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FINAL PROJECT REPORT
Part A

AUTOMATED ENVIRONMENTAL CONTROL OF AN ACOUSTIC TEST FACILITY

by

Dennis L. Luckinbill, Project Director
Charles Everett

Contract No. NAS 9-10385

NASA Manned Spacecraft Center
Houston, Texas

January 1971

Engineering Complex

TENNESSEE TECHNOLOGICAL UNIVERSITY
COOKEVILLE, TENNESSEE

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NOMENCLATURE

t	time
$W(t)$	weighting function of a linear system
s	complex constant, $s = \sigma + j\omega$
$Y(s)$	transfer function of a linear system
$Y(j\omega)$	frequency-response function of a linear system
$\phi_{xx}(\tau)$	autocorrelation function of a random process, $x(t)$
$G_{xx}(\omega)$	power spectral density function of a random process, $x(t)$
$\delta(\tau)$	Dirac delta-function or impulse function
T_c	time constant
$u(t)$	step function
$\overline{y(t)^2}$	mean-square value of $y(t)$
$F_n(s)$	transfer function of n^{th} wave shapers
$G_n(s)$	transfer function of n^{th} 1/3 octave filters
B	bandwidth of a filter
f_c	center frequency of a filter
L_1	linear time differential operator with respect to t_1
$D_1{}^n$	n^{th} order time differential operator with respect to t_1
H	linear differential operator which is the product of L_1 and its adjoint operator

Chapter I

INTRODUCTION

The acoustic environment generated and encountered by large rocket boosters during lift-off and aerodynamic flight is the object of considerable research today. In an acoustical test facility, random noise is generated to simulate such an environment. NASA--Manned Spacecraft Center in Houston, Texas--has built and operates the Spaceflight Acoustics Laboratory (SAL) for this purpose. At present, the SAL operates in an open loop with no means of automatically controlling the noise spectrum generated inside the acoustic test chamber. It is the purpose of this paper to develop mathematical models of the present open loop system and a closed loop system using a digital computer for control. A computer control algorithm is investigated for optimal or near optimal control of the noise spectrum inside the test chamber and a digital simulation of the closed loop system is programmed to evaluate the overall system, using the proposed computer control algorithm.

Two general areas of literature have been investigated. One area is controllers in general, and the other area is the consideration of random inputs to a system from the statistical point of view. Many papers have been published on optimum and adaptive-predictive controllers [1,2,3,4,5,6,10,11,12,13,14,15], as well as design of control systems with digital computer control [7,8,19,20]. Pastel [16] and Mori [17] have given papers analyzing the problem of random inputs to systems. In addition, books by Lee [18] and Laning and Battin [21] have been written about this problem. However, most books and papers treat the random input as noise or an unwanted disturbance added to another input such as a step, ramp, or sine wave input into the system. The output due to the random signal is unwanted and the problem is to filter the unwanted noise output from the desired output which is due to the actual signal driving the system. However, the problem considered in this paper is a random signal driving the system and creating an acoustical noise spectrum inside the test chamber. Methods to control and shape this spectrum in an optimum or near optimum manner are examined by use of a digital simulation.

I. DESCRIPTION OF THE PROBLEM

The Spaceflight Acoustics Laboratory's (SAL) acoustic control system is shown in Figure I.1. A random noise generator capable of producing either a "white" noise (equal energy per cycle) or "pink" noise (equal energy per octave) from 20.0 Hertz to 10,000.0 Hertz, produces an electrical random signal. This electrical signal is placed into twenty-four different wave shapers with transfer functions $F_n(s)$ where n varies from 1 to 24. Each wave shaper has a similar magnitude versus frequency plot, but each with a different center frequency. The center frequencies are given in Table 1.

The electrical output from each wave shaper then goes into a digital attenuator. Each attenuator can be varied from 0.0 decibels which corresponds to an output to input ratio of 1.0, to -60.0 decibels which corresponds to an output to input ratio of 0.001 in increments of 0.5 decibels. These attenuators are adjusted to shape the acoustic noise spectrum inside the test chamber. The dashed lines in Figure I.1

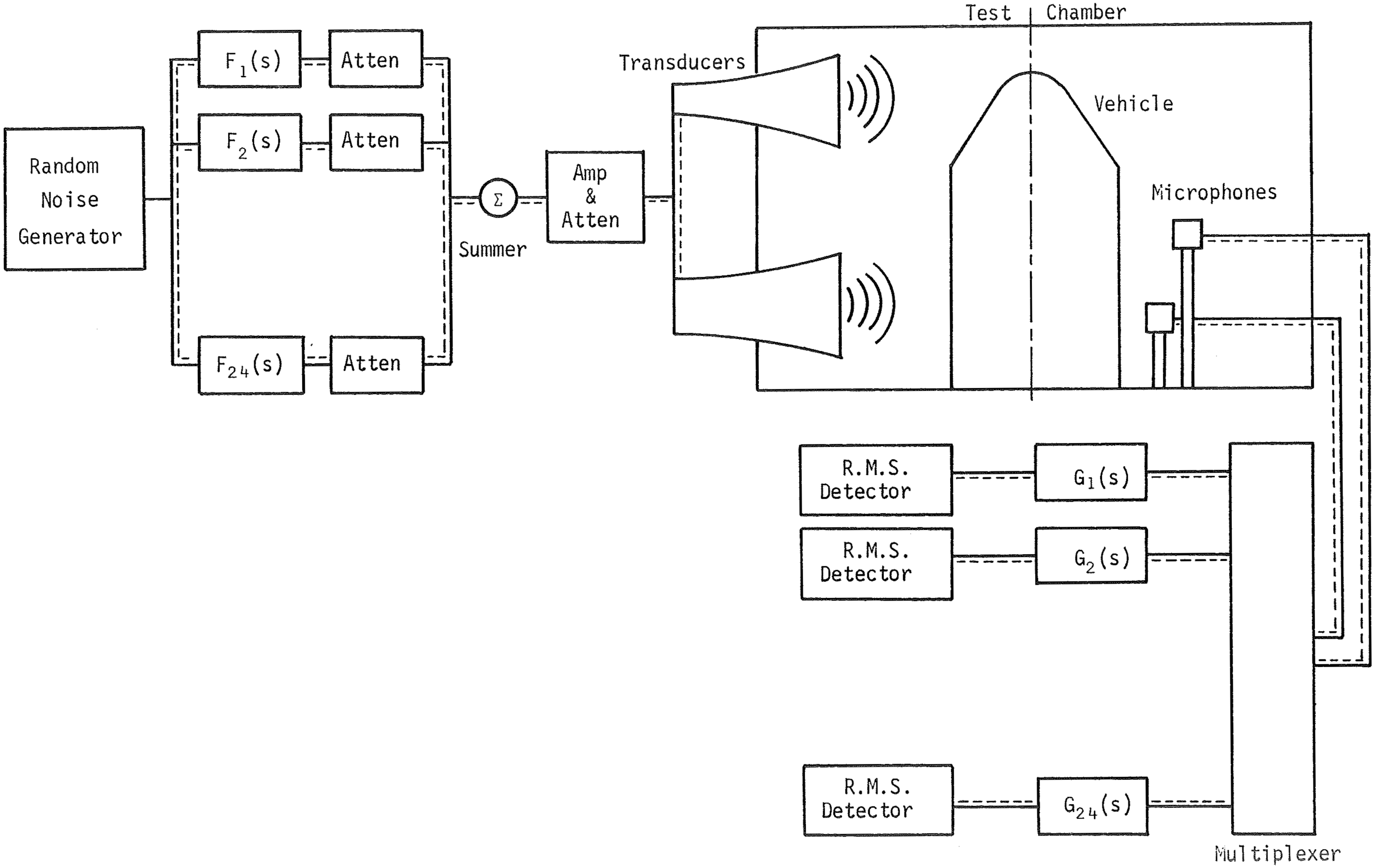


Figure I.1: Open Loop Acoustic Control System

Table 1

Center Frequencies for 1/3 Octave Filters

n	Center Frequency in Hertz
1.	25.0
2.	31.5
3.	40.0
4.	50.0
5.	63.0
6.	80.0
7.	100.0
8.	125.0
9.	160.0
10.	200.0
11.	250.0
12.	315.0
13.	400.0
14.	500.0
15.	630.0
16.	800.0
17.	1000.0
18.	1250.0
19.	1600.0
20.	2000.0
21.	2500.0
22.	3150.0
23.	4000.0
24.	5000.0

show the electrical paths of the various signals.

The output signal of the attenuators is electrically summed and then placed into an overall amplifier and attenuator. Also, incorporated with this overall attenuator is a signal clipper to reduce any electrical signal spikes present to a maximum value. This amplifier and attenuator fix the overall gain of the summed signals.

The output of the overall amplifier and attenuator drives an electro-pneumatic transducer which converts the random noise from an electrical signal to acoustical noise. These electro-pneumatic transducers, referred to as acoustical horns, are placed inside the test chamber with the vehicle to be tested. Microphones are placed around the vehicle to convert the acoustical random noise energy back into an electrical form.

The electrical output from these acoustic microphones is multiplexed into twenty-four 1/3 octave filters with center frequencies as shown in Table 1. These 1/3 octave filters with transfer functions $G_n(s)$ where n varies from 1 to 24, have magnitude versus frequency plots as shown in Figure I.2. These plots show that the filters are down -20.0 decibels at 1/3 octave on each side of the center frequency and down -3.0 decibels at each 1/6 octave on each side of the center frequency. The bandwidth of these filters is the frequency range with an attenuation of less than -3.0 decibel on each side of the center frequency.

In an open loop mode, the electrical output from the 1/3 octave filters is placed into analog r.m.s. (root-mean-square) detectors. Readings are taken from the analog r.m.s. detectors and the attenuators are adjusted manually until the desired acoustic noise spectrum is produced.

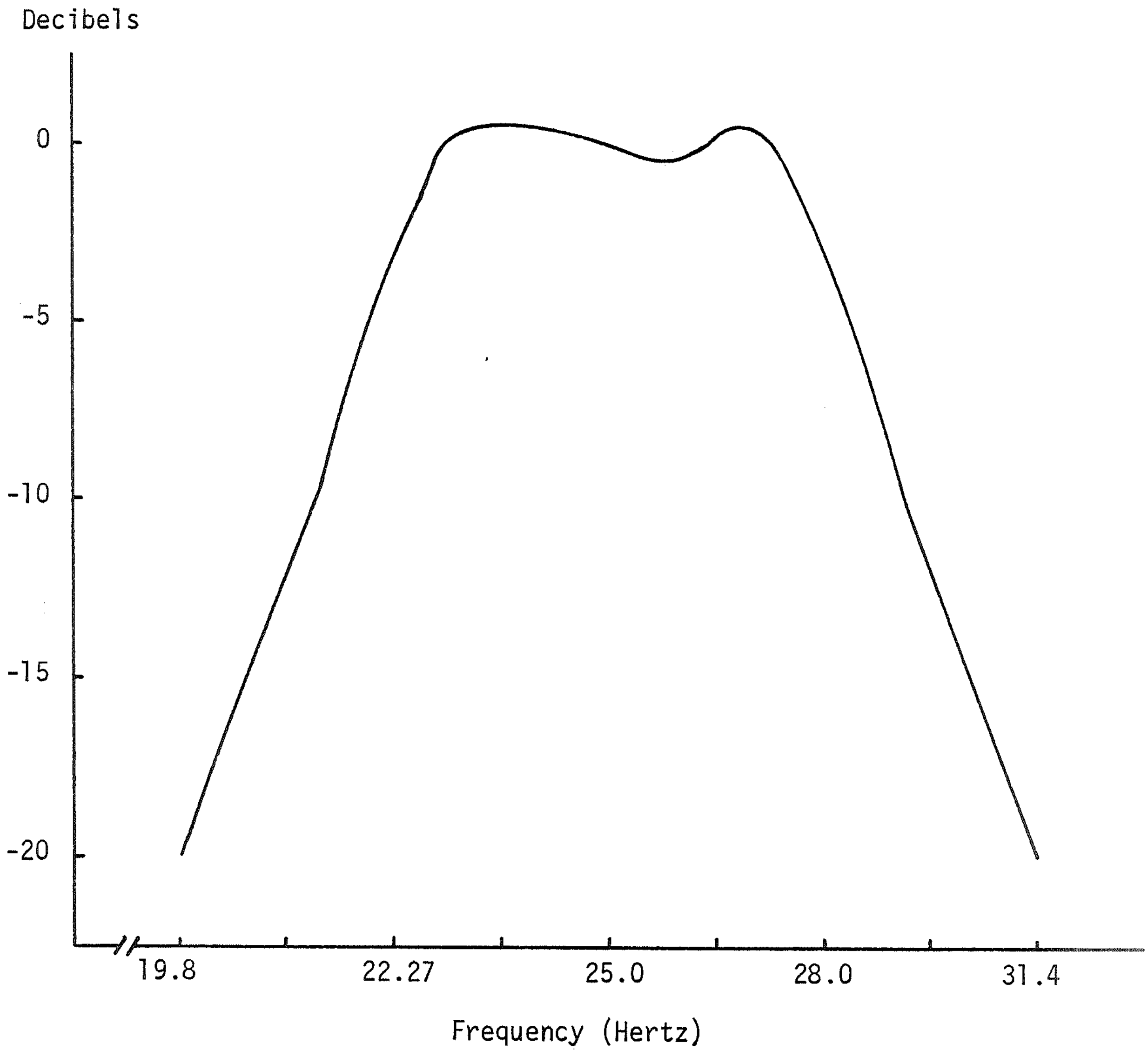


Figure I.2: Magnitude vs. Frequency Plot of
B & K 1/3 Octave Filter

It is not possible to set the random noise generator and digital attenuators to fixed levels and maintain the desired noise spectrum from test to test since the acoustic properties of the test chamber change from test to test. Therefore, it is desirable to use a digital feedback control system for the test facility such that the acoustic noise spectrum could be shaped in minimum time by automatically controlling the digital attenuators. This feedback control configuration is shown in Figure I.3. The r.m.s. values are then calculated by the digital computer and the digital attenuators are set by the computer, according to a computer control algorithm. The desired r.m.s. values for each 1/3 octave channel are denoted by R_n where n varies from 1 to 24.

The purpose of this paper is to examine a computer control algorithm to automatically determine the digital attenuator settings necessary for the correct shape of the acoustical noise spectrum. In addition, procedures for determining a r.m.s. value of the output of the 1/3 octave filter using the digital computer will be examined.

It will be assumed that the acoustical test chamber will have only a variable gain for each 1/3 octave frequency range involved, with a negligible time lag or time delay. This initial assumption would also consider the acoustical horns and microphones to have no distortion, which means they have a flat frequency response plot. Therefore, for this preliminary work, the acoustical test chamber may be disregarded and the test facility may be considered as shown in Figure I.4. This figure is essentially the same as Figure I.3 but with the acoustical test chamber replaced with a gain K.

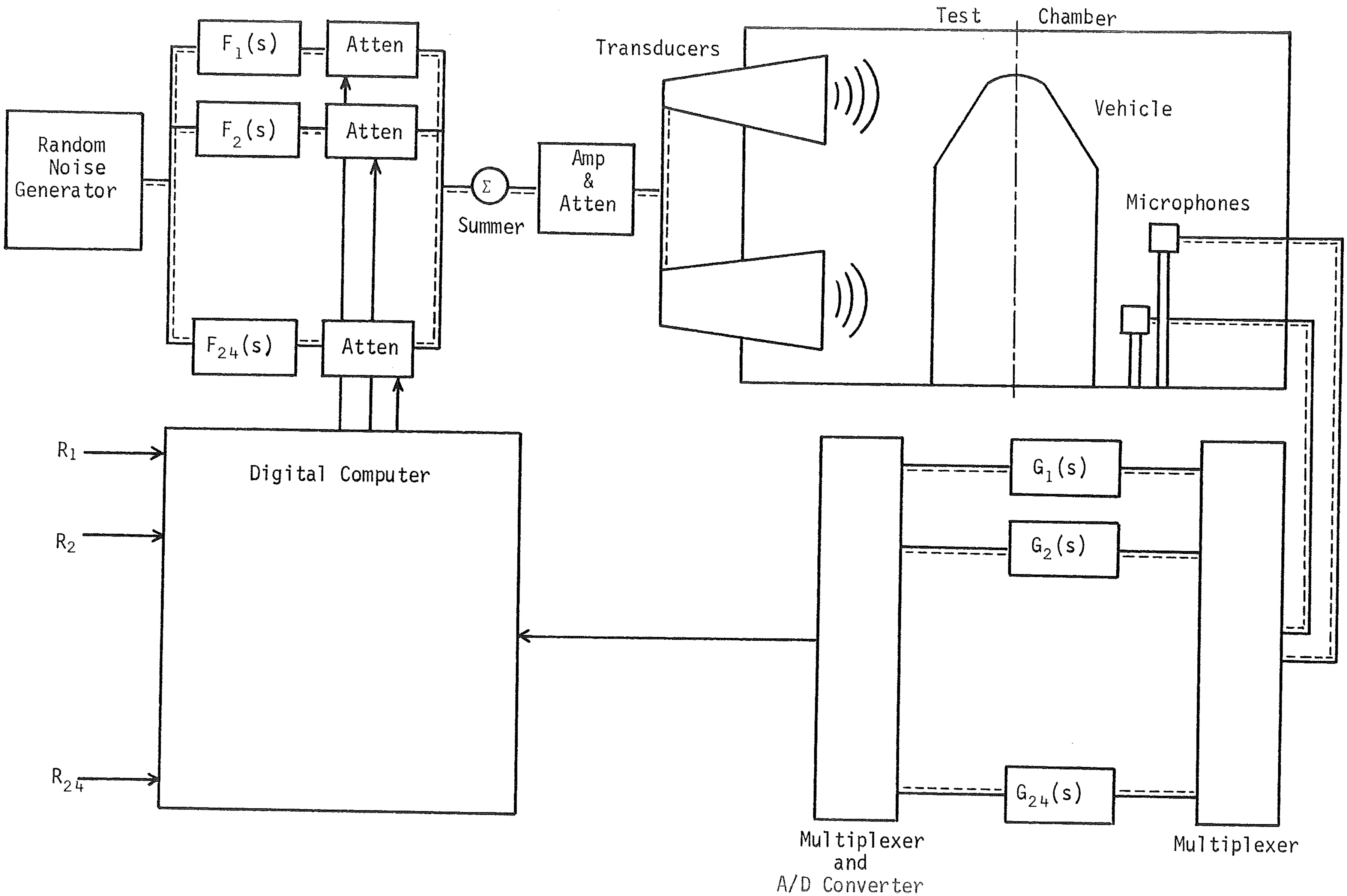


Figure I.3: Closed Loop Acoustic Control System

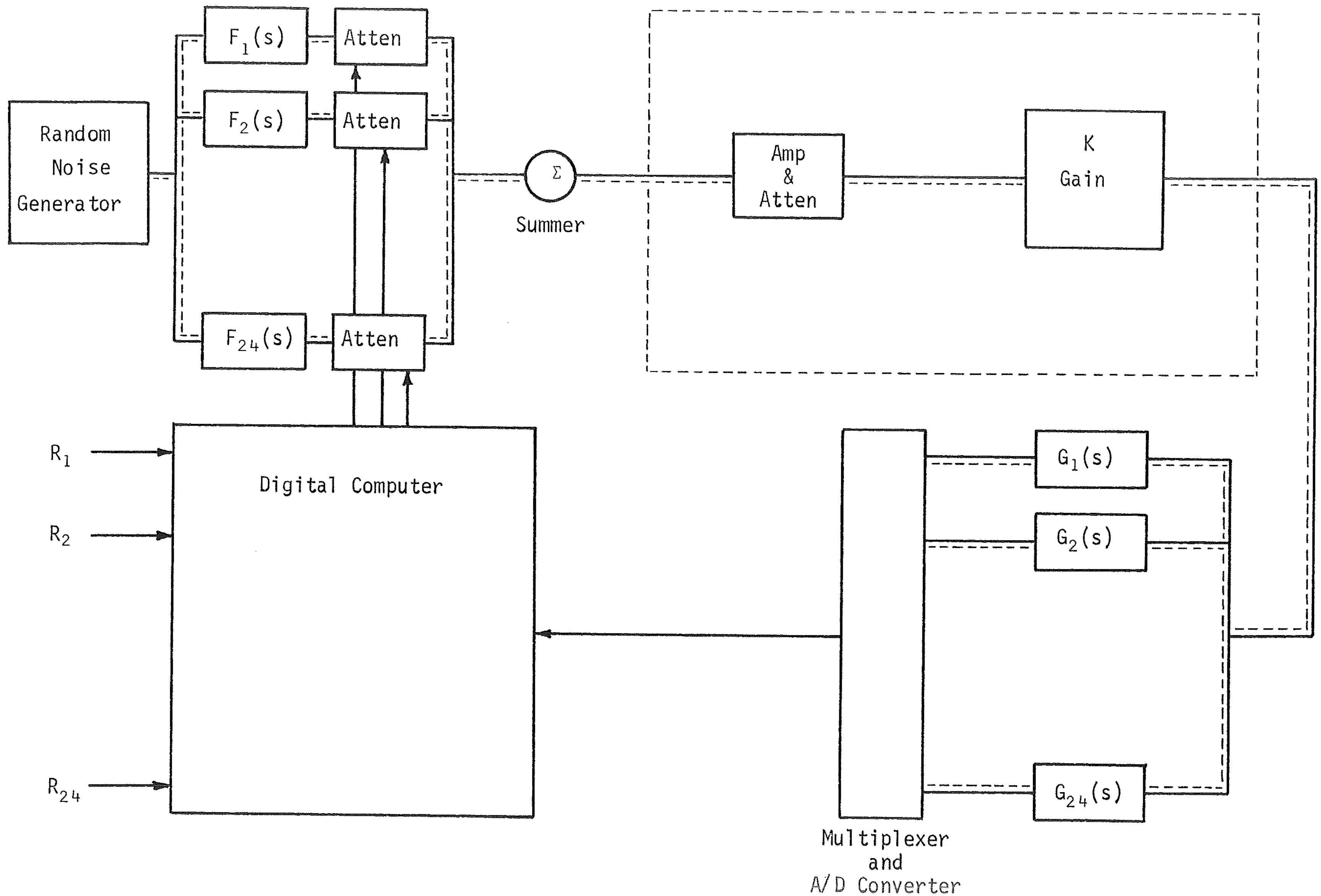


Figure I.4: Approximated System for Simulation

Chapter II

RANDOM PROCESSES

In this chapter it is shown that the output of the ideal mean-square detector and sixth-order linear differential equation is equivalent to the solution of a twelfth-order linear differential equation. The steady-state mean-square output of a first-order linear differential equation is developed using the power spectral density function of the input and the weighting function of the linear equation. Then the time history of the mean-square output is derived using the impulse response of the system and the autocorrelation function of the input. A shaping filter is added to the above system. This means that a white noise is put into a linear system, and the steady-state output is switched into the first-order linear system previously discussed. It is shown how this shaping filter alters the steady-state output of the system and the time response of the system. Finally, the linear differential equation for the mean-square output of a n^{th}-order differential equation is derived together with the necessary boundary conditions for any stationary input with an autocorrelation function $\phi_{xx}(\tau)$.

