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BUCKLING OF BORON/ALUMINUM AND GRAPHITE/RESIN FIBER COMPOSITE ANISOTROPIC PANELS

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TECHNICAL PAPER proposed for presentation at
Fall Meeting of the Society of Aerospace Material
and Process Engineers
Huntsville, Alabama, October 5-7, 1971

E-6455

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Abstract

Theoretical results are presented for the buckling of anisotropic plates. The plates are subjected to simple and combined inplane loading. The plates are made from fiber composite material of boron/aluminum or high modulus graphite/resin. The results are presented in nondimensional form as buckling load versus fiber orientation angle for various plate aspect ratios.

1. INTRODUCTION

Feasibility studies for the Space Shuttle have indicated that the use of advanced fiber composite structural components can result in a considerable increase in payload in the shuttle system. Boron/aluminum and graphite/resin fiber composites are leading contenders for shuttle applications because these composites offer high stiffness-to-density and high strength-to-density ratios. Panels made from these materials will have to meet both material strength and buckling requirements. This paper will deal with a theoretical investigation of the buckling of flat rectangular panels made from boron/aluminum and Thornel-75/Epoxy fiber composites.

Several papers have appeared recently dealing with the buckling of anisotropic plates.[1-8] However, design data are not available for flat panels subjected to compressive loads and made from advanced fiber composites such as boron/aluminum and graphite/resin. The method described in Reference 3 has proved efficient in buckling studies of boron/epoxy plates. This method is used herein to generate design data for boron/aluminum plates. Some data for Thornel-75 graphite/epoxy/resin plates are also generated for comparison purposes. Data were generated for plate aspect ratios of 1/2, 1, 2, and 4.

The panels considered are anisotropic and simply supported. They are subjected to combined inplane (normal and shear) loads (Fig. 1). The material is a unidirectional composite with the fiber direction oriented at an arbitrary angle to the load direction (Fig. 1). The analytical algorithm used is the assumed mode method in conjunction with the Galerkin method. A computer code was developed based on the Galerkin method and the code was used to generate the theoretical design data presented herein. A brief description of the analytical method is given in the report. The numerical algorithm used to solve the resulting eigenvalue problem and a listing of the computer program with sample cases are given in appendices B and C of Reference 11.

2. BRIEF DESCRIPTION OF UNDERLYING THEORY

The underlying theory for buckling loads of anisotropic plates is discussed in Reference 3 with pertinent discussions in References 6-8. Briefly, this theory consists of expressing the potential energy of a plate in terms of displacement variables. Taking the variation of the potential energy function yields the field equation and the corresponding boundary conditions. The resulting system then is solved by the assumed mode technique in conjunction with the Galerkin method.

The resulting equation after the variation of the energy function is given as follows:

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$$\int_0^a \int_0^b [D_{11}w,_{xxxx} + 2(D_{12} + 2D_{33})w,_{xxyy} + D_{22}w,_{yyyy}$$

$$+ 4D_{13}w,_{xyxx} + 4D_{23}w,_{xyyy}$$

$$-(N_x w,_{xx} + 2N_{xy}w,_{xy} + N_y w,_{yy})]\, \delta w\, dy\, dx$$

$$+\int_0^b (D_{11}w,_{xx} + D_{12}w,_{yy} + 2D_{13}w,_{xy}) \Big|_0^a \delta w,_x dy$$

$$+\int_0^a (D_{21}w,_{xx} + D_{22}w,_{yy} + 2D_{23}w,_{xy}) \Big|_0^b \delta w,_y dx = 0$$

where D represents plate bending stiffness about the structural axis; w represents the out-of-plane displacement; (,) represents partial differentiation with respect to the variables following the comma; $\overline{N}$ represents the in-plane applied load whose directions are identified by the subscript(s); δ denotes the variation of the function following it; and a and b are the plate dimensions. See also Figure 1. Note, the area integral in Equation (1) represents the field equation and the line integrals represent the boundary conditions.

The assumed buckling mode described in Reference 3 is represented by a Fourrier double sine series. This mode satisfies the imposed boundary conditions, but, it does not satisfy the natural boundary conditions if the material and structural axes do not coincide. However, the mode is forced to satisfy the natural boundary conditions approximately through the Galerkin method as discussed in Reference 3.

Substituting the assumed mode in Fquation (1), applying the Galerkin method and carrying out the algebra, results in a set of linear equations which represent the eigenvalue problem of the plate. This system is coupled for either a combination of shear and normal loads and/or noncoincident material and structural axes.

The eigenvalue problem is solved using the Power method which is a highly effective iterative numerical technique in seeking the largest eigenvalue of the system. The indicial equations which were used to generate this system and the Power method are given in appendix B of Reference 11 and in Reference 3.

3. BRIEF DESCRIPTION OF COMPUTER PROGRAM

The numerical algorithm described in appendix B[11] has been developed into a computer code which is rather simple and can be generated from the information supplied in appendix B[11] or Reference 3. A listing of the computer program is given in appendix C,[11] with sample cases for both input and output.

