BELLCOMM, INC.
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B71 01045

SUBJECT: Computationally Convenient Forms for Conic Section Equations Case 103-9

DATE: January 8, 1971

FROM: D. H. Novak

ABSTRACT

The three-dimensional vector forms of selected conic section relationships are described. These relationships are of general interest for application to a wide variety of trajectory mechanics problems; in particular, they have been used in a newly developed n-body trajectory simulation based on the Virtual Mass concept (Reference 1). The kinematic relationships are derived directly from the basic two-body equation of motion in order to stimulate greater appreciation of the geometric interpretation of the vector orbital elements. A new "universal form" for time of flight is presented and the simplicity of the recursive computational procedure is detailed. A program is described for solving the Kepler problem in either of two possible modes: an approximate solution in a free-running trajectory propagation, or an exact iteration to a precise time for accurate event simulation.

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MEMORANDUM FOR FILE

1. INTRODUCTION

Classically, conic section equations were derived in polar coordinates in the plane of the orbital motion. These forms served astronomers well for a period of about two hundred years as they performed laborious orbital calculations by hand. Curiously, however, these same forms are not well suited to the computation of precision trajectories in electronic digital computers. The basic causes of the difficulty lie in the presence of trigonometric functions in the conic forms and in the transformation equations between an arbitrary reference frame and the orbital plane. When computing by hand, it is most efficient to look up trigonometric functions in pre-computed tables and interpolate to the specific arguments desired. The digital computer, however, operates most efficiently by evaluating the functions when needed by means of truncated infinite series expansions. Measured by the standards of exceedingly short times required for most operations, the computer takes a comparatively long time to evaluate trigonometric functions. Thus, it pays to avoid them if the computation does not become too complicated in the process. Another difficulty peculiar to automatic computation using the polar coordinate forms is that when the eccentricity or the orbital inclination is zero, certain other elements are not defined. Whereas a human being can "shift computational gears" with relative ease in such cases, rather elaborate logical tests must be programmed to enable the digital computer to handle all such possibilities.

The Kepler problem seeks to determine the final position on a known orbit corresponding to a specified time of flight from a given initial position. It would seem that Kepler's problem would find frequent use in trajectory propagation using the Encke procedure. In a strict sense, however, the classical Kepler problem is not required in the majority of cases. A propagation along a known trajectory is desired, but only rarely is it necessary to control an integration step to a precise pre-determined time interval. All that is required is an accurate a posteriori determination of the time associated with a final position which satisfies the orbital equations with high accuracy, but which only approximates a prespecified time interval.

This memorandum presents conic section relationships in forms which are particularly useful for trajectory calculations in a digital computer. Aside from the main guiding principle of achievement of high accuracy, emphasis is placed upon derivation of forms which will make for efficient computation in the majority of cases. Some inefficiency can be tolerated for the few times necessary to iterate to exact times in the true Kepler problem.

The kinematic relationships are derived in Cartesian coordinates in the next section. A simple universal recursion formula for computation of time of flight is derived in Section 3. Finally, Section 4 describes how the basic building blocks are used in a subroutine called KEPLER to solve the Kepler problem. This subroutine has two modes of operation as used in the Virtual Mass trajectory program described in Ref. 1: (1) free-running for efficient trajectory propagation, and (2) accurate iteration to a prespecified time.

2. VECTOR FORMS FOR CONIC EQUATIONS

One of the most inefficient aspects of the use of the classical conic section equations derives from the fact that they are written in two-dimensional polar coordinates in the plane of motion. Thus, there is the constant need to transform back and forth between the general three-dimensional coordinate system and the instantaneous plane of motion relative to the reference body. In addition, the presence of trigonometric functions in the in-plane forms makes for difficulty in digital computer evaluation. Although the representation of the two-body results in three-dimensional vector notation is not new, force of longstanding habit seems to perpetuate the use of the older, less efficient, forms. This memorandum documents the use of the more efficient conic section vector forms in the Virtual Mass program (Refs. 1 and 2) in the hope of stimulating more widespread adoption. The kinematic relationships are derived directly from the basic equation of motion. This is for the sake of completeness and to provide pedagogical appreciation for the geometric interpretations of the vector orbital elements.

