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SWATHING PATTERNS OF EARTH-SENSING SATELLITES AND THEIR CONTROL BY ORBIT SELECTION AND MODIFICATION

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SUMMARY

A generalized investigation of the swathing patterns produced by earth observation satellites in circular, sun-synchronous orbits reveals an extensive array of orbits with interesting and varied pattern-generating properties. The study focuses on repeating-type patterns, which occur well-distributed over the useful range of orbital altitudes and repeat cycle periods, and in particular on "minimum-drift" repeating patterns. The latter are regularly distributed around a series of zero-drift altitudes and are advantageous because of their uniform swath progressions and one-day intervals between adjacent swaths. A major choice or compromise in pattern selection is between extent of geographic coverage (~5 to 100%) and frequency of re-observation of covered areas (1 to ~20 days). It is feasible, however, to modify the swathing patterns of operating satellites by altering their orbits, with a propellant budget probably not exceeding a few percent of the spacecraft weight.

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INTRODUCTION

In surveying the possible choices of swathing patterns for earth-sensing satellites, several features are seen to be advantageous for a variety of applications. Among these are:

- Full earth coverage
- Viewing under advantageous and consistent lighting conditions
- Viewing adjacent areas with minimum time lapse
- Repetitive viewing of areas of interest
- Adequate sensor resolution
- Adequate lifetime

These criteria figured in the choice of the operational orbit to be employed by the initial Earth Resources Technology Satellites (ERTS) (Ref. 1). This orbit is of approximately 493 n.m. altitude and 99° inclination. The 493 n.m. altitude produces an orbital period such that the satellite falls slightly short of completing its fourteenth revolution in 24 hours, with the result that the subsatellite swaths are displaced somewhat westward on each succeeding day. By prescribing the altitude (and period) precisely, the subsatellite swaths are made contiguous, so that the entire earth's surface, within the latitudes overflown, is covered after a sufficient time (18 days, in this case). The $\sim 99°$ inclination is determined explicitly by the requirement that the orbit be sun-synchronous, i.e., that the orbit plane revolve at the same rate as the earth-sun line. This provision maintains, except for unavoidable seasonal variations in sun aspect, the desired sub-satellite illumination conditions established originally by proper orientation of the initial orbit. It should be noted in addition that the high inclination angle also permits the near-total earth coverage obtained (for a 99° orbit, latitudes above 81° N. and S. are not overflown).

Thus, by judiciously selecting the orbit for the early ERTS missions, a number of desired swathing pattern features will be obtained: nearly full earth coverage every 18 days, viewing of adjacent areas with a one-day time lapse, and control of lighting conditions. The selected orbit has also been examined and found satisfactory with respect to sensor resolution and lifetime.

For subsequent missions, other swathing pattern features may be desirable, such as the capability to view some particular area of interest more frequently, perhaps daily. In fact, it may be desirable and feasible with more sophisticated follow-on earth observation satellites to modify the operational orbit according to needs which become apparent during the mission.

With such possibilities in mind, the present analysis is aimed at examining the features of swathing patterns and their associated orbits in a general way, and in making a preliminary estimate of the feasibility and usefulness of various modes and the requirements for transferring from one mode to another.

GENERATION OF SWATHING PATTERNS

Basically, the free parameter at our disposal in controlling swathing patterns is simply orbital altitude. This is true because altitude determines orbital period which, together with the fixed angular rotation rate of the earth, determines explicitly the subsatellite track. It is assumed in this discussion that, for consistency in sensing conditions, only circular, sun-synchronous orbits are being considered.

To begin the analysis, consider a circular, sun-synchronous orbit of arbitrary altitude. The subsatellite track produced by a spacecraft in this orbit will be developed over successive revolutions as shown in Fig. 1. The subsatellite point is assumed to begin its motion at time t = 0 and at point P_0, on the equator and at an arbitrary meridian. After one complete revolution, designated Rev. 1, the subsatellite point is again on the equator (at point P_1) and moving southward, with t = T, where T is the orbital period. The westward longitudinal displacement of P_1 from P_0, produced by the eastward rotation of the earth under the satellite orbit, is:

$$\Delta\lambda = \frac{T}{D} \times 360° \qquad (1)$$

where D is the earth's period of rotation (one day = 24 hours).

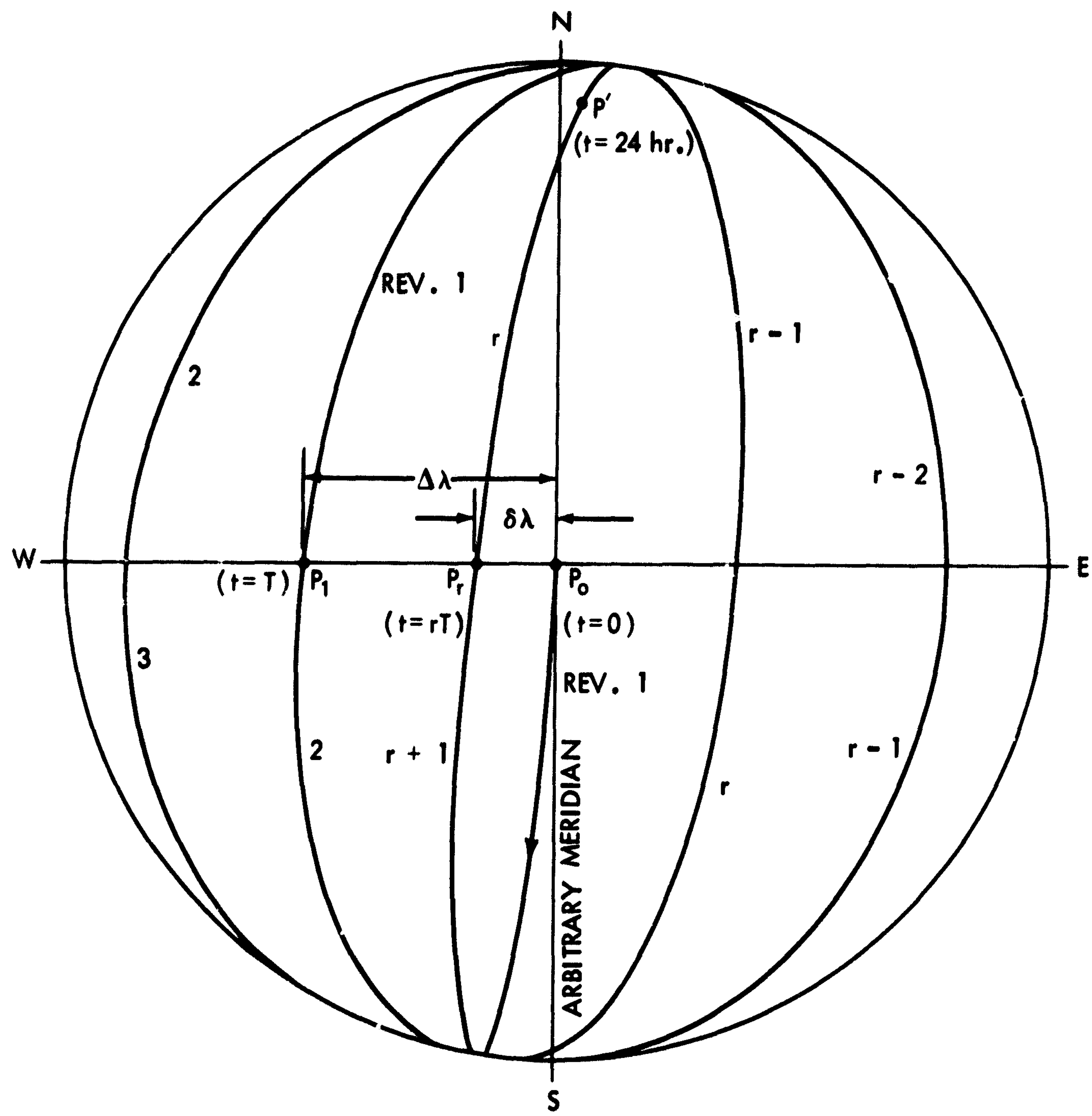


Figure 1. Development of Subsatellite Tracks

As t approaches one day, the tracks approach P_0 from the east. The last southward equatorial crossing east of P_0 is the beginning of Rev. r. When $t = 24$ hours exactly, the earth will have made one complete revolution under the "fixed" orbit.* Point P_0 will again be directly under the orbital path, but in general, the subsatellite point will be some distance short of its southward equatorial crossing, as at Point P' in Fig. 1. When $t = rT$, the track of Rev. r terminates on the equator at point P_r, $\delta\lambda$ west of P_0. The parameter $\delta\lambda$, the westward longitudinal displacement of the swath pattern of a given day from that of the preceding day, is referred to hereafter as the "daily drift." It can be expressed quantitatively in terms of the time excess of r orbit periods relative to the length of the day:

$$\delta\lambda = \frac{rT - D}{D} \times 360° \tag{2}$$

<u>Repeating Patterns</u>

Suppose for a moment that the track of revolution r had returned to point P_0 ($\delta\lambda = 0$) – the "one-day repeater" mode. The requirement for this result is that the satellite make r revolutions while the earth is rotating through one revolution, i.e., that the orbital period is a submultiple of the length of the day:

$$rT = D \tag{3}$$

---

*To be more precise, the earth has turned slightly more than one revolution in inertial space during the 24 hours, as required by the $\sim 1°$ rotation of the earth-sun line during that time. To maintain the desired constant angular relationship with the earth-sun line, the orbit has precessed through an equal compensating angle (the sun-synchronism property).

