PLASMON NEUTRINOS EMISSION IN A STRONG MAGNETIC FIELD. II: LONGITUDINAL PLASMONS

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ABSTRACT

The decay of a longitudinal plasmon into two neutrinos is studied in the presence of a strong magnetic field. Contrary to the transverse case, for longitudinal plasmons the existence of a new mode, entirely dependent on the magnetic field, greatly enhances the energy loss at high densities. Denoting by Q_H and Q_O the neutrino energy losses with and without magnetic field respectively, the situation is as follows: at $H \simeq 10^{11}$ gauss and $T \geq 10^9$ °K, $Q_O \gg 10^5 Q_H$ for $\rho < 10^{11}$ g cm^{-3}, while $Q_H \gg 10^{10} Q_O$ for $\rho > 10^{11}$ g cm^{-3}. A second physically interesting feature is the anisotropic character of the energy loss which is highly peaked along the field lines, giving rise a shorter cooling time in that direction than in any other.

Introduction:

In a previous paper (Canuto, Chiuderi and Chou, 1970) (hereafter referred to as Paper I) the effect of an intense magnetic field on the decay rate of a transverse plasmon into a neutrino-antineutrino pair was considered. It was shown that the effect is usually negligible in the regions of astrophysical interest. This is mainly due to the fact that the magnetic field enters into the formulas only through the cyclotron frequency $\omega_c = \frac{eH}{mc} = \frac{H}{H_q}\frac{mc^2}{\hbar}$ which is very small in comparison with the plasma frequency, ω_p, $\frac{\omega_c}{\omega_p} \simeq 25 \frac{H}{H_q} \rho_6^{-\frac{1}{2}}$.

In this paper we will study the analogous problem of the decay of a longitudinal plasmon into a neutrino-antineutrino pair in the presence of a strong magnetic field. As we shall show, there is a new longitudinal plasma mode, which reduces to $\omega = \omega_c$ for $\theta = 0$, greatly enhancing the energy loss due to neutrinos emission. Since the technique of the computation closely follows that of Paper I, we will give here only an outline of the general procedure, referring to Paper I for more details.

1. The Longitudinal Plasma Modes

The longitudinal plasmon vector potential $A_\mu^{(1)}$ can be written

$$A_\mu^{\ell}(\vec{k}) = \sum_{\vec{k}} N_\gamma^{1/2}\left\{a(\vec{k})e^{ikx} + a^\dagger(\vec{k})e^{-ikx}\right\}e_\mu^{\ell}(\vec{k}) \quad (1)$$

where

$$N_\gamma = \frac{4\pi\hbar c^2}{\Omega\,\omega^2}\left|\frac{\mathcal{T}r\,\lambda_{ij}^{\ell}}{\frac{\partial\Lambda^{\ell}}{\partial\omega}}\right| \tag{2}$$

λ_{ij}^{ℓ} and Λ^{ℓ} are the cofactors and the determinant of the modified Maxwell operator Λ_{ij}^{ℓ} given by Melrose (1968):

$$\begin{aligned} \Lambda_{ij}^{\ell} &= \Lambda^{\ell}\,\kappa_i\kappa_j \;; \qquad \kappa_i = \frac{k_i}{|\vec{k}|} \\ \Lambda^{\ell} &= \kappa_i\Lambda_{ij}\kappa_j = \kappa_i\,\epsilon_{ij}\,\kappa_j \end{aligned} \tag{3}$$

and Λ_{ij} is the Maxwell operator discussed in Paper I, namely,

$$\Lambda_{ij} = \left(\frac{ck}{\omega}\right)^2\left[\kappa_i\kappa_j - \delta_{ij}\right] + \epsilon_{ij} \tag{4}$$

ϵ_{ij} being the dielectric tensor of the medium. As discussed in Paper I we shall choose a purely classical ϵ_{ij}, in the form given by Stix (1962).

$$\epsilon_{ij} = \begin{vmatrix} S & -i\mathcal{D} & 0 \\ i\mathcal{D} & S & 0 \\ 0 & 0 & \mathcal{P} \end{vmatrix} \tag{5}$$

$$S = \frac{1}{2}(R+L)\,, \quad \mathcal{D} = \frac{1}{2}(R-L)$$

$$R = 1 - \frac{\omega_p^2}{\omega^2}\left(\frac{\omega}{\omega-\omega_c}\right),$$
$$L = 1 - \frac{\omega_p^2}{\omega^2}\left(\frac{\omega}{\omega+\omega_c}\right),$$
$$P = 1 - \frac{\omega_p^2}{\omega^2},$$
$$\omega_p^2 = \frac{4\pi N e^2}{m}, \qquad \omega_c = \frac{eH}{mc}$$

N is the electron number density and the ion component has been neglected.

