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ON THE ATTITUDE STABILITY OF THE SAS A SPACECRAFT

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DEFINITION OF PARAMETERS

I_T = Spacecraft Transverse Moment of Inertia (19.57 slug-ft^2)

I_z = Spacecraft Polar Moment of Inertia (21.19 slug-ft^2)

I_W = Momentum Wheel Polar Moment of Inertia (0.007488 slug-ft^2)

I_{WT} = Momentum Wheel Transverse Moment of Inertia (0.003744 slug-ft^2)

B_W = Momentum Wheel Damping Constant (0.005 ft-lb-s/rad)

ω_f = Natural Frequency of the Wheel Inertia Rim (98 rad/s)

B_D = Damper Damping Constant (7.38×10^{-5} ft-lb-s/rad)

K_W = Momentum Wheel Spring Constant (52.82 ft-lb/rad)

K_D = Damper Spring Constant (4.5×10^{-5} ft-lb/rad)

l = Displacement Between Damper and Center of Mass Along the z-Axis (1.148 ft)

m = Damper End Mass (0.01475 slugs)

r_o = Displacement Between Damper Hinge Point and Center of Mass Along a Transverse Axis (0.0820 ft)

r_1 = Length of Damper Pendulum (0.629 ft)

$\dot{T}_W$ = Rate of Energy Dissipation in the Momentum Wheel

$\dot{T}_D$ = Rate of Energy Dissipation on the Main Spacecraft Body

ω_s = Relative Angular Rate of Momentum Wheel (209 rad/s)

$\left.\begin{matrix}\omega_x\\ \omega_y\\ \omega_z\end{matrix}\right\}$ = Spacecraft Body Angular Rates

θ = Nutation Angle (Angle between the spacecraft polar axis and the spacecraft momentum vector)

λ = Spacecraft Nutational Frequency

H = Momentum Wheel Angular Momentum

ON THE ATTITUDE STABILITY OF THE SAS A SPACECRAFT

by

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INTRODUCTION

The Small Astronomy Satellite (SAS) is a dual-spin system being designed and developed for Goddard Space Flight Center by the Johns Hopkins Applied Physics Laboratory. Basically, the system consists of a high speed rotor (momentum wheel) housed inside the main spacecraft body. The momentum wheel speed is maintained constant by the use of a synchronous hysteresis motor. The body is spun up about its longitudinal axis and the spin rate (1/12 rpm) is maintained by the use of magnetic torque coils. A nutation damper is designed intentionally to be an energy dissipator; however, if additional energy dissipation exists due to nonrigid elements in the momentum wheel, the attitude stability of the entire spacecraft becomes questionable. The purpose of this analysis is, therefore, to identify and model energy dissipation for SAS A in the body as well as in the momentum wheel and then to analyze the effect of energy dissipation on system stability.

STABILITY CRITERIA

In Reference 1, Likins has shown that the necessary and sufficient conditions for dual-spin stability are expressed by

$$\frac{\dot{T}_W}{\lambda_W} + \frac{\dot{T}_D}{\lambda_D} < 0, \tag{1}$$

where $\dot{T}_D$ and $\dot{T}_W$ are energy dissipation rates for the main body (due to the on-board damper) and for the momentum wheel, respectively. The frequencies λ are expressed as follows:

$$\lambda_D = \left(\frac{I_z + I_W}{I_T} - 1\right)\omega_z + \left(\frac{I_W}{I_T}\right)\omega_s \tag{2a}$$

$$\lambda_W = \left(\frac{I_z + I_W}{I_T} - 1\right)\omega_z + \left(\frac{I_W}{I_T} - 1\right)\omega_s. \tag{2b}$$

Because $\dot{T}_D$ and $\dot{T}_W$ are negative quantities, λ_D and λ_W must both be positive for absolute stability under all conditions. It can be seen that if either λ_D or λ_W is negative (λ_W is negative for SAS A) and the corresponding energy dissipation rate is large enough, Equation 1 might exceed zero. If this condition should occur, the nutational motion would build up, and, eventually, the spacecraft would tumble. For the SAS A design, λ_D is positive; however λ_W is negative. The following paragraphs will show how expressions for $\dot{T}_W$ and $\dot{T}_D$ are derived for SAS A. In reality, the rate of energy dissipation for the momentum wheel will affect the rate of energy dissipation for the body damper and vice versa. In this analysis, $\dot{T}_W$ and $\dot{T}_D$ will be computed independently of each other and will then be combined, with the use of Equation 1, to give the system stability. This is valid, as it can be shown that even for an assumed nutation angle of 10 deg, the maximum flexing angle α_{max} of the inertia ring (Figure 1) is at least an order of magnitude smaller than the nutation angle.