The vector equation of motion of a body in an inverse-square central force field is

$$\ddot{\bar{r}} = -\frac{\mu\bar{r}}{r^3} \tag{1}$$

where μ is the prod[illegible] of the mass and the Universal Gravitational constant. Cross multiplication by $\bar{r}$ causes the right side to vanish identically:

$$\bar{r} \times \ddot{\bar{r}} = - \frac{\mu}{r^3} \bar{r} \times \bar{r} \equiv 0$$

Integrating the result gives

$$\bar{H} = \bar{r} \times \dot{\bar{r}} \tag{2}$$

This constant of integration $\bar{H}$ is orthogonal to the plane of motion and is equal in magnitude to the angular momentum or twice the areal rate. The restriction of the motion to a plane and the validity of Kepler's second law are seen to be simple analytical consequences of the assumption of a central force field (the exponent of the law of attraction plays no role).

Now, form the vector product of Eqn. (2) and Eqn. (1) divided by $-\mu$:

$$- \frac{1}{\mu} \bar{H} \times \ddot{\bar{r}} = \frac{(\bar{r} \times \dot{\bar{r}}) \times \bar{r}}{r^3}$$

It can easily be shown that the right side is $\frac{d}{dt}\left(\frac{\bar{r}}{r}\right)$ and hence that this equation can be integrated to yield

$$\bar{e} = - \frac{\bar{r}}{r} - \frac{\bar{H} \times \dot{\bar{r}}}{\mu} \tag{3}$$

The significance of the integration constant $\bar{e}$ is not immediately obvious. The inner product of Eqns. (2) and (3) reveals that

$$\bar{e} \cdot \bar{H} = - \frac{\bar{r}}{r} \cdot \bar{r} \times \dot{\bar{r}} - \frac{\bar{H} \times \dot{\bar{r}}}{\mu} \cdot \bar{H} \equiv 0 \tag{4}$$

or, $\bar{e}$ is orthogonal to $\bar{H}$ and hence lies in the plane of motion. The dot product of Eqn. (3) and $\bar{r}$ gives

$$\bar{e} \cdot \bar{r} = - \frac{\bar{r}}{r} \cdot \bar{r} - \frac{\bar{H} \times \dot{\bar{r}}}{\mu} \cdot \bar{r}$$

Interchanging the dot and cross in the last term on the right and substituting from Eqn. (3) results in

$$\bar{e}\cdot\bar{r} = - r + \frac{H^2}{\mu} \qquad (5)$$

The orbit in space is specified as the intersection of the surface defined by Eqn. (5) with the plane normal to $\bar{H}$. Writing the dot product on the left as er cos θ, where θ is the angle between $\bar{e}$ and $\bar{r}$, the classical in-plane representation of the conic section in polar form results:

$$r = \frac{p}{1 + e\cos\theta}$$

where $p \triangleq H^2/\mu$. Thus, the magnitude of $\bar{e}$ is the eccentricity of the conic section and, for the sign convention of Eqn. (3), the vector points along the major axis toward periapsis. It seems appropriate, therefore, to call $\bar{e}$ the "eccentricity vector".

Observe that the constants $\bar{H}$ and $\bar{e}$ are completely determined in any three-dimensional coordinate system by Eqns. (2) and (3), having given the position $\bar{r}$, the velocity $\dot{\bar{r}}$, and the central mass μ. The simple computer subroutine CONIC, described in Refs. 1 and 2, does precisely that. These vector orbital elements define the geometry of the orbit just as do the classical orbital elements a, e, i, Ω, and ω. Of course, six elements are defined by the three components each of $\bar{H}$ and $\bar{e}$, but the identical satisfaction of the orthogonality condition given by Eqn. (4) implies that, in fact, there are only five independent elements.

There are certain computational difficulties associated with the classical forms when i = 0 (then Ω and ω are not defined) and when e = 0. It will be seen, however, that although $\bar{e}$ can vanish, no problems are encountered with the use of the vector orbital elements $\bar{e}$ and $\bar{H}$.

