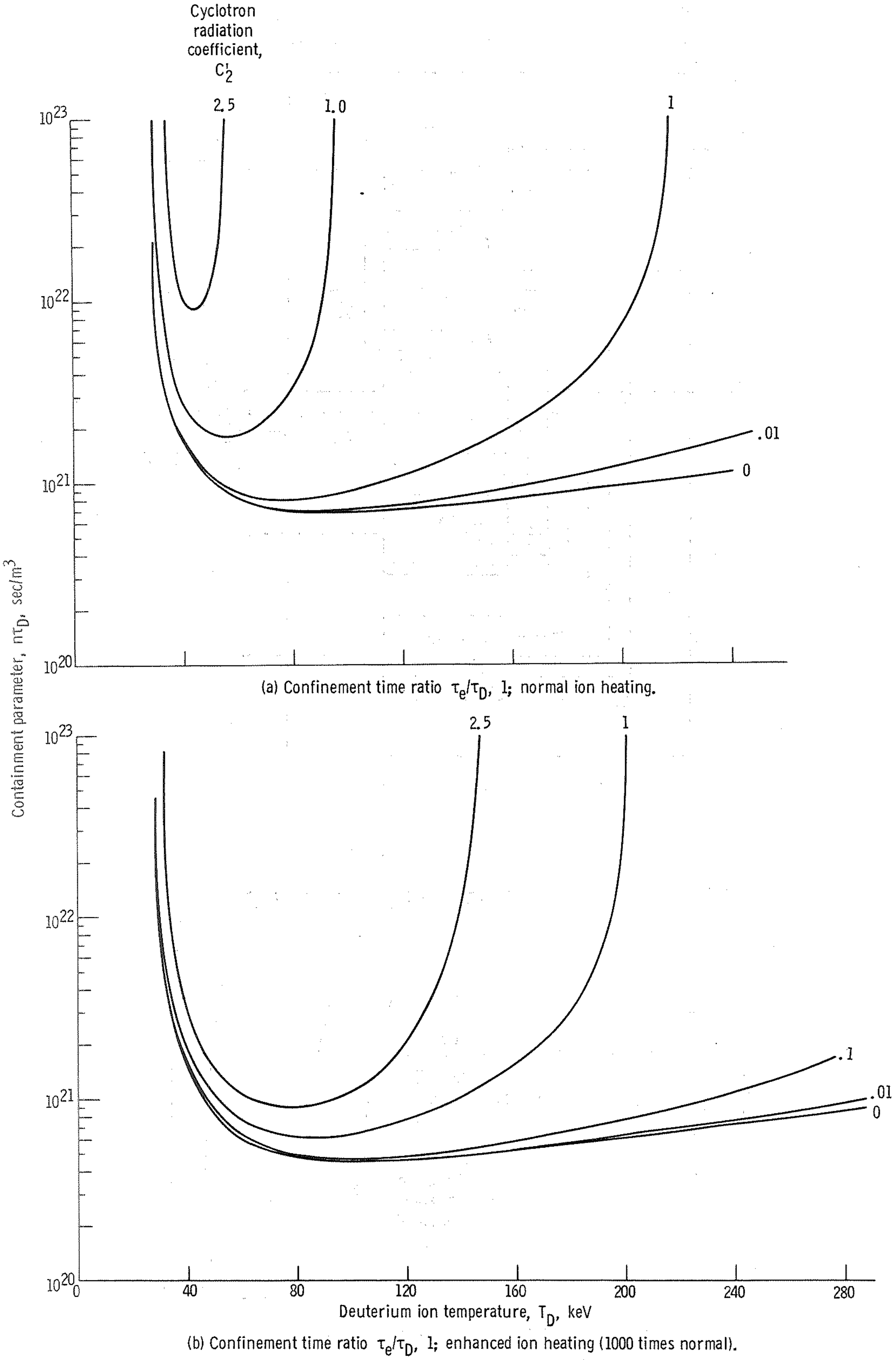
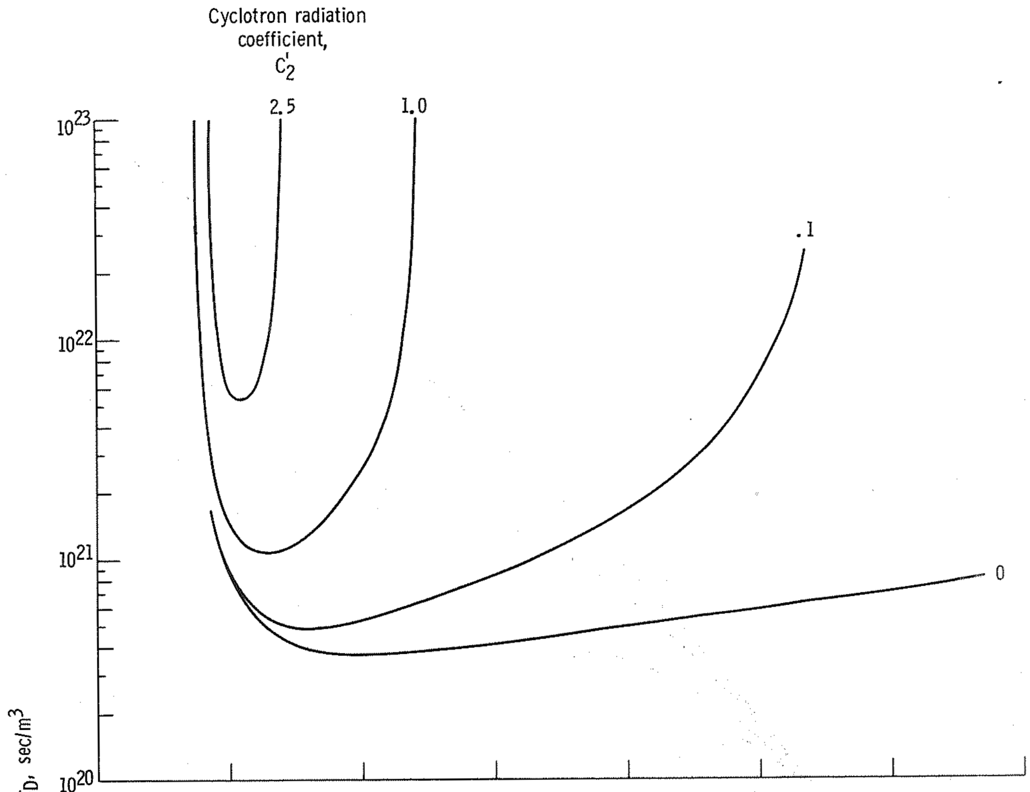


Figure 2. - Required reflectivity, Γ, for cyclotron radiation as function of external beta, β_0.

