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CRACK SHAPES AND STRESS INTENSITY FACTORS FOR EDGE-CRACKED SPECIMENS

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ABSTRACT

A simple stress intensity factor expression is given for a deep edge crack in a plate in tension. The shapes of cracks opened by tension or bending are approximated by conic sections, and the conic section coefficients related to plate geometry by very simple empirical equations. The magnitude of the crack displacement is a function of applied load, plate geometry, and the elastic constants of the plate material. The shape of a loaded crack in a semi-infinite plate is, approximately, a portion of an ellipse whose semimajor axis is about three times the crack length. As the crack length (relative to the plate width) increases, the crack shape becomes parabolic, then hyperbolic, the acuity of the hyperbola increasing with the relative crack length.

INTRODUCTION

The problem of a crack in the edge of a strip or a plate under normal tension or bending is of interest to materials engineers as well as to structural analysts. For maximum utility in both fields, expressions for the stress intensity factor are needed over the widest possible range of the ratio of crack length to plate width. The effects of loading and geometrical parameters on the shape of the opened crack are of fundamental interest.

STRESS INTENSITY FACTOR FOR A DEEP EDGE CRACK IN A TENSION SPECIMEN

Wilson [1] has shown that collocation results for bend and compact tension specimens can be extrapolated to $a/W = 1.0$ by expressing them in the form of appropriate dimensionless parameters. A similar construction is also possible for the edge crack in pure tension. When the dimensionless parameter

$$\frac{KB}{P} \frac{(W - a)^{3/2}}{(W + 3a)}$$

is computed from collocation results [2,3] and plotted (Fig. 1), it is seen to rapidly approach a value of 0.5 with increasing a/W. The expression

$$K = \frac{P}{B} \frac{(W + 3a)}{2(W - a)^{3/2}} \tag{1}$$

is within 1 percent of the referenced collocation results for $a/W \geq 0.3$.

It is of interest to note that (1) for the tension specimen and Wilson's expressions for the bend and compact tension specimens converge as $a \rightarrow W$. In these equations a common dimensionless parameter is

$$\frac{KB(W - a)^{3/2}}{M} = \begin{cases} \dfrac{1 + 3a/W}{a/W} & \text{(tension specimen)} \\ 4 & \text{(bend specimen)} \\ \dfrac{5 + 3a/W}{1 + a/W} & \text{(compact tension specimen)} \end{cases}$$

where M is the nominal bending moment (taken to be $P(W + a)/2$ for the compact specimen or $Pa/2$ for the pure tension specimen). This parameter, computed from collocation results and from the extrapolation equations, is plotted in Fig. 2 where the convergency can be readily seen. This indicates that the stress state for a very deep crack approaches one of pure bending regardless of the manner of loading.

THE SHAPE OF AN EDGE CRACK OPENED BY PURE TENSION OR BENDING

It is generally well known that the opening-mode elastic stress field near the tip of a crack may be completely described by the stress intensity factor and appropriate coordinate functions. It is less well known that the stress intensity factor also describes the displacements of the crack surfaces near the crack tip. That is, near the crack tip the stress intensity factor describes not only the stresses in the uncracked region but also the deflections in the cracked region. If only the first term of the displacement function [4] is considered, a flat crack opens under load into a parabola whose tip radius is (exactly)

$$r = \frac{4}{\pi} \left(\frac{1 - \nu^2}{E}\right)^2 K^2 \tag{2}$$

where E is Young's modulus and ν is Poisson's ratio. Since the tip radius is a significant feature of the entire crack profile, the consideration of crack shape may prove useful.

No analytical expressions are available for the shape of an edge crack in a finite-width plate under normal tension or bending. The shape of an edge crack in a semi-infinite plate under remote normal tension is given by Wigglesworth [5] in the form of an infinite series. Using boundary collocation, Gross [3] computed opening displacements under normal tension or bending for edge cracks 30, 50, and 70 percent of the plate width. The crack shapes for these cases could be described by a polynomial of sufficiently high order in two variables (distance from crack tip and relative crack length). However, such a function would probably be cumbersome to use, and the influence of the individual coefficients on the crack shape would be difficult to visualize.

