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Abstract

Data from a series of pitot tube rocket soundings of the atmosphere are examined. The experiments consist of four soundings carried out over a two-day period at Fort Churchill, Canada in January and February 1967 by Smith, Theon and Horvath.

Temperature data from the four soundings were averaged, and a smooth curve was drawn through the points. A hydrostatically determined atmosphere based on the above lapse rate was calculated. Deviations from the above mean atmosphere were calculated, and a wavelike structure was observed. The waves grow very slowly below approximately 80 km, and acquire a more rapid growth rate at higher altitudes. The density and temperature variations have a wavelength of 10-20 km and are 180 degrees out of phase. The pressure variation is somewhat irregular. However, in regions where pressure variation is well behaved it is out of phase with temperature and density by about 90 degrees.

Gravity wave theory was used to compute wave patterns like those observed experimentally. A highly damped upward propagating wave gives good results. Turbulence provides the most effective damping mechanism, but it is found that generally expected levels of turbulence are too low to provide the needed damping.

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I. Experimental Evidence

Data from a series of pitot tube rocket experiments were studied. The experiments consisted of four soundings carried out over a two-day period at Fort Churchill, Canada in January and February 1967. The data are published by Smith et al. (1967). The pitot tube technique is described by Horvath et al. (1962).

Smith, Katchen, and Theon (1968) have compared the results of the pitot tube experiment with data from rocket grenade studies and found temperature agreement to within a few degrees Kelvin. The differences between the two types of experiment are generally less than 3 degrees K below 60 km, and less than 5 degrees below 90 km. The grenade technique is based on measurements of the speed of sound, and is therefore directly related to temperature structure. The grenade technique thus gives an independently obtained temperature profile, which compares favorably with the temperature profile deduced from the pitot tube experiments.

The temperature data from the four soundings were averaged and a smooth curve drawn through the points. A hydrostatically determined atmosphere based on the above lapse rate was calculated. Experimental data were then plotted as $\rho/\bar{\rho}$, $P/\bar{P}$, $T-\bar{T}$ versus altitude where the barred quantities refer to the mean atmosphere calculated above. A characteristic plot is shown in Figure 1.

Temperature and density are 180 degrees out of phase. The density and temperature variations have a wavelength of 10-20 km. The pressure variation seems somewhat irregular. However, in the regions where pressure variation is well behaved it is out of phase with temperature and density variation by about 90 degrees. These phase relations are what one would expect for internal gravity waves. The waves grow very slowly below approximately 80 km, and acquire a more rapid growth rate at higher altitudes. The other three experimental plots exhibit similar behavior.

II. Numerical Calculations

The numerical calculations are based on the gravity wave treatment developed by Volland (1969). Essentially, a matrix method is used to obtain periodic plane wave solutions of a linearized version of the following set of equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

$$\rho \frac{D\vec{v}}{Dt} + \nabla P - \rho \vec{g} = 0$$

$$\rho C_v \frac{DT}{Dt} + P \nabla \cdot \vec{V} - \nabla(\kappa \nabla T) = 0$$

$$P - \mu \rho T = 0$$

Inclusion of the heat transfer term in the energy equation provides a mechanism of dissipation since destruction of thermal gradients leads to the creation of entropy. It is recognized that both molecular transfer and turbulence result in comparable values for viscosity and conductivity. Therefore, viscosity should be included in both momentum and energy equations. However, such inclusion would greatly complicate the mathematics. Thus a compromise has been made in favor of a more tractable mathematical model which includes some damping, even though the effective conductivity employed may not be physically realistic.

For the non-conducting limit, the above analysis leads to the well known gravity wave equations described by Hines (1960).

The analysis is for an isothermal atmosphere, but may be applied to a series of isothermal slabs. Atmospheric gradients are assumed small, and the ray approximation for vertical wave propagation is used.