As will be seen in Section 4, a position can be determined on an orbit at a given time increment from an initial position. Often it is desired to know the velocity associated with this new position vector. Specifically, therefore, we need a form for computing the velocity $\dot{\bar{r}}$ at a position $\bar{r}$ on a given orbit $\bar{H}$, $\bar{e}$. First, observe that since $\bar{H}$ is orthogonal to $\dot{\bar{r}}$,

$$\frac{\bar{H}}{H} \times \dot{\bar{r}}$$

is a vector in the plane of motion, perpendicular to the velocity vector and equal to it in magnitude. The cross product of this resulting vector by the same unit normal to the plane gives the original velocity identically:

$$\dot{\bar{r}} \equiv \left(\frac{\bar{H}}{H} \times \dot{\bar{r}}\right) \times \frac{\bar{H}}{H} = \frac{\bar{H}}{H^2} \times (-\bar{H} \times \dot{\bar{r}})$$

Substituting for the expression in parentheses from Eqn. (3), one obtains the desired equation

$$\dot{\bar{r}} = \frac{\bar{H}}{H^2} \times \mu\left(\bar{e} + \frac{\bar{r}}{r}\right) = \left(\frac{\mu}{H^2}\right) \bar{H} \times \left(\bar{e} + \frac{\bar{r}}{r}\right) \qquad (6)$$

3. CONIC ARC TIME OF FLIGHT

The preceding section showed how to express, in terms of the vector orbital elements $\bar{e}$ and $\bar{H}$, the shape of the complete orbit resulting from a given instantaneous position and velocity combination. It also showed how to find the velocity, given an arbitrary position on a specified orbit. None of the relationships, however, tells anything about the time-history of motion along the orbital path. That is the subject of this section.

The time of flight was classically shown to be a transcendental function of the initial and final positions and the influential orbital elements. The deceptively simple-looking forms involving inverse trigonometric and hyperbolic functions or logarithmic functions present mechanization difficulties for numerical evaluation in digital computers. Logical tests must be devised to determine whether to use the elliptic, parabolic or hyperbolic forms, and computational inaccuracies result from small differences between relatively large numbers, division by small quantities, etc. Some authors (see Ref. 3) have unified the three separate forms into a single "universal" equation.

Such contrivances, however, still seem to suffer computational complexity and, in certain instances, numerical evaluation difficulties.

Here the same thing will be done, except that we will cast our universal form in terms of the vectors $\bar{r}_1$, $\Delta\bar{r} = \bar{r}_2 - \bar{r}_1$, $\bar{e}$ and $\bar{H}$. It may be possible to develop a direct derivation in the final form from the vector orbital geometry. The procedure followed here, however, is the indirect method used in the initial development. We start with the classical expressions involving Kepler's equation and transform everything to the vector notation adopted. The treatment begins with the elliptic case. Later an outline will be given of the analogous hyperbolic case to show the influence upon the final universal form.

For an ellipse with semi-major axis a and semi-minor axis b, the total enclosed area is πab. Dividing the areal rate H/2 by the total area gives the frequency $\frac{H}{2\pi ab}$. Multiplying this by 2π gives the mean angular rate ω_M of the orbital motion as

$$\omega_M = \frac{H}{ab} \tag{7}$$

In terms of the given orbital parameters, the semi-major and semi-minor axes are, respectively:

$$\left.\begin{aligned} a &= \frac{H^2}{\mu(1 - e^2)} \\ b &= \frac{H^2}{\mu\sqrt{1 - e^2}} \end{aligned}\right\} \tag{8}$$

The eccentric anomaly E maps displacement (from periapsis) on the true elliptic orbit onto a circumscribing circular orbit. This eccentric anomaly can be expressed in terms of the vector orbital elements $\bar{e}$ and $\bar{H}$. To this end, it is useful to define an in-plane unit normal to the major axis as

$$\bar{n} = \frac{\bar{H} \times \bar{e}}{He} \tag{9}$$

In terms of this geometry, it turns out that

$$\left.\begin{aligned} \sin E &= \frac{\bar{n}\cdot\bar{r}}{b} \\ \cos E &= e + \frac{\bar{e}\cdot\bar{r}}{ea} \end{aligned}\right\} \tag{10}$$

