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Final Technical Report on Grant NGR-19-012-001

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The purpose of this work was to develop a model for the description of nucleon-nucleon and pion-nucleon collisions at high energy and to incorporate this model into a complete Monte-Carlo description of a Nucleor Cascade.

In the initial phase of the work an attempt was made to use the statical model originally proposed by Fermi[1]. This attempt[2] was prompted by the success of the author (in an earlier work) to describe proton-antiproton annihilations at rest with a thermodynamic version of Fermi's model. The failure of this simple model to correctly describe the angular distribution of the particles produced in high energy nucleon-nucleon collisions led to its rejection.

The next phase of the work consisted of a study of a thermodynamic model developed by Hagedorn[3]. It was found that in the energy region where reliable experimental results are available that the Hagedorn model yields rather good agreement with experiment. The Hagedorn model was thus adopted as the basic model for the description of high energy particle collisions.

The effort in this research was then directed toward calculating, based on the Hagedorn model, the momentum and particle number distributions necessary for a Monte-Carlo description of a Nucleor Cascade. The required momentum distributions, for the case of proton-proton collisions, were readily available from Hagedorn's[3] calculations. The momentum distributions for the pion-proton case were worked out by the author in collaboration with Dr. Hagedorn. It is worth noting here that the distributions for the pion-proton case needs further experimental verification in order to insure their validity. The particle number distributions, within the framework of the Hagedorn model, are quite a problem.

This problem has not been satisfactorily resolved as of this writing. It turns out that, within the energy region where reliable experimental results are available, the particle number distributions are to a good approximation Poisson. One can then use "as a stop gap measure" the Poisson distribution together with the Hagedorn model momentum distributions to form a complete nucleor model for high energy particle collisions. The author is continuing work on the particle number distribution. When a satisfactory calculation is made, it will be made available to the Cosmic Ray Branch of NASA's Manned Spaced-Craft Center in Houston.

The details of how to incorporate the Hagedorn model momentum distributions into a Monte-Carlo description of a Nucleor Cascade program are supplied in the appendix to this report.

References:

1. E. Fermi, Progress of Theoretical Physics 5,570 (1950)
2. Ph. D. Dissertation by the Author (Unpublished)
3. R. Hagedorn and J. Ranft, Supplement to Nuovo Cimento 6,169 (1968)

Appendix to Technical Report on Grant

NGR-19-012-001

I. Momentum and angular distribution tables

The following momentum and angular distribution tables need to be calculated and saved. A description of how these tables are to be used is given in section 2 below.

DMIJK(P,E) $\equiv$ Momentum distribution tabular value.

Where

I = 1 for nucleon (proton or neutron) table.

= 2 for π^+ and π^0 table.

= 3 for π^- table.

J = 1 for newly created particles.

= 2 for "Through going" particles.

K = 1 for "Thermodynamic contribution".

= 2 for "Isobar contribution".

= 3 for sum of "thermodynamic contribution" and "Isobar contribution".

E = Laboratory energy of the incident particle.

P = Magnitude of a secondary particle of type "I".

DAIJK(θ,E) $\equiv$ Angular distribution tabular value. Where I,J,K, and E are defined as above and $\theta \equiv$ angle that the momentum of the secondary makes with the collision axis.

For definitions of the W(p) and Q(E) functions used below see section III of this appendix.

1. Through Going Protons (I=1, J=2)

$$\text{DM123(P,E)} = 2\pi P^2 \Delta P \int_0^{\pi} W_p(\vec{P}) \sin\theta \, d\theta$$

$$\text{DA123}(\Theta,\text{E}) = 2\pi \sin\theta \Delta\theta \int_0^{P_{max}} W_p(\vec{P}) P^2 \, dP$$

$P_{max} = \gamma o \beta o M_p$; M_p = 938 Mev; See Section III for $\gamma o, \beta o$ values.