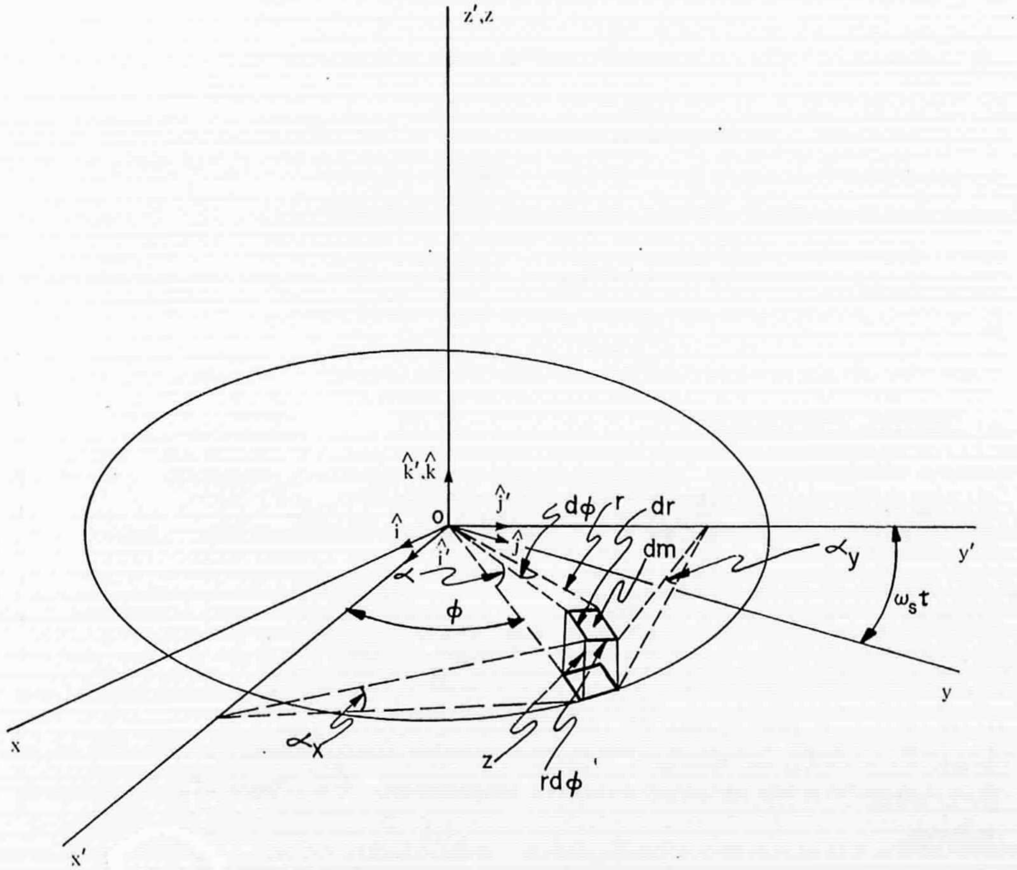

Figure 1—Motion of incremental mass on the momentum wheel rim due to flexibility of wheel structure.

