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FORMULAS FOR nth ORDER DERIVATIVES OF HYPERBOLIC AND TRIGONOMETRIC FUNCTIONS

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16. Abstract

Formulas for the derivatives of any order are derived in the form of finite series for the hyperbolic and trigonometric cotangent, tangent, cosecant, and secant. These formulas are useful for the evaluation of Fourier sine and cosine integrals commonly expressed in terms of the derivatives. The coefficients in the series have a simple recursive property which facilitates their calculation.

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FORMULAS FOR nth ORDER DERIVATIVES OF HYPERBOLIC AND TRIGONOMETRIC FUNCTIONS

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SUMMARY

Formulas are derived and presented in the form of finite series for derivatives of any order of the hyperbolic cotangent, tangent, cosecant, and secant. The coefficients have a simple recursive property which facilitates their computation. A representative derivation and proof by mathematical induction are given for the hyperbolic cotangent. An application of the formulas to the evaluation of certain Fourier sine and cosine integrals is demonstrated. A method for obtaining formulas for the derivatives of the corresponding trigonometric functions is also presented.

INTRODUCTION

A method for evaluating cubic lattice sums which arise in the theory of magnetism has recently been developed (ref. 1). This method used infinite series expansions which are partially summed by means of the Laplace transform (ref. 2) and result in certain Fourier sine and cosine integrals.

These integrals and several other Fourier sine and cosine integrals are presented in standard tables of integrals (refs. 3 to 5 and references therein) in terms of derivatives of hyperbolic functions such as ctnh and tanh. The general case involves the derivative of nth order. Examples of these are (ref. 1)

$$\int_0^\infty \frac{x^{2n} \sin ax}{e^x - 1}\, dx = (-1)^n \frac{d^{2n}}{da^{2n}}\left(\frac{\pi}{2} \operatorname{ctnh} \pi a - \frac{1}{2a}\right) \qquad (1)$$

$$(a > 0)$$

$$\int_0^\infty \frac{x^{2n+1} \cos ax}{e^x - 1} dx = (-1)^n \frac{d^{2n+1}}{da^{2n+1}} \left(\frac{\pi}{2} \text{ctnh } \pi a - \frac{1}{2a}\right) \tag{2}$$

$$(a > 0)$$

In reference 6, the transforms of these integrals are listed as functions of Riemann zeta functions in the form of infinite series, which are inconvenient to evaluate. Formulas for the higher derivatives in equations (1) and (2) do not appear in any of the standard references on mathematical tables and formulas (refs. 7 to 11) or treatises on applied mathematics (refs. 12 to 18). Direct computation of a higher order derivative becomes inconvenient in the absence of a general formula. Furthermore, in the problem mentioned previously, the integrals in equations (1) and (2) appear as the nth terms in infinite series. These series are more easily handled with the nth term expressed in a more analytical form.

In this report, general formulas are derived which give the derivatives of the hyperbolic cotangent to any order in the form of finite series. Numbers are defined for the coefficients of the series which have a simple recursive property and are easily calculated; thus, the formulas are particularly suitable for numerical evaluation by desk-top computers with built-in programs for hyperbolic and trigonometric functions. Parallel formulas are also presented for the hyperbolic functions tanh, sech, and csch and for the trigonometric functions ctn, tan, sec, and csc. A representative induction proof for the formulas is given in the appendix.

DERIVATION OF FORMULAS

The formula for the derivative of arbitrary order of the hyperbolic cotangent (ctnh) is derived as follows.

Let $A_0 = \text{ctnh } y = u'u^{-1}$, where $u = \sinh y$ and $u' = \cosh y$. Then $u'' = u$ and $(u')^2 = 1 + u^2$. For successively higher derivatives, where $A_n = (d/dy)A_{n-1}$, carefully rearranging terms in the following way makes it possible to discern a recursive pattern and thus write down the general term, which will subsequently be proved:

$$A_0 = u'u^{-1}$$

$$A_1 = -u^{-2}$$

$$A_2 = (2!)u'u^{-3}$$

$$A_3 = -(3!)u^{-4} - (2^2)u^{-2}$$

$$A_4 = (4!)u'u^{-5} + (2^2)(2)!u'u^{-3}$$

$$A_5 = -(5!)u^{-6} - (2^2 + 4^2)(3!)u^{-4} - (2^2)^2u^{-2}$$

$$A_6 = (6!)u'u^{-7} + (2^2 + 4^2)(4!)u'u^{-5} + (2^2)^2(2!)u'u^{-3}$$

$$A_7 = -(7!)u^{-8} - (2^2 + 4^2 + 6^2)(5!)u^{-6} - \left[(2^2)^2 + 4^2(2^2 + 4^2)\right](3!)u^{-4} - (2^2)^3u^{-2}$$

$$A_8 = (8!)u'u^{-9} + \left[\sum_{l_1=0}^{3} (2l_1)^2\right](6!)u'u^{-7} + \left[\sum_{l_2=0}^{2} (2l_2)^2 \sum_{l_1=0}^{l_2} (2l_1)^2\right](4!)u'u^{-5} + (2^2)^3(2!)u'u^{-3} \qquad (3)$$

Let us define numbers $W_{2n,k}$ such that

$$W_{2n,0} = 1$$

$$W_{2n,1} = \sum_{m=0}^{n} (2m)^2$$

$$W_{2n,k} = \sum_{m=0}^{n} (2m)^2 W_{2m,k-1}$$

$$= (2n)^2 W_{2n,k-1} + W_{2(n-1),k} \qquad (4)$$

The series for $k = 1$ represented by $W_{2n,1}$ is well known and has the sum $2n(n + 1)(2n + 1)/3$ (ref. 19). The derivatives of A_0 can be written in terms of the $W_{2n,k}$ and we have, for example,

$$A_8 = (8!)u'u^{-9} + W_{6,1}(6!)u'u^{-7} + W_{4,2}(4!)u'u^{-5} + W_{2,3}(2!)u'u^{-3} \qquad (5)$$

The even derivatives for arbitrary n can then be written

$$A_{2n} = (2n)!u'u^{-(2n+1)} + W_{2(n-1),1}\left[2(n-1)\right]!u'u^{-(2n-1)}$$

$$+ W_{2(n-2),2}\left[2(n-2)\right]!u'u^{-(2n-3)} + \ldots + W_{4,n-2}(4!)u'u^{-5} + W_{2,n-1}(2!)u'u^{-3}$$

$$= \sum_{k=0}^{n-1} W_{2(n-k),k}\left[2(n-k)\right]!u'u^{-2(n-k)-1} \tag{6}$$

The odd derivatives can be similarly written:

$$A_{2n+1} = 1\sum_{k=0}^{n} W_{2(n-k+1),k}\left[2(n-k)+1\right]!u^{-2(n-k+1)} \tag{7}$$

In terms of hyperbolic functions,

$$\frac{d^{2n}}{dy^{2n}} \operatorname{ctnh} y = \operatorname{ctnh} y \sum_{k=0}^{n-1} W_{2(n-k),k}\left[2(n-k)\right]!(\operatorname{csch} y)^{2(n-k)} \tag{8}$$

$$(n \geq 1)$$

$$\frac{d^{2n+1}}{dy^{2n+1}} \operatorname{ctnh} y = -\sum_{k=0}^{n} W_{2(n-k+1),k}\left[2(n-k)+1\right]!(\operatorname{csch} y)^{2(n-k+1)} \tag{9}$$

$$(n \geq 0)$$

Since

$$\frac{d^k}{dy^k}\frac{1}{y} = \frac{(-1)^k k!}{y^{k+1}} \tag{10}$$

The Fourier sine and cosine integrals in equations (1) and (2) can be written explicitly as

$$\int_0^{\infty} \frac{x^{2n} \sin ax}{e^x - 1} dx = \frac{(-1)^n}{2} \pi^{2n+1} \frac{d^{2n}}{dy^{2n}} \left(\operatorname{ctnh} y - \frac{1}{y} \right)$$