The dispersion relation can be written formally as follows:

$$\Lambda^{\ell} = A = S \sin^2\theta + P \cos^2\theta \tag{6}$$

or

$$tg^2\theta = -\left(\frac{P}{S}\right) \tag{7}$$

Inserting the explicit expressions of P and S in Eq. (7) we have

$$tg^2\theta = -\frac{(\omega^2-\omega_c^2)(\omega^2-\omega_p^2)}{\omega^2(\omega^2-\omega_p^2-\omega_c^2)} \,. \tag{8}$$

The possible longitudinal modes are then given by ($\omega_h^2 = \omega_p^2 + \omega_c^2$):

$$\omega_\pm^2 = \frac{1}{2}\left\{ \omega_h^2 \pm \left[\omega_h^4 - 4\,\omega_p^2\omega_c^2\cos^2\theta\right]^{1/2} \right\}$$

which at $\theta=0$ and $\theta=\frac{\pi}{2}$ becomes:

$$\theta=0: \quad \omega_-^2 = \omega_c^2, \qquad \omega_+^2 = \omega_p^2$$

$$\theta=\frac{\pi}{2}: \quad \omega_-^2 = 0, \qquad \omega_+^2 = \omega_h^2 = \omega_p^2 + \omega_c^2$$

since $\omega_h^2 \simeq \omega_p^2$, the ω_+ mode is practically independent of H and isotropic, and is therefore the one studied by Adams, Ruderman and Woo (1963). All the quantities referring to this mode will be hereafter denoted by a subscript zero.

On the other hand the mode ω_- at $\theta=0$ depends solely on H and this is precisely the mode which dominates for high densities. Analogously, the presence of this mode will be characterized by a subscript H.

2. Computation of the Energy Loss

The computation of the energy loss proceeds in the same way as for the transverse case, with only slight modifications. We shall not elaborate on the details but will indicate the results appropriate to the longitudinal case. The energy loss is given by the formula

$$Q = \sum_{spin} \frac{\Omega^2}{(2\pi\hbar)^6} \iint d^3p_1\, d^3p_2 \frac{|S|^2}{\Omega T}(E_1 + E_2) \tag{9}$$

where $\vec{p}_1$, E_1 and $\vec{p}_2$, E_2 are the neutrino and antineutrino final momentum and energy. The relevant S-matrix for the neutrino-antineutrino decay of a longitudinal plasmon is:

$$S = \frac{g}{e\sqrt{2}} \frac{N_\gamma^{1/2}}{\hbar c} (2\pi)^4 \delta^4\left(\frac{p_1+p_2}{\hbar} - k\right)\left[\Pi_{\alpha\beta}(\vec{k},\omega)\, e_\alpha\right] \times \tag{10}$$

$$\left[\overline{u}(p_2)\gamma_\beta(1+\gamma_5)v(p_1)\right],$$

where all the symbols have the same meaning as in Paper I. The polarization tensor $\Pi_{\alpha\beta}(\vec{k},\omega)$ is defined as

$$\Pi_{\alpha\beta}(\vec{k},\omega) = \frac{e^2}{\hbar c}\frac{1}{(2\pi\hbar)^4}\int d^4p\, \mathrm{Tr}\left\{\gamma_\alpha G(p)\gamma_\beta G(p+k)\right\} \tag{11}$$

and it is related to the dielectric tensor $\epsilon_{\alpha\beta}$ through the relations (Tsytovich, 1961)

$$\begin{aligned}
\Pi_{ij} &= \frac{i}{4\pi}\left(\frac{\omega}{c}\right)^2\left[\epsilon_{ij} - \delta_{ij}\right], \\
\Pi_{i4} &= i\left(\frac{\omega}{c}\right)^{-1}\Pi_{ij}k_j, \\
\Pi_{44} &= -\left(\frac{\omega}{c}\right)^{-2}\Pi_{ij}k_i k_j.
\end{aligned} \tag{12}$$

