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ENERGY EXCHANGES DURING THE PARTIALLY INELASTIC IMPACT OF TWO MASSES HAVING COLLINEAR VELOCITIES

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ABSTRACT

An expression was developed for the kinetic energy transformed into work, heat, and sound during the impact of a model of a spherical reactor containment vessel and a concrete block. The expression is a function of the masses, Newton's coefficient of restitution, and the initial velocities of the two masses. The energy transformed is shown to be constant for two given masses with a given coefficient of restitution impacted at a constant relative velocity between the masses. The total energy transformed into work, heat, and sound is shown to be less than the kinetic energy of the smaller mass due to the relative velocity between the masses.

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SUMMARY

The kinetic energy transformed into work, heat, and sound during the impact of a model spherical reactor containment vessel and a reinforced concrete block was investigated. An expression was developed for the kinetic energy transformed in terms of the ratio of the two masses, Newton's coefficient of restitution, the mass of the model, and the initial velocities of the two masses.

The equation obtained showed that the total energy transformed into damage to the test model could not be greater than the kinetic energy of the smaller mass (the spherical model) due to the relative velocity between the two masses.

Two methods of testing were considered: (1) A model sphere was accelerated to test velocity and was impacted into a stationary concrete block, and (2) the concrete block was accelerated to test velocity and was impacted into a stationary model sphere. The equation for the energy transformed was the same for both cases. Thus, the equation showed that the total energy transformed into work, heat, and sound during impact is constant for two given masses with a given coefficient of restitution (of the same materials and construction) tested at the same relative velocity.

INTRODUCTION

Safety requirements for a mobile nuclear propulsion system dictate that the nuclear reactor must be contained within a spherical containment vessel in the event of a severe accident. Impact tests between model containment vessels and reinforced concrete blocks have been conducted to verify that severe impacts could result in large deformations of the containment vessel without rupture or leakage of the vessel (ref. 1).

The purpose of this report is to examine and to compare the energy changes that occur for two methods of impact testing. A model test involves the impact of a spherical model containment vessel and a much heavier reinforced concrete block.

In the first method of testing, the model or ball is in motion and the block is stationary. The model containment sphere is mounted on a rocket sled which is locked to, and constrained to follow the steel tracks of the rocket test run as shown in figure 1. Rockets accelerate the sled with the model up to test velocity. A short distance ahead of the impact site the sled carrying the ball is deflected into a pit and is destroyed. The spherical model is released from the ball sled and flies at impact test velocity into the face of a stationary reinforced concrete block.

The ball may rebound off the block with residual kinetic energy. The initial kinetic energy in the model sphere is reduced during the impact by the amount of kinetic energy transferred to the block and by energy transformed into ball damage, block damage, heat, and sound. The maximum change in kinetic energy in the ball occurs when all of the kinetic energy due to its initial velocity is either transferred or transformed during the impact.

This test procedure has the disadvantage that instrumentation mounted on the model or sled is destroyed during the impact.

In the second test procedure, the block is in motion and the ball is stationary. The stationary model spherical containment vessel is positioned on a styrofoam pedestal between the rails of the rocket test run at the impact site. The block is mounted on a rocket sled. The sled and block are accelerated up to impact test velocity. The block impacts with the model at impact test velocity and is subsequently stopped with a water brake. A schematic of this test is shown in figure 2. A photograph of the model and block are shown before the test in figure 3.

In this case, the change in kinetic energy of the block results in kinetic energy in the ball, damage to the ball, damage to the block, plus the energy transformed into heat and sound. Instrumentation for the model is mounted away from the model at a distance from the impact site and is not damaged in the impact test. Figure 4 is a photograph of the hollow sphere model after impact. Figure 5 shows a block damaged during an impact test.

In the first method of testing, the maximum total change in energy is limited by the initial kinetic energy in the ball. In the second method, the much larger mass of the block has much more energy at test velocity. Greater energy changes occur in the second method of testing than in the first method when the masses of the ball and block are the same for a given impact velocity.

The problem to be investigated involves the kinetic energy transformed into block damage, ball damage, heat, and sound for the two methods of testing. Are the methods equivalent? Is the damage to the block and the model the same for both methods of testing?

SYMBOLS

e coefficient of restitution

E kinetic energy, J

K mass ratio, m_2/m_1

m mass, kg

u initial velocity, ms^{-1}

v final velocity, ms^{-1}

V $u_1 - u_2$, ms^{-1}

Subscripts:

1 ball

2 block

c constant

D damage

H heat

s sound

T transformed

ANALYSIS

The procedure for the analysis is to write an equation for energy changes that occur during the impact. Equations for the conservation of momentum and for Newton's coefficient of restitution can be solved for the velocity after impact of each of the masses in terms of the initial velocities. With the velocities known before and after the impact, the changes in the kinetic energy can be calculated for each of the masses. These equations can be substituted into the equation for energy changes that occur during impact to obtain an equation for the total energy transformed into ball damage, block damage, heat, and sound. The relation obtained will be examined to compare the two methods of testing with respect to similitude.

