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MODEL STUDIES OF HELMHOLTZ RESONANCES IN ROOMS WITH WINDOWS AND DOORWAYS

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16. Abstract

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SUMMARY

The cavity resonance of a room enclosed by large windows and open doors can be set into motion if the windows should suddenly be subjected to an impulsive load, say, a sonic boom. To determine the conditions and possible damaging consequences of such a resonance, a study has been conducted which utilises the method of expressing the windows, air and common doorway of two typically joined rooms in terms of equivalent lumped elements. The resultant dynamic system is treated as a series of coupled Helmholtz resonators and has as its mathematical description a set of coupled, second order differential equations. Solutions to these equations have been generated on an analog computer for several types of impulsive loading conditions. Experiments were also performed on actual scaled modelsto guide the computer study. Results describe the conditions for which a maximum of coupling occurs between the window and room resonances.

PART I: EXPERIMENTAL AND SIMULATED MODEL STUDIES

1. INTRODUCTION

In recent full scale tests [1,2] subjecting buildings to sonic booms, it was observed that windows backed with open rooms contained in their response spectra certain low frequency components which could be identified with the Helmholtz resonances of the rooms. Since conditions favouring such resonances are quite common in modern dwellings which incorporate large plate glass windows in their designs, it is becoming increasingly important that the consequences of the coupling mechanism between a window and its resonant cavity be better understood. If a large window is backed with a closed room, the primary effect of the room air is to appear as a stiffness to the window, the extent of which depends upon the window area and room volume. If, however, the same window is backed with an open room, any sudden motion of the window could initiate a Helmholtz resonance in the room. Under the condition of resonance, the room air would appear to the window as a time varying stiffness and could conceivably act to amplify the window's response. The present study was undertaken to determine the particular room conditions under which possible dynamic amplification of the window response would occur. The approach used to develop a simplified description of the problem was to express the window, room air, and open doorway in terms of equivalent lumped elements. An appropriate combination of these lumped elements yields a description of the real system in terms of coupled, second order, differential equations.

2. LUMPED ELEMENT REPRESENTATION

A good approximation to the dynamical behaviour of a glass window which is mounted in a rigid support frame is a simply supported flat plate which has uniformly distributed mass and elasticity. If the centre deflections of the plate are less than 0.6 of the plate thickness, its dynamic response to any given load is given by linear plate theory. In cases where the plate's centre deflection exceeds this amount, Crandall and Kurzweil [3] have noted that the errors accumulated by extending the linear theory into the non-linear load-deflection regions are in the direction of conservatism. For the uniform pressure loading assumed in this particular investigation, the centre deflection of the plate, W_{max}, due to the sum contribution of each individual plate mode, is given in terms of the static loading, q_o, the flexural rigidity, D, and the plate area, ab, by the equation [4]

$$W_{max} = \frac{16 q_o}{\pi^6 D} \sum_{m=1} \sum_{n=1} \frac{(-1)^{\frac{m+n}{2} - 1}}{mn \left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2} \quad . \tag{1}$$

This exact solution provides a basis to which the equivalent deflections of the lumped element spring can be compared.

In general, lumped element representation is achieved by requiring that the parameters defining the lump element possess mechanical properties which are equivalent to those of the members they represent. For this case, arranging for equivalence between the plate and lumped element parameters is a somewhat arbitrary process; but, for purposes of comparing experiment with theory, a convenient place to start is to require the plate and its equivalent lumped element to have identical deflections. This choice not only determines an equivalent area for the lumped element but also provides it with the same velocities and accelerations at the plate centre. The latter identity results from the requirement that a lumped element must always have the same natural frequency as the system it represents. Two further requirements are needed to fix the relationships between the plate and lumped element parameters. If both systems have the same potential and kinetic energies while executing free vibrations, the lumped element parameters of area, A', mass, m', and stiffness, k', may be written as

$$A' = \frac{4ab}{\pi^2} \tag{2}$$

$$m' = m/4 \tag{3}$$

$$k' = \frac{\pi^4 D(a^2 + b^2)^2}{4a^3 b^3} \tag{4}$$

where m is the total mass of the plate (see for example Lowery [5]). If such an equivalent lumped element is subjected to the same uniform static load, q_o, as the real plate deflection given in equation (1), the equivalent deflection of the spring of the lumped element will be

$$W'_{max} = \frac{16\, q_o a^4 b^4}{\pi^6 D(a^2 + b^2)^2} .$$

A comparison of the expression for W'_{max} with that given by the exact solution in equation (1) shows a discrepancy between the two of only 2½%, an amount which is within the working limits of this particular study. In moving from static to time dependent loading, contributions to the window's centre deflections from higher order modes would of course be weighted differently depending on the frequency content of the particular load. However, in this study, the volume displacing modes which could cause Helmholtz resonances in open rooms are of primary interest and, since contributions from the higher order modes are small compared to the volume displaced by the fundamental (1,1) mode, their effect can be neglected. Thus, even for time dependent loadings, the equivalent lumped element representing the actual window can be considered to be a good approximation.

Consider next the lumped element representation of the air enclosed by the window and a room. The technique of representing the air in a room in terms of a lumped element is restricted to cases where the pressure disturbances occur nearly uniformly throughout a room. This restriction can be satisfied by requiring that the wavelength of the window's fundamental mode be large when compared to a typical dimension of the room. Under this condition the action of the air pressure on the fundamental mode of the window has the effect of opposing the window motion with a force directly proportional to the window deflection. This force can thus be thought of as a stiffness acting in series with the bending stiffness of the window. If the cyclic pressure changes in the room air are small and occur adiabatically, the acoustic pressure acting to inhibit the window motion can be given in terms of the air density, ρ_o, the speed of sound in air, c_o, the equivalent window area, A', the window deflection, x, and the room volume, V, by the equation [6]

$$P = \frac{\rho_o c_o^2 A' x}{V} \quad . \tag{5}$$

The total force, F, of the room air acting on the window is therefore

$$F = \frac{\rho_o c_o^2 (A')^2 x}{V} \tag{6}$$

and the equivalent stiffness, k", of the room air becomes

$$k'' = \frac{\rho_o c_o^2 (A')^2}{V} \quad . \tag{7}$$

Consider last the lumped element representation of the air mass contained in a door opening. If the room air is thought of as the stiffness of the Helmholtz resonator, the air contained in the doorway can be thought of as its mass. The requirement is that the area of the doorway be small compared to the area of the wall which contains it so that the air enclosed by the doorway can behave as an incompressible slug of fluid. A proper measure of the inertia of this air mass depends largely on the geometry of the room construction about the doorway and hence would vary considerably for each different design. For this particular study, the doorway will be represented by a circular tube which terminates in the planes of the two walls it joins. Since the volume of air oscillating in the tube extends slightly beyond its ends [7], it is necessary to correct the tube length, L, by the amount $K\sqrt{A/\pi}$, where K is an empirically determined constant and A is the cross-sectional area of the tube. The effective tube length, L', is therefore

$$L' = L + K\sqrt{\frac{A}{\pi}} \quad . \tag{8}$$

The effective air mass, m_e', can then be expressed as

$$m_e' = \rho_o A L' \quad . \tag{9}$$

3. MATHEMATICAL MODEL

Using the lumped element representations developed in Section 2, the equations of motion in the room arrangement shown in Figure 1a can be constructed with the following conventions. If the system is treated simply as having two degrees of freedom, the coordinates for the motion of the equivalent window mass and the effective air mass in the connecting tube can be given as x_1 and x_2 respectively. Damping losses in the system will be assumed to originate only in the bending stresses in the window and from the air friction in the connecting tube. Both of these losses will be considered velocity dependent and thus the forces associated with the window and tube damping can be given as $c_1\dot{x}_1$ and $c_2\dot{x}_2$ respectively. To write an equation of motion for each of the two masses, m_1' and m_2', consider first a force balance on m_1' under the influence of a suddenly applied force, F(t). The applied force will be balanced by the inertial force, $m_1'\ddot{x}_1$, the damping force, $c_1\dot{x}_1$, the bending stiffness force, $k_1'x_1$, and the room pressure, P_1, applied over the equivalent window area, A_1'. The resulting equation is given as

$$F(t) = m_1'\ddot{x}_1 + c_1\dot{x}_1 + k_1'x_1 + A_1'P_1 \quad (10)$$

where

$$P_1 = \frac{\rho_o c_o^2}{V_1} (A_1'x_1 - A_2x_2) \quad . \quad (11)$$

Similarly an equation of motion can be written for the mass m_2' where the exciting force now becomes the room pressure, P_1, applied over the tube area, A_2. The balancing forces will be the inertial force, $m_2'\ddot{x}_2$, the damping force $c_2\dot{x}_2$, and the room pressure, P_2, applied over the tube area, A_2. The resulting equation is

$$P_1A_2 = m_2'\ddot{x}_2 + c_2\dot{x}_2 + P_2A_2 \quad (12)$$

where

$$P_2 = \frac{\rho_o c_o^2 A_2 x_2}{V_2} \quad . \quad (13)$$

Substitution of the pressure terms into equations (10) and (12) yields

$$F(t) = m_1'\ddot{x}_1 + c_1\dot{x}_1 + k_1'x_1 + (A_1')^2 \frac{\rho_o c_o^2 x_1}{V_1} - A_1'A_2 \frac{\rho_o c_o^2 x_2}{V_1} \quad (14)$$

and

$$0 = m_2'\ddot{x}_2 + c_2\dot{x}_2 + \frac{A_2^2 \rho_o c_o^2 x_2}{V_2} - A_1'A_2 \frac{\rho_o c_o^2 x_1}{V_1} + \frac{A_2^2 \rho_o c_o^2 x_2}{V_1} \quad . \quad (15)$$

