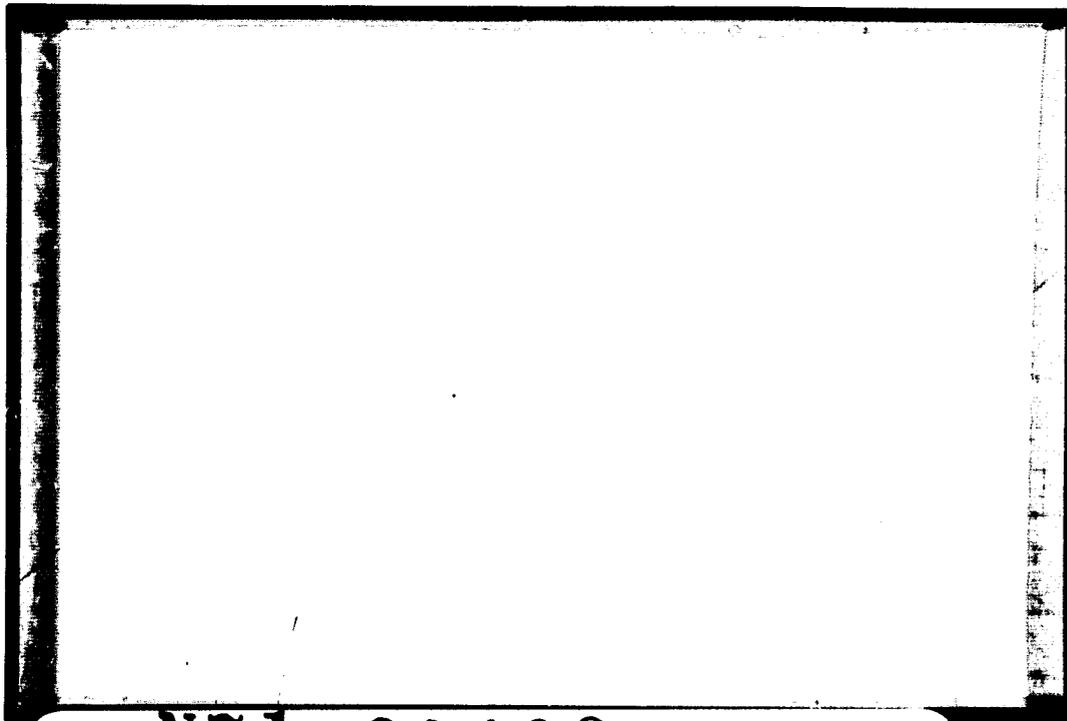


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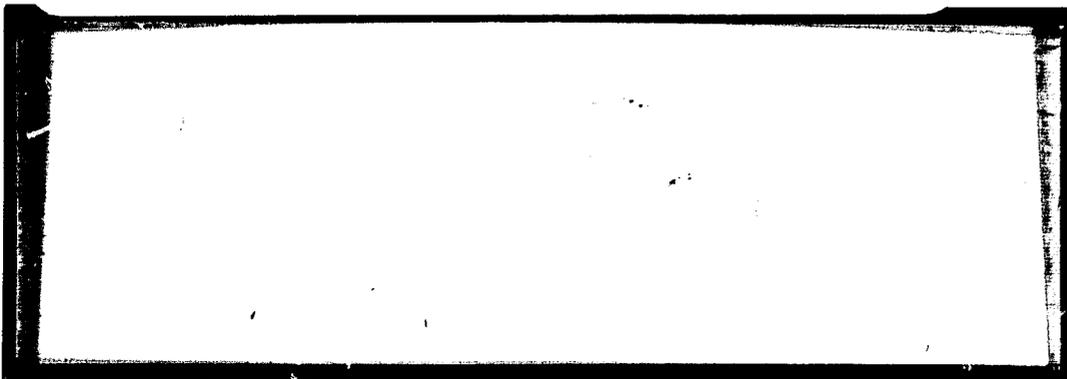
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AUBURN UNIVERSITY  
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TECHNICAL REPORT NO. 1

RECEIVER ANTENNAS FOR APPLICATION IN  
A TELEVISION BROADCAST RELAY SYSTEM

Prepared by

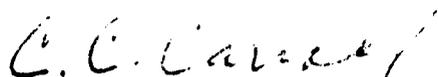
SATELLITE COMMUNICATIONS LABORATORY  
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## FORWARD

This is a technical report of a study conducted by the Electrical Engineering Department under the auspices of the Auburn Research Foundation toward the fulfillment of the requirements prescribed in NASA Contract NAS8-24818. An investigation of suitable home UHF receiver antennas for use in a synchronous satellite relay system is presented.

## ABSTRACT

Various types of antennas are considered for use a UHF home receiving antennas involving a synchronous satellite relay link. A brief survey of present UHF and their application in this system is presented. Spiral and helical antennas are considered and designed for this mode of operation.

#### ACKNOWLEDGEMENT

The authors wish to express their appreciation to the many members of the group from the Auburn University Electrical Engineering Department, both student and faculty, who contributed to the overall preparation of the study.

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RECEIVER ANTENNAS FOR APPLICATION IN  
A TELEVISION BROADCAST RELAY SYSTEM

J. M. Loomis III and E. R. Graf

INTRODUCTION

Numerous proposals have been made, primarily to the National Aeronautics and Space Administration,<sup>1,2</sup> to use the existing Saturn system in synchronous orbit to relay UHF (470-890 MHz) television signals for home reception.

Home T.V. antennas suitable for application with a broadcast system of this type are considered in this study. Those antennas presently employed with home receivers have two major drawbacks, firstly, they are designed to receive only horizontal polarization and secondly, they may present a sizeable wind resistance when rotated in a modified direction.

A number of other systems, spiral and helical antennas, are available which on a mass-produced basis would be as inexpensive and possess better antenna characteristics. These antennas provide a higher gain and will accept circular polarization. Circular polarization is a definite advantage due to the Faraday effect which can cause a total loss of signal (see Appendix I). The design criterion for this study include:

1. Gain
2. Bandwidth

3. Polarization
4. Operational Ease
5. Size
6. Expense

The gain for all antennas considered is taken relative to an isotropic point source and is given in dB. The beamwidth should be large enough so that minimal pointing will be necessary to achieve a strong signal, yet small enough to provide acceptable gain. Since the satellite is stationary there is no need for a multi-lobed antenna; here only one main beam is necessary. The bandwidth should cover the entire spectrum used for UHF television transmission. The polarization will be considered with relationship to the Faraday effect. The antennas should be small enough to be easily handled and able to withstand normal wind and weather conditions. Expense will be considered in relation to present antenna systems and is compared on a mass-produced basis. The antennas should be internally matched to either 300 ohm twin lead or 50 ohm coaxial cable to facilitate the owners installation.

Throughout this study electromagnetic scaling is used in the absence of antenna data in the frequency spectrum under consideration.

## I. UHF ANTENNA SURVEY

A variety of home UHF television antennas are currently being produced. These antennas were designed for surface television reception, that is, designed to receive a horizontally polarized wave from a nearby transmitter. For the most part, these antennas present a low wind resistance, however orientation in a vertical position will increase wind resistance considerably.

The following currently produced antennas examined were taken from Jasik's Antenna Engineer's Handbook.<sup>3</sup> They are: 1. the triangular dipole in front of a screen, 2. stacked-V antenna, 3. corner reflector, 4. Yagi, and 5. log periodic. Gain versus frequency is plotted on the following pages (Figures 1-1 through 1-5). The solid line indicates the gain relative to a horizontally polarized source and the dotted line denotes the gain assuming a maximum 3 dB polarization loss due to Faraday rotation.

The triangular dipole in front of a screen (Figure 1-1), the stacked-V antenna (Figure 1-2), and the corner reflector (Figure 1-3), if turned upward, would maintain their wind resistance characteristics. The Yagi (Figure 1-4) and the log periodic (Figure 1-5), if turned upward, would increase their wind resistance; however, it should not be a major hindrance. All of these antennas have their lowest gain at the low end of the frequency spectrum (570 MHz.) and the highest gain near 800 MHz.

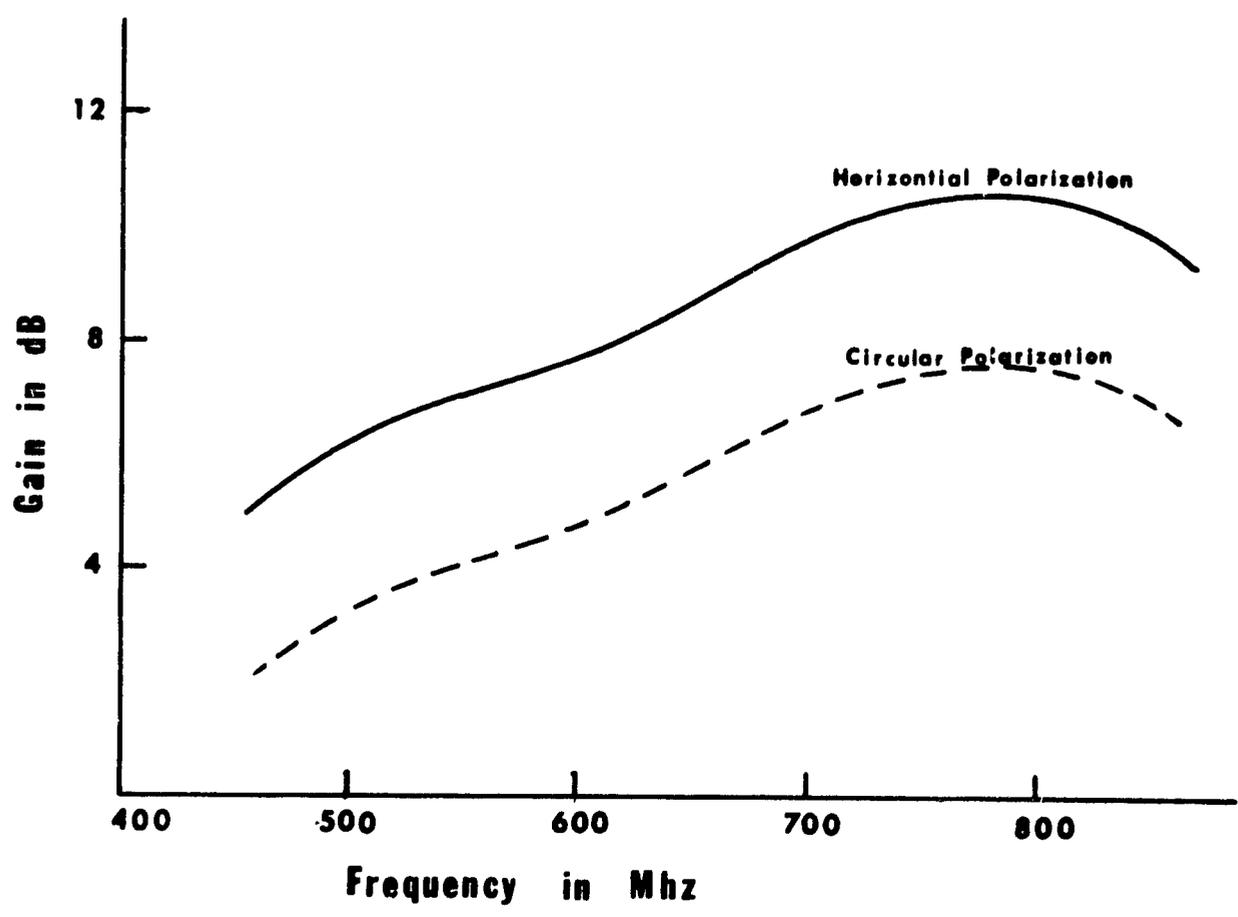
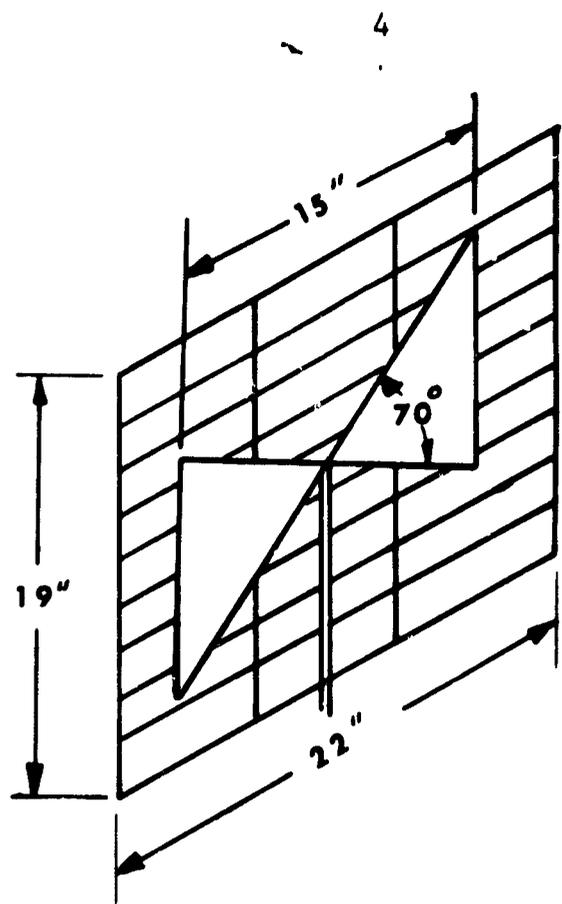


Figure 1-1-- Triangular dipole in front of a screen.

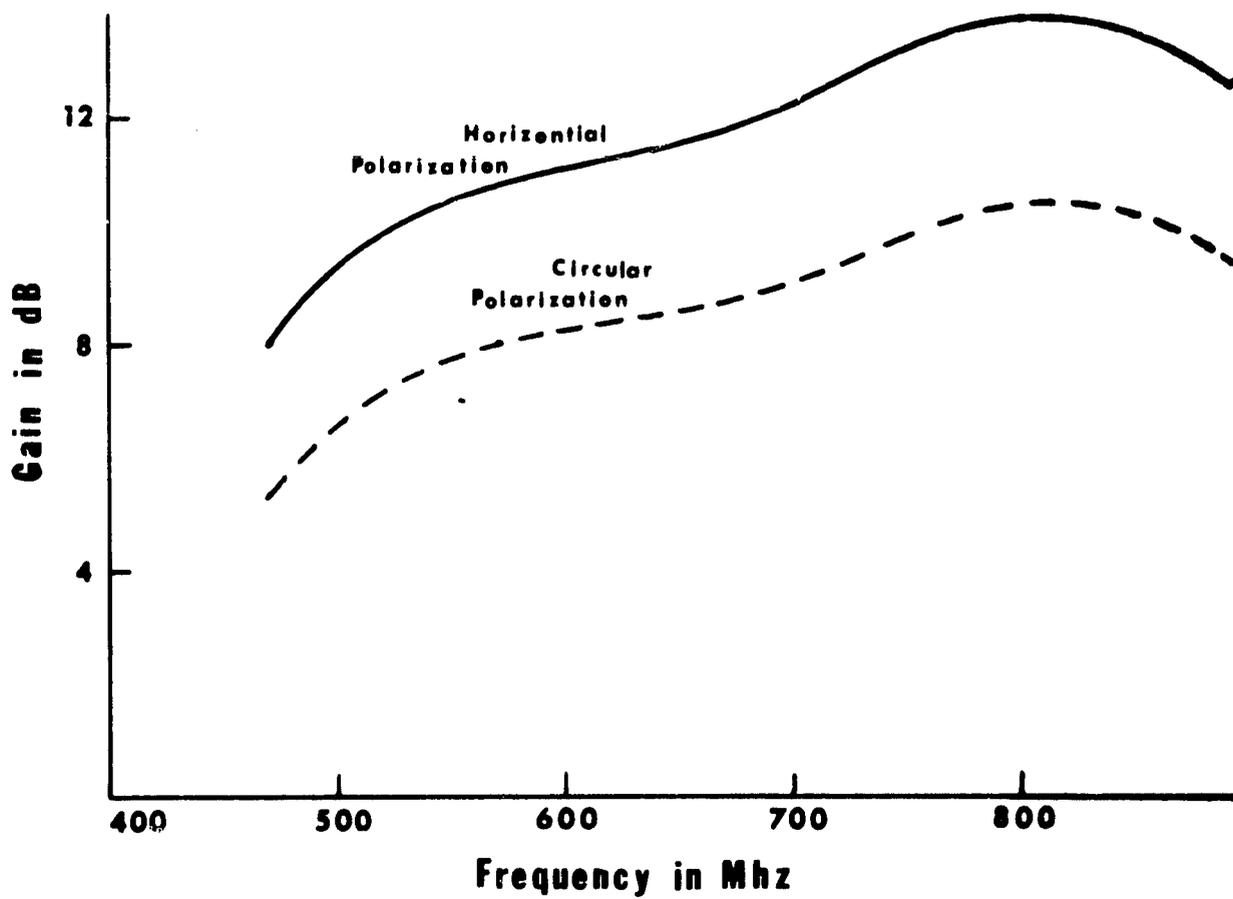
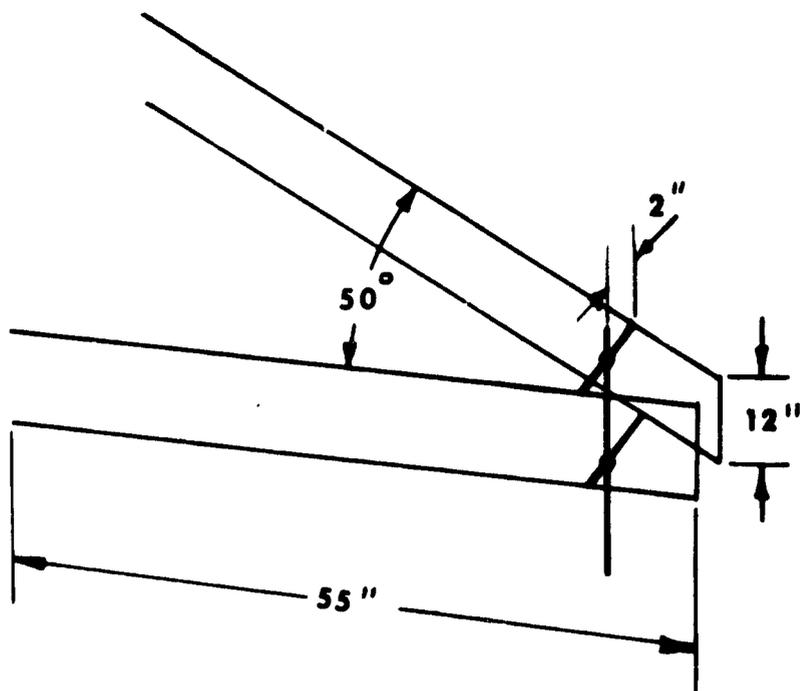


Figure 1-2 -- The stacked-V antenna.

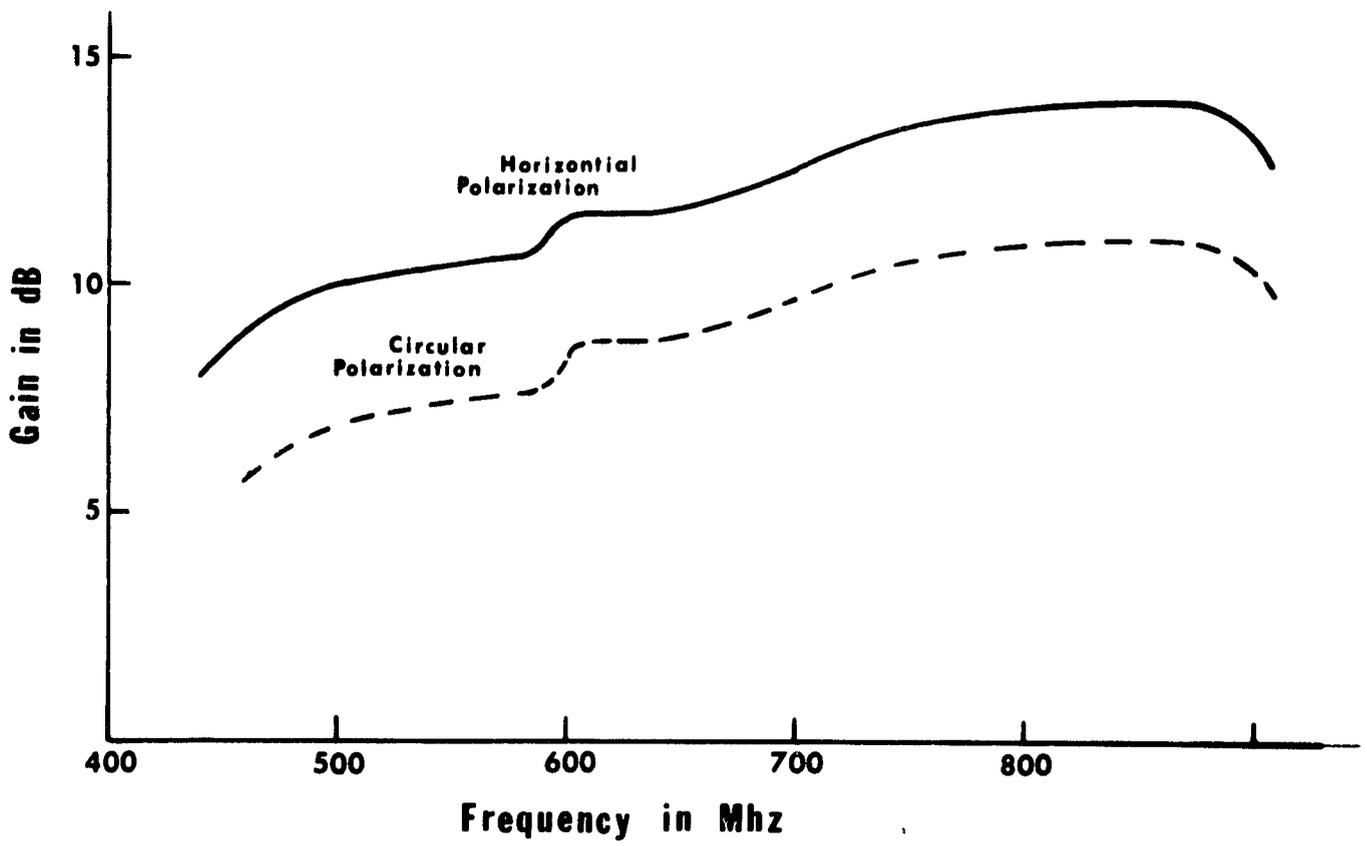
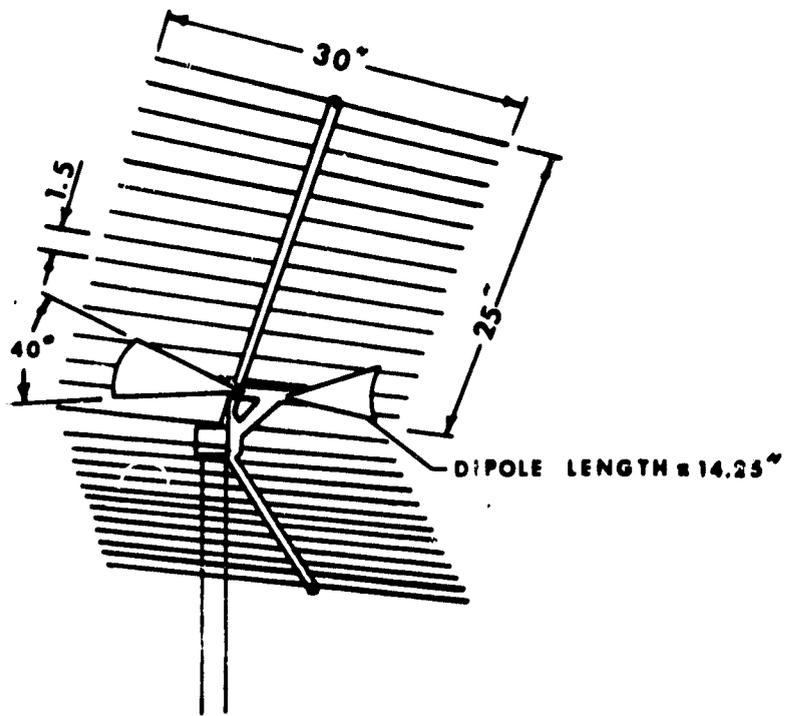


Figure 1-3 -- The corner reflector.

