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THEORY AND APPLICATION OF AN EDGE GRADIENT SYSTEM FOR GENERATING OPTICAL TRANSFER FUNCTIONS

by Edwin Klingman

George C. Marshall Space Flight Center
Marshall Space Flight Center, Ala. 35812

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16. Abstract

A system has recently been developed that is capable of measuring optical transfer functions quite accurately. This report contains the theory, system description, computer program, and instructions necessary to use this system for practical measurements. Results of the measurements made by the system are also discussed.

The Modulation Transfer Functions generated by the system have been found extremely useful for purposes of comparing loss of resolution through optically contaminated quartz flats. Absolute measurements have not been made, however results indicate quite good accuracies are obtainable. For comparative measurements, the system is superb.

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SUMMARY

A system recently developed to generate optical transfer functions (OTFs) using the edge gradient analysis (EGA) technique is described herein. The basic equations and the system components and functions are presented and discussed. The computer program and its input are given and discussed in detail, as well as interface and operating instructions. Results of measurements made by the system are presented and discussed. No real effort was made at calibration since the system was built to satisfy a need for comparison of OTFs under various conditions of contamination; the system is excellent in this respect.

INTRODUCTION

Edge gradient analysis techniques have been developed[1,2] and are under consideration for use in determining the optical degradation of contaminated lens. Such a program is necessary for interpretation of the data obtained in space flight optical experiments of the ATM type. The EGA measurements are being performed by the Optical Physics Branch, Space Sciences Laboratory, Science and Engineering, Marshall Space Flight Center. Analysis techniques for handling the data are being developed by the Mathematical Physics Branch of the same laboratory. This report discusses the data analysis of the EGA measurements.

EDGE GRADIENT THEORY

Until recently, the standard method of measuring the resolving power of lens systems has been the bar chart. Although this technique has the advantages of cheapness and simplicity, it is not necessarily reproducible by other observers, because of the inherent subjectivity involved. As the requirements for standardized measurements arose

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1. M. A. Berkovitz: Edge Gradient Analysis OTF Accuracy Study. MTF Seminar Proceedings. Perkin-Elmer Corporation, Boston, Massachusetts, March 21 and 22, 1968.

2. John Williams: The Optical Transfer Function and Its Applications. NASA-MSFC IN-SSL-P-70-5.

with television systems and aerial photography, new methods were invented to determine system resolution. One particularly simple method is that of EGA. The basic setup is shown in Figure 1.

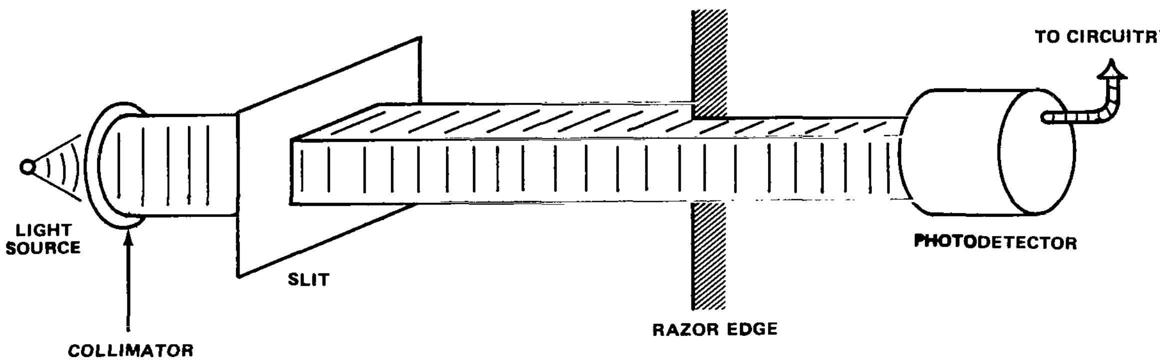


Figure 1. Edge gradient detector.

With geometric optics, the intensity that fell on the photodetector as the razor edge moved across the beam varied as shown in Figure 2a. With diffraction-limited systems, the result appeared as shown in Figure 2b. Any perturbations on the system would degrade the image still further.

A. GEOMETRICAL OPTICS

B. DIFFRACTION LIMITED OPTICS

Figure 2. Illuminance versus edge position.