To consider gravity wave propagation through regions of wind shear, define the frequency of a moving fluid parcel, or the 'intrinsic frequency' as:

$$\Omega = \omega - \vec{k} \cdot \vec{U}$$

where

$\vec{k}$ is wave number

$\vec{U}$ is wind velocity

ω is wave frequency in a quiescent atmosphere

The above value of intrinsic frequency can be substituted into the equations describing gravity wave propagation in a quiescent atmosphere, and a solution obtained. Jones (1969) discusses the conditions under which the above procedure is valid. These conditions are generally well met, except when Ω approaches either zero or the Brunt-Vaisala frequency.

A mean temperature profile was fed into the computer program which then calculated a mean atmosphere based on the hydrostatic equation and the ideal gas law, and various characteristic quantities at 1 km vertical intervals. The atmosphere was assumed to be homogeneous horizontally.

Values of the circular frequency, ω, and the horizontal wavelength were selected. The range of frequencies considered was 10^{-3} sec^{-1} to 10^{-2} sec^{-1}. The horizontal wavelength, λ_x was varied between 25 km and 500 km.

Gossard (1962) observed gravity waves in the troposphere. He found the period to vary between 15 minutes and 2 hours. The horizontal wavelength for waves of 15 minute period was 19 km and for waves of 2 hours period it was 150 km. The periods investigated in the present study range from 19.5 minutes to 1.94 hours, thus being approximately the same as those observed by Gossard. However, the horizontal wavelength range was between 48.5 km, and 372.5 km i.e., about 2.5 times the wavelengths reported by Gossard. The difference is due to the longer vertical wavelengths which are indicated by the present study.

The initial amplitude of the elementary wave was adjusted to match the density deviation in the 20 km to 40 km altitude region.

A series of values of damping were selected. A very high value of damping was found to be necessary to give a wave growth comparable to what was observed experimentally.

Most calculations assume a quiescent atmosphere, but several wind distributions are also studied. A wind varying sinusoidally with altitude is found to give the most interesting results.

Figure 2 shows calculated gravity wave propagation in a quiescent atmosphere. The effective thermal diffusivity is assumed to be constant with altitude. Figure 3 shows a case where the thermal diffusivity increases linearly with altitude. Figure 3 also shows the pattern obtained when a plane gravity wave propagates into a sinusoidal wind field. The modulation is of the same type as observed in the experiments. Observed winter wind patterns at Fort Churchill indicate that the background wind chosen for the calculation is not unrealistic. (NASA TR R-316)

III. Effective Thermal Diffusivity

When energy transfer is due to purely molecular processes, then the thermal diffusivity, κ, is defined as:

$$\kappa = \frac{k}{\rho C_p}$$

k is thermal conductivity

ρ is density

C_p is heat capacity at constant pressure

The effective thermal diffusivity, K, is defined in the same manner, except that the molecular thermal conductivity is replaced by an "effective conductivity". Processes contributing to "effective conductivity" include conduction, convection and turbulent mixing, radiative transfer, and possibly, chemical effects.

It can be readily shown (Eberstein, 1969) that of the above processes, turbulence is the most important in the altitude range of the experiment.

A simple model of turbulent thermal diffusivity is presented below.

Define a turbulent thermal diffusivity K, such that

$$\frac{K}{\kappa} = \left(\frac{K}{\kappa}\right)_0 \left(\frac{Re - Re_{crit}}{Re_0}\right)$$

where κ is the molecular thermal diffusivity, and Re is Reynold's number. The Reynold's number is generally defined as:

$$Re = \frac{LV}{\nu}$$

where L is some suitable length dimension, V is a suitable velocity and ν is the kinematic viscosity. Following general astronomical practice, further justified by Blamont and DeJager (1961), L is taken as the atmospheric scale height. Blamont and DeJager used a turbulent velocity, estimated from their measurements, for V. In the present study it will be assumed that

$$\tilde{V} = \beta \bar{V}$$

where $\tilde{V}$ is the turbulent velocity, $\bar{V}$ is the mean wind velocity, and β is the ratio of r.m.s. turbulent fluctuation velocity to mean velocity of the flow, and is generally in the order of 10%.