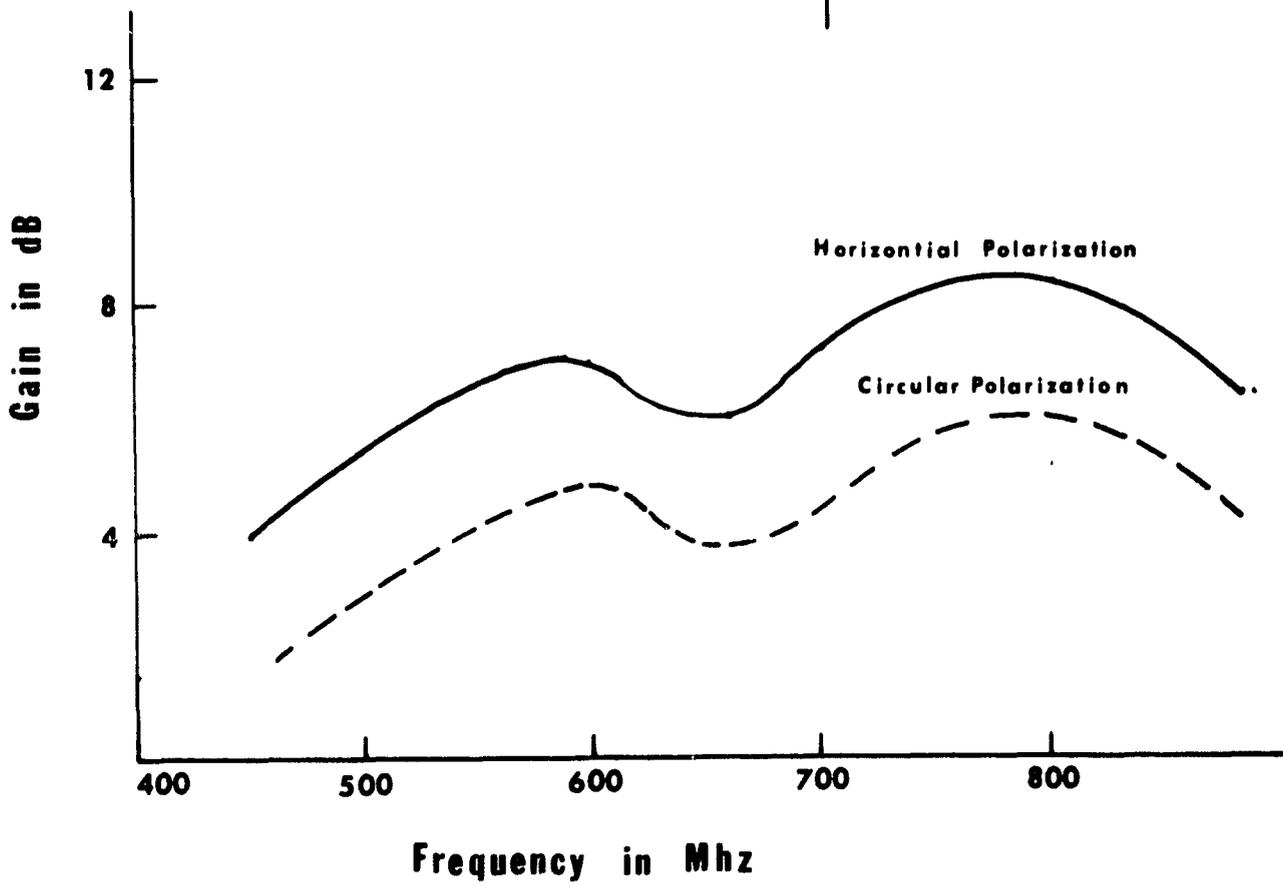
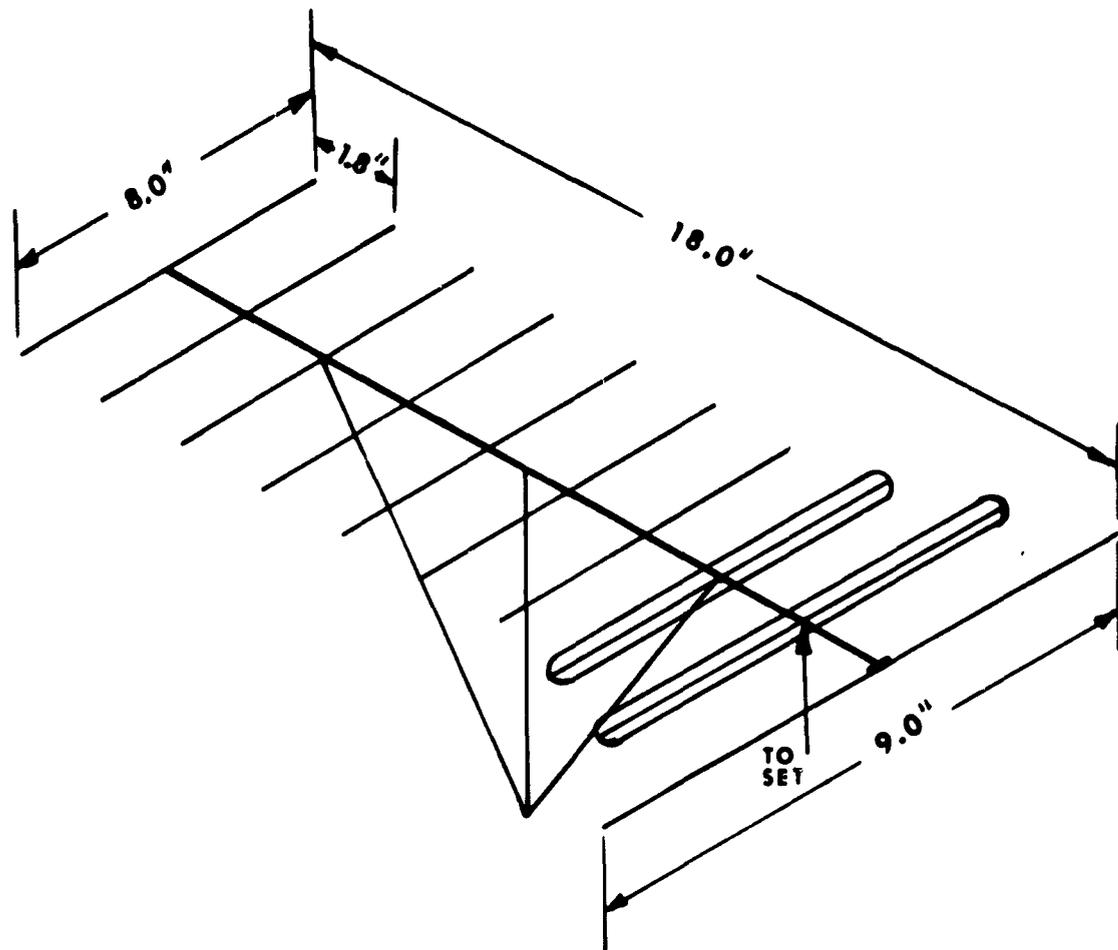


Figure 1-4-- Yagi.

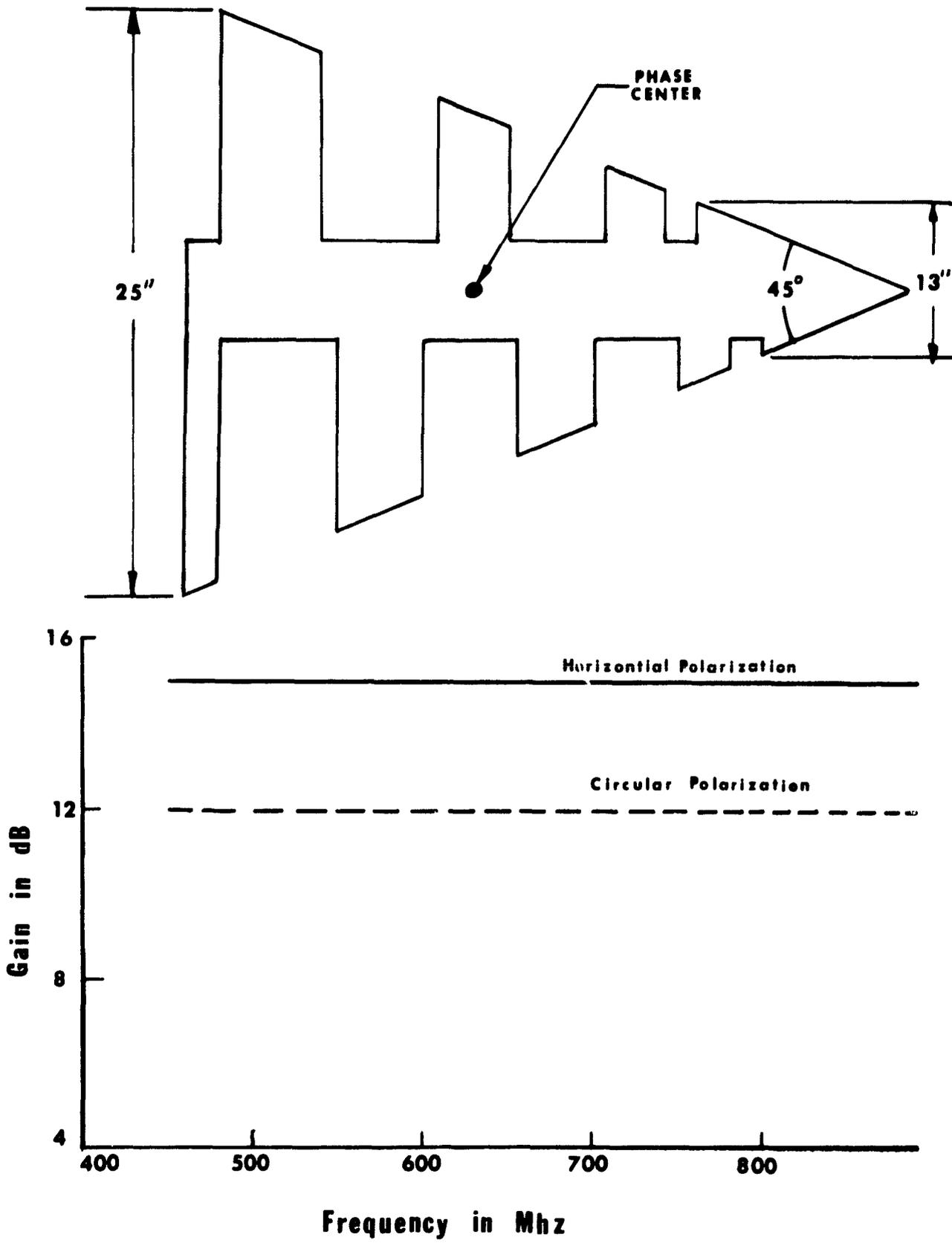


Figure 1-5-- Log-periodic antenna.

Another possible antenna that meets the requirements is the parabolic reflector (Figure 1-6). However for the desired gain this antenna is relatively large and requires a rather accurate construction to meet specifications.<sup>4</sup> A wideband feed antenna is also necessary. For the above reasons this antenna is expensive and imposes unreasonable financial restraints on the consumer.

Spiral and helical antennas, compared with the antennas briefly described above, are better suited for applications in the UHF television system under consideration and are discussed in detail in the remainder of this work

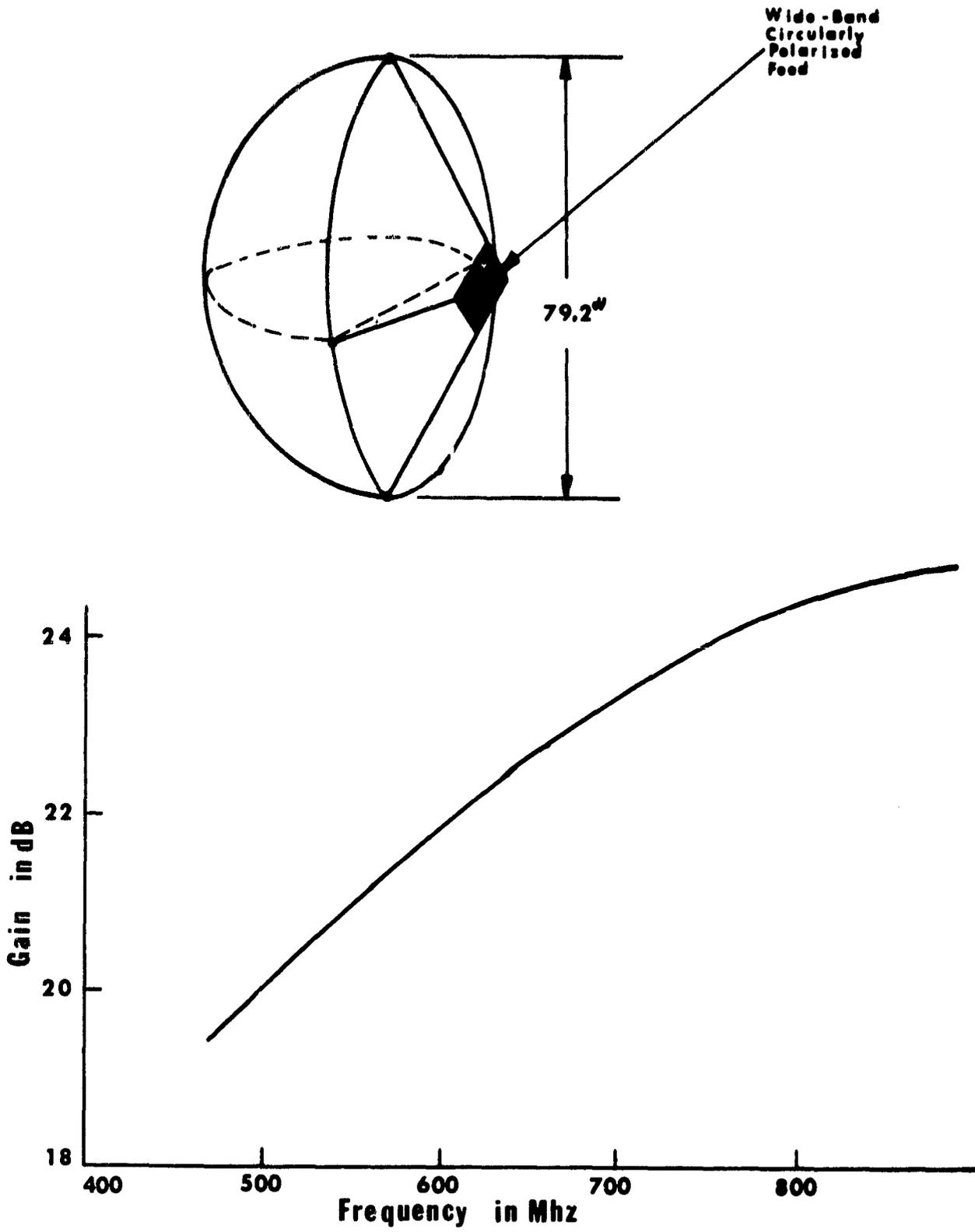


Figure 1-6-- Parabolic reflector.

## II. SPIRAL ANTENNAS

Spiral antennas present a good solution to the widebanded, circularly polarized antenna requirement. It is a two-dimensional structure that can be made by a photoetching technique or by a metal stamping process. There are three varieties of spirals that have received wide attention; these are the logarithmic or equiangular spiral, the Archimedean or arithmetic spiral and its rectangular counterpart. All these antennas have essentially the same characteristics and since the dual-arm Archimedean spiral has the greatest symmetry, this form will be discussed. Many of the results, however, are applicable to other configurations.<sup>5</sup>

The Archimedean spiral (Figure 2-1) arms are defined by the equation,

$$r = r_0 + a\phi$$

and where  $r$  = radius of arm at any particular point

$r_0$  = initial radius

$a$  = constant

$\phi$  = angle measured from x axis.

The dual-arm spiral has arms excited by currents having a 180 degree phase relationship and will radiate from a band of mean diameter equal to  $\lambda/\pi$ .<sup>6</sup> The radiation pattern is bidirectional and

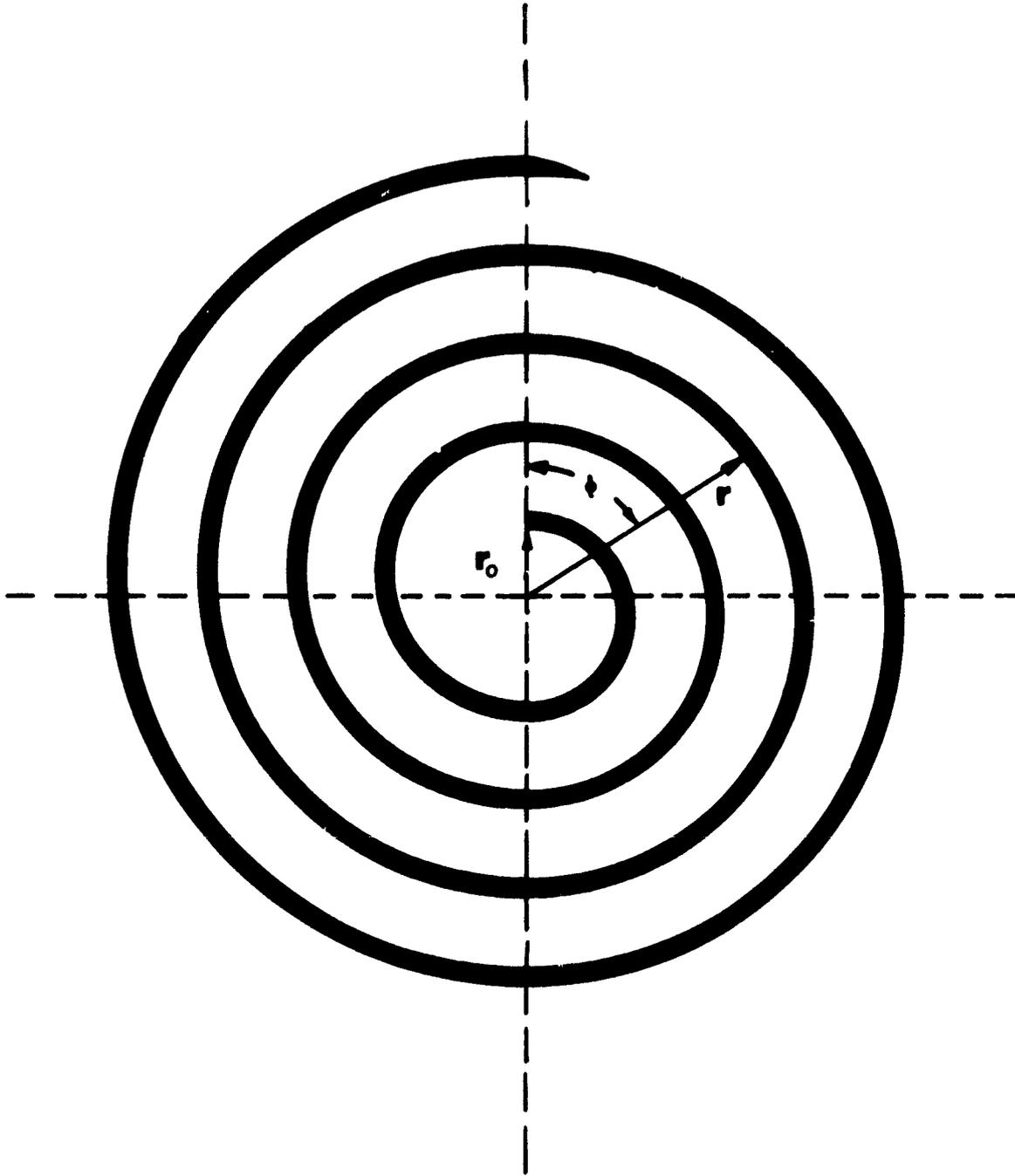


Figure 2-1-- The Archimedean spiral.

circularly polarized according to the winding sense of the spiral. This is defined as the normal mode and is the one in which this spiral is intended to operate.

Since the application in which the spiral is intended to operate requires a single main lobe, the spiral can be mounted at the mouth of a cavity. The task is then to establish the optimum values for the spiral diameter, cavity depth, spiral rate of growth, conductor width and spacing, and type of feed structure.