ENERGY DISSIPATION FROM THE MOMENTUM WHEEL (T_W)

The only source of energy dissipation for the momentum wheel to be considered here is the flexibility of the inertia ring support. Because of the structural damping in the inertia ring support, energy will be dissipated as the support both flexes and rotates without ball-bearing deformation. If the flexing angle α is assumed to be small, this source of energy dissipation can be modeled as a linear two-degree-of-freedom damping system which is subjected to an inertial torque. The equations of motion for this energy dissipation from the rotating wheel can be written as (see Appendix A)

$$I_{WT}\ddot{\alpha}_x + B_W\dot{\alpha}_x + [K_W + I_{WT}\omega_s^2]\alpha_x = \lambda H \tan\theta \sin(\omega_s - \lambda)t \tag{3}$$

$$I_{WT}\ddot{\alpha}_y + B_W\dot{\alpha}_y + [K_W + I_{WT}\omega_s^2]\alpha_y = -\lambda H \tan\theta \cos(\omega_s - \lambda)t, \tag{4}$$

where the constants B_W and K_W, which refer to the damping (rate) constant and the spring constant of the flexible rotating wheel, respectively, were determined experimentally.

The necessary experiment was performed by the Johns Hopkins University Applied Physics Laboratory. The values of B_W and K_W were determined for both air and vacuum conditions. The values used in this analysis were determined in air and represent worst-case values (maximum destabilizing effect). With the wheel stationary and with the spin axis vertical, an impulse was applied to the outer rim, or inertia ring. Instrumentation was set up to provide a record of the vertical displacement of the inertia rim as a function of time. The motion of the ring indicated the response was essentially that of a second-order system. With this assumption, the natural frequency ω_f is found from the response curve, and K_W is computed:

$$K_W = I_{WT}\omega_f^2. \tag{5a}$$

Also, from the response curve, $\zeta\omega_f$ is determined, by noting the time at which the initial displacement has been reduced by a factor of $1/\epsilon$; then B_W is computed as

$$B_W = 2I_{WT}\zeta\omega_f. \tag{5b}$$

In terms of the constants B_W and K_W, the steady-state solution of Equation 3 can be expressed as follows:

$$\alpha_x = \frac{\lambda H \tan\theta}{[(K_W + 2I_{WT}\omega_s\lambda)^2 + B_W^2\omega_s^2]^{1/2}} \sin(\omega_s t - \psi). \tag{6}$$

From Equation 3, the average energy dissipation rate due to a torque about the x'-axis can be computed as

$$\dot{T}_{Wx} = -B_W \dot{\alpha}^2_{\text{avg}}$$

$$= \frac{-B_W \lambda^2 H^2 \omega_s^2 \tan^2 \theta}{2[(K_W + 2I_{WT}\omega_s\lambda)^2 + B_W^2\omega_s^2]}. \tag{7}$$

Similarly, the average energy dissipation rate $\dot{T}_{Wy}$ due to a torque about the y'-axis can be derived from Equation 4. Thus, the total average energy dissipation rate for the momentum wheel is finally obtained:

$$\dot{T}_W = \dot{T}_{Wx} + \dot{T}_{Wy}$$

$$= -K_1 \tan^2 \theta, \tag{8}$$

where

$$K_1 = \frac{B_W \lambda^2 \omega_s^2 H^2}{[(K_W + 2I_{WT}\omega_s\lambda)^2 + B_W^2 \omega_s^2]}.$$

ENERGY DISSIPATION CAUSED BY THE PENDULOUS DAMPER (T_D)

As in the previous case, with an initial nutation angle of amplitude θ for the spacecraft, the equation of motion for the pendulous damper can be written as (see Appendix A)

$$I_m \ddot{\beta} + B_D \dot{\beta} + K_D \beta = -\frac{(\mu\lambda H \tan\theta)}{I_T} \sin \lambda t. \tag{9}$$

The steady-state solution of Equation 9 is given by

$$\beta = -\frac{\mu\lambda H \tan\theta}{I_T[(K_D - I_m\lambda^2)^2 + B_D^2\lambda^2]^{1/2}} \sin(\lambda t - \gamma). \tag{10}$$

