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THE PROPAGATION OF COHERENT RADIATION IN A CYLINDRICAL PLASMA COLUMN

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16. Abstract

Refraction plays a very significant role in the absorption of laser energy in high temperature plasmas. The geometry considered here is a long cylindrical magnetically confined plasma. The laser pulse is assumed to enter the plasma from the ends. It is then necessary that the radiation propagate the full length of the plasma so that it will be absorbed. The refraction of the laser radiation in the plasma column then depends on the radial distribution of the density. Several representative density profiles are considered and solutions for the propagation are presented. It is shown that a profile with the maximum density on the axis is very difficult to heat over long distances, but if the density has a minimum on the axis then the plasma column acts like a "light pipe" and traps the laser beam. In this case the effective absorption length of the laser beam is also reduced.

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I. INTRODUCTION

The problem to be discussed arises in the application of high powered lasers to the attainment of controlled thermonuclear reactions, CTR. In the theoretical development of this area Steinhauer and Ahlstrom[1] considered the problem of heating a one-dimensional non-uniform plasma with a laser. This study elucidated the parameters and the fundamental character of the heating phenomena in a stationary plasma. The term stationary plasma refers to the limit in which the duration of the laser pulse is much shorter than the acoustic time based on the length of the plasma heated and its final temperature.

Dawson[2], Kidder[3], and Hertzberg[4] have proposed the use of a long wavelength gas laser to heat large volumes of plasmas which are magnetically confined. Recent developments of high power pulsed CO_2 lasers[5] with $\lambda = 10.6\mu$ show that gas laser development is becoming competitive with the solid state lasers in the production of high energy pulses. The significant advantage in using the long wavelength as first pointed out by Dawson, is that the laser energy can be absorbed by plasmas at densities which can be confined in magnetic fields that are currently feasible. Vlases and Ahlstrom[6] have studied the thermodynamic, the absorption, and the refraction problems associated with using a long wavelength laser pulse to heat a long magnetically confined DT. A necessary condition for the application of the laser is the absorption of the radiation. The inverse bremsstrahlung mechanism has been extensively studied[7,8,9] and in addition nonlinear[10] and resonance phenomena[11,12] have been examined. Thus, the necessary condition of the absorption of the radiation has been studied essentially independent of the geometry of the plasma.

General Refraction Problem

When the geometry of a particular scheme is investigated it becomes obvious that refraction effects can play a dominant role. The density gradients in the plasma may refract the radiation out of the plasma before it has traveled far enough to be absorbed. The equation for the index of refraction in a gas composed of atoms, ions, and electrons is

$$\tilde{n} = 1 + (K_o + K_o'\lambda^{-2})n_o + (K_i + K_i'\lambda^{-2})n_i - 4.5 \times 10^{-22} n_e \lambda^2$$

where $n_{o,e,i} \sim cm^{-3}$ and $\lambda \sim$ microns. This equation applies when the frequency $\tilde{\omega}$ is much greater than the plasma frequency ω_p and the cyclotron frequency. For $\lambda > 0.5\mu$ and a typical gas the contribution to the atoms and the ions can be combined so that

$$\tilde{n} \approx 1 + 10^{-23}(n_o + n_i) - 4.5 \times 10^{-22} n_e \lambda^2$$

Thus it is clear that for $\lambda > 0.5\mu$ and ionization greater than 10% that $\tilde{n} - 1 < 0$ where as for an un-ionized gas $\tilde{n} - 1 > 0$. Therefore, in an un-ionized gas the refraction deflects the beam towards the higher density but in a fully ionized plasma the index of refraction decreases with increasing density so the refraction deflects the beam towards the lower density. Since all plasmas have $n_e \rightarrow 0$ at their boundaries, refraction could play a dominant role in the application of laser radiation to the production of CTR.

The simple equation relating the radius of curvature of a ray, R, to the index of refraction gradient is

$$\frac{1}{R} = \frac{\sin \alpha}{\tilde{n}} \left| \text{grad } \tilde{n} \right|$$

where α is the angle between the ray and the direction of grad $\tilde{n}$. It is particularly interesting to note that $R \rightarrow 0$ at the plasma critical density, i.e., where the laser frequency equals the plasma frequency. For most laser heated plasmas, the maximum density, ρ_m, will be greater or equal to the critical density, ρ_c.

In this paper only the problem of a cylindrical plasma with azimuthal symmetry irradiated in the axial direction is discussed. The results developed here indicate effects that would be important in other geometries as well, and the general trends can be carried over.

