BEARING CAPACITY OF LUNAR SURFACE MATERIALS

By GLEN L. MARTIN[1], Bozeman, Mont.

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Abstract.—Slope stability analysis, applied to failed slopes in lunar craters, suggests that the static mass-bearing capacity of the lunar surface may range from 860 to as much as 9,400 g/cm^2. The analysis includes seismic acceleration as one of the variables. Such accelerations may have a pronounced effect on the stability of lunar slopes.

Several methods have been proposed for indirectly evaluating the bearing capacity of lunar surface materials. One, proposed by Jaffe (1965), applies slope stability analyses to lunar crater walls to estimate the strength parameters of the soil-like material. The resulting strength parameters are then used to evaluate bearing capacity.

Jaffe, working with Ranger 7 photographs, determined a "lower bound for the mass bearing capacity" of lunar surface materials. Although he assumed that the crater slopes selected for analysis had a factor of safety of unity, there was no photographic evidence of slope failure. Jaffe reported that the analyzed slopes had a "soft appearance suggesting substantial dust cover." Such slopes may have reached their configuration by surface erosion rather than failing in accord with the classical "slip circle" assumptions.

In addition, Jaffe assumed that gravity is the only force responsible for the failure of lunar slopes. It is generally accepted that seismic and seepage forces contribute to the failure of terrestrial slopes, and there is no reason to believe that seismic forces are not involved in forming the lunar landscape. Jaffe also did not allow for a "factor of safety" in determining the probable lunar bearing capacity.

[1] Associate Professor of Civil Engineering and Engineering Mechanics, Montana State University, Bozeman, Mont., and Research Civil Engineer, U.S. Geological Survey, Menlo Park, Calif. This work was done under National Aeronautics and Space Administration contract R 66.

The purpose of this paper is to furnish insight into the complexity of indirectly determining lunar bearing capacity by slope-stability techniques, and, also to present what seems to be a rational approach to the problem.

METHODS OF SLOPE STABILITY ANALYSIS

There are many methods of slope stability analysis for determining the factor of safety of a slope against failure. The four more commonly used are: (1) the wedge method, (2) the classical method of slices, (3) Bishop's adaptation to the method of slices, and (4) the finite-element method. In their present form of development, all four assume a linear slope (no curvature normal to the profile of the slope).

The wedge method is valid where a low-strength plane that can be defined controls the failure geometry of the slope. The presence of such a plane in lunar materials has not been established; thus, this method is not applicable.

The method of slices has proven to be an acceptable method of analysis and is presently in a better stage of development than the other three methods mentioned above. It was selected for use in this study.

Bishop's adaptation to the method of slices (Bishop, 1955), which uses effective-stress techniques, yields results which better define the failure state in some terrestrial materials, predominantly those that are cohesive (Schuster, 1967). This more complex method of solution requires much more time and effort, and does not seem justified for lunar near-surface materials because they probably have relatively small values of unit cohesion.

The finite-element method does not yet lend itself to the determination of strength characteristics, by trial-and-error methods, that can in turn be used in bearing-capacity determinations.

All the aforementioned methods assume a slope with linear contours. However, the contours of slopes of lunar craters tend to be concentric with the rim, and a method for rationally considering the effects of such concave surfaces has not yet been devised. The assumption of a linear slope, when the slope is in reality concave, should result in higher values for the strength parameters than really exist. This results from the fact that the slope concavity itself is a source of "strength."

DEVELOPMENT OF STABILITY CHARTS

Because all the variables that must be known to make unique analyses of the slope stability of lunar near-surface materials are not known, stability charts have been formulated which make multiple analyses a relatively simple task.

The variables involved in a slope stability analysis employing the method of slices are:

1. Surface geometry of the slope.
2. Subsurface geometry of the materials making up the slope.
3. Unit weight of the materials making up the slope.
4. Pore pressures in the materials (pore-liquid or pore-gas pressures).
5. Strength characteristics of the materials (usually expressed in terms of the angle of internal friction and the unit cohesion).
6. Induced seismic forces on the slope.
7. Factor of safety of the slope (defined by Bishop, 1955, p. 7).

