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FINAL TECHNICAL REPORT

LUNAR ORBITER SELENODESY STUDIES

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I INTRODUCTION

The initial period for Contract NSR 05-007-083 was July 1, 1966 to June 30, 1968, and the supplemental period was July 1, 1968 to June 30, 1969. The latter was extended to December 31, 1969. However, work on the main task, analysis of lunar orbiter tracking data, was actually started at NASA-Goddard Space Flight Center (GSFC) in 1963, and not terminated at UCLA until September 1970.

The effort at GSFC was carried out in connection with the Anchored IMP project. It was terminated in April 1967, after it became evident that Anchored IMP had not attained an orbit close enough to the moon to be useful selenodetically. The work at UCLA after December 31, 1969 was supported by the University of California, and suspended in September 1970 with the departure of the principal programmer working on the analysis of lunar orbiter tracking data. The decision has now been made that it would not be worthwhile to resume the effort. Although it is felt that lunar satellite orbits have not been exploited to their fullest potentiality, approaches different from that developed under this contract appear to offer greater hope of solving the main remaining problem, determination of the far side gravitational field.

Work under Contract NSR 05-007-083 fell under four headings, which correspond to the subjects of Sections II - V of this report.

(1) Analysis of lunar orbiter tracking data constituted about 80 percent of the effort.

(2) Interpretation of the lunar gravitational field constituted slightly more than 10 percent. This work was transferred to NASA Contract NGL 05-007-002 in May, 1969.

(3) Error analysis of laser ranging to the moon constituted between 5 and 10 percent. This work has been continued under NASA Contract 05-007-283, which began April 1, 1970.

(4) Miscellaneous efforts constituted less than 2 percent. They included re-examination of Brown's lunar theory to determine the effect of the lunar oblateness and statistical analysis of topographic heights on the moon.

II ANALYSIS OF LUNAR ORBITER TRACKING DATA

The main guiding principle of this analysis was to make the minimum adaptation of techniques which had been sucessful in analyzing earth satellite orbits to determine variations in the earth's gravitational field (KAULA, 1963a, 1966a, 1968). These analyses of earth satellite orbits were begun at GSFC in 1960, continued at UCLA under Contract NSR 05-007-060, and terminated early in 1967 because (1) the difference in technique from that at SAO was no longer great enough to warrant a separate effort; and (2) the amount of routine technical work required was greater than appropriate for a university research project.

The distinctive characteristics of the earth satellite orbit analysis were:

(1) Use of an analytic intermediary orbit, as well as analytic formulation of gravitational field and luni-solar perturbations and partial derivatives.

(2) Formation of the observation equations in terms of right ascension & declination for camera observations, and range-rate for Doppler observations (rather than in terms of osculating Kepler elements).

(3) Combination of data from several arcs of the same satellite by the technique of partitioned normals (KAULA, 1966b, p. 105), and from several satellites by saving the normals and applying weights to their combination.

The programs used in the earth satellite orbit analysis had three stages:

(1) S & C: selection and conversion. At this stage, redundant data were omitted or compressed by using polynomial fits to residuals with respect to an orbit. The data were referred to coordinate axes appropriate to the epochs of the orbital arcs, and reformulated so as to be convenient input for the succeeding stages.

(2) SOA-P: Satellite orbit analysis, preliminary. At this stage, orbital arcs were determined iteratively until good enough fits were obtained that any further correction could be safely assumed linear. Arrays to be used for partial derivatives with respect to spherical harmonic coefficients of the gravitational field were then calculated.

(3) SOA-F: Satellite orbit analysis, final. At this stage, the data were reanalyzed to determine not only constants of integration, but also spherical harmonic coefficients of the gravitational field and shifts of tracking station location. All exercises of statistical options -- relative weighting of different satellites, relative weighting of different tracking types, weighting according to phases of angles critical to perturbations, a priori sigmas for parameters, etc. -- were concentrated at this stage.

Two main differences in the nature of the lunar satellite orbit analysis necessitated corresponding modifications in the programs: (1) the tracking stations were located on a different body than that about which the satellite orbited; and

(2) the perturbations of the satellite orbit are much larger relative to the central term.

Tracking from stations located on the earth to satellites orbiting around the moon gained the advantages of much greater tracking coverage, and of insensitivity of the gravitational field determination to errors in tracking station position. However, it had the disadvantage of requiring incorporation of the lunar ephemeris. Furthermore, the synchronous rotation of the moon prevented observation of the short-term perturbations by the farside gravitational field.

The modifications made were:

(1) partial derivatives with respect to station location in SOA-F were removed;

(2) the lunar ephemeris tape was read in S & C and tracking station position & velocity were calculated in a selenocentric coordinate system referred to the lunar equator and Cassini point. These station positions & velocities were saved and used in the formation of observation equations in SOA-P and SOA-F.

The data used were range rates from the Deep Space Instrumentation Facilities (DSIF) at Goldstone, Madrid, Johannesburg, and Woomera to the Lunar Orbiter Satellites I through V. The format of the data was as given by LORELL et al. (1965). The conversion of Doppler count to range-rate was as described by WARNER et al. (1964, pp. 15-20). The units used for the converted range rate were such that the lunar radius $\underline{R}$, mass $\underline{M}$, and gravitational constant $\underline{G}$ were all unity: i.e., a time unit of 1035 seconds. The ephemeris tape used was the Jet Propulsion Laboratory DE-19 (DEVINE, 1967).

There were practical difficulties entailed in converting the ephemeris and data tapes from 7-track (adaptable to JPL's IBM 7094) to 9-track (adaptable to UCLA's IBM 360); in converting ephemeris tape interpolation programs using some system subroutines not available on the IBM 360; and in obtaining modifications of constants relating to the data conversion from those given in WARNER et al. (1964). However, it is felt that these difficulties did not have any serious effect on the results, compared to either (1) the system bugs associated with UCLA's conversion from an IBM 7094 to IBM 360-40, 360-75, and 360-91 in turn; or (2) the inadequacies in modelling the large orbital perturbations.

Perturbations of lunar satellites are larger than those of earth satellites for four reasons: (1) the magnitudes of the variations in the gravitational field, expressed by the spherical harmonic coefficients $\overline{C}_{\ell m}$, $\overline{S}_{\ell m}$, are larger, as inferred from analysis of physical librations and predicted by the equal shearing stress assumption (KAULA, 1963b); (2) the moon rotates much more slowly, and has a smaller oblateness, thus leading to slower pericenter motion and more small divisor buildup; (3) the earth is much larger relative to the moon than the moon is relative to the earth, so that the ratio $[\underline{G}\underline{m}^*\underline{r}^2/\underline{r}^{*3}]/[\underline{G}\underline{M}/\underline{r}]$ is larger by a factor of about 100; and (4) the satellites can come much closer to the body, because there is no atmosphere.

The dominant means by which the SOA program determined variations of the earth's gravitational field was from perturbations not averaged out in a single revolution of the orbit, and appearing mainly as oscillations at a rate of m cycles per day for a tesseral harmonic of order m. These oscillations are essentially linear with respect to an intermediary which is non-linear in the principal perturbing effect, the oblateness $\underline{J_2}$. The theory used for this intermediary was BROUWER's (1959).

In the case of the moon, it is desirable to use data spanning at least a month, in order to observe the analogous m-cycle/month perturbations from lunar tesseral harmonics. Probably only in this manner can the farside variations be obtained. In the first approximation, the tesseral harmonic effects can probably be taken as linear, as well as the solar effects and the short periodic earth effects. However, the long periodic perturbations by the earth and the second degree harmonics of the moon's gravity field have significant non-linear interactions. It was decided that the modification most consistent with the guiding principle of minimum change was to replace the long periodic part of the Brouwer theory by the numerical integration of an intermediary in terms of Kepler elements. For such an intermediary, integration steps on the order of .05 day are feasible, since the most prominent perturbations have rates of 2, 4, etc. cycles/month.

It was assumed that the disturbances possibly giving rise to significant non-linear long term effects comprised (1) the earth, expressed as Legendre functions of degrees two and three; (2) the zonal harmonics of the moon $\underline{J}_2$ and $\underline{J}_3$; and (3) the equatorial ellipticity of the moon, $\underline{J}_{22}$. Disturbing functions in terms of the osculating Keplerian elements of the satellite and third body are given by KAULA (1961) for variations in the planetary gravitational field and by KAULA (1962) for the third body effects. However, two minor modifications are desirable in order to cope with the theoretical difficulties of obtaining a correct force function for the numerically integrated intermediary. Firstly, to perform the "bookkeeping" associated with the mutual interaction of short period terms giving rise to long term effects, it is convenient to adopt a subscript convention such that all terms of like argument are combined with a positive coefficient on the mean anomaly term in the argument. Secondly, to account for the indirect effects of the perturbation of the moon by sun using the lunar theory of BROWN (1905), either in tabular form or in an abbreviated analytical form such as that given by BATTIN (1964, p. 380), it is more convenient to express the position of the earth in selenocentric inertial coordinates $\{\underline{r}^*, \varphi^*, \lambda^*\}$ referred to the lunar equator and Cassini point at epoch.

The initial disturbing function $\underline{R}$ thus becomes:

$$R = \sum_{n=-3}^{3} \sum_{k=0}^{3} A_{onk} \begin{Bmatrix} \cos \\ \sin \end{Bmatrix}_{n-k\,\text{odd}}^{n-k\,\text{even}} \{n\omega + k(\Omega - \lambda^*)\}$$
$$+ \sum_{j=1}^{\infty} \sum_{n=-3}^{3} \sum_{k=-3}^{3} A_{jnk} \begin{Bmatrix} \cos \\ \sin \end{Bmatrix}_{n-k\,\text{odd}}^{n-k\,\text{even}} \{n\omega + jM + k(\Omega - \lambda^*)\}, \qquad (1)$$

where, with subscript correspondences

$$\left.\begin{aligned} l &= [5-(-1)^n]/2 \\ m &= |k| \\ p &= [l-(-1)^k n]/2 \\ q &= (-1)^k (j-n), \end{aligned}\right\} \qquad (2)$$

the coefficients A_{jnk} are, from KAULA (1961, Eq. (27)) and KAULA (1962, Eqs. (2) - (3)),

$$\begin{aligned} A_{jnk} = \Bigg[& \frac{m^* a^l}{r^{*l+1}} P_{lm}(\sin\varphi^*)(2-\delta_{om}) \frac{(l-m)!}{(l+m)!} H_{lpq}(e) \\ & + \left\{ \delta_{21}\delta_{2m} \frac{\mu}{a}\left(\frac{a_e}{a}\right)^2 J_{22} - \delta_{21}\delta_{om}\frac{\mu}{a}\left(\frac{a_e}{a}\right)^2 J_2 \right. \\ & \left. - \delta_{3l}\delta_{om}\frac{\mu}{a}\left(\frac{a_e}{a}\right)^3 J_3 \right\} G_{lpq}(e) \Bigg] F_{lmp}(i) \end{aligned} \qquad (3)$$