Kepler's equation transforms the eccentric anomaly placement on the circumscribing circular orbit to an angular positioning M commensurate with the time rate of movement at the mean orbital rate given by Eqn. (7):

$$M = E - e \sin E \tag{11}$$

Thus, a time increment from position $\bar{r}_1$ to $\bar{r}_2$ is very simply derived from the mean anomaly difference as

$$\Delta t = \frac{M_2 - M_1}{\omega_M} = \frac{E_2 - E_1 - e(\sin E_2 - \sin E_1)}{\omega_M} \tag{12}$$

The eccentric anomaly difference above must be evaluated as an inverse trigonometric function from

$$\sin(E_2 - E_1) = \sin E_2 \cos E_1 - \cos E_2 \sin E_1$$

Substitution from Eqns. (10), appropriately evaluated, gives

$$\sin(E_2 - E_1) = \frac{\bar{n}\cdot\bar{r}_2}{b}\left[e + \frac{\bar{e}\cdot\bar{r}_1}{ea}\right] - \frac{\bar{n}\cdot\bar{r}_1}{b}\left[e + \frac{\bar{e}\cdot\bar{r}_2}{ea}\right]$$

Writing $\bar{r}_2 = \bar{r}_1 + \Delta\bar{r}$, this can be expressed as

$$\sin(E_2 - E_1) = \frac{\bar{n}\cdot\Delta\bar{r}}{b} e + \frac{(\bar{n}\cdot\Delta\bar{r})(\bar{e}\cdot\bar{r}_1) - (\bar{n}\cdot\bar{r}_1)(\bar{e}\cdot\Delta\bar{r})}{eab}$$

Noting that the second term on the right involves the magnitude of $\bar{r}_1 \times \Delta\bar{r}$ and substituting for $\bar{n}$ from Eqn. (9) leads finally to the desired relation:

$$E_2 - E_1 = \sin^{-1}\left[\frac{\Delta\bar{r}}{b} \times \frac{\bar{H}}{H}\cdot\left(\frac{\bar{r}_1}{a} + \bar{e}\right)\right] \tag{13}$$

The difference of sines of E_2 and E_1 in Eqn. (12) can easily be shown to give

$$e(\sin E_2 - \sin E_1) = \frac{\Delta\bar{r}}{b} \times \frac{\bar{H}}{H}\cdot\bar{e} \tag{14}$$

From Eqns. (7), (13) and (14), the time in Eqn. (12) can be written as

$$\Delta t = \frac{Hb}{\mu(1 - e^2)}\left\{\sin^{-1}\left[\frac{\Delta\bar{r}}{b} \times \frac{\bar{H}}{H}\cdot\left(\frac{\bar{r}_1}{a} + \bar{e}\right)\right] - \frac{\Delta\bar{r}}{b} \times \frac{\bar{H}}{H}\cdot\bar{e}\right\} \tag{15}$$

If the infinite series expansion for the inverse sine is used, this becomes:

$$\Delta t = \frac{\Delta\bar{r} \times \bar{H}\cdot\bar{r}_1}{H^2} + \frac{HbZ^3}{\mu(1 - e^2)}\left[\frac{1}{2\cdot 3} + \frac{1\cdot 3}{2\cdot 4\cdot 5} Z^2 + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7} Z^4 + \ldots\right] \tag{16}$$

where

$$Z = \frac{\Delta\bar{r}}{b} \times \frac{\bar{H}}{H}\cdot\left(\frac{\bar{r}_1}{a} + \bar{e}\right) \tag{17}$$