The equations of motion for the equivalent masses m_1' and m_2' are thus given in a set of coupled, second order, differential equations. Note that the form of these equations differs slightly from that of the usual coupled oscillator equations. In this case, the coefficients of the displacement terms contain different combinations of areas and room volumes. The degree of coupling between the systems components thus depends largely on the choice of relative room volumes and area ratios. Solutions to these coupled equations were generated on an E.A.I.580 analog computer. However, to direct the computer study towards a particular problem, a series of experiments were first conducted on a real system having a design similar to that shown in

Figure 1a. The experimental results provided a basis to which the computer solutions could be compared and also allowed for a check to be made on the use of the equivalent mass, stiffness and area approximations.

4. PHYSICAL MODEL

The physical model shown in Figure 1b, consisted of two airtight, identical rooms joined by a common connecting tube. A flanged opening was machined in the side of one of the rooms to accommodate a window which was secured with modelling clay. Each room was constructed of 9 mm thick Zylonite and measured 200 x 300 x 400 mm. These dimensions were chosen to represent a typical room based on a 1:20 scaling ratio. The acoustic pressure in each room was monitored by a ½" condenser microphone. The Perspex connecting tube had a wall thickness of 4 mm, an inside diameter of 47 mm, and ranged in length from 45 mm to 212 mm. The tube lengths were chosen such that the ratio of the contained air mass to the adjacent room stiffness would restrict the range of Helmholtz frequencies to wavelengths that would be large compared to the longest dimension of the room. Meeting this restriction ensures that the room pressure will be nearly everywhere uniform and hence the lumped element approach is a valid one. Each tube end was indexed so as to form an airtight joint with the adjoining wall. Before the model was assembled, the effective length of the tube was determined by the following method. A single Helmholtz resonator was constructed by separating the second room together with the connecting tube from the first room containing the window. A flange was then placed around the free end of the connecting tube to produce end conditions which were identical to those obtained when the tube was still joined to the wall of the first room. The simple resonator thus consisted of the room air cavity which could communicate with the atmosphere through the cylindrical neck of the connecting tube. Resonance was achieved simply by snapping one's fingers near the free end of the connecting tube. The Helmholtz frequency of this system was determined by analysing the transient response of the room pressure. The angular frequency of this system is given as

$$\omega^2 = \frac{c_o^2 \; A_2}{(L + K\sqrt{A_2/\pi})V} \qquad (16)$$

where K is the empirical constant being sought after. For a series of six different tube lengths, the average K had a value of 1.72 with the maximum deviation from this mean not exceeding 3%. The effective mass contained in the connecting tube was thus given as

$$m_2' = A_2 \rho_o (L + 1.72 \sqrt{A_2/\pi}) \quad . \tag{17}$$

The window was constructed of 1 mm thick Vybar and measured 140 mm x 240 mm. Since the window was to be excited electromagnetically, a small square of iron shim stock was bonded to its centre. On the side opposite this location, a 2 gm accelerometer was mounted to monitor the window's accelerations. Recall from Section 2 that the effective area corresponding to the fundamental mode was a factor of $4/\pi^2$ times the actual area, A. To check experimentally this number, the window was excited in its frame with and without the presence of the first room. With the room present, the additional stiffness of the room air caused the frequency of the fundamental mode to shift upward 17.1%. Using the theoretical factor of $4/\pi^2$ in calculating the increase in window frequency due to the room air stiffness, $\rho_o c_o^2 \frac{(4A/\pi^2)^2}{V}$, the frequency shift should have been 17.0%. This near agreement convincingly demonstrates the validity of the lumped element approach to this particular problem.

The electro-magnet used to excite the window was a standard Post Office 3000 relay coil which has a D.C. resistance of 50 ohms. The coil was activated directly from a D.C. power supply which was interrupted with a low noise switching transistor. A single pulse of a rectangular wave generator was used to trigger the switching transistor. A monitor of the current in the coil indicated that the pulse shape of the associated electromagnetic force was nearly a ramp function having a pulse width equal to that of the rectangular triggering pulse.

5. EXPERIMENTAL RESULTS

Experiments were conducted on the model described in Section 4 to determine the extent of mutual coupling between the impulsively excited window and the associated Helmholtz resonances in the adjoining rooms. Changes in the room's resonant frequencies were achieved by varying the length of the connecting tubes. For each tube length, the window acceleration and corresponding room pressure fluctuations were recorded on a four channel storage oscilloscope. A typical photograph of the data corresponding to a tube length of 167 mm is shown in Figure 2a. For this particular series the window responded at nearly a single frequency, thus allowing a good approximation of the displacement to be written as acceleration/(angular frequency)^2. If the air movement in the tube produces identical pressure changes in each room, an expression relating window acceleration of room pressure for this particular case may be obtained from equation (11) in the form

$$P_1 \doteq \frac{\rho_o c_o^2 A'}{V} \left(\frac{\ddot{x}_1}{\omega^2}\right) - P_2 \quad . \tag{18}$$

Attempts were made to verify this expression quantitatively from Figure 2a at several points in time along the response curves. The results obtained fell easily within the limits of the transducer calibration errors of $\pm$ 0.5 dB thus providing a good check on part of the governing equations developed in Section 3.

Information on the spectral content of each of the system's response data was obtained with the on-line analysis scheme used by the I.S.V.R. Data Analysis Centre. The data was processed with a Marconi Myriad II digital computer which was programmed for Fourier (transient) analysis. Presentations of the results were made with an on-line incremental plotter. Typical plots of the Fourier representation of the system's responses in the frequency domain are shown in Figures 2b, 2c and 2d. This particular set of plots corresponds to the real time response data shown in Figure 2a. The simplicity of the frequency response curves facilitated the use of the standard bandwidth method to determine the amount of damping in each of the system's components. This information was essential for obtaining values for the coefficients in equations (14) and (15). In general, the damping of the window was unaffected by the change in the room resonance frequencies caused by altering the length of the connecting tubes. However, the damping in the connecting tube as indicated by the Room 2 pressure response was affected by this change and showed a small but linear increase in magnitude by a factor of 0.25 as the tube length was increased by a factor of 4.7. This linear rise in damping is probably due to the increased frictional forces along the walls of the tube since viscous damping is directly a function of surface area.

The frequency response curves also provided useful information about the extent of coupling which occurs between the system's components. As the connecting tube lengths were shortened thus increasing the rooms' resonant frequencies, the spectra of the window's acceleration showed an increased presence of the frequencies associated with the rooms' resonances. An example of this condition is shown in Figure 3 which is a set of response spectra corresponding to a connecting tube length of 45 mm. Note, however, that an expected similar change in the content of the Room 2 pressure spectrum (Figure 3d) is not to be found. The magnitude of the component associated with the window frequency in this plot is not substantially different from that shown in the lightly coupled case observed in Figure 2d. This behaviour is yet without explanation.

6. COMPUTER SIMULATION

A comparative study of the problem was conducted on an E.A.I.580 analog computer. Equations (14) and (15) were suitably scaled with the magnitude scaling factors being based on the following estimated maximum values of the variables :

$$x_1 = 0.01 \text{ cm} ; \quad \dot{x}_1 = 5.0 \text{ cm/s} ; \quad \ddot{x}_1 = 50 \text{ cm/s}^2 ;$$
$$x_2 = 0.05 \text{ cm} ; \quad \dot{x}_2 = 30 \text{ cm/s} .$$

A time scaling factor of $\beta = 500$ was chosen so that, with the computer in its fast mode of operation, transient solutions could be obtained in real time for immediate spectral analysis. With the computer in its slow mode the output could be recorded on a pen recorder.