Squaring S, Eq. (9), integrating over the neutrino final momenta and summing over the neutrino spins, we easily get the following expression for the energy loss per unit solid angle and per unit volume:

$$\begin{aligned}
Q_{0,H}(\theta) = &\frac{1}{12\pi^3}\left(\frac{g^2}{\hbar\alpha}\right)\int d\omega\int_0^{\omega/c}|\vec{k}|^2 d|\vec{k}|\left[\frac{\omega^2}{c^2} - |\vec{k}|^2\right] \times \\
&(-1)^{\delta_{\beta 4}}\Pi_{\alpha\beta}\Pi^*_{\rho\beta}e_\alpha e^*_\rho\left|\frac{\lambda^\ell_{ss}}{\omega\frac{\partial\Lambda^\ell}{\partial\omega}}\right| f(\omega)\,\delta(\omega-\omega_\pm),
\end{aligned} \tag{13}$$

where

$$f(\omega) = \left\{\exp\left(\frac{\hbar\omega}{kT}\right) - 1\right\}^{-1},$$

and we have explicitly introduced the identity

$$\int \delta(x)\, dx = 1$$

to recall that ω is a solution of the dispersion equation, Eq. (8).

3. Parallel and Perpendicular Propagation

For $\theta=0$ and $\omega = \omega_+ = \omega_p$ we have, after some lengthy algebra:

$$Q_o(0) = \frac{1}{384\pi^5} \frac{g^2}{\hbar\alpha} \frac{\omega_p^2}{c^6} f(\omega_p) \int_0^{\omega_p/c} |\vec{k}|^2 d|\vec{k}| \left[\omega_p^2 - c^2|\vec{k}|^2\right]^2$$

$$= \overline{Q}\, \omega_p^9\, f(\omega_p) \qquad (14)$$

where

$$\overline{Q} = \frac{g^2}{\alpha \hbar c^9} \frac{1}{5040\pi^5}$$

Integrating over the solid angle, we obtain the formula given by Adams, Ruderman and Woo (1963), Inman and Ruderman (1964) and Zaidi (1965). For $\theta=0$ and $\omega_- = \omega_c$, we get:

$$Q_H(0) = \frac{7}{2} \overline{Q}\, \omega_p^2\, \omega_c^7\, f(\omega_c) \qquad (15)$$

At this point we would like to comment on the modifications needed when using Eq. (15) in the relativistic domain, since the cyclotron frequency becomes velocity dependent. Calling ω_{CR} the relativistic cyclotron frequency we have

$$\omega_{CR} = \frac{eH}{mc}\left[1-\frac{v^2}{c^2}\right]^{1/2} \equiv \frac{\omega_c}{\gamma} \tag{16}$$

Each particle in the plasma now has its own ω_{CR} and we have to average over the velocity distribution. For the completely degenerate electron gas we are treating here, this operation has the effect of replacing ω_c in Eq. (15) by $< \omega_{CR} > \to \omega_c/\mu$ where μ is the chemical potential of the electron gas in units of mc^2. Therefore, in the relativistic domain, Eq. (15) now reads

$$Q_H(0) = \frac{7}{2}\overline{Q}\,\omega_p^2\left(\frac{\omega_c}{\mu}\right)^7 f\left(\frac{\omega_c}{\mu}\right) \tag{17}$$

For $\theta = \pi/2$ we have essentially one mode, $\omega = \omega_h$ since the other one, $\omega = 0$, gives, of course, a vanishing contribution to the energy loss. Then for $\theta = \pi/2$, $\omega = \omega_h$ we have

$$Q_0\left(\frac{\pi}{2}\right) = \overline{Q}\,\omega_p^2\omega_h^5\left[\omega_p^2 + \frac{11}{4}\omega_c^2\right] f(\omega_h) \tag{18}$$

Here again ω_c has to be replaced by ω_c/μ in the relativistic domain.