In (1) and (3), $\{a, e, i, M, \omega, \Omega\}$ are the Keplerian elements of the satellite. In (3), m^* is the mass of the earth times the gravitational constant; μ is the mass of the moon times the gravitational constant; a_e is the equatorial radius of the moon; the δ_{ij}'s are Kronecker deltas; $P_{\ell m}(\sin\varphi^*)$ is Legendre's Associated function; the inclination function $F_{\ell mp}(i)$ is (KAULA, 1961, Eq. (20)):

$$\begin{aligned} F_{lmp}(i) = & \sum_t \frac{(2l-2t)!}{t!(l-t)!(l-m-2t)!\,2^{2l-2t}} \sin^{l-m-2t} i \\ & \times \sum_{s=0}^{m} \binom{m}{s} \cos^s i \sum_c \binom{l-m-2t+s}{c} \binom{m-s}{p-t-c} (-1)^{c-\kappa}, \end{aligned} \qquad (4)$$

where κ is the integer part of $(\ell-m)/2$, t is summed from 0 to the lesser of p or κ, and c is summed over all values making the binomial coefficients non-zero; and the eccentricity functions $H_{\ell pq}(e)$ and $G_{\ell pq}(e)$ are most succinctly defined as the Hansen functions:

$$\left.\begin{aligned} H_{lpq}(e) &= X_{(l-2p+q)}^{l,(l-2p)}(e) = X_j^{l,n}(e) \\ G_{lpq}(e) &= X_{(l-2p+q)}^{-l-1,(l-2p)}(e) = X_j^{-l-1,n}(e). \end{aligned}\right\} \tag{5}$$

The Hansen functions are valid for any $e < 1$. The particular formulation used is that given by TISSERAND (1889, p. 256); an alternative formulation is given by PLUMMER (1960, p. 46).

To remove the short period terms on the second line of Eq. (1) by the customary von Zeipel technique (BROUWER and CLEMENCE, 1961, ch. 17; KAULA, 1966b, Sec. 3.5), we have, using Delaunay variables,

$$\begin{aligned} L &= (\mu a)^{\frac{1}{2}} \\ G &= [\mu a(1-e^2)]^{\frac{1}{2}} \\ H &= [\mu a(1-e^2)]^{\frac{1}{2}} \cos i \\ h &= \Omega - \lambda^*, \end{aligned} \tag{6}$$

the force function:

$$F = \frac{\mu^2}{2L^2} + H\dot{\lambda}^* + R. \tag{7}$$

For the transformation

$$F(L, G, H, M, \omega, h) \rightarrow F'(L', G', H', -\omega, h) \tag{8}$$

we get

$$\left.\begin{aligned}
&\frac{\mu^2}{2L^2} + H\dot{\lambda}^* && = F_0' \\
&-\frac{\mu^2}{L^3}\frac{\partial S_1}{\partial M} + R + \dot{\lambda}^*\frac{\partial S_1}{\partial h} - \frac{\partial S_1}{\partial t} && = F_1' \\
&\frac{1}{2}\frac{3\mu^2}{L^4}\left(\frac{\partial S_1}{\partial M}\right)^2 + \frac{\partial R}{\partial L}\frac{\partial S_1}{\partial M} + \frac{\partial R}{\partial G}\frac{\partial S_1}{\partial \omega} && \\
&-\frac{\mu^2}{L^3}\cdot\frac{\partial S_2}{\partial M} + \dot{\lambda}^*\frac{\partial S_2}{\partial h} - \frac{\partial S_2}{\partial t} + \frac{\partial R}{\partial H}\frac{\partial S_1}{\partial h} && = F_2' + \frac{\partial F_1'}{\partial \omega}\frac{\partial S_1}{\partial G} + \frac{\partial F_1'}{\partial h}\frac{\partial S_1}{\partial H}
\end{aligned}\right\} \tag{9}$$

Substituting (1) in (9), and using the determining function $\underline{S_1}$ to remove short period terms, we obtain for the force function $\underline{F}'$ of the intermediary to be integrated numerically,

$$F_1' = \sum_{n=-3}^{3}\sum_{k=0}^{3} A_{onk}\left\{\begin{matrix}\cos\\ \sin\end{matrix}\right\}_{n-k\text{ odd}}^{n-k\text{ even}}\{n\omega + kh\} \tag{10}$$

$$S_1 = \frac{L^3}{\mu^2}\int (R - F_1')\,\mathrm{d}M$$

$$= \frac{L^3}{\mu^2}\sum_{j=1}^{\infty}\frac{1}{j}\sum_{n=-3}^{3}\sum_{k=-3}^{3} A_{jnk}\left\{\begin{matrix}\sin\\ -\cos\end{matrix}\right\}_{n-k\text{ odd}}^{n-k\text{ even}}\{n\omega + jM + kh\} \tag{11}$$

$$F_2' = \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{3\mu^2}{2L^4}\left(\frac{\partial S_1}{\partial M}\right)^2 + \frac{\partial R}{\partial L}\frac{\partial S_1}{\partial M} + \frac{\partial R}{\partial G}\frac{\partial S_1}{\partial \omega} + \frac{\partial R}{\partial H}\frac{\partial S_1}{\partial h}\right]\mathrm{d}M$$

$$= \frac{L^3}{2\mu^2}\sum_{j,n,n',k,k'}\left\{\frac{3}{2L}A_{jnk} + \frac{\partial A_{jnk}}{\partial L} + \frac{n'}{j}\cdot\frac{\partial A_{jnk}}{\partial G} + \frac{k'}{j}\frac{\partial A_{jnk}}{\partial H}\right\}$$

$$\cdot A_{jn'k'}\left\{(-1)^{n'-k'}\begin{matrix}\cos\\ \sin\end{matrix}\right\}_{(n-n')-(k-k')\text{ odd}}^{(n-n')-(k-k')\text{ even}}\{(n-n')\,\omega + (k-k')\,h\} \tag{12}$$

It would be more convenient to express $\underline{F_2'}$ in a form similar to $\underline{F_1'}$ in (10),

$$F_2' = \sum_{g=-6}^{6}\sum_{s=0}^{6} B_{gs}\left\{\begin{matrix}\cos\\ \sin\end{matrix}\right\}_{g-s\text{ odd}}^{g-s\text{ even}}\{g\omega + sh\}, \tag{13}$$

Such a form is particularly effective if $\sin\varphi^*$ in (3) is replaced by 0 and if $\underline{r}^*$ is replaced by $\underline{a}^*$ in these non-linear terms, since then the small coefficients $\underline{B}_{gs}$ will be for practicable purposes constant over the entire lifetime of some satellites, or at least constant over a time short -- say, weeks -- compared to a full cycle of pericenter ω for satellites of large eccentricity and semi-major axis. Working out the algebra of the subscript combinations yields

$$\left.\begin{array}{ll} 1)\ \text{for}\ k' \le k: & k' = k - s,\ n' = n - g \\ 2)\ \text{for}\ k' > k: & k' = k + s,\ n' = n + g \end{array}\right\} \tag{14}$$

So

$$\begin{aligned} B_{gs} = \frac{L^3}{2\mu^2} \sum_{j,n,k} \Bigg[& \left\{ \frac{3}{2L} A_{jnk} + \frac{\partial A_{jnk}}{\partial L} \right\} \{ A_{j(n-g)(k-s)} \\ & + (1 - \delta_{ok}) A_{j(n+g)(k+s)} \} + \frac{\partial A_{jnk}}{\partial G} \left\{ \frac{n-g}{j} A_{j(n-g)(k-s)} \right. \\ & \left. + \frac{n+g}{j} (1 - \delta_{ok}) A_{j(n+g)(k+s)} \right\} + \frac{\partial A_{jnk}}{\partial H} \left\{ \frac{k-s}{j} A_{j(n-g)(k-s)} \right. \\ & \left. + \frac{k+s}{j} (1 - \delta_{ok}) A_{j(n+g)(k+s)} \right\} \Bigg] \left\{ \begin{array}{ll} 1 & g-s\ \text{even} \\ (-1)^{n-k+1} & g-s\ \text{odd} \end{array} \right. \end{aligned} \tag{15}$$

Although the Delaunay elements are more convenient to derive the contribution to the long-term variations of the orbit from mutual interaction of short period terms, for purposes of integration of the intermediary the use of a disturbing function

$$R' = F_1' + F_2' \tag{16}$$

in the Lagrangian planetary equations is equally valid. The form of the complete disturbing function $\underline{R}$ in Eq. (1) is such that a representation in Keplerian elements is more convenient. Using (6) and the Kepler law $\mu = n^2 a^3$,

$$\frac{\partial(a, e, i)}{\partial(L, G, H)} = \begin{Bmatrix} 2/na, & 0 & 0 \\ (1-e^2)/na^2 e, & -\sqrt{1-e^2}/na^2 e & 0 \\ 0 & \cot i/na^2\sqrt{1-e^2} & -1/na^2\sqrt{1-e^2}\sin i \end{Bmatrix} \tag{17}$$

whence, also substituting from (6),

$$\begin{aligned} B_{gs} = \frac{1}{2n} \sum_{j,n,k} \Bigg[& \left\{ \frac{3}{2na^2} A_{jnk} + \frac{2}{na} \cdot \frac{\partial A_{jnk}}{\partial a} + \frac{\sqrt{1-e^2}}{na^2 e} \left(\sqrt{1-e^2} - \frac{n-g}{j} \right) \frac{\partial A_{jnk}}{\partial e} \right. \\ & \left. + \frac{1}{jna^2\sqrt{1-e^2}\sin i} ([n-g]\cos i - [k-s]) \frac{\partial A_{jnk}}{\partial i} \right\} A_{j(n-g)(k-s)} \\ & + \{1-\delta_{ok}\} \left\{ \frac{3}{2na^2} A_{jnk} + \frac{2}{na} \frac{\partial A_{jnk}}{\partial a} + \frac{\sqrt{1-e^2}}{na^2 e} \left(\sqrt{1-e^2} - \frac{n+g}{j} \right) \frac{\partial A_{jnk}}{\partial e} \right. \\ & \left. + \frac{1}{jna^2\sqrt{1-e^2}\sin i} ([n+g]\cos i - [k+s]) \frac{\partial A_{jnk}}{\partial i} \right\} A_{j(n+g)(k+s)} \Bigg] \\ & \cdot \begin{bmatrix} 1 \\ (-1)^{n-k+1} \end{bmatrix}_{g-s\ \mathrm{odd}}^{g-s\ \mathrm{even}} \end{aligned} \tag{18}$$

In (15) and (18), the $\underline{j}$ summation is from 1 to ∞, while the other limits are:

$$\begin{gathered} \max(-3, -3-g) \le n \le \min(3, 3+g) \\ -3 \le k \le 3. \end{gathered} \tag{19}$$

