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ORIENTATION AND RESONANCE LOCKS FOR SATELLITES IN THE ELLIPTIC ORBIT

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ABSTRACT

In order to achieve the maximum strength of higher rotational locks for satellites in the elliptic orbits, the condition of satellite orientation during the process of deployment is established.

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ORIENTATION AND RESONANCE LOCKS FOR SATELLITES IN THE ELLIPTIC ORBIT

INTRODUCTION

Chernousko (Chernousko, 1963) has discussed the resonance locks in the rotation of satellite in orbits of arbitrary eccentricity. Based on the results of Chernousko, Lutze and Abbitt (Lutze and Abbitt, 1969) have evaluated the lock strength of resonance rotation. In order to obtain such locked-in motions the satellite must be oriented initially in a way such that its attitude can be controlled by the field of gravity. In this article, we shall present a formula to describe the orientation of satellite relative to the center of force. We shall also examine the condition of satellite orientation which allows the maximum lock strength of resonance rotation to occur.

MAXIMUM LOCK STRENGTH OF RESONANCE ROTATION

The strength of a rotational lock is defined by (Lutze and Abbitt, 1969; Chernousko, 1963)

$$\Phi_k (e) = \int_0^{\pi} \frac{1 + e \operatorname{Cos} f}{\pi (1 - e^2)^{3/2}} \cdot \operatorname{Cos} (kM - 2f)\, df \tag{1}$$

where

$$M = 2 \tan^{-1} \left[\left(\frac{1 - e}{1 + e} \right)^{1/2} \tan \frac{f}{2} \right] - \frac{e (1 - e^2)^{1/2} \operatorname{Sin} f}{1 + e \operatorname{Cos} f}$$

In Equation (1), f is the true anomaly, e is the orbital eccentricity and

$k = 2P_0 / P_r$ where P_0 and P_r are orbital and rotational periods respectively.

The maximum lock strength occurs when the derivative of Equation (1) with respect to e vanishes. Therefore the condition for a maximum lock strength is

$$\int_0^{\pi} \left[\frac{3e + (1+2e^2)\cos f}{\pi (1-e^2)^{5/2}} \operatorname{Cos}(kM-2f) - \frac{1+e\cos f}{\pi(1-e^2)^{3/2}} k \frac{dM}{df} \operatorname{Sin}(kM-2f) \right] df = 0 \qquad (2)$$

The results of Equation (2) and the corresponding values of maximum Φ_k (e) are given in Table 1.

Table 1

k	2	3	4	5	6	7	8
e	0	0.40	0.55	0.60	0.65	0.70	0.75
Max. Φ_k (e)	1.00	0.95	1.13	1.33	1.53	1.76	1.97

Table 1 shows that for each resonance rotation a particular value of e will provide the strongest lock. These results indicate that the maximum lock strength of resonance rotation and the orientation of satellite on an elliptical orbit are closely linked.

ORIENTATION

Let us consider an ideally oriented satellite which rotates with a resonance spin rate but does not oscillate about its equilibrium attitude. The angle between the least moment of inertia of the satellite and the radius vector is defined as the angle of orientation

$$\phi = \psi - f \qquad (3)$$

where ψ is the angle of rotation. The angular velocity of orientation is

$$\frac{d\phi}{dt} = \frac{2\pi}{P_r} - \frac{df}{dt} \tag{4}$$

By applying Kepler's formula, the instantaneous orbital angular velocity of the satellite around the center of force is

$$\frac{df}{dt} = \frac{2\pi\,(1 + e\cos f)^2}{P_0\,(1 - e^2)^{3/2}} \tag{5}$$

From Equations (4) and (5) the satellite orientation relative to the center of force as a function of the true anomaly can be found by integration.

$$\phi(f) = \int_{f_0}^{f} \left[\frac{(1 - e^2)^{3/2}}{2\,(1 + e\cos f)^2}\, k - 1\right] df \tag{6}$$

For initial condition $t = 0$, $\phi = 0$ when $f_0 = 0$, the results of Equations (5) and (6) by integration are

$$t = \left[\mathrm{Sin}^{-1}\left(\frac{e + \cos f}{1 + e\cos f}\right) - \frac{e\,(1 - e^2)^{1/2}\sin f}{1 + e\cos f} - \frac{\pi}{2}\right]\frac{P_0}{2\pi} \tag{7}$$

$$\phi = \frac{1}{2}\left[\mathrm{Sin}^{-1}\left(\frac{e + \cos f}{1 + e\cos f}\right) - \frac{e\,(1 - e^2)^{1/2}\sin f}{1 + e\cos f} - \frac{\pi}{2}\right] k - f \tag{8}$$

In the case of the planet Mercury, $e = 0.206$ and $k = 3$. From Equations (7) and (8), the orientation of the planet Mercury relative to the Sun during perihelion passage is illustrated in Figure 1. In Figure 1, it is shown that the angle ϕ oscillates with an amplitude of the order of 0.6 degree in a cycle of 14 days.