The output of the edge gradient scanner will have the form of the edge scan shown in Figure 2a. If $E(x)$ is the distribution of illuminance per unit distance on the edge scan, then the spread function $S(x)$ is given by $S(x) = dE(x)/dx$. The OTF is the Fourier transform of the spread function, i.e.,

$$O(k) = \int_{-\infty}^{+\infty} S(x)\, e^{i2\pi kx}\, dx \quad .$$

Here, the spatial frequency k is numerically the reciprocal of the distance between corresponding points on adjacent elements of a repetitive pattern and is expressed in cycles per unit length. Thus it is seen to be related to the number of lines that the system is capable of resolving. Such a figure of merit can be produced objectively, thereby eliminating the inherent subjectivity of an observer reading a bar chart. The ability to detect small changes in the resolution of the system makes a technique of this type particularly valuable for examining the effects of contamination in optical systems.

By use of de Moires' theorem, the complex OTF can be written in terms of its real and imaginary components,

$$O(k) = R(k) + i\, I(k) \quad ,$$

where

$$R(k) = \int_{-\infty}^{+\infty} S(x) \cos(2\pi kx)\, dx$$

and

$$I(k) = \int_{-\infty}^{+\infty} S(x) \sin(2\pi kx)\, dx \quad .$$

These integrals are seen to be the sine and cosine Fourier transforms of the spread function S(x). In terms of the modulation transfer function (MTF) and the phase transfer function (ΦTF), the OTF can be written as

$$O(k) = M(k)\, e^{i\Phi(k)} \quad .$$

The magnitude M(k) is given by

$$M(k) = \sqrt{R^2(k) + I^2(k)}$$

and the phase $\Phi(k)$ by

$$\Phi(k) = \tan^{-1}\left(\frac{R(k)}{I(k)}\right) \quad .$$

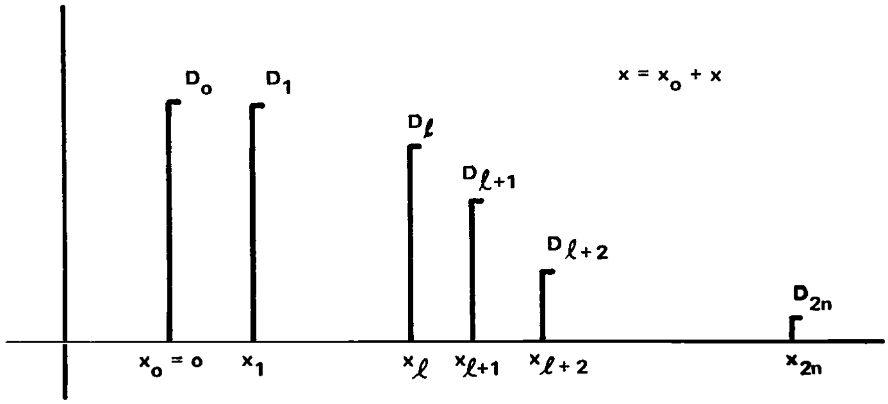
Then the problem is to obtain M(k) and $\Phi(k)$ from an edge scan. This will be done numerically as explained in the next section. A procedure of this type essentially samples the curve in Figure 3a at discrete intervals to generate the set of data points shown in Figure 3b. The continuous function E(x) is thus approximated by the data points $D_j \equiv E(x_j)$ at each point x_j, j = 1, 2, 3, . . . , 2n (Fig. 4). Because of the smoothness of the functions involved, it was found to be sufficient to use a two-point formula for determination of the spread function S(x), i.e.,

$$\left.\frac{dE}{dx}\right|_{x=x_{j+1}} \sim \frac{D_{j+1} - D_j}{x_{j+1} - x_j}$$

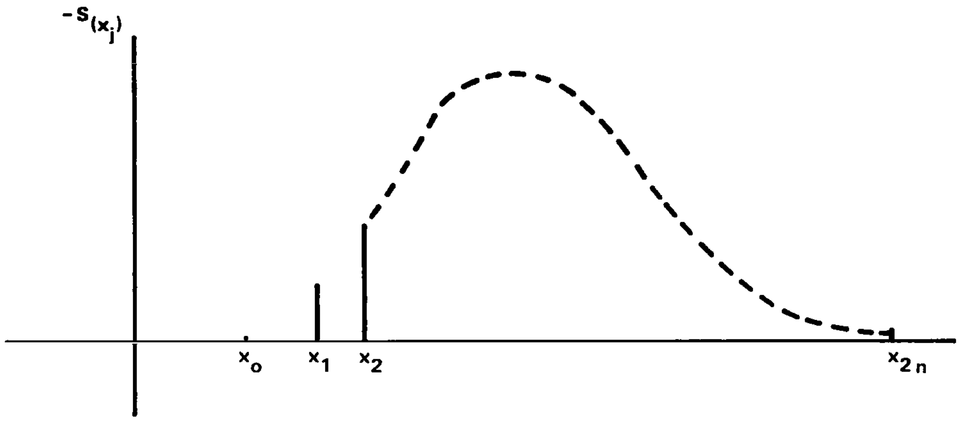
A. OUTPUT OF SCANNER B. SAMPLED OUTPUT OF SCANNER