The final scaled equations may then be written

$$[\ddot{x}_1/50\beta] = \frac{2}{m_1'} \{-A[\dot{x}_1/5] - B[100\, x_1] - C[100\, x_1] + D[20\, x_2] + E[1]\} \qquad (19)$$

$$[\ddot{x}_2/\beta] = \frac{1}{m_2'} \{-F[\dot{x}_2/30] - G[20\, x_2] - H[20\, x_2] + I[100\, x_1]\} \qquad (20)$$

where the coefficients have the values

$$A = c_1/20\beta \; ; \quad B = k_1'/10^4\beta \; ; \quad C = (A_1')^2\rho_o c_o^2/10^3\beta v_1 \; ;$$

$$D = A_1'A_2\rho_o c_o^2/2.10^3\beta v_1 ; \quad E = 1/10^3\beta \; ; \quad F = 30\, c_2/\beta \; ;$$

$$G = A_2^2\rho_o c_o^2/20\,\beta v_2 \; ; \quad H = A_2^2\rho_o c_o^2/20\beta v_1 \; ; \quad I = A_1'A_2\rho_o c_o^2/100\beta v_1 \; .$$

The quantities m_1' and m_2' were isolated so that individual potentiometers could be set up on the computer to represent them.

A scaled version of equation (11) is also required to give the pressure in the first room. The scaled pressure equation may be written

$$[P_1/100] = J[100\, x_1] - K[20\, x_2] \qquad (21)$$

where $J = \rho_o c_o^2 A_1'/10^4 v_1$

$K = \rho_o c_o^2 A_2/2.10^3 v_1$.

The pressure in the second room was assumed to be proportional to the voltage representing x_2, as shown by equation (13). Values for the quantities involved in the above coefficients were obtained from the experiments conducted on the physical model.

Instead of applying an external forcing function, F(t), an initial velocity was imparted to the window in the form of an initial condition on the appropriate integrator in the computer. Provided the duration of the force is small compared with the natural period of the oscillator under examination, an initial velocity can be thought of as equivalent to an exciting force. For a system initially at rest, the velocity, v, acquired under the action of an impulsive force, is $v = \int_0^\varepsilon F(t)\, dt/m$, where m is the mass of the system and ε is the duration of the force. This expression may be written: $v = \bar{F}\varepsilon/m$, where $\bar{F}$ is the equivalent steady force operating for a time ε.

As an initial check on the program, the frequencies corresponding to the uncoupled window and room resonances were measured with an electronic counter. Fine tuning of any frequency that was not in agreement with the experimental values was made by a small adjustment of the potentiometers representing the masses, m_1' and m_2'. A critical test of the validity of the governing equations rested on the question whether or not the spectra of the simulated response data was the same as that of the experimental response data. To facilitate such a comparison, the method of on-line digital computer analysis which was used in the experimental study was applied. An initial velocity in the form of a step function was applied to the window mass, m_1', and the corresponding responses of the window acceleration and room pressures were consecutively recorded and analysed on the digital computer. This type of analysis was performed for a number of different values of m_2' and c_2 corresponding to the different connecting tube lengths used in the experiment. A set of typical results from this series is shown in Figure 4 which gives the response data in both the time and frequency domain. This particular set of simulated response data corresponds to the set of actual response data shown in Figure 2. An overall comparison between the simulated and actual response data is given in tabulated form in Table 1.

7. COMPARISON OF ACTUAL AND SIMULATED EXPERIMENTAL RESULTS

The results of both the actual and simulated experiments supported the notion that the transient response of a window influences and is influenced by the Helmholtz resonances of the open room which it encloses. A measure of the mutual influence or coupling indicated that the dynamics of the window, room and door combination behave much like that given by the system's equivalent lumped element representation. A review of Figures 2 and 4 and Table 1 shows for the most part good agreement between theory and experiment in terms of phase, amplitude and frequency content. A discrepancy is apparent, however, between the spectral content of the actual and simulated response of the Room 2 pressure. By some yet unexplained mechanism, the slug of fluid contained in the coupling tube acts to filter out the component at the window frequency. Since the tube lengths are appreciable in comparison to the wavelength of the pressure disturbances they transmit, it would seem reasonable to assume that the motion of the fluid in the coupling tube deviates somewhat from that of its idealised incompressible model. A velocity probe in this region within the coupling tube would help to resolve this question.

PART II: COUPLING STUDIES ON SIMULATED MODELS

1. FREQUENCY COINCIDENCE BETWEEN THE WINDOW AND ROOM RESONANCES

The near agreement between the model's actual response data and that of its mathematical representation suggested that the computer study could be extended within reasonable limits to include other conditions as well. One condition for the dynamic amplification of the window's response would be the case where the frequency of the rooms' Helmholtz resonance was coincident with that of the window's fundamental mode. To obtain such a coincidence of frequencies with the simulated model, the rooms' Helmholtz frequency was progressively increased by altering the m_2' and c_2 potentiometers. The window displacement and corresponding room pressures were recorded and analysed for each potentiometer setting prior to and at coincidence. The condition of frequency coincidence produced no appreciable change in the maximum window displacement for a given forcing function. However, the maximum amplitude of the pressure in Room 2 exhibited a considerable increase. This amount was a factor of 3.2 above that obtained when the room frequency was 1/4 of the window frequency. This increased coupling was also evident in the frequency spectra of the window's acceleration. As the condition of frequency coincidence was approached, the Fourier component identified with the rooms' Helmholtz frequency increased proportionately in magnitude, the extent of which can be seen over a short range of frequency ratios in Table 1. It is interesting to note that no similar change occurred in Room 2 pressure spectra. Approaching the condition of frequency resonance had little effect on the relative magnitudes of the Fourier components corresponding to the window and Helmholtz room resonances. Recall that this same effect was also noticed in the experiments on the physical model.

Although the condition of frequency coincidence produced no dramatic changes in the maximum displacement of the window, the possible side effects of the substantial increases in the maxima of the corresponding room pressures bear mention. At coincidence, the maxima of the room pressures were nearly of the same magnitude with the second room's being slightly higher than the first room's. In terms of sound pressure level, the magnitude of these maximum room pressures measured approximately 100 dB. On a full scale basis, this is the sound pressure level that would result from a 0.5 mm displacement of a window (2 x 3 m) backed by two adjoining rooms (each 4 x 6 x 8 m). Although the low frequency ranges of room Helmholtz resonances would usually fall below the hearing range of humans, the intensity level at which they occur could be quite severe and could easily produce startle effects among the rooms' inhabitants.

2. MODEL OF TWO WINDOWS IN A SINGLE ROOM

Since rooms usually contain more than one window, it is important to know on a quantitative basis how efficiently the enclosed room air couples the responses of two similar windows. Of particular interest would be a measure of the extent of coupling as a function of room volume and nearness of window frequencies. To develop appropriate coupled equations to describe these conditions, the same procedure as described in Part I was followed. By using lumped element representation, equations describing the case of two windows enclosing a single room were developed in the form

$$F(t) = m_1'\ddot{x}_1 + c_1\dot{x}_1 + k_1'x_1 + \frac{(A_1')^2\rho_o c_o^2 x_1}{V_1} - \frac{A_1'A_2'\rho_o c_o^2 x_2}{V_1} \tag{22}$$

$$0 = m_2'\ddot{x}_2 + c_2\dot{x}_2 + k_2'x_2 + \frac{(A_2')^2\rho_o c_o^2 x_2}{V_1} - \frac{A_1'A_2'\rho_o c_o^2 x_1}{V_1} \tag{23}$$

The constant coefficients in the above equations represent those same quantities as given in Part I, but now subscripts 1 and 2 are representative of the first window and second window respectively.

To guide the computer study, the physical model used in Part I was modified to a single room arrangement having two windows mounted in opposite walls. Since the dimensions and materials of the room and window components were unchanged, the constant coefficients associated with each component were also considered to be unchanged. Thus in setting up equations (22) and (23) for the computer study, the values for the constant coefficients initially were chosen to be the same as those obtained from the frequency analyses of the various responses in Part I. For this series of computer simulations, the forcing function, F(t), was generated externally in the form of a single rectangular pulse which was suitably scaled both in magnitude and period.

