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DISCRIMINATION OF BRIEF EMPTY TIME INTERVALS

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by

Ramona Carbotte and A. B. Kristofferson

Technical Report No. 21

March 1971

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DEPARTMENT OF PSYCHOLOGY

McMASTER UNIVERSITY

HAMILTON, ONTARIO

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ABSTRACT

This paper presents experimental tests of three quantitative models of psychological time. In two of these, psychological time is a continuous variable, and a statistical decision theory analysis is used to obtain predicted performance measures. The third is a finite state model. It assumes that there is a fixed time base underlying the measurement (coding) process, independent of sensory input. A measure of an interval is the number of time points occurring within it. When asked to choose the longer of a pair of intervals, the observer is in a state of uncertainty when the coding of the two intervals have the same value. In this case, it is assumed that the stimulus pattern has been stored and can be recoded a second time.

The first experiment involved forced choice discrimination of adjacent intervals, defined by a pattern of three brief auditory signals, with a base duration of 156 ms. The second also involved adjacent intervals, but the observers made same-different judgments. In the third, the intervals were separated by two seconds; discrimination functions were obtained for two base durations (100 and 200 ms), using a forced choice procedure.

The continuous models are shown to be inadequate, while the counting model accounts for the results of the three experiments reasonably well. Estimates of the period of the time base are of the order of 25, 50 and 100 ms.

Introduction: Quantitative theories of time perception

There are probably many ways in which an individual codes time information, depending on the order of magnitude and the type of time intervals being dealt with. One type of interval is called "filled" because the interval is defined by the duration of a stimulus pulse which remains present throughout the interval. Two filled intervals of different durations actually differ in the total amount of stimulus energy which they contain as well as in their durations, and discrimination between them may depend upon either time or energy or both. Although time, like space, cannot be isolated from energy input, it does have a quantitative aspect independent of the energy conveying it. An interval bounded by two brief signals s_1 and s_2 is referred to as an empty interval, and two such patterns differing only in the time between onsets of s_1 and s_2 differ in their time input but not in their energy content. The experiments to be reported here deal with empty intervals, and the three models to be tested all assume that the discrimination is based solely upon the time information contained in intervals.

A. Creelman's model

Among the few quantitative theories proposed for time discrimination is one formulated by Creelman (1962). It assumes a mechanism which counts the discharges of a very large number of independent units firing during the interval T to be measured. The probability density distribution for the number of counts (N) from this random source can be closely approximated by a normal distribution with mean and variance λT , when λT is large. The parameter λ is a constant reflecting the rate of firing of the pulse source. The response strategy is assumed to be one which associates the longer interval with the

larger count. In order to obtain a predicted performance measure, consider the difference ΔN in counts obtained in the two intervals presented on each trial in a forced choice (FC) task. It is assumed that the count from the first interval is always subtracted from the count obtained in the second interval. The mean of this difference distribution will be $\lambda \cdot \Delta T$ for the S_2 patterns ($T, T + \Delta T$) and $-\lambda \Delta T$ for the S_1 patterns ($T + \Delta T, T$), where ΔT is the difference between the two durations. In both cases the variance of the difference distribution is $2 \lambda T + \lambda \cdot \Delta T$. When ΔN is greater than some criterion value the decision is to choose the second interval as the longer of the pair. The corresponding measure of sensitivity is

$$d' = \frac{2 \lambda^{\frac{1}{2}} \Delta T}{(2T + \Delta T)^{\frac{1}{2}}} \quad (1)$$

Creelman modifies this measure by including a factor $(1 + KT)^{-\frac{1}{2}}$ to take into account memory loss of the count of the first interval while the count during the second is being obtained. He also adds a term σ_v^2 to the variance $(2 \lambda T + \lambda \Delta T)$ to account for any uncertainty in the beginning and ending of the signals marking the durations. This parameter is an inverse function of signal-noise ratio. The resulting expression for the detectability of a given duration difference ΔT is then

$$d' = \frac{1}{(1+KT)^{\frac{1}{2}}} \cdot \frac{2 \lambda^{\frac{1}{2}} \Delta T}{(2T + \Delta T + \sigma_v^2)} \quad (2)$$

for the forced choice situation.

He tested this model with filled auditory intervals separated by .8 sec, and concluded that his model provided a good description of the data over the range of T used: from .04 to 1.0 sec. However, for one experiment the data seem to be poorly fit by the predicted functions. This is one which jointly varies three variables (T , ΔT , and signal voltage, v_s) in such a way that the ratio $\Delta T/T$ was constant for all T and the energy in the increment ΔT also was kept constant. As T and ΔT were increased, v_s was decreased. He assumes that the only effect of decreased signal intensity is to increase the ambiguity in onset and offset of the durations, and so to increase σ_v^2 . It is possible, though, that λ might also be affected by signal voltages, reaching an asymptotic value as v_s increases beyond a certain level. He has not directly tested what seems to be a basic underlying assumption, that the discrimination of the intervals is made independently of the total energy contained in them.

B. Quantal counting models

Kristofferson (1965) has proposed a counting model of a very different nature (from Creelman's) for duration discrimination. In three quite different types of experiments he has obtained behavioral constants of the order of 50 ms, highly correlated (within individuals) with each other and with the half period of the alpha frequency (1967). He has formulated a theory of psychophysical time involving a very stable internal periodic process, independent of ongoing sensory events. This periodic process, or clock, is looked upon as generating time points which mark off equal units, or quanta, of time. The time points have an important role in controlling the gating of incoming sensory input

and in regulating the flow of information through the central nervous system. He has suggested that another possible function for this clock is in coding time information over some range of durations. An interval could be coded in terms of the number of time points occurring during it. This count is independent of the total amount of energy input; it depends only on when the interval begins and ends, and not on how it is defined to the observer. In testing this model he used filled intervals defined by the time between successive offsets of a visual and an auditory stimulus. No conclusions were drawn about the adequacy of the model, but it did look promising both as a means of investigating duration discrimination, and as a possible extension of his temporal quantum theory.

The model suggested by Kristofferson can be tested in a situation involving adjacent empty intervals, T_1 and T_2 , defined by 3 very short signals $s_1 - s_2 - s_3$ chosen so as to be easily detected by the observer. On any trial, either $T_1 > T_2$ or $T_2 > T_1$, and the S is to indicate which is the longer interval. If there is a fixed time base underlying the measurement process, the coding of the two intervals will not be independent, and this dependence is reflected in the shape of the predicted discrimination functions, $P(C)$ vs ΔT .

The basic assumptions of the quantal counting models are as follows. The brief successive signals $s_1 - s_2 - s_3$ each define a point in time for the subject in such a way that the internal intervals which are produced (I_1 and I_2) are equal to the corresponding external intervals T_1 and T_2 on every presentation. This strong assumption is likely to require revision but it is used here to simplify the argument. The intervals I_1 and I_2 are considered as being superimposed on a stable time base which is not affected by ongoing sensory events, with time points occurring at constant intervals of q ms. (Fig. 1). If the

durations of I_1 and I_2 are expressed in terms of these q-units, we have

$$I_1 = (m+b_1)q \text{ with } m \text{ an integer and } 0 \leq b_1 < 1$$

$$I_2 = (n+b_2)q \text{ with } n \text{ an integer and } 0 \leq b_2 < 1.$$

If I_1 begins anywhere within the last $b_1 \cdot q$ ms of an ongoing quantum, or ends anywhere within the first $b_1 \cdot q$ ms of a quantum, $m+1$ time points will occur during I_1 since there will be m complete quanta within I_1 . Otherwise m time points will occur during I_1 , bounding $m-1$ complete quanta. (See fig. 1.)

Assume that for later comparison the intervals I_1 and I_2

Fig. 1 about here

are coded in terms of the number of time points occurring within each interval.

Then I_1 is coded as t_1 , with

$$P(t_1 = m) = 1 - b_1$$

$$P(t_1 = m + 1) = b_1$$

This gives a quantal, probabilistic internal representation of external time.

When I_1 and I_2 are successive, where I_2 begins will depend on where I_1 begins, with respect to the time base. Thus the coding of I_2 will depend on that of I_1 , or equivalently, on the total duration T_D of T_1 and T_2 . Kristofferson assumed that the coding of the second interval was independent of that of the first; the intervals to be discriminated were separated by approximately 8 sec.

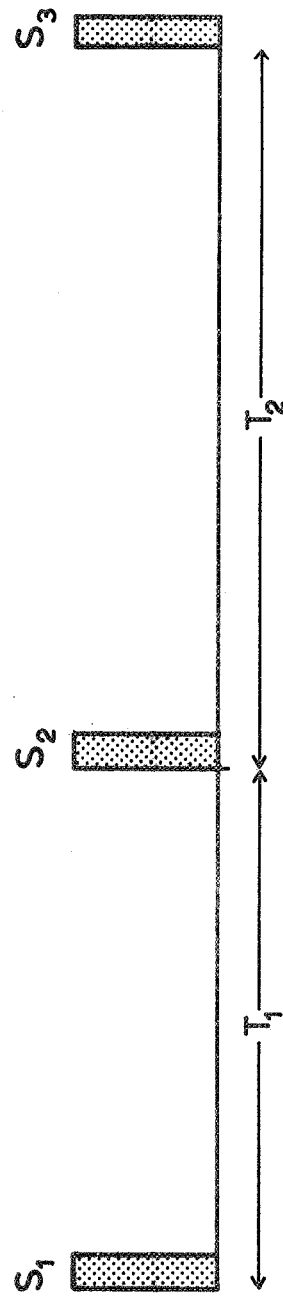


Fig.1: A pattern of 3 signals S_1 - S_2 - S_3 defining 2 successive time intervals, T_1 and T_2 , superimposed on a hypothetical internal time base.

In deciding whether T_1 or T_2 is the longer interval, the integers t_1 and t_2 resulting from the coding process are compared and a correct choice is made when there is a difference of at least one between these values. When $t_1 = t_2$, the choice is arbitrary. If the response alternatives are denoted by 1 ("T₁ is longer") or 2 ("T₂ is longer"), the decision process is summarized by

$$P(1 \mid t_1 > t_2) = 1$$

$$P(1 \mid t_1 < t_2) = 0$$

$$P(1 \mid t_1 = t_2) = k' \quad \text{with} \quad 0 \leq k' \leq 1$$

These coding and decision processes can be schematically summarized by the two probability tree diagrams shown in Fig. 2. S_1 is a stimulus pattern in which $T_1 > T_2$ and S_2 is one in which $T_2 > T_1$. If $P(S_1)$ ($P(S_2)$) is the

Fig. 2 about here

probability of pattern S_1 (S_2) occurring on a trial, the probability $P(C)$ of correctly choosing the longer interval over a series of trial is

$$\begin{aligned} P(C) &= P(1 \mid S_1) \cdot P(S_1) + P(2 \mid S_2) \cdot P(S_2) \\ &= \frac{1}{2} [\alpha + k' \gamma + \alpha' + (1 - k') \gamma'] \quad \text{when } P(S_1) = P(S_2) = \frac{1}{2}. \end{aligned}$$

However, given S_2 , we can consider it as being the S_1 pattern reversed. The number of time points in I_2 depends on where I_2 ends, which in turn "determines" where I_1 ends; i.e., determines the number of points in I_1 . Thus the calculation

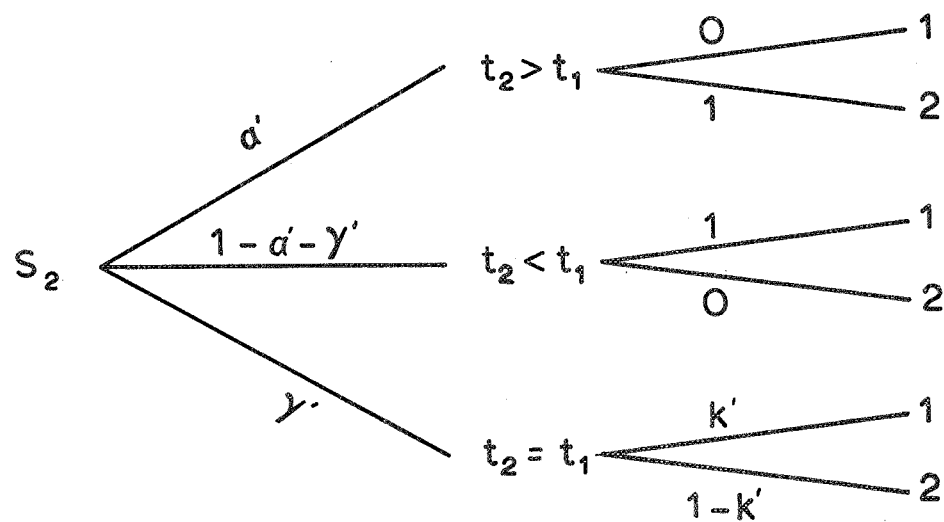
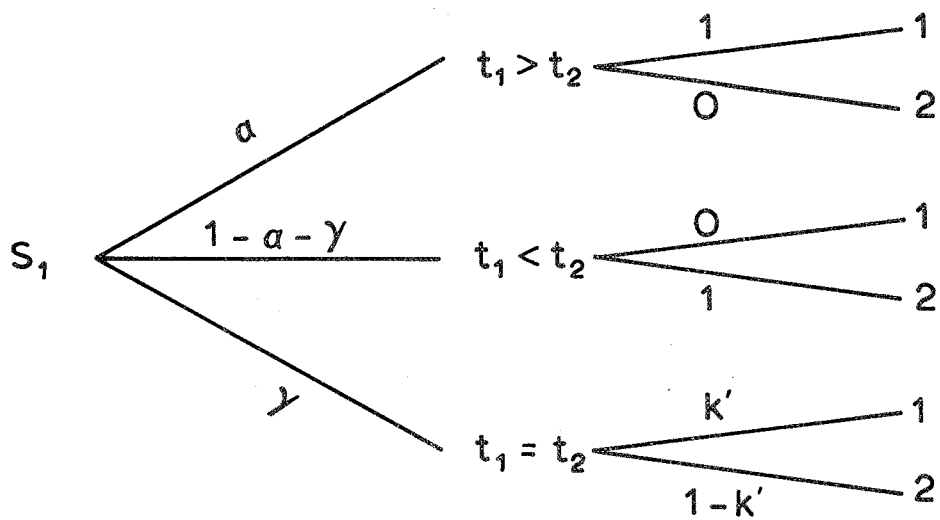


Fig.2: The quantal counting model: a schematic representation of the outcomes of the coding and decision processes.

of α' and γ' given S_2 could be considered as equivalent to the calculation of α and γ given S_1 . Hence, $\alpha' = \alpha$ and $\gamma' = \gamma$, and we have

$$P(C) = \alpha + \gamma/2$$

For a series of (T_1, T_2) pairs, let the shorter interval in each pair have the fixed value T_s and let the longer interval be variable T_v . Then S_1 is the pair (T_v, T_s) and S_2 is (T_s, T_v) . For any given difference, $\Delta T = T_v - T_s$, α and γ are obtained by considering how many time points can occur during I_1 and I_2 , and the relative probabilities of these numbers. Assuming a given particular value of q , we have

$$I_s = (s + b_s)q \quad 0 \leq b_s \leq 1$$

$$I_v = (v + b_v)q \quad 0 \leq b_v \leq 1$$

$$I_D = I_s + I_v = (d + b_d)q \quad 0 \leq b_d \leq 1$$

with s , v , and d being integers. Several cases must be considered. If $v = s+2$, T_v will always be discriminated as longer than T_s since I_v will contain either $s + 2$ or $s + 3$ time points while I_s can contain only s or $s + 1$ time points. For both $v = s$ and $v = s + 1$, the cases $b_v + b_s \leq 1$ and $b_v + b_s \geq 1$ have to be taken separately. The calculations are described in greater detail in Appendix I.

The resulting parametric equations for the predicted duration discrimination functions are

$$P(C) = \frac{1}{2} \cdot (1 - b_s + b_v) \quad \Delta T/q = b_v - b_s \quad \text{for } b_s \leq b_v \leq 1$$

$$\begin{aligned}
P(C) &= \frac{1}{2} \cdot (2 - b_s) & \Delta T/q &= 1 - b_s + b_v & \text{for } 0 \leq b_v < 1 - b_s \\
P(C) &= \frac{1}{2}(1 + b_v) & \Delta T/q &= 1 - b_s + b_v & \text{for } 1 - b_s \leq b_v < 1 \\
P(C) &= 1 & & & \text{for } \Delta T/q \geq 2 - b_s
\end{aligned} \tag{3}$$

These functions are shown in Fig. 3a for three values of b_s : 0, 1/3, 3/4. For all values of b_s except $b_s = 0$, there is a region of ΔT where $P(C)$ is independent of ΔT . The higher the level of $P(C)$ at which it occurs, the longer it is; it begins when $\Delta T = (1 - b_s)q$, and has length $(1 - b_s)q$. Before and after this level segment, the functions are linear with slope independent of b_s or b_v . $P(C)$ reaches 1 for ΔT having some value between q and $2q$ ms: $P(C) = 1$ when $\Delta T = (2 - b_s)q$ ms.

Figure 3 about here

It is of interest to compare (3) to the relation between $P(C)$ and ΔT if the count in T_2 were independent of the count in T_1 . In this case the parametric equations are

$$\begin{aligned}
P(C) &= \frac{1}{2}(1 + b_v - b_s) & \Delta T/q &= b_v - b_s & b_s \leq b_v < 1 \\
P(C) &= \frac{1}{2}(2 - b_s + b_v \cdot b_s) & \Delta T/q &= 1 + b_v - b_s & 0 \leq b_v < 1
\end{aligned} \tag{4}$$

Figure 3b shows the functions for three values of b_s : 0, 1/2, 3/4. These are equivalent to the equations derived by Kristofferson (1965). The increase

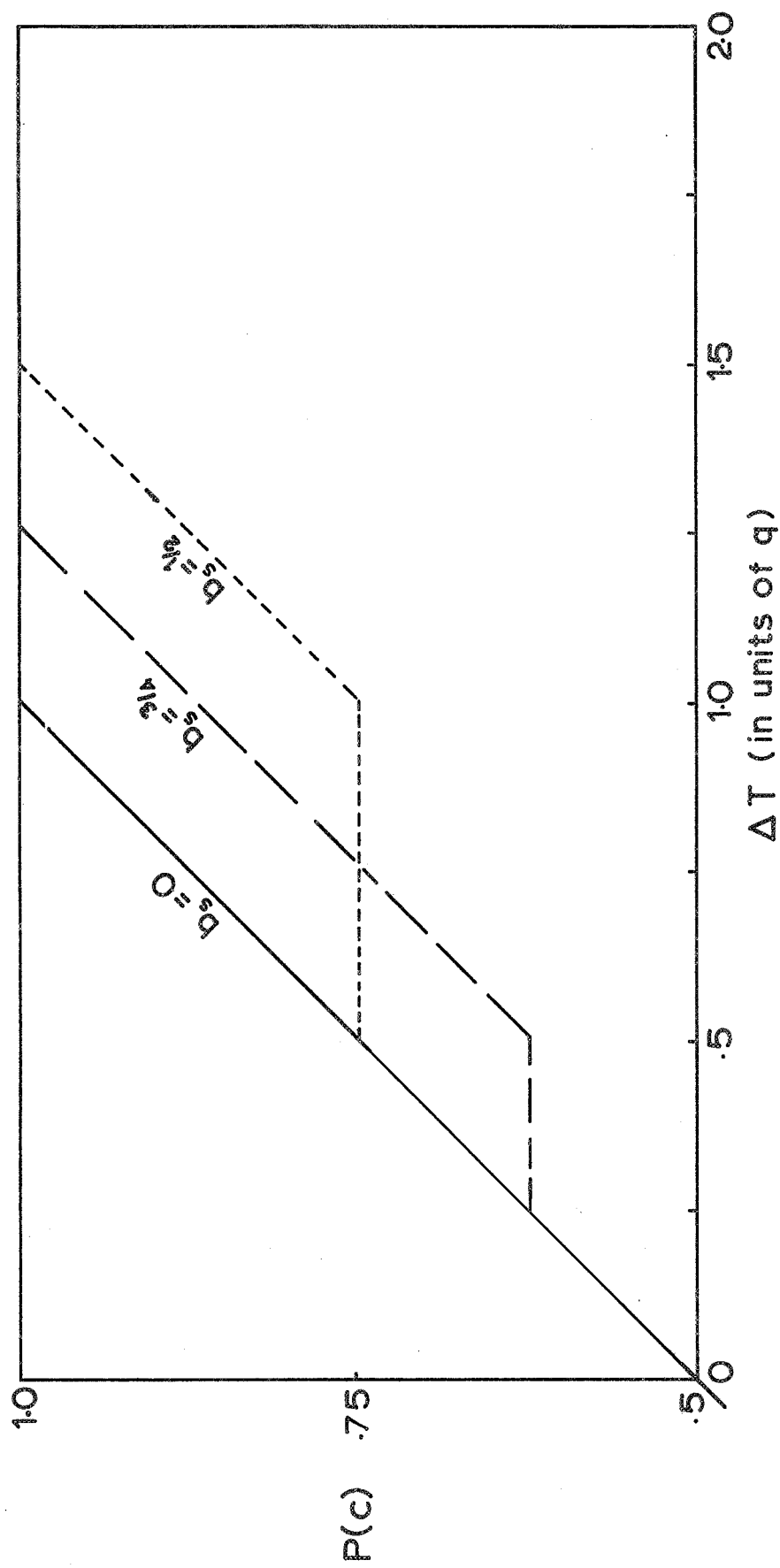


Fig. 3(a): Predicted duration discrimination functions; coding of T_2 dependent on coding of T_1 .

