N71-31221
NASA TM X-67882

NASA TECHNICAL MEMORANDUM

NASA TM X-67882

DESIGN OF EXPERIMENTS AS "MULTIPLY TELESCOPING" SEQUENCES OF BLOCKS

by Arthur G. Holms
Lewis Research Center
Cleveland, Ohio

TECHNICAL PAPER proposed for presentation at
The 131st Annual Meeting of the American Statistical Association
Colorado State University, Fort Collins, Colorado, August 23-26, 1971

DESIGN OF EXPERIMENTS AS "MULTIPLY TELESCOPING" SEQUENCES OF BLOCKS

by Arthur G. Holms

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio

E-6311

ABSTRACT

A blocked two-level factorial experiment was designed to measure the elevated-temperature time-dependent corrosion effects of liquid metal on immersed structural materials. Several types of block effects were postulated. The experiment was designed to be "multiply telescoping" so that orthogonal estimates of important parameters could be obtained in the presence of two or more types of block effects, even if the number of blocks introducing the several effects were not specified in advance. The rationale is given for selecting defining contrast and treatment generators to provide the options of multiple telescoping. The rationale is also given for identifying parameters aliased together and confounded with blocks.

INTRODUCTION

Davies and Hay (1950) have noted that experiments could be designed as sequences of orthogonal blocks of two-level fractional-factorial experiments such that observations either from the first block or from a small number of blocks could be used to estimate the coefficients of a simple model. Then, at the option of the experimenter, new blocks of the sequence could be performed such that all acquired observations would be used cumulatively to estimate models of successively greater generality; and the coefficient estimators would be orthogonal to the block effects. General rules for doing this were given by Daniel (1956). A large catalog of the defining contrasts for appropriate plans together with an evaluation of their properties was provided by Addelman (1969). The name "telescoping" has been applied to similar plans in a smaller catalog that also listed the treatments and the aliased parameters (Holms (1968)). In these papers a single influence such as an influence correlated with time was assumed to have caused the block effects.

The concept of designing for two independent block effects (double confounding) was developed by Holms and Sidik (1971a). Designing for more than two independent block effects was introduced by Holms and Sidik (1971a) under the name of "multiple telescoping." In that paper, rules were given for identifying parameters aliased together and for indentifying parameters confounded with block effects in terms of some given group of defining contrasts. However, the problem of actually generating appropriate groups of defining contrasts was left untreated.

The purpose of the present paper is to discuss multiple telescoping in detail. The discussion will be illustrated by the design of an experiment to observe the elevated temperature corrosion effect of liquid metals on the immersed structural material. Rules are developed for identifying parameters aliased together and for indentifying parameters confounded with block effects that are simpler to use than those that had been developed (Holms and Sidik (1971a)). Other rules for use when the blocks performed differ from the design options have been given by Holms (1971b).

STRUCTURE OF MULTIPLE TELESCOPING

Model Equations

In brief, the purpose of a two-level fractional factorial experiment is to estimate the coefficients of an equation. The equation together with assumptions about the experimental error are called the model. The factorial experiment provides estimates of the treatment parameters (coefficients) of an equation of the form

$$
\begin{aligned}
Y = \beta_I + \beta_A x_A + \beta_B x_B + \dots + \beta_g x_g \\
+ \beta_{AB} x_A x_B + \beta_{AC} x_{AC} + \dots + \beta_{g-1, g} x_{g-1} x_g \\
+ \beta_{ABC} x_A x_B x_C + \dots + \beta_{g-2, g-1, g} x_{g-2} x_{g-1} x_g \\
+ \dots + \epsilon \qquad (1)
\end{aligned}
$$

where $x_A, \dots x_g$ are the independent variables, ϵ is the random error, and the β's are the treatment parameters.

Notation for Treatments and Defining Contrasts

The independent variables can be standardized (transformed) so that the upper level is represented by $x_A = +1$, $x_B = +1$, etc. and the lower level by $x_A = -1$, $x_B = -1$, etc. A combination of levels of the independent variables is called a treatment. The notation for the treatments is illustrated by the elements of the rows of table I(a) that are under single letter column headings. The first column of table I(a) gives the familiar notation for treatments described by Davies (1960). The third column of table I(a) and columns to the right give the linear combinations of the observations, which on division by the number of items in the column, estimate treatment parameters of Eq. (1) that are subscripted to match the column headings.