Since D is a constant and r is limited to integral values, T will be limited to a series of discrete values in the one-day repeater mode. The admissible values of T will also be constrained by practical considerations, such as restrictions on altitude imposed by requirements of sensor resolution and orbital lifetime. If we admit, for example, altitudes in the range 100 to 1,000 n.m., we obtain the five values of T plotted along the N = 1 line in Fig. 2. N is the period of the pattern repetition cycle, one day for the present one-day repeater mode. The relationship between T and h in Fig. 2 is the basic expression for the period of a circular orbit:

$$T = 2\pi \sqrt{\frac{(\rho_0 + h)^3}{K}} \tag{4}$$

where ρ_0 = radius of earth

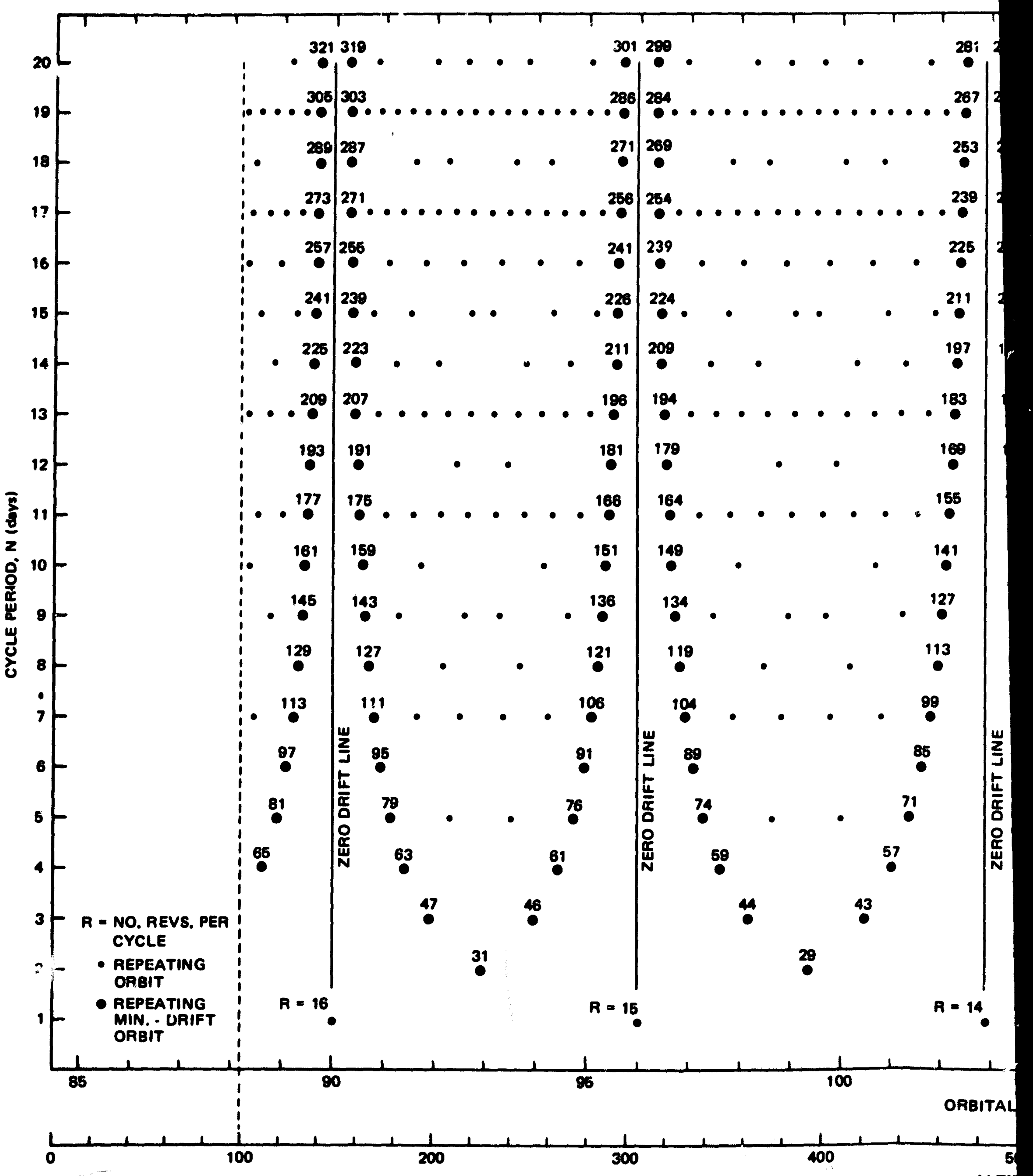
h = orbital altitude

K = gravitational parameter of earth (GM).

To find orbits which produce repeat cycle periods of N days, where N is an integral number, we replace D by DN in Eq. (3) and write:

$$T = D \frac{N}{R} \tag{5}$$

The symbol R is used in Eq. (5) to indicate the integral number of satellite revolutions in a repeat cycle of N days (rather than r, the revolution in progress or terminating when t = D). The basic effect of Eq. (5) is to restrict repeating orbit



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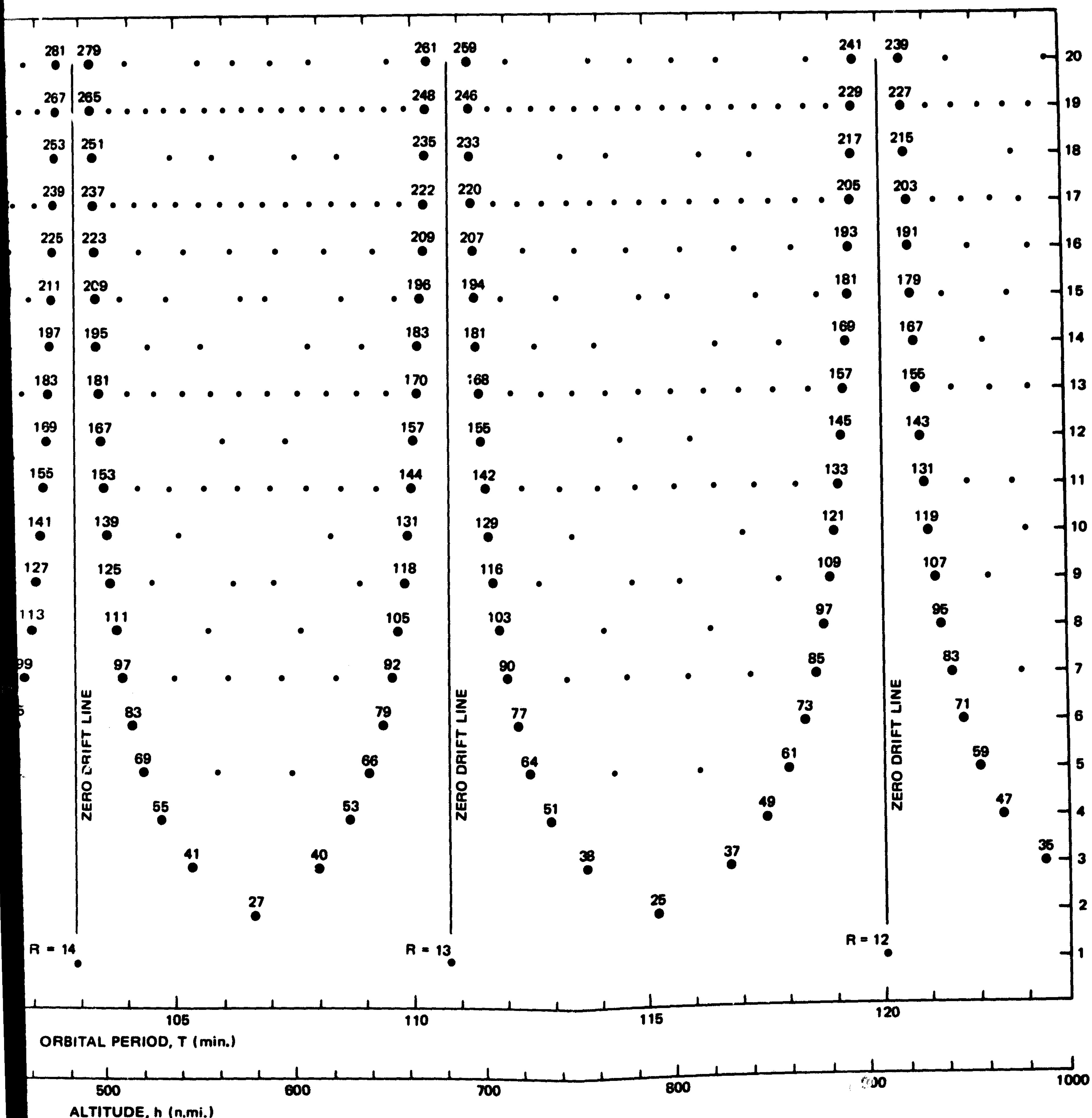

Figure 2. Array of Orbits Which Produce Repeating Swath Patterns

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periods to values which are submultiples of the cycle period (DN). Thus Eq. (3) is a special case of Eq. (5), with N = 1.