A variety of stability charts for the rapid solution of slope stability problems for slopes of isotropic materials have been published. None of these charts, however, incorporate seismic acceleration as a variable. In addition, most charts do not incorporate angles of internal friction greater than 20° or 30°.

Stability charts, with seismic acceleration as a variable, were formulated as follows:

1. A slope geometry, defined by straight lines with the upper boundary horizontal (fig. 1), was assumed. The material making up the slope was assumed to be isotropic, pore pressures were assumed to be nonexistent in the slope, and the circular failure arc was assumed to go through the toe of the slope (because the opposite wall of the crater would prohibit deep-seated failure). The angle that

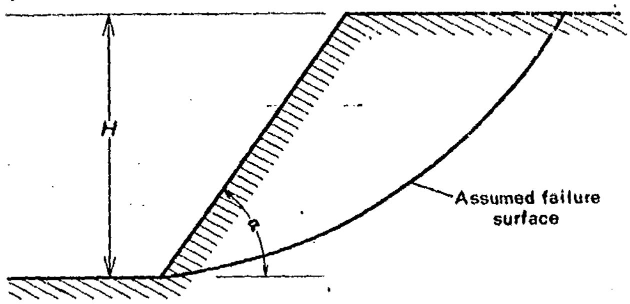


FIGURE 1.—Assumed slope geometry of a hypothetical lunar crater. Symbols are explained in text.

the slope makes with the horizontal was defined as α, and the height of the slope was defined as H.

2. A method-of-slices program on a digital computer was used to determine the minimum factor of safety of the slope by systematically varying

(a) α, the slope angle,
(b) ϕ, the angle of internal friction of the soil material,
(c) A, the horizontal seismic acceleration in terms of the acceleration of gravity (g), and
(d) N_m, stability number, equal to $c/\rho g H$, where
c = cohesion of the soil (dynes/cm^2),
ρ = soil mass density (g/cm^3),
g = acceleration due to gravity (cm/sec^2), and
H = critical height (cm) of an inclined slope that can be maintained by the soil at slope angle α, with horizontal acceleration A.

Typical results of the stability analyses described above are shown in figure 2. The example is for α=40° and ϕ=35°, with each of the curves representing a different value of horizontal acceleration.

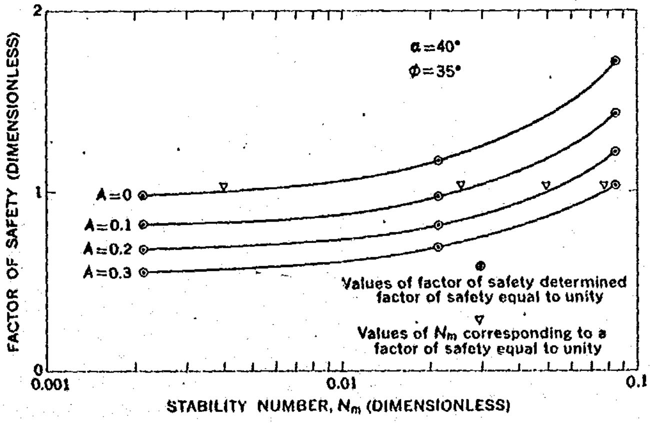


FIGURE 2.—Results of multiple stability analyses with varying parameters for the assumed slope geometry in figure 1. Symbols are explained in text.

3. For each slope angle, α, relationships similar to those shown (fig. 2) were determined for each of several angles of internal friction.

4. The stability number, N_m, corresponding to a factor of safety equal to unity was determined for each of the selected acceleration values. (For a factor of safety equal to unity, the stresses tending to cause slope failure are exactly offset by the strength of the slope.)

5. From the results of 4 above, the relationship between the stability number and seismic acceleration was determined (see fig. 3, which is valid for a slope angle of 40°).

The resulting stability chart (fig. 3) is valid for a single value of slope angle, α, and for a factor of safety equal to unity. The previously stated assumptions with regard to slope geometry, material, pore pressures, and failure surface, are, of course, inherent in the stability chart.

APPLICATION OF STABILITY CHARTS TO THE LUNAR SURFACE

Craters showing evidence of slope failure either by apparent slumping of the crater walls or by apparent concentric tension cracks near the crater rims were observed on Lunar Orbiter II photographs (see fig. 4).