4. Propagation at Arbitrary Angles

The general expression for the energy loss Eq. (13) can be considerably simplified by expanding it in powers of (ω_c/ω_p). To the lowest order in ω_c/ω_p Eq. (13) becomes

$$Q_H(\theta) = \frac{7}{2}\overline{Q}\,\omega_p^2\,\omega_c^7\,f(\omega_-)\,\cos^7\theta \tag{19}$$

where $\omega_-^2 = \omega_c^2 \cos^2\theta$

If furthermore $T \gtrsim 10^9$ the energy loss per unit mass becomes

$$Q_H(\theta)/\rho = 8.24\,T\left(H/H_q\right)^6 \cos^6\theta \quad \mathrm{erg\ g^{-1}\ sec^{-1}} \tag{20}$$

In the relativistic domain the preceding formula goes into:

$$\frac{1}{\rho}Q_H(\theta) = 8.24\,T\,\rho_6^{-2}\left(H/H_q\right)^6 \cos^6\theta \quad \mathrm{erg\ g^{-1}\ sec^{-1}} \tag{21}$$

Eq. (21) can be immediately integrated over the solid angle giving for the total luminosity.

$$\frac{1}{\rho}Q_H = 14.8\,T\,\rho_6^{-2}\left(H/H_q\right)^6 \quad \mathrm{erg\ g^{-1}\ sec^{-1}} \tag{22}$$

5. Discussion and Numerical Results

The presence of a magnetic field gives rise to two modes of propagation which have been called ω_+ and ω_-. The ω_+ mode is practically independent of H, and isotropic in the regions of astrophysical interest. In fact all the relevant quantities referring to this mode contain the magnetic field through the combination $(\omega_c/\omega_p)^2$ which has to be compared to unity (see Fig. 18).

The ω_- mode gives rise to expressions depending strongly on the field and on the angle θ. Moreover the density dependence of the energy losses Q_H and Q_O is completely different. In fact, remembering that $\omega_p \sim \rho^{\frac{1}{2}}$ and that for high densities $\mu \sim \rho^{1/3}$, we see from Eqs. (14) and (22) that

$$\frac{1}{\rho} Q_0 \sim \rho^{7/2} \; e^{-aT^{-1}\rho^{1/2}}$$

while

$$\frac{1}{\rho} Q_H \sim \rho^{-2}$$

Therefore the energy loss in the presence of a magnetic field can be many orders of magnitude greater than the corresponding loss without field for sufficiently high densities. The energy loss per unit mass at $\theta = 0$ has been computed from Eqs. (14) and (17), dividing them by the density, for $H/Hq = 10^{-2}$ and $H/Hq = 1$ and several temperatures ($Hq = \frac{m^2c^3}{e\hbar} = 4.414 \times 10^{13}$ G). The results have been plotted in Figs. 1 and 2 as a function of $\rho_6 = 10^{-6}\rho/\mu_e$, $\mu_e = Z/A$. In Fig. 3 we have reported the total energy losses as deduced from Eqs. (14) and (22) for $T = 10^9$ and $H = 10^{-2}$ Hq.

As already pointed out whatever sizable effect the magnetic field is responsible for, it always introduces some degree of anisotropy. In this case, the two factors concur, since the mode $\omega = \omega_-$ which alone is responsible for the total luminosity at $\rho >> 10^{11}\,gcm^{-3}$ is strongly reached around $\theta = 0$. When the remaining neutrinos processes (Urca, photoneutrino synchrotron, and pair creation) are evaluated, a complete picture of the weak interaction process in magnetic neutron stars will then be possible and the important question of the cooling time (Canuto, 1970) by neutrinos could then be reinvestigated.