The inputs to the code are: composite system indentification, fiber volume ratio, orientation angle, plate aspect ratio, and flexural rigidities (bending stiffness). The outputs are the number of terms in the assumed mode series expansion required for convergence, the relative error between the two last iteration cycles, the buckling load and topo-plot data of the buckled shape of the plate normalized with respect to the largest deflection.

The alogrithm runs into difficulty when the shear buckling load of a plate is sought. In this particular case, the difficulty is overcomed by including normal loads which are a very small fraction of the shear load. Further discussion on why these difficulties arise are described in Reference 3.

4. THFORETICAL DESIGN DATA

The theoretical design data generated herein are based on the schematic illustrated in Figure 1. Note, that in this figure, the type of loading condition, the plate geometry, and the fiber orientation are defined. The x-y coordinate reference system is referred to as the structural axes system. The fiber direction co-ordinate system which is located at the angle θ from the structural axes system is referred to as the material axes system. The loading conditions are identified by N_x, N_y, and N_{xy} is noted in the figure.

The bending stiffnesses required in calculating the buckling loads were calculated using the computer code described in Reference 9. Typical values of the elastic constants of the plate along its material axes are given in Table 1 for boron/aluminum and Thornel-75/resin composites with a fiber volume ratio of 0.5. The flexural rigidities are computed as functions of the orientation angle using the data in Table 1.

4.1 BUCKLING OF BORON/ALUMINUM AND THORNEL-75/RESIN PANELS

Buckling loads for a single loading condition for panels from boron/aluminum and Thornel-75/epoxy are illustrated in Figure 2, where the specific buckling stress has been plotted as a function of the orientation angle for an aspect ratio of 2. The schematic in the figure illustrates the type of load condition as well as the orientation angle. As can be seen, in this figure, boron/aluminum composites are more efficiently utilized than Thornel-75/resin composites in structures which are critical in buckling as measured by the specific buckling stress.

Buckling load comparisons where the panels are loaded in the y-direction are illustrated in Figure 3. In this figure, it is seen that the boron/aluminum composite panel is considerably stronger in buckling than the corresponding Thornel-75/epoxy panel. In this plot, the nondimensional buckling load parameter is plotted as a function of the orientation angle for the fixed panel aspect ratio of 2.

4.2 BUCKLING LOADS FOR INDIVIDUAL LOADING CONDITIONS

Design data for boron/aluminum panels which are subjected to compressive load in the x-direction are illustrated in Figure 4 for various aspect ratios and orientation angles. In this figure the nondimensional buckling load parameter is plotted as a function of the orientation angle for various aspect ratios. The important point to be noted from this figure is that the buckling load is independent, or almost independent, of orientation angle in panels where the aspect ratio is approximately greater than one. Another point to be noted is that the buckling load depends only moderately on the orientation angle in panels of aspect ratio less than one.

A cross-plot of Figure 4 is illustrated in Figure 5. The nondimensional load is plotted as a function of panel aspect ratio for various orientation angles. The insensitivity of the buckling load as a function of orientation angle in panels with aspect ratio greater than about one is clearly illustrated in this figure. The dashed line symbol is used to represent these curves to emphasize that buckling load values were computed only at the aspect ratios 1/2, 1, 2, and 4.

Buckling loads for panels which are loaded in the y-direction only are given in Figure 6. As can be seen, in this figure, the buckling loads are almost independent of the orientation angle in panels with aspect ratios of 1/2 and greater. The buckling load on the other hand, is very sensitive to the aspect ratio in panels with aspect ratios of approximately less than two.

Buckling loads for a boron/aluminum composite panel loaded with shear only are illustrated in Figure 7. The points to be noted in this figure are the following:

(1) There is a mild buckling load dependence on the orientation angle for panel aspect ratios of less than about one.
(2) The buckling load is relatively independent of orientation angle for panel aspect ratios of greater than about one.
(3) The buckling load is very sensitive to the aspect ratio less than two, and this dependence becomes rather insignificant for panel aspect ratios greater than two.

4.3 BUCKLING LOADS FOR TWO EQUAL SIMULTANEOUS LOADINGS

Buckling loads, when the panel is loaded with equal loads in the x- and y-direction, are shown in Figure 8. In this figure, the nondimensional buckling load parameter is plotted as a function of orientation angle for various panel aspect ratios.

The results in this figure show that the buckling load is slightly dependent on the orientation angle for panel aspect ratios less than one, and practically independent of orientation angle for aspect ratios greater than one. The buckling load is sensitive to panel aspect ratio for aspect ratios less or equal to two. This dependence becomes insignificant for panel aspect ratios greater than two. The curves of the buckling load as a function of the independent variables indicated in Figure 8, parallel the curves of the buckling loads indicated in Figures 4 and 6, for the individual loadings.

Buckling loads for the case when the panel is loaded in the x-direction combined with shear are shown in Figure 9. The curves of the buckling load for this loading condition is parallel to that of the individual cases (Figs. 4 and 7). Buckling loads for the case when the panel is loaded in the y-direction combined with shear are illustrated in Figure 10. The buckling load in this figure seems to be practically independent of orientation angle for the aspect ratios investigated. However, it is quite sensitive to the panel aspect ratio for aspect ratios less than two.