Unfavorable Density Profile

The most obvious geometry where $\rho(r)$ is parabolic with ρ_{max} at $r = 0$ is considered first. The refraction effects are detrimental because of the unfavorable density gradient. It is shown that heating this type of plasma is very difficult due to the refraction losses.

If the incident laser beam is focused then it is shown that there exists a solution where the laser beam can be made to propagate indefinitely even in an unfavorable density gradient. Realistically some deviation from the ideal case would occur in practice. When a realistic deviation is allowed then it is shown that the propagation distance is improved by a factor of approximately three over the unfocused case.

Favorable Density Profile

The other possibility which exists is that the laser beam is propagated in a region where the density gradient is favorable, i.e., leads to self focusing. It is shown that for a favorable density variation a parallel incident beam is trapped as in a light pipe. Solutions for the wave fronts in a parabolic density profile are presented. These solutions based on geometrical optics show that the wave fronts are a series of ellipsoids which collapse into a filament on the axis, then expand outward, turn around and collapse into a filament again, with the process being repeated over and over. During this process envelopes of the wave fronts, caustics, are formed during each collapse phase.

There are a number of ways in which such a density profile could be produced. Two possible approaches are considered in this paper. For example, if a dynamic pinch is used to create the initial plasma then the laser energy addition could be accomplished during the collapse phase of the pinch. A second approach would be to use a low energy laser pulse to modify an initial unfavorable density distribution so that there is a density minimum along the axis. A one dimensional perturbation solution is presented which shows how the density distribution of a plasma would be affected by a small energy addition over times longer than an acoustic time based on the diameter of the plasma. This solution shows that it may indeed be feasible to use a tailored laser pulse so that large amounts of energy could be added to the plasma.

Finally it is shown that in a light pipe configuration of the plasma the effective absorption length of the plasma can be reduced by as much as an order of magnitude or more.

II. DEFOCUSING IN A PARABOLIC DENSITY PROFILE

The geometry considered is a long cylindrical plasma column with azimuthal symmetry and a density maximum on the axis with a monotonic decrease of the density in the radial direction. A laser beam is incident on the plasma column in the axial direction where it is assumed that the diffraction effects can be neglected and the laser beam is in the TEM_{00} mode with the intensity distribution approximated by a parabola. The general configuration is shown in figure (1). It is accurate to use geometrical optics as long as the plasma density is less than the critical density and caustics are not generated. The equations for the intensity and the absorption must be modified near the critical density and caustics if the details are desired in these localized regions.

The Eikonal equation for a cylindrically symmetric geometry is

$$\frac{1}{c^2(r)} = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial r}\right)^2$$

where u corresponds to time and c is the phase velocity. This equation can be reduced to a differential equation for the rays

$$c\frac{d^2r}{dx^2} + \left[1 + \left(\frac{dr}{dx}\right)^2 \right] \frac{dc}{dr} = 0 \qquad (1)$$

where the two boundary conditions are that

$$r(0) = r_o$$
$$\frac{dr}{dx}(0) = 0 \tag{2}$$

The integration of equation (1) leads to the ray equation

$$x = \int_{r_o}^{r} \frac{c\,dr}{[\ c^2(r_o) - c^2\ ]^{1/2}} \tag{3}$$

The phase velocity is given by

$$c = c_o(1 - \zeta)^{-1/2}$$

where c_o is the speed of light in a vacuum and $\zeta = \frac{\omega_p^2}{\omega^2} = \frac{\rho}{\rho_c}$, the ratio of the density of the plasma to the critical density for the particular laser wave length. Thus equation (3) becomes

$$x = \sqrt{1 - \zeta_o} \int_{r_o}^{r} \frac{dr}{\sqrt{\zeta_o - \zeta}} \tag{4}$$

where $\zeta_o = \zeta(r_o)$ and $\zeta = \zeta(r)$.

Now in order to evaluate the integral in equation (4), a parabolic density variation is assumed.

$$\zeta = \zeta_m \ [\ 1 - \alpha(\tfrac{r}{a})^2\ ] \tag{5}$$

$\alpha = 1$ corresponds to $\zeta = 0$ at the edge of the plasma, $r = a$; and $\alpha < 1$ corresponds to $\zeta = (1-\alpha)\zeta_m$ at $r = a$. Thus, for $\alpha < 1$ the density profile is flatter as shown in figure (2).

