NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Memorandum 33-488

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Smithsonian Astrophysical Observatory

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PREFACE

The work described in this report was performed by the Project Engineering Division of the Jet Propulsion Laboratory.

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ABSTRACT

This report surveys Saturn's system of rings with emphasis on the establishment of physically reasonable ring models. The dimensions of the planet and principal ring features are determined from published measurements. Photometry of the A and B rings is reviewed, revising some published results, and commenting on derived parameters in light of recent photometry of snow, ice, and other materials. Models for Rings A, B, and C are presented with supporting discussion and consideration is given to the existence of particles closer to the planet than Ring C and further away than Ring A.

SATURN'S RINGS — A SURVEY

by A. F. Cook,* F. A. Franklin* and F. D. Palluconi†

INTRODUCTION

This statement attempts a survey of the optical properties of the System of Saturn's Rings with a particular view of establishing physically reasonable ring models.

In section 1, we review published measures of the dimensions of the system to obtain the most likely values both of the extent of the system and certain of its features.

Section 2, presents a review of ring photometry, revising some published results and then commenting upon derived parameters in light of recent photometry.

In subsection 3.1, we discuss several models of Rings A and B, indicating what problems remain to be solved in order to establish their physical soundness. We conclude this subsection by mentioning one model that seems to satisfy all observational requirements. Subsection 3.2 briefly outlines a potential investigation to establish a steady state theory of Ring C and give representative radii and number densities for the particles. In subsection 3.3, we adopt what is probably the most likely model discussed in 3.1 and briefly mention the calculation that has been coupled with it to establish a ring profile, i.e., the light intensity in the ring as a function of distance from Saturn. This leads us to make some comments with regard to the existence of material outside the most conspicuous parts of the ring system. The final subsection 3.4, discusses observation of a new ring interior to Ring C and several mechanisms for producing a gap observed between this new ring and Ring C.

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1. PHYSICAL DIMENSIONS OF THE SATURN SYSTEM

In this section we shall try to obtain the most likely set of dimensions of features of the Saturn system from the various measurements to be found in the literature. We shall not include a discussion of the orbital parameters of the satellites. For such, the reader is referred to high quality measures and reduction of G. Struve (1933), which have been re-reduced with the inclusion of recent measures by Kozai (1957/58). His results are substantially the same as those of Struve.

Almost all measures of the dimensions of the disk and ring were made over 50 years ago; they are of decidely mixed quality. In principle, to carry out such measurements is not difficult, but several observational effects operate in ways that can greatly reduce precision. Chief amongst these is irradiation, the apparent enlarging of a bright area when viewed against a dark background. The importance of this effect was apparently not fully realized by most observers. In subsequent tabulations in this text, we shall rely only upon these measures in which an attempt was made to remove irradiation. In our view, this means, in general, consideration of only the following papers: H. Struve (1894), See (1902), and Lowell and Slipher (1915a,b). We shall compare these measures with the more recent ones of Dollfus (1969, 1970).

1.1 *Micrometer Measures of the Ring System*

See (1902) was apparently the first to attempt an experimental determination of irradiation corrections in the Saturn system. Unfortunately, there are several errors and inconsistencies in his final tabulation so that we must spend a little time in further examination. The night-time measures are reported in See (1901), they are re-quoted, (1902), but a small correction of 0ʺ03 in the diameters has been applied in the latter for which no explanation has been provided. This correction is too small to warrant consideration and we shall use for the night-time set the (1902) values. See compared day and night measures for the outer distance of Ring A and for the equatorial diameter of the disk, but did not make the appropriate comparison for the other ring boundaries. Irradiation being what it is, this is not the best technique. In one case, he applied the irradiation correction backwards, hence this new compilation. For the outer radius of Ring A, the correction for irradiation as determined by See 0.″15; for the equatorial radius of the disk, 0.″28. For the Saturnian system, these represent upper and lower limits of the corrections, since they depend upon the brightness of an element of solid angle. We shall assume presently that the inner boundary of Ring C requires no correction. For the other boundaries, we suggest the following corrections to See's night-time measures: for the inner radii of Rings A and B, 0.″20, outer radius of Ring B, 0.″25 (See's suggestion). These values conform to the relative brightness of the various regions.

Table Ia

Feature	Diameter (arc secs)
Outer A	40.27
Inner A	34.76
Cassini Division, Width	0.42
Inner B	25.93
Width of C	2.75

See's (1902) Night-Time Measures in Arc Seconds
Reduced to a Distance of 9.5388 AU

We then obtain the following radii, corrected for irradiation and coded as indicated.

Table Ib

Feature	Radii (arc secs)	
Outer A	19.98	M
Inner A	17.58	NC
Cassini Division, Width	0.87	NC
Outer B	16.71	NC
Inner B	13.18	NC
Inner B	12.94	M
Inner C	10.23	see text

M: day-time measure by See (1902)

NC: night-time measure, corrected for irradiation as mentioned

See's (1902) Measures Corrected by Authors for Irradiation Reduced to a Distance of 9.5388 AU

Why the day and night measures of the inner boundary of Ring B have essentially the same value remains a mystery and this is the outstanding sore-thumb of this tabulation. See has measured only the width of Ring C. It is reasonable to suppose that the inner boundary of Ring C is unaffected by irradiation. Using the mean of the two measured widths, day and night, of Ring C and the inner boundary of Ring B uncorrected for irradiation, we obtain the quoted value for the inner boundary of Ring C given in Table 1b. We feel that this table provides the most likely set of values from See's published papers.

A more complete attempt to remove the effects of irradiation from measures of ring dimensions was carried out by Lowell and Slipher (1915 a,b). These observers measured the complete ring system in day-light or bright twilight. The following tabulation is derived from irradiation-free measures only, using a mean of values given by the two observers, weighted according to the number of settings on a given feature. The method of weighting is inconsequential.

Table II

Feature	Radius (arc secs)
Outer A	19.84
Inner A	17.63
Outer B	16.86
Inner B	13.21
Inner C	10.96

Measures of Saturn's Ring System Derived by Authors from Irradiation-free Observations of Lowell and Slipher (1915 a,b) at 9.5388 AU.

The least satisfactory measures among this set are those of the inner boundary of Ring C. The fewest number of settings are made on this feature and they were performed by only one of the two observers. Slipher's measures, giving an inner radius for Ring C of 10".80, apparently were not felt to be completely free of irradiation. Because Ring C is so faint, night-time measures of its inner boundary, if properly made, could be free of irradiation. Suppose, for example, that an observer measures the two separations between (1) the preceding inner boundary of the ellipse of Ring C and the preceding equatorial limb of the planet and (2) the preceding inner boundary of Ring C and the following equatorial limb.

The mean of these two measures gives a value for the inner radius of Ring C that is unaffected by the irradiation of the disk. This, in fact, was the measuring procedure used by a number of the earlier observers and their results are, for this quantity, entitled to some consideration. Thirty measures on 7 nights on each of the two sides of the planet by Dyson and Lewis (1895) give a radius of 10".38. Similar measures by Barnard (1896) on 11 nights give 10".26 and by Hall (1885) on 2 nights yield 10".26.

For the inner boundary of Ring B we have a further determination, Cook and Franklin (1958) which results from a reduction of the eclipse of Iapetus by the rings as observed by Barnard (1890). This value is 13".22 ±0".06. Recent measures of Dollfus (1969) are listed in Table III for four boundaries. These observations were made with a double image micrometer which Dollfus (1954) has designed to reduce uncertainties arising from various optical effects and seeing. It is not, however, clear to us that such a micrometer eliminates the effects of irradiation. The images that it forms in a telescope would be substantially fainter than those without it, so that irradiation would presumably be reduced for all ring features and remain a problem only for the brightest, e.g., the outer boundary of Ring B.

Table III

Feature	Radius (arc secs)
Outer A	19.72
Inner A	17.45
Outer B	17.05
Inner B	13.34

The Measures of Dollfus (1969) at 9.539 AU

It is clear from the discussion thus far that all sets of observations have some short-comings and are not of uniform internal quality. It is, therefore, not easy to obtain a properly weighted mean and our final tabulation is admittedly somewhat subjective. We have formed means from Table Ib, averaging the two values for the inner radius of Ring B, Tables II and III, giving double weight to those of Table II. For Dollfus' measures, Table III, we give double weight to the outer radius of Ring A, which has the most measures (8) and the lowest probable error of all ring measures. To the other three ring dimensions we have assigned unit weight. There seems to be some uncertainty with regard to Dollfus' value for the radius of outer B: His three individual measurements (we have dropped a discordant fourth that appears to be misprint) do not correspond to the quoted mean. It is also among the largest values measured for this feature; we retain it however, with unit weight. Measures of the inner radius of B, Cook and Franklin (1958), and the inner radius of Ring C, Barnard (1896), Dyson and Lewis (1895) and Hall (1885) are included in the means, where Hall's are given 1/2 weight. The results are tabulated as follows:

Table IV

Feature	Radius (arc secs)	Extreme Value
Outer A	19.85	20.30
Inner A	17.57	17.38
Outer B	16.87	17.09
Inner B	13.21	12.81
Inner C	10.5	10.2

Final Means for Ring Elements at 9.5388 AU

It is also clear that, of all the ring dimensions, the inner boundary of Ring C shows the greatest uncertainty. It seems to us, that the measurements more likely indicate a vacant space, or region of very low particle density, stretching from 10″.5 to the planet's equatorial radius, whose value we shall next discuss. Barnard (1890), during the eclipse of Iapetus by the shadow of the rings, saw no diminution in the brightness of the satellite until it reached 10″.4 from Saturn, (Cook and Franklin, 1958).

We hesitate to put any well-defined uncertainties on the measures given in Table IV. With the exception of the inner radius of Ring C, an uncertainty of ±0″.1 is very probably an upper limit. We do include in Table IV the appropriate extreme value of a boundary given by any one of the observers heretofore mentioned.

1.2 *Measures of the Diameter of the Disk*

The comments made in Section 1 with regard to the effects of irradiation on measured ring dimensions apply to a still greater degree to observations of the planet itself. In addition, measures of the equatorial diameter must be corrected for phase and measures of the polar diameter for the elevation of the earth above the planet's equatorial plane. In general, measures of the latter quantity are less frequent than those of the former. Most determinations of the radii of the planet result from direct micrometer measures, but eclipses of the satellites by the planet's shadow provide another method that has proved quite useful.