The equations of motion applicable to the intermediary are (see, e.g., KAULA, 1966b, p. 29):

$$\left.\begin{aligned}
\frac{da}{dt} &= \frac{2}{na}\cdot\frac{\partial R}{\partial M}\\
\frac{de}{dt} &= \frac{1-e^2}{na^2 e}\frac{\partial R}{\partial M} - \frac{(1-e^2)^{\frac{1}{2}}}{na^2 e}\frac{\partial R}{\partial\omega}\\
\frac{di}{dt} &= \frac{\cos i}{na^2(1-e^2)^{\frac{1}{2}}\sin i}\frac{\partial R}{d\omega} - \frac{1}{na^2(1-e^2)^{\frac{1}{2}}\sin i}\frac{\partial R}{\partial\Omega}\\
\frac{dM}{dt} &= n - \frac{1-e^2}{na^2 e}\frac{\partial R}{\partial e} - \frac{2}{na}\frac{\partial R}{\partial a}\\
\frac{d\omega}{dt} &= -\frac{\cos i}{na^2(1-e^2)^{\frac{1}{2}}\sin i}\cdot\frac{\partial R}{\partial i} + \frac{(1-e^2)^{\frac{1}{2}}}{na^2 e}\frac{\partial R}{\partial e}\\
\frac{d\Omega}{dt} &= \frac{1}{na^2(1-e^2)^{\frac{1}{2}}\sin i}\frac{\partial R}{\partial i},
\end{aligned}\right\} \qquad (20)$$

where, from (16), (10), and (13),

$$\left.\begin{aligned}
\frac{\partial R}{\partial\{a,e,i\}} &= \sum_{g,s}\left\{\frac{\partial A_{ogs}}{\partial\{a,e,i\}} + \frac{\partial B_{gs}}{\partial\{a,e,i\}}\right\}\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}^{g-s\text{ even}}_{g-s\text{ odd}}\{g\omega + s(\Omega-\lambda^*)\}\\
\frac{\partial R}{\partial\{\omega,\Omega\}} &= \sum_{g,s}\{g,s\}\{A_{ogs}+B_{gs}\}\begin{Bmatrix}-\sin\\ \cos\end{Bmatrix}^{g-s\text{ even}}_{g-s\text{ odd}}\{g\omega + s(\Omega-\lambda^*)\}\\
\frac{\partial R}{\partial M} &= 0
\end{aligned}\right\} \qquad (21)$$

While the evaluation of $\underline{B}_{gs}$ and its derivatives is laborious to program, as mentioned, it is not time-consuming in an orbit calculation, having to be done only once or at wide intervals.

Brown's theory gives the longitude, ℓ and latitude, β of the moon in a geocentric system referred to ecliptic and vernal equinox. To obtain the selenocentric position of the earth referred to equator and Cassini point for use in calculating $\underline{A}_{jnk}$, the signs are reversed and rotations from the vernal equinox to the Cassini point through the longitude of the node Ω_M plus about the Cassini point through the inclination of the ecliptic to the lunar equator, $\underline{I}$, about 1°32′.1:

$$\left.\begin{matrix}\cos\varphi^*\cos\lambda^*\\ \cos\varphi^*\ \sin\lambda^*\\ \sin\varphi^*\end{matrix}\right\} = \mathrm{R}_1(-I)\,\mathrm{R}_3(\Omega_{\mathrm{M}})\left\{\begin{matrix}\cos\beta\cos l\\ -\cos\beta\ \sin l\\ -\ \sin\beta\end{matrix}\right. \tag{22}$$

where

$$\mathrm{R}_1(-I)\mathrm{R}_3(\Omega_{\mathrm{M}}) = \left\{\begin{matrix}\cos\Omega & \sin\Omega & 0\\ -\cos I\sin\Omega & \cos I\cos\Omega & -\sin I\\ -\sin I\sin\Omega & \sin I\cos\Omega & \cos I\end{matrix}\right\} \tag{23}$$

The range of latitude β is $\pm.09$ and of distance $\underline{r}^*$ is $\pm.05\underline{a}^*$. Hence the neglect of these variations in calculating the non-linear coefficients $\underline{B}_{gs}$ should make a difference of about 1% in those coefficients. In the linear coefficients $\underline{A}_{onk}$ these variations are definitely significant and were, of course, taken into account in the calculation of the $\underline{A}_{onk}$.

An extensive comparison of the integrator against a Cowell numerical integration was carried out. In this manner, the integration interval of .04 days was decided.

At the time this analysis was undertaken, it was complementary to those by LORELL (1967) and MICHAEL & TOLSON (1967). The integration and formulation of partial derivatives was analytic, similar to Lorell's, rather than numerical, like Michael's. The data was analyzed in the form of range-rate like Michael's, rather than Kepler elements, like Lorell's.

For the analysis, adequate data were defined as (1) at lease seven days in orbit without disturbance by operation of the attitude jets; (2) no breaks in tracking of more than three days; and (3) close enough to the moon to be significantly perturbed. Satellites for which there were good data for more than about half a month could further be analyzed in terms of either several short arcs (S) of 6 - 9 days or long arcs (L) of 11-21 days. Table 1 summarizes the data.

Table 1 Lunar Satellite Orbits

Group	Semi-Major Axis Lunar Radii	Eccentricity	Inclination deg.	Satellites	Arc Length	No. Arcs	Starting MJD	Ending MJD
1.	1.57	.32	12.	Ia	S	1	39351.0	39359.0
				Ib	S	3	39385.3	39406.9
					L	1		
2.	1.55	.33	17.	IIc1	S	2	39467.7	39482.3
					L	1		
				IIc2	S	2	39510.8	39523.2
					L	1		
3.	1.56	.32	21.	IIIb	S	2	39559.2	39576.0
					L	1		
				IIIc	S	1	39618.4	39625.1
4.	1.13	.04	21.	IIIe1	L	1	39592.0	39607.0
				IIIe2	L	1	39678.0	39689.0
				IIIe3	S	2	39738.5	39754.5
					L	2	39731.0	39773.0
5.	2.16	.52	84.	IVc	S	2	39559.2	39576.0
					L	1		
6.	1.46 - 1.63	.30	85.	Vc	L	1	39760.3	39774.2
				Vd	S	2	39799.4	39817.2
					L	3	39781.0	39844.3

Of the 19 data sets listed in Table 1, 11 were successfully analyzed in SOA-P. Success appeared to be defined roughly as residuals less than about 10^{-5} D^2, where D was the arc length in days. The successful fits were IaS, IbS, IIc2S, IIc2L, IIIbS, IIIbL, IIIe3L, IVcS, IVcL, VdS, VdL. Arcs for which success could not be attained were IIc1, IIIe1, and VcL.

Arcs analyzed in SOA-F were IaS, and a combination of IaS and IIcS. These analyses were carried through spherical harmonic indices ℓ,m of 12,5. However, none of them attained results which could be called reasonable.

At no time was performance on the computer attained smoothly enough to enable a proper data analysis in which different weighting alternatives were tested out, as had been the habit in analyzing earth satellite orbits. Approximately 90% of the time was occupied by what the programmer believed to be program, system, or tape reading difficulties. The principal defect in organizing the analysis was probably the lack of a researcher intermediary between the principal investigator and the programmer. Such a researcher could be intimately involved in the programming & debugging, as had been found necessary to make progress in the earth satellite analyses. However, the large computer programs were too complex for anyone working part time, such as a graduate student, to become familiar with them.

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III INTERPRETATION OF THE LUNAR GRAVITATIONAL FIELD

Analyses of the initial results for the gravitational field suggested that:

(1) the moon is closer to isostatic equilibrium than the earth;

(2) the variations of the gravity field are positively correlated with those of the topography.

The results of MULLER & SJOGREN (1968) showed that (1) is correct, but that (2) is not. Rather, the most pronounced correlation is of gravity highs with topographic lows, the ringed maria. Work of this subject in the last year of the contract was devoted mainly to explaining these "mascons".

1. Observational Data

The striking positive gravity anomalies associated with ringed maria determined by MULLER & SJOGREN (1968) are of undoubted reality, since each 2° interval north-south orbit trace was an independent determination. The anomaly accelerations so estimated are probably smaller in absolute magnitude than the true values, because they are derived from Doppler residuals with respect to a separate set of orbital constants of integration for each trace, which certainly absorbed some of the irregular effects. There is also some damping because of the satellite altitude, and some distortion because of the approximate technique of reduction to uniform altitude. However, it is difficult to imagine how the anomalies could be too big at all, or too small by more than a factor of, say, two.

2. Inferred Excess Mass

In table 1 are summarized the principal data for the five mass concentrations which are the largest. The total excess mass ΔM is calculated by the half-space modification of Gauss' theorem (see, e.g., GARLAND, 1965, p. 137):

$$\Delta M = \frac{1}{2\pi G} \int_{Area} \Delta g \, dA \qquad (1)$$

where G is the gravitational constant, 6.67×10^{-8} cgs.

It is perhaps worthwhile pointing out that the lunar mass concentrations are <u>not</u> extraordinarily large by terrestrial standards. On the earth, there are at least 20 "mascons" bigger that Mare Imbrium, and the largest (Indonesia - Philippines, Kerguelen - Crozet plateaus, Iceland - North Atlantic) are at least seven times as big as Imbrium (KAULA, 1969). This fact re-emphasizes that in the moon we see a muted landscape, as has already been indicated by other criteria such as the magnitude of spherical harmonic coefficients of the lunar gravitational field and the elevation differences of lunar topography compared to those predicted by the "equal stress implication" hypothesis. (KAULA, 1963). Another indicator of mildness is the rms mean anomaly for the 83 $10^\circ \times 10^\circ$ squares in fig. 1: $\pm$ 33 milligals, appreciably less than the $\pm$ 96 milligals predicted by equal-stress scaling from terrestrial data (6 times $\pm$ 16 milligals from KAULA, 1959).

A source of frustration in studying the moon is the lack of good elevation data. But from the analyses available of photography from the earth (BALDWIN, 1963; MEYER & RUFFIN, 1965; MILLS & SUDBURY, 1968; RUNCORN & SHRUBSALL, 1968), it appears highly probable that the ringed maria are topographic lows. In this respect the lunar mass concentrations are strikingly different from terrestrial mass concentrations, which are almost always associated with topographic highs, as shown by KAULA (1969) from data, and as reasoned by UREY (1968) from consideration of magma fractionation.

In the past, it has been suggested that the ringed maria are mass excesses on the basis of several morphological characteristics: the concentric rilles, the "drowning" of craters on the side toward the center, the isostatic adjustment suggested by the low depth-diameter ratio, etc. (O'KEEFE, 1968a; KAULA, 1968, p. 331; RUNCORN & SHRUBSALL, 1968).