Appropriate changes can be made in the basic definitions to treat the hyperbolic case:

$$\omega_M = \frac{\mu(e^2 - 1)}{Hb}$$

$$a = \frac{H^2}{\mu(e^2 - 1)}$$

$$b = \frac{H^2}{\mu\sqrt{e^2 - 1}}$$

$$\sinh H_p = \frac{\bar{n}.\bar{r}}{b}$$

$$\cosh H_p = \bar{e} - \frac{\bar{e}\cdot\bar{r}}{ea}$$

$$M = e \sinh H_p - H_p$$

Here the hyperbolic parameter H_p replaces the eccentric anomaly E of the elliptic case. Following through in a manner similar to the first case, we can derive the analogue to Eqn. (15) as

$$\Delta t = \frac{Hb}{\mu(e^2 - 1)} \left\{ \frac{\Delta\bar{r}}{b} \times \frac{\bar{H}}{H}\cdot\bar{e} - \sinh^{-1}\left[\frac{\Delta\bar{r}}{b} \times \frac{\bar{H}}{H}\cdot\left(\bar{e} - \frac{\bar{r}_1}{a}\right)\right]\right\}$$

Expanding the inverse hyperbolic sine in an infinite series gives the counterpart to Eqn. (16) as

$$\Delta t = \frac{\Delta\bar{r} \times \bar{H}\cdot\bar{r}_1}{H^2} + \frac{HbZ^3}{\mu(e^2 - 1)}\left[\frac{1}{2\cdot 3} - \frac{1\cdot 3}{2\cdot 4\cdot 5} Z^2 + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7} Z^4 - \ldots\right] \quad (18)$$

where

$$Z = \frac{\Delta\bar{r}}{b} \times \frac{\bar{H}}{H}\cdot\left(\bar{e} - \frac{\bar{r}_1}{a}\right) \quad (19)$$

This expansion is convergent for $Z^2 < 1$; thus, there is some limit upon the hyperbolic arc length. The condition on Z is always met in the elliptic case. If the expressions for a and b are substituted into these two forms and the algebra is followed through, they can be combined into another candidate for the "universal form":

$$\Delta t = \frac{\Delta \overline{r}}{H} \times \frac{\overline{H}}{H} \cdot \overline{r}_1 + \mu W^3 \sum_{N=1}^{\infty} \left\{ \left[\operatorname{sgn}(1-e)^{N+1} \right] \left[\prod_{n=1}^{N} \left(1 - \frac{.5}{n}\right) \right] \cdot \frac{Z^{2(N-1)}}{2N+1} \right\} \quad (20)$$

where

$$\left.\begin{aligned} W &= \frac{\Delta \overline{r}}{H} \times \frac{\overline{H}}{H} \cdot \left[\overline{e} + \frac{\mu(1-e^2)}{H^2} \overline{r}_1 \right] \\ Z^2 &= |1 - e^2| \left(\frac{\mu W}{H} \right)^2 \qquad (Z^2 < 1) \end{aligned}\right\} \quad (21)$$

It is clear from the forms of Eqns. (20) and (21) that there are no computational problems for eccentricities near or equal to 0 and 1. In fact, the parabolic case e = 1 can be shown to reduce to the familiar form involving only two terms. Perhaps the most interesting feature of Eqn. (20) is the geometrical significance of the first term. One cyclic permutation of the factors in the triple scalar product reveals that this is nothing more than the triangular area formed by the two position vectors divided by the orbital areal rate. One is therefore afforded a graphical feel for how well-suited this method is to the computation of times for short arcs. In such cases the triangular area is a very good first approximation to the total area of the segment. All that remains for the infinite series of terms is to add on the small region between the chord line and the curved arc. Numerical experience has shown that only a relatively few terms are required to do this with high accuracy.

Finally, Eqn. (20) is especially well formulated for a simple recursion scheme for numerical evaluation in a digital computer. Figure 1 shows a computational flow diagram for the procedure as implemented in the subroutine CONICT. Details of the programming aspects of this subroutine are given in Ref. (2). Some FORTRAN notation is used in the diagram where it helps to