Blamont and DeJager (1961) found that the atmosphere undergoes a turbulent to laminar transition at about 100 km altitude. At this point, the Reynold's number based on fluctuation velocity is 2,000.

In the present study a mean wind structure having the following characteristics is assumed:

$$V = Z^{1/2} \qquad 0 \leq Z \leq 5$$

$$V = Z \qquad Z \geq 5$$

where V is mean wind speed in meters per second, and Z is altitude in kilometers. The mean velocity is then used to calculate the Reynold's number, resulting in a transition Re of 20,000. This result agrees with Blamont, and DeJager's value if one substitutes $\tilde{V} = 0.1\,\bar{V}$.

Experimental data presented in the Meteorological Rocket Network Data Reports (1967), and the upper atmosphere wind measurements of Bedinger (1968) show that the assumed wind profile is reasonable. Winds have great temporal variability however, and, any stationary model must be considered with great caution. The wind variability, of course, suggests large deviations in the turbulence level, and consequently in damping factor.

The reference value K_0 was taken from early Eiffel tower measurements reported by Brunt (1952) for an altitude of some 150 meters. There, $K_0 = 10^5$ $cm^2\ sec^{-1}$. Since κ_0 is 0.2 $cm^2\ sec^{-1}$, the ratio, $(K/\kappa) = 5 \times 10^5$. More recently, Hosler (1969) determined vertical diffusivity from radon profiles. Average derived vertical diffusivities for a 90 meter height interval range from 3.1×10^5 $cm^2\ sec^{-1}$ during unstable convective periods to 8.3×10^3 cm^2 sec^{-1} during stable inversion periods. The above measurements were carried out at a tower site near Washington, D. C. The Eiffel tower measurements reported by Brunt gave 10^5 $cm^2\ sec^{-1}$ which is close to Hosler's value for unstable convective periods.

In general, the author's simple damping estimate represents a reasonable upper limit to steady state turbulent transport of thermal energy. Comparison of the author's estimate with that of other investigators in Figure 4 shows the present estimate to be somewhat high.

The curve of Johnson and Wilkins (1965, 1966) is based on estimated limits of steady downward heat transport in the atmosphere. Colegrove, Johnson and Hanson (1966) estimate an average eddy diffusion coefficient based on molecular and atomic oxygen transport. Some turbulent mass diffusivities are also included for comparison. The mass transfer data are from the work of Booker (1956) and Zimmerman and Champion (1963). The agreement is reasonable. Zimmerman and Champion's results also show approach to molecular behavior around 100 km altitude in agreement with the results of Blamont and DeJager.

An "experimental damping curve" is also shown. This curve was attained by calculating the turbulent thermal diffusivity required to give the gravity wave pattern shown in Figure 3. It is seen that the needed damping is considerably greater than any steady state estimates. However, the experimental damping estimates are below the diffusion coefficients estimated by Hodges (1969) for gravity waves with non-linear terms which generate internal instabilities that break down into turbulence. The above comparison suggests that gravity waves generate their own turbulence field. This conclusion is justified by observing that a gravity wave having a 30% density deviation generates a wind of the order of 100 m sec^{-1}.

Conclusions

Semi-quantitative agreement between an atmospheric wavelike structure and gravity wave theory has been demonstrated.

The most important damping mechanism is turbulence. However, naturally occurring turbulence does not appear adequate to provide the experimentally required damping. It is suggested that gravity wave activity creates its own, much stronger turbulence field. Thus is would be important to measure atmospheric turbulence both at times of low and high gravity wave activity. Unfortunately, Volland's equations are such that one would expect the effective conductivity to take on unrealistically high values. Thus the suggestion that gravity waves generate strong turbulence fields, while compatible with the results of the present study, remains to be rigorously confirmed.

Modulation of gravity waves by the wind field is important. Thus, it would be interesting to simultaneously measure the thermodynamic parameters and the background windfield.