The effect of antenna diameter on gain is shown in Figure 2-2. It should be noted that the spiral asymptotically approaches its maximum gain at an approximate diameter of  $\lambda/2$ . By noting the data in Figure 2-3<sup>7</sup>, this dimension also provides an axial ratio of less than 2dB. The cavity depth as related to gain is shown in Figure 2-4 and is a maximum near  $\lambda/4$ , which would compare with the behavior of a dipole over a ground plane.

The cavity depth seems to provide the major hindrance to an extremely large operating bandwidth. In terms of a tolerable reduction in gain at the band edges, say 3 dB, the useable antenna bandwidth is limited to approximately 3:1 (ratio of  $f_{\max}$  to  $f_{\min}$ ). The conductor width and spacing does not have a great effect on the useable bandwidth or impedance of the antenna.<sup>8</sup> The feed structure requires special attention, however; a number of methods to match balanced and unbalanced systems over a relatively wide frequency range are available.<sup>9,10</sup>

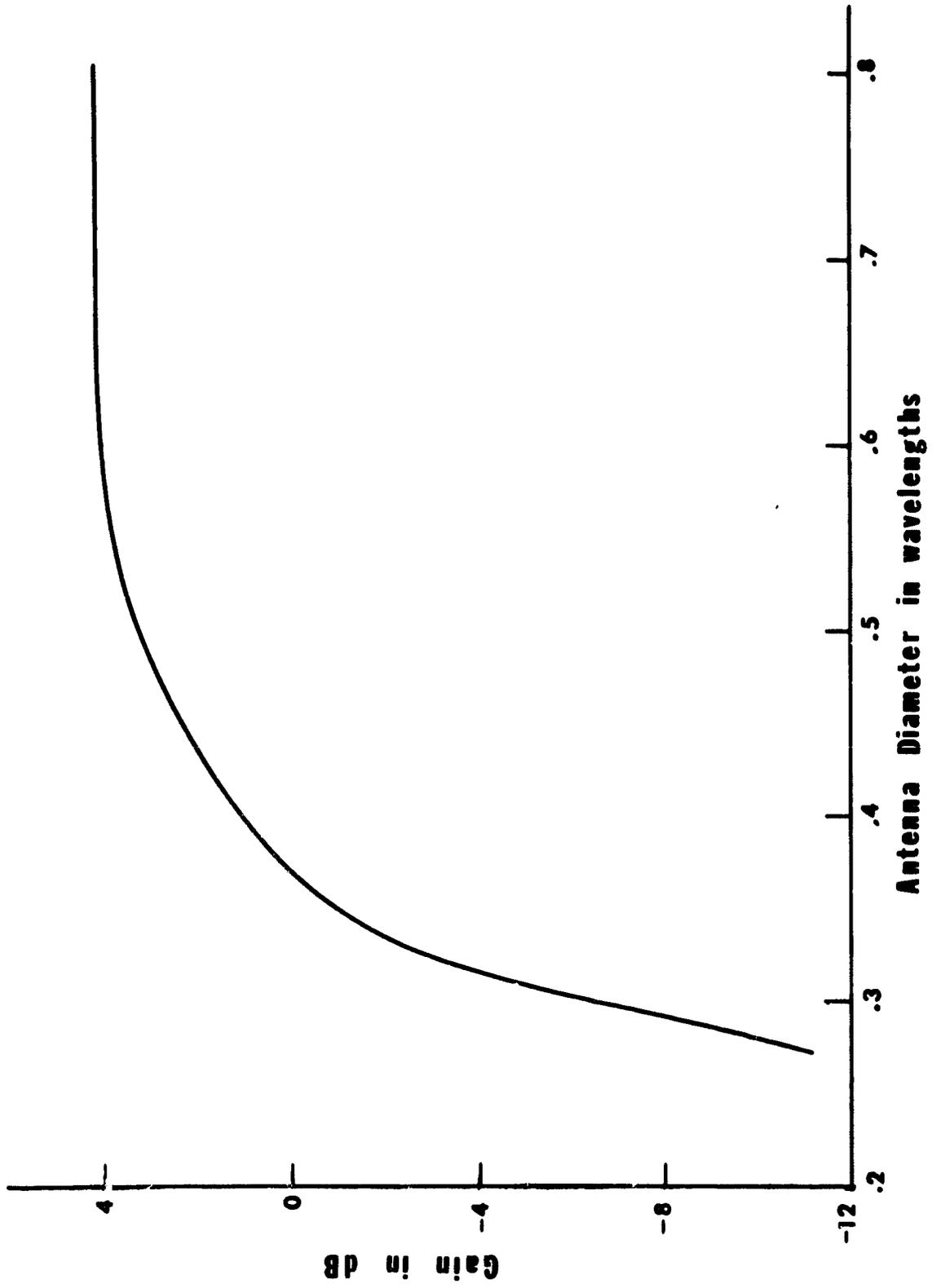


Figure 2-2-- The effect of antenna diameter on gain

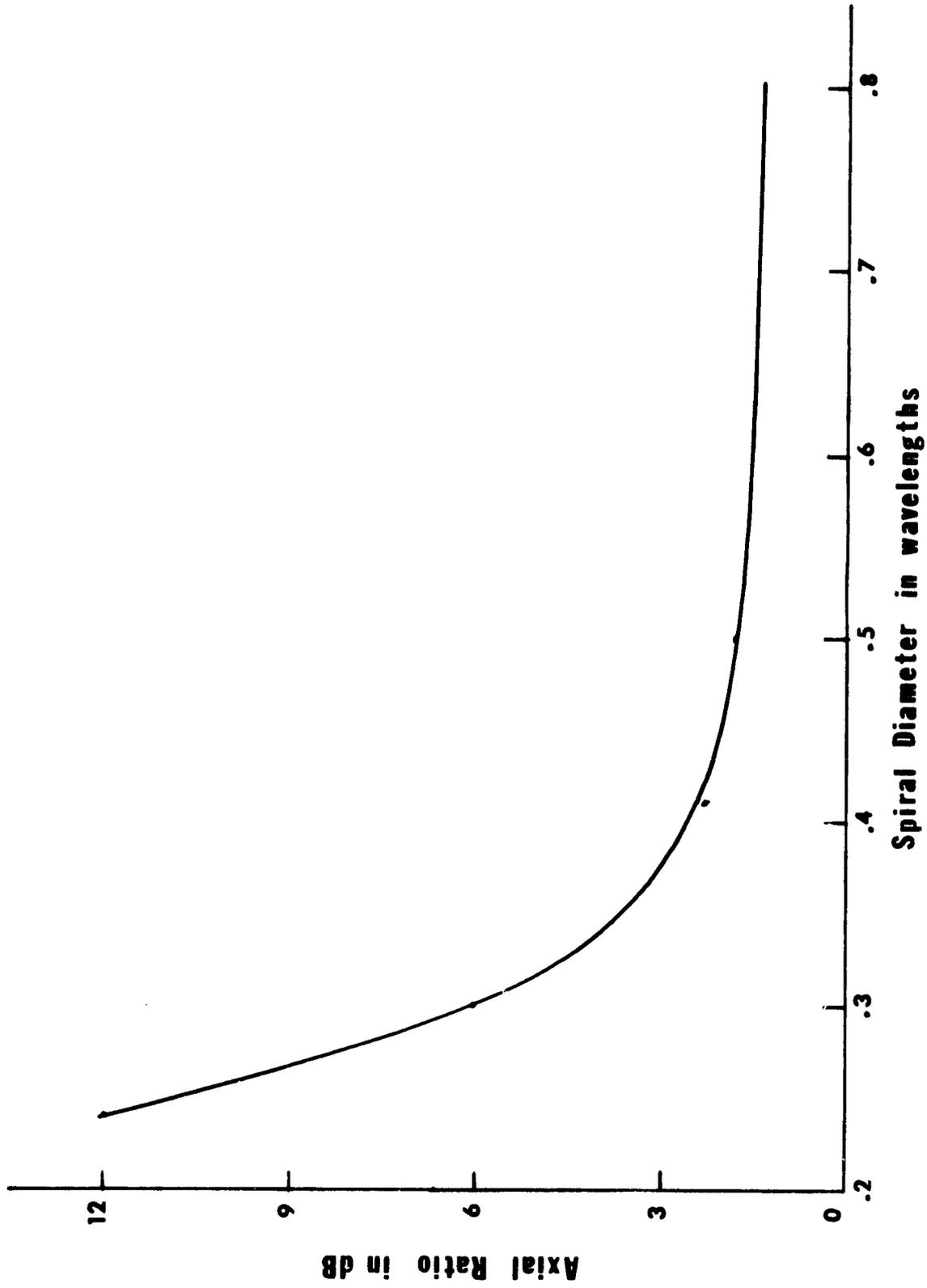


Figure 2-3-- On-axis axial ratio as a function of spiral diameter.

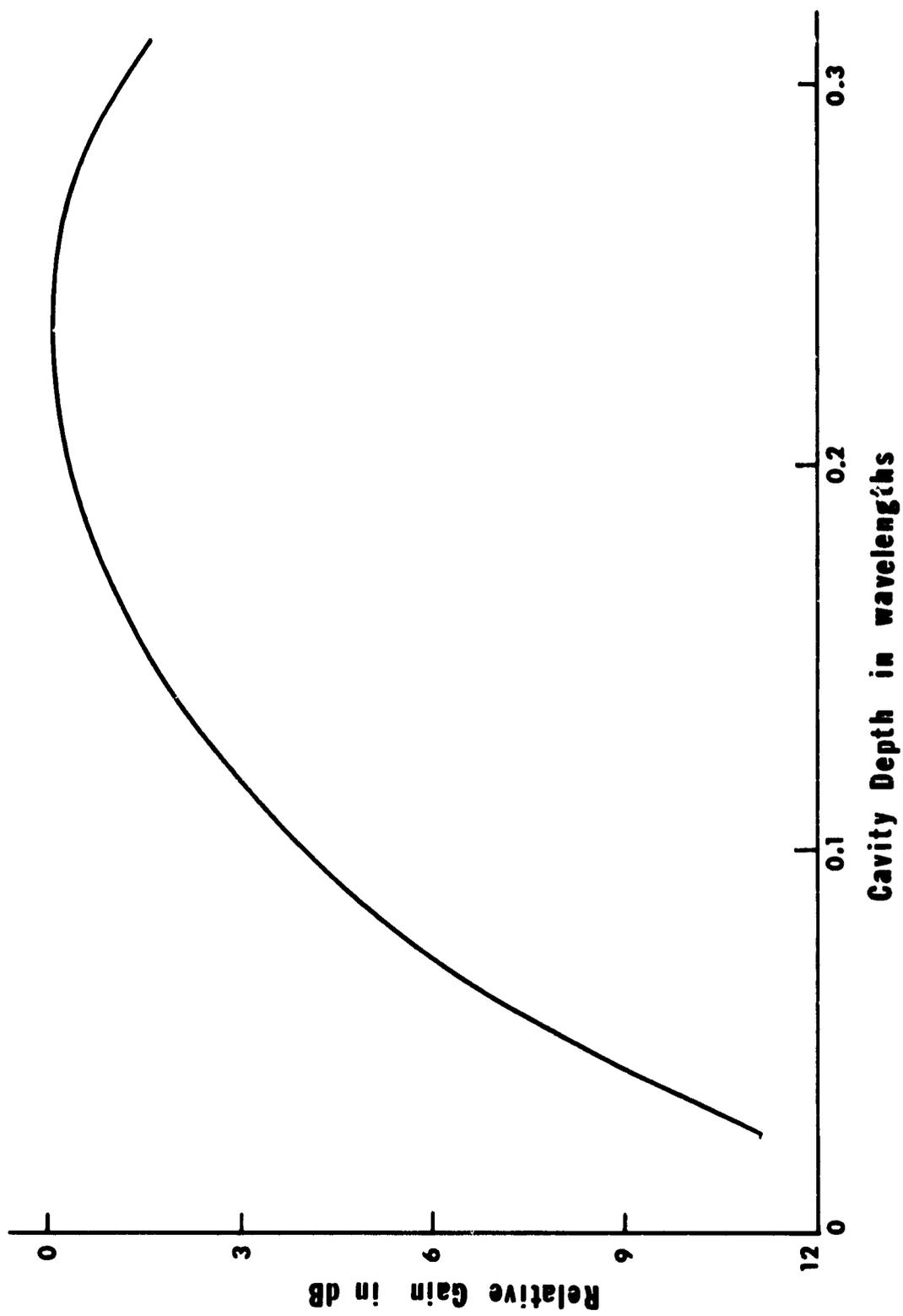


Figure 2-4-- Cavity depth as a function of gain for a dual arm spiral.

The axial ratio requirement results in criterion which force further bandwidth reductions. However, bandwidths in excess of 2:1, which are large enough to cover the entire UHF television spectrum are entirely feasible.

An Archimedean spiral with the conductor arm width equal to the space between the arms and a diameter of 28.0" will make a good UHF television antenna. The axial ratio (Figure 2-5) is less than 3 dB over the entire frequency band. The VSWR (Figure 2-6) referred to a 50 ohm line is less than 1.5 and the beamwidth (Figure 2-7) is between 60 and 80 degrees. Typical patterns are shown in Figure 2-8. These patterns are measured in both the  $\psi = 0$ -degree and  $\phi = 90$ -degree planes.

The Archimedean spiral can be approximated by a series of semi-circles. The patterns of such an approximate spiral correlate very well with those of the Archimedean spiral.<sup>11</sup> For this reason and the relatively simple means of construction, the spiral is a most inexpensive antenna.

The spiral does, however, have an important disadvantage in that the gain is low compared to other UHF antennas with similar radiation characteristics. The gain can be increased by making a conical spiral.<sup>12,13</sup> This type spiral loses its simplicity of construction. Since the conical helix, discussed in the next section, offers the same radiation properties as well as a simpler construction, the conical spiral will not be pursued further in this study.

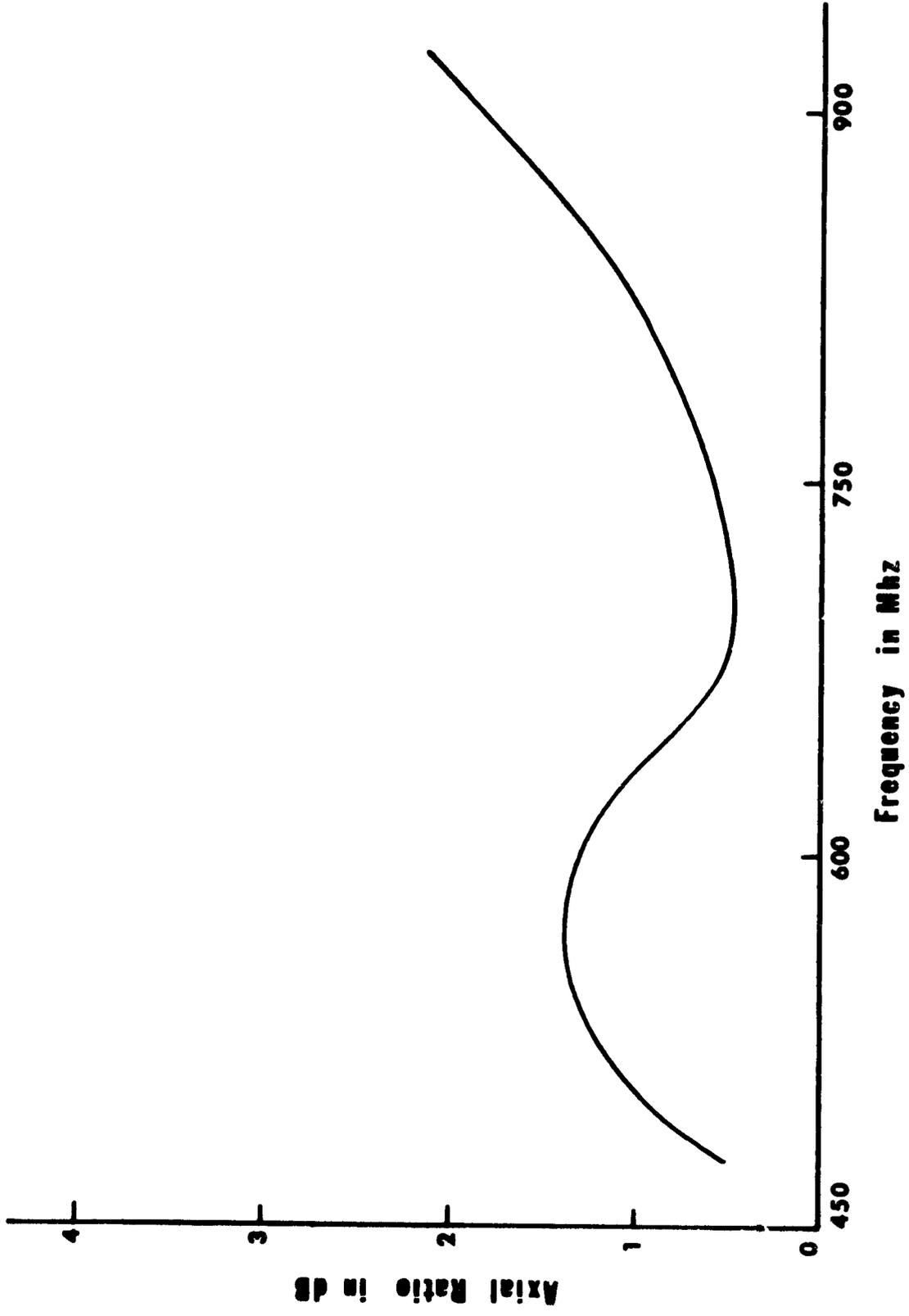


Figure 2-5-- Axial ratio as a function of frequency for a 28"-diameter Archimedean spiral.

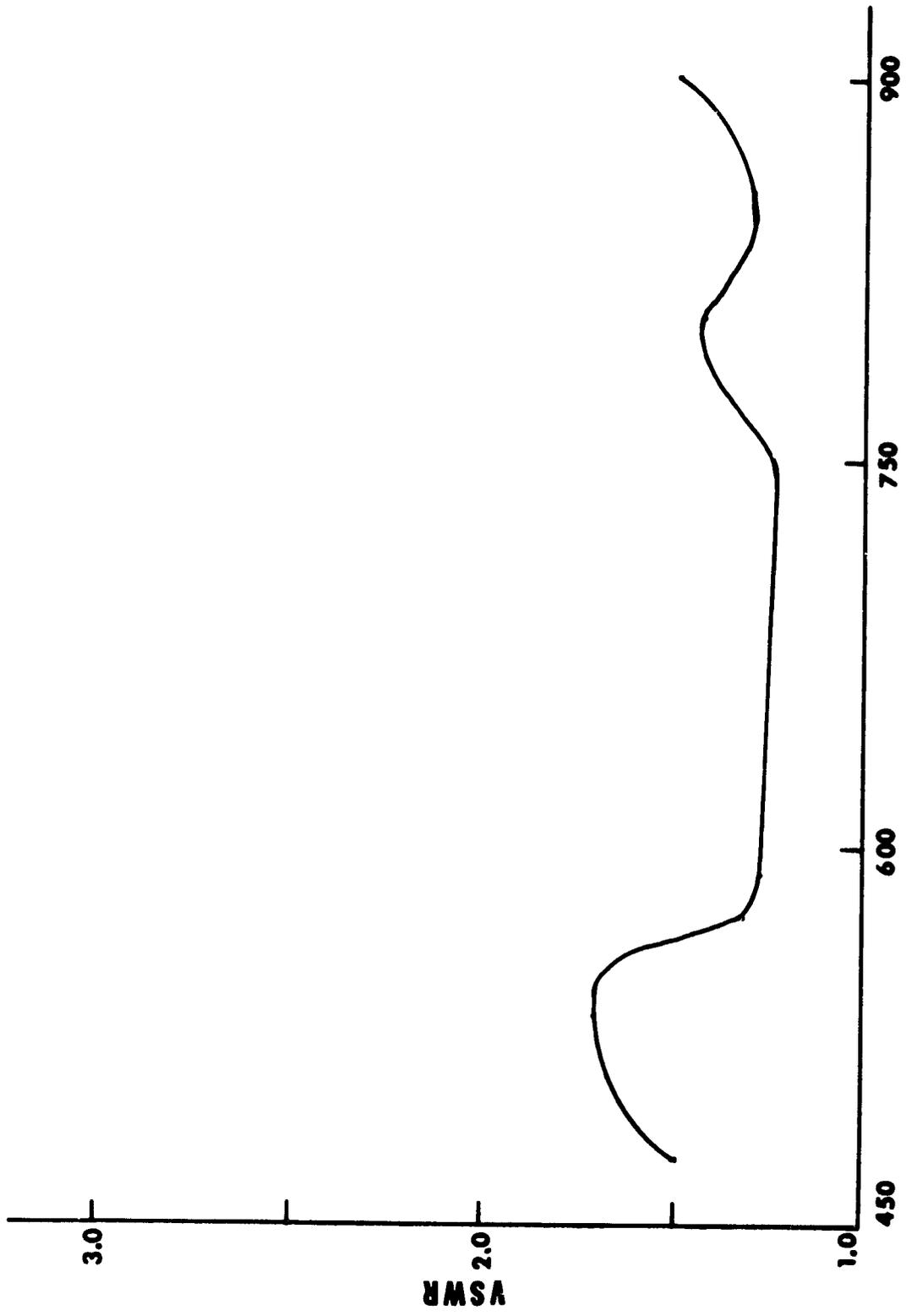


Figure 2-6--- VSWR referred to a 50-ohm line versus frequency for a 28"-diameter Archimedean spiral.