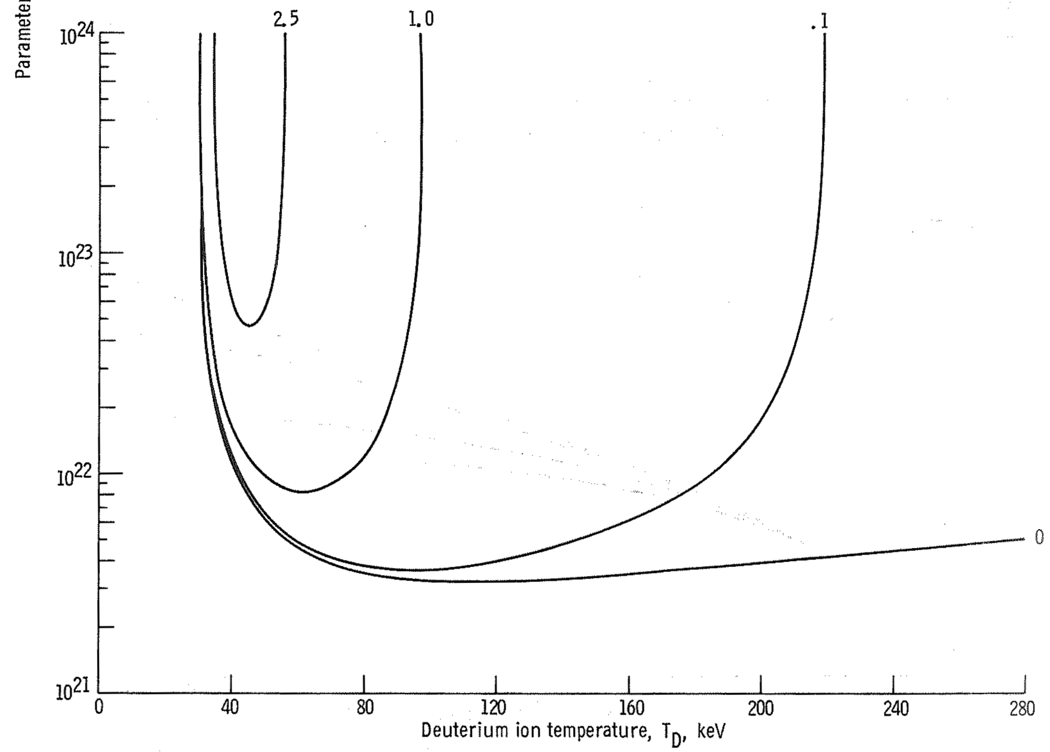


(b) Confinement time ratio τ_e/τ_D, 1; enhanced ion heating (1000 times normal).

Figure 3. - Containment parameter $n\tau$ for a steady-state deuterium - helium-3 reactor.

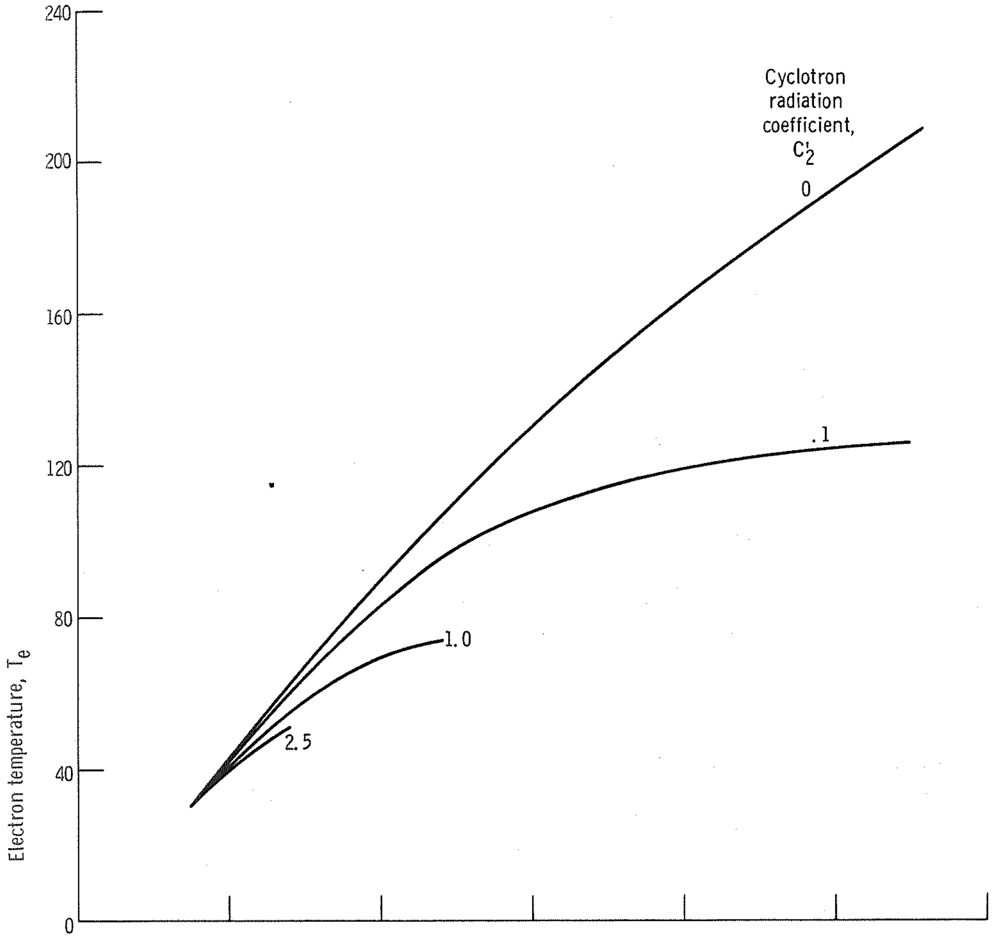


(c) Confinement time ratio τ_e/τ_D, 10; normal ion heating.