Using the previous time and magnitude scaling, the scaled form of equations (22) and (23) can be written as

$$\left[\frac{\ddot{x}_1}{50\beta}\right] = \frac{2}{m_1'}\{-A\left[\frac{\dot{x}_1}{5}\right] - B[100x_1] - C[100x_1] + D[100x_2] + E[1]\} \qquad (24)$$

$$\left[\frac{\ddot{x}_2}{50\beta}\right] = \frac{2}{m_2'}\{-F\left[\frac{\dot{x}_2}{5}\right] - G[100x_2] - H[100x_2] + I[100x_1]\} \qquad (25)$$

where

$$A = F = c/20\beta$$
$$B = G = k'/10^4\beta$$
$$C = D = H = I = \rho_o c_o^2 (A')^2/10^4 V\beta$$
$$E = 1/10^3\beta \quad .$$

Since the maximum bending stress in a window is directly proportional to its maximum centre deflection, it was decided that a useful measure of coupling between windows would be given in expressing the maximum displacement of the second window as a fractional part of the maximum displacement of the first window, i.e. $(x_2)_{max}/(x_1)_{max}$. The functional dependence of this ratio on room size and nearness of window frequencies was determined by varying the potentiometers corresponding to the room volume, V, and the mass of the second window, m_2', respectively. The results are given in Figures 5 and 6.

Similar tests were also performed on the actual model. The single data point which was obtained fell reasonably close to the corresponding point on the simulated curves.

The inverse relationship between the coupling of the two identical windows and the room volume is more gentle than might be expected. An increase in room size by a factor of 3 only decreased the coupling by 1/4. However, as a precautionary note, it should be recalled that an assumption in the simulation is that the pressure disturbance be everywhere uniform in the room. As the room volume is made larger and larger, this condition might be difficult to realize physically and thus Figure 5 should be interpreted with care.

For the given room volume, V_o, the degree of coupling between two windows having nearly the same frequencies can be considered to be quite strong. Even where the frequencies differ by as much as a factor of ± 0.4, the second window responds at nearly 1/2 of the first window's maximum displacement. With such strong coupling, it is easy to imagine certain situations where, say, an external impulse reaches a window before or after it has been excited internally through the coupling action of another window. Then, depending on the phase relative to the window motion, the external force would either act to decrease or increase the maximum displacement of the window by an amount proportional to the richness of its spectral content at the window's resonant frequency.

3. MODEL OF TWO MUTUALLY JOINED ROOMS EACH CONTAINING A WINDOW

Another common arrangement of room-window combinations is the case where two rooms, each containing a window, are joined by a common door or hallway. As shown in the earlier cases, under certain conditions, the transient motion of a window can excite the Helmholtz resonance in the open room it encloses. If a second room, also containing a window, shares the same opening, e.g. a doorway, the pressure disturbances in the second room due to the Helmholtz resonances could cause its window to respond. Of particular interest would be the extent of this response as a function of the nearness of the frequency of the window to that of the Helmholtz room resonance.

To study this condition, equations were set up according to the procedure developed in Part I in the form

$$F(t) = m_1'\ddot{x}_1 + c_1\dot{x}_1 + k_1'x_1 + \frac{\rho_o c_o^2 (A_1')^2 x_1}{V_1} - \frac{\rho_o c_o^2 A_1' A_2 x_2}{V_1} \qquad (26)$$

$$0 = m_2'\ddot{x}_2 + c_2\dot{x}_2 + \frac{\rho_o c_o^2 A_2^2 x_2}{V_1} + \frac{\rho_o c_o^2 A_2^2 x_2}{V_2} - \frac{\rho_o c_o^2 A_1' A_2 x_1}{V_1} - \frac{\rho_o c_o^2 A_2 A_3' x_3}{V_2} \qquad (27)$$

$$0 = m'_3\ddot{x}_3 + c_3\dot{x}_3 + k'_3x_3 + \frac{\rho_o c_o^2 (A'_3)^2 x_3}{V_2} - \frac{\rho_o c_o^2 A_2 A'_3 x_2}{V_2} \qquad (28)$$

where the subscripts 1, 2, and 3 are representative of the window in the first room, the air slug contained in the connecting chamber, and the window in the second room respectively. Since the second window was identical to the first window and all else was unchanged, the values for the constant coefficients in the above equations were the same as those obtained experimentally in Part I. The forcing function, $F(t)$, was generated externally in the form of a single rectangular pulse which was suitably scaled both for magnitude and for period. Using the same time and magnitude scaling as in Part I, the scaled form of the equations (26), (27) and (28) can be written as

$$\left[\frac{\ddot{x}_1}{50\beta}\right] = \frac{2}{m'_1}\{-A\left[\frac{\dot{x}_1}{5}\right] - B[100x_1] - C[100x_1] + D[20x_2] + E[1]\} \qquad (29)$$

$$\left[\frac{\ddot{x}_2}{\beta}\right] = \frac{1}{m'_2}\{-F\left[\frac{\dot{x}_2}{20}\right] - G[20x_2] - H[20x_2] + I[100x_1] + J[100x_3]\} \qquad (30)$$

$$\left[\frac{\ddot{x}_3}{50\beta}\right] = \frac{2}{m'_3}\{-K\left[\frac{\dot{x}_3}{5}\right] - L[100x_3] - M[100x_3] + N[20x_2]\} \qquad (31)$$

where

$$A = K = c/20\,\beta$$
$$B = L = k'/10^4\beta$$
$$C = M = \rho_o c_o^2 (A')^2/10^4\, V\beta$$
$$D = N = \rho_o c_o^2 (A_2)(A')/2 \times 10^3\, V\beta$$
$$E = 1/10^3\beta$$
$$F = 30\, c_2/\beta$$
$$G = H = A_2^2 \rho_o c_o^2/20\, V\beta$$
$$J = I = \rho_o c_o^2 A' A_2/100\, V\beta \quad .$$

A useful expression for examining the extent of coupling between windows in separate rooms joined with a common doorway is given as the ratio of the two windows' maximum displacements, i.e. $(x_3)_{max}/(x_1)_{max}$. A measure of this ratio was determined first for the condition of holding the window frequency constant while changing that of the rooms' Helmholtz resonance, and second, for the condition at a fixed room Helmholtz frequency with differing ratios of window frequencies. These changes were made on the computer circuit by adjusting the potentiometers corresponding to the masses m'_2 and m'_3. The results of these studies are shown in Figures 7 and 8.

To check experimentally the computer study, the physical model described in Part I was modified accordingly by adding a window to the second room. The model thus consisted of two identical rooms with two identical windows joined by connecting tubes of variable lengths.

Figure 7 shows that two identical windows can couple quite strongly even when each is located in separate rooms which are connected by a common hallway. The strength of the coupling is directly a function of the proximity of the windows' resonant frequency to that of the rooms' Helmholtz resonances. Near frequency coincidence the second window reaches nearly the same maximum displacement as the first window. If, during the maximum response cycle of the second window, an external impulsive load was applied, e.g. a reflected sonic boom, the maximum total displacement could be considerable.

Figure 8 shows the dependence of the coupling on the proximity of the two window frequencies for two fixed frequencies of room Helmholtz resonances. As is the previous case involving two windows located in the same room, the coupling remains quite high over a wide range of frequency ratios. Even where these ratios differ by as much as a factor of ± 0.30, the second window responds at nearly 1/2 the maximum displacement of the first window.

CONCLUSIONS

In connected rooms which are enclosed by large windows, motion of the windows in their fundamental modes influences and is influenced by the Helmholtz resonances of the rooms.

Mathematical descriptions of such systems which utilise equivalent lumped element representations produce response data which closely resemble those obtained from experiments performed on physical models.

A coincidence of frequencies between the window and room cavity resonance produced no increase in the maximum response of the window for a given loading. However, the room pressure reached a maximum which was 3.2 times higher than that corresponding to the case where the two frequencies differed by a factor of 4.

When two similar, large windows share the same room, the motion of one can cause the other to respond with nearly the same maximum amplitude.

With two similar, large windows located in different rooms joined by a common doorway, the initiation of the Helmholtz room resonance by motion of the window in one room can cause the window in the adjoining room to respond at nearly the same maximum amplitude.