Equation (1) defines the total transformed energy term.

$$\Delta E_T = \Delta E_{1D} + \Delta E_{2D} + \Delta E_H + \Delta E_s \qquad (1)$$

This equation states that the total kinetic energy transformed is equal to the sum of the energy components transformed into ball damage, block damage, heat and sound. The change in kinetic energy during impact may then be expressed

$$\Delta E_1 = \Delta E_T + \Delta E_2 \tag{2}$$

This equation applies to the first method of testing with the ball in motion. It states that the change in the kinetic energy of the ball is equal to the energy transformed plus the increase in the kinetic energy of the block.

With the block in motion the energy balance becomes

$$\Delta E_2 = \Delta E_T + \Delta E_1 \tag{3}$$

This equation shows that the change in kinetic energy of the block is always greater than the change in kinetic energy of the ball for an inelastic impact.

The mass of the block is several times the mass of the ball. This may be written

$$m_2 = Km_1 \tag{4}$$

where K is the mass ratio. Thus, the block moving at an impact velocity V has K times as much initial kinetic energy as the ball has when the ball impacts the stationary block at the same velocity V.

Equation (2) shows that the change in kinetic energy of the block is one component of the change in kinetic energy of the ball for the moving ball test procedure. In this case, the change in kinetic energy of the block is always less than or equal to the change in kinetic energy of the ball.

Equation (3) shows that the change in kinetic energy of the ball is one component of the change in kinetic energy of the block for the moving block test procedure. In this case, the change in kinetic energy of the block is always greater than or equal to the change in kinetic energy of the ball.

Conservation of momentum yields

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \tag{5}$$

Substitution of equation (4) into equation (5) reduces the equation to

$$u_1 + Ku_2 = v_1 + Kv_2 \tag{6}$$

where u_1 is the velocity of the ball and u_2 is the velocity of the block before impact; and v_1 is the velocity of the ball and v_2 is the velocity of the block after impact. The coefficient of restitution as defined by Newton is (ref. 2).

$$e = \frac{v_2 - v_1}{u_1 - u_2} \tag{7}$$

Solving equations (6) and (7) for the velocities after impact yields (ref. 3).

$$v_1 = \frac{1 - Ke}{K + 1} u_1 + \frac{K(1 + e)}{K + 1} u_2 \tag{8}$$

$$v_2 = \frac{1 + e}{K + 1} u_1 + \frac{K - e}{K + 1} u_2 \tag{9}$$

Thus, the velocities of both masses after impact are functions of the velocities before impact, the mass ratio, and the coefficient of restitution.

The change in kinetic energy of the ball is given by

$$\Delta E_1 = \frac{m_1}{2} (u_1^2 - v_1^2) \tag{10}$$

Substitution from equation (8) into (10) yields an equation for the change in energy of the ball involving only the velocities of the masses before impact.

$$\Delta E_1 = \frac{K}{(K + 1)^2} \left(\frac{m_1}{2}\right)(1 + e)(u_1 - u_2)[(2 + K - Ke)u_1 + K(1 + e)u_2] \tag{11}$$

Following the same procedure for the block, we have

$$\Delta E_2 = \frac{Km_1}{2} (v_2^2 - u_2^2) \tag{12}$$

and

$$\Delta E_2 = \frac{K}{(K + 1)^2} \left(\frac{m_1}{2}\right)(1 + e)(u_1 - u_2)[(1 + e)u_1 + (2K + 1 - e)u_2] \tag{13}$$

Substitution of equations (11) and (13) into equation (2) after simplification yields an equation for the total transformed energy.

$$\Delta E_T = \frac{K}{K + 1} (1 - e^2)\left[\frac{m_1}{2} (u_1 - u_2)^2\right] \tag{14}$$

This analysis has assumed that the initial velocity of the block (u_2) was less than the initial velocity of the ball (u_1). Different equations would be obtained if the block is assumed to have the greater velocity. The only difference, however, would be a change in the sign of equations (10) through (13). The order of the velocities in the velocity difference term of equation (14) would be reversed, but this change would not affect the value obtained for the total energy transformed.

With the proper interpretation of the signs obtained for the changes in energy, the equations presented may be applied to both cases. Either of the two masses involved in an impact may have the greater velocity.