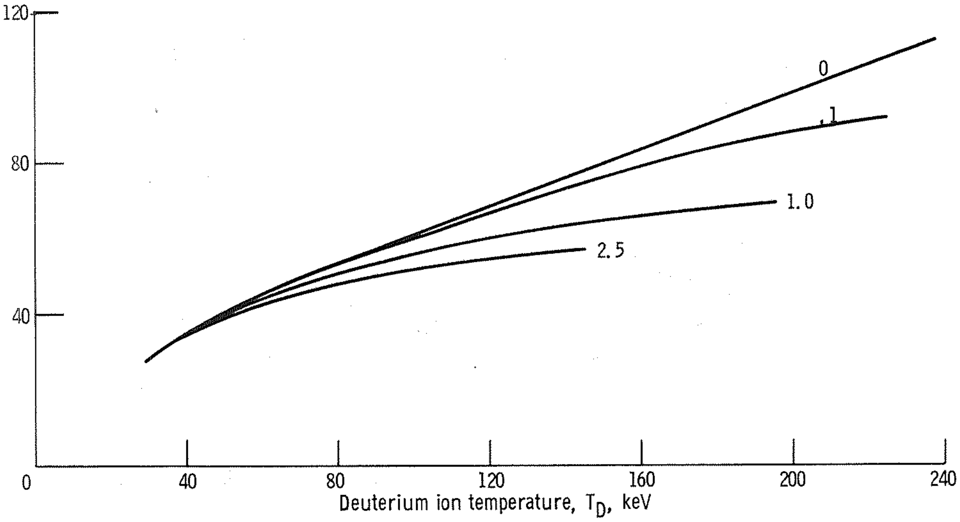


(d) Confinement time ratio τ_e/τ_D, 0.1; normal ion heatings.

Figure 3. - Concluded.

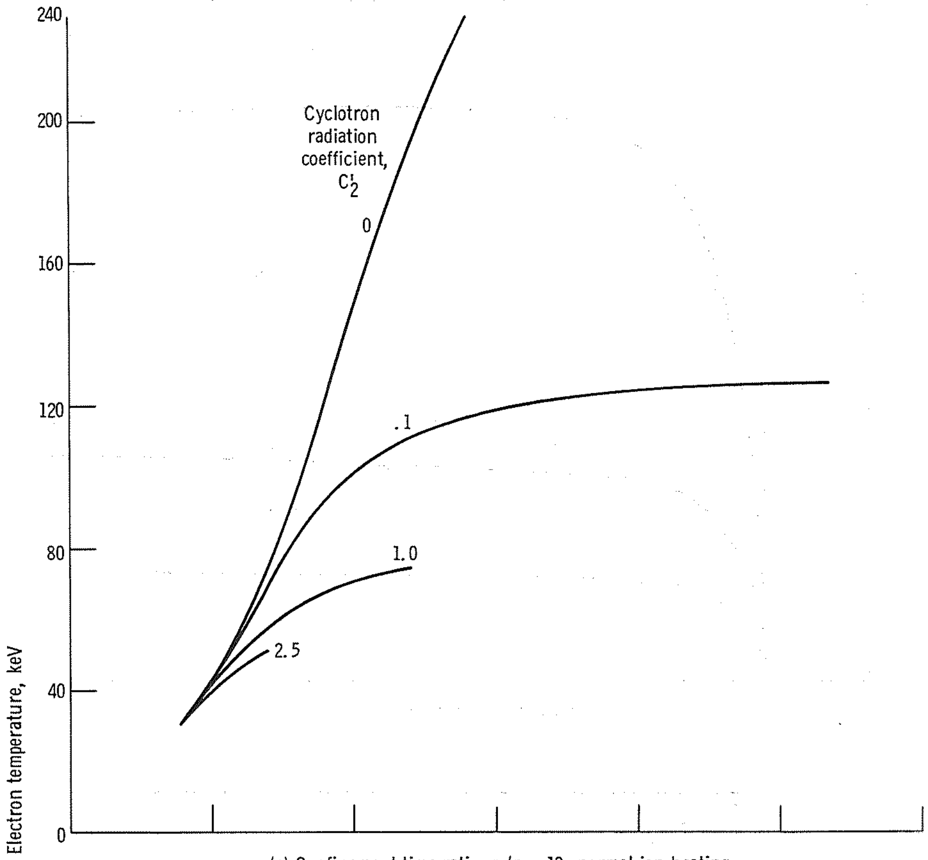


(a) Confinement time ratio τ_e/τ_D, 1; normal ion heating.

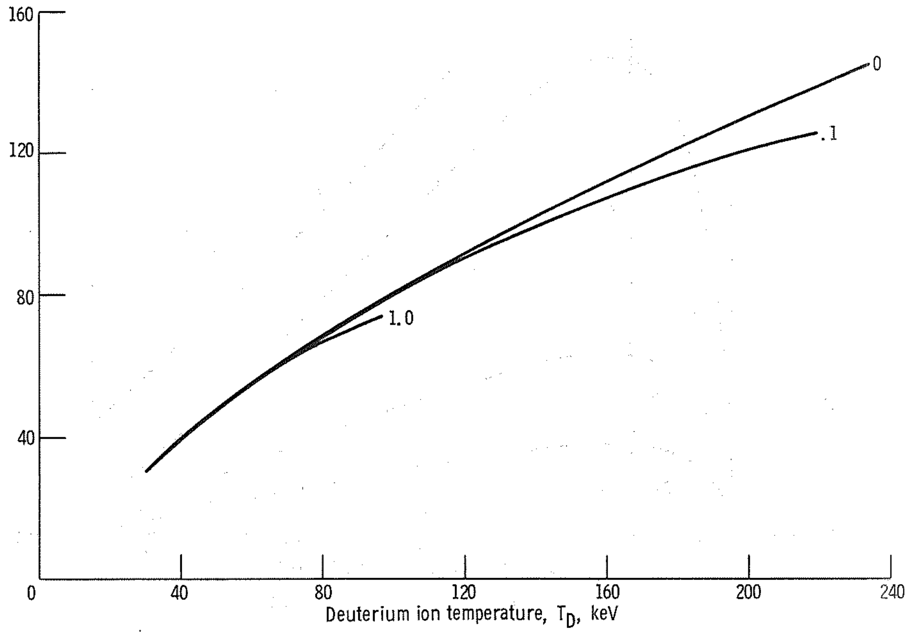


(b) Confinement time ratio τ_e/τ_D, 1; enhanced ion heating (1000 times normal).