For micrometer measures, we again rely on day-light measures of See (1902) and Lowell and Slipher (1915 a,b). Their values for the equatorial diameter are 17″.24 and 17″.26 respectively; both are in the visual and hence apply to ∿ 5500°A. The Lowell and Slipher measures were made only on two nights, but essentially at opposition so that corrections for phase were negligible. Both these papers also contain measures at night and both show irradiation to enlarge the equatorial diameter by 0."5. Night-time measures by others, i.e., Barnard (1896), Dyson and Lewis (1895), and Hall (1885) agree with the night measures of See and Lowell and Slipher to a much greater extent than do corresponding measures of ring dimensions. Presumably, so would their day-light measures had they been made. We suspect, therefore, that a value of the equatorial diameter of the planet of 17."25 is probably accurate to ±0."05.

H. Struve (1894) reported extensive micrometer measurements, particularly to determine the orbital elements of Saturn's satellites. Useful by-products of his investigations are dimensions of Saturn's disk. Struve was among the first to call attention to the problem of irradiation and frequently observed with a slightly illuminated field which would partially remove its effects. To what extent irradiation actually remained a problem in his observations is uncertain. We shall briefly review Struve's measures of the equatorial diameter because his determinations have been made in two distinctly different ways and because his measures of the polar diameter, where irradiation is less important, are the most extensive in the literature.

From the mean of 93 observations of the equatorial diameter, with respect to 2 satellites, Struve gives the value 17."47. Struve himself indicates that this value must be considered to be an upper limit. His reasons are as follows: (1) The micrometer wires used for determining the position of a satellite relative to the disk were occasionally not set on the limb of the planet but at an equal distance on the disk from the limb; (2) mean values derived on nights of better seeing gave, for one quoted example, smaller diameters than the final average by the amount 0."13. Struve's discussion, therefore, suggests that the most accurate equatorial diameter from these measures is less than his quoted value of 17."47.

Struve has also obtained values of Saturn's equatorial diameter from observations of eclipses of satellites. The mean of 9 events yields an equatorial diameter of 17"50. This type of observation is also beset with a number of problems, one of which Struve fails to mention. This is the "augmentation" of the diameter of the shadow cast by a planet because of its atmosphere. Observers of lunar eclipses have found it necessary to increase the effective diameter of the earth by ∿ 1.5% in order to account for eclipse phenomena. Presumably a similar correction, though of unknown magnitude, applies for Saturn. It is therefore, reasonable to suppose that Struve's value derived from eclipses is too high by a few percent. Recent measures by Dollfus (1969) place the equatorial diameter at 17"33 ±0"07. We shall, therefore, adopt as final mean the value 17"29 ±0.07, as the most likely equatorial diameter.

The situation with regard to the polar diameter is less satisfactory. Struve's measures, which gave an equatorial diameter of 17"47 yield a value of 15"64 for the polar diameter. Residuals in satellite positions, made relative to the Rhea and Titan, suggest a correction of -0"27 to this value. The eclipses give a value of 15"78, and an observation of the shadow of Titan passing over the planet's disk yields 15"65. Other micrometer measures, all of which lack corrections for irradiation, give the values 16"22, (Barnard, 1896) and 16"79, (Dyson and Lewis, 1895), where irradiation corrections would probably be ∿ 0"3 or less. The measures of Dollfus (1969) give 15"47 ±0"05 and, in view of the above remarks, we shall adopt this value, unaltered by an averaging, as the polar diameter of Saturn.

2. PHOTOMETRIC PROPERTIES OF SATURN'S RINGS

2.1 *Optical Thickness; Results Derived From Occultations of Stars and the Eclipse of Iapetus*

Occultations of objects bright enough or of sufficient contrast to provide reliable estimates of the optical thickness of the rings are rare events. Two papers, Bobrov (1956) and Cook and Franklin (1958) contain to our knowledge, a complete summary of available information to date.* Consequently, we shall not undertake a further summary, but simply state and appraise results.

Consideration of occultations, the eclipse of Iapetus as observed by Barnard (1890), and the visibility of the disk of Saturn through Ring A, but not through Ring B, led Bobrov (1956) to the set of average representative values given in Table V.

Table V

	Ring A	Ring B	Ring C
τ	0.5	1.0	0.1

Representative Optical Depths, Bobrov (1956)

Unfortunately, it is not an easy matter to make magnitude estimates of a star behind a bright sheet of material. Indeed, as Bobrov points out, values of τ derived from the occultation of Leipzig I 4091 by Ring B are inconsistent with the invisibility of the disk of the planet through Ring B.

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* *Observations of the occultation of (Strolling Astronomer, Vol. 20, p. 76, 1966/67) BD-19°5925, July 23, 1962, were not such as to yield any new information. See also Bobrov (1970), pp. 398-399.*

Observations of occultations could very probably best be made visually using a polarizing photometer equipped with a traveling prism. The prism would allow a comparison star to be moved in the field of the telescope relative to Saturn and so be superposed upon a portion of the ring in the same relative configuration as the occulted star, by such, or similar means, a visual comparison between the occulted and comparison stars becomes a precise measurement. Unfortunately, no observations of this type have ever been made. We urge that occultations be predicted and observed in this way.

From a reduction of the eclipse observations of Iapetus, Cook and Franklin (1958) show that τ for Ring C reaches 0.18 at that ring's outer boundary. This paper also provides a value of τ for Ring B at its inner boundary of 0.58, with a lower limit of 0.45.

From time to time there are visual reports of a fourth ring (Ring D) external to Ring A. We shall review the observations relating to its presumed existence a little later and now only comment that τ for Ring D can be no more than 0.01.

2.2 *Optical Thickness: Results from Ring Photometry and Photometric Properties of the Two Bright Rings*

We continue this discussion with a review of photometry of the ring system as it can supply additional estimates of the optical thickness in addition to other parameters. We shall base this review largely upon the observations presented by Franklin and Cook (1965). In the course of the present complete re-examination of the data discussed in that paper, we have found that an area factor and distance correction were inadvertently omitted in plots giving the ring to disk brightness ratios. The factor which corrects for both of these effects is $\frac{\pi}{2}(1-0.019\alpha^{\circ})$ where α is the phase angle in degrees. This factor must be divided into the vertical scales of Figures 8 and 9, Franklin and Cook (1965), which give, respectively, the ring to disk brightness ratios in (V) and (B). The effect of this correction in magnitudes is, fortunately, nearly constant with phase angle. Thus, the entries of Column 5 of Table II of Franklin and Cook (1965) that give the (V) magnitude of the total ring alone as a function of α must all be increased, i.e., made fainter, by a magnitude increment of 0.202 ±0.011, where the "error" gives the amount by which the extreme values depart from the average figure. The corresponding positive correction to the (B) magnitudes given in Column 6 of Table II, Franklin and Cook (1965), is 0.192 ±0.013. If we neglect the "error" term, then we must make the following comments with regard to the results of that paper:

(1) Figures 11 and 12, which present the (V) and (B) phase curves of the ring remain basically unchanged in shape but the above corrections, 0.202 (V) and 0.192 (B) must be added to their vertical scales.

(2) To all the entries of Table III, which gives the brightness in magnitudes/arc sec^2 of 5 characteristic ring elements, must be added to the corrections 0.20 (V) and 0.19 (B).

(3) The discussion of the opposition surge or pip given in that paper requires, on the basis of the corrections mentioned here, no change. We shall presently review those results and the models they led to in the light of recent work.

(4) The effect of the two corrections is to revise the photometric parameters, τ and the geometric albedo, p, of Rings A and B. The following paragraphs will now lead to a revision of these values; values of τ are essentially unaffected, p's do change substantially.

Here we follow the general methods as outlined by Franklin and Cook (1965). Adding the mentioned corrections of 0.20 (V) and 0.19 (B) to the 5 Ring elements given in that paper we obtain:

Table VI

	A_1	A_2	B_1	B_2	B_3
V	7.40	6.89	6.55	6.67	7.00
B	8.25	7.74	7.40	7.52	7.85

Ring Element Magnitudes in Magnitude/Arc Sec^2 at Opposition June 25/26, 1959 in the Visual and Blue

These quantities are in magnitudes/arc sec^2 at opposition on June 25/26, 1959 and thus, apply at an Earth-Saturn distance of 9.050 AU. Included in these values is the non-linear surge in brightness near opposition, which amounts to 0.23 (V) and 0.28 (B). To obtain the brightness of the ring elements at zero phase and exclusive of the pip, add 0.23 and 0.28 to the tabular values.

The luminous solar input incident on the rings corresponds in isotropic total reflection to a magnitude/arc sec^2 of 5.40 (V) and 6.03 (B). These values are based upon an absolute solar magnitude, $V_{\odot}$ = -26.81 and $(B-V)_{\odot}$ = 0.63, Harris (1961). If we deduct the solar values from, and add the amplitude of the pip to the appropriate entries of Table VI, we obtain the magnitude by which the various ring segments are fainter than the incident solar radiation. We wish to account for this difference. At present a theory of multiple scattering that might be applied to this problem has not been developed.* Even such a theory would require as input the scattering function of a single particle. Our policy here, as in the 1965 paper, is to try to represent the observations by computing the brightness at $\alpha = 0°$ resulting from single scattering and then adding to it the contribution of the higher orders obtained for the theoretically well-known case of isotropic scattering. Values of the 2 geometric albedos, in (V) and (B), and the optical thickness which best represent the 5 Ring elements in the two colors are given in Table VII.

Table VII.

Ring Element	p_V	p_B	τ
A_1	0.82	0.65	0.17
A_2	0.82	0.65	0.37
B_1	0.82	0.65	1.0
B_2	0.82	0.65	0.61
B_3	0.82	0.65	0.32

The Geometric Albedo in the Visual and Blue, p_V, p_B, and the Optical Depth τ Deduced for Ring Elements.

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* *Currently being examined by J. B. Pollack at Cornell.*

Lumme (1970) has found, on the basis of different photometry p_V = 0.82, τ (Ring B) = 1.25 and τ (Ring A) = 0.3.