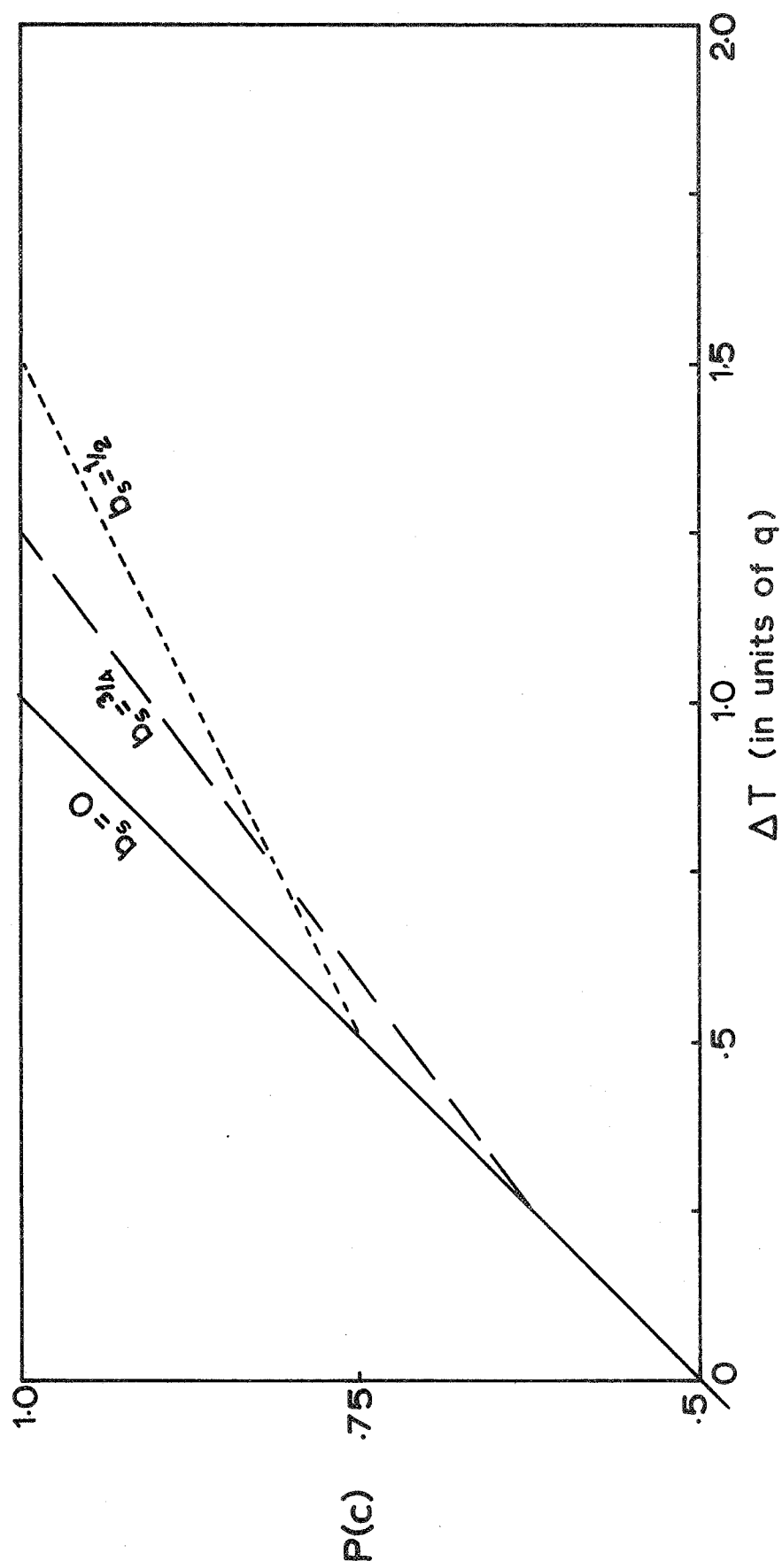


Fig. 3(b): Predicted duration discrimination functions; coding of T_2 independent of coding of T_1 .

in $P(C)$ for $0 \leq \Delta T/q < 1 - b_s$ is identical to that for the dependent case, and the rise of $P(C)$ as it increases from .5 to 1 spans the same distance on the ΔT axis. However, there is now no level region, and the slope of the second segment does depend on b_s . In both the dependent and independent cases, changes in the base duration, T_s , produce substantial changes in the functions because of the change in the parameter b_s .

C. The quantal onset-offset model

In the quantal counting model, psychological time is dealt with as a discrete variable. Creelman's model treats it as a continuous variable, with a statistical decision theory type of analysis for linking the time measurement process with the decision process, and then predicting performance. However, an alternative model yielding time as a continuous variable has been proposed by Allan, Kristofferson and Weins (1970). Whenever a fixed interval T is presented to the observer, the size of its corresponding psychological representative I takes on some value within a certain specifiable range. The variability in I is assumed to be due entirely to variability in the delays between onset of T and onset of I , and offset of T and offset of I . The nature of the timing process is left unspecified, except that it yields measures on a continuous variable I , with

$$I = T + d_{\text{off}} - d_{\text{on}}$$

being the measure of the internal interval corresponding to T . The delay in signaling the onset (offset) for the timing of I is denoted by d_{on} (d_{off}). If it is assumed that these delays are independent, and that they are uniformly distributed random variables with $0 \leq d_{\text{on}}, d_{\text{off}} \leq q$, the probability distribution

for I is triangular. It is defined over the range $T - q \leq I \leq T + q$ with $E(I) = T$ and $\text{var}(I) = q^2/6$.

For two adjacent intervals defined by the time between the onsets of three brief stimuli $s_1 - s_2 - s_3$, the measures I_1 and I_2 of the intervals T_1 and T_2 are dependent in that the delay in registering the offset of I_1 is equal to the delay in registering the onset of I_2 . For the two adjacent intervals in Fig. 4,

$$I_1 = T_1 + d_2 - d_1$$

$$I_2 = T_2 + d_3 - d_2$$

Figure 4 about here

The observer stores (without memory loss) the value of I_1 while I_2 is being measured, and he then compares these values. If the difference $\Delta I = I_2 - I_1$ is greater than a criterion value, c , he says that the second interval was longer; otherwise he chooses the first as longer. On each trial

$$\Delta I = T_2 - T_1 + d_3 + d_1 - 2d_2$$

with $E(\Delta I) = \Delta T$, and $\Delta T - 2q \leq \Delta I \leq \Delta T + 2q$.

To obtain the probability distribution for ΔI , first define the random variable X by

$$X = d_3 + d_1 - 2d_2$$

When d_1 , d_2 , and d_3 are independent, $E(X) = 0$ and $\text{var}(X) = q^2/2$. The probability

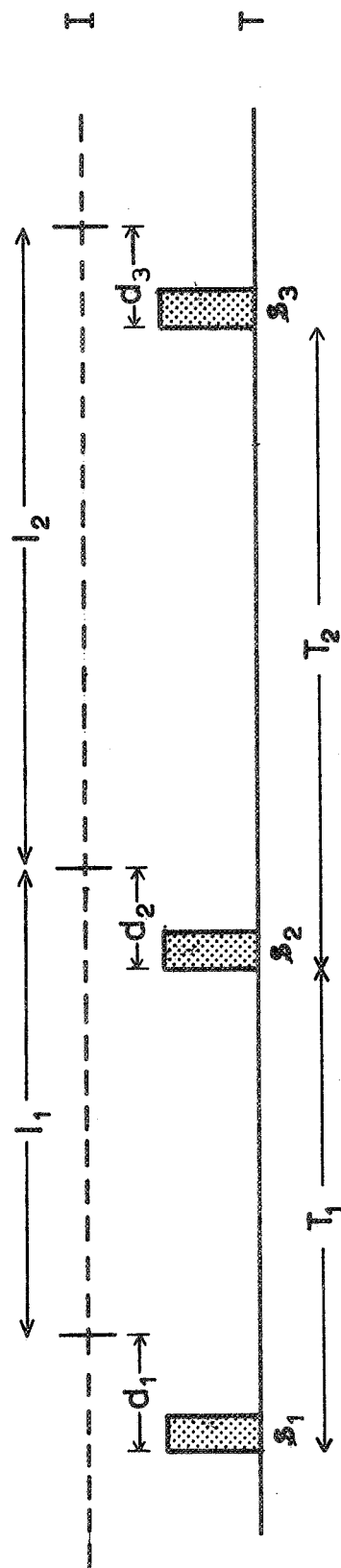


Fig.4: A hypothetical relation between external (T) and internal (I) time; d_1 and d_2 are the delays in registering the onsets of T_1 and T_2 , d_2 and d_3 are delays in registering the offsets of T_1 and T_2 .

distribution for X (derived in Appendix II) consists of 3 parabolic segments

$$\begin{aligned} f_1(X) &= (2q + X)^2 / 4q^3 & \text{for } -2q \leq X \leq -q \\ f_2(X) &= (2q^2 - X^2) / 4q^3 & \text{for } -q \leq X \leq q \\ f_3(X) &= (2q - X)^2 / 4q^3 & \text{for } q \leq X \leq 2q \end{aligned} \quad (5)$$

The expressions for the cumulative distribution giving the area under this distribution (the $Q^{(3)}$ curve) as a function of the distance of a point c from the mean, with all distances measured in units of q , are also given in Appendix II. A table of areas under this curve is then used in exactly the same way as a table of areas under the standardized normal curve; q corresponds to the standard deviation σ , and $(\Delta I - \Delta T)/q$ is analogous to a z-score, $(X - \mu)/\sigma$.

The corresponding probability distribution for ΔI (given the occurrence of an S_2 pattern) is now obtained by setting $X = \Delta I - \Delta T$ in equations (5) above. (For S_1 , set $X = \Delta I + \Delta T$.) For a particular value of ΔT , once we have $P(1 | S_1)$ and $P(2 | S_2)$ we obtain the distance of c from the means of both distributions in units of q by using the table of areas under the $Q^{(3)}$ curve. Figure 5 illustrates the relation between the empirical probabilities $P(1 | S_1)$ and $P(2 | S_2)$, and the areas under the difference distributions. It is assumed that the S's response is "2" whenever ΔI is greater than c , and is "1" otherwise. When pattern S_1 occurs, the corresponding value of ΔI on

Figure 5 about here

which the decision is based will be drawn from the distribution with mean $-\Delta T$

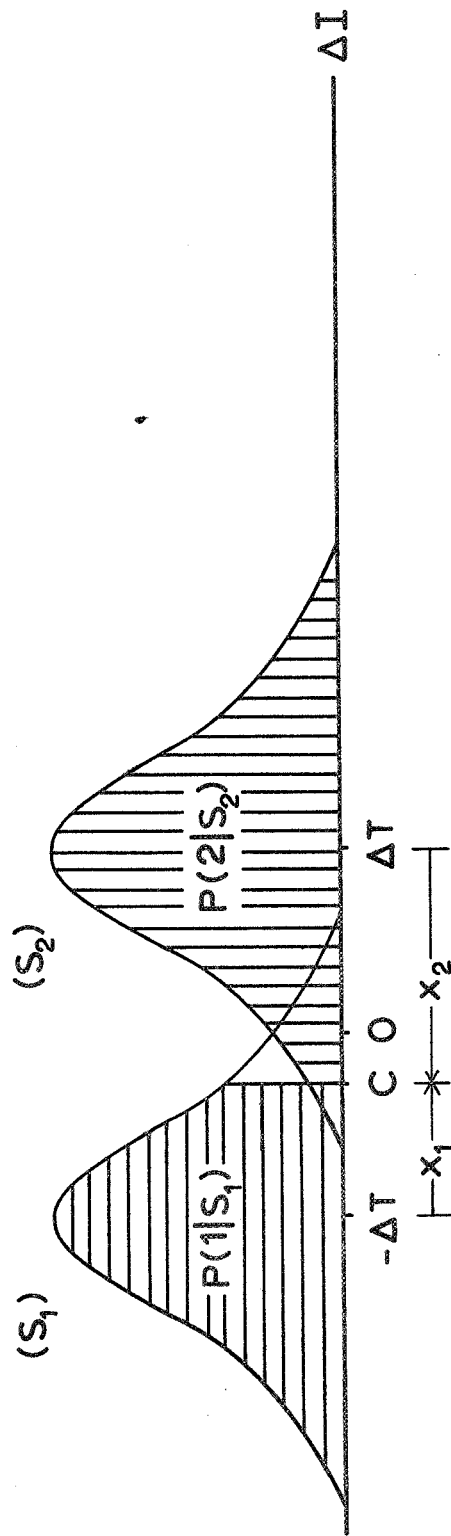


Fig.5: Probability distributions for the difference ΔI of the internal measures of intervals T_1, T_2 , conditional on the occurrence of pattern S_1 or S_2 . All distances are expressed in units of q .

and defined over the range $(-\Delta T - 2q) \leq \Delta I \leq (-\Delta T + 2q)$. The area under the S_1 distribution to the left of c will be the probability of correctly saying that T_1 was the longer interval, when S_1 is presented over a series of trials. Similarly, for S_2 the set of possible values of ΔI has the same probability distribution but now with mean ΔT and defined over $(\Delta T - 2q) \leq$

$\Delta I \leq (\Delta T + 2q)$. $P(2 | S_2)$ is the probability of correctly choosing T_2 as the longer interval, and corresponds to the probability of obtaining a value of ΔI greater than c over a series of S_2 trials.

Let d_q denote the distance between the means of these distributions. In Fig. 5, x_1 (x_2) is the directed distance of the criterion c from the mean of the (S_1) ((S_2)) distributions. Then

$$d_q = x_1 - x_2 = 2 \cdot \Delta T / q \quad (7)$$

Thus the model predicts a linear relation between ΔT and d_q , with the slope giving an estimate of q . Moreover, from each value of ΔT we can get an estimate of the location of c with respect to the zero point of the ΔI axis. Once we have a best estimate of q and c , we can calculate the predicted discrimination function. For $q = 25$ and $c = 0$, this is shown in Fig. 6. It differs considerably from that predicted by the counting model, as it shows a curvilinear rise, and then a slow approach to 1. as ΔT approaches $4q$. Finally, the model predicts no change in the discrimination function with a change in base duration, since the difference distributions depend only on ΔT and q .

Figure 6 about here

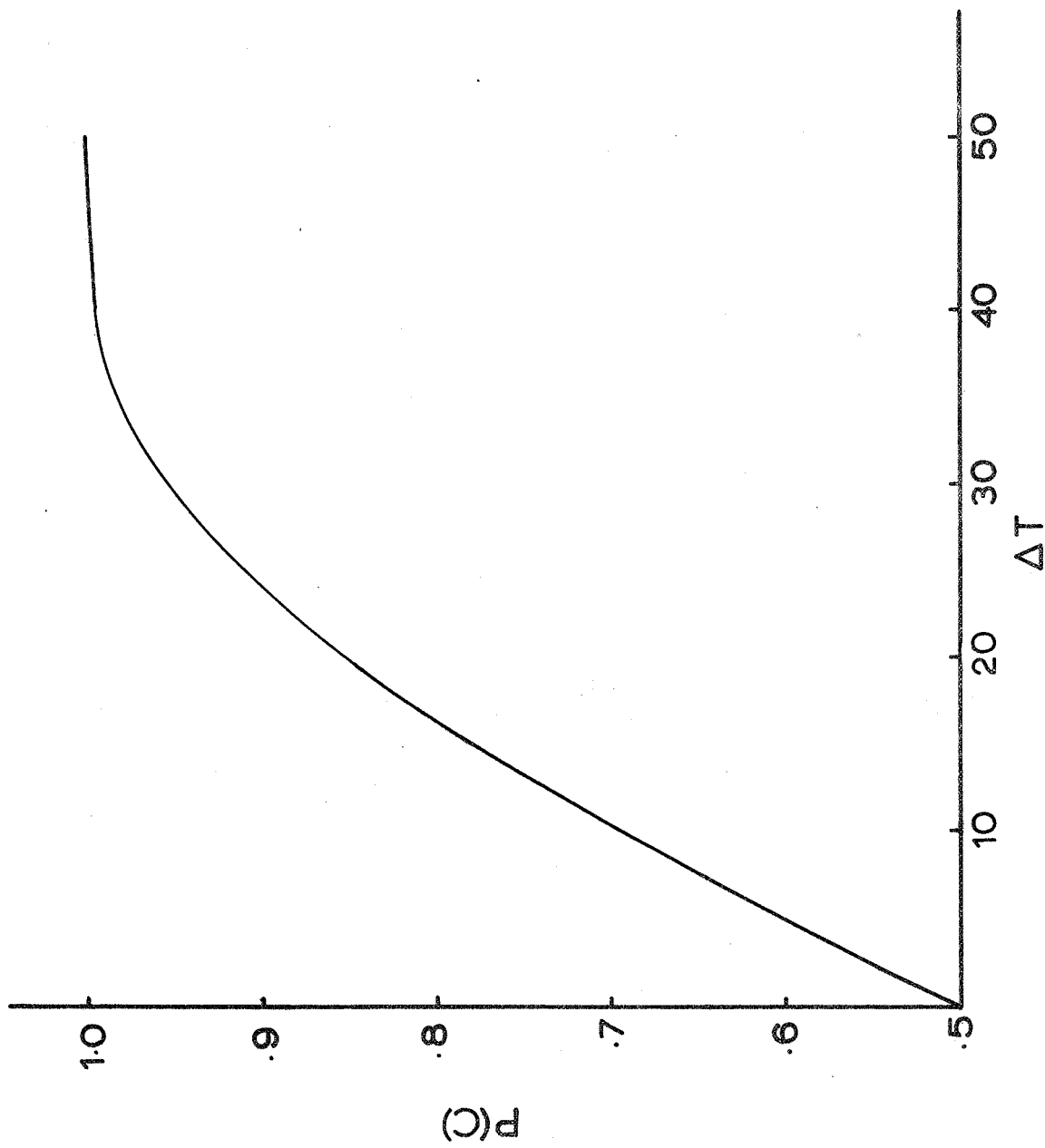


Fig.6. Predicted discrimination function from the onset-offset model, for $q = 25$ ms.

Experiment I: Adjacent Empty Intervals

The first experiment was designed to determine the span and general shape of the duration discrimination function for adjacent empty intervals, given a base duration $T_s = 156$ ms. When $q = 48$ ms, $s = 3$ and $b_s = \frac{1}{4}$; this base duration would result in relatively small numbers (3,4) for the possible counts in T_s if the quantal counting model were valid. It should be kept in mind that q is a parameter which would have to be estimated for each subject individually, and that only qualitative features of the $P(C)$ vs ΔT functions can be predicted from this model. Small changes in q give considerable changes in b_s , which would be reflected in the actual shape of the function.

The first set of ten T_v values (series I) were chosen so that the interval pairs span a ΔT range of 13 to 105 ms. A second series (series II) with $5 \leq \Delta T \leq 59$ ms was run in order to obtain more points on the lower part of the function. For each subject the sessions were continued until he had at least eight successive sessions with approximately the same daily overall error rate $p(e)$. The experimental duration discrimination functions are based on at least 210 trials per point, excluding those sessions before $p(e)$ had stabilized.

All ten values of ΔT occurred in each session; each pair (T_{vi}, T_s) and (T_s, T_{vi}) for $i = 1, \dots, 10$, occurred the same number of times, with the trials on which each would occur being randomly selected at the beginning of each session. Each one hour session was divided into six blocks, with one minute rest periods between blocks. The four paid observers were usually run for one session per day. The auditory signals were presented binaurally over earphones, and Ss were seated in a sound attenuated room. The equipment has been fully described elsewhere (Abel, 1970).

Each of the signals $s_1-s_2-s_3$ was a 3 ms sinusoidal pulse of 2000 cps, intensity .4 rms volts, and rise-decay time of 1 ms. On each trial there occurred a 250 ms auditory warning signal, a delay of 2 seconds, and then the signal pattern. The response time was limited to $4-(T_v+T_s)$ seconds, and if a correct response was made within that time, feedback was given in the form of two 125 ms tones. The next trial began 4 seconds later.

After series I and II, subjects were run under several different conditions intended as exploratory tests of the effects of various changes in procedure on reducing the overall error rate, and on the shape of the discrimination function. One S (KL) ran a series with a 15 ms increase in base duration. Only 6 values of ΔT were used, so that a discrimination function with 240 trials per point was obtained in 6 sessions.