Figure 4. - Electron and ion temperatures for a steady-state deuterium - helium-3 reactor.

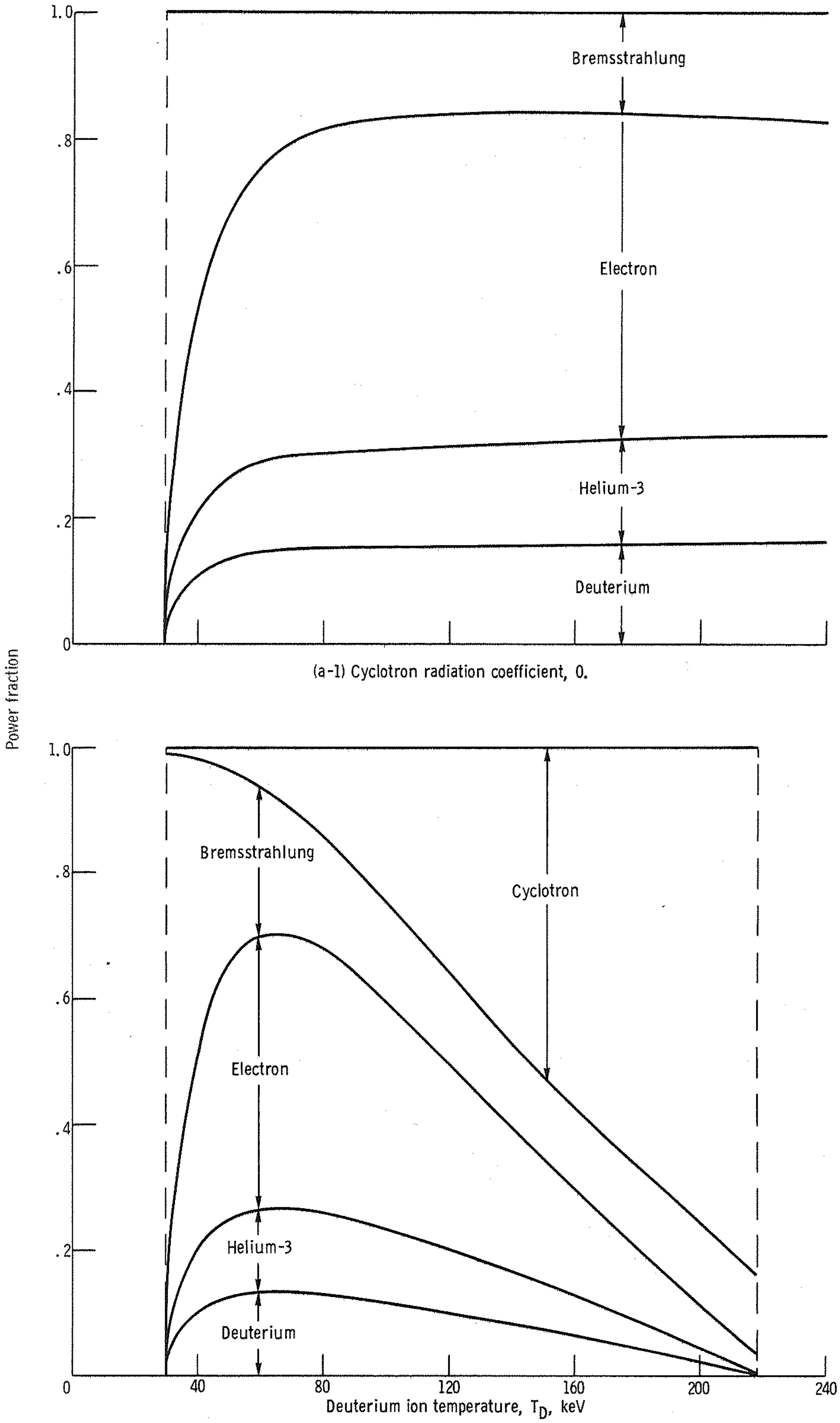


(c) Confinement time ratio τ_e/τ_D, 10; normal ion heating.



(d) Confinement time ratio τ_e/τ_D, 0.1; normal ion heating.

Figure 4. - Concluded.

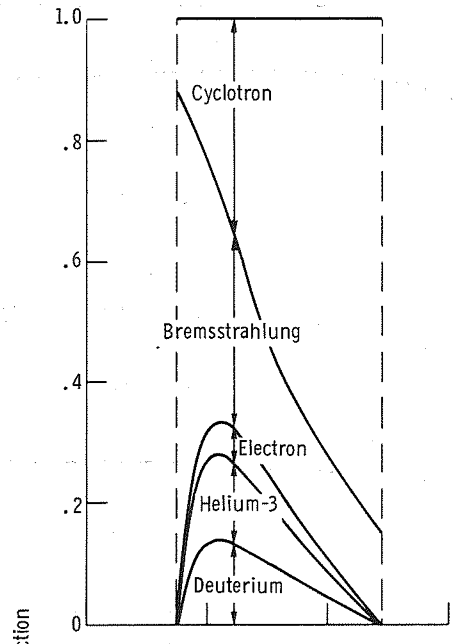


(a-1) Cyclotron radiation coefficient, 0.