2. Newly created π^+ and π^0 (I=2, J=1)

$$\text{DM211(P,E)} = 2\pi P^2 \Delta P \int_0^{\pi} \frac{W_{\pi^-}(\vec{P})}{Q_{\pi}(E)} \sin\theta \, d\theta$$

$$\text{DM212(P,E)} = 2\pi P^2 \Delta P \int_0^{\pi} \left\{ \begin{matrix} \text{Isobar decay term} \\ \text{See section III} \end{matrix} \right\} \sin\theta \, d\theta$$

$$DA211(\theta,E) = 2\pi \sin\theta\Delta\theta \int_0^{P_{max}} \frac{W_{\pi^-}(\vec{P})}{Q_\pi(E)} P^2 dP$$

$P_{max} = \gamma_o\beta_o M_\pi$; M_π = 140 Mev: See Section III for γ_o, β_o values.

$$DA212(\theta,E) = 2\pi \sin\theta\Delta\theta \int_0^{P_{max}} \left\{\begin{matrix}\text{Isobar decay term} \\ \text{See Section III}\end{matrix}\right\} P^2 dP$$

3. Newly Created π^- (I=3, J=1)

$$DM311(P,E) = 2\pi P^2 \Delta P \int_0^{\pi} \frac{W_{\pi^-}(\vec{P})}{Q_\pi(E)} \sin\theta d\theta$$

$$DA311(\theta,E) = 2\pi \sin\theta\Delta\theta \int_0^{P_{max}} \frac{W_{\pi^-}(\vec{P})}{Q_\pi(E)} P^2 dP$$

$P_{max} = \gamma_o\beta_o M_\pi$; M_π = 140 Mev; See Section III for γ_o, β_o values.

4. Through Going π^+ and π^o (I=2, J=2)

$$DM221(P,E) = 2\pi P^2 \Delta P \int_0^{\pi} W^*_{\pi^-}(\vec{P}) \sin\theta d\theta$$

$$DM222(P,E) = 2\pi P^2 \Delta P \int_0^{\pi} \left\{ \begin{matrix} \text{Isobar decay term} \\ \text{See section III} \end{matrix} \right\} \sin\theta d\theta$$

$$DA221(\theta,E) = 2\pi \sin\theta \Delta\theta \int_0^{P_{Max}} W^*_{\pi^-}(\vec{P}) P^2 dP$$

$$DA222(\theta,E) = 2\pi \sin\theta \Delta\theta \int_0^{P_{Max}} \left\{ \begin{matrix} \text{Isobar decay term} \\ \text{See Section III} \end{matrix} \right\} P^2 dP$$

$P_{max} = \gamma o \beta o M\pi$; $M\pi$ = 140 Mev; See Section III for $\gamma o, \beta o$ values.

5. Through Going π^- (I=3, J=2)

$$DM321(P,E) = 2\pi P^2 \Delta P \int_0^{\pi} W_{\pi^-}^*(\vec{P}) \sin\theta \, d\theta$$

$$DA321(\theta,E) = 2\pi \sin\theta \Delta\theta \int_0^{P_{Max}} W_{\pi^-}^*(\vec{P}) P^2 \, dP$$

$P_{max} = \gamma o \beta o M\pi$; $M\pi = 140$ Mev; See Section III for $\gamma o, \beta o$ values.

II. Precription for picking momenta and angles from Distribution Tables

1. Pick an energy value, E, such that

   (1) Distribution tables have been calculated for E

   (2) $E_i \varepsilon [10^{-1/2}E, 10^{1/2}E)$ where E_i is the incident particles energy.

2. Through Going Nucleon case

   (1) Pick a number X from the uniform distribution

$$f(x) = \begin{cases} \frac{1}{k} & \text{for } o<x<k \\ 0 & \text{otherwise} \end{cases}.$$

Where $K = \sum_{p=0}^{P_{Max}} DM123(p,E)$.

   (2) Pick a momentum value, P, such that

$$X < \sum_{P=0}^{P} DM123(p,E) < \sum_{P=0}^{P+\Delta P} DM123(p,E)$$

   (3) Pick a value, Θ , such that

$$X < \sum_{\theta'=0}^{\theta} DA123(p,E) < \sum_{\theta'=0}^{\theta+\Delta\theta} DA123(p,E)$$

3. Newly created π^+ and π^0 case

   (1) Pick a number X from the uniform distribution

$$f(x) = \begin{cases} \frac{1}{k} & \text{for } o<x<k \\ 0 & \text{otherwise} \end{cases} .$$

$$\text{Where } K = Q\pi(E_i) \sum_{P=0}^{P_{Max}} DM211(p,E) + \sum_{P=0}^{P_{Max}} DM212(p,E).$$