Results

Table 1 shows $P(2/S_1)$, $P(2/S_2)$, and $P(C)$ for series I and II. The corresponding discrimination functions are plotted in fig. 7. There are several common qualitative features in these functions. For three Ss, $P(C) = 1$ for ΔT ranging from 70 to 100 ms, but $P(C)$ has risen from .5 to .9 in half of this distance. Thus there is a longer upper portion of the curve where $P(C)$ slowly approaches 1. The slopes decrease steadily and $P(C)$ does not appear to be a linear function of ΔT along any part of the range except for one S(KL). For this S, the 15 ms change in T_s had no effect; the resulting function overlaps his series II function. In most cases there is a small region where $P(C)$ seems to change very little. For 3 Ss there is one such region around $\Delta T = 20$ to 30 ms. For the fourth, the function is level at the upper end of the series II range, from $\Delta T = 45$ ms onwards.

Table 1 about here

Figure 7 about here

It can be seen from Table 1 that there is a very consistent tendency for choosing T_2 as the longer interval when the difference between the intervals is small. The only exception is HS in series I. Also, in all cases except one, there seems to be a reversal of this "bias" as ΔT becomes larger. Define $P(2)$ by $\frac{1}{2}[P(2/S_2) + P(1/S_1)]$, and $P(1)$ in a similar way. The difference $P(2) - P(1)$ decreases from fairly large positive values to zero or small negative values when ΔT is somewhere in the range 20 to 35 ms. This region corresponds roughly to where the "kinks" occur, i.e., to the region of non-monotonicity in the functions.

Theoretical analysis

(a) Creelman's model

Equation (1) predicts that d' vs $\Delta T/(2T + \Delta T)^{1/2}$ should be a linear relation, as does equation (2) if $\sigma_v^2 = 0$. Fig. 2 shows a plot of these values for series II. Creelman takes $\sigma_v^2 = 0$ for voltages in a range where performance is uninfluenced by increments in signal voltage; $v_s = .4$ rms assumed to be within this range. For two Ss, the points are quite well fit by a straight line over the entire range of ΔT except for the kinks occurring around $\Delta T = 25$ ms. For the other two, only the lower half of the plot is linear. After $\Delta T = 25$ ms, MR shows a systematic deviation while for PC the relation becomes non-monotonic.

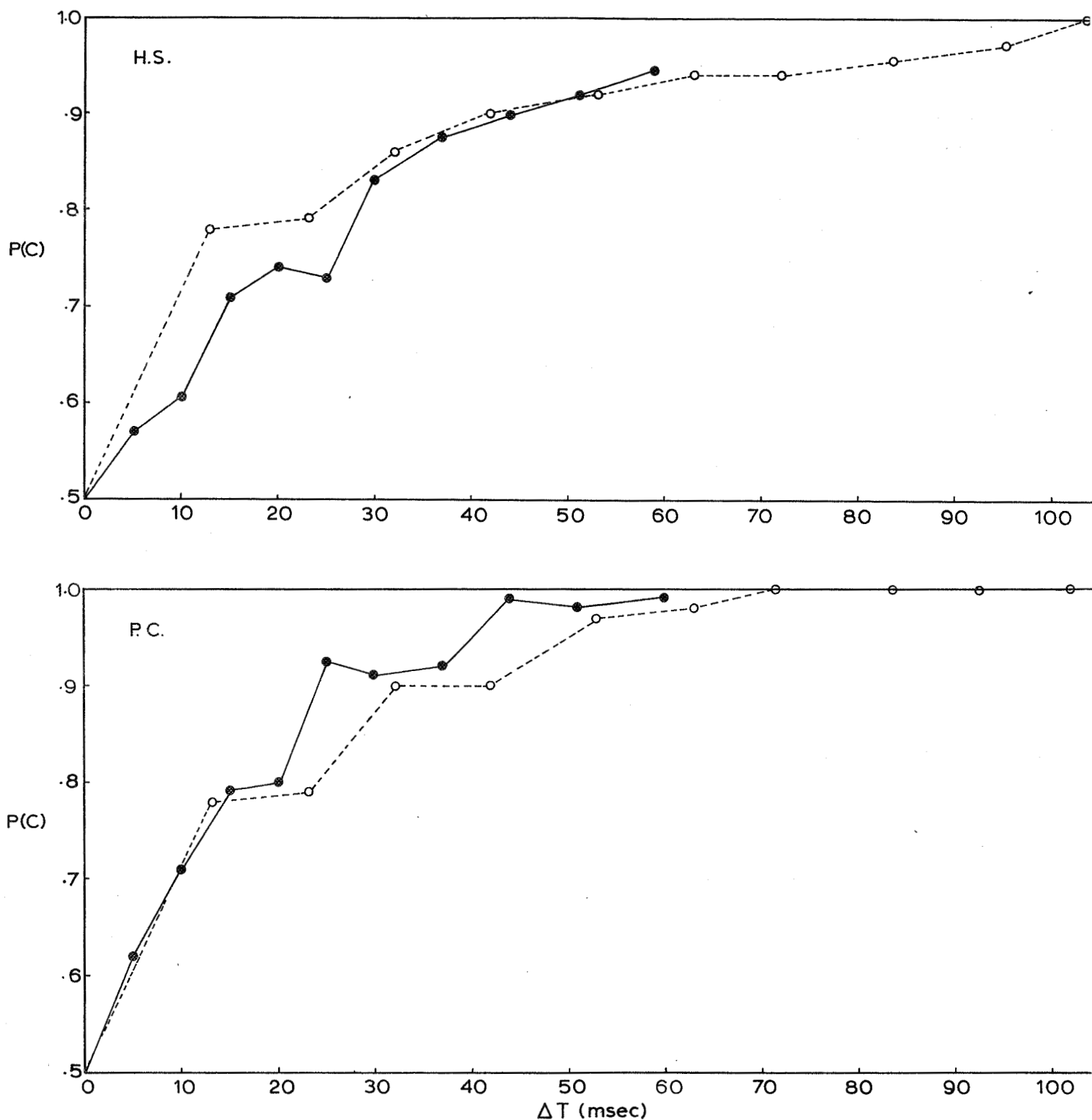


Fig. 7. Duration discrimination functions for adjacent intervals. Series I (o---o) and series II (●—●). $T_s=156$ ms.

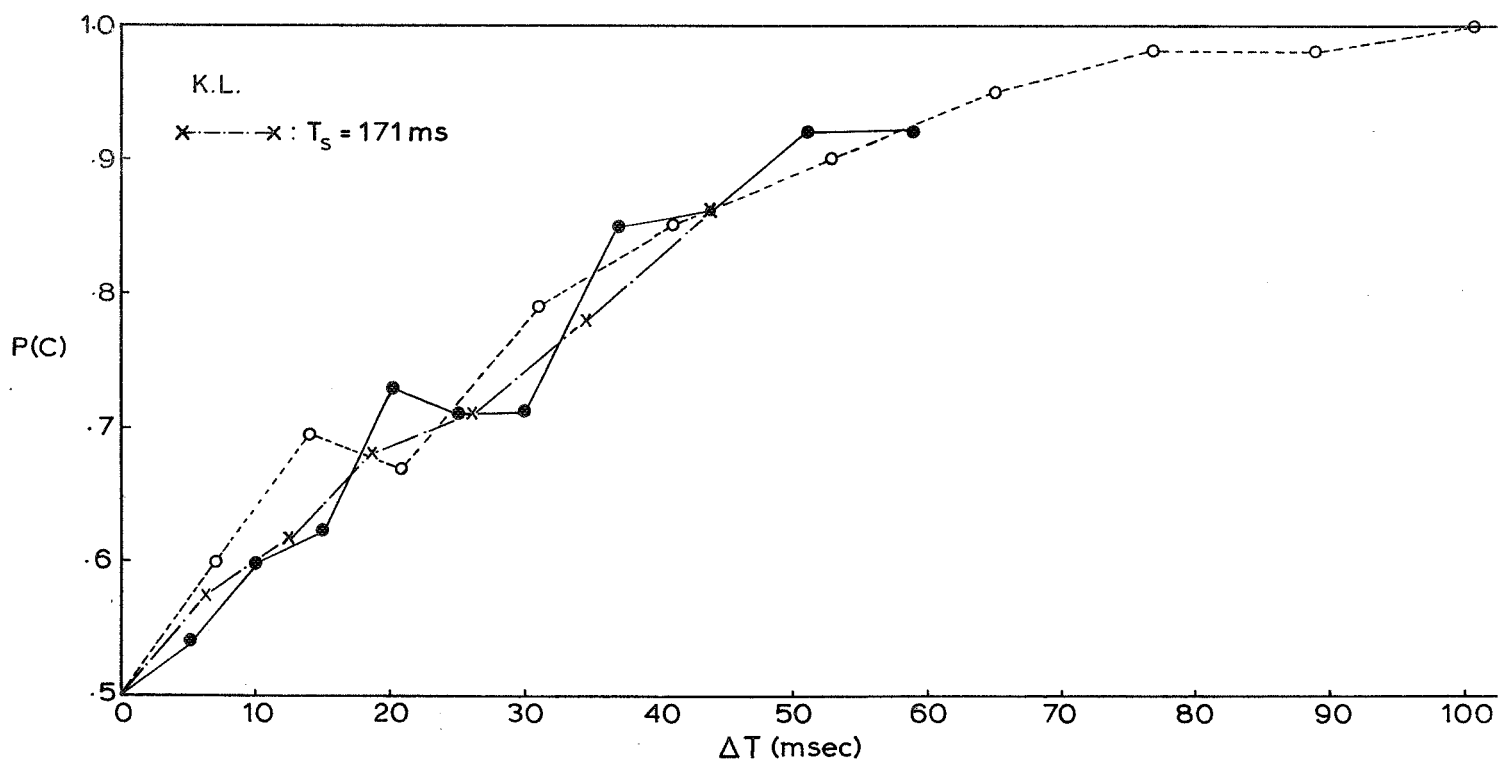
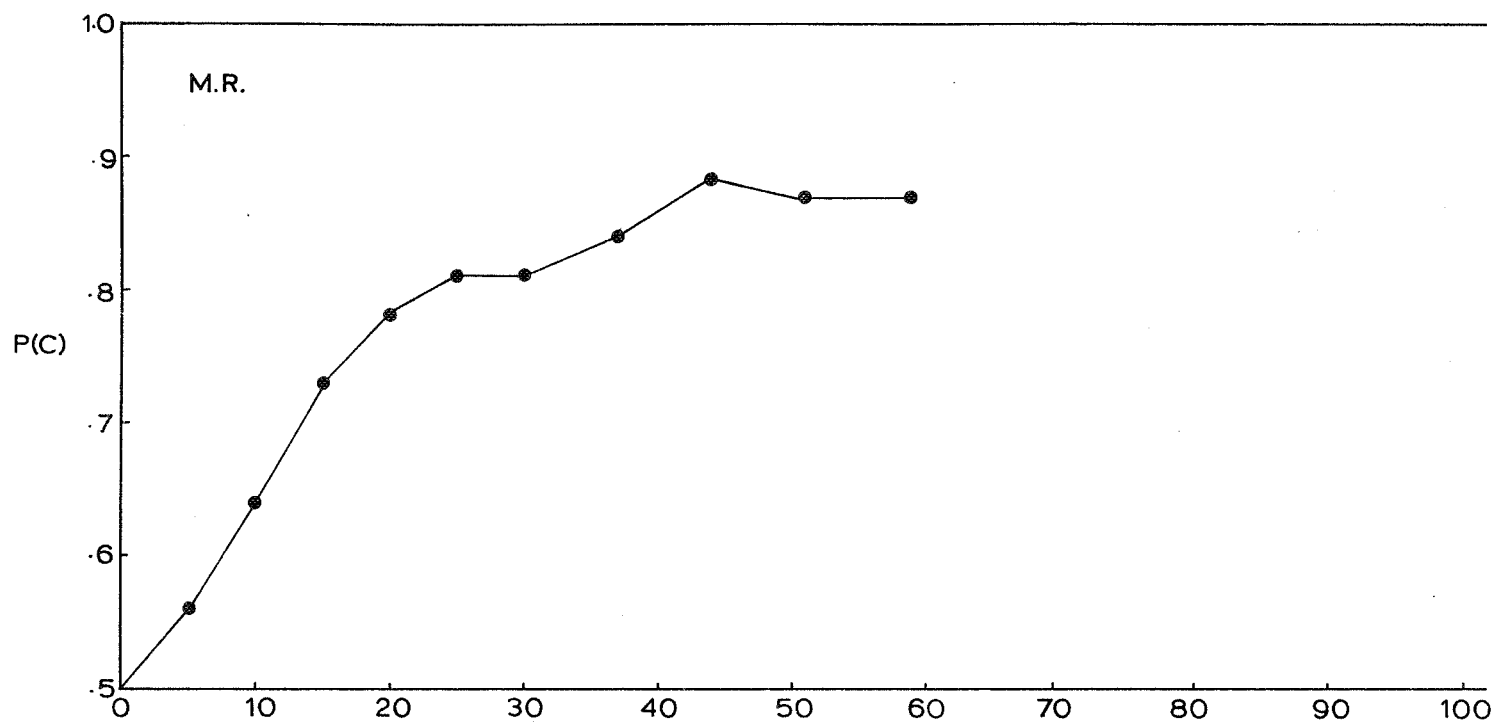


Fig. 7. Duration discrimination functions for adjacent intervals. Series I (o---o) and series II (●—●). $T_s=156$ ms.

The shift, as ΔT increases, of the relative difference between the probability of a T_2 response as compared to that of a T_1 response creates a difficulty for any model of performance using SDT. This change in response bias would correspond to a shift of a decision criterion from some negative value to some positive value on the ΔI axis. Since all values of ΔT are presented intermixed within every session, the decision criterion should be the same for all values of ΔT . It is assumed that the only information that the observer has, on which to base his decision, is some value of ΔI . However, it seems we would have to assume that he also extracts some information regarding the absolute size of ΔI . If ΔI is large he uses a different criterion than he does when ΔI is small. The criterion location for a given value of ΔT is estimated by

$$c = \frac{1}{2} \cdot [z(P(2 | S_2)) + z(P(2 | S_1))].$$

The units in which this distance is expressed change as ΔT changes since, according to the Creelman model, the variance of the ΔI distribution is an increasing function of ΔT . In order to make comparisons between the values of c for different values of ΔT , we will use as a basic unit of distance the standard deviation of the ΔI distribution when $\Delta T = 5$. Hence we multiply c by the factor

$$\frac{(\lambda \cdot (2T + \Delta T))^{1/2}}{(\lambda \cdot (2T + 5))^{1/2}} = \left[\frac{312 + \Delta T}{317} \right]^{1/2}$$

Figure 8b and Table 2 give the criterion location as a function of ΔT for those Ss showing a shift in the difference $P(2) - P(1)$, in series II. There appears

to be a general trend for c to increase from negative to positive values.

Table 2 about here

Figure 8 about here

(b) The quantal onset-offset model

Figure 9 shows d_q as a function of ΔT . It is reasonably linear for only two Ss; the straight lines (fit by eye) yield estimates of q of 55 ms for KL and 44 ms for HS. After $\Delta T = 25$ ms, the function for MR shows a systematic deviation, and that for PC becomes quite variable. A line corresponding to $q = 25$ ms fits the points in the lower half of the ΔT range for PC. As with the Creelman model, we would see a systematic shift of the criterion from negative to positive values with respect to the zero point of the ΔI axis.

Figure 9 about here

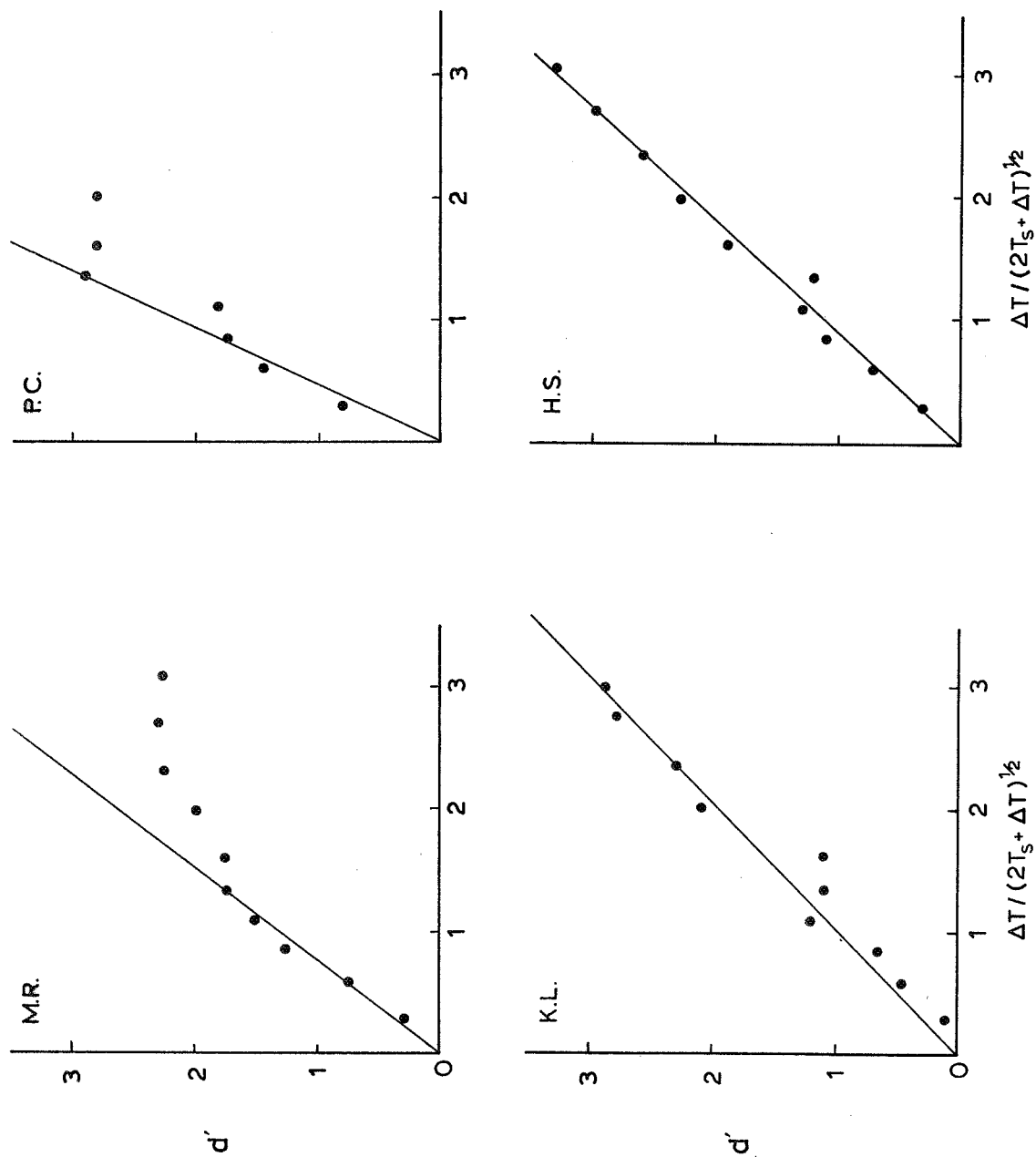


Fig. 8a. Creelman's model: d as a function of $\Delta T / (2T_s + \Delta T)^{1/2}$ for adjacent intervals, series II.

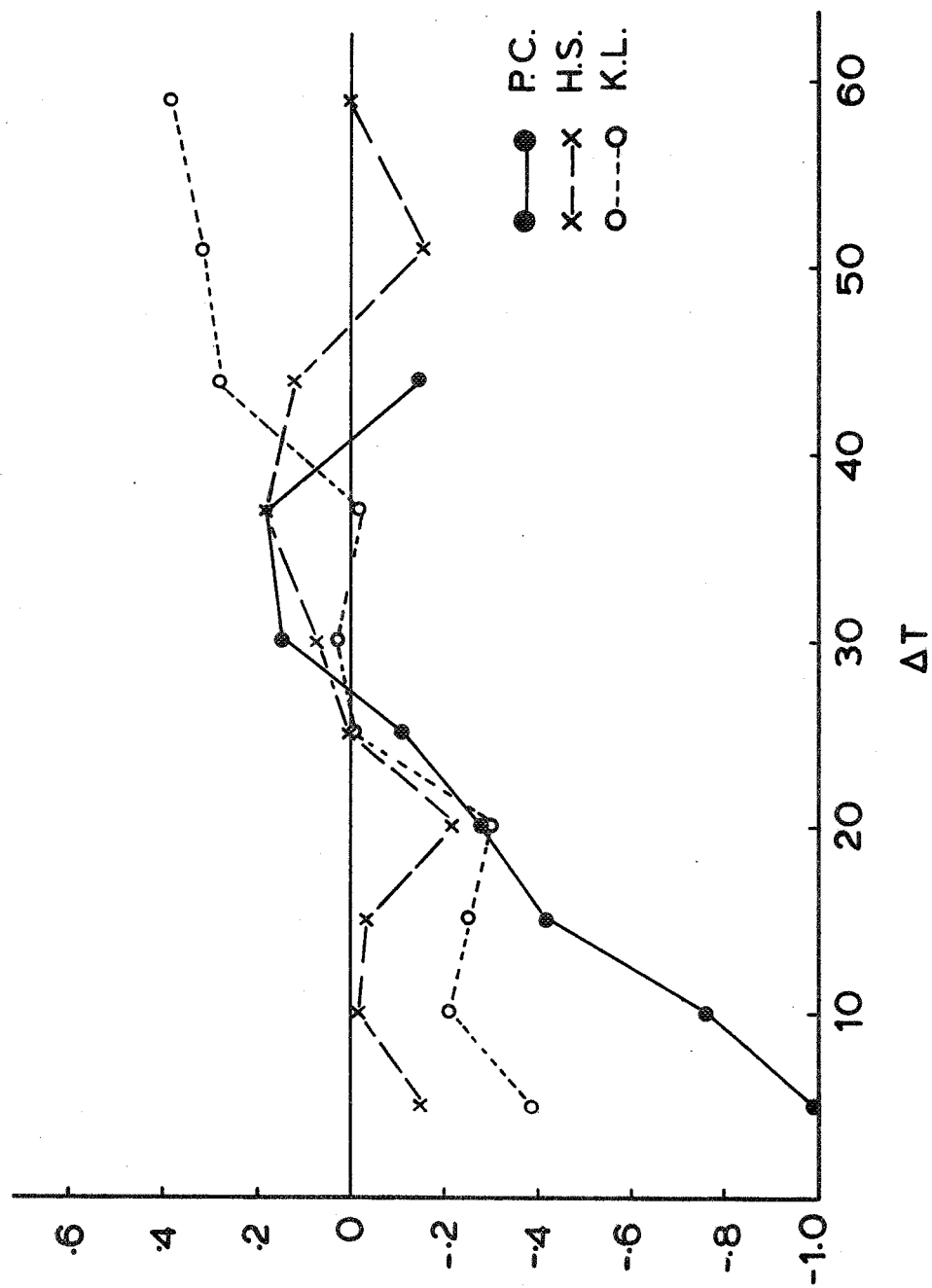


Fig. 8(b). Criterion location (Creelman's model) as a function of ΔT .