As can be seen from equations (A.12) and (A.13) in Appendix A, the mean-square value of the output of a linear system can be found if the autocorrelation function or power spectral density function of the random process input is known. It is only a matter of raising the mean-square value to the one-half power to find the r.m.s. (root-mean-square) value of the output.

A first-order and second-order linear system will now be considered with a white noise input and an expression is found for the transient response of the mean-square value of the output. By raising this expression to the one-half power, the transient response for the r.m.s. value of the output will be given.

Consider a first-order system as shown in Figure II.1

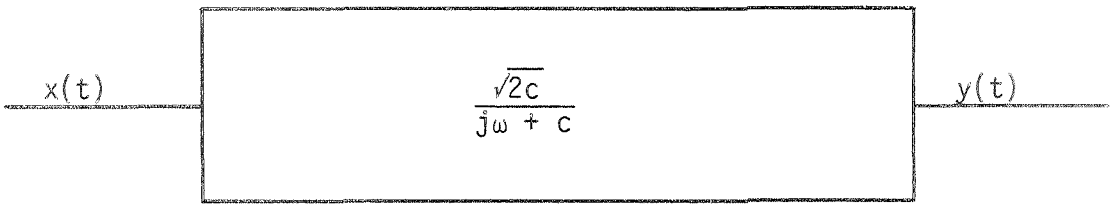


Figure II.1: First-Order System with a White Noise Input

The system is initially at rest and at time $t = 0$, the system receives a white noise input $x(t)$ with $G_{xx}(\omega) = G_0/\pi$ and $\phi_{xx}(\tau) = G_0\delta(\tau)$. From equation (A.13),

$$\overline{y(\infty)^2} = \int_0^\infty |Y(j\omega)|^2 \, G_{xx}(\omega)\, d\omega$$

$$= \int_0^\infty \left|\frac{\sqrt{2c}}{j\omega + c}\right| G_0/\pi \, d\omega = G_0 \qquad (2.1)$$

Equation (2.1) gives the steady-state value of the mean-square value of the output, $y(t)$.

By equation (A.12), the mean-square value of the output, $y(t)$, can be found as a function of time,

$$\overline{y(t)^2} = \int_0^t \int_0^t W(t-\tau_1)W(t-\tau_2)\phi_{xx}(\tau_1-\tau_2)\,d\tau_1\,d\tau_2$$

The weighting function, $W(t)$, of $Y(j\omega) = \sqrt{2c}/(j\omega+c)$ is

$$W(t) = \sqrt{2c}\, e^{-ct}$$

and

$$\phi_{xx}(\tau_1-\tau_2) = G_0\ \delta(\tau_1-\tau_2)$$

The mean-square value of the output is

$$\overline{y(t)^2} = \int_0^t \int_0^t W(t-\tau_1)W(t-\tau_2)G_0\ \delta(\tau_1-\tau_2)d\tau_1 d\tau_2$$

$$= G_0 \int_0^t W(t-\tau)^2\ d\tau = G_0[1 - e^{-2ct}] \tag{2.2}$$

For the first-order system given in Figure II.1 and the time response of the mean-square value of $y(t)$ given in equation (2.2), a model could be constructed such that a step input is applied and the time response of $z(t) = \overline{y^2(t)}$ is observed. By choosing the correct first-order system for the model, the transient response of $z(t)$ of the model with a step input will have the same time constant as the transient response of the mean-square value of $y(t)$ in Figure II.1 with a white noise input. Such a first-order model is given in Figure II.2.

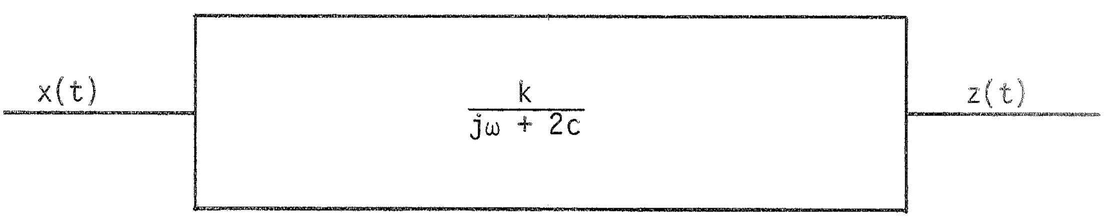


Figure II.2: Model of First-Order System with a White Noise Input

Let the step input in Figure II.2 be $x(t) = 2cu(t)$ so that the time response of $z(t)$ will be

$$z(t) = k(1 - e^{-2ct}) \tag{2.3}$$

The transient response of z(t) given in equation (2.3) with a step input will have the same time constant as the transient response of the mean-square value of y(t) as given in equation (2.2) with a white noise input. Be raising both sides of equation (2.2) to the one-half power, the transient response of the r.m.s. value of y(t) can also be obtained with a white noise input. Again by raising both sides of equation (2.3) to the one-half power, the transient response of the r.m.s. value of the output of a first-order system with a white noise input can be modeled.

If the power spectral density of the white noise input to Figure II.1 is changed by passing the white noise through an attenuator before it is put into the system, the final value of the mean-square output is proportioned by a new gain factor, k'. In contrast, it may be noted that by changing the amplitude of the step input into the system in Figure II.2 used to model the mean-square output of the system in Figure II.1 with a white noise input, a change in the final value of z(t) is also observed. Thus a change in the amplitude of the white noise input to the system in Figure II.1 with the mean-square output response shown in equation (2.2) can be modeled by changing the amplitude of the step input to the system in Figure II.2 and using its response z(t) as the model of the mean-square response given in equation (2.2).

However, for the more general case of nonzero boundary conditions, as indicated later in this chapter, the more general model would be a second order differential equation with an impulse function, $\delta(t)$ as the input.

Consider the following system as shown in Figure II.3.

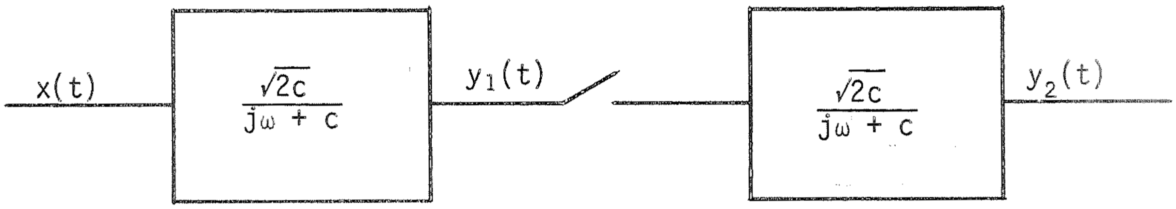


Figure II.3: Second-Order System with a White Noise Input

The input x(t) is again a white noise with $G_{xx}(\omega) = G_0/\pi$ and $\phi_{xx}(\tau) = G_0\delta(\tau)$. The switch is open and the mean-square output of the $y_1(t)$ has reached its steady-state value. At time $t = 0$, the switch is closed and the mean-square value of $y_2(t)$ can be found. From equation (A.13)

$$\overline{y_2(\infty)^2} = \int_0^\infty G_{yy}(\omega)\,d\omega$$

$$= \int_0^\infty \left|\frac{2c}{(j\omega+c)^2}\right|^2 G_0/\pi \; d\omega = G_0/c \qquad (2.4)$$

From equation (A.12) the mean-square value of the output, $y_2(t)$, can be found as a function of time,

$$\overline{y_2(t)^2} = \int_0^t \int_0^t W(t-\tau_1)W(t-\tau_2)\phi_{xx}(\tau_2-\tau_1)\,d\tau_1 d\tau_2$$

Laning and Battin [21] have shown the above equation can be written as

$$\overline{y_2(t)^2} = F(t)^2 + 2c \int_0^t F(\tau)\,d\tau$$

since

$$F(\tau) = \sqrt{2c}\;\tau\; e^{-ct}$$

and

$$F(t)^2 = 2ct^2e^{-2ct}$$

$$\overline{y_2(t)^2} = F(t)^2 + 2c\int_0^t F(\tau)\ d\tau$$

$$= G_0[2et^2e^{-2ct} + 2c\int_0^t 2c\tau^2e^{-2ct}\ d\tau]$$

$$= G_0/c - G_0e^{-2ct}/c - G_02t\ e^{-2ct} \qquad (2.5)$$

It can be observed that this is also the response of a second order linear differential equation to a step input.

It will be shown that for a system that can be modeled with a linear differential equation, the mean-square value of the output also obeys a linear differential equation. Consider the linear system in Figure II.4

$$x(t) \longrightarrow \boxed{\frac{1}{a_nD^n + a_{n-1}D^{n-1} + \ .\ .\ . + a_1D + a_0}} \longrightarrow y(t)$$

Figure II.4: Nth-Order System with a Random Process Input

where

$\phi_{xx}(t_1,t_2)$ = autocorrelation function of the input, $x(t)$

$\phi_{yy}(t_1,\ t_2)$ = autocorrelation function of the output, $y(t)$

$\phi_{xy}(t_1,t_2)$ = cross-correlation function of the input-output

$$D^n = \frac{d^n}{dt^n}$$

Papoulis [24] has shown that $\phi_{xy}(t_1,t_2)$ satisfies the differential equation (in t_2),

$$a_n D_2^n \phi_{xy}(t_1,t_2) + a_{n-1} D_2^{n-1} \phi_{xy}(t_1,t_2) + \ldots + a_0 \phi_{xy}(t_1,t_2)$$
$$= \phi_{xx}(t_1,t_2) \tag{2.6}$$

where

$$D_2^n = d^n/(dt_2)^n$$

and

$$\phi_{xy}(t_1,0) = D_2\phi_{xy}(t_1,0) = \ldots = D_2^{n-1}\phi_{xy}(t_1,0) = 0$$

$\phi_{yy}(t_1,t_2)$ satisfies the same type of differential equation with t_2 considered a constant

$$a_n D_1^n \phi_{yy}(t_1,t_2) + a_{n-1} D_1^{n-1} \phi_{yy}(t_1,t_2) + \ldots + a_0 \phi_{yy}(t_1,t_2)$$
$$= \phi_{xy}(t_1,t_2) \tag{2.7}$$

where

$$\phi_{yy}(0,t_2) = D_1 \phi_{yy}(0,t_2) = \ldots = D_1^{n-1} \phi_{yy}(0,t_2) = 0$$

by letting $t = t_1-t_2$, $dt = -dt_2$ if t_1 is held constant and $dt = dt_1$ if t_2 is held constant.

Now define the operators

$$L_{t_1} = \sum_{i=1}^{n} A_i \frac{d^i}{dt^i} = \sum_{i=1}^{n} A_i D^i \tag{2.8}$$

and

$$L_{t_2} = \sum_{j=1}^{n} A_j (-1)^j \frac{d^j}{dt^j} = \sum_{j=1}^{n} A_j (-1)^j \; D^j \tag{2.9}$$

Eliminating $\phi_{xy}(t_1,t_2)$ from equations (2.6) and (2.7) and using the operators defined in equations (2.8) and (2.9),

$$\sum_{i=1}^{n} D^i \sum_{j=1}^{n} (-1)^j \; D^j \; [\phi_{yy}(t_1,t_2)] = \phi_{xx}(t_1,t_2) \tag{2.10}$$

If $x(t)$ is stationary then $\phi_{xx}(t_1,t_2) = \phi_{xx}(t_1-t_2)$. To find the mean-square value of the output $y(t)$, let $t_1=t_2$ in the autocorrelation function of the output. It can be shown that the product of two series,

$$\sum_{i=0}^{n} a_i \; D^i \; \sum_{j=0}^{n} b_j \; D^j = \sum_{k=0}^{2n} \sum_{j=0}^{k} (a_{k-j} b_j) \; D^k \tag{2.11}$$

From equations (2.8) and (2.9), $b_j = (-1)^j a_j$. Substituting this into equation (2.11)

$$\sum_{k=0}^{2n} \sum_{j=0}^{k} (a_{k-j} b_j) \; D^k = \sum_{k=0}^{2n} \sum_{j=0}^{k} (-1)^j \; A_j \; A_{k-j} \; D^k$$

$$= \sum_{k=0}^{2n} D^k \sum_{j=0}^{k} (-1) \; A_j \; A_{k-j} \tag{2.12}$$

If k is odd then,

$$\sum_{j=0}^{k} (-1)^j \; A_j \; A_{k-j} = 0 \tag{2.13}$$

Substituting equation (2.13) into equation (2.12) and letting k' = k/2

$$\sum_{k=0}^{2n} D^k \sum_{j=0}^{k} (-1)\ b_j\ b_{k-j} = \sum_{k'=0}^{n} D^{2k'} \sum_{j=0}^{2k'} (-1)^j\ b_j\ b_{k'-j}$$

let k = k' and define

$$C_k = \sum_{j=0}^{2k} (-1)^j\ b_j\ b_{k-j}$$

Define the new operator H which is the product of the operator L and its adjoint operator

$$H = \sum_{k=0}^{n} C_k\ D^{2k}$$

Equations (2.6) and (2.7) reduce to

$$H\ [\phi_{yy}(t)] = \phi_{xx}(t) \qquad (2.14)$$

Equation (2.14) shows that the mean-square value of the output of a system which can be represented by a linear differential equation, as shown in Figure II.4, can be related to the autocorrelation function of the input by a linear differential operator. It may also be noted that the new operator H is the product of the operator L_t and its adjoint operator.

One difficulty with equation (2.14) is the determination of enough boundary conditions to make the differential equations well posed. If the pair of equations (2.6) and (2.7) is written in terms of the variable t

$$\sum_{i=0}^{n} A_i D^i\ \phi_{xy}(t) = \phi_{xx}(t)$$

$$\sum_{i=0}^{n} (-1)^i\ A_i D^i\ \phi_{yy}(t) = \phi_{xy}(t) \qquad (2.15)$$

with boundary conditions

$$D^i \phi_{xy}(-t_1) = 0 \qquad i=0,1, \ldots ,n-1$$

$$D^i \phi_{yy}(t_2) = 0 \qquad i=0,1, \ldots ,n-1$$

then the problem is well posed.

If $\phi_{xx}(t)$ is the output of a wave shaper, the steady state value of $\phi_{xx}(t)$ is used as the input to equations (2.14) or (2.15).

The results of this chapter are helpful in at least two ways. First they provide an analytical base for understanding the time response of the simulation during the first one-second period. During this period, the response of the simulated mean-square detector is identical to the mean-square output discussed in this chapter. Secondly, they justify the use of a linear model to replace the filter and mean-square detector for further analytical study and thus a linear model for use in identification, the system to be identified would be a linear system such as equation (2.14) with a steady-state autocorrelation function input which would be a step rather than an impulse function. The boundary conditions could be obtained from the simulations presented here.

Chapter III

DIGITAL SIMULATION OF SINGLE CHANNEL 25.0 HERTZ 1/3 OCTAVE SYSTEM

A single channel system model will be constructed using only the 25.0 Hertz wave shaper and filter as shown in Figures III.1 and III.2. In this configuration there is no input of narrowband random noise from the other wave shapers and, therefore, no interplay from other narrowband random noise of different center frequencies.

The Spaceflight Acoustics Laboratory (SAL) obtained digitized random data for use in the digital simulation in the following manner. The random noise generator's electrical output was fed into the 25.0 Hertz wave shaper, and the output of the wave shaper was digitally recorded on magnetic tape. The 25.0 Hertz wave shaper has a narrow bandwidth, which means that B/f_c is a small number where B is the bandwidth measured at the -3.0 decibel points relative to the magnitude at the center frequency, f_c. The output of the wave shaper is narrowband random noise which looks very much like a sinusoidal wave of frequency f_c, but with slow and random amplitude and phase variations. For modeling and simulation purposes, the random noise generator and wave shaper, $F_1(s)$, are substituted with the data output provided by SAL.

The 25.0 Hertz 1/3 octave filter was modeled from a magnitude versus frequency plot of a B & K 1/3 octave filter provided by the Spaceflight Acoustics Laboratory as shown in Figure I.2. The work of modeling the 1/3 octave filter was performed by Atkins [23] and the

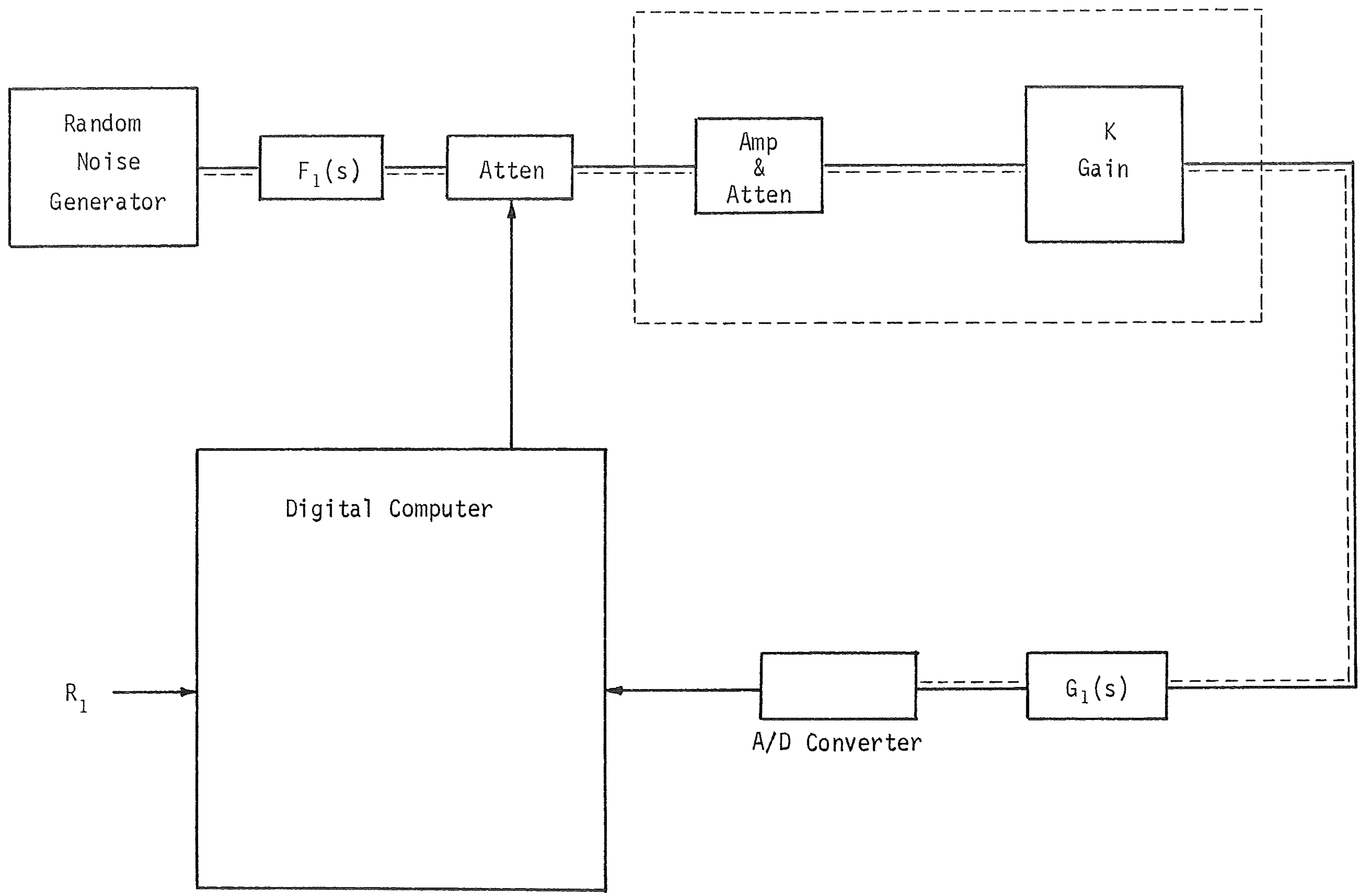
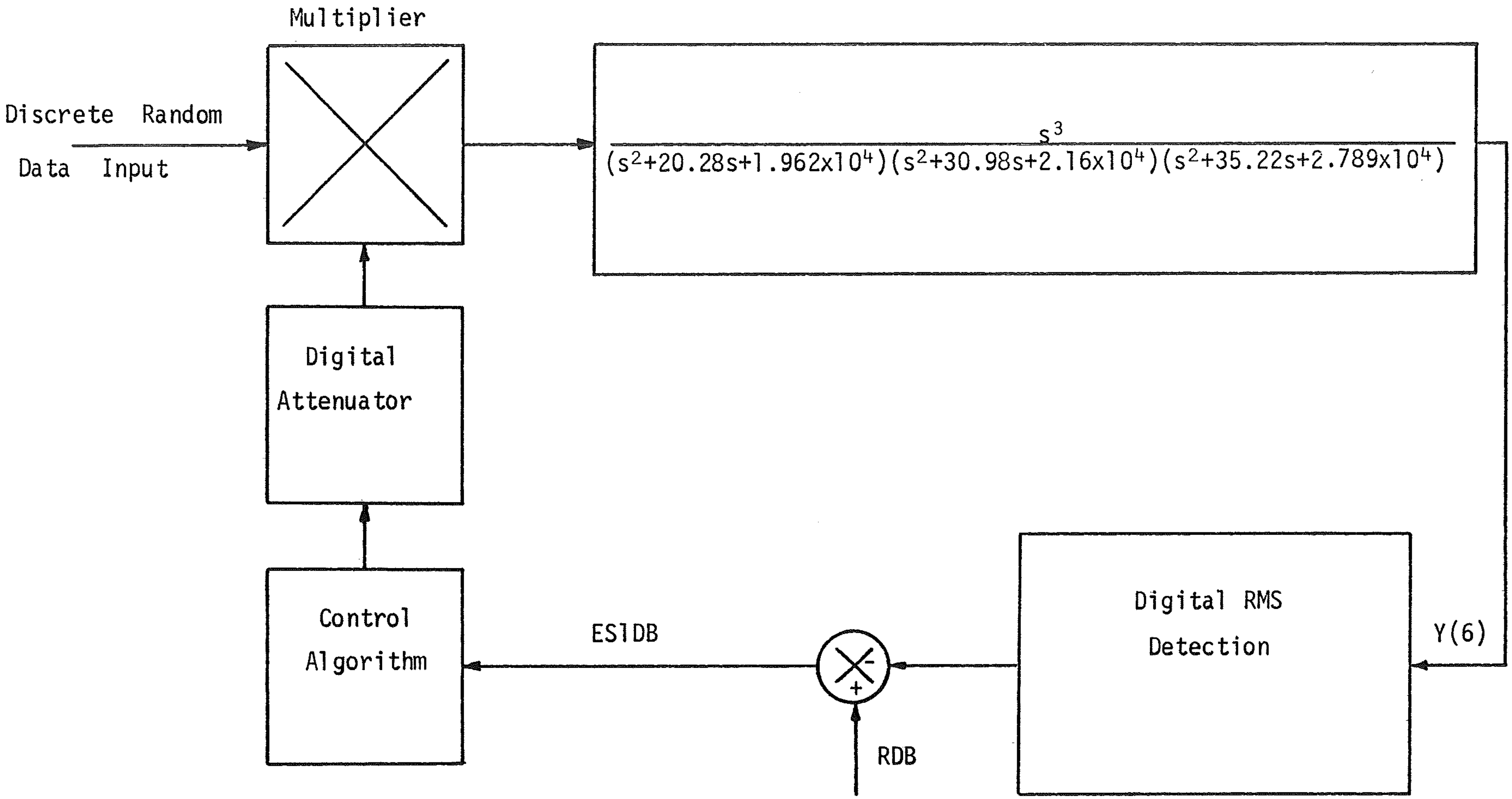


Figure III.1: Approximated Single 1/3 Octave Channel



RDB = 140 Decibels

Fiugre III.2: Block Diagram of Digital Simulation of Approximated System transfer function found for the 25.0 Hertz center frequency 1/3 octave filter was

$$G_1(s) = \frac{s^3}{(s^2+20.28s+1.962\times10^4)\ (s^2+30.98s+2.16\times10^4)\ (s^2+35.22s+2.789\times10^4)}$$

The magnitude versus frequency plot of $G_1(s)$ is given in Figure III.3.