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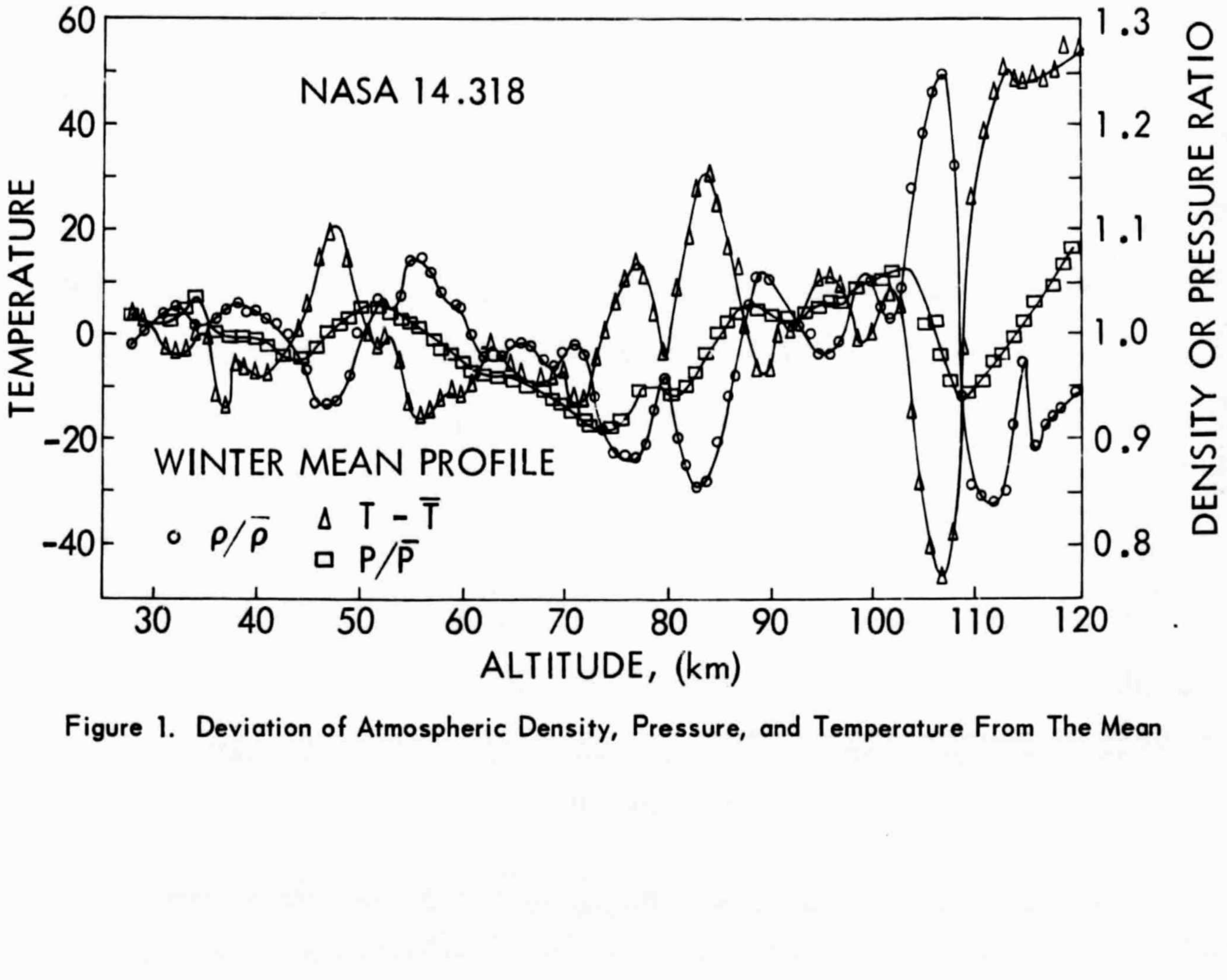

Figure 1. Deviation of Atmospheric Density, Pressure, and Temperature From The Mean

