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ABSTRACT

The IMP 4 orbit (Explorer 34) has a resonant perturbation on the time of perigee passage due to the third harmonic of the earth's gravitational field. The amplitude of the perturbation is 9 minutes, and the period is about 240 days. It appears that the effect can provide a significant test of the current values of the tesseral harmonics of third order. Other highly eccentric orbits may be expected to demonstrate this effect.

COMMENSURABILITIES ON THE IMP 4 ORBIT

The IMP 4 (Explorer 34) satellite has an unexpected resonance with the tesseral term J_{33} in the earth's gravitational field. The resonance occurred because the orbital period of 4 1/3 days was nearly commensurate with the earth's rotational period. Normally, a highly eccentric orbit like IMP 4 does not show strong secular effects due to the earth's gravitational harmonics. The major effects on the highly eccentric orbit derive from the lunar and solar perturbations. In fact, a commensurability with the earth's rotation would not be maintained on most orbits with the eccentricity of IMP 4 because the lunar perturbation on the mean anomaly would alter the orbital period by as much as 3 or 4 hours in an incoherent manner. But on the IMP 4 orbit, the lunar perturbation on the mean anomaly was much smaller because the satellite orbit had a high inclination to the lunar orbit (near 90^{o}). This special circumstance permitted the commensurability with the earth's rotation.

The satellite was launched on May 24, 1967 at 14:21 UT. Its initial orbital elements are (as used in this paper) a=17.8855, e=941870, i=$67\overset{\circ}{.}202$, ω=$179\overset{\circ}{.}981$, Ω=$167\overset{\circ}{.}8964$, M.A.=$0\overset{\circ}{.}0122$ at t=14:21 UT. Perigee rose from 385 days to a maximum height of nearly 4000. km. Then perigee declined gradually until the 705th day in orbit when the satellite passed through the atmosphere at a height just over 100 km. The atmosphere severely degraded the orbital period, and the satellite reentered and was lost on the subsequent pass. (On the perigee pass before the first atmospheric entry the height had been over 300 km, well above the region where the atmosphere will significantly alter an orbit.) There are therefore 162 orbits where the geodetic resonance may be seen and tracking data was obtained during this entire period.

COMMENSURABILITY

The orbit has a period of about 4 1/3 days so that in 13 days, or 3 orbits, the ground track repeats. Actually the track is not quite stationary, the long term repetition period being about 240 days. The effect is demonstrated by the longitude of the perigee subsatellite point, shown in Fig. 1. The points fall on three lines, because there is a net rotation of the earth by 1/3 of a day every time the satellite returns to perigee. The slope of the line is the drift rate, which arises because the orbit is not exactly 4 1/3 days. The repetition period implied by the drift rate is about 240 days. Thus the tesseral terms containing multiples of 3 in the earth's gravitational field are selectively built up by the resonance.

The curve in the longitude lines is due to an unidentified perturbation, possibly a short period solar perturbation of the line of apsides. The calculations agree with the data with regard to this effect, so that it is real and is accurately computed by the model.

PERTURBATION DUE TO THE COMMENSURABILITY

The effect of the resonance is observable in the time of perigee passage. This is not surprising since the earth's gravitational harmonics normally have a strong short period perturbation on the mean anomaly near perigee. That is, the harmonic terms are proportional to the r^{-3} power and therefore perturbational effects diminish rapidly as the distance from the earth increases. On the IMP 4, the resonance caused terms like J_{33} to build up.

The observed times of perigee passage are derived from the World Maps (ref. 1). The World Maps consist of arcs which span about a month. These arcs are obtained from the orbit-determination process and therefore are not raw residuals. It is believed that the net result of the processing is to produce a particularly accurate quantity in the times of perigee passage due to the geometry of highly eccentric orbits.

The time of perigee passage was computed from an initial position and velocity vector which had been previously derived by fitting the time of perigee passages over the entire orbital history. The initial conditions which gave the best fit overall were used to compute an ephemeris without interruption for further differential corrections during the computation. The method employed has been described previously (ref. 2).