The -20.0 decibels points on the magnitude versus frequency plots of the 1/3 octave filters with center frequencies given in Table 1 occur at the next center frequency above and below the center frequency whose magnitude plot is being discussed. This is shown for three magnitude versus frequency plots in Figure III.4.

A mathematical definition of root-mean-square which is used is:

$$R(t) = \left[\frac{1}{T} \int_{t-T}^{t} y(\zeta)^2 \, d\zeta \right]^{1/2}$$

where $R(t)$ is the r.m.s. value of $y(\zeta)$ over the time interval T. For the purpose at hand, it is desired that the r.m.s. value be based on only a limited number of past values of $y(\zeta)$ to give a best estimate of the present r.m.s. value of $y(\zeta)$. The definition of the r.m.s. value of $y(\zeta)$ to be used is

$$R(t) = \left[\frac{1}{t-(t-T)u(t-T)} \int_{(t-T)u(t-T)}^{t} y(\zeta)^2 \, d\zeta \right]^{1/2}$$

where $u(t)$ is the unit step function.

The discrete analog of the above definition is

$$R(t_n) = \left[\frac{1}{n-(n-N)u(n-N)} \sum_{i=(n-N)u(n-N)}^{n} y(t_i)^2 \right]^{1/2} \qquad (3.1)$$

where $R(t_n)$ is the discrete r.m.s. value of $y(t_i)$ and N is the maximum

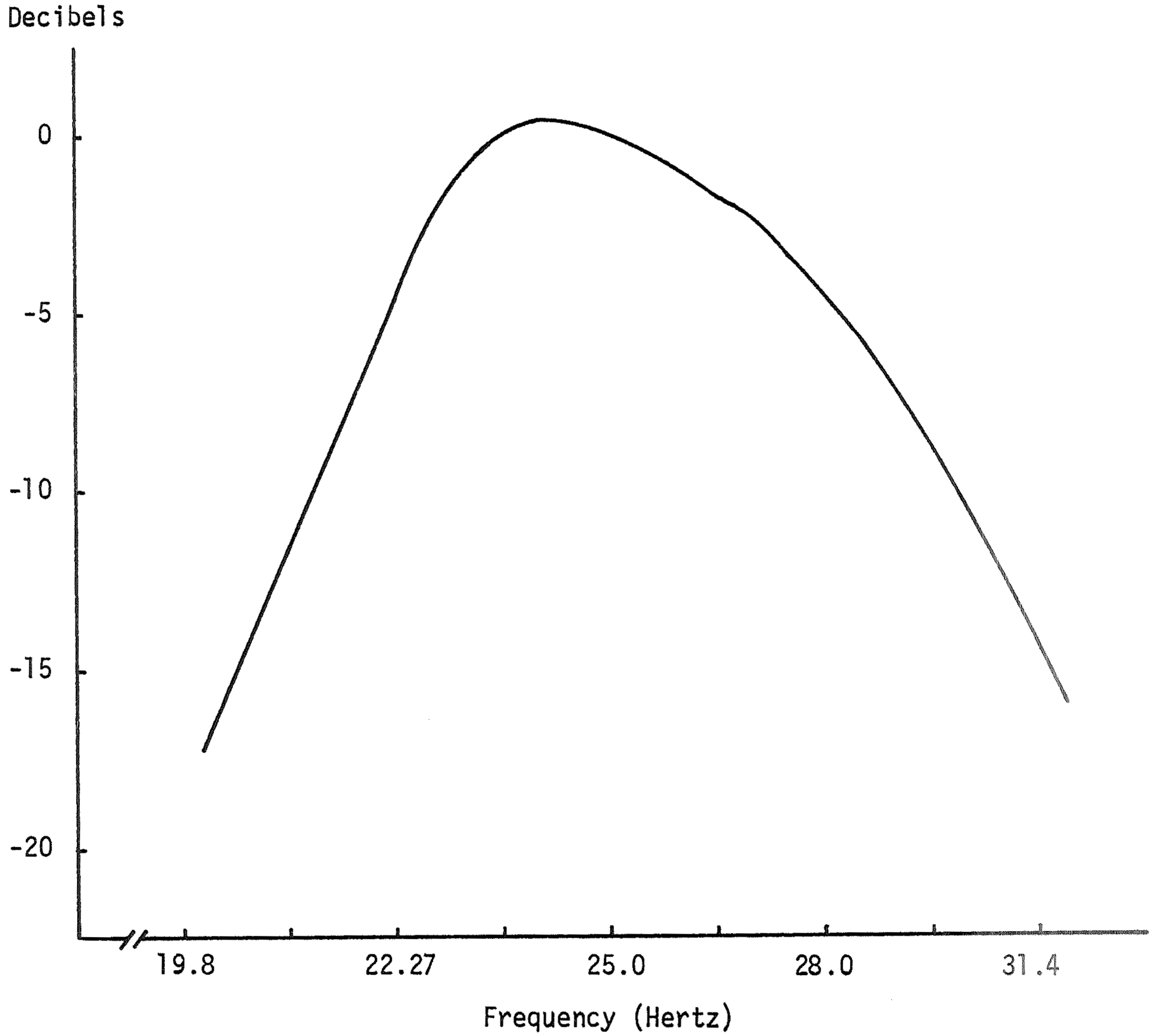


Figure III.3: Magnitude vs. Frequency Plot for Model of 1/3 Octave Filter

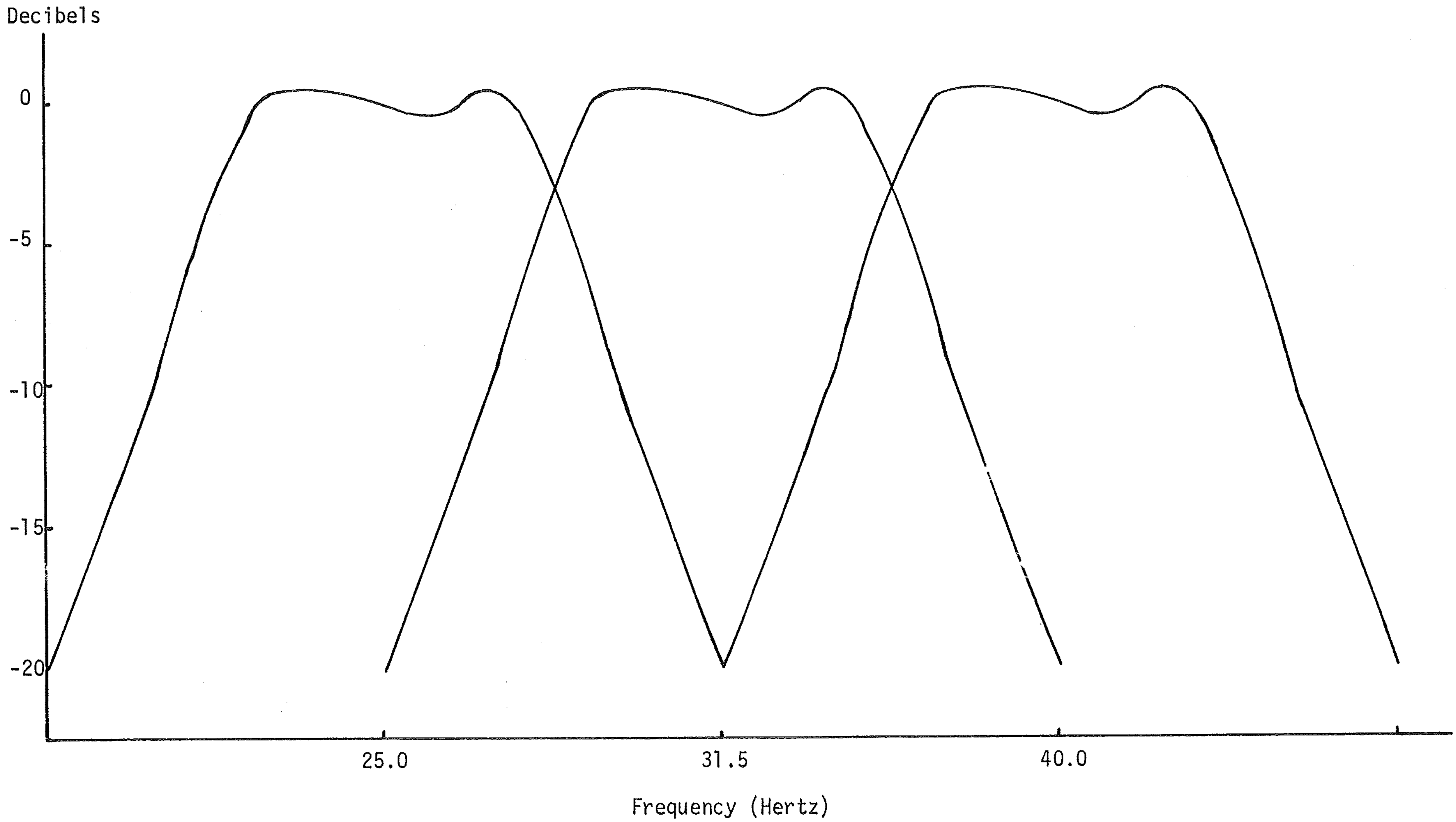


Figure III.4: Magnitude vs. Frequency Plots for Three 1/3 Octave Filters number of discrete values of $y(t_i)^2$ which the summation will contain.

Consider now the following double summation and notation. Let

$$R(t_k)^2 = \frac{1}{(k-\hat{k})\ 10} \sum_{j=\hat{k}}^{k} \sum_{i=1+10(j-1)}^{10j} y(t_i)^2 \qquad (3.2)$$

where $\hat{n} = (n-N)u(n-N)$, where $k = 1, 2, 3, \ldots$ and is the greatest-integer function $[|n/10|]$ and $\hat{k}$ is the greatest-integer function $[|\hat{n}/10|]$. Thus

$$R(t_k) = \left[\frac{1}{(k-\hat{k})\ 10} \sum_{j=\hat{k}}^{k} \sum_{i=1+10(j-1)}^{10j} y(t_i)^2 \right]^{1/2} \qquad (3.3)$$

where $k = 1, 2, \ldots$ as defined above. Equation (3.3) gives a new r.m.s. value of $y(t_i)$ for every ten new values of $y(t_i)$, whereas, equation (3.1) gives a new r.m.s. for every new sample of $y(t_i)$.

In the digital simulation the r.m.s. value will be calculated as in equation (3.3). Ten values of $y(t_i)$ will be taken by the r.m.s. detector at one time and a r.m.s. output reading will be given. Ten more new values of $y(t_i)$ will be taken and a new r.m.s. value will be given. A maximum number of 250 samples will be used in calculating the r.m.s. value where the number 250 corresponds to N in equation (3.3). This manner of storing ten new values of $y(t_i)$ at a time saves computer storage, and basing the r.m.s. values only on the number of samples available up to a maximum of 250 past values gives a best estimate of the present r.m.s. value of $y(t_i)$.

A digital simulation of the system, shown in Appendix C, was made on the IBM 360 digital computer using Fortran IV programming language. Figure III.2 gives a block diagram of this simulation.

The random data taken from the output of the 25.0 Hertz wave shaper , $F_1(s)$, by SAL was used as the input for the 25.0 Hertz 1/3 octave filter since the acoustical test chamber is being replaced by a gain in this preliminary work. The output of the 1/3 octave filter is used to calculate a r.m.s. value, denoted RMS, by the method previously described. The r.m.s. value is used to obtain a r.m.s. value in decibels, denoted RMSDB, by the following method.

$$RMSDB = 20.0 \ Log_{10}(RMS)$$

This decibel quantity is used to simulate a quantity that is used to determine the sound pressure level inside the actual test chamber.

The r.m.s. output in decibels is used to calculate an error, ES1DB = RDB - RMSDB, where RDB = 140 decibels and is the desired sound pressure level. This choice of the value RDB is an arbitrary one and similar simulation results would be expected of other values of RDB. This error is used by the proposed computer algorithm to obtain a change in the attenuator setting denoted by DBCH.

The digital attenuator is simulated by the computer as a number that can vary from 0.0 decibels to -60.0 decibels in integer increments of 0.5 decibel steps. In the digital simulation, this number is known as ATSET. The random input to the 1/3 octave filter is multiplied by a corresponding number designated as ATGN, where ATGN is given by:

$$ATGN = 10.0^{(ATSET/20.0)}$$

A RUNGE-KUTTA integration subroutine was used to solve the sixth order differential equation used to simulate the 1/3 octave filter. An integration step size of 0.004 seconds was used since a 25.0 Hertz sinusoidal has a period of 0.04 seconds; and this would give ten discrete output values per cycle, which would give a good approximation of the actual curve.

Chapter IV.

PROPOSED CONTROL FOR A SINGLE CHANNEL SYSTEM

Certain requirements to be met by the Spaceflight Acoustics Laboratory require that the final attenuator setting be made by the end of the first second after system activation. A maximum of five adjustments are to be made at 0.2 second intervals. These adjustments are to be made on the digital attenuators in the form of a change in the settings from 0.0 decibels to -60.0 decibels. A factor involved in selecting the 25.0 Hertz 1/3 octave filter for this preliminary investigation and simulation is that the response of the 25.0 Hertz 1/3 octave channel is the slowest of all center frequencies involved, the 25.0 Hertz center frequency being the lowest. Thus, to find a satisfactory control algorithm for the 25.0 Hertz 1/3 octave channel would insure meeting the maximum time requirements placed on the system.

The error, ES1DB = RDB - RMSDB, where RDB = 140 decibels is the desired output of the system in decibels; and RMSDB is the actual r.m.s. output of the system in decibels, will be sampled at 0.2 second intervals. The proposed computer control algorithm will use this error, ES1DB, in computing the attenuator setting changes.

The computer control algorithm is a proportional type in that a certain portion of the error, ES1DB, is used to make a change in the attenuator setting. In Fortran IV computer language, the control algorithm is:

DBCH = 0.5 x IFIX(ES1DB x CONT + SIGN(0.5, ES1DB))

DBCH denotes the change in the attenuator setting to be made and ES1DB is the error as given above. The IFIX statement changes its argument to an integer. The SIGN (0.5, ES1DB) statement gives the number 0.5 with the sign (plus or minus) of ES1DB attached. If the argument of IFIX, ES1DBxCONT + SIGN (0.5, ES1DB), is a number other than an integer, the IFIX statement "rounds" or truncates this number into an integer. If the argument of IFIX is already an integer, then this integer is the number the IFIX statement would give for evaluation. The CONT is an arbitrary constant such that it is multiplied by the error, ES1DB, to obtain a number proportional to the error. Due to the simulation being made on a digital computer, the first control time will be 0.204 seconds, using 0.004 seconds as the integration step size.

The main problem encountered is that of the initial attenuator settings. These can vary from -60.0 decibels to 0.0 decibels. Computer programs were evaluated with initial attenuator settings from -60.0 decibels to 0.0 decibels in 10.0 decibel increments, with the constant CONT set at values of 0.3, 0.4, 0.5, 0.6, and 0.7 for these initial attenuator settings.

Several other computer control algorithms were used with the digital simulation for evaluation. One consisted of using a 25.0 Hertz sine wave as the input to the system and using a constant attenuator setting such that the r.m.s. output in decibels was the desired 140.0 in steady-state. This output data was used in the following manner. The random data was then used as the input to the system and the output due to the random data was compared to the output due to the 25.0 Hertz sine wave previously obtained. A computer algorithm was used to make this comparison between the two outputs and to then make a change in the attenuator setting based upon this comparison. However, due to the varied initial attenuator settings, this computer control algorithm gave generally unsatisfactory results.

Other control algorithms evaluated were a "predictive" type control and various types of integral controllers using previous errors, ES1DB = 140.0 - RMSDB, summed for the attenuator setting changes. These also exhibited generally poor response for selecting the correct attenuator setting.

An on-off type of control was evaluated using the computer simulation in the following manner. The attenuator setting was set to 0.0 decibels for minimum attenuation of the random signal input. To nullify the inertial effects in the system output,when the r.m.s. value of the output of the 25.0 Hertz 1/3 octave filter was a certain range in decibels of the desired r.m.s. output, the attenuator was set at -60.0 decibels, which is the maximum value of attenuation of the random signal used to drive the system. After the attenuator was set at -60.0 decibels for a short period of time, a computer control algorithm such as a proportional type could be used to determine the correct attenuator setting. One advantage of the on-off control is that the system is brought up in minimum possible time; and for the system simulation, the 140 decibels output occurred in approximately 0.2 seconds. One possible disadvantage is that the acoustical test chamber could experience some destructive effects for a sustained minimum attenuator setting.

In actual simulations poor results were obtained because no satisfactory switching curve could be found.

Chapter V

RESULTS AND CONCLUSIONS OF THE DIGITAL SIMULATION

Figure V.1 shows the system simulation with a 25.0 Hertz sine wave input with no feedback control and the attenuator setting at 0.0 decibels. The attenuator setting remains constant throughout this particular simulation, and the resulting plot is of the r.m.s. value of the output in decibels, designated RMSDB, versus time. By examining Figure V.1, a slight rise occurs abruptly at approximately 1.1 seconds. This is due to the method of calculation of the r.m.s. value in that ten early values are deleted and ten updated ones are added to a summation used in calculating the r.m.s. value of the output of the 1/3 octave filter. The r.m.s. value from 0.0 seconds to approximately 0.4 seconds is significantly lower than the remaining r.m.s. values, thus yeielding larger r.m.s. calculations when they are deleted.