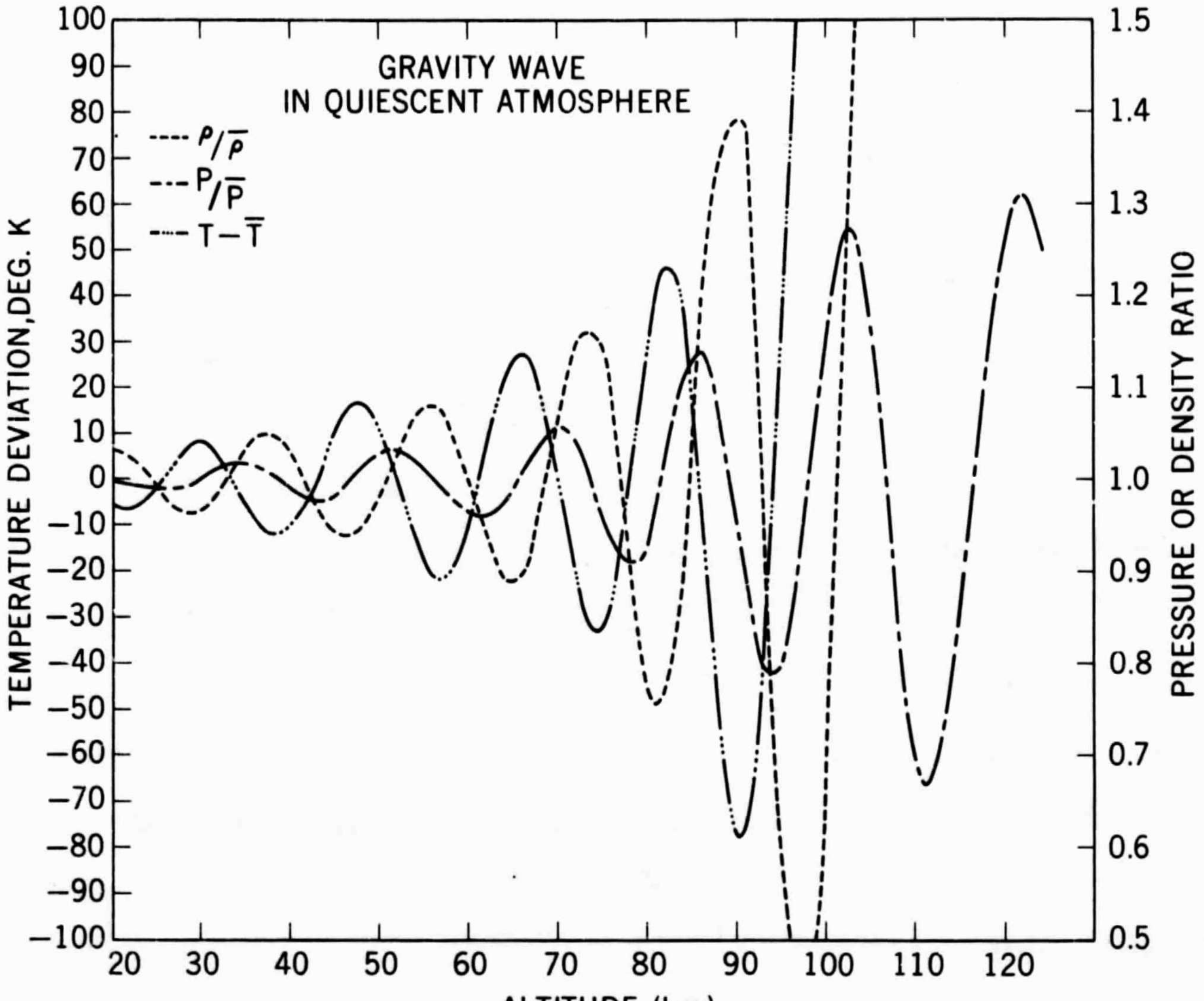

Figure 2. Calculated Gravity Wave Propagation In A Quiescent Atmosphere