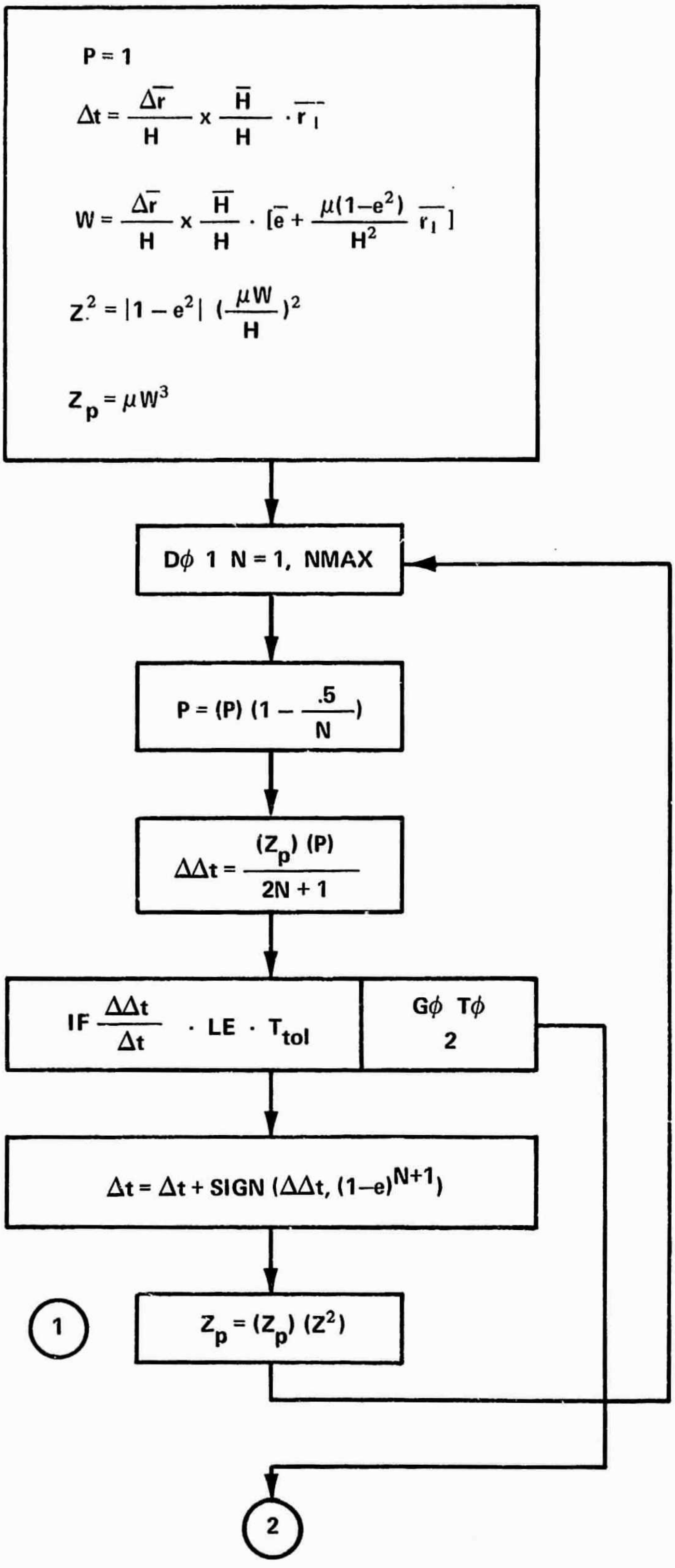

FIGURE 1 - FLOW-DIAGRAM OF CONIC ARC TIME OF FLIGHT

clarify the logical procedure. Specifically, the conventional FORTRAN interpretation is intended when the same symbol appears on both sides of an equation. Note particularly the simplicity of the computations in the recursion loop.

4. KEPLER'S PROBLEM

As remarked in the Introduction, most often in simple trajectory propagation it is not necessary to compute state vectors at precisely prescribed times. It is only when some special event must be simulated, such as a velocity correction or a navigation observation, that an accurate iteration to some condition is required. Accordingly, a procedure is needed which will rapidly compute a time step approximating a desired increment, but which can, when required, compute a predetermined time increment with high accuracy.