The random input provided by SAL was used as the input to the system simulation in an open loop configuration with a constant attenuator setting of -13.0 decibels as shown in Figure V.2. Again, the r.m.s. output of the 1/3 octave filter in decibels is plotted versus time. This figure shows that for the random input provided by SAL, the correct attenuator setting in this particular digital simulation is approximately -13.0 decibels for the desired output of 140 decibels, plus or minus 5.0 decibels. It will be noted from Figure V.2 that the response of the r.m.s. output from approximately 0.5 seconds to approximately 3.0 seconds is very

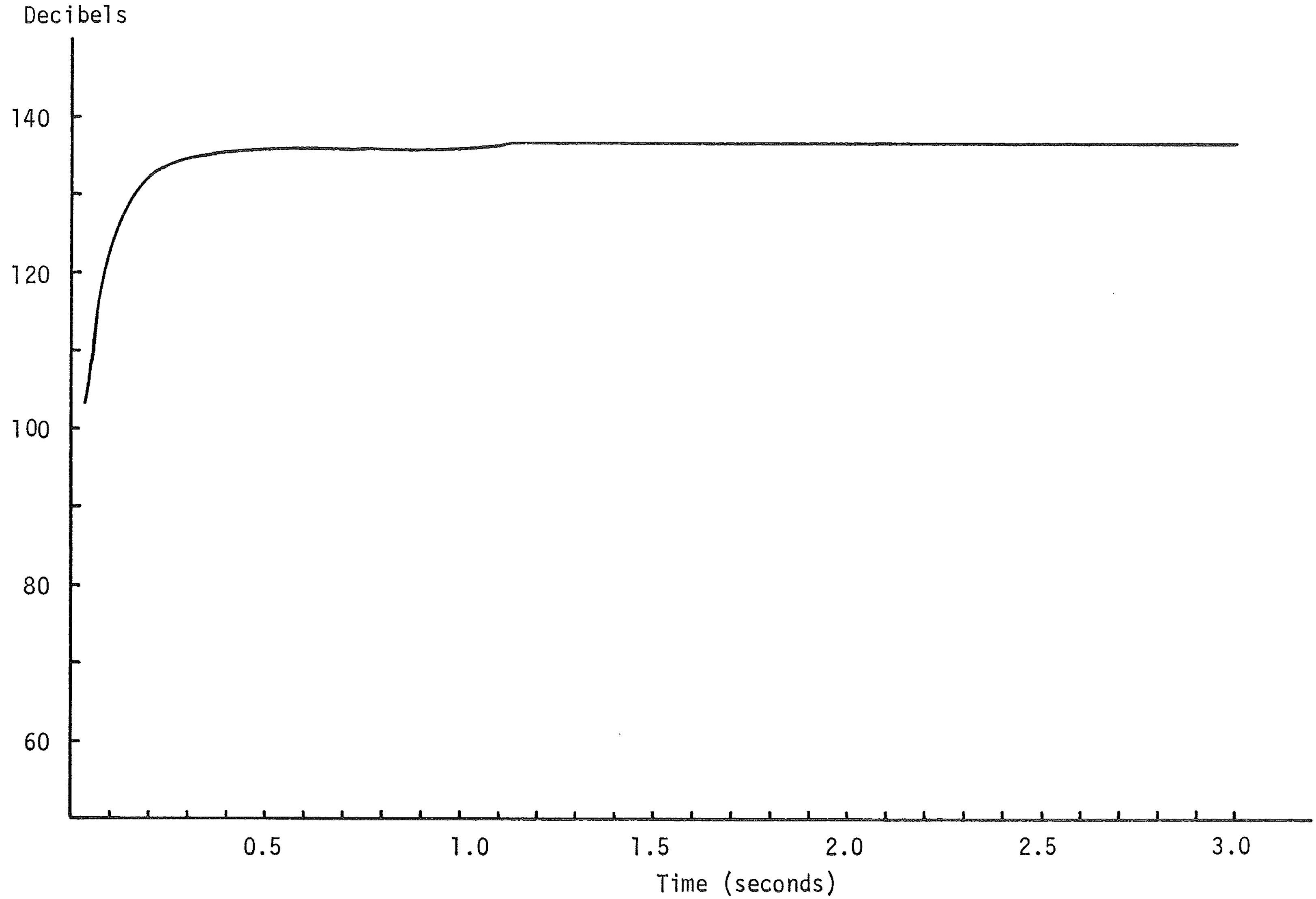


Figure V.1: Open Loop Time Response of 25.0 Hertz Simulated System with a 25.0 Hertz Sinusoidal Input

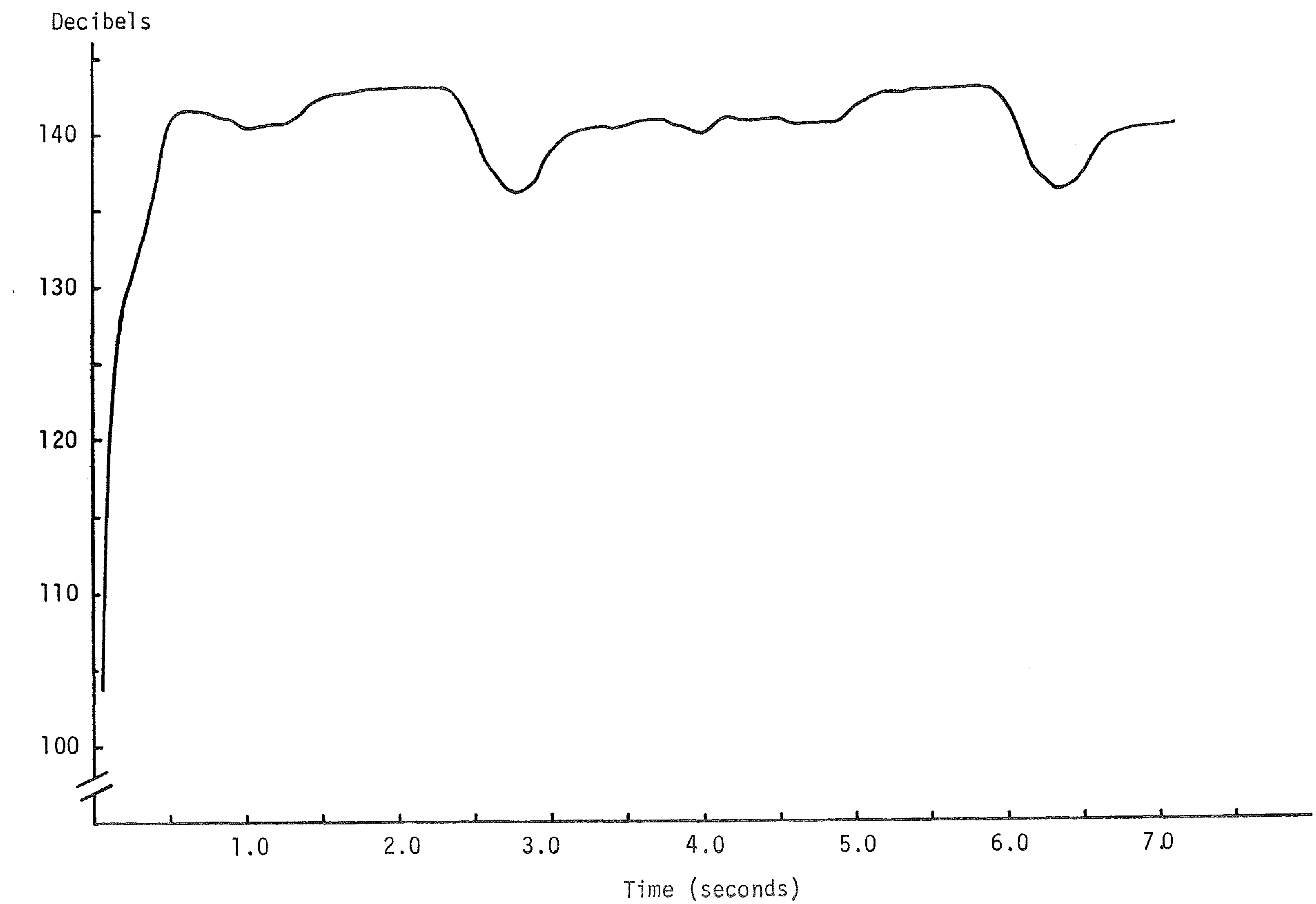


Figure V.2: Open Loop Time Response for a Narrowband Random Noise Input similar to the response from approximately 3.0 seconds to 7.0 seconds. This is because the same random data was used again since there was only an approximate 3.0 seconds of actual random data provided. An approximate settling time can be obtained from Figure V.1 and Figure V.2.

As stated before, a desirable requirement of the system in the closed loop mode is that the final attenuator setting be made at the end of the first second after the system is activated. A maximum of five adjustments spaced at 0.2 second intervals are to be made in this one-second time frame by the proposed computer control algorithm

$$DBCH = 0.5xIFIX(ESIDBxCONT + SIGN(0.5,ES1DB))$$

Since the control algorithm is essentially a proportional error feedback control, CONT was given values of 0.3, 0.4, 0.5, 0.6, and 0.7 in these simulations; and the attenuator responses were plotted. The attenuator responses are given in Appendix B for the different values of CONT and different initial attenuator settings.

For a particular initial attenuator setting, the greater the value of CONT the more evident is an oscillation existing in the attenuator response. For this particular digital simulation and random data input, a setting of -13.0 decibels is the approximate setting for an r.m.s. output of 140 decibels. For CONT = .3, the attenuator response is close to being critically damped and will arrive at the correct attenuator setting in approximately 3.0 seconds. In Table II is given all the attenuator responses for CONT = 0.7 with initial attenuator settings of -60.0 decibels to 0.0 decibels in 10.0 decibel increments. It is interesting to note that in one second the attenuator response is approximately at the correct setting for any given initial condition.

Table 2

Attenuator Settings Versus Time for
a Value of CONT = 0.7

Time(sec)	Initial Attenuator Settings						
0.0	0.0	-10.0	-20.0	-30.0	-40.0	-50.0	-60.0
0.204	-0.5	-7.0	-13.5	-20.0	-26.5	-33.0	-39.5
0.408	-4.0	-7.5	-11.5	-15.5	-19.5	-23.5	-27.5
0.612	-9.0	-10.0	-12.0	-13.5	-15.5	-17.5	-19.5
0.816	-13.5	-12.5	-12.5	-12.0	-12.0	-12.5	-13.5
1.02	-18.0	-14.5	-12.5	-10.5	-9.0	-8.0	-8.0
1.224	-22.5	-16.5	-12.5	-9.0	-6.5	-5.0	-5.0
1.428	-26.5	-18.5	-13.5	-10.0	-8.0	-6.5	-6.5
1.632	-27.5	-19.0	-14.5	-12.0	-11.0	-10.0	-10.0
1.836	-26.0	-19.0	-15.5	-14.0	-14.0	-13.5	-13.5
2.04	-24.0	-19.0	-16.5	-16.5	-17.0	-17.0	-17.0
2.244	-22.0	-19.0	-17.5	-18.5	-20.0	-20.5	-20.5
2.448	-19.0	-18.0	-18.0	-20.0	-22.5	-23.5	-23.5
2.652	-14.0	-15.0	-16.5	-19.0	-22.0	-23.5	-23.5
2.856	-10.5	-12.0	-14.0	-16.5	-19.5	-21.0	-21.0
3.0							

Although this should not lead to any general conclusions, one might find a similar characteristic in the actual system.

The above procedure for finding the correct value of CONT that for any initial attenuator setting would give approximately the correct attenuator setting within a given number of adjustments for a single channel system is essentially trial-and-error. However, when the simulation is made of a multiple channel system, a different procedure may be necessary. In the present single channel simulation, it is assumed that the overall amplifier and attenuator are set such that the only necessary adjustments made are on the digital attenuators of which there are the same number as there are wave shapers. In the multiple channel system simulation with each 1/3 octave channel having a different desired r.m.s. output, it may prove advantageous to use the overall amplifier and attenuator to adjust all 1/3 octave channels by approximately the same amount so that the largest desired r.m.s. value for a 1/3 octave channel is at its correct attenuator setting. The 1/3 octave channels will then be adjusted by the individual digital attenuators to the correct setting relative to the one 1/3 octave channel already set by the overall amplifier and attenuator.

The attenuator settling time of this system simulation can be defined as the time required for the attenuator setting to reach values such that the difference between the maximum and minimum of these values is within a given tolerance. The one-second time requirement is not met in this mode of operation and the computer control algorithm is allowed to operate throughout the simulation. From the attenuator responses in Appendix B, the settling time increases as the value of CONT increases. For a given initial attenuator setting, increasing the value of CONT increases the amplitude of the oscillations in the attenuator responses and more time is required for the oscillations to decrease.

Chapter VI

FUTURE WORK

In this work, the acoustic test chamber including the acoustic horns and microphones are considered to be linear and to only contribute a gain to the 25.0 Hertz 1/3 octave system simulation. In actual practice nonlinearities possibly due to the response characteristics of the acoustic horns and microphones, interference of the acoustic signals due the the actual configuration of the test chamber and vehicle inside, and other factors will have to be included in the system simulation. Also, the acoustic time response will have to be taken into consideration in the simulation since this will be dependent on the relative location of the acoustic horns and microphones and the reverberation characteristics of the room. The linear model of the system considered in this paper is useful in that a control algorithm found satisfactory for the linear model will be of value in devising a control algorithm for the nonlinear simulation.

Chapter II gives a method for analytically obtaining the mean-square response of the output of a linear system with a random input. This permits a system with a random input to be modeled by the response of another system with an input such as a step function. Identification models based upon the system simulation have been devised by Sumaria [23] which model the system's response characteristics with a first-order, second-order, and third-order system. Optimal or near optimal control algorithms could be constructed using these identification models of the system. Preliminary evidence indicates that the control algorithm obtained based on the system simulation can be refined using the identification model techniques.

The next step suggested after the 25.0 Hertz 1/3 octave single channel simulation is to obtain three transfer functions for the 25.0, 31.5, and 40.0 Hertz center frequencies and simulate the system as before, but with three center frequencies involved. From Figure III.4, it can be seen that for a particular magnitude versus frequency plot, its magnitude is down 20.0 decibels at the next center frequencies. This means one can expect a small amount of interaction among the 1/3 octave filters involved since all output of the digital attenuators are to be summed and then placed into each 1/3 octave filter as in Figure I.4. Therefore, it is recommended that the various nonlinearities and the three 1/3 octave channels be included in future simulations and that the identified linear models be used for obtaining better algorithms.

BIBLIOGRAPHY

1. Van Der Grinten, P.M.E.M., "Finding Optimum Controller Settings," Control Engineering, pp. 51-66, December, 1963.

2. Kershow, W. V., "Adaptive Control by Plant Identification," Control Engineering, pp. 103-110, September, 1965.

3. Kim, M. and N. Ula, "Self-Adjusting Controllers for Linear Systems with Correlated Disturbances," Journal of the Franklin Institute, Vol. 283, No. 2, pp. 118-138, February, 1967.

4. Koepcke, R. W., "A Discrete Design Method for Digital Control," Control Engineering, pp. 83-89, June, 1966.

5. Kelly, C. R., "Closing the Loop with Predictive Controllers," Control Engineering, pp. 75-78, May, 1968.

6. Kim, M. and G. U. Ramos, "Implementation of Optimum Sampled-Data Control," IEEE Transactions on Automatic Control, Vol. AC-11, No. 1, pp. 274-277, April, 1966.

7. Cundall, C. M. and V. Latham, "Designing Digital Computer Control Systems--Part 1--Short Sampling Periods," Control Engineering, pp. 82-86, October, 1962.

8. Cundall, C. M. and V. Latham, "Designing Digital Computer Control Systems--Part 2--Long Sampling Periods," Control Engineering, pp. 109-113, January, 1963.

9. Lee, Y. S., "A Time-Optimal Adaptive Control System via Adaptive Switching Hypersurface," IEEE Transactions on Automatic Control, Vol. AC-12, No. 4, pp. 367-372, August, 1967.

10. Nightingale, J. M., "Practical Optimizing Systems. II - Discrete Parameter Adjustment," Control Engineering, pp. 93-98, November, 1965.

11. Sarachik, P. E., and G. M. Kranc, "Optimal Control of Discrete Systems with Constrained Inputs," Journal of the Franklin Institute, Vol. 277, No. 3, pp. 237-255, March, 1964.

12. Porcelli, G. and K. A. Fegley, "Optimal Design of Digitally Compensated Systems by Quadratic Programming," Journal of the Franklin Institute, Vol. 282, No. 5, pp. 303-317, November, 1966.

13. Murphy, W. J., "Optimal Stochastic Control of Discrete Linear Systems with Unknown Gain," IEEE Transactions on Automatic Control, Vol. AC-13, No. 4, pp. 338-343, August, 1968.

14. Karnopp, D., "On the Optimization of Saturating Servomechanisms Subjected to Random Disturbances," Journal of the Franklin Institute, Vol. 285, N. 6, pp. 475-482, June, 1968.

15. Lim, Y. S., "Asymptotic Solutions of an Optimal Servo Problem," IEEE Transactions on Automatic Control, Vol. AC-13, No. 1, pp. 45-50, February, 1968.

16. Pastel, M. P., "Analyzing Nonlinear Systems with Random Inputs," Control Engineering, p. 113-117, February, 1962.

17. Mori, M., "Statistical Treatment of Sampled-Data Control Systems for Actual Random Inputs," Transactions of the ASME, Vol. 80, pp. 444-456, February, 1958.

18. Lee, Y. W., Statistical Theory of Communications, John Wiley and Sons, Inc., New York and London, 1960.

19. Slaughter, J. B., "Compensating for Dynamics in Digital Control," Control Engineering, pp. 109-114, May, 1964.

20. Carlson, A. B., Communication Systems: An Introduction to Signals and Noise in Electrical Communications, McGraw-Hill Book Company,

21. Laning, J. H. and R. H. Battin, Random Processes in Automatic Control, McGraw-Hill Book Company, Inc., New York, 1956.

22. Atkins, A., Future M.S. Thesis, Tennessee Technological University, 1971.

23. Sumaria, V., "Identification of a Linear System From Sampled Noisy Data," M.S. Thesis, Tennessee Technological University, August, 1970.

24. Papoulis, A., Probability, Random Variables, and Stochastic Processes, McGraw-Hill Book Company, New York, St. Louis, San Francisco, 1965.

APPENDICES

APPENDIX A

DEFINITIONS FOUND IN RANDOM PROCESSES

The following are definitions found in random processes.

1. A random process, E, is a sample space or ensemble comprised of functions of time as the fundamental elements.

2. A random process, E, is said to be stationary if every translation in time carries E into itself in such a manner that probability is preserved.

3. A stationary random process is said to possess the ergodic property, provided that for every random variable V(x), the relation

$$E\,[V\{x(t)\}] = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} V\,[x(t+\tau)]\, d\tau$$

where $E\,[V\{x(t)\}]$ is the expectation of the function x(t), is valid for all functions x(t), with the possible exception of a set of functions of zero probability.

4. A system is said to be linear provided that for every pair of inputs, $x_1(t)$, $x_2(t)$ and corresponding outputs $y_1(t)$, $y_2(t)$, the input $Ax_1(t) + Bx_2(t)$ produces as output $Ay_1(t) + By_2(t)$ where A and B are arbitrary constants. For a linear system it is possible to superimpose the effects produced by any number of outputs.

5. A linear system is characterized by its response to a unit-impulse excitation. The response of the linear system to the unit-impulse is called the weighting function and is denoted by W(t). The output of the system y(t), is obtained by the convolution integral

$$y(t) = \int_0^\infty W(\tau)x(t-\tau)\, d\tau \qquad (A.1)$$

where $W(\tau)$ is the weighting function and $x(t)$ is the input to the linear system.

6. A characterization which is often used for stable constant-coefficient systems is the frequency-response function, $Y(j\omega)$, i.e., the response of the linear system to a sinusoidal input. Using complex notation, the input to the system is $x(t) = e^{j\omega t}$. The output is obtained from equation (A.1) and is given by

$$y(t) = \int_0^\infty W(\tau)x(t-\tau)\ d\tau$$

$$= \int_0^\infty W(\tau)e^{j\omega(t-\tau)}d\tau$$

$$= e^{j\omega t}\int_0^\infty W(\tau)e^{-j\omega\tau}\ d\tau \tag{A.2}$$

It is seen that the output of the system differs from the input only by a constant complex factor

$$Y(j\omega) = \int_0^\infty W(\tau)\ e^{-j\omega\tau}\ d\tau \tag{A.3}$$

The quantity $Y(j\omega)$ is called the frequency-response function.

By computing the Laplace transform of the weighting function, $W(t)$, rather than the Fourier transform, a quantity called the transfer function, $Y(s)$, is obtained

$$Y(s) = \int_0^\infty W(t)e^{-st}\ dt \tag{A.4}$$

where s is a complex constant, $s = \sigma + j\omega$.

7. An autocorrelation function of a random process, $x(t)$, is

$$\phi_{xx}(\tau) = \overline{x(t)x(t+\tau)} = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x(t)x(t+\tau)\ dt \tag{A.5}$$

for $\tau = 0$,

$$\phi_{xx}(0) = \overline{x(t)x(t)} = \overline{x(t)^2} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)^2 \, dt$$

8. The power spectral density function of a random process, x(t), is

$$G_{xx}(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} \phi_{xx}(\tau) e^{-j\omega\tau} \, d\tau \qquad (A.6)$$

Since $\phi_{xx}(\tau)$ is an even function,

$$G_{xx}(\omega) = \frac{2}{\pi} \int_{0}^{\infty} \phi_{xx}(\tau) \cos \omega\tau \, d\tau$$

And since $G_{xx}(\omega)$ is the Fourier transform of $\phi_{xx}(\tau)$

$$\phi_{xx}(\tau) = \frac{1}{2} \int_{-\infty}^{\infty} G_{xx}(\omega) e^{j\omega\tau} \, d\omega = \int_{0}^{\infty} G_{xx}(\omega) \cos \omega\tau \, d\omega \qquad (A.7)$$

9. Input-output relations for autocorrelation functions and power spectral density functions.

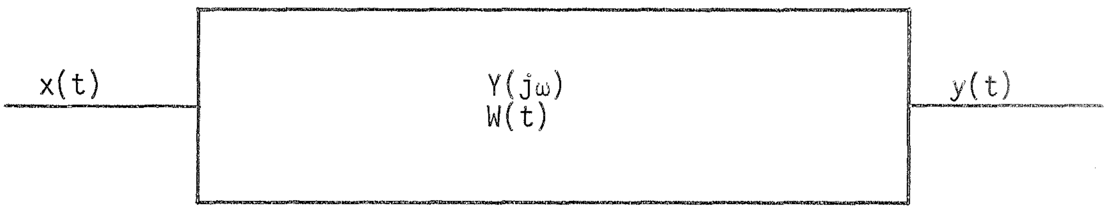


Figure A.1: A Linear System with a Random Process Input

The <u>autocorrelation function of the output of a linear system</u>, as in Figure A.1, $\phi_{yy}(\tau)$, is found in terms of the autocorrelation function of the input $\phi_{xx}(\tau)$ and the weighting function of the system $W(\tau)$.

The <u>power spectral density function of the output</u>, $G_{yy}(\omega)$, is found in terms of the frequency response function of the system, $Y(j\omega)$.