For a given admissible range of T values, such as that allowed by the altitude constraint cited previously ($100 < h < 1000$ n.m.), Eq. (5) shows that the admissible values of R grow numerically larger (linearly) as N increases. Hence, the number of integral R values occurring within the admitted range increases linearly with N. This effect is shown in Fig. 2, which displays graphically the array of orbits within the above altitude range which satisfy Eq. (5), with N and R restricted to positive integral values.

Another effect which should be noted is reflected in Fig. 2 as an apparent omission of certain points, notably at even values of N. These omissions occur when the fraction N/R in Eq. (5) is reducible to lower terms. In such cases, the situation is identical to that for the reduced N/R, and in fact, the associated swathing pattern is characteristic of the reduced N. Hence, points corresponding to reducible N/R fractions are omitted as extraneous.

An advantage to moving to longer repeat cycle periods (increasing N) is that greater coverage of the earth is obtained. To illustrate, when N = 1 (the one-day repeater mode), the widely spaced swath pattern shown in Fig. 3 is generated during the first day, and on each subsequent day the same pattern is simply retraced. When N = 2, as in Fig. 4, we get a similar pattern during the first day, but the swath of Rev. r + 1 (Rev. 15 in Fig. 4) falls midway between those

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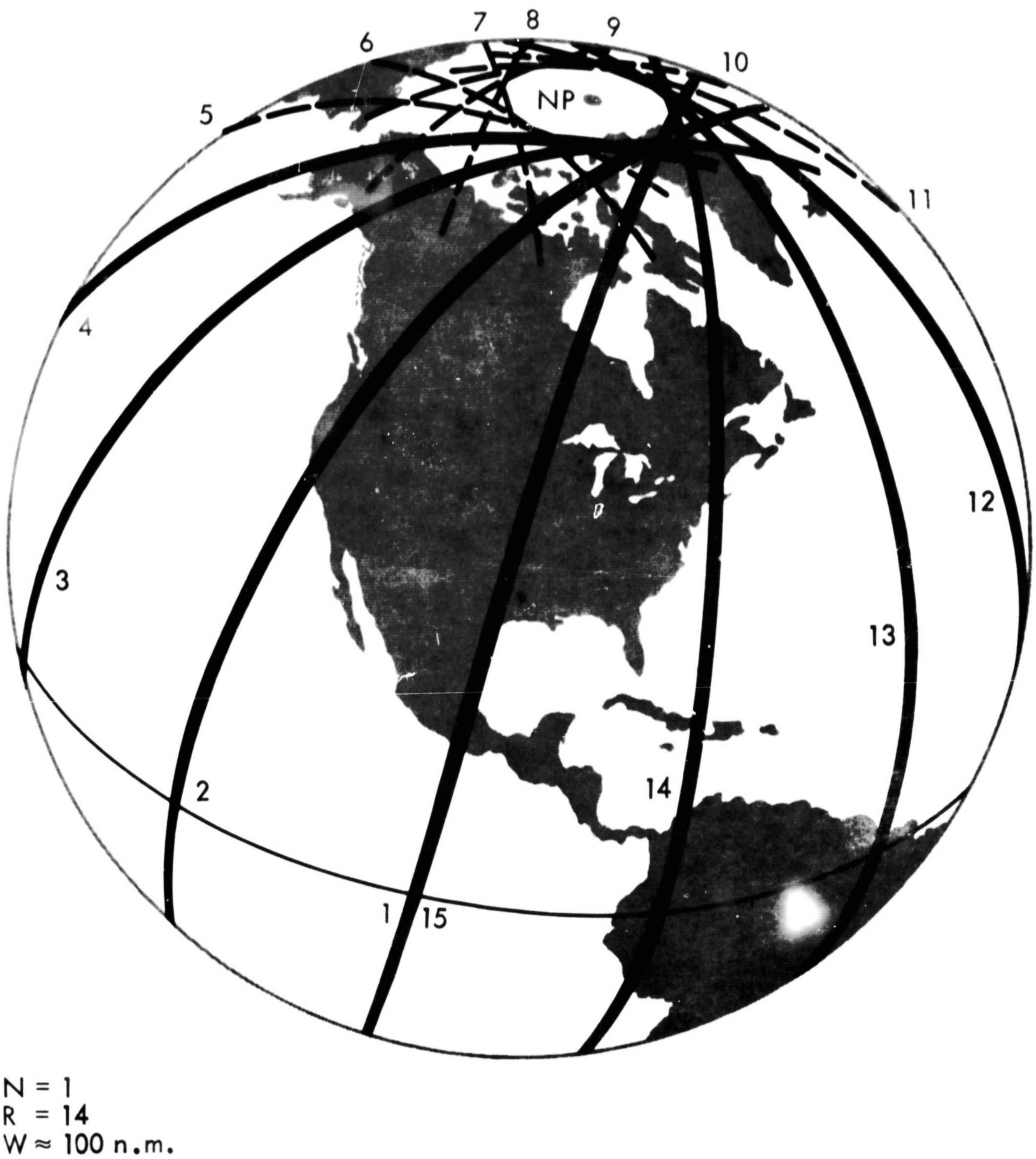
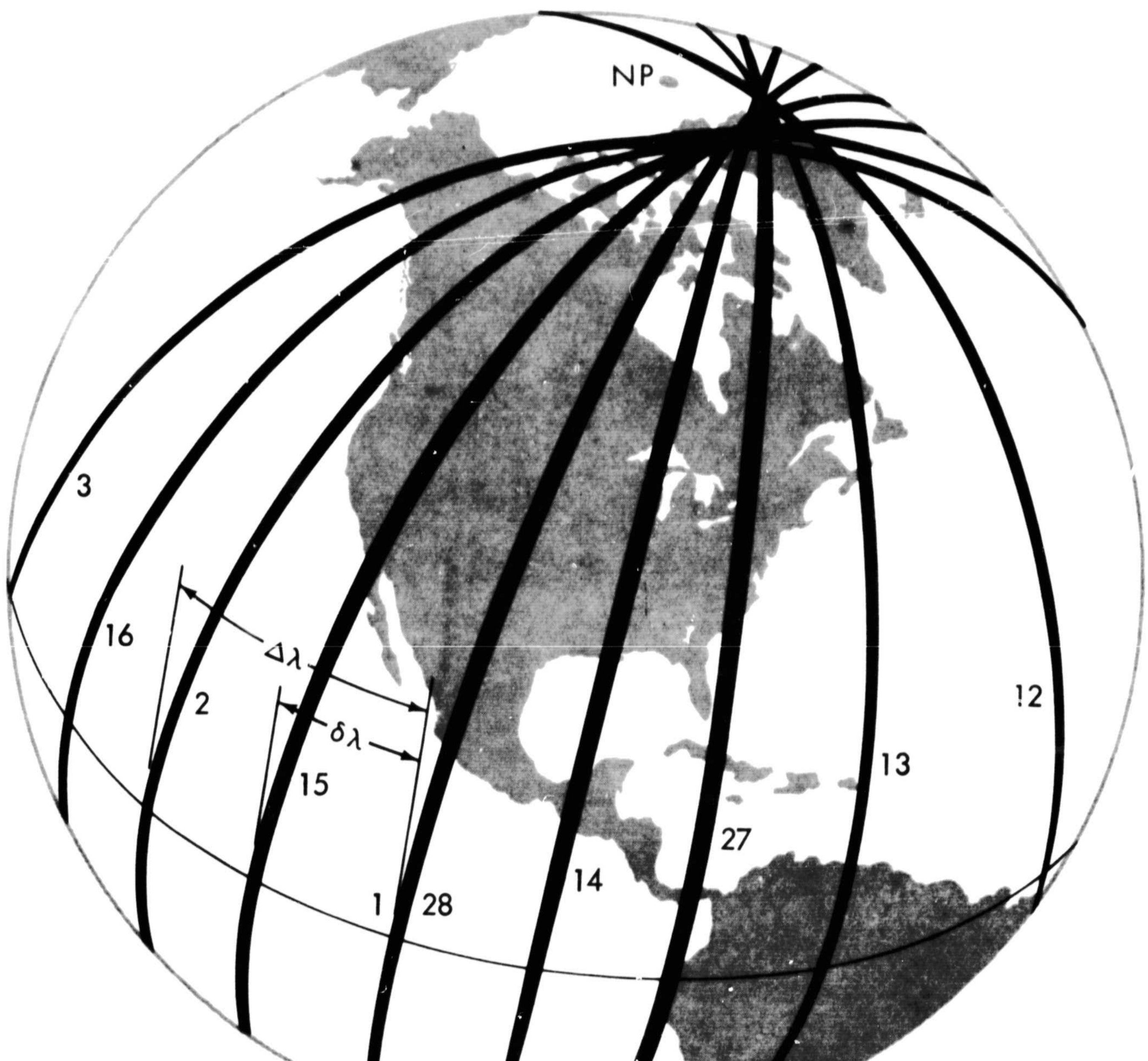


N = 1
R = 14
W ≈ 100 n.m.