The first requirement is to write an expression for the final position $\bar{r}_2$ which is a rigorous solution of the given orbit equation and which involves an approximation to the desired time of flight from the initial position $\bar{r}_1$. Since the orbit is presumed known (i.e., $\bar{e}$ and $\bar{H}$ are known vectors), the velocity $\dot{\bar{r}}_1$ at the position $\bar{r}_1$ can be computed by Eqn. (6). Therefore, $\bar{r}_2$ can be expressed as the following linear combination of the initial state vectors:

$$\bar{r}_2 = B\left[\bar{r}_1 + (\Delta\tau)\ \dot{\bar{r}}_1\right] \triangleq B\bar{\sigma} \tag{22}$$

The geometry is illustrated in Fig. 2 and shows that $\Delta\tau$ determines the time (or true anomaly) increment

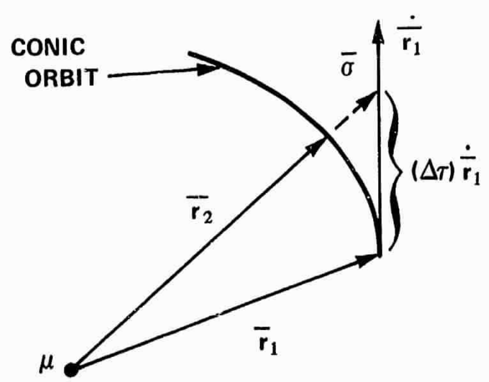

FIGURE 2 - GEOMETRY OF FINAL POSITION DETERMINATION

and B ensures satisfaction of the orbital equation. Once $\Delta\tau$ is given, B is easily computed, since Eqn. (22) must satisfy Eqn. (5):

$$B = \frac{H^2}{\mu(\bar{e}.\bar{\sigma} + \sigma)} \tag{23}$$

The problem, therefore, is reduced to that of relating $\Delta\tau$ to the desired time increment Δt_d. A simple second order relationship is assumed:

$$\Delta\tau = \Delta t_d + \kappa(\Delta t_d)^2 \tag{24}$$

where the constant term is 0 and the linear coefficient is 1, for it must be true that $\Delta\tau \to \Delta t_d$ as $\Delta t_d \to 0$. The coefficient, κ, of the second order term is initially estimated. If there are no prior data available, the starting value of κ is taken to be zero. In the case where similar successive steps are being computed along an orbit (as described in Ref. 1), the value of κ for the preceding step can be used as the current first estimate.

Once the value of κ is selected, $\Delta\tau$ is computed by Eqn. (24). The vector $\bar{\sigma}$ can then be calculated from the definition in Eqn. (22). Equation (23) fixes the value of B, and finally Eqn. (22) gives $\bar{r}_2$. Knowledge of $\bar{r}_1$, $\bar{r}_2$, $\bar{e}$ and $\bar{H}$ enables one to employ Eqn. (20) to determine the actual time increment Δt to this particular estimate of $\bar{r}_2$. If this actual Δt is substituted into Eqn. (24) for Δt_d, one can then solve explicitly for the correct value of κ associated with the $\Delta\tau$ and Δt:

$$\kappa = \frac{\Delta\tau - \Delta t}{(\Delta t)^2} \tag{25}$$

In the free-running mode, where one is not interested in iterating precisely to the desired Δt_d, the actual Δt corresponding to $\bar{r}_2$ is accepted and one proceeds to the next step.

For the next step, the value of κ just computed from Eqn. (25) is used with the new Δt_d in Eqn. (24). However, when it is necessary to iterate accurately to the desired Δt_d, the actual Δt is compared with Δt_d. If they do not agreed to within a specified tolerance, the new estimate of κ from Eqn. (25) is fed back into Eqn. (24) with the same desired Δt_d to obtain a better estimate of the $\Delta\tau$ required. The cycle is repeated until Δt matches Δt_d within the tolerance.

The computation flow diagram is depicted in Fig. 3 for subroutine KEPLER. The two modes of calculation are controlled by the setting of the logical switch LØØP. When LØØP = . TRUE., the DØ loop is honored and the program iterates accurately to the desired time increment. When LØØP = . FALSE., a transfer out of the DØ loop occurs on the first pass through and the free-running, but accurately computed, increment is accepted. Reference 2 gives complete programming details of KEPLER, including a FORTRAN listing.