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Frequency Ratio of Two Major Fourier Components in Response Spectra (Helmholtz Freq. / Window Freq.)		Magnitude Ratio of Two Major Fourier Components in Response Spectra (Comp. at Helmholtz Frequency / Comp. at Window Frequency)					
		Window Acceleration		Pressure Room 1		Pressure Room 2	
Exp.	Sim.	Exp.	Sim.	Exp.	Sim.	Exp.	Sim.
0.71	-	0.72	-	1.05	-	3.85	-
0.61	0.60	0.25	0.19	1.18	1.97	5.40	2.05
0.55	0.54	0.14	0.10	1.01	1.11	6.53	1.71
0.52	0.52	0.09	0.07	0.81	0.81	6.01	1.60
0.49	0.49	0.07	0.07	0.81	0.79	7.50	2.06

TABLE 1: COMPARISON OF SPECTRAL DATA BETWEEN EXPERIMENT AND SIMULATION

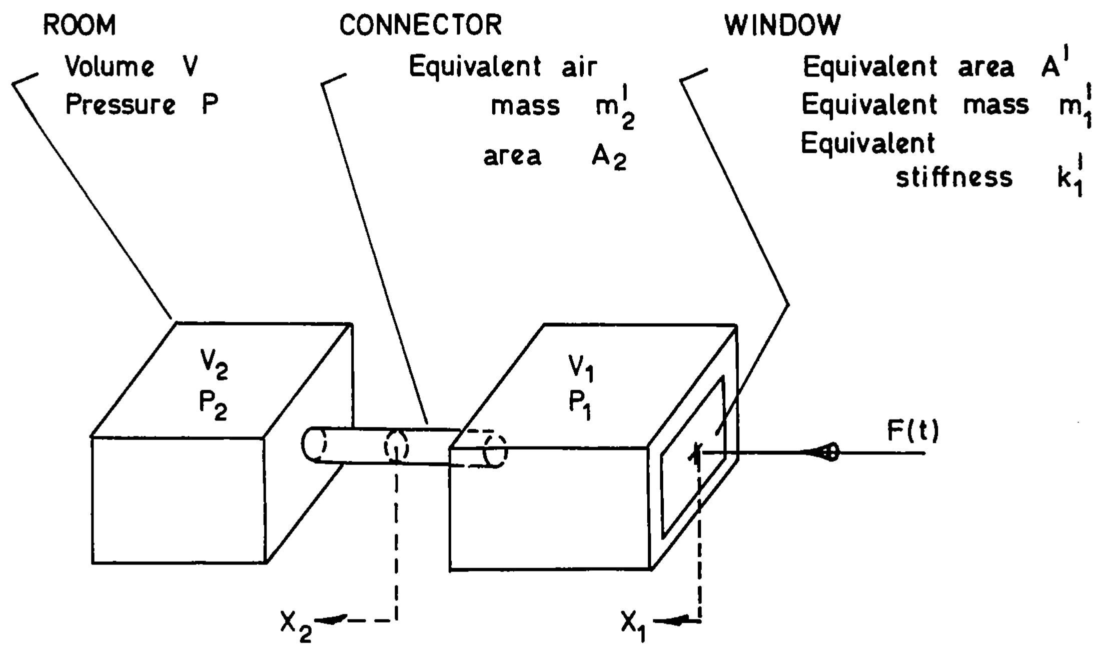


Fig. 1(a) Arrangement of rooms, window and connecting chamber.

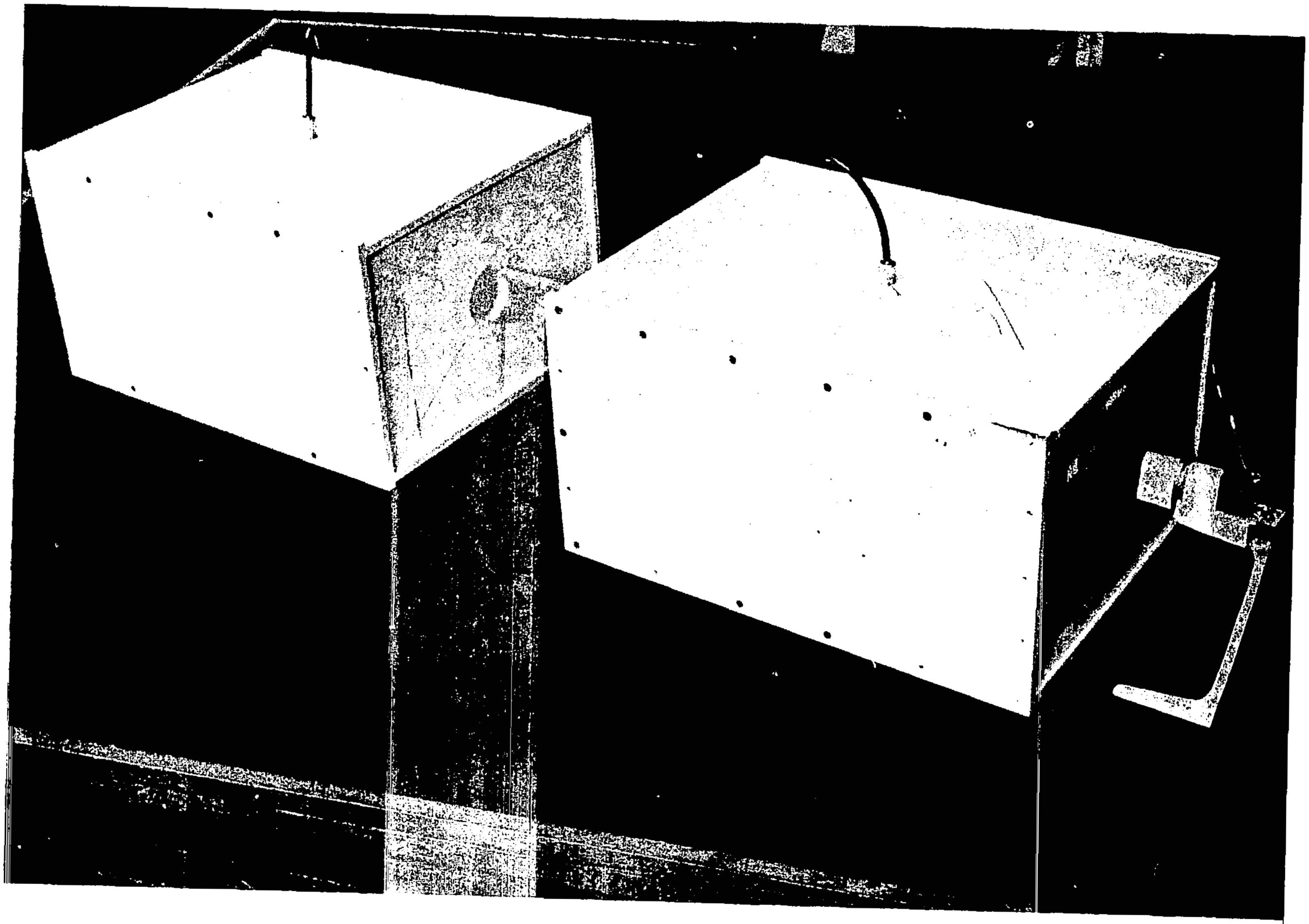


Fig. 1b Photograph of physical model.