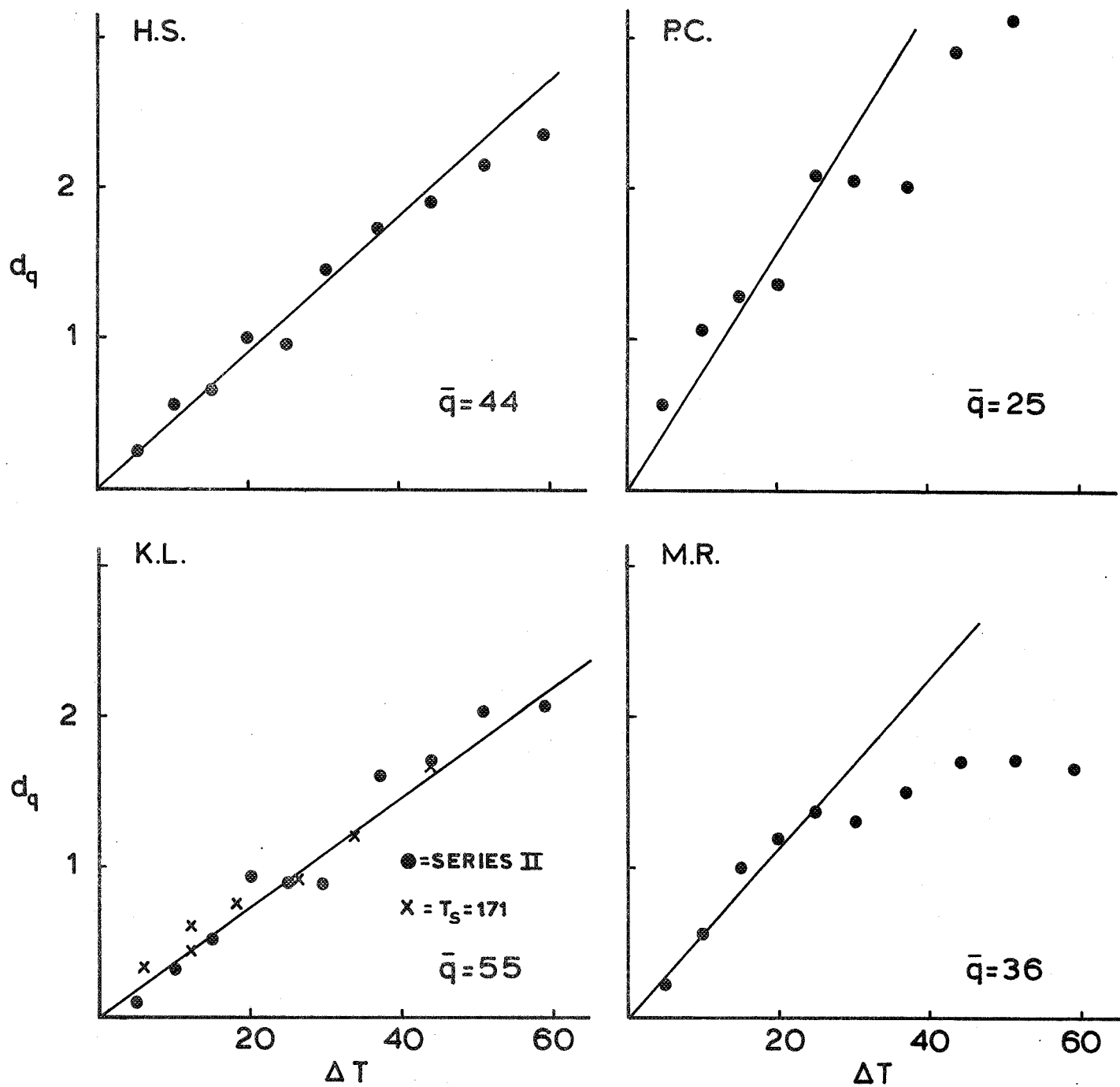


Fig. 9. Quantal onset-offset model: d_q as a function of ΔT for adjacent intervals, series II.

(c) Quantal counting models: the two-look model

Both versions of the quantal counting model are clearly inadequate, since for three Ss the data are not at all well described by the straight line segments predicted. For the fourth, there was no change in his functions when T_s was changed by an amount sufficient to substantially alter the parameter b_s if q were approximately 50 ms. However, the quantal counting model can be easily modified into one generating curved duration functions which do have features corresponding to those obtained in this experiment.

When the coded values of T_1 and T_2 are equal ($t_1 = t_2$) the observer is left in an uncertain state as to whether there is actually a difference between T_1 and T_2 . Assume that when he is in this uncertain state he somehow can take a "second look" at the pattern and code T_1 and T_2 a second time, the second coding being independent of the first. The pattern would have to be stored for a short time with no loss in information while the first coding is being completed. Fig. 10 summarizes the coding and decision processes occurring for an S_1 pattern. A similar diagram

Figure 10 about here

would hold for the S_2 pattern. If the results of the second coding are independent of those of the first, the probabilities α and γ have the same values for the second look as for the first. For $S_1 = (T_v, T_s)$

$$P(1|S_1) = \alpha + \gamma\alpha + \gamma^2 k'.$$

Similarly, given $S_2 = (T_s, T_v)$,

$$P(2|S_2) = \alpha + \gamma\alpha + \gamma^2 (1-k'),$$

so that if $P(S_1) = P(S_2) = \frac{1}{2}$,

$$P_2(C) = \alpha + \gamma\alpha + \frac{\gamma^2}{2} \quad (8)$$

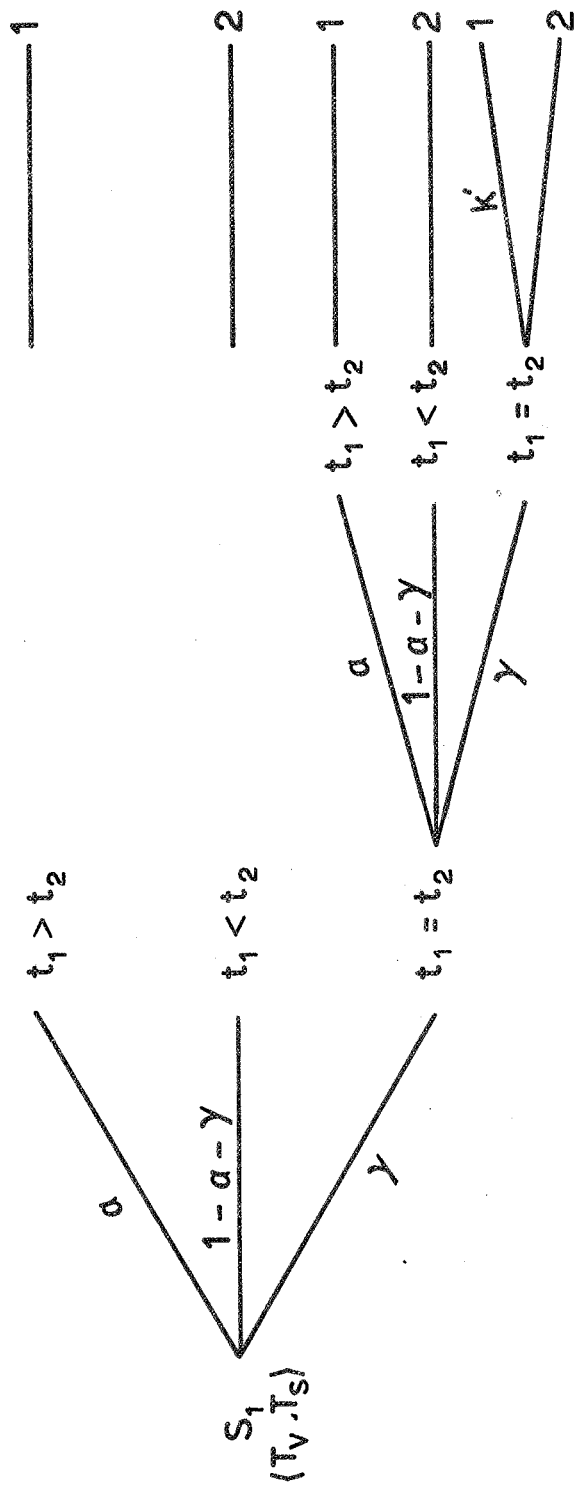


Fig.10 A schematic representation of the two-look model.

Using the values of α and γ obtained in the 'one-look' case, the duration discrimination functions now have the parametric equations

$$(a) \quad P_2(C) = \frac{1}{2} (1-b_s)^2 + b_v(2-b_v) \quad)$$

$$\Delta T/q = b_v - b_s$$

$$\text{for } b_s \leq b_v < 1-b_s$$

$$(b) \quad P_2(C) = \frac{1}{2} (b_v^2 - b_s^2 + 1)$$

$$\Delta T/q = b_v - b_s$$

$$\text{for } 1-b_s \leq b_v < 1$$

(9)

$$(c) \quad P_2(C) = \frac{1}{2} (2-b_s^2)$$

$$\Delta T/q = 1+b_v - b_s$$

$$\text{for } 0 \leq b_v < 1-b_s$$

$$(d) \quad P_2(C) = \frac{1}{2} (1+2b_v - b_v^2)$$

$$\Delta T/q = 1+b_v - b_s$$

$$\text{for } 1-b_s \leq b_v < 1$$

These are plotted in Fig. 11a for $b_s = 1/4$ and $3/4$. The corresponding functions obtained when the coding of T_2 is assumed independent of the coding of T_1 are shown in Fig. 11b, for $b_s = 1/4$ and $3/4$. They consist of two curved segments, with the parametric equations

$$(a) \quad P_2(C) = \frac{1}{2} (1-b_s)^2 + b_v(1-b_s^2) - b_v(1-2b_s)$$

$$\Delta T/q = b_v - b_s$$

$$\text{for } b_s \leq b_v < 1$$

$$(b) \quad P_2(C) = 1 - \frac{1}{2} b_s^2 (1-b_v)^2$$

$$\Delta T/q = 1+b_v - b_s$$

$$\text{for } 0 \leq b_v < 1$$

The span of these functions is still between q and $2q$. In the dependent situation, a horizontal segment of variable length (depending on the value of b_s) occurs unless $b_s = 0$; the higher the value of $P(C)$ at which it occurs the longer it will be. The rise of $P(C)$ is quite fast for the lower part of the ΔT range, but it decreases considerably as ΔT increases. These features correspond to features of the functions for HS and MR shown in fig. 9, if q is of the order of approximately 50 ms. For PC a much smaller value of q would be needed. There is still a substantial difference between the predicted functions for various values of b_s , in the dependent case; a relatively small change in base duration would provide a good direct test of the model. However, if t_1 and t_2 are independent, there is much less difference between the functions for various base durations. If a two-look function were fit to the data for KL, q would have to be of the order of 100 ms. In this case a 15 ms change in base duration changes b_s by only .15. The failure to obtain a change in his discrimination functions might be due to either one of two reasons; t_1 and t_2 are independent, or the change in b_s is not large enough.

Figure 11 about here

From the relations leading to equation (8), we get $P(2) - P(1) = 2\gamma^2(\frac{1}{2} - k')$. This difference depends on both $k' = P(1 | t_1 = t_2)$, and $\gamma = P(t_1 = t_2)$, and will change in sign only if k' changes from some value less than $\frac{1}{2}$ to some value greater than $\frac{1}{2}$ (or vice versa). Further, although γ is not generally a monotonic decreasing function of ΔT it does decrease linearly with ΔT over the range $0 \leq \Delta T \leq (1 - b_s)q$ if b_s is $\leq \frac{1}{2}$, for t_1 and t_2 dependent. This would be the case for $45 \leq q \leq 52$, for example. Thus the model could account for a substantial change in $P(2) - P(1)$, but not for a reversal in sign.

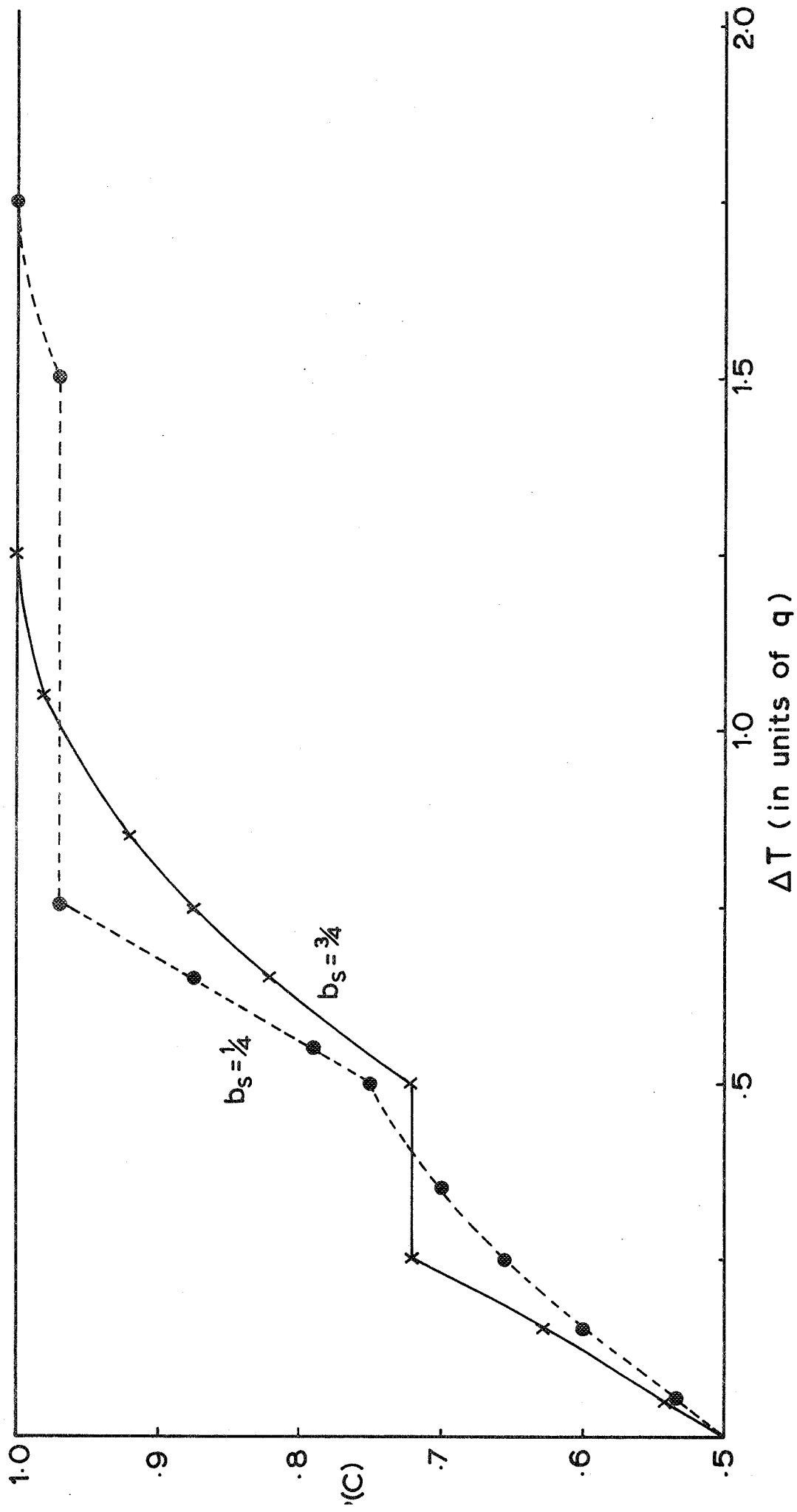


Fig.11(a). Predicted two-look duration discrimination functions for adjacent intervals; t_2 dependent on t_1 .

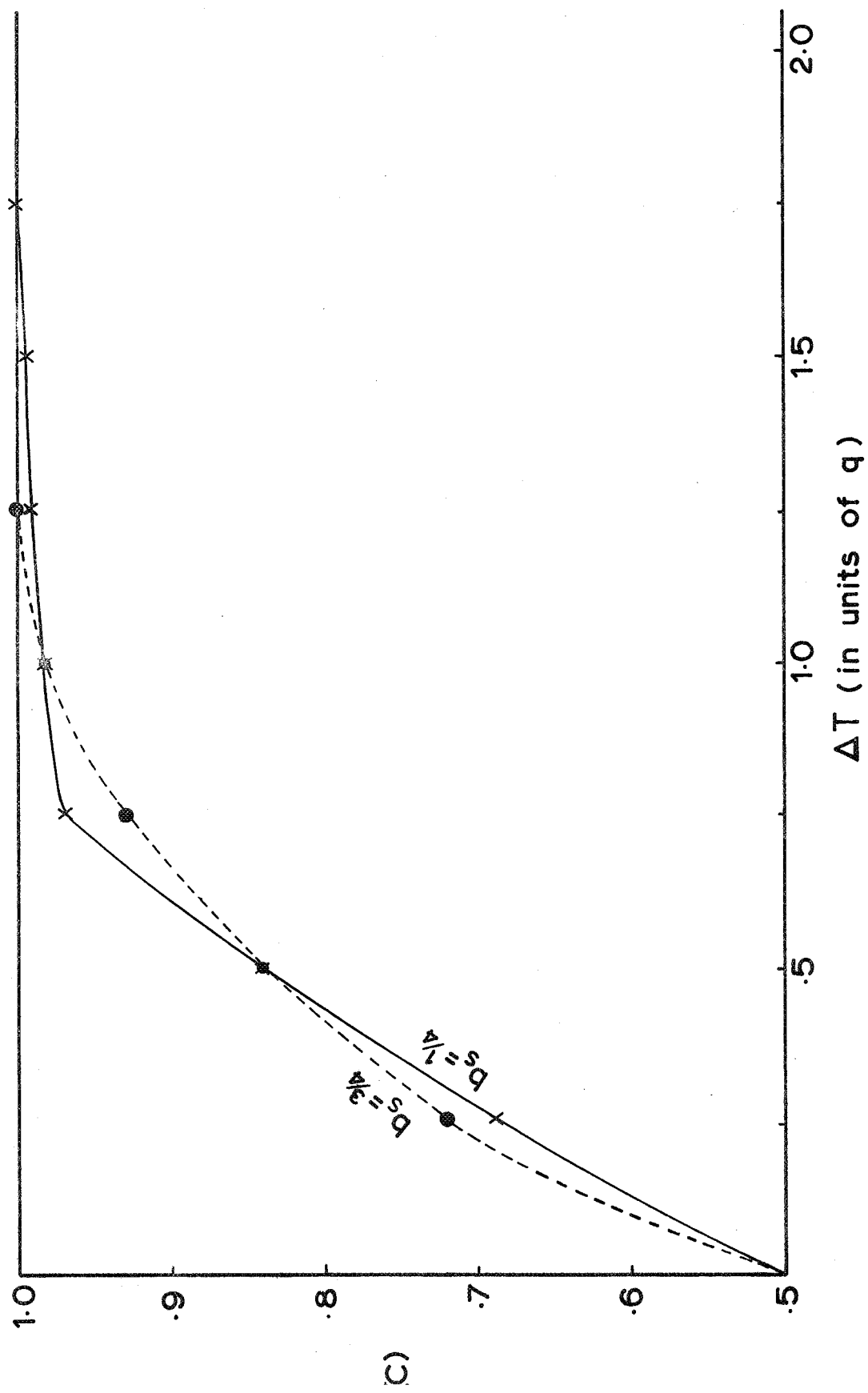


Fig. 11(b). Two-look duration discrimination functions, t_2 independent of t_1 .