For the autocorrelation function of the output of the system in Figure A.1, $\phi_{yy}(\tau)$ the output y(t) is given by equation (A.1),

$$y(t) = \int_0^\infty W(\tau)x(t-\tau)\, d\tau$$

Replace t and τ by (t_1, τ_1) and (t_2, τ_2), respectively to obtain the two equations

$$y(t_1) = \int_0^\infty W(\tau_1)x(t_1-\tau_1)\, d\tau_1$$

$$y(t_2) = \int_0^\infty W(\tau_2)x(t_2-\tau_2)\, d\tau_2$$

Then

$$y(t_1)y(t_2) = \int_0^\infty W(\tau_1)x(t_1-\tau_1)d\tau_1 \int_0^\infty W(\tau_2)x(t_2-\tau_2)\, d\tau_2$$

and

$$y(t_1)y(t_2) = \int_0^\infty W(\tau_1)d\tau_1 \int_0^\infty W(\tau_2)x(t_1-\tau_1)x(t_2-\tau_2)\, d\tau_2 \quad (A.8)$$

The mathematical expectation of both sides of equation (A.8) is computed averaging under the integral sign in the right-hand member. Since x(t) and y(t) are stationary random processes,

$$E\,[y(t_1)y(t_2)] = \phi_{yy}\,(t_2-t_1)$$

and

$$E\,[x(t_1-\tau_1)x(t_2-\tau_2)] = \phi_{xx}\,(t_2-t_1-\tau_2+\tau_1)$$

From equation (A.8), the autocorrelation function of the output, $\phi_{yy}(\tau)$, is

$$\phi_{yy}(\tau) = \int_0^\infty W(\tau_1)d\tau_1 \int_0^\infty W(\tau_2)\phi_{xx}(\tau-\tau_2+\tau_1)d\tau_2 \qquad \text{(A.9)}$$

where $\tau = t_2 - t_1$.

The power spectral density of the output of the system, $G_{yy}(\omega)$, is from equation (A.6)

$$G_{yy}(\omega) = \frac{1}{\pi}\int_0^\infty \phi_{yy}(\tau)e^{-j\omega\tau}d\tau$$

Substituting the expression found for $\phi_{yy}(\tau)$ in equation (A.9) and rearranging the terms gives

$$G_{yy}(\omega) = \frac{1}{\pi}\int_0^\infty e^{-j\omega\tau}\,d\tau \int_0^\infty W(\tau_1)d\tau_1 \int_0^\infty W(\tau_2)\phi_{xx}(\tau-\tau_2+\tau_1)\,d\tau_2$$

$$= \frac{1}{\pi}\int_0^\infty W(\tau_1)d\tau_1 \int_0^\infty W(\tau_2)d\tau_2 \int_0^\infty \phi_{xx}(\tau-\tau_2+\tau_1)e^{-j\omega\tau}d\tau$$

$$= \frac{1}{\pi}\int_0^\infty W(\tau_1)e^{j\omega\tau_1}\,d\tau_1 \int_0^\infty W(\tau_2)e^{-j\omega\tau_2}\,d\tau_2 \int_\infty^\infty \phi_{xx}(\tau-\tau_2+\tau_1)$$

$$e^{-j\omega(\tau-\tau_2+\tau_1)}d\tau$$

$$= \int_0^\infty W(\tau_1)e^{j\omega\tau_1}\,d\tau_1 \int_0^\infty W(\tau_2)e^{-j\omega\tau_2}\,d\tau_2\,\frac{1}{\pi}\int_0^\infty \phi_{xx}(t)e^{j\omega t}\,dt \qquad \text{(A.10)}$$

where a change of variable, $t = \tau - \tau_2 + \tau_1$, occurs in the last term of equation (A.10). From equation (A.6),

$$G_{xx}(\omega) = \frac{1}{\pi}\int_0^\infty \phi_{xx}(t)e^{-j\omega t}\,dt$$

and from equation (A.3),

$$Y(j\omega) = \int_0^\infty W(\tau_2)e^{-j\omega\tau_2}\, d\tau_2$$

Since the weighting function takes on only real values, the complex conjugate of the frequency response function is

$$Y(j\omega)^* = \int_0^\infty W(\tau_1)e^{j\omega\tau_1}\, d\tau_1$$

Substituting $G_{xx}(\omega)$, $Y(j\omega)$, and $Y(j\omega)^*$ into equation (A.10) gives the power spectral density function of the output, $G_{yy}(\omega)$,

$$G_{yy}(\omega) = Y(j\omega)^*\, Y(j\omega)\, G_{xx}(\omega) = |Y(j\omega)|^2\, G_{xx}(\omega) \qquad \text{(A.11)}$$

10. In Figure A.1 it is assumed the random process input and output have zero mean. The mean-square value of the output $y(t)$ is from equation (A.9)

$$\overline{y(\infty)^2} = \phi_{yy}(0) = \int_0^\infty W(\tau_1)d\tau_1 \int_0^\infty W(\tau_2)\, \phi_{xx}(\tau_1-\tau_2)\, d\tau_2$$

$$\overline{y(t)^2} = \int_0^t \int_0^t W(t-\tau_1)W(t-\tau_2)\, \phi_{xx}(\tau_1-\tau_2)d\tau_1 d\tau_2 \qquad \text{(A.12)}$$

and from equation (A.7)

$$\phi_{yy}(\tau) = \int_0^\infty G_{yy}(\omega)\, \cos\, \omega\tau\, d\omega$$

Thus for $\tau = 0$

$$\phi_{yy}(0) = \int_0^\infty G_{yy}(\omega)\, d\omega$$

Equation (A.11) gives

$$G_{yy}(\omega) = |Y(j\omega)|^2 G_{xx}(\omega)$$

Then

$$y(\infty) = \int_0^\infty |Y(j\omega)|^2 \, G_{xx}(\omega) \, d\omega \tag{A.13}$$

11. A random process, x(t), possessing a constant power spectral density is referred to as a <u>white noise</u>. For a white noise, the power spectral density function, $G_{xx}(\omega) = G_0/\pi$, a constant. The autocorrelation function is

$$\begin{aligned}\phi_{xx}(\tau) &= \frac{1}{2}\int_0^\infty G_{xx}(\omega) e^{j\omega\tau} d\omega \\ &= \frac{1}{2}\int_0^\infty G_0/\pi \; e^{j\omega\tau} d\omega \\ &= G_0 \; \delta(\tau)\end{aligned}$$

where $\delta(\tau)$ is the impulse function.

APPENDIX B

PLOTS OF ATTENUATOR RESPONSES OBTAINED FROM THE DIGITAL SIMULATION

Plots of the attenuator responses for values of CONT equal to 0.3, 0.4, 0.5, 0.6, and 0.7 using the computer control algorithm, DBCH = 0.5 x IFIX (ES1DB x CONT + SIGN (0.5, ES1DB)), in the digital simulation are shown on the following pages. Various initial attenuator settings are used.

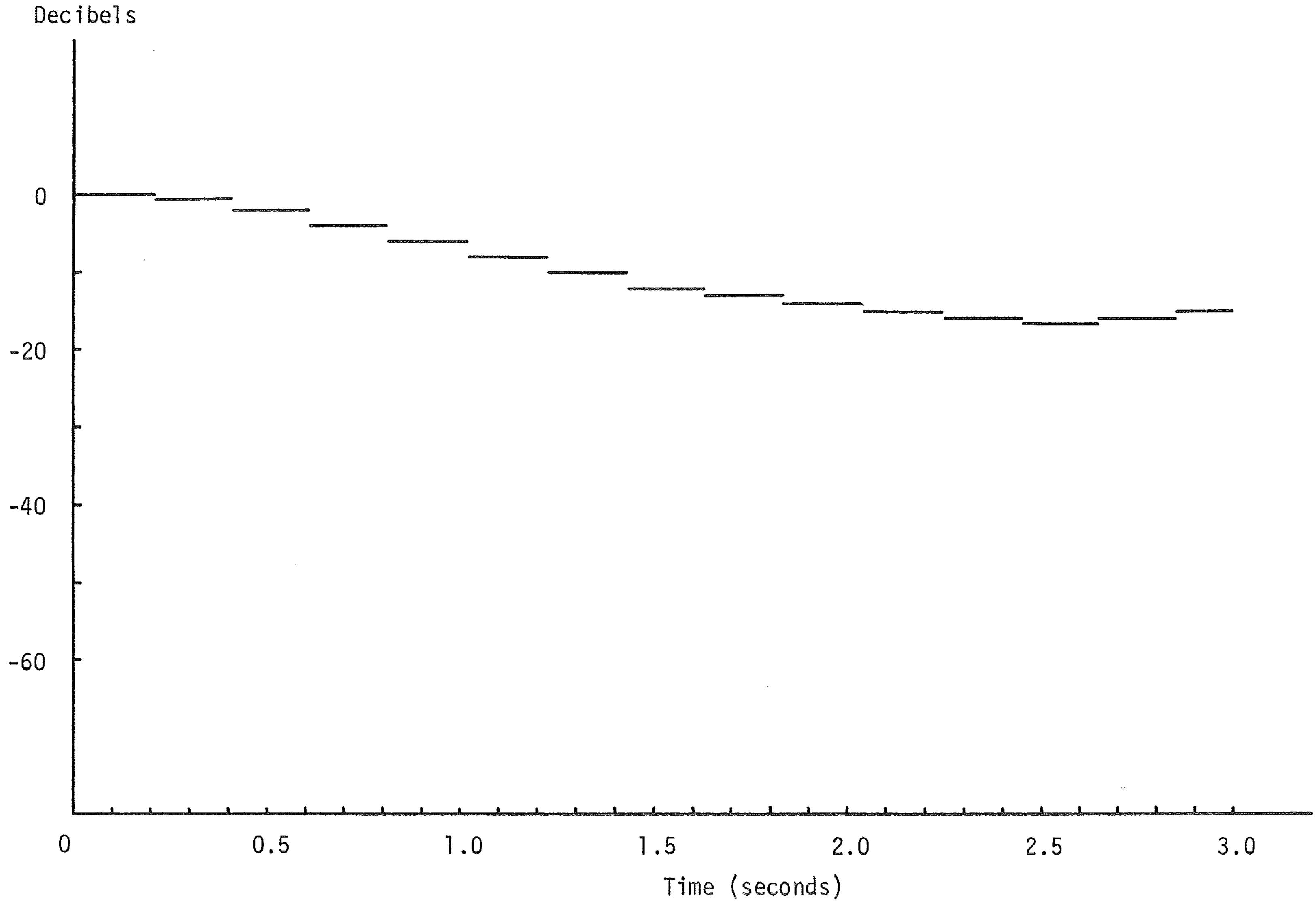


Figure B.1: Closed Loop Attenuator Response for an Initial Attenuator Setting of 0.0 Decibels with Cont = 0.3

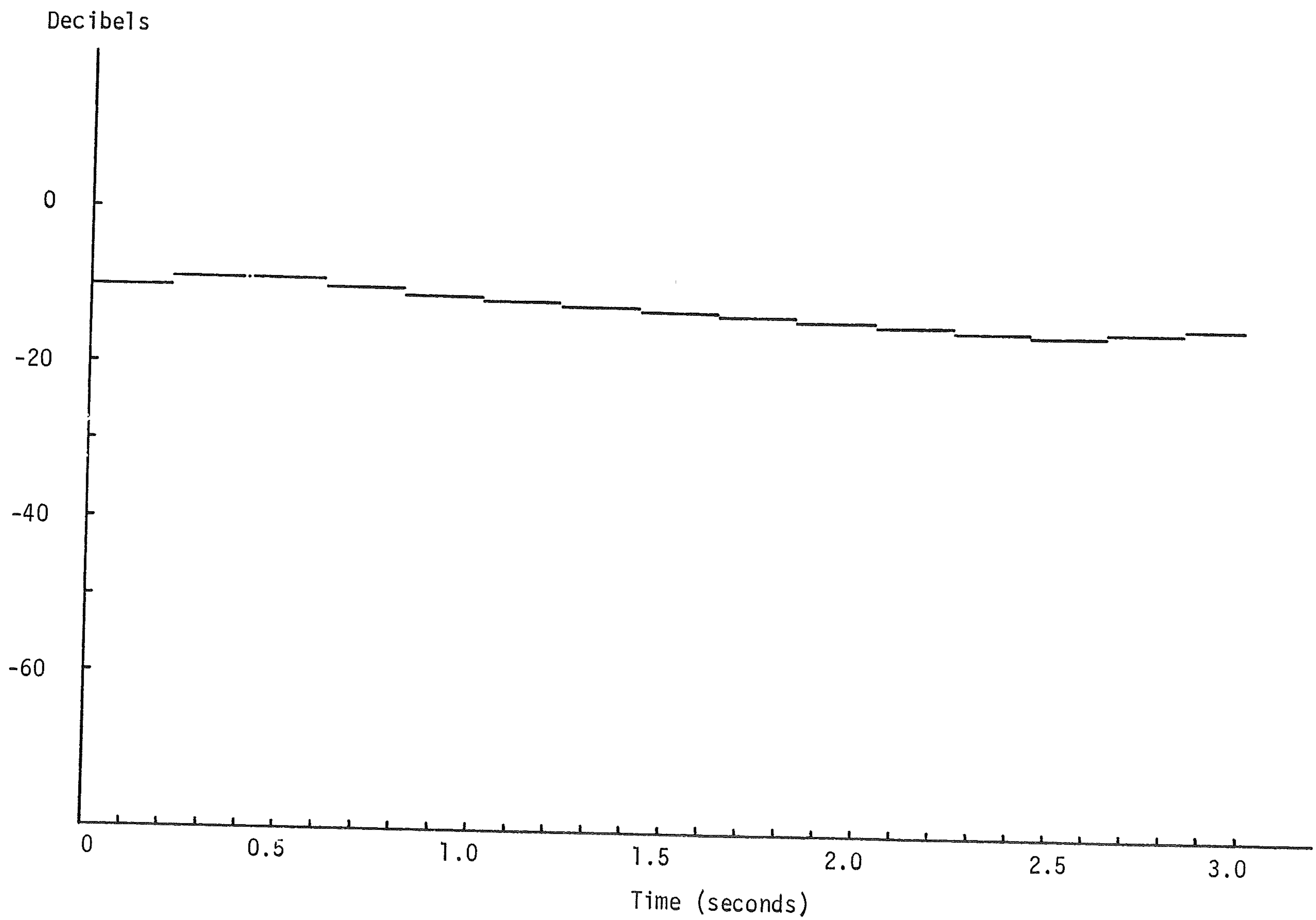


Figure B.2: Closed Loop Attenuator Response for an Initial Attenuator Setting of -10.0 Decibels with Cont = 0.3

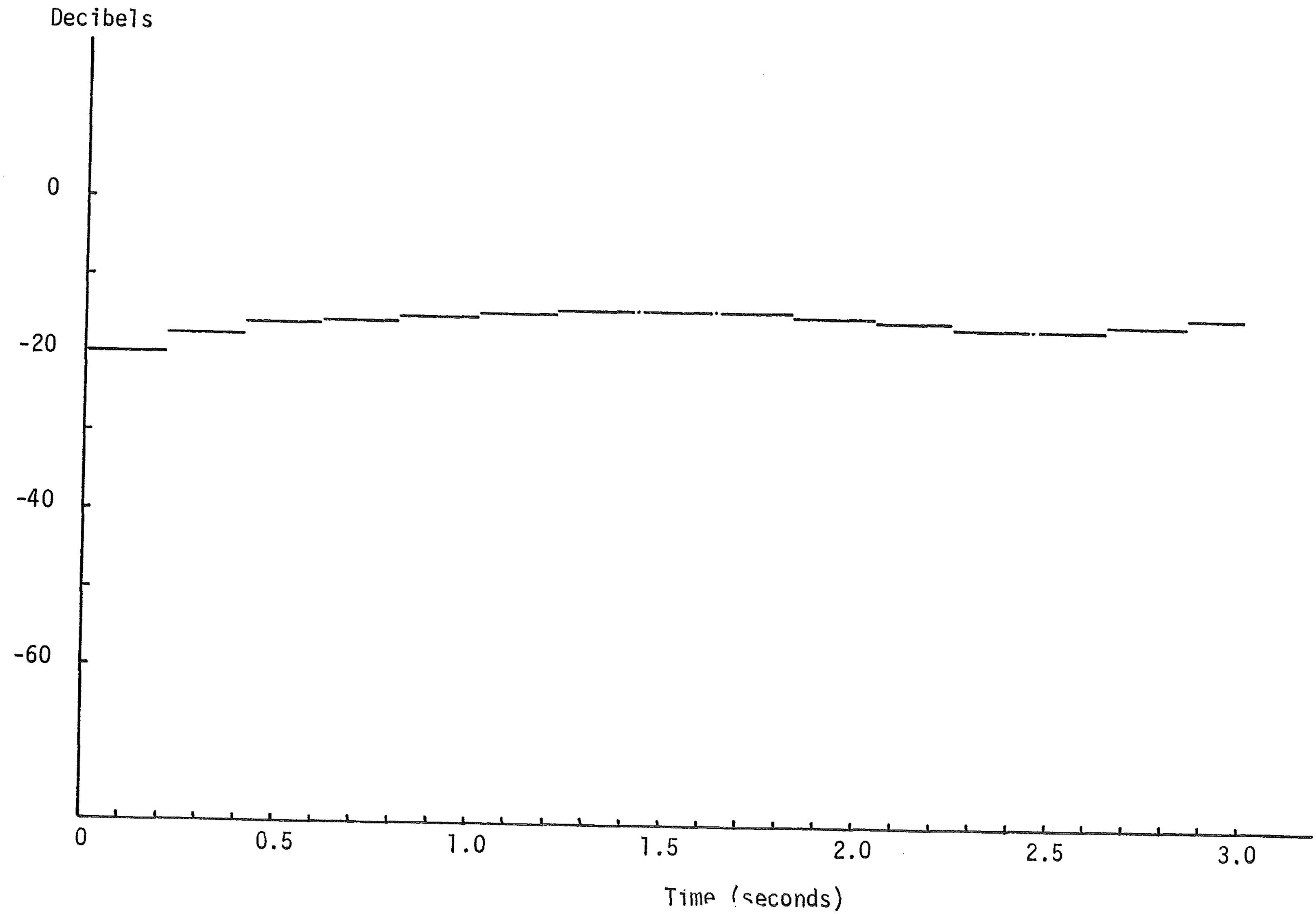


Figure B.3: Closed Loop Attenuator Response for an Initial Attenuator Setting of -20.0 Decibels with Cont = 0.3

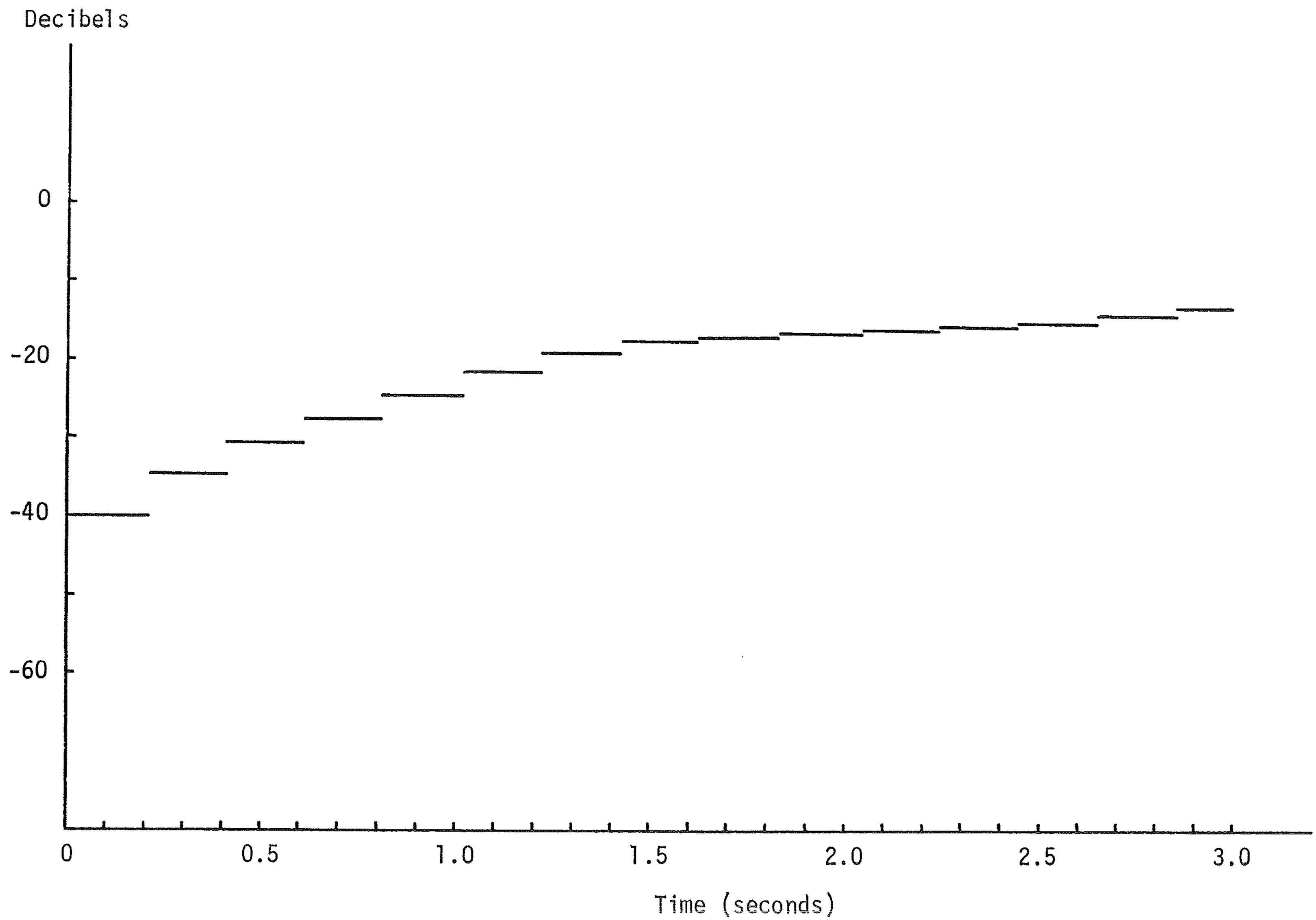


Figure B.4: Closed Loop Attenuator Response for an Initial Attenuator Setting of -40.0 Decibels with Cont = 0.3