NOTE: NORTHBOUND (nighttime) SWATHS ARE OMITTED

Figure 3. Swath Pattern of One-Day Repeater Orbit



N = 2
R = 27
W ≈ 100 n.m.

Figure 4. Swath Pattern of Two-Day Repeater Orbit

of Revs. 1 and 2. This requires that the daily drift $\delta\lambda = \Delta\lambda/2$. The effect is to rotate the entire second-day pattern by $\Delta\lambda/2$ relative to the first-day pattern, resulting in a symmetrical swathing pattern with twice the coverage (at the equator) of the N = 1 mode. If we increase the cycle period further to N = 4, the daily swathing patterns are rotated through $\Delta\lambda/4$ each day, and we get the quadrupled coverage shown in Fig. 5. It has become clear that in this series of patterns

$$\delta\lambda = \frac{\Delta\lambda}{N} \qquad (N > 1) \tag{6}$$

Minimum Drift Patterns

As we proceed to higher values of N, however, we find that some orbits in the N - T array of Fig. 2 obey Eq. (6) and some do not. Specifically, it is only the orbits closest to the "zero drift" lines (vertical lines at the one-day repeater altitudes) which obey Eq. (6). These may be termed "minimum drift" orbits because all other orbits in the array exhibit higher drift rates than those specified by Eq. (6). An important property of the minimum drift orbits is that they produce a uniform, unidirectional progression of swaths of minimum day-to-day displacement (for a given N). Other orbits in the array produce similar geographic coverages over the same cycle periods, but the day-to-day progression of the swaths is not uniform and the time interval between adjacent swaths will generally be greater than one day.

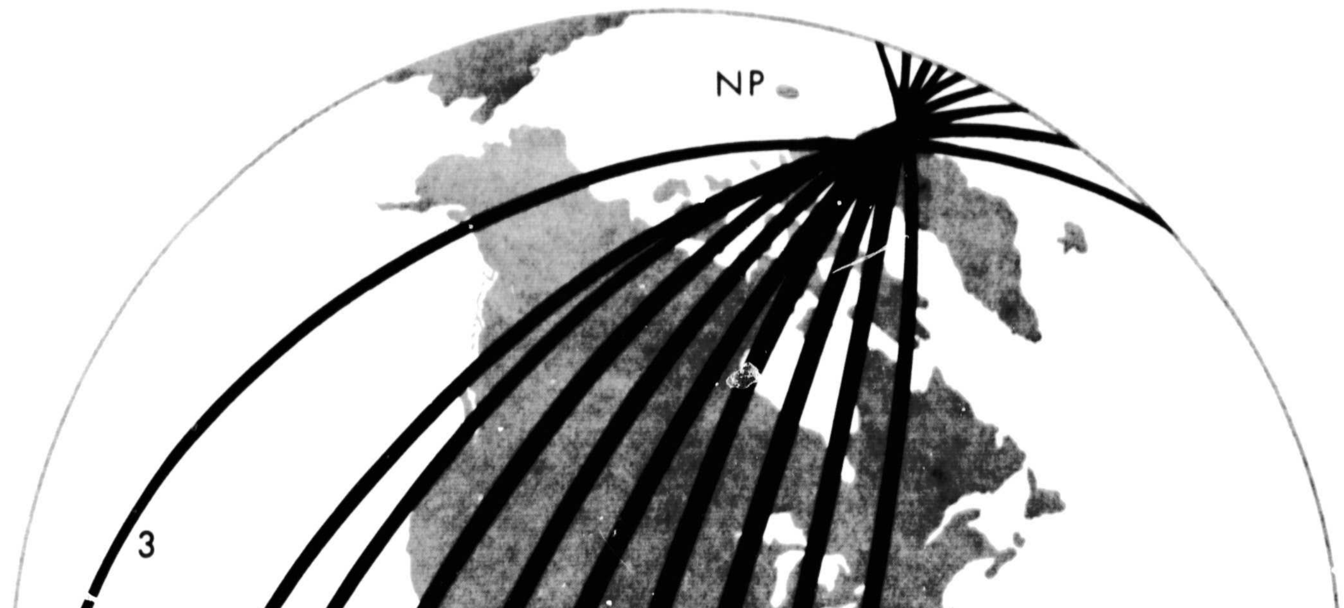


Figure 5. Swath Pattern of Four-Day Repeater Orbit

Minimum-drift orbits are indicated in Fig. 2 by large dots. They are arrayed along a series of hyperbolic curves in Fig. 2, symmetrically about the zero drift lines. Note that this family includes the $R = 27$ and $R = 55$ members, which generate the patterns of Figs. 4 and 5, respectively. In general, minimum drift orbits are identified by the relation

$$R = R_1 N \pm 1 \quad (N > 1) \tag{7}$$

where R_1 is the number of revolutions per day of the adjacent one-day repeater orbit. The sign choice in Eq. (7) determines whether a particular orbit point falls right or left of the adjacent zero-drift line and also the apparent direction of drift (plus for eastward and minus for westward). Drift direction can also be determined from the magnitude of the daily drift $\delta\lambda$ from Eq. (2) relative to the quantity $\Delta\lambda/2$. These characteristics are summarized in the following table:

PROPERTIES OF MINIMUM-DRIFT ORBITS

Revolutions/Cycle	$R = R_1 N - 1$	$R = R_1 N + 1$
Apparent Drift Direction	Westward	Eastward
Position Relative to Adjacent Zero Drift Line (in N-T Array, Fig. 2)	Right	Left
$\delta\lambda$ Range (Eq. 2)	$0 < \delta\lambda < \Delta\lambda/2$	$\Delta\lambda/2 < \delta\lambda < \Delta\lambda$

Fractional Coverage and Overlap

Returning to the subject of earth coverage illustrated by Figures 3, 4, and 5, we find that the useful limit to the increasing-N strategy for improving coverage is reached when the swaths become contiguous or overlap slightly at the equator. Such patterns provide full earth coverage except for the extreme polar regions not overflown ($\phi > \pi - i$). Contiguity is obtained when the daily drift (minimum drift orbits) equals the equatorial swath intercept:

$$\delta \lambda = \frac{W}{60 \sin i} \tag{8}$$

where W is the swath width and i is the orbit inclination. The factor 60 converts W in nautical miles on earth's surface, or subtended angle in minutes at earth's center, to subtended angle in degrees.

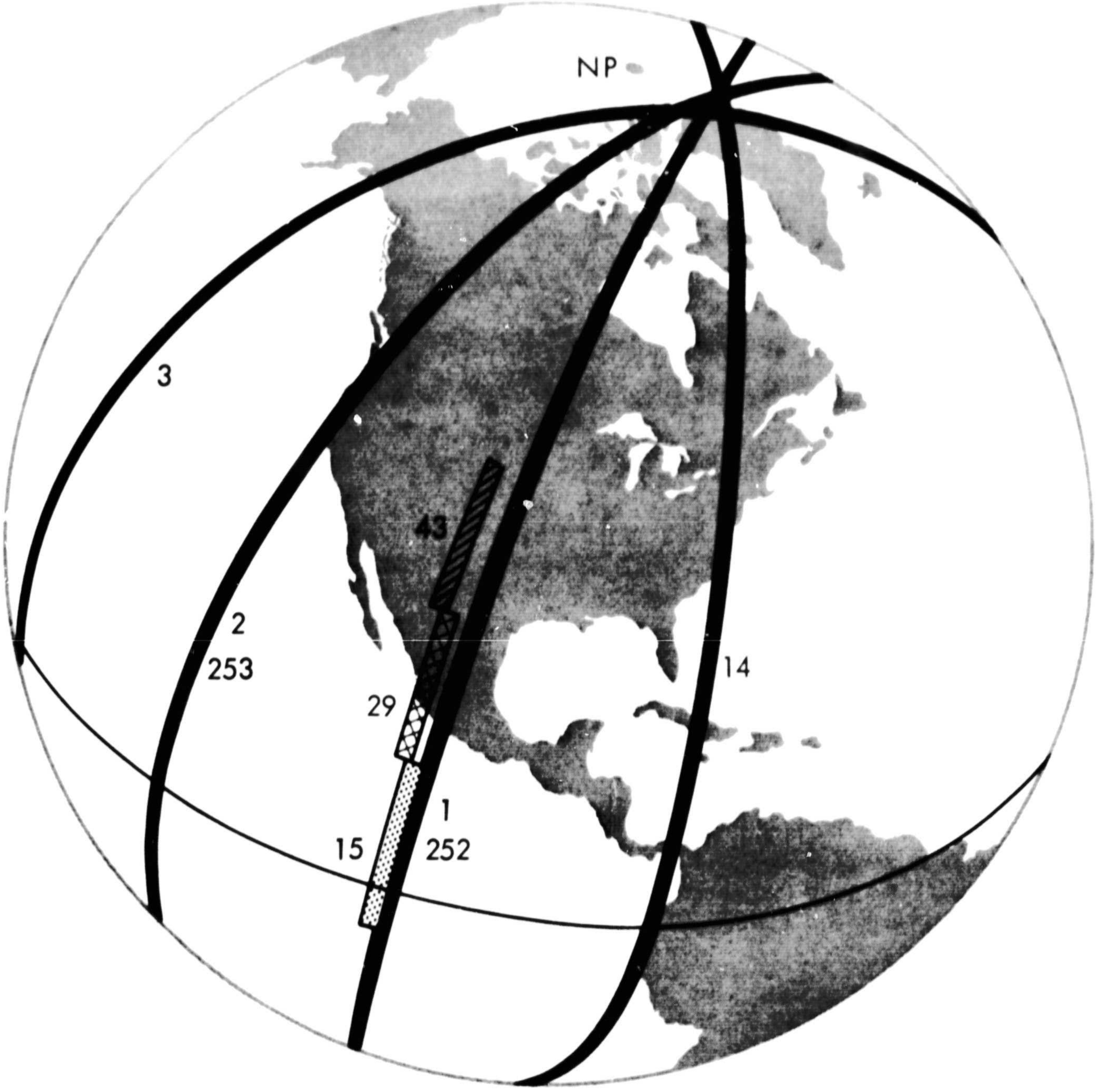
A pattern with a convenient amount of overlap is illustrated in Fig. 6. Here the pattern-determining parameters correspond closely to the nominal ERTS case: N = 18, h = 493 n.m., W = 100 n.m., and the daily swath overlap is about 17%.