Figure 3. Conversion of output of scanner into forms suitable for numerical treatment.

although more points could be used if it were considered necessary. The system that was used [the multichannel analyzer (MCA)] automatically provides a uniform grid; i.e., $x_{j+1} - x_j = \Delta x$ for all j's. This allows the use of a Simpson's rule integration procedure whereby the points can be handled sequentially and the integral can be calculated by using a running sum. The formula to be used is



A. SAMPLED ILLUMINANCE CURVE



B. COMPUTED POINTS ON SPREAD FUNCTION

Figure 4. System internal data.

$$\int_{x_o}^{x_{2N}} f(x)\,dx = \frac{\Delta x}{3}\left[f_o + 4(f_1 + f_3 + f_{2n-1}) + 2(f_2 + f_4 + \ldots f_{2n-2}) + f_{2n}\right],$$

where

$$f_{i+1} = \frac{D_{i+1} - D_i}{x}\begin{Bmatrix}\sin\\ \cos\end{Bmatrix}(2\pi k x_{i+1})$$

and

$$f_o \equiv 0 \quad .$$

This last condition is assured by always starting on the flat part of the curve where the slope is zero.

SYSTEM DESCRIPTION

The block diagram shown in Figure 5 depicts the system used to generate MTFs. The components shown in the dotted lines will be described in detail.

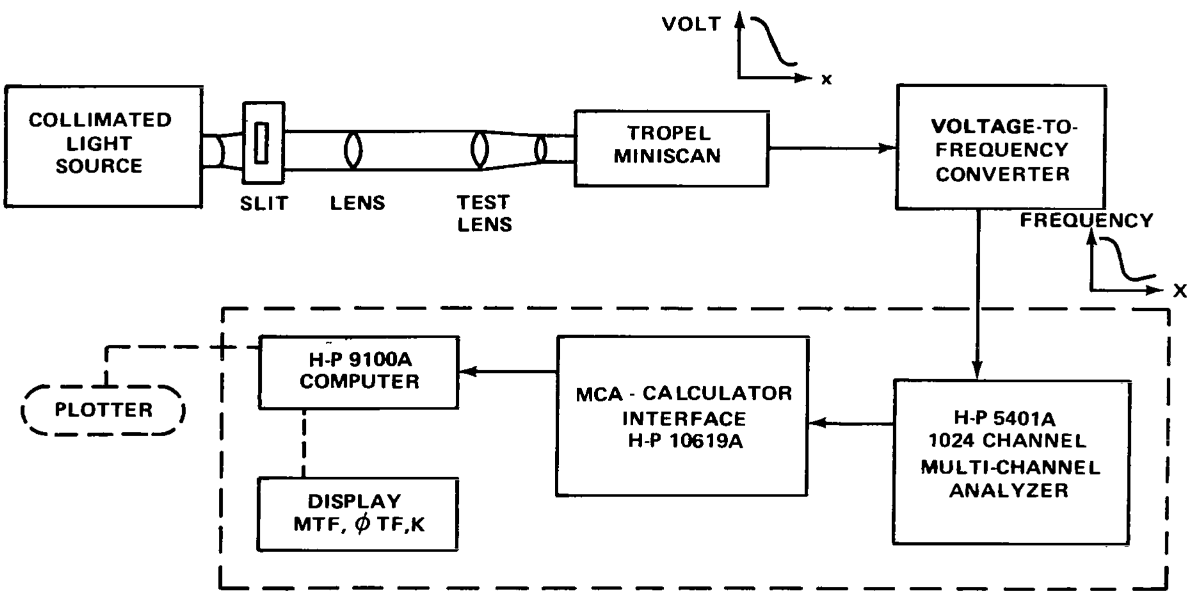


Figure 5. Block diagram of the system.

The optical arrangement is such that the test lens images a slit of (incoherent) light on the Tropel Miniscan. The miniscan contains a razor edge, as shown in Figure 1, that scans across the beam. The resulting change in the intensity seen by the photodetector behind the edge results in the voltage-versus-position curve as shown. This voltage is used to drive a V to F converter whose output is a frequency-versus-position curve that duplicates the input voltage curve. This frequency is then inputted to an

MCA that repetitively accounts for fixed increments of time. This accomplishes the sampling of the frequency curve as indicated in Figure 3. One scan is performed by the miniscan, and the sampled output is held in storage in the MCA. The MCA is interfaced to a mini-computer, and the data points are transferred channel by channel upon request. These points are used in the numerical procedure described earlier.