4.4 BUCKLING LOADS FOR THREE EQUAL SIMULTANEOUS LOADINGS

Buckling loads for panels which are loaded in the x and y-directions combined with shear are shown in Figure 11. The schematic in this figure indicates the type of loadings and their respective ratios. The nondimensional buckling load is plotted as a function of orientation angle for various panel aspect ratios.

Comparing corresponding curves from Figures 8 and 11, it is seen that the addition of the shearing load decreases the buckling load of the panel only slightly. The point to be noted then is, that a panel subjected to compressive loads in the x and y-direction will resist almost an equal amount of shearing load for approximately the same buckling load.

4.5 BUCKLING LOAD FOR TWO OR THREE UNEQUAL SIMULTANEOUS LOADINGS

Buckling loads for panels which are subjected to unequal loads in the x and y-directions are shown in Figure 12. The type of loading condition and respective loading magnitudes are illustrated in the sketch given in the figure. In this figure, the nondimensional buckling load parameter is plotted as a function of the orientation angle for various aspect ratios. The curves of the buckling load for this type of loading condition parallel those of the cases with equal loading condition, as was previously discussed (see Fig. 8). One additional point to be noted, is that, the buckling loads of panels with aspect ratios of greater than about two, remain almost invariant as a function of aspect ratio when the orientation angle is greater than about 45^{o}.

Buckling data for panels loaded with unequal combined loading conditions are illustrated in Figure 13. The loading condition for the panel is illustrated in the schematic in the figure. The buckling load is plotted as a function of orientation angle for various aspect ratios. A point to note, in this figure, is that, at some orientation angles, panels with aspect ratios of greater than two could have greater buckling loads than panels with aspect ratio of two. As can be seen, the buckling load for panels with aspect ratio of approximately two is lower than the panel with aspect ratio of four when the orientation angle is approximately less than 45^{o}.

All the buckling data presented and discussed indicate that the buckling load is insensitive to orientation angle for panels with high aspect ratios. The buckling load is mildly sensitive to the orientation angle in panels with low aspect ratios. This observation leads to the important conclusion that the buckling loads of boron/aluminum composite anisotropic plates can be approximately determined using classical orthotropic theory. This conclusion is indeed a useful one, since the buckling of orthotropic plates has been extensively treated in Reference 5.

5. POSSIBLE EXTRAPOLATIONS OF DESIGN DATA

The design data presented and discussed were based on a fixed fiber volume ratio of 0.5. The data presented herein can be used to extrapolate buckling loads for plates made from composites with different fiber volume ratio. The results will be very close if the variation of the fiber volume ratio is within approximately ±20 percent of the 0.5 value which was used in generating the design data.

The extrapolation recommended above is justified since the nondimensional buckling load is nondimensionalized with respect to both composite longitudinal modules and thickness. It is well known that both composite moduli and thickness depend on the fiber volume ratio[10] and that this dependence is approximately linear in the fiber volume ratio range 0.4 to 0.6. In this sense, then, the extrapolation using the design data presented herein for fiber volume ratios within ±20 percent of 0.5 should yield reasonable results.

6. CONCLUSIONS

The discussion of the theoretical design data presented leads to the following conclusions:

(1) Design data for the buckling of unidirectional boron/aluminum panels with fibers oriented at any angle to the load direction have been generated. and are reported herein.
(2) Specific buckling stress comparisons showed that boron/aluminum composite panels are more efficient than high modulus graphite/resin composites panels in general.
(3) The buckling load of boron/aluminum unidirectional panels is practically independent of fiber direction at high aspect ratio values. At these aspect ratios, the plate can be assumed to have its material axes coincide with its structural axes. Consequently, classical buckling theory of orthtropic plates can be used to predict the buckling load.
(4) The buckling loads of boron/aluminum unidirectional panels is only modestly dependent on fiber direction at plate aspect ratios of less than one.
(5) Boron/aluminum composite panels loaded by normal in-plane loads which are near the critical load can carry considerable shear load before they buckle.
(6) The buckling loads of boron/aluminum panels are practically independent of aspect ratio and loading condition at aspect ratios greater than about two.
(7) The buckling loads of panels with fiber volume ratio within approximately ±20 percent from 0.5 can be extrapolated from the design data presented herein, using the appropriate panel thickness and the appropriate composite longitudinal modulus.

7. REFERENCES

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BIOGRAPHY

Dr. Christos C. Chamis

Dr. Christos C. Chamis is presently with the Structures and Polymers Section of the NASA-Lewis Research Center, Cleveland, Ohio where he has been since 1968. He received his B.S. in Civil Engineering (1960) from Cleveland State, M.S. (1962) and Ph.D. (1967) in Engineering Mechanics from Case Western Reserve University, where he was a member of the Engineering Design Center. His current research is in the area of analysis, design and optimization of composite structural components. He is also involved in the analysis and design of testing methods for advanced composites. His experience in structural fiber composites dates back to 1962 when he was with the Engineering Analysis group of B. F. Goodrich Research Center.