To convert the geometric albedos to Bond albedos, we require a knowledge of the phase integral, q. This parameter is determined chiefly by the surface structure of the scattering centers and clearly cannot be predicted theoretically for the ring particles. Observationally, it cannot be obtained unless measurements extend over at least 50° in α. Thus, our only option is to infer its value indirectly. The phase variation of a body and its phase integral are related in the sense that a large phase variation implies a small q. For both the Moon and Mercury q can be obtained observationally; its value being 0.585 and 0.563 in the two cases. Also, in the range $0° < \alpha < 10°$, both these objects show a phase variation of approximately the same amount as the ring particles, 0.036 mag/degree. For this reason we adopt q = 0.57 for the ring particles and this assumption leads to Bond, or diffuse, albedos of 0.47 (V) and 0.37 (B). The two quantities, p_V and p_B and consequently the Bond albedos, $A_{(V)}$ and $A_{(B)}$ are the only quantities to be materially affected during this revision. Although q = 0.57 seems to us the most reasonable value, it must probably also be regarded as a minimum value. Larger q's would increase the Bond albedo. We shall return to the question of the albedo in just a few paragraphs.

The optical thickness given in Table VII is to some extent fixed by its value for the ringlet, B_1. The major drawback of this procedure is caused by the increasing importance, at $\tau \geq 1$, of the higher order scattering which we have dealt with inexactly, though reasonably. For τ = 1 in the ringlet B_1, higher order scattering contributes ∿20% of the total radiation field. If we let $\tau \to \infty$ for B_1 and consider the particles to scatter isotropically, then we obtain albedos that are only 5% lower than the above two.

It now seems highly likely, Pilcher et al. (1970) that frozen H_2O forms a major constituent of the ring. We are therefore, led to ask whether albedos of 0.47 and 0.37 are reasonable. In a sense, this question can immediately be answered in the negative, because all forms or pure snow show nearly constant reflectivities from ~5500°A to ~4400°A, while these albedos show a 20% drop. It is clear that the photometry cannot be so much in error and that the frozen H_2O must be contaminated or be only one component of the ring material. The originally posed question though still stands: do the usual forms of snow have albedos of essentially the amount as measured in V?

2.3 *Photometric Properties of Snow*

Veverka (1970) has recently discussed the photometric properties of snow. His review of the literature is thorough but his analysis is slightly limited by the assumption that snow is a Lambert reflector, although experiment indicates otherwise. We reproduce his Figures 5-4 and 5-5 as Figures 1 and 2, with further comment later. These figures display the ratio f of the direct back reflectivity at a given angle of incidence i and reflection ε, to the normal reflectivity. Veverka's three types of snow are: I, freshly fallen snow; II, wind-packed snow; and III, frozen rain crust. The best fits to the observations appear to be given by the following representations:

Type I		$f(i) = 0.83 + 0.17 \cos 2i$	(1)
Type II		$f(i) = 0.675 + 0.325 \cos 2i$	(2)
Type III	$0 \le i \le \pi/3$	$f(i) = 0.92 + 0.08 \cos 2i$	
	$\pi/3 \le i \le \pi/2$	$f(i) = 0.52$	(3)

The fits (1), (2), and (3) imply for the laws of reflection, the following expressions for the bi-directional

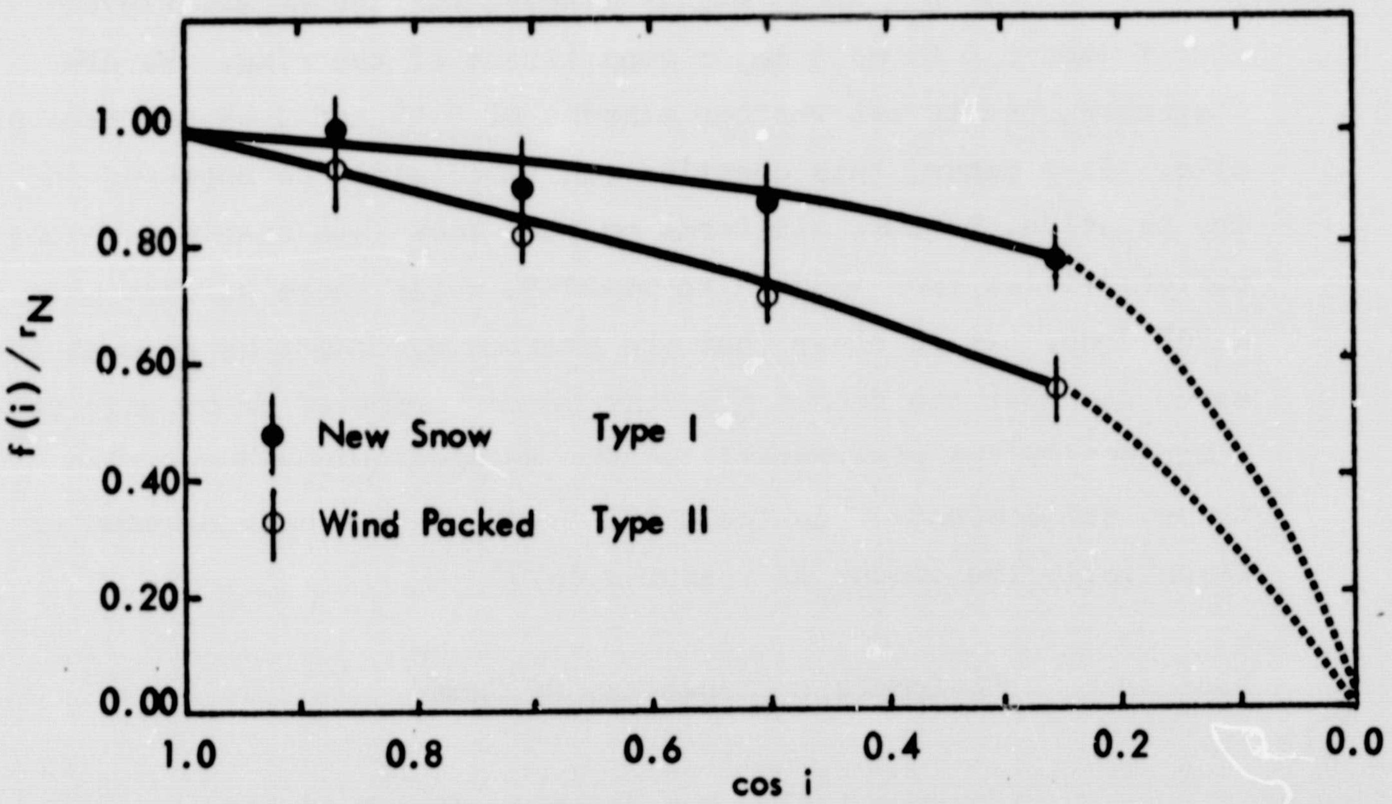

Figure 1. The Normalized Reflectivity of Snow, Types I and II. (From Veverka (1970) Figure 5-4)

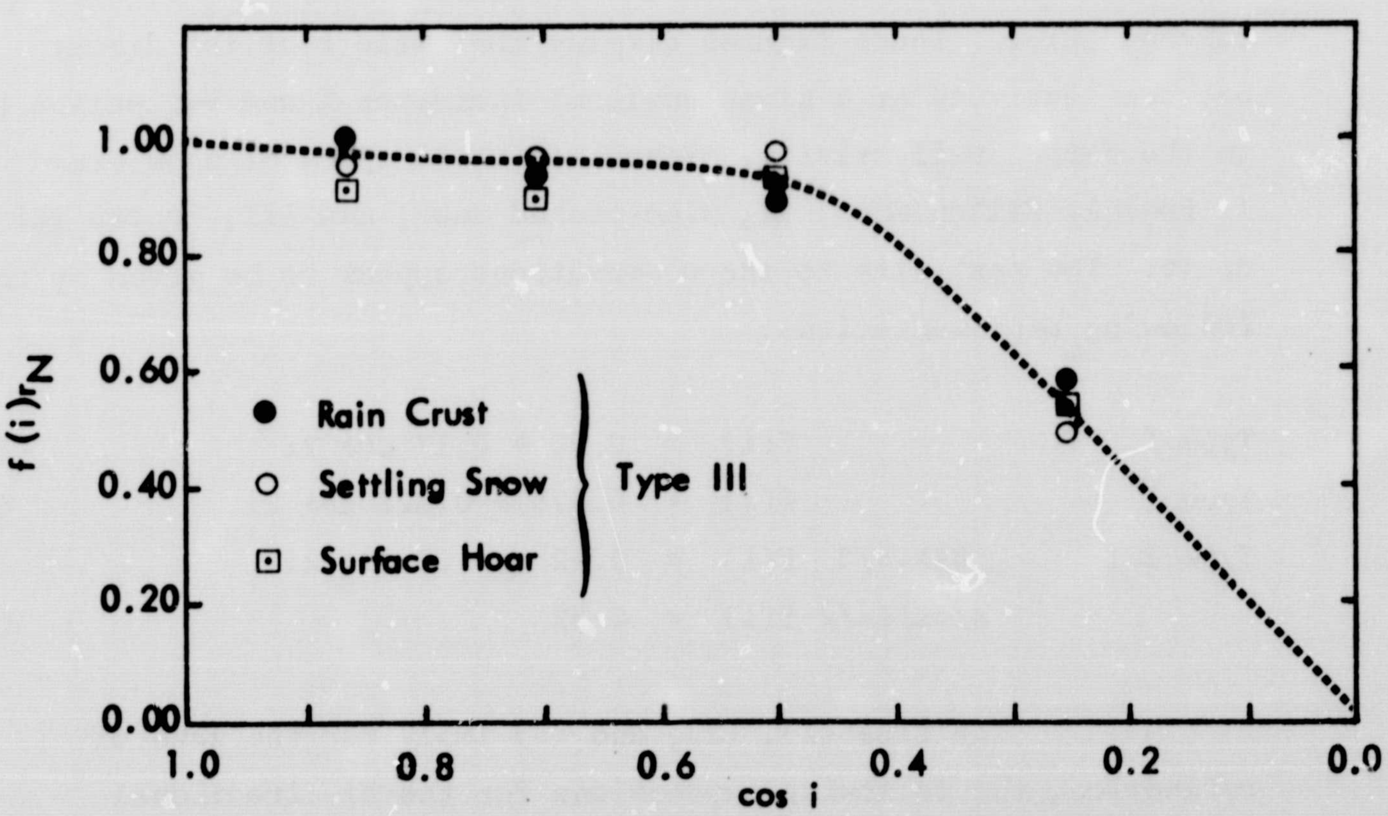

Figure 2. The Normalized Reflectivity of Snow, Type III. (From Veverka (1970) Figure 5-5)

reflectivity, $A(i,\varepsilon)$, where the principle of reciprocity is maintained:

Types I and II

$$A(i,\varepsilon) = r_N(1-\gamma+\gamma\cos 2i)^{\frac{1}{2}}(1-\gamma+\gamma\cos 2\varepsilon)^{\frac{1}{2}} \quad (4)$$

Type III

$0 \le i \le \pi/3, \quad 0 \le \varepsilon \le \pi/3$ $$A(i,\varepsilon) = r_N(1-\gamma+\gamma\cos 2i)^{\frac{1}{2}}(1-\gamma+\gamma\cos 2\varepsilon)^{\frac{1}{2}} \quad (5)$$

$0 \le i \le \pi/3, \quad \pi/3 \le \varepsilon \le \pi/2$ $$A(i,\varepsilon) = r_N b^{\frac{1}{2}}(1-\gamma+\gamma\cos 2i)^{\frac{1}{2}}$$

$\pi/3 \le i \le \pi/2, \quad 0 \le \varepsilon \le \pi/3$ $$A(i,\varepsilon) = r_N b^{\frac{1}{2}}(1-\gamma+\gamma\cos 2\varepsilon)^{\frac{1}{2}}$$

$\pi/3 \le i \le \pi/2, \quad \pi/3 \le \varepsilon \le \pi/2$ $$A(i,\varepsilon) = r_N b, \quad (6)$$

where for Type I $\gamma = 0.17$, Type II $\gamma = 0.325$, Type III $\gamma = 0.08$, and $b = 0.52$. In these expressions r_N denotes the normal reflectivity,

$$r_N = A(0,0). \quad (7)$$

We next desire the geometric albedo, p, for which $i = \varepsilon$, and we integrate over the illuminated hemisphere to find:

$$p = 2 \int_0^{\pi/2} A(i,i) \cos^2 i \sin i \, di,$$

whence for:

Types I and II

$$p = \frac{2}{3} r_N \left(1 - \frac{4}{5}\gamma\right) \quad (8)$$

Type III

$$p = \frac{2}{3} r_N \left(\frac{7}{8} - \frac{47}{80}\gamma + \frac{1}{8} b\right) \quad (9)$$

We next require the diffuse reflectivity and this involves an integration over the hemisphere of the directions of reflection:

$$A_L(i) = 2\int_0^{\pi/2} A(i,\varepsilon)\, cos\, \varepsilon\, sin\, \varepsilon\, d\varepsilon \tag{10}$$

whence for Types I and II we obtain:

$$A_L(i) = \frac{[1 - (1-2\gamma)^{3/2}]}{3\gamma}\, r_N\, (1 - \gamma + \gamma\, cos\, 2i)^{\frac{1}{2}} \tag{11}$$

and for Type III:

$$0 \le i < \pi/3 \quad A_L(i) = \left[\frac{1-(1-\frac{3}{2}\gamma)^{3/2}}{3\gamma} + \frac{1}{4} b^{\frac{1}{2}}\right] r_N (1-\gamma+\gamma\, cos\, 2i)^{\frac{1}{2}}$$

$$\pi/3 < i \le \pi/2 \quad A_L(i) = r_N\left[\frac{1-(-\frac{3}{2}\gamma)^{3/2}}{3\gamma}\, b^{\frac{1}{2}} + \frac{1}{4} b\right] \tag{12}$$

The diffuse reflectivity at normal incidence is:

$$R_N = A_L\, (0), \tag{13}$$

and the diffuse albedo is:

$$A_D = 2\int_0^{\pi/2} A_L\, (i)\, cos\, i\, sin\, i\, di \tag{14}$$

whence we find:

Types I and II

$$A_D = \left[\frac{1 - (1-2\gamma)^{3/2}}{\gamma}\right]^2 r_N \tag{15}$$

Type III

$$A_D = \left[\frac{1 - (1-\frac{3}{2}\gamma)^{3/2}}{3\gamma} + \frac{1}{4} b^{\frac{1}{2}}\right]^2 r_N \tag{16}$$

where the Bond albedo is identically equal to the diffuse albedo.

Finally, we require the brightness as a function of phase angle. Let the flux illuminating a sphere be πF in a parallel beam. The brightness is the total intensity reflected per unit cross-section and per unit solid angle of reflection. Let this brightness be denoted $B_p(\alpha)$, where α is the phase angle. We introduce the "photometric longitude" λ, measured in the plane containing the Earth, Sun, and a ring particle; it is measured from the Sun toward the Earth and the "photometric latitude" is measured from this plane. We then have:

$$B_p(\alpha) = \frac{F}{\pi} \int_{\alpha-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} A(i,\varepsilon) \cos^2 \phi \cos \lambda \cos(\lambda-\alpha) \cos \phi \, d\phi d\lambda \tag{17}$$

where:

$$\cos i = \cos \phi \cos \lambda \tag{18}$$

$$\cos \varepsilon = \cos \phi \cos (\lambda-\alpha) \tag{19}$$

The geometric albedo is, by definition:

$$p \equiv B_p(0) \tag{20}$$

and we define the phase function as the ratio:

$$\Phi(\alpha) = \frac{B_p(\alpha)}{B_p(0)} = \frac{B_p(\alpha)}{p} \tag{21}$$

The phase integral is the integral of ϕ over all directions:

$$q = 2 \int_0^{\pi} \Phi(\alpha) \sin \alpha \, d\alpha \tag{22}$$

and the Bond albedo is then:

$$A_B = pq \tag{23}$$

It follows that, given the diffuse albedo from Equations (15) or (16) and the geometric albedo from Equations (8) or (9) we can obtain the phase integral from:

$$q = \frac{A_B}{p} = \frac{A_D}{p} . \qquad (24)$$

As an example, consider the following tabulation of the three types of snow:

Table VIII

Type	r_N	R_N	A_B	p	p/r_N	q
I	0.82	0.74	0.68	0.57	0.58	1.44
II	0.70	0.57	0.46	0.35	0.49	1.34
III	0.64	0.58	0.53	0.40	0.62	1.32

Photometric Properties of Snow

The normal reflectivities, r_N, are the only observed values; all other quantities are derived from the above formulation. The quantity of chief importance for us is the Bond Albedo, A_B. It thus appears that there do exist snows with albedos similar to those obtained for the rings. Thus, the rings are best explained as composed of something like freshly fallen snow with a reddening reflection of dust. The expected temperature of the rings is so low that we anticipate the formation of vitreous ice, (Fletcher, 1970), which would most nearly resemble Type III. Possibly the Bond albedo of vitreous ice is somewhat higher so that a reddening admixture of dust may be present. The reader will note that the quantities in Table VIII refer to a smooth surface of snow, not the rough surface implied by the observed phase function of the rings. The Bond albedos are then the only quantities that can be directly compared with observations.

3. REMARKS ON RING MODELS A AND B

3.1 *Models of Rings A and B*

In discussing photometry of the ring system before we completed our study, (Cook and Franklin, 1965), Bobrov in a series of papers, see Bobrov (1940), (1954), showed that the data allowed the reasonably precise evaluation of only one quantity which related to the dynamical state of the ring as a whole. This quantity was the fraction of the total ring volume which the particles themselves occupy. This result indicated that this fraction, called the volume density, D, was of the order of 10^{-3}. Basically, his interpretation, which dates back to Seeliger (1887), views the phase curve of the ring as a consequence of mutual shadowing. Near opposition, when the light source and the observer are within a few minutes of arc of one another as seen from Saturn, the ring system appears brightest. Outside of opposition, the shadows of foreground particles impinge on those in the background and the ring brightness falls. Our measures, in general, confirmed the early photometry; but also indicated that the phase curve of the ring was color dependent. The interpretation of this phenomenon led us two possible ring models. One could account for the wave-length dependence if (1), the average particle radius was sufficiently small, about 300μ; or (2), still allowing the particles to be of an indeterminate size, if they possessed small surface features which could produce the observed color dependence in much the same way as the 'glory' is produced. Although, the observed wave-length dependence is near the limit of measurement, these two possible models do not appear to require any further internal modification at this time. By way of summary, we should like to emphasize that both of these models use the shadowing mechanism proposed by Seeliger to explain at least a part of the opposition surge.

Now, two important comments, or questions, must be posed with regard to the above models: (1) Can the rings maintain a thickness much larger than a particle radius and what is the mechanism by which this is done, if it is done; and (2) What are the implications of the photometry of laboratory samples as measured by Oetking (1966) and Hapke (1966)? We shall discuss (1) first. It is clear that a central assumption of any model that employs Seeliger type shadowing to account for the opposition surge supposes that the ring particles exist in a medium that is several and perhaps many particle radii in thickness. It is also clear that either all ring particles must pass through the plane of the ring system twice during a complete orbit of the primary, i.e., about 4 times per day or that mutual electrostatic repulsion, for example, must "float" the particles at low relative velocities. Thus, in the first case, even if the fraction of the total volume of the ring occupied by particles is as low as 10^{-3}, collisions between particles on any relevant time scale must be frequent events. Qualitatively, then, it would seem very likely that the vertical thickness of the rings would rapidly diminish, for all collision models except perfectly elastic ones. It is even quite conceivable that a layer only one particle diameter thick would be the eventual result.*

Gravitational instabilities within the ring will not produce the needed kinetic energy to extend the rings vertically. The demonstration of this conclusion is readily derived from the discussion of Cook and Franklin (1964). If the particles are approximately spherical and lie in the plane of the rings, we may regard them as point masses all in one plane. Cook and Franklin treated a continuum of such points as a first case. They showed that oscillations with axial symmetry about Saturn are the most unstable and that all wave-numbers above the limit:

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* *Note that a ring thickness of $\sim$ 1 km, has been recently well-established observationally, by Kiladze (1967) and by Focas and Dollfus (1969). Such a thickness means that Model II of Franklin and Cook (1965) must be abandoned.*

$$m'' \geq \frac{1}{2\pi\,\bar{a}} \quad \frac{S}{R\bar{a}^2} \tag{25}$$

would grow exponetially with time. There m" denotes the wave number, $\bar{a}$ the distance from Saturn of mass S and R is the surface density of the ring. However, we do not have, in the rings, a continuum of points, a fact that means self-gravitational effects will be heavily diluted as the wave-length becomes short compared to the interparticle distance. The problem probably should be discussed again to include this effect. In the meantime, we can say that a wave-length, m''^{-1}, equal to twice the interparticle distance, d_p, will suffer little dilution, but one half of this value will suffer considerable dilution and much smaller wave-lengths will be so badly diluted that instability will not occur.