The ray equation, (4), using (5) reduces to

$$\frac{r}{r_o} = \cosh \left\{ \frac{x}{a} \left[\frac{\alpha\zeta_m}{1 - \zeta_m\ (1 - \alpha\ r_o^2/a^2)} \right]^{1/2} \right\} \tag{6}$$

Equation (6) can then be used to determine the distance that the laser beam propagates along the column. A measure of this distance is the value of x for which one half the laser beam energy has left the plasma column, $x_{1/2}$. Figure (3) shows the variation of $x_{1/2}/a$ with α for three values of ζ_m . For each of the values of ζ_m a value of the absorption length for $\lambda = 10.6\ \mu$ and $T_e = 10$ KeV is also given. It is clear that the radius of the plasma would have to be very large for any significant portion of the energy to be absorbed at 10 KeV and that α should be very small.

It should be also be pointed out that the case $\alpha < 1$ could be considered as a full parabolic plasma profile, $\alpha = 1$, with a smaller diameter laser beam D_L . For example, if D_P is the plasma diameter then $D_L/D_P = {}^1/_3$ corresponds to $\alpha \doteq {}^1/_{10}$ and $D_L/D_P = {}^1/_{10}$ corresponds to $\alpha \doteq {}^1/_{100}$. As long as the characteristic thermal conduction time in the lateral direction is much less than the longitudinal acoustic time, the whole plasma column would be heated.

Finally, for this case it is straightforward to make a perturbation calculation of the percentage of the energy absorbed at any given temperature. Figure (4) shows that for a CO_2 laser and temperatures greater than 100 eV a significant fraction of the energy is not absorbed and at 10 KeV the percentage absorbed is negligible. These calculations were done for a $a = 1.0$ cm and $\alpha = 0.10$. For larger plasma diameters and smaller values of α the picture would of course be more favorable.

III. FOCUSED LASER BEAM

Now suppose for the parabolic density profile, the incident laser beam is focused by a lens such that in the absence of the plasma the focal point would be at $x = b$ as shown in figure (5). Before solving this case two limits are obvious. If $b/a \to \infty$ then this is the unfocused case discussed in the previous section. If $b/a \to 0$ then the radiation will traverse the plasma column and go out the other side. Solutions for finite values of b/a should give increased propagation distances; in fact it would seem reasonable to search for a solution such that the radiation is trapped.

The boundary conditions are now

$$r(0) = r_o$$

$$\frac{dr}{dx}(0) = -\frac{r_o}{b} \qquad (7)$$

The solution for this case has three forms:

$$\frac{x}{a} = \lambda \left\{ \sinh^{-1} \frac{r}{a|\beta|} - \sinh^{-1} \frac{r_o}{a|\beta|} \right\} \text{ for } \beta^2 > 0 \qquad (8)$$

$$\frac{x}{a} = \gamma \left\{ \ln \frac{r}{a} - \ln \frac{r_o}{a} \right\} \qquad \text{for } \beta^2 = 0 \qquad (9)$$

$$\frac{x}{a} = \gamma \left\{ \cosh^{-1} \frac{r}{a|\beta|} - \cosh^{-1} \frac{r_o}{a|\beta|} \right\} \text{ for } \beta^2 < 0 \qquad (10)$$

where

$$\beta^2 = \frac{1}{1 + \frac{b^2}{r_o^2}} \left\{ \frac{1}{\alpha} \left(\frac{1}{\zeta_m} - 1 \right) - \frac{b^2}{a^2} \right\}$$

and

$$\gamma^2 = \frac{1}{1 + \frac{r_o^2}{b^2}} \left\{ \frac{1}{\alpha} \left(\frac{1}{\zeta_m} - 1 \right) + \frac{r_o^2}{a^2} \right\}$$

For $\beta^2 > 0$ the ray paths are overfocused so that they cross over the axis and exit from the plasma on the other side. For $\beta^2 < 0$ the ray paths are underfocused. For $\beta^2 = 0$ all the ray paths approach the axis asymptotically. This condition then requires

$$\frac{b^2}{a^2} = \frac{1}{\alpha} \left(\frac{1}{\zeta_m} - 1 \right) \qquad (11)$$

It should be noted that in heating a plasma, this type of solution would would only be useful if the characteristic thermal conduction time in the lateral direction is much less than the longitudinal acoustic time so that the whole plasma is heated.