(2) Pick a momentum value, P, such that

$$X < Q\pi(E_i) \sum_{P=0}^{P} DM211(p,E) + \sum_{P=0}^{P} DM212(p,E) < Q \sum_{P=0}^{P+\Delta P} + \sum_{P=0}^{P+\Delta P} .$$

(3) Pick a value, Θ , such that

$$X < Q\pi(E_i) \sum_{\theta'=0}^{\theta} DA211(p,E) + \sum_{\theta'=0}^{\theta} DA212(p,E) < Q \sum_{\theta'=0}^{\theta+\Delta\theta} + \sum_{\theta'=0}^{\theta+\Delta\theta} .$$

4. Newly created π^- case

(1) Pick a number X from the uniform distribution

$$f(x) = \begin{cases} \frac{1}{k} & \text{for } o<x<k \\ 0 & \text{otherwise} \end{cases} .$$

$$\text{Where } K = Q\pi(E_i) \sum_{P=0}^{P_{Max}} DM311(p,E).$$

(2) Pick a value, P, such that

$$X < Q\pi(E_i) \sum_{P=0}^{P} DM311(p,E) < Q\pi(E_i) \sum_{P=0}^{P+\Delta P} DM311(p,E) .$$

(3) Pick a value, Θ , such that

$$X < Q\pi(E_i) \sum_{\theta'=0}^{\theta} DA311(p,E) < Q\pi(E_i) \sum_{\theta'=0}^{\theta+\Delta\theta'} DA311(p,E)$$

5. Through Going π^+ and π^0 case

(1) Pick a number X from the uniform distribution

$$f(x) = \begin{cases} \frac{1}{k} & \text{for } o<x<k \\ 0 & \text{otherwise} \end{cases}$$

Where $K = \sum_{P=0}^{P_{MAX}} [DM221(p,E) + DM222(p,E)]$.

(2) Pick a value, P, such that

$$X < \sum_{P=0}^{P} [DM221(p,E) + DM222(p,E)] < \sum_{P=0}^{P+\Delta P} [DM221(p,E) + DM222(p,E)]$$

(3) Pick a value, Θ , such that

$$X < \sum_{\theta'=0}^{\Theta} [DA221(\Theta,E) + DA222(\Theta,E)] < \sum_{\theta'=0}^{\Theta+\Delta\theta'} [DA221(\Theta,E) + DA222(\Theta,E)]$$

6. Through Going π^- case

(1) Pick a number X from the uniform distribution

$$f(x) = \begin{cases} \frac{1}{k} & \text{for } o<x<k \\ 0 & \text{otherwise} \end{cases}$$

Where $K = \sum_{P=0}^{P_{MAX}} DM321(p,E)$

(2) Pick a momentum value, P, such that

$$X < \sum_{P=0}^{P} DM321(p,E) < \sum_{p=0}^{P+\Delta P} DM321(p,E)$$

(3) Pick an angular value, Θ , such that

$$X < \sum_{\theta'=0}^{\theta} DA321(\theta',E) < \sum_{\theta'=0}^{\theta+\Delta\theta} DA321(\theta',E)$$

III. Hagedorn Model Momentum Distributions

1. Newly created π^-

$$W_{\pi^-}(\vec{P}) = Q_{\pi}(E)\int_{-1}^{1}\left\{ O \xrightarrow{\pi^-} \right\} d\lambda \quad .$$

$$O \xrightarrow{\pi^-} \equiv F(\lambda)\, L(\beta)\left\{ f_{\pi^-}(\xi', \lambda)\right\} \quad .$$

$$F(\lambda) \equiv (1-|\lambda|)\, e^{-a|\lambda|}\left\{\frac{1}{a^2}(a-1+e^{-a})\right\}^{-1} \quad .$$

$$L(\beta)\left\{f_{\pi}(\xi')\right\} = f_{\pi}\left[\gamma(\xi - \beta P_{\parallel})\right]\cdot \frac{\gamma(\xi-\beta P_{\parallel})}{\xi} \quad .$$