DISCUSSION OF RESULTS

Let the kinetic energy of the ball due to the relative velocity of the ball with respect to the block be denoted by E_{12}. Then E_{12} is given by the quantity in brackets in equation (14). Let the impact velocity, $V = u_1 - u_2$. The kinetic energy of the ball due to the relative velocity between the ball and the block becomes

$$E_{12} = \frac{m_1}{2} V^2 \tag{15}$$

For a given m_1 and V, E_{12} is equal to E_{1c}, a constant. When values are selected for the mass ratio, K and the velocities u_1 and u_2, the energy changes given by equations (11), (13), and (14) become functions of the coefficient of restitution, e. For each value of e ($0 \leqslant e \leqslant 1$), the equations may be evaluated to obtain energy changes equal to a factor N times the kinetic energy of the ball, E_{1c}.

$$\Delta E = N E_{1c} \tag{16}$$

Substitution of E_{1c} into equations (11), (13), and (14) yields the following equations.

$$\Delta E_1 = \frac{K}{(K+1)^2}\left(\frac{1+e}{u_1 - u_2}\right)[(2 + K - Ke)u_1 + K(1 + e)u_2]E_{1c} = N_1E_{1c} \tag{17}$$

$$\Delta E_2 = \frac{K}{(K+1)^2}\left(\frac{1+e}{u_1 - u_2}\right)[1 + e)u_1 + (2K + 1 - e)u_2]E_{1c} = N_2E_{1c} \tag{18}$$

$$\Delta E_T = \frac{K}{K+1}(1 - e^2)E_{1c} = N_TE_{1c} \tag{19}$$

Some graphs of special cases are shown in figures 6 to 8. A value of 10 is used for the mass ratio K. The coefficient N in equations (17) to (19) is plotted on the ordinate, and the coefficient of restitution e which varies from 0. to 1. is plotted on the abscissa.

Figure 6 shows the energy changes that occur during an impact of two masses having a mass ratio of 10 with collinear velocities. The larger mass is stationary, but is assumed to be on rollers so that it can receive kinetic energy from the impact.

When the coefficient of restitution is 0. the impact is said to be perfectly plastic. The ball impacts the block, and both masses continue to move at a lower but the same velocity. The plastically impacted ball retains some of its original kinetic energy. The graph shows that the total energy transformed into work, heat, and sound for this case is a maximum. But, the total energy transformed is always less than the initial kinetic energy of the ball at impact velocity.

As the coefficient of restitution increases to 1.0, the amount of energy transformed drops to 0. For this value, the impact is theoretically perfectly elastic. Kinetic energy is transferred from the ball to the block, and no energy is transformed into work (damage), heat, and sound.

Figure 7 is a graph of the energy changes that occur when a moving block impacts a stationary ball. Again the ratio of the mass of the block to the mass of the ball is 10. The equation for the total energy transformed in figure 7 is the same as in figure 6. In this case the block having the larger mass provides the total energy transformed plus the change in the kinetic energy of the ball. The kinetic energy lost by the block increases monotonically as the coefficient of restitution varies from 0. to 1. At a value of 1. the impact is perfectly elastic. The change in the kinetic energy of the block is 3.3 times the kinetic energy in the ball moving at the initial block velocity. In this case no block energy is transformed into work heat or sound. The graph shows that as in figure 6 the energy transformed can never be greater than the kinetic energy of the ball moving at an impact velocity equal to the relative velocity between the two impacting masses.

Both the ball and the block are in motion in figure 8. The relative velocity between the masses is the same V. The kinetic energy of the ball is $4\left(\frac{m_1}{2} V^2\right)$. This is four times the initial kinetic of the ball in figure 6.

In figure 6 the ball impacted a stationary block. The maximum change in kinetic energy occurred for the ball at a value of e = 0.1. For this case the ball gives up all of its kinetic energy due to the relative velocity between the ball and the block.

The block in motion impacts with a stationary ball is the case graphed in figure 7. The change in energy of the block is always greater than the maximum change in energy in figure 6.

Both masses are in motion in figure 8. The relative velocity of the impact is the same as in figures 6 and 7. The ball in figure 8 has a higher level of kinetic energy than the ball in figure 6. Now the ball in figure 8 has the most change in energy which is several times the kinetic energy for the ball due to the relative velocity of the impact as shown by the higher values for N. For the elastic impact, $e = 1.0$ and $N = 3.97$. Almost all of the initial kinetic energy of the ball has been transferred to the block, and no energy is transformed into work, heat or sound.