TABLE I. - THEORETICAL UNIDIRECTIONAL COMPOSITE PROPERTIES AT FVR = 0.5

Property	Boron/Aluminum		Thornel-75/Epoxy	
Longitudinal modulus	24.2×10^6 N/cm^2	35.0×10^6 psi	26.0×10^6 N/cm^2	37.8×10^6 psi
Transverse modulus	16.8×10^6 N/cm^2	24.3×10^6 psi	6.9×10^6 N/cm^2	1.0×10^6 psi
Shear modulus	8.0×10^6 N/cm^2	11.6×10^6 psi	0.44×10^6 N/cm^2	0.63×10^6 psi
Poisson's ratio	0.24	0.24	0.25	0.25
Density	2.62 g/cm^3	0.095 lb/in.^{3}	1.55 g/cm^3	0.056 lb/in.^{3}

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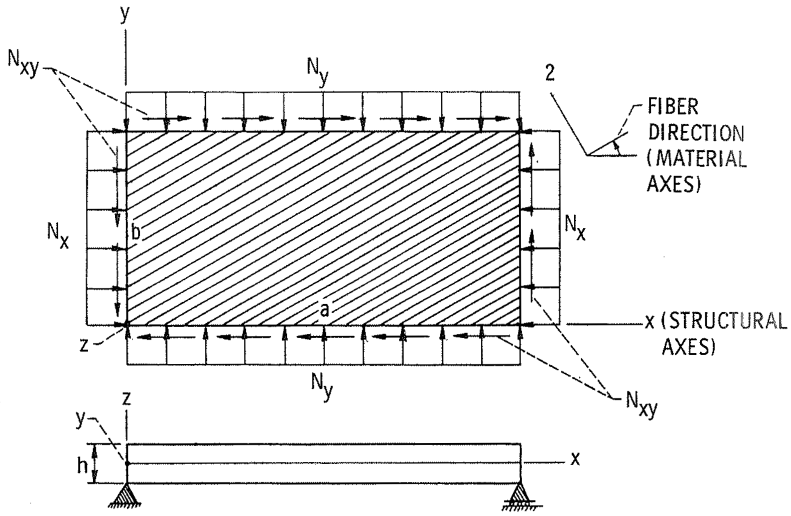

Figure 1. - Fiber-composite plate geometry and loading - all four edges simply supported (aspect ratio = a/b).

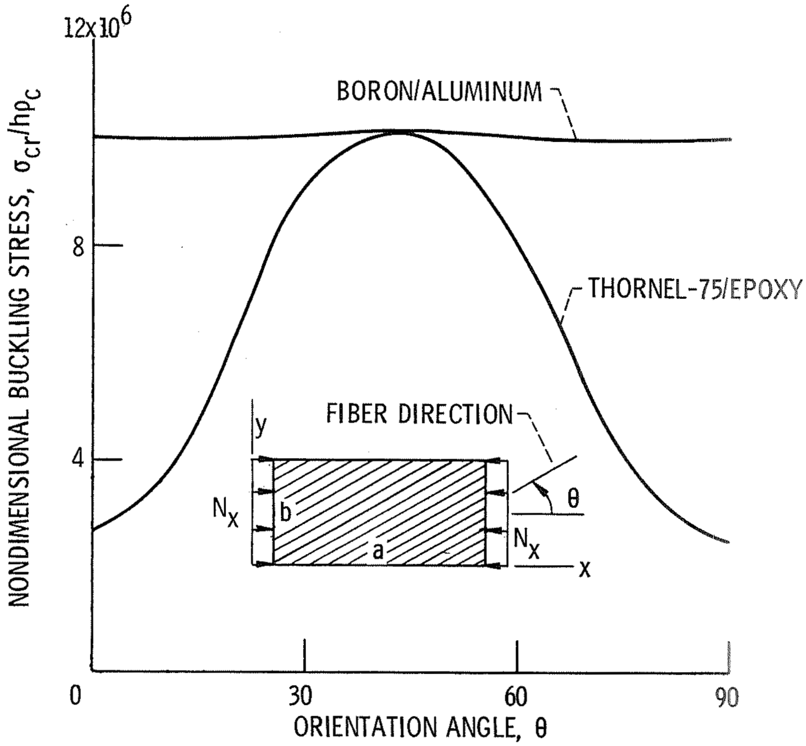

Figure 2. - Specific buckling stress of two fiber composites plates - simply-supported four edges. FVR = 0.5. (a/b) = 2.