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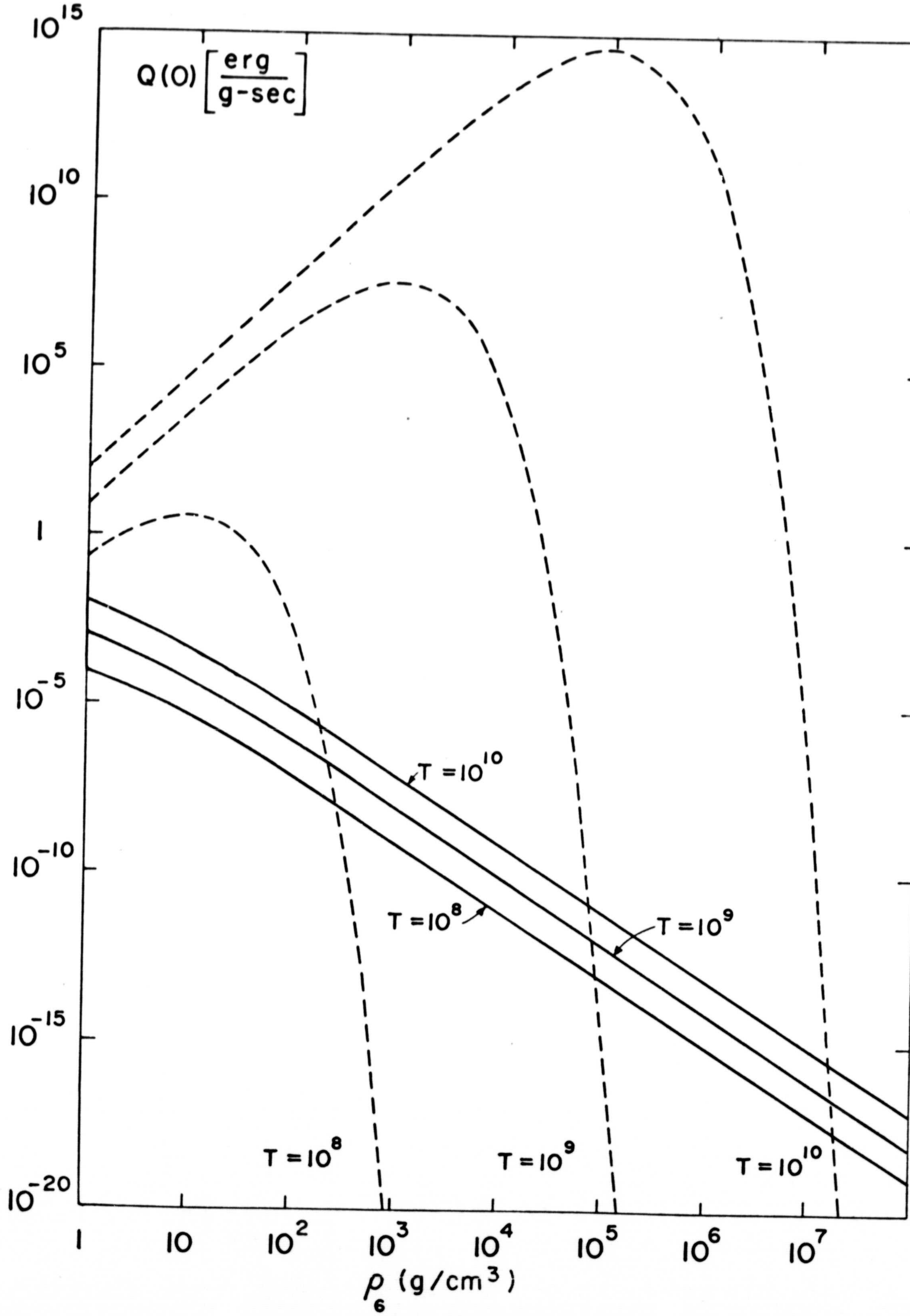
FIGURE CAPTIONS