As a practical matter it would be impossible to satisfy equation (11) exactly. Thus it is of interest to examine the effect of an error in e.g. α . Equations (8) and (10) give

$$\frac{x_{1/2}}{a} \sim \ln \frac{\text{const.}}{|\alpha - \alpha_c|}$$

where α_c is the correct value for equation (11) to be satisfied. It is easily seen that only a small error in specifying α leads to values $x_{1/2}/a$ close to those obtained in the unfocused case. Figure (6) shows the variation of $x_{1/2}/a$ vs α for $\alpha - \alpha_c = 0.10\alpha_c$ and $0.01\alpha_c$.

IV. REFRACTION IN A FAVORABLE DENSITY PROFILE

It is clear that if the density profile has a minimum the rays will be refracted into this region. It may even be possible to trap the rays in a minimum density region. The configuration considered is the same as that of Section II except now it is assumed that a density minimum exists.

In this case it is both convenient and interesting to use the equations in their characteristic form so that

$$\frac{dr}{ds} = \pm \frac{2}{c_o} [\zeta(r_o) - \zeta(r)]^{1/2} \qquad (12)$$

and

$$\frac{dx}{ds} = \frac{2}{c_o} (1 - \zeta(r_o))^{1/2} \qquad (13)$$

Consider the density variation shown in figure (7). From equation (12) it is clear that any ray entering the column with $r_1 \leq r_o \leq r_2$ and $dr/dx\ (0) = 0$ will be trapped in this minimum region. Such a configuration could then be called a "light pipe" since the light is trapped in the plasma column.

In order to study the behavior of the radiation in a minimum region in more detail, it is instructive to assume a specific density profile and solve equations (12) and (13) for the ray paths and the wave fronts. The density variation chosen is

$$\zeta(r) = \zeta_m + (1 - \zeta_m) \left(\frac{r}{a} \right)^2 \qquad (14)$$

This is a parabolic curve with ζ_{min} at $r = 0$ and $\zeta = 1.0$ or $\rho = \rho_c$ at $r = a$. The radius of the laser beam is again assumed to be equal to a.

The equation for the rays is then

$$r = r_o \cos \left[\frac{x/a}{(1-r_o^2/a^2)^{1/2}} \right] \qquad (15)$$

and the equation for the wave fronts is

$$1 = \frac{(x/a)^2}{\sigma^2} + \frac{(r/a)^2}{(\cos\ \sigma)^2} \qquad (16)$$

where $\sigma = \dfrac{2\ s\ (1-\zeta_m)^{1/2}}{c_o a}$. Note that s ~ time. The general proof has already shown that all the rays would be trapped in this case which is confirmed by equation (15). For this parabolic density profile all the rays are cosine functions where the wave length of the oscillating rays goes to zero as $r_o \to a$. The wave fronts are ellipsoids where the initial manifold is the plane $x = 0$, $r \leq a$. The longitudinal axis of the elliptical wave front grows linearly with time and the length of the radial axis oscillates with time. Several characteristic wave fronts are shown in figure (8). The wave fronts alternately collapse and expand in the favorable density variation region. This behavior leads to a series of envelopes or caustics and filaments along the axis where according to geometrical optics infinite intensities would be produced. The geometry of the caustics is shown in figure (9).

V. CREATION OF THE FAVORABLE DENSITY PROFILE

There are many ways in which the favorable density profile could be generated. The collapse phase of a pinch type device and the use of selective volume heat addition are discussed. In reference 6 it has been suggested that the laser heating of a dense, relatively cold θ-pinch plasma is a very attractive possibility for the production of CTR. The θ-pinch produces a quasi-static plasma with an unfavorable density profile. However, if the laser were fired into the θ-pinch just before the implosion, the density profile would then be like the solid line in figure (10). This is a favorable profile, having a strong density minimum on the axis. Hence the laser beam would be trapped and remain in the plasma. However, such an approach would require a laser pulse shorter than the time for the imploding wave to collapse through a distance equal to the radius of the beam.

If the laser were fired into the θ-pinch just after the implosion, then the density profile would correspond to the reflected shock from the implosion. This density profile is shown as the dashed line in figure (10). This approach would require that the duration of the laser pulse be less than the propagation time of the reflected shock wave across the plasma.