Because the surface bearing capacity is desired, strength characteristics of materials at the surface and at very shallow depths must be approximated. For this reason, small-diameter craters were selected for analysis because the failure of their slopes is limited by their depths. Thus, the analysis applies to near-surface materials.

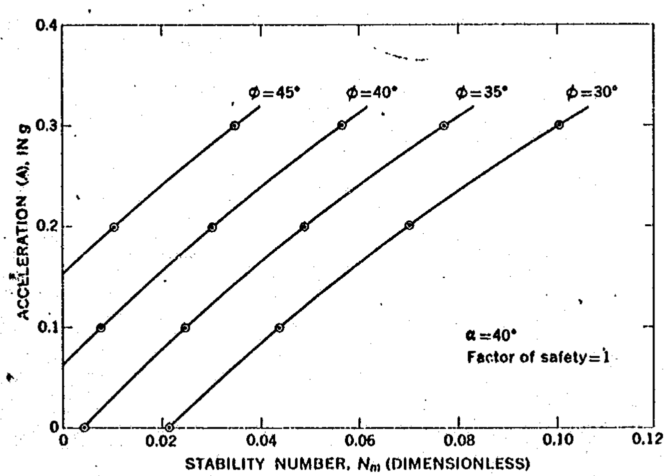


FIGURE 3.—Stability chart for assumed slope geometry (fig. 1) with a slope angle of 40° plotted for four angles of internal friction. Symbols explained in text.

Had craters without the evidence of failure described above been selected, there would be no assurance that the factor of safety would not be considerably greater than unity. Analysis of slopes in such craters could give a very misleading and conservative value for the strength of the soil material.

Very large diameter deep craters would, of course, include deeper, probably more competent materials, and the strength values determined by slope stability techniques would be higher than would actually exist in the near-surface materials.

The limitation imposed by the Lunar Orbiter photographs was that of detection resolution. Had greater detection resolution been available, shallower craters could have been included in the analyses, and the "average" strength characteristics determined would then be more indicative of the near-surface material.

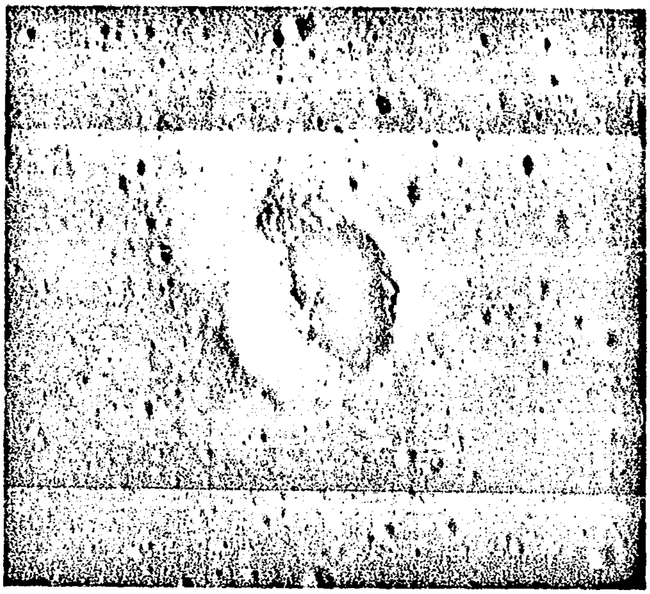
Profiles of selected craters (figs. 4, 5), obtained by means of photoclinometric techniques developed by Glen R. Taylor and J. J. Lambiotte, of Langley Research Center, and by Kenneth Watson (1968), were used to determine slope geometry, the representative slope angle before failure, α, and the slope height, H, for each crater. The results are shown in table 1.

Strength characteristics were then determined as follows: (1) The appropriate chart, selected on the basis of slope geometry, was consulted. (2) The corresponding stability number, N_m, was determined from the chart, with the assumption of a rational value for the angle of internal friction, ϕ, and an assumed seismic acceleration, A.