$$a = 5.635 \quad . \quad \beta = \frac{\lambda}{|\lambda|}\left[\frac{1}{\gamma}\sqrt{\gamma^2-1}\right] \quad .$$

$$\gamma = 1 + (\gamma_0 - 1)|\lambda| \quad . \quad \gamma_0 = \sqrt{\frac{\gamma_L+1}{2}} \quad . \quad \gamma_L = \frac{E_L}{M_p}$$

$$M_p = 938\,\text{Mev} \quad . \quad \xi = \sqrt{p^2 + m_\pi^2} \quad . \quad P_{\parallel} = p\cos\theta \quad .$$

$$M_\pi = 140\,\text{Mev} \quad . \quad \theta \equiv \text{Direction of } \vec{p} \quad .$$

$$f_\pi(\xi',\lambda) = \begin{cases} \frac{V}{8\pi^3}\left[e^{\frac{\xi'}{T(\lambda)}} - 1\right]^{-1} & \text{for } \xi' \leq (\xi'_\pi)_{Max} \\ 0 & \text{for } \xi' > (\xi'_\pi)_{Max} \end{cases} .$$

$$T(\lambda) = \tau T(\gamma_0, \gamma) = \tau T[\xi(\gamma_0, \gamma)] .$$

$$\tau = 1.027 \quad . \quad \xi(\gamma_0, \gamma) = \frac{M_p}{V}\left(\frac{\gamma_0}{\gamma}\right)$$

$$V = v V_0 = \frac{4}{3}\pi(1.656)(M_\pi)^{-3}$$

$$M_p = 938 \text{ Mev} . \quad M_\pi = 140 \text{ Mev}$$

$T(\xi)$ will be supplied as a tabulated function. (See pg. 300 of reference 3)

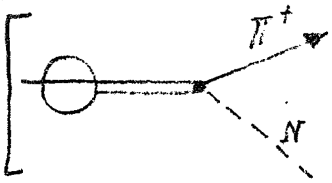
$$(\xi')_{Max} = \frac{M_\gamma^2 + M_\pi^2 - M_p^2}{2 M_\gamma} .$$

$$M_\gamma = \gamma E_{cm} - \sqrt{\gamma^2 E_{cm}^2 - E_{cm}^2 + M_p^2} .$$

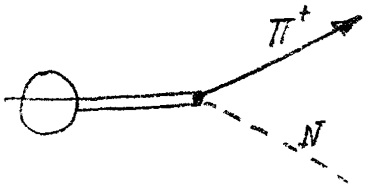
$$E_{cm} = 2 \gamma_0 M_p$$

$q_\pi(E) = (0.848) E_L^{\frac{1}{4}} [Gev]^{\frac{1}{4}}$ (See page 33 of Hagedorn Model II[3]).

2. Newly created π^0 and π^+

$$W_{\pi^+}(\vec{P}) = W_{\pi^-}(\vec{P}) + \int_{-1}^{1} \Bigg[$$  $$\Bigg] d\lambda \quad .$$

The second term is the isobar decay term.



$$\equiv \frac{F_0(\lambda)}{N_B(\lambda)} L(\beta) \sum_{N^*} A_{N^*} f^*_{\pi, N^*}(\xi', \lambda) \quad .$$

$$F_0(\lambda) = \frac{(1-\lambda)\exp[-b\lambda] + \lambda d \exp[-c(1-\lambda)]}{\left(\frac{1}{b^2}\right)(b-1+e^{-b}) + \left(\frac{b}{c^2}\right)(c-1+e^{-c})}$$

$$b = 20.8 \quad , \quad c = 2.4 \quad , \quad d = 7.1$$

$$N_B(\lambda) = 4V\left(\frac{M_p T}{2\pi}\right)^{\frac{3}{2}} \exp\left[-\frac{M_p}{T}\right]\left\{1 + \frac{a_B}{4M_p^{3/2}} \exp\left[\frac{M_p}{T}\right] E_1\left(M^* \frac{T_0 - T}{T_0 T}\right)\right\}.$$

$$E_1(x) = \int_x^\infty t^{-1} e^{-t} dt \quad . \qquad a_B = 2 \times 10^3 (\text{Mev})^{\frac{3}{2}} .$$