Another approach which is very promising is to modify an existing unfavorable density profile by some mechanism of selective volume energy addition. The added energy would produce a pressure imbalance that could lead to redistribution of the plasma and a favorable density profile for trapping the laser beam. The obvious means of depositing the energy is the laser, but it has just been shown that the laser beam is quickly refracted out of the plasma column for any value of ζ of practical interest for fusion. However, it should be noted that a massive redistribution of the density can be achieved by doubling the plasma energy. If the initial temperature is 100 eV then this represents only 1% of the energy required to achieve 10 KeV. If the heating from 100 eV to 200 eV were done in a way that produces a favorable density profile, then the remainder of the heating (to 10 KeV) could be done very efficiently, since there would be no refraction losses. Inefficiencies in the heating to 200 eV could be tolerated since this represents only 1% of the total energy required.

Figure (3) shows that if the initial plasma column has $n_{e_{max}} = 5 \times 10^{18}/cm^3$ and $\alpha = 0.10$ then for a CO_2 laser, $\lambda = 10.6\mu$, one half the laser beam would have left the column in three plasma radii. However, if $\mu = 1.06$ corresponding to a Nd^+ laser then $\frac{x^1/_2}{a} \approx 100$.

Figure (11) shows that 80% of the energy from the Nd^+ laser pulse would be absorbed. Therefore the heating could be done with two laser pulses, a small energy short duration 1.06μ pulse to tailor the density profile and a large energy 10.6μ pulse to achieve the final temperature.

An estimate of the energy required for the density tailoring can be made by examining the density profile created by a cylindrical laser beam in a uniform plasma. The case where $\sigma\rho/\rho_o << 1$ was studied by Raîser.[13] Based on his work, it can be shown that for laser pulses longer than the time for an acoustic wave to traverse the beam, $t_p c_s/a >> 1$, the density change is given by

$$\frac{\delta\rho}{\delta_o} = 1.73 \times 10^5 \frac{J_o/A}{T_e^{5/2}} \frac{\zeta}{\sqrt{\zeta-1}} \left(- \frac{I(r)}{I_{max}}\right) \qquad (17)$$

where $I(r)$ is the beam intensity profile, and I_{max} is the maximum intensity. Here it has been assumed that the absorption length is long compared to the radius of the beam, and linearized fluid mechanics is used to evaluate the motion. The density profile that would be produced by a laser beam with a parabolic intensity profile is shown in figure (12). For practical values, e.g., J_o = 1 joule at 1.06μ, $A = 3\ mm^2$, $\zeta = {}^1/_2$ and $T_e = 10^2$ eV; then $\delta\rho/\rho_o = 0(1)$. This calculation is valuable only as an order of magnitude estimate but it does indicate that a sizable density "hollow" can be produced by a moderately small laser pulse.

It is interesting to note that once a favorable density profile has been produced the additional energy absorption due to an incident Gaussian laser beam will continue to push mass towards the outer edge of the beam. This problem is very similar to the thermal blooming problem[14,15] which is experienced in the propagation of a laser beam through the atmosphere. The fundamental and convenient difference is that in the case of a fully ionized plasma the laser beam is trapped because $\tilde{n} - 1 \sim -n_e$.

VI. ABSORPTION LENGTH WITH A FAVORABLE DENSITY PROFILE

It is of interest to compare the absorption length in a uniform plasma to the absorption length in the plasma with a favorable density profile. Naturally, the absorption length in the "light pipe" is not a clearly defined quantity. The rays of the laser beam that enter the central region where the density is lowest will experience weaker absorption than those rays which enter near the periphery - at a higher density. Furthermore, all rays which do not enter at the density minimum will oscillate back and

and forth across the plasma as they move down its length. Thus, at any point in the column one finds a strange conglomeration of rays, whose intensity has been damped by widely varying amounts, and whose position seems to have no relationship to the origin of the rays.

The appropriate way to measure an "effective" absorption length for such a plasma is to calculate how the overall laser power diminishes in moving down the plasma column. The approach is to find first how each bundle of rays has diminished in intensity. Consider an annular bundle of rays of equal intensity.

$$dW(x,r_o) = I(r_o) \cdot 2\pi r_o dr_o \exp\{ -\int_o^x K[r(x,r_o)]ds\}$$

Where $dW(x,r_o)$ is the power at x of the radiation that entered the plasma between the circles $r = r_o$ and $r = r_o + dr_o$. $I(r_o)$ is the intensity profile of the laser; $K(r)$ is the absorption coefficient, and ds is an incremental distance along the ray which began at r_o. The total power in the cross section at x is found by integrating dW over all r_o.