The energy transformed into work, heat, and sound is given by equations (14) and (19). If a spherical reactor containment vessel impacts into the earth, the value of the mass ratio in the equations approaches infinity, and the ratio of $K/(K + 1)$ approaches a value of 1. As a collision approaches a perfectly plastic impact, the value of the coefficient of restitution approaches 0. The term $(1 - e^2)$ in the equations approaches the value 1. Thus, equations (14) and (19) show that the maximum kinetic energy transformed into work, heat, and sound is limited to the kinetic energy of the smaller mass based on the relative velocity between the masses.

The three special cases investigated and graphed show that the kinetic energy exchanged during an impact of two masses may be several times the kinetic energy of the smaller mass due to the relative or impact velocity. The equation for the kinetic energy transformed into work, heat, and sound is the same for all cases. It is therefore concluded that the damage to the ball and the block is the same for either method of testing whether the block impacts a stationary ball, or the ball impacts the stationary block when the relative velocity between the masses, the mass ratio, and the coefficient of restitution are the same.

As far as the method of testing is concerned, similitude is attained when the mass ratio, the coefficient of restitution, and the relative velocity are the same.

CONCLUDING REMARKS

Energy exchanges during the partially inelastic impact of two masses having collinear velocities were investigated. Newton's coefficient of restitution and the principle of the conservation of momentum were applied to obtain expressions for the velocities after the impact. Expressions for the change in the kinetic energy of each mass were used in an energy balance to obtain an equation for the total kinetic energy transformed into work, heat, and sound.

The equation for the kinetic energy transformed was found to be a function of the ratio of the two masses, the coefficient of restitution and the kinetic energy of the smaller mass due to the relative velocity between the two masses. The energy transformed is independent of which mass has velocity or is at rest as long as the relative velocity is constant. The maximum amount of kinetic energy transformed in an impact is equal to or less than the kinetic energy of the smaller mass due to the relative velocity between the masses.

Two test procedures were considered. In the first procedure, the spherical model or ball is accelerated up to test velocity and impacted into the face of a stationary concrete block. In the second procedure, the concrete block is accelerated up to test velocity and impacted into a stationary ball. Although the change in the kinetic energy of the block in the second test procedure is much greater than the change in kinetic energy for the block in the first test procedure, the total energy transformed into work (block and ball damage), heat, and sound is the same for both test procedures when the relative velocity of the impact is the same.

In comparing two methods of testing, the total kinetic energy transformed into work, heat, and sound is the same when the mass ratio, the coefficient of restitution, and the relative velocity of impact are the same for both methods.

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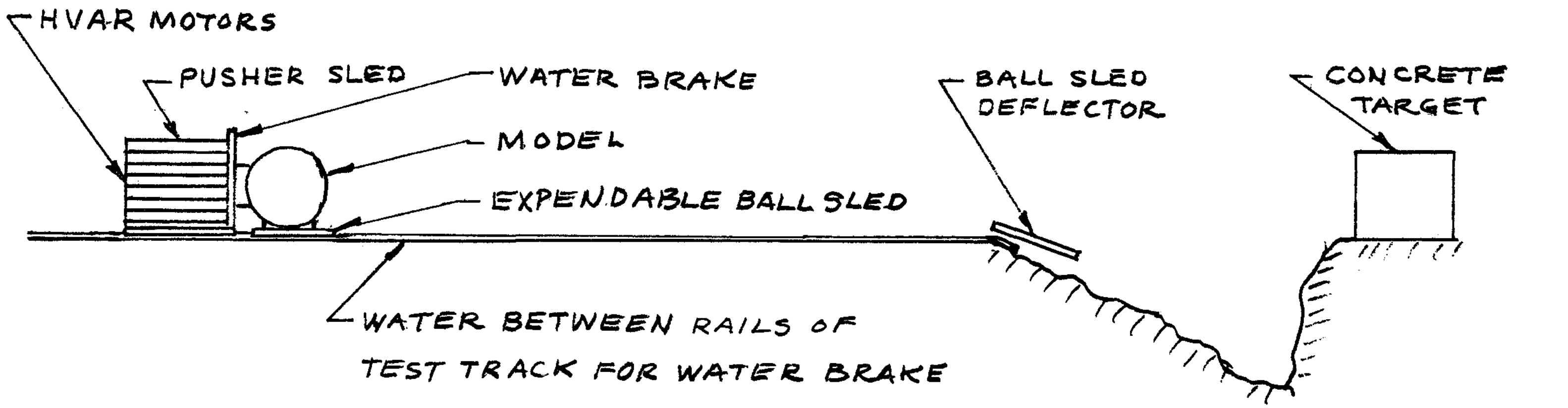

FIGURE 1. - SCHEMATIC OF IMPACT TEST OF SPHERICAL MODEL REACTOR CONTAINMENT VESSEL AND A STATIONARY CONCRETE BLOCK.