For minimum drift orbits in general, the fractional coverage (f) obtained at the equator can be calculated using a modification of Eq. (8):

$$f = \frac{W/60 \sin i}{\delta \lambda} \tag{9}$$

The required value of $\delta \lambda$ can be obtained from Eq. (6) and the result expressed in convenient form by means of Eqs. (1) and (5) as follows:

$$f = \frac{WR}{21{,}600 \sin i} \tag{10}$$



N = 18
R = 251
W ≈ 100 n.m.

Figure 6. Swath Pattern of "Full-Coverage" Orbit

Here, the swath width W is in nautical miles as before; the factor 21,600 (arc minutes per revolution) converts it into a fraction of a great circle. Values of f greater than unity indicate a fractional overlap of (f - 1).

The linear variation of f with R is shown graphically for convenient reference in Fig. 7, for W = 50, 100, and 150 n.m. The data are based on a constant inclination angle i = 99°, which is approximately correct (cf. Fig. 11) for the altitudes of present interest.

ORBIT SELECTION

Having determined various properties of the swathing patterns of circular, sun-synchronous orbits, the principal task remaining is to select, from a rather large array of orbits, such as that shown in Fig. 2, the particular orbit which is best suited to an individual earth survey application. The decisions required will frequently fall into the following categories, which are listed below along with some considerations which may be useful in guiding the choices.

Altitude Region

Good sensor resolution and high scanning speed (short period orbits) both favor low altitude orbits. On the other hand, factors such as atmospheric drag, orbit perturbations by harmonics of the geopotential, and station contact time favor the choice of higher altitudes. Altitudes in the vicinity of that chosen for the ERTS missions (~500 n.m.) appear to provide a reasonable compromise of fairly wide applicability.

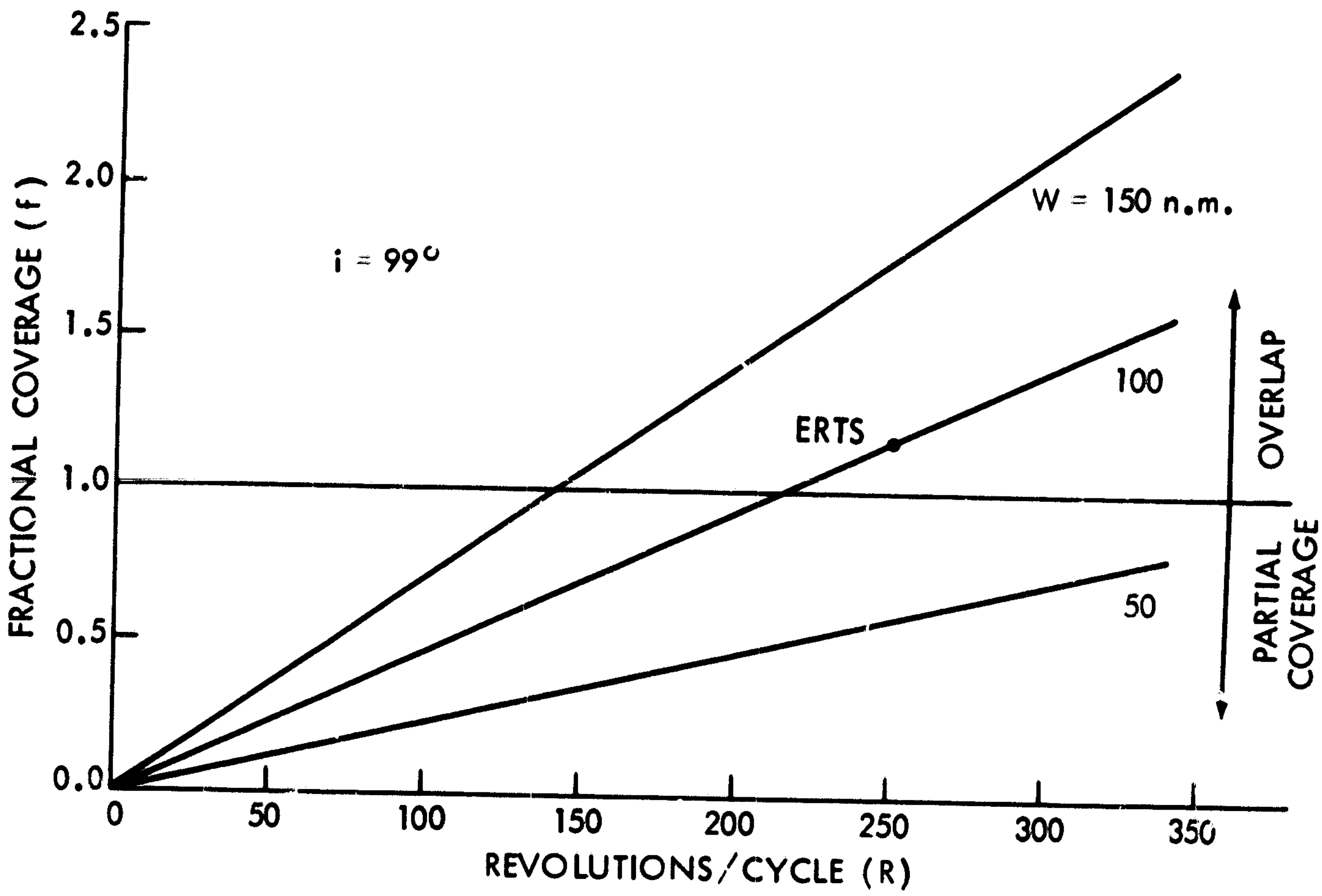


Figure 7. Coverage of Repeating Swath Parameters

Repeat Cycle Period

It is logical to assume that repeating-type swathing patterns, such as those generated by orbits in the N - T array of Fig. 2 will be preferred for earth observation satellites in all cases. The basic reason is that repeating patterns provide a regular, easily predictable pattern of coverage and an opportunity to make direct comparisons between data sets taken at regular intervals for any location covered. Another reason is that repeating patterns are available in such a distribution that choosing a repeating-type pattern is not particularly restrictive in relation to other selection criteria.

As to the particular cycle period to be chosen for a given application, the essential question is whether the application requires frequent, repetitive coverage of the same geographical areas, or whether progressive, near-global coverage is required. These two types of coverage require small and large values, respectively, of the repeat cycle period (N). The actual value of N required for "full" coverage depends on the swath width, W, as indicated by Fig. 7 (R depends on N per Eq. (5)). Some applications may call for a compromise between the two types of coverage described, in which case an intermediate value of N would be selected. In such instances, the gaps between covered strips would be kept reasonably small while reducing the time lapse between successive re-surveys to a few days.

Swathing Sequence and Drift Direction

For obtaining assemblages of data which are as nearly concurrent in time as possible, it is desirable to place the pattern of swaths for a given day as close as possible to that of the preceding day. This is accomplished by choosing a "minimum-drift" orbit, as described earlier. In the case of the ERTS orbit, choosing the particular 18-day repeater orbit (R = 251) which is also a minimum-drift orbit minimizes the daily westward advance of the swath pattern and yields the desired contiguity (with overlap). Some other 18-day repeater orbit would have finally produced "full" coverage also, but not day-to-day contiguity. The same general behavior is exhibited for other values of N ($N > 2$), so minimum-drift orbits will usually provide the most desirable coverage pattern.