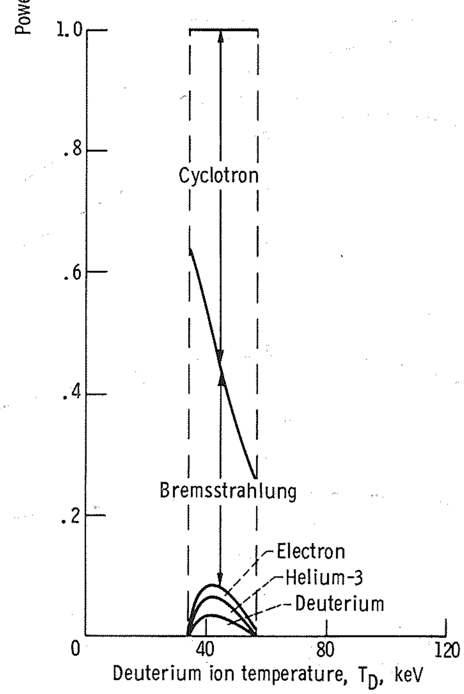
(a-2) Cyclotron radiation coefficient, 0.1.

(a) Confinement time ratio τ_e/τ_D, 1; normal ion heating.

Figure 5. - Distribution of output power from a steady-state deuterium - helium-3 reactor.



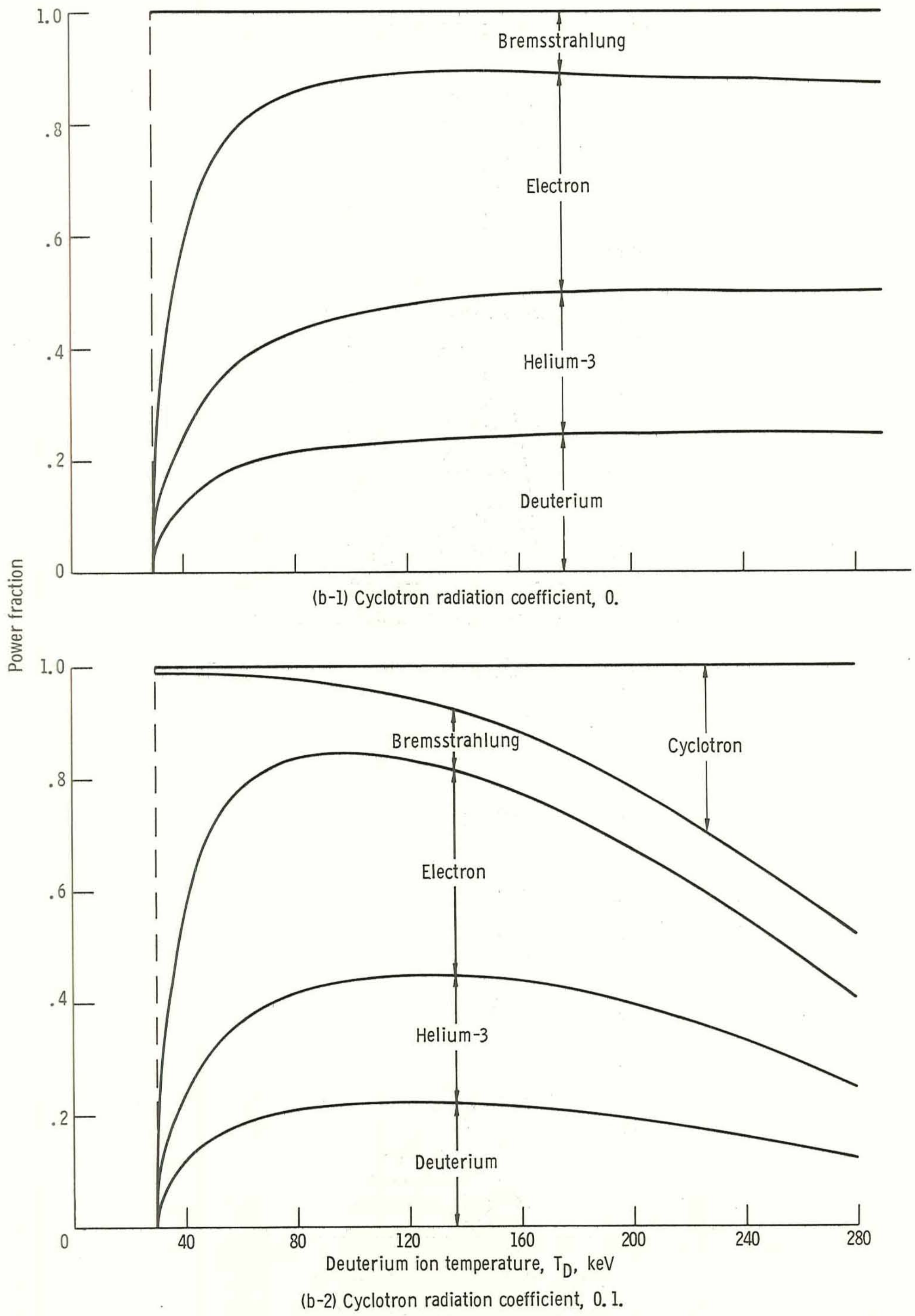
(a-3) Cyclotron radiation coefficient, 1.0.



(a-4) Cyclotron radiation coefficient, 2.5.

(a) Concluded.

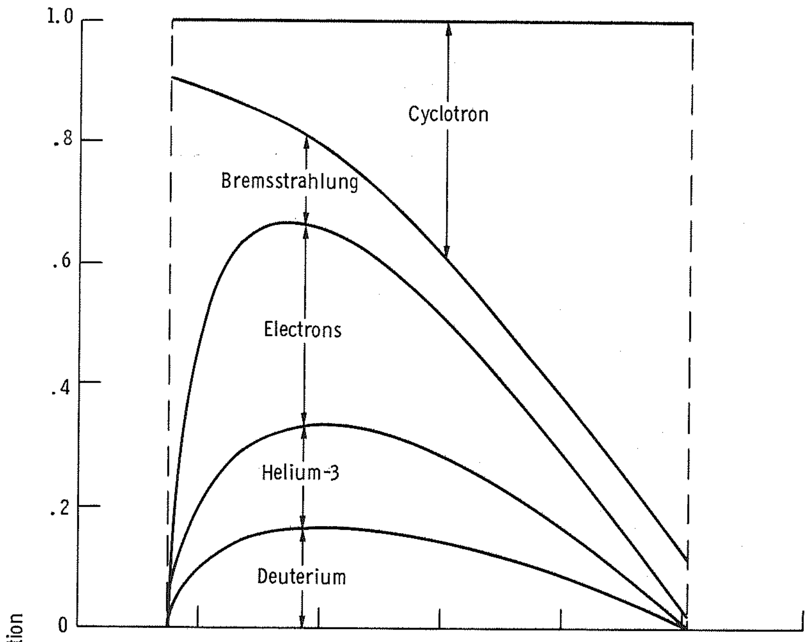
Figure 5. - Continued.