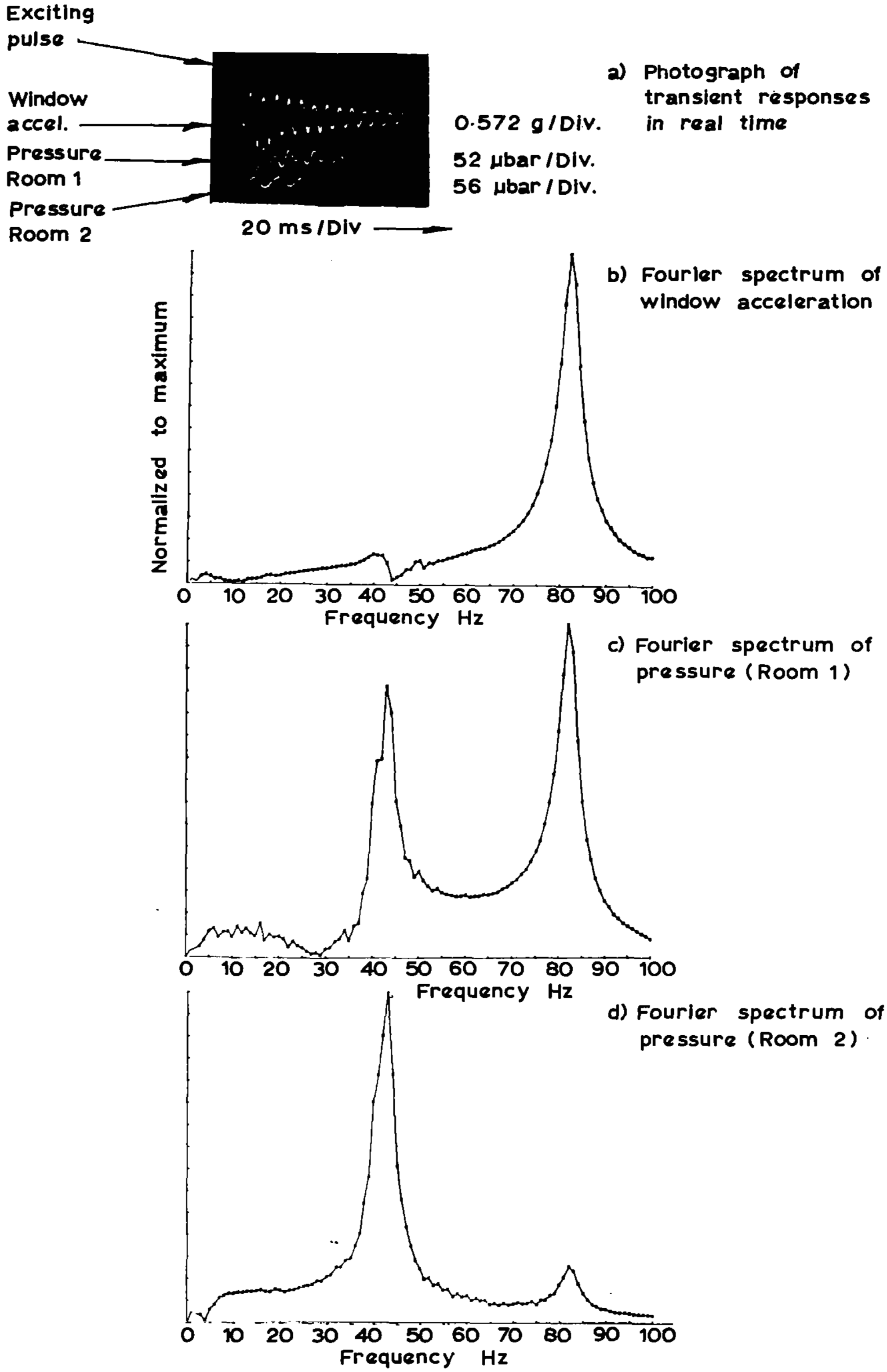


Fig. 2 Transient responses of experimental model. (Room joined by connecting tube of length, L = 167 mm).

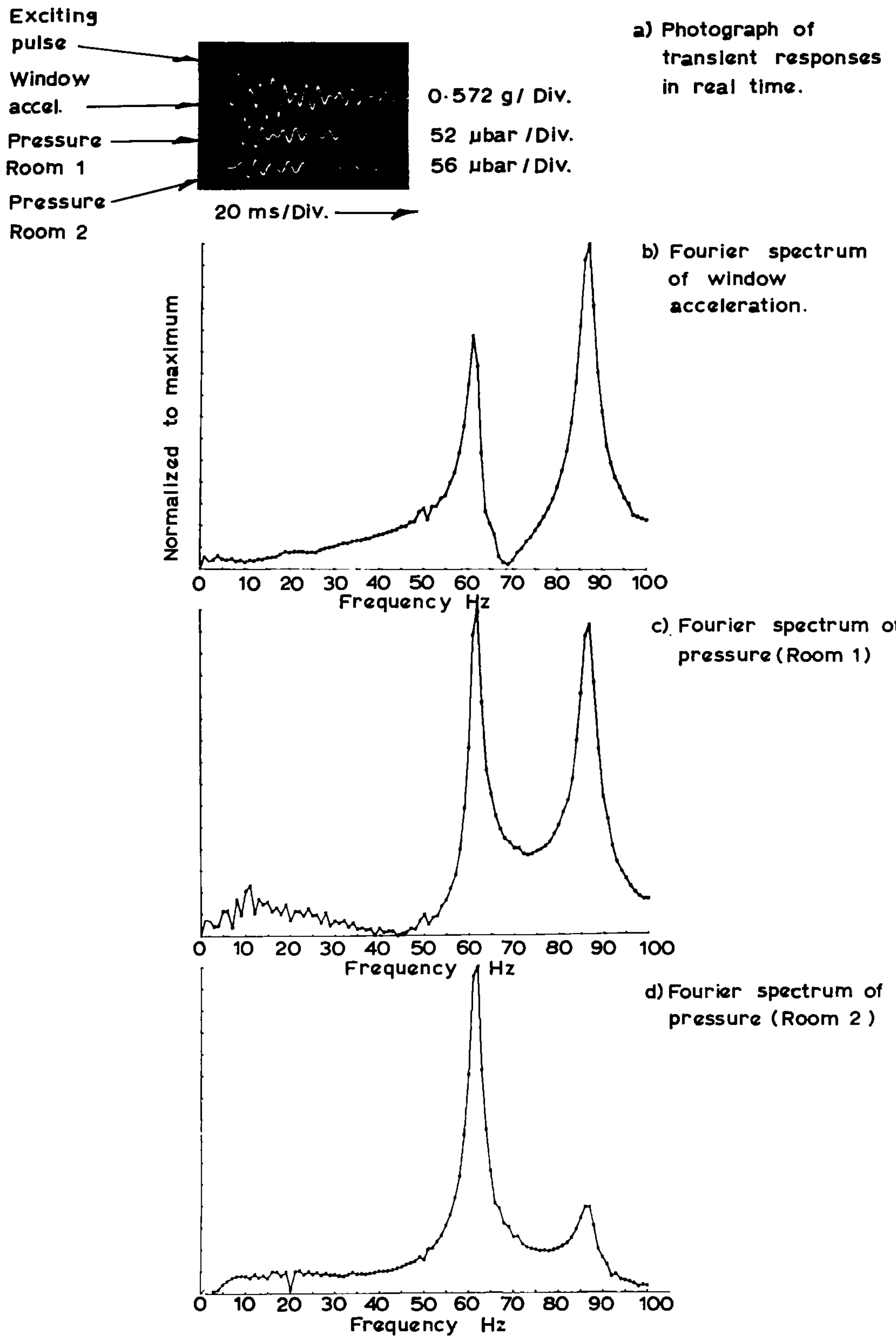


Fig. 3 Transient responses of experimental model. (Rooms joined by connecting tube of length. L = 45 mm)