The corresponding value of unit cohesion was determined by multiplying the stability number, N_m, by the mass density, ρ, the acceleration of gravity, g, and the height of the slope, H. The unit cohesion, c, thus determined would be the minimum required to maintain the stability of the slope under the imposed conditions.

Unit cohesions obtained using the above procedure, with various assumptions for both angle of internal friction, ϕ, and seismic acceleration, A, for selected slopes identified in table 1, are shown in table 2. The density ρ, of the soil-like material is assumed to be 1.5 g/cm^3. Craters with the flatter slopes are excluded because stability charts have not yet been prepared for slopes of less than 25°.

The obverse method of using the example stability chart is to assume a unit cohesion, angle of internal friction, and density. Then, for a slope of a given height and slope angle, the acceleration necessary to cause failure may be found. For example, a slope of 40° (fig. 3) underlain by cohesionless material with an angle of internal friction of 40° would fail if subjected to a horizontal seismic acceleration of 0.06 *g*.



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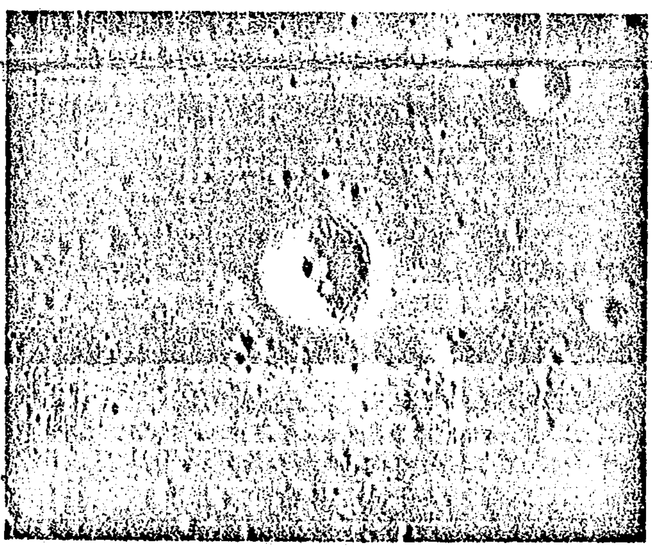

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FIGURE 4.—Lunar Orbiter photographs showing examples of slope failure. Profiles for these craters are shown in figure 5. *A*, apparent slumping of crater wall (Lunar Orbiter Site II P-2, photograph H-36). *B*, apparent concentric tension crack near crater rim (Lunar Orbiter Site II P-2, photograph H-39).

VALUES OF UNIT COHESION

The value of unit cohesion that is obtained depends greatly on the value of horizontal acceleration that is assumed. A rational value for the horizontal acceleration is not known, but lunar horizon-

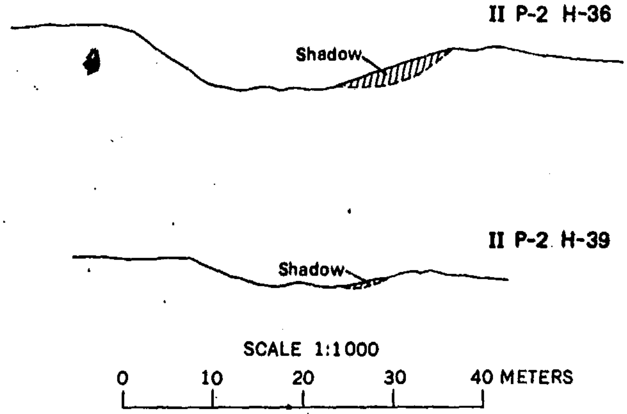

FIGURE 5.—Profiles of craters in figure 4. No vertical exaggeration. Numbers and letters refer to site, photograph, and framelet designations in table 1.