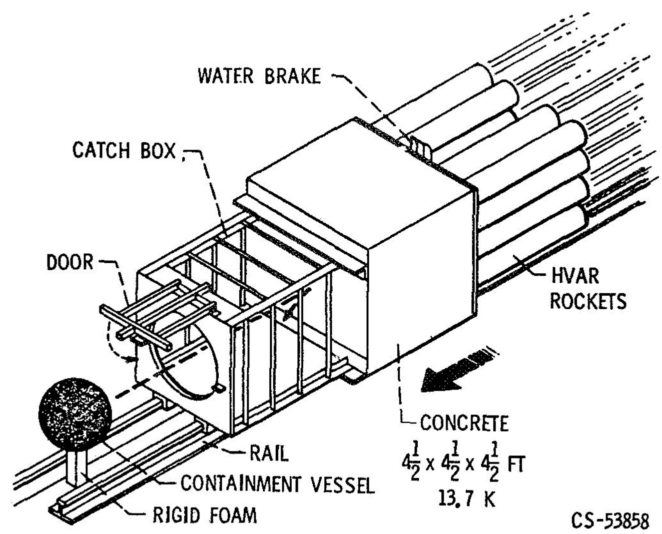

Figure 2. - Schematic of an impact test of a moving concrete block and a stationary model of a spherical reactor containment vessel.

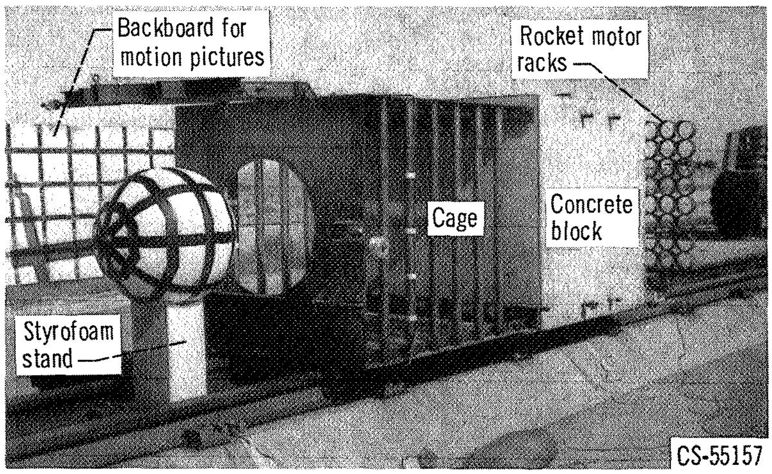

Figure 3. - Test equipment for moving block impact with a stationary model of a spherical reactor containment vessel.

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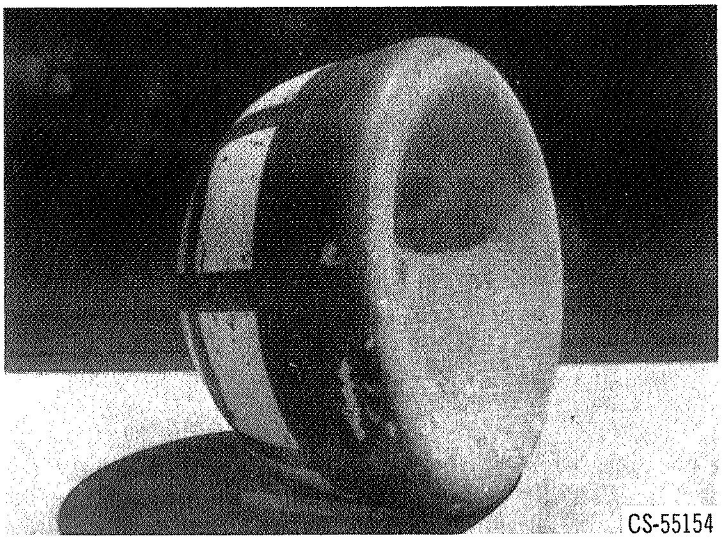

Figure 4. - Hollow sphere after impact at 119 m/sec (392 ft/sec).