The influence of the tesseral terms can be seen using a computation based on a model which does not contain the tesseral harmonics and comparing with the data. The difference between the data and the computation forms a sinusoid (Fig. 2). The period of the sinusoid is about 242 days, in accord with the period of the resonance drift.

The geopotenial terms (1, m, p, q) affecting this orbit are calculated from the index formula

$$\ell-2p+q=13i \qquad m=3i,\ i=1,2,3\ldots$$

The dominant terms on this orbit are (3, 3, 1, 12), (4, 3, 2, 13), (5, 3, 3, 14), (6, 3, 3, 13) and (6, 6, 3, 26). A calculation by Wagner (ref 3) based on the analysis of Kaula (ref 4) established that the (3, 3) and (4, 3) terms with a beat period of 235 days have amplitudes of several minutes (of time), in agreement with the observed effect in Fig. 2. (Table 1.) Because the orbital elements, including perigee height, vary so much during the lifetime due to the solar and lunar perturbation, the calculation was performed for 2 sets of orbital elements. One set is the initial orbital elements following launch, the other is 200 days later when the perigee height has been raised to 3145. km. The final column in Table 1 indicates the reduction of the amplitude of the perturbation in perigee time due to the raising of perigee.

The resonance effect is further demonstrated by computing the orbit with the (3,3) tesseral term included using $10^6 J_{33}=-.2164$ and $\lambda_{33}=22^o.8$. The difference between the data and the computation performed including the J_{33} is shown in Fig. 3. The amplitude of the difference in time of perigee passage is substantially reduced when the tesseral model is used. There is still a sinusoidal tendency left in the difference curve, indicating that the J_{33} term and higher order resonant harmonics could be further improved. Even at the .01 minute level there is no evidence of scatter in the data, indicating that the time of perigee passage is being measured with a high order of accuracy and that the uncertainties in the model are larger than the random noise in this measurement.

CONCLUSIONS

There is a resonant perturbation in the time of perigee passage of IMP 4 due to the J_{33} and higher degree harmonics of tesseral order 3 in the earth's gravitational field. The amplitude of the perturbation is 9 minutes. Therefore it may be possible to measure the resonant harmonics from this orbit, even though it is not a near earth satellite orbit.

Future highly eccentric orbits should be examined for commensurability effects. The orbital period of this satellite was 104.33 hours, resonant about the 4 1/3 day period, with the additional 1/3 hour causing the drift rate. The nearest fractional day in the sequence 1/2, 1/3, 1/4... is 4 1/4 days, or

102 hours, that is, there is another resonant period within 2 hours. On the other side, there is a resonant period at 108 hours, due to the 1/2 day fraction. Therefore for distant satellites there is a strong probability that some strong commensurability effects will occur, provided that the lunar perturbation does not have a large effect on the mean anomaly. This condition will occur when the satellite orbit is somewhat inclined to the lunar plane, or when the semimajor axis is a moderate fraction of the moon's. The commensurability can be determined by examining the ground track of the longitude of perigee, as in Fig. 1.

The measurement of the time of perigee passage of IMP 4 is accurate enough to provide a stringent test of the tesseral terms of third degree in the model of the earth's gravitational field. The geometry of a highly eccentric orbit causes the time of perigee passage to be determined with exceptional accuracy. It may be that laser or other precision tracking along the perigee arc of a highly eccentric orbit would yield precision determination of the solid-earth lunar and solar tides.

ACKNOWLEDGEMENTS

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Table 1

Estimated Effects of the Resonant Terms on IMP 4 Orbit

	Case I: Initial orbital elements hp=200 km, e=.9419, i=67.20, $\dot{M}$=.05899 rad/hr			Case II: hp=3145 km, e=.9151, i=66.81, $\dot{M}$=.06017 rad/hr (orbital elements 200 days after launch)			
(l, m, p, q)	Beat Period (Days)	Δ Central Angle (°)	Δ Perigee Time (min)	Beat Period (Days)	Δ Central Angle (°)	Δ Perigee Time (min)	% reduction of perigee effect
3, 3, 1, 12	235	.462	8.4	242	.209	3.6	57
4, 3, 2, 13	236	.211	3.7	242	.059	1.0	73
5, 3, 3, 14	236	.073	1.3	242	.013	.2	85
6, 3, 3, 14	236	.042	.7	242	.005	.1	86
6, 6, 3, 26	118	.023	.4	121	.003	.1	75