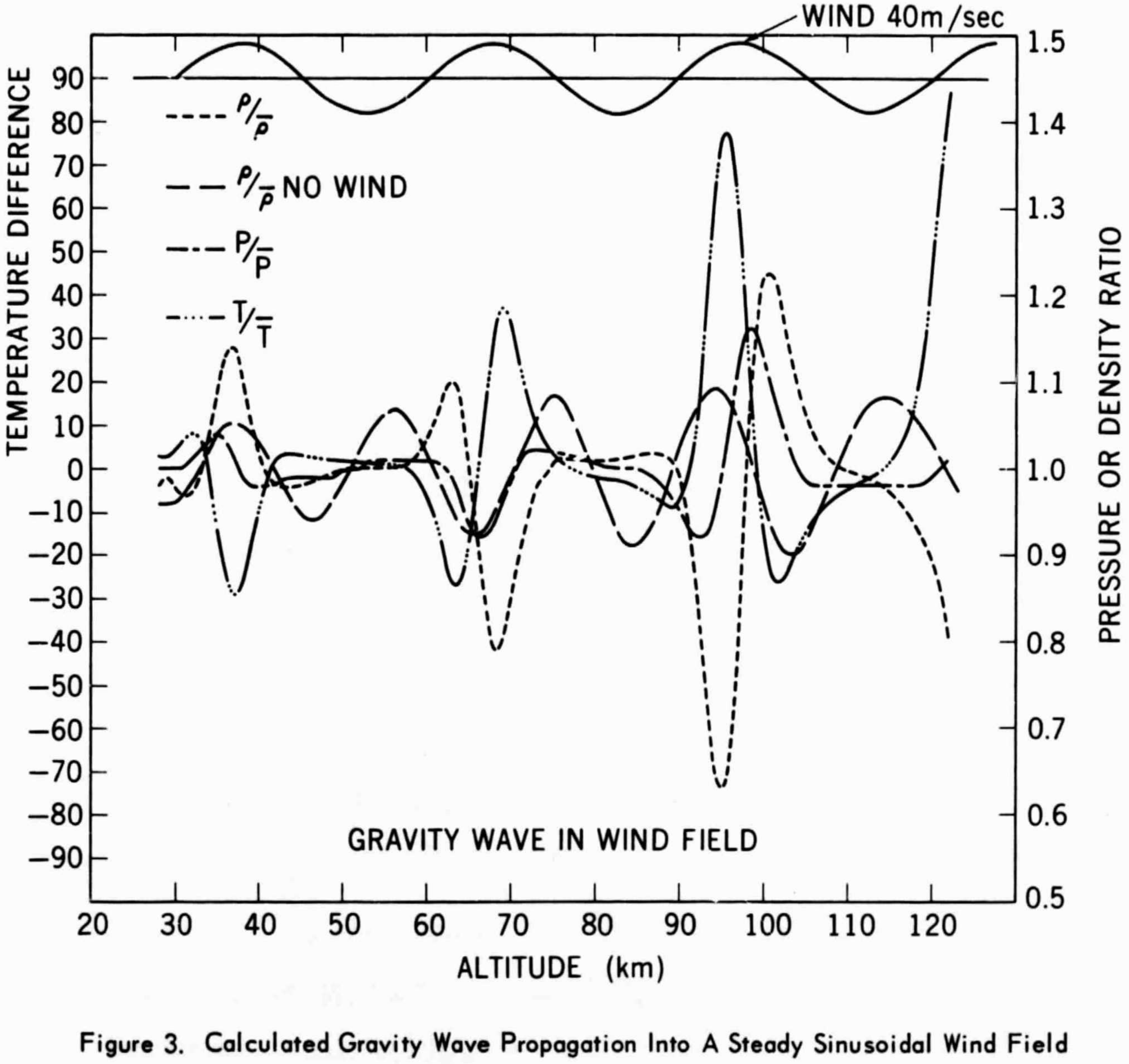

Figure 3. Calculated Gravity Wave Propagation Into A Steady Sinusoidal Wind Field