Summary

Both of the models which treat time as a continuous variable are supported by the data of only two of the four Ss. However, in these cases there is the further difficulty that there seems to be a systematic criterion change as ΔT increases, contrary to the assumptions of the statistical decision theory approach used in deriving a predicted measure of performance.

The quantal counting model as described in the introduction is inadequate for all four Ss, but the "two-look" version of this model looks promising. It is obtained by adding the assumption that the pattern is coded a second time when the outcome of the first coding is such that there is uncertainty as to which is the larger interval. The functions predicted in this case share several qualitative features with the functions obtained, and would be able to account for the decrease in the difference $P(2) - P(1)$ over the lower part of the ΔT range, which was seen with all four Ss.

Experiment II: Same Different Judgments of Adjacent Empty Intervals

The shift in the bias seen in the forced choice discrimination of adjacent intervals gave rise to speculation as to whether the S uses some information regarding the absolute size of the difference between the two intervals in making the decision as to which is the larger. One possibility is that this information is obtained from the rhythmic structure of the pattern of three auditory signals. As ΔT becomes larger, there is a definite feeling of rhythm associated with these patterns, at least at the base duration chosen for experiment I. If this subjective rhythm were being used in some way, it might be reflected in an improved performance (as compared to forced choice performance) when the S is asked to decide only whether or not the intervals are equal, without specifying the longer one. That is, he might detect that the intervals are different, rather than the same, even though he cannot determine which one is the longer. Detecting that they are different might lead to placing the criterion in a different place on the ΔI axis. In this experiment T_D and ΔT were kept fixed within any one session. The stimulus alternatives were chosen from the set

$$\begin{array}{ll} S_0 = (T_s, T_s) & (T_1 = T_2) \\ S_1 = (T_s + \Delta T/2, T_s - \Delta T/2) & (T_1 > T_2) \\ S_2 = (T_s - \Delta T/2, T_s + \Delta T/2) & (T_1 < T_2) \end{array}$$

with the probability of an S_0 pattern always being $P(S_0) = \frac{1}{2}$. The total duration, $T_D = 2T_s$, was constant in any one session.

Data were obtained from 2 Ss (HS and KL) who had participated in experiment I and from one naive S (RM). The procedure and events on each trial were identical to those in experiment I, except that Ss were now instructed to press key 1 if the intervals were equal and to press key 2 if they were different. Feedback was given when the response was correct. The data in Table 3 summarize performance from four different conditions.

Table 3 about here

A. Three stimulus alternatives per session.

In this series, S_0 , S_1 , and S_2 were presented in each session, with $P(S_0) = \frac{1}{2}$, $P(S_1) = \frac{1}{4}$, and $P(S_2) = \frac{1}{4}$. T_s was 156 ms for all 3 Ss; ΔT was either 30 or 50 ms. The ordered triad $P(D | S_2)$, $P(D | S_1)$, $P(D | S_0)$ represents the probability of the response "different" for each of the three stimulus types presented, summarized over several sessions. The data summary indicates the total number of trials involved. $P(C)$ is obtained from the combination:

$$P(C) = \frac{1}{2}P(S | S_0) + \frac{1}{4}P(S | S_1) + \frac{1}{4}P(D | S_2) \quad (11)$$

The main finding is the strong, consistent asymmetry in treatment of S_1 and S_2 . For all 3 S's there seems to be a marked tendency to associate the response "different" with the S_2 pattern, while S_1 and S_0 seem to be confused. $P(D | S_1)$ is not much greater than $P(D | S_0)$ for HS and RM at $\Delta T = 30$ ms. At $\Delta T = 50$ ms, KL initially treated S_1 and S_2 equivalently (.76, .76, .52), but he then showed a shift in performance to (.71, .48, .41).

B. Two stimulus alternatives per session

In order to directly compare discriminability between S_0 and S_2 with that between S_0 and S_1 , sessions were run with $P(S_1) = P(S_0) = \frac{1}{2}$ and $P(S_2) = P(S_0) = \frac{1}{2}$. For both HS and KL, it was found that $P(C | S_1 \text{ and } S_0)$ was about 20% less than $P(C | S_2 \text{ and } S_0)$.

C. Learning effects

After obtaining the "baseline" levels of performance in series A and B, Ss continued with sessions of S_0 and S_1 patterns to see whether they could improve in discriminating between S_0 and S_1 with extended practice. At the beginning of each session, KL and RM were given a sample of 10 S_0 trials, to define to them what an S_0 ("same") pattern actually sounded like. The third S (HS) received S_0 samples at $\Delta T = 50$ ms, but not at $\Delta T = 30$. The data shown in Table 3 are from sessions after a practice series. The notes in the final column indicate whether or not S_0 samples were presented in these test sessions also.

HS showed no change after 3 practice sessions with $\Delta T = 30$; $P(C)$ was still at .5. However, at $\Delta T = 50$ performance shifted from (.87, .46, .26), to (.9, .8, .24) after practice. For RM as well, the biggest improvement was in $P(D | S_1)$; at $\Delta T = 30$, he changed from (.78, .45, .33), to (.84, .75, .35) after 3 sessions of practice on S_0 and S_1 .

KL also showed a substantial change: from (-, .61, .41) to (-, .78, .24), with $P(C | S_1 \text{ \& } S_0)$ after practice being equal to $P(C | S_2 \text{ \& } S_0)$ before practice. But when he was retested with three stimulus patterns, performance was dramatically altered. S_0 and S_2 were now more confusable than S_0 and S_1 ... a reversal of

the original situation. After three more sessions of practice with S_0 samples, and three stimulus alternatives present, he still did not recover his original level of performance (.58, .80, .37). It remains to be seen whether the effect would occur with other Ss.

D. Change in total duration

For HS, T_D was changed from 312 to 284 ms; performance with three stimulus patterns per session was only slightly different. However, there was a much more substantial change when T_D was increased by 140 ms: from (.80, .36, .33) at 360 ms to (.52, .46, .41) at 500 ms. The quantity changed most is $P(D | S_2)$.

For KL, as T_D increased from 312 ms to 450 ms, performance changed from (.58, .80, .37) to (.38, .63, .36). In this case, $P(D | S_1)$ and $P(D | S_2)$ decreased by comparable amounts. After further practice with S_2 samples, performance was (.55, .45, .36).

For both Ss, it seems as if S_1 and S_2 patterns could be treated more similarly at the larger total durations. Further, the magnitude of the difference $P(D | S_2) - P(D | S_1)$ seems to depend on ΔT and T_D ; both these time parameters would influence the confusability of S_1 and S_2 with S_0 . But KL's shift in performance after 4 sessions with $\Delta T = 50$, $T_D = 312$ shows that such changes occur sometimes with no change in ΔT or T_D .

Theoretical considerations

The asymmetry between Ss treatment of S_1 and S_2 poses a problem for both the Creelman and quantal onset-offset models in their original form, or for any model using statistical decision theory to characterize the decision process. These models involve overlapping distributions of ΔI , the difference

between the internal representations of the time intervals defined by the stimulus patterns. It is assumed that in the same-different task, the S fixes two criteria, β_1 and β_2 . Whenever $\beta_1 < \Delta I < \beta_2$, his decision is that the intervals are the same. If ΔI lies outside these bounds, his decision is that $T_1 \neq T_2$. Figure 12 represents the decision situation.

Figure 12 about here

The data obtained from sessions with 3 stimulus alternative present are not sufficient to give an estimate of d'_{01} , the distance between the means of the (S_0) and (S_1) distributions. (Similarly for d'_{02} .) The location of β_1 (β_2) with respect to μ_1 (μ_2) is obtained from $P(D | S_1)$ ($P(D | S_2)$). But the location of β_1 and β_2 with respect to μ_0 is not possible until we know the relative areas in the shaded portions of the tails of the (S_0) distribution beyond β_1 and β_2 ; we have only the sum of these areas which is $P(D | S_0)$. (See fig. 12)

The data from sessions involving the $S_1 - S_0$ and the $S_2 - S_0$ alternatives do yield estimates of d'_{01} and d'_{02} . These are shown in Table 4; $d'_{c,01}$ and $d'_{c,02}$ refer to values obtained by using the quantal onset-offset model. In both cases, d'_{02} is much larger than d'_{01} . For HS the ratio d'_{02}/d'_{01} is about 10, while for KL it is 3; both models predict that this ratio should be 1.

Table 4 about here

The effect of a change in T_D with ΔT kept constant also suggests

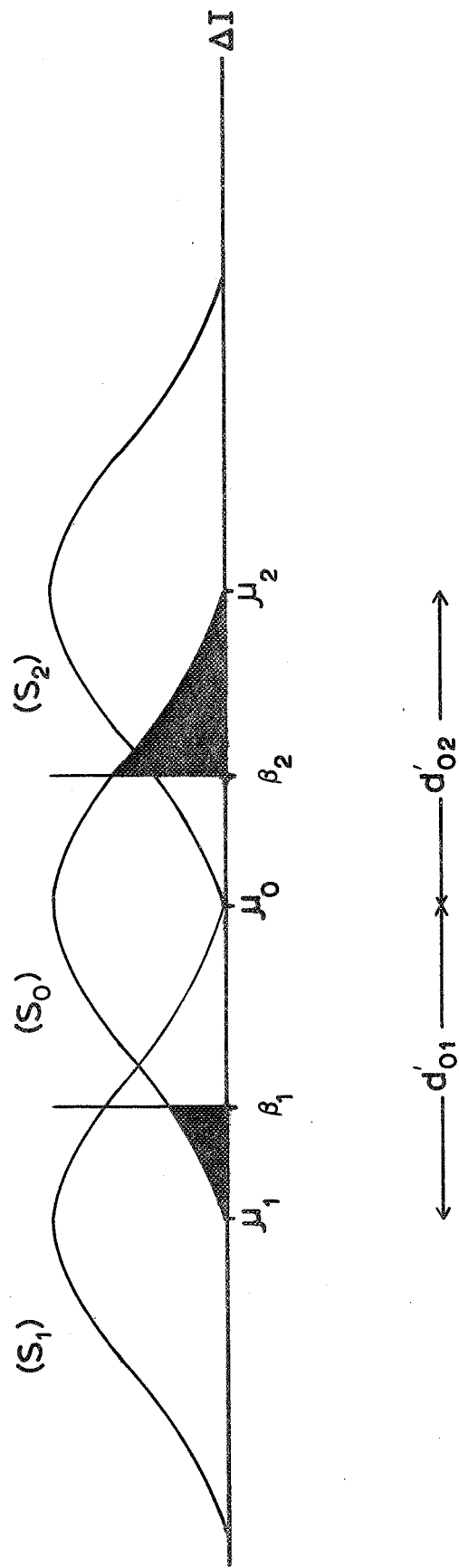


Fig.12. Representation of the decision process in the same-different situation.

that the onset-offset delay model is not adequate, unless $T_s = 180$ or $T_s = 250$ lies outside the range within which it would apply. Over a certain range of T_s there should be no effect on d'_{01} or d'_{02} corresponding to a given ΔT . On the other hand, Creelman's model would predict a decrease in $d' = d'_{01} + d'_{02}$. Using the simpler formula (1), the predicted change in d' as T_s increases from 180 ms to 250 ms is approximately

$$d'_{(250)} = \frac{(2(180) + \Delta T)^{\frac{1}{2}}}{(2(250) + \Delta T)^{\frac{1}{2}}} \cdot d'_{(180)} \cong .85 d'_{(180)}$$

However, the data are not sufficient to yield an estimate of d' at either value of T_s so we cannot establish whether the change in performance obtained actually does correspond to a change of this magnitude in d' .

It should be kept in mind that these models assume a response strategy which is based on the difference of the measures of the two intervals, and on the use of two criteria. If it were very difficult to use two criteria reliably, this should be reflected in a substantial improvement in $P(D | S_1)$ in the $S_0 - S_1$ sessions, as compared to $P(D | S_1)$ in the $S_0 - S_1 - S_2$ sessions where only one criterion would be necessary. This was not seen with HS. On the other hand, Ss might use only the information in the first interval, and classify the pattern according to whether the first interval is shorter than a certain critical value. A procedure which would be informative with respect to this question, is one which would use three S_0 patterns:

$$S_0 = (T_s, T_s) \quad S_{01} = (T_s + \Delta T/2, T_s + \Delta T/2) \quad S_{02} = (T_s - \Delta T/2, T_s - \Delta T/2) .$$

If the response strategy were as just suggested, $P(D | S_{02})$ would be much larger than the $P(D | S_{01})$.

One way of using the q-counting type of model to account for the asymmetry in performance on S_1 and S_2 found in this experiment is to assume that the decision process involves two bias parameters:

$$P(\text{"different"} | t_1 = t_2) = k'$$

$$P(\text{"different"} | t_1 > t_2) = k$$

$$P(\text{"different"} | t_1 < t_2) = 1$$

A probability tree diagram summarizing the coding and decision processes is given in Fig. 12. The parameters α_i , β_i , γ_i ($i = 0, 1$) depend on ΔT and q , and on whether or not t_2 is dependent on t_1 . The calculations are described in Appendix I. From the diagram we obtain the following set of 3 equations, any 2 of which (a and b, say) give us an estimate of k' and k .

$$P(D | S_0) = \alpha_0 k + \beta_0 + \gamma_0 \cdot k' \quad (a)$$

$$P(D | S_1) = \alpha_1 k + \beta_1 + \gamma_1 \cdot k' \quad (b) \quad (12)$$

$$P(S | S_2) = \alpha_1 + \beta_1 k + \gamma_1 \cdot k' \quad (c)$$

Figure 13 about here

To test the model we need data from sessions in which three stimulus alternatives are present, since we want the same values of k and k' to be in effect when we predict the value of $P(S | S_2)$ from (c). For certain assumed values of q ,

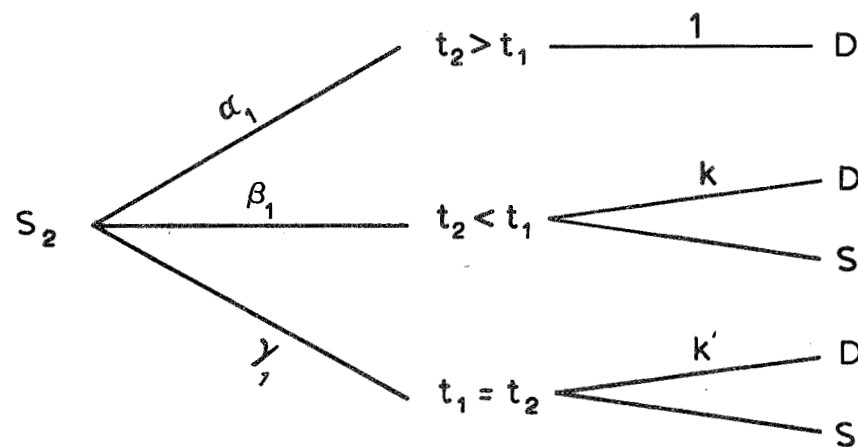
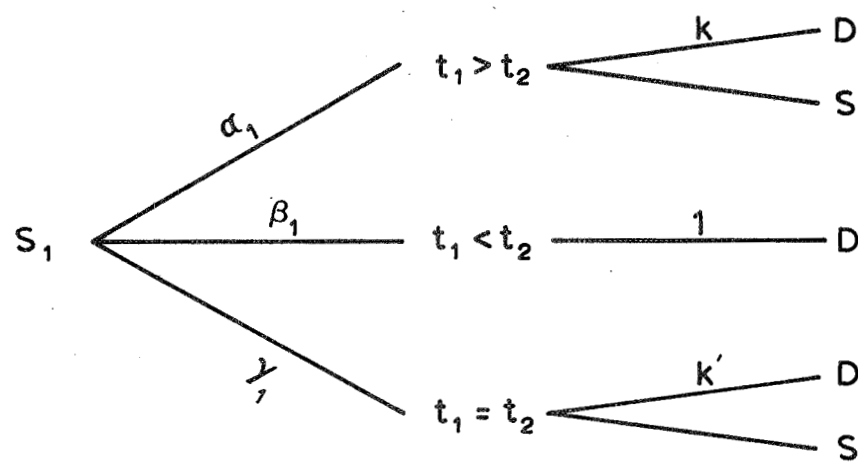
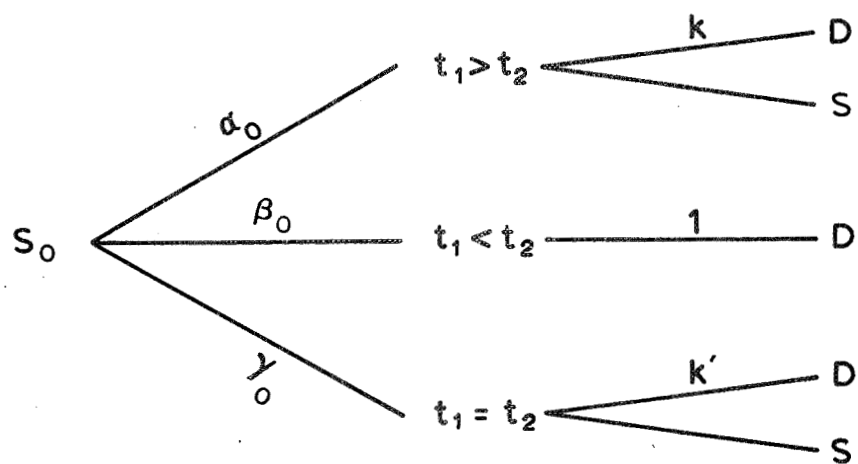


Fig.13. Schematic for the quantal counting model in the same-different situation.

the corresponding calculated values of k , k' and $P(D | S_2)$ are shown in Table 5, for t_1 and t_2 dependent. The obtained value of $P(D | S_2)$, for comparison, is the first (underlined) number in the corresponding performance triad.

There are several indirect restrictions on q ; the solutions to (12) must be such that

- (1) $0 \leq k, k' \leq 1$
- (2) the predicted value of $P(D | S_2)$ must be close to the experimental value.
- (3) given a small change in T_D , the same values of k , k' , and q should predict performance, and
- (4) after further practice with S_0 samples, any performance change should be reflected in a change in the bias parameters only.

The values of q shown in Table 5 have been selected from those which best satisfy all these conditions. For both HS and KL, who had a large increase in T_D , q had to be doubled to get $P(D | S_2)$ into the range of the obtained value. The problem of establishing the uniqueness of these solutions still remains, since for one given triad there may be two or more values of q which satisfy conditions (1) and (2). However, this set does not necessarily coincide with that providing acceptable solutions when T_D is increased. A range of q from 20 to 35 ms was explored for the three Ss, and another from 46 to 60 ms. Within these ranges, a value of q between 25 and 26 ms gives the best solutions for HS and RM, while q must be somewhere around 51 or 52 for KL.

Table 5 about here

It is of interest that these values of q must be doubled to give

acceptable solutions with the longer total durations. The question arises as to how q might depend on the total duration of the patterns being dealt with. Accuracy in performance could be substantially increased by using a time base with a smaller period (half the "fundamental"), provided there is no memory loss of the number of time points in the intervals being measured, or, there might be a critical number of time points beyond which double the "fundamental" period is used as a unit of measurement of the intervals being dealt with.

The two-look assumption has not been used for the same-different situation, since in this case there is no outcome of the coding process which corresponds to the state of "uncertainty" in the forced choice task. That is, there is a direct correspondence between the response alternatives defined to the observer and the outcomes of the coding process.