We thus can state three problems, the first two of which the moon has in common with the earth:

(i) How was mass transferred to make the maria mass excesses? Was it a) the infalling body which created the maria; or b) some sort of transport on the surface: an erosion and sedimentation; or c) an internal transfer, a flow consequent upon a pressure generated by heat inhomogeneities, as in the earth, or by some more passive means peculiar to the moon?

(ii) How has the mass excess been supported since it was transferred? Has it been essentially passive -- the strength of a lithosphere much thicker than exists on earth -- or has it been dynamic: convection currents, etc.?

(iii) Why are the ringed maria denser? Are there mechanisms aside from those associated with mass transfer which may be significant? This greater density need not exist at the surface, or at any other specific level; it is an average condition applying to the entire vertical section.

3. Impact Formation

The distinct margins of the ringed maria make it evident that the impacting of another body is their principal cause. A difficulty which then immediately presents itself is the distinct ellipticity of the ringed maria Imbrium, Serenetatis, and Crisium, since hypervelocity impacts are essentially explosions with symmetrical consequences. Perhaps this ellipticity is one of the strongest indicators that the formations of these maria were primordial events: 1) low enough impact velocities to allow some shaping by oblique impact would be likely to occur only with the infall of another earth satellite; and 2) only very ancient features would be expected to suffer enough subsequent plastic deformations to change their horizontal dimensions appreciably (cf. ÖPIK, 1961).

Estimating the energy required to form craters the size of the ringed maria is an appreciable extrapolation exercise.

At present, the largest artificial crater is about 900 meters in radius, formed by a 15 MT (6×10^{23} ergs) surface burst (VORTMAN, 1968). VORTMAN obtained results for surface bursts on coral limestone which indicate that radii R scale as higher than the cube-root of the energy and depths D as less:

$$\underset{\text{meters}}{R} = 30.0\ \underset{\text{kilotons}}{E^{0.387}} \tag{2}$$

$$D = 12.5\ E^{0.215} \tag{3}$$

For volume V_c, he obtained:

$$\underset{m^3}{V_c} = 1.11 \times 10^4\ \underset{\text{kilotons}}{E^{1.0}} \tag{4}$$

The 1.0 exponent is surprising; one would expect a slightly lower exponent, due to higher fallback with larger bursts. Neglecting compression and brecciation, and using 2.6 gms cm^{-3} for the bulk density of limestone, we get for a throwout mass T:

$$\underset{\text{grams}}{T} = 2.9 \times 10^{10}\ \underset{\text{kilotons}}{E} \tag{5}$$

For buried explosions, the largest artificial crater is about 186 meters in radius, formed by a 100 kT (4×10^{21} ergs) burst buried 193 meters deep in desert alluvium (CHABAI, 1965). This result agrees rather closely with eq. (2). Although for smaller bursts greater effectiveness is obtained with burial, it does not appear to make so much difference with large nuclear bursts. Buried explosions yield radii which scale at a power of 0.333 or lower. Both VORTMAN and CHABAI reason that for still larger energies cube-root scaling should be approached for both surface and buried bursts. On the other hand, W.N. HESS (personal communication) points out that since the range of throwout is a function of its initial velocity which in turn is dependent on impact velocity, eventually for a large enough burst a significant fraction of throwout will fall back into the crater.

If we assume cube root scaling gives a result equal to eq. (2) at 15 MT, we get:

$$\underset{\text{meters}}{R} = 50.4\ \underset{\text{kilotons}}{E^{0.333}} \tag{6}$$

Converting units, we get:

$$\text{max. } \underset{\text{ergs}}{E} = 3.27 \times 10^{23}\ \underset{\text{km}}{R^{3.00}} \tag{7}$$

$$\text{min. } E = 3.38 \times 10^{23}\ R^{2.58} \tag{8}$$

Or, substituting $MV^2/2$ for E, for the impacting mass M:

$$\text{max. } \underset{\text{grams}}{M} = 6.54 \times 10^{13}\ \underset{\text{km}}{R^{3.00}}\ \underset{\text{km/sec}}{V^{-2}} \tag{9}$$

$$\text{min. } M = 6.76 \times 10^{13}\ R^{2.58}\ V^{-2} \tag{10}$$

In table 2 we give the masses calculated from eqs. (4) - (10) for impact velocities which might be characteristic of

1) other satellites of the earth: 2.5 km/sec ~ $\left(2\ GM_M/R_M\right)^{\frac{1}{2}}$;
2) asteroids: 15 km/sec ~ $\left(GM_S/a_E\right)^{\frac{1}{2}}/2$;
3) parabolic comets: 43 km/sec ~ $\left(2\ GM_S/a_E\right)^{\frac{1}{2}}$.

Using eq. (5) and converting units, we get for the throw-out mass T:

$$\min.\ \underset{\text{gm}}{T} \approx 3.5\ \underset{\text{gm}}{M}\ V^2 \tag{11}$$

This amount will be on the low side for the moon, since eq. (5) applies to a surface burst with an atmosphere and a higher gravitational acceleration g. GAULT et al. (1963) obtain results for M ≤ 1 gm which would be consistent with:

$$T \approx 100\ M\ V^{3.2}$$

There probably should also be an exponent of less than 1.0 on the infall mass M.

4. Age Estimation from Crater Density

A problem which still exists is the relatively young age of the maria inferred from their crater density and the rate of infall of bodies to the earth. Such an age estimate must be based on craters which are too large to reach a saturation

TABLE 1 MASS CONCENTRATIONS AND ASSOCIATED RINGED MARIA

Name		Diameter EW 10^3 km	Diameter NS 10^3 km	Peak Anomaly cm^2/sec	Area of Mare 10^6 km^2	Area of Positive Anomaly 10^6 km^2	Implied Excess Mass Total 10^{18} gm	Implied Excess Mass Per Unit Area 10^3 gm/cm^2
Imbrium	Outer	1.21	1.00	0.23	0.95	0.50	700.	140.
	Inner	0.59	0.60		0.27			
Serenetatis		0.64	0.73	0.18	0.36	0.33	400.	120.
Crisium		0.58	0.39	0.10	0.18	0.17	200.	120.
Nectaris	Inner	0.32	0.30	0.10	0.08	0.20	170.	85.
	Outer	0.72	0.63		0.36			
Humorum		0.36	0.36	0.05	0.10	0.15	65.	43.

TABLE 2 MASSES OF INFALLING BODIES IMPLIED BY RINGED MARIA DIMENSIONS

Name		Mean Radius km	Vel. 2.5 km/sec Min.-Mass 10^{18} gm	Vel. 2.5 km/sec -Max. 10^{18} gm	Vel. 15 km/sec Min.-Mass 10^{18} gm	Vel. 15 km/sec -Max. 10^{18} gm	Vel. 43 km/sec Min.-Mass 10^{18} gm	Vel. 43 km/sec -Max. 10^{18} gm
Imbrium	Outer	550	127.	1740.	3.54	48.4	0.43	5.90
	Inner	300	27.	280.	0.74	7.9	.09	0.96
Serenetatis		340	37.	410.	1.03	11.5	.12	1.40
Crisium		240	15.	140.	0.42	4.0	.05	0.49
Nectaris	Outer	340	37.	410.	1.03	11.5	.12	1.40
	Inner	155	5.	40.	0.11	1.1	.01	0.13
Humorum		180	7.	60.	0.18	1.7	0.02	0.21

density. SHOEMAKER et al. (1962) gave for craters more than 1 mile in diameter:

$$\underset{(10^6\ km^2)^{-1}}{F} = 3.2 \times 10^3\ \underset{km}{R}^{-1.71} \qquad (12)$$

as an average for all maria, which is about the same as for all the ringed maria <u>except</u> Crisium, which is appreciably less. To convert this crater frequency to an age there are needed firstly, a crater-size-energy relationship, and secondly, an estimate of frequency of infalling bodies as a function of size. Modifications of both these items since SHOEMAKER et al. (1962) tend to reduce the age estimates. The higher than 1/3 power scaling suggested by eq. (2) reduces the size of body required to create a given sized crater. And more recent estimates of infall rates into the earth are appreciably higher (SHOEMAKER & LOWRY, 1967; MC CROSKY, 1968). These data apply, of course, to smaller bodies than required to create 1 km craters. MC CROSKY's results give, applying a factor of 0.474 for the smaller capture cross-section of the moon (assuming 10 km/sec approach velocity);

$$\underset{(10^6\ km^2\ 10^9\ year)^{-1}}{f} = 1.2 \times 10^{13}\ \underset{gm}{M}^{-0.62}, \quad M < 2.5 \times 10^6\ gm \qquad (13)$$

Using V = 15 km/sec in eq. (9) to substitute for M:

$$\underset{(10^6\ km^2\ 10^9\ year)^{-1}}{f} = 49.4 \times 10^3\ \underset{km}{R}^{-1.86} \qquad (14)$$

which gives an age of only 65×10^6 years. Using a velocity appropriate for overtaking orbits, as suggested by WETHERILL (1968a), would reduce eq. (14) by a factor of only two. It thus seems likely that (a) the cumulative frequency F in eq. (12) is too low because of obliterating effects; and (b) the exponent in eq. (13) should give a faster drop-off with mass than does -0.62 (obviously the exponent must eventually drop below -1.0, if there is not to be an infinity of total infalling mass).

A radius of crater for which obliteration by subsequent infalls should be negligible is 5 km (KAULA, 1968, p. 316). With the overtaking velocity of 9 km/sec in eq. (9), this corresponds to a mass of 10^{14} gms. If eq. (12) is the accumulation for 4.5×10^9 years, then the frequency for $M \geq 10^{14}$ gms would be $46(10^6\ \text{km}^2\ 10^9\ \text{yr})^{-1}$. An exponential law connecting this value to a value agreeing with eq. (13) for a mass of 2.5×10^6 gms is:

$$\underset{(10^6\ \text{km}^2\ 10^9\ \text{year})^{-1}}{f} = 2.71 \times 10^{17}\ \underset{\text{gm}}{M}^{-1.13},\quad M > 2.5 \times 10^6\ \text{gm} \tag{15}$$

which is not at all implausible.

5. Energy Sources affecting Maria

Under this heading are included both the infalling bodies and the conditions which lead to heating in the outer parts of the early moon and hence promote isostatic adjustment, outgassing of water, fractionation, and, possibly, lava flow.

Asteroids, or planetesimals, are generally favored as the infalling bodies because of their greater density and their apparently more common occurrence in the inner solar system. However, comets are not to be ruled out. The masses of 10^{16} to 6×10^{18} grams listed as corresponding to cometary velocity in table 2 correspond to the range of masses estimated for Halley's comet (WHIPPLE, 1963). An objection to comets is the small capture radius of the moon for such rapidly moving bodies. But if comets are necessary to slice off the earth's atmosphere to make tektites, then we have had two 2×10^{30} erg, or 10^{17} - 10^{18} gram, events in the last million years, and four in the last 34 million (KAULA, 1968, pp. 388 - 390). We have no notion of comet frequency in the early solar system, other than it is likely that they were much more abundant. KOPAL (1965) suggested that comets created the maria because they would constitute a diffuse source of high energy per gram resulting more in melting of a broad layer than in cratering and throwout. The lack of tensile strength in comets is not of significance in cratering, since for hypervelocity impacts all materials are of negligible strength (BJORK, 1961). The effect of the lower density would be to make the velocity stagnation point shallower, resulting in a higher diameter/depth ratio.