An arrangement illustrating a performance of the treatments of table I(a) in four blocks is given by table I(b). The resulting confounding of treatment parameters with block effect parameters is illustrated in table II, which will be discussed subsequently.

Other discussion of notation occurs in Davies (1960) and Holms and Sidik (1971a).

Crossed Classification of Block Effects

Two assumptions are fundamental to planning an experiment for multiple telescoping. One is that the block effects may be classed as crossed. With crossed block effects, each type of block effect can be identified separately from every other type of block effect. Interactions among the block effects can then be defined and estimated. The other assumption is that no block variables interact with treatment variables. If they did, then such block variables would be handled as treatment variables. Such handling would increase the number of parameters to be estimated but would require no basic change in the theory, because crossed classification is assumed for both block and treatment variables.

Maximum Number of Types of Block Effects

The construction of a plan for multiple telescoping beings with a principal block that is a $1/2^h$ replicate of the full 2^g experiment. As such it contains 2^{g-h} treatment combinations and becomes a full 2^g factorial experiment if expanded by doubling h times. Every such doubling could be accompanied by a new type of block effect, so that as many as h types of block effects can be postulated. The number of independent defining contrasts for the 2^{g-h} replicate is h so that the maximum number of block effect types is equal to the number of generators of the smallest principal block.

Defining Contrasts

The properties of experiments consisting of the contemplated stopping points of telescoping sequences are mainly determined by the distribution of the word lengths of the defining contrasts of the smallest fraction of the experiment. (Construction of the experiment is simplified by making the smallest fraction a principal block.) Addelman (1969) has tabulated some of the properties that can be achieved. The particular sequence of expansions he tabulated for a given family of defining contrasts could constitute one possible path through the many options of multiple telescoping. The other paths of multiple telescoping deserve investigation.

In many cases, the defining contrasts tabulated by Addleman could serve the purposes of multiple telescoping. In other cases (such as the present corrosion experiment) the smallest block must be smaller than his smallest block and a larger number of contrast generators are needed.

Block Effect Sources and Block Parameters

Block effects from sources i, j, k, ... can be represented by first degree parameters μ_i, μ_j, μ_k, ... and by block interaction parameters μ_{ij}, μ_{ik}, ..., μ_{ijk}, ... Half replicates of the experiment can then be defined that would not introduce the i-source block effects, or would not introduce the j-source block effects, or would not introduce some other source of block effects. Thus for example, if the half replicate were not performed that would introduce the j source block effects, then all block parameters μ_j, μ_{ij}, μ_{ijk}, ...

having a subscript j are zero. A particular defining contrast of a half replicate can then be associated with such a particular source of block effects.

GENERATION OF DEFINING CONTRASTS

Physical Constraints on the Corrosion Experiment

The experiment requires the observation of the elevated temperature corrosion effect of a liquid metal on specimen materials as a function of six independent variables. Four specimens can be installed in a furnace and four furnaces can be installed in a vacuum chamber. The experimenter wanted the option of using one or two vacuum chambers. Because just four specimens can be loaded into a furnace, the basic block size consists of four treatments per block. The treatments are assumed to be assigned at random to individual units within the blocks.

The general arrangement of the experiment is shown by table III. The paired numbers (i, j) are row block headings that identify the four furnaces. The two vacuum chambers are identified by $l = 1$ and $l = 2$. Two successive loadings of each chamber are labeled $k = 1$ and $k = 2$. If only one vacuum chamber were used for all four loadings, the loadings would be identified by the paired numbers (k, l). A basic assumption is that both treatment effects and block effects are each crossed classification effects. (If two vacuum chambers were used and a different set of furnaces used in the second vacuum chamber, the furnaces would be "nested" within vacuum chambers in both a physical and statistical sense. The design and analysis of "nested" multiply telescoping experiments has not been investigated.)

The planned experiment has three independent sources of block effects, namely, (1) differences among the four furnaces, (2) differences between the two vacuum chambers, and (3) differences between the first and the second loading of a vacuum chamber. Because the full replicate consists of four doublings from the smallest block, four independent sources of block effects could have been tolerated (as represented by the four symbols i, j, k, and l).