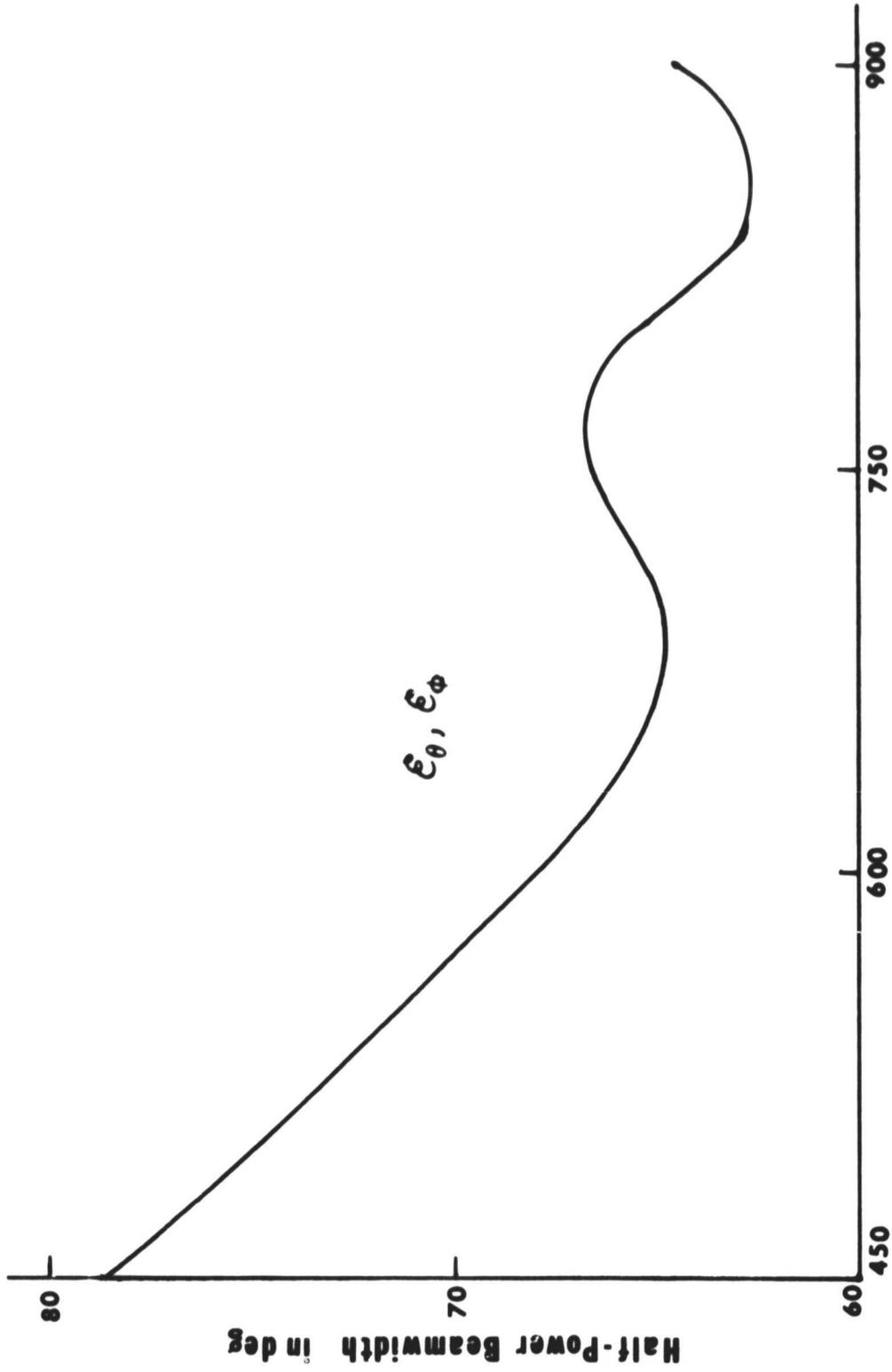
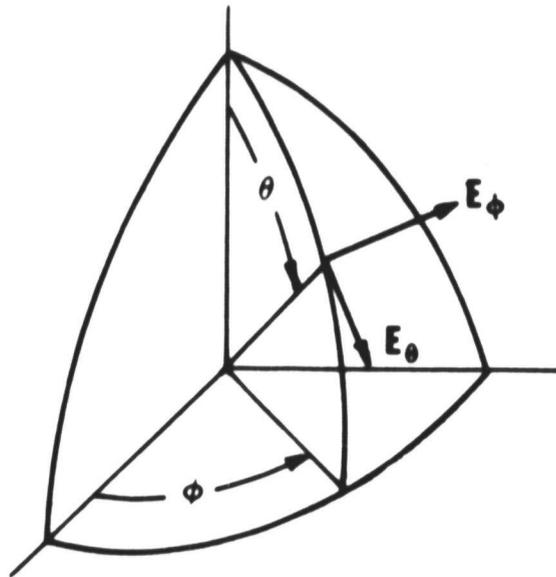


Figure 2-7-- Beamwidth as a function of frequency for a 28"-diameter Archimedean spiral.



### Geometry of Radiation-Pattern Measurements

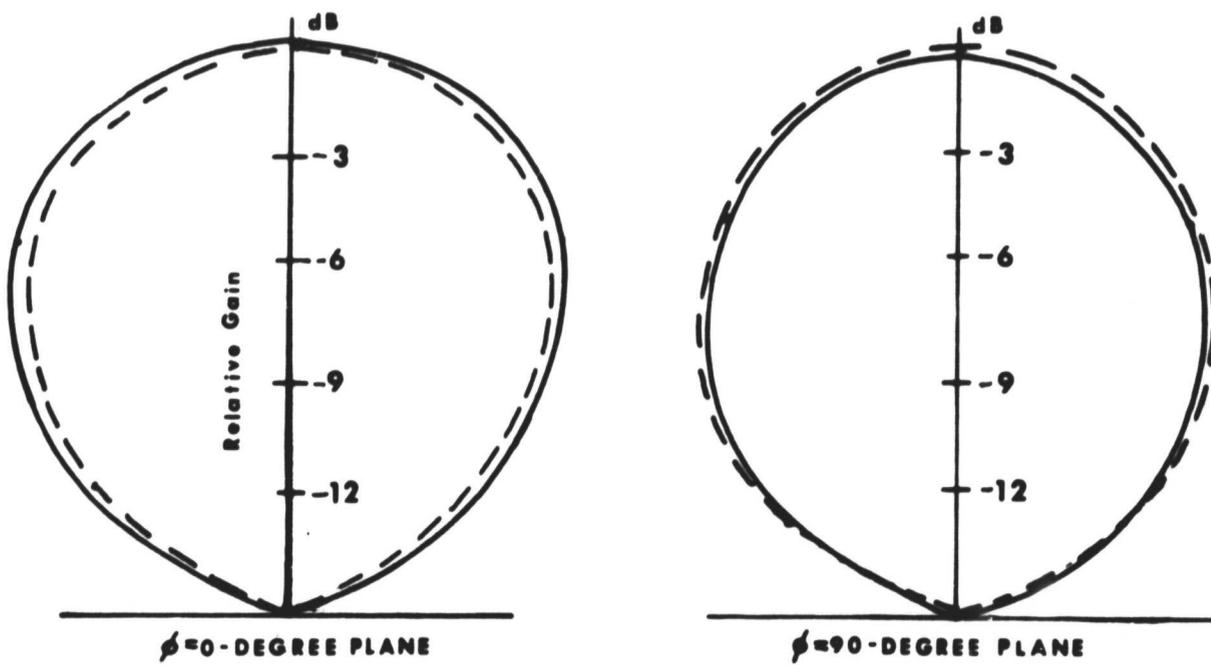


Figure 2-8-- Typical  $E_\theta$  and  $E_\phi$  patterns for a 28"-diameter Archimedean spiral.

### III. HELICAL ANTENNAS

Cylindrical helical antennas, as first described by Kraus,<sup>14</sup> present another solution to the wideband antenna requirement. The natural adjustment of phase velocity so that fields from each turn add nearly in phase in the axial direction accounts for the persistence of the axial mode of radiation over a 1.7 to 1 range in frequency.<sup>15</sup> The terminal impedance, as well as circular polarization, are relatively constant over the same frequency range. Since the cylindrical helix will not cover the entire UHF television spectrum two variations will also be discussed, the conical helix (Figure 3-9) and the multifilar helix (Figure 3-14). The conical helix retains many of the same characteristics as the cylindrical helix except it possesses a wider bandwidth. The multifilar helix, which is slightly more complicated, has a broader bandwidth as well as higher gain possibilities.

A helix may be readily excited in the axial mode if the helix circumference is approximately one wavelength. A simple method to excite the helix is to connect one end of the helix to the inner conductor of a coaxial transmission line and terminate the outer conductor in a ground plane. This ground plane may be either flat or conical in shape and should be at least one-half wavelength in diameter.<sup>16</sup>

The cylindrical helix is defined by the following parameters as shown in Figure 3-1.

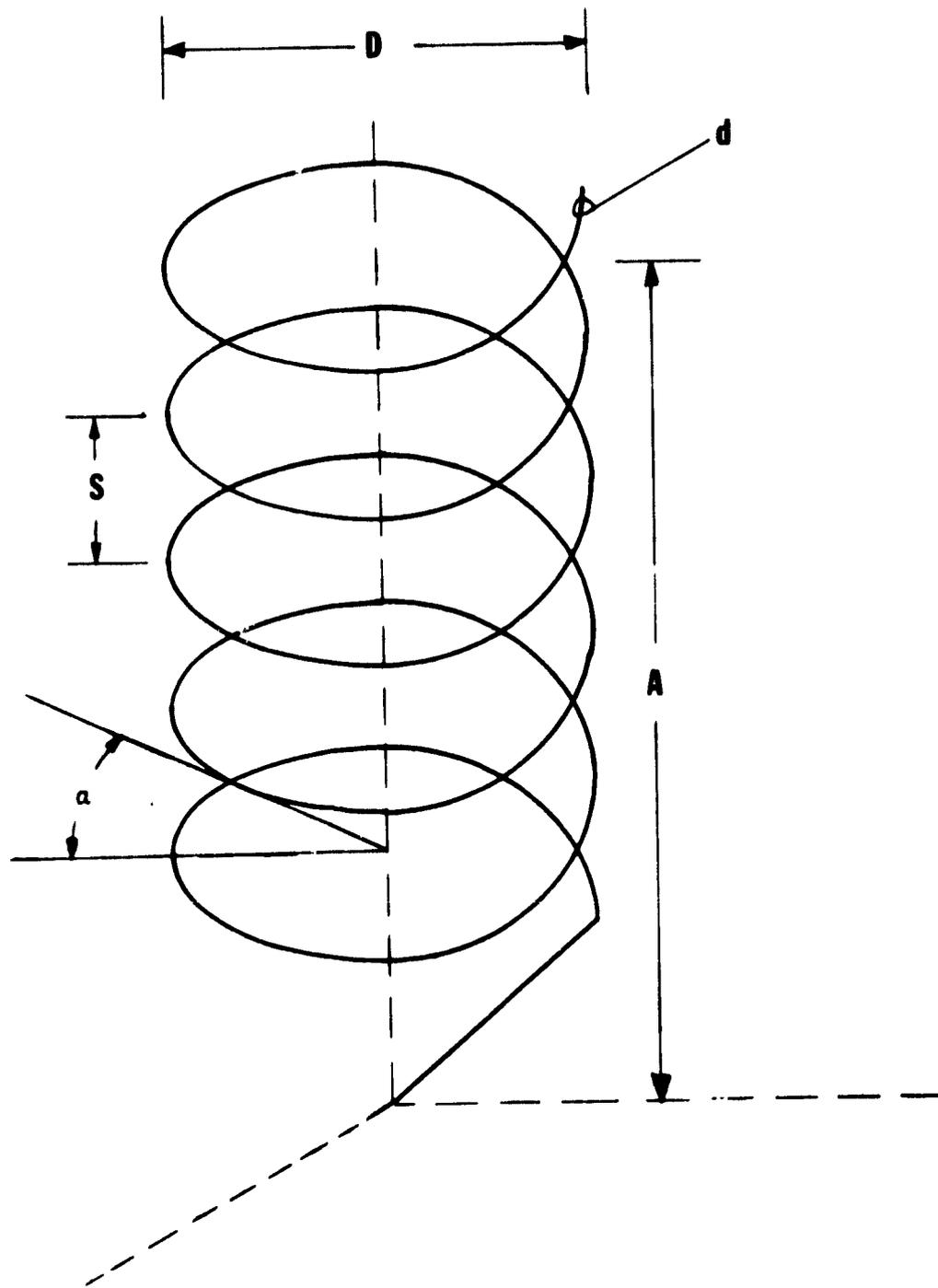


Figure 3-1-- The cylindrical helix.

D = diameter of helix

C = circumference of helix =  $\pi D$

S = spacing between turns (center to center)

$\alpha$  = pitch angle  $\arctan S/\pi D$

L = length of one turn

n = number of turns

A = axial length

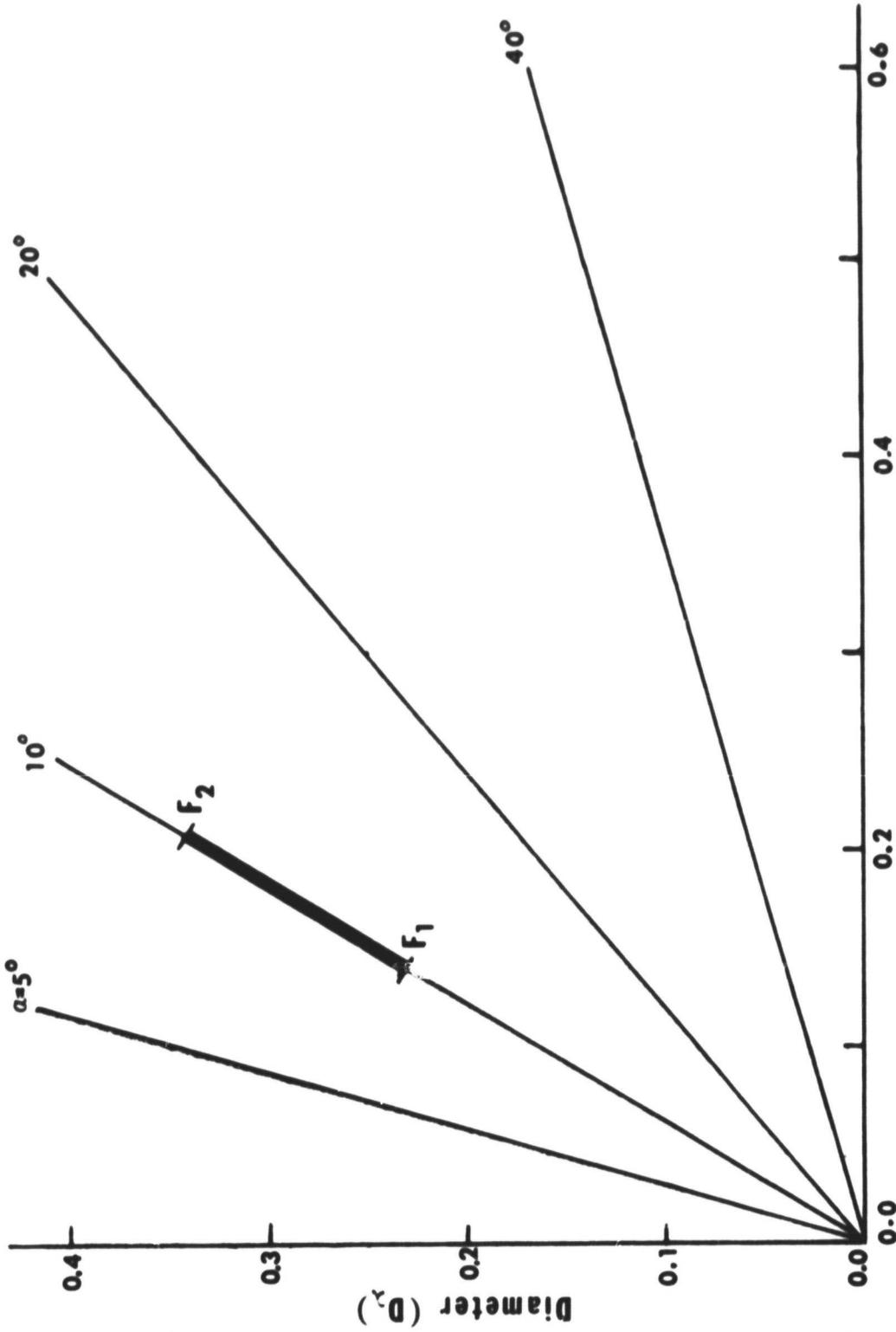
d = diameter of helix conductor

$\phi$  = angle with respect to helix axis

A subscript  $\lambda$  signifies the dimension is measured in free space wavelengths. All the parameters listed above affect the properties of the antenna, however these parameters, in general, are not critical and therefore the helix is one of the simpler types of antennas.

As shown in Figure 3-2 the relationship between D and S moves along a constant pitch-angle line as a function of frequency. If  $F_1$ , is the lower frequency limit of the axial mode and  $F_2$  the upper frequency limit, the range of dimensions for a 10-degree helix would be suggested by the heavy line on the diameter-spacing chart of Figure 3-2. The center frequency  $F_0 = (F_1 + F_2)/2$  and is approximately 680 MHz for UHF television reception.

The properties of a helical beam antenna are primarily a function of pitch angle. The angle resulting in a maximum frequency range ( $F_2 - F_1$ ) is said to be the "optimum" pitch angle. To determine the optimum pitch angle, the pattern, impedance, and polarization



**Spacing Between Turns ( $S_\lambda$ )**  
 Figure 3-2-- Diameter-spacing chart showing the relationship of  $D_\lambda$  to  $S_\lambda$  for various pitch angles ( $\alpha$ ).

characteristics may be compared on a diameter spacing chart as in Figure 3-3. The three contours indicate the region of satisfactory performance as determined by measurements on helicies of various pitch angle as a function of frequency.<sup>17</sup> A satisfactory pattern is considered to be one with a major lobe in the axial direction with relatively small minor lobes. Inside the pattern contour, the patterns are of this form and have half-power beamwidths of less than 60 degrees and as small as 30 degrees. Inside the impedance contour the terminal impedance is relatively constant and is nearly a pure resistance of 100 to 150 ohms. Inside the axial ratio contour, the axial ratio in the direction of the helix axis is less than 1.25-1.

It is apparent that if the pitch angle is small or large the frequency range is decreased. A pitch angle from 12 to 14 degrees would appear to be "optimum" for helicies about 1.6 wavelengths long at the center frequency. Since the properties of the helix change slowly in this region there is nothing critical about this value.

The expressions for the various parameters of the axial helix are given below. These relations apply to helicies for  $12^\circ < \alpha < 15^\circ$ ,  $3/4 < C_\lambda < 4/3$  and  $n > 3$ .

The pattern is given by<sup>19</sup>

$$E = \left( \sin \frac{90^\circ}{n} \right) \frac{\sin n\psi/2}{\sin (\psi/2)} \cos \phi \quad (3-1)$$

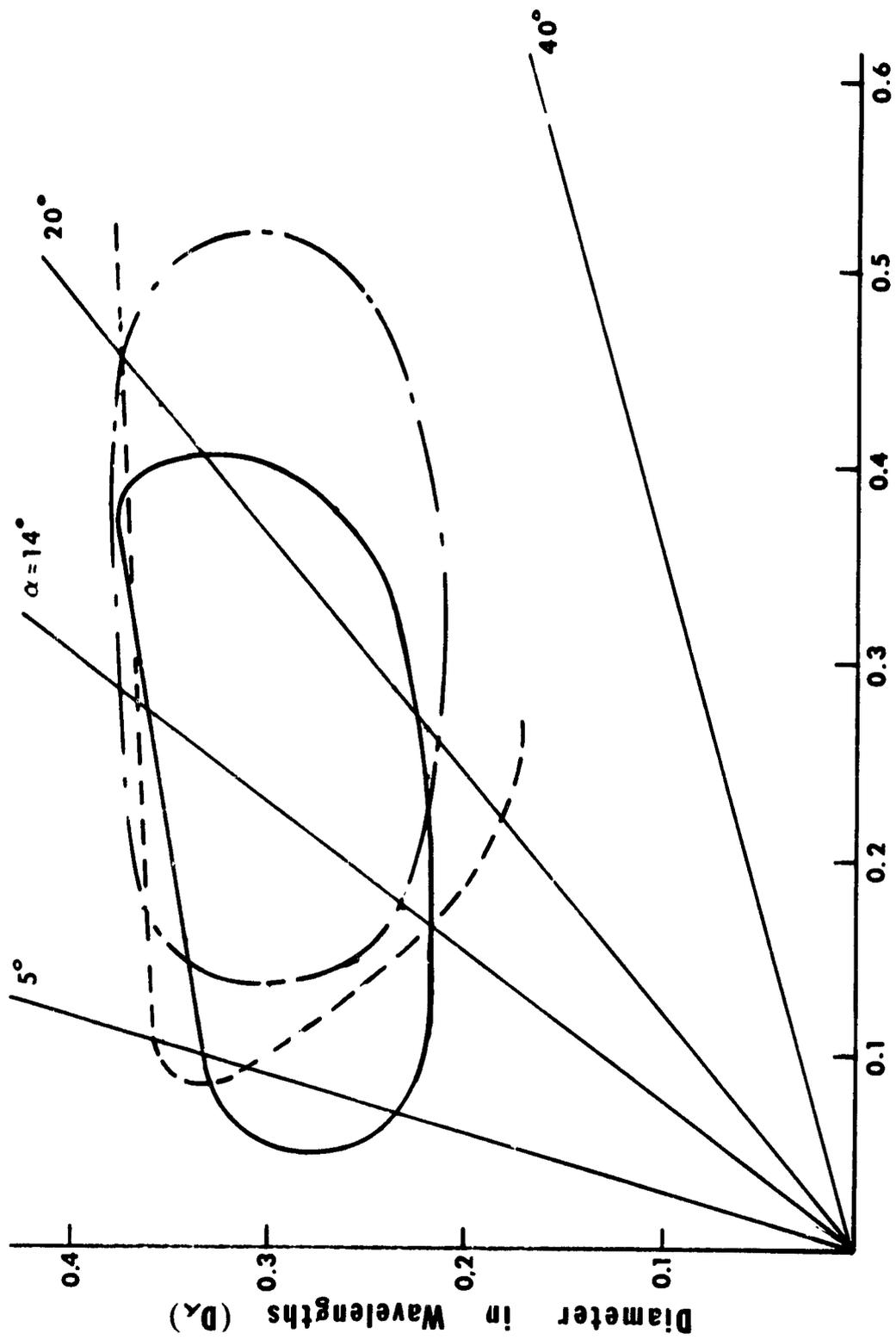


Figure 3-3-- Diameter-spacing chart with measured performance contours for the axial mode of radiation.