In the following section it will be assumed that the edge scan data have been loaded into the channels of the MCA so that the frequency at a given position is proportional to the illuminance at that position as measured by the scanning apparatus. The MCA in the multichannel scaling mode (MCS) counts pulses with no analysis of the amplitude of these pulses. The instrument counts the number of pulses in the input pulse train during a gating interval, stores this count in one channel of memory, and counts the number of pulses in the next gating interval for the next memory channel. This process is repeated sequentially in order to generate a series of numbers in memory that represent the frequency changes with time. These numbers can then be numerically analyzed to produce the MTF. To analyze these numbers most efficiently, it is desirable to interface the MCA with the computing unit that performs the numerical operations, as discussed in the next section.

Interfacing the Hewlett-Packard 9100A to the MCA

At this point it is necessary to discuss the specific system components in detail. Obviously, the interconnections and procedures will vary from system to system. The components used are listed below.

Hewlett-Packard Model 5401A MCA System which consists of the following:

5410A Power Supply/Interface

5416A Analog-to-Digital Converter

5421A Digital Processor

H-51-180AR Oscilloscope Mainframe

5431B Display Plug in

Hewlett-Packard MCA-Calculator Interface 10619A

Hewlett-Packard 9100A Calculator

The system interconnections are as follows:

1. Connect the 50-pin ribbon connector of the 10619-60003 cable assembly to 5410A rear-panel connector J5, labelled SERIAL OUTPUT.

2. Connect the short BNC cable to the EXT START connector on the rear panel of Digital Processor 5421A.

3. Connect the plastic-hooded connector of the 10619-60003 cable to the calculator rear-panel mating connector.

Operation of the interfaced system will now be described further because the system does not operate exactly as specified by the MCA-Calculator Interface Operating and Service Manual.

Data transmission from the MCA to the calculator is initiated by a FORMAT command from the calculator keyboard or stored program. A channel of data (6 digits) from the MCA is sent to the calculator, followed by a CONTINUE entry. The next FORMAT command causes the next channel of MCA data to be entered, etc.

Two special entries are supposed to be available with the 5401A MCA, selected by installing plug-in jumpers on the 10619-60002 Readout Control Card. One jumper enables selection of the 5401A starting address as initial data entry; the other enables a SET FLAG - CONTINUE to be entered after final data entry, which causes a conditional branch in the calculator program. These jumpers were installed, but neither of the special entries was found to be present. This is possibly because the digital processor is a model 5421A rather than a 5422A although the interface manual indicates that the 5421A is acceptable. With the 5421A Digital Processor, the first FORMAT command and all subsequent FMT commands return the contents of the channels as data. After the final data entry, the MCA will stop.

Several interesting aspects of operation were noted that are not mentioned in the Interface Operating Manual. Assume that the MCA is set to return channels 100 to 250 as shown in Figure 6.

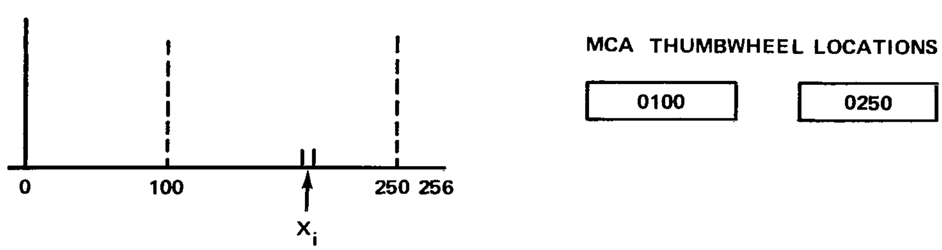


Figure 6. MCA set to return channels 100 to 250.

If the MCA thumbwheels are set to bracket the region 100 to 250, the first FORMAT command will return the data in channel 100. The next FORMAT command will return the data from channel 101, etc. This sequence will continue until either channel 250 is reached and the MCA stops or the program logic branches to a section of the program with no FORMAT commands. If for some reason it is desirable to stop at location x_i, there are several ways to do this. A programmed STOP will stop the program at the appropriate address, and a manual CONTINUE command will start the program running so that the next data location will be x_{i+1}. If the PAUSE key is held down, the program will stop at the next PAUSE statement in the program or at the next FORMAT command. Again the CONTINUE command resumes at the following data location. If the program is halted with a manual STOP command entered from the keyboard, both the program and the MCA will stop (the green light on the digital processor will go OFF). When a CONTINUE command is sent, the MCA will return to the initial data location (channel 100 in this case) and begin again from this point.