$$M^* = 1236 \text{ Mev} \qquad T_0 = 130 \text{ Mev} \quad .$$

$$f^{*}_{\pi^+, N^*}(\mathcal{E}', \lambda) = \frac{Z_{N^*} V M^*}{16\pi^3 \mathcal{E}' p' P_{\pi^+}} T^2 \left\{ \left(1 + \frac{\mathcal{E}^{(-)}}{T}\right) \exp\left[-\frac{\mathcal{E}^{(-)}}{T}\right] - \left(1 + \frac{\mathcal{E}^{(+)}}{T}\right) \exp\left[-\frac{\mathcal{E}^{(+)}}{T}\right] \right\} .$$

$$\mathcal{E}^{(\pm)} = \frac{M^*}{M^2_{\pi^+}} \left(\mathcal{E}' \mathcal{E}_{\pi^+} \pm p' P_{\pi^+} \right) .$$

$$p' = \sqrt{\mathcal{E}'^2 - M^2_{\pi^+}}$$

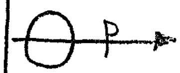
$$P_{\pi^+} = \frac{1}{2M^*} \left\{ \left[M^{*2} - (M_{\pi^+} + M_j)^2 \right] \left[M^{*2} - (M_{\pi^+} - M_j)^2 \right] \right\}^{\frac{1}{2}} .$$

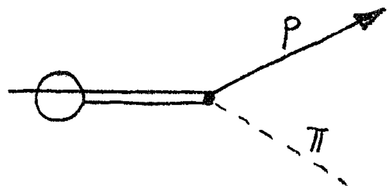
$$\mathcal{E}_{\pi^+} = \frac{1}{2M^*} \left(M^{*2} + M^2_{\pi^+} - M_j^2 \right)$$

Newly Created π Table

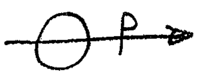
A_{N^*}	M^* (Mev)	Z_{N^*}	M_j (Mev)
57.3	1236	8	M_p = 938.3
1,733.	1525	4	M_n = 939.6
65,460.	2200	8	M_n = 939.6

3. "Through Going" Nucleons (proton and Neutron)

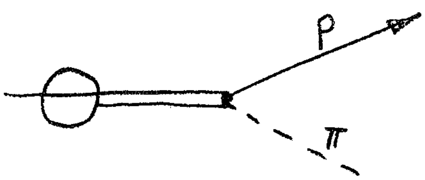
$W_p(\vec{P}) = \int_{-1}^{1} \Big[$ 

$+$ 

$\Big] d\lambda$.



$\equiv \dfrac{F_0(\lambda)}{N_B(\lambda)} L(\beta) \left\{ f_N(\xi' \lambda) \right\}$.



$\equiv \dfrac{F_0(\lambda)}{N_B(\lambda)} L(\beta) \left\{ \sum A_{N^*} f^*_{p, N^*}(\xi', \lambda) \right\}$.

$F_0(\lambda) \equiv$ See π^+ case.

$N_B(\lambda) \equiv$ See π^+ case.

$$f_N(\mathcal{E}',\lambda)=\begin{cases}\frac{2V}{8\pi^3}\left[e^{\frac{\mathcal{E}'}{T}}+1\right]^{-1} & \text{for } \mathcal{E}'\le(\mathcal{E}_p')_{Max}\\ 0 & \text{for } \mathcal{E}'>(\mathcal{E}_p')_{Max}\end{cases}.$$

$$(\mathcal{E}_p')_{Max}=\gamma\,(\gamma_o M_p-\beta P_{\parallel})$$

See π^- case for a relation between γ and λ.

See π^- case for a relation between γ_0 and $\gamma_L = \frac{E_L}{M_p}$.

$$L(\beta)\left\{f_N(\mathcal{E}',\lambda)\right\}=$$ See π^- case.

Proton table

A_{N^*}	M^* (Mev)	Z_{N^*}	M_j (Mev)
57.3	1236	12	M_π = 140
1,733.	1525	8	M_π = 140
65,460.	2200	16	M_π = 140

$f_{p,N^*}(\mathcal{E}',\lambda)$ is the same as $2f_{\pi^+,N^*}(\mathcal{E}',\lambda)$ except $\pi^+ \to p$.