where

$$\phi = 360^\circ \left[ S_\lambda (1 - \cos\phi) + \frac{1}{2n} \right]$$

The gain in dB is<sup>20</sup>

$$G = 11.8 + 10 \log \left[ C_\lambda^2 (nS_\lambda)^{\frac{1}{2}} \right] \text{ dB}, \quad (3-2)$$

the half-power beamwidth

$$B = \frac{52}{C_\lambda \sqrt{nS_\lambda}} \text{ degrees}, \quad (3-3)$$

the terminal resistance has been shown to be approximately

$$R = 140C_\lambda \quad (3-4)$$

and the axial ratio is approximately

$$\text{AR} = \frac{2n + 1}{2n} . \quad (3-5)$$

The relationships are plotted in Figures 3-4 through 3-9 for a helical antenna whose dimensions are

$$\alpha = 14^\circ$$

$$D = 5.8''$$

$$n = 6$$

$$S = 4.4''$$

$$A = 26.4''$$

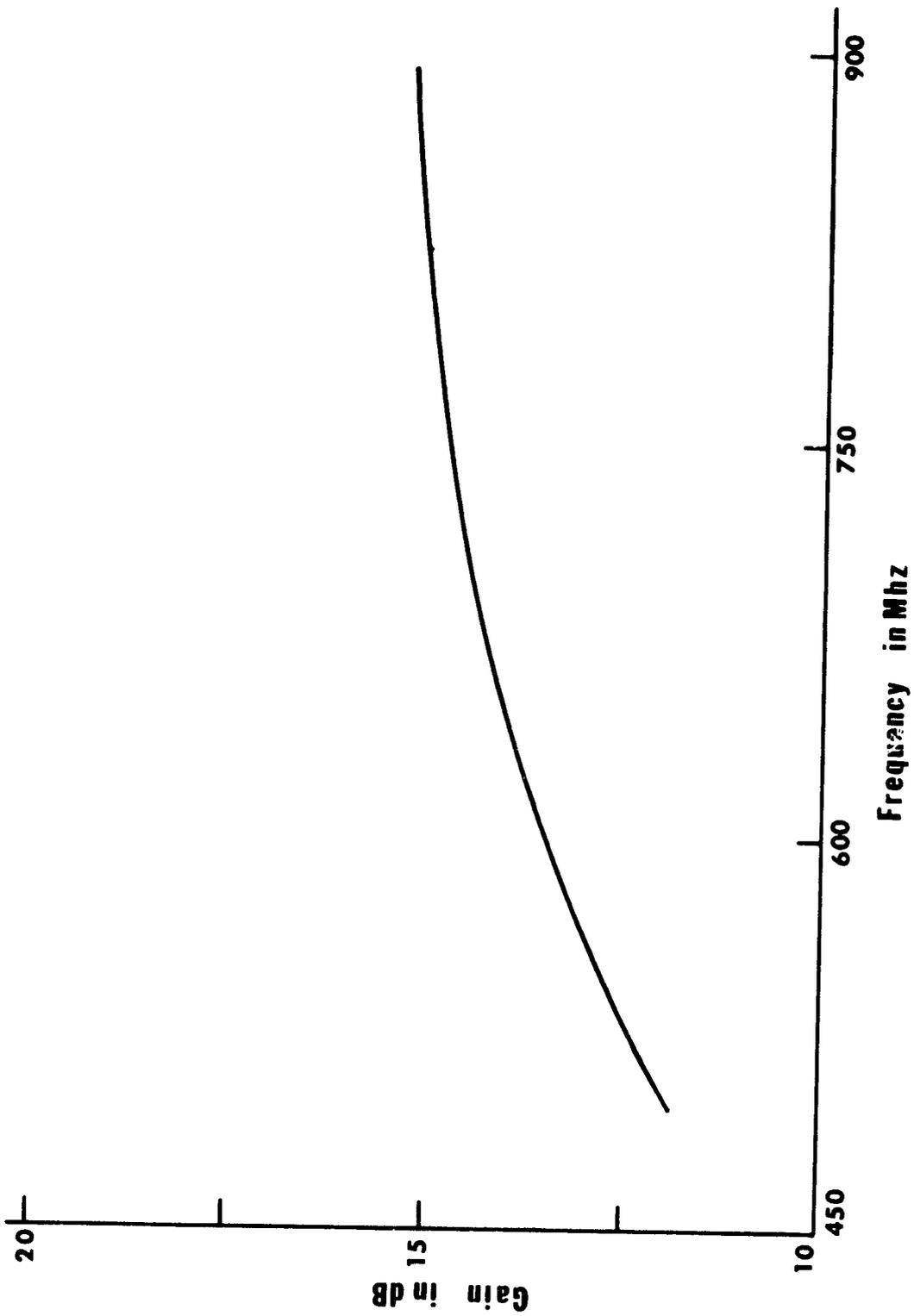


Figure 3-4-- Gain as a function of frequency for a helical antenna as defined by Equation 3.6.

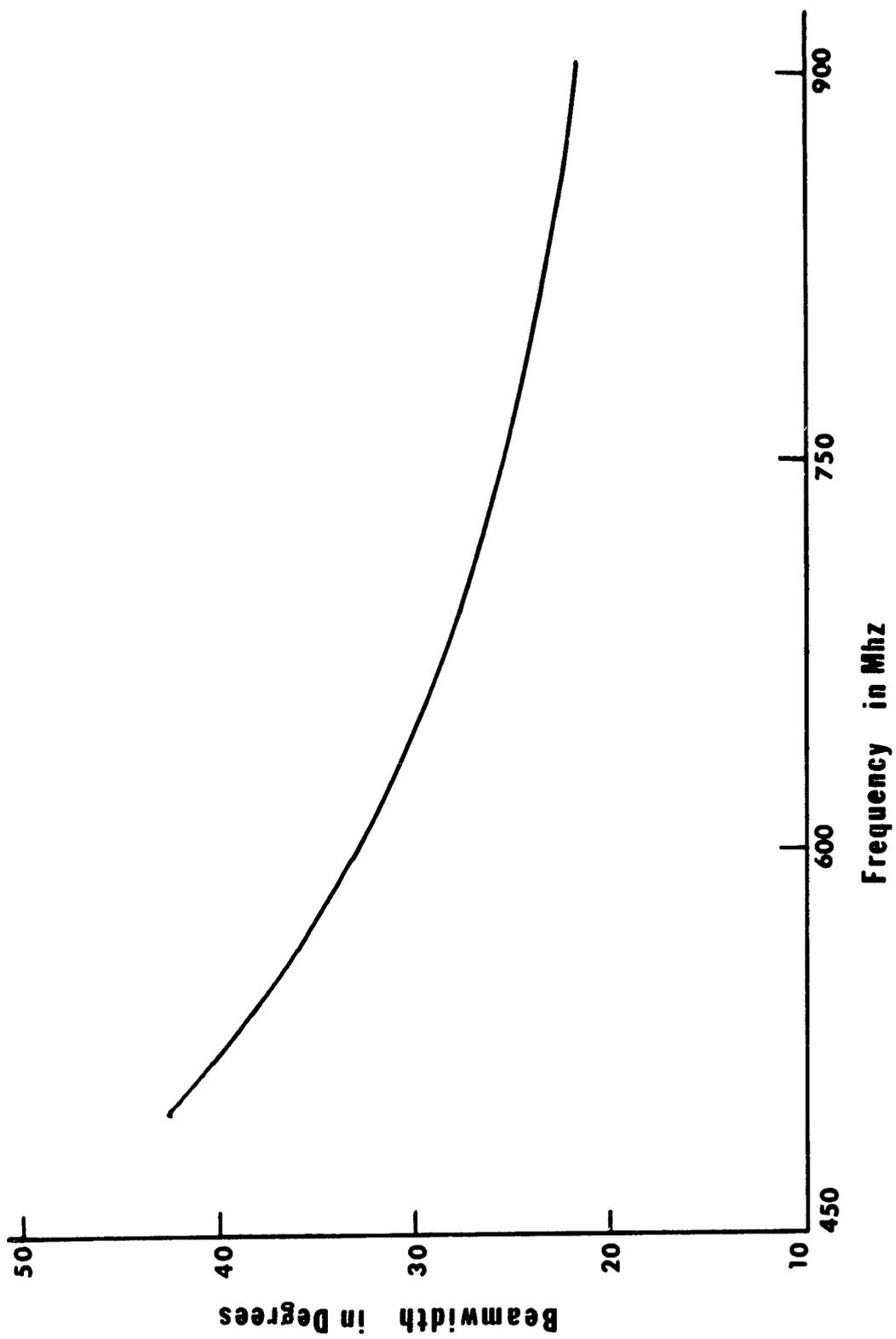


Figure 3-5-- Half-power beamwidth as a function of frequency as defined by Equation 3.6.

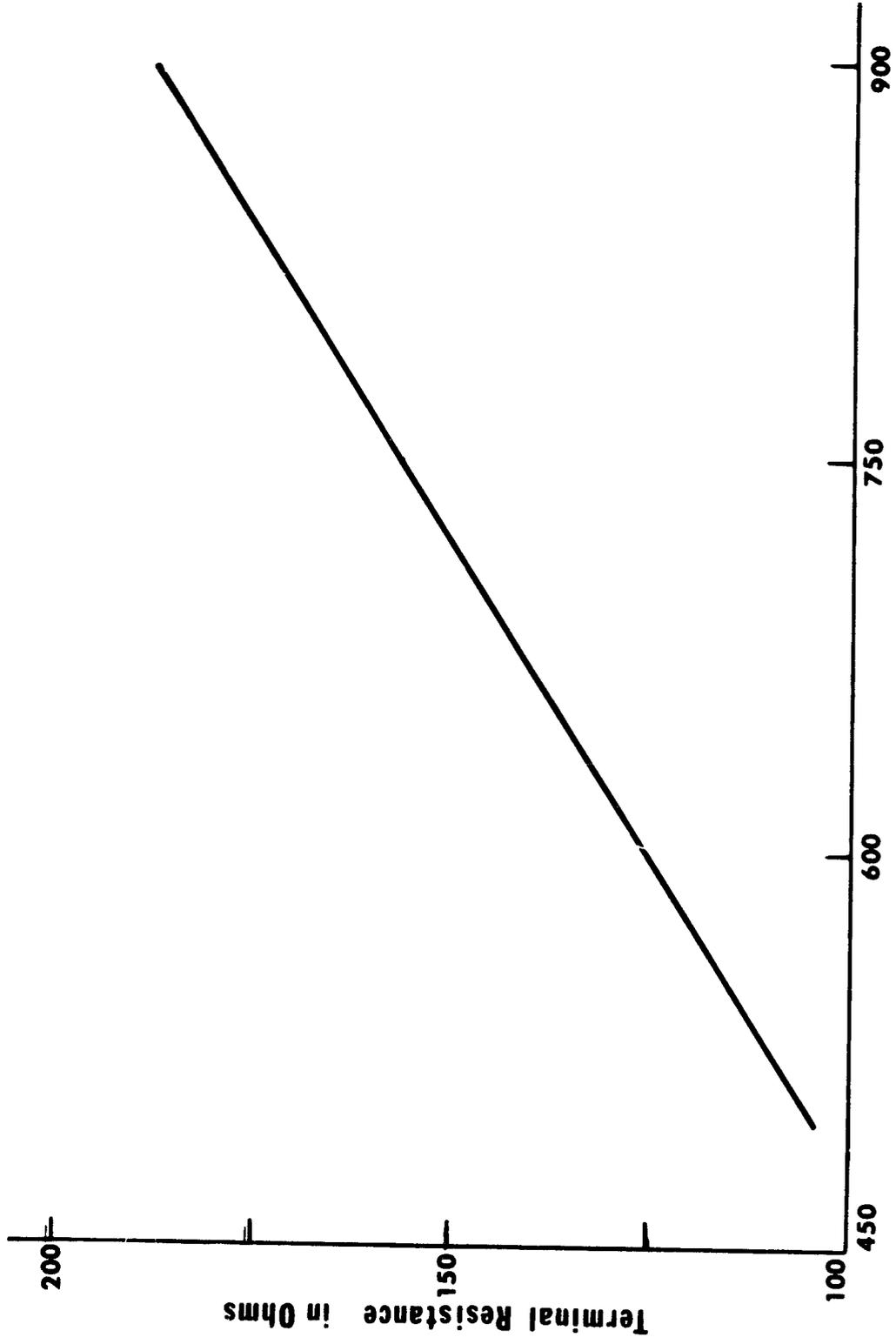
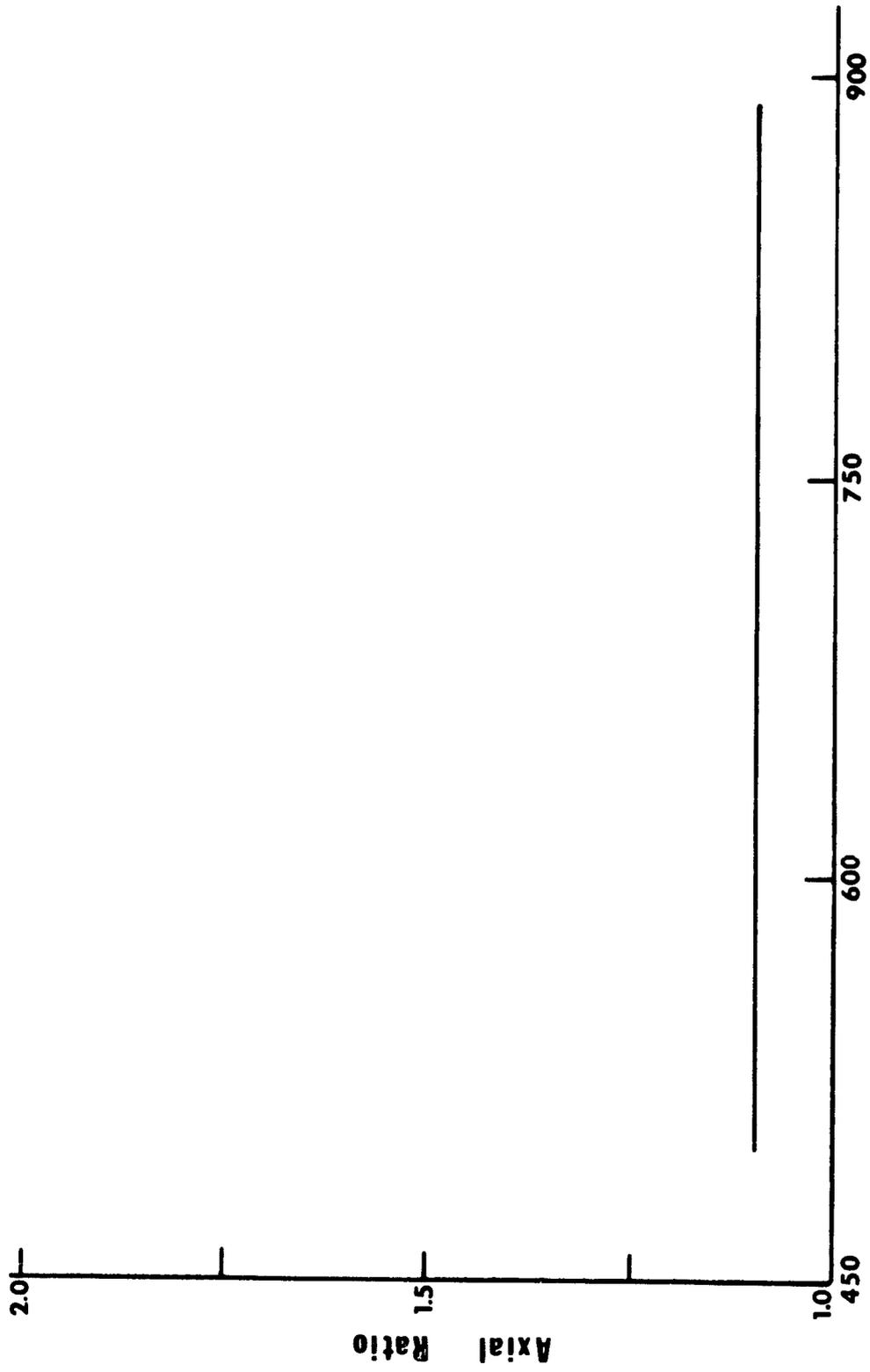


Figure 3-6-- Impedance as a function of frequency for a helical antenna defined by Equation 3.6.



**Figure 3-7-- Axial ratio as a function of frequency for a helical antenna defined by Equation 3.6.**

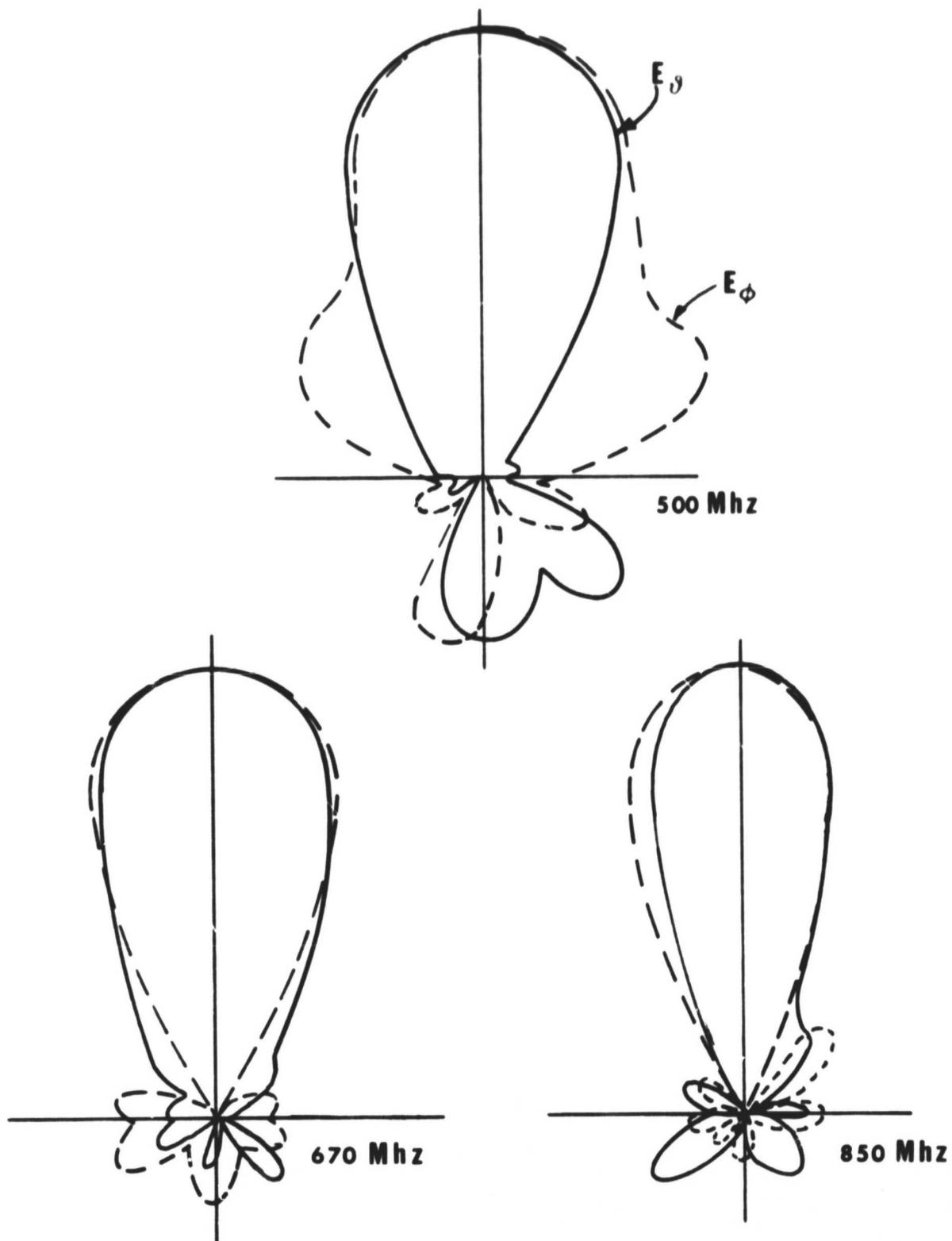


Figure 3-8-- Typical  $E_\phi$  and  $E_\theta$  for a helical antenna defined by Equation 3.6.

$$d = 0.1'' \quad (3-6)$$

This would enable the helix to operate from 490 (Channel 13) to 870 (Channel 82). The patterns are plotted for three frequencies 500, 670, and 850 MHz; the axial ratio would be approximately 1.1 over the entire spectrum.

From the parameters plotted in Figures 3-4 through 3-9, it is apparent that the helix would be suitable for a high gain, circularly polarized UHF antenna. The antenna can be cheaply built and the impedance can be held relatively constant over its bandwidth. However, the helical antenna as discussed here has one serious drawback, the bandwidth will not extend over the entire UHF television spectrum. Therefore a slight variation is necessary if one would wish to provide a large enough bandwidth. The helix could be slightly tapered (conical helix).

The conical helix<sup>18</sup> will provide a bandwidth sufficient to cover the entire UHF television spectrum. This increased bandwidth, however will be accompanied by a small loss of simplicity, a slight reduction in gain, and an increase in beamwidth for the same length antenna.

The conical helix is shown in Figure 3-9, its parameters are similar to that of a cylindrical helix, the notable exceptions being

$D_0$  = smallest diameter of the helix

$D_\ell$  = largest diameter of the helix

$D_x$  = diameter at any point  $x$  along the helix axis

$\theta$  = angle with respect to the vertical axis at which the helix is tapered.