Although these crack shapes may not be exact conic sections, it is expedient to model them as such. Then only a single conic section coefficient needs to be correlated with the relative crack length. The general equation of a conic section with the origin at the vertex can be written as

$$\left(\frac{\eta}{\eta_o}\right)^2 = \frac{2}{2+m}\left(\frac{y}{a}\right) + \frac{m}{2+m}\left(\frac{y}{a}\right)^2 \tag{3}$$

where the notation is given in Fig. 3 and m is the conic section coefficient. The physical interpretation of the coefficient m is as follows:

$m = -1$	an ellipse, $(\eta/\eta_o)^2 = 1 - (1 - y/a)^2$
$-1 < m < 0$	a portion of an ellipse (semimajor axis, $-a/m$)
$m = 0$	a parabola, $(\eta/\eta_o)^2 = y/a$
$0 < m < \infty$	an hyperbola (origin at $y = -a/m$)
$m = \infty$	a pair of straight lines, $(\eta/\eta_o) = \pm y/a$

This can be seen in (Fig. 3), where (3) is plotted for three values of m and the limiting case $m = \infty$.

After suitable differentiation of (3), the expression

$$\eta_o^2 = ar(2 + m) \tag{4}$$

is obtained for the conic section model, where r is the crack tip radius (radius of curvature at $y = 0$). This equation shows the interrelationship between mouth displacement, tip radius, and the conic section coefficient. Any two of these three terms are sufficient to define the conic section. Equation (4) may be put in another useful form using (2) and the form equation

$$K = Y\sigma\sqrt{a}$$

where σ is P/BW in tension or $6M/BW^2$ in bending, M is a bending moment opening the crack, and Y is the dimensionless stress intensity factor (calibration factor). Thus (4) becomes

$$\frac{E\eta_o}{(1 - \nu^2)\sigma a} = 2Y \sqrt{\frac{2 + m}{\pi}} \tag{5}$$

where the dimensionless mouth displacement is given in terms of the calibration factor and the conic section coefficient.

The conic section model (3) was compared with the available crack profiles [3,5] as follows. Using collocation values [6] or the exact solution [5] for the dimensionless mouth displacements and the calibration factor expressions of Brown and Srawley [7], Wilson [1], and (1), conic section coefficients were computed from (5). These are plotted against the relative crack length in Fig. 4 (the curves shown will be discussed later), and may be seen to increase with increasing relative crack length. Using these calculated coefficients in (3), relative crack displacements (η/η_o) were computed at $y/a = 0.1, 0.2, 0.3, \ldots, 0.9$. These were within 3.2 percent of corresponding displacements determined from Wiggleworth's solution [5], Gross's collocation results [3], and unpublished collocation results (B. Gross, NASA Lewis Research Center; $a/W = 0.2, 0.4,$ and 0.6). Thus the shape of edge cracks opened by bending or tension can be very closely approximated by the conic section model.

The effect of relative crack length on crack shape can be seen in Fig. 5. In a semi-infinite plate the crack shape is (approximately) a portion of an ellipse whose semimajor axis is about three times the crack length. The Wigglesworth solution is also shown here for comparison. As the relative crack length increases, the crack shape becomes parabolic, then hyperbolic, the acuity of the hyperbola increasing with the relative crack length. As would be expected, this trend is more pronounced for the case of pure bending than for pure tension.

SOME SIMPLE APPROXIMATE EXPRESSIONS

As can be seen in Fig. 4, the simple expression

$$m = -0.3 + 15(a/W)^n \tag{6}$$

(where n is 2.3 for bending or 3.3 for tension) is a fairly good approximation over the range of the available solutions ($0 \leq a/W \leq 0.7$). Extrapolation of (6) beyond $a/W = 0.7$ is somewhat questionable at present, since collocation results are not available for comparison. Crack displacements are likely to be very sensitive to geometry in this range, and (6) may be overly simple. It was shown earlier that the stress state for very deep cracks ($a \rightarrow W$) approaches one of pure bending regardless of the manner of loading. If the stress state is unique, the crack profile should also be unique. Thus the conic section coefficients for tension and bending should converge as $a \rightarrow W$. In this respect the form of (6) is proper.