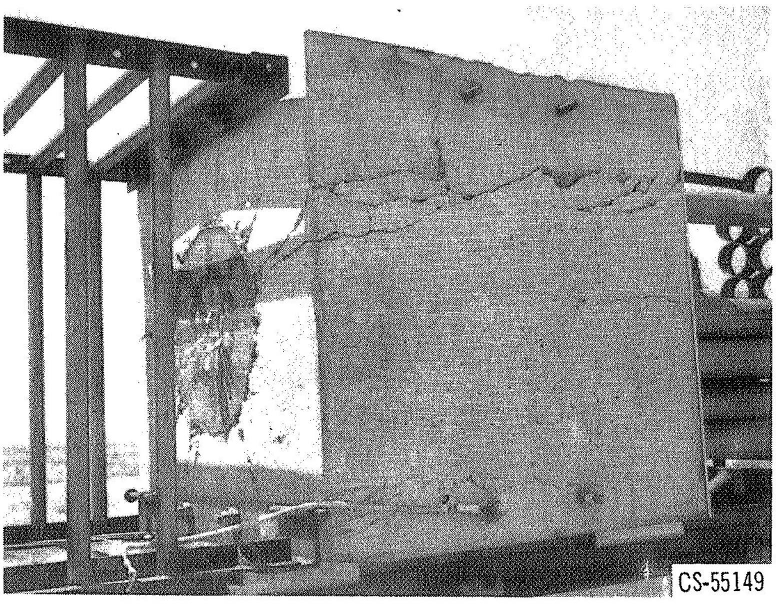

Figure 5. - 6800 Kg(1500 lb) reinforced concrete block damaged in an impact with a stationary model of a spherical reactor containment vessel.