Defining Contrasts for the Principal Block

A single block of the corrosion experiment can be thought of as a 2^{g-h} experiment where $2^{g-h} = 4$. Then with six independent variables, $g = 6$ and h must be 4. That is, the number of contrast generators for the smallest fractional replicate is 4.

One way of beginning the synthesis of the four needed generators is to use the letters A, B, C, and D singly as generators. The complete group of defining contrasts is then obtained by multiplying these letters together in all possible combinations as shown by table IV. Using only the letters A, B, C, and D omits the variables x_E and x_F from the experiment. These variables are included by attaching combinations of E and F to the generators A, B, C, and D.

An unfortunate constraint on the experiment is that the specimen temperature cannot be varied from specimen to specimen within a furnace. If the temperature is x_A, then x_A is a constant within the block represented by a furnace. Thus the main effect of x_A must always be confounded with some kind of a block effect, and therefore the single letter A must appear among the defining contrasts of table IV.

Thus the letters E and F cannot be suffixed to the letter A of table IV, but these letters can be added to the letters B, C, and D in an arbitrary manner, except that the distribution of the word lengths might be a consideration. (In general, short word lengths are undesirable because they confound low order treatment parameters with block effects, or else they give poor resolution levels to the fractional replicates.) The letters E and F were added as shown in table IV. They then participate in the same multiplication combinations as the A, B, C, and D as shown in table IV.

Defining Contrasts for Expansions of Experiment and for Block Effects

From the block of size four, the experiment can be expanded in stages with the important treatment parameter estimates orthogonal to the block parameter estimates, provided the stages consist of doublings. With each of the doublings, an independent source of block effects is assumed to be introduced. Furthermore, the defining contrasts will be a subgroup of the defining contrasts before the doubling. Assuming the presence of the maximum number of block effect sources (one new source with each of the four doublings from the principal block to the full replicate) let the sources be represented by the discrete variables i, j, k, and l. These variables take the value 1 if the doubling has not occurred (block effect not introduced) and take the value 2 if the doubling has occurred (block effect introduced). The relations of these sources to the blocks of treatments are then shown by the i, j, k, and l row and column headings of table III.

The subgroups of defining contrasts that will represent successive doublings of the experiment must be chosen from the 16 defining contrasts of the first block (table IV), and this can be done to minimize the aliasing and confounding of the lower order treatment parameters. The probability is highest that the experiment will stop at the half replicate consisting of the four furnaces operated for just two loadings of one vacuum chamber. That replicate is defined by $l = 1$ in table III. So that such an experiment will have the highest possible resolution level, (Box and Hunter (1961)), its defining contrast should be the longest length word (-ABCDE) of table IV.

Because there is some small probability of stopping at the quarter replicate consisting of just one vacuum chamber loading ($k = 1$ and $l = 1$), the contrast defining doubling with respect to k should also be a long word of table IV. ABEF was so chosen, and its product with -ABCDE, namely, -CDF, should also be a long word of table IV.

The experimenter's prior probabilities of some of the block effects were relatively high, -50 percent for a direct effect of differences among furnaces and 20 percent for a difference between vacuum chambers. But the prior probabilities for stopping at fractional replicates not already discussed were quite small, so that the matching of other defining contrasts of table IV to the block sources should not be done anticipating such stopping points, but should be done according to the prior probabilities of block effects. The high prior probabilities of furnace block effects suggest that the treatment parameters confounded with furnace block effects should have small prior probabilities of being nonnegligible, namely they should be the higher order interactions. Thus the defining contrasts that define expansions of the experiment with respect to i, j, and ij should be some of the longer words of table IV. The words chosen for i and j in table IV must be words such that their product (representing furnace block effect ij) will also be a longer word. The words chosen were -ABC, -BDF, and ACDF.

The four independent defining contrasts just chosen to represent doublings with respect to i, j, k, and l are so identified in table IV. Each such contrast by itself is a defining contrast for a half replicate experiment. Any two of these contrasts provide the generators of a quarter replicate experiment. Thus many different regular fractions of the experiment from a half replicate on down to a 1/8 replicate may be generated by using combinations of these four generators. In such cases, the defining contrasts so generated will be subgroups of the group generated by all four half replicate generators (as listed for the 1/16 replicate by table IV).