We substitute the values of e and the corresponding values of k from Table 1 into Equation (8). The angle of orientation, ϕ, at different positions, f, are shown in Figure 2. Figure 2 shows that the satellite tends to keep the same face to the center of force during perigee passage. For higher rotational locks ($k = 3, 4, 5, \ldots$) the value of ϕ oscillates about the radius vector with an amplitude of $\pi/8$ for $-5/8\pi \leq f \leq 5/8\pi$. This specifies the condition of satellite orientation under which maximum strength of higher rotational locks may be induced.

CONCLUDING REMARKS

Equation (8) describes the orientation of satellites relative to the force center. The maximum strength of higher rotational locks for satellite in the elliptic orbit may be obtained by orienting its attitude to the force center at $f = -5/8\pi$, 0 and $5/8\pi$ in the process of deployment. The result of this analysis is also applicable to the calculation of insolation on the surface of the planet Mercury.

ACKNOWLEDGMENT

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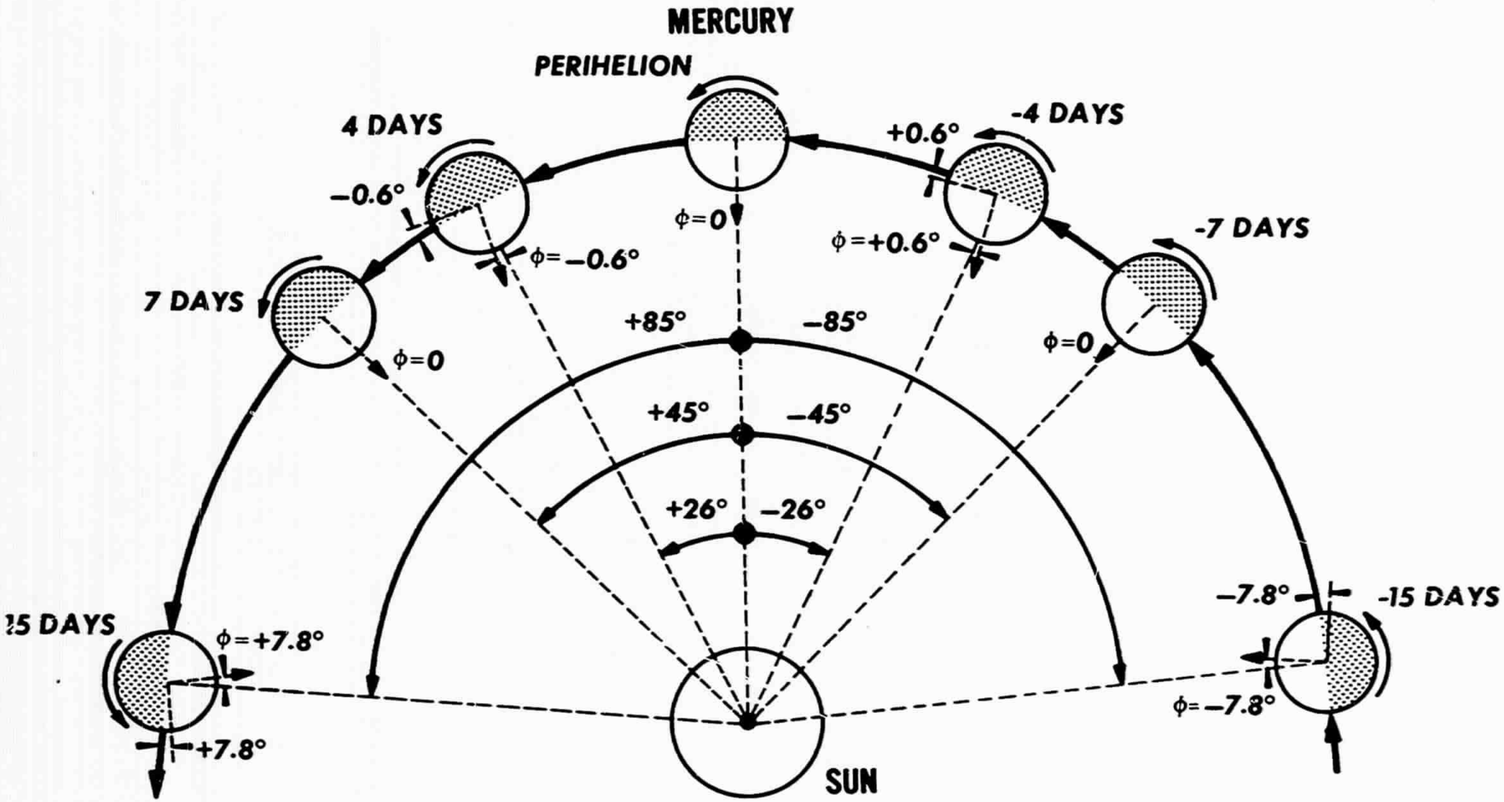