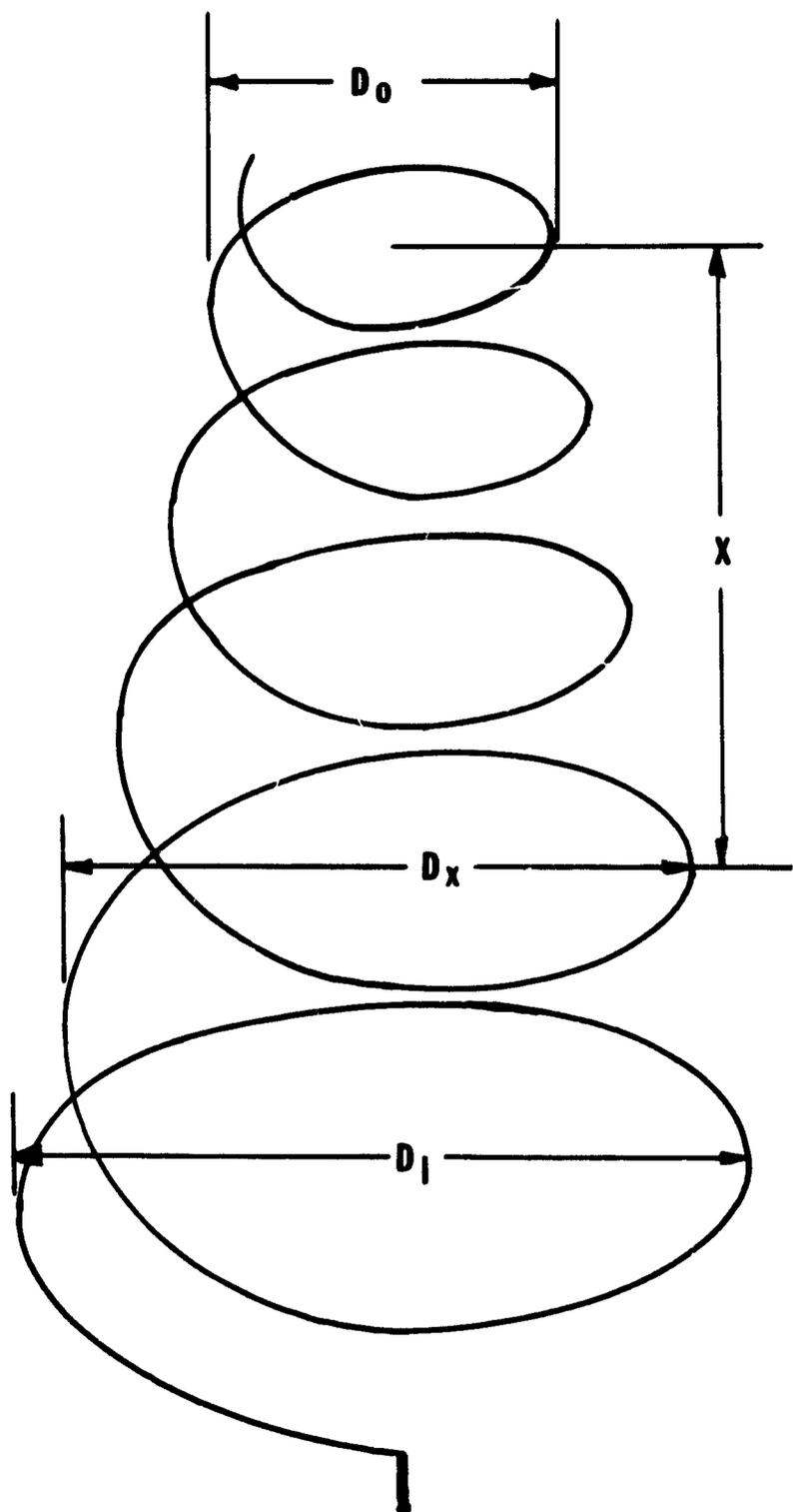


Figure 3-9-- The conical helix.

$D_x$  is related to  $D_0$  by the relation

$$D_x = D_0 + x \tan \theta. \quad (3-7)$$

As in the case of the cylindrical helix the subscript  $\lambda$  indicates measurements in free space wavelengths.

The pitch angle,  $\alpha$ , of the conical helix is usually kept constant, the diameter,  $D$ , the circumference,  $C$ , and the spacing,  $S$ , will vary for each turn. The conical helix operates in much the same way as the cylindrical helix. At all the frequencies within a given operational bandwidth, there exists on the helix a group of turns upon which approximately a full wave occurs. This is the group of turns whose circumferences vary between  $0.75$  and  $1.33\lambda$ . At the lower frequencies of the band these turns are found near the maximum values of  $D_x$ , at higher frequencies in the region of minimum values of  $D_x$ , at intermediate frequencies in the central part of the helix.

Chatterjee<sup>19</sup> has shown that the degree of directivity for conical helices is not determined by the total number of turns or the total length of the helix, but only by the number of turns and length of the group in which  $C$  is between  $0.75$  and  $1.33\lambda$ . From this information a conical helix has been designed to provide an adequate bandwidth from 450 to 900 MHz. The dimensions are given below:

$$D_0 = 3.62''$$

$$D_\ell = 9.8''$$

$$\alpha = 6^\circ$$

$$A = 24''$$

$$\theta = 14.5^\circ$$

$$n = 10$$

(3-8)

Using the same formulas as for the cylindrical helix for beamwidth and gain, these antenna parameters are plotted in Figures 3-10 and 3-11. The impedance, as determined experimentally, is plotted in Figure 3-12. The patterns are plotted in Figure 3-13 for three frequencies (500-670-850 MHz).

From the curves and patterns shown in Figures 3-10 through 3-13 an extension in length and a slight tapering of a helical antenna will provide much the same characteristics as the non-tapered helix and will also increase the bandwidth enough to cover the entire UHF television spectrum. This is accomplished with a slight loss of simplicity.

Another helical-type antenna is the multifilar helix (Figure 3-14). This antenna and the data presented in this study is taken from Gerst and Worden.<sup>20</sup> This multifilar helix has the unique property that polarization, bandwidth and gain can be controlled independently. The term quadra-filar refers to the number of windings wound in either direction; oppositely wound helices start at each of four feedpoints.

The multifilar helix offers a number of advantages as compared to the conventional helix. The multifilar helix has more gain and a larger bandwidth; if the full bandwidth is not used the antenna can be designed so the impedance remains more nearly constant over the bandwidth required.

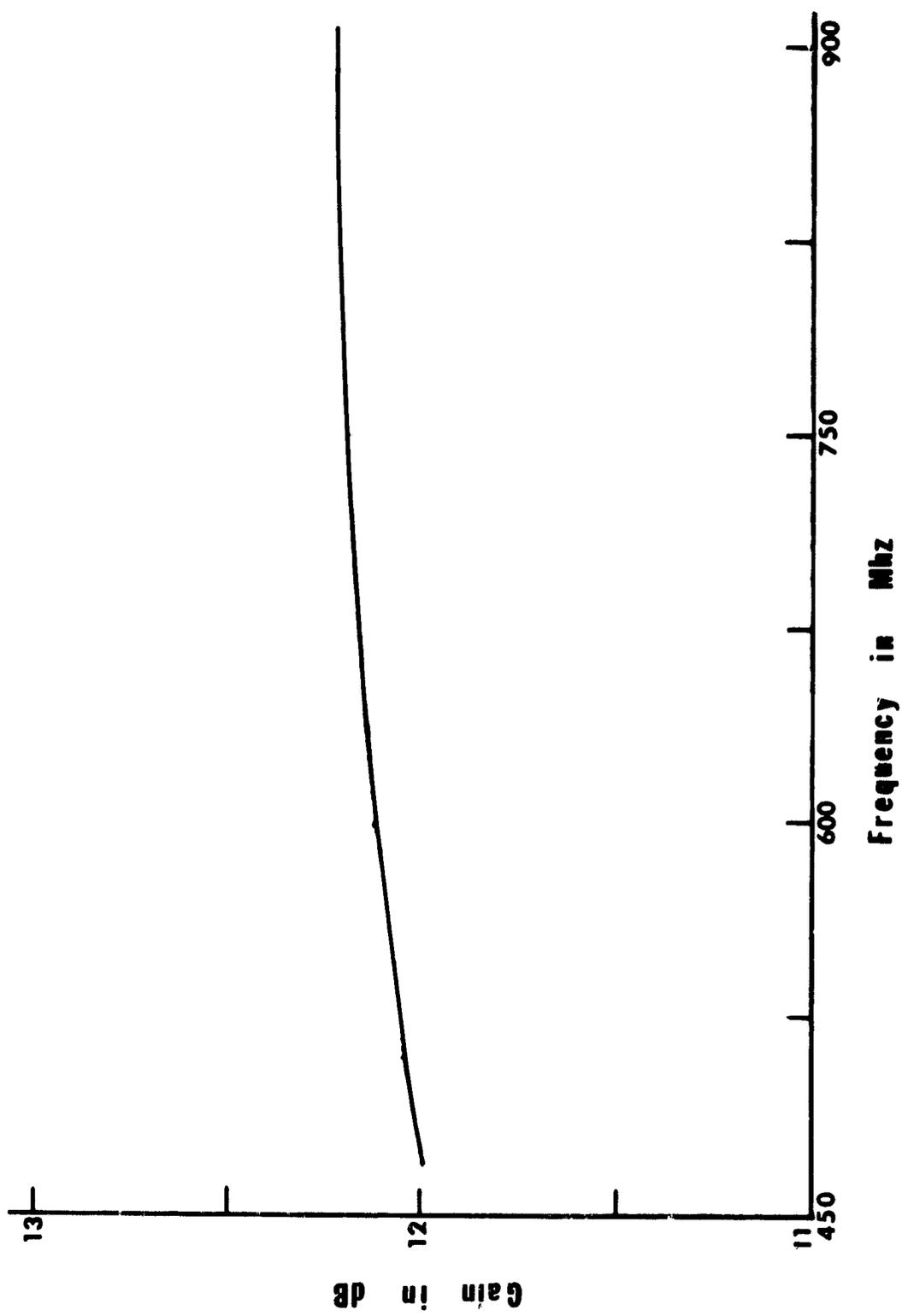


Figure 3-10-- Gain as a function of frequency for a conical helix defined by Equation 3.8.

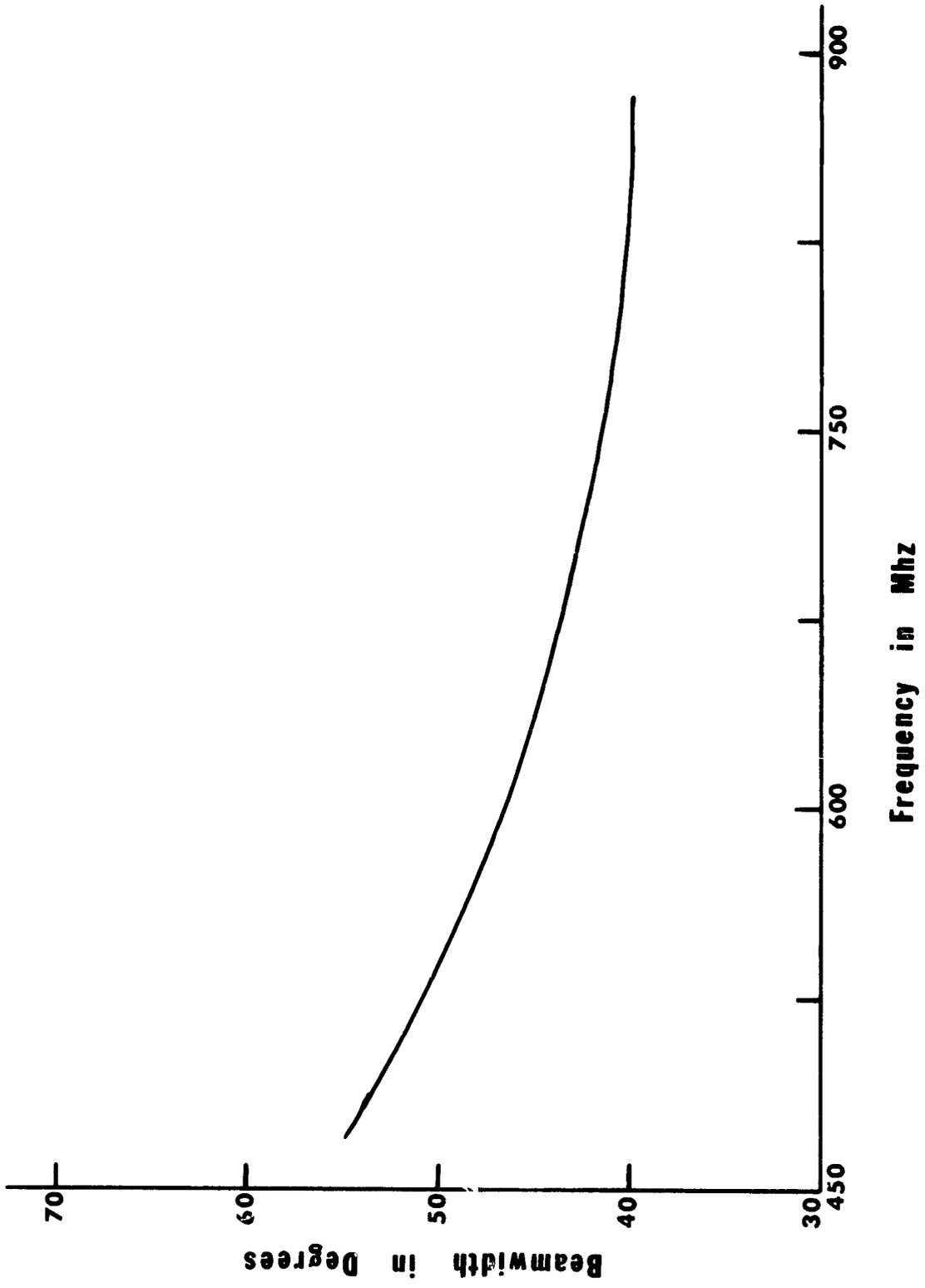


Figure 3-11-- Beamwidth as a function of frequency for a conical helix defined by Equation 3.8.

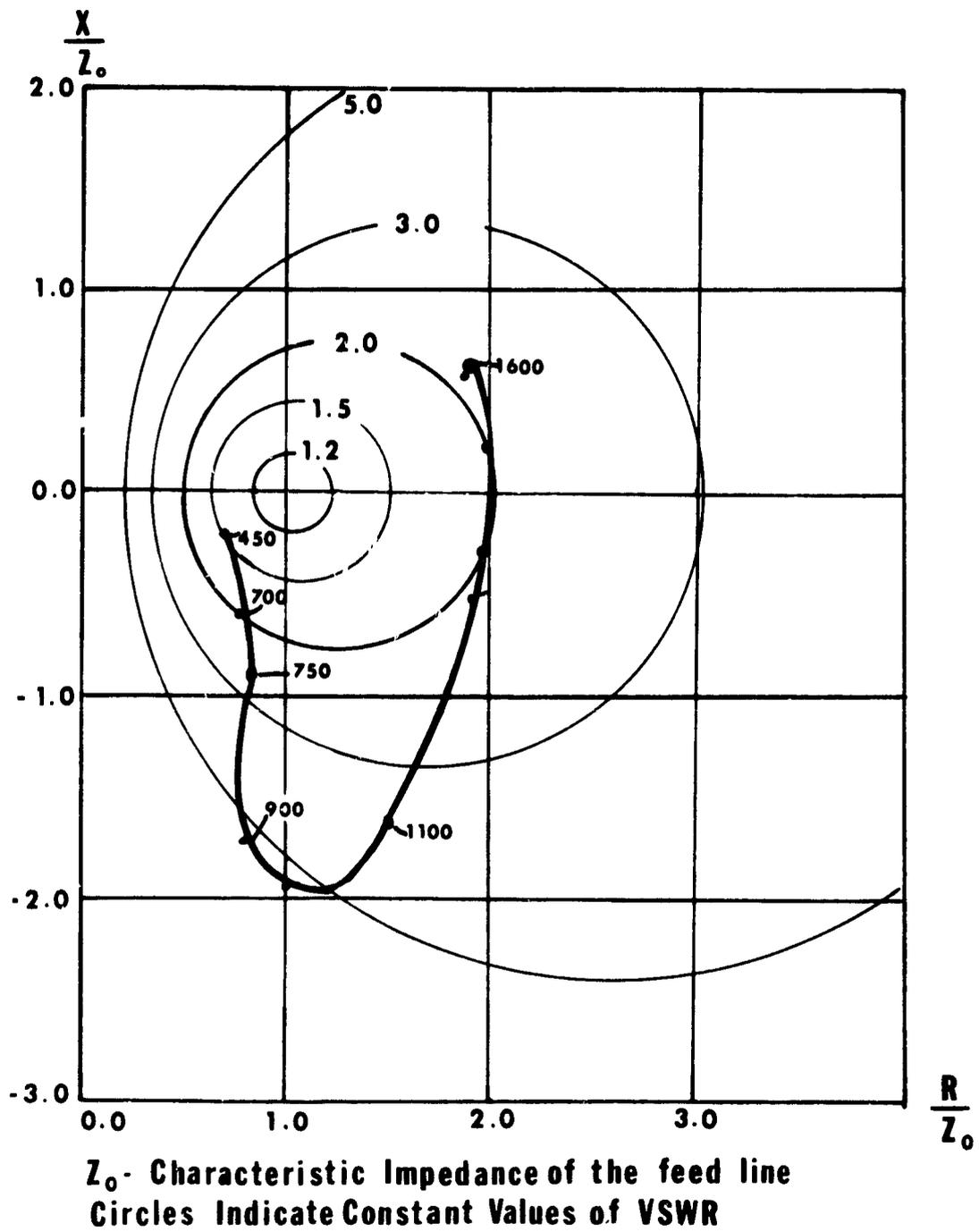


Figure 3-12-- Impedance as a function of frequency for a conical helix defined by Equation 3.8.

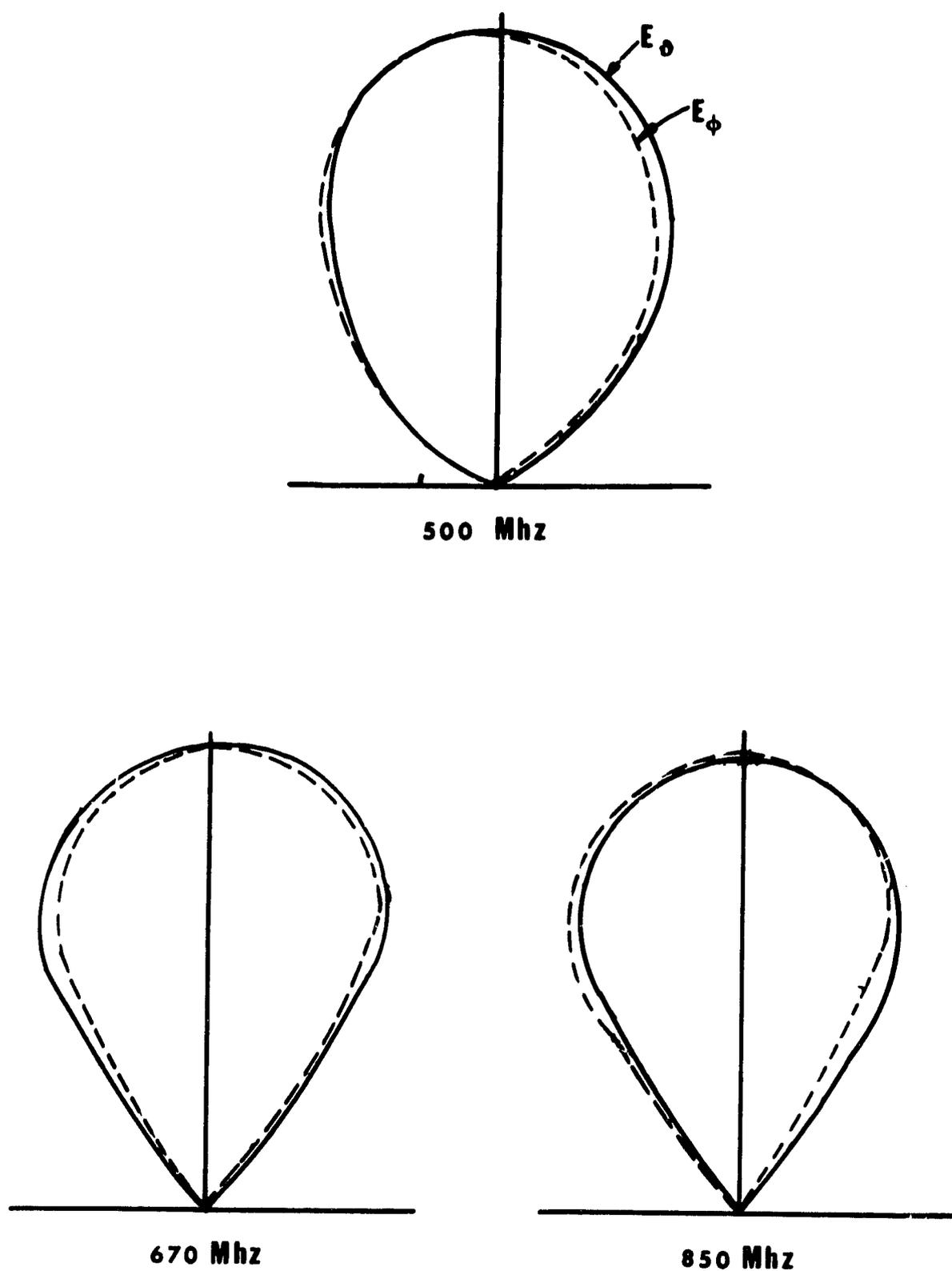


Figure 3-13--  $E_\phi$  and  $E_\theta$  patterns for a conical helix defined by Equation 3.8.