Δ (Central Angle) = Δ (M. A.) + $\Delta\omega$ + $(\Delta\Omega)$ cosi

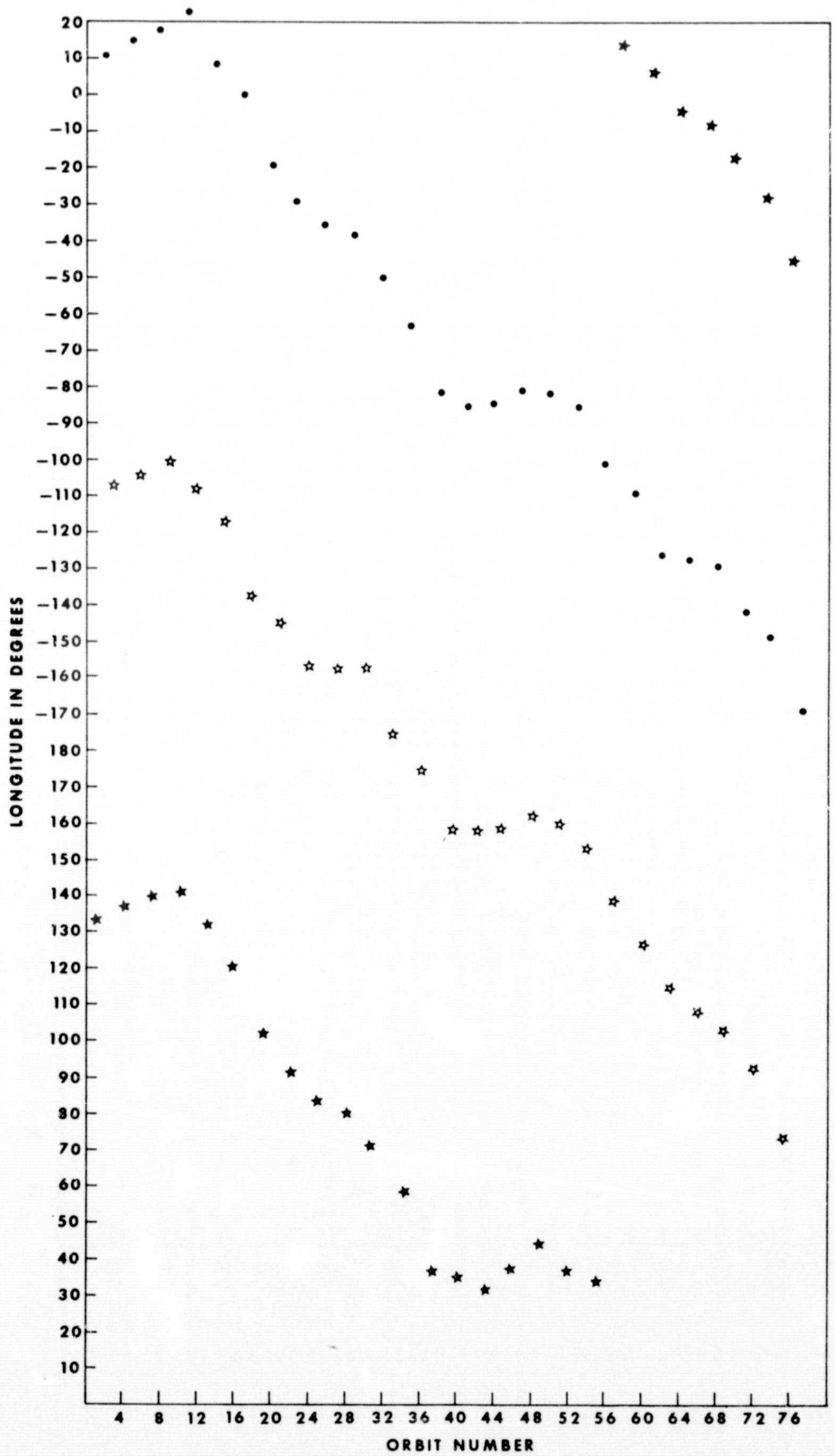