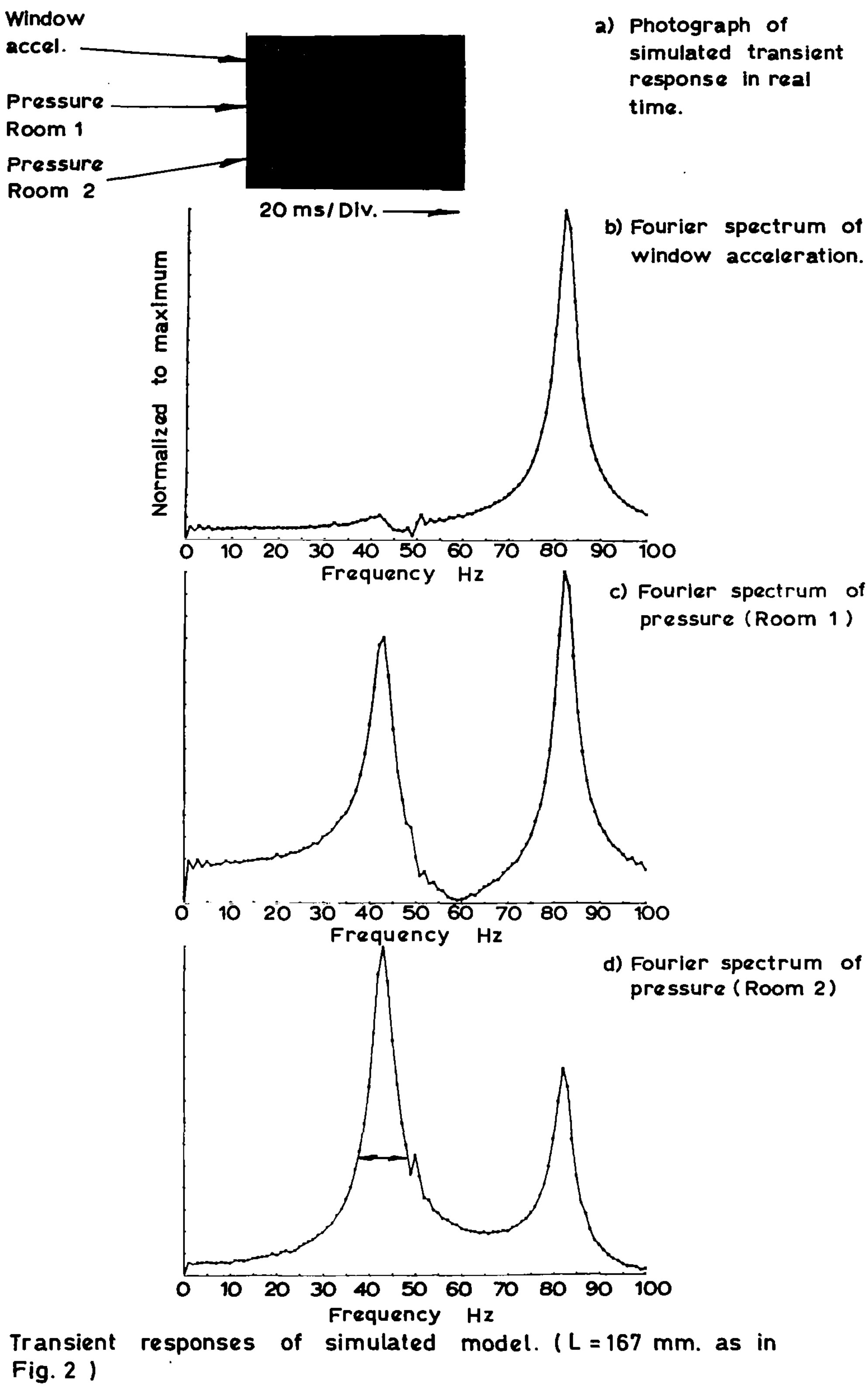


Fig. 4 Transient responses of simulated model. (L = 167 mm. as in Fig. 2)

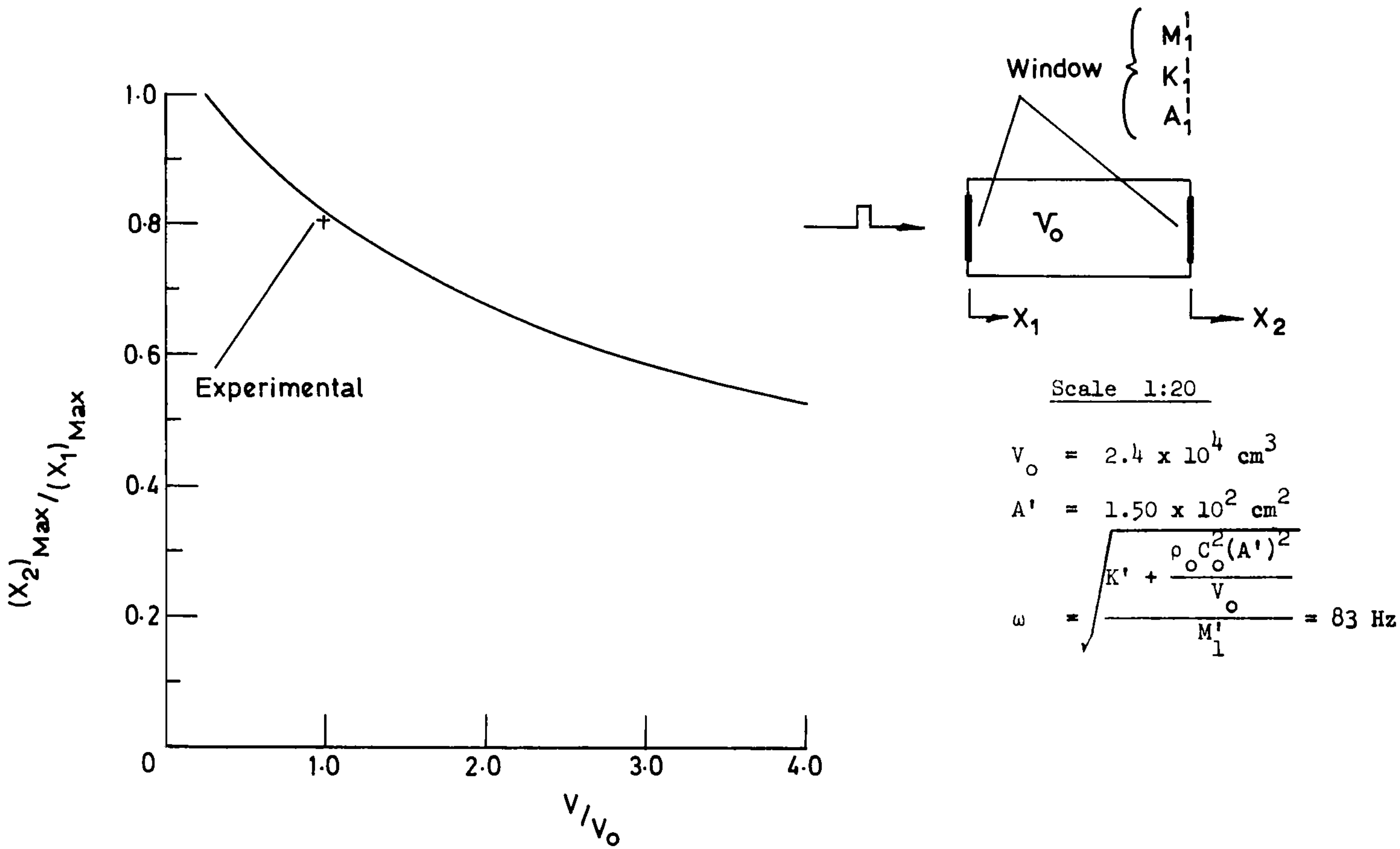
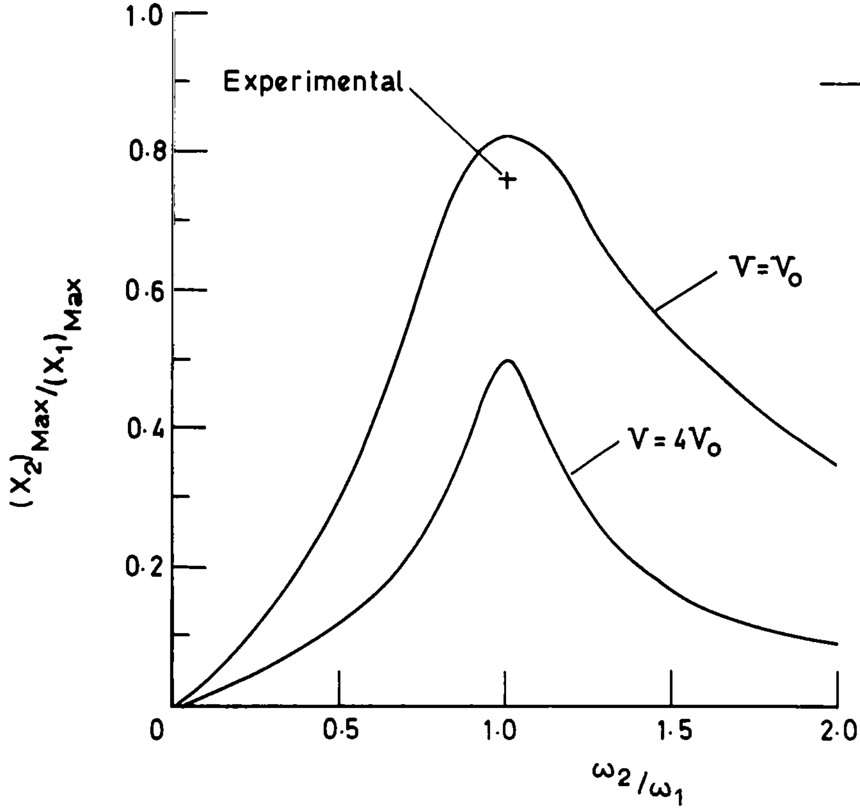
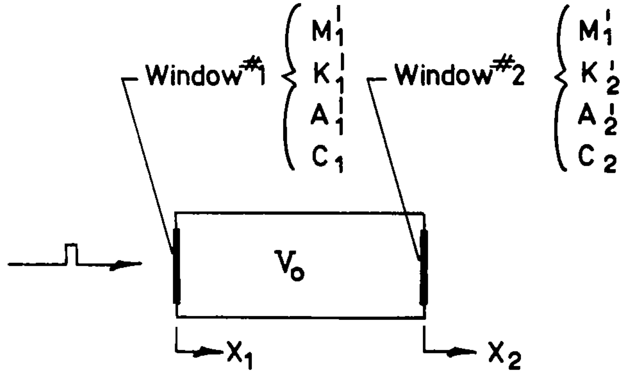


Fig. 5 Coupling between two identical windows as a function of room size.