The Computer Program Used to Calculate the Optical Transfer Function

The program used to compute the MTF and the ΦTF as functions of spatial frequency uses a simple two-point formula to compute the spread function and a 2n-point Simpson's rule integration to sum the integral, where 2n is the number of channels from which data are obtained. The six storage registers of the 9100A Calculator are used to store the following data:

Storage Register	Information Stored
a	j^{th} data point D_j
b	spatial frequency $k = k_o + \Delta k$
c	normalizing factor for MTF
d	j^{th} channel location x_j
e	sum imaginary integral
f	sum real integral

The flow diagram for the 9100A Program to calculate the MTF and ΦTF is shown in Figure 7, and the program is listed in Table 1.

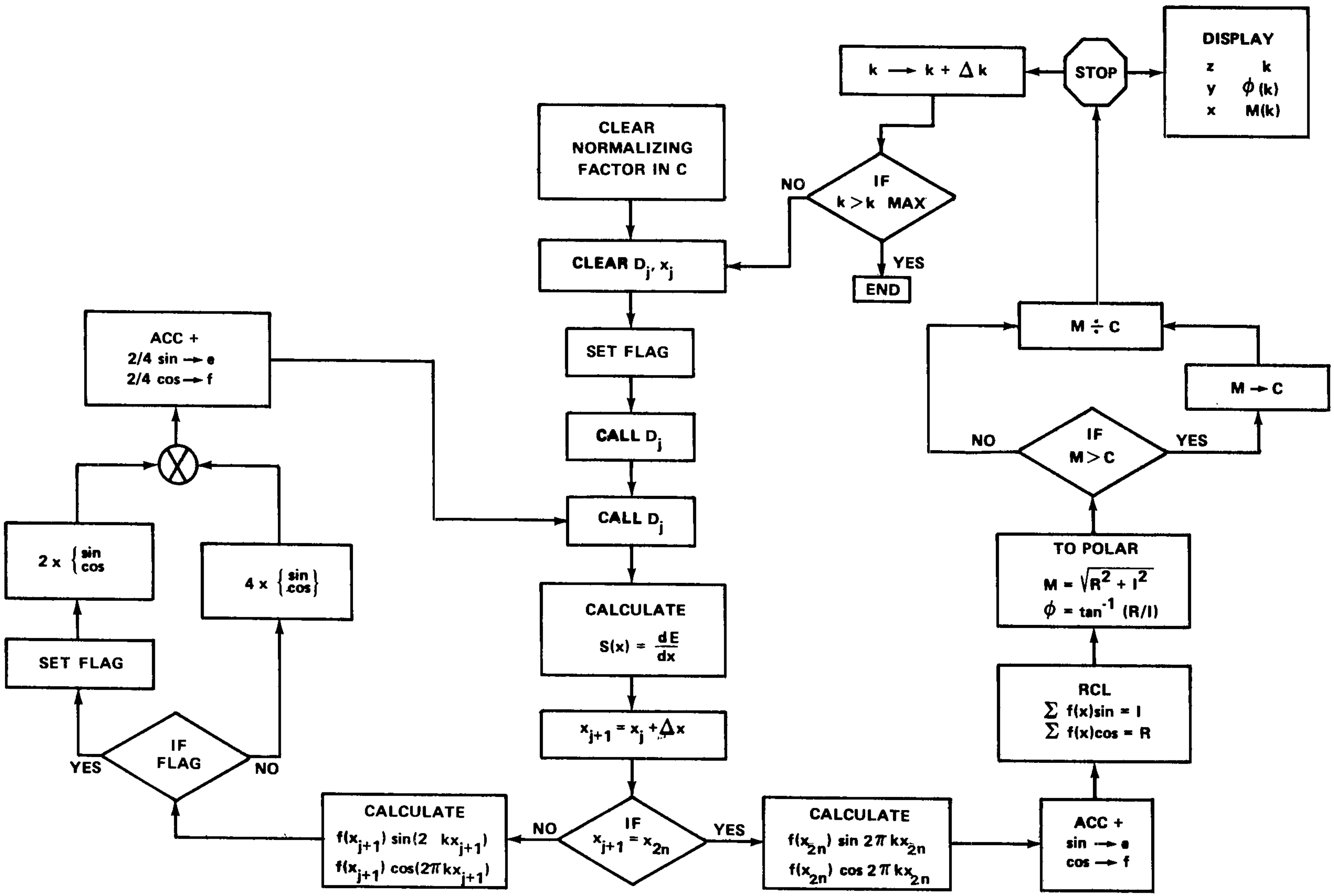


Figure 7. Flow chart for calculation of OTF from MCA data.