For the present, we adopt d_p^{-1} for m", i.e., d_p^{-1} is probably very near the critical, or most unstable, wave number. We substitute this into the above inequality along with

$$R = \frac{4}{3}\pi\,\rho_p\,r_p^{\,3}\,N_p, \tag{26}$$

where ρ_p is the density of a particle of radius r_p and N_p is the number of particles per unit area. We also have the optical thickness:

$$\tau = \pi\,r_p^{\,2}\,N_p \tag{27}$$

Finally, we employ

$$d_p = \frac{1}{m''} = \frac{1}{N_p^{\,1/2}} \tag{28}$$

After a little manipulation we obtain:

$$\tau \geq \left[\frac{3}{8\pi^{1/2}}\left(\frac{S}{\bar{a}^{\,3}}\right)\frac{1}{\rho_p}\right]^{2/3} \approx 0.32 \tag{29}$$

The effect of dilution mentioned above can only be to raise the value of τ to larger numbers. Only in Ring B is τ significantly greater than this limit and hence only there could this sort of instability develop. From Cook and Franklin's (1964) Equation (27) for the angular frequency of an instability with $\theta = 0$ (radial wave) we expect any such instability to grow for $\tau \simeq 1$ in an enfolding time of about 7000 sec or 2 hours. The velocities generated would be small compared to a characteristic one given by the inter-particle distance ($\sim 10^5$ cm, from the observations of the edge on ring thickness, multiplied by the orbital angular frequency, 1.4×10^{-4}, i.e., small compared to 14 cm sec^{-1}). The efficiency of transfer of kinetic energy to oscillations perpendicular to the plane of the rings will not only be reduced by a factor of $(1-\alpha)$, where α is the accommodation coefficient for snow balls colliding at a few cm sec^{-1} but also by another due to the fact that the particles are nearly spherical. Allowance for these two factors probably sets the vertical velocity of ring particles at a few cm sec^{-1}, and maybe much less. Division of such a number by the angular frequency of vertical oscillations, twice the orbital frequency, yields a vertical amplitude of $\sim$ 0.1 km or less. We conclude that such displacements play no significant or observable role in Ring B and we have seen that they do not occur in Ring A.

Can we find other mechanisms to provide a finite ring thickness for particles larger than those of Franklin and Cook's (1965) Model II, which as we have seen, must be discarded on observational grounds, that is, for particles larger than $\sim$ 0.1 cm in radius? Consider the two possibilities:

(1) Collisions, coupled with friction from the gradient of orbital angular velocity as an energy source might produce a steady state such that the ring stabilizes at a finite thickness, greater than a particle diameter.

(2) Electrostatic repulsion between the particles balances gravitational forces and produces a ring of finite thickness.

Both of these possibilities can be discarded, thanks again to the $\sim$ 1 km observed ring thickness. The first case has been discussed by Cook and Franklin (1964) who showed that in Ring B one might obtain a vertical thickness about five times the value due to thermal velocities. For Franklin and Cook's Model II not to apply, the ring particles must be, in radius $\gtrsim$ 0.1 cm. Assuming a minimum density of 1/20 gm cm^{-3} for the particles and a kinetic temperature of 100°K, the kinetic motions of such particles would lead to a thickness of the ring of $\sim$ 0.1 cm which might be enlarged to $\sim$ 0.5 cm or so by friction. This value can neither produce the low fraction of the volume filled required by Model I, nor be reconciled with observed ring thickness.

The second case, electrostatic repulsion, has also been discussed by Cook and Franklin (1965). Very briefly, the result showed that electrostatic repulsion could only be a likely mechanism if the ring particles had radii $\lesssim$ 0.1 cm. For larger particles the necessary charges and ion densities in the ring become unrealistic. For the case of particles 0.1 cm in radius, the predicted ring thickness was again much less than the 1 km value observed.

In the previous discussions we have made frequent use of the observation that the ring thickness is $\sim$ 1 km. We have made this datum a requirement that must be satisfied by any ring model. It is important to remember, however, that there remains the possibility that the ring is enveloped in a tenuous atmosphere or even that particles outside Ring A exist in orbits of relatively high inclinations. Since the space density of such particles is much reduced from its value in Rings A or B, the collision frequency would also drop so that orbits with inclinations slightly different from zero might exist. Near ring plane passage either of these effects could suggest a ring of finite thickness.

Finally, we are led to a last possible source of vertical motions, perturbations of ring particles by the inner satellites of Saturn. It seems likely, Franklin and Colombo (1970) that the radial structure of the ring system is governed by resonances between the local orbital frequency of the ring particles and the orbital frequency of Mimas. At and near a resonance, ring particles are perturbed into eccentric orbits that could bring them into collision with adjacent particles. The above paper considered a simple case of a single layer of particles, so spaced that collisions did not occur. Thus, the model postulated a prior evolution to a collisionless state and is therefore unable to consider the vertical distribution of particles. We are currently beginning an investigation of this question as a case of the numerical N body problem. Collisions, with the accommodation coefficient as a parameter will be included, but results cannot be expected for several months. At present it is not possible to assess accurately the likelihood that this mechanism can maintain a finite ring thickness, but our suspicions are pessimistic on energy considerations. Ultimately, the energy must come from the potential energy of a satellite in the field of Saturn, the satellite being Mimas, or possibly Mimas and Tethys. The ring quite certainly has a mass greater than that of Mimas. All depends upon the collision frequency and the degree of elasticity of the collisions which can only be considered in a numerical model. One does expect a 'thickening' of the ring vertically near a resonance, but this may be a very local phenomenon.

The above discussion has lead us to doubt the existence of rings many particle radii in thickness. If we are forced to accept monolayer rings, we are confronted with the need to explain the observed phase curve in some other terms than Seeliger-type shadowing. First, though we are prompted to ask whether we can explain the opposition surge as a consequence of 'microscopic' shadowing, resulting from possible intricacies of surface structure. Essentially, we are still trying to use a Seeliger-type of shadowing, but we postulate that it is produced by an elaborate surface structure rather than arising between particles.

The appropriate theory for application to this problem has been developed by Irvine (1966). His formulation applies only for single scattering and so must be used with some caution in this problem. However, photometry of the rings indicates that for $\tau \leq 1$ single scattering accounts for 75 to 80% of the observed brightness. Thus, we can approximately treat the problem by requiring that any phenomenon be represented by using only $\sim$ 80% of the total light.

The slope of the phase curves in the region of phase angle $\alpha = 2^\circ$ to 6° can be fitted on Irvine's model with a fraction of the volume filled of $\sim$ 1/3, e.g., the surfaces of the particles show an intricate needle-structure, with 2/3 of the volume adjacent to the needles empty. Such curves, however, do not at all accurately predict the shape of the opposition surge. At much lower fractions of the volume filled, about 1/50, Irvine's curves do resemble the observed ones throughout the entire range $\alpha = 0^\circ$ to 6°, but we regard the existence of particles with such low densities near their surfaces with great suspicion.

There remains, in our opinion, one exit from the difficulties we have outlined; or, in the present context, one way of accounting for the phase curves with a single layer model. This is the second question mentioned on page 23. We refer the reader to the papers of Oetking (1966) and Hapke (1966) who found from a study of various laboratory samples that an opposition surge was nearly always present, and that failure to detect it in the past was the result of the inability of observers to measure at very small phase angles. The effect could not be associated in any obvious way with such quantities as particle size, albedo, composition or degree of compaction of the sample. We thus simply conclude that this effect may well be exhibited by ring particles and consequently, offers a possible, though physically unexplained, means of accounting for all or part of the opposition surge, and its color dependence.

Our final conclusion is, alas, necessarily rather vague. To us it seems as though the most consistent model for Rings A and B is a single layer of particles, probably near 1 km in diameter which exhibit Oetking's effect. The particles, at least in the brightest part of Ring B, are essentially in contact, probably rolling on one another. This model received further support from recent work (Franklin et al. 1971) which shows that a mass of Ring B $\stackrel{\sim}{\geq} 6 \times 10^{-6}$ of Saturn's (or a mean ring density, $\rho \stackrel{\sim}{\geq} 0.1$ gm/cm^3) can augment the outer boundary of Ring B by $\sim 0\overset{''}{.}2$ and thus account for the fact that the observed Cassini division is asymmetrically placed with respect to the resonance at 1/2 Mimas' period. On the other hand a ring model (Bobrov, 1970) that interprets the opposition surge wholly in terms of shadowing (i.e., no Oetking effect) gives for D, the fraction of the ring volume occupied by macroscopic particles, the value $D = 10^{-2}$. If the ring particles have a density of 1 gm/cm^3, the implied mean ring density of $\rho = 0.01$ gm/cm^3 is too small by a factor of 10 to account for the Cassini division displacement. Whether the rings actually exist in a mono-layer, or, do show a thickness at least a little greater than a particle diameter can only be answered by a model in which the sources of particle motions perpendicular to the ring plane and the effects of collisions are included.

3.2 *Models of Ring C*

The role of meteoroidal bombardment of Saturn's rings was first pointed out by Bandermann and Wolstencroft (1969) and discussed quantitatively by Cook and Franklin (1970). The existence of this phenomenon can be coupled with the photometric behavior of Ring C as follows: the optical thickness of Ring C just inside Ring B, according to Cook and Franklin (1958), is about equal to that in the outer part of Ring A [Franklin and Cook (1965)] however, the geometric albedo of Ring C is far less than that of Ring A. It follows that the particles in Ring C are much poorer back scatterers than those in Rings A and B. Presumably, the Ring C particles are good forward scatterers, i.e., they are small particles. Thus, the temptation to identify the particles of Ring C as small bits of ice spalled from Ring B by meteoroidal bombardment is overwhelming.