Scale 1:20

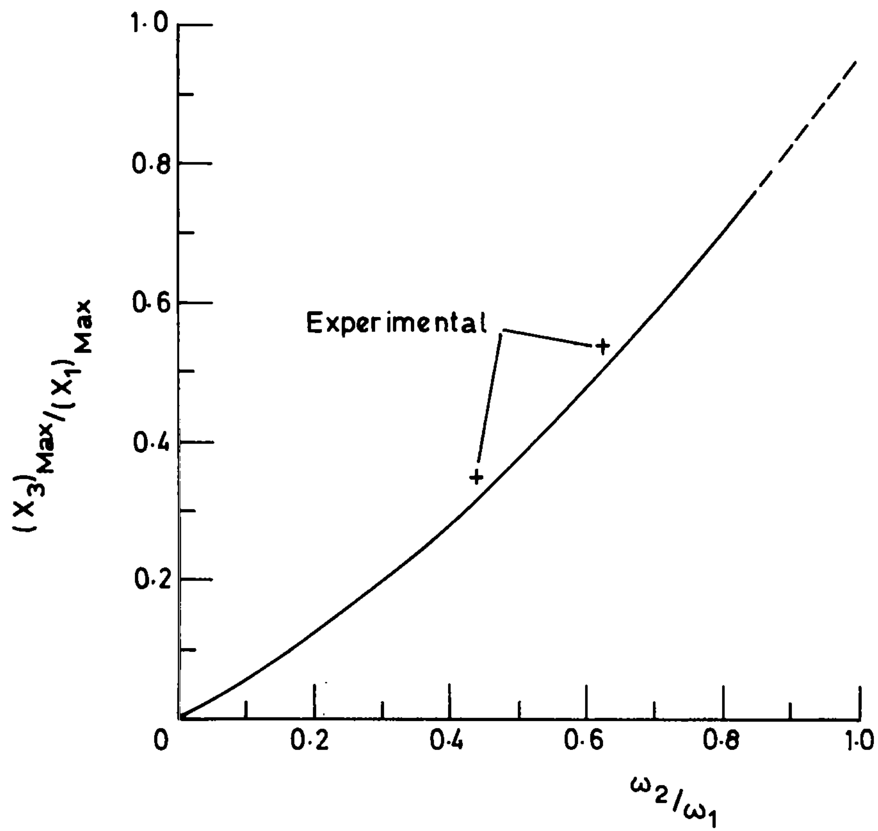
$$V_o = 2.4 \times 10^4 \text{ cm}^3$$

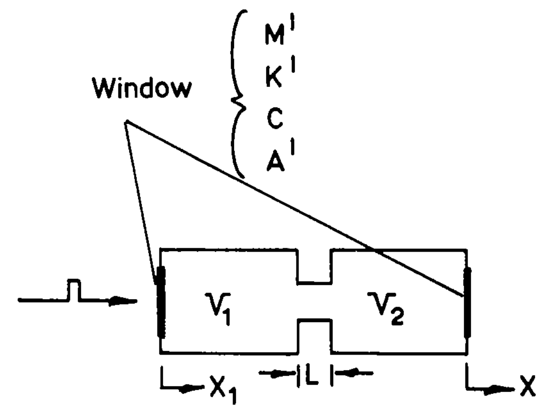
$$A_1' = A_2' = 1.5 \times 10^2 \text{ cm}^2$$

$$\omega_1 = \sqrt{\frac{K_1' + \frac{\rho_o C^2 (A_1')^2}{V_o}}{M_1'}} = 83 \text{ Hz}$$

$$\omega_2 = \sqrt{\frac{K_2' + \frac{\rho_o C_o^2 (A_2')^2}{V_o}}{M_2'}}$$

Fig. 6 Coupling between two windows of different frequency.





Scale 1:20

$V_1 = V_2 = 2.4 \times 10^4 \text{ cm}^3$

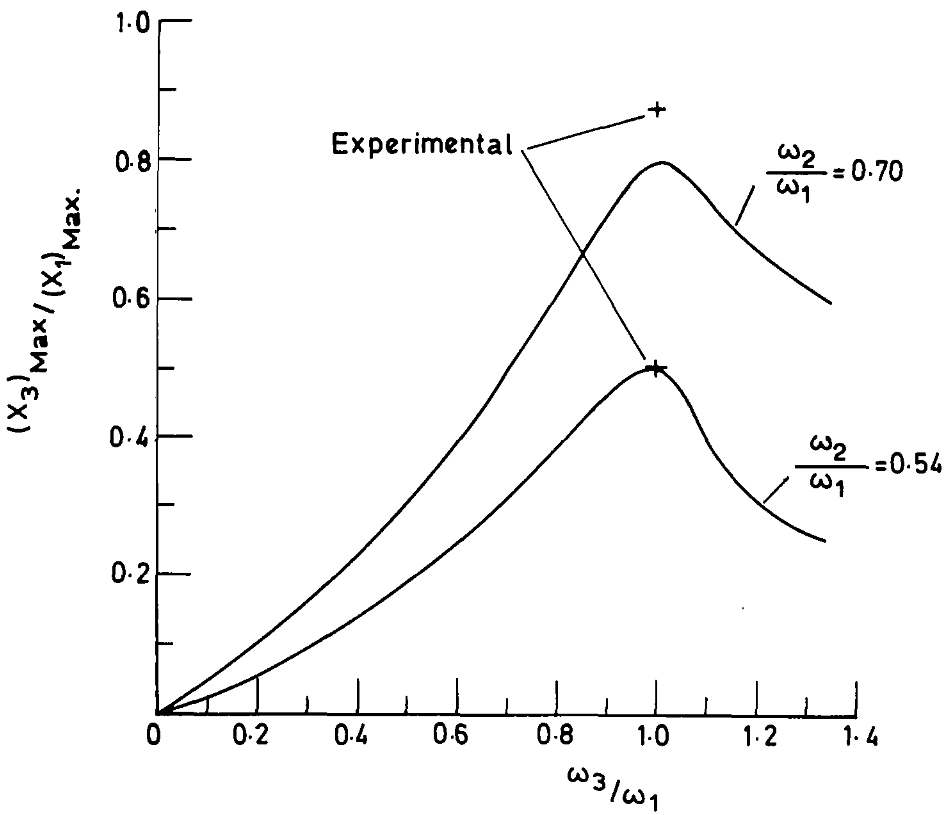
$A' = 1.5 \times 10^2 \text{ cm}^2$

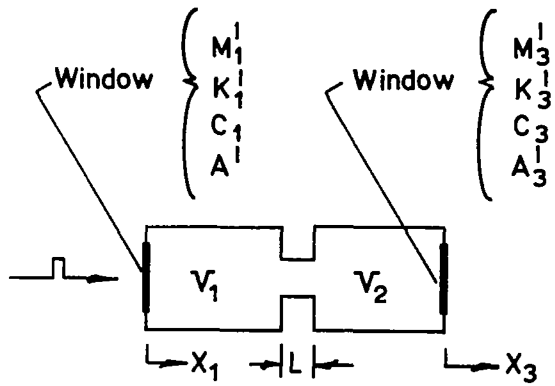
$A_2 = 18 \text{ cm}^2$

$$\omega_1 = \omega_3 = \sqrt{\frac{K' + \frac{\rho_o C_o^2 (A')^2}{V_o}}{M'}}$$

$$\omega_2 = C_o \sqrt{\frac{A_2}{L}\left[\frac{1}{V_1} + \frac{1}{V_2}\right]}$$

Fig. 7 Coupling between two identical windows in separate rooms joined by a common doorway.





Scale · 1:20

$V_1 = V_2 = 2.4 \times 10^4 \text{ cm}^3$

$A_1' = A_3' = 1.5 \times 10^2 \text{ cm}^2$

$A_2 = 18 \text{ cm}^2$

$$\omega_1 = \sqrt{\frac{K_1' + \frac{\rho_o C_o^2 (A_1')^2}{V_1}}{M_2'}} = 83 \text{ Hz}$$

$$\omega_2 = C_o \sqrt{\frac{A_2}{L}\left[\frac{1}{V_1} + \frac{1}{V_2}\right]}$$

$$\omega_3 = \sqrt{\frac{K_3' + \frac{\rho_o C_o^2 (A_3')^2}{V_2}}{M_3'}}$$

Fig. 8 Coupling between two windows of different frequencies in separate rooms joined by a common doorway.

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