Figure 1. Orientation of Mercury Relative to the Sun During Perihelion Passage

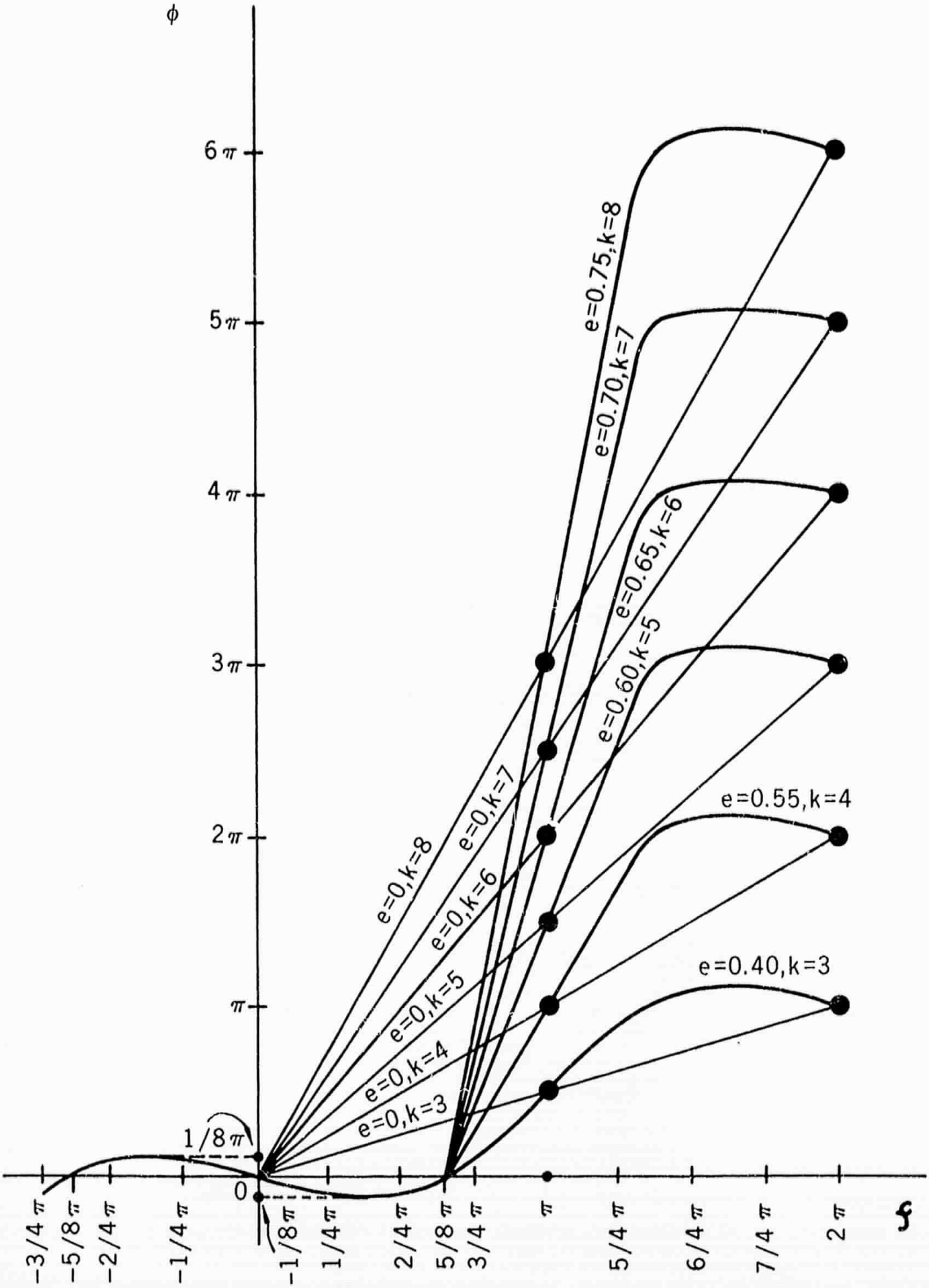

Figure 2. Orientation of Satellite for Higher Rotational Locks