We shall briefly outline the steps which we plan to take to discuss the question quantitatively. The meteoroidal bombardment of Ring B must not only throw spall about, but also release a certain amount of gas, a small portion of which must be photoionized by solar radiation before the molecules are reaccreted on the ring. These ions may be expected to lock on to Saturn's magnetic field. This field, whose existence we must presently only assume, almost certainly has an angular velocity less than that for particles in circular orbit about Saturn in the inner part of Ring B. A deceleration of gas and ions and consequently a tendency to spiral toward the planet follows. Some of the gas and ions will depart inward from Ring B and this cloud will also impose a drag on the spalled particles, pulling some of their aposaturnia closer to the planet than the inner edge of Ring B. After a time there will be a build-up of spalled ice particles in the region of Ring C. Transfer by collision of further spalled particles from Ring B will then take place. Meteoroidal bombardment of the particles in Ring C will tend to vaporize the small ice particles completely and generate gas locally in Ring C, producing a drag upon the small particles, much as outlined above. The profile of Ring C presumably will be then determined by the competing processes of injection of newly spalled particles from Ring B and destruction of small particles by meteoroidal bombardment while the particles slowly spiral toward Saturn under the net drag imposed by the gas and ions. This study could prove important because it will allow us to extend the observed Ring C profile inward, closer to the planet, with some confidence. It may also allow us to make some predictions about the alleged Ring D, outside of Ring A, and maybe to infer something about Saturn's magnetic field.

For the present, let us employ the observed profile of the optical thickness of Ring C with assumptions regarding the possible particle radii. The optical thickness curve was deduced from an eclipse of Iapetus by Ring C as observed by Barnard (1890). We reproduce here in Figure 3 results of these observations. The particles in Ring C are probably vitreous ice which is formed by crystallization from water vapor at temperatures below 113°K, (Fletcher, 1970). Taking these to be solid at a density of 0.92 g cm^{-3} and spherical in shape, we calculate numbers of particles per cm^2 column through the rings for equivalent spherical particles of radii 10μ, 100μ, and 1000μ. In view of their scattering properties, we find it difficult to believe that the particles have radii outside of this range. The following table presents some results for an optical thickness of 0.10. Values can be rescaled in direct proportion for other optical thicknesses.

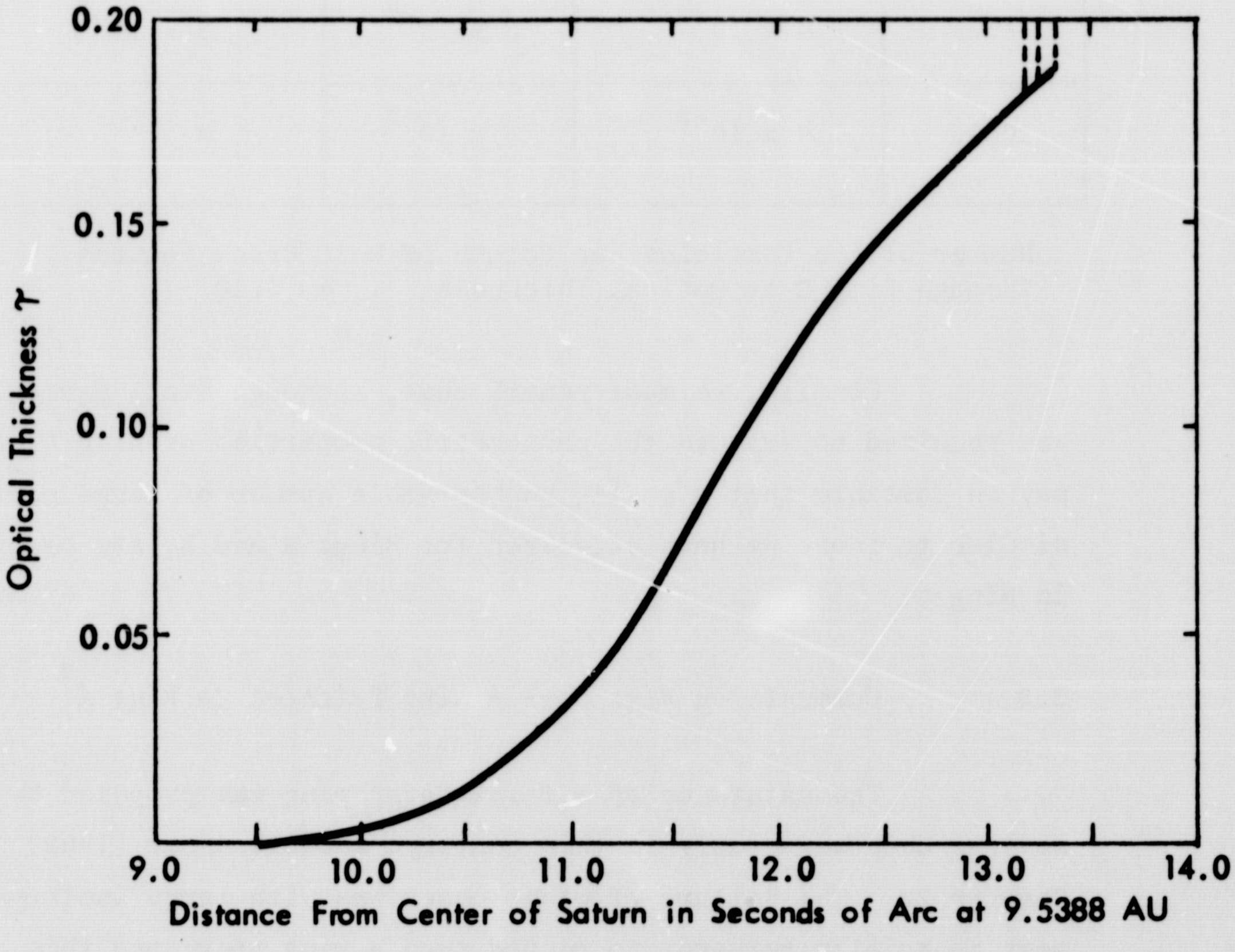

Figure 3. The Optical Thickness of Ring C Plotted Against Distance From the Planet's Center

Table IX

Radius of Particle	Mass of Particle gms	Cross Section cm^{-2}	No. of Particles per Unit Area cm^{-2}
10μ	4×10^{-9}	3×10^{-6}	3×10^{4}
100μ	4×10^{-6}	3×10^{-4}	3×10^{2}
1000μ	4×10^{-3}	3×10^{-2}	3

Number of Ice Particles per Column of Unit Cross Section Through Ring C at Optical Thickness, $\tau = 0.10$

Finally, we must remark that, although small particles are required to explain the photometric properties of Ring C, it may be possible that a small, unobservable number of large particles, similar to those we have suggested for Rings A and B, may be present in Ring C.

3.3 *Comments on Ring D -- A Ring Exterior to Ring A**

The existence of a faint outer ring was proposed by several observers early in this century, see Alexander (1962) Chapter 28. The failure of other observers with large instruments, most especially Barnard, to record such a ring prevented this possible discovery from gaining wide-spread acceptance. Those observers who did claim to have observed Ring D saw it as a dark projection, about 1 arc sec wide, against the planet's disk. In recent times, Cragg (1954) has made the same claim, except that he and another observer independently, report seeing Ring D both in projection against the disk and as a bright area against the sky.

* *This section and the final one are additions to the main body of this review written to report new material about to be published or very recently published.*

Still more recently, Feibelman (1967) has used microdensitometer traces of plates, exposured up to 30 minutes, to show that some faint light in the ring plane does exist at distances from the planet some two times that of the outer boundary of Ring A. These observations were made near ring plane passage of the Earth. Thus, there appears to be some observational evidence for:

(1) a faint ring immediately exterior to Ring A.

(2) a possible faint ring extending to much greater distances, essentially, if Feibelman is correct, to the orbit of Enceladus.

We feel that this second claim must be viewed with deep reservation. Rosino and Stagni (1969) failed to locate this extension on good quality plates taken essentially at the same time.

It is well-known that Ring C and the Cassini Division show bright knots or condensations when the Earth and the Sun are on opposite sides of the ring plane. Let us first suppose that this phenomenon has never been seen in the Ring D region. The knots in Ring C have been followed by Barnard as close to Saturn as $10''.56$, where the optical thickness is ~ 0.01. Hence, we suppose that τ for Ring D ≤ 0.01.

Franklin and Colombo (1970) obtained a density profile of the conspicuous parts of the ring system on the assumption that ring particles were so spaced that collisions did not occur. In that paper, the oblateness of Saturn and perturbations by the inner satellites were included. This collisionless model required, for example, that near a resonance the areal density of particles must drop rapidly and the resulting computed profiles of the boundaries of, and gaps in the ring bears a distinct resemblance to the observed ring profile.

A few sample orbits were also obtained more distant from the planet than the outer boundary of Ring A, in the hypothetical Ring D region. Such orbits showed high eccentricities which suggested that the areal density of material, in non-colliding orbits, must be several orders of magnitude below the areal density of similar particles in Ring A. It is now clear, however, that the calculation upon which this conclusion was based corresponded to a special class of orbits of high eccentricity, i.e., orbits of low eccentricity do, in fact, exist in this region. This leads us now to the prediction that there may exist, exterior to Ring A, several concentrations of particles lying between the resonances: 2/3 P_{Mimas} and 3/4 P_M; 3/4 P_M and 4/5 P_M. These concentrations are shown in Figure 4, where the scale on the abscissa is drawn such that $R_{Mimas} = 1.00$ and the ordinate scale gives the surface number density, N, of ring particles. N is inversely proportional to ΔR, which is the minimum radial separation of adjacent particles such that collisions will not occur. As an example ΔR is approximately 200 meters at the Ring A maximum, with $R_{Mimas} = 1.86 \times 10^5$ km. Further details regarding the assumptions and the computation involved in obtaining Figure 4 can be found in Franklin and Colombo (1970) and Franklin et al. (1971). The first of these concentrations, between 2/3 P_M and 3/4 P_M is reduced in radial extent by perturbations associated with 1/2 of the period of Enceladus. Crosses on Figure 4 give limits imposed by Enceladus. The next, between 3/4 P_M and 4/5 P_M is somewhat narrower and shows a density decrease from the first by a factor of about 4-5, while the first shows a maximum density about one order less than is shown by Ring A.