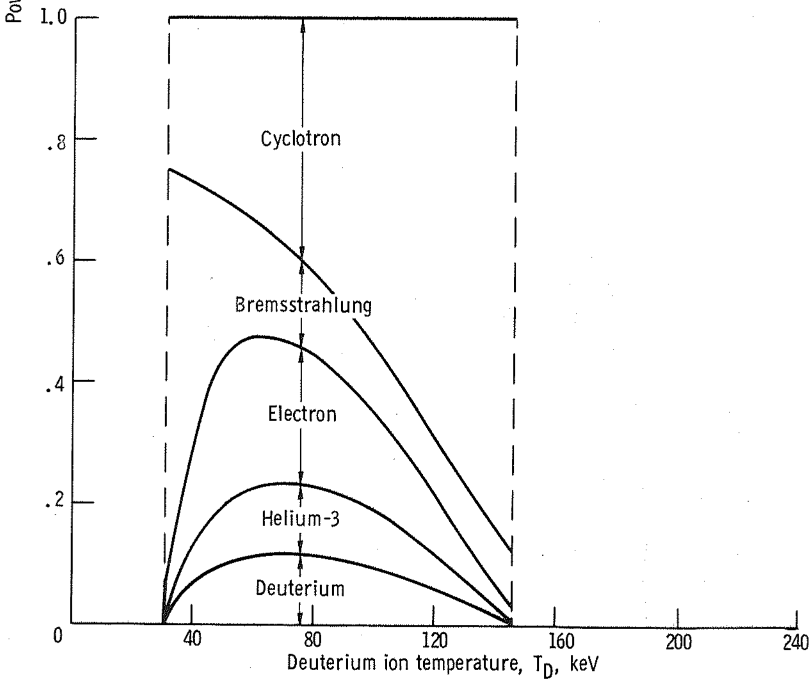
(b-2) Cyclotron radiation coefficient, 0.1.

(b) Confinement time ratio τ_e/τ_D, 1; enhanced ion heating (1000 times normal).

Figure 5. - Continued.



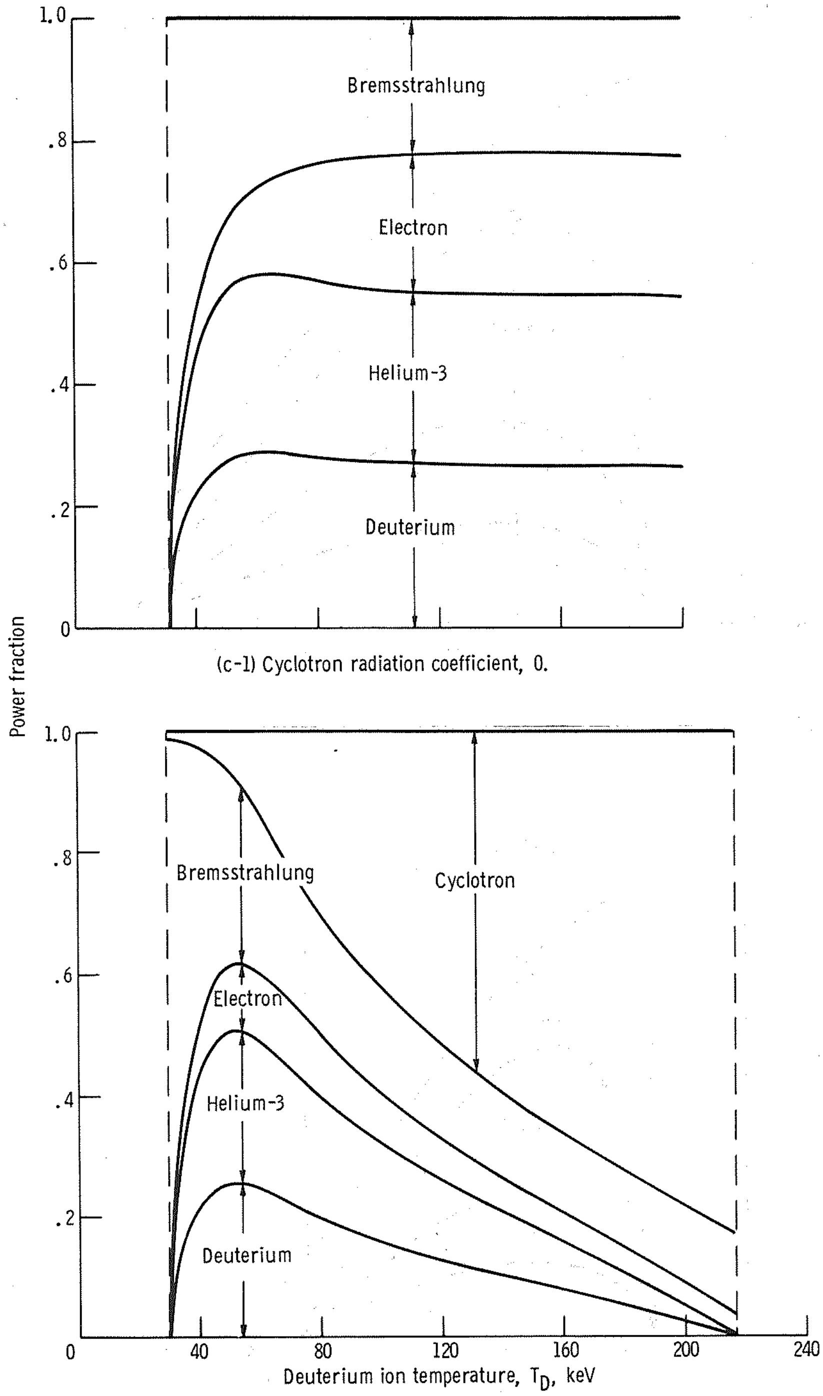
(b-3) Cyclotron radiation coefficient, 1.0.