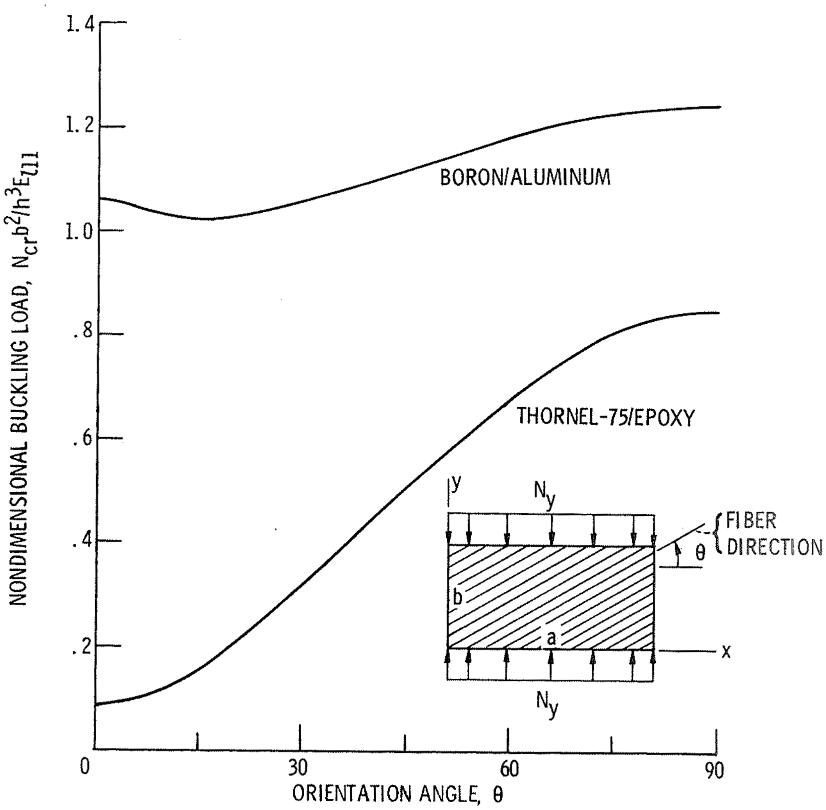

Figure 3. - Buckling loads for two fiber composite plates - simply supported four edges. FVR = 0.5. (a/b) = 2.

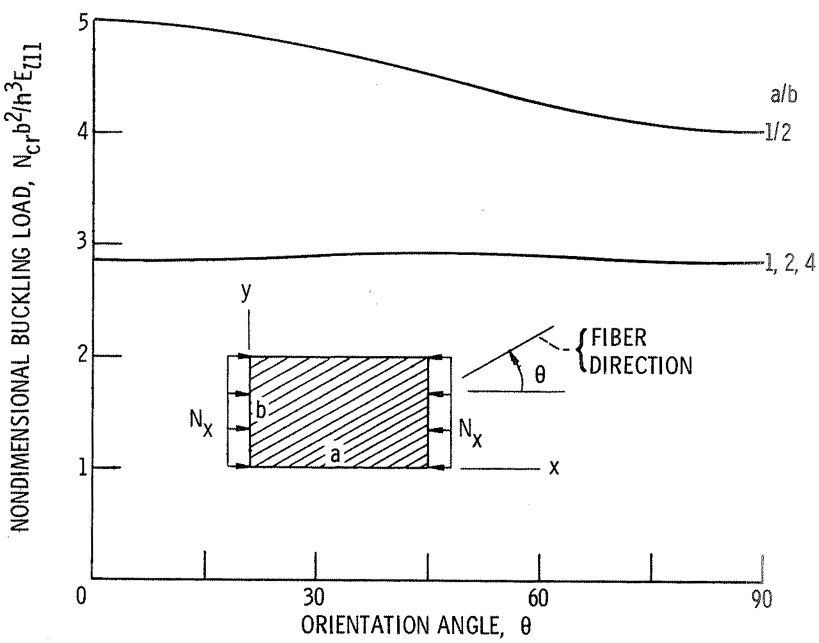

Figure 4. - Buckling loads for boron/aluminum composite plates - simply supported four edges with FVR = 0.5 and subjected to normal (N_x) compressive load.

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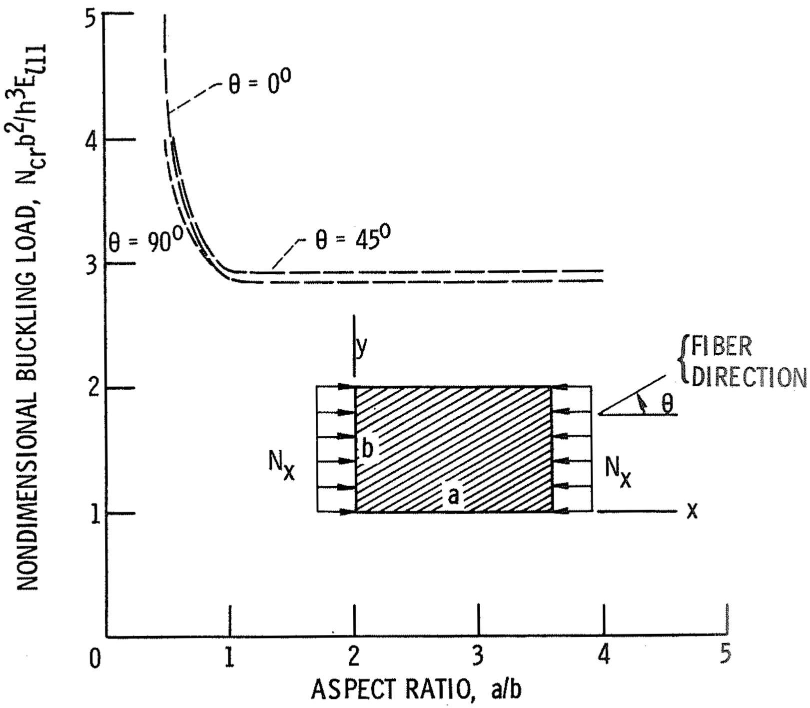

Figure 5. - Buckling loads for boron/aluminum composite plates with FVR = 0.5. Simply supported four edges and subjected to normal (N_x) load.