Whence, the average energy dissipation rate due to the pendulous damper can be computed as

$$\dot{T}_D = -B_D \dot{\beta}^2_{\text{avg}}$$

$$= -K_2 \tan^2 \theta, \tag{11}$$

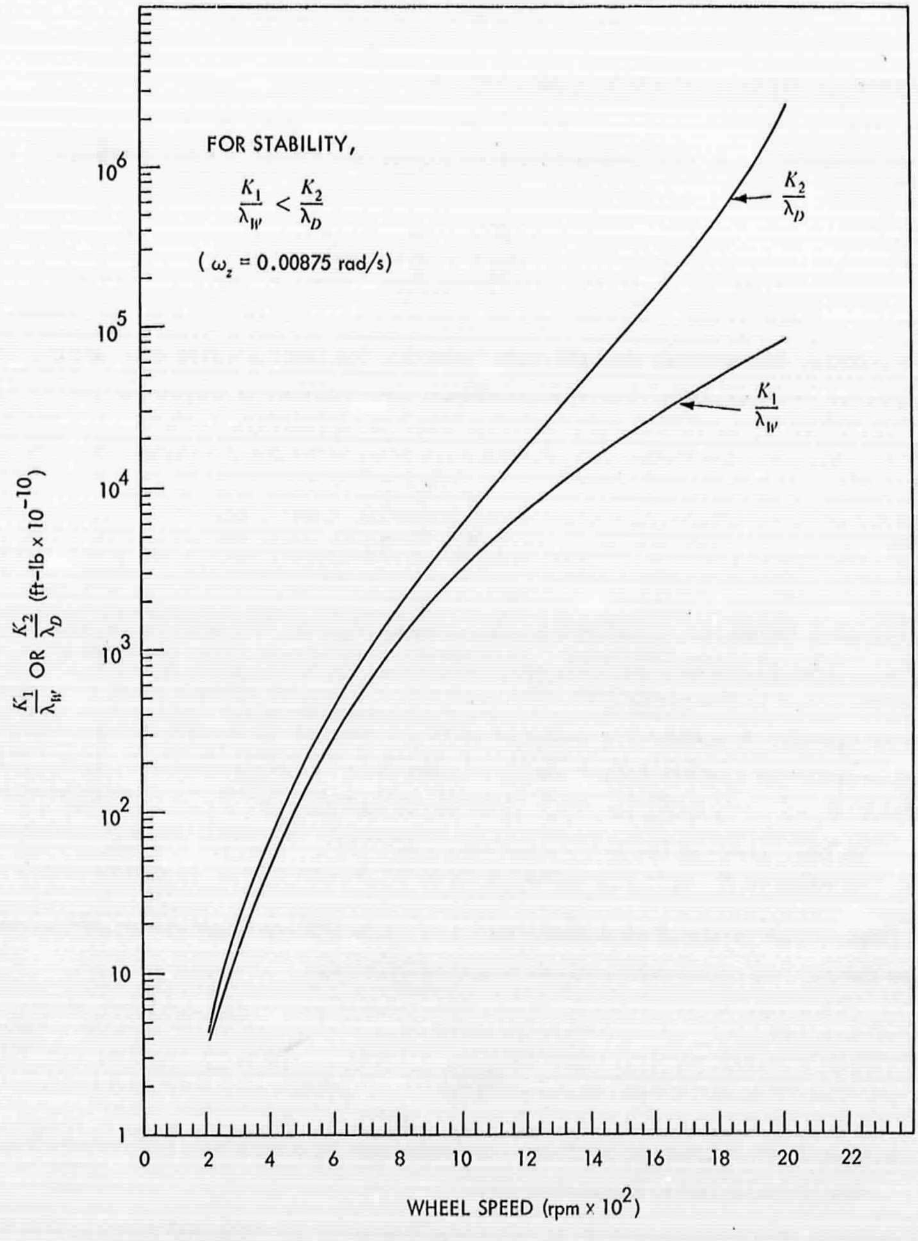

Figure 2—Stability results as a function of wheel speed.

where

$$K_2 = -\frac{B_D \mu^2 \lambda^4 H^2}{2I_T^2 [(K_D - I_m \lambda^2)^2 + B_D^2 \lambda^2]}.$$