No basis for choosing between westward and eastward drift directions has been developed herein, although the daily drift parameter ($\delta\lambda$) was taken (arbitrarily) as positive westward, and the examples used all exhibited westward drifts. The choice in actual cases is a matter to be decided on the basis of detailed mission analyses, considering such factors as operational convenience and orbital perturbations due to harmonics of the geopotential.

PROVIDING FOR ORBIT MODIFICATION

As noted in the INTRODUCTION, it may be desirable to modify the swathing patterns of future earth observation satellites during their period of flight operations, possibly based on requirements which develop during the flight. Results of the preceding analysis indicate that such modifications will be quite feasible to perform, provided a modest propulsion capability is incorporated into the spacecraft.

Pattern Modification

It appears that the pattern modifications likely to be needed in practice can be accomplished by orbital transfers between nearby altitudes of the same drift direction, as between minimum-drift orbits along the same curve in Fig. 2 (including the one-day repeater orbit). The maximum altitude difference within the westward-drifting group right of the R = 14 line is 96.4 n.m., with the most likely transition probably between a "full-coverage" orbit ($N \approx 18$) and the one-day repeater (maximum frequency re-covering of a particular area). The propulsion requirements (Δv's) for such transfers can be approximated as the sum of the two impulsive velocity increments, Δv_l and Δv_h, applied at the lower and higher altitudes respectively. Since the orbital inclination required to maintain sun-synchronism is a function of altitude,

$$i = \cos^{-1} [C(\rho_0 + h)^{7/2}] \tag{11}$$

where C is a combined constant (see Appendix), each altitude change requires a specific adjustment of inclination. Total velocity requirements for the combined altitude and inclination changes, assuming that all inclination-changing is done at the higher altitude, are shown in Fig. 8, as a function of lower altitude and

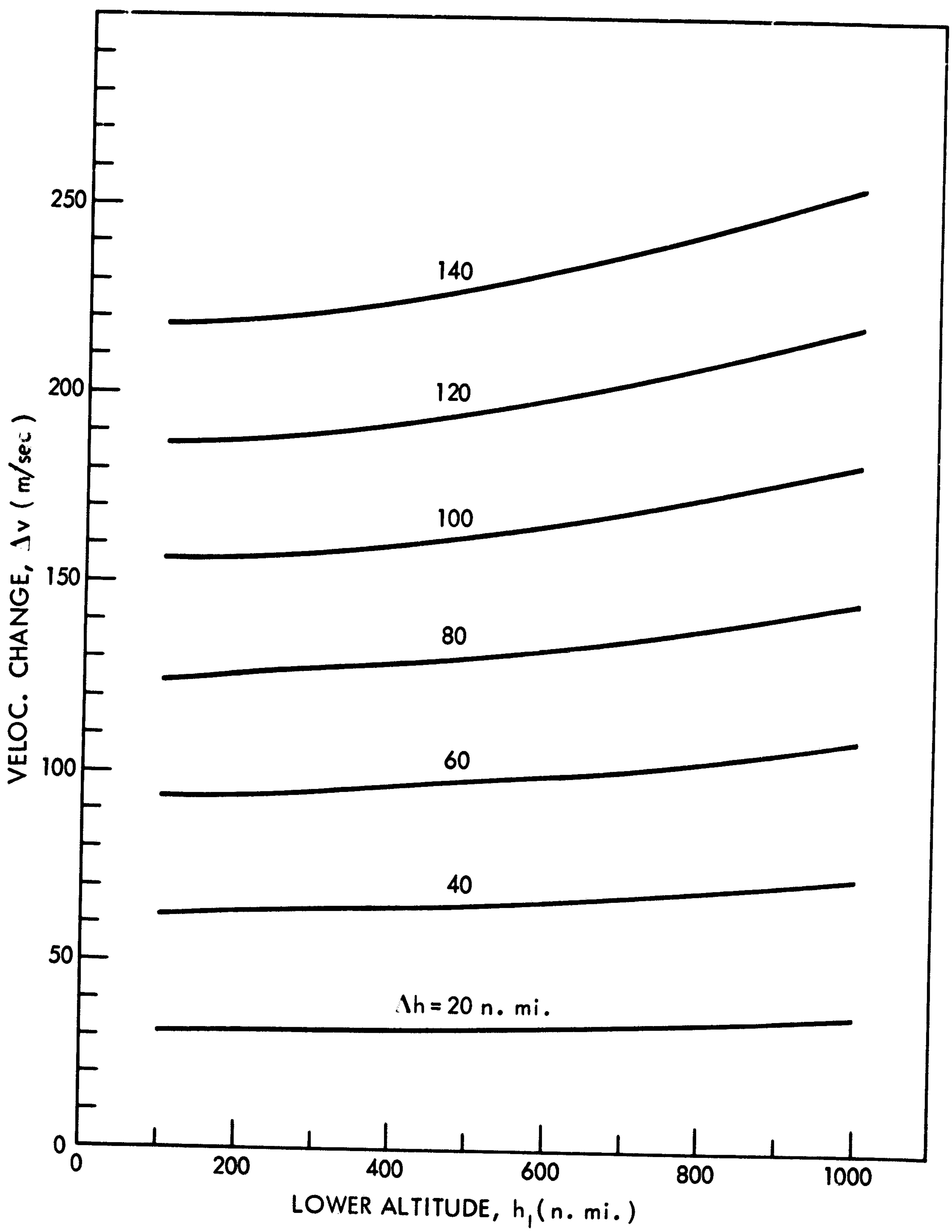


Figure 8. Velocity Requirements for Combined Altitude and Inclination Change

altitude change. The Δv values apply for either upward or downward transfers. Details of the underlying analysis are given in the Appendix.

Having obtained the required velocity increments from Fig. 8, the associated vehicle mass ratios for various values of specific impulse can be determined from Fig. 9. As an example, a transfer over the full 96.4 n.m. intragroup difference cited above involves a $\Delta v \approx 155$ m/sec according to Fig. 8 (h_1 = 483 n.m.), for which Fig. 9 indicates a mass ratio of 1.054 at 300 sec. I_{sp}.

Phasing

A final consideration which so far has not been addressed is that of phasing. As an example, suppose that an earth observation satellite is in the R = 251 orbit (Fig. 2) when a situation develops which requires daily observation of a particular location. The most economical way of handling the phasing problem would be to wait for the existing pattern to drift over the site, and then transfer to the adjacent one-day repeater orbit (R = 14). This would involve a wait of between 0 and 18 days, depending on the location of the site with respect to the current orientation of the daily pattern.

If the phasing time using existing drift is found to be excessive, the drifting process can be accelerated by transferring temporarily to an intermediate rapid-phasing orbit. For example, the drift direction could be reversed if appropriate by transferring to the R = 253 orbit. Conversely, the existing westward drift rate could be doubled by transferring to the R = 125 (reduced 250) orbit. Either