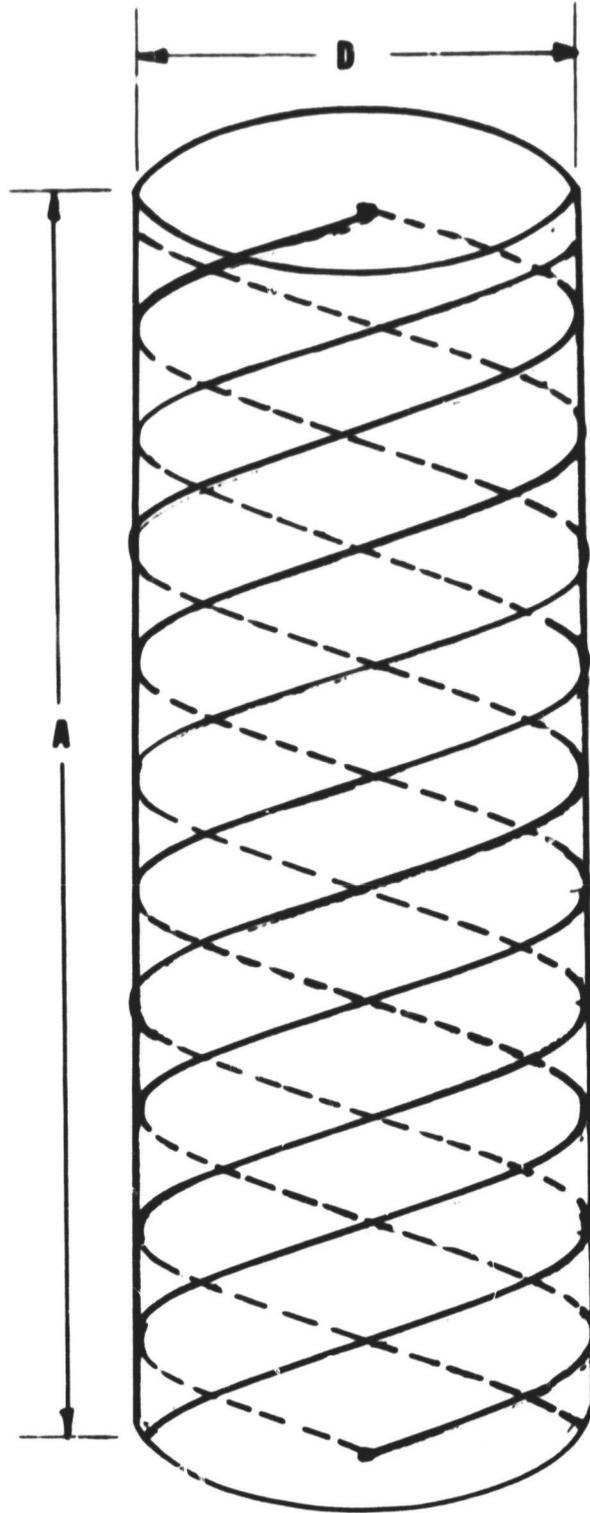


Figure 3-14-- The quadrafilar helix.

The maximum theoretical bandwidth is proportional to  $N + 1$  where  $N$  is the number of windings in one direction. The reason for the increased bandwidth can be explained by an argument in which the helix is related to a broadband loop antenna. A loop can be broadbanded by cutting it into segments and feeding each segment with signals of proper phase. Each successive feedpoint in the loop must have a slightly different phase-difference being  $2\pi/N$ ;  $N$  is the number of segments. Since the phase is fixed only at the feed points on the loop, errors occur at points distant from the segment feed point. Also the phase error increases as the frequency increases. Dividing the loop into more parts cuts the phase variation while increasing the number of fixed phased points, this insures a more broadband operation.

A unifilar helix is sectioned so that there is only one conductor at any single value  $\theta$ ,  $\theta$  being measured from any arbitrary point on the circumference. However, a  $N$ -winding helix consists of  $n$  segments, each  $1/N^{\text{th}}$  of a complete turn. Since the feed points in the helix are phased in the same way as in a broadband loop it can be expected that added windings will help overcome phase variations and make the helix more broadbanded.

This analysis is only an approximate method and does not take into consideration the pitch angle,  $\alpha$ . The pitch angle affects the propagation of various modes along the helical axis and therefore must be taken into consideration if maximum radiation is to be achieved.

The bounds for axial radiation for a N-filar helix are the three curves (Figure 3-15) labeled  $C_{\lambda(\max)}$ ,  $C_{\lambda(N)}$ ,  $C_{\lambda(\min)}$  where

$$C_{\lambda(\max)} = \frac{\cos\alpha}{1 - \sin\alpha}$$

$$C_{\lambda(N)} = \frac{N}{2} \cot\alpha \quad (3-9)$$

$$C_{\lambda(\min)} = \frac{\cos\alpha}{1 + \sin\alpha}$$

From these curves the necessary pitch angle,  $\alpha$ , and the antenna diameter,  $D$ , for a given design frequency can be found. All the values of  $C$  are measured in the area between the  $C_{\lambda(\min)}$  and  $C_{\lambda(\max)}$  curves. For a given pitch angle and value of  $N$ , the lowest possible operating frequency is always fixed by the intersection of a vertical line from the horizontal axis and the  $C_{\lambda(\min)}$  curve. The highest value of  $C_{\lambda}$  is determined by intersection of the same vertical line with either the  $C_{\lambda(\max)}$  or appropriate  $C_{\lambda N}$  curve whichever is reached first.

The next step is to design for bandwidth. The bandwidth ( $F_{\max}/F_{\min}$ ) is related to the pitch angle by the expression.

$$BW = \frac{N}{2} \frac{(\cos\alpha)(\cot\alpha)}{1 - \sin\alpha} \quad (3-10)$$

This is plotted in Figure 3-16. Since the required bandwidth is

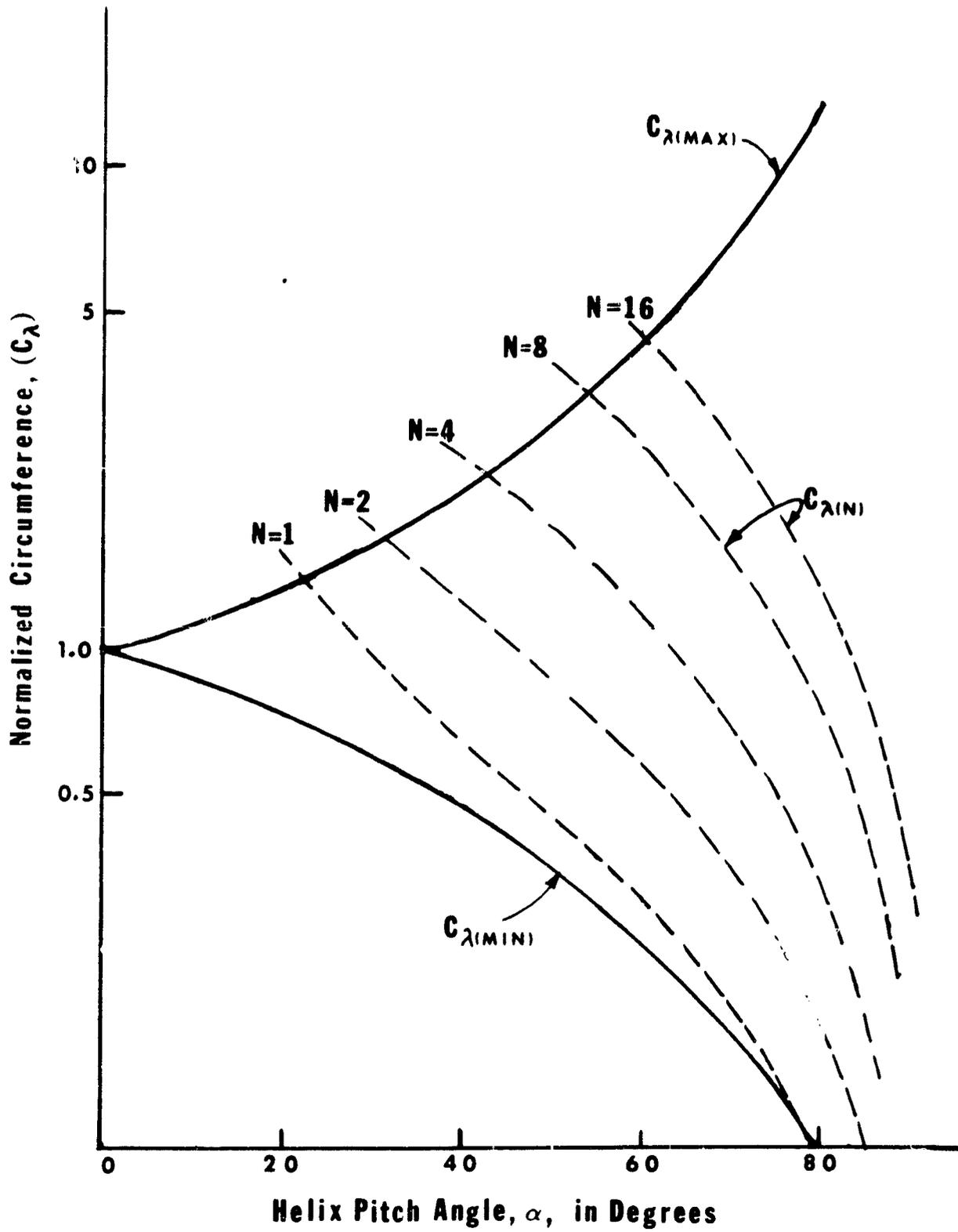


Figure 3-15-- Bounds of axial radiation for a N-filar helix.

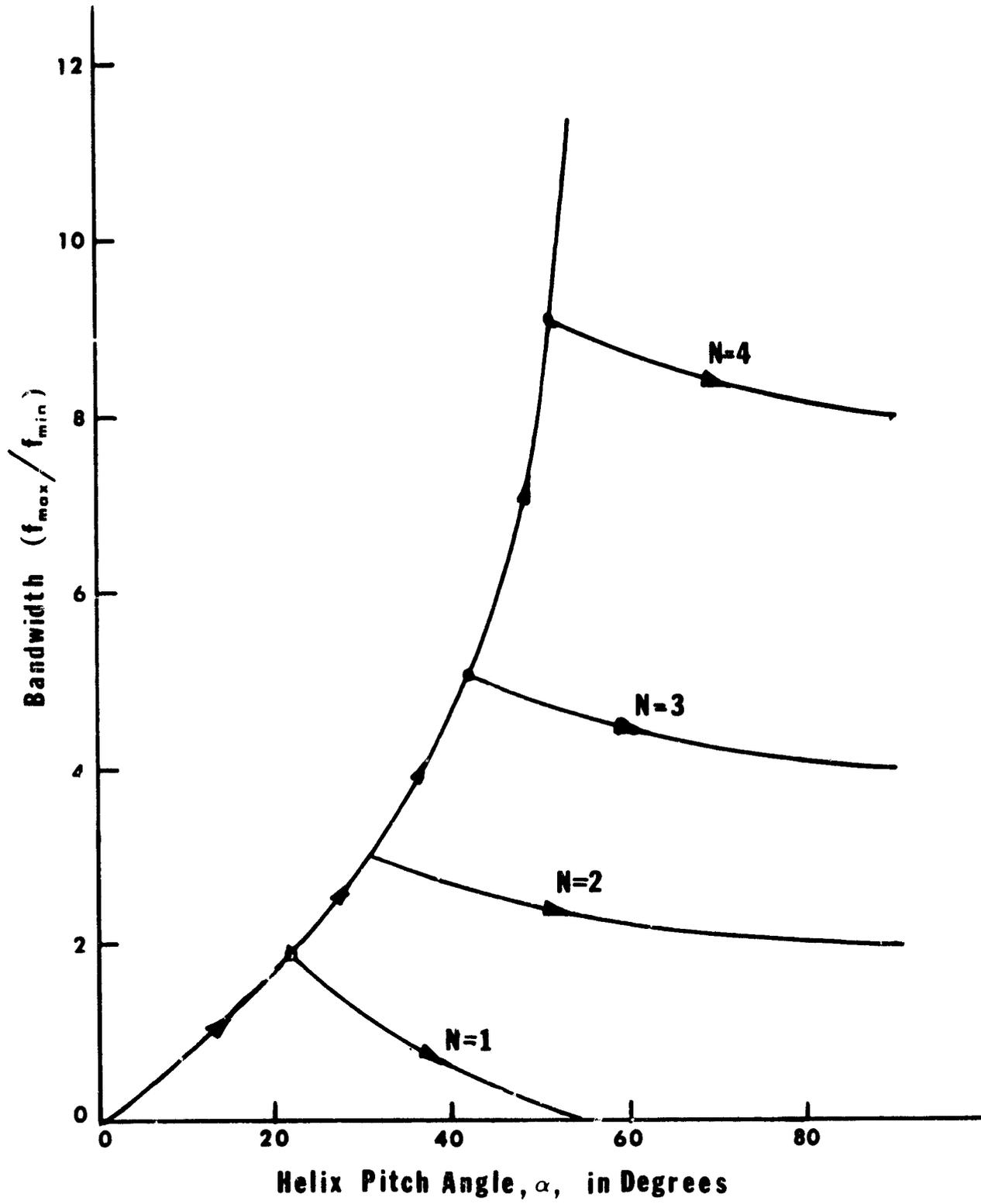


Figure 3-16-- Theoretical bandwidth for a given number of windings in one direction.

constant from 450-900 MHz, this curve has little meaning to us except to insure a large enough bandwidth to cover the entire UHF television spectrum. For this reason a bandwidth of 4 to 1 would be a good choice; this choice would also insure a higher gain.

The pattern of the multifilar helix can be approximated from that of an equal amplitude endfire array. The resulting radiation pattern is  $(\sin x)/x$  curve given by

$$E_{\phi} = \frac{\sin[A_{\lambda}(1 - \sin\phi)\pi]}{A_{\lambda}(1 - \sin\phi)\pi} \quad (3-12)$$

The patterns are plotted for a quadra-filar helix with the following dimensions (see Figure 3-17)

$$\begin{aligned} A &= 39'' \\ \alpha &= 43 \text{ degrees} \\ C &= 23.8'' \\ D &= 7.56'' \end{aligned} \quad (3-13)$$

Although the bandwidth is much greater than is needed, the gain is increased if the higher end of the bandwidth is used. The gain is plotted in Figure (3-18).

The polarization of the quadra-filar antenna can be made circular if all four feeds are used, and each feed is spaced 90 degrees apart in time and position. The impedance has been found to be approximately 150 ohms from each of the four input windings to ground. The quadra-filar antenna like the spiral is a balanced system and

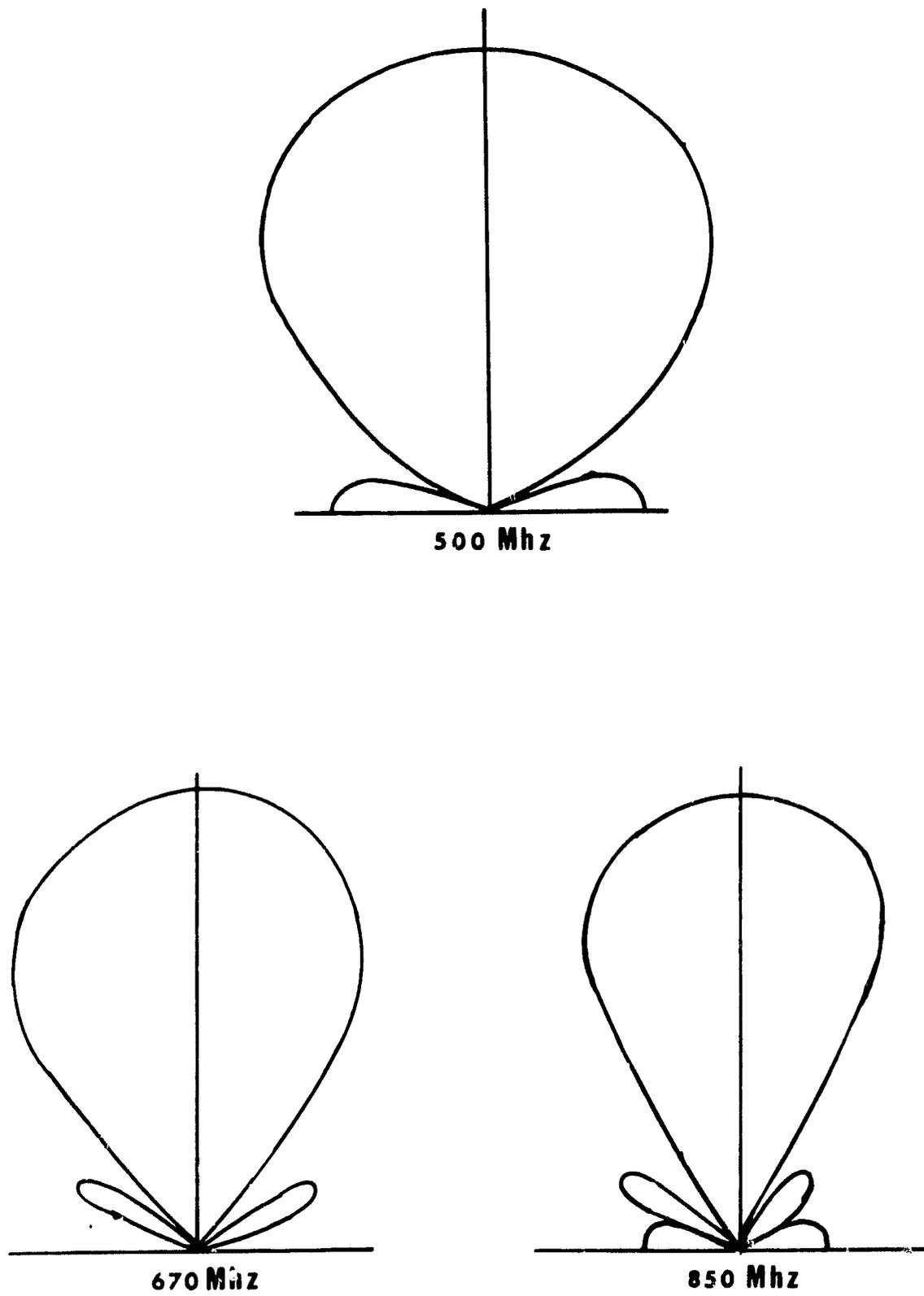


Figure 3-17--  $E_{\phi}$  patterns for a quadrafilar helical antenna as defined by Equation 3-13.

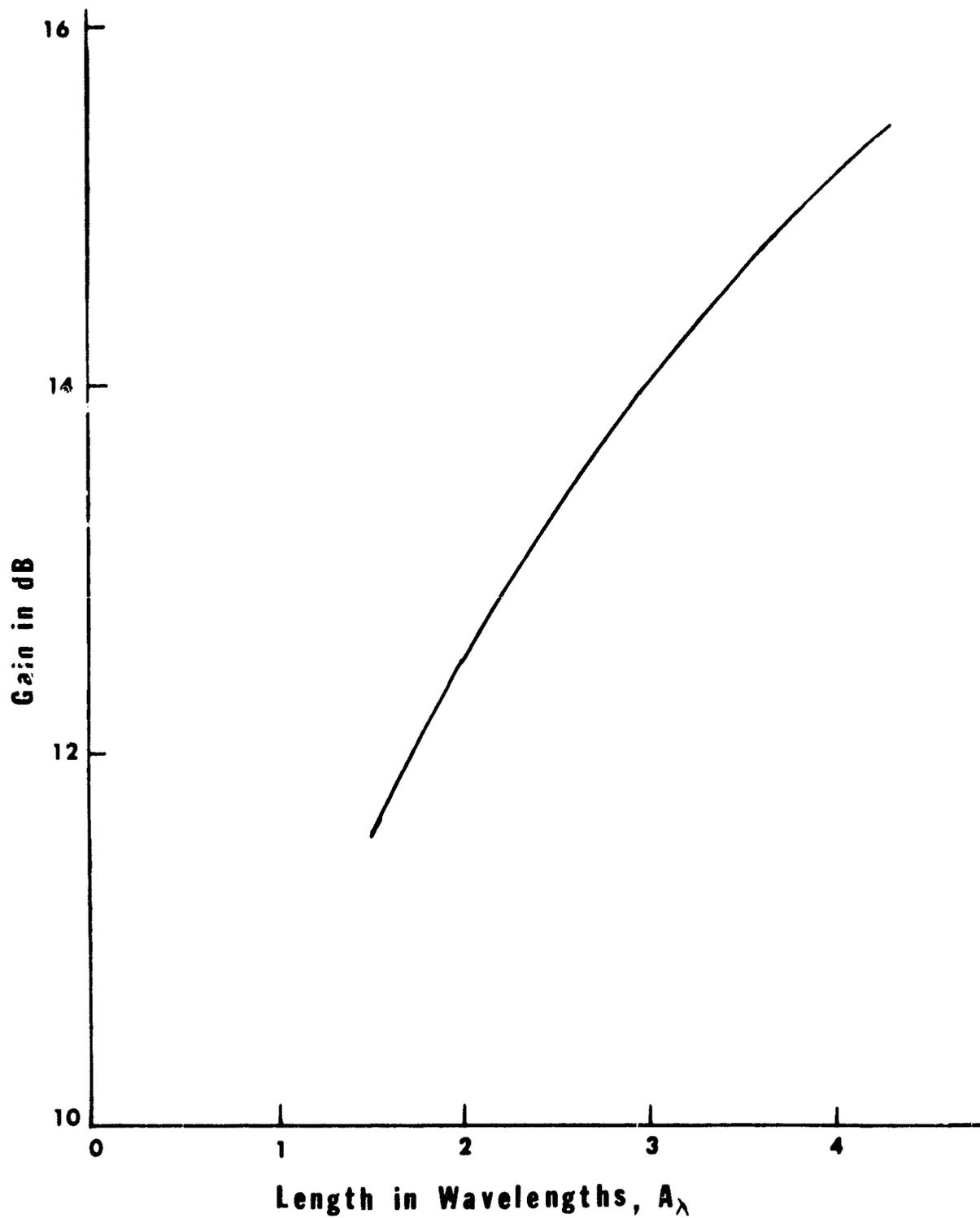


Figure 3-18-- Gain as a function of axial length for a quadrafilar antenna.

therefore requires a balun. Inasmuch as the balun used for the spiral (see Chapter II) has a bandwidth of 2:1 it can be used for the quadrifilar antenna with good results.

This antenna would provide the necessary increase in bandwidth and a slight increase in gain over the cylindrical helix first considered. The multifilar helix has also shown remarkably stable polarization and impedance characteristics over this same bandwidth.

#### IV. CONCLUSIONS

UHF television antennas that are currently being produced have two disadvantages in that they are linearly polarized and may create a formidable wind resistance when operated in a different orientation. For the most part they are inexpensive and provide adequate gain for satellite reception.