DETERMINATION OF TREATMENT GENERATORS

Treatment generators are selected by the rule of even numbers (Davies (1960)). The selection was made from the display of table V. It lists all the treatments of the full factorial experiment as row headings, and, as column headings it lists the independent defining contrasts of the half replicate experiments first selected for that purpose in table IV. Also listed in table V are the numbers of letters that occur in common between the treatments and the defining contrasts. According to the rule of even numbers, the treatment generators of the principal block must have only even numbers in common with all of the defining contrast generators. Scanning table V shows that the first two treatments meeting these conditions are the first two treatment generators listed in table VI. Any given generator that doubles the size of the experiment must have an even number of letters in common with all the defining contrast generators, except for the generator that disappears on doubling the experiment with the given treatment generator. The first four such treatment generators of table V are indicated by the occurrences of three even numbers underlined together with one odd number not underlined. The associations between such treatment generators and the contrast generator they eliminate by the doubling is shown in table VI. The use of these treatment generators to generate the detailed plan of the experiment is shown by table III.

IDENTIFICATION OF CONFOUNDED PARAMETERS

Illustrative Example with Four Independent Variables

The basic method for identifying the treatment parameters that are confounded with block parameters will be developed through a discussion of the hypothetical experiment (Holms and Sidik (1971a)) with treatment levels shown by table I(a). The full factorial experiment of table I(a) was assumed to be subdivided for double telescoping as shown by table I(b).

The groups of defining contrasts for the contemplated replicates will be symbolized by $C(i, j)$, where the discrete variables i and j give the numbers of rows and columns of blocks respectively, in the replicate. As discussed by Holms and Sidik (1971a), the column contrasts of table I(b) that have uniform algebraic signs over the full or fractional replicates identify defining contrasts as follows:

$$C(1, 1) = I, \ -ABC, \ -ABD, \ CD$$

$$C(1, 2) = I, \ -ABD$$

$$C(2, 1) = I, \ -ABC$$

$$C(2, 2) = I.$$

The interrelations of the preceding lists of defining contrasts with possible block parameters will be established by comparing the defining contrasts of table I(b) with postulated block parameters. The postulated block parameters are listed in the first column of table II, namely, (1) a grand mean, μ_0, (2) an effect resulting from doubling over rows, μ_i, (3) an effect from doubling over columns, μ_j and (4) a row column block interaction effect, μ_{ij}.

In the case of just the principal block, (i = 1 and j = 1), table I(b) shows that the contrasts I, -ABC, -ABD, and CD all have the same sign, and thus the estimator of the grand mean for this block also estimates a linear combination of treatment parameters with these contrasts as subscripts. These treatment parameters are all aliased together and confounded with the grand mean as indicated under C(1, 1) of table II.

If the experiment is now doubled by adding the block for which j = 2, then the contrast under ABD (table I(b)) is of uniform sign so that β_{ABD} is confounded with the grand mean, β_I, as represented by the intersection of row μ_0 and column c(1, 2) of table II. Furthermore, the contrast ABC has opposite signs for the two blocks, so that the estimate of the parameter β_{ABC} is also an estimate of the difference between the blocks. The same statement can be made with respect to the CD contrast of table I(b) so that β_{CD} and β_{ABC} are both (table II) confounded with μ_j. Similar statements also apply to the C(2, 1) replicate. For the full replicate, the column under I of table I(b) estimates the grand mean and that parameter is called μ_0. The contrast under ABD has a change of sign from i = 1 to i = 2 and therefore estimates the treatment parameter β_{ABD} confounded with the block parameter μ_i. The contrast under ABC changes sign when j changes from j = 1 to j = 2 and therefore estimates the confounded combination of μ_j and β_{ABC}. The contrast under CD has positive signs for both i = 1, j = 1 and i = 2,

$j = 2$, but it has negative signs for both $i = 1$, $j = 2$, and $i = 2$, $j = 1$. Therefore the contrast CD estimates the treatment parameter β_{CD} confounded with the block interaction parameter μ_{ij}. Thus the first two columns of table II show which of the defining contrasts of the smallest fractional replicate become estimators of the postulated block effects in the full replicate.