$$= \frac{(-1)^n}{2} \pi^{2n+1} \left\{ \operatorname{ctnh} y \sum_{k=0}^{n-1} W_{2(n-k),k} [2(n-k)]! (\operatorname{csch} y)^{2(n-k)} - \frac{(2n)!}{y^{2n+1}} \right\} \quad (11)$$

$$\int_0^{\infty} \frac{x^{2n+1} \cos ax}{e^x - 1} dx = \frac{(-1)^{n+1}}{2} \pi^{2n+2} \left\{ \sum_{k=0}^{n} W_{2(n-k+1),k} [2(n-k)+1]! (\operatorname{csch} y)^{2(n-k+1)} - \frac{(2n+1)!}{y^{2n+2}} \right\} \quad (12)$$

where $y = \pi a$. These integrals are thus easily and conveniently evaluated for any n to any degree of accuracy. Equations (11) and (12) have been checked numerically for $n = 0$ to 5 and $a = 1$ to 5.

Formulas for the higher derivatives of tanh, sech, and csch, which may be derived in a similar way, are tabulated in the next section. A method is also described for obtaining the higher derivatives of the corresponding trigonometric functions from the formulas for the hyperbolic functions.

The coefficients in the derivatives of sech, csch, sec, and csc consist of the sums of odd numbers, $W_{2n+1,k}$, where

$$W_{2n+1,0} = 1$$

$$W_{2n+1,1} = \sum_{m=0}^{n} (2m+1)^2$$

$$W_{2n+1,k} = \sum_{m=0}^{n} (2m+1)^2 W_{2m+1,k-1}$$

$$= (2n+1)^2 W_{2n+1,k-1} + W_{2n-1,k} \quad (13)$$

A representative proof by mathematical induction for the formulas is given in the appendix.

Tables for the numbers $W_{2n,k}$ and $W_{2n+1,k}$ are most conveniently generated using $W_{2,k} = 2^{2k}$, $W_{4,k} = \sum_{m=0}^{k} 2^{2(k+m)}$, and $W_{2n,k} = (2n)^2 W_{2n,k-1} + W_{2(n-1),k}$ for even numbers and $W_{1,k} = 1$ and $W_{2n+1,k} = (2n+1)^2 W_{2n+1,k-1} + W_{2n-1,k}$ for odd numbers.

TABULATION OF HIGHER DERIVATIVES

All the formulas presented in this section may be derived in the manner outlined for the hyperbolic cotangent in the previous section, the formula for which is repeated here for the sake of completeness.

Hyperbolic Functions

$$\frac{d^{2n}}{dy^{2n}} \operatorname{ctnh} y = \operatorname{ctnh} y \sum_{k=0}^{n-1} W_{2(n-k),k} \left[2(n-k)\right]!(\operatorname{csch} y)^{2(n-k)} \tag{14}$$

$$\frac{d^{2n+1}}{dy^{2n+1}} \operatorname{ctnh} y = -\sum_{k=0}^{n} W_{2(n-k+1),k} \left[2(n-k)+1\right]!(\operatorname{csch} y)^{2(n-k+1)} \tag{15}$$

$$\frac{d^{2n}}{dy^{2n}} \tanh y = \tanh y \sum_{k=0}^{n-1} (-1)^{n-k} W_{2(n-k),k} \left[2(n-k)\right]!(\operatorname{sech} y)^{2(n-k)} \tag{16}$$

$$\frac{d^{2n+1}}{dy^{2n+1}} \tanh y = \sum_{k=0}^{n} (-1)^{n-k} W_{2(n-k+1),k} \left[2(n-k)+1\right]!(\operatorname{sech} y)^{2(n-k+1)} \tag{17}$$

$$\frac{d^{2n}}{dy^{2n}} \operatorname{csch} y = \sum_{k=0}^{n} W_{2(n-k)+1,k} \left[2(n-k)\right]!(\operatorname{csch} y)^{2(n-k)+1} \tag{18}$$

$$\frac{d^{2n+1}}{dy^{2n+1}} \text{csch } y = - \text{ctnh } y \sum_{k=0}^{n} W_{2(n-k)+1,k} \left[2(n-k)+1\right]!(\text{csch } y)^{2(n-k)+1} \tag{19}$$

$$\frac{d^{2n}}{dy^{2n}} \text{sech } y = \sum_{k=0}^{n} (-1)^{n-k} W_{2(n-k)+1,k} \left[2(n-k)\right]!(\text{sech } y)^{2(n-k)+1} \tag{20}$$

$$\frac{d^{2n+1}}{dy^{2n+1}} \text{sech } y = - \tan y \sum_{k=0}^{n} (-1)^{n-k} W_{2(n-k)+1,k} \left[2(n-k)+1\right]!(\text{sech } y)^{2(n-k)+1} \tag{21}$$