If the principal source of internal energy in the moon has been the usual assumption of chondritic U, Th, K content with initial uniform distribution, then the temperature gradient in the outer 200 kilometers is about 5°K/km (LEE, 1968). With this model, it is unlikely that even the biggest mare-creating impacting asteroid could punch its way deep enough to reach material at melting temperature in the manner suggested by RONCA (1966). We need to obtain some sort of non-uniformity of heating in the formation of the moon, both laterally and radially. The obvious source is the gravitational energy released in the final stages of infall of planetesimals, as suggested by RINGWOOD (1966). In this manner could be obtained the differentiation needed to account for the alpha-scattering data as well as to enable the mass excesses to coincide with topographic deficiencies. A confinement of heating to the outer layers would also enable cooling in time for the maria subsequently to sustain appreciable departures from isostatic equilibrium.

6. Gravity Anomaly-Mass Relationships and Isostasy

A feature of diameter L on a sphere of radius a will have a spherical harmonic expression whose principal term is of degree $\ell = a/(2\pi L)$. For the amplitude δV_ℓ of the potential perturbation at radius r arising from a surface density distribution of degree ℓ and amplitude σ_ℓ at depth D, from KAULA (1968, p. 67):

$$\delta V_\ell = 4\pi G \frac{(a-D)^{\ell+2}}{r^{\ell+1}} \cdot \frac{\sigma_\ell}{2\ell+1} \qquad (16)$$

where G is the gravitational constant 6.67×10^{-8} cgs. For the gravity anomaly Δg we take the derivative and set $r = a$:

$$\Delta g_\ell = 4\pi G \frac{\ell+1}{2\ell+1}\left(\frac{a-D}{a}\right)^{\ell+2} \sigma_\ell \qquad (17)$$

(The Bruns term $2\delta V_\ell/r$ is neglected, since in this problem heights above an equipotential were not used). Assuming $L \ll a$, $D \ll a$, we can write:

$$\Delta g_\ell \approx 2\pi G\left[1 - (\ell+2)\frac{D}{a}\right]\sigma_\ell \qquad (18)$$

If a surface distribution at radius r_1 is isostatically compensated at radius r_2, then their respective contributions to hydrostatic pressure are equal:

$$g(r_1)\ \sigma_\ell(r_1) = g(r_2)\ \sigma_\ell(r_2) \qquad (19)$$

In a homogeneous moon, $g(r) \sim r$; the increase in mass of the compensation over the surface matter would thus be small compared to the wavelength effect $(\ell+2)D/a$ in eq. (18), and hence we can write that for the anomaly arising from an isostatically compensated layer σ_ℓ:

$$\Delta g_\ell \approx 2\pi G \ell \frac{D}{a} \sigma_\ell \qquad (20)$$

Or, substituting $a/(2\pi L)$ for ℓ, $\Delta\rho h$ for σ_ℓ, and numbers for the constants, from eq. (18)

$$\text{uncompensated } \Delta g \approx .04 \, \Delta\rho \, h \qquad (21)$$
$$\text{cm/sec}^2 \quad (\text{gm cm}^{-3}) \text{km}$$

$$\text{compensated } \Delta g \approx .02 \frac{D}{L} \Delta\rho \, h \qquad (22)$$
$$\text{cm/sec}^2 \quad (\text{gm cm}^{-3}) \text{km}$$

Hence if isostasy has prevailed, either the excess mass ΔM must be several times that calculated from eq. (1), or else the compensation must be at a depth which is more than the width of the feature, which is physically implausible.

The relaxation time t_r for a feature of width L and density $\Delta\rho$ on a half space of viscosity η is (MC KENZIE, 1967):

$$t_r = \frac{2\eta}{\Delta\rho g L} \qquad (23)$$

For a crater created by a mass M of density ρ_M in a crust of density ρ_c, we can put roughly:

$$\Delta\rho = \rho_c - \left(\rho_M - \rho_c\right)\frac{\rho_c}{\rho_M} \cdot \frac{M}{T} \qquad (24)$$

where M and T are, respectively, impacting and throwout mass, related by eq. (1). Substituting numbers and 2R = L, we get:

$$t_r \approx 0.6 \times 10^{-24} \left[1 + .3\frac{\delta\rho}{\rho_M} V^{-2}\right] \cdot \eta \cdot R^{-1} \qquad (25)$$
$$10^9 \text{ yr} \qquad \text{km/sec(gm/cm/sec) km}$$

where $\delta\rho = \rho_M - \rho_c$.

If we assume a form for the temperature T and pressure P dependence of viscosity similar to that suggested by diffusion creep theory:

$$\eta = a\,T\,\exp\left(\frac{b + cP}{T}\right) \tag{26}$$

and fit it to viscosity values inferred from post glacial uplift (MC KENZIE, 1967), we get

$$\left.\begin{aligned} a &= 1.78 \times 10^{-22} \\ b &= 1.10 \times 10^{5} \\ c &= 2.47 \times 10^{2} \end{aligned}\right\} \tag{27}$$

for T in °K and P in kbar. Using (270 + 5 D)°K (LEE, 1968) for T and .05 D for P, we get on the moon

$$\eta \approx [1 + .0185\,D]\,10^{\left\{-19.3 + \frac{1}{.0056 + .0001\,D}\right\}}, \quad D < 300 \text{ km} \tag{28}$$

where D is depth in kilometers. According to eq. (28), the viscosity of 3×10^{27} needed to cause appreciable relaxation of a 500 km radius depression in 4.0×10^{9} years is reached at a depth of about 150 km. Hence we should expect the ringed maria to show signs of isostatic adjustment, but not craters of less than, say, 100 km diameter, _if_ (i) lunar materials have a temperature dependence of viscosity similar to earth materials; (ii) the moon has a U, Th, K content not significantly less than chondritic; (iii) the moon started cold.

7. Related Phenomena

As mentioned, the lunar mass concentrations raise problems of mass transfer, mass support, and densification. Satisfying solutions for these problems would also explain or take into account:

(i) The low mean density of the moon, 3.34 gms/cm^3, suggesting that, relative to the earth, the moon has either retained oxides and volatiles, or it has lost iron. Volatile and oxide retention in turn implies a capture origin (UREY, 1963), while loss of iron implies a fission origin (RINGWOOD, 1966). Combination of these hypotheses is possible, of course. A pure fission origin now seems unlikely on the dynamical grounds that backward integration of the moon's orbit under any plausible tidal friction uniformly distributed over the earth always yields a definite inclination with respect to the earth's equator (GOLDREICH, 1966).

(ii) The probable near homogeneity of the moon. The most recent analysis of Lunar Orbiter tracking data for the gravitational harmonics obtain a J_{20}, $[C - \frac{1}{2}(A+B)]/MR^2$, of 2.071×10^{-4} and a J_{22}, $\frac{1}{4}(B-A)/MR^2$, of 0.224×10^{-4} (MICHAEL <u>et al.</u>, 1970). These values used with the $(C-A)/C$ of 6.29×10^{-4} yield a C/MR^2 negligibly different from homogeneous, 0.40.

(iii) The differing crater density of the maria and uplands. Obviously the maria could have been formed in the declining stages of an early period of intense infall of planetesimals following soon after the formation of the moon itself. But it is not to be ruled out that processes long after the initial formation of the maria have obliterated many craters.

(iv) The smoothness of the maria: fragmentation, water, or low-viscosity lava?

(v) The occurrence of many sinuous rilles near the margins of circular maria (PEALE et al., 1968; SCHUBERT et al., 1970). The sinuous rilles are, of course, the principal evidence of water near the surface of the moon. Surface water in the moons' early history has previously been suggested to account for the smooth maria surface as well as the rilles (GILVARRY, 1964; UREY, 1967). Some sinuous rilles have indications of being relatively new, and hence a satisfactory hypothesis for the mare should explain the recent localization of sub-surface water near their margins.

(vi) The elemental abundances estimated by the alpha-scattering experiments on Surveyors V - VII (TURKEVICH et al., 1968). The lower Mg and higher Aℓ and Si abundances indicate fractionation with respect to chondritic meteorites. The high abundances of metals with refractory oxides, Aℓ and (probably) Ca, further make it difficult to accept a fission origin of the moon, (as suggested by RINGWOOD, 1966), and indicate a magmatic differentiation similar to that forming basalt. To suggest that the alpha-scattering results are affected mainly by achondritic meteorite material from elsewhere is to push this differentiation off to a smaller body where it is even harder to explain.

(vii) The low adhesion of lunar surface material to the Surveyor magnets; suggesting some magnetite, Fe_3O_4, but no free iron (DE WYS, 1968).

(viii) The low depth/diameter ratio of the maria, suggesting either isostatic uplift since the original formation of the crater, or filling in with other material.

(ix) The low electrical conductivity of a major portion of the moon, inferred from its slight effect on transients in the interplanetary magnetic field (NESS, 1969). This low conductivity in turn implies temperatures less than 800°C.

(x) The absence of strike-slip faulting or other indicators of horizontal motion on the moon.

(xi) The thermal enhancement of 5° to 10°C in most ringed maria found by post-eclipse and post-sunset infra-red observations (SAARI & SHORTHILL, 1967). The ringed maria do not, however, contain an exceptionally large number of "hot spots". This phenomenon indicates that the thermal conductivity of the maria surface is higher than on other areas undisturbed by recent large meteorite impact.

8. Mass Transfer Mechanisms

To create mass excesses, at least one of three types of transfer must have occurred: an infall of a body from outside the moon; or a lateral transfer of matter on the surface of the moon; or a lateral transfer of matter inside the moon. Because the mass excesses coincide with topographic lows, all of these mechanisms also require a chemical differentiation, either outside or inside the moon.

(i) Denser infalling body (MULLER & SJOGREN, 1968; UREY, 1968, UREY & MAC DONALD, 1971). In this hypothesis, the chemical differentiation took place outside the moon: the infalling body must have lost volatiles or silicates relative to the moon. As indicated by comparing the masses in tables 1 and 2, the body must have hit at relatively low velocity, in turn implying that it was also an earth satellite. Furthermore, the throwout mass given by eq. (11) must necessarily be wrong; UREY & MAC DONALD (1971) emphasize the lack of evidence of such large scale throwout. But this hypothesis must be considered speculative until the detailed calculations for such large impacts are thought through and carried out.