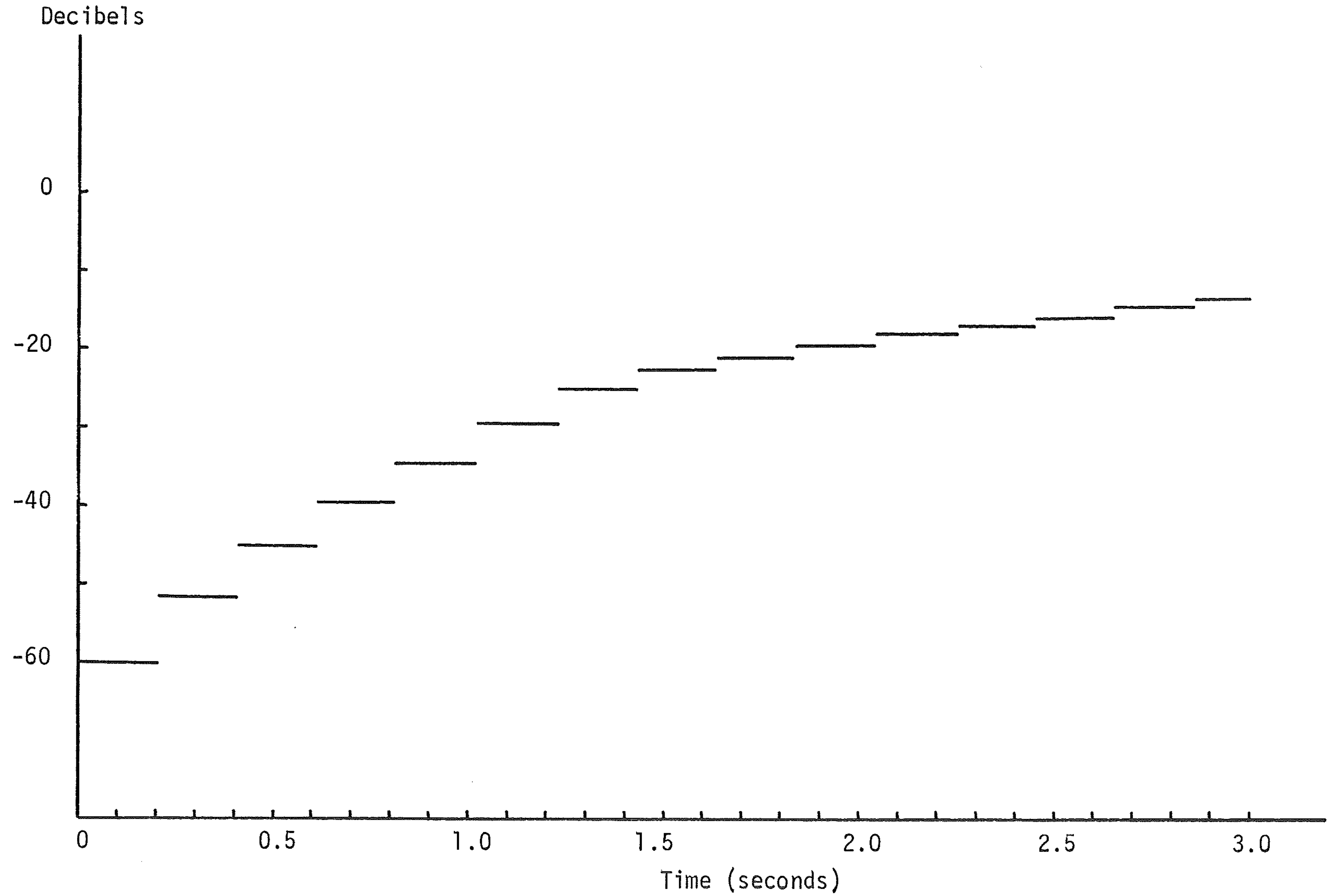


Figure B.5: Closed Loop Attenuator Response for an Initial Attenuator Setting of -60.0 Decibels with Cont = 0.3

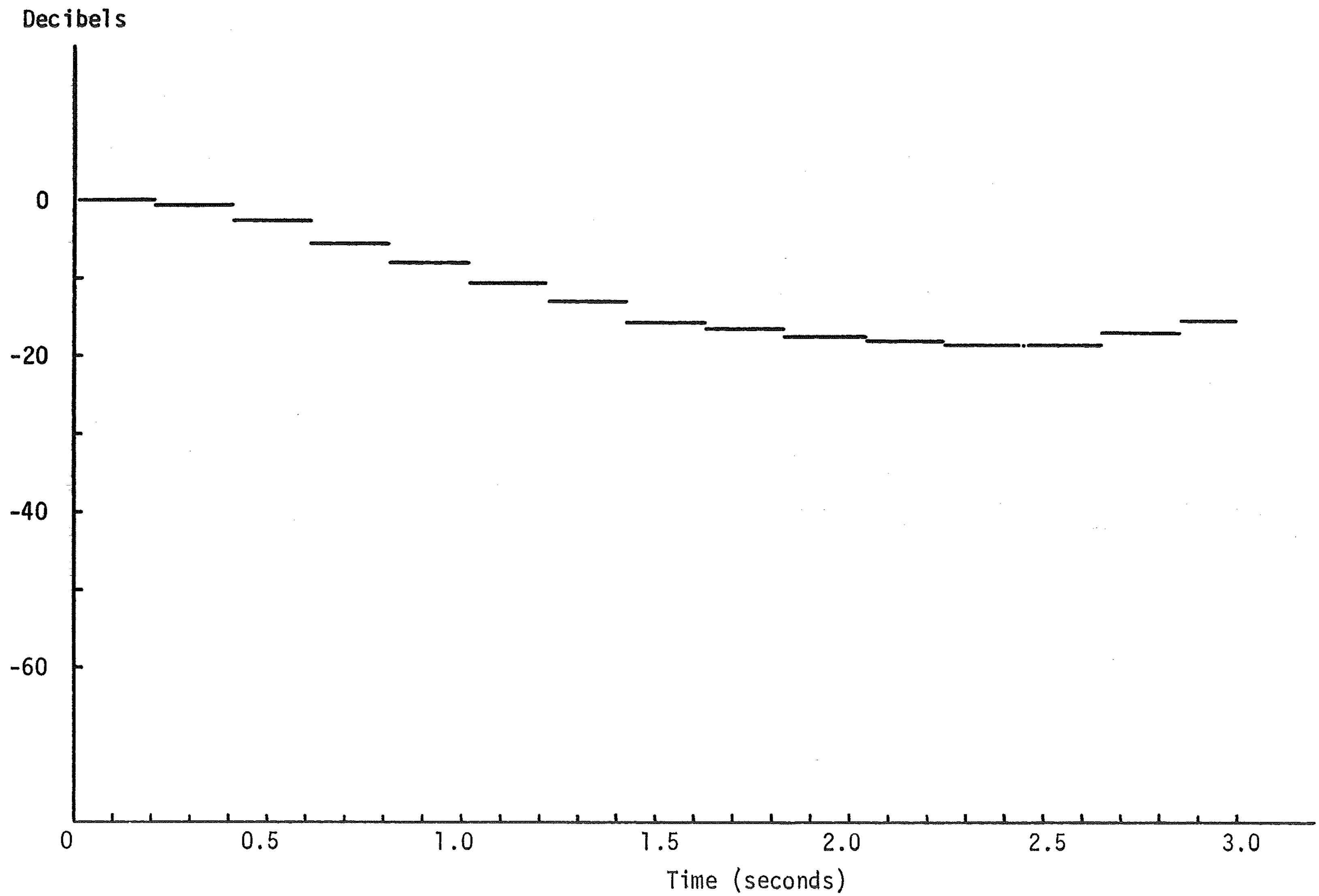


Figure B.6: Closed Loop Attenuator Response for an Initial Attenuator Setting of 0.0 Decibels with Cont = 0.4

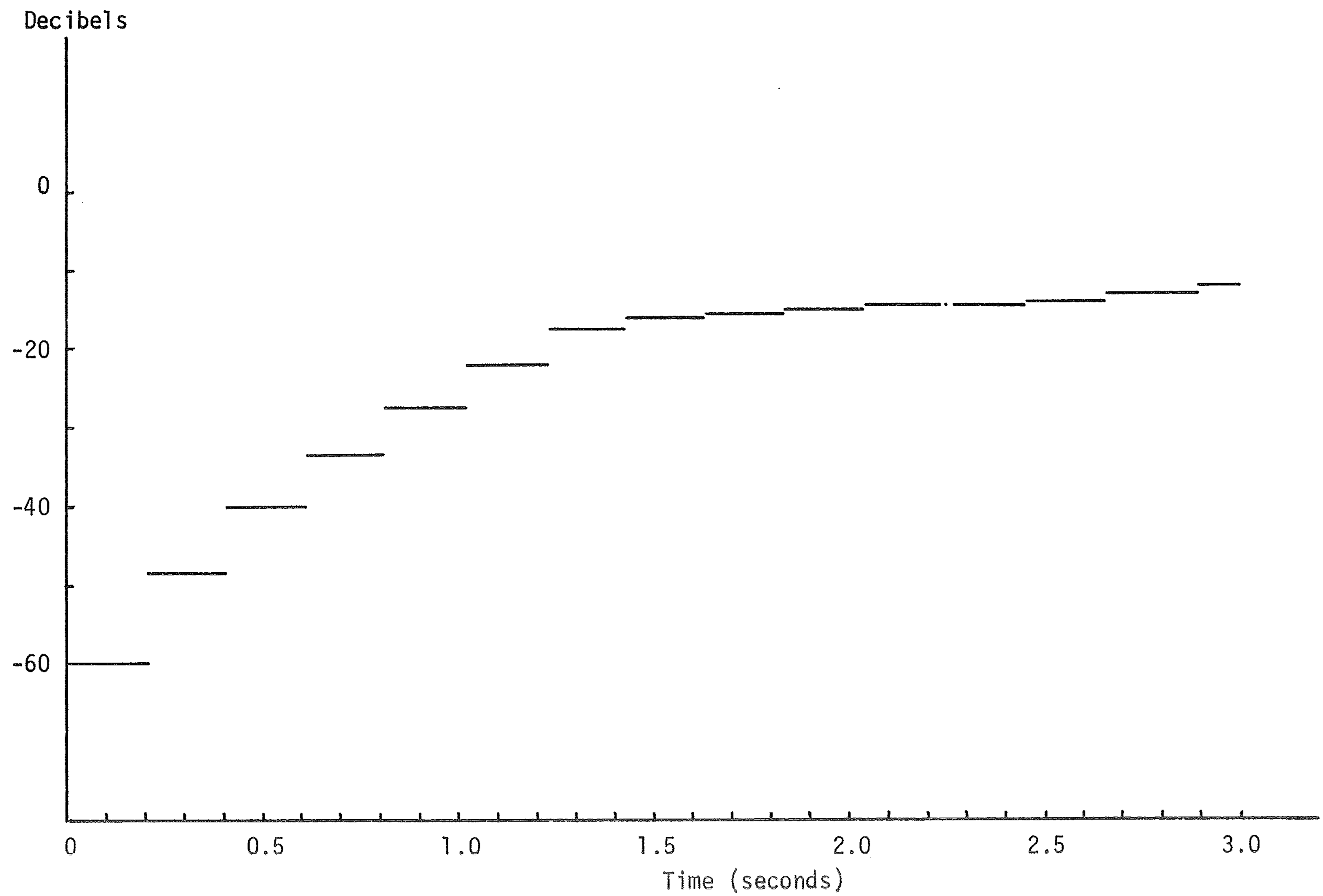


Figure B.7: Closed Loop Attenuator Response for an Initial Attenuator Setting of -60.0 Decibels with Cont = 0.4

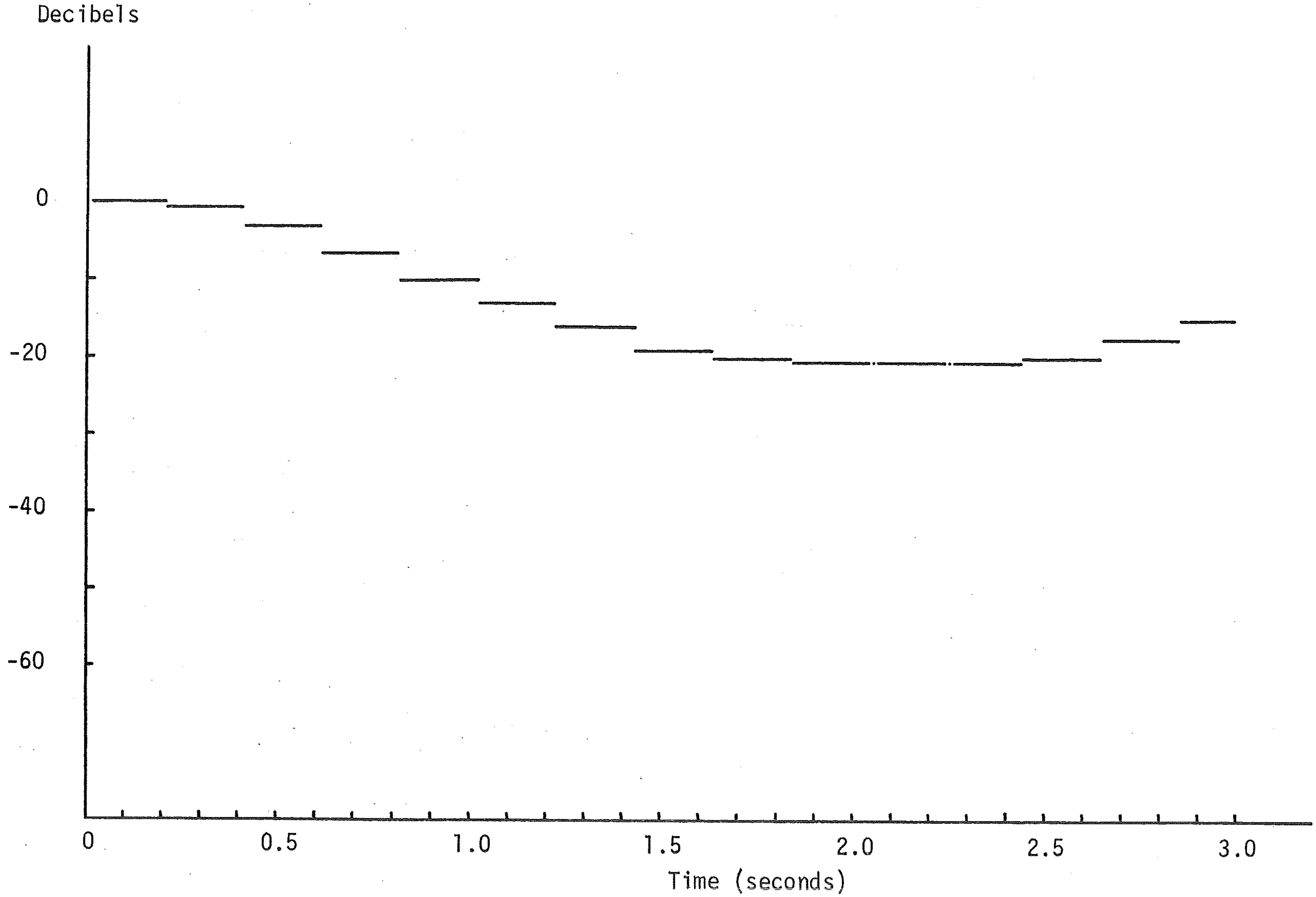


Figure B.8: Closed Loop Attenuator Response for an Initial Attenuator Setting of 0.0 Decibels with Cont = 0.5

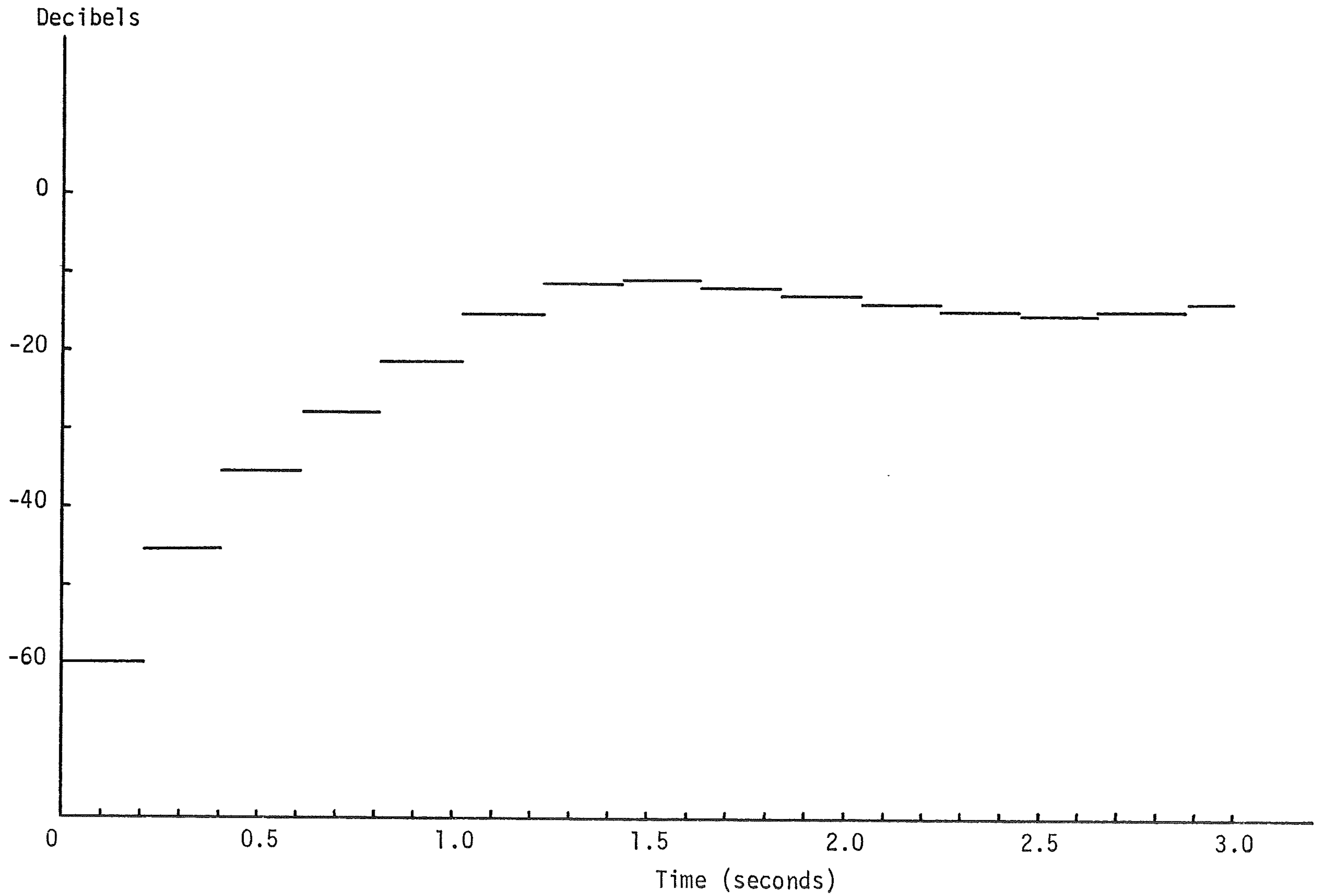


Figure B.9: Closed Loop Attenuator Response for an Initial Attenuator Setting of -60.0 Decibels with Cont = 0.5

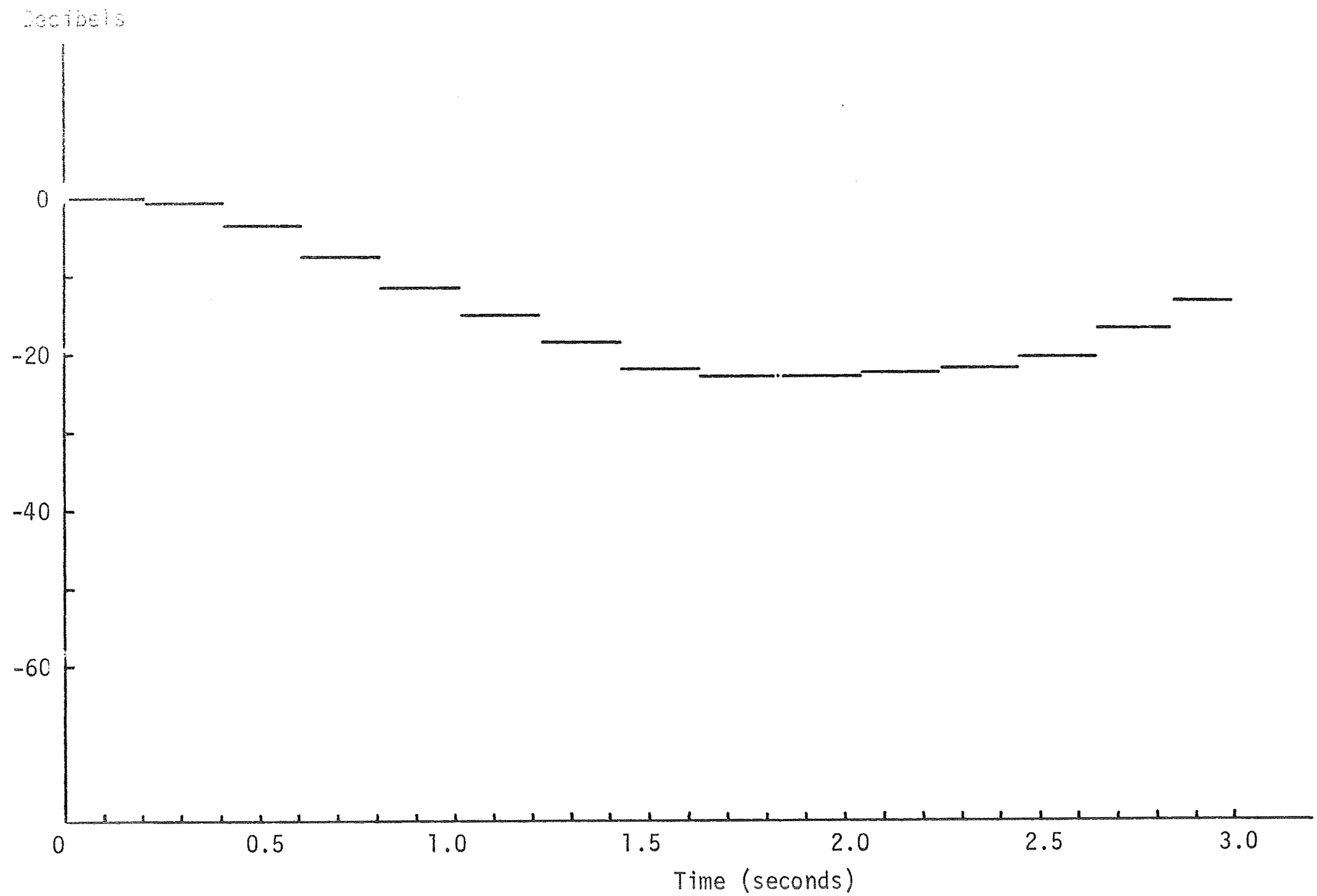


Figure B.10: Closed Loop Attenuator Response for an Initial Attenuator Setting of 0.0 Decibels with Cont = 0.6