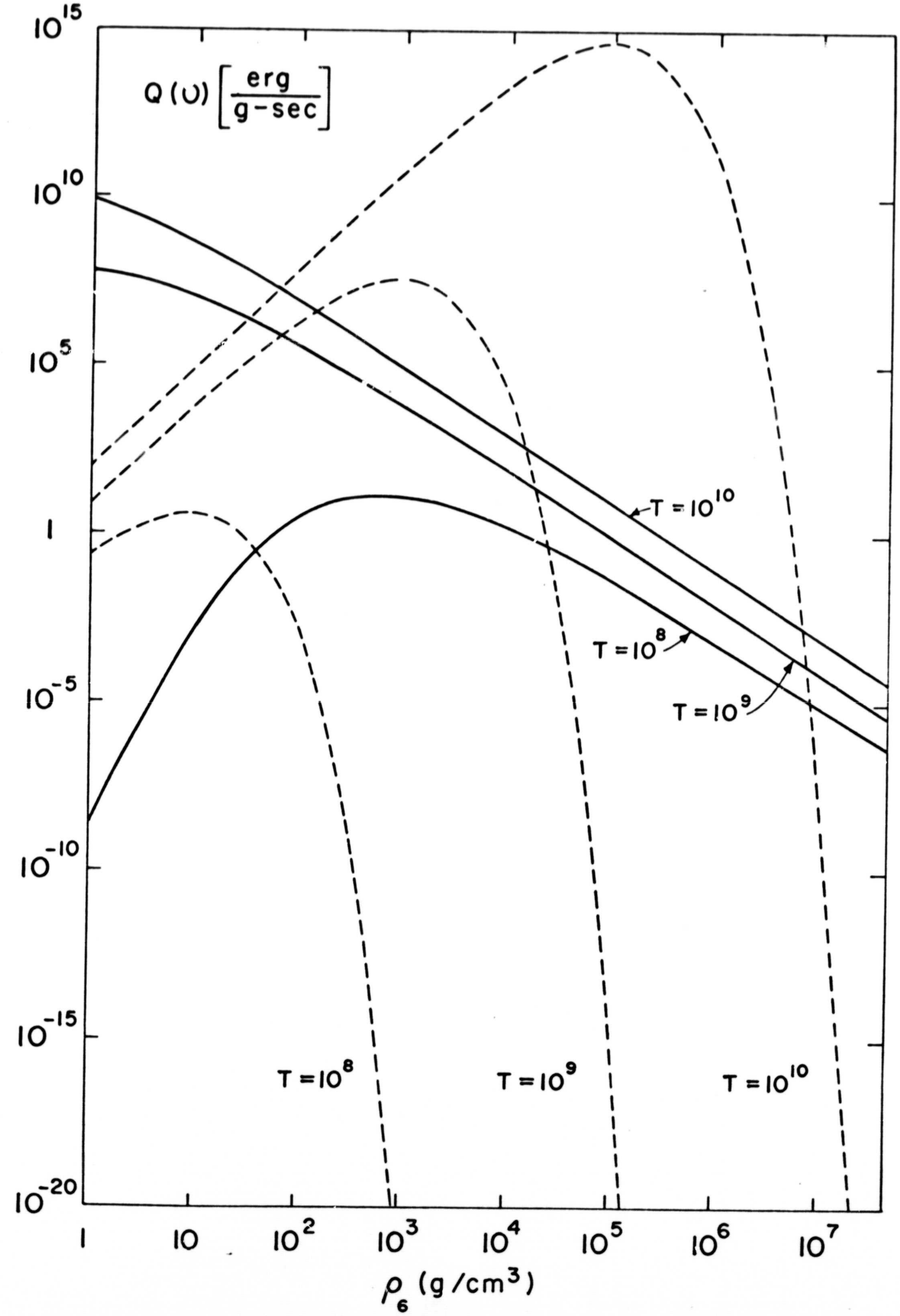
Figure 1. Energy loss per unit mass and unit solid angle at $\theta = 0$, as a function of ρ_6 and different temperatures. The solid curve refers to the mode $\omega = \omega_c$, and $H/Hq = 10^{-2}$ ($Hq = 4.414 \times 10^{13}$ G). The dashed curve refers to the mode $\omega = \omega_p$, which is independent of the field.

Figure 2. Same as in Figure 1 for $H = Hq$.