$$W(x) = 2\pi \int_o^a I(r_o) \cdot \exp [- \int_o^x Kds] \, r_o dr_o \tag{18}$$

The effective absorption length can be calculated by performing the integral (18) for a particular density profile. Weak oscillatory terms will arise in the calculation of W(x) but these should be neglected giving $\bar{W}(x)$. The effective absorption length is then given by

$$l_{eff} = -\left(\bar{W}(x)/\frac{d\bar{W}(x)}{dx}\right)_{x=0} \tag{19}$$

Calculation of $\ell_{eff}/c_o\tau$ was performed for the favorable parabolic density equation (14) using two different intensity profiles; $I(r_o) \sim 1 - (r/a)^2$ which approximates the TEM_{00} mode, and $I(r_o) \sim (r/a)^2[1-(r/a)^2]$ which approximates the TEM_{00+01} mode. c_o is the speed of light in a vacuum and τ is the electron-ion collision time. The results are presented in figure (13) where they are compared to the absorption length in a uniform plasma at a density equal to the minimum density of the favorable profile. It is seen that there is a significant enhancement of the absorption length. In fact there is an actual maximum absorption length which occurs approximately when the minimum density is half the critical density. There is at least an order of magnitude decrease in the absorption length for all densities less than about 0.4 of the critical density.

Thus, it is seen that the light pipe not only traps the laser radiation, but reduces the absorption lengths considerably from the often inconveniently long lengths which arise in a uniform density plasma.

VII. SUMMARY AND CONCLUSIONS

The problem of propagating a laser beam along a long cylindrically symmetrical column of fully ionized plasmas has been considered using geometrical optics. For the case where the density has a maximum on the axis and decreases monotonically in the radial direction the ray paths are refracted out of the plasma column. The solutions show that it is very difficult to heat a long plasma column with this unfavorable density profile. By focusing the incident laser beam it is theoretically possible to keep the light from being refracted out of the plasma column. However, this solution can be compared to finding a neutral stability point in the middle of an unstable region. So that any deviation leads to a very rapid refraction of the laser beam out of the plasma.

A "light pipe" effect is found if there is a density minimum. For the general case of a density minimum it is shown that a parallel incident beam is trapped by the plasma column. A special case where the density variation is parabolic is considered, and the solution shows that the wave fronts are a series of ellipsoids which first collapse into a filament on the axis, then expand out against the increasingly density, then reflect and again collapse into a filament on the axis. This process is repeated over and over again until the energy is either absorbed or is propagated out the end of the plasma column. During each collapse phase of the propagation, envelopes of the wave fronts are formed where the intensity becomes very large. The intensity is also very large in the filaments on the axis. The use of geometrical optics in these localized regions is not correct for calculating either the intensity or the absorption.

Finally it is shown that for the "light pipe" solutions there is a significant decrease in the effective absorption length due to the rays oscillating back and forth across the plasma. It is clear from these solutions that there are very significant advantages to the light pipe configuration for the production of a thermonuclear plasma.

These studies are very encouraging and dictate the desirability and the necessity of the additional studies which are proceeding.

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FOOTNOTES

1. L. C. Steinhauer and H. G. Ahlstrom, Phys. Fluids, to be published.
2. J. M. Dawson, private communication.
3. R. E. Kidder, private communication.
4. A. Hertzberg, private communication.
5. A. E. Hill, Applied Phys. Letters, 12, 9, pgs. 324-326, 1 May 1968.
6. G. C. Vlases and H. G. Ahlstrom, Amer. Phys. Soc. Bull., Vol. 14, No. 11, pg. 1023.
7. L. Spitzer, Jr., Physics of Fully Ionized Gases, (Interscience, New York, 1962), Chapter 5.
8. J. M. Dawson and C. R. Oberman, Phys. Fluids, 5, 517 (1962), 6, 394 (1963).
9. H. Hora, Institute für Plasmaphysik, Garching, Report 6/23 (1964), Report 6/27.
10. H. Hora, Phys. Fluids, 12, 182 (1969).
11. J. Dawson, P. Kaw, and B. Green, Phys. Fluids, 12, 875 (1969).
12. P. K. Kaw and J. M. Dawson, Phys. Fluids, 12, 2586 (1969).
13. Yu P. Raîser, Soviet Physics JETP, 25, 208 (1967).
14. S. A. Akhmanov, D. P. Krindach, A. V. Migulin, A. P. Sukhorukov, and R. V. Khokhlov, IEEE Journal Quant. El. 4, 568 (1968).
15. F. G. Gerbhardt and D. C. Smith, Appl. Phys. Letters, 14, 52 (1969).

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