Finally, it would be surprising if the moon were not hit by some bodies of velocity about 10 km/sec and size large enough to make the ringed maria. Mare Orientale appears most likely to have been caused by such an impact, but even it is a moderate mass concentration (MULLER & SJOGREN, 1968).

(ii) Lateral transfer on the surface: isostatic adjustment followed by sedimentation (GILVARRY, 1969). In this hypothesis, the chemical differentiation required is the prior separation of a lunar crust of lower density. Then if the pre-mare crater was isostatically compensated by the upthrust of a dense root and froze because of cooling off, any subsequent sedimentation would be an excess load. This model is inconsistent if an H_2O atmosphere is invoked for the sedimentation, since temperatures high enough in the outer parts to promote sufficient outgassing are needed <u>after</u> temperatures have become low enough at greater depths to prevent creep. The notion of an atmosphere of longer decay time than the cooling time of a lithosphere is hard to accept.

The slower "sedimentation" mechanisms of meteorite bombardment plus possibly electrostatic effects (GOLD, 1966) seem more plausible. However, the calculations of ROSS (1968) indicate that this mechanism is inadequate to account for the 1 km or so layer required to account for the peak anomaly of 0.2 cm/sec^2, by modification of eq. (17):

$$h \approx \frac{\Delta g}{2\pi G\rho} \tag{29}$$

We need better estimates of how secondary fragments from meteorite impacts are redistributed relative to the topography over the eons.

(iii) <u>Internal lateral transfer: dynamic</u>. This process is the manner in which major mass excesses are created on earth: thermal or chemical inhomogeneities generate pressure excesses which force up the mass excesses. The process is very active, as evidenced by the strong correlation of major mass concentrations with recent geology (KAULA, 1969). For this reason, it is certainly of lesser importance on the moon, perhaps negligible. If the moon were hot enough internally for convection currents, etc. to occur, then it would be surprising to find the major mass excesses correlated so closely with ancient features of such obviously external cause as the ringed maria, as well as not to find any evidence of horizontal motion on the moon's surface.

(iv) Internal lateral transfer: passive. If a lighter crust has been differentiated, then the creation of a crater will result in the upthrusting of a denser root by simple isostasy. The problem then is to find a mechanism by which to generate an extra pressure to force up the additional mass required by the gravimetry. WISE & YATES (1970) show how the existence of higher terrain around an isostatically compensated crater could result in the forcing up of a mass excess over the crater provided that a) there existed structural weaknesses around the crater margin permitting magma flow (very plausible); and b) the crust was thick enough that the magma forced up was of crustal density, so that there was a drop of the center of mass of the entire section involved. This hypothesis requires a rather thick crust of more than 40 km in order to account for the major mass concentrations, as well as negative anomalies around each mass excess. Such anomalies appear to exist in the data, but the manner of inferring the gravity anomalies as residuals to a best fitting orbit makes it possible that the negatives north and south of each mass concentration are not real. A perhaps more serious objection is that the bulk of the lithosphere has to be strong enough to support the mass excesses (and hence of less than 800°C temperature) at the same time as there exist hot spots (of more than 1100°C) in the vicinity sufficient to create the magma.

We therefore would like to find a means of creating additional pressure. If the outer parts of the moon were made hotter during the process of formation than the interior, due to greater energy of infalling matter, then subsequently the outer parts would cool much more rapidly. The consequent thermal contraction would exert appreciable pressure on the interior. It would also result in considerable tension in the outer shell, so that the obvious objection is that the shell would crack to relieve the pressure on a much more local scale than the mass concentrations. But the process is extremely effective, so much so that only a very minor portion is needed to account for the mass concentrations. Assuming negligible compressibility, we have

$$\Delta M = 4\pi \; R^3 \; \alpha \; \rho \; \delta T \qquad (30)$$

where ΔM is the total mass excess, R is the moon's radius, α is the coefficient of thermal expansion, and δT is the difference in temperature drop of the interior and the shell. Using 1.5×10^{21} grams for ΔM (from table 1), and 2×10^{-5} for α, we get $0.35°C$, or only 1/1000 of the differential temperature drop which could very plausibly have occurred.

Hence if the relief by cracking of pressure arising from thermal contraction is only 99.9% efficient, then there could easily remain sufficient residual pressure to force up the dense roots below the ringed maria far enough beyond isostatic equilibrium to create the mass concentrations. The actual process, involving materials of viscosity in excess of 10^{22} poises, would differ appreciably from that of a fluid under pressure in a rigid container which has been weakened at a few points, but the principle is similar. Localized temperature differences or extrusive activity are not necessary features of the process; lava flows would help, though, to lower the temperature enough to facilitate subsequent support by elastic strength.

PEALE & LINGENFELTER (personal communication) have recently suggested that the mass concentrations were pulled up, rather than pushed up, by the attraction of the earth after the moon had been slowed down to synchronous rotation. For tide height h we have

$$\begin{aligned} h &\approx U_2/g \\ &\approx [GM\, R_2/r^3]/[Gm^*/R^2], \end{aligned} \tag{31}$$

whence for the earth-moon distance

$$r \approx \left[\frac{MR^4}{m^* h}\right]^{1/3} \tag{32}$$

A height h of 1 km should be sufficient (more than twice the maximum mass per unit area in table 1, divided by 3.0), which yields about 14 earth radii for r by eq. (32). (As any student of tides should realize, this mechanism does <u>not</u> give any preference for mass concentrations on the side toward the earth).

9. Mass Support Mechanisms

To support the mass concentrations in the 4.5×10^9 years or so since their creation, there exist essentially only two mechanisms: passive elastic strength, requiring a lithosphere able to sustain about 50 bars shear at 100 km depth (CONEL & HOLSTROM, 1968), or dynamic convection currents, requiring sufficient temperature inhomogeneities to generate comparable viscous stresses. As discussed in Sec. 6, the thermal regime of the moon seems to allow a lithosphere on the order of 150 km thick, while, as discussed in Sec. 8, there is a lack of evidence of any dynamic processes anywhere nearly as closely correlated with surface features as the mass concentrations.

10. Densification Mechanisms

Aside from the processes of either infall of a denser body or upthrust of a denser root necessarily associated with the mass transfer mechanisms discussed in Sec. 8, there are several possible ways in which the mass concentrations may have become denser without the influx of additional matter. They are worth examining as additional considerations pertaining to the early history of the moon.

(i) Magmatic fractionation and lava flow (BALDWIN, 1968; O'KEEFE, 1968b). The impact of a large body has a heating effect and creates structural weaknesses so that the maria become likely sites for lava flow, either at the time of impact or subsequently.

On the earth, areas of lava flow are mass excesses only if they contain activity in recent geologic time -- say the last 5×10^6 years, and are almost always associated with topographic excesses (KAULA, 1969), contrary to what seems to be the case for the lunar ringed maria, as previously mentioned. Hence it is even more necessary on the moon than on the earth that the lava be denser than other surface rocks because it is drawn from deeper rock.

The oxygen/metal ratios obtained by the Surveyor alpha-scattering measurements (see table 3) indicate that the normal lunar surface may be denser than basalt. To be yet denser, the ringed maria material would have to be either more basic or more ferrous than terrestrial lavas. A possible alternative is that the oxygen abundances obtained by alpha-scattering are systematically low, as suggested by the apparent absence of free iron indicated by the Surveyor magnet experiment (DE WYS, 1968). If the high $A\ell$, Si, and Ca abundances are correct, then the oxygen/metal ratio will be about the same as average basalt, even assuming zero carbon and hydrogen content (see table 3).

(ii) Devolatilization of hydrated minerals. The hydrous mineral serpentine when heated to about 500°C breaks down, allowing water to escape (BOWEN & TUTTLE, 1949):

$$\underset{\text{Serpentine}}{2Mg_3Si_2O_5(OH)_4} \rightarrow 3Mg_2SiO_4 + SiO_2 + 4H_2O$$

$$\rightarrow \underset{\text{Forsterite}}{2Mg_2SiO_4} + \underset{\text{Enstatite}}{2MgSiO_3} + \underset{\text{Water}}{4H_2O} \qquad (33)$$

TABLE 3: ATOMIC ABUNDANCE RATIOS AND DENSITIES

Material		Oxygen/Metal	Fe/(Fe+Mg)	Density g/cm^3
Minerals				
Magnetite	Fe_3O_4	1.33	1.0	5.2
Olivine: Forsterite	Mg_2SiO_4	1.33	0.0	3.22
Olivine: Fayalite	Fe_2SiO_4	1.33	1.0	4.39
Hematite	Fe_2O_3	1.50	1.0	5.2
Pyroxene: Diopside	$CaMgSi_2O_6$	1.50	0.0	3.22
Pyroxene: Enstatite	$MgSiO_3$	1.50	0.0	3.21
Pyroxene: Ferrosilite	$FeSiO_3$	1.50	1.0	3.96
Feldspar: Anorthite	$CaA\ell_2Si_2O_8$	1.60	---	2.76
Feldspar: Albite	$NaA\ell Si_3O_8$	1.60	---	2.63
Serpentine	$Mg_3Si_2O_5(OH)_4$	1.80	0.0	2.6
Goethite	$FeO(OH)$	2.00	1.0	4.3
Quartz	SiO_2	2.00	0.0	2.62
Rocks				
Dunite	mainly Olivine	~ 1.33	.09	3.25
Peridotite	Olivine & Pyroxene	~ 1.39	.15	3.2
Lunar Surface, Alpha Scattering:	Observed	< 1.35	~ 0.7	
		< 1.28	~ 0.6	
		< 1.38	~ 0.3	
	Adjusted	1.49	~ 0.6	
		1.55	~ 0.5	
		1.50	~ 0.25	
Ca-rich achondrite, Alpha Scattering		1.51	0.50	3.0
Conventional		1.52	0.45	
Basalt, Alpha-Scattering		1.50	0.63	2.8
Conventional		1.57	0.52	
Gabbro		1.55	0.39	3.0

Serpentine has a density of about 2.6 gm cm^{-3}, while a mixture of forsterite and enstatite would have a density of about 3.2 gm cm^{-3}. On the earth the more common interaction is to form talc ($Mg_3Si_4O_{10}(OH)_2$), which is stable up to about 800°C. In any case, if a large impact occurred leading to extensive devolatilization of hydrated rocks, an appreciable densification would be effected.

It may be objected that the dehydrated impact products would have a low density because of brecciation, as occurs in terrestrial craters (BEALS et al., 1963). But in a crater as large as a ringed maria this would remain true only for the uppermost layer; the voids would be compressed out of most of the material. A brecciated surface layer would react to the isostatic uplifting by considerable crumbling and sliding about of the fragmented material, obliterating many small craters and thus perhaps accounting for the smooth appearance of the maria. The concentration of sinuous rilles around the margins of ringed maria (PEALE et al., 1968; SCHUBERT et al., 1970) also suggests that the main parts of the maria have been thoroughly dehydrated, while the margins are an intermediate zone in which release of water is promoted by heat flow raising the permafrost level; fractures in the rocks; lateral flow of water caused by the higher pressure under the uplands; and closer approach of the permafrost to the surface because of the lower topographic elevation. The loss of water by the maria may even lead to a temporary atmosphere with resulting sedimentation to smooth the surface, on a much more modest scale than proposed by

GILVARRY (1964) (who implicitly assumes that the outgassing of the moon has been as efficient as that of the earth).