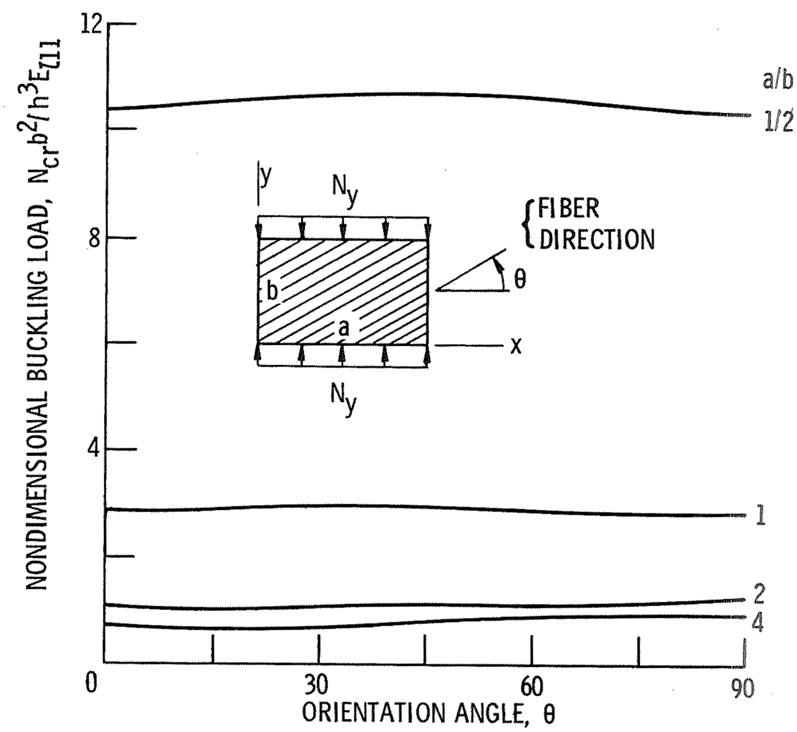

Figure 6. - Buckling loads for boron/aluminum composite plates with FVR = 0.5, simply supported four edges and subjected to normal (N_y) compressive load.

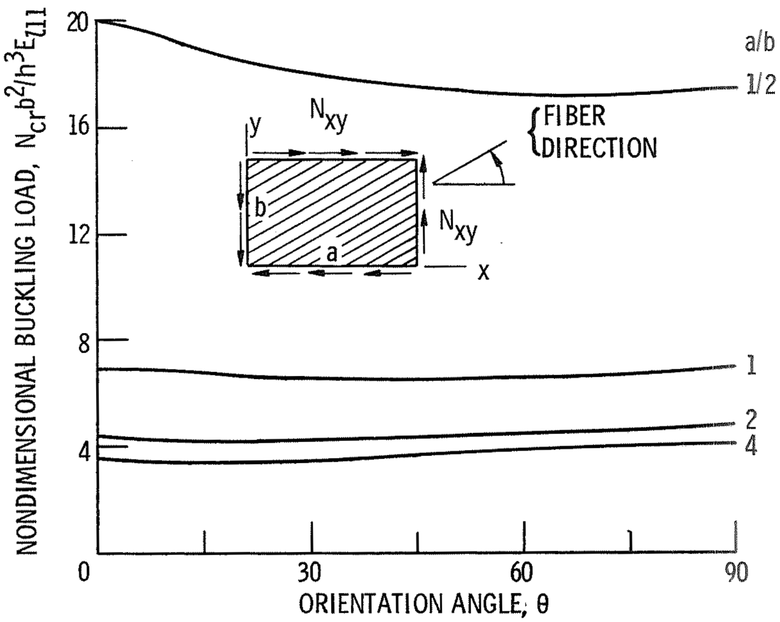

Figure 7. - Buckling loads for boron/aluminum composite plates with FVR = 0.5, simply supported four edges and subjected to shear (N_{xy}) load.