NUMERICAL RESULTS AND CONCLUSIONS

As a stability criterion, substitution of Equations 8 and 11 into Equation 1 now yields

$$-\frac{K_1}{\lambda_W} - \frac{K_2}{\lambda_D} < 0. \tag{12}$$

Equation 2b indicates that the parameter λ_W for the SAS A is negative for all wheel speeds of $\omega_s > 0.007$ rpm. For this condition, the stability criterion (Equation 12) reduces to

$$\frac{K_1}{\lambda_W} < \frac{K_2}{\lambda_D}. \tag{13}$$

Figure 2 shows the quantities K_1/λ_W and K_2/λ_D as functions of the momentum wheel speed ω_s. This plot indicates that, although the SAS A spacecraft will not be unstable for any wheel speed in the range 200-2000 rpm,* the degree of stability will be different at different speeds. A convenient measure of this degree of spacecraft stability may be obtained in terms of a safety factor which is computed by taking the ratio of the maximum allowable K_1 ($K_1 = 0.0452$ for SAS A) to the actual value of K_1 at the operating wheel speed selected. Thus, at the chosen operating speed of 2000 rpm for the SAS A momentum wheel, the value of K_1 is found to be 15.78×10^{-4} ft-lb/s, and the safety factor is about 30.

Thus, it can be concluded that there is no potential problem of instability in the SAS A due to the flexing of the momentum wheel inertia ring.

REFERENCES

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2. Bainum, P. M., Fuechsel, P. G., and MacKison, D. L. "On the Motion and Stability of a Dual-Spin Satellite with Nutation Damping", AIAA Guidance, Control and Flight-Mechanics Conference, Held in Princeton, New Jersey, August 1969, Paper No. 69-857, American Institute of Aeronautics and Astronautics, New York, N.Y.

*In actual practice, the spacecraft may be shown to be stable for the entire range of wheel speeds between 0 and 2000 rpm.

Appendix A

Equations of Motion

Energy Dissipation by the Momentum Wheel

Consider an elementary mass dm on the rim of the momentum wheel (Figure 1) which is subjected to inertial forces. If $\mathbf{i}'$, $\mathbf{j}'$, and $\mathbf{k}'$ represent unit vectors for the momentum wheel with 0 as the center of mass for the whole spacecraft, so that

$$\mathbf{r} = \mathbf{i}' r \cos\phi + \mathbf{j}' r \sin\phi + \mathbf{k}' z, \tag{A1}$$

then the y' and z' components of the inertial forces may be found, using small-angle approximations, as

$$\begin{aligned} dF_2 &= -dm[(\omega_z + \omega_s)^2 r \sin\phi] \\ dF_3 &= dm[\ddot{z} + \dot{\omega}_x' r \sin\phi - \dot{\omega}_y' r \cos\phi \\ &\qquad + \omega_x'(\omega_z + \omega_s) r \cos\phi \\ &\qquad + \omega_y'(\omega_z + \omega_s) r \sin\phi], \end{aligned} \tag{A2}$$

where $dm = \rho hrdrd\phi$, ρ being the mass per unit area of a wheel rim of any arbitrary thickness h.

In order to find the moment about the x'-axis due to the above forces, note that $z = (r \sin\phi)\alpha_x$; thus, the total moment about the x'-axis due to the forces acting on all the elementary masses on the wheel rim is given by

$$\begin{aligned} M_x &= \iint (r \sin\phi) dF_3 - \iint z dF_2 \\ &= \left(2\rho h \int_{d_1}^{d_2} r^3 dr\right) \left\{ \left(\int_0^{\pi} \sin^2\phi d\phi\right) [\ddot{\alpha}_x + \dot{\omega}_x' + \omega_y'(\omega_z + \omega_s) + (\omega_z + \omega_s)^2 \alpha_x] \right. \\ &\quad \left. + \left(\int_0^{\pi} \sin\phi \cos\phi d\phi \, [-\dot{\omega}_y' + \omega_x'(\omega_z + \omega_s)]\right) \right\} \\ &= I_{WT}[\ddot{\alpha}_x + (\omega_z + \omega_s)^2 \alpha_x + \dot{\omega}_x' + \omega_y'(\omega_z + \omega_s)], \end{aligned} \tag{A3}$$

where $I_{WT} = m(d_1 + d_2)/4$, m is the mass of the wheel rim and d_1 and d_2 are, respectively, the inner and outer radius of the rim.