The results from this experiment suggest the inclusion of a second bias parameter in the two-look model. [The same results are obtained if instead we set $k = P(1 | t_2 > t_1)$.] Setting $k = P(2 | t_1 > t_2)$, the resulting expression for $P_2(C)$ is

$$2 P_2(C) = P + (1 - P) \quad \text{where } P = 2\alpha + 2\gamma\alpha + \gamma^2 \quad (13)$$

or

$$2 P_2(C) = P(1 - k) + k$$

The calculation of P can be made for each value of ΔT once we have selected a value for q . The original formula (8) in the two-look model is a special case of (13) with $k = 0$. P is a quantity varying between 1 and 2, so the term $k(1-P)$ is always negative, except at $\Delta T = 0$ where $P = 1$. Thus the predicted value of $P_2(C)$ is never 1. This prediction is not supported by the data from

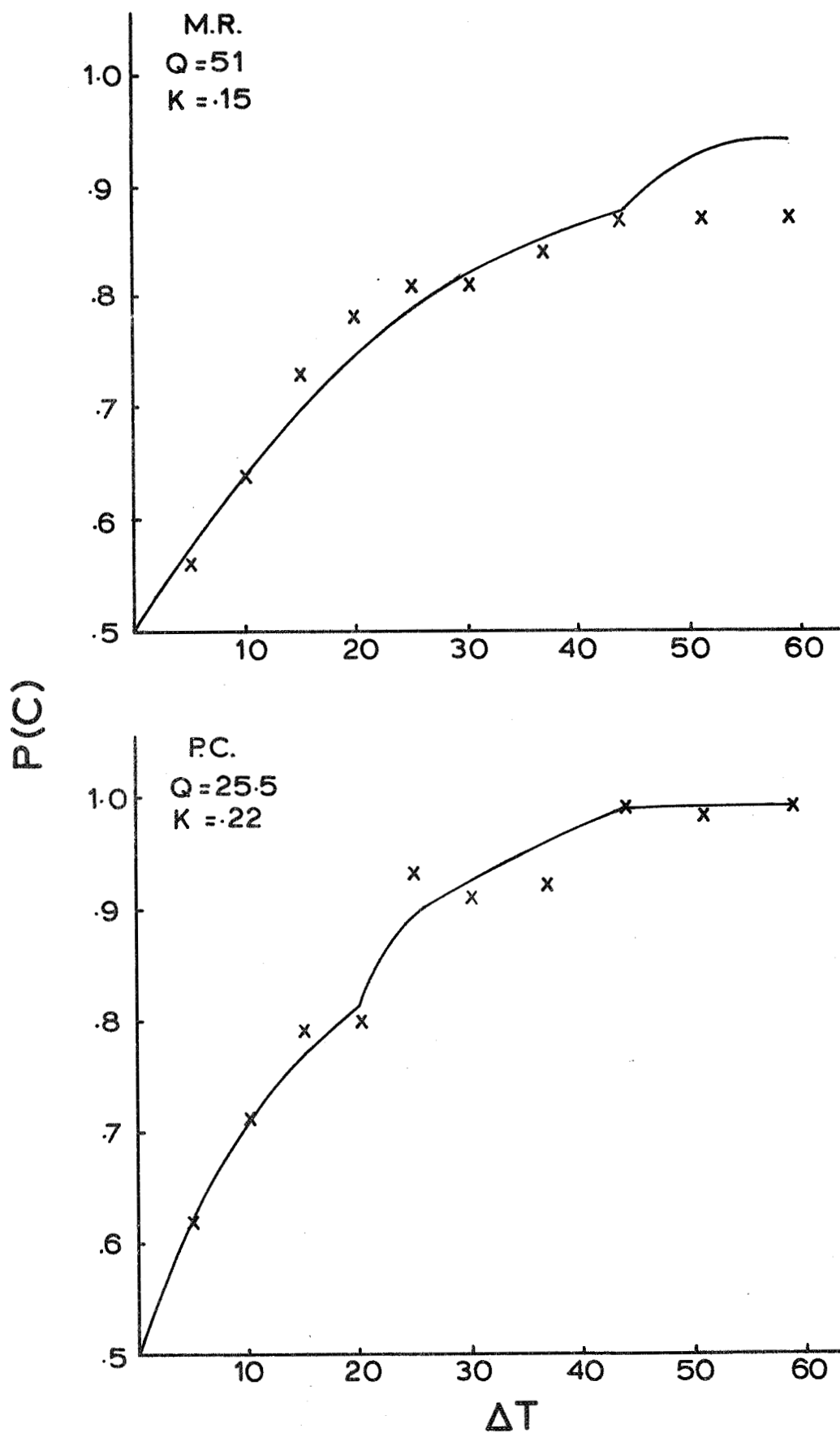


Fig.14. Predicted two-look, two-bias discrimination functions compared to adjacent interval data, series II.

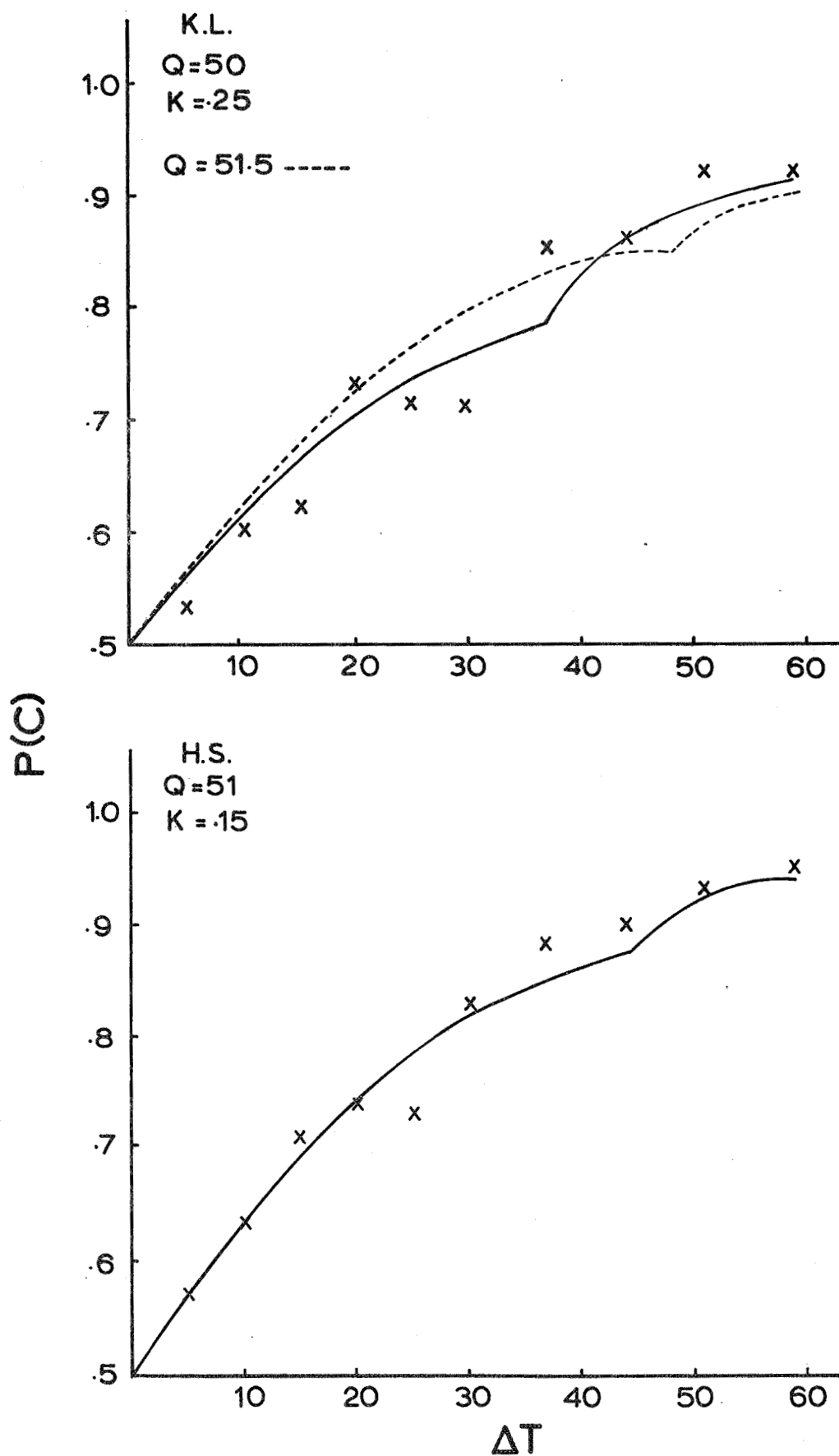


Fig.14. Predicted two-look, two-bias discrimination functions compared to adjacent interval data, series II.

series I in experiment I, where 3 Ss did obtain $P(C) = 1$ when ΔT was large.

So we set $k = P(2|t_1 - t_2 = 1)$ and $k' = P(t_1 - t_2 = 1)$, and now obtain

$$2P_2(C) = P - k(1 + \gamma)(\alpha + \alpha' + \gamma - 1) \quad (14)$$

As long as $\alpha = \alpha'$, (14) is equivalent to (13); this will be the case until $\Delta T = (1 - b_s)q$. Hence for the lower portion of the ΔT range, $2P(C)$ plotted as a function of P should be linear with slope $1-k$, and should pass through $(1,1)$. If $45 \leq q \leq 52$, $b_s < \frac{1}{2}$ and this range is at least 22 ms long. This means that from the first 4 or 5 data points of series II we can estimate k and so compare predicted to obtained performance, for the entire range of ΔT . Figure 14 shows two-look, two-bias curves with q of the order of 50 ms for 3 Ss. For one S, q has to be of the order of 25 ms. In all four cases, the data are quite well fit by the predicted functions. The range of q is restricted by the condition that we must obtain a reasonably linear plot of $2P(C)$ vs P , for $\Delta T \leq (1-b_s)q$, with most points lying on or below the diagonal from $(1, 1)$ to $(2, 2)$. Points lying above this diagonal correspond to performance better than that predicted with $k = 0$. Another restriction on q is that for those Ss who have taken part in both experiments I and II, there should be some correspondence between the values of q obtained in both situations. For KL this is so; but for HS, q in the FC task is twice the value of q obtained from the SD task. Once again we can raise the question as to whether q depends upon the context for making a duration discrimination.

Figure 14 about here

Discussion

It was initially intended to compare performance for two tasks, forced choice and same-different. This comparison cannot be legitimately made yet since data was not obtained for a FC task using the same stimulus alternatives as in the SD task. One basis of comparison might be the $P(C)$ measure. Comparing $P(C)$ from experiment I with $P(C)$ from this experiment, both HS and KL show a substantial decrement. In series II of experiment I, $P(C)$ was $\cong .83$ at $\Delta T = 30$, for HS; $P(C)$ was $\cong .9$ at $\Delta T = 50$ for KL. However, this comparison is only suggestive. Including S_0 would change the context for making the discrimination between S_1 and S_2 ; the confusability of S_0 and S_1 decreases $P(D | S_1)$ and may also influence $P(1 | S_1)$ in the FC task. It seems more likely though, that we would see this confusion reflected in a higher value for $P(1 | S_0)$ than for $P(2 | S_0)$, if random feedback were given on the S_0 trials.

In addition to the problem of a basis of comparison for the FC and SD situations, there is also the problem of whether S_1 and S_2 can come to be treated more equivalently with the right kind of training. If not, the original versions of those models of time perception using statistical decision theory will have to be modified to deal with adjacent intervals. On the other hand we now do have a version of the quantal counting model which can take into account this asymmetry in the treatment of S_1 and S_2 . The bias parameters should be sensitive to instructions regarding response strategy and payoffs. The asymmetry is an intriguing finding which needs to be replicated.

Experiment III: Separated Empty Intervals

This experiment looks at the shape of discrimination functions for intervals separated by 2 seconds, for two base durations. The procedure was the same as in experiment I. The stimulus alternatives were again $S_1 = (T_v, T_s)$ or $S_2 = (T_s, T_v)$ with $\Delta T = T_v - T_s$ having any one of 5 values in each session. Sixteen sessions were run at each value of T_s , divided into eight with $T_s = 200$ ms, sixteen with $T_s = 100$ ms, and a final set of eight at 200 ms. Two Ss (PV and KL) did not complete the entire series, but both functions are based on at least 300 trials per point. For the other 3 Ss, the functions are based on nearly 800 trials per point. PL and V. were new Ss; the other three had participated in experiment II. The intensity of the signals remained unchanged but the durations were increased slightly to 6 ms.

Results

The discrimination functions (Fig. 15) are smooth sharply rising curves, with decreasing slope as ΔT approaches 30 ms for 4 of the 5 Ss. The one linear function (KL) is very similar to the performance seen from this S in experiment I with adjacent intervals. For HS, performance is better at both base durations in this case than it was with adjacent intervals.

The effect of a change in base duration is not consistent over all Ss. Three show better performance on all values of ΔT when T_s is 100 ms as compared to $T_s = 200$ ms: one (KL) shows no difference; and for V, the direction of the difference between the two functions depends on ΔT .

Table 6 shows performance at both base durations. Comparing $P(1/S_1)$ with $P(2/S_2)$, there seems to be no definite bias trends. $P(2/S_2)$ is greater

than $P(1/S_1)$ in only four out of ten cases. For 3 Ss the differences $P(1/S_1) - P(2/S_2)$ have the same sign for both base durations. The most striking bias effect is seen with KL, for whom $P(1/S_1)$ is less than .5 until ΔT is > 30 ms.

Figure 15 about here

Table 6 about here

Theoretical analysis

Again we find that the quantal onset-offset model is not adequate in describing the data. The model predicts no change in performance when base duration is changed, since the ΔI distributions depend only on ΔT and q . However, a change in performance was definitely seen for three Ss. The table of cumulative areas under the distribution with $\Delta I = 0$ is calculated from the formulae used in McKee et al (1970) where this model is used for the forced choice situation. Using this table to obtain d_q , we find that the d_q vs ΔT relation is linear for $T_s = 100$ for all 5 Ss, but is non-linear when $T_s = 200$ in 3 cases. Further, for 3 Ss the points for $T_s = 200$ deviate systematically from the straight line fitting the points for $T_s = 100$. (Fig. 16) It may be that $T_s = 200$ is outside the range in which this model applies.

The simple version of the Creelman model is also somewhat inadequate. The plot of d' as a function of $(\Delta T/2T_s + \Delta T)^{1/2}$ is shown in Fig. 17; for only two Ss are the data from both base durations reasonably well fit by the same

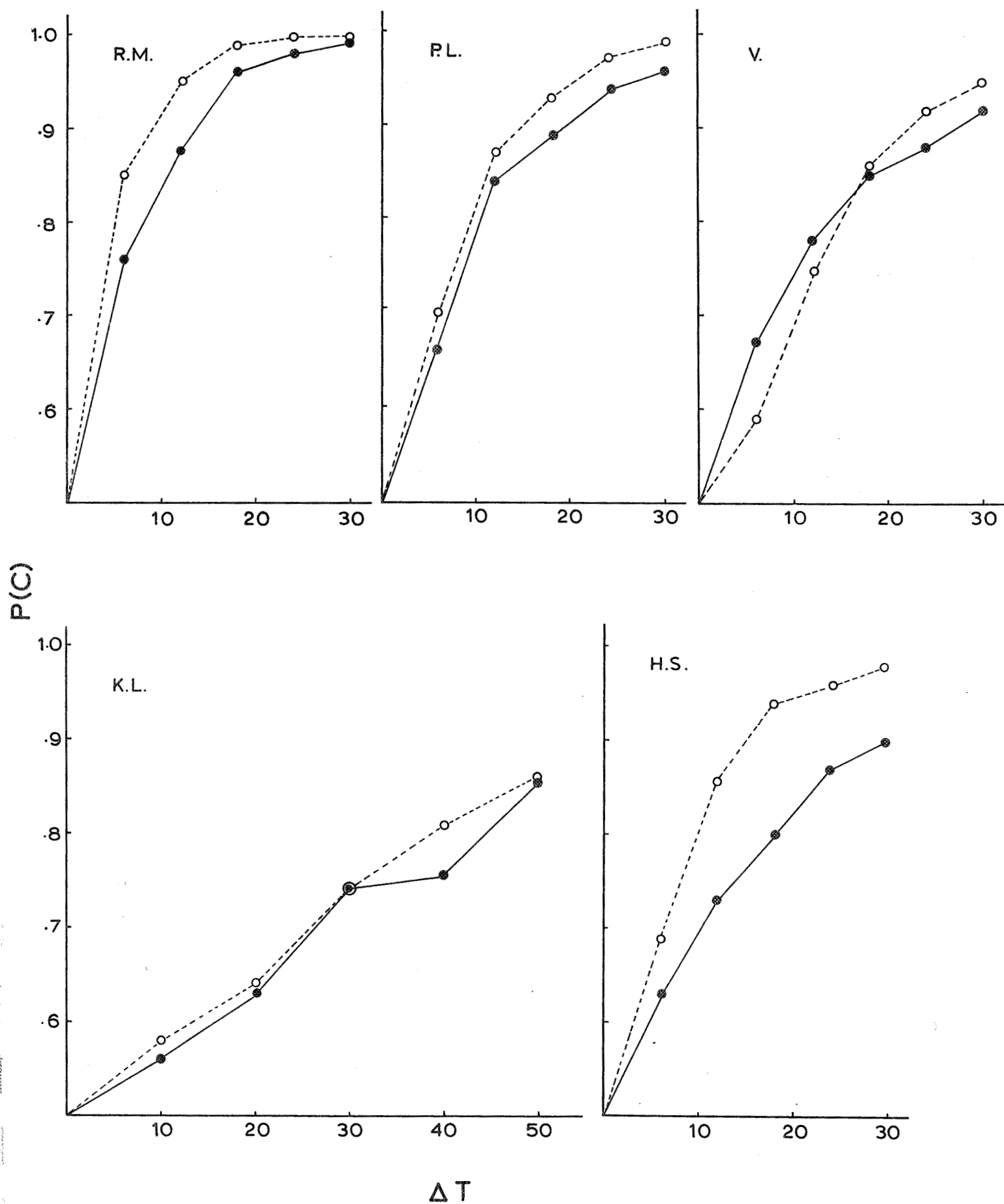


Fig.15. Duration discrimination functions for separated intervals. $T_s=100$ (o---o) and $T_s=200$ (●—●).

zero-intercept straight line. In the other cases, the points corresponding to data from one base duration lie consistently below the line fitting the points from the other. A plot of $d'_{(100)}$ vs $d'_{(200)}$ is shown in Fig. 18 for all Ss. Equation (1) predicts that all points should lie slightly below a line with a slope of $\sqrt{2}$, but definitely above one with a slope of 1. For $0 \leq \Delta T \leq 50$,

$$\sqrt{2} \geq d'_{(100)}/d'_{(200)} = \left[\frac{400 + \Delta T}{200 + \Delta T} \right]^{\frac{1}{2}} \geq \sqrt{1.80}$$

Including the factor $(1 + KT_s)^{-\frac{1}{2}}$ would enhance the predicted ratio. The obtained ratios for at least 2 Ss (V and KL) are consistently less than predicted, and that for a third (HS) is much larger than predicted.

Figure 16 about here

Figure 17 about here

Figure 18 about here

In applying the two-look, two-bias model to the data from this experiment, we assume that the coding of T_2 is independent of the coding of T_1 when calculating the quantities α , α' and δ . For KL, the plots of $2P(C)$ vs P resulting from $q = 51$ are not at all well fit by a straight line but with $q = 102$ there is considerable improvement. For HS, with $q = 25$ the plots of $2P(C)$ vs P are linear at both base durations but they do not have the same slope; at $T_s = 100$, $k = 0$, but at $T_s = 200$, $k = .25$. However, if we use $q = 50$ for $T_s = 200$, we obtain $k = 0$. The predicted functions for HS shown in Fig. 19 are those for $T_s = 100$ ($q = 25$) and $T_s = 200$ ($q = 50$). For the other three Ss whose data can be reasonably well fit by this model, only the predicted functions

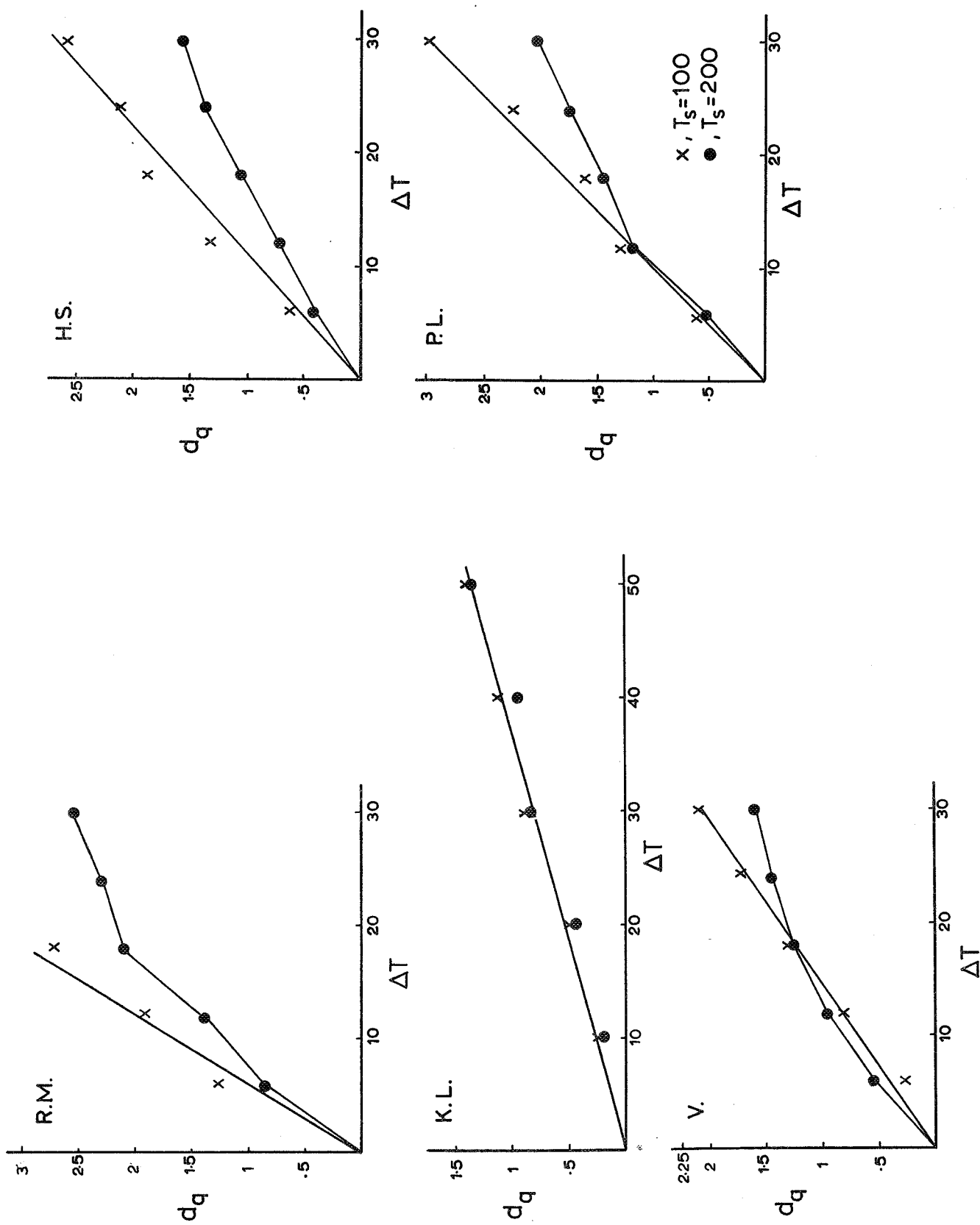


Fig.16. Quantal onset-offset model: d_q as a function of ΔT , for separated intervals: $T_s=100$ (x—x) and $T_s=200$ (●—●).