X-ray analyses of type I and type II carbonaceous chondrites indicates that they are in large part hydrated layer-lattice silicates similar to serpentine or chlorite $[(Mg,Al,Fe)_{12}(Si,Al)_8O_{20}(OH)_{18}]$ (DUFRESNE & ANDERS, 1963; MASON, 1962). It is tempting to try to construct a lunar surface out of carbonaceous chondrite less hydrocarbons. But as indicated in table 4, a subsequent fractionation is needed to get rid of the excess magnesium and iron. Such a fractionation could result in a "mantle" largely of olivine, iron, plus perhaps some pyroxene. See WETHERILL (1968b) for further discussion.

In any case, any significant hydration of lunar surface materials requires an appreciably higher oxygen/metal ratio than indicated by the Surveyor alpha-scattering results.

(iii) Throwout or volatilization of lighter silicates. If the moon already had a differentiated crust significantly lighter than its interior, then the bulk of the material thrown out upon the formation of the pre-mare crater would be appreciably less dense than the material which rose in its place upon isostatic adjustment.

Volatilization requires temperatures in excess of 1500°K, and hence seems unlikely to be significant.

(iv) Reduction of lunar material by the infalling body. (WASSON, personal communication). If the infalling body contained appreciable hydrocarbons -- such as type I carbonaceous chondrites -- then there could be reactions with lunar surface

TABLE 4: HYPOTHETICAL ATOMIC ABUNDANCES

Derived from Type I Carbonaceous Chondrites

	Na	Al	Ca	S	Mg	Ni	Fe	Si
Carbonaceous Chondrite I	.13	.09	.07	.50	1.00	.04	.88	1.00
Volatilized	.11			.49				
"Crust" ≈ Lunar Surface by Alpha Scattering	.02	.09	.07	.01	.04		.05	.24
"Mantle" ≈ 90% Olivine + 10% Ni-Fe & Iron Oxide					.96	.04	.83	.76

OR

	Na	Al	Ca	S	Mg	Ni	Fe	Si
Volatilized	.11			.495				
"Crust" ≈ Alpha Scattering	.01	.045	.035	.005	.02		.025	.12
"Mantle" ≈ 5% Plagioclase + 30% Pyroxene + 40% Olivine + 25% Ni-Fe & Iron Oxide	.01	.045	.035		.98	.04	.855	.88

material of the type:

$$\frac{1}{4}Fe_3O_4 + H_2 \rightarrow \frac{3}{4}Fe + H_2O \qquad (34)$$

requiring on the order of 1000°K (KAULA, 1968, p. 430). Similar reactions for other oxides require appreciably higher temperatures, and hence the iron abundance in the lunar surface would have to be a lot higher than indicated by the alpha-scattering experiment for this mechanism to be an important way of increasing density.

(v) Phase transitions by shock compression. This mechanism was suggested by GOLD (1966) to account for the radar reflection properties of Tycho. A shock of 100 kb is required to convert silica into stishovite, density 4.3 gm cm^{-3}, while shock pressures in the range 110 - 200 kb are required to convert enstatite and forsterite into periclase, spinel, and stishovite mixtures of about 10 percent greater density (KAULA, 1968, p. 21). The mass shocked to this pressure in numerical calculations for a 5×10^{22} erg event by BJORK (1961) was about 10^{11} grams. Scaling proportionate to energy obtains masses about the same as those given in table 1 -- or anomalous masses only one tenth as much.

(vi) Thermal contraction. For a temperature anomaly ΔT there will be a density anomaly $\Delta\rho$:

$$\Delta\rho = -\rho\alpha\Delta T \qquad (35)$$

where α is the coefficient of thermal expansion, about 2×10^{-5} $^{\circ}C^{-1}$ for silicates. There will be some down-bending of isotherms under maria, resulting from their lower elevation. But such an effect could hardly be more than $5^{\circ}C$, or 3×10^{-4} gm/cm^3, and would be counteracted at a relatively shallow depth _if_ the maria have a thicker layer of fragmented material, as suggested above in Sec. 8 (ii). However, _if_ the higher thermal conductivity of the maria suggested by the infra-red observations persists to an appreciable depth, then there could be some significant effects on the density.

11. Density Reduction Mechanisms

We must not overlook the possibility that the ringed maria are denser because they lack something which would make the rest of the moon less dense. Significant reversals of the processes in Sec. 10 which appear to be possible are:

(i) Hydration of minerals. The locations of sinuous rilles (SCHUBERT _et al._, 1970) indicates the terrain below the ringed maria to be dehydrated, and the continued presence of water elsewhere. Above the $500^{\circ}C$ isotherm, there could be appreciable occurrence of serpentinization, the reverse of eq. (33). This process has more limited occurrence on earth (HESS, 1955), apparently because the earth has been more completely outgassed than we would expect the moon to be. Serpentinization (or other processes occuring if CO_2 is also present: see GREENWOOD, 1967) below the permafrost would not be inconsistent with the low oxygen/metal ratio at the surface indicated by the alpha-scattering measurements. Hydration of iron oxides is also a possibility.

A counter-indicator to any significant interstitial water in the moon is the low electrical conductivity inferred from the magnetometry (O'KEEFE, personal communication).

(ii) Thermal expansion. The presence of a thicker insulating layer, as suggested by the infra-red eclipse observations, as well as a higher proportion of thé radiogenic heat sources associated with the lighter silicates as a consequence of the fractionation process, could combine to make the uplands appreciably warmer than the ringed maria. But it seems difficult to attain the sharp localization of the mass excesses in this way

12. Discussion and Conclusions

The mass concentrations, like the alpha scattering abundances, indicated a differentiated surface layer on the moon: it is easier to suggest mechanisms to create the mass excesses if there can be lateral variations in the degree of fractionation; isostatic compensation of surface depressions by denser subcrustal material; etc. The possibility of creating crustal material in a relatively short time is confirmed by the ancient age of much of the earth's crust and by the existence of achondritic meteorites.

Such an early differentiation of a lunar crust requires a heat source near the surface early in the moon's history:

either the energy of infalling bodies, or an hyperactive early sun, or an extremely large atmosphere (UREY, 1963). Of these, the energy of infalling bodies is the simplest. If the moon was accreting, rather than losing, mass, then a major fraction of the impact velocities must have been low enough that the proportion of throwout completely lost does not exceed the infalling mass: i.e., the major part of the moon must collect out of materials at "satellite" velocities on the order of 2 km/sec. A dense atmosphere would greatly facilitate this mass accretion, while an opaque atmosphere is essential for the retention of the heat generated by the infalling meteorites. Using a specific heat of 0.25 cal/$^{\circ}$C/gm and a heat of fusion of 100 cal/gm, and assuming as much as 50 percent of the energy going into heating, the infall velocity would have to exceed 2 km/sec to raise the temperature of the impacting body alone to melting. Hence it is likely the melting and consequent fractionation was mild and irregularly distributed. The presence of the sinuous rilles indicates that it was indeed mild enough to allow the retention of water in places, which in turn suggests that serpentinization may be significant.

The bodies which caused the mass concentrations came later than those which formed most of the moon's surface, and hence could very well have come from a different place, and hence at much higher velocity. The spectacular concentric features of Mare Orientale, which has strong but incomplete indications of being a mass concentration in MULLER & SJOGREN's analysis, are certainly the consequence of "asteroidal" or higher velocities (VAN DORN, 1968). Hence there could be available appreciable additional energy to account for some of the mare features, such as the smooth floor and apparent dehydration.

Pertinent to the origin of the moon, the mass concentrations must be counted as another evidence that, at least, much of the outer parts did not come from the earth. The evidence which now appears to indicate most strongly that the lunar interior fissioned from the earth is the mean temperature of less than 800°C inferred from the satellite magnetometry behind the moon. It is hard to imagine other means than the fractionation by volatility which such an event would entail to account for the loss of U, Th, K by the lunar material.

A scenario proposed as a working hypothesis:

(i) In accretion, the earth acquired a massive opaque atmosphere which raised temperatures sufficiently to cause iron to melt, resulting in formation of the core and rotational instability. A cloud of volatilized silicates was thrown off with the atmosphere. Subsequently material condensed into dust out of this cloud while the gases largely escaped. In this process, there was fractionation according to volatility, with Fe, Aℓ, Ca, U, Th retained in the earth; Mg, Si in the dust; and Na, K, S in the lost gases (RINGWOOD, 1966).

(ii) The dust cooled appreciably and then combined into one or more planetesimals. These satellites constituted the second force necessary for any reasonable probability of the earth capturing in orbit additional material from the solar nebula. This material captured from the nebula was rich in volatiles, perhaps like carbonaceous chondrites.

(iii) The planetesimals and captured material combined to form a proto-moon while there was still an appreciable amount of chondritic-like material in orbit, as dust or small bodies.

(iv) Additional material fell into the moon; as time progressed, the distribution of velocities of this material shifted from one with a peak around 2 km/sec "satellite" velocities to one toward 10 km/sec "asteroidal" velocities.

(v) The infalling material and the associated atmosphere generated enough heat near the lunar surface to cause fractionation into a basalt-like crust and a largely olivine mantle. This heating was mild enough to permit retention of considerable water, and its rise from the interior.

(vi) In the terminal stages of lunar formation, much higher velocity impacts by asteroids or comets occurred, resulting in creation of large craters, some melting of lunar surface material, and raising the temperature of the crust enough to cause outgassing of water to some depth. This outgassed water may have formed a small atmosphere with limited erosion and sedimentation resulting.

(vii) Subsequent cooling of the outer several 10's of kilometers below 800°C stopped isostatic creep and allowed sufficient tensile strength to develop so that pressures in excess of hydrostatic were created. These processes pushed material up in places of structural weakness beyond isostatic equilibrium. This overcompensation of the ringed maria took place by the rise of a high density root, so that an appreciable depression remained at the surface when balance was attained.

(viii) Further dropping of the temperature permitted extensive serpentinization where water was available -- i.e., elsewhere than the ringed maria.

(ix) Some mass excess was added to the ringed maria by sedimentation resulting from downhill motion of fragmented material. Either this material has been somehow sintered to attain a higher thermal conductivity than the uplands, or, more likely (and consistent with occurrence of hot spots at new craters), it retains a higher conductivity because it has not been exposed so long to solar wind effects.