Trigonometric Functions

Analogous formulas for the corresponding circular functions can be simply obtained by making the following substitutions:

$$\left.\begin{aligned} \text{ctnh } y &= i\text{ctn } iy \\ \tanh y &= - i\tan iy \\ \text{csch } y &= i\csc iy \\ \text{sech } y &= \sec iy \\ \frac{d^m}{dy^m} &\equiv i^m \frac{d^m}{d(iy)^m} \end{aligned}\right\} \tag{22}$$

Then, for example

$$\frac{d^{2n}}{dy^{2n}} \text{ctn } y = \text{ctn } y \sum_{k=0}^{n-1} (-1)^k W_{2(n-k),k} \left[2(n-k)\right]!(\csc y)^{2(n-k)} \tag{23}$$

$$(n \geq 1)$$

CONCLUDING REMARKS

Previous untabulated formulas for the derivatives to any order of certain hyperbolic and trigonometric functions have been derived and presented. These formulas are given in the form of finite series, the coefficients of which have a simple recursive property and thus are easily calculated. At least one use for these formulas is in the evaluation of Fourier sine and cosine integrals such as those mentioned in the report.

As n increases, the coefficients become very large. In actual applications, however, there may be multiplying factors which may somewhat offset this trend. Such is the case for the problem in the theory of magnetism referred to in the INTRODUCTION. For example, factors multiplying the integrals in equations (1) and (2) are of the form

$$\frac{\left[(2n)!\right]^2}{\left[2^{2n}(n!)^2\right]^3}$$

For general reference purposes, a complete listing of the 16 formulas for the nth order derivatives is not necessary. The four formulas in equations (15), (17), (18), and (20), together with a definition of the numbers $W_{m,k}$, are sufficient. The other formulas are then easily gotten by either a single differentiation or simple substitution using equation (22).

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, May 4, 1971,
129-02.

APPENDIX - PROOF BY MATHEMATICAL INDUCTION OF FORMULAS FOR DERIVATIVES OF HYPERBOLIC COTANGENT

A detailed proof by mathematical induction of the formula for the odd derivatives of ctnh y, $d^{2n+1}\text{ctnh}\ y/dy^{2n+1}$, is given here to verify its validity for all n. The formula for $d^{2n}\text{ctnh}\ y/dy^{2n}$ is consequently also verified. Similar proofs by mathematical induction can be constructed for the tanh, sech, and csch formulas.

The derivatives of $d^{2n+1}\text{ctnh}\ y/dy^{2n+1}$ for $n = 0, 1$ are

$$\frac{d}{dy}\,\text{ctnh}\ y = -\,\text{csch}^2\ y = -\,u^{-2} \tag{A1}$$

$$\begin{aligned}\frac{d^3}{dy^3}\,\text{ctnh}\ y &= -\,(3!)\text{csch}^4\ y - (2^2)\text{csch}^2\ y \\ &= -\,(3!)u^{-4} - (2^2)u^{-2}\end{aligned} \tag{A2}$$

where $u = \sinh y$. These cases are readily verified by direct calculation.