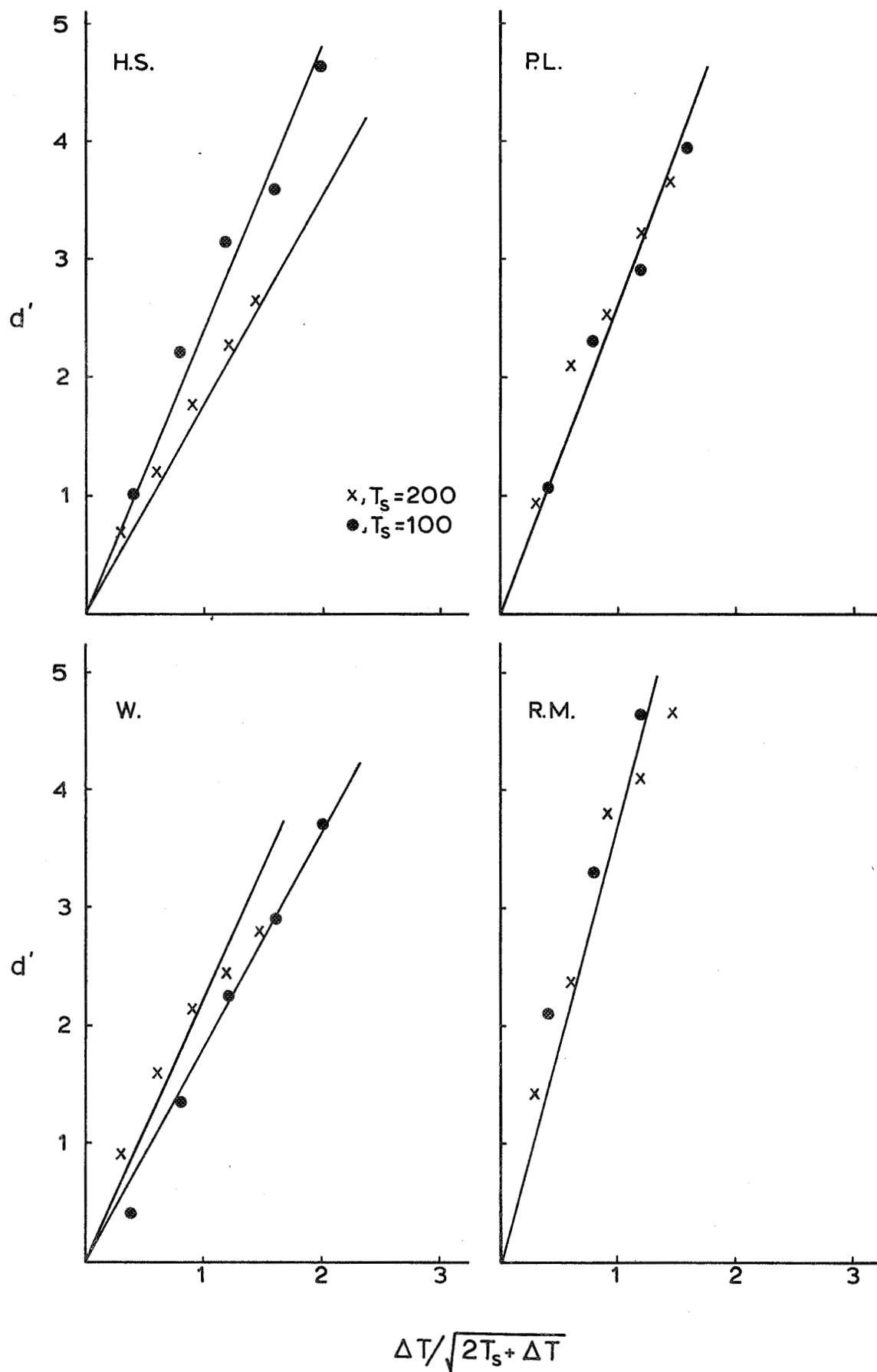


Fig. 17. Creelman's model: d' as a function of $\Delta T / (2T_s + \Delta T)^{1/2}$ for separated intervals.

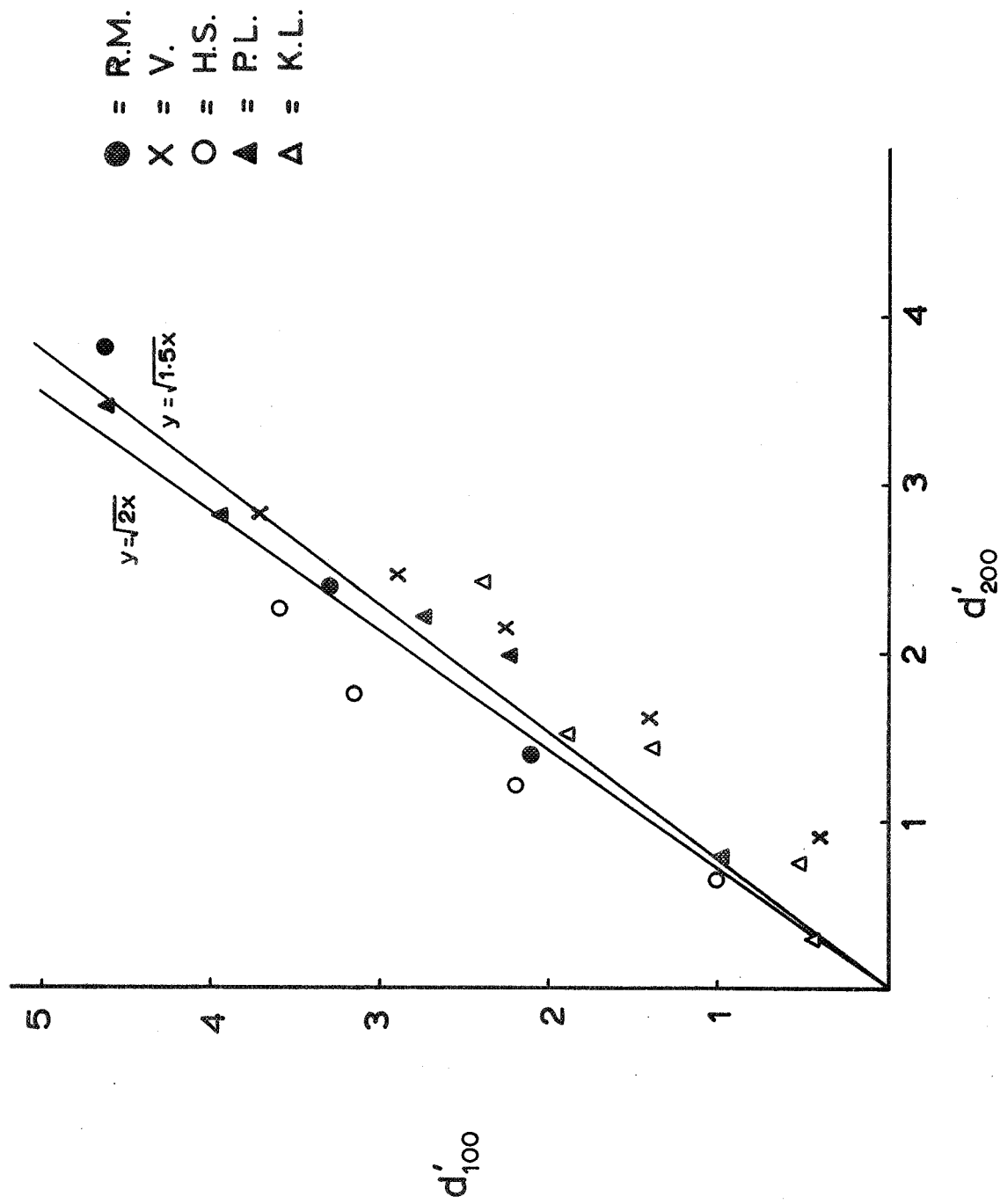


Fig. 18. Creelman's model: d'_{100} vs d'_{200} (separated intervals).

for $T_s = 100$ are shown. In these cases, there is very little difference between the functions at the two base durations; generally the function for $T_s = 200$ is slightly below that for $T_s = 100$. For the two new Ss (PL and V) q must be of the order of 25 ms. For RM performance is much better than the model predicts with q around 25 ms; a much smaller q would be needed.

Figure 19 about here

Summary and Conclusions

Of the three models described in the introduction, we have found those two which deal with time as a continuous variable (Creelman's and the quantal-onset-offset model) to be inadequate for the adjacent interval situation with both same-different and forced-choice judgments. These models require that the intervals be treated symmetrically; i.e., that it should not matter whether the longer interval is presented first or second in the pattern. However, there apparently is some kind of dependence between the intervals when the signal which marks the offset of the first also denotes the onset of the second. The centre pulse serves two functions, and the subjective rhythmic quality of the stimulus patterns may reflect a further complicating factor. It may be more reasonable to make the assumption of independence of the measurement of the two intervals when they are separated by some interval considerably longer than their total duration. Although the discrimination of separated empty intervals may provide a simpler stimulus situation in which to test various theories of time discrimination, Creelman's model is still not adequate, and the quantal onset-offset model can also be rejected. That is, these continuous models are inadequate as they have been formulated and applied in this case; successful modifications may (or may not) be possible.

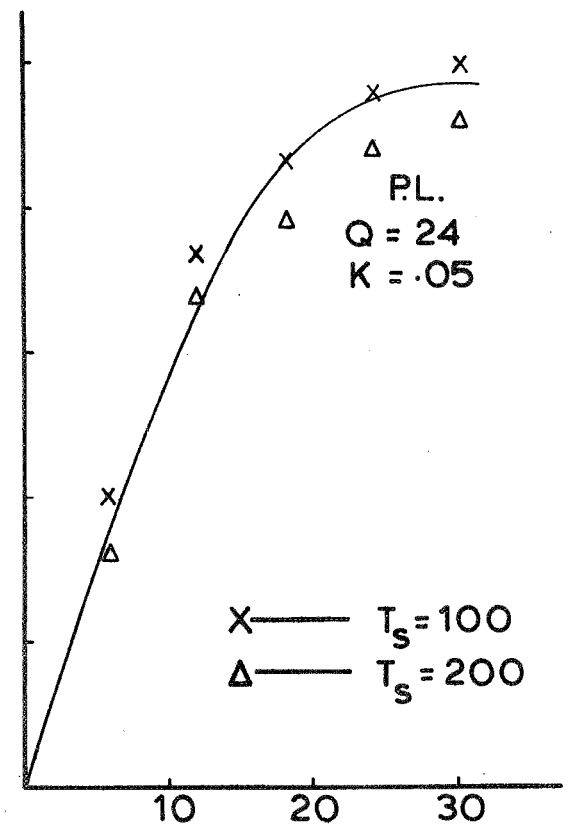
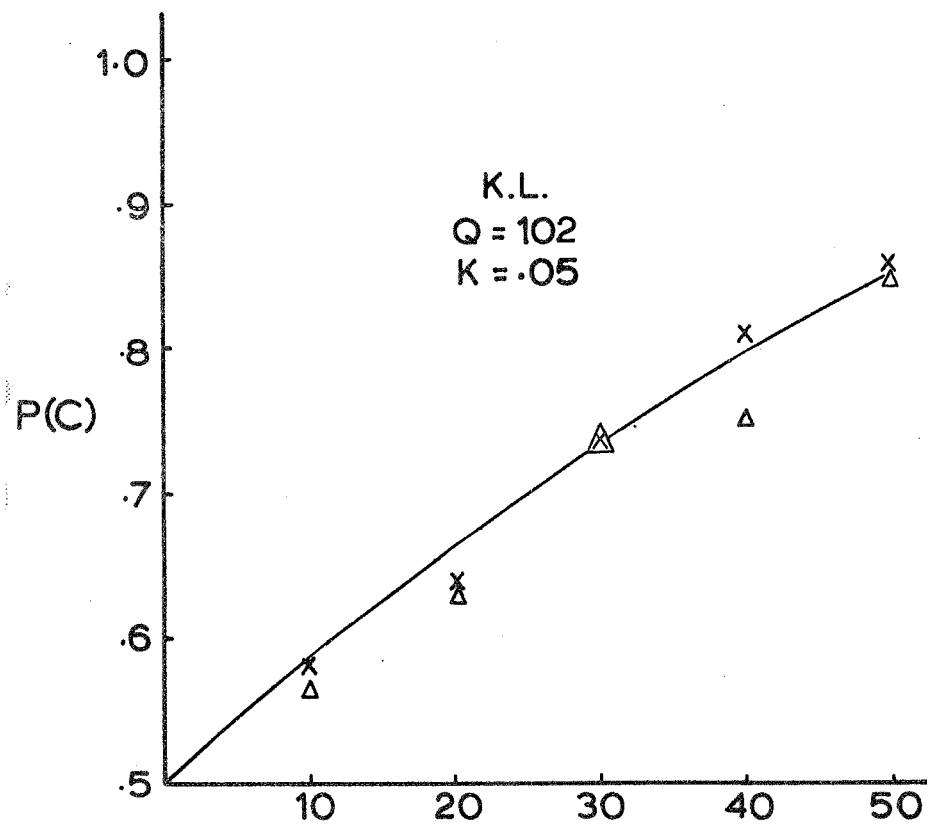
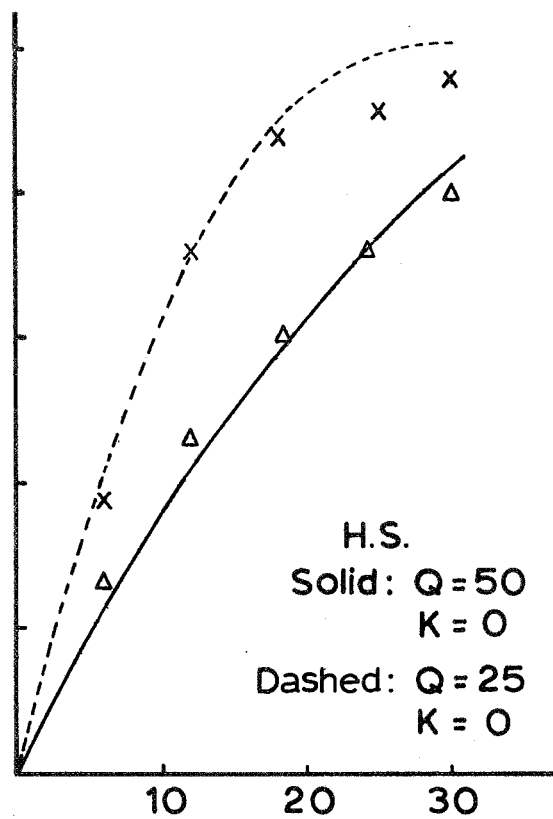
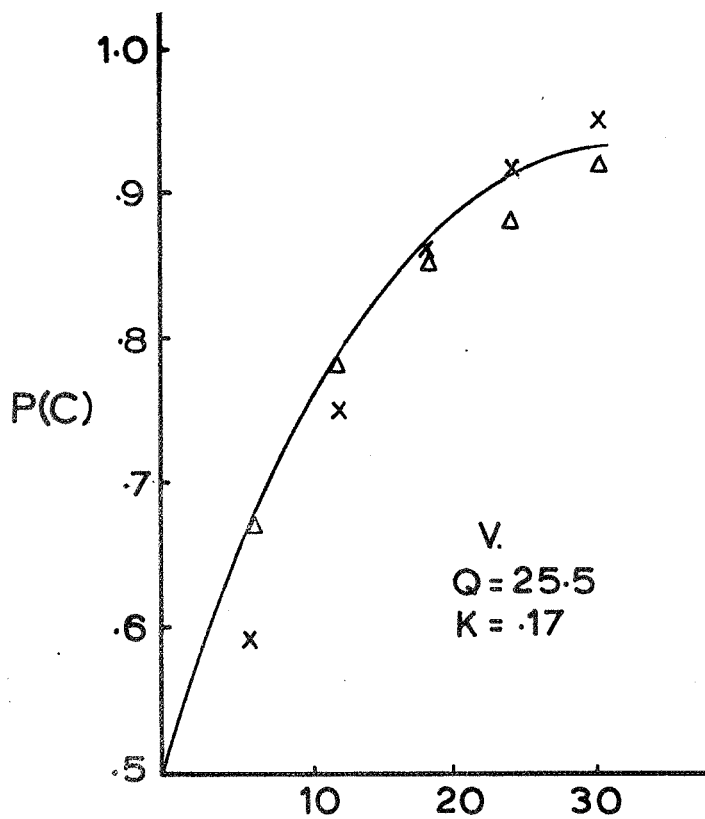


Fig.19. Predicted two-look,two-bias discrimination functions compared with separated interval data.

The class of models generated from the quantal counting assumption involving an internal time base independent of ongoing sensory events treats time as a discrete variable. This set of models has dealt quite successfully with the results of the three experiments reported here -- adjacent as well as separated intervals. One important assumption is made for the forced choice situation: that a second look at the intervals to be discriminated occurs when the results of the first measurement leave the observer in a state of uncertainty as to which is the longer interval. Such a state of uncertainty occurs in those situations where the S is presented with a pair of intervals on each trial, and the response alternatives are such that a longer (or shorter) interval must be selected.

Estimates of the value of the period of the time base were of the order of 25, 50 and 100 ms. An observer might use either q or $2q$ as a unit of measurement, depending on the base duration and the type of judgment to be made. These estimates are of considerable interest since they may be related to estimates of the psychophysical time quantum obtained in very different types of experiments (Kristofferson, 1967).

Appendix I: Sample calculations - quantal counting model

Assuming a given particular value of q , we have

$$I_s = (s+b_s)q \quad 0 \leq b_s < 1$$

$$I_v = (b+b_v)q \quad 0 \leq b_v < 1$$

$$I_D = I_s + I_v = (s+v+b_s+b_v)q = (d+b_d)q \quad 0 \leq b_d < 1$$

with s , v , and d being integers. Several cases must be considered. If $v=s+2$, I_v will contain either $s+2$ or $s+3$ time points while I can contain only s or $s+1$ time points. I_v will always contain at least one more time point than I_s and so will always be chosen as the longer interval. For both $v=s$ and $v=s+1$, the cases $b_v+b_s < 1$ and $b_v+b_s \geq 1$ have to be taken separately, since d will be $s+v$, or $s+v+1$.

As an example of the calculations, take $v=s$ and $b_v+b_s > 1$. Then $d=2s+1$ and $b_d=b_s+b_v-1$. Given $S_1 = (T_v, T_s)$, T_v contains either s or $s+1$ time points, with

$$P(t_1=s) = 1-b_v$$

T_D contains either $2s+1$ or $2s+2$ time points, with

$$P(t_D=2s+2)=b_d$$

Figure A1 represents the various outcomes possible for any trial and their associated probabilities, for this particular case.

Fig. A1 about here

Let $P(i;j)$ denote the probability of i points occurring in T_1 and j points in T_2 :

$$P(i;j) = P(t_2=j \mid t_1=i) \cdot P(t_1=i)$$

Then $\alpha = P(t_2 > t_1) = P(s+1; s)$ (1)

and $\gamma = P(t_2 = t_1) = P(s; s) + P(s+1; s+1)$ (2)

But $P(s; s) + P(s; s+1) = P(t_1 = s) = 1 - b_v$ (3)

and $P(s+1; s) + P(s+1; s+1) = P(t_1 = s+1) = b_v$ (4)

Looking at the total number of points within I_1 and I_2 ,

$$P(s; s) = P(t_D = 2s) = 0 \quad (5)$$

$$P(s+1; s+1) = P(t_D = 2s+2) = b_d \quad (6)$$

$$P(s+1; s) + P(s; s+1) = P(t_D = 2s+1) = 1 - b_d \quad (7)$$

From (3) and (5) $P(s; s+1) = 1 - b_v$ (8)

and using (6) and (4), $P(s+1; s) = b_v - b_d$ (9)

As a check, the sum of (8) and (9) is $1 - b_d$, as in (7).

Going back to (1) and (2)

$$\gamma = b_d = b_v + b_s - 1,$$

$$\alpha = b_v - b_d = 1 - b_s$$

and hence

$$P(c) = \gamma + \alpha/2 = (1 - b_s + b_v)/2$$

for $\Delta T = (b_v - b_s)q$, as long as

$$1 - b_s \leq b_v \leq 1$$

The other three cases are done in the same way.