A quick general method for identifying the treatment parameters confounded with block parameters is illustrated by table II. The defining contrasts for the contemplated fractional replicates are given by the subscripts of the treatment parameters aliased with μ_o (listed in the μ_o row of table II). Then for a block effect that can exist, the defining contrast of the full replicate for that block parameter (in the second column of table II) multiplies the defining contrasts of the particular fractional replicate (subscripts in the μ_o row) to give the subscripts of the treatment parameters confounded with the particular block parameter. For example, the C(1, 2) replicate has defining contrasts I, -ABD as shown by the subscripts of the treatment parameters in the μ_o row. Doubling the C(1, 1) replicate to the C(1, 2) replicate is done with respect to the j source of block effects, so that the μ_j parameter is postulated. In the full replicate the parameter μ_j has defining contrast -ABC (second column of table II). The products of -ABC with the defining contrasts of the C(1, 2) replicate (namely I and -ABD) are -ABC and CD, and thus the treatment parameters confounded with μ_j are $-\beta_{ABC} + \beta_{CD}$ as listed under C(1, 2) of table II.

Identification of Confounded Parameters in the Corrosion Experiment

The groups of defining contrasts will identify the treatment parameters confounded with block parameters as was just done with four independent variables. As a start, block parameters are postulated to represent the physical plan of the experiment. Individual furnaces might degrade with time, either gradually or suddenly. Therefore all conceivable interaction effects might exist between the furnace blocks and the time blocks. Thus besides the furnace effects μ_i, μ_j, μ_{ij}, and the time effects μ_k, μ_l, μ_{kl}, the furnace-time interactions μ_{ik}, μ_{il}, μ_{jk}, μ_{jl}, μ_{ijk}, μ_{ijl}, μ_{ikl}, μ_{jkl}, μ_{ijkl} are postulated. $C(i, j, k, l)$ will symbolize the groups of defining contrasts defining the replicates at the contemplated stopping points. Those variables among i, j, k, l that are identified with a generator of a given replicate will remain at their low level, whereas doubling with respect to a source of block effects will be represented by the presence of the variable at its high level. The association between the variables representing sources of block effects and the generators of fractional replicate defining contrasts was discussed when describing table VI.

The scheme for identifying the aliased combinations of model parameters that are confounded with block parameters is basically the same as that displayed for four independent variables by table II, and is given for the corrosion experiment by table VII. The first column lists the block parameters and the second column gives the experimenter's prior probabilities that such block effects exist. The four defining contrasts shown in table VI for expansions with respect to i, j, k, and l are listed in the third column of table VII. These four defining contrasts are then multiplied together as required by the multiplications of i, j, k, and l in the first column. The resulting products are the contrasts that estimate the block effect parameters of the full replicate that were listed in the first column. As would be expected, these contrasts also occur in the first column of table IV. These contrasts are also the subscripts of the treatment parameters estimated by the contrasts, therefore such treatment parameters are confounded with the associated block parameters in the full replicate.

The defining contrasts for the larger fractional replicates, as generated from table VI, are given in the fourth through thirteenth columns in the μ_0 row of table VII. Under these replicates, the block effects of the full replicate must disappear according to the fraction of the full replicate that is not performed. For the fraction that is performed, the block effect contrast of the full replicate experiment multiplies the fractional replicate defining contrasts to give the subscripts of the treatment parameters that are aliased together and confounded with the block effect parameters (as listed in the columns of table VII).

ALTERNATIVE IDENTIFICATION PROCEDURES

An identification procedure alternative to that illustrated by tables II and VII was given by Holms and Sidik (1971a). The identification of aliased parameters estimated by Yates' method from fractional replicates other than principal fractions has been discussed by Nelder (1963) and also by Holms (1971b).

CONCLUDING REMARKS

Methods were developed for selecting contrast and treatment generators to provide the options of multiple telescoping. The main principles developed were:

1. The maximum number of distinct types of block effects that can be

postulated is equal to the number of doublings of the experiment size occurring from the smallest block to the stopping stage. In expanding from a single block to a full replicate, the number of distinct types of block effects is then equal to the number of defining contrast generators of the single block.

2. For the intended particular fractional replicate options, the identification of treatment parameters confounded with block parameters can be done as follows. For a block effect that can exist, the defining contrast that estimates the associated block parameter in the full replicate is used to multiply all the defining contrasts of the particular fractional replicate. The results are the subscripts of the treatment parameters that are confounded with the particular block parameter.

REFERENCES

Addelman, S. (1969). Sequences of Two-Level Fractional Factorial Plans. Technometrics 11, 477-509.