Figure 1. Ground Track of the Longitude at Perigee Passage

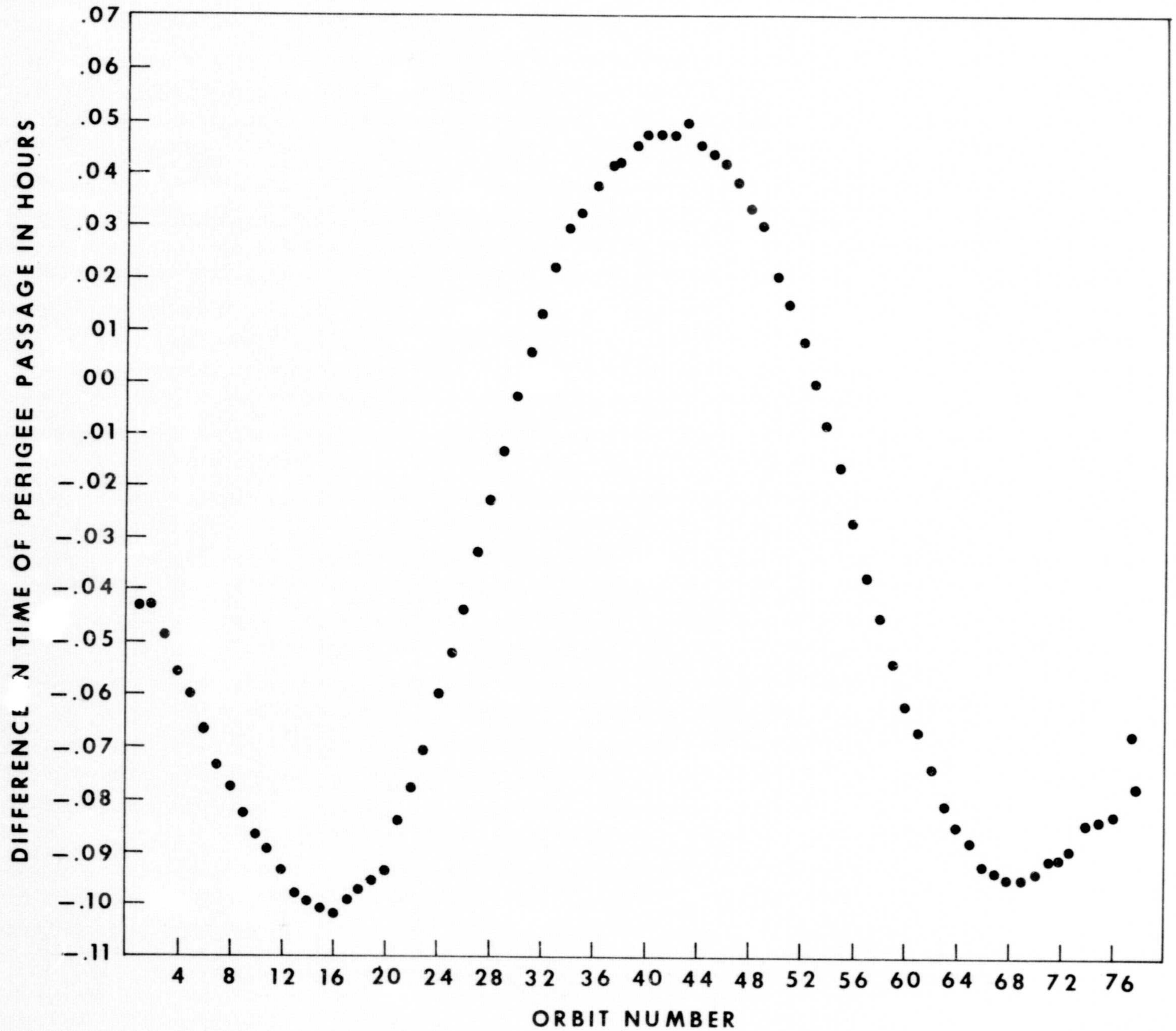

Figure 2. Effect of the Tesseral