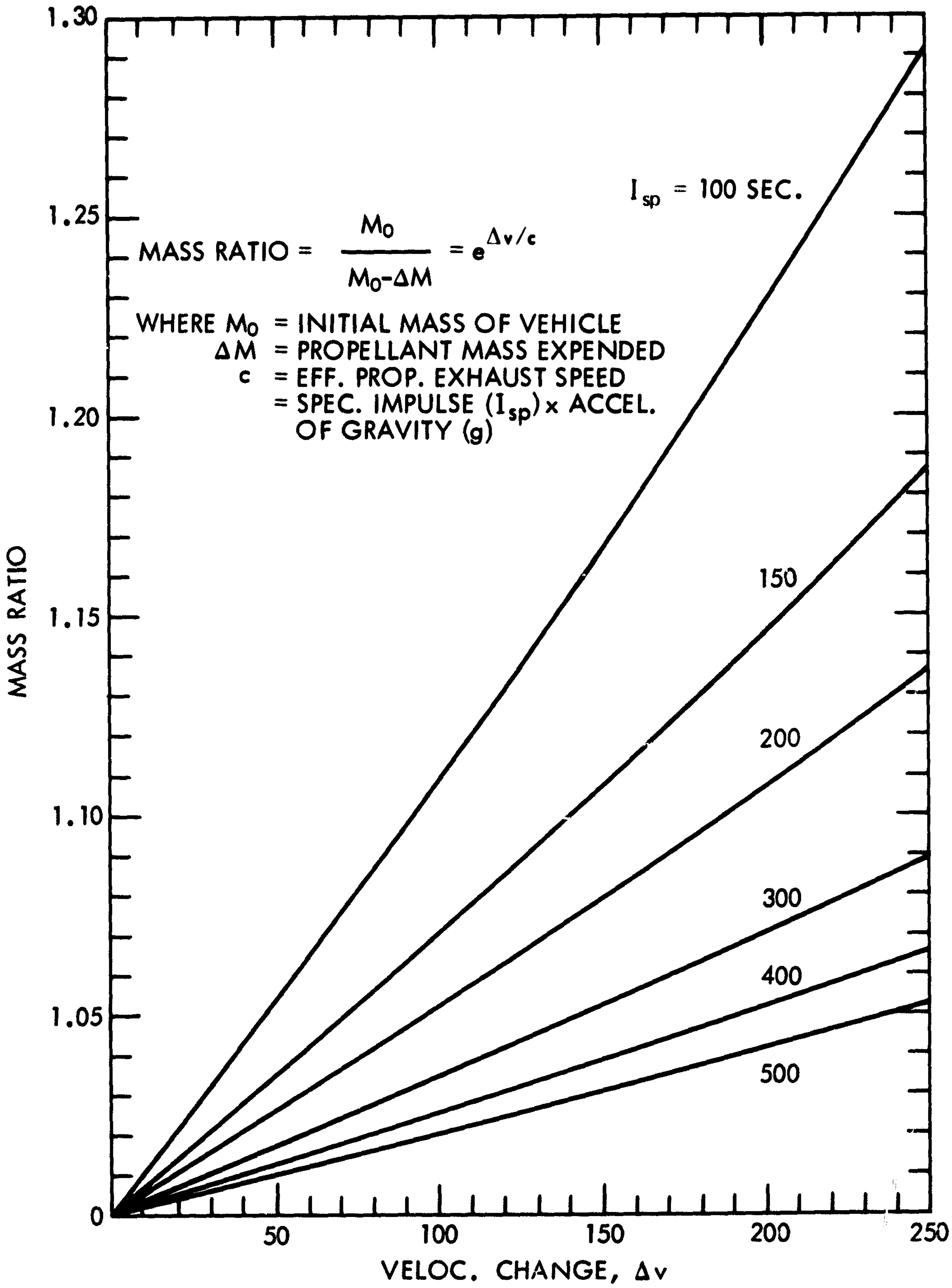


Figure 9. Mass Ratio vs. Velocity Change at Various I_{sp} Levels

of these maneuvers would require an altitude change of 10.4 n.m. and a total additional propulsive velocity change of about 30 m/sec., according to Fig. 8. Note that this Δv is twice the one-way requirement, since a reverse maneuver is required as soon as the accelerated phase change has been accomplished.

Many other examples and variations can be hypothesized, but in practice the proper strategy must be determined on an individual basis as a trade-off between time considerations and propulsion capabilities.

CONCLUSIONS

An extensive array of repeating swath patterns is available to the planner of earth observation satellite missions. These vary from frequent-reobservation, low-coverage patterns to nearly full-coverage, infrequent (e.g., 18 days) reobservation patterns, all available in convenient distribution over the useful range of altitudes.

Within the general family of repeating orbits, groups of "minimum drift" orbits occur which have the advantage of yielding uniformly progressing swath patterns, with the time interval between adjacent swaths limited to one day. Sets of orbits which produce these patterns occur symmetrically grouped (eastward and westward drifting) around a series of one-day repeater or zero-drift altitudes: 148.2 n.m., 306.1 n.m., 482.7 n.m. . . (see Fig. 2).

In general, the preferable method for modifying the swathing pattern of an earth-sensing satellite from a long-cycle, high-coverage mode to a short-cycle mode for frequent re-observation is to transfer to the adjacent one-day repeater orbit. In the 14 rev/day region, this maneuver requires an altitude change of only 10.4 n.m. (from 493.1 to 482.7 n.m.).

If the rather dispersed coverage pattern of the one-day repeater orbit is not acceptable and a reduction in the frequency of re-observation can be tolerated, transfer to one of the "several-day-repeater" orbits would be indicated.

The velocity increments and mass ratios associated with these transfers are modest. Assuming a specific impulse of 300 sec., for example, the "preferred" 18-day to one-day cycle transfer cited above involves a mass ratio of only 1.006. Even the unlikely "worst case" transfer between the one-day and two-day repeater orbits requires only 1.054 in mass ratio.

Thus the question of providing a swathing pattern change capability in future earth observation satellites hinges primarily on the practicability of providing the basic vectored thrusting capability. The added weight of propellants can probably be limited to a few percent of the total spacecraft weight.

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NOMENCLATURE

SYMBOL	DEFINITION	UNITS
a	semi-major axis of orbit	m
C	combined constant $\left(\frac{2\dot{\Omega}}{3\rho_0^2 J_2 \sqrt{K}}\right)$	$m^{-7/2}$
D	length of day (24 hr = 1440 min)	min
e	eccentricity of orbit	–
f	fractional coverage at equator	–
h	altitude	n.m.
i	inclination of orbit	deg.
I_{sp}	specific impulse	sec.
J_2	geopotential coefficient	–
K	gravitational parameter of earth	m^3/sec^2
N	period of pattern repetition cycle (an integral number)	days
r	sequential number of last revolution beginning east of P_0 (during first day)	–
R	integral number of satellite revolutions during period N	–
R_1	integral number of satellite revolutions during one day (one-day repeater orbit)	–
t	time	min.
T	period of orbit	min.
W	swath width	n.m.

SYMBOL	DEFINITION	UNITS
$\Delta\lambda$	interval, in geographic longitude, between consecutive southward equator crossings of subsatellite track	deg.
$\delta\lambda$	displacement, in geographic longitude, of P_r (equatorial terminus of Rev. r) west of P_0. This is the westward longitudinal displacement of the swath pattern of a given day from that of the preceding day, referred to as the "daily drift."	deg.
Δv	impulsive velocity change	m/sec.
ρ	radial distance from center of earth	n.m.
ϕ	geographic latitude	deg.
$\dot{\Omega}$	nodal regression rate of orbit	rad/sec

SUBSCRIPTS

a	apogee
c	circular orbit
h	higher orbit
l	lower orbit
o	earth surface
p	perigee

APPENDIX

PROPULSION REQUIREMENTS FOR TRANSFER BETWEEN SUN-SYNCHRONOUS ORBITS

The basic purpose of the transfer maneuvers considered here is to modify the subsatellite track patterns of circular, sun-synchronous orbits by changing their orbital period. The period change is accomplished simply by transferring to a new altitude, but the necessity to maintain sun-synchronism requires an accompanying plane change. The procedure employed here is based on the standard 2-impulse, Hohmann-type transfer, with the higher altitude impulse modified to incorporate the plane change.

The basic geometry of the maneuver is shown in Fig. 10, along with the associated velocity vector diagrams. The vehicle is initially in the lower orbit (radius ρ_l), moving with the local circular velocity v_{cl}. To transfer to the higher orbit, Δv_l is applied, which increases the vehicle velocity to v_{llh}, the perigee velocity of the transfer orbit. The vehicle coasts to the higher orbit, at which point its velocity has decreased to v_{hlh}. The second impulse Δv_h then injects the vehicle into the higher orbit. The impulse Δv_h is determined so as to increase the velocity v_{hlh} to v_{ch}, the circular velocity at that altitude, while rotating the velocity vector through the required plane change angle Δi (see velocity vector triangle, Fig. 10). The vehicle is then in circular orbit at the

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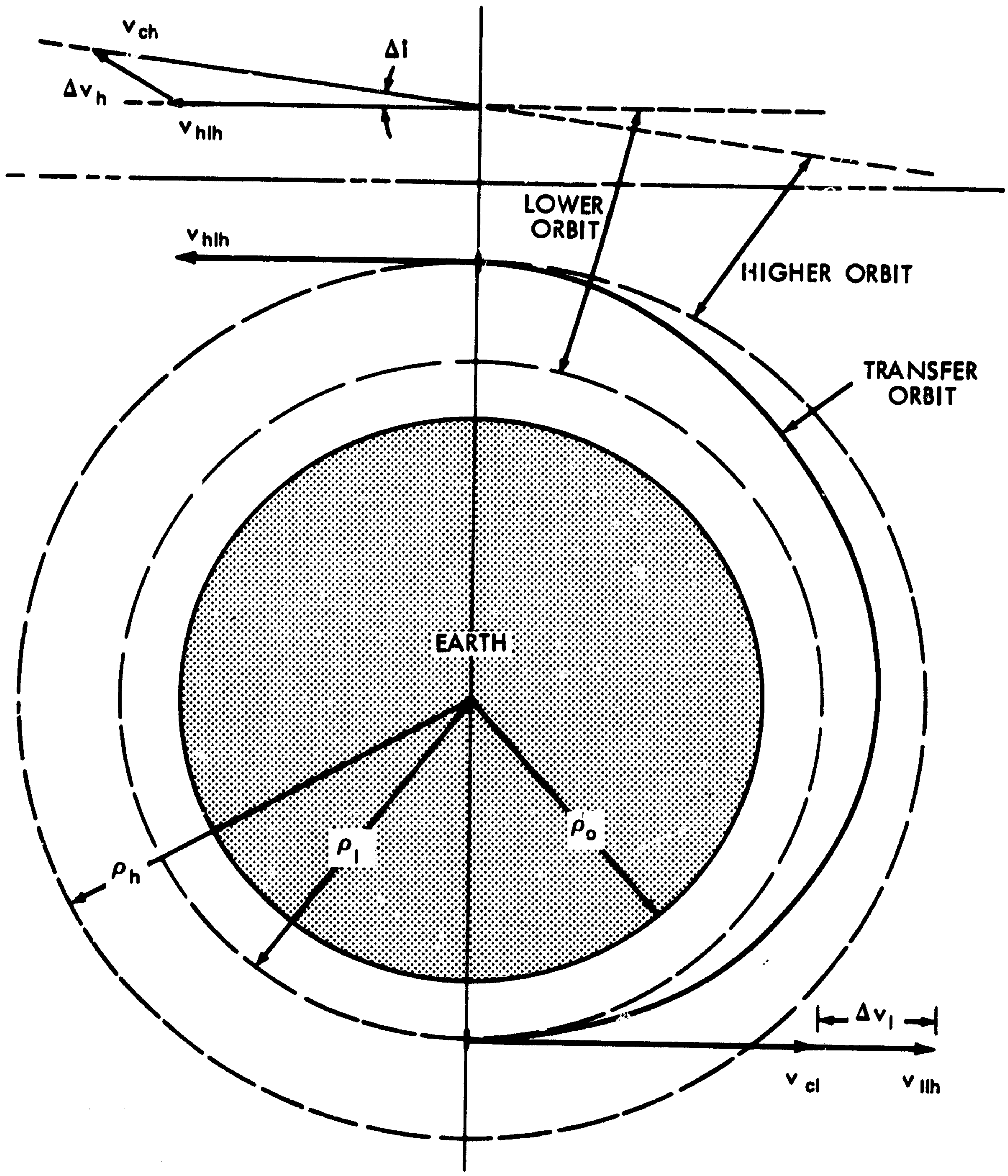