Equation (3) gives only the relative crack displacements. An expression for the mouth displacement, $\eta_o = f(a/W)$, is needed to determine absolute values. This could be obtained by fitting a polynomial in a/W to available collocation solutions [3,6]. However, a simple approximation can be made by substituting (6) into (5). The dimensionless mouth displacement so calculated is compared with collocation values [6] in Fig. 6, and the maximum error is 3.7 percent (bending) or 2.9 percent (tension) for $a/W \leq 0.7$. Extrapolation beyond $a/W = 0.7$ is not recommended.

SUMMARY

A simple stress intensity factor expression is given for an edge crack deeper than 30 percent of the width of a plate in tension. The shapes of edge cracks opened by tension or bending can be closely approximated by conic sections. The conic section coefficient can be related to the relative crack length by very simple empirical equations. The magnitude of the crack displacements is a function of applied load, plate geometry, and the elastic constants of the plate material.

The shape of a loaded crack in a semi-infinite plate is, approximately, a portion of an ellipse whose semimajor axis is about three times the crack length. As the crack length (relative to the plate width) increases, the crack shape becomes parabolic, then hyperbolic, the acuity of the hyperbola increasing with the relative crack length. This trend is more pronounced for the case of pure bending than for pure tension.

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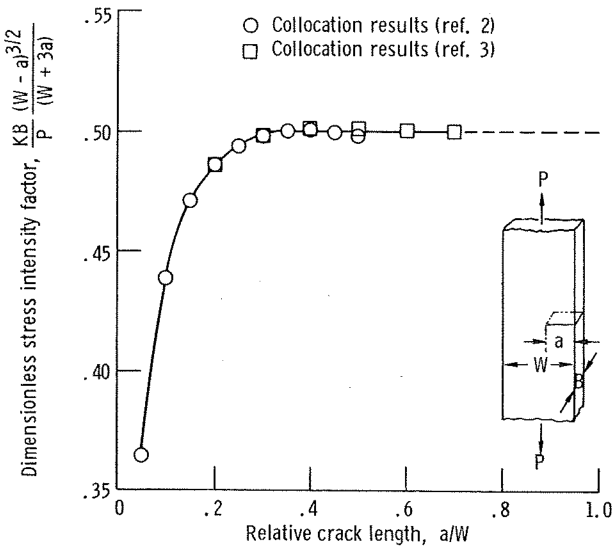

Figure 1. - Dimensionless stress intensity factor from boundary collocation results.

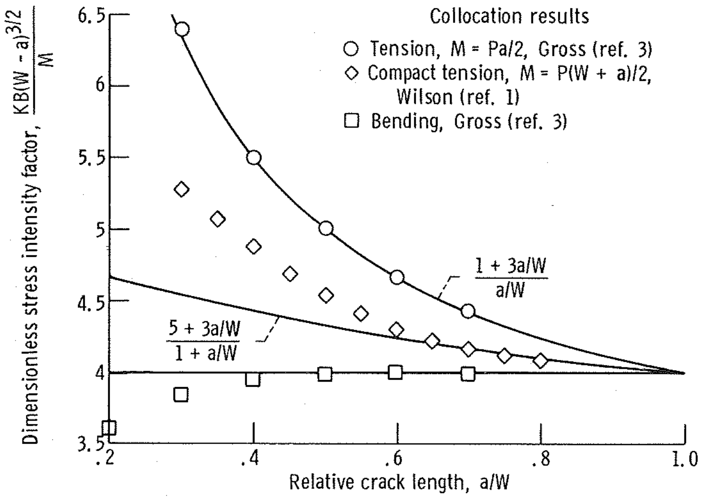

Figure 2. - Dimensionless stress intensity factor.

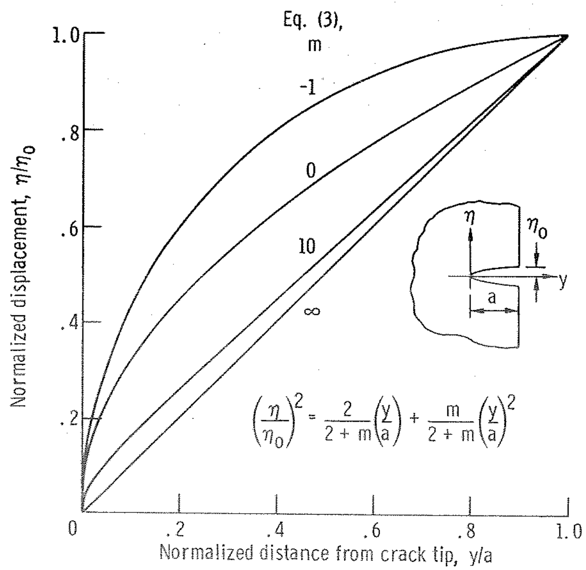

Figure 3. - Conic sections, eq. (3).