When T_1 and T_2 are independent,

$$P(i; j) = P(t_1 = i) \cdot P(t_2 = j)$$

When $v = s$, $\alpha = P(s+1; s) = b_v(1 - b_s)$ and

$$\gamma = P(s+1;s+1) + P(s;s) = b_v \cdot b_s + (1-b_v)(1-b_s)$$

$$\text{thus } P(c) = \alpha + \gamma/2 = (1+b_v-b_s)/2 \text{ for } \Delta T/q = b_v - b_s.$$

Appendix II: Probability distributions - onset-offset delay model

Given 3 uniformly distributed random variables, x_1, x_2, x_3 with the probability distribution functions

$$g_1(x_1) = 1/q \quad 0 \leq x_1 \leq q$$

$$g_2(x_2) = 1/q \quad 0 \leq x_2 \leq q$$

$$g_3(x_3) = 1/q \quad 0 \leq x_3 \leq q$$

we want to calculate the probability distribution function for the linear combination

$$X = x_1 + x_2 - 2x_3$$

X is defined over the range $-2q \leq X \leq 2q$.

For any value A within this range,

$$\begin{aligned} P(X=A) &= \sum_{m,n} P(x_1=m \text{ and } x_2=n \text{ and } x_3=\frac{1}{2}(x_1+x_2-A)) \\ &= \sum_{m,n} P(x_1=m) \cdot P(x_2=n) \cdot P(x_3 = \frac{1}{2}(x_1+x_2-A)) \\ &= \frac{1}{q^3} \cdot \sum \text{all points } (m,n) \text{ satisfying conditions 1,2,3} \\ &\quad \text{simultaneously.} \end{aligned}$$

$$(1) \quad 0 \leq m \leq q$$

$$(2) \quad 0 \leq n \leq q$$

$$(3) \quad 0 \leq \frac{m+n-A}{2} \leq q.$$

All points satisfying (1) and (2) lie within the shaded square shown in fig. A2.

All points satisfying (3) lie between the lines $n=2q+A-m$ and $n=A-m$. Hence all points satisfying conditions (1), (2) and (3) simultaneously are in that portion of the square lying between the pair of lines. As A increases, the "n-intercepts" of the lines move up on the n -axis, and the overlap of the two areas changes.

As A increases from $-2q$ to $-q$, the overlap is $\frac{1}{2}(2q+A)^2$.

When $-q \leq A \leq 0$, the overlap is $q^2 - \frac{1}{2}A^2 = \frac{1}{2}(2q^2 - A^2)$.

For $0 \leq A \leq q$, the overlap is $\frac{1}{2}(2q^2 - A^2)$.

For $q \leq A \leq 2q$, the overlap is $\frac{1}{2}(2q-A)^2$.

Thus the probability distribution function for X consists of three segments,

$$\begin{aligned} f_1(X) &= \frac{1}{2} \cdot (2q-X)^2 / 2q^3 & q \leq X \leq 2q \\ f_2(X) &= \frac{1}{2} \cdot (2q^2 - X^2) / 2q^3 & -q \leq X \leq q \\ f_3(X) &= \frac{1}{2} \cdot (2q+X)^2 / 2q^3 & -2q \leq X \leq -q \end{aligned} \quad (4)$$

The additional factor of $\frac{1}{2}$ has been included as a normalization constant, in order to have the areas under this distribution function (the $Q^{(3)}$ curve) sum to unity.

To calculate the cumulative distribution, we express all distances in units of q . Set $x=X/q$, and $c=c'/q$. Then these areas are

$$\begin{aligned} \int_{-2}^{c'} \frac{1}{4}(2-x)^2 dx &= 1/12 \cdot (2+c')^3 & \text{for } -2 \leq c' \leq -1 \\ \int_{-1}^{c'} \frac{1}{4}(2-x^2) dx + 1/12 &= \frac{1}{4}(2c' - \frac{c'^3}{3}) + \frac{1}{2} & \text{for } -1 \leq c' \leq 1 \\ \int_1^{c'} \frac{1}{4}(2-x)^2 dx + 11/12 &= 1 - (2-c')^3 / 12 & \text{for } 1 \leq c' \leq 2 \end{aligned} \quad (5)$$

It is important to keep in mind that c' is a directed distance from the mean of the $Q^{(3)}$ distribution.

Fig. A2 about here

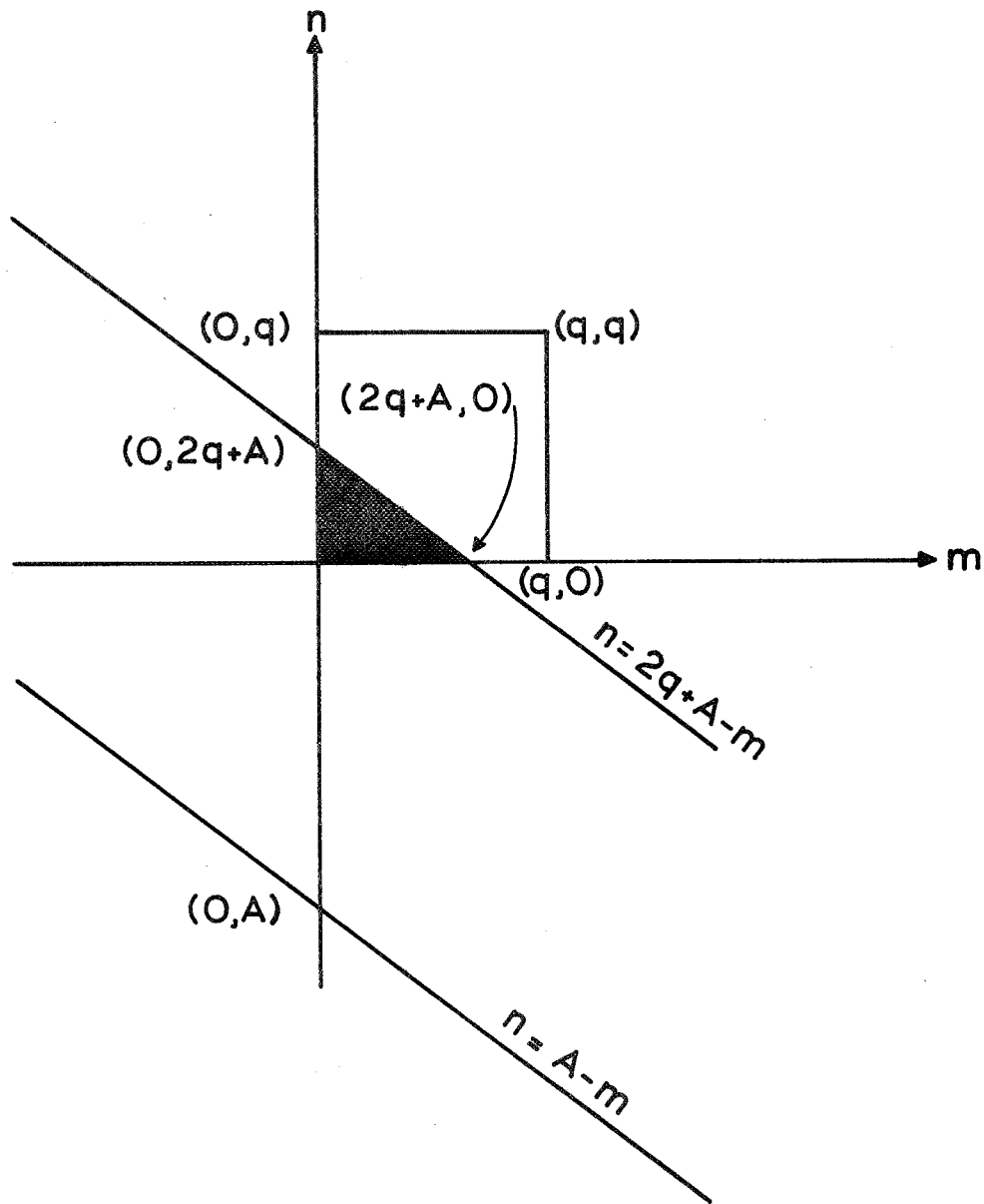


Fig.A-2. Shaded area represents the total number of points (m,n) satisfying simultaneously the three conditions, $0 \leq m \leq q$, $0 \leq n \leq q$, and $0 \leq \frac{m+n-A}{2} \leq q$ where A can take on any value between $\pm 2q$

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TABLE 1 Summary of Performance : Adjacent Intervals

HS: Series I					HS: Series II				
T	P(2/S ₂)	P(2/S ₁)	P(C)	d'	T	P(2/S ₂)	P(2/S ₁)	P(C)	d'
13	.72	.16	.78	1.57	5	.62	.5	.57	.30
23	.78	.20	.79	1.64	10	.64	.63	.63	.69
32	.84	.13	.86	2.12	15	.70	.71	.71	1.10
42	.92	.13	.90	2.53	20	.81	.74	.74	1.32
53	.97	.13	.92	3.01	25	.73	.73	.73	1.22
63	.93	.04	.94	3.22	30	.81	.83	.83	1.92
72	.98	.09	.94	3.39	37	.84	.88	.88	2.33
84	.99	.08	.96	3.72	44	.88	.90	.90	2.58
93	1.00	.07	.97	3.79	51	.95	.93	.93	2.98
104	1.00	.02	1.00	4.37	59	.95	.95	.95	3.28
PC: Series I					PC: Series II				
13	.89	.39	.76	1.51	5	.92	.72	.62	.82
23	.83	.25	.79	1.62	10	.93	.51	.71	1.44
32	.91	.09	.91	2.68	15	.90	.32	.79	1.75
42	.90	.06	.92	2.83	20	.88	.27	.80	1.78
53	.97	.01	.98	4.20	25	.94	.09	.93	2.89
63	.97	.01	.99	4.20	30	.90	.06	.91	2.83
72	.99	.01	.99	4.64	37	.89	.06	.92	2.78
84	1.00	.00	1.00	-	44	.99	.02	.99	4.37
93	1.00	.00	1.00	-	51	.95	.00	.98	-
104	1.00	.00	1.00	-	59	.98	.00	.99	-

TABLE 1 Summary of Performance : Adjacent Intervals

KL: Series I					KL: $T_s = 171$				
ΔT	$P(2/S_2)$	$P(2/S_1)$	$P(C)$	d'	ΔT	$P(2/S_2)$	$P(2/S_1)$	$P(C)$	d'
7	.82	.63	.60	.58	6	.84	.71	.57	.44
14	.87	.48	.70	1.18	12	.76	.48	.62	.76
21	.79	.44	.67	.96	18	.80	.44	.68	.99
31	.77	.19	.79	1.62	26	.71	.28	.71	1.14
41	.89	.18	.85	2.14	34	.68	.13	.78	1.60
53	.89	.09	.90	2.58	44	.82	.40	.86	2.20
65	.92	.02	.95	3.45					
77	.96	.00	.98	-					
89	.96	.02	.98	3.8					
101	.99	.01	.99	4.64					
KL: Series II					MR: Series II				
5	.67	.63	.53	.11	5	.73	.63	.56	.28
10	.67	.49	.60	.44	10	.76	.49	.64	.76
15	.72	.47	.62	.66	15	.79	.34	.73	1.22
20	.76	.31	.73	1.21	20	.84	.30	.78	1.52
25	.71	.29	.71	1.11	25	.84	.23	.81	1.73
30	.70	.28	.71	1.10	30	.87	.27	.81	1.74
37	.86	.15	.85	2.12	37	.85	.17	.84	1.99
44	.81	.08	.86	2.28	44	.90	.16	.87	2.27
51	.88	.05	.92	2.82	51	.89	.14	.87	2.31
59	.88	.03	.92	2.92	59	.91	.18	.87	2.26

TABLE 2: Criterion location for the Creelman model
(in standard deviation units)

T	PC	HS	KL
5	-1.00	-.16	-.39
10	- .77	-.02	-.21
15	- .42	-.03	-.25
20	- .29	-.23	-.28
25	- .11	0	0
30	.15	.09	.03
37	.18	.18	-.02
44	.15	.12	.28
51	*	-.16	.31
59	*	0	.39

*: No estimate of d' or c is available, since either $P(1/S_1)$ or $P(2/S_2)$ is 1.

TABLE 3 SUMMARY OF SAME - DIFFERENT DATA

A. Three stimulus alternatives per session

	ΔT	T_D	$P(D/S_2)$	$P(D/S_1)$	$P(D/S_0)$	$P(S/S_0)$	$P(C)$	
KL								
days 2 - 4	50	312	$\frac{136}{179} = .76$	$\frac{137}{180} = .76$	.52	$\frac{172}{361} = .48$	.62	
days 5 - 6	50	312	$\frac{83}{117} = .71$	$\frac{56}{116} = .48$	.41	$\frac{136}{232} = .59$	.59	
HS								
days 2 - 4	30	312	$\frac{154}{180} = .84$	$\frac{77}{180} = .43$	.36	$\frac{230}{362} = .64$	.64	
days 5 - 6	30	312	$\frac{101}{118} = .86$	$\frac{58}{119} = .49$	.34	$\frac{160}{241} = .66$	.67	
*	50	312	$\frac{117}{132} = .87$	$\frac{61}{132} = .46$	.26	$\frac{201}{273} = .74$	.70	
RM								
days 2 - 3	50	312	$\frac{87}{94} = .93$	$\frac{78}{107} = .73$	.15	$\frac{185}{217} = .85$	.84	
days 4 - 6	30	312	$\frac{173}{222} = .78$	$\frac{100}{221} = .45$	.33	$\frac{300}{449} = .67$	.64	S_0 samples

*a change in T , was made towards the end of the experiment.

TABLE 3 (continued)

B. Two stimulus alternatives per session

	T	T _D	P(D/S ₂)	P(D/S ₁)	P(D/S ₀)	P(S/S ₀)	P(C)	
KL S ₀ - S ₁	50	312	-	$\frac{210}{243} = .61$	.41	$\frac{202}{341} = .59$	.60	
S ₀ - S ₂	50	312	$\frac{71}{94} = .76$	-	.21	$\frac{74}{94} = .79$	.77	
HS S ₀ - S ₂	30	312	$\frac{95}{121} = .79$	-	.27	$\frac{89}{122} = .73$	.76	
S ₀ - S ₁	30		-	$\frac{77}{245} = .31$	.27	$\frac{176}{241} = .73$	.52	

C. Change in total duration								
KL	50	450	$\frac{46}{120} = .38$	$\frac{74}{117} = .63$	.36	$\frac{133}{240} = .64$	.57	S ₀ samples
	50	450	$\frac{30}{55} = .55$	$\frac{30}{65} = .45$	.36	$\frac{72}{113} = .64$	.57	S ₂ samples
HS	30	284	$\frac{115}{135} = .85$	$\frac{38}{108} = .35$	.37	$\frac{140}{221} = .63$	.62	
	30	360	$\frac{156}{194} = .80$	$\frac{76}{211} = .36$	.33	$\frac{277}{413} = .67$	.63	
	30	500	$\frac{76}{147} = .52$	$\frac{69}{148} = .47$	.41	$\frac{170}{290} = .59$	.54	

TABLE 3 - SUMMARY OF SAME - DIFFERENT DATA

(continued)

D. Performance after $S_0 - S_1$ practice

		ΔT	T_D	$P(D/S_2)$	$P(D/S_1)$	$P(D/S_0)$	$P(S/S_0)$	$P(C)$	
KL	$S_0 - S_1$	50	312	-	$\frac{232}{299} = .78$	.24	$\frac{224}{296} = .76$	.77	
	$S_0 - S_1 - S_2$	50	312	$\frac{101}{206} = .49$	$\frac{155}{205} = .76$	.40	$\frac{230}{382} = .60$	.61	
		50	312	$\frac{78}{136} = .58$	$\frac{103}{131} = .80$	.37	$\frac{168}{267} = .63$	.66	S_0 samples (in testing)
HS	$S_0 - S_1$	30	312	-	$\frac{90}{235} = .38$	.39	$\frac{139}{228} = .61$	.50	
	$S_0 - S_1 - S_2$	50	312	$\frac{60}{67} = .90$	$\frac{56}{70} = .8$	.24	$\frac{103}{135} = .76$	.81	S_0 samples (in testing)
RM	$S_0 - S_1$	30	312	-	$\frac{89}{125} = .71$	.30	$\frac{80}{115} = .70$	.71	S_0 samples
	$S_0 - S_1 - S_2$	30	312	$\frac{65}{67} = .84$	$\frac{57}{75} = .76$	.35	$\frac{96}{148} = .65$	.73	S_0 samples

Table 4 : Estimates of d'_{01} and d'_{02} from the two stimulus alternative data, for the quantal onset - offset (Q00) and the Creelman (C) models.

	ΔT	performance	(Q00)	d'_{01}	d'_{02}	(C)	d'_{01}	d'_{02}
KL	50	(-, .61, .41)		.40	-		.51	-
		(.76, -, .21)		-	1.17		-	1.51
HS	30	(-, .31, .27)		.09	-		.10	-
		(.79, -, .27)		-	1.10		-	1.42

Table 5 : Estimates of k , k' , and $P(D/S_2)$ from the three stimulus alternative data. (T2 dependent on T1)

Performance	T_D	ΔT	q	k'	k	$P(D/S_2)$
HS (<u>.84</u> , .43, .36)	312	30	25	-.01	.51	.84
			26	.36	.48	.73
HS (<u>80</u> , .36, .33)	360	30	25	.07	.43	.82
			26	.26	.41	.74
HS (<u>.52</u> , .47, .41)	500	30	52	.07	.9	.52
			51	.3	.74	.57
KL (<u>.76</u> , .76, .52)	312	50	50	.37	.1	.76
			51	.46	1.009	.76
KL (<u>.71</u> , .48, .41)	312	50	50	.29	.60	.73
			51	.36	.60	.71
(<u>.55</u> , .45, .36)	450	50	112	.35	.78	.50
			110	.30	.84	.49
RM (<u>.78</u> , .45, .33)	312	30	25	-.081	.551	.83
			26	.33	.54	.72
(<u>.84</u> , .76, .35)	312	30	25	-.23	.95	.80
			26	.35	1.06	.73
						after practice & S_0 samples

Table 6 : Summary of Performance : Separated Intervals

$T_s = 100$											$T_s = 200$										
RM	ΔT	$P(1/S_1)$	$P(2/S_2)$	$P(C)$	d'	ΔT	$P(1/S_1)$	$P(2/S_2)$	$P(C)$	d'	RM	ΔT	$P(1/S_1)$	$P(2/S_2)$	$P(C)$	d'					
	6	.849	.855	.85	2.12	6	.801	.713	.76	1.40		6	.801	.713	.76	1.40					
	12	.960	.942	.95	3.30	12	.903	.857	.88	2.36		12	.903	.857	.88	2.36					
	18	.994	.991	.99	4.64	18	.967	.962	.96	3.80		18	.967	.962	.96	3.80					
	24	1.	1.	1.	-	24	.975	.977	.98	4.10		24	.975	.977	.98	4.10					
	30	1.	1.	1.	-	30	.987	.989	.99	4.64		30	.987	.989	.99	4.64					
HS	6	.674	.708	.69	1.00	6	.621	.635	.63	.66		6	.621	.635	.63	.66					
	12	.827	.890	.86	2.18	12	.730	.721	.73	1.19		12	.730	.721	.73	1.19					
	18	.920	.961	.94	3.15	18	.775	.835	.80	1.76		18	.775	.835	.80	1.76					
	24	.939	.982	.96	3.60	24	.820	.912	.87	2.26		24	.820	.912	.87	2.26					
	30	.968	.998	.98	4.64	30	.876	.932	.90	2.64		30	.876	.932	.90	2.64					
V	6	.520	.639	.58	.41	6	.698	.650	.67	.91		6	.698	.650	.67	.91					
	12	.659	.832	.75	1.36	12	.790	.777	.78	1.58		12	.790	.777	.78	1.58					
	18	.815	.907	.86	2.26	18	.889	.816	.85	2.14		18	.889	.816	.85	2.14					
	24	.882	.962	.92	2.92	24	.839	.929	.88	2.46		24	.839	.929	.88	2.46					
	30	.922	.988	.95	3.72	30	.887	.938	.92	2.78		30	.887	.938	.92	2.78					
PL	6	.722	.671	.7	1.02	6	.782	.553	.66	.9		6	.782	.553	.66	.9					
	12	.917	.819	.87	2.32	12	.929	.743	.84	2.11		12	.929	.743	.84	2.11					
	18	.923	.928	.93	2.87	18	.912	.867	.89	2.47		18	.912	.867	.89	2.47					
	24	.979	.972	.98	3.93	24	.944	.945	.94	3.19		24	.944	.945	.94	3.19					
	30	.986	1.	1.	-	30	.972	.956	.96	3.63		30	.972	.956	.96	3.63					
KL	10	.471	.694	.58	.43	10	.411	.705	.56	.32		10	.411	.705	.56	.32					
	20	.469	.815	.64	.50	20	.460	.804	.63	.74		20	.460	.804	.63	.74					
	30	.615	.868	.74	1.44	30	.638	.851	.74	1.40		30	.638	.851	.74	1.40					
	40	.702	.915	.81	1.92	40	.613	.893	.75	1.51		40	.613	.893	.75	1.51					
	50	.775	.945	.86	2.41	50	.643	.977	.85	2.41		50	.643	.977	.85	2.41					