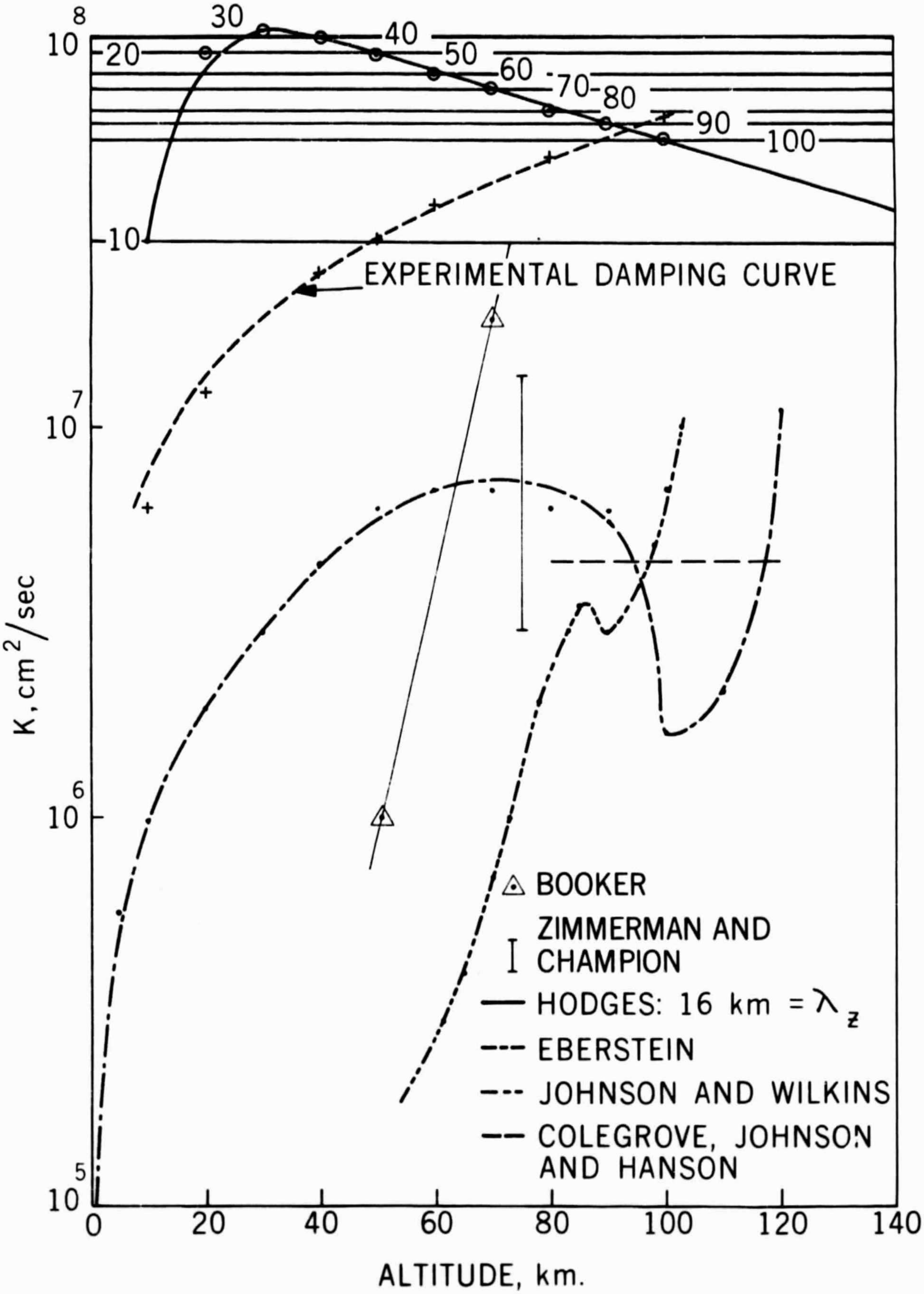

Figure 4. Estimates of Eddy Diffusivity