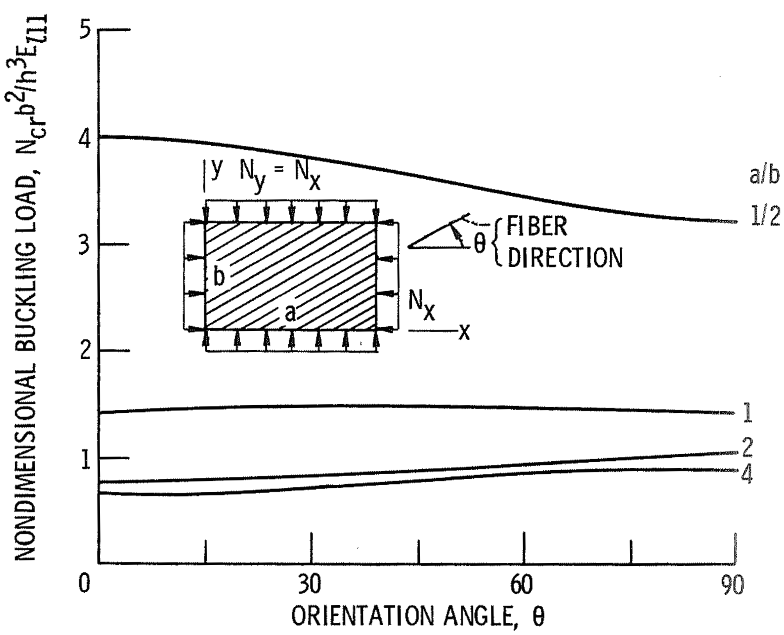

Figure 8. - Buckling loads for boron/aluminum composite plates with FVR = 0.5, simply supported four edges and subjected to combined normal compressive ($N_x = N_y$) loads.

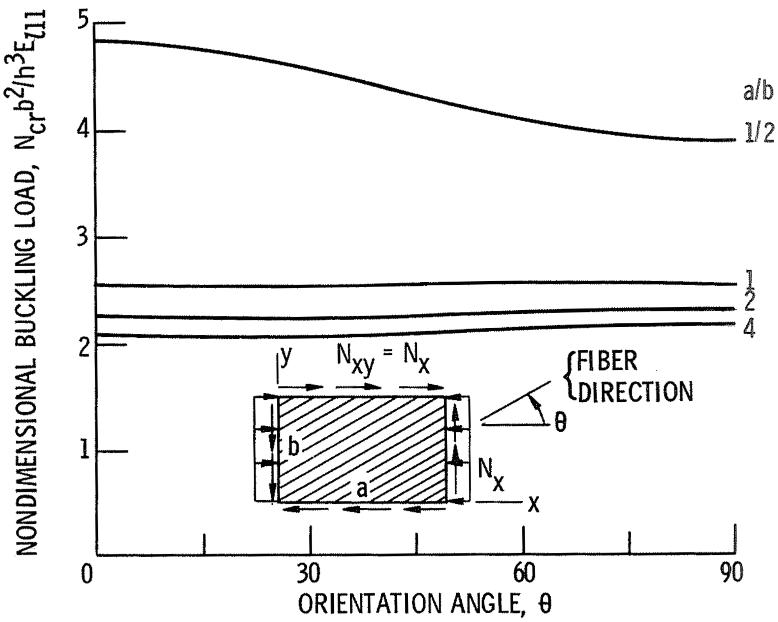

Figure 9. - Buckling loads for boron/aluminum composite plates with FVR = 0.5, simply supported four edges and subjected to combined normal (N_x) and shear ($N_{xy} = N_x$) loads.

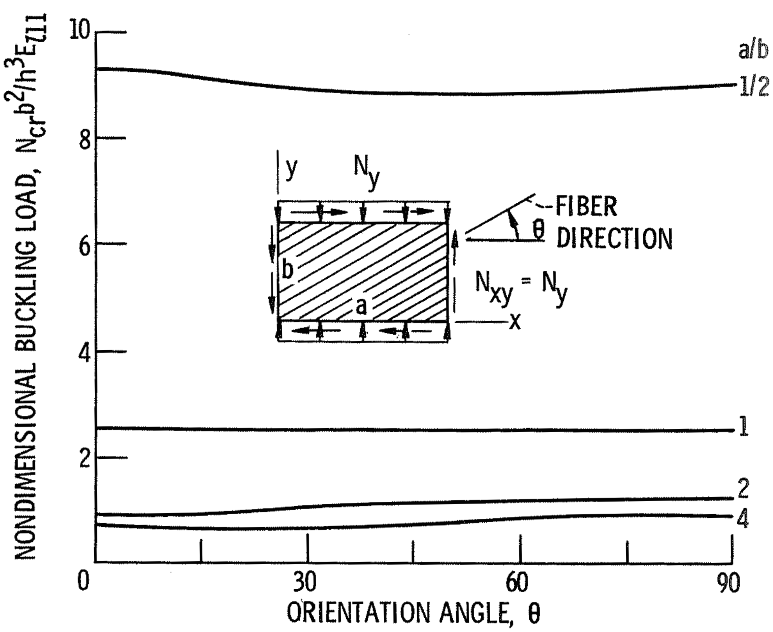

Figure 10. - Buckling loads for boron/aluminum composite plates with FVR = 0.5, simply supported four edges and subjected to combined normal (N_y) and shear ($N_{xy} = N_y$) loads.