Assume the formula true for $n = m - 1$. Then

$$\frac{d^{2m-1}}{dy^{2m-1}}\,\text{ctnh}\ y = -\sum_{k=0}^{m-1} W_{2(m-k),k}\left[2(m-k) - 1\right]!\,u^{-2(m-k)} \tag{A3}$$

By direct differentiation,

$$\frac{d^{2m}}{dy^{2m}}\,\text{ctnh}\ y = \sum_{k=0}^{m-1} W_{2(m-k),k}\left[2(m-k)\right]!\,u'u^{-2(m-k)-1} \tag{A4}$$

$$\frac{d^{2m+1}}{dy^{2m+1}} \operatorname{ctnh} y = -\sum_{k=0}^{m-1} W_{2(m-k),k}\left[2(m-k)+1\right]!(1+u^2)u^{-2(m-k+1)}$$

$$+\sum_{k=0}^{m-1} W_{2(m-k),k}\left[2(m-k)\right]!u^{-2(m-k)}$$

$$= -\sum_{k=0}^{m-1} W_{2(m-k),k}\left[2(m-k)+1\right]!u^{-2(m-k+1)}$$

$$-\sum_{k=0}^{m-1} W_{2(m-k),k}\left\{\left[2(m-k+1)\right]! - \left[2(m-k)\right]!\right\}u^{-2(m-k)} \qquad \text{(A5)}$$

Consider the first sum.

$$\sum_{k=0}^{m-1} W_{2(m-k),k}\left[2(m-k)+1\right]!u^{-2(m-k+1)}$$

$$= W_{2m,0}(2m+1)!u^{-2(m+1)} + \sum_{k=1}^{m-1} W_{2(m-k),k}\left[2(m-k)+1\right]!u^{-2(m-k+1)} \qquad \text{(A6)}$$

In the second sum,

$$\left[2(m-k)+1\right]! - \left[2(m-k)\right]! = \left[2(m-k)\right]^2\left[2(m-k)-1\right]! \qquad \text{(A7)}$$

The second sum is then

$$\sum_{k=0}^{m-1} \left[2(m-k)\right]^2 W_{2(m-k),k}\left[2(m-k)-1\right]!u^{-2(m-k)}$$

$$= \sum_{k=0}^{m-2} \left[2(m-k)\right]^2 W_{2(m-k),k}\left[2(m-k)-1\right]!u^{-2(m-k)} + (2^2)W_{2,m-1}u^{-2}$$

$$= \sum_{k=1}^{m-1} \left[2(m-k+1)\right]^2 W_{2(m-k+1),k-1}\left[2(m-k)+1\right]!u^{-2(m-k+1)} + W_{2,m}u^{-2} \qquad \text{(A8)}$$

where the dummy index k is replaced by $k - 1$ and $W_{2,m} = (2^2)W_{2,m-1}$ by definition. Recombining the two series gives

$$\frac{d^{2m+1}}{dy^{2m+1}} \operatorname{ctnh} y = -W_{2m,0}(2m+1)!u^{-2(m+1)}$$

$$- \sum_{k=1}^{m-1} \left\{\left[2(m-k)+1\right]^2 W_{2(m-k+1),k-1} + W_{2(m-k),k}\right\}\left[2(m-k)+1\right]!u^{-2(m-k+1)}$$

$$- W_{2,m}u^{-2} \qquad \text{(A9)}$$

By the definition of the numbers $W_{2n,k}$,

$$W_{2m+1,0} = W_{2m,0} = 1$$

and

$$\left[2(m-k)+1\right]^2 W_{2(m-k+1),k-1} + W_{2(m-k),k}$$

$$= \left[2(m-k+1)\right]^2 W_{2(m-k+1),k-1} + \sum_{l=0}^{m-k} (2l)^2 W_{2l,k-1}$$

$$= \sum_{l=0}^{m-k+1} (2l)^2 W_{2l,k-1}$$

$$= W_{2(m-k+1),k} \qquad \text{(A10)}$$

Then

$$\frac{d^{2m+1}}{dy^{2m+1}} \operatorname{ctnh} y = -W_{2(m+1),0}(2m + 1)!u^{-2(m+1)}$$

$$- \sum_{k=0}^{m-1} W_{2(m-k+1),k}\left[2(m - k) + 1\right]!u^{-2(m-k+1)} + W_{2,m}u^{-2}$$

$$= -\sum_{k=0}^{m} W_{2(m-k+1),k}\left[2(m - k) + 1\right]!u^{-2(m-k+1)} \qquad \text{(A11)}$$

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