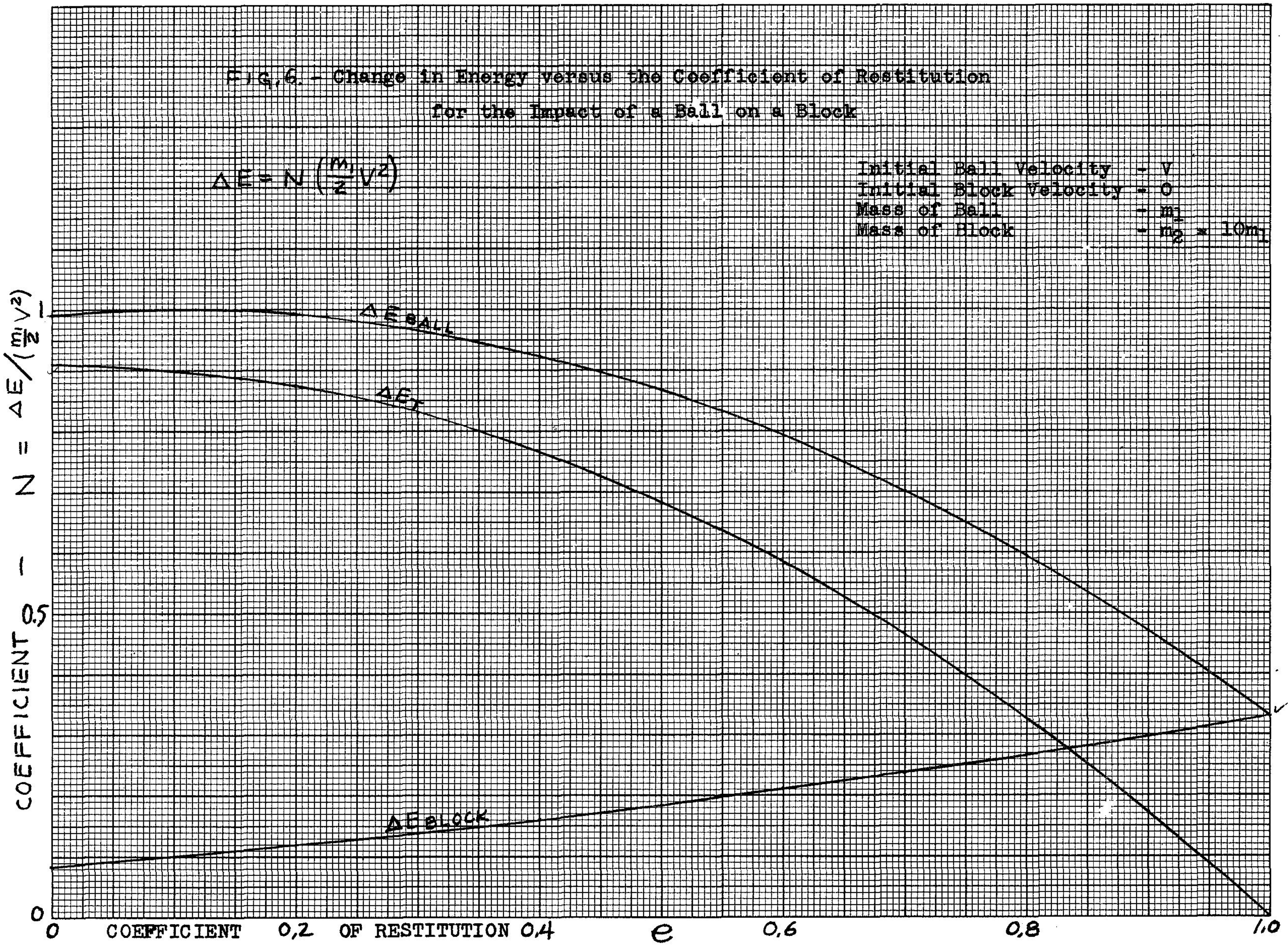

FIG. 6. - Change in Energy versus the Coefficient of Restitution for the Impact of a Ball on a Block

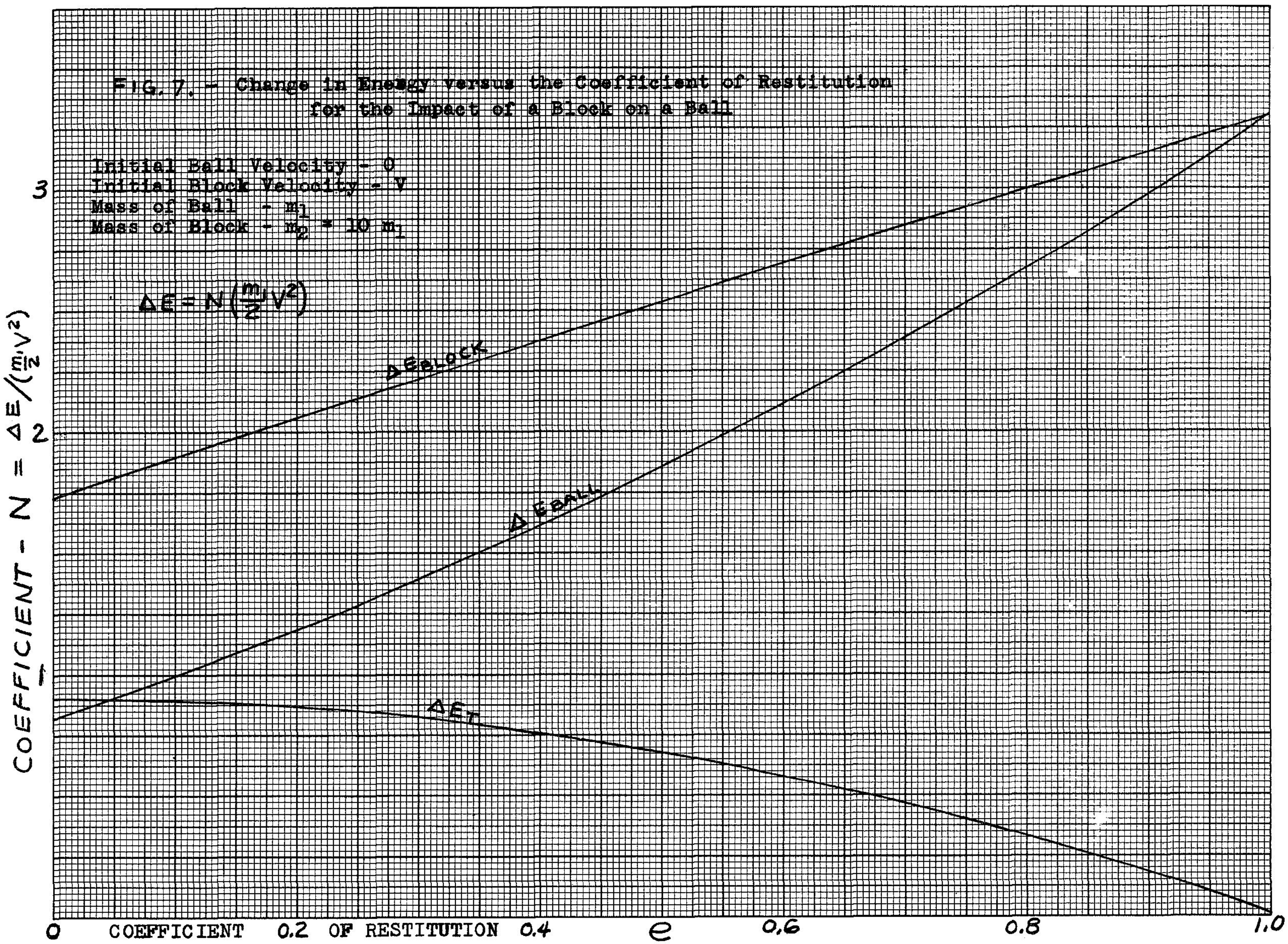

FIG. 7. - Change in Energy versus the Coefficient of Restitution for the Impact of a Block on a Ball