$F(\lambda)\equiv$ See π^- case.

$$N_F(\lambda) = Z_F V\left(\frac{M_F T(\lambda)}{2\pi}\right) \exp\left[-\frac{M_F}{T(\lambda)}\right]\left\{1 + \frac{a_F}{Z_F M_F^{3/2}} \exp\left[\frac{M_F}{T(\lambda)}\right] E_1\left(\frac{M_F^*(T_0 - T)}{T\,T_0}\right)\right\}$$

F	Z_F	M_F	a_F	M_F^*
K	2	496 Mev	4 X 10^3 Mev$^{3/2}$	725 Mev
Y	6	1171 Mev	1.7 X 10^3 Mev$^{3/2}$	1382 Mev
$\bar{B}$	4	940 Mev	2 X 10^3 Mev$^{3/2}$	1236 Mev
π	3	140 Mev	2 X 10^3 Mev$^{3/2}$	549 Mev

$E_1(x) \equiv$ See π^+ case. $T(\lambda) \equiv$ See π^- case. $T_0 = 160$ Mev

4. "Through Going" π^-

$$W_{\pi^-}^{-*}(\vec{P}) = \int_0^1 \left[\bigcirc\!\!\xrightarrow{\ \pi^-\ }\right] d\lambda \, .$$

$$\bigcirc\!\!\xrightarrow{\ \pi^-\ } \;\equiv\; \frac{F_0(\lambda)}{N_\pi(\lambda)} L(\beta)\left\{f_{\pi^-}(\varepsilon;\lambda)\right\} .$$

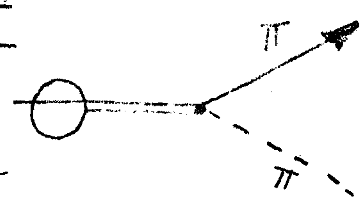
$F_0(\lambda) \equiv$ See through going proton case.

$L(\beta)\left\{ f_{\pi^-}(\varepsilon', \lambda) \right\}$ See newly created π^- case.

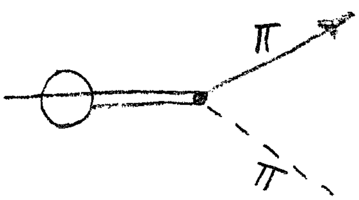
$N_\pi(\lambda) \equiv$ See page (16) of this appendix.

5. "Through Going" π^+, π^0

$$W^*_{\pi^+}(\vec{P}) = W^*_{\pi^-}(\vec{P}) + \int_0^1 \Big[$$



$$\Big] d\lambda .$$



$$\equiv \frac{F_0(\lambda)}{N_\pi(\lambda)} L(\beta)\left\{ \sum_{A_{\pi^*}} A_{\pi^*} f^*_{\pi,\pi^*}(\varepsilon', \lambda) \right\} .$$

$$f^*_{\pi,\pi^*}(\varepsilon', \lambda) = \frac{2 Z_{\pi^*} V M^*}{16\pi^3 \varepsilon' p' p_\pi} \left\{ \left(1 + \frac{\varepsilon^{(-)}}{T}\right) \exp\left[-\frac{\varepsilon^{(-)}}{T}\right] - \left(1 + \frac{\varepsilon^{(+)}}{T}\right) \exp\left[-\frac{\varepsilon^{(+)}}{T}\right] \right\} .$$

$$\mathcal{E}^{(\pm)} = \frac{M^*}{M_\pi^2}\left(\mathcal{E}'\mathcal{E}_\pi \pm P'P_\pi\right) . \quad P' = \sqrt{\mathcal{E}'^2 - M_\pi^2} . \quad \mathcal{E}_\pi = \frac{1}{2M^*}\left(M^{*2} + M_\pi^2 - M_j^2\right) .$$

$$P_\pi = \frac{1}{2M^*}\left\{\left[M^{*2} - (M_\pi + M_j)^2\right]\left[M^{*2} - (M_\pi - M_j)^2\right]\right\}^{\frac{1}{2}}$$

Through going π Table

A_{π^*}	M^* (Mev)	Z_{π^*}	M_j (Mev)
8.3×10^3	765	6	140
1.5×10^3	1069	1	140
1.5×10^2	1263	5	140