Box, G. E. P. and Hunter, J. S. (1961). The 2^{k-p} Fractional Factorial Designs. I Technometrics 3, 311-351.

Daniel, C. (1956). Fractional Replication in Industrial Research. Proceedings of the Third Berkely Symposium on Mathematical Statistics and Probability 5, University of California Press, 87-98.

Davies, O. L., Ed. (1960). The Design and Analysis of Industrial Experiments. Second ed., Hafner Publishing Co.

Davies, O. L. and Hay, W. A. (1950). The Construction and Uses of Fractional Factorial Designs in Industrial Research. Biometrics 6, 233-249.

Holms, A. G. (1968). Designs of Experiments as Telescoping Sequences of Blocks. Proceedings of the Thirteenth Conference on the Design of Experiments in Army Research Development and Testing. ARO-D Report 68-2, 379-414.

Holms, A. G. and Sidik, S. M. (1971a). Designs of Experiments as "Doubly Telescopying" Sequences of Blocks with Application to a Nuclear Reactor Experiment. Technometrics, to be published.

Holms, A. G. (1971b). Design of Experiments as "Multiply Telescoping" Sequences of Blocks with Application to Corrosion by Liquid Metals. NASA TN D-6452.

Nelder, J. A. (1963). Identification of Contrasts in Fractional Replicates of 2^n experiments. Appl. Stat. 12, 38-43.

TABLE I. - 2^4 EXPERIMENT

(a) Full replicate in one block

Treatment	Response	Matrix of independent variables															
		I	A	B	AB	C	AC	BC	ABC	D	AD	BD	ABD	CD	ACD	BCD	ABCD
(1)	y_1	+1	-1	-1	+1	-1	+1	+1	-1	-1	+1	+1	-1	+1	-1	-1	+1
a	y_2	+1	+1	-1	-1	-1	-1	+1	+1	-1	-1	+1	+1	+1	+1	-1	-1
b	y_3	+1	-1	+1	-1	-1	+1	-1	+1	-1	+1	-1	+1	+1	-1	+1	-1
ab	y_4	+1	+1	+1	+1	-1	-1	-1	-1	-1	-1	-1	-1	+1	+1	+1	+1
c	y_5	+1	-1	-1	+1	+1	-1	-1	+1	-1	+1	+1	-1	-1	+1	+1	-1
ac	y_6	+1	+1	-1	-1	+1	+1	-1	-1	-1	-1	+1	+1	-1	-1	+1	+1
bc	y_7	+1	-1	+1	-1	+1	-1	+1	-1	-1	+1	-1	+1	-1	+1	-1	+1
abc	y_8	+1	+1	+1	+1	+1	+1	+1	+1	-1	-1	-1	-1	-1	-1	-1	-1
d	y_9	+1	-1	-1	+1	-1	+1	+1	-1	+1	-1	-1	+1	-1	+1	+1	-1
ad	y_{10}	+1	+1	-1	-1	-1	-1	+1	+1	+1	+1	-1	-1	-1	-1	+1	+1
bd	y_{11}	+1	-1	+1	-1	-1	+1	-1	+1	+1	-1	+1	-1	-1	+1	-1	+1
abd	y_{12}	+1	+1	+1	+1	-1	-1	-1	-1	+1	+1	+1	+1	-1	-1	-1	-1
cd	y_{13}	+1	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1	+1
acd	y_{14}	+1	+1	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1
bcd	y_{15}	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1
abcd	y_{16}	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1