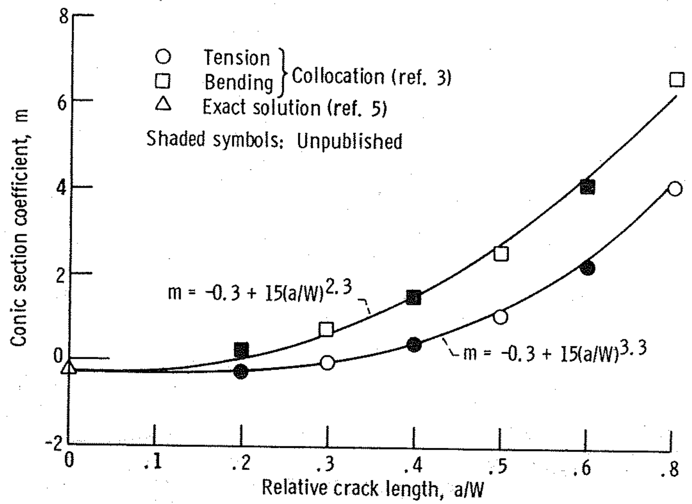

Figure 4. - Effect of relative crack length on the conic section coefficient.

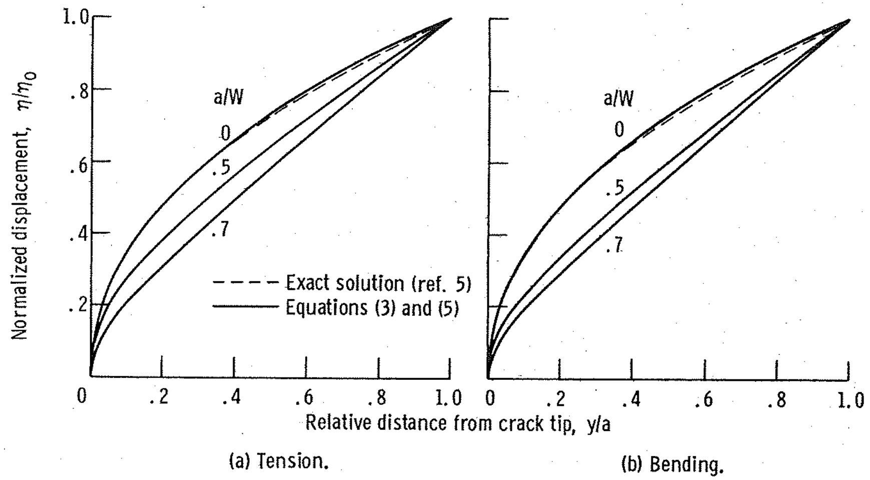

Figure 5. - Normalized crack shapes from equations (3) and (5) and an exact solution.

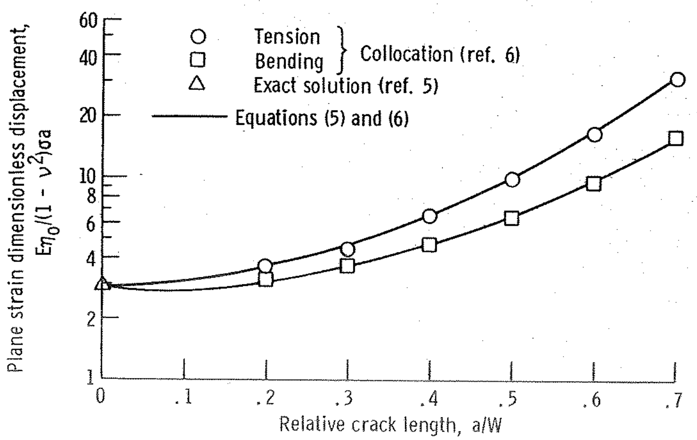

Figure 6. - Effect of relative crack length on the plane-strain crack mouth displacement.