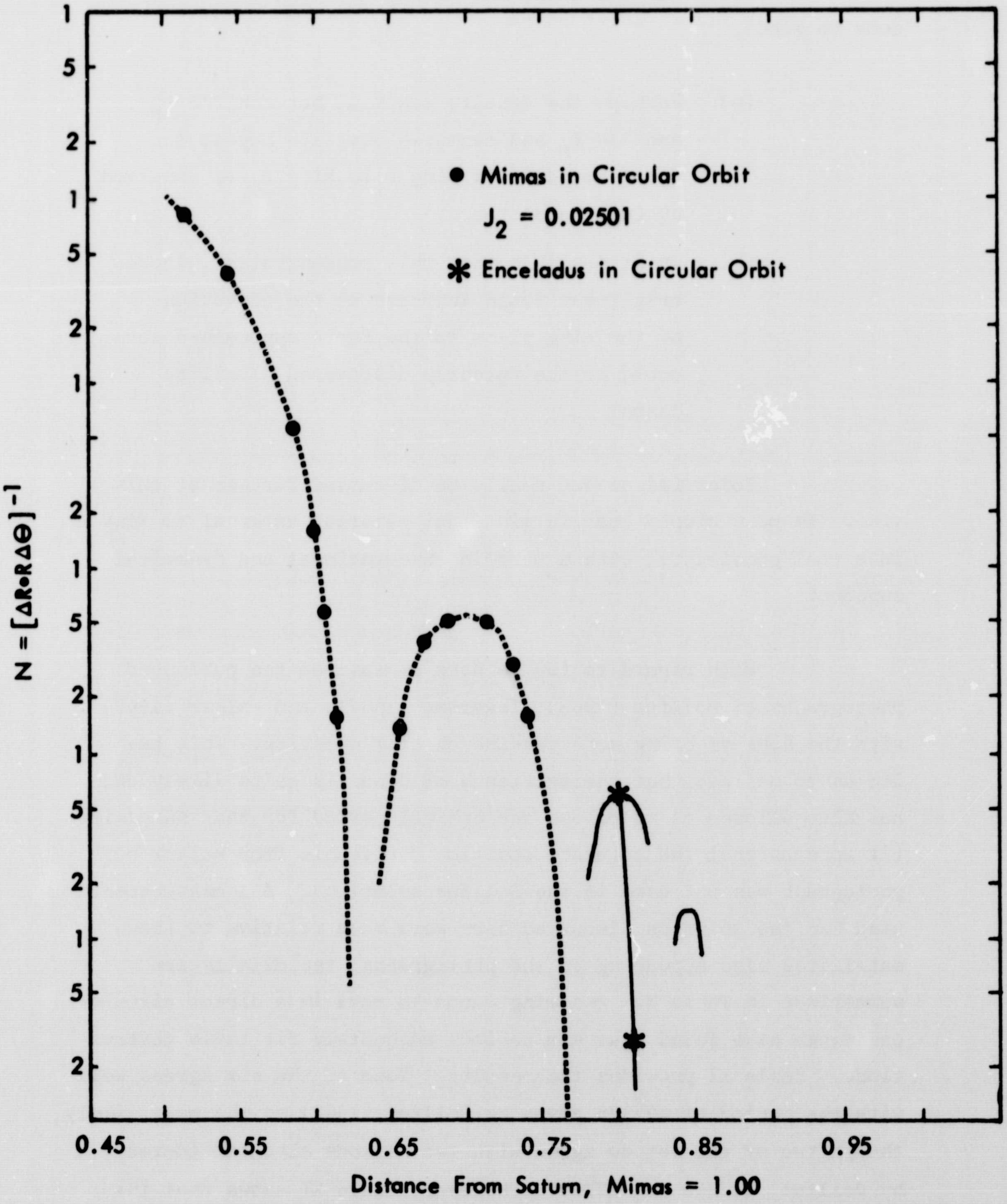

Figure 4. Predicted Surface Density Profile of the Ring Obtained with the Perturber, Mimas, in a Circular Orbit.

With regard to the first concentration, two questions come to mind:

(a) Perhaps the density maximum, between 2/3 P_M and 3/4 P_M and centered some 1".5 beyond the outer boundary of Ring A is Ring D, as observed by Cragg and others.

(b) Is it possible that this concentration, whose brightness would increase as the elevation of the ring plane to the Earth approached zero, could be the recently discovered satellite Janus?

Point (a) cannot really be discussed further at this time. We must simply bear in mind that material external to Ring A is a real possiblity, with a certain observational and dynamical support.

With regard to (b) we have remeasured the published photographs of Dollfus (1968), Texereau (1967), and Walker (1967) with the hope of being more precise on this question. This has led us to believe that the existence of Janus is quite likely and has also allowed us to revise the orbital period for that satellite. (It appears that the accurate position deriveable from Walker's photograph was not used in the Dollfus solution). All measurements used for the solution discussed here were made relative to known satellites also appearing on the photographs; the details are summarized in Table X. Assuming Janus to move in a direct circular orbit, we have found that six periods adequately fit these observations. Table XI provides the results. None of the six agrees well with the period of $17^h.975$ given by Dollfus (1968) as the most likely, though two of the set do agree with two periods also considered by Dollfus, $18^h.263$ and $17^h.697$. However, Table XI shows that the period that best fits the three published observations is $19^h.565$.

Table X

Observer	Time of Middle of Exposure (UT)	Exposure Time	No. of Satellites appearing on photograph (excluding Janus)	Position of Janus at mean distance (9.5389 AU)
Dollfus (1)	15 Dec. 1966 $18^h\ 22^m\ 30^s$	21 min	4	21".8±0".3(E)
Walker (2)	18 Dec. 1966 $01^h\ 35^m\ 47^s$	30 sec	5	19".5±0".1(E)
Texereau (3)	29 Oct. 1966 $02^h\ 55^m$	5 min	2	16".0(W)

Information Obtained from Three Photographs of the Satellite Janus

If we can accept the model proposed by Franklin and Colombo (1970), this value receives some support from a recent photograph of remarkable quality taken by Guerin (1970). This photograph recorded, to our knowledge for the first time two "divisions", or narrow regions where the intensity gradient is very steep in Ring A, which are the visually observed components of the Encke division. Measures we have made on pre-publication copies of this photograph show that the outer of these two lies at 3/5 of the period of Mimas, substantiating the treatment of Franklin and Colombo (1970). The inner lies at 0.691, or 18".60 on Figure 4. This value corresponds closely to 2/3 of the period $19^h.565$, or 18".59. Except for the period given by Case II (b), which is nearly the same as Case II (a), none of the other four periods predicts a feature in this region. It is also possible to associate a pronounced feature in Ring B with 1/2 of this period (but with none of the other four). Many observers (Alexander, 1962) have noted a narrow region of rapid intensity change near the center of Ring B; a measurement of the ring profile of Dollfus (1970) places it at $15''.42$, and 1/2 of the $19^{h}.565$ period corresponds to $15''.36$. Except for a possible double minimum near the inner boundary of Ring B, noted by Dollfus (1970) but apparently not by others (Alexander, 1962), it is the most conspicuous feature in Ring B. Three quarters of the period $19^{h}.565$ lies outside of Ring A.

In conclusion, we feel that the certain existence of material external to Ring A has not been reliably demonstrated, nor, however, has the possibility been excluded. The existence of Janus is most likely and it seems now quite impossible to suppose, as it did seem to us at one time, that Janus might not be a satellite at all, but rather an outer ring, visible at the time of ring plane passage.

Table XI*

Comparison of Observed (obs) and Calculated (calc) Orbit Parameters for the Satellite Janus for Different Possible Periods, P.

Case I: Assume $M_1 < 0$, $M_2 > 0$, then

O_{bs}	r_{obs}	M_{obs}	M_{calc}	r_{calc}	n_o
(1)	21".8(E)	-17°.1	-12.0	22".3	n_o = 488°.10/day
(2)	19.5(E)	+31.3	+31.0	19.6	a_o = 22".81
(3)	16.0(W)	+135.5	+134.9	16.1	P = $17^h.701$

Case II: Assume $M_1 > 0$, $M_2 < 0$, (a)

(1)	21".8(E)	+26.6	27°.1	21.7	n_o = 441°.61/day
(2)	19.5(E)	-36.9	-36.9	19.5	a_o = 24".38
(3)	16.0(W)	+229.0	+228.8	16.1	P = $19^h.565$

(b)

(1)	21".8(E)	+26°.3	23°.2	22".3	n_o = 443.58/day
(2)	19.5(E)	-36.7	-36.3	19.6	a_o = 24".31
(3)	16.0(W)	+131.2	+131.0	15.9	P = $19^h.478$

CaseIII: Assume $M_1 > 0$, $M_2 > 0$

(1)	21".8(E)	+20°.5	22°.3	21".5	n_o = 473.74/day
(2)	19.5(E)	+33.1	32.2	19.7	a_o = 23".27
(3)	16.0(W)	+133.3	133.3	16.0	P = $18^h.238$

Case IV: Assume $M_1 < 0$, $M_2 < 0$ (a)

(1)	21.8(E)	-22°.2	-23°.9	21".5	n_o = 465°.22/day
(2)	19.5(E)	-34.1	-33.6	19.6	a_o = 23".55
(3)	16.0(W)	+132.8	+133.0	16.1	P = $18^h.572$

(b)

(1)	21.8(E)	-22°.6	-20°.4	22".1	n_o = 463°.32/day
(2)	19.5(E)	-34.3	-34.5	19.5	a_o = 23".61
(3)	16.0(W)	+227.3	+227.0	16.1	P = $18^h.648$

*The mean anomaly, M is measured from eastern elongation.
Observers (Obs.) (1)≡Dollfus; (2)≡Walker; (3)≡Texereau.
M_1 and M_2 refer to observations (1) and (2).
r is radial distance from Saturn at mean distance (9.5388 AU).
a_o is the semi-major axis and n_o the mean daily motion of Janus.