(b) With double telescoping among four blocks

Row i	Column j	Treatment	Matrix of independent variables															
			I	A	B	AB	C	AC	BC	ABC	D	AD	BD	ABD	CD	ACD	BCD	ABCD
1	1	(1)	+1	-1	-1	+1	-1	+1	+1	-1	-1	+1	+1	-1	+1	-1	-1	+1
		acd	+1	+1	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1
		bcd	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1
		ab	+1	+1	+1	+1	-1	-1	-1	-1	-1	-1	-1	-1	+1	+1	+1	+1
1	2	ad	+1	+1	-1	-1	-1	-1	+1	+1	+1	+1	-1	-1	-1	-1	+1	+1
		c	+1	-1	-1	+1	+1	-1	-1	+1	-1	+1	+1	-1	-1	+1	+1	-1
		abc	+1	+1	+1	+1	+1	+1	+1	+1	-1	-1	-1	-1	-1	-1	-1	-1
		bd	+1	-1	+1	-1	-1	+1	-1	+1	+1	-1	+1	-1	-1	+1	-1	+1
2	1	d	+1	-1	-1	+1	-1	+1	+1	-1	+1	-1	-1	+1	-1	+1	+1	-1
		ac	+1	+1	-1	-1	+1	+1	-1	-1	-1	-1	+1	+1	-1	-1	+1	+1
		bc	+1	-1	+1	-1	+1	-1	+1	-1	-1	+1	-1	+1	-1	+1	-1	+1
		abd	+1	+1	+1	+1	-1	-1	-1	-1	+1	+1	+1	+1	-1	-1	-1	-1
2	2	a	+1	+1	-1	-1	-1	-1	+1	+1	-1	-1	+1	+1	+1	+1	-1	-1
		cd	+1	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1	+1
		abcd	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1
		b	+1	-1	+1	-1	-1	+1	-1	+1	-1	+1	-1	+1	+1	-1	+1	-1

TABLE II. - TREATMENT PARAMETERS CONFOUNDED WITH BLOCK PARAMETERS

Block effects		Replicate,[a] C(i, j)			
Parameter	Defining contrast	C(2, 2)	C(2, 1)	C(1, 2)	C(1, 1)
		Treatment parameters			
μ_o	I	β_I	$\beta_I-\beta_{ABC}$	$\beta_I-\beta_{ABD}$	$\beta_I-\beta_{ABD}-\beta_{ABC}+\beta_{CD}$
μ_i	-ABD	$-\beta_{ABD}$	$-\beta_{ABD}+\beta_{CD}$		
μ_j	-ABC	$-\beta_{ABC}$		$-\beta_{ABC}+\beta_{CD}$	
μ_{ij}	CD	β_{CD}			

[a]Obtained by doubling to level i with respect to the i source of block effects and by doubling to level j with respect to the j-source of block effects.

TABLE III. - PLAN OF EXPERIMENT

Block effect source		Furnace number, r	Vacuum chamber, l			
			1		2	
			Loading, k			
			1	2	1	2
			Loading number, c			
j	i		1	2	3	4
1	1	1	(1) bcde bcf def	ac abde abf acdef	abd ace acdf abef	bcd e df bcef
	2	2	abcd ae adf abcef	bd ce cdf bef	c bde bf cdef	a abcde abcf adef
2	1	3	ab acde acf abdef	bc de f bcdef	d bce bcdf ef	acd abe abdf acef
	2	4	cd be bdf cef	ad abce abcdf aef	abc ade af abcdef	b cde cf bdef

TABLE IV. - DEFINING CONTRASTS

OF SMALLEST BLOCK C(1, 1, 1, 1)

Defining contrast	Block effect source
I	
A	
-B EF	
-C EF	
D E	
AB EF	k
AC EF	
-AD E	
BC	
-BD F	j
-CD F	
-ABC	i
ABD F	
ACD F	
BCD E	
-ABCD E	l

TABLE V. - SELECTION OF TREATMENT GENERATORS

Treatment	Independent defining contrasts				Treatment	Independent defining contrasts			
	-ABC	-BDF	ABEF	-ABCDE		-ABC	-BDF	ABEF	-ABCDE
(1)	0	0	0	0	f	0	1	1	0
a	1	0	1	1	af	1	1	2	1
b	1	1	1	1	bf	1	2	2	1
ab	2	1	2	2	abf	2	2	3	2
c	1	0	0	1	cf	1	1	1	1
ac	2	0	1	2	acf	2	1	2	2
bc	2	1	1	2	bcf	2	2	2	2
abc	3	1	2	3	abcf	3	2	3	3
d	0	1	0	1	df	0	2	1	1
ad	1	1	1	2	adf	1	2	2	2
bd	1	2	1	2	bdf	1	3	2	2
abd	2	2	2	3	abdf	2	3	3	3
cd	1	1	0	2	cdf	1	2	1	2
acd	2	1	1	3	acdf	2	2	2	3
bcd	2	2	1	3	bcdf	2	3	2	3
abcd	3	2	2	4	abcdf	3	3	3	4
e	0	0	1	1	ef	0	1	2	1
ae	1	0	2	2	aef	1	1	3	2
be	1	1	2	2	bef	1	2	3	2
abe	2	1	3	3	abef	2	2	4	3
ce	1	0	1	2	cef	1	1	2	2
ace	2	0	2	3	acef	2	1	3	3
bce	2	1	2	3	bcef	2	2	3	3
abce	3	1	3	4	abcef	3	2	4	4
de	0	1	1	2	def	0	2	2	2
ade	1	1	2	3	adef	1	2	3	3
bde	1	2	2	3	bdef	1	3	3	3
abde	2	2	3	4	abdef	2	3	4	4
cde	1	1	1	3	cdef	1	2	2	3
acde	2	1	2	4	acdef	2	2	3	4
bcde	2	2	2	4	bcdef	2	3	3	4
abcde	3	2	3	5	abcdef	3	3	4	5