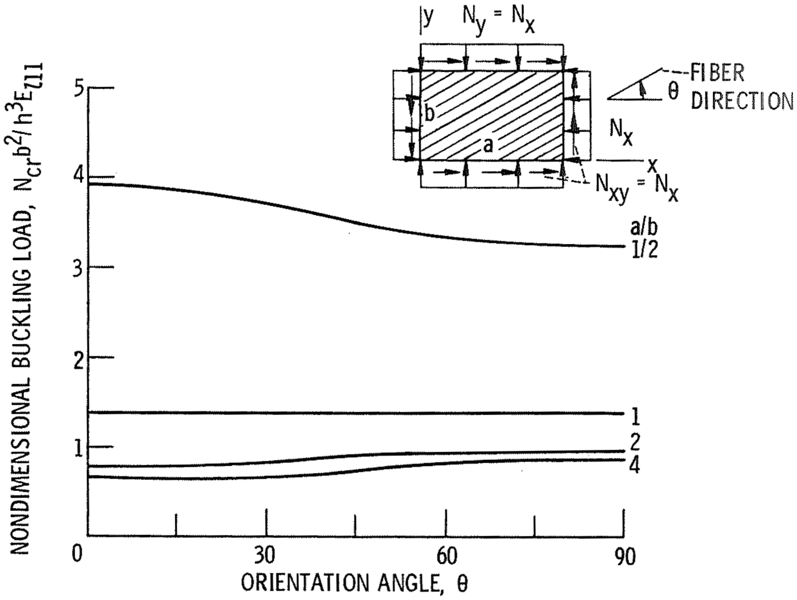

Figure 11. - Buckling loads for boron/aluminum composite plates with FVR = 0.5, simply supported four edges and subjected to combined normal ($N_y = N_x$) and shear ($N_{xy} = N_x$) loads.

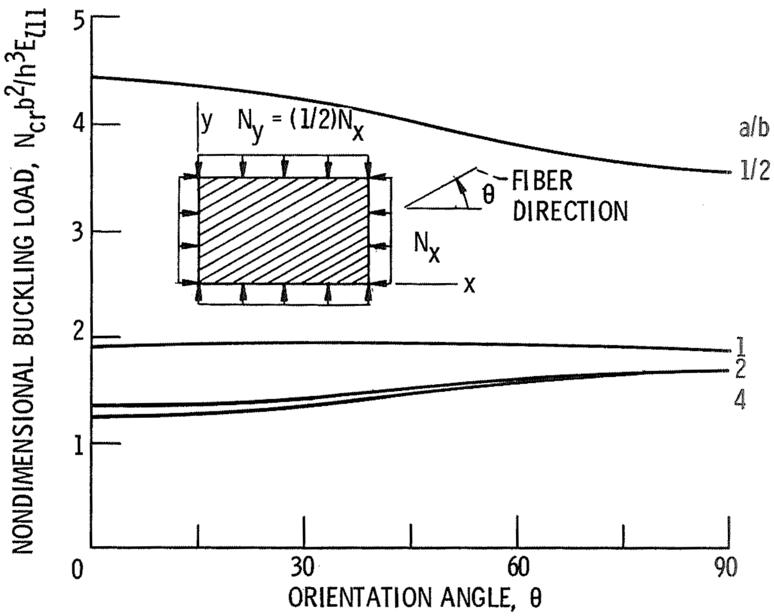

Figure 12. - Buckling loads for boron/aluminum composite plates with FVR = 0.5, simply supported four edges and subjected to combined normal ($N_y = (1/2)N_x$) loads.

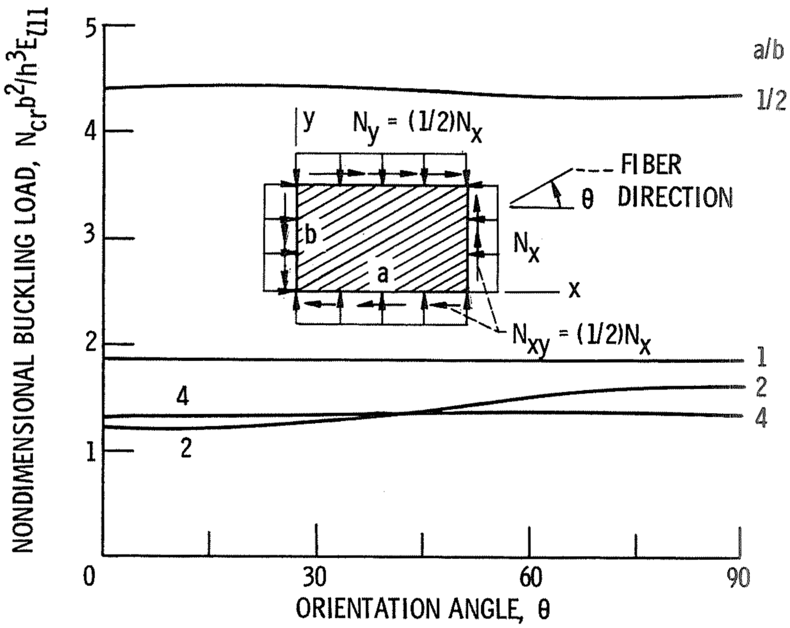

Figure 13. - Buckling loads for boron/aluminum composite plates with FVR = 0.5, simply supported four edges and subjected to combined normal ($N_y = (1/2)N_x$) and shear ($N_{xy} = (1/2)N_x$) loads.