The torque given by Equation A3 must equal the sum of the damping and the restoring (spring) torques arising from the values of B_W and K_W for the momentum wheel. Thus,

$$I_{WT}\ddot{\alpha}_x + B_W\dot{\alpha}_x + [K_W + I_{WT}(\omega_z + \omega_s)^2]\alpha_x = -I_{WT}[\dot{\omega}_x' + \omega_y'(\omega_z + \omega_s)], \quad \text{(A4)}$$

or, in terms of the angular rates ω_x and ω_y of the spacecraft body,

$$\begin{aligned} I_{WT}\ddot{\alpha}_x &+ B_W\dot{\alpha}_x + [K_W + I_{WT}(\omega_z + \omega_s)^2]\alpha_x \\ &= -I_{WT}\left\{[\dot{\omega}_y - (\omega_z + 2\omega_s)\omega_x] \sin \omega_s t + [\dot{\omega}_x + (\omega_z + 2\omega_s)\omega_y] \cos \omega_s t\right\} \\ &\cong -I_{WT}[(\dot{\omega}_y - 2\omega_s\omega_x) \sin \omega_s t + (\dot{\omega}_x + 2\omega_s\omega_y) \cos \omega_s t], \end{aligned} \quad \text{(A5)}$$

(since $\omega_z \ll \omega_s$).

Now if, as an approximation, the spacecraft is assumed to be freely nutating with a constant amplitude θ (though, in reality, θ will be a slowly-varying time function because of the presence of dissipation), we have

$$\begin{aligned} \omega_x &= \omega_o \cos \lambda t \\ \omega_y &= \omega_o \sin \lambda t, \end{aligned} \quad \text{(A6)}$$

where

$$\omega_o = (H/I_T) \tan \theta,$$

and $H\ (= I_z\omega_z + I_W\omega_s)$ is the z component of the total angular momentum vector.

Thus, by the use of Equation A6, the torque equation given by Equation A5 can be reduced to

$$\begin{aligned} I_{WT}\ddot{\alpha}_x &+ B_W\dot{\alpha}_x + (K_W + I_{WT}\omega_s^2)\alpha_x \\ &= -\frac{I_{WT}}{I_T} H \tan\theta\,(\lambda - 2\omega_s) \sin(\omega_s - \lambda)t \cong \lambda H \tan\theta \sin(\omega_s - \lambda)t, \end{aligned} \quad \text{(A7)}$$

since $\omega_z \ll \lambda \ll \omega_s$,

$$\lambda = \frac{(I_z - I_T)\omega_z + I_W\omega_s}{I_T} \cong \frac{I_W\omega_s}{I_T},$$

and, for the momentum wheel,

$$I_W = 2I_{WT}.$$

Similarly, if the moments about the y'-axis (for which $z = \alpha_y r \cos\phi$) are considered and Equation A2 is used, it can be shown that

$$I_{W'T}\ddot{\alpha}_y + B_W\dot{\alpha}_y + (K_W + I_{WT}\omega_s^2)\alpha_y = -\lambda H \tan\theta \cos(\omega_s - \lambda)t. \tag{A8}$$

Pendulous Damper

For the case of a single pendulous damper, the equation of motion as derived by Bainum et al. (Reference 2) can be expressed as

$$I_m\ddot{\beta} + B_D\dot{\beta} + K_D\beta = \mu(\dot{\omega}_x - \omega_y\omega_z), \tag{A9}$$

where $I_m = mr_1^2$, $\mu = mr_1 l$ and β is the damper angle. It is assumed that $(m/M) \ll 1$, $\dot{\omega}_z \cong 0$, and $r_o \cong 0$.

Again, with the angular rates of the spacecraft body as given by Equation A6, the equation of motion for the pendulous damper can be reduced to

$$\begin{aligned} I_m\ddot{\beta} + B_D\dot{\beta} + K_D\beta &= -\mu(\lambda + \omega_z)\omega_o \sin\lambda t \\ &\cong -\frac{\mu\lambda H \tan\theta}{I_T}\sin\lambda t, \end{aligned} \tag{A10}$$

(since $\omega_z \ll \lambda$).