We thus favor an excess internal pressure associated with the cooling of the moon as the principal mechanism of creating the mass concentrations, since it would require such a tiny fraction of the thermal contraction effect and would still operate when there was a lithosphere several 10's of kilometers thick. An auxiliary effect important in making the ringed maria denser, without transferring mass, is dehydration, leading not only to higher density but also to higher thermal conductivity.

The observations which seem most poorly explained by the above scenario are the evidences of downbending of the crust under a superficial load, which are most pronounced in Mare Humorum: the craters "drowned" on the side toward the center, the concentric rille systems. Erosion and sedimentation mechanisms seem inadequate. It is thus likely that the lava mechanism proposed by WISE & YATES (1970) operated to produce some of these evidences without accounting for the entire mass concentration, and hence without requiring an excessively thick crust.

The constitution of the mass concentrations by a low velocity impacting body of density around 4.0 gm cm^{-3}, as proposed by UREY & MAC DONALD (1971), cannot be ruled out because of our ignorance of the mechanics of such impacts. But it is an unattractive hypothesis because of its complexity, being much dependent on the details of formation of the earth-moon system.

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IV ERROR ANALYSIS OF LASER RANGING TO THE MOON

Numerical error analyses were made in which the covariance matrices, and thence sigmas and correlation coefficients, resulting from fictitious series of observations were calculated. The purpose was to test the effects of:

(1) changing the maximum zenith angle of observation;

(2) adding a southern hemisphere telescope;

(3) not determining the ranging bias separately;

(4) adding a second reflector on the moon;

(5) extending the observations over 18.6 years, a full cycle of the node;

(6) utilizing only one telescope instead of two in the northern hemisphere.

Parameters which were solved for included:

(1) orbital elements: the six Kepler elements at epoch ($\underline{a}$, $\underline{e}$, $\underline{I}$, $\underline{M}$, ω, Ω), plus scale;

(2) lunar forced libration parameters: equatorial inclination $\underline{I}$, and $f = (C-B)/(C-A)$ (using the 12 leading Fourier components of its effect);

(3) moon tide Love numbers;

(4) Fourier components of wobbles in the earth's rotation axis;

(5) Fourier components of variations in the earth's rotation;

(6) telescope coordinates;

(7) telescope horizontal drift;

(8) reflector coordinates.

Error sources which were allowed for in the model:

(1) instrumental random error;

(2) constant bias;

(3) zenith angle dependent random error;

(4) zenith angle dependent bias;

(5) sun-moon angle dependent random error;

(6) sun-moon angle dependent bias;

(7) cloudy weather, random day-to-day.

The standard case calculated assumed:

(1) observations three times a day for 437 days, weather permitting, at the extremes of allowable zenith angle and the meridian;

(2) maximum zenith angle of 60°;

(3) minimum sun-moon angle of 30°;

(4) two telescopes:

Maui (20°N, 204°E, 60% clear weather) and

Pic-du-Midi (43°N, 0°, 40% clear weather);

(5) one reflector:

Mare Tranquillitatis (2°N, 34°E);

(6) random error

$\pm(10. + 10.\ \sec \underline{Z} + 12.\ \cos^6 \psi/2)$ where $\underline{Z}$ is zenith angle and ψ is sun-moon angle;

(7) a priori sigma of bias

$\pm(5. + 5.\ \sec \underline{Z} + 6.\ \cos^6 \psi/2)$cm;

(8) a priori sigmas of lunar orbit elements:

$\Omega = \pm\ 5. \times 10^{-6}$

$\underline{I} =\ \ 5. \times 10^{-6}$ radians

ω = $100. \times 10^{-6}$

$\underline{a}$ = 10^{-10} earth radii

$\underline{e}$ = $5. \times 10^{-6}$

$\underline{M}$ = $15. \times 10^{-6}$

scale = 2.5×10^{-6} earth radii

The Fourier components of the wobble and rotation which were solved for were for the periods 13.251, 13.665, 14.106, and 27.330 days.

The results are given in the table.

The main conclusions are:

(1) observations should be made daily, if possible;

(2) for the specified error model, the maximum zenith angle of 60° is reasonable;

(3) an additional station in the southern hemisphere would improve significantly determination of the action elements of the orbit ($\underline{a}$, $\underline{e}$, $\underline{I}$), telescope coordinates, and monthly oscillations in the earth's rotation & wobble;

(4) independent calibration of bias is desirable;

(5) a second reflector on the moon would significantly improve determination of the lunar equator;

(6) continuation of tracking over 18.6 years would yield an order-of-magnitude improvement in determination of the orbit;

(7) at least two tracking stations well separated in longitude are needed for strong determination of the orbit and of variations in the earth's rotation.

TABLE SIGMAS RESULTING FROM NUMERICAL ERROR ANALYSES OF LASER RANGING TO THE MOON

Run No.	Distinction
1	Standard (see Text)
2	Higher zenith angle: 75°
3	Adding telescope at Sydney
4	No a priori sigmas, orbit & bias
5	Adding reflector, Sinus Medii
6	Reflector at Sinus Medii only
7	Observations 2/month, 18.6 yrs.
8	Lower zenith angle: 45°
9	Telescope at Maui only

Parameter	Unit	Sigma for Run No.								
Orbit		1	2	3	4	5	6	7	8	9
Node	10^{-6}	.19	.24	.05	.20	.139	.19	.022	.37	1.07
Incl.	10^{-6}	.028	.029	.005	.027	.017	.027	.005	.045	.12
Perigee	10^{-6}	.20	.27	.07	.21	.14	.19	.021	.37	1.09
S.M. Axis	10^{-6}	4.92	7.05	2.03	4.94	3.05	4.73	1.71	5.28	20.35
Eccen.	10^{-6}	.12	.13	.06	.12	.09	.12	.02	.22	.54
Moon										
Eq. Incl.	10^{-6}	.42	.38	.25	.41	.09	.40	.23	.57	.79
[C-B]/[C-A]	10^{-6}	7.84	10.93	5.88	7.81	5.81	7.75	2.51	7.61	9.79
Wobble										
13.25d	cm	3.	3.	2.	3.	2.	3.	9.	3.	5.
13.66d	cm	4.	4.	3.	4.	3.	4.	156.	9.	21.
14.10d	cm	3.	3.	2.	3.	2.	3.	8.	3.	5.
27.33d	cm	8.	7.	4.	8.	6.	8.	25.	26.	182.

		1	2	3	4	5	6	7	8	9
<u>Rotation</u>										
13.25d	ms	.015	.025	.012	.015	.010	.014	.045	.014	.030
13.66d	ms	.120	.178	.069	.113	.038	.114	.212	.130	.374
14.10d	ms	.016	.026	.012	.016	.011	.016	.045	.015	.031
27.33d	ms	.29	.36	.09	.29	.23	.27	.14	.47	1.71
<u>Track Sta.</u>										
Loc.	m	.5	.6	.3	.5	.4	.5	1.4	1.9	2.8
Drift	cm/yr	19.	19.	8.	19.	9.	19.	.8	27.	32.
<u>Reflector</u>										
Coord.	m	.25	.30	.19	.28	.19	.30	.08	1.04	1.00

Further analyses should be made to take into account restrictions in the number of hours per day the telescope is available and to utilize the probably more accurate determinations of wobble & rotation by radio telescope interferometry.

V MISCELLANEOUS

1. Statistical Analysis of Topography

An autocovariance analysis was made of topographic elevations determined from earth-based photography by the U.S. Army Map Service and the USAF Aeronautical Chart & Information Center. The power spectrum obtained from this analysis showed good agreement for degrees 3 and higher, but disagreement for degrees 0, 1, and 2 -- suggesting serious discrepancies between the scale and orientation provided by the AMS and ACIC control systems.

2. Investigation of Lunar Secular Motions

Cassini's law and the physical librations give values for $(C-A)/C$ and $(A-B)/C$. To obtain the physically more interesting quantity C/MR^2, however, we need two dimensionless coefficients J_2 and J_{22} of the lunar gravitational field, definable as:

$$\left.\begin{aligned} J_2 &= [C - \tfrac{1}{2}(A+B)]/MR^2 \\ J_{22} &= (B-A)/4MR^2 \end{aligned}\right\}$$

Prior to lunar satellites, the moon's own orbit was the only phenomenon sensitive to these terms, principally through the rates of precession $\dot{\Omega}$ of the node (the point of intersection of the orbit and the ecliptic), and $\dot{\omega}$ of the perigee (the point of closest approach to the earth). The "observed" rates are 1 revolution per 18.6 years, or -6,967,943.6 seconds of arc per century for $\dot{\Omega}$, and 1 revolution per 8.9 years, or +14,643,534.5 seconds per century for $\dot{\omega}$. All but about 800 seconds per century of this motion is caused by the sun; the planets contribute about 200 seconds per century, and the earth's oblateness J_2 contributes about 600 seconds per century, thus leaving a residual of less than 30 seconds per century that must be accounted for by the moon. Until the advent of artificial earth satellites the earth's oblateness was too poorly known to permit one to distinguish the lunar effects. But a calculation at that time (1959) yielded an implausibly high value for C/MR^2 of 0.56, which suggest that the moon is closer to a hollow (0.67) than to a homogeneous (0.40) sphere. It was therefore concluded that the solar contribution to $\dot{\Omega}$ postulated in the lunar theory of Brown was in error.

But the revision of Brown's theory by ECKERT (1965) left an even larger residual in $\dot{\Omega}$, -28 seconds per century, which resulted in the even more unreasonable value of 0.64 for C/MR^2. This "hollow moon paradox" persisted for some years until VAN FLANDERN (1968) pointed out that error lay not in Brown's theoretical $\dot{\Omega}$, but rather in his "observed" $\dot{\Omega}$, which he calculated as the difference of two quantities (BROWN, 1915):

$$\dot{\Omega} = \frac{L_{1910} - L_{1750}}{1910 - 1750} - \frac{F_{1910} - F_{1840}}{1910 - 1840}$$

where *L* is the moon's longitude and *F* is the argument of latitude, the angle along the orbit from the node to the moon. In using these values over different time bases, Brown was making the implicit assumption that his "clock", the earth's rotation, had a uniform rate. But we now know that the earth's rotation varies, and on the basis of Brown's own values for the difference between the observed and calculated longitude of the moon, the mean rate for 1840 to 1910 is about 10 seconds per century less than the mean rate for 1750 to 1910 (MUNK & MAC DONALD, 1960, p. 202). The application of this correction brings the value for C/MR^2 down to a comfortable 0.41.

However, a closer examination of Brown's discussion (BROWN, 1915) indicates that the latitude data are for 1847 to 1901, rather than 1840 to 1910. Because of the abrupt change in the earth's rotation in 1901, this reduces the mean rate $\dot{F}$ even further, so that the inferred C/MR^2 is .06. It seems probable that the observations are not as accurate as estimated by Brown. Furthermore, there remains doubt as to the actual value of the earth's J_2 used by him.

References for Section V

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