The spiral antenna combines circular polarization, adequate beamwidth, and easy operation with an extremely low profile. The spiral antenna, however, has a lower gain than many of the presently manufactured antennas, but its circular polarization increases its effective gain. The economy and low profile of the spiral still makes it a good choice if adequate signal strength is available from the satellite.

Helical antennas considerably increase the gain as compared to the spiral antenna. This increase in gain is accompanied by an increase in wind resistance and a slight loss of simplicity. All the helical antennas, cylindrical, conical, and multifilar, have essentially the same characteristics. The conical helix loses a small amount of gain to provide a bandwidth wide enough to cover the entire UHF spectrum. The multifilar helix provides good gain and bandwidth characteristics and is probably the best choice for a UHF television antenna, it is however, the most expensive of the helical types.

The tables that follow (Table 4-I and 4-II) summarize the antennas considered in this study. The various antennas are compared on the basis of their gain at three different frequencies, beamwidth, polarization, operational ease, wind resistance and size, and expense.

The gains of the linear polarized antennas are given considering a 3 dB loss due to Faraday rotation. Operational ease is defined as easy if rooftop mounting is not required, and moderate if rooftop mounting is required. The wind resistance and size are considered to be low if the vertical projection of the antenna is less than one square foot, moderate if the vertical projection is between one square foot and four square feet, and large if the vertical projection is greater than four square feet. Expense is considered on the basis of mass-produced antennas. An antenna will be considered inexpensive if its cost is less than ten dollars, moderate if the antenna would cost more than ten dollars but less than thirty dollars, and expensive if more than thirty dollars.

	GAIN IN dB			MAIN BEAMWIDTH	POLARIZATION	
	500	660	820			
1. Triangular dipole in front of screen	5	6	8	-	linear	
2. Stacked - V	7	9	11	-	linear	
3. Corner Reflector	7	9	10	-	linear	
4. Yagi	5	6	8	-	linear	
5. Log Periodic	12	12	12	-	linear	
6. Parabolic Reflector	20	21	23	13-20	circular	
7. Spiral	8	9	10	60-80	circular	
8. Cylindrical Helix						
$\alpha = 14^\circ$	A					
D = 5.8"	4'	12	14	15	25-45	circular
S = 4.4"	6'	16	17	19	20-40	circular
	8'	19	21	22	18-35	circular
9. Conical Helix						
$D_o = 3.6''$	A					
$D_\ell = 9.8''$	4'	12	12	12	40-55	circular
$\alpha = 6^\circ$	6'	15	15	15	30-45	circular
10. Quadra-filar Helix						
$\alpha = 43^\circ$	A					
D = 12.5	4'	12	13	15	40-50	circular
	6'	13	15	17	30-45	circular
	8'	15	17	18	25-40	circular

Table 4-I -- Comparison of UHF antennas as to gain, beamwidth and polarization.

		OPERATIONAL EASE	WIND RESISTANCE & SIZE	EXPENSE
1.	Triangular dipole in front of screen	easy	low	inexpensive
2.	Stacked - V	moderate	moderate	moderate
3.	Corner Reflector	moderate	moderate	moderate
4.	Yagi	moderate	moderate	expensive
5.	Log Periodic	moderate	large	expensive
6.	Parabolic Reflector	moderate	large	expensive
7.	Spiral	easy	low	inexpensive
8.	Cylindrical Helix			
	$\alpha = 14^\circ$	A		
	D = 5.8"	2'	easy	low
	S = 4.4"	4'	moderate	moderate
		6'	moderate	moderate
9.	Conical Helix			
	$D_o = 3.6''$	A		
	$D_\ell = 9.8''$	4'	moderate	moderate
	$\alpha = 6^\circ$	6'	moderate	moderate
10.	Quadrifilar Helix			
	$\alpha = 43^\circ$	A		
	D = 12.5"	2'	easy	low
		4'	moderate	moderate
		6'	moderate	expensive

Table 4-II -- Comparison of UHF antennas as to operational ease, wind resistance and expense.

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## APPENDIX A

### FARADAY ROTATION

When an electromagnetic wave propagates through the ionosphere in the presence of the earth's magnetic field, the axis of polarization of a linearly polarized wave is altered. In a system employing linearly polarized antennas, the effect tends to decouple the transmitter and receiver and produces an apparent loss of power. If the ionosphere remained constant, we would need only to rotate the axis of polarization of either antenna until maximum radiation is obtained. However, this is not the case, the ionosphere is subject to changes due to diurnal variations, seasonal variations, solar cycle variations, and geographical variations.<sup>21</sup>

When the electromagnetic wave impinges on the ionosphere with the axis of polarization in any direction other than parallel to the magnetic field, the wave can be broken up into two components, one perpendicular and one parallel to the magnetic field. The two component waves propagate at different phase velocities. Upon leaving the ionosphere the differing components add together to yield a polarization at a modified angle ( $\theta$ ). This rotation depends on

1. The frequency of the incident wave
2. The physical characteristics of the medium.

The rotation for the quasi-longitudinal case ( $\phi$  small) is given by

$$\theta = \frac{e^3 \lambda^3}{8\pi^2 c} \int_{h_1}^{h_2} NB \cos\phi \, dh$$

where

$e$  = charge of particle, coulomb.

$m$  = mass of particle, kg

$\epsilon_0$  = permittivity of vacuum ( $8.85 \times 10^{-12}$  farad/meter)

$c$  = velocity of light ( $3 \times 10^8$  m/sec)

$N$  = number of particles/m<sup>3</sup>

$B$  = magnetic flux density, webers/m<sup>2</sup>

$\phi$  = angle between  $B$  and direction of wave propagation

$h$  = distance of wave propagation, m.

If we take a typical case we can approximate values for  $N$  and  $B$ . For the case of a synchronous satellite and a fixed receiving station  $\phi$  remains constant. The equation now becomes

$$\theta = \frac{e^3 \lambda^2}{8\pi^2 c^3 \epsilon_0 m^2} NB \cos\phi \int_0^h dh$$

Based on the work of Harold Pratt,<sup>23</sup> we assume

$$H = 30 \text{ webers/m}^2$$

$$\text{and } N = 2.8 \times 10^{12} \text{ electrons/m}^3.$$

The above were assumed to exist between 230 and 370 km and zero elsewhere.

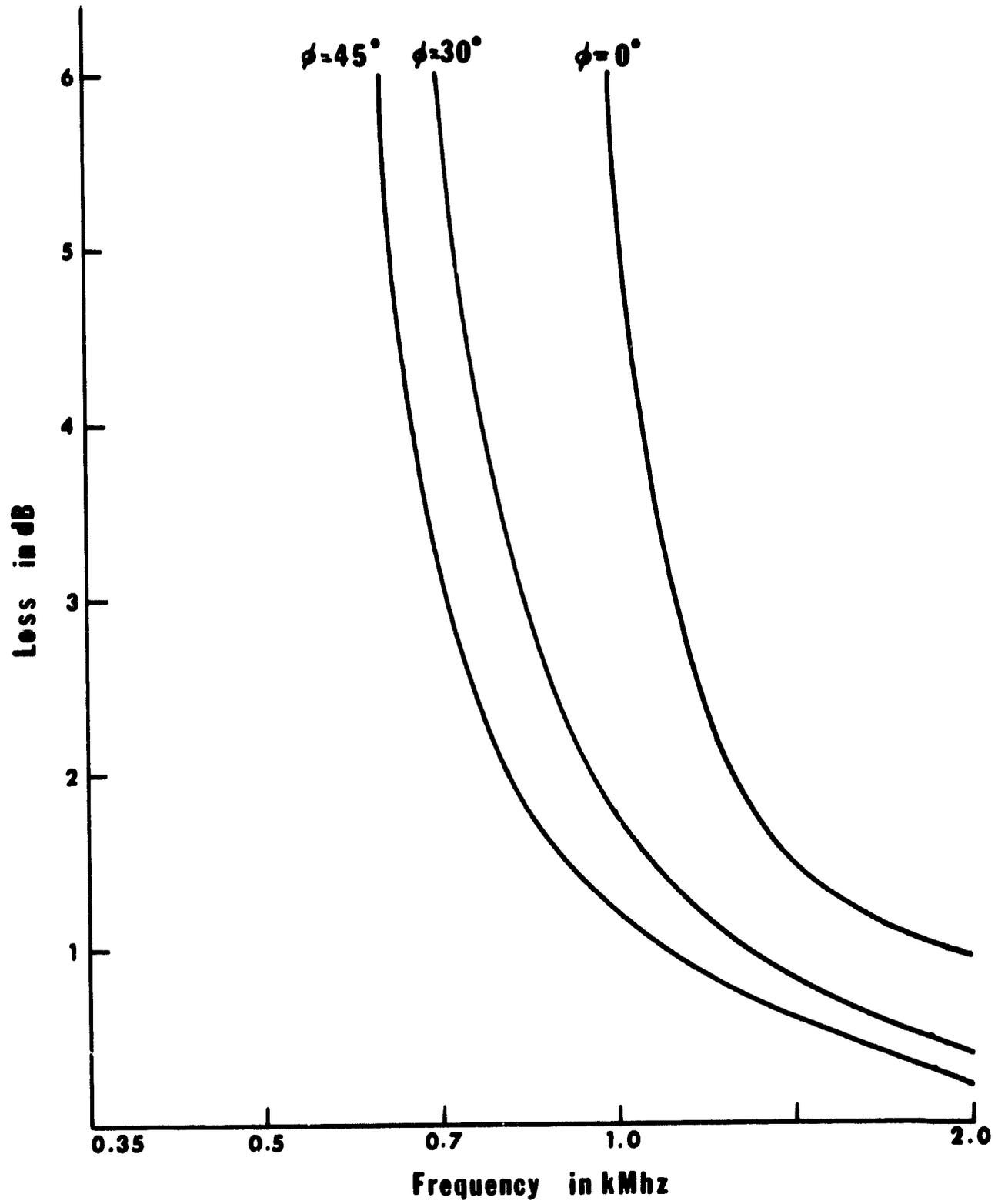
Therefore if we assume that the polarization of the transmitting and receiving antennas are the same the apparent power loss is given by

$$L = -20 \log \cos \theta$$

where L = power lost in dB.

The loss is plotted versus frequency for various values of  $\phi$  in Figure A-1-1.

However it should be pointed out that if one of the antennas is circularly polarized the maximum loss is 3 dB and if both antennas are circularly polarized there is no apparent loss of power.



A-1.-1. Faraday loss as a function of frequency.

## APPENDIX B

### PATTERN FACTORS

The pattern factors are given below for the various antennas considered in this study.

Spiral antenna patterns given assume a semicircular approximation to the Archimedean spiral.<sup>24</sup> They are

$$A_{\theta} |_{\phi = 0} = .2C \cos \theta \sum_{k=1}^m \left\{ -\frac{1}{2} C_{k,0} J_0(x_k) \cos x_1 \right. \\ \left. + \sum_{\ell=1}^{\infty} [\beta_{k,2\ell-1} J_{2\ell-1}(x_k) \sin x_1 - C_{k,2\ell} J_{2\ell}(x_k) \cos x_1] \right\},$$

$$A_{\theta} |_{\phi = 90^\circ} = 2C \cos \theta \sum_{k=1}^m \left\{ \frac{1}{2} D_{k,0} J_0(x_1) + \sum_{\ell=1}^{\infty} (-1) D_{k,2\ell} J_{2\ell}(x_1) \right\},$$

$$A_{\phi} |_{\phi = 0^\circ} = 2C \sum_{k=1}^m \left\{ D_{k,0} J_0(x_k) \cos x_1 \right. \\ \left. + \sum_{\ell=1}^{\infty} [E_{k,2\ell-1} J_{2\ell-1}(x_k) \sin x_1 + D_{k,2\ell} J_{2\ell}(x_k) \cos x_1] \right\}$$

$$\text{and } A_{\phi} |_{\phi = 90^\circ} = 2C \sum_{k=1}^m \left\{ \frac{1}{2} C_{k,0} J_0(x_k) + \sum_{\ell=1}^{\infty} (-1) C_{k,2\ell} J_{2\ell}(x_k) \right\}$$

where: A = vector potential

a = radius of the semicircle

$$B_{k,2\ell-1} = \frac{2(-1)^k (-1)^\ell (2k-1) (d_k^2 + (2\ell-1)^2)}{d_k^2 + (2\ell-2)^2 (d_k^2 + (2\ell)^2)} \cdot [-e_1 + e_2 + e_3 - e_4]$$

$$\beta = 2\pi/\lambda$$

$$C = \frac{I_0 a}{4\pi R} e^{j(\omega t - \beta R)}$$

$$C_{k,2\ell} = \frac{2(-1)^k (-1)^\ell (2k-1)(-d_k^2 + 4\ell^2 - 1)}{(d_k^2 + (2\ell-1)^2)(d_k^2 + (2\ell-1)^2)} \cdot [e_1 + e_2 - e_3 - e_4]$$

$$C_{k,o} = \frac{(-2)(-1)^k (2k-1)^2}{1 + (2k-1)^2 d^2} [e_1 + e_2 - e_3 - e_4]$$

$$D_{k,2\ell} = \frac{-2(-1)^k (-1)^\ell (2k-1)d(d_k^2 + 4\ell^2 + 1)}{(d_k^2 + (2\ell-1)^2)(d_k^2 + (2\ell-1)^2)} [e_1 + e_2 + e_3 + e_4]$$

$$D_{k,o} = \frac{-2(-1)^k (2k-1)^2 d}{1 + (2k-1)^2 d^2} [e_1 + e_2 + e_3 + e_4]$$

$$E_{k,2\ell-1} = \frac{2(-1)^k (-1)^\ell (2k-1)^2 d(d_k^2 + (2\ell-1)^2 + 1)}{d_k^2 + (2\ell-2)^2(d_k^2 + (2\ell)^2)} [-e_1 + e_2 - e_3 + e_4]$$

$$d = \gamma a = (\alpha - j\beta)a$$

$$d_k = (2k-1)d$$

$$e_1 = e^{-\pi k^2 d}$$

$$e_2 = e^{\pi(k-1)^2 d}$$

$$e_3 = e^{-\pi(2m^2 - k^2)d}$$

$$e_4 = e^{-\pi(2m^2 - (k-1)^2)d}$$

$$e_4 = e$$

$I_0$  = current on the spiral

$J$  = Bessel functions

$m$  = number of semicircles in each half the balanced spiral

$R, \theta, \phi$  = are regular spherical coordinates

$$x_k = (2k-1)\beta a \sin \theta.$$

The above equations apply if the principal planes ( $\phi = 0^\circ$  and  $\phi = 90^\circ$ ) are considered.

Another useful component of  $\bar{A}$  is on the axis of polarization;  $\bar{A}$  evaluated at  $\theta = 0^\circ$ . It makes no difference whether  $A_\phi$  or  $A_\theta$  is used because the results are the same with a 90-degree rotation of the coordinate system. At  $\theta = 0^\circ$ ,  $x_k = y = 0$  which means that all terms involving Bessel functions of higher order than zero will be equal to zero. Then

$$A_\theta |_{\theta = 0^\circ} = C \sum_{k=1}^m (D_{k,0} \sin \phi - C_{k,0} \cos \phi);$$

and for the far field

$$\bar{E} = -j\omega\mu\bar{A}.$$

Cylindrical helical antenna patterns<sup>25</sup> are given as

$$E = \left( \sin \frac{90^\circ}{n} \right) \frac{\sin n\alpha/2}{\sin \alpha/2} \cos \phi$$

where

$n$  = number of turns

$$\alpha = 360 \left[ S_z (1 - \cos \phi) + \frac{1}{2n} \right] \text{ in degrees}$$

$\phi$  = angle with respect to the z-axis

$S_z$  = Spacing between turns in wavelengths.

The pattern for the conical helix is <sup>26</sup>

$$\begin{aligned} E_{\theta} = & E_0 \sum_{m=1}^n a_m I_m \exp(-jV_m) \sum_{-\infty}^{\infty} j^n J_n(d_{\theta_m}) \\ & \times \left[ (1 - jk_m) \left( \frac{\sin x'_m}{x'} e^{-jn\theta_m} - \frac{\sin x\pi}{x} e^{jn\theta_m} \right) \right. \\ & + (1 + jk_m) \left( \frac{\sin y'_m \pi}{y'} e^{-jn\theta_m} - \frac{\sin y\pi}{y} e^{jn\theta_m} \right) \\ & + k_m \{ (f'_x + f'_y) e^{-jn\theta_m} - (f_x + f_y) e^{jn\theta_m} \} \\ & \left. - jD \{ (F'_x + F'_y) e^{-jn\theta_m} - (F_x + F_y) e^{jn\theta_m} \} \right] \end{aligned}$$

$$\begin{aligned} E_{\tau} = & E_0 \sum_{m=1}^n a_m I_m \exp(-jV_m) \sum_{-\infty}^{\infty} j^n J_n(d_{\theta_m}) \\ & \times \left[ \cos \theta \{ (-j - k_m) \left( \frac{\sin x'_m \pi}{x'} e^{-jn\theta_m} - \frac{\sin x\pi}{x} e^{jn\theta_m} \right) \right. \\ & \times (-j + k_m) \left. \left( \frac{\sin y'_m \pi}{y'} e^{-jn\theta_m} - \frac{\sin y\pi}{y} e^{jn\theta_m} \right) \right\} \\ & - k_m \cos \theta \{ (f_{x'} - f_{y'}) e^{-jn\theta_m} - (f_x - f_y) e^{jn\theta_m} \} \\ & + D \cos \theta \{ (F_{x'} - F_{y'}) e^{-jn\theta_m} - (F_x - F_y) e^{jn\theta_m} \} \end{aligned}$$

$$\begin{aligned}
& + 2 \sin\theta \tan\alpha \left( \frac{\sin v' \pi}{v'} e^{-jn\theta_m} - \frac{\sin v \pi}{v} e^{jn\theta_m} \right) \\
& + jk_m (f_{v'} e^{-jn\theta_m} - f_v e^{jn\theta_m}) - jD (F_{v'} e^{-jn\theta_m} \\
& + F_v e^{jn\theta_m}) - 2E_o \ell j \sin\theta \frac{1 - \cos\beta\ell (1 - \cos\theta)}{\beta\ell (1 - \cos\theta)} ]
\end{aligned}$$

where:

$A_m$  = axial length to turn  $m$

$\beta$  =  $2\pi / \lambda$

$\alpha$  = pitch angle

$\theta$  = angle between the  $z$ -axis and the line at which the helix is tapered

$b_{\theta_m} = D_m \tan\alpha \cos\theta$

$C' = \beta D_m \sec\alpha \frac{k_m}{2} - \frac{1}{2} D_m k_m \beta \tan\alpha \cos\theta = C - \delta$

$D' = \beta D_m \sec\alpha \frac{k_m}{2} - \frac{1}{2} D_m k_m \beta \tan\alpha \cos\theta = D - \delta$

$E_o = \frac{j\omega \mu I_o \exp(-j\beta R_o)}{4\pi R_o^2}$

$f_x = j\pi \frac{\cos x\pi}{x} + j \frac{\sin x\pi}{x}$

$F_x = \pi \frac{2\sin x\pi}{x} + 2\pi \frac{\cos x\pi}{x^2} - 2 \frac{\sin x\pi}{x^3}$

$d_{\theta_m} = \beta D_m \sin\theta$

$g_m = \beta D_m \sec\alpha$

$D_m$  = diameter at turn  $m$

$I_0$  = current at the base of the antenna (magnitude)

$J_n$  = Bessel function of order  $n$

$m$  = turn number

$$n_{\theta m} = \beta A_m \cos$$

$n$  = number of turns

$$p_{\theta m} = \beta D_m k_m \sin$$

$R_0$  = distance from the helix at which the field is to be calculated

$$x = n - g_m + b_{\theta m} + 1 = v + 1$$

$$y = n - g_m + b_{\theta m} - 1 = v - 1$$

$$x' = n - b_{\theta m} - g_m + 1 = v' + 1$$

$$y' = n - b_{\theta m} - g_m - 1 = v' - 1$$

The  $E_\phi$  and  $E_\theta$  components of the electric field at a distant point can be computed from these equations. The expressions are long and complicated, but fortunately, for practical computation, a number of approximating assumptions can be made since some of the terms are small compared to others. For example if the pitch angle,  $\alpha$ , is small all terms involving  $\tan \alpha$  can be neglected. In most cases only the 0<sup>th</sup> and 1<sup>st</sup> order Bessel functions need be retained. The higher order terms need consideration only for fields due to bigger turns and for large pitch angles.

As was shown in Section III that the pattern for the multifilar helix can be approximated by

$$E_\phi = \frac{\sin[A_\lambda (1 - \sin\phi)\pi]}{A_\lambda (1 - \sin\phi)\pi}$$

where

$A_\lambda$  = axial length in wavelengths

$\phi$  = angle with respect to the z-axis.

## APPENDIX C

### LIST OF SYMBOLS

A - Axial length of a helix

AR - Axial ratio (ellipicity)

$\alpha$  - Pitch angle of a helix =  $S/\pi D$

B - Beamwidth

B - Magnetic flux =  $\mu_0 H$

c - Velocity of light =  $3 \times 10^8$  meters/sec

C - Circumference of helix =  $\pi D$

d - Diameter of the helix conductor

D - Diameter of a helix

$D_\ell$  - Largest diameter of a conical helix

$D_o$  - Smallest diameter of a conical helix

$D_x$  - The diameter of a conical helix at any point x

e - Charge of a particle

$\epsilon_o$  - Permittivity of a vacuum ( $8.85 \times 10^{-12}$   $\frac{\text{farads}}{\text{m}}$ )

G - Gain relative to an isotropic point source

h - Distance of wave propagation

H - Magnetic flux density

L - Length of one turn on a helix

$\lambda$  - Subscript to denote measurements made in wavelengths

m - Mass of a particle

$n$  - Number of turns of a helix

$N$  - Number of particles/m<sup>3</sup>

$N$  - Number of windings on a multifilar helix

$r$  - Radius of a spiral at any particular point

$r_0$  - Initial radius of a spiral antenna

$S$  - Spacings between turns on a helix

$\mu_0$  - Permeability of free space