5. SUMMARY

Conic section computational algorithms have been developed in terms of vector orbital elements. The fact that these vector elements are never undefined and that the forms do not involve trigonometric functions make them particularly well suited for digital computer applications. The time of flight has been expressed in a power series expansion which is evaluated by a very simple recursion formula. This expansion is universally applicable for elliptic, parabolic, and hyperbolic arcs.

A subroutine (KEPLER) has been designed to solve the Kepler problem in either of two modes. The approximate mode determines a final position which is an accurate solution to the orbit, but which only approximates the prespecified desired time of flight from the initial position. An accurate determination is then made of the actual time associated with the flight to this final position. This solution is usually all that is required in a free running trajectory propagation. However, if it is necessary to compute to a precise point in time, the exact mode is utilized. With this option, the final position determination is iterated until the time matches that desired.

These conic routines constitute the basic computational elements of the Virtual Mass program (Ref. 1). This

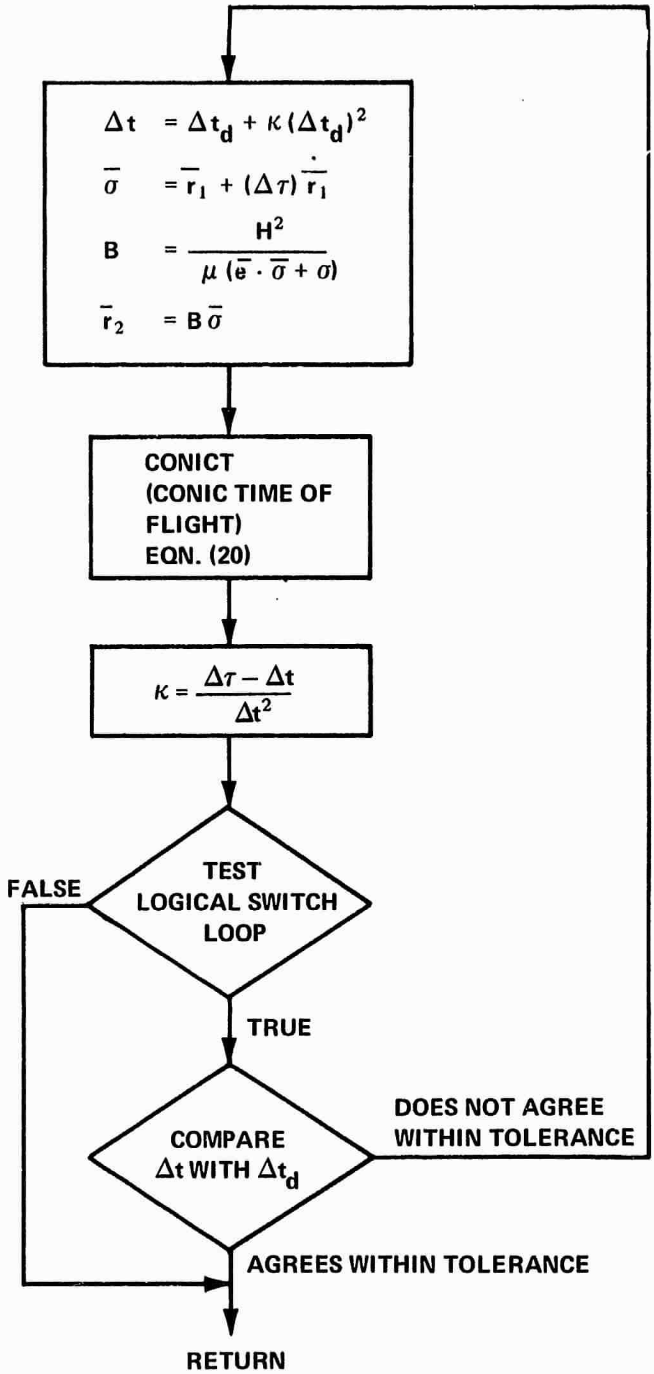

FIGURE 5 - COMPUTATION FLOW LOGIC FOR KEPLER

program, which generates numerical solutions to the N-Body problem, is a simple, self-starting, fast and highly accurate method of numerically integrating a spacecraft trajectory.

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