Harmonic On Time of Perigee Passage

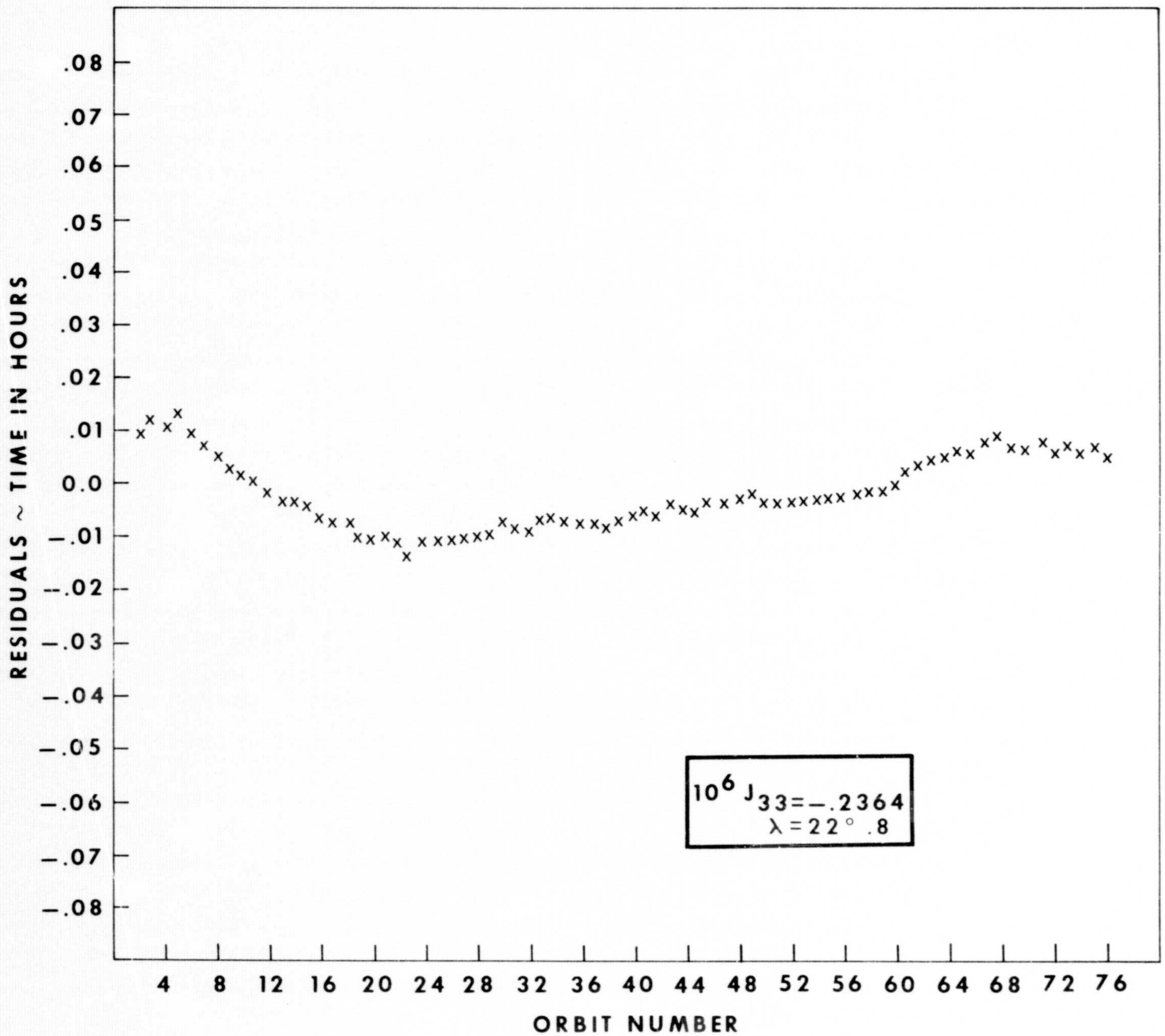

Figure 3. Residuals With Tesseral Harmonic Included