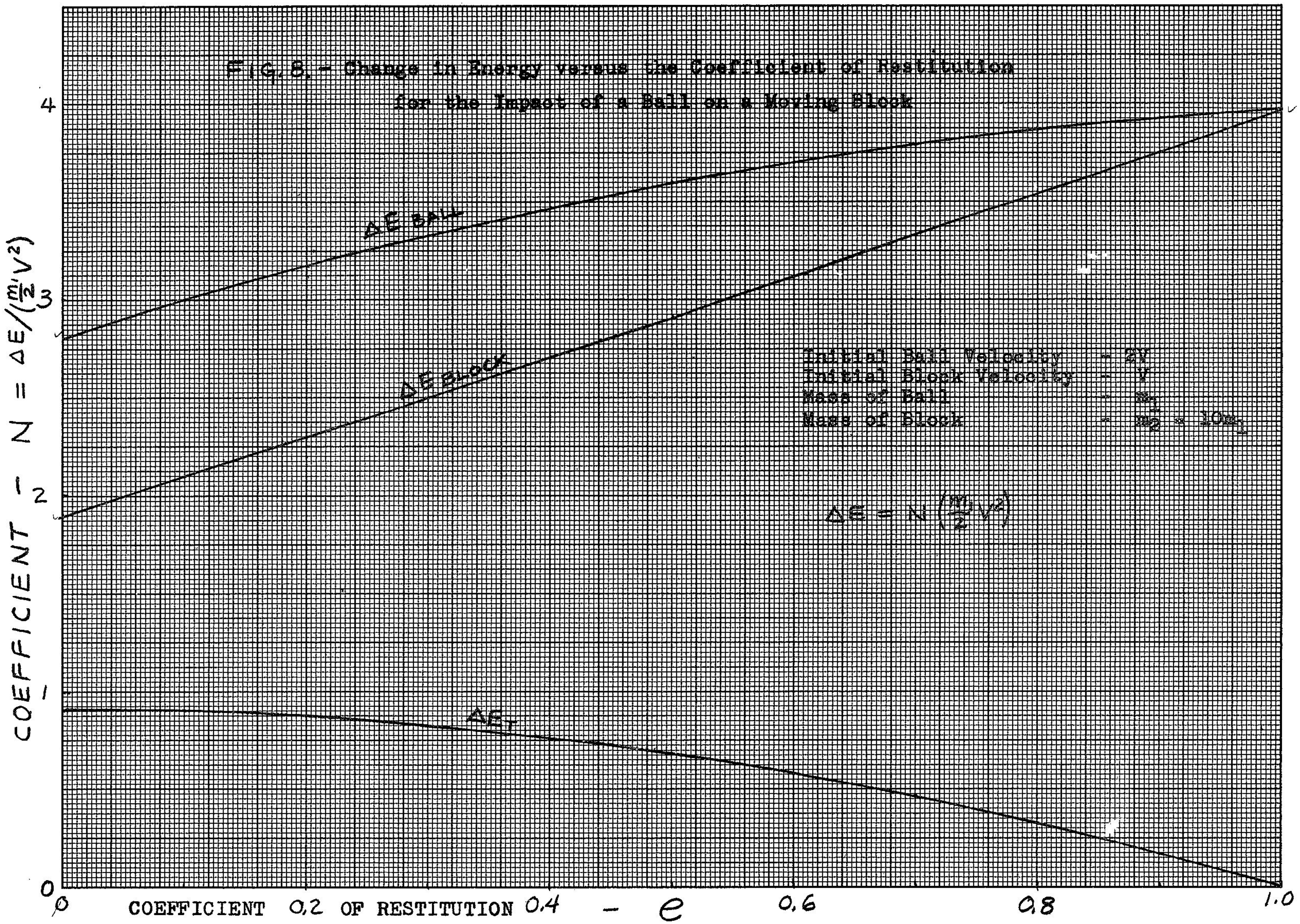
FIG. 8. - Change in Energy versus the Coefficient of Restitution
for the Impact of a Ball on a Moving Block
COEFFICIENT - N = ΔE/(m1/2 V²)
0
1
2
3
4
COEFFICIENT OF RESTITUTION - e
0
0.2
0.4
0.6
0.8
1.0
ΔE BALL
ΔE BLOCK
ΔE_T
Initial Ball Velocity - 2V
Initial Block Velocity - V
Mass of Ball - m1
Mass of Block - m2 = 10m1
ΔE = N (m1/2 V²)