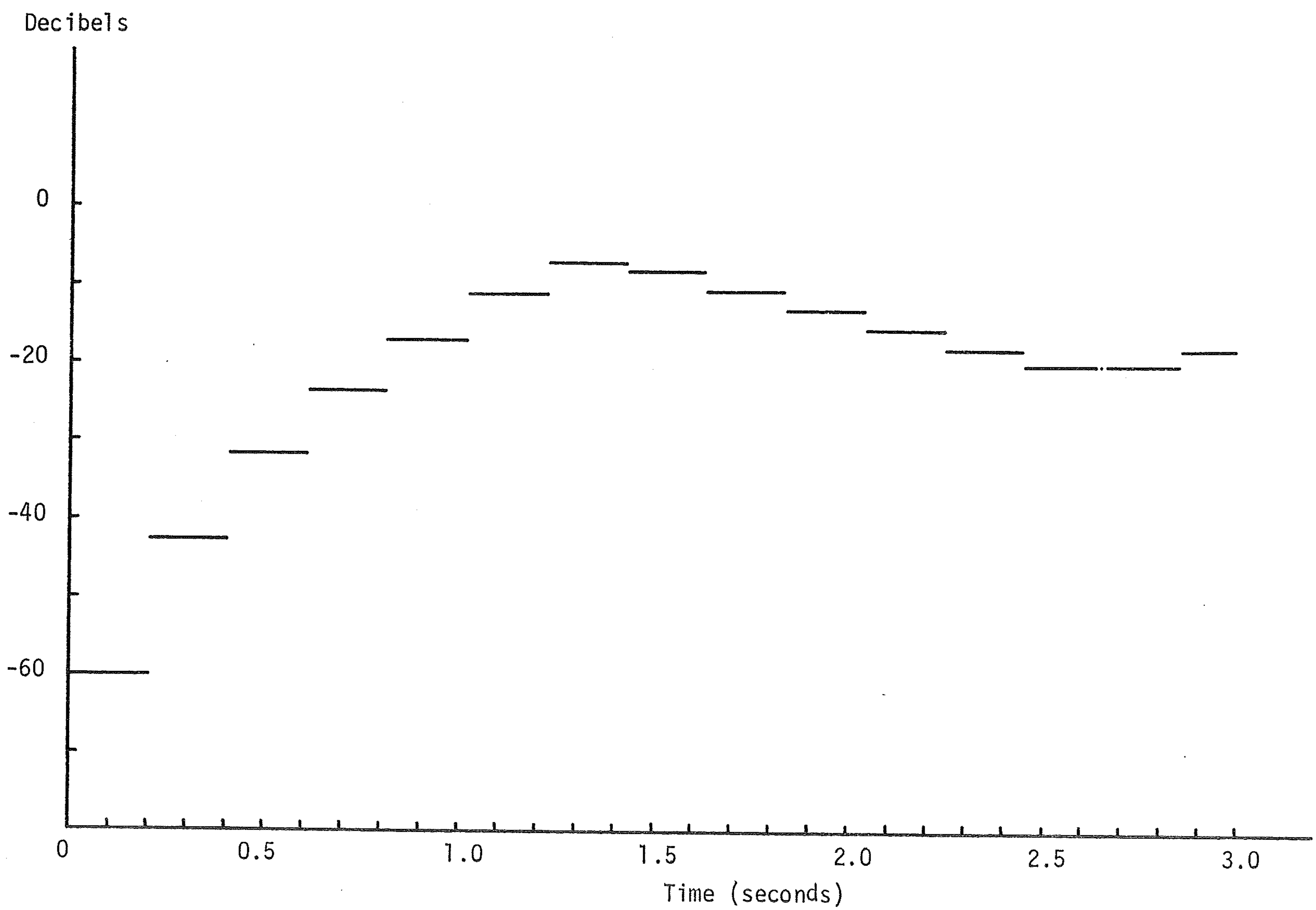


Figure B.11: Closed Loop Attenuator Response for an Initial Attenuator Setting of -60.0 Decibels with Cont = 0.6

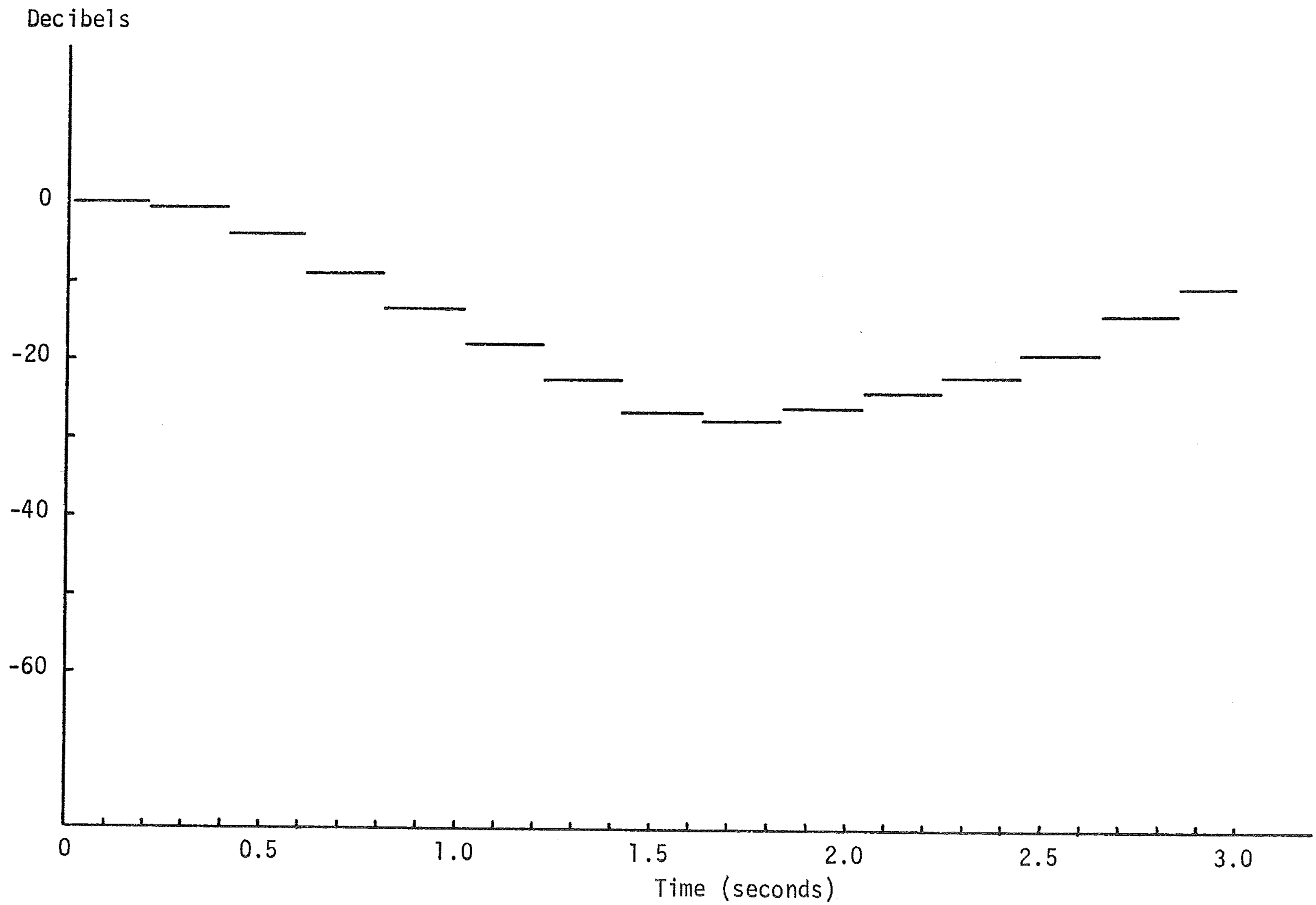


Figure B.12: Closed Loop Attenuator Response for an Initial Attenuator Setting of 0.0 Decibels with Cont = 0.7

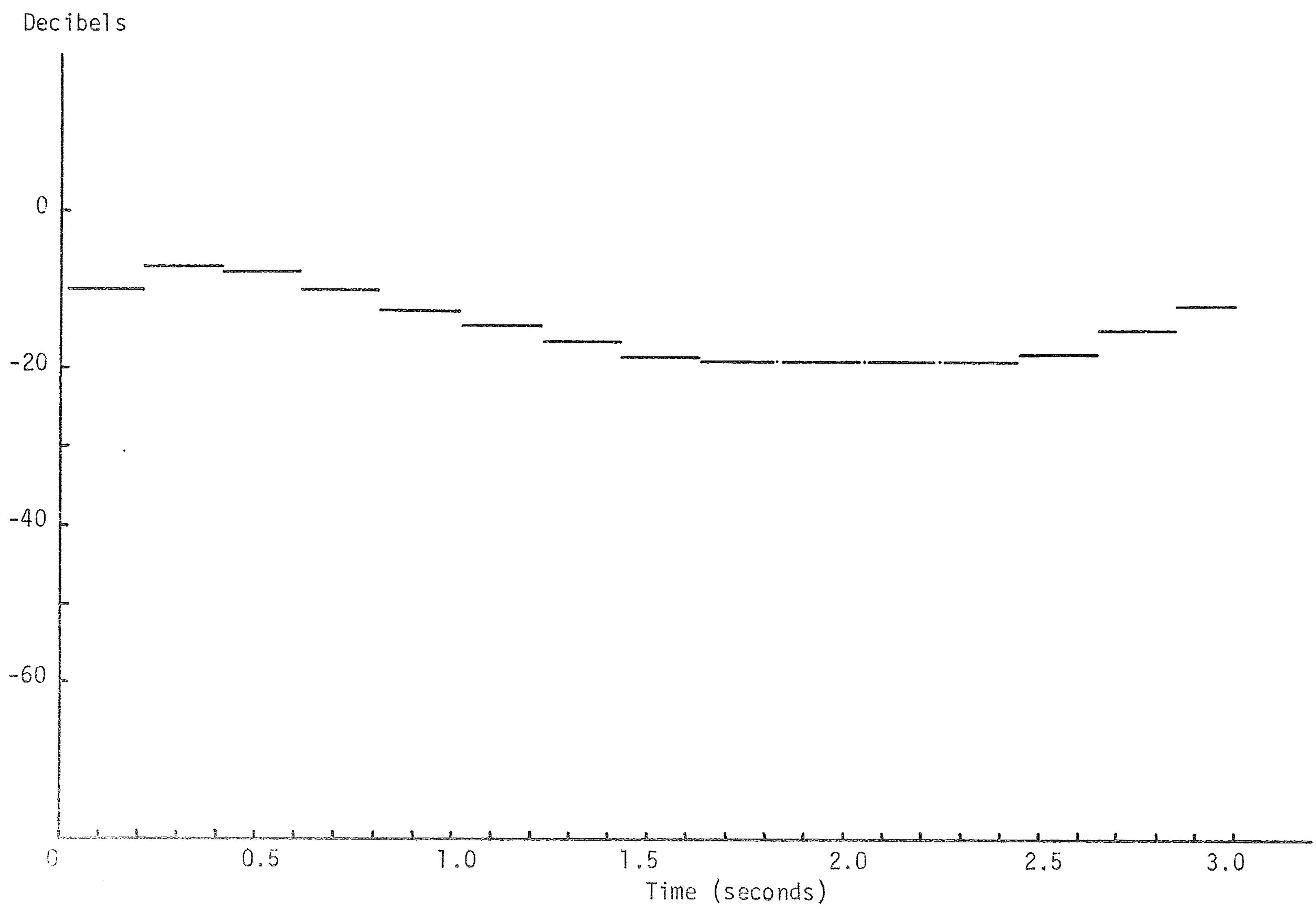
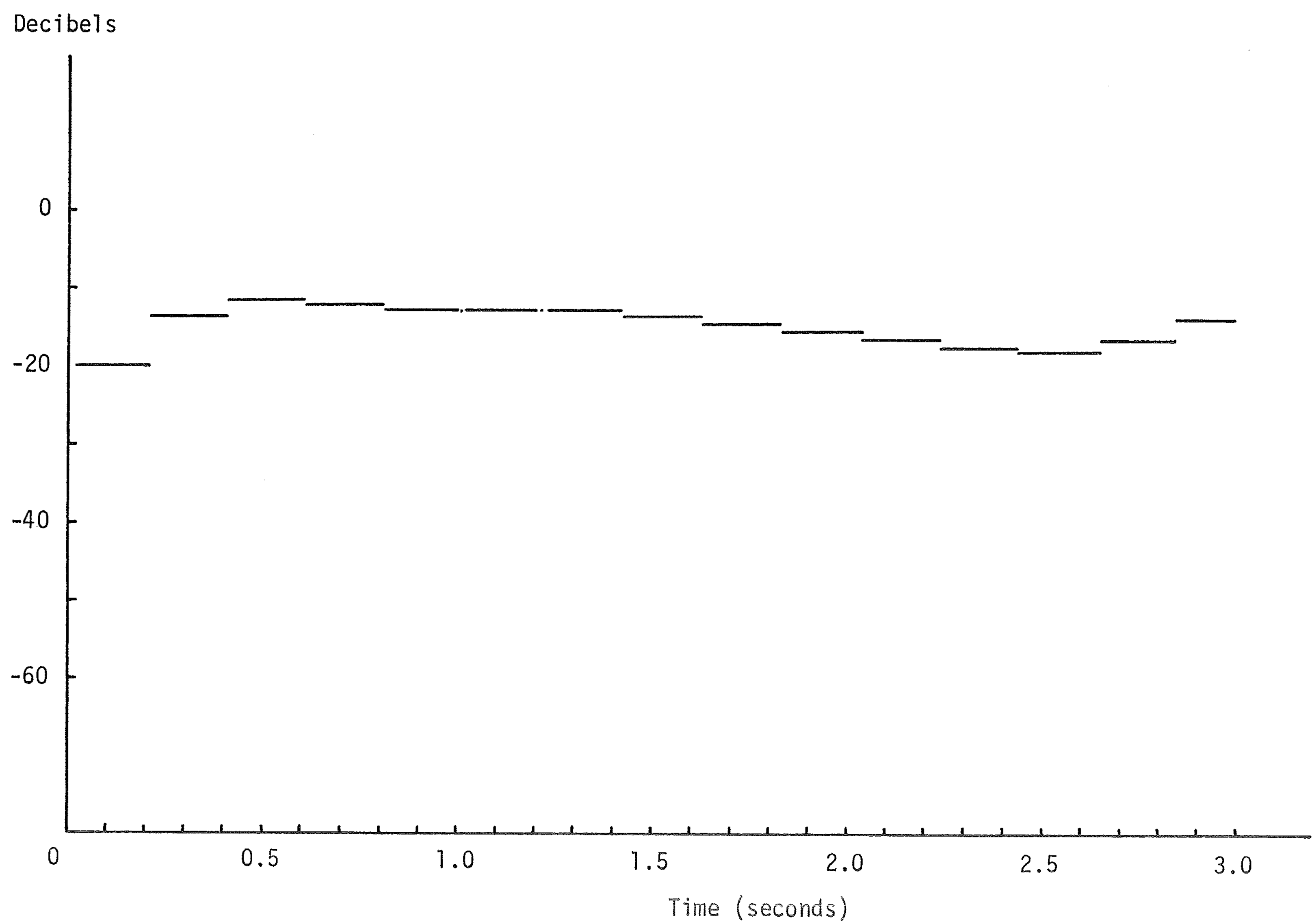


Figure B.13: Closed Loop Attenuator Response for an Initial Attenuator Setting of =10.0 Decibels with Cont = 0.7



Fi ure B.14: Closed Loop Attenuator Response for An Initial Attenuator Setting of -20.0 Decibels with Cont = 0.7

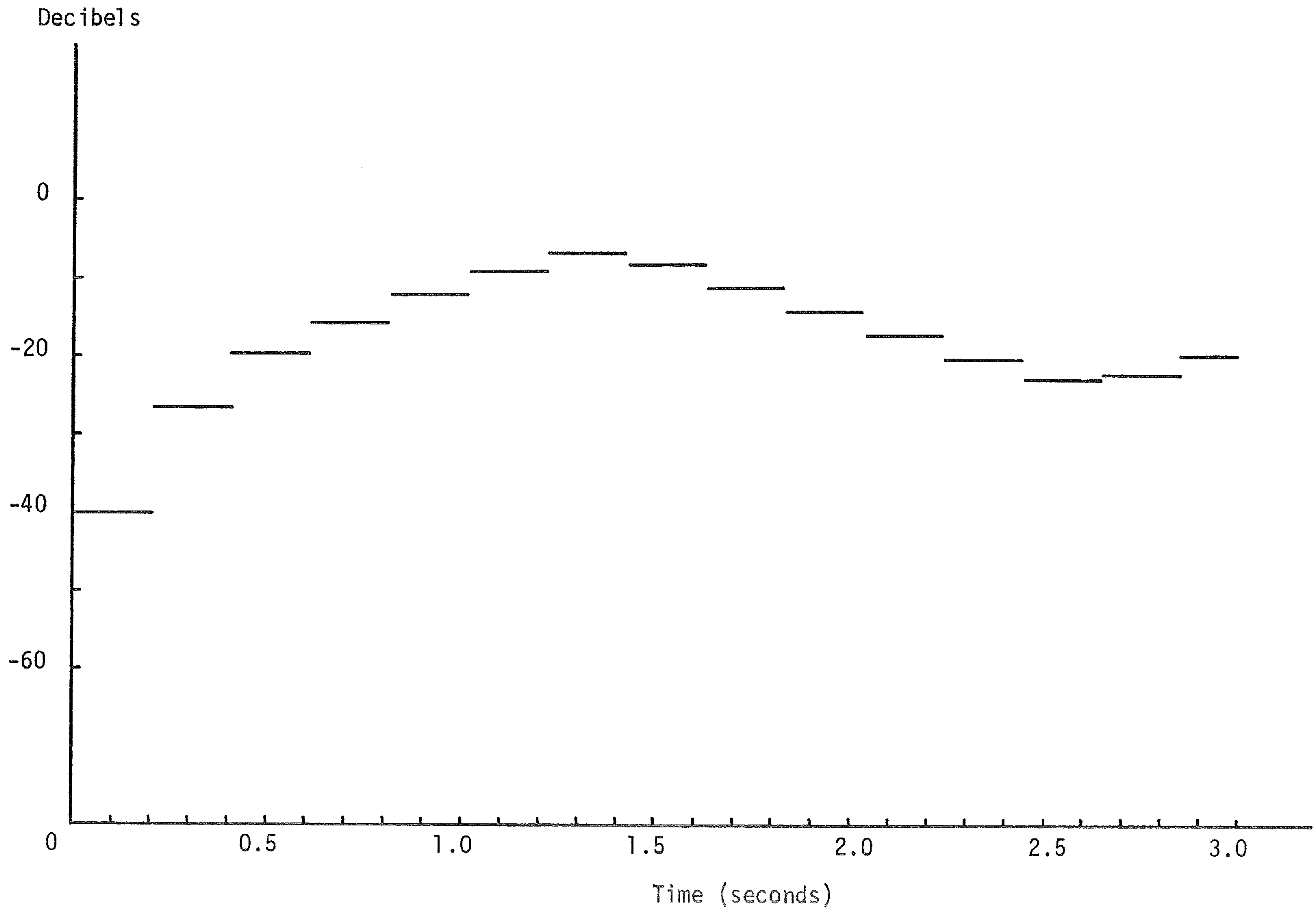


Figure B.15: Closed Loop Attenuator Response for an Initial Attenuator Setting of -40.0 Decibels with Cont = 0.7