TABLE 1.—*Identification, slope angle, and slope height of selected lunar craters*

Identification			Slope angle, α (degrees)	Slope height, H (meters)
Site	Photograph	Framelet		
II P-2	H-36	142	35	12.0
II P-2	H-37	223	25	11.5
II P-2	H-38	425	25	14.5
II P-2	H-39	483	20	6.5
II P-2	H-42	913	30	23.5
II P-6B	H-84	403	40	14.3
II P-6B	H-87	792	20	5.0
II P-6B	H-87	796	20	8.0
II P-11	H-164	884	30	7.0
II P-11	H-164	924	45	15.0

TABLE 2.—*Probable values of unit cohesion for lunar surface materials*

[Density, ρ, assumed to be 1.5 g/cm³]

Framelet	Assumed angle of internal friction, Φ (degrees)	Assumed seismic acceleration, A (cm/sec²)	Stability number, N_M	Unit cohesion, c, required for stability (dynes/cm²)
425	35	0.5	0.1030	36,600
	40	.5	.0710	25,200
884	30	.5	.1605	27,600
	35	.5	.1340	23,000
924	25	0	.0470	17,300
	25	.25	.1170	43,000
223	35	.5	.1030	29,000
	40	.5	.0710	20,000
142	30	0	.0073	2,150
	30	.25	.0705	20,800
	35	.25	.0440	13,000
913	30	.5	.1605	92,500
	35	.5	.1340	77,200
403	30	0	.0214	7,500
	30	.2	.0715	25,100
	35	0	.0041	1,440
	35	.2	.0490	17,200

tal accelerations would probably be as great, if not greater, than those experienced on the earth (up to 0.3 g).

Upper and lower bounds for unit cohesion would probably be about 92,000 and 1,000 dynes/cm², respectively (table 2). These extremes, along with the listed angles of internal friction, compare favorably with those reported from Surveyor experiments (Scott and others, 1967, p. 61–93; Scott and Roberson, 1968b, p. 121–161, 1968a, p. 171–79; Choate and others, 1968).

DETERMINING EXTREME VALUES OF BEARING CAPACITY

Although many other rational bearing-capacity equations are now in use, the Terzaghi equation, for general shear conditions, best suits the purpose of this study (Scott and O'Keefe, 1967). The use of other equations would give slightly smaller or greater values for bearing capacity, but in consideration of all the previous assumptions it seems doubtful that the selection of another equation would be justified for purposes of accuracy alone. The Terzaghi equation for the ultimate bearing capacity, $q_{ultimate}$, of a surface footing, circular in plan view, is

$$q_{ultimate} = 1.3cN_c + 0.3b\rho gN_\alpha, \tag{1}$$

where c = unit cohesion of the foundation material (dynes/cm²),

b = diameter of the circular footing (cm),

ρ = mass density of the foundation material (g/cm³), and

N_c and N_α are dimensionless numbers which are functions of the angle of internal friction of the foundation material.

The ultimate bearing capacity must be divided by an appropriate factor of safety. The appropriate factor of safety is dependent on the assurance of uniform foundation materials and the confidence that the engineer can place in the data used in the analysis. The factor of safety for determining the bearing capacity of terrestrial materials commonly ranges from 1.5 to 3. A factor of safety of 3 seems appropriate in considering lunar materials. The allowable bearing capacity, $q_{allowable}$, would then be $q_{ultimate}/3$, or

$$q_{allowable} = 0.433cN_c + 0.1\,b\rho gN\alpha. \tag{2}$$

The allowable bearing capacity may be expressed as a mass-bearing capacity by dividing each term by the acceleration of gravity, g, or

$$q_{mass\text{-}allowable} = 0.433\frac{cN_c}{g} + 0.1\,b\rho N_\alpha. \tag{3}$$

For the calculation of static mass-bearing capacity on the lunar surface for a structure founded on a circular footing 1 meter in diameter, the allowable mass-bearing capacity would then range from 860 g/cm² (ϕ=35°, c=1,440 dynes/cm², table 2) to 9,409 g/cm² (ϕ=30°, c=92,500 dynes/cm², table 2) or 1.4 to 15.3 newtons/cm². These values are comparable to those predicted on the basis of Surveyor data (Scott and Roberson, 1968a, p. 171–179; Choate and others, 1968, p. 129–169).

CONCLUSIONS

From the available information, the static mass-bearing capacity on the lunar surface should be at least 860 g/cm². The upper limit could conceivably be 9,409 g/cm², but is probably less.

Because seismic activity may be influential in forming the lunar landscape, its effects should be ignored when using slope-stability techniques to evaluate lunar bearing capacity.

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