TABLE 1. H-P 9100A PROGRAM USED TO CALCULATE OTF FROM MCA DATA

Address	Instruction	Comment
00	CLEAR x → ( ) c	initialize normalizing factor
03	CLEAR x → ( ) a x → ( ) d	initialize address and data storage
08	SET FLAG FMT	returns first data point from MCA to x-register
	x → ( ) a	load first data point in a-register
0c	FMT	returns j^{th} data point from MCA to x-register
0d	STOP/PAUSE/CONT	slack point
10	↑ y ⇌ ( ) a	replace D_j by D_{j+1}
	x ⇌ y −	calculate dE
15 16 17	ENTER EXP CHNG SIGN 4	dx = 0.0001
	÷	$S(x) = \frac{dE}{dx}$
  1b 1c 1d 20	d ↑ ENT. EXP CHNG SIGN 3 + y → ( ) d	$x_{j+1} = x_j + \Delta x$

TABLE 1. (Continued)

Address	Instruction	Comment
23	#	
24	#	number of channels times $\Delta x = x_{2n}$
25	#	
26	IF x = y	$x_{j+1} = x_{2n}$
27	4	
28	d	
	π	
	X	
	2	
	X	$\cos(2\pi k x_{j+1})$
	b	
30	X	
	↓	
	↑	
	COS	
34	ROLL ↓	
	SIN	
	ROLL ↓	
	X	
	ROLL ↑	
	X	
3a	IF FLAG	
3b	4	
3c	4	
	SET FLAG	Simpson's rule integration
	2	4 × odd terms
	GO TO	2 × even terms
42	4	
43	5	
44	4	
45	X	
46	ROLL ↑	
	X	
	↓	
	ACC +	
	GO TO	

TABLE 1. (Continued)

Address	Instruction	Comment
46	0	
4c	C	
4d	STOP/PAUSE/CONT	slack point
50	π	
	X	
	2	
	X	
	b	
	X	
	COS	
	ROLL ↓	
	SIN	
	ROLL ↓	
	X	
	ROLL ↑	
60	X	
	ACC +	
	RCL	R(k) in f, I(k) in e
	TO POLAR	M(k), Φ(k) in x- and y-registers
65	STOP/PAUSE/CONT	slack point
	↑	normalize M(k) to zero frequency
	c	
	IF x < y	
	y → ( )	
	c	
	c	
	÷	
	b	
70	ROLL ↓	
71	STOP	DISPLAY z k spatial frequency y Φ(k) ΦTF x M(k) MTF

NOTE: Manual STOP should be depressed at this point to return the MCA to the starting address for the next run with k incremented.

TABLE 1. (Concluded)

Address	Instruction	Comment
72	ROLL ↓	
	n ln x	calculate Δk (choose appropriate number N)
	+ $y \rightarrow (\ )$ b	increment $k' = k + \Delta k$
78	2	
79	5 k_{max}	
7a	0	could easily be omitted with no real inconvenience
	IF $x > y$	
7c	0	
7d	3	
80	STOP	
81	END	

NOTE: With the H-P 9100B, an n-point derivative could easily be calculated; however, all results indicate that the two-point formula is sufficient.

DETERMINATION OF Δx

The spatial frequency k is normally expressed in lines per millimeter; therefore, the increment Δx should also be expressed in millimeters. The miniscan used in the OTF was driven with a 6-rpm drive resulting in a 1 μm/s scan. For most cases the MCA sample time rate was 100 ms per channel. This results in a value for Δx of

$$\frac{1\mu}{s} \times \frac{100\ \text{ms}}{\text{channel}} = 0.0001\ \frac{\text{mm}}{\text{channel}} = \Delta x \quad .$$

The general formula for determining the numerical value of Δx is:

$$\text{Miniscan rate}\left(\frac{\mu\text{m}}{\text{s}}\right) \times \text{MCA sample time rate}\left(\frac{\text{ms}}{\text{channel}}\right) = \Delta x\ (\text{mm}) \quad .$$

EXPERIMENTAL RESULTS

Results are obtained with relatively little effort with use of the OTF generator. No great amount of work was expended in attempting to calibrate the instrument. This was because only comparative MTFs were required at the time. The ability of the instrument to generate MTFs for detecting changes in resolution caused by contamination is indicated in Figures 8 and 9. Figure 8 illustrates the curves obtained from one lens under varying conditions that affect the resolution of the lens. Figure 9 shows changes in the MTF as a function of distance from a dirty window. Figure 10 is a typical sampled curve representing the output of the edge scan unit as stored in the MCA.