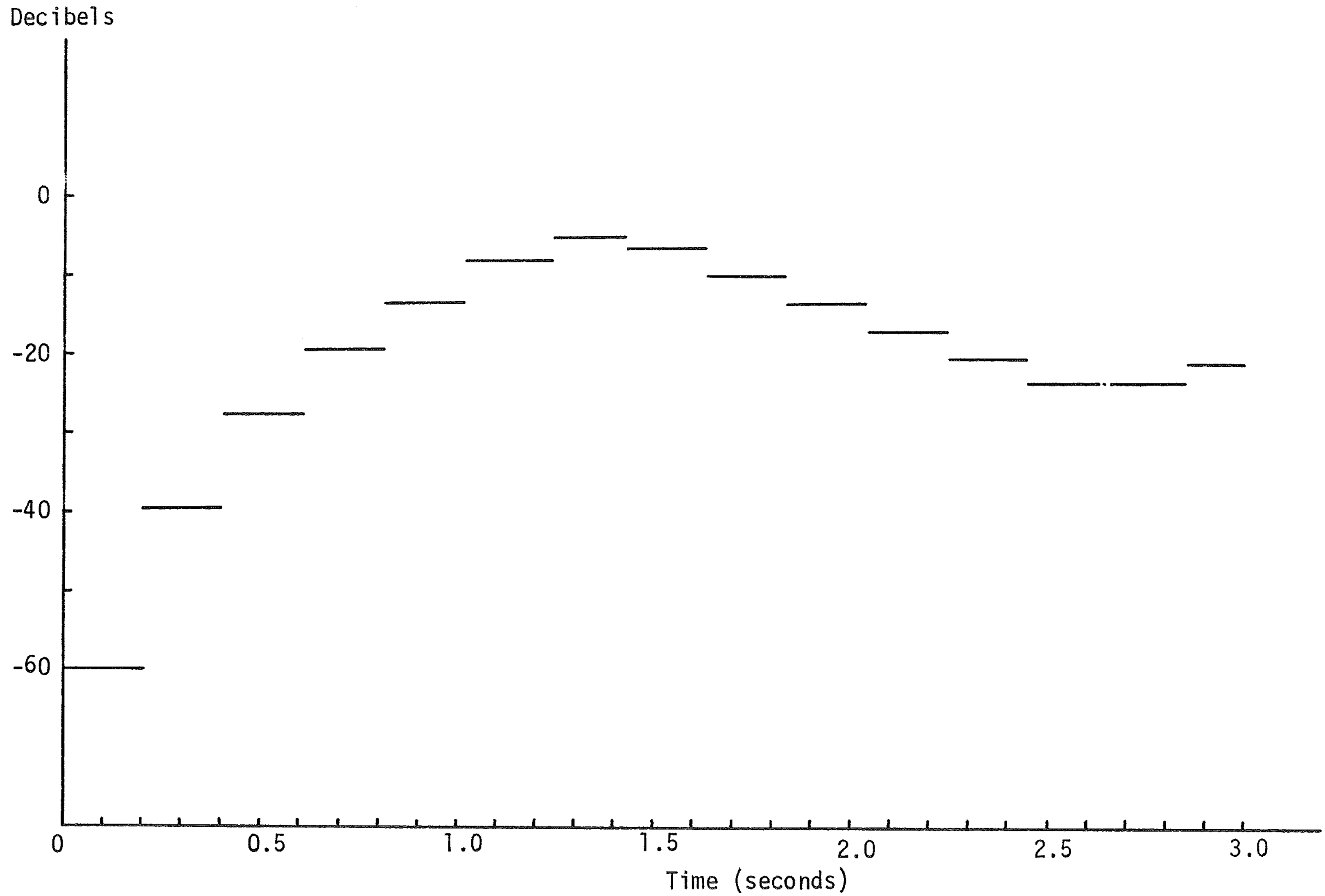


Figure B.16: Closed Loop Attenuator Response for an Initial Attenuator Setting of -60.0 Decibels with Cont = 0.7

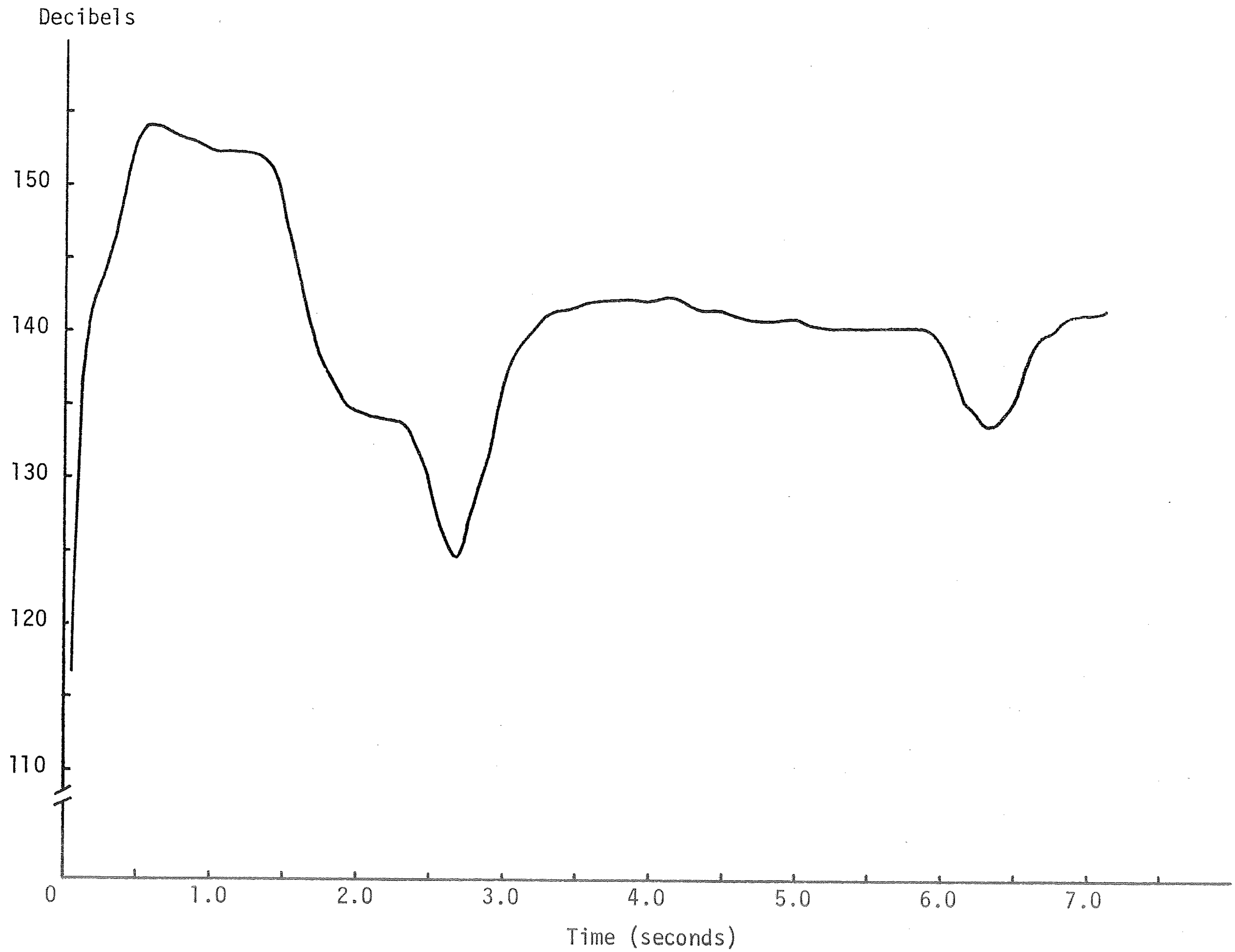


Figure B.17: Closed Loop R.M.S. Time Response for an Initial Attenuator Setting of 0.0 Decibels with Cont = 0.7

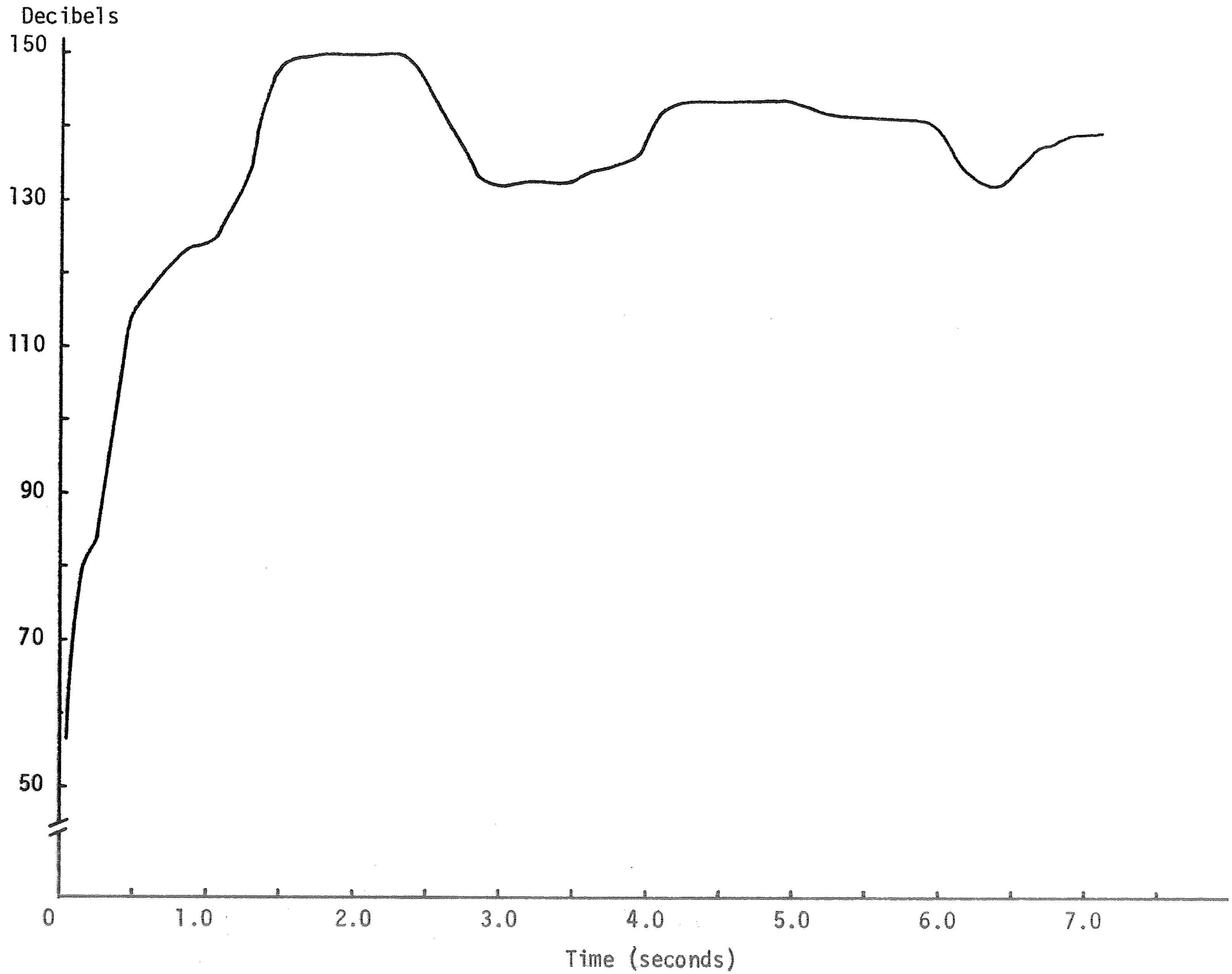


Figure B.18: Closed Loop R.M.S. Time Response for an Initial Attenuator Setting of -60.0 Decibels with Cont = 0.7

APPENDIX C

LISTING OF THE DIGITAL COMPUTER SIMULATION OF THE SINGLE CHANNEL 25.0 HERTZ 1/3 OCTAVE SYSTEM

A listing of the digital computer simulation of the single channel 25.0 Hertz 1/3 octave system is shown on the following pages. Fortran IV programming language was used, and the simulation was performed on the IBM 360 digital computer.

```
C       MAIN PRCGRAM
  101 FORMAT(5X,10G10.4)
  152 FORMAT(2X,I5,8(3X,G12.6))
  201 FORMAT(2F8.4)
  252 FORMAT(5G15.6)
  352 FORMAT(5X,'STEP',2X,'TIME',10X,'ES1DB',10X,'ES2DB',10X
     1,'RMSDB',10X,'RMS1DB',9X,'CONT',11X,'ATSET',10X,'ATGN')
      INTEGER IBIG(10),STEP
      REAL Y(60),LF,RBIG(1270),RANVEC(1000)
      DIMENSICN XTIME(200),YRMSCB(200)
      CCMMCN/RBLK/R,E,OM,H,EPS
     1            ,SN,X,C,B,TIME
     2            ,A(10),LF(20)
     3         ,TSTOP,SCM,VEC(100)
     4        ,RMS,RMSDB,RMS1
     5        ,ES1DB,ES1RMS,ES2DB,ES2RMS,RMS1DB
     6        ,DBCH,ATSET,ATGN,ATCUT,AMPAT
     7        ,RANSN,TIMRAN,RDB,RRMS,CONT
     8        ,SVEC(1000),BVEC(100),TSOR,RM,ESDBAV,SUMDB,ARMSDB
     9        ,RANSIG,SUMERS,DBMAX,DBMIN,TC
      COMMCN/IBLK/LIMIT,NEC,NWRITE,NTD,NTD2
     1        ,NTD3,IICP,STEP,MR,LTD
      EQUIVALENCE (LIMIT,IBIG),(R,RBIG)
      DATA RANVEC/1000*0.C/
      DO 100 I=1,10
  100 IBIG(I) = C
      DC 102 I=1,1270
  102 RBIG(I) = C.0
    1 READ (5,252,END=300) IN1,IN2
      WRITE (6,101) IN1,IN2
      IF (IN1) 10,10,2
    2 IBIG(IN1)=IN2
      GO TC 1
   10 CCNTINUE
      NTD=NEQ
      NTD2=NTD+NEQ
      NTD3=NTD2+NEQ
   11 READ (5,252) IN1,RN1
      WRITE (6,101) IN1,RN1
      IF (IN1) 20,20,12
   12 RBIG(IN1)=RN1
      GC TC 11
   20 CCNTINUE
   21 READ (5,252) IN1,RN1
      IF (IN1) 30,30,22
   22 Y(IN1)=RN1
      Y(IN1+NEQ)=RN1
      Y(IN1+NTD2)=RN1
      GC TC 21
   30 CCNTINUE
      IF(LTD.GT.0) GC TO 91
```

```
      IA = 1
   51 READ(5,201) TIMRAN,RANSN
      IF(TIMRAN) 91,91,52
   52 RANVEC(IA) = RANSN
      IA = IA+1
      GC TC 51
   91 CONTINUE
      IMO = IA
      TIME = 0.0
      SCM = 0.0
      TSOR = C.C
      RM = 0.C
      ES1DB = 0.0
      ES2CB = 0.0
      RMSCB = C.C
      RMS1DB = C.0
      DBCH = 0.0
      ATGN = 0.0
      LC = 0
      IICP = 1
      DC 104 I=1,60
  104 Y(I) = 0.0
      CO 1C6 I=1,1000
  1C6 SVEC(I) = C.0
      DC 108 I= 1,100
  108 BVEC(I) = C.0
      STEP = C
      LCOUNT=C
      CALL SLCPE
      WRITE (6,352)
      WRITE(6,152) STEP,TIME,ES1DB,ES2DB,RMSDB,RMS1DB,ATSET
     1,Y(IICP+5),CONT
   50 STEP=STEP+1
      TIME = STEP*H
      LCOUNT=LCCUNT+1
C
C     DIGITAL SIMULATICN CF FEEDBACK CONTROL SYSTEM
C
C     CHANGES IN ATTENUATCR SETTINGS ARE CALCULATED AT 0.204
C     SECCND INTERVALS.
      KNT = MCD(STEP,51)
      IF(KNT.NE.0) GO TO 18
C
C     ERRCR CALCULATED IN CB.
      ES1CB = RDB-RMSDB
C
C     CONTROLLER LOGIC. DB CHANGE IN ATTENUATCR IS CALCULATED.
      DBCH = C.5*IFIX(ES1CB*CCNT+SIGN(0.5,ES1CB))
C
C     ATTENUATOR SETTING IN DB .
      ATSET = ATSET+DBCH
```

```
      IF(ATSET.GE.0.0) ATSET = 0.C
      IF(ATSET.LE.-60.0) ATSET = -60.0
   18 CCNTINUE
C
C     ATTENUATCR GAIN.
      ATGN = 10.0**(ATSET/20.0)
C
C     RANCOM SIGNAL INPUT TO ATTENUATOR FROM WAVE SHAPER
      MKT = MCD(STEP,IMO)
      IF(MKT.NE.O) LKT = MKT
      IF(MKT.EQ.O) LKT = IMO
C
C     ATTENUATCR CUTPUT .
      RANSIG = RANVEC(LKT)
      ATCUT = RANSIG*ATGN
C
C     ALL ATTENUATOR OUTPUTS ARE SUMMED, AND PUT THROUGH AN
C     CVERALL AMPLIFIER ANC ATTENUATOR. THIS SETS AN OVERALL
C     SYSTEM GAIN.
      AMPAT = ATCUT*1000CCCC.C
C
C     SIMULATICN CF 1/3 CCTAVE FILTER ( 25 HZ CENTER FREQ. )
      CALL DCIT(Y)
C
C     RMS VALUE CALCULATEC, 250 SAMPLES USED.  TEN SAMPLES AT
C     A TIME.
      NR = MCC(STEP,250)
      IF(NR.NE.C) MR=NR
      IF(NR.EQ.C) MR = 250
      IF(STEP.GE.10.AND.STEP.LE.250) CT = TIME/H
      IF(STEP.LT.10.OR.STEP.GT.25C) CT = 25C.C
      KA = MCD(STEP,10)
      IF(KA.NE.C) KO = KA
      IF(KA.EQ.O) KO = 10
      KCO = MR/1C
      X = Y(IICP+5)**2
      TSOR = TSCR+X
      IF(KC.NE.10) GC TC 19
      RM = RM + TSOR-BVEC(KOO)
      BVEC(KCC) = TSOR
      RMS = SQRT(RM/CT)
C
C     RMS VALUE IN DB CALCULATEC.
      IF(RMS.GT.10.0) RMSCB = 2C.C*ALOG10(RMS)
      TSCR = C.C
   19 CCNTINUE
C
C     RMS VALUE CALCULATEC,25C SAMPLES USED. CNE SAMPLE AT
C     A TIME.
      SCM = SCM+X-SVEC(MR)
      SVEC(MR) = X
```

```
      RMS1 = SQRT(SOM/CT)
C
C     RMS VALUE IN DB CALCULATED.
      IF(RMS1.GT.10.0) RMS1DB = 20.0*ALOG10(RMS1)
C
      IF(LCOUNT.NE.NWRITE) GO TO 60
      WRITE(6,152) STEP,TIME,ES1DB,ES2DB,RMSDB,RMS1DB,ATSET
     1,Y(IICP+5),CONT
      LCOUNT=0
   60 CONTINUE
      IF(LKT.NE.IA) GO TO 50
      LTD = LTD+1
      IF(LTD.EQ.1) GO TO 50
  300 STOP
      END
      SUBROUTINE SLOPE
      INTEGER IBIG(10),STEP
    1 FORMAT(20A4)
      REAL Y(60),LF,RBIG(1270),RANVEC(1000)
      COMMON/RBLK/R,E,OM,H,EPS
     1             ,SN,X,C,B,TIME
     2             ,A(10),LF(20)
     3         ,TSTOP,SCM,VEC(100)
     4        ,RMS,RMSDB,RMS1
     5        ,ES1DB,ES1RMS,ES2DB,ES2RMS,RMS1DB
     6        ,DBCH,ATSET,ATGN,ATCUT,AMPAT
     7        ,RANSN,TIMRAN,RDB,RRMS,CONT
     8        ,SVEC(1000),BVEC(100),TSOR,RM,ESDBAV,SUMDB,ARMSDB
     9        ,RANSIG,SUMERS,DBMAX,DBMIN,TC
      COMMON/IBLK/LIMIT,NEC,NWRITE,NTD,NTD2
     1        ,NTD3,IICP,STEP,MR,LTD
      EQUIVALENCE (LIMIT,IBIG),(R,RBIG)
      RETURN
      ENTRY RHS(Y)
C
C     TRANSFER FUNCTION OF 1/3 OCTAVE FILTER WITH CENTER
C     FREQUENCY OF 25.0 HERTZ.
      LF(1) = Y(2)
      LF(2) = -A(2)*Y(2)-A(1)*Y(1)+A(7)*AMPAT
      LF(3) = Y(4)
      LF(4) = -A(4)*Y(4)-A(3)*Y(3)+A(8)*Y(2)
      LF(5) = Y(6)
      LF(6) = -A(6)*Y(6)-A(5)*Y(5)+A(9)*Y(4)
      RETURN
      END
      SUBROUTINE DOIT(Y)
      INTEGER IBIG(10),STEP
      REAL Y(60),LF,RBIG(1270),RANVEC(1000)
      COMMON/RBLK/R,E,OM,H,EPS
     1             ,SN,X,C,B,TIME
     2             ,A(10),LF(20)
```

```
     3         ,TSTOP,SOM,VEC(1CC)
     4         ,RMS,RMSDB,RMS1
     5         ,ES1DB,ES1RMS,ES2DB,ES2RMS,RMS1DB
     6         ,DBCH,ATSET,ATGN,ATOUT,AMPAT
     7         ,RANSN,TIMRAN,RDB,RRMS,CONT
     8         ,SVEC(1000),BVEC(1CO),TSCR,RM,ESDBAV,SUMDB,ARMSDB
     9         ,RANSIG,SUMERS,DBMAX,DBMIN,TC
      COMMCN/IBLK/LIMIT,NEQ,NWRITE,NTD,NTD2
     1         ,NTD3,IICP,STEP,MR,LTD
      KCUNT=0
      IICPA=IICP
      IICPP=IICP
      IN1=IICP
   50 IN2=IICPA
      IN3=IICPP
      CALL RHS(Y(IN2))
C     LF ARRAY RETURNED BY ABCVE
      IF (KCUNT.GT.0) GC TC 55
      IICPA=IICP+NTD
      IF (IICPA.GT.NTD3) IICPA=1
      IN2=IICPA
   55 SUM1=0.0
      SUM2=0.C001
      DC 60 I=1,NEQ
      TEMP=Y(IN1)+LF(I)*H
      SUM1=SUM1+ABS(Y(IN3)-TEMP)
      SUM2=SUM2+ABS(TEMP)
      Y(IN2)=TEMP
      IN1=IN1+1
      IN2=IN2+1
   60 IN3=IN3+1
      IF(KCUNT.GT.0) GO TO 65
      IICPP=IICP-NTD
      IF(IICPP.LT.1) IICPP=NTD2+1
      IN3=IICPP+NEQ
   65 ITEMP=IICPA
      IICPA=IICPP
      IICPP=ITEMP
      KCUNT=KCUNT+1
      IF((SUM1/SUM2).GT.EPS.AND.KCUNT.LT.LIMIT) GO TO 70
      IICP=IICPP
      RETURN
   70 DC 75 I=1,NEQ
      IN1=IN1-1
      IN2=IN2-1
      IN3=IN3-1
   75 Y(IN3)=(Y(IN1)+Y(IN2))*0.5CODO
      GC TC 50
      END
```