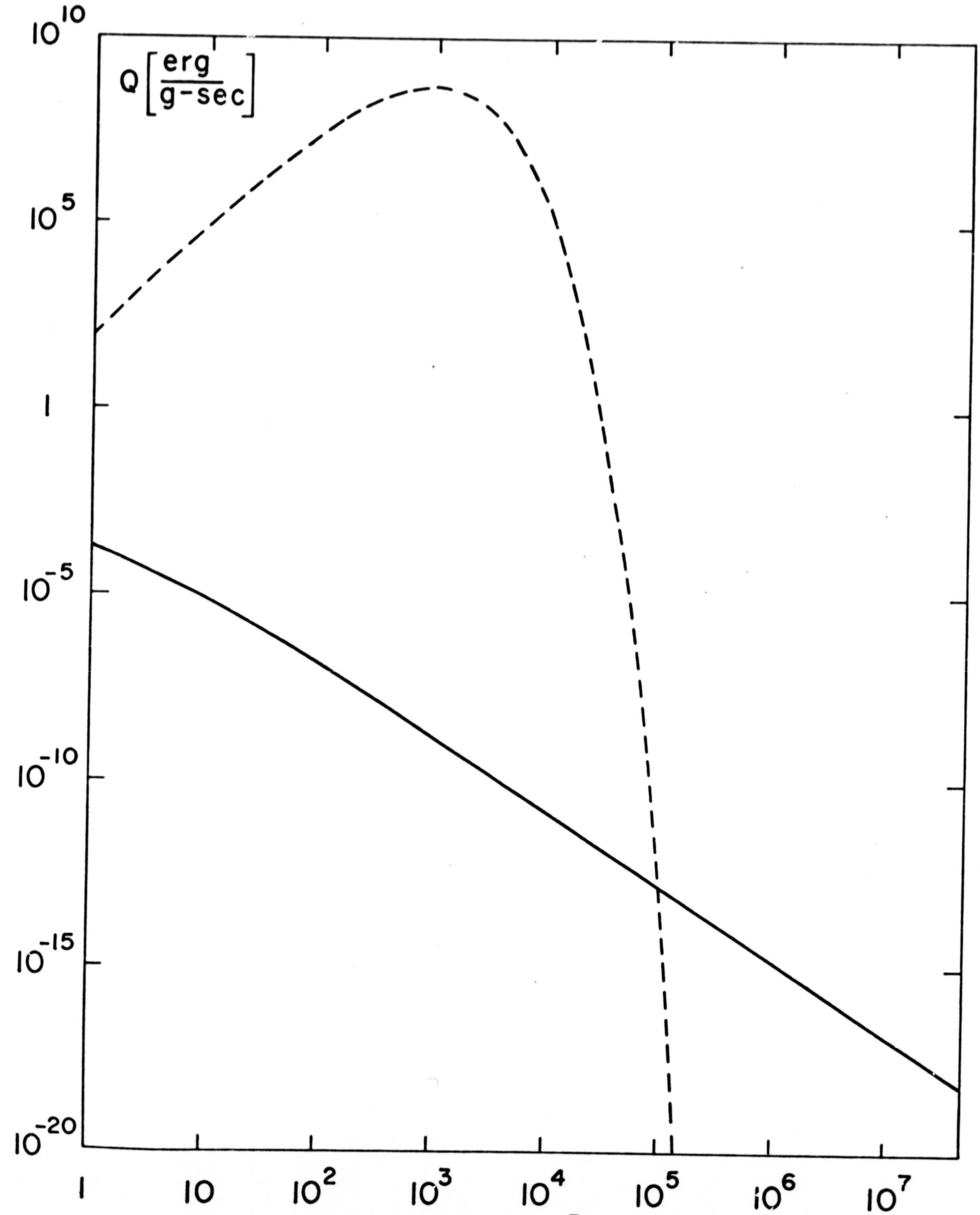
Figure 3. Total energy losses with (solid line) and without magnetic field vs. ρ_6 for $T = 10^9$ °K and $H/Hq = 10^{-2}$.



$Q(0)\left[\frac{erg}{g\text{-}sec}\right]$
10^{15}
10^{10}
10^{5}
1
10^{-5}
10^{-10}
10^{-15}
10^{-20}
$T=10^{10}$
$T=10^{8}$
$T=10^{9}$
$T=10^{8}$
$T=10^{9}$
$T=10^{10}$
1
10
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
10^{7}
ρ_6 (g/cm^3)



Q (υ) [erg / g-sec]
10^{15}
10^{10}
10^{5}
1
10^{-5}
10^{-10}
10^{-15}
10^{-20}
T = 10^{10}
T = 10^{8}
T = 10^{9}
T = 10^{8}
T = 10^{9}
T = 10^{10}
1
10
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
10^{7}
ρ_6 (g / cm^3)



$Q\left[\frac{erg}{g\text{-}sec}\right]$
10^{10}
10^{5}
1
10^{-5}
10^{-10}
10^{-15}
10^{-20}
1
10
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
10^{7}
ρ_6 (g/cm^3)