Figure 10. Geometry of Combined Altitude and Inclination Change

new altitude and inclination. Although the maneuver just described is an upward transfer, inspection of Fig. 10 reveals that the procedure is reversible – by reversing the sequence and direction of the propulsive impulses.

The various quantities referred to above are obtainable from the following standard equations:

$$v_{ci} = \sqrt{\frac{K}{\rho_i}} \qquad (i = 1, h) \tag{12}$$

$$v_{i1h} = \sqrt{K\left(\frac{2}{\rho_i} - \frac{1}{a}\right)} \qquad (i = 1, h) \tag{13}$$

$$a = \frac{\rho_1 + \rho_h}{2} \tag{14}$$

$$\Delta v_1 = v_{11h} - v_{c1} \tag{15}$$

The second impulse Δv_h is obtained by applying the law of cosines to the velocity triangle in Fig. 10:

$$\Delta v_h = \sqrt{v_{h1h}^2 + v_{ch}^2 - 2\, v_{h1h}\, v_{ch} \cos \Delta i} \tag{16}$$

The inclination angle i is derived from the sun-synchronism requirement as the inclination which produces a nodal regression rate equal to the mean angular rate of the earth-sun line. An approximate expression for the regression rate based on the second harmonic of the geopotential is given in Ref. 2:

$$\dot{\Omega} = - \frac{J R^2 \mu^{1/2}}{a^{7/2} (1 - e^2)^2} \cos i \tag{17}$$

Converting to the present nomenclature by the relations $J = 3/2\ J_2$, $R = \rho_0$, and $\mu = K$, and noting that for circular orbits, $a = \rho = \rho_0 + h$ and $e = o$, Eq. (17) can be rewritten:

$$i = \cos^{-1} [C \rho^{7/2}] \tag{18}$$

where

$$C = \frac{2\dot{\Omega}}{3 \rho_0^2 J_2 \sqrt{K}} = 1.50948 \text{ m}^{-7/2} \tag{19}$$

The above numerical value for C is derived from the following physical data taken from Ref. 3:

$\dot{\Omega} = 1.99107 \times 10^{-7}$ rad/sec (from 31.5569×10^6 sec/trop. yr).

$\rho_0 = 6.37816 \times 10^6$ m.

$J_2 = 1.0827 \times 10^{-3}$

$K = 3.98601 \times 10^{14}\ m^3/sec^2$

The variation of the inclination i with altitude ($h = \rho - \rho_0$) over the present range of interest is plotted in Fig. 11, based on Eq. (18).

Having now determined the necessary input quantities, the desired total transfer propulsion requirement can be evaluated via Eqs. (15) and (16) as:

$$\Delta v = \Delta v_1 + \Delta v_h \tag{20}$$

This is the quantity plotted in Fig. 8, using $h_1 = \rho_1 - \rho_0$ and $\Delta h = \rho_h - \rho_1$.

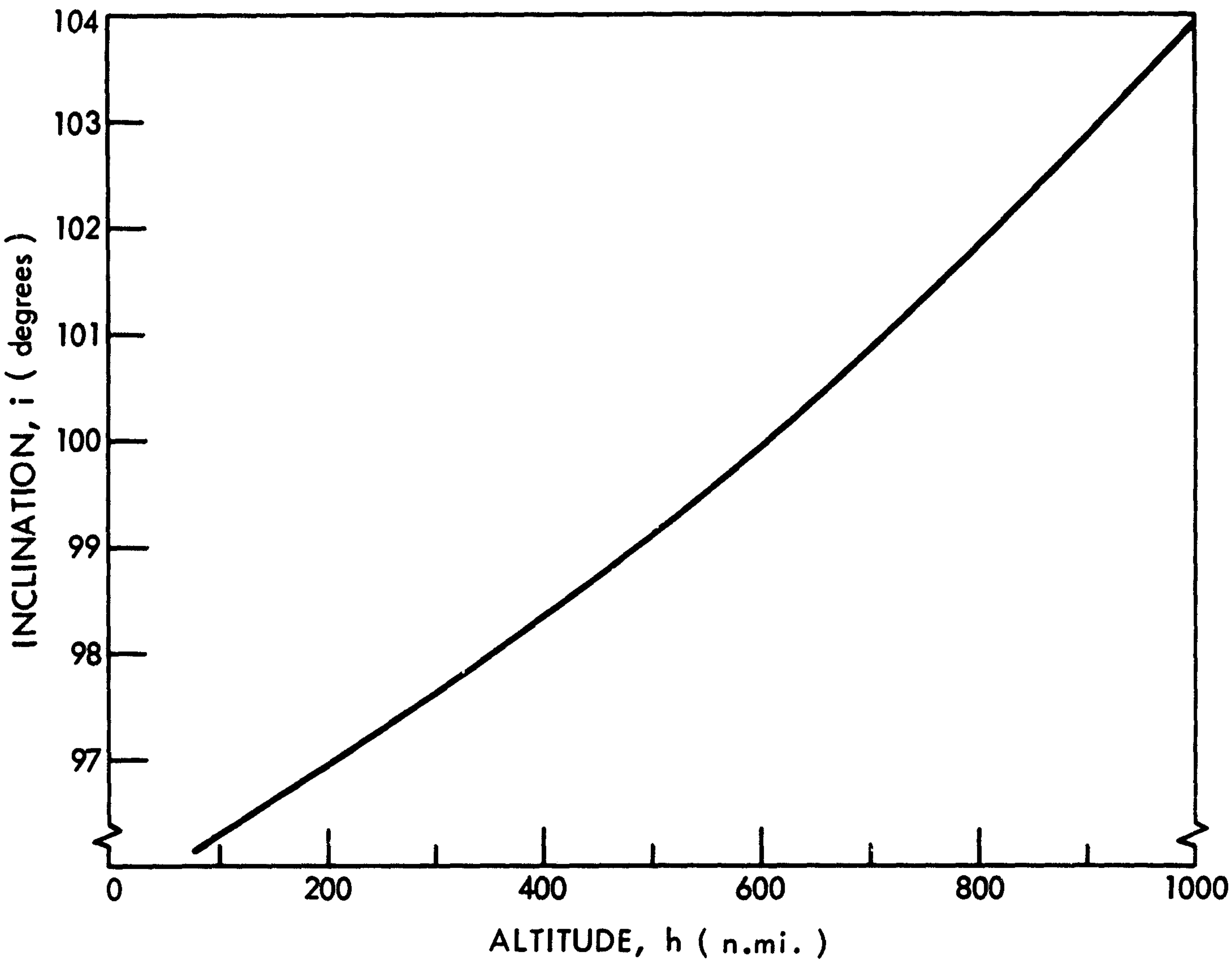

Figure 11. Inclination of Circular, Sun-Synchronous Orbits

The quantity $\cos\Delta i$ in Eq. (16) can be reexpressed in algebraic form if more convenient computationally by means of the following development:

$$
\begin{aligned}
\cos \Delta i &= \cos (i_h - i_l) \\
&= \cos i_h \cos i_l + \sin i_h \sin i_l \\
&= C \rho_h^{7/2} \times C \rho_l^{7/2} + \sqrt{1 - C^2 \rho_h^7} \sqrt{1 - C^2 \rho_l^7} \\
&= C^2 \rho_h^{7/2} \rho_l^{7/2} + \sqrt{(1 - C^2 \rho_h^7)(1 - C^2 \rho_l^7)} \qquad (21)
\end{aligned}
$$