While checking out the system, an accident occurred that illustrates the Fourier character of the MTF very well. The MCA was turned off over night and somehow lost two channels of data, resulting in two sharp (negative) spikes on a normal edge scan curve. The MTF of this curve extended hundreds of times further along the spatial frequency axis with nonnegligible response all the way out. This, of course, is in agreement with the known harmonic analysis of such curves. Although this type of data is totally unrepresentative of the type of data to be analyzed with this system, it was assumed that the extreme nature of the data resulted in a good qualitative check on the behavior of the system.

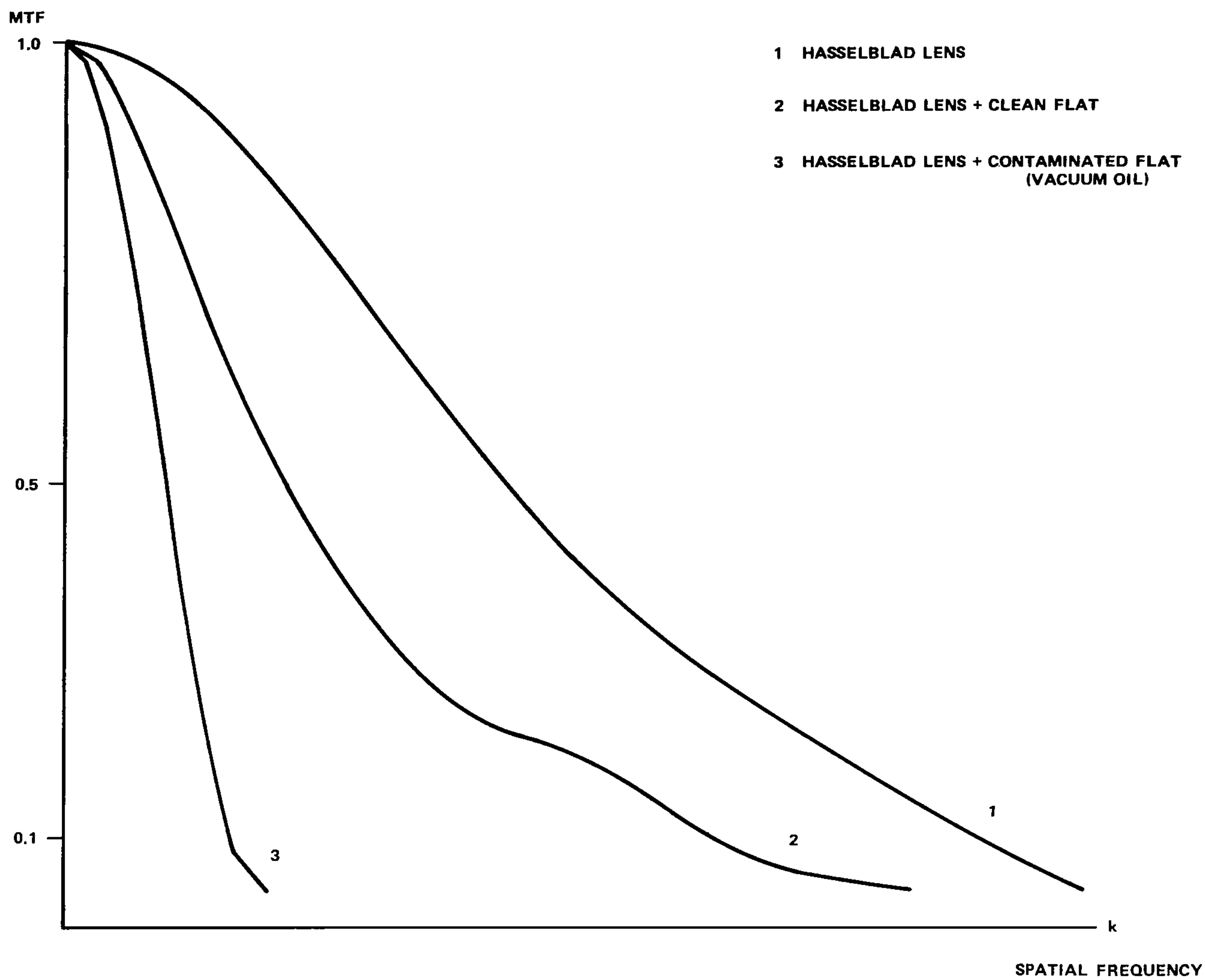


Figure 8. MTF versus contamination curves.