(b-4) Cyclotron radiation coefficient, 2.5.

(b) Concluded.

Figure 5. - Continued.

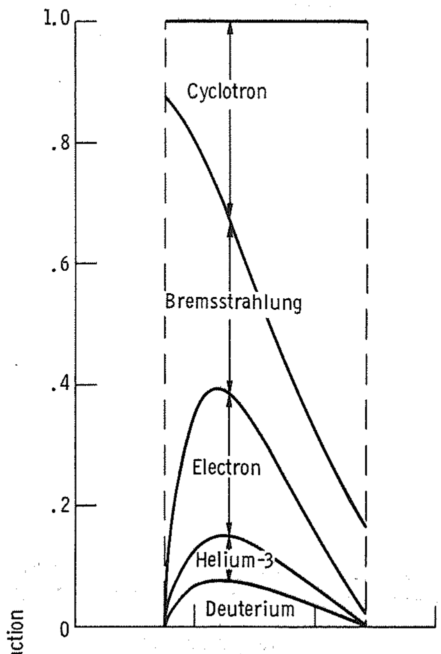


(c-1) Cyclotron radiation coefficient, 0.

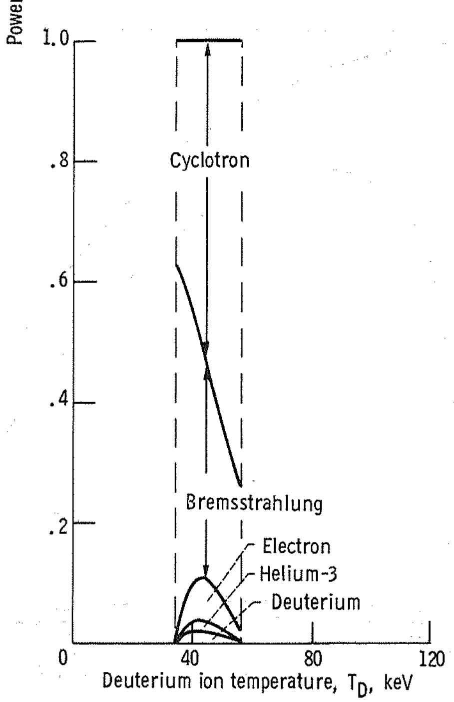
(c-2) Cyclotron radiation coefficient, 0.1.

(c) Confinement time ratio τ_e/τ_D, 10; normal ion heating.

Figure 5. - Continued.



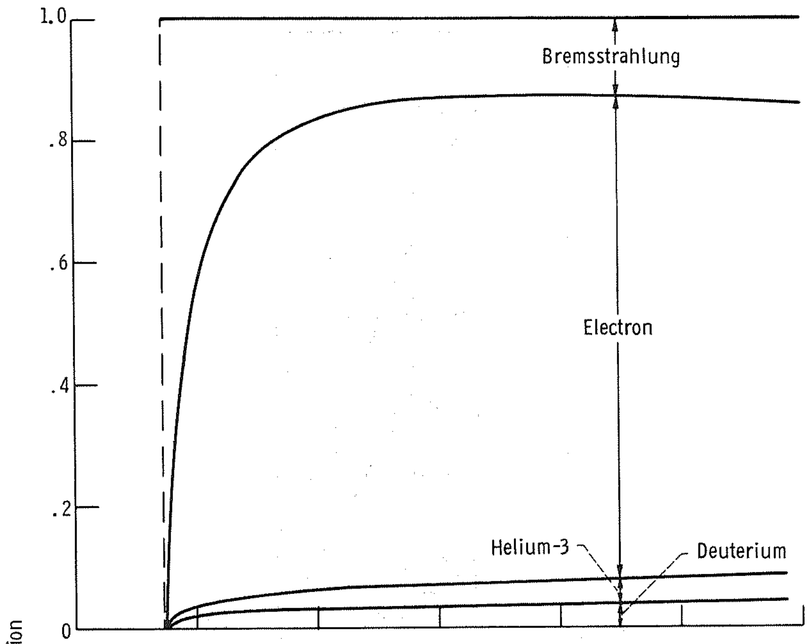
(c-3) Cyclotron radiation coefficient, 1.0.



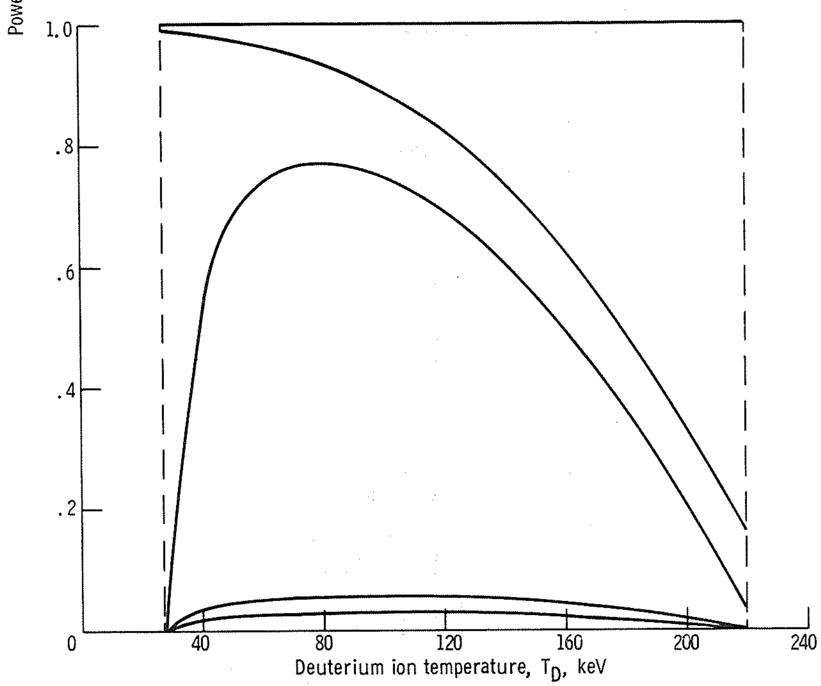
(c-4) Cyclotron radiation coefficient, 2.5.