3.4 Comments on a Ring Interior to Ring C

Recently Guerin (1970) has published an extraordinary photograph that presents new data on the ring system. He apparently has succeeded in photographing not only Ring C and its inner boundary, but has also recorded a ring interior to Ring C. The division or gap, which seems quite well-defined, between Ring C and the new interior ring presents a new challenge. Measures of the position of this new gap place it at 10"8 ±0"2 from Saturn. As we have already remarked, Franklin and Colombo (1970) have attempted to account for the major features of the ring profile as a consequence of resonance phenomena. We are thus led to inquire whether this new feature can be explained in terms of a resonance with one of the satellites. To this end, we have explored three possibilities. The first, a resonance between the local orbital frequency of a ring particle and 4 times that of Mimas, can quickly be dismissed. The perturbations associated with this resonance depend upon the square of the orbital eccentricity of Mimas and are too small to be of any consequence. A second possibility relates to a resonance between the mean motion of a particle near the observed gap and the moving tidal bulge raised by Mimas on Saturn. Once again calculations show that such a bulge is probably too small to perturb greatly a ring particle in the critical region. (The height of the tide raised on Saturn by Mimas is less than 30 cm).

One final mechanism, however, does seem very promising. At 11"2 from Saturn the oblateness of the primary causes the orbits of ring particles to precess at rates which equal the mean motion of the most massive satellite of the Saturn system, Titan. Calculations show that the resonance between these two motions would lead to a gap, defined in the same way as by Franklin and Colombo (1970), of at least ∿ 0"3 in width and centered at 11"2 from Saturn. This seems to be displaced somewhat from the location which we have measured on Guerin's photograph. We await further photographs and measurements with much interest.

To conclude: the presence of material interior to Ring C rests upon a single observation, but it is a modern one that will be further checked in the near future. A preliminary theoretical reconnaissance shows the gap in this region to be comprehensible. Thus, the chance that a spacecraft traversing this region would encounter ring particles must be regarded as likely.

REFERENCES

Alexander, A. F. O'D (1962), The Planet Saturn, the MacMillan Company, New York, 474 pages.

Bandermann, L. W., and Wolstencroft, R. D. (1969), "The Erosion of Particles in the Rings of Saturn", Abstract in Bulletin American Astronomical Soc., Vol. 1, page 233.

Barnard, E. E. (1890), "Observations of the Eclipse of Iapetus", Royal Astronomical Soc. Monthly Notices, Vol. 50, pp. 107-110.

Barnard, E. E. (1896), "Micrometrical Measures of the Ball and Ring System of the Planet Saturn", Royal Astronomical Soc. Monthly Notices, Vol. 56, pp. 163-172.

Barnard, E. E. (1902), "On the Dimensions of the Planets and Satellites", Astronomishe Nachrichten, Vol. 157, pp. 261-268.

Bobrov, M. S. (1940), "On the Physical Interpretation of the Phase Curve of Saturns Rings", Astronomicheskii Zhurnal, Vol. 17, pp. 1-.

Bobrov, M. S. (1954), "On the Structure of Saturn's Rings. II", Astronomicheskii Zhurnal, Vol. 31, pp. 41-50.

Bobrov, M. S. (1956), "On the Problem of the Thickness of Saturn's Rings", Astronomicheskii Zhurnal, Vol. 33, pp. 161-172.

Bobrov, M. S. (1970), "Physical Properties of Saturn's Rings", in Surfaces and Interiors of Planets and Satellites, ed. by A. Dollfus, Academic Press, pp. 377-458.

Cook, A. F. II, and Franklin, F. A. (1958), "Optical Properties of Saturn's Rings: I. Transmission", Smithsonian Contributions to Astrophysics, Vol. 2, pp. 377-383.

Cook, A. F., and Franklin, F. A, (1964), "Rediscussion of Maxwell's Adams Prize Essay on the Stability of Saturn's Rings", Astronomical J., Vol. 69, pp. 173-200.

Cook, A. F., and Franklin, F. A. (1965), "On the Structure of Saturn's Rings", Astronomical J., Vol. 70, pp. 135-136.

Cook, A. F., and Franklin, F. A. (1970), "The Effect of Meteoroidal Bombardment on Saturn's Rings", Astronomical J., Vol. 75, pp. 195-205.

Cragg, T. A. (1954), mentioned in article in Sky and Telescope, Vol. 13, p. 382.

Dollfus, A. (1970), "New Optical Measurements of the Diameters of Jupiter, Saturn, Uranus, and Neptune", Icarus, Vol. 12, pp. 101-117.

Dollfus, A. (1969), "New Optical Measurements of the Diameters of Jupiter, Saturn, Uranus, and Neptune", in the "Results About Major Planets", by Dollfus, A., Prinz, R., and Focas, J. H., Final Scientific Report, Vol. 1, U. S. Air Force, Cambridge Research Laboratories, Report No. AFCRL-69-0530 (I).

Dollfus, A. (1968), "The Discovery of the Tenth Satellite of Saturn", L'Astronomie, June-July, pp. 253-262.

Dollfus, A. (1954), "Le Diametre des Satellites de Jupiter, de Titan, et de Neptune", Comptes Rendus, Vol. 238, pp. 1475-1478.

Dyson, F., and Lewis, T. (1895), "Diameters of Saturn and His Rings", Royal Astronomical Soc. Monthly Notices, Vol. 56, pp. 14-17.

Feibelman, W. A., (1967), "Concerning the 'D' Ring of Saturn", *Nature*, Vol. 24, pp. 793-894.

Fletcher, N. H. (1970), *Chemical Physics of Ice*, Cambridge University Press, 271 pp.

Focas, J., and Dollfus, A (1969), "Optical Properties and Thickness of the Saturn Rings Observed Edgewise in 1966", *Comptes Rendus*, Serie B - Sciences Physiques, Vol. 268, pp. 100-103.

Franklin, F. A., and Cook, A. F. (1965), "Optical Properties of Saturn's Rings. II. Two Color Phase Curves of the Two Bright Rings", *Astronomical J.*, Vol. 70, pp. 704-720.

Franklin, F. A., and Colombo, G. (1970), "A Dynamical Model for the Radial Structure of Saturn Rings", *Icarus*, Vol. 12, pp.338-347.

Franklin, F. A., Colombo, G., and Cook, A. F. (1971), "A Dynamical Model for the Radial Structure of Saturn's Rings. II", submitted to *Icarus*.

Guerin, P. (1970), "The New Ring of Saturn", *Sky and Telescope*, Vol. 40, page 88.

Hall, A. (1885), "Saturn and Its Ring", Washington Observations, for 1885, U. S. Naval Observatory, Appendix II, 22 pages.

Hapke, B. (1966), "Comments on Paper by Philip Oetking, 'Photometric Studies of Diffusely Reflecting Surfaces with Applications to the Brightness of the Moon'", *J. Geophysical Research*, Vol. 71, page 2515.

Harris, D. L. (1961), "Photometry and Colorimetry of Planets and Satellites", in *Planets and Satellites*, edited by Kuiper, G. P. and Middlehurst, B. M., University of Chicago Press, pp. 272-342.

Irvine, W. M. (1966), "The Shadowing Effect in Diffuse Reflection", J. Geophysical Research, Vol. 71, pp. 2931-2937.

Kiladze, R. I. (1967) "Photographic Observations of Saturn's Rings at the Transit of the Earth Through Their Plane", Astronomical Circular, No. 439, Publishing Office Astronomical Communication of the USSR Academy of Science, one page.

Kozai, Y. (1957/58), "On the Astronomical Constants of the Saturian Satellites System", Annals of the Tokyo Astronomical Observatory, Second Series, Vol. V, pp. 73-106.

Kozai, Y. (1957/58), "Titan and Rhea, Saturnian Satellites 1927-1947", Annals of the Tokyo Astronomical Observatory, Second Series, Vol. V, pp. 107-127.

Lowell, P., and Slipher, E. C. (1915), "Measures of Saturn-Ball, Rings, and Satellites", Lowell Observatory Bulletin 66, pp. 73-80.

Lowell, P., and Slipher, E. C. (1915), "Saturn's Rings", Lowell Observatory Bulletin 68, pp. 85-115.

Lumme, K. (1970), "On Photometric Properties of Saturn's Rings", Astrophysics and Space Science, Vol. 8, pp. 90-101.

Oetking, P. (1966), "Photometric Studies of Diffusely Reflecting Surfaces with Applications to the Brightness of the Moon", J. Geohpysical Research, pp. 2052-2513.

Pilcher, C. B., Chapman, C. R., Lebofsky, L. A., and Kieffer, H. H. (1970), "Saturn's Rings: Identification of Water Frost", Science, Vol. 167, No. 3923, pp. 1372-1373.

Rosino, L., and Stagni, R. (1969), "Observations of Saturn During the Period of Theoretical Disappearance of the Ring", Societa Astronomica Italiana Memorie, Vol. 40, pp. 191-204.

See, T. J. J. (1901), "Researches on the Equatorial Diameter of Saturn, and on the Dimensions of His System of Rings", Astronomische Nachrichten, Vol. 154, pp. 269-286.

See, T. J. J. (1902), "Micrometrical Measures of the Equatorial Diameter of Saturn and His System of Rings", Astronomische Nachrichten, Vol. 157, pp. 389-400.

Seeliger, H. von (1887), "Zur Theorie d. Beleuchtung d. Grossen Planeten, insbesondere d. Saturn", Abhandlungen Bayer. Akad. Wiss., Kl. II 16, 114 pp.

Struve, G. (1933), "Neue Untersuchungen im Saturnsystem", IV and V, Veroffentlichungen de Universitatssternwarte zu Berlin-Babelsberg, Vol. 6, 61 pages IV, 44 pages V.

Struve, H. (1894), "On the Dimensions of Saturn's Disc", Royal Astronomical Soc. Monthly Notices, Vol. 54, pp. 452-470.

Texereau, J. (1967), "Observing Saturn's Edgewise Rings, October, 1966", Sky and Telescope, Vol. 33, pp. 226-227.

Veverka, J. G. (1970), Icarus, in preparation, See also Dissertation, Harvard University, 1970, "Photometric and Polarimetric Studies of Minor Planets and Satellites".

Walker, R. L., Jr. (1967), "A Tenth Satellite of Saturn?", An anonymous article in Sky and Telescope, February, which makes reference to Walker, pages 71 and 93.

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