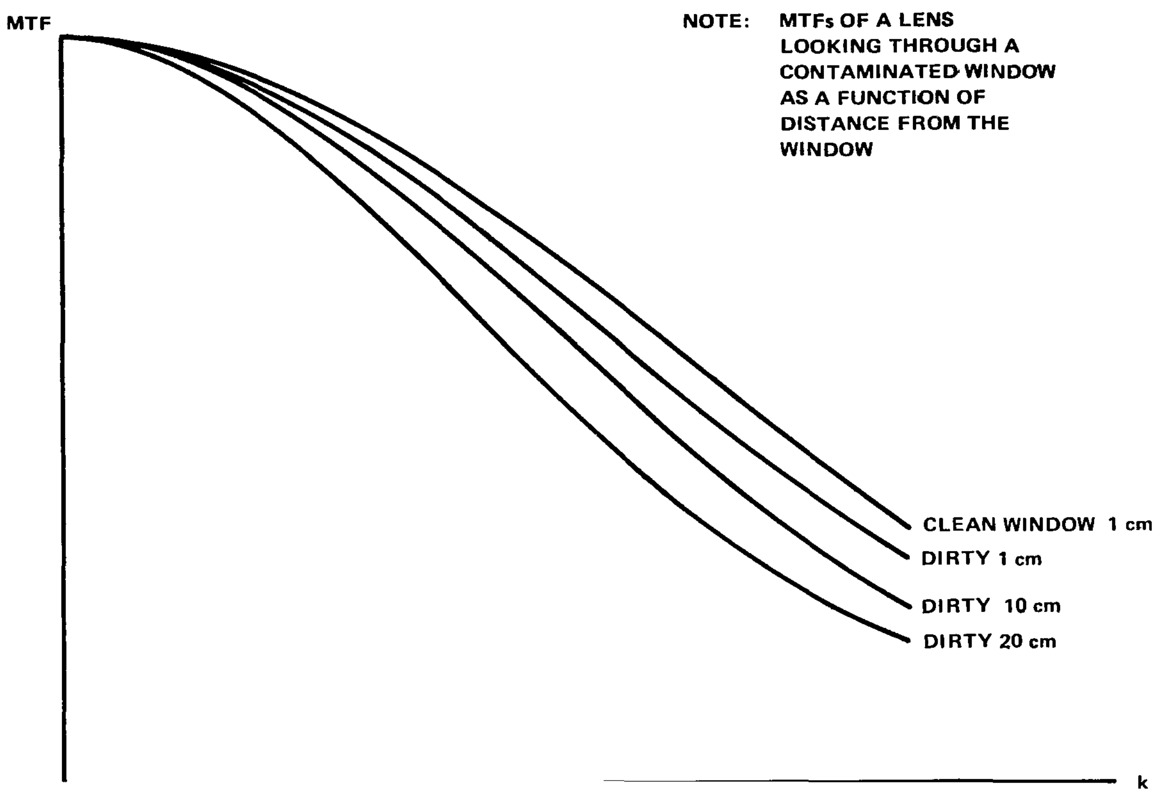


Figure 9. MTF versus distance curves.

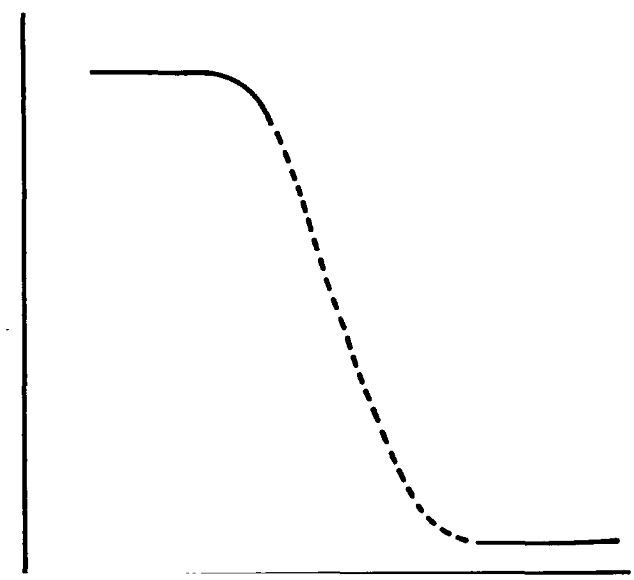


Figure 10. Sampled output of edge scanner as stored in multichannel scaler.

CONCLUSION

The MTFs calculated using the OTF generator (no use has been made of the ΦTFs) were quite successful in detecting small changes in the resolution of optical systems caused by contamination in the system. It is believed that this analysis technique will contribute measurably to the optical contamination program.

George C. Marshall Space Flight Center
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