(c) Concluded.

Figure 5. - Continued.



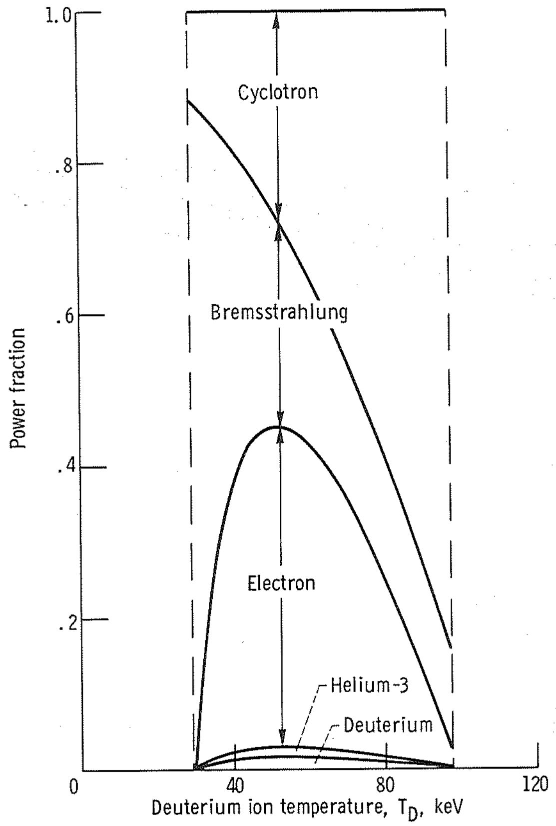
(d-1) Cyclotron radiation coefficient, 0.



(d-2) Cyclotron radiation coefficient, 0.1.

(d) Confinement time ratio τ_e/τ_D, 0.1; normal ion heating.

Figure 5. - Continued.



(d-3) Cyclotron radiation coefficient, 1.0.

(d) Concluded.

Figure 5. - Concluded.

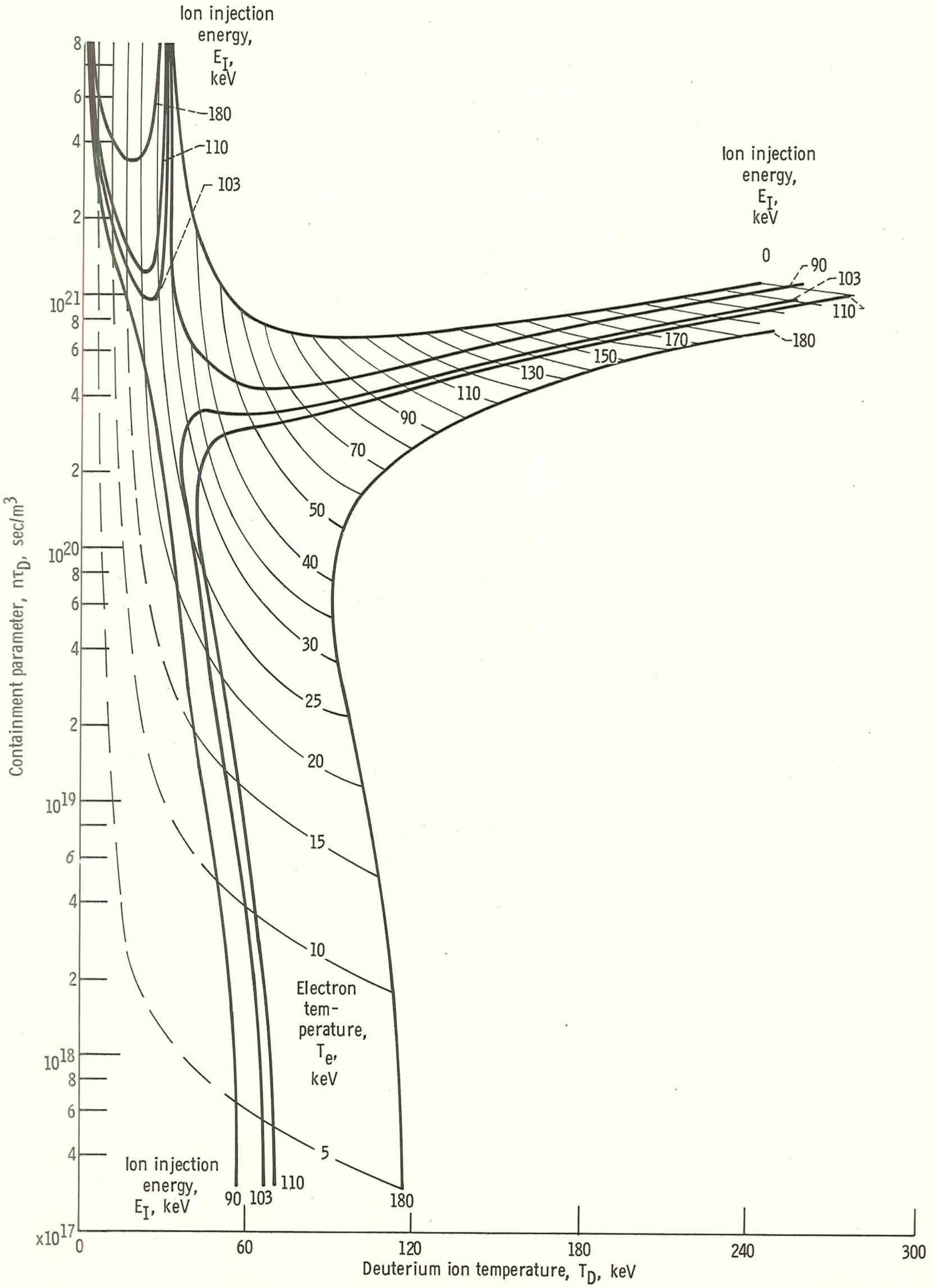


Figure 6. - Containment parameter $n\tau$ for a steady-state deuterium - helium-3 reactor. Zero cyclotron radiation.

NASA-Langley, 1971 —— 25 E-6104

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