TABLE VI. - ASSOCIATION OF TREATMENT AND CONTRAST

GENERATORS WITH BLOCK EFFECT SOURCES

Treatment generator	Block effect source introduced with doubling by treatment generator	Defining contrast eliminated by doubling with treatment generator
bcde		
bcf		
abcd	i	-ABC
ab	j	-BDF
ac	k	ABEF
abd	l	-ABCDE

TABLE VII. - TREATMENT PARAMETERS CONFOUNDED WITH BLOCK PARAMETERS

Block effects			Fractional replicate,[a] C(i, j, k, l)									
Parameter	Prior probability	Defining contrasts	C(1, 2, 2, 2)	C(2, 1, 2, 2)	C(2, 2, 1, 2)	C(2, 2, 2, 1)	C(1, 1, 2, 2)	C(1, 2, 1, 2)	C(1, 2, 2, 1)	C(2, 1, 1, 2)	C(2, 1, 2, 1)	C(2, 2, 1, 1)
			Stopping probability									
			0.02	0.02	0.02	0.34	0.01	0.01	0.01	0.01	0.01	0.05
			Subscripts of treatment parameters									
μ_o	1.00	I	I -ABC	I -BDF	I ABEF	I -ABCDE	I -ABC -BDF ACDF	I -ABC ABEF -CEF	I -ABC -ABCDE DE	I -BDF ABEF -ADE	I -BDF -ABCDE ACEF	I ABEF -ABCDE -CDF
μ_i	0.50	-ABC		-ABC ADCF	-ABC -CEF	-ABC DE				-ABC ACDF -CEF BCDE	-ABC ACDF DE -BEF	-ABC -CEF DE ABDF
μ_j	0.50	-BDF	-BDF ACDF		-BDF -ADE	-BDF ACEF		-BDF ACDF -ADE BCDE	-BDF ACDF ACEF -BEF			-BDF -ADE ACEF BC
μ_k	0.10	ABEF	ABEF -CEF	ABEF -ADE		ABEF -CDF	ABEF -CEF -ADE BCDE		ABEF -CEF -CDF ABDF		ABEF -ADE -CDF BC	
μ_l	0.20	-ABCDE	-ABCDE DE	-ABCDE ACEF	-ABCDE -CDF		-ABCDE DE ACEF -BEF	-ABCDE DE -CDF ABDF		-ABCDE ACEF -CDF BC		
μ_{ij}	0.50	ACDF			ACDF BCDE	ACDF -BEF						ACDF BCDE -BEF -A
μ_{ik}	0.10	-CEF		-CEF BCDE		-CEF ABDF					-CEF BCDE ABDF -A	
μ_{il}	0.10	DE		DE -BEF	DE ABDF					DE -BEF ABDF -A		
μ_{jk}	0.10	-ADE	-ADE BCDE			-ADE BC			-ADE BCDE BC -A			
μ_{jl}	0.10	ACEF	ACEF -BEF		ACEF BC			ACEF -BEF BC -A				
μ_{kl}	0.10	-CDF	-CDF ABDF	-CDF BC			-CDF ABDF BC -A					
μ_{ijk}	0.10	BCDE				BCDE -A						
μ_{ijl}	0.10	-BEF			-BEF -A							
μ_{ikl}	0.10	ABDF		ABDF -A								
μ_{jkl}	0.10	BC	BC -A									
μ_{ijkl}	0.10	-A										

[a]C(i, j, k, l) is the fractional replicate which, if doubled with respect to i, j